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Is There Value in Complicating Volatility Management?

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Abstract

Volatility managed portfolios take less risk when volatility is high, and more risk when volatility is low. Moreira and Muir (2017) employ a simple methodology which scales factor exposure by the inverse of realized variance. Daniel and Moskowitz (2013) propose a more complex method which scales factor exposure by forecasted Sharpe ratio, and demonstrate theoretically that this is superior to just using variance. We examine this proposition by employing their strategy on the stock market factors studied in Moreira and Muir's paper. To isolate the performance impact of individual return and forecast methods, we also create strategies for all combinations of return and variance forecast in Moreira and Muir's paper and Daniel and Moskowitz's.

Both the simple and more complex methods produce large alphas and increased Sharpe ratios over buy-and-hold strategies for a wide range of factors. More interestingly, complicating volatility management beyond that of Moreira and Muir (2017) only has a modest impact on alphas and Sharpe ratios. The more complex variance forecast is not much better than a random walk forecast, and forecasting returns with the more complex methodology does generally not improve performance. The notable exception is Daniel and Moskowitz's (2013) momentum factor. Complex volatility management does, however, entail other desirable properties. It results in less volatile weights, which reduce transaction and liquidity costs. It also generates more desirable return distributions with improved skewness and kurtosis, which reduce downside risk.

Contents

Abbreviations

Abbreviation	Description
MKT	Fama-French's (1993) market factor. Value-weighted index.
SMB	Fama-French's (1993) size factor.
HML	Fama-French's (1993) value factor.
MomF	Fama-French's (2012) momentum portfolio.
MomD	Daniel and Moskowitz's (2013) momentum portfolio.
RMW	Fama-French's (2015) profitability factor,
CMA	Fama-French's (2015) investment factor.
BAB	AQR's betting-against-beta factor based on Frazzini and Pedersen (2014).
IA	Hou, Xue and Zhang's (2014) investment factor.
ROE	Hou, Xue and Zhang's (2014) return on equity factor.

Tables

Figures

1. Introduction

Volatility managed portfolios take less risk when volatility is high, and more risk when volatility is low. Thereby, they seek to take advantage of variations in the risk-return tradeoff. In their 2017 paper, Moreira and Muir employ a simple, constant volatility management strategy on a selection of factors. Their method scales exposure by the inverse of realized variance in a similar vein to Barroso and Santa-Clara (2015). The performance of several of these factors improves considerably, yielding better Sharpe ratios and significant abnormal returns which survive controls for asset pricing models and transaction costs (Moreira & Muir, 2017).

For the goal of maximizing in-sample unconditional Sharpe ratio, Daniel and Moskowitz (2013) demonstrate theoretically that a dynamic volatility management strategy is superior to a constant one for factors with time-varying Sharpe ratios. They also show empirically that this applies in practice to their momentum factor. A dynamic volatility management strategy scales exposure to a factor by its forecasted Sharpe ratio.

All the factors in Moreira and Muir's (2017) paper have time-varying Sharpe ratios, as shown in Figure 1. Consequently, the use of a dynamic volatility management strategy on the factors studied in Moreira and Muir's paper should in theory improve their performance further. We seek to do this by employing Daniel and Moskowitz's (2013) dynamic volatility management strategy on the stock market factors examined in Moreira and Muir's paper. These are the market, size, value, momentum, profitability, return on equity, investment, and betting-against-beta factors[1].

To establish a benchmark of simple volatility management performance against which we will compare more complex methods, we start by detailing the volatility management method used by Moreira and Muir (2017). From this point forward, we may refer to this method as *Moreira and Muir's method* for the sake of readability. The volatility managed factors obtained are first regressed on the unmanaged factors, and subsequently with added controls for Fama and French's market, size and value factors (1993). We may hereafter refer to Fama and French's market, size and value factors as *Fama-French's three factor model*. Seven out of ten volatility managed factors produce statistically significant abnormal returns in both regression specifications. With the exception of Daniel and Moskowitz's (2013) and Fama-

[1] We study two different momentum portfolios, Fama-French's (2012) momentum and Daniel and Moskowitz's (2013) momentum. We also study two investment portfolios, Fama-French's (2015) investment factor and Hou, Xue and Zhang's (2014) investment factor. Further details are outlined in the data section.

French’s (2012) momentum portfolios, which generate respective annual alphas of 20 and 10 percent, the magnitudes of the annual alphas range from one to six percent with additional controls for Fama-French’s three factors. These results illustrate the efficacy of simple volatility management, and are in accordance with those of Moreira and Muir.

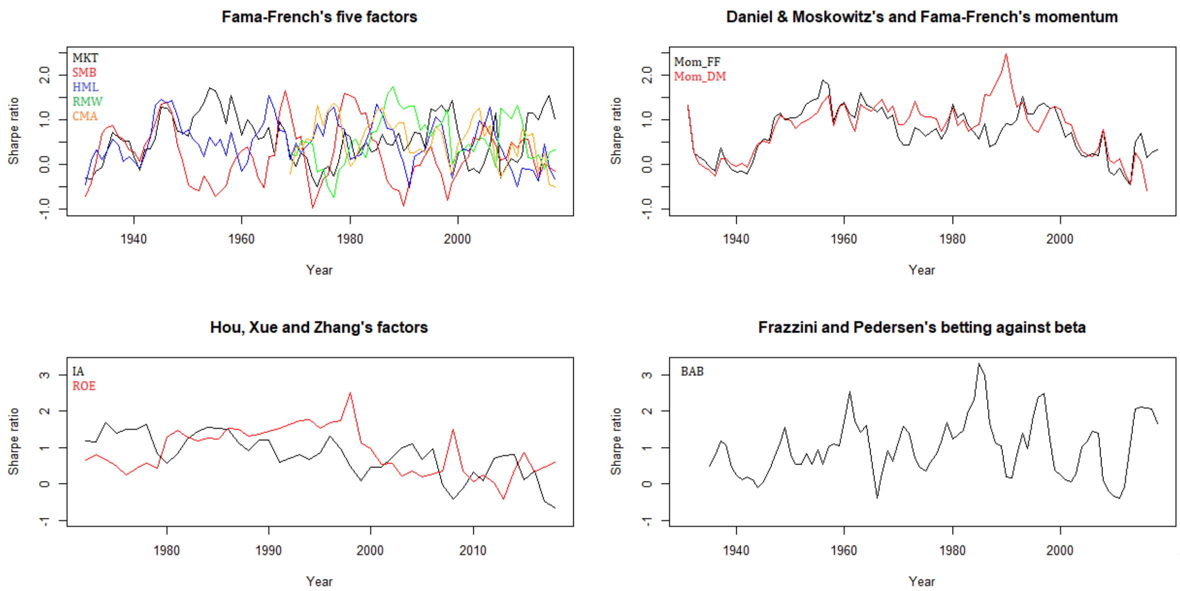

Figure 1: Sharpe ratios over a five-year rolling period for the market, size, value, momentum, profitability, return on equity, investment, and betting-against-beta factors studied in Moreira and Muir (2017), in addition to Daniel and Moskowitz’s momentum portfolio (2013).

Having established a benchmark against which we can compare more complex methods, we proceed by employing the more complex, dynamic volatility management method of Daniel and Moskowitz (2013) on the same set of factors. This constitutes an expansion of the paper, as their method was originally only employed on momentum. The difference from Moreira and Muir’s method is that Daniel and Moskowitz forecast return in addition to variance, and that they forecast variance in a more complex manner by means of a GJR-GARCH process. For readability, we may from now on refer to the dynamic volatility management method in Daniel and Moskowitz (2013) as *Daniel and Moskowitz’s method.*

We test the efficacy of Daniel and Moskowitz’s method in the same manner as we did with Moreira and Muir’s, by regressing its managed factors on the unmanaged factors with and without Fama-French’s three factors as additional controls. This yields similar results with significant abnormal return for seven out of ten factors when controlling for Fama-French’s three factors. As one might expect, it is the factor for which the method is designed, momentum, which sees the largest improvement with annual alphas of 21 percent for Daniel and

Moskowitz's (2013) momentum and 10 percent for Fama-French's (2012) momentum. The remaining five factors generate annual abnormal returns of three to six percent.

Having found that both Moreira and Muir's and Daniel and Moskowitz's method appear to yield considerable improvements on the unmanaged factors, we next seek to examine whether the latter, more complex method outperforms the former and more simple. To test whether Daniel and Moskowitz's method performs better than Moreira and Muir's, we regress Daniel and Moskowitz's managed factors on Moreira and Muir's managed factors and the unmanaged factors, with and without Fama-French's three factors as additional controls.

The market, value and return on equity factors generate significant abnormal returns. Additionally, there is weaker evidence at the ten percent significance level that the size and two momentum factors generate abnormal returns. The magnitude of the improvement ranges from one to three percent annually. Thus, there seems to be some evidence that Daniel and Moskowitz's method improves on Moreira and Muir's.

To further assess the difference in performance, we also compute the Sharpe ratios of the unmanaged and managed factors. We find that some form of volatility management yields higher Sharpe ratios than unmanaged factors for all ten factors but Fama-French's (2015) investment factor. Further, Daniel and Moskowitz's more complex method generates a higher Sharpe ratio than Moreira and Muir's for eight out of the ten factors. While the improvements in Sharpe ratio from the unmanaged to the managed factors can be large at up to 58 ppt., the differences in Sharpe ratio between Moreira and Muir's method and Daniel and Moskowitz's are more moderate with improvements of up to seven ppt.

Although Daniel and Moskowitz's method appears to moderately outperform Moreira and Muir's, there are issues with its real-time implementability. We find that Daniel and Moskowitz (2013) use future information embedded in the coefficients of both their return and variance forecasts. We therefore propose a modified version of Daniel and Moskowitz's method. Instead of using the full sample to forecast return and variance, we employ expanding window forecasts with a ten-year training period. Modifying their method in this manner ensures that only information available ex ante is used.

This raises the question of whether the use of future information affects the performance of Daniel and Moskowitz's method. To test this, we regress the modified version of the strategy on the original, the unmanaged factor and Fama-French's three factor model. Due to the use of a ten-year training period in the modified version, we need to match the samples by cutting the first ten years of observations for the original method to get an accurate comparison. The resulting alphas are never statistically significantly different from zero, meaning that the original version does not seem to benefit from the use of future information in this sample.

We previously established that Daniel and Moskowitz's original method appears to moderately outperform Moreira and Muir's for some factors. Since both the variance and return forecasts are different in Moreira and Muir's method and Daniel and Moskowitz's, we cannot determine whether differences in performance stem from variations in return or variance forecast. Next, we seek to examine what drives these differences in performance. Although we found no difference in performance between our modification of Daniel and Moskowitz's method and the original, we employ our modified version for the remainder of the thesis as it is implementable in real-time.

To examine what drives the differences in performance between Moreira and Muir's method and our modified version of Daniel and Moskowitz's, we create a strategy for each combination of return and variance forecast. Daniel and Moskowitz (2013) and Moreira and Muir (2017) collectively contain two different variance forecasts, and three different return forecasts. The variance and return forecasts can thus be combined in six unique ways, yielding six unique volatility management strategies. Four of these are new combinations, which we coin hybrid strategies. The remaining two are effectively our modified Daniel and Moskowitz method and Moreira and Muir's method. Adding these four new strategies allows us to isolate the performance impact of different variance and return forecasts.

For every factor, we report the Sharpe ratio of the six strategies. Eight out of ten factors benefit from volatility management of some kind. More importantly, there are generally small differences in Sharpe ratio between the simple method of Moreira and Muir and the five more complex methods. Daniel and Moskowitz's (2013) momentum and Hou, Xue and Zhang's (2014) investment factor benefit the most from more complex volatility management methods, with respective improvements in Sharpe ratio of seven and six ppt. In the case of Daniel and Moskowitz's momentum it is the complex variance forecast which improves performance, while it is the return forecast which adds to performance for the investment factor. For the remaining cases where a more complex method outperforms Moreira and Muir's method and the unmanaged factor, the magnitude of the improvement is 4 ppt. or less. There is no clear pattern in whether the return or the variance forecast drives these mild improvements.

Thus, the improvements in Sharpe ratio of complicating volatility management beyond that of Moreira and Muir's method generally appear to be modest. To further examine these differences, we report the alphas generated by regressing each hybrid strategy and our modified version of Daniel and Moskowitz's method on the unmanaged factor, Moreira and Muir's managed factor and Fama-French's three factor model. The results of these regressions give further indication that there are generally either no or modest gains in complicating volatility management. There are only three out of 50 cases where a more complex method than Moreira and Muir's produces significant abnormal return.

Unsurprisingly, two out of three cases are for Daniel and Moskowitz's (2013) momentum factor, the factor for which the complex return and variance forecast are designed. They both generate an annual abnormal return of roughly 3.5 percent. The remaining case is for Hou, Xue and Zhang's (2014) investment factor. It sees an abnormal return of roughly one percent. There is also weaker evidence at the ten percent significance level that two other hybrid methods outperform Moreira and Muir's, one for Daniel and Moskowitz's momentum and one for the return on equity factor. Since only three out of 50 complex strategies outperform Moreira and Muir's method, it is difficult to deduce which combination of return and variance forecast, if any, is best suited to outperforming a simple volatility management strategy.

In more general terms, these findings indicate that it is hard to materially increase Sharpe ratios and abnormal returns by complicating volatility management beyond Moreira and Muir's method, which uses a random walk forecast for variance and a time constant to scale factor exposure. In light of Daniel and Moskowitz's proof that the optimal portfolio weight is proportional to Sharpe ratio and not just variance (2013), this suggests that the more complex variance forecast is not much better than a random walk forecast. It also indicates that it has proved difficult to forecast returns with Daniel and Moskowitz's methodology.

However, complicating volatility management strategies beyond that of Moreira and Muir (2017) may generate other desirable properties. We show that the more complex methods generally entail less volatile weights. To the extent that transaction and liquidity costs are non-zero, this indicates that the performance of the complex methods may be somewhat better in relative terms than what our initial results suggest.

Additionally, more sophisticated volatility management seems to generate more desirable return distributions with higher skewness and lower kurtosis than Moreira and Muir's method. It thus appears that complex volatility management is better at reducing downside risk. This seems like an appropriate feature given that the initial purpose of Daniel and Moskowitz's method was to reduce momentum crash risk (2013).

It is also worth noting that the strategies explored in this thesis use the same methodology for all factors. There may be benefit in tailoring different variance and return forecasts to the properties of the different factors, as opposed to using a one-size-fits-all approach. The fact that complex volatility management performs better than Moreira and Muir's method for the factor for which it was originally designed, Daniel and Moskowitz's (2013) momentum, indicates that this may be a fruitful area for future research.

Our thesis consists of four main parts: Data, empirical analysis, discussion and conclusion. The data section describes the sources of the factors used, while the empirical analysis presents our methods and results. The discussion addresses a selection of issues, while the final section concludes.

2. Data

The market (MKT), size (SMB), value (HML), momentum (MomF), profitability (RMW) and investment (CMA) factors are obtained from Kenneth French's website (2019). Daniel and Moskowitz's (2013) momentum factor (MomD) is obtained from Kent Daniel's website (2019)[2]. Daniel and Moskowitz's momentum portfolio is formed differently from Fama-French's (2012), using the 10th and 90th percentiles of past performance as opposed to the 30th and 70th, as the respective cut-off points for winners and losers. Hou, Xue and Zhang's (2014) investment (IA) and return on equity (ROE) factors were kindly provided by Lu Zhang via e-mail[3]. Frazzini and Pedersen's (2014) betting-against-beta (BAB) factor is gathered from AQR's webpage (2019). We include monthly and daily data for all above factors. This gives us daily and monthly return series for ten factors, the periods of which are detailed in Table 1.

Table 1. Periods of daily and monthly factor return series.

Factor	Daily	Monthly
MKT	1926/07/01 -2018/12/31	1926/07 - 2018/12
SMB	1926/07/01 -2018/12/31	1926/07 - 2018/12
HML	1926/07/01 -2018/12/31	1926/07 - 2018/12
MomF	1926/11/03 - 2018/12/31	1926/12 - 2018/12
MomD	1927/01/03 - 2013/03/28	1927/01 - 2013/03
RMW	1963/07/01 - 2018/12/31	1963/07 - 2018/12
CMA	1963/07/01 - 2018/12/31	1963/07 - 2018/12
BAB	1930/12/01 - 2018/12/31	1930/12 - 2018/12
IA	1967/01/03 - 2017/12/29	1967/01 - 2017/12
ROE	1967/01/03 - 2017/12/29	1967/01 - 2017/12

[2] There are several momentum portfolios which are formed differently available on Kent Daniel's webpage (2019). We use the same portfolios as Daniel and Moskowitz (2013). These are the daily and monthly series of momentum portfolios sorted on total return with breakpoints computed from all firms, not just NYSE firms.

[3] Lu Zhang's email address is: zhang.1868@osu.edu.

3. Empirical analysis

In the empirical analysis we first analyze the performance of Moreira and Muir's constant volatility management method for the factors listed in the data section. This is done is subsection A. In subsection B we employ Daniel and Moskowitz's dynamic method on the same factors to see if it improves performance further. Next, we modify Daniel and Moskowitz's method so that it only takes ex ante information as input and becomes implementable in real-time. This is laid out in subsection C. Finally, in subsection D, we create a strategy for each combination of return and variance forecast. This allows us to examine what drives the differences in performance between Moreira and Muir's method and our modified version of Daniel and Moskowitz's.

A. Moreira and Muir's method

Our empirical analysis starts by constructing volatility managed portfolios using Moreira and Muir's method. To avoid confusion, the following diagram describes the terminology we will use throughout this paper.

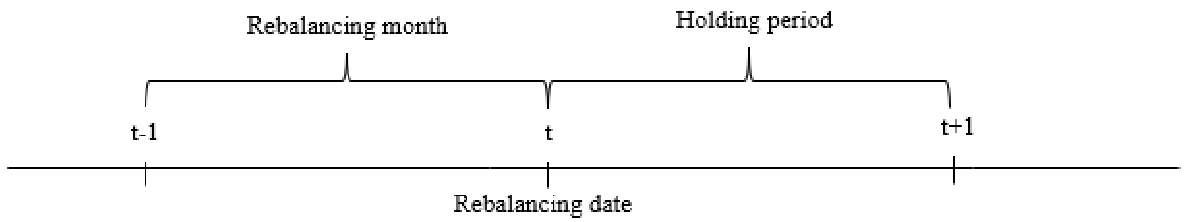

Figure 2: Notation and terminology

On each rebalancing date, exposure to every factor is scaled by the inverse of its realized variance and a time constant. The volatility managed holding period return is thus given by the following expression:

$$f_{t+1}^{MM} = \frac{c}{RV_t(f)} f_{t+1} \tag{1}$$

Where f_{t+1} is the holding period excess return of the buy-and-hold factor portfolio, $RV_t(f)$ is the realized variance, and c is a constant that scales exposure to the factor. The parameter c is set such that the full sample standard deviation of the managed portfolio equals that of the buy-and-hold portfolio[4]. The superscript "MM" is used in f_{t+1}^{MM} to indicate that the factor is volatility managed using Moreira and Muir's method.

The realized variance is the monthly variance of the 22 daily returns leading up to and including the rebalancing date. It is given by:

$$RV_t(f) = \sum_{d=1/22}^{1} \left(f_{t-1+d} - \frac{\sum_{d=1/22}^{1} (f_{t-1+d})}{22} \right)^2 \tag{2}$$

Future monthly variance is thus effectively modelled as a random walk without drift. After completing this portfolio construction, each factor has a monthly time series of unmanaged returns, $\boldsymbol{f}$, and one of volatility managed returns, $\boldsymbol{f^{MM}}$.

Univariate regressions of $\boldsymbol{f^{MM}}$ on $\boldsymbol{f}$ are then conducted factor by factor. The model specification is given by:

$$f_{t+1}^{MM} = \alpha + \beta f_{t+1} + \epsilon_{t+1} \tag{3}$$

A significant and positive α implies that the volatility managed factor has a higher Sharpe ratio than the unmanaged factor. If the factor in question is systematic and contains pricing information for a large set of assets and strategies, a positive alpha also implies that the mean-variance frontier is expanded by the volatility managed portfolio (Moreira & Muir, 2017). In addition to the univariate regressions, we also control for the Fama-French three factor model and the unmanaged factor. The results are presented in Table 2.

Despite using a bigger sample than Moreira and Muir[5] (2017), the results are very similar to what they report. Several of the factors have significant alphas, both in the univariate case and controlling for Fama-French's three factor model. This speaks to the efficacy of adjusting exposure according to realized variance.

Having presented the replication of Moreira and Muir's method, we will turn to the alternative volatility management strategy of Daniel and Moskowitz (2013).

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[4] The choice of c does not affect the Sharpe ratio of the volatility managed portfolio. Therefore, the use of future data to compute it does not bias the results.

[5] See Table A1 in appendix A for the replication of Moreira and Muir's (2017) results where our sample matches theirs.

Table 2

Volatility-managed alphas with Moreira and Muir's method

In Panel A, we run monthly time series regressions of volatility managed returns à la Moreira and Muir (2017) on the unmanaged returns for each factor, $f_{t+1}^{MM} = \alpha + \beta f_{t+1} + \epsilon_{t+1}$. In Panel B, Fama-French's three factor model is used as an additional control in the regressions from Panel A. The samples are 1926-2018 for MKT, SMB, HML and MomF; 1963-2018 for RMW and CMA; 1967-2017 for ROE and IA; 1927-2013 for MomD and 1930-2018 for BAB. All factors are annualized by scaling monthly returns by 12 and standard errors are robust for heteroscedasticity.

Panel A: Univariate Regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(11)	(10)
	σ	σ	σ	σ	σ	σ	σ	σ	σ	σ
	MKT	SMB	HML	MomF	MomD	RMW	CMA	ROE	IA	BAB
MKT	0.61^{***}									
	(0.06)									
SMB		0.61^{***}								
		(0.08)								
HML			0.57^{***}							
			(0.07)							
MomF				0.48^{***}						
				(0.07)						
MomD					0.53^{***}					
					(0.06)					
RMW						0.60^{***}				
						(0.08)				
CMA							0.69^{***}			
							(0.05)			
ROE								0.66^{***}		
								(0.06)		
IA									0.72^{***}	
									(0.05)	
BAB										0.59^{***}
										(0.05)
Alpha (α)	4.59^{***}	-0.50	1.68^{*}	12.0^{***}	23.5^{***}	2.51^{***}	0.35	5.06^{***}	1.61^{***}	6.33^{***}
	(1.53)	(0.88)	(0.98)	(1.65)	(2.96)	(0.82)	(0.64)	(0.97)	(0.62)	(0.97)
N	1109	1109	1109	1105	1034	665	665	610	610	1056
R^2	0.37	0.37	0.33	0.23	0.28	0.36	0.48	0.43	0.51	0.35
RMSE	50.7	30.3	34.3	49.3	88.0	20.9	17.3	22.8	15.8	30.6

Panel B: Alphas controlling for Fama-French's three factors

Alpha (α)	5.14^{***}	-0.28	2.37^{**}	10.0^{***}	20.4^{***}	3.17^{***}	-0.083	5.42^{***}	1.09^{*}	5.82^{***}
	(1.54)	(0.86)	(0.99)	(1.54)	(2.83)	(0.83)	(0.65)	(0.99)	(0.61)	(0.95)

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

B. Daniel and Moskowitz's method

There are three main differences between the volatility management of Moreira and Muir (2017) and that of Daniel and Moskowitz (2013). First, Daniel and Moskowitz forecast variance and return, as opposed to just variance. Second, they forecast variance differently from the random walk used by Moreira and Muir. Finally, Daniel and Moskowitz's method is intended for the momentum factor, while Moreira and Muir's method is intended for a broad selection of factors.

The volatility managed holding period return using Daniel and Moskowitz's method is given by:

$$f_{t+1}^{DM} = \frac{1}{2\lambda}\left(\frac{\hat{\mu}_t}{\hat{\sigma}_t^2}\right) f_{t+1} \tag{4}$$

Here, $\hat{\mu}_t$ is the return forecast, $\hat{\sigma}_t^2$ is the variance forecast, and λ is a constant that scales exposure to the factor. We use the superscript "DM" to indicate that the factor is volatility managed using Daniel and Moskowitz's method.

The variance forecast is done by first fitting a GARCH model originally proposed by Glosten, Jagannathan and Runkle (1993), to each factor's daily returns. It is defined as:

$$f_t = \mu + \epsilon_t \tag{5}$$

Where $\epsilon_t \sim \mathcal{N}(0, \sigma_{G,t}^2)$, and $\sigma_{G,t}^2$ is governed by the process:

$$\sigma_{G,t}^2 = \omega + \beta\sigma_{G,t-1}^2 + \left(\alpha + \gamma I(\epsilon_{t-1} < 0)\right) \epsilon_{t-1}^2 \tag{6}$$

$I(\epsilon_{t-1} < 0)$ is a dummy that takes the value of one if $\epsilon_{t-1} < 0$ and zero otherwise. The parameter set $\{\hat{\mu}, \hat{\omega}, \hat{\beta}, \hat{\alpha}, \hat{\gamma}\}$ is estimated using maximum likelihood over the full sample of daily returns for each factor[6]. This is used to compute $\sigma_{G,t}^2$, of which we take the root to obtain the GJR-GARCH volatility, $\sigma_{G,t}$. The realized standard deviation of the six months (126 days) preceding the start of the rebalancing month, $\sigma_{126,t}$, is then computed. Next, the realized volatility of the 22 days following the rebalancing date is created, $\sigma_{22,t+1}$.

[6]See Table B3 in appendix B for the maximum likelihood estimates, $\{\hat{\mu}, \hat{\omega}, \hat{\beta}, \hat{\alpha}, \hat{\gamma}\}$ for each factor.

We thus have a daily time series of GJR-GARCH volatility, $\sigma_{G,t}$, six months realized volatility, $\sigma_{126,t}$, and next month's realized volatility, $\sigma_{22,t+1}$. These are filtered to contain rebalancing dates only, changing their frequency from daily to monthly. After preparing the data, the following regression is run on the full sample of each factor[7]:

$$\sigma_{22,t+1} = \alpha + \beta_1 \sigma_{126,t} + \beta_2 \sigma_{G,t} + \epsilon_t \tag{7}$$

The coefficients $\{\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2\}$ are extracted and used to forecast variance on every rebalancing date:

$$\hat{\sigma}_t^2 = \left(\hat{\alpha} + \hat{\beta}_1 \sigma_{126,t} + \hat{\beta}_2 \sigma_{G,t}\right)^2 \tag{8}$$

Equation 8 gives the denominator used in Daniel and Moskowitz's (2013) volatility management strategy from Equation 4. This is the first component of Daniel and Moskowitz's method. The second is the numerator, which is the return forecast.

We start the process of forecasting returns by creating a monthly bear market indicator, $I_{B,t}$. It equals one if the cumulative market return[8] in the 24 months leading up to the rebalancing date is negative, and zero otherwise. Next, we compute the realized market variance of the six months (126 days) preceding the start of the rebalancing month, $\sigma_{m,t}^2$. This daily series is filtered to only include rebalancing dates, transforming its frequency to monthly. Next, an interaction term between the realized market variance and the bear market indicator is generated, $(\sigma_{m,t}^2 \times I_{B,t})$.

The entire process thus yields three monthly time series: $I_{B,t}$, $\sigma_{m,t}^2$ and $(\sigma_{m,t}^2 \times I_{B,t})$. The following two regressions are then run on the full sample for each factor:

$$f_t = c + \delta_1 I_{B,t-1} + \delta_2 \sigma_{m,t-1}^2 + \delta_3 \left(\sigma_{m,t-1}^2 \times I_{B,t-1}\right) + \epsilon_t \tag{9}$$

$$f_t = \nu + \gamma \left(\sigma_{m,t-1}^2 \times I_{B,t-1}\right) + \epsilon_t \tag{10}$$

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[7] See Table B4 in appendix B for the regression output from Equation 7 for each factor.

[8] The monthly market return series used to create the monthly bear market indicator, $I_{B,t}$, is the value-weighted market index obtained from the CRSP database (Wharton Research Data Services, 2019).

We use the estimated coefficients $\{\hat{c}, \hat{\delta}_1, \hat{\delta}_2, \hat{\delta}_3\}$ and $\{\hat{\nu}, \hat{\gamma}\}$ to forecast returns in the following two ways:

$$\hat{\mu}_t^* = \hat{c} + \hat{\delta}_1 I_{B,t-1} + \hat{\delta}_2 \sigma_{m,t-1}^2 + \hat{\delta}_3\left(\sigma_{m,t-1}^2 \times I_{B,t-1}\right) \tag{11}$$

$$\hat{\mu}_t = \hat{\nu} + \hat{\gamma}\left(\sigma_{m,t-1}^2 \times I_{B,t-1}\right) \tag{12}$$

Both specifications given by Equation 11 and Equation 12 are shown in Daniel and Moskowitz's paper (2013)[9], but they use Equation 11 when volatility managing their momentum portfolio. However, one must keep in mind that Equation 11 was conceived by Daniel and Moskowitz for the purpose of forecasting returns for their momentum portfolio. Consequently, there is no reason to expect that it will work for all ten factors included in this thesis. Indeed, $\hat{\delta}_1$and $\hat{\delta}_2$ are not significantly different from zero at the 95 percent confidence level for the majority of the factors in this thesis. However, $\hat{\gamma}$, is statistically significant at the 95 percent confidence level for the majority of factors for Equation 12[10].

We find a Sharpe ratio of 1.17 in the sample 1927-2013 when volatility managing Daniel and Moskowitz's (2013) momentum both when using equations 11 and 12 to forecast returns. This is close to the 1.18 reported by Daniel and Moskowitz, and suggests that the results are insensitive to the choice of return forecasting method.

Due to the lack of statistical significance when using Equation 11 and the identical performance in terms of Sharpe ratio, we choose to forecast returns using the simpler, univariate specification in Equation 12 for all factors in our thesis.

To recapitulate, Daniel and Moskowitz's volatility management (2013) produces two monthly time series, $\hat{\sigma}_t^2$ and $\hat{\mu}_t$. These are combined in $f_{t+1}^{DM} = \frac{1}{2\lambda}\left(\frac{\hat{\mu}_t}{\hat{\sigma}_t^2}\right) f_{t+1}$ to compute the monthly time series of volatility managed returns à la Daniel and Moskowitz, $\boldsymbol{f^{DM}}$. As in Moreira and Muir (2017), the constant λ is chosen so that the full sample volatility of $\boldsymbol{f^{DM}}$ is equal to that of the unmanaged factor returns, $\boldsymbol{f}$[11].

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[9] Equations 11 and 12 correspond to columns five and four, respectively, in Table 7 of Daniel and Moskowitz's paper (2013).

[10] See Table B1 in appendix B for the regression output from equations 9 and 10 for each factor.

[11] As in the case of Moreira and Muir's method, the choice of λ has no bearing on the Sharpe ratio of Daniel and Moskowitz's method.

Then we run univariate regressions of $\boldsymbol{f}^{\boldsymbol{DM}}$ on $\boldsymbol{f}$ for all ten factors. We also report alphas after adding additional controls in the form of Fama-French's three factors. The results are presented in Table 3. As was the case using Moreira and Muir's volatility management method, several factors produce significant alphas both in the case of the univariate regressions and with additional controls for Fama-French's three factors. Also similar to Moreira and Muir's method, MomF and MomD benefit the most from volatility management. They produce annual alphas of 10 and 21 percent, respectively, controlling for Fama-French's three factor model.

Table 3

Volatility-managed alphas with Daniel and Moskowitz's method

In Panel A, we run monthly time-series regressions of volatility managed returns à la Daniel and Moskowitz (2013) on the unmanaged returns for each factor $f_{t+1}^{DM} = \alpha + \beta f_{t+1} + \epsilon_{t+1}$. In Panel B, Fama-French's three factor model is used as an additional control in the regressions from Panel A. The samples are 1927-2018 for MKT, SMB, HML and MomF; 1964-2018 for RMW and CMA; 1967-2017 for ROE and IA, 1927-2013 for MomD and 1931-2018 for BAB. All factors are annualized by scaling monthly returns by 12 and standard errors are robust for heteroscedasticity.

Panel A: Univariate Regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	σ	σ	σ	σ	σ	σ	σ	σ	σ	σ
	MKT	SMB	HML	MomF	MomD	RMW	CMA	ROE	IA	BAB
MKT	0.74***									
	(0.058)									
SMB		0.82***								
		(0.045)								
HML			0.63***							
			(0.081)							
MomF				0.40***						
				(0.094)						
MomD					0.53***					
					(0.06)					
RMW						0.67***				
						(0.083)				
CMA							0.77***			
							(0.047)			
ROE								0.66***		
								(0.073)		
IA									0.84***	
									(0.050)	
BAB										0.70***
										(0.054)
Alpha (α)	4.35***	0.86	2.17**	13.9***	23.5***	1.97**	0.27	5.77***	0.96**	5.11***
	(1.31)	(0.63)	(0.92)	(1.92)	(3.42)	(0.78)	(0.57)	(1.08)	(0.47)	(0.94)
N	1102	1102	1102	1098	1034	657	657	602	602	1049
R^2	0.544	0.679	0.391	0.158	0.28	0.444	0.596	0.441	0.714	0.488
RMSE	43.3	21.7	32.7	51.8	88.0	19.5	15.3	22.8	12.0	27.1

Panel B: Alphas controlling for Fama-French's three factors

Alpha (α)	4.99***	0.69	2.98***	10.1***	20.6***	2.71***	-0.14	5.77***	0.64	4.36***
	(1.31)	(0.63)	(0.92)	(1.61)	(3.11)	(0.77)	(0.58)	(1.06)	(0.48)	(0.91)

Standard errors in parentheses. * *$p < 0.10$,* ** *$p < 0.05$,* *** *$p < 0.01$.*

To see if Daniel and Moskowitz's more complex method yields better performance than the simpler method of Moreira and Muir, we run two regressions. In the first regression, we run the factors managed by Daniel and Moskowitz's method, $\boldsymbol{f^{DM}}$, on the unmanaged factors, $\boldsymbol{f}$, and the factors managed according to Moreira and Muir's method, $\boldsymbol{f^{MM}}$. It is thus given by the following expression:

$$f_{t+1}^{DM} = \alpha + \beta_1 f_{t+1}^{MM} + \beta_2 f_{t+1} + \epsilon_{t+1} \tag{13}$$

In the second regression we use the same specification as in the first, but add Fama-French's three factor model as an additional control:

$$f_{t+1}^{DM} = \alpha + \beta_1 f_{t+1}^{MM} + \beta_2 f_{t+1} + \beta_3 MKT_{t+1} + \beta_4 SMB_{t+1} + \beta_5 HML_{t+1} + \epsilon_{t+1} \tag{14}$$

The results are presented in Table 4. Without controlling for Fama-French's three factor model, Daniel and Moskowitz's volatility management method generates statistically significant abnormal returns at the five percent level for SMB, both momentum portfolios and ROE. There is also weak evidence at the ten percent significance level in the case of HML.

Adding controls for Fama-French's three factor model renders the alphas of both momentum portfolios and the SMB factor only weakly significant with a ten percent significance level. This implies that the alphas reported in Panel A for these factors partly reflect exposure to the risk factors embedded in Fama-French's three factor model, rather than actual abnormal return. With controls for Fama-French's three factors, there is no evidence that Daniel and Moskowitz's method improves on Moreira and Muir's for the RMW, CMA, IA and BAB factors.

The factors which Daniel and Moskowitz's method does improve on are MKT, HML and ROE. It is worth noting that there is evidence at the five percent significance level that MKT and HML produce abnormal return with controls for Fama-French's three factors, but not without. This appears to be due to HML acting as a hedge for MKT managed with Daniel and Moskowitz's method, and MKT acting as a hedge for HML managed according to Daniel and Moskowitz's method[12]. Surprisingly, there is only weak evidence that Daniel and Moskowitz's method improves on the factor for which it was designed, momentum, but seems to improve on a few for which it was not intended to be used.

[12] The market factor when volatility managed according to Daniel and Moskowitz's method loads negatively on HML, and HML when volatility managed in the same way loads negatively on MKT.

Table 4

Testing whether Daniel and Moskowitz's method improves on Moreira and Muir's

In Panel A, we run the factors managed by Daniel and Moskowitz's method on the unmanaged factors and the factors managed according to Moreira and Muir's method. In Panel B, we add controls for Fama-French's three factors. A significantly positive alpha implies that Daniel and Moskowitz's method produces abnormal returns in excess of systematic risk exposure that is not explained by exposure to Moreira and Muir's method either, indicating that Daniel and Moskowitz's method outperforms Moreira and Muir's. The samples are 1927-2018 for MKT, SMB, HML and MomF; 1964-2018 for RMW and CMA; 1967-2017 for ROE and IA, 1927-2013 for MomD and 1931-2018 for BAB. All factors are annualized by scaling monthly returns by 12 and standard errors are robust for heteroscedasticity

Panel A: Alphas controlling for Moreira and Muir's method and the unmanaged factor

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	MKT	SMB	HML	MomF	MomD	RMW	CMA	ROE	IA	BAB
Alpha (α)	1.23	1.08^{**}	0.98^{*}	4.94^{***}	5.85^{***}	-0.50	0.15	1.66^{**}	-0.12	0.093
	(0.81)	(0.50)	(0.55)	(1.31)	(1.59)	(0.36)	(0.34)	(0.67)	(0.33)	(0.55)
N	1102	1102	1102	1098	1027	657	657	602	602	1049
R^2	0.829	0.773	0.779	0.527	0.784	0.875	0.865	0.787	0.881	0.800
RMSE	26.5	18.3	19.7	38.9	48.5	9.26	8.85	14.1	7.75	16.9

Panel B: Alphas with additional controls for Fama-French's three factors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Alpha (α)	1.70^{**}	0.82^{*}	1.42^{***}	2.27^{*}	2.76^{*}	-0.096	0.090	1.35^{**}	-0.074	-0.29
	(0.79)	(0.49)	(0.53)	(1.22)	(1.54)	(0.36)	(0.35)	(0.68)	(0.34)	(0.53)

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Next, we compute the annualized Sharpe ratios of every factor's unmanaged returns and those obtained using Moreira and Muir's and Daniel and Moskowitz's volatility management methods. We use the formula below, where $E[f_t]$ is the expected excess return and $\sigma_t(f_t)$ is the standard deviation of factor f, both over the full sample.

$$SR_f = \sqrt{12}\frac{E[f_t]}{\sigma(f_t)} \tag{15}$$

The annualized appraisal ratios are also computed using the same formula as Moreira and Muir (2017):

$$AR_f = \sqrt{12}\frac{\alpha_f}{RMSE_f} \tag{16}$$

Where α_f is the univariate alpha of the volatility managed version of factor f, using Moreira and Muir's method or that of Daniel and Moskowitz. RMSE is the root mean squared error, given by:

$$RMSE_f = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\hat{f}_t - f_t\right)^2}$$

Where $\hat{f}_t$ is the monthly returns predicted by the model, and f_t is the observed values. The results of equations 15 and 16 are compiled in Table 5.

Table 5 shows that the Sharpe ratios of all factors but CMA are improved by volatility management, either using Moreira and Muir's or Daniel and Moskowitz's methods. Particularly strong improvements occur for MomF, MomD, ROE and BAB. This indicates that the improved performance of volatility managed portfolios relative to unmanaged portfolios is persistent across a wide range of factor strategies. These findings are in line with those found by Moreira and Muir (2017).

Comparing performance across volatility management methods, we find that Daniel and Moskowitz's method yields higher Sharpe ratios than Moreira and Muir's for eight out of ten factors. These are the MKT, SMB, HML, CMA, MomF, MomD, ROE and IA factors. However, the improvements are generally quite modest, with the largest increases in Sharpe ratio being seven and five percentage points for both the momentum portfolios and ROE, respectively. This indicates that Daniel and Moskowitz's method does improve on Moreira and Muir's across a wide range of factors, but that the gains are generally modest.

The fact that the gains in Sharpe ratio are modest seems to fit well with the overall results of Table 4. As evidenced by Panel B in Table 4, Daniel and Moskowitz's method only improves on three out of ten factors relative to Moreira and Muir's method at the five percent significance level. For these factors, the magnitude of the improvement is small with an annual alpha of 1.7 percent as the maximum. Overall, abnormal returns in Panel B of Table 4 seem to be associated with gains in Sharpe ratio in Table 5 from using Daniel and Moskowitz's method over Moreira and Muir's.

It is worth noting that Daniel and Moskowitz's method is meant for their momentum portfolio. We therefore forecast returns for all factors using a model that was specifically tailored to respond to the properties of momentum portfolios (Daniel & Moskowitz, 2013).

As such, it seems plausible that the use of different return forecasting models for different factors could further improve the performance of Daniel and Moskowitz's method. The same argument does not apply to the variance forecast, as it is not tailored for momentum specifically, but made for general use (Glosten , Jagannathan, & Runkle, 1993). Even if Daniel and Moskowitz's (2013) return forecast is not suited for factors other than their momentum portfolio, it is possible that their variance forecast does benefit the volatility management of other factors.

However, there are issues with regards to the real-time implementability of Daniel and Moskowitz's volatility management method. For a strategy to be implementable in real time, it must only use information available on the rebalancing date to adjust exposure on the rebalancing date.

The variance forecast in Daniel and Moskowitz's method relies on the coefficients $\{\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2\}$ extracted from the regression in Equation 7, which is estimated once on the full sample. The forecasted volatility parameter, $\sigma_{G,t}$ also relies on a GJR-GARCH process using the full sample.

Similarly, the return forecast relies on $\{\hat{\nu}, \hat{\gamma}\}$ obtained from the regression in Equation 10, which is estimated once on the full sample. This means that they are effectively forecasting variance and returns using future information contained in $\{\hat{\alpha}, \hat{\beta}_1, \hat{\beta}_2\}$, $\sigma_{G,t}$ and $\{\hat{\nu}, \hat{\gamma}\}$. Thus, their proposed strategy is not implementable in real time.

The goal of the next subsection is to modify Daniel and Moskowitz's method so that it only uses information available ex ante in forecasts of variance and return, and thus can be implemented in real time. This will allow us to tell how much of the improvement found using Daniel and Moskowitz's method comes from the use of future information, and how much is attributable to the efficacy of the methodology.

Table 5
Sharpe ratios and appraisal ratios of volatility managed portfolios

For every unmanaged and managed factor we include the annualized Sharpe ratio $SR_f = \sqrt{12}\frac{E[f_t]}{\sigma(f_t)}$ and the annualized appraisal ratio $AR_f = \sqrt{12}\frac{\alpha_f}{RMSE_f}$ in parentheses. The samples are 1926-2018 for MKT, SMB, HML and MomF; 1963-2018 for RMW and CMA; 1967-2017 for ROE and IA, 1927-2013 for MomD and 1931-2018 for BAB.

Factor	Unmanaged	Moreira and Muir (2017)	Daniel and Moskowitz (2013)
MKT	0.43	0.51	0.55
		(0.31)	(0.35)
SMB	0.23	0.10	0.27
		(-0.06)	(0.14)
HML	0.37	0.35	0.41
		(0.17)	(0.23)
RMW	0.41	0.58	0.53
		(0.42)	(0.35)
CMA	0.50	0.39	0.42
		(0.07)	(0.06)
MomF	0.49	0.98	1.05
		(0.84)	(0.93)
MomD	0.59	1.10	1.17
		(0.93)	(0.93)
ROE	0.74	1.07	1.15
		(0.77)	(0.88)
IA	0.74	0.76	0.77
		(0.35)	(0.28)
BAB	0.75	1.01	0.99
		(0.72)	(0.65)

C. Modified Daniel and Moskowitz method

The volatility managed holding period return using our modified, ex ante Daniel and Moskowitz method is given by the following expression:

$$f_{t+1}^{DMX} = \frac{1}{2\lambda}\left(\frac{\hat{\mu}_{X,t}}{\hat{\sigma}_{X,t}^2}\right) f_{t+1} \quad (17)$$

We use the superscript "DMX" to indicate that the factor is volatility managed using Daniel and Moskowitz's method with only information known ex ante. Similarly, subscript "X" means that the return and variance forecasts are made using ex ante information only. To make Daniel and Moskowitz's method implementable in real time, we use expanding window forecasts of variance and return with a ten-year training period. This is illustrated below. Since we use an expanding window forecast, our estimation period grows as the rebalancing date, T, approaches the end of the full sample

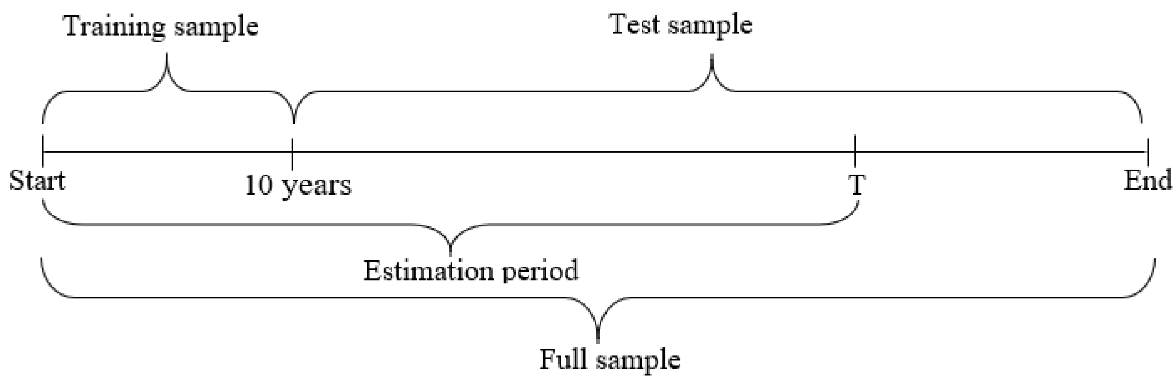

Figure 3: Division of sample into training and test period.

We use the same GJR-GARCH process as before, but now refit the model on an expanding window for each rebalancing date in the test sample:

$$f_t = \mu_t + \epsilon_t \quad (18)$$

Where $\epsilon_t \sim \mathcal{N}(0, \sigma_{G,t}^2)$, and $\sigma_{G,t}^2$ is governed by the process:

$$\sigma_{G,t}^2 = \omega_t + \beta_t \sigma_{G,t-1}^2 + \left(\alpha_t + \gamma_t I(\epsilon_{t-1} < 0)\right) \epsilon_{t-1}^2 \tag{19}$$

We still forecast GJR-GARCH variance daily, but the difference from before is that we now get a new parameter set every month, $\{\hat{\mu}_t, \widehat{\omega}_t, \hat{\beta}_t, \hat{\alpha}_t, \hat{\gamma}_t\}$ instead of a fixed set of parameters. This yields a daily series of ex ante GJR-GARCH volatility forecasts given by:

$$\sigma_{GX,t} = \sqrt{\widehat{\omega}_t + \hat{\beta}_t \sigma_{G,t-1}^2 + \left(\hat{\alpha}_t + \hat{\gamma}_t I(\epsilon_{t-1} < 0)\right) \epsilon_{t-1}^2} \tag{20}$$

Otherwise, we use the same six-months realized volatility, $\sigma_{126,t}$, as specified in subsection B. Instead of using future information in the form of next month's realized volatility, $\sigma_{22,t+1}$, we modify it to instead be the realized volatility of the rebalancing month, $\sigma_{22,t}$.

We thus have a daily time series of ex ante GJR-GARCH volatility, $\sigma_{GX,t}$, six-months realized volatility, $\sigma_{126,t}$ and realized volatility in the rebalancing month, $\sigma_{22,t}$. The time series $\sigma_{GX,t}$ and, $\sigma_{126,t}$ are lagged by one month (22 days), producing $\sigma_{GX,t-1}$ and, $\sigma_{126,t-1}$. Next, we filter $\{\sigma_{22,t}, \sigma_{GX,t-1}, \sigma_{126,t-1}\}$ to only contain rebalancing dates, changing the frequency of the data from daily to monthly.

The realized volatility in the rebalancing month, $\sigma_{22,t}$, is then regressed on the lagged six-months realized volatility and the lagged ex ante GJR-GARCH volatility.

$$\sigma_{22,t} = \alpha + \beta_1 \sigma_{126,t-1} + \beta_2 \sigma_{GX,t-1} + \epsilon_t \tag{21}$$

This is done for all rebalancing dates on an expanding window in the test period. We thus obtain a new set of coefficients every month, $\{\hat{\alpha}_t, \hat{\beta}_{1,t}, \hat{\beta}_{2,t}\}$. These reflect the relationship between past volatility and contemporaneous volatility. To forecast next month's variance, we combine $\{\hat{\alpha}_t, \hat{\beta}_{1,t}, \hat{\beta}_{2,t}\}$ with the contemporaneous $\sigma_{126,t}$ and $\sigma_{GX,t}$ variables. The formula is defined as follows:

$$\hat{\sigma}_{X,t}^2 = \left(\hat{\alpha}_t + \hat{\beta}_{1,t} \sigma_{126,t} + \hat{\beta}_{2,t} \sigma_{GX,t}\right)^2 \tag{22}$$

This constitutes the denominator in our modified Daniel and Moskowitz method, used in Equation 17. The next step is to forecast returns using only information known ex ante.

As previously discussed, it is the fact that the regressions in equations 9 and 10 are run on the full sample which biases Daniel and Moskowitz's (2013) return forecast. Thus, we need to refit the regressions every rebalancing date on an expanding window, again using a training period of ten years. This way, the regression coefficients will not contain future information.

None of the variables used in the original return forecast are biased by the use of future information. Therefore, there is no need to modify the monthly time series consisting of the bear market indicator, $I_{B,t}$, the six months market variance, $\sigma^2_{m,t}$, and the interaction term combining them both $(\sigma^2_{m,t} \times I_{B,t})$.

For every rebalancing date, we run the following regressions. They are equivalent to equations 9 and 10.

$$f_t = c + \delta_1 I_{B,t-1} + \delta_2 \sigma^2_{m,t-1} + \delta_3 \left(\sigma^2_{m,t-1} \times I_{B,t-1}\right) + \epsilon_t \tag{23}$$

$$f_t = \nu + \gamma \left(\sigma^2_{m,t-1} \times I_{B,t-1}\right) + \epsilon_t \tag{24}$$

We get one set of coefficients per rebalancing date, $\{\hat{c}_t, \hat{\delta}_{1,t}, \hat{\delta}_{2,t}, \hat{\delta}_{3,t}\}$ and $\{\hat{\nu}_t, \hat{\gamma}_t\}$. Returns are then forecasted using equivalent specifications as in equations 11 and 12:

$$\hat{\mu}^*_{X,t} = \hat{c}_t + \hat{\delta}_{1,t} I_{B,t-1} + \hat{\delta}_{2,t} \sigma^2_{m,t-1} + \hat{\delta}_{3,t} \left(\sigma^2_{m,t-1} \times I_{B,t-1}\right) \tag{25}$$

$$\hat{\mu}_{X,t} = \hat{\nu}_t + \hat{\gamma}_t \left(\sigma^2_{m,t-1} \times I_{B,t-1}\right) \tag{26}$$

These correspond to equations 11 and 12, but only use information which is available ex ante. Again, we choose to employ the simpler, univariate forecast[13]. Therefore, Equation 26

[13]To assess the implications of our choice, we have made an alternative version of Table 6 in Table B2 of appendix B. The only difference is that we let our modified Daniel and Moskowitz method use the multivariate return forecast instead of the univariate version in Table 6. Comparing the two tables, we see that the alphas generated are very similar in magnitude and significance. The only material exception is the IA factor which seems to benefit more from the multivariate forecast than the univariate one. Overall, these results indicate that performance as measured by alpha is largely insensitive to the choice of return forecast method.

constitutes the numerator in our modified Daniel and Moskowitz method. The constant, λ, is chosen in the same way as in subsection B.

To summarize, our modified version of Daniel and Moskowitz's volatility management method produces a monthly time series of variance forecasts, $\hat{\sigma}^2_{X,t}$ and a monthly time series of return forecasts, $\hat{\mu}_{X,t}$. These are combined in $f^{DMX}_{t+1} = \frac{1}{2\lambda}\left(\frac{\hat{\mu}_{X,t}}{\hat{\sigma}^2_{X,t}}\right) f_{t+1}$ to compute the monthly time series of ex ante volatility managed returns à la Daniel and Moskowitz (2013), $\boldsymbol{f^{DMX}}$.

Next, we run univariate regressions of $\boldsymbol{f^{DMX}}$ on $\boldsymbol{f}$ for all ten factors and report alphas after controlling for Fama-French's three factors in addition to the unmanaged factor. The results are presented in Table 6.

Table 6

Volatility-managed alphas with the modified Daniel and Moskowitz method

In Panel A, we run monthly time-series regressions of volatility managed returns using our modified Daniel and Moskowitz method on the unmanaged returns for each factor, $f_{t+1}^{DMX} = \alpha + \beta f_{t+1} + \epsilon_{t+1}$. In Panel B, Fama-French's three factor model is used as an additional control in the regressions from Panel A. The samples are 1937-2018 for MKT, SMB, HML and MomF; 1973-2018 for RMW and CMA; 1977-2017 for ROE and IA, 1937-2013 for MomD and 1941-2018 for BAB. All factors are annualized by scaling monthly returns by 12 and standard errors are robust for heteroscedasticity.

Panel A: Univariate Regressions

	(1) σ MKT	(2) σ SMB	(3) σ HML	(4) σ MomF	(5) σ MomD	(6) σ RMW	(7) σ CMA	(8) σ ROE	(11) σ IA	(10) σ BAB
MKT	0.78***									
	(0.04)									
SMB		0.80***								
		(0.06)								
HML			0.77***							
			(0.05)							
MomF				0.51***						
				(0.06)						
MomD					0.63***					
					(0.07)					
RMW						0.60***				
						(0.09)				
CMA							0.77***			
							(0.06)			
ROE								0.68***		
								(0.08)		
IA									0.75***	
									(0.06)	
BAB										0.74***
										(0.05)
Alpha (α)	1.71	0.23	0.68	6.80***	19.9***	0.31	0.39	6.10***	1.21*	4.05***
	(1.12)	(0.65)	(0.67)	(1.50)	(2.86)	(0.99)	(0.61)	(1.15)	(0.63)	(0.86)
N	981	981	981	981	912	542	542	488	488	934
R^2	0.60	0.64	0.59	0.26	0.40	0.36	0.59	0.46	0.57	0.55
RMSE	34.1	20.9	21.8	41.5	71.8	22.1	15.2	22.4	14.7	23.1

Panel B: Alphas controlling for Fama-French's three factors

Alpha (α)	1.70	0.03	1.07	5.85***	17.6***	0.62	0.04	5.47***	0.76	3.46***
	(1.15)	(0.69)	(0.69)	(1.67)	(2.90)	(1.17)	(0.62)	(1.16)	(0.62)	(0.86)

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

At first glance, it seems that restricting the information used for return and variance forecasts reduces alphas for nearly all factors. Only four factors have significantly positive alphas at the five percent level using our modified Daniel and Moskowitz method, as opposed to seven using the original method. These are the momentum portfolios, ROE and BAB. Comparing Panel B in Table 6 to Panel B in Table 3, it thus seems that the performance of Daniel and Moskowitz's method is substantially weakened when restricting it to ex ante information.

Note however, that the factors in Table 6 have ten years shorter samples due to the test period in our modified method. As such, the results are not directly comparable between the original Daniel and Moskowitz method in Table 3 and our modified Daniel and Moskowitz method in Table 6.

For a more accurate comparison, we cut the samples of the volatility managed factors which use the original Daniel and Moskowitz method to match those of our modified method. Then we regress $\boldsymbol{f^{DMX}}$ on $\boldsymbol{f}$ and $\boldsymbol{f^{DM}}$, as given below:

$$f_{t+1}^{DMX} = \alpha + \beta_1 f_{t+1}^{DM} + \beta_2 f_{t+1} + \epsilon_{t+1} \tag{27}$$

We also add controls for Fama-French's three factors, yielding the following specification:

$$f_{t+1}^{DMX} = \alpha + \beta_1 f_{t+1}^{DM} + \beta_2 f_{t+1} + \beta_3 MKT_{t+1} + \beta_4 SMB_{t+1} + \beta_5 HML_{t+1} + \epsilon_{t+1} \tag{28}$$

The regression output for both specifications is presented in Table 7.

Panel A of Table 7 shows that there is no difference in performance between our modified version of Daniel and Moskowitz's method and the original for all but the BAB factor, despite the original method's use of future information. Adding controls for Fama-French's three factor model in Panel B eliminates this one remaining difference. This implies that the alpha reported in Panel A for the BAB factor was due to exposure to systematic risk contained in Fama-French's three factor model. Thus, it removes the somewhat surprising initial result that the BAB factor performs better using the method which restricts information to that available ex ante.

In total, by looking at Panel B, there appears to be no difference between the modified and original method. Surprisingly, the use of future information does not seem to matter much for performance. All variations in alpha between Panel B in Table 3 and Panel B in Table 6 seem to be due to the change in sample, not the change in method.

As there is no evidence that the original and modified Daniel and Moskowitz methods differ in terms of performance after the ten-year training period, one would expect differences in performance to stem from the training period. The MKT, HML and RMW factors no longer seem to benefit from volatility management when the first ten years are cut out of the sample. This indicates that Daniel and Moskowitz's original method works well in the first ten years of the sample for these factors, relative to the unmanaged factors. For the MKT and HML factors, the training period encompasses a highly volatile time period in the form of the Great Depression. It seems likely that the difference between unmanaged and managed strategies in such climates will be particularly large. Thus, the omission of the Great Depression from MKT and HML's samples seems like a plausible explanation for why these factors no longer generate abnormal returns when volatility managed using our modified Daniel and Moskowitz method.

It could be the case that the use of future information is insignificant, so that the modified and original version of Daniel and Moskowitz's method are equivalent in the first ten years of the sample. There could also be differences between the two in either direction, though one would expect future information to benefit rather than impede performance. Since we use all available data for all factors, we cannot test for the importance of future information in the first ten years of the sample.

Although we found no difference in performance between our modification of Daniel and Moskowitz's method and the original, we employ our modified version for the remainder of the thesis as it is implementable in real-time.

We have seen that volatility management improves performance for a wide range of factors, both when using Moreira and Muir's simple method and Daniel and Moskowitz's more complicated one. The goal of the next subsection is to analyze where the improvements of volatility managed factors relative to unmanaged factors come from, and what drives differences in performance between Moreira and Muir's method and our modified version of Daniel and Moskowitz's method.

Table 7

Testing the importance of Daniel and Moskowitz's (2013) use of future information

Panel A shows the alphas generated by regressing the factors managed using our modified version of Daniel and Moskowitz's method on those managed using their original method and the unmanaged factors. Panel B adds controls for Fama-French's three factors. Significant alphas indicate that the use of future information in Daniel and Moskowitz's original method matters for its performance, relative to our modified method which only uses ex ante information. The samples are 1937-2018 for MKT, SMB, HML and MomF; 1973-2018 for RMW and CMA; 1977-2017 for ROE and IA, 1937-2013 for MomD and 1941-2018 for BAB. All factors are annualized by scaling monthly returns by 12 and standard errors are robust for heteroscedasticity

Panel A: Alphas controlling for Daniel and Moskowitz's original method and the unmanaged factor

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	MKT	SMB	HML	MomF	MomD	RMW	CMA	ROE	IA	BAB
Alpha (α)	0.051	-0.66	-0.27	0.74	0.70	-0.42	-0.23	0.40	0.33	0.97**
	(0.60)	(0.42)	(0.31)	(1.19)	(0.92)	(0.91)	(0.40)	(0.39)	(0.36)	(0.47)
N	981	981	981	981	912	542	542	488	488	934
R^2	0.906	0.821	0.910	0.505	0.933	0.456	0.827	0.907	0.881	0.900
RMSE	16.6	14.7	10.1	33.9	23.9	20.3	9.84	9.31	7.70	10.8

Panel B: Alphas with additional controls for Fama-French's three factors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Alpha (α)	0.11	-0.31	-0.10	1.02	0.75	-0.29	-0.35	0.57	0.13	0.66
	(0.62)	(0.45)	(0.31)	(1.40)	(0.92)	(1.07)	(0.41)	(0.40)	(0.37)	(0.48)

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

D. Hybrid strategies

Since both the variance and return forecasts are different in Moreira and Muir's and our modified version of Daniel and Moskowitz's method, we cannot determine whether differences in performance stem from the differences in return or variance forecast. To isolate the impact of each individual component, we create a strategy for each combination of return and variance forecasts in Moreira and Muir's method and our modified version of Daniel and Moskowitz's method. This allows us to pinpoint where changes in performance come from for each factor, i.e. whether they come from changes in variance forecast or changes in return forecast.

There are three return forecast methods. The first is the time constant, c, from Moreira and Muir's paper[14] (2017). The second is our modified Daniel and Moskowitz (2013) univariate forecast from Equation 26, $\hat{\mu}_{X,t}$, given by:

$$\hat{\mu}_{X,t} = \hat{\nu}_t + \hat{\gamma}_t\left(\sigma^2_{m,t-1} \times I_{B,t-1}\right)$$

[14] Moreira and Muir (2017) do not refer to the time constant as a return forecast, but mathematically it serves much of the same purpose as Daniel and Moskowitz's (2013) return forecasts. We have therefore coined it a return forecast in some instances to simplify the explanation of our hybrid strategies.

Finally, there is our modified Daniel and Moskowitz (2013) multivariate return forecast from Equation 25, $\hat{\mu}^*_{X,t}$, given by:

$$\hat{\mu}^*_{X,t} = \hat{c}_t + \hat{\delta}_{1,t} I_{B,t-1} + \hat{\delta}_{2,t} \sigma^2_{m,t-1} + \hat{\delta}_{3,t} \left(\sigma^2_{m,t-1} \times I_{B,t-1} \right)$$

There are two variance forecast methods. The first of which is Moreira and Muir's (2017) realized variance from Equation 2, RV_t:

$$RV_t(f) = \sum_{d=1/22}^{1} \left(f_{t-1+d} - \frac{\sum_{d=1/22}^{1} (f_{t-1+d})}{22} \right)^2$$

The other is our modified version of Daniel and Moskowitz's (2013) variance forecast from Equation 22, $\hat{\sigma}^2_{X,t}$:

$$\hat{\sigma}^2_{X,t} = \left(\hat{\alpha}_t + \hat{\beta}_{1,t} \sigma_{126,t} + \hat{\beta}_{2,t} \sigma_{GX,t} \right)^2$$

Combining these return and variance forecasts yields six different strategies for adjusting exposure according to variance and return, detailed in Table 8 below.

Table 8

Six unique volatility management strategies

Formulas for the weights used in the six volatility management strategies, which combine different forecasts of variance and return. The bottom right cell corresponds to Moreira and Muir's volatility management method, while the middle left cell is our modified Daniel and Moskowitz method. The remaining four are new volatility management strategies which we coin hybrid strategies. The hybrid strategies and the modified version of Daniel and Moskowitz's method comprise what we call the complex volatility management strategies.

	Variance forecast method	
Return forecast method	$\hat{\sigma}^2_{X,t}$	RV_t
$\hat{\mu}^*_{X,t}$	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}^*_{X,t}}{\hat{\sigma}^2_{X,t}}\right)$	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}^*_{X,t}}{RV_t}\right)$
$\hat{\mu}_{X,t}$	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}_{X,t}}{\hat{\sigma}^2_{X,t}}\right)$	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}_{X,t}}{RV_t}\right)$
c	$\frac{c}{\hat{\sigma}^2_{X,t}}$	$\frac{c}{RV_t}$

The bottom right cell corresponds to Moreira and Muir's volatility management method, while the middle left cell is our modified Daniel and Moskowitz method. The remaining four are new volatility management strategies which we coin hybrid strategies. Since we already computed the monthly time series of every return and variance forecast, all that remains is to combine them according to Table 8. This yields six monthly time series of volatility managed returns for each of the ten factors, giving a total of 60 return series. The annualized Sharpe ratio of each strategy for every factor is presented in Table 9.

Table 9 shows that for the factors SMB, HML, MKT and CMA there is either no benefit to be had from any form of volatility management, or modest gains. All types of volatility management are worse than an unmanaged strategy in the case of SMB and HML, while there are modest gains of four ppt. or less to be had by volatility managing MKT and CMA.

Some form of volatility management is considerably better than an unmanaged strategy for the factors MomD, MomF, ROE, BAB, RMW and IA. Relative to unmanaged strategies, the biggest improvements in absolute terms come from managing the two momentum portfolios and the ROE factor, with gains of 40 to 50 ppt. The remaining three factors, RMW, BAB and IA, experience a gain of 15 to 22 ppt.

However, only a selection of these six factors materially benefit from more complex volatility management than what is proposed by Moreira and Muir (2017). These are MomD and IA with gains of seven and six ppt., respectively. All of the gain for MomD comes from forecasting variance with Daniel and Moskowitz's method. It is worth noting that Fama-French's (2012) momentum does not receive the same benefit. As Daniel and Moskowitz's (2013) momentum portfolio is formed using more fine-grained cutoff points for winners and losers, this could indicate that the benefits of complex volatility management are generated in the extremes of the spectrum of winners and losers.

Interestingly, all of the gain for IA comes from predicting factor return using the multivariate forecast proposed by Daniel and Moskowitz (2013). The remaining four, MomF, RMW, ROE and BAB, do either not benefit from more complex volatility management methods than Moreira and Muir's, or only see modest gains of up to four ppt. There is no clear pattern in whether the return or the variance forecast drives these mild improvements.

It comes as a surprise that the return forecast does not add material value to the factor for which it was designed, MomD, but does add value for a factor for which it was not devised, IA. While there seems to be a theoretical foundation for its use in the former case, to our knowledge that is not the case for the latter. Given our previous discussion in subsection B about the univariate and multivariate return forecasts, given by equations 12 and 11 respectively, it is also surprising that it is the multivariate return forecast which adds value. Recall that the majority of the coefficients for the multivariate return forecasts were not statistically significant at the five percent level.

Table 9

Sharpe ratios of six volatility management strategies

We create one volatility management strategy for each of the six combinations of variance forecast, $(\hat{\sigma}^2_{X,t}, RV_t)$, and return forecast, $(c, \hat{\mu}_{X,t}, \hat{\mu}^*_{X,t})$, for each of the ten factors. Here, $(\hat{\sigma}^2_{X,t}, RV_t)$, are our modified version of Daniel and Moskowitz's (2013) variance forecast and Moreira and Muir's (2017) realized variance, respectively. The return forecasts, $(c, \hat{\mu}_{X,t}, \hat{\mu}^*_{X,t})$, are Moreira and Muir's time constant, our modified univariate Daniel and Moskowitz return forecast and our modified multivariate Daniel and Moskowitz return forecast, respectively. This yields six monthly time series of volatility managed returns for each of the ten factors, a total of 60 return series. For each of them, we calculate the annualized Sharpe ratio. The samples are 1937-2018 for MKT, SMB, HML and MomF; 1973-2018 for RMW and CMA; 1977-2017 for ROE and IA, 1937-2013 for MomD and 1941-2018 for BAB.

			Variance forecast method	
Factor	Unmanaged Sharpe ratio	Return forecast method	$\hat{\sigma}^2_{X,t}$	RV_t
MKT	0.49	$\hat{\mu}^*_{X,t}$	0.46	0.41
		$\hat{\mu}_{X,t}$	0.49	0.44
		c	0.52	0.48
SMB	0.21	$\hat{\mu}^*_{X,t}$	0.19	0.09
		$\hat{\mu}_{X,t}$	0.19	0.12
		c	0.14	0.09
HML	0.43	$\hat{\mu}^*_{X,t}$	0.38	0.37
		$\hat{\mu}_{X,t}$	0.40	0.36
		c	0.42	0.38
MomF	0.55	$\hat{\mu}^*_{X,t}$	0.79	0.98
		$\hat{\mu}_{X,t}$	0.77	0.98
		c	0.90	0.99
MomD	0.69	$\hat{\mu}^*_{X,t}$	1.16	1.08
		$\hat{\mu}_{X,t}$	1.18	1.12
		c	1.18	1.11
RMW	0.42	$\hat{\mu}^*_{X,t}$	0.25	0.25
		$\hat{\mu}_{X,t}$	0.30	0.38
		c	0.56	0.64
CMA	0.57	$\hat{\mu}^*_{X,t}$	0.61	0.61
		$\hat{\mu}_{X,t}$	0.49	0.48
		c	0.53	0.52
ROE	0.81	$\hat{\mu}^*_{X,t}$	1.17	1.15
		$\hat{\mu}_{X,t}$	1.24	1.20
		c	1.23	1.20
IA	0.61	$\hat{\mu}^*_{X,t}$	0.70	0.76
		$\hat{\mu}_{X,t}$	0.65	0.71
		c	0.64	0.70
BAB	0.85	$\hat{\mu}^*_{X,t}$	0.92	1.02
		$\hat{\mu}_{X,t}$	1.04	1.12
		c	1.03	1.10

To further explore these findings, we regress the volatility managed return series of the modified Daniel and Moskowitz method, Moreira and Muir's method, and the four hybrid strategies on the unmanaged factor return and Fama-French's three factors. The alphas of these regressions are presented in Table 10.

MKT, SMB, HML and CMA did not materially benefit from any form of volatility management in terms of Sharpe ratio. Neither of them has significant alphas at the five percent significance level. The other factors all have at least one positive and significant alpha. The factors which had higher improvement in Sharpe ratio generally also have higher alphas.

In summary, the results from Table 10 seem to corroborate the findings from Table 9. The same factors which did not benefit from volatility management in terms of Sharpe ratio, MKT, SMB, HML and CMA, did not get any positive and significant alphas. The remaining factors, i.e. both momentum portfolios, RMW, IA, ROE and BAB, all had increased Sharpe ratios from volatility management. For these there is little variation in alpha generated by going from the simple strategy proposed by Moreira and Muir (2017), to any of the more sophisticated strategies.

To further examine the performance impact of complicating volatility management beyond that of Moreira and Muir (2017), we regress the factors managed according to our modified Daniel and Moskowitz method and the hybrid strategies, $\boldsymbol{f^C}$, on the unmanaged factors, $\boldsymbol{f}$, and the factors managed according to Moreira and Muir's method, $\boldsymbol{f^{MM}}$. Here the superscript "C" stands for "Complex". The regression specification is given by:

$$f_{t+1}^{C} = \alpha + \beta_1 f_{t+1}^{MM} + \beta_2 f_{t+1} + \epsilon_{t+1} \qquad (29)$$

We then add Fama-French's three factor model as an additional control, yielding the following regression specification:

$$f_{t+1}^{C} = \alpha + \beta_1 f_{t+1}^{MM} + \beta_2 f_{t+1} + \beta_3 MKT_{t+1} + \beta_4 SMB_{t+1} + \beta_5 HML_{t+1} + \epsilon_{t+1} \qquad (30)$$

If α is statistically significantly positive, it means that the complex method generates return which cannot be entirely explained by Moreira and Muir's method, the unmanaged factor and Fama-French's three factor model. These alphas are computed for the five complex strategies for every factor and presented in Table 11.

Table 10

Alphas for six volatility management strategies

We create one volatility management strategy for each of the six combinations of variance forecast, $(\hat{\sigma}^2_{X,t}, RV_t)$ and return forecast, $(c, \hat{\mu}_{X,t}, \hat{\mu}^*_{X,t})$, for each of the ten factors. Here, $(\hat{\sigma}^2_{X,t}, RV_t)$, are our modified version of Daniel and Moskowitz's (2013) variance forecast and Moreira and Muir's (2017) realized variance, respectively. The return forecasts, $(c, \hat{\mu}_{X,t}, \hat{\mu}^*_{X,t})$, are Moreira and Muir's time constant, our modified univariate Daniel and Moskowitz return forecast and our modified multivariate Daniel and Moskowitz return forecast, respectively. We regress the volatility managed return series of each of these strategies on the unmanaged factor return and Fama-French's three factors. The alphas of these regressions are presented in this table with standard errors in parenthesis. The samples are 1937-2018 for MKT, SMB, HML and MomF; 1973-2018 for RMW and CMA; 1977-2017 for ROE and IA, 1937-2013 for MomD and 1941-2018 for BAB. All factors are annualized by scaling monthly returns by 12, and standard errors are robust for heteroscedasticity.

Factor	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}_{X,t}}{\hat{\sigma}^2_{X,t}}\right)$	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}^*_{X,t}}{\hat{\sigma}^2_{X,t}}\right)$	$\frac{c}{\hat{\sigma}^2_{X,t}}$	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}_{X,t}}{RV_t}\right)$	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}^*_{X,t}}{RV_t}\right)$	$\frac{c}{RV_t}$
MKT_α	1.70 (1.15)	1.91 (1.27)	1.85* (1.04)	1.78 (1.40)	1.82 (1.46)	1.97 (1.34)
SMB_α	0.03 (0.69)	0.04 (0.85)	-0.09 (0.67)	-0.55 (0.82)	-0.73 (0.90)	-0.58 (0.81)
HML_α	1.07 (0.69)	0.96 (0.70)	1.13* (0.66)	0.89 (0.82)	1.12 (0.80)	0.86 (0.79)
RMW_α	0.62 (1.17)	1.08 (1.27)	2.56*** (0.91)	1.51 (1.16)	1.10 (1.19)	3.45*** (0.98)
CMA_α	0.039 (0.62)	0.99 (0.63)	0.41 (0.64)	0.28 (1.54)	1.16 (0.71)	0.56 (0.73)
$MomF_\alpha$	5.85*** (1.67)	6.16*** (1.65)	7.58*** (1.46)	9.79*** (1.49)	10.1*** (1.49)	9.72*** (1.43)
$MomD_\alpha$	17.6*** (2.90)	17.3*** (2.83)	17.3*** (2.59)	19.4*** (2.98)	18.5*** (2.90)	18.5*** (2.71)
ROE_α	5.47*** (1.16)	5.55*** (1.19)	5.35*** (1.05)	5.91*** (1.21)	6.01*** (1.22)	5.78*** (1.15)
IA_α	0.76 (0.62)	1.55** (0.68)	0.75 (0.63)	1.25* (0.70)	2.05*** (0.72)	1.31* (0.70)
BAB_α	3.46*** (0.86)	3.42*** (1.06)	3.59*** (0.81)	5.61*** (0.88)	5.49*** (1.02)	5.55*** (0.88)

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 11

Do complex methods outperform Moreira and Muir's method?

We create one volatility management strategy for each of the six combinations of variance forecast, $(\hat{\sigma}^2_{X,t}, RV_t)$, and return forecast, $(c, \hat{\mu}_{X,t}, \hat{\mu}^*_{X,t})$, for each of the ten factors. Here, $(\hat{\sigma}^2_{X,t}, RV_t)$, are our modified version of Daniel and Moskowitz's (2013) variance forecast and Moreira and Muir's (2017) realized variance, respectively. The return forecasts, $(c, \hat{\mu}_{X,t}, \hat{\mu}^*_{X,t})$, are Moreira and Muir's time constant, our modified univariate Daniel and Moskowitz return forecast and our modified multivariate Daniel and Moskowitz return forecast, respectively. We regress the returns of factors volatility managed using our complex strategies on the unmanaged factor, the factor managed using Moreira and Muir's method and Fama-French's three factors. If the alphas produced are statistically significantly positive, it implies that the more complex volatility management methods outperform the simpler Moreira and Muir method. The samples are 1937-2018 for MKT, SMB, HML and MomF; 1973-2018 for RMW and CMA; 1977-2017 for ROE and IA, 1937-2013 for MomD and 1941-2018 for BAB. All factors are annualized by scaling monthly returns by 12, and standard errors are robust for heteroscedasticity.

Complex strategy	(1) MKT	(2) SMB	(3) HML	(4) MomF	(5) MomD	(6) RMW	(7) CMA	(8) ROE	(9) IA	(10) BAB
$\frac{1}{2\lambda}\left(\frac{\hat{\mu}_{X,t}}{\hat{\sigma}^2_{X,t}}\right)$	0.33	0.24	0.45	1.34	3.74**	-0.46	-0.35	1.01	-0.27	-0.32
	(0.64)	(0.59)	(0.35)	(1.53)	(1.75)	(1.07)	(0.38)	(0.65)	(0.32)	(0.57)
$\frac{1}{2\lambda}\left(\frac{\hat{\mu}^*_{X,t}}{\hat{\sigma}^2_{X,t}}\right)$	0.51	0.11	0.38	1.42	2.66*	0.15	0.66	0.80	0.56	0.55
	(0.78)	(0.84)	(0.46)	(1.51)	(1.60)	(1.21)	(0.50)	(0.73)	(0.48)	(1.08)
$\frac{c}{\hat{\sigma}^2_{X,t}}$	0.58	0.28	0.51	1.73	3.44**	-0.26	-0.050	1.01*	-0.33	-0.30
	(0.54)	(0.38)	(0.32)	(1.18)	(1.36)	(0.34)	(0.24)	(0.56)	(0.29)	(0.44)
$\frac{1}{2\lambda}\left(\frac{\hat{\mu}_{X,t}}{RV_t}\right)$	-0.25	-0.13	0.018	-0.17	1.01	0.055	-0.79	0.17	-0.012	0.23
	(0.28)	(0.55)	(0.14)	(0.29)	(1.07)	(1.04)	(0.69)	(0.26)	(0.20)	(0.29)
$\frac{1}{2\lambda}\left(\frac{\hat{\mu}^*_{X,t}}{RV_t}\right)$	-0.22	-0.53	0.30	0.27	-0.36	-0.0093	0.74	0.083	0.85**	1.21
	(0.45)	(0.86)	(0.33)	(0.67)	(1.10)	(1.10)	(0.49)	(0.33)	(0.40)	(1.03)

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

There are three complex methods which seem to outperform Moreira and Muir's. Daniel and Moskowitz's (2013) momentum portfolio looks to benefit the most. Employing Daniel and Moskowitz's variance forecast and either their univariate return forecast or Moreira and Muir's (2017) time constant, generates respective annual alphas of 3.7 and 3.4 percent.

As was the case with Sharpe ratios in Table 9, Fama-French's (2012) momentum does not receive the same benefit as Daniel and Moskowitz's (2013). This is a further indication that the benefits of complex volatility management are generated in the extremes of the spectrum of winners and losers.

Hou, Xue and Zhang's (2014) investment factor, IA, also benefits modestly from more complex volatility management. Combining a multivariate return forecast with Moreira and Muir's (2017) realized variance generates an annual alpha of 0.85 percent. Additionally, there is weaker evidence at the ten percent significance level that two more complex methods outperform Moreira and Muir's, a further case for MomD and one for ROE.

More importantly, only three out of 50 complex strategies generate statistically significant alphas at the five percent level. Since so few strategies outperform Moreira and Muir's method, it is difficult to deduce which combination of return and variance forecast, if any, is best suited to outperforming a simple volatility management strategy. Indeed, it suggests that there is rarely considerable value gained by complicating volatility management strategies beyond that of Moreira and Muir (2017).

We previously found that Daniel and Moskowitz's original method moderately outperforms Moreira and Muir's, and that there is no difference in performance between the original and modified versions of Daniel and Moskowitz's method. One might therefore expect our modified Daniel and Moskowitz method to outperform Moreira and Muir's, and it may seem dissonant that there are so few instances where that is the case.

However, one must keep in mind that the modified version of Daniel and Moskowitz's method has a ten-year shorter sample than the original due to its ten-year training period. Thus, our initial finding that Daniel and Moskowitz's original method outperforms Moreira and Muir's seems to depend on the inclusion of this period.

Overall, these findings indicate that it is hard to materially increase Sharpe ratios and abnormal returns by complicating volatility management beyond that of Moreira and Muir (2017), which uses a random walk forecast for variance and a time constant to scale factor exposure. As Daniel and Moskowitz present proof that the optimal portfolio weight is proportional to Sharpe ratio and not just variance (2013), this suggests that the more complex variance forecast is not much better than a random walk forecast. It also indicates that it has proved difficult to forecast returns with Daniel and Moskowitz's methodology.

On the other hand, the most considerable improvement is generated where we would expect it to be, namely the factor for which Daniel and Moskowitz's method was designed, MomD. This gives reason to believe that there may be promise for complex volatility management if one tailors different variance and return forecasts to the properties of the different factors, as opposed to using a one-size-fits-all approach.

Thus far we have only evaluated performance by Sharpe ratio and abnormal return. Even though complex volatility management generally does not appear to yield large improvements over Moreira and Muir (2017) along these measures, it may generate other desirable properties. This will be looked at in the next section.

4. Discussion

In this part of the thesis, we first seek to highlight some methodological issues. Then we will discuss the sensitivity of our findings, notably to frictions such as transaction and liquidity costs. Finally, we will consider how the higher order moments skewness and kurtosis vary across different volatility management strategies.

Moreira and Muir's and Daniel and Moskowitz's volatility management methods can be critiqued for their use of the time constants, c and λ. In both cases they are calculated using the full sample standard deviation of unmanaged returns, thus using future information. As we explained in subsections A and B, this does not affect the Sharpe ratio of the managed factors. It does, however, affect the magnitude of the weights invested in the factor, and may thereby impact transaction and liquidity costs. Depending on the magnitude of the proposed weights, they may in the worst case make the investment infeasible due to leverage and shorting constraints.

Faced with such frictions, Sharpe ratio becomes a function of the magnitude of the weights, and thus also of the time constants c and λ. As such, if these frictions exist, one can no longer get away with choosing c and λ in the same way without biasing the performance measures of the managed portfolios by the use of future information. This is because c and λ will propose weights which are artificially stable when computed on the full sample, as opposed to only using information available ex ante.

While we acknowledge these methodological issues, Moreira and Muir (2017) find that their strategy is robust to transaction costs for those factors where good measures of such costs are available. If the weights proposed by the more complex volatility management strategies are less volatile than those proposed by Moreira and Muir's method, then these strategies are likely to entail less trading. Thus, they are also likely to survive transaction costs and to incur less liquidity costs[15].

To examine whether this is the case, we report the standard deviation of the weights proposed by Moreira and Muir's method and the more complex strategies in Table 12. The standard deviations of the weights proposed by Moreira and Muir's method are in column seven, while columns two through six contain those of the more complex strategies. The complex methods generally propose weights which are considerably less volatile than Moreira

[15] We acknowledge that this is a relatively unsophisticated way of examining this question. Transaction and liquidity costs are not only functions of the amount of trading. To fully take account of transaction and liquidity costs, a more thorough empirical analysis would be required.

and Muir's, except for MomF. This suggests that the performance of these strategies would also survive transaction costs, and that they seem likely to incur less liquidity costs.

On closer inspection, we see that the driver of the reduced volatility of the weights is switching from a random walk forecast of variance, RV_t, to the more sophisticated variance forecast from Daniel and Moskowitz (2013), $\hat{\sigma}^2_{X,t}$. If anything, adding Daniel and Moskowitz's return forecasts seems to increase the volatility of the weights. Thus, it seems that the best strategy to implement when faced with such frictions, would be the one in column four which uses the sophisticated variance forecast and no return forecast. It does not seem particularly surprising that the general-purpose variance forecast works for several factors, while the return forecasts tailored to momentum do not seem to work for other factors than momentum. Notably, the return forecasts neither seem to work for momentum.

More generally, when taking into account frictions, the complex volatility management methods seem more likely to outperform Moreira and Muir's simple method due to their less volatile weights. More complex methods thus seem to have some desirable properties in real-world application since they require less trading activity. This adds some nuance to our previous findings, which indicate that there is generally little value added in complicating volatility management beyond Moreira and Muir's method. Next, we will examine whether complex volatility management adds further benefit by improving the higher order moments of skewness and kurtosis.

In the empirical analysis we evaluate the performance of different volatility management strategies using Sharpe ratio and estimates of abnormal return. However, the initial purpose of volatility management was to reduce momentum crash risk (Daniel & Moskowitz, 2013; Barroso & Santa-Clara, 2015). The extent to which this was achieved was partly assessed by evaluating the higher order moments of skewness and kurtosis of the volatility managed momentum portfolios, relative to the unmanaged momentum factors.

To examine the effects of volatility management through an additional lens, we will do the same exercise on all factors examined in this thesis. We present the skewness and kurtosis of all volatility management methods for all factors in Table 13. A decrease in kurtosis and an increase in skewness reduces downside risk (Barroso & Santa-Clara, 2015), as it skews the return distribution towards positive values while simultaneously thinning down the tails.

Some form of volatility management improves the skewness of the monthly return distribution for all factors, and improves kurtosis for seven out of ten factors. More importantly, one or more complex methods improve skewness relative to Moreira and Muir's method for eight out of ten factors, and improves kurtosis for nine out of ten factors. Finally, at least one complex method improves both skewness and kurtosis compared to Moreira and Muir's method for five out of ten factors. These are the MKT, RMW, CMA and both momentum factors.

Thus, complex volatility management skews the return distribution towards positive values while simultaneously thinning down the tails for several factors, thereby reducing downside risk relative to Moreira and Muir's method. This is further indication that there may be benefit in complicating volatility management beyond that of Moreira and Muir (2017), despite the general lack of considerable improvement in Sharpe ratio and abnormal return.

Table 12

The volatility of weights for six volatility management strategies

We create one volatility management strategy for each of the six combinations of variance forecast, $(\hat{\sigma}^2_{X,t}, RV_t)$, and return forecast, $(c, \hat{\mu}_{X,t}, \hat{\mu}^*_{X,t})$, for each of the ten factors. Here, $(\hat{\sigma}^2_{X,t}, RV_t)$, are our modified version of Daniel and Moskowitz's (2013) variance forecast and Moreira and Muir's (2017) realized variance, respectively. The return forecasts, $(c, \hat{\mu}_{X,t}, \hat{\mu}^*_{X,t})$, are Moreira and Muir's time constant, our modified univariate Daniel and Moskowitz return forecast and our modified multivariate Daniel and Moskowitz return forecast, respectively. We compute the standard deviations of the weights of each strategy and tabulate them below. The samples are 1937-2018 for MKT, SMB, HML and MomF; 1973-2018 for RMW and CMA; 1977-2017 for ROE and IA, 1937-2013 for MomD and 1941-2018 for BAB.

Factor	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}_{X,t}}{\hat{\sigma}^2_{X,t}}\right)$	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}^*_{X,t}}{\hat{\sigma}^2_{X,t}}\right)$	$\frac{c}{\hat{\sigma}^2_{X,t}}$	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}_{X,t}}{RV_t}\right)$	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}^*_{X,t}}{RV_t}\right)$	$\frac{c}{RV_t}$
MKT	0.74	0.77	0.63	1.17	1.19	1.04
SMB	0.58	0.80	0.68	0.85	0.96	0.96
HML	0.74	0.77	0.63	1.17	1.19	1.04
MomF	4.99	4.87	3.65	1.51	1.60	1.52
MomD	0.83	0.94	0.85	1.12	1.22	1.20
RMW	0.82	1.11	0.88	0.94	1.25	1.12
CMA	0.73	0.81	0.79	2.04	1.02	1.00
ROE	0.74	0.85	0.73	0.95	1.01	0.93
IA	0.80	0.82	0.78	0.88	0.90	0.89
BAB	0.68	0.83	0.68	1.02	1.11	1.06

Table 13

Skewness and kurtosis of six volatility management strategies

Panel A shows the skewness of returns for the four hybrid strategies in addition to the modified version of Daniel and Moskowitz's method, Moreira and Muir's and the unmanaged factor. Panel B shows the kurtosis of returns for the four hybrid strategies in addition to the modified version of Daniel and Moskowitz's method, Moreira and Muir's and the unmanaged factor. This is done for the market, size, value, momentum, investment, profitability, return on equity and betting-against-beta factors. The samples are 1937-2018 for MKT, SMB, HML and MomF; 1973-2018 for RMW and CMA; 1977-2017 for ROE and IA, 1937-2013 for MomD and 1941-2018 for BAB.

Factor	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}_{X,t}}{\hat{\sigma}^2_{X,t}}\right)$	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}^*_{X,t}}{\hat{\sigma}^2_{X,t}}\right)$	$\frac{c}{\hat{\sigma}^2_{X,t}}$	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}_{X,t}}{RV_t}\right)$	$\frac{1}{2\lambda}\left(\frac{\hat{\mu}^*_{X,t}}{RV_t}\right)$	$\frac{c}{RV_t}$	Unmanaged factor
				Panel A: Skewness of returns			
MKT	-0.38	3.18	0.20	0.61	-1.14	0.47	-0.53
SMB	-0.37	-0.15	0.36	1.54	1.04	0.69	0.77
HML	-0.43	0.38	-0.03	0.51	0.70	1.98	0.69
MomF	0.01	0.25	0.46	0.65	0.94	0.64	-1.72
MomD	-0.07	0.45	0.35	1.50	1.17	-0.34	-1.38
RMW	-0.11	0.44	0.47	0.59	0.87	0.48	-0.34
CMA	0.56	1.04	0.37	0.20	1.18	-0.56	0.38
ROE	1.40	0.51	1.13	-0.94	1.05	1.32	-0.66
IA	0.03	0.68	0.64	0.32	0.57	0.99	0.22
BAB	0.25	0.68	0.93	0.97	1.12	0.79	-0.63
				Panel B: Kurtosis of returns			
MKT	2.56	53.25	22.16	1.61	29.11	3.05	3.18
SMB	2.85	9.55	9.12	10.25	5.89	3.97	6.99
HML	2.32	3.15	14.86	1.26	2.62	14.85	5.38
MomF	9.20	3.24	8.71	2.40	3.46	3.86	13.66
MomD	9.68	3.05	2.22	9.91	6.70	6.40	7.85
RMW	8.05	7.31	2.60	2.34	4.65	14.73	11.95
CMA	6.01	11.31	2.17	31.21	5.42	5.61	1.79
ROE	12.00	6.65	8.51	22.01	5.74	8.39	4.74
IA	3.39	79.24	11.98	1.58	3.24	8.38	1.77
BAB	9.35	71.16	7.87	39.91	6.76	5.28	4.38

5. Conclusion

Both Moreira and Muir's and the more complex volatility management methods produce large alphas and increased Sharpe ratios over buy-and-hold strategies for a wide range of factors. More interestingly, complicating volatility management beyond the simple method of Moreira and Muir only has a modest impact on alphas and Sharpe ratios. The notable exception is Daniel and Moskowitz's (2013) momentum factor.

In light of Daniel and Moskowitz's proof that the optimal portfolio weight is proportional to Sharpe ratio and not just variance (2013), this suggests that the more complex variance forecast is not much better than a random walk forecast. It also indicates that it has proved difficult to forecast returns with Daniel and Moskowitz's methodology.

Complex volatility management does, however, entail other desirable properties. It results in less volatile weights, which reduces transaction and liquidity costs. It also generates more desirable return distributions with improved skewness and kurtosis, which reduce downside risk. This seems like an appropriate feature given that the initial purpose of Daniel and Moskowitz's method was to reduce momentum crash risk (2013).

The strategies explored in this thesis use the same methodology for all factors. There may be benefit in tailoring different variance and return forecasts to the properties of the different factors, as opposed to using a one-size-fits-all approach. The fact that complex volatility management outperforms Moreira and Muir's method for the factor for which it was originally designed, Daniel and Moskowitz's (2013) momentum, indicates that this may be a fruitful area for future research.

References

AQR. (2019, January 30). *Betting Against Beta: Equity Factors Data, Monthly.* Retrieved from aqr: https://www.aqr.com/Insights/Datasets/Betting-Against-Beta-Equity-Factors-Monthly

Barroso, P., & Santa-Clara, P. (2015). Momentum has its moments. *Journal of Financial Economics*, 111-120.

Daniel , K., & Moskowitz, T. J. (2013). Momentum Crashes. *Swiss Finance Insititure Research Paper no. 13-61*.

Daniel, K. (2019, January 30). *Data/Appendices.* Retrieved from kentdaniel: http://www.kentdaniel.net/data.php

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 3-56.

Fama, E. F., & French, K. R. (2012). Size, value and momentum in international stock returns. *Journal of Financial Economics*, 457-472.

Fama, E. F., & French, K. R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, 1-22.

Frazzini, A., & Pedersen, L. H. (2014). Betting Against Beta. *Journal of Financial Economics*, 1-25.

French, K. R. (2019, January 30). *Current Research Returns*. Retrieved from http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Glosten , R. L., Jagannathan, R., & Runkle, E. D. (1993, December). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return. *The Journal of Finance*, pp. 1779-1801.

Hou, K., Xue, C., & Zhang, L. (2014). Digesting Anomalies: An Investment Approach. *Review of Financial Studies*, 650-705.

Moreira, A., & Muir, T. (2017). Volatility-Managed Portfolios. *The Journal of Finance*, 1611-1644.

Wharton Research Data Services. (2019, Feb 20). *Wharton Research Data Services.* Retrieved from https://wrds-web.wharton.upenn.edu/wrds/process/wrds.cfm

Appendix A: Replicating Moreira and Muir (2017)

Table A1

Replication of Moreira and Muir's method

In Panel A, we run monthly time-series regressions of volatility managed returns à la Moreira and Muir (2017) on the unmanaged returns for each factor, $f_{t+1}^{MM} = \alpha + \beta f_{t+1} + \epsilon_{t+1}$. In Panel B, Fama-French's three factor model is used as an additional control in the regressions from Panel A. The samples are 1926-2015 for MKT, SMB, HML and MomF; 1963-2015 for RMW and CMA; 1967-2015 for ROE and IA and 1930-2012 for BAB. All factors are annualized by scaling monthly returns by 12 and standard errors are robust for heteroscedasticity.

Panel A: Univariate Regressions

	(1) σ MKT	(2) σ SMB	(3) σ HML	(4) σ MomF	(5) σ RMW	(6) σ CMA	(7) σ ROE	(8) σ IA	(9) σ BAB
MKT	0.60***								
	(0.06)								
SMB		0.61***							
		(0.08)							
HML			0.57***						
			(0.08)						
MomF				0.48***					
				(0.07)					
RMW					0.60***				
					(0.08)				
CMA						0.69***			
						(0.05)			
ROE							0.68***		
							(0.06)		
IA								0.71***	
								(0.05)	
BAB									0.57***
									(0.05)
Alpha (α)	4.45***	-0.51	1.78*	12.5***	2.61***	0.45	5.14***	1.84***	5.90***
	(1.56)	(0.91)	(1.02)	(1.71)	(0.88)	(0.68)	(1.02)	(0.65)	(1.00)
N	1065	1065	1065	1060	621	621	575	575	975
R^2	0.371	0.369	0.322	0.228	0.351	0.467	0.440	0.496	0.349
RMSE	50.9	30.9	34.9	50.2	21.6	17.8	23.3	16.2	30.1

Panel B: Alphas controlling for Fama-French three factors

	(1) σ MKT	(2) σ SMB	(3) σ HML	(4) σ MomF	(5) σ RMW	(6) σ CMA	(7) σ ROE	(8) σ IA	(9) σ BAB
Alpha (α)	5.04***	-0.29	2.48**	10.4***	3.44***	-0.083	5.60***	1.27**	5.43***
	(1.57)	(0.90)	(1.03)	(1.60)	(0.89)	(0.65)	(1.04)	(0.65)	(0.98)

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Appendix B: Replicating Daniel and Moskowitz (2013)

Daniel and Moskowitz's (2013) return forecasts

Table B1 contains the regression output of the univariate and multivariate return forecasts of Daniel and Moskowitz's original method. Table B2 reports the alphas generated by the modified Daniel and Moskowitz's method using the multivariate return forecast. Since the alphas generated are very similar to those in Table 6 where the univariate forecast is used, it suggests that the performance of the modified Daniel and Moskowitz method is insensitive to the choice of return forecast method.

Table B1

Daniel and Moskowitz's (2013) return forecasts

Panel A shows the output of the univariate regression from Equation 10 used to forecast returns by Daniel and Moskowitz (2013): $f_t = \nu + \gamma\left(\sigma^2_{m,t-1} \times I_{B,t-1}\right) + \epsilon_t$. Panel B shows the output from their multivariate regression from Equation 9: $f_t = c + \delta_1 I_{B,t-1} + \delta_2 \sigma^2_{m,t-1} + \delta_3\left(\sigma^2_{m,t-1} \times I_{B,t-1}\right) + \epsilon_t$. The variables have a monthly frequency. The samples are 1927-2018 for MKT, SMB, HML and MomF; 1964-2018 for RMW and CMA; 1967-2017 for ROE and IA, 1927-2013 for MomD and 1931-2018 for BAB.

Panel A: Regression output for the univariate return forecast

$$f_t = \nu + \gamma\left(\sigma^2_{m,t-1} \times I_{B,t-1}\right) + \epsilon_t$$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	MKT	SMB	HML	MomF	MomD	RMW	CMA	ROE	IA	BAB
$\hat{\gamma}$	-0.000	0.002***	-0.000	-0.005***	-0.097***	-0.000	-0.000	-0.003***	-0.000	-0.001
	(-0.03)	(3.03)	(-0.51)	(-5.31)	(-5.63)	(-0.06)	(-0.11)	(-4.23)	(-0.15)	(-1.16)
$\hat{\nu}$	0.007***	0.001	0.0038***	0.009***	0.020***	0.003***	0.003***	0.007***	0.004***	0.007***
	(3.85)	(1.20)	(3.50)	(6.09)	(7.04)	(2.96)	(3.57)	(6.35)	(5.01)	(6.91)

Panel B: Regression output for the multivariate return forecast

$$f_t = c + \delta_1 I_{B,t-1} + \delta_2 \sigma^2_{m,t-1} + \delta_3\left(\sigma^2_{m,t-1} \times I_{B,t-1}\right) + \epsilon_t$$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	MKT	SMB	HML	MomF	MomD	RMW	CMA	ROE	IA	BAB
$\hat{\delta}_3$	-0.001	-0.000	-0.001	-0.004*	-0.009**	-0.001	-0.004**	-0.003*	-0.003**	0.000
	(-0.28)	(-0.07)	(-0.64)	(-1.94)	(-2.28)	(-0.60)	(-2.52)	(-1.65)	(-2.00)	(0.27)
$\hat{\delta}_2$	0.001	0.001	-0.000	-0.001	-0.002	0.001	0.002	-0.001	0.001	-0.001
	(0.64)	(0.83)	(-0.16)	(-0.62)	(-0.76)	(0.37)	(1.53)	(-0.62)	(0.83)	(-0.55)
$\hat{\delta}_1$	-0.002	0.007**	0.005	-0.001	0.005	0.003	0.011***	0.002	0.001***	-0.003
	(-0.37)	(2.04)	(1.18)	(-0.11)	(0.54)	(0.77)	(3.48)	(0.56)	(3.38)	(-0.85)
$\hat{c}$	0.006***	-0.000	0.004**	0.001***	0.021***	0.002	0.001	0.007***	0.003**	0.008***
	(2.60)	(-0.06)	(2.41)	(4.91)	(5.55)	(1.63)	(0.75)	(4.56)	(2.14)	(5.38)

t-statistics in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table B2

Volatility-managed alphas with the modified Daniel and Moskowitz method using the multivariate return forecast

In Panel A, we run monthly time-series regressions of volatility managed returns using our modified Daniel and Moskowitz method with the multivariate return forecast on the unmanaged returns for each factor, $f_{t+1}^{DMX} = \alpha + \beta f_{t+1} + \epsilon_{t+1}$. In Panel B, Fama-French's three factor model is used as an additional control in the regressions from Panel A. The samples are 1937-2018 for MKT, SMB, HML and MomF; 1973-2018 for RMW and CMA; 1977-2017 for ROE and IA, 1937-2013 for MomD and 1941-2018 for BAB. All factors are annualized by scaling monthly returns by 12 and standard errors are robust for heteroscedasticity.

Panel A: Univariate Regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(11)	(10)
	σ	σ	σ	σ	σ	σ	σ	σ	σ	σ
	MKT	SMB	HML	MomF	MomD	RMW	CMA	ROE	IA	BAB
MKT	0.72***									
	(0.054)									
SMB		0.65***								
		(0.057)								
HML			0.75***							
			(0.065)							
MomF				0.47***						
				(0.059)						
MomD					0.59***					
					(0.061)					
RMW						0.31***				
						(0.075)				
CMA							0.67***			
							(0.061)			
ROE								0.61***		
								(0.075)		
IA									0.67***	
									(0.063)	
BAB										0.57***
										(0.064)
Alpha (α)	1.68	0.57	0.58	7.38***	20.1***	0.96	1.57**	6.04***	1.84***	4.29***
	(1.23)	(0.81)	(0.68)	(1.52)	(2.80)	(1.14)	(0.68)	(1.17)	(0.69)	(1.18)
N	981	981	981	981	912	542	542	488	488	934
R^2	0.521	0.428	0.559	0.218	0.354	0.097	0.449	0.368	0.455	0.329
RMSE	37.5	26.3	22.5	42.5	74.3	26.1	17.5	24.3	16.5	28.1

Panel B: Alphas controlling for Fama-French's three factors

Alpha (α)	1.91	0.040	0.96	6.16***	17.3***	1.08	0.99	5.55***	1.55**	3.42***
	(1.27)	(0.85)	(0.70)	(1.65)	(2.83)	(1.27)	(0.63)	(1.19)	(0.68)	(1.06)

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Daniel and Moskowitz's (2013) variance forecasts

Table B3 shows the maximum likelihood estimates of the GJR-GARCH parameters detailed in equations 5 and 6, for all ten factors. We use the R-package "rugarch" created by Alexios Ghalanos to compute them.

$$f_t = \mu + \epsilon_t$$

Where $\epsilon_t \sim \mathcal{N}(0, \sigma^2_{G,t})$, and $\sigma^2_{G,t}$ is governed by the process:

$$\sigma^2_{G,t} = \omega + \beta\sigma^2_{G,t-1} + \left(\alpha + \gamma I(\epsilon_{t-1} < 0)\right)\epsilon^2_{t-1}$$

Table B4 shows the output of the regression from Equation 7 used to forecast variance by Daniel and Moskowitz (2013).

Table B3

Maximum likelihood estimates of the GJR-GARCH parameters

Glosten, Jagannathan and Runkle's (1993) model is run on the daily time series of return for each factor, producing a set of maximum likelihood estimates of the GJR-GARCH parameters for each factor. The samples are 1927-2018 for MKT, SMB, HML and MomF; 1964-2018 for RMW and CMA; 1967-2017 for ROE and IA, 1927-2013 for MomD and 1931-2018 for BAB.

	(1) MKT	(2) SMB	(3) HML	(4) MomF	(5) MomD	(6) RMW	(7) CMA	(8) ROE	(9) IA	(10) BAB
$\hat{\mu}$	0.000375	0.000021	0.000085	0.000382	0.000855	0.000116	0.000087	0.000316	0.000162	0.000316
	(8.3048)	(0.87438)	(3.65243)	(14.2105)	(14.4838)	(5.42551)	(3.7709)	(11.5843)	(6.53361)	(11.5843)
$\hat{\omega}$	0.000001	0.000000	0.000000	0.000000	0.000001	0.000000	0.000000	0.000000	0.000000	0.000000
	(4.1685)	(0.96923)	(1.04791)	(1.5900)	(1.7099)	(0.91835)	(1.3548)	(1.1679)	(0.63835)	(1.1679)
$\hat{\alpha}$	0.036107	0.079880	0.094655	0.146755	0.110629	0.050048	0.072473	0.062769	0.057613	0.062769
	(10.7879)	(13.4443)	(16.7936)	(18.4166)	(9.6366)	(25.6606)	(30.5783)	(30.3044)	(23.6690)	(30.3044)
$\hat{\beta}$	0.894835	0.900253	0.902455	0.861815	0.895763	0.944164	0.936114	0.941677	0.948406	0.941677
	(176.672)	(144.177)	(164.737)	(130.920)	(84.1376)	(369.949)	(332.537)	(331.571)	(341.882)	(331.571)
$\hat{\gamma}$	0.106460	0.035537	0.002542	-0.019140	-0.016395	0.003357	-0.025839	-0.018177	-0.019477	-0.018177
	(16.1847)	(7.92282)	(0.57878)	(-3.4161)	(-3.2748)	(0.87850)	(-6.0407)	(-4.3379)	(-5.0841)	(-4.3379)

t-statistics in parentheses.

Table B4

Regression output for Daniel and Moskowitz's (2013) variance forecast

Daniel and Moskowitz's (2013) variance forecast is applied to every factor by regressing the factor's volatility in the next month , σ_{22t+1}, on its six-month realized volatility, $\sigma_{126,t}$ and its GJR-GARCH forecasted volatility, $\sigma_{G,t}$. The regression specification is: $\sigma_{22t+1} = \alpha + \beta_1\sigma_{126,t} + \beta_2\sigma_{G,t} + \epsilon_t$. The time series are monthly, and the samples are: 1927-2018 for MKT, SMB, HML and MomF; 1964-2018 for RMW and CMA; 1967-2017 for ROE and IA, 1927-2013 for MomD and 1931-2018 for BAB

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	MKT	SMB	HML	MomF	MomD	RMW	CMA	ROE	IA	BAB
$\hat{\beta}_1$	0.2920^{***}	0.3550^{***}	0.3556^{***}	0.2210^{***}	0.2189^{***}	0.1180^{***}	0.1701^{***}	0.0894^{***}	0.0859^{*}	0.3960^{***}
	(11.10)	(12.14)	(11.88)	(7.94)	(7.04)	(2.92)	(4.03)	(2.18)	(1.83)	(10.66)
$\hat{\beta}_2$	0.6071^{***}	0.4990^{***}	0.5230^{***}	0.6080^{***}	0.6603^{***}	0.7720^{***}	0.7170^{***}	0.7840^{***}	0.7980^{***}	0.3960^{***}
	(24.86)	(18.90)	(18.87)	(23.98)	(21.47)	(19.30)	(17.40)	(19.45)	(16.86)	(8.84)
$\hat{\alpha}$	0.0005^{*}	0.0005^{***}	0.0004^{***}	0.0008^{***}	0.0008^{**}	0.0003^{***}	0.0003^{***}	0.0004^{***}	0.0004^{***}	0.0011^{***}
	(1.95)	(4.02)	(3.41)	(5.01)	(2.36)	(3.59)	(3.46)	(3.87)	(3.62)	(6.57)

t-statistics in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$