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Decision Making in Norwegian Dairy Farming Using Mathematical Programming

Maximising Farmer's Gross Margin Under Subsidy Regulations

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Abstract

Norwegian dairy farming is characterised by increased consolidation and a wide array of governmental support schemes. The farming industry can utilise business analytics to assist the complex decision making facing Norwegian farmers. In this thesis, we develop a mixed integer linear programming model that maximises the farmer's gross margin under farming activity and subsidy constraints.

We use the optimisation model to study a small, a medium, and a large dairy farm located in Jæren, Norway. We find that all of the three farms have pure dairy cow herds in the optimal solution. Subsidy's total share of income is 30.2% lower for the large farm than for the small farm. We also find that the marginal subsidy amount drops from NOK 2.07 for the small farm, to NOK 0.89 for the medium farm, to NOK 0.52 for the large farm, representing a decrease of 74.8%. Additionally, we found that a milk quota reduction from 135 000 litres to 105 000 litres for the small farm results in such a large reduction of profitability that the farm is better off renting a larger quota to obtain maximal milk production. Lastly, we found that a milk price reduction of 30% only has implications for the large farm, which replaces eight dairy cows with eight suckler cows.

This thesis further discusses the use of continuous variables in a farming optimisation model, as well as milk quota valuations and variable cull rates. In addition to the optimisation model, the work done in this thesis also includes a separate input handling system. This sets some basis for further work to develop a fully integrated whole-farm decision support system for Norwegian dairy farming.

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1 Introduction and Scope of Research

In the 2018 annual report of the farmer owned cooperative TINE SA, president and CEO Gunnar Hovland proclaims “We have to take on board the opportunities provided by technology throughout the entire value chain” (TINE SA, 2019). Therefore, in 2018, MIMIRO – a company aiming to utilise agricultural data and develop a farm management system to “increase farmers’ productivity and profitability” (TINE SA, 2019) – was established, jointly owned by TINE and Felleskjøpet. Farm management systems are often referred to as agricultural decision support systems – abbreviated to AgriDSS to distinct it as a branch of DSS. In the early 70’s, Morton introduced the concepts entailed in DSS under the term “management decision systems” (Morton, 1971). Since then, the development of DSSs has come a long way , and can for the purpose of this thesis be characterised as responsive computer programs integrating data and mathematical models to assist the end-user with analysis and decision making.

Agricultural farms have a complex and diverse nature, as there are many components involved in managing a specific type of farm, and the farmer is often faced with several options regarding how he can operate his farm. In Norway, agricultural farms are getting fewer in numbers and larger in size (Statistics Norway, 2019, 2018). This represents a need for increased supervision on the farms, as having a larger farm opens up for diversifying into several activities simultaneously, and thus more questions as to how the farm should or could be managed arises. One objective of the Norwegian government is to have a diverse agriculture with a varied farm structure and geographically dispersed production (LMD, 2018b). As an incentive for farmers to help the government reach this objective, amongst several other objectives, the government provides subsidies to farmers. In return, farmers facing unfavourable production environments are able to secure a viable income while continuing to operate farms located in areas that otherwise would be uncompetitive. Since dairy and beef production is the largest productions in Norwegian agriculture (NAA, 2019a), this thesis will propose a farm management tool for dairy farmers, laying the premises for further development of a fully integrated decision support system for Norwegian farmers.

The main purpose and scope of research of this thesis is to utilise business analytics to develop a farm management tool to assist Norwegian dairy farmers' decision making. This is achieved by creating an optimisation model using Mathematical Programming. This optimisation model can be used by farmers to help them make better decisions by facilitating an exchange of information between the farmer and the model, assist with information processing and expand the farmer's perception of complex and unstructured problems. More specifically, we propose a Linear Programming model where the objective-function is to maximise the gross margin of the farm by optimising the allocation of the farm's resources. The optimisation model will take farm characteristics as input data from the user. Data of animal feed requirements and plans are provided by TINE, while the price of milk, beef, fodder and grains is regulated by an agricultural agreement and traded through the farmer-owned cooperatives TINE SA, Nortura SA and Felleskjøpet SA (LMD, 2018a). This agricultural agreement also determines the subsidies awarded to Norwegian agriculture, and is thus included in the model. Furthermore, based on agricultural farm data from the Norwegian Agriculture Agency (NAA) we use experimental data to create three farms of different sizes in terms of farmland and animal stalls to investigate differences in resource allocation and subsidies as part of the total gross margin, succeeded by a scenario analysis on milk quota and milk prices. The solution of the optimisation model returns the optimal structure of livestock to keep at the farm, how to allocate farmland between crop and grazing activities, milk yield per dairy cow, slaughter age of bulls and suckler cows, and sale of calves. Additionally, the model provides the farmers with insights into the subsidy's contributions to total margin, which can be useful if the farmer expects the subsidy scheme to change in the future. Lastly, the farming optimisation model can be used to estimate the change in profitability resulting from a change in quota size which helps to estimate the price a farmer is willing to pay for quota, or the price a farmer is willing to sell the quota for.

First, a literature review of agricultural farm management tools is given, where the contributions of the proposed optimisation model is substantiated.

1.1 Related Work and Contributions

In the literature, there are many models seeking to improve decision making in farming, and the scope of research is broad. Some focus on feeding, grazing, and harvesting regimes (e.g. Flaten et al., 2012, 2015; Sommerseth, 2018; Doole et al., 2013; Ashfield et al., 2014; Crosson et al., 2006; Dowson et al., 2019; Uyeh et al., 2018; Higgins et al., 2019). Others evaluate economic and environmental trade-offs (e.g. Doole and Romera, 2015; Villalba et al., 2019), production under milk quota (e.g. Hansen, 2009; Kristensen, 1989; Hennessy et al., 2012), and ranking and replacement of dairy cows (e.g. Kristensen, 1989; Kristensen et al., 2006; Shahinfar et al., 2014; Sommerseth, 2018; Heikkilä et al., 2008). Most of the methodologies described in the literature involves simulation and optimisation models, and for more exhaustive reviews the reader is directed to Reidsma et al. (2018), Stygar and Makulska (2010), and Janssen and van Ittersum (2007).

Although there is a large scientific interest in contributing to agricultural decision support tools (Reidsma et al., 2018), the use by farmers has been limited (Rose et al., 2016). Rossi et al. (2014) and Matthews et al. (2008) ascribes the low uptake to the "problem of implementation", due to technical limitations of the models and a low acceptance rate amongst the end users. Rose et al. (2016) suggests characteristics which must be implemented in the development of an agricultural support tool to enhance its effectiveness. The characteristics can be summarised as (i) ease of use, (ii) relevance to farmers' existing practices, (iii) performance, (iv) cost, (v) trust, (vi) compliance with market and legislation, (vii) the need for prerequisite knowledge, and (viii) required infrastructure. Further, Lundström and Lindblom (2018) emphasises the need for shifting from a goal-oriented focus towards considering how a particular support tool can be utilised together with farmers' situated knowledge. Hence, the emphasis of a support tool must be on *support*. Farmers often operate in complex and unstructured settings, which are subject to change and not necessarily have apparent solutions. Nevertheless, the role of the support tool is not to replace the decision-maker, but to facilitate an exchange of information between the user and the model to assist the user with information processing and expand his perception of the problem at hand.

The main contribution of this thesis is to develop an optimisation model for Norwegian dairy farming while addressing the challenges pointed out by Rose et al. (2016) and Lundström and Lindblom (2018). We believe this will result in a model with the potential to assist farmers' decision making and make a greater impact in their practices. Thus, the optimisation model proposed in this thesis will integrate user submitted input data with public data, with the ability to present useful statistics and analyses. Designed like this, it is not the answer of the objective-function (goal-oriented thinking) which is interesting, but rather its ability to present and explore farm practices. In the development of the optimisation model, we will focus on four of the characteristics proposed by Rose et al. (2016): (ii) relevance to farmers' existing practices; (iii) performance; and (vi) compliance with market and legislation. Altogether, this will constitute a farm management tool where theoretical research is transformed into convenient and intuitive applications for the farmer.

1.1.1 Mathematical Programming in Agriculture

Mathematical programming is an optimisation method extensively applied in the literature to analyse farming systems. The primary applications has been to assist decision-makers by evaluating current and alternative practices, assessing agricultural policy-changes, innovations or other experimental designs. Usually, the objective function maximises the farm's gross margin, often succeeded by sensitivity analysis on milk and beef prices (Reidsma et al., 2018; Stygar and Makulska, 2010). In the literature there are many examples of mathematical programming models for agriculture, including stochastic (e.g. Dowson et al., 2019; Fornés, 2019), discrete (e.g. Breen et al., 2019), linear (e.g. Flaten et al., 2015; Hansen, 2009), non-linear (e.g. Doole et al., 2013; Doole and Romera, 2015; Fornés, 2019), and dynamic (e.g. Dowson et al., 2019).

As the optimal allocation of farm resources is part of our scope of research, linear programming was identified as the most appropriate modelling technique. The methodology will be explained in detail in chapter 4.

1.1.1.1 Linear Programming

In the literature, several farm support tools utilising linear programming has been developed. Dowson et al. (2019) develops a multi-stage stochastic linear optimisation model called POWDer to analyse a New Zealand dairy farm case study over 52 weeks, each week being one stage in the model. It is therefore a finite-horizon, discrete-time stochastic model. The model maximises operating profit by choosing the optimal activities at each stage, solved with Gurobi. The model is a combination of three models; a grass growth model, an animal model, and a milk price model. The model is able to decide the quantity of palm kernel to feed and when to dry off cows. They found that no optimal strategy exists for all cases, but depends on the combination of economic and weather uncertainties. Further, the impact of a reduced stocking rate (dairy cows per decare of farmland) was analysed. The operating profit improved even if environmental and economic benefits associated with a reduction in stocking number was excluded. A drawback of the model is that animals will consume all they can, and only by grazing on pasture and eating palm kernel. Also, beef cattle is not included.

A linear programming model was combined together with a simulation model by Rodias et al. (2019) to schedule distribution of liquid manure to various crops by minimising operation cost subject to constraints for seeding, field cultivating, available manure, working hours as well as restriction on which weeks a tractor can be operated. By introducing another tractor, the annual cost was reduced by almost 4% compared to the base scenario with just one tractor. A second and third scenario was studied with extra fields having higher nitrogen demands next to the farm, with one and two tractors available. Compared to the base case, savings of 2.5% and approximately 6% was obtained. While this is a fine optimisation for logistics management, it does not involve a whole-farm model, and thus has a narrow use-case.

Klootwijk et al. (2016) and Van Middelaar et al. (2013) studies pure dairy farms in the Netherlands. Klootwijk et al. (2016) evaluates the economic and environmental impact of a policy introduced in the Netherlands to limit phosphate production after the country abolished the milk quota system. A whole-farm linear programming model was used to evaluate the effect of the policy change on pure dairy farms, where they investigated changes in farm structure, income, nitrogen and phosphate surpluses, and GHG emissions. The objective-function maximised labour income. The optimisation was proceeded with scenario analysis on shed capacity, milk yield per cow and crop yields, prices for manure disposal and processing, the price of acquiring more land, and the price of milk. The model formulation is not provided, so it is hard to say more about how the model works. Van Middelaar et al. (2013) analyses feeding strategies to reduce GHG emissions on Dutch dairy farms by increasing the use of maize silage at the expense of grass and grass silage in the dairy cow's diet. Their objective-function maximises labour income of the household, by optimising land use and animal diets. The farm's fixed resources, links between activities and environmental policies are constraints of the model. Despite the fact that the research on Dutch dairy farms by Klootwijk et al. (2016) and Van Middelaar et al. (2013) involves many aspects of interest to us, it is not conducted on a farm with the opportunity to have beef cattle as a competitor to dairy cows.

A model developed by Flaten et al. (2015) maximises total gross margin of two livestock farms in Norway under three different harvesting regimes in grass silage production. They propose that no single harvesting regime is always best, but a three-cut system producing highly digestible silage is more profitable when much land is available. Their model maximises total gross margin of farms with 150 000 l milk quota, a housing capacity of 25 cows and farmland varying from 10 to 30 hectares, with 20 hectares as the basis. A similar study was conducted on dairy goat farms in mountainous areas of Norway by Flaten et al. (2012). Although both of these papers are great research on Norwegian dairy farms, little emphasis is given other subjects than farmland management and grass production. Hansen (2009) investigates another concern of Norwegian dairy farming, which is the effects of purchasing and renting milk quota. A linear programming model called TINE Optimal is used. He proposes that farmers with low or moderate milk yield per cow can increase their profitability by increasing their milk quota which in turn will increase the farms total milk delivery. This is then achieved by feeding cows with higher forage quality – as long as the cow still has potential for reaching a higher milk yield. Further, scenarios of different changes in restrictions such as the number of cows, cowshed capacity and farmland are investigated to find binding and slack variables. While the model used has many interesting activities included, the research only considers quota purchase and increased milk production.

Collins et al. (2013) uses linear programming to investigate the benefits of crop diversification and rotation on Sri Lankan farms by maximising the money available to the farmer based on the farmer's initial amount of money, the cost of seeds, fertilizer and pesticide, and the monetary value of the crops grown. To handle the non-linearity arising from including crop rotation in the formulation, dummy variables are applied. Another case of crop planning is demonstrated by Yano and Sakawa (2013) with a multiobjective linear programming model, maximising profit and minimising working hours subject to farmland constraints.

Crosson et al. (2006) introduced another linear programming model with focus on feeding and nutritional requirements, called the Grange Beef Model. This model also maximises gross margin, however only to identify optimal beef production, and not a dairy or mixed dairy and beef farm system. A linear programming model on beef farms, namely Opt'INRA, was developed by Veysset et al. (2005) to explore farmers' economic consequences of transitioning to organic farming due to new constraints introduced by European policy-changes for the Massif Central, France.

Visagie et al. (2004) optimised a crop-dairy farm using mixed integer linear programming. The study identifies the optimal mix of crops and the number of animals the farm needs to keep in the presence of crop production risk. As a measure of risk, the deviation of income from the expected value was used. They found strategies depending on crop rotation principles generally preferred to strategies that follow mono-crop production practices. For none of the risk levels specified was mono-crop systems (wheat and medics) a part of the optimal solution. For profit maximisation and risk minimisation, diversification of a crop-livestock structure was concluded the best option.

Like much of the literature presented, the aforementioned research all has narrow use-cases, and do not involve a whole-farm model with activities for crops as well as both dairy and beef cattle management. Older research involving linear programming in agriculture is conducted by Conway and Killen (1987), Morrison et al. (1986) and Butterworth (1985). Even though all three are whole-farm models, Conway and Killen (1987) studies dairy and grassland management while emphasising the impact of milk quotas, Morrison et al. (1986) considers if farmland should be used for crop production or grazing, and Butterworth (1985) focuses on crops and beef cattle. Summarised, most of the models mentioned in this section focus on crop production or the management of dairy cattle or beef cattle separately. Also, many of the models that are developed for livestock concerns feed and nutrition management, often with a trade-off analysis between economical profit and environmental degradation. Although these topics are important to consider for any farmer, there is little research on cases of Norwegian livestock farming. Norwegian dairy farms are special, as most of the dairy cows are of the Norwegian Red breed (NDHRS, 2019), which is a dual-purpose dairy and beef cow, meaning it has both a high milk yield while also producing a considerably amount of beef (Sommerseth, 2018). Additionally, Norwegian farms are encouraged to have a diversified farm structure (LMD, 2018b), and are provided with substantial governmental support to both dairy cows and suckler cows.

The main contribution in this thesis is therefore the development of a linear programming model as a management tool for Norwegian dairy farms, with the opportunity to specialise in beef production instead. The model will maximise the farm's gross margin under farm constraints and subsidy regulations, optimising the farm's resource allocation. The research constitutes a comparison of three farms on the optimal structure of livestock, succeeded by scenario analysis of a reduction in milk quota and price. Additionally, the governmental support schemes are included in the constraints of the model, and the current policy contribution to the farm's total gross margin is analysed accordingly. Thus, this linear programming model can be viewed as an extension to the model and research of Hansen (2009), which is where this thesis is positioned in the flow of literature.

1.2 Structure

This thesis is divided into 8 chapters. This chapter has presented the scope of our research, a literature review of farm management tools, and the contributions of this thesis. Chapter 2 introduces the reader to relevant background regarding Norwegian agriculture in general and Norwegian dairy farming in particular. Chapter 3 briefly describes the construction of our optimisation model. Chapter 4 presents a thorough review of the mathematical programming methodology applied in the development of the optimisation model, before the data supplied to the model is presented in chapter 5. In chapter 6, three farms of different sizes are analysed. Implications of the results and assumptions are then discussed in chapter 7 together with recommendations for future research and extensions. The final chapter summarises our work and lists the main conclusions drawn from the thesis.

2 Background

In this chapter we will give a brief overview of Norwegian agricultural farms, with focus on dairy farming. The background is followed by presenting government objectives for Norwegian agriculture, before we provide a thorough description of how the industry is regulated through several subsidy schemes. The subsidies most relevant to dairy and beef farmers are presented.

2.1 Norwegian Agricultural Farming

Norwegian agricultural holdings are becoming fewer and larger for every year (Statistics Norway, 2019). As illustrated in Figure 2.1, the average decares of farmland per farm has increased by 69.41% since the millennium – from 148.8 decares in 2000 to 248.7 decares in 2018. In the same period, the number of agricultural farms has decreased by 42.19%, totalling 39 621 agricultural farms in 2018. This indicates Norwegian agriculture has faced large structural changes.

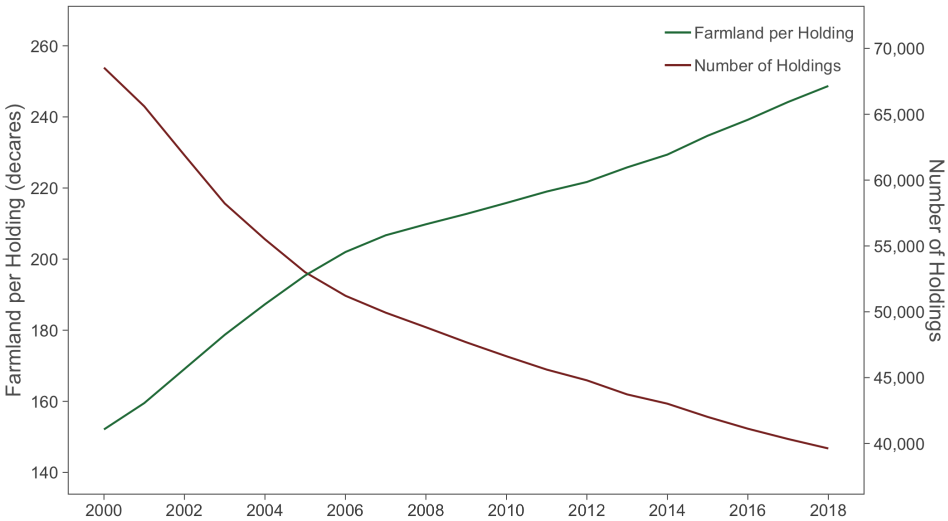


Figure 2.1: Average agricultural area per farm (decares) and number of farms, from 2000 to 2018 (Statistics Norway, 2019)[1].

For a closer look at the development of agricultural farms, Figure 2.2 presents the number of farms by decares of agricultural area from 2008 to 2018. Over the period, most farms

[1]This constitutes all kinds of agricultural holdings, not just livestock holdings.

have had $100 - 199$ decares of farmland. This is also the group that has encountered the largest reduction since 2008 – losing 4 218 farms. Furthermore, there are 843, 2 087, 2 632 and 837 fewer farms in the groups with $0 - 49, 50 - 99, 200 - 299$ and $300 - 499$ decares of farmland, respectively. However, the group of largest farms, having 500 or more decares of farmland, has recruited 1 413 farms. This makes up a net reduction of 9 204 agricultural farms from 2008. In total there are respectively 5 370, 6 573, 10 263, 6 566, 6 194 and 4 655 holdings with $0 - 49, 50 - 99, 100 - 199, 200 - 299, 300 - 499$ and 500 or more decares of farmland in 2018. All in all, Figure 2.1 and Figure 2.2 depicts that Norwegian agriculture now consists of fewer farms than before, with a greater portion of the farms having more decares of farmland (Statistics Norway, 2019).

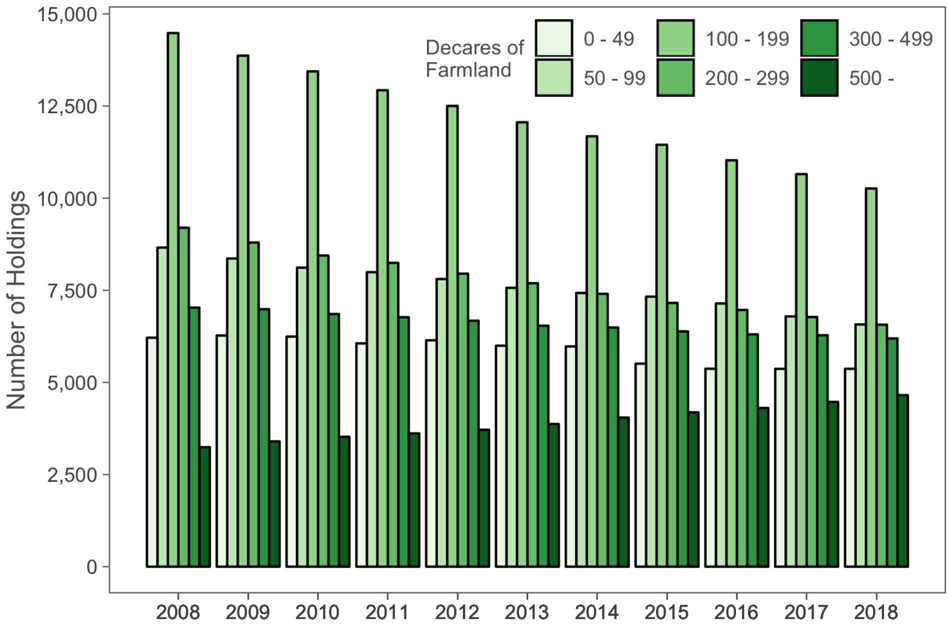
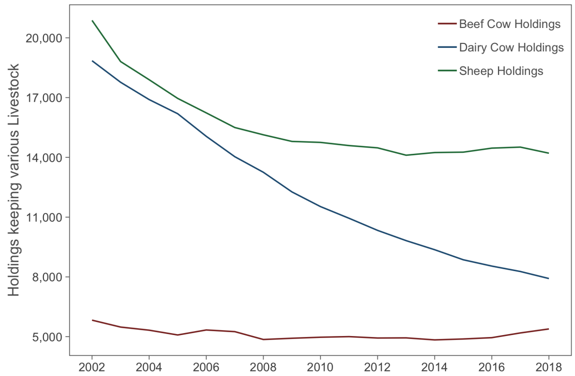


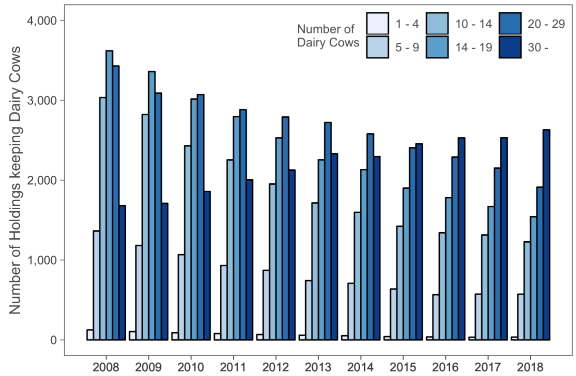
Figure 2.2: Holdings by decares of agricultural land, from 2008 to 2018 (Statistics Norway, 2019).

In 2018, there are 27 539 holdings keeping various livestock, whereas 7 918 farms have dairy cows, 5 388 farms have beef cows and 14 209 farms have sheep (Statistics Norway, 2018). For farms keeping dairy cows and sheep, this is a reduction of 58.00% and 31.93% since 2002, while for farms keeping beef (suckler) cows the reduction is only 7.57%. Thus, there are relatively more holdings keeping beef cows in 2018 compared to earlier years. See Figure 2.3a.

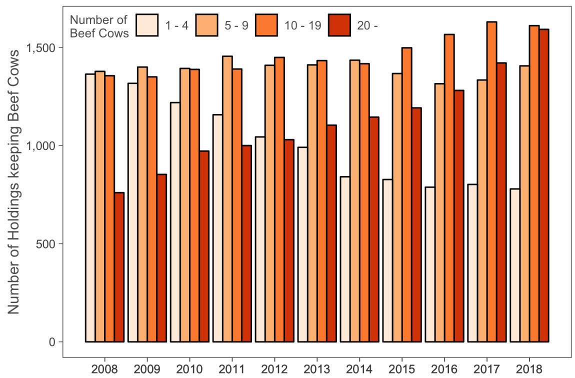
Further examination of farms keeping dairy cows, suckler cows and sheep in Figure 2.3



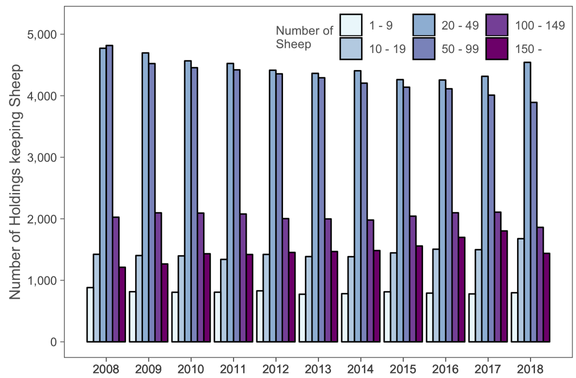
(a) Number of holdings with livestock.



(b) Holdings keeping dairy cows, by herd size.



(c) Holdings keeping suckler cows, by herd size.



(d) Holdings keeping sheep, by herd size.

Figure 2.3: Holdings of cattle and sheep by herd size, from 2008 to 2018 (Statistics Norway, 2018).

reveals the distribution of the particular herd sizes from 2008 to 2018. There is a clear pattern that there are fewer farms with a herd of less than 30 dairy cows (Figure 2.3b), as all herd size intervals are decreasing over the time period except for the herd sizes of 30 or more dairy cows, which is increasing. Additionally, since 2015, the number of farms by dairy cow herd size has been chronological. In 2018, there are 35, 572, 1 227, 1 542, 1 912 and 2 630 farms with $1-4, 5-9, 10-14, 14-19, 20-29$ and 30 or more dairy cows in the herd – meaning 33.22% of farms with dairy cows now has a herd of 30 or more dairy cows. In the period from 2008 to 2011, dairy cow herds with $10-14, 15-19$ and $20-29$ animals was more common (Statistics Norway, 2018).

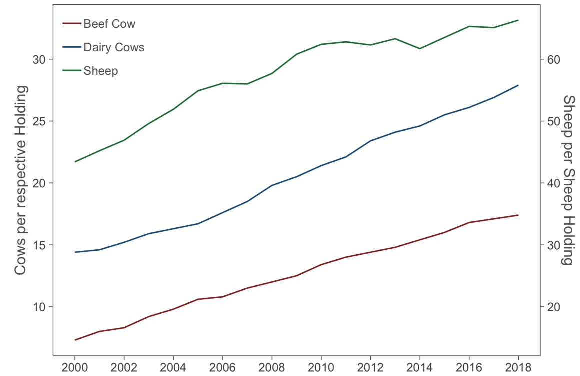
For suckler cow herds (Figure 2.3c), the general tendency is similar to dairy cow herds in that there are larger suckler cow herds in 2018 than before. Herds of $1-4$ suckler cows is the only interval that has decreased since 2008, while herds of 20 or more beef cows has experienced the largest increase. In 2018, there are 779, 1 406, 1 611 and 1 592 suckler cow farms with herds of $1-4, 5-9, 10-19$ and 20 or more animals, barely making herds of

$10 - 19$ suckler cows the most common. For sheep (Figure 2.3d), there has not been a considerable shift in the distribution of herd sizes from 2008 to 2018. The number of farms with herds of $20 - 49$ sheep has decreased by 925 farms, though it is still amongst the largest number of farms together with herds of $50 - 99$ sheep. The number of farms with $10 - 19$ and 150 or more sheep has increased with 480 farms in total. All in all the herds has stayed around the same levels throughout the period, and in 2018 there was 798, 1 677, 4 543, 3 892, 1 861 and 1 438 farms with herds of $1 - 9, 10 - 19, 20 - 49, 50 - 100, 100 - 149$ and 150 or more sheep (Statistics Norway, 2018).

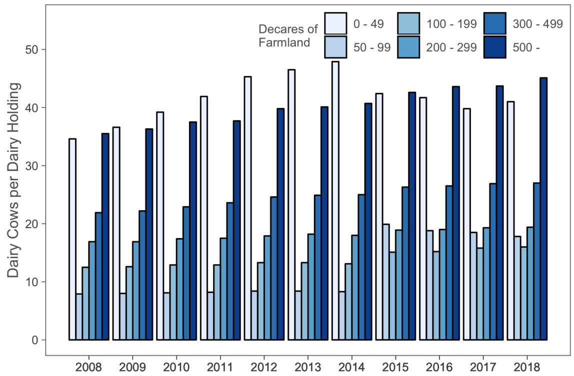
The number of suckler cows per holding keeping suckler cows has increased by 138.36%, from 7.3 animals in 2000 to 17.4 animals in 2018 (Figure 2.4a). For dairy cows, the number of animals per farm has gone from 14.4 in 2000 to 27.9 in 2018, making it a 93.75% increase. The average number of sheep per farm from 2000 to 2018 has increased by 22.9 animals, which amounts to a 53.76% increase up from 43.4 to 66.3 animals per farm (Statistics Norway, 2018).

When investigating the average number of livestock per farm by decares of agriculture area (Figure 2.4), we see that in 2018, farms with 500 or more decares of farmland also has the most dairy cows (45.1), suckler cows (29.2) and sheep (127.5) per holding of the respective livestock. However, the farms with the least decares of farmland had the most dairy cows per farm in the period from 2009 to 2014, peaking at 47.9 dairy cows per dairy cow holding with $0 - 49$ decares of farmland in 2014. This also represents the greatest shift of structure for the three types of holdings, as for suckler cow and sheep farms, the farms with 500 or more decares of farmland has had the highest number of animals per farm throughout the period. In 2018, the smallest dairy farms still have 41 dairy cows on average. A difference between the three types of holdings are, while the number of suckler cows and sheep per farm have more or less been chronologically ordered by decares of farmland over the period, both the smallest and the largest dairy cow farms has had the highest number of dairy cows per farm (Statistics Norway, 2018).

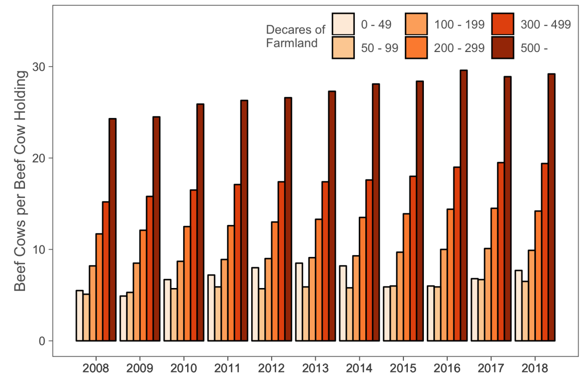
Summarised, Norwegian agricultural farms in general and farms holding livestock in particular has decreased in number but increased in size. The farms are not only larger than before in terms of decares of agricultural land, but also when it comes to the number of animals per farm. Most of the dairy cow farms has a herd of 30 or more animals, and



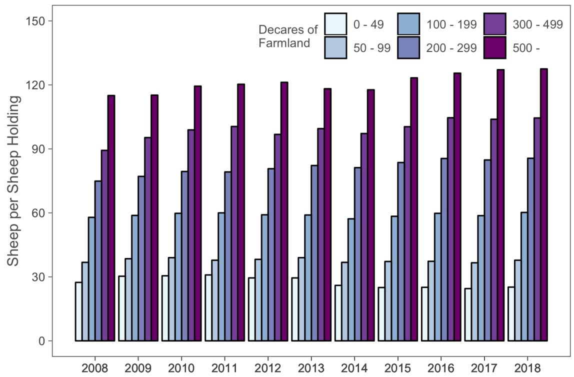
(a) Dairy cows, beef cows and sheep per respective holding.



(b) Dairy cows per holding keeping dairy cows, by agricultural land (decares).



(c) Beef cows per holding keeping beef cows, by agricultural land (decares).



(d) Sheep per holding keeping sheep, by agricultural land (decares).

Figure 2.4: Cows and sheep per holding by decares of agricultural land, from 2008 to 2018 (Statistics Norway, 2018).

herds of fewer animals are decreasing while herds of 30 or more animals are increasing. For farms with suckler cows, most farms has a herd of 5 or more animals, and herds of 20 or more animals has increased the most. Suckler cow farms with only $1-4$ animals are decreasing. Holdings keeping sheep has experienced a slight increase of herds with $10-19$ and 150 or more animals, but most farms still has $20-99$ animals. More intuitively, the more decares of farmland a farm is in possession of, the more livestock it has. This is especially true for farms with suckler cows and sheep, where the farms with the most decares of farmland has many more animals per farm than smaller farms. However, for farms keeping dairy cows, the farms with the least decares of farmland are so to say on par with the farms with the most decares of farmland when it comes to number of dairy cows per farm.

As illustrated here, in Norwegian agriculture, farms seem to be merging together, thus becoming fewer and larger. Intuitively, larger farms are more complex than smaller farms, as they have more farmland and larger herds. It follows that the consolidation calls for more management, as more decisions have to be made, such as what activities the land should be used for (growing crops for sale vs grazing), what livestock to have on the farm and how many, etc. Therefore, the development in Norwegian agricultural farms highlights the importance of assisting the decision maker with a farm management tool like the optimisation model proposed in this thesis. Such a model will improve the farmer's ability to process the increase in information a larger farm represents, and assist him in exploring how to better manage his farm.

2.1.1 Norwegian Dairy and Meats Production

Dairy production is the largest single production in Norwegian agriculture (NAA, 2019b). Yet, the number of dairy cows has decreased significantly in recent decades. As of 1 January, 2018, there are just below 220 000 dairy cows in Norway, compared to over 300 000 at the millennium – a reduction of 27.12% (Statistics Norway, 2018). The Norwegian Agriculture Agency (NAA) reports that there are 8 149 farms with dairy cow milk quota in 2018, down from 17 601 farms in 2003 (NAA, 2019b). Over the same period, the average litres of dairy cow milk quota per farm has increased by 118.56%. In 2018, the average quota amounted to 196 134 litres per farm (see Figure 2.5). Given the increase in dairy cows per farm and reduction of farms keeping dairy cows (Statistics Norway, 2018), the increase in litres of milk quota per farm, as well as the decrease in farms with milk quota, is to be expected.

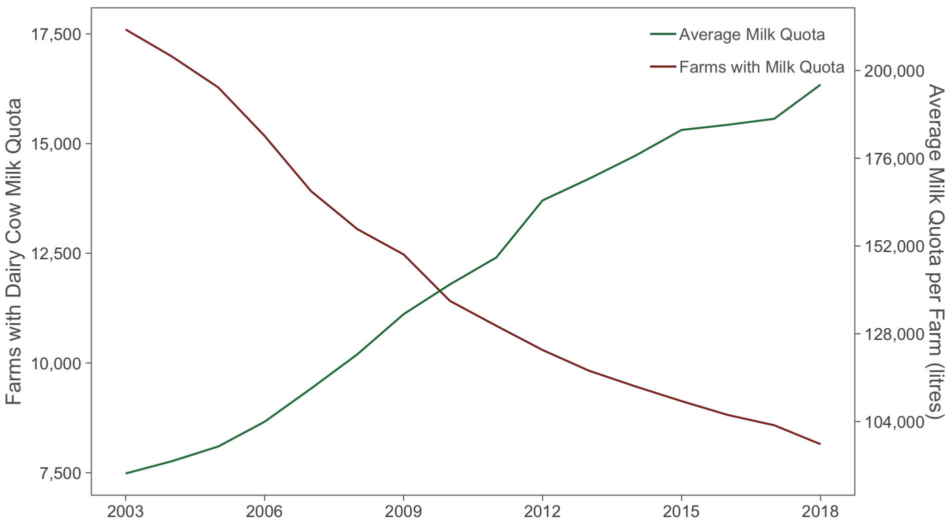
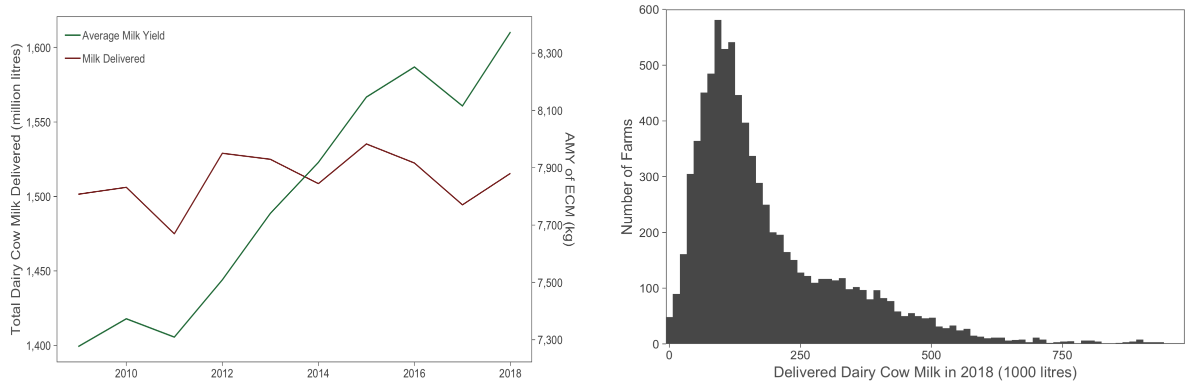


Figure 2.5: Average dairy cow milk quota per farm (litres) and number of farms with dairy cow milk quota, from 2003 to 2018 (NAA, 2019b).

Despite fewer dairy cows in Norwegian agriculture, the total litres of dairy cow milk delivered has held the same level over the last decade – amounting to approximately 1 500 million litres every yer (NAA, 2019a). Considering the decrease in number of dairy cows while still maintaining a stable level of milk production, this suggests Norwegian dairy production has improved it's productivity. Figure 2.6a supports this, as we see a rise in average milk yield (AMY) per dairy cow from just under 7 300 kg of energy corrected milk (ECM) in 2009 to over 8 300 kg of ECM in 2018 (NDHRS, 2019). We also observe that 2011 and 2017 are the only years with a decline in productivity. The average litres of cow milk delivered per dairy farm in 2018 was 181 000 litres – 15 000 litres below the average quota size. However, the median is just below 135 000 litres, meaning there are some farms producing a lot more than what is common – as illustrated by the long right tail in figure 2.6b. The 1st quartile is at 83 000 litres, while the 3rd quartile is at 242 000 litres. The farm with the highest production delivered 977 000 litres of dairy cow milk in 2018 (NAA, 2019a).



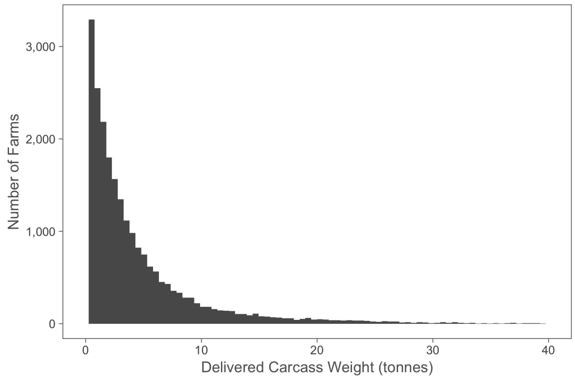
(a) Dairy cow milk delivery and AMY of ECM. **(b)** Delivered dairy cow milk in 2018.

Figure 2.6: Total dairy cow milk delivery (million litres) and AMY per dairy cow of ECM (kg), from 2009 to 2018 (NAA, 2019a; NDHRS, 2019). Distribution of delivered milk in 2018 (1 000 litres) (NAA, 2019a).

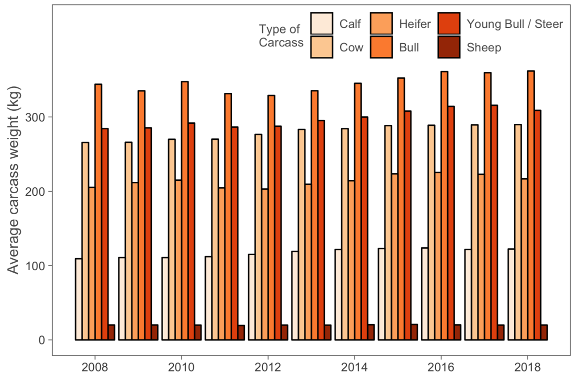
The productivity in Norwegian meat production has also undergone similar improvements. Although the total number of cattle and sheep in 2018 has since year 2000 decreased from 985 000 to 874 521 and 1 131 734 to 1 008 230, respectively, there has been an increase recent years (Statistics Norway, 2018). The total meat delivered (i.e. cattle & sheep carcasses) in 2018 amounted to 115 000 tonnes, an 8.0% increase from 2005. Thus, despite having fewer farms delivering beef and sheep meats, the delivered carcass weight per farm has increased every year since 2014, averaging at over 4 600 kg per farm in 2018 (see Figure 2.7c). However, when investigating farms delivering carcasses in 2018 (figure 2.7a, we see most firms deliver a relative low carcass weight. The median delivered carcass weight is just below 2 500 kg, while the 1st quartile is at 950 kg and the 3rd quartile is almost 5 500, not very far from the mean. The farm delivering the most meat is a cattle farm with more than 265 000 kg of young bull and steer carcasses. In total, this farm delivers 366 000 kg of cattle carcasses and no sheep carcasses.

Further, even though there are slightly more cattle and sheep than before, an increase in average carcass weight of cattle helps explaining the increased total delivery. We see that the average carcass weight of cattle has increased during the period from 2008 and 2018. In 2018, the average carcass weight of calves, cows, heifers, bulls, and young bulls / steers was 122, 290, 217, 362 and 309 kg, respectively. The average carcass weight of sheep has varied between 19 and 21 kg over the same period (NAA, 2019a)[2].

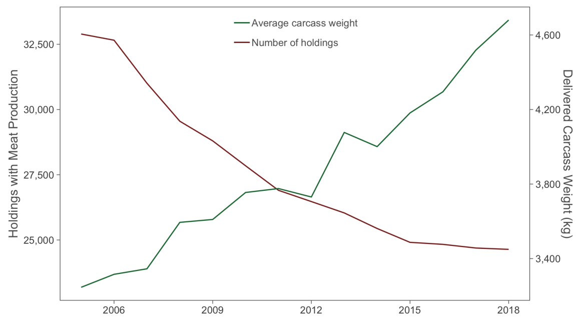
[2] All kinds of sheep averaged together.



(a) Delivered cattle and sheep carcasses (tonnes) per farm in 2018.



(b) Average carcass weight (kg) of cattle and sheep.



(c) Number of holdings delivering cattle and/or sheep carcasses versus average carcass weight (kg) per holding.

Figure 2.7: Delivered cattle and sheep carcasses (NAA, 2019a).

Summarised, Norwegian dairy and meat production consists of fewer but more productive farms than before. Dairy farms have larger milk quotas and larger herds, and each member of the herd continues to improve it's milk yield. Similarly, the meat production also consists of fewer farms delivering beef and sheep meats, but more carcasses are delivered per farm, and the average carcass weight of cattle has increased slightly since 2008. In accordance with increased productivity, it is natural to question if the increase is sustainable for a given farm. Higher yielding cows and heavier cattle requires more feed, which means a farm must have more farmland, or purchase more fodder, to sustain a higher production. Additionally, every farmer's milk delivery is constrained by his milk quota. It is therefore necessary for farmers to investigate how this quota is utilised. However, it may not be obvious what is best of increasing yield of the dairy cows a farmer already has to reach the quota, or if he should try to get a larger dairy cow herd, or if he should sell or rent out eventual surplus quota, or do other changes. It might even be that the farmer is better off with other cattle than dairy cows, in order to sell more beef instead. The productivity increase in Norwegian dairy and meats production is another example to why an optimisation model is useful. Although farmers tend to have great situated knowledge of their farm, an optimisation model can help them by providing insight into scenarios they may not be so certain about, as well as a tool for evaluating their current practices.

2.2 Regulation of Norwegian Agriculture

To protect Norwegian agriculture and to ensure self-sufficiency and national food security, amongst other objectives for Norwegian agriculture (LMD, 2018b), the Norwegian government provides the farmers large subsidies, import and other market regulations (OECD, 2018), compensating the disadvantages faced by farmers due to unfavourable production environments making it difficult to compete in an open market, such as "harsh climate, extensive areas of rugged terrain and short growing seasons" (Lien et al., 2018). It is likely these supportive schemes have influenced the structural development of Norwegian agriculture seen in section 2.1.

Every year, the farmer organisations Norges Bondelag and Norsk bonde- og småbrukarlag and the Norwegian Ministry of Agriculture and Food (LMD) negotiates and enters into a regulative agreement for Norwegian agriculture (Jordbruksavtalen). For the remaining of the thesis this agreement will be referred to as the Agricultural Agreement. The Agricultual Agreement defines economically supportive measures to reach certain political objectives for the Norwegian agriculture that are not exhaustively regulated by law, parliamentary resolutions or regulations (LMD, 2018a). The measures defined by the Agricultural Agreement involves price and market regulations, production and price subsidies, as well as means for welfare and development support. Norwegian agricultural objectives are set by the parliament (LMD, 2018b), where the main objectives are food security and preparedness, agriculture throughout the country, increased added value, and a sustainable agriculture with less greenhouse gas emissions. In Table 2.1, the objectives are summarised. Furthermore, these objectives shall be reached through research, innovation and competence, an efficient agricultural and food management, and by safeguarding Norwegian interests and secure progress in international processes (LMD, 2018b).

Table 2.1: Political objectives for Norwegian agriculture and food (LMD, 2018b, translated).

Food security and preparedness	**Agriculture throughout the country**	**Increased added value**	**Sustainable agriculture with less GHG emissions**
Secure consumers safe food	Facilitate use of land & pasture resources	Utilise market-based production possibilities	Reduce pollution from agricultural activity
Increase food preparedness	Possibilities for settlement and employment	A competitive and cost-effective value chain for food	Reduced GHG emissions, increased CO^2 absorption and good climate adaptions
Good animal & plant health	A diverse agriculture with a varied farm structure and geographically dispersed production	An effective and profitable utilisation of the farms' resources	Sustainable farms and protection of agricultural land and resource basis
Good animal welfare	Secure recruitment throughout the country	Develop Norway as a food nation	Maintain the cultural landscape and biodiversity
Increase usage of biological resources by focusing on breeding, research and education	An ecological sustainable reindeer herding	Facilitate the farmer's income opportunities and ability to invest in the farm	
		Sustainable forestry and competitive forest and wood-based value chains	

2.2.1 Subsidies of Norwegian Farms

OECD (2018) reports the support to Norwegian farmers accounted for 57% of gross farm revenues in 2015 to 2017, three times higher than the OECD average. Furthermore, the effective commodity prices received by the farmers were 84% above world market prices on average. In 2019, the total amount of subsidies to farmers are budgeted to approximately MNOK 15 878 (LMD, 2018a). Of this, about MNOK 3 569 are price subsidies and MNOK 9 024 are production subsidies. The remaining MNOK 3 285 are for different supportive measures regarding welfare (MNOK 1 518), development (MNOK 274), compensation (MNOK 43), extraordinary operating expenses (MNOK 24), market regulation (MNOK 302) and grants to the Agricultural Development Fund (MNOK 1 124).

The principal support mechanisms defined by the Agricultural Agreement (2018a) are provided based on output, headage, and acreage, often with a regional dimension, so that farmers with an unfavourable production environment are awarded more. A number of other supplementing schemes are in place that, for example, reimburse farmers for hiring replacement labour during leave or illness, compensate farmers in the event of natural disasters or losses due to predators, or facilitate organic production. The Agricultural Development Fund provides a wide range of support schemes for investments made on the farm, while a regional environmental program aims to reduce pollution and preserve biodiversity and cultural heritage. The Agricultural Agreement (2018a) also sets target prices for traded commodities. The farmer co-operatives TINE SA, Nortura SA and Felleskjøpet SA are responsible for keeping the average price of respectively milk, meats, and grains at or below their target price. Failure to do so results in an equivalent reduction the following year, with the possibility to include a maximum price and ultimately reducing import tariffs, should this maximum price be exceeded two weeks in a row. There is no minimum price guaranteed, meaning every farmer carries the complete economic risk.

In the following we will elaborate on price and production subsidies relevant for farms keeping cattle and sheep.

2.2.1.1 Price Subsidies

Price subsidies are output-based, awarded as an extra sum per unit of sold milk and meat, as well as for wool, grains for human consumption, and impairment of grains. A fixed price per unit of grains for human consumption, meats of sheep and lambs, and wool is given regardless of where the farm is located. The rates are NOK/kg 0.413 of grains and NOK/kg 3.81 of sheep and lamb meats. Lambs with less than 13 kg of carcass weight are not eligible for price subsidy. For wool, the rate is NOK/kg 40. Additionally, a quality based payment per unit of calf and heifer beef is given depending on the quality of the beef. The rates are NOK/kg 2.50 for regular quality beef and NOK/kg 7.50 for good quality beef. A maximum of 50 500 tonnes of beef are eligible for quality based payments nationwide. In 2018, 89 483 tonnes of beef carcasses was delivered in total. This is over the maximum limit for quality based payments. However, some of the delivered beef may not meet the quality requirements (NAA, 2019a). For lambs, a quality based payment per head is given depending on the scale of the abattoir and quality of the meat. The rates are NOK/head 450 for good quality meat at high-scale abattoirs, and NOK/head 409 for all qualities at abattoirs that slaughtered less than 2 000 sheep and lambs the previous year. Also, an extra payment of NOK/head 40 are given for organic produced lambs with both good quality slaughtered at high-scale abattoirs, and for all qualities at abattoirs with low-scale production. A total of 1 075 700 lamb carcasses are eligible for quality based support every year (LMD, 2018a). The limit of 1 075 700 carcasses is shared between lambs and kids. In 2018, 1 125 400 lamb carcasses was delivered. It is likely some failed to meet the quality requirements (NAA, 2019a).

Furthermore, regional deficiency payments are rewarded based on the geographical location of the farm. The regional price subsidy rates for meats and milk are summarised in Table 2.2. Regional price subsidies can be awarded for a maximum of 1 536 million litres milk and 186 000 tonnes of meats, nationwide. In 2018, 1 516 million litres of dairy cow milk and 115 314 tonnes of beef, sheep, and lamb meats were delivered. Even if this is under the limit, the limits are shared with milk delivery from goats and meats from goats, pigs, poultry and horses – making it possible that the farmers received smaller subsidy rates [3]. If there are too many applicants or animals eligible for subsidies, the rates are reduced accordingly for everyone (Sommerseth, 2018). Potatoes for human consumption are awarded a flat rate of NOK/kg 1.40 – for an unrestricted quantity – to farms located in Northern Norway (i.e. the counties Nordland, Troms and Finnmark) (LMD, 2018a).

The regional price subsidy is divided into 10 regions $A - J$ for milk, and 5 regions $1 - 5$ for meats. Note that beef and sheep meats are not awarded regional price subsidies in region 4 and 5, in addition to the five specified counties[4]. Farms located in the most

[3] In total, 1 535 million litres milk and 350 676 tonnes of meat was delivered in 2018 (NAA, 2019a).
[4] Only pork is awarded a regional price subsidy in region 4, 5 and the specified counties.

Table 2.2: Regional price subsidy rates for milk (NOK/litre) and meats (NOK/kg) (LMD, 2018a).

Milk		**Meats**		
Region	Rate	Region	Animal	Rate
A	0.00	1	Cattle & sheep	0.00
B	0.12	2	Cattle & sheep	5.25
C	0.37	3	Cattle & sheep	8.05
D	0.53	4	Cattle	11.80
E	0.60	4	Sheep	13.90
F	0.69	4		
G	0.97	5	Cattle	12.40
H	1.18	5	Sheep	14.40
I	1.76	5		
J	1.85	Agder, Hordaland, Sogn og Fjordane, Møre og Romsdal		

favorable regions – i.e. milk region A and beef region 1 – are not provided with any regional price subsidies. This particular area concerns farms in Jæren, Rogaland. In general, more subsidies are awarded the further north in the country a farm is located, and to mountainous areas. Farms located in regions (J, 5) receives the highest regional price subsidy rates. This constitutes all of Finnmark except for Alta (LMD, 2018a).

2.2.1.2 Production Subsidies

Production subsidies are awarded as an extra sum based on the number and kind of livestock kept at the farm, the number and type of grazing livestock, decares of cultural landscape, decares and use of agricultural land, regional environmental programs, and ecological production. For production subsidies awarded for livestock, there are two application rounds – with one deadline in the spring (15 March), and one deadline in the autumn (15 September). The animals are counted in advance of the deadlines, before 1 March and 1 September, respectively. Half of the production subsidy rates are awarded for the animals counted on either date (LMD, 2018a). Headage payments for livestock follows a degressive scheme, except for other cattle which are awarded a flat rate of NOK/head 770. The rates for dairy cows are 4 168, 2 562, 1 100, and NOK/head 770 if the farm has $1 - 14, 15 - 30, 31 - 50$ or 51 or more dairy cows, respectively. For suckler cows, NOK/head 3 880 are awarded if you have less than 51 suckler cows, and

NOK/head 770 are awarded if you have 51 or more suckler cows. The accumulated subsidy awarded for $1-60$ dairy cows, suckler cows and other cattle is shown in Figure 2.8. For sheep, NOK/head 1 462 are awarded for the first 125 sheep, and NOK/head 538 for every additional sheep[5]. The headage payments are restricted to a maximum of NOK 560 000 per farm. This limit also applies to a small/medium enterprise subsidy introduced later on. In addition, only a maximum of 340 100 cows, 648 100 other cattle and 1 053 000 sheep (and goats) are eligible for subsidy nationwide (LMD, 2018a). There was 310 600 cows (dairy and suckler), 563 900 other cattle and 1 008 200 sheep in 2018 (NAA, 2019a).

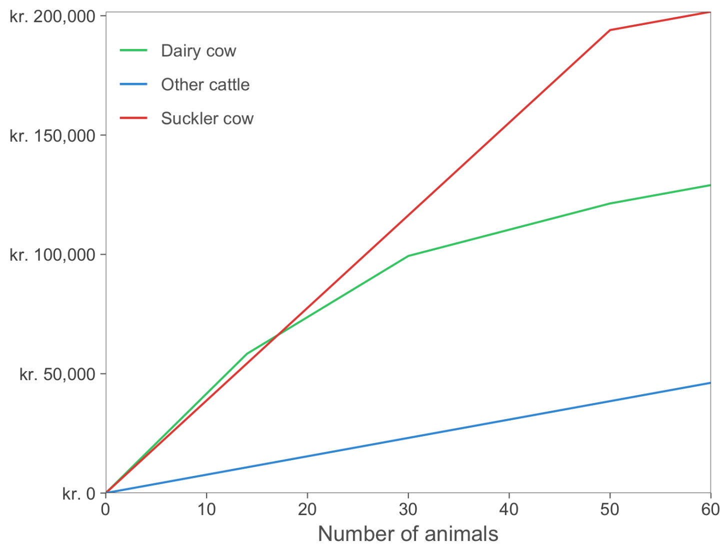
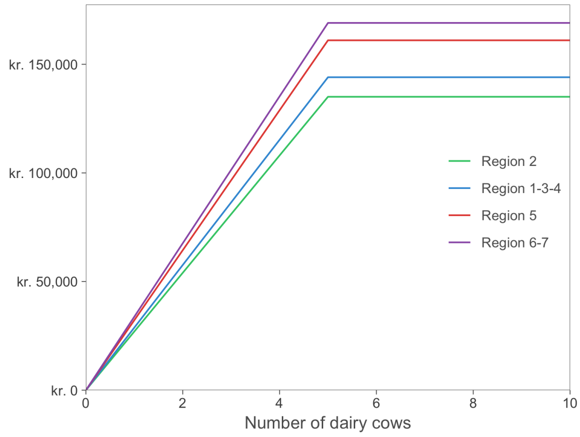


Figure 2.8: Headage payments for up to 60 dairy cows, suckler cows and other cattle (LMD, 2018a).

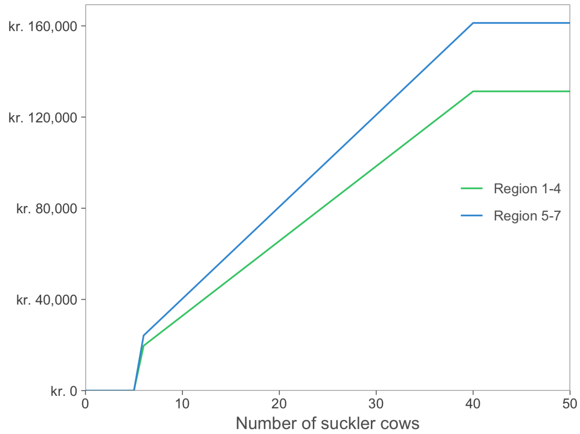
Operational subsidies are awarded for farms with dairy cows and suckler cows for production regions $1-7$ as regional deficiency payments. If a farm has fewer than 5 dairy cows, the farm is are awarded a flat rate per head depending on the farm's location. Farms with 5 or more dairy cows receive one single subsidy depending on the farm's location. As illustrated in Figure 2.9a, this subsidy is constructed such that having 5 dairy cows with the flat rate per head equals the subsidy given for having 5 or more animals within the same region. Thus, the maximum operational subsidy awarded from this regional deficiency scheme is; NOK/farm 144 050 located in regions 1, 3, and 4; NOK/farm 135 050 located in region 2; NOK/farm 161 050 located in region 5; and NOK/farm 169 050 located in regions 6 and 7. Thus, for farms with dairy production,

[5]The rates for sheep differ if a farm also has goats. Then, NOK/head 868 is awarded for the first 126 sheep, and NOK/head 194 for every extra sheep.

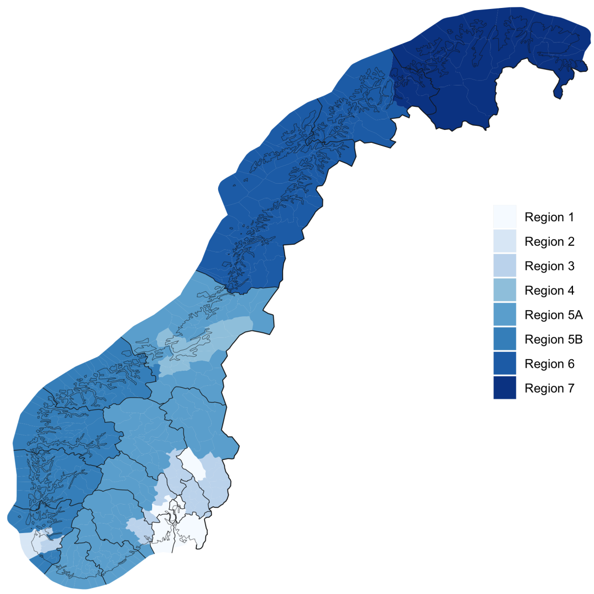
farms in the northern part of the country are awarded the most operational subsidies (LMD, 2018a).



(a) Regional production subsidies for milk production.



(b) Regional production subsidies for beef production.



(c) Map of regions for production subsidies.

Figure 2.9: Regional subsidies for dairy and beef production (LMD, 2018a).

Operational support for suckler cows resembles the one for dairy cows, only that a farm needs at least 6 suckler cows to be awarded any subsidy. A flat rate per head is given from keeping $6 - 39$ suckler cows, and a fixed rate per farm if you surpass 39 suckler cows, depending on the location of the farm. We see in Figure 2.9 that farms in Northern Norway are awarded the most operation subsidies for keeping suckler cows. The maximum subsidy awarded from this subsidy scheme is NOK/farm 161 280 located in regions 5, 6, 7, and NOK/farm 131 280 located in the other regions. A total of 27 100 farms can receive operational subsidies for dairy and beef production (LMD, 2018a). As presented in section

2.1, there was 27 539 holdings keeping livestock in 2018, but less than 8 000 and 5 500 farms had dairy and/or suckler cows, respectively. Therefore, there are room for plenty more applicants. However, the farmers that already applies for dairy and beef production produce so much and has so many animals that the limit on how many animals that can be subsidised for headage payments, or how much milk and beef that can receive price subsidies, is so to say reached. If those limits are binding, every farmer will get reduced rates if new farmers enters the industry by obtaining dairy or suckler cows.

Further, a subsidy for small and medium sized dairy farms (SME) is awarded to farms keeping less than 51 dairy cows. This scheme is also degressive, in that the rate is NOK/head 1 400 for the first 23 dairy cows, and then a negative NOK 1 150 for every extra head. By design, only farms with less than 51 dairy cows will be receive this support, encouraging smaller farms to produce dairy cow milk. Thus, a farm with 23 dairy cows will receive the maximum amount of NOK 32 200. See Figure 2.10 (LMD, 2018a).

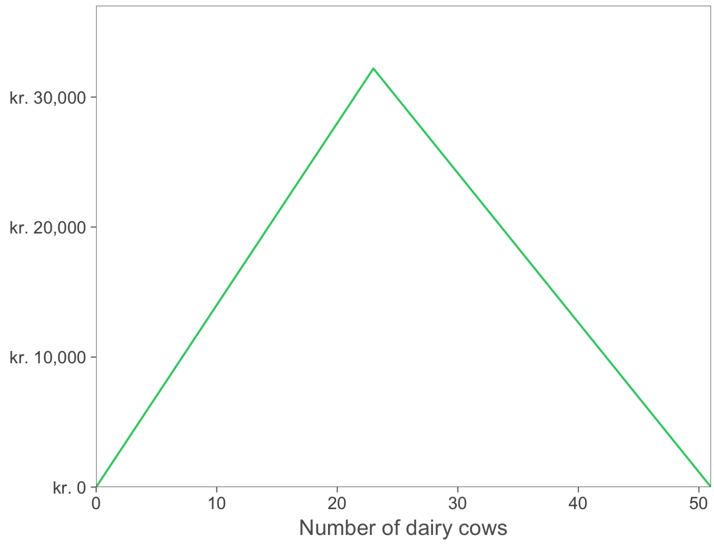
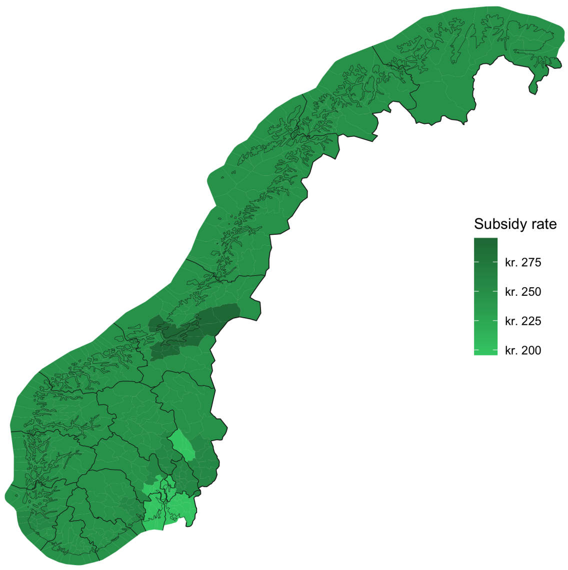


Figure 2.10: Production subsidies for small and medium sized dairy cow farms (LMD, 2018a).

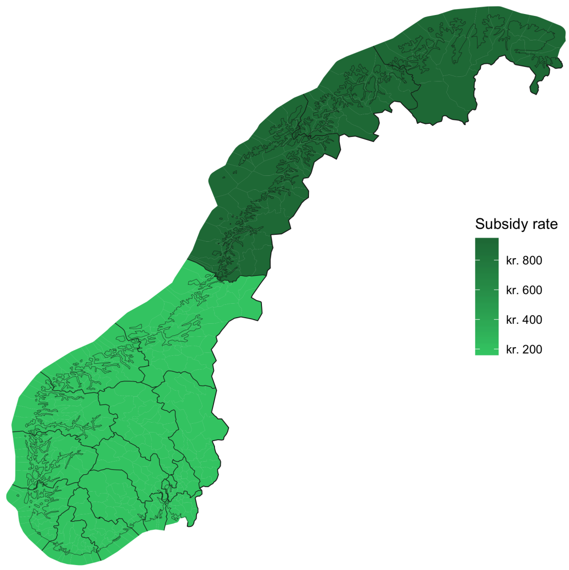
A final livestock production payment is awarded for grazing livestock. A base payment of NOK/head 370 of cattle and NOK/head 40 of sheep and lamb is given for grazing on pasture. For grazing on rangeland, NOK/head 678 of cattle and NOK/head 195 of sheep and lamb is awarded if they graze for a minimum of 5 weeks. Only sheep born last year or earlier and lambs born this year are eligible for grazing subsidies. The support for sheep grazing on rangeland is calculated as a weighted average of the number of animals let out and harvested from the rangeland. In the calculation, sheep that are let out weighs 70%, while sheep harvested after the minimum required weeks weighs 30%, lowering the penalty of losing sheep (LMD, 2018a). For example, if 100 sheep are let out to graze on rangeland, but only 90 of them are harvested after minimum 5 weeks, the farmer receives $(100 * 0.7 + 90 * 0.3) * 195 = 97 * 195 =$ NOK 18 915.

Other acreage-based (AK) support schemes are given as a lump-sum payment of NOK 162 per decare of agricultural land as an incentive to preserve the cultural landscape, as well as regional deficiency payments per decare based on current use of the land area. Agricultural land is here defined as any area within the three categories; (1) cultivated land; (2) surface cultivated land; and (3) pasture land. Agricultural land used to grow forage or crops of forage products – e.g. pasture land, (surface) cultivated meadows, hay and silage – and crops of grains and potatoes, are all covered by the regional deficiency scheme, where farms in less favourable production environments are subsidised more. Illustrative maps for the regional acreage subsidy rates per decare of crops with grains and potatoes are shown in Figure 2.11. Farms in Northern Norway (region 5, 6, and 7) receive NOK/decare 930 of agricultural land with potato crops, as opposed to NOK/decare 178 for farms in located in other regions. For grains, farms in regions 1, 2, 3, and 4 receive respectively 198, 258, 258, and NOK/decare 293 of agricultural land with grain crops, while the farms Northern Norway get NOK/decare 247 (LMD, 2018a).

To decide how many decares of agricultural land a farm has that are qualified to receive support as forage area, the following limitation is made (LMD, 2018a). First, find how many decares of pasture land that are eligible for subsidies. This will be the minimum of decares of pasture land the farm de facto possesses, and the number of livestock kept at the farm multiplied with a corresponding limitation factor for pasture land for the given livestock and region the farm is located in. Next, multiply this area by 0.6. Second, find how many decares of forage area that relates to the farm's livestock husbandry. This is the minimum of (a) the actual decares of cultivated and surface cultivated forage area the farm possesses, plus the eligible pasture land found in the previous step, and (b) the number of livestock kept at the farm multiplied with a corresponding limitation factor for forage area for the given livestock and region – not equivalent to the previous factors. Lastly, find how many decares of forage area that relates to the farm's sale of forage. This



(a) Regional production subsidies for land with potato crops.



(b) Regional production subsidies for land with grain crops.

Figure 2.11: Regional production subsidies per decare of agricultural land with crops of potatoes and grains (LMD, 2018a).

is found as the minimum of decares of cultivated and surface cultivated forage area the farm actually possesses, and kg's of forage sold multiplied with a corresponding limitation factor for forage area for the given forage product and region. The total decares of forage area eligible for subsidy is then the sum of the second and third step, thus, at most constituting the sum of de facto decares of cultivated and surface cultivated land, plus 60% of the pasture land. Farms located in both region 1 and 2 are not awarded production subsidies for forage area, while the rates per decare are 85, 105, 268, 303, 303 and 349 for regions 3, 4, 5A, 5B, 6 and 7, respectively (see Figure 2.12).

2.2.1.3 Subsidy Recap

The price and production subsidies presented are the ones most relevant for farms specialising in dairy and meats production. In Table 2.3 is a short summary of the subsidies presented in this chapter. Further on we will describe how these subsidies are included in the optimisation model in section 3.2. However, there are several other support schemes not included in this thesis which could be relevant for cattle and sheep farmers. These schemes include flat rated headage payments for farms holding listed livestock, farms with organic production or in a transition to organic production, as well as support for hiring extra labour in case of leave or vacation, and support through regional

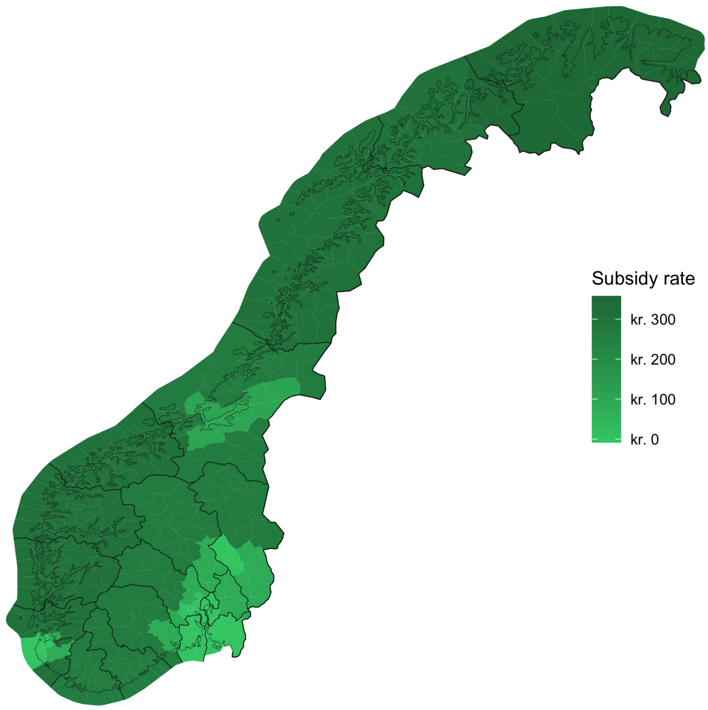


Figure 2.12: Regional production subsidies per decare of agricultural land classified as forage (roughage) area (LMD, 2018a).

environmental programs governed by each county. Regional deficiency schemes are also in place for the sale of eggs, vegetables, fruits, and berries, as well as the transportation of grains, feed, and organic grains. Additionally, imports of feed may also be price subsidised. Thus, it is clear that one can model an even more general farm management tool for Norwegian farmers by including these schemes, plus subsidies for other livestock such as pigs, goats, poultry, and their corresponding outputs. This would constitute an optimisation model valuable to even more farmers.

Table 2.3: Summary of the subsidies included in the optimisation model (LMD, 2018a).

	Price subsidies	
Type	Production activity	Comment
Flat rate per unit	Meats, grains and wool	Additional amount depends on quality of meats
Regional deficiency payments	Milk and meats	See Table 2.2
	Production subsidies	
Type	Production activity	Comment
Degressive headage payments	Cattle and sheep	See Figure 2.8
Operational	Dairy and suckler cows	Regional deficiency scheme. See Figure 2.9
SME	Dairy cows	See Figure 2.10
Grazing	Pasture and rangeland	Depends on the type of livestock. For rangeland, the animal must graze for at least 5 weeks
Other acreage-based support (AK)	Agricultural land and crops grown	Flat rate per decare of agricultural land. Regional deficiency payments depending on use of the land. May be limited by a calculated maximum area. See Figure 2.11 and 2.12

3 Description of the Problem

The goal of this thesis is to create a mathematical optimisation model that can assist dairy farmers in their decision making. A crucial task in developing the model is to formulate real life dairy farming activities in mathematical terms. We aim to make our model as realistic as possible, while still taking some assumptions to avoid making the model too complex. This chapter will outline our strategy of translating dairy farming activities and subsidy schemes into mathematical formulations.

3.1 Dairy Farming Activities

Essentially, a dairy farm consists of agricultural land and animals. Since we are studying dairy farms in this thesis, we include dairy cows, suckler cows, bulls, and sheep as our livestock categories. Many dairy farms have only dairy cows, but suckler cows might be present if the milk production capacity is fully utilised, or if suckler cows are more profitable. Sheep does not compete with cows for stall places in the shed, and may only be present if a farm has its own sheep places. In the model, livestock categories are created based on a three-layer hierarchy, as shown in Figure 3.1.

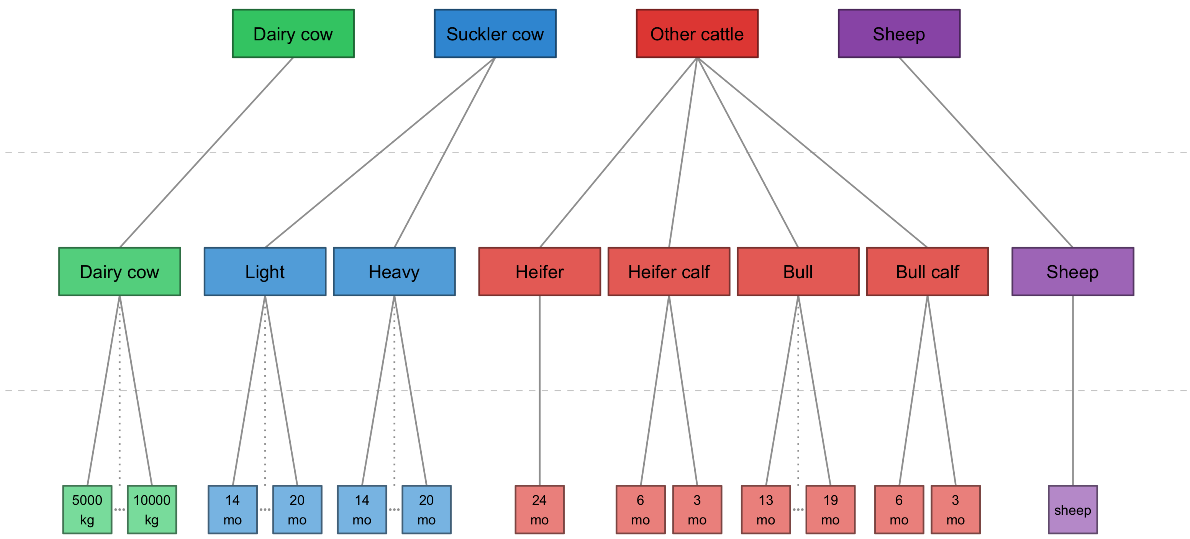


Figure 3.1: Livestock hierarchy. Animal types are divided into a three-level hierarchy. The top level is used in calculating subsidies, the middle level to calculate stall places, the low level to calculate margins.

The purpose of this hierarchical division is that the different hierarchies are used in different calculations. When calculating subsidies, we are only interested in the major livestock categories, corresponding to the top level in the hierarchy. When describing relationships between animals, we are interested in the categories in the middle hierarchy. Lastly, we are interested in the lowest hierarchy when calculating livestock margins.

The main output product of a dairy farm is milk. In Norway, the milk production by each farm is restricted by milk quotas. Each farm has their own quota which specifies how many litres of milk they can produce annually. The model makes sure that the total milk production does not exceed the quota. In reality, it is possible to produce outside of the quota, but this results in a fee charged per litre of milk produced outside the quota (NAA, 2019b). It is generally more profitable to purchase or rent more quota than producing outside the quota. Therefore, we have not added the option to produce more than the level of the quota.

Each farm includes a shed, which is the building where livestock are housed and fed. Livestock are usually fed outside on the pasture during the summer season and inside the shed during the winter season. The shed consists of several stalls, one for each animal. Thus, the number of stalls in the shed limits how many animals can be on the farm. The farm model has to make sure that the number of animals does not exceed the number of stalls in the farm. The model divides stalls into four categories – stalls for cows, stalls for calves under 6 months, stalls for cattle over 6 months, and stalls for sheep. The configuration of stalls puts restrictions on which animals the farm can have. Generally, if the number of stalls for cows is relatively large compared to other stalls, the farm will have the ability to only feed up heifers, and thus selling bulls at early age. This is because the cow capacity is not the limiting factor affecting how many animals the farm can have – the stalls for calves and heifers will be filled up before the stalls for cows. On the other hand, if the number of stalls for cows is low relative to the other kinds of stalls, the farmer can feed up bulls to adult age. Now the farm has free capacity for heifer/bull stalls, while the capacity for cows is fully utilised.

Agricultural land is used to grow crops, both cash crops – which is grown for sale – and subsistence crops – which is grown to feed the farm's own livestock. In the summer, livestock are fed by grazing outside on the farmland, and in the winter livestock are fed from forage that was harvested the previous growing season. In addition, animals are fed supplementary concentrates and proteins to reach nutritional requirements. In the optimisation model, farmland is divided into pasture land and cultivated land. Pasture land is the part of the farmland which is only used for grazing livestock or harvesting forage, and not to produce crops for sale. Cultivated land can be used both for grazing livestock and to produce crops for sale. Since livestock only consume a limited amount of forage, there might be surplus crops on the farm. This surplus crop can be sold for profit. The crop types included in the model are forage, grains, and potatoes, but the model is easily extendable to include other types of crops. The choice of which cash crop to grow depends on the margin of the crop. In addition, different crops are awarded different subsidy rates which also influence the choice of which cash crop to grow.

The final element of the farm model is the relationship between different animals. Dairy cows go through three stages during their life, from calves to heifers, and from heifers to dairy cows. Calves are defined as animals younger than six months. Heifers are defined as female cattle older than six months which have not yet calved (given birth). Dairy cows are adult female cattle that have calved at least once, and are producing milk. The periods of each segment of a dairy cow's life are different. The period for calves is set to six months. The period for heifer is the period from calf to the first calving and in Norway the average calving age is 25.8 months (NDHRS, 2019), meaning that the period for heifer is 19.8 months. The period for dairy cows, referred to as productive life, varies from farm to farm and the average productive life is 20.6 months. This means that at the time of slaughter a dairy cow is 46.4 months old. In reality, the productive life of dairy cows can be longer. The theoretical lifespan of a dairy cow is much longer than four years and the decision to slaughter before expected lifespan is due to economic reasons.

Since we separate an animal's life into different stages, we have to make sure that the relationship between the number of animals in each stage is logical. In the model, the number of animals should neither increase nor decrease, we need a steady number of animals. The model takes care of this by stating that the number of heifers calving and turning into dairy cows should equal the number of dairy cows slaughtered. Similar conditions are made for all other animal categories. This logic is also applied to the birth of new calves.

3.2 Subsidies

Norwegian dairy farmers receive government subsidies based on different measures of their performance. The subsidy structure was described in detail in the preceding chapter. A crucial task in the development of a dairy farming optimisation model is to formulate the subsidy schemes in mathematical terms.

The optimisation model developed in this thesis will handle the subsidies in two ways. Some subsidies are included directly into the margin parameters that are taken as inputs in the model. Other subsidies are determined in the optimisation model. Generally, subsidies that are given as a linear function of the number of animals, milk delivered, or meat delivered, are included in the margin parameter. These include price subsidies, where the subsidy rate is added directly on top of the milk or meat price. In cases where the subsidy amount depend on the total number of animals the farm has in its possession, the subsidy structures are included as constraints in the optimisation model. This is true for production subsidies for livestock, operations, and SME subsidies. The optimisation model will maximise gross margin while taking into account the degressive shape of these subsidy structures. Subsidies for acreage and cultural landscape are also included in the model. This is because the these subsidies are determined based on both the number of animals and the actual acreage of the agricultural land.

4 Optimisation Model

In this chapter, we formulate our dairy farming optimisation model. We are using mathematical programming techniques, a set of tools commonly used in business analytics research, to construct our model. A brief overview of mathematical programming is provided before we describe the optimisation model in detail.

4.1 Mathematical Programming

Mathematical programming is a branch of operational research concerning the optimal allocation of scarce resources from a set of possible and competing activities. The allocation is limited by a set of constraints imposed by the nature of the problem being studied. A large variety of different mathematical programming models exist, and some characteristics defining the various models are displayed in Figure 4.1. The linear programming model is the most well-known and widely used mathematical program.

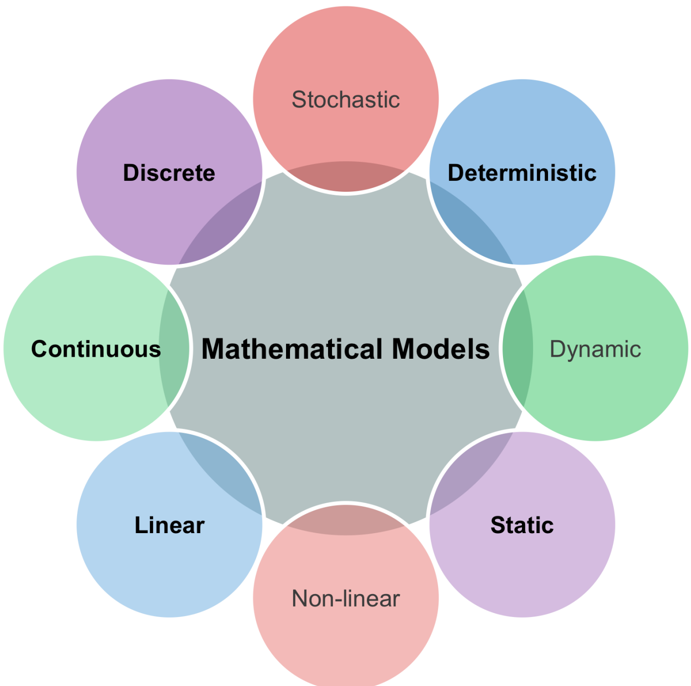


Figure 4.1: Mathematical Models with a sample of key characteristics. Adapted from Fornés (2019).

A linear programming model is defined as a model where all mathematical equations uses linear functions exclusively. A non-linear programming model has at least one non-linear function. Continuous decision variables, discrete decision variables, or a combination of both can be used in a mathematical model. The distinction between them are elaborated further below. Deterministic and stochastic refers to how the coefficients are generated. Deterministic coefficients are constant while stochastic coefficients are random variables. A dynamic mathematical model incorporates time into the model, while a static model studies a current-state scenario. The elements in Figure 4.1 in bold text (deterministic, static, linear, continuous, and discrete) are the elements that fit the farming optimisation model developed in this thesis.

4.1.1 Linear Programming

Linear programming (LP) problems can be formulated in the following way:

$$\begin{aligned} \min_{x} \quad & c^T x \\ \text{s.t.} \quad & Ax \geq b \\ & x \geq 0 \end{aligned}$$

Here, x is an n-dimensional column vector of decision variables, c^T is an n-dimensional row vector of objective coefficients, A is an $m \times n$ matrix of constraint coefficients, and b is an m-dimensional column vector of right-hand side limits (Luenberger and Ye, 2008).

4.1.1.1 Mixed Integer Linear Programming

Mixed Integer Linear Programs (MILP) are subsets of linear programs where one or more of the decision variables have to take integer values. Binary variables are a special case of integer variables where the value can only be zero or one. A general mixed integer linear program is defined as follows:

$$
\begin{aligned}
\min_{x} \quad & c^T x \\
\text{s.t.} \quad & Ax \geq b \\
& x \geq 0 \\
& x_i \in \mathbb{Z} \quad \forall i \in \mathcal{I}
\end{aligned}
$$

The notation is the same as for the standard linear programming model with the addition of the integer decision variables x_i. If all x-variables have to be integer, the problem turns into a pure integer linear program (ILP). If all x-variables have to be either zero or one, we have a zero-one integer program.

A program where one set of the decision variables are continuous and another set are binary is a case of the mixed integer linear programs. The optimisation model developed in this thesis is such a model. The decision variables are both continuous and binary. Binary variables are used to express two different states, where the value 1 represents one state and the value 0 the other state.

Mathematical programming models are solved using a computer software. A wide range of programming languages can be used to formulate a mathematical program. The formulation needs to be coupled with a solver that performs the actual optimisation procedure and obtains the optimal solution. There are several solvers available, such as CPLEX, GUROBI and MINOS. In this thesis, we use AMPL (Fourer et al., 2003) to formulate our mathematical program and CPLEX (IBM, nd) as the solver.

In the next section the full optimisation model formulation is shown and all components of the model are explained in detail.

4.2 Optimisation Model

4.2.1 Sets

$$O^{ma} = \text{Animals major}$$

$$O^{mi}_k = \text{Animals minor, } k \in O^{ma}$$

$$O^{ti}_{k,m} = \text{Animals tiny, } k \in O^{ma},\ m \in O^{mi}_k$$

$$O^a = \bigcup_{k \in O^{ma}} \bigcup_{m \in O^{mi}_k} O^{ti}_{k,m}$$

$$P = \text{Stall space categories}$$

$$L = \text{Land usage}$$

$$S^{dc} = \text{Subsidy interval for livestock – dairy cow}$$

$$S^{sc} = \text{Subsidy interval for livestock – suckler cow}$$

$$S^{oc} = \text{Subsidy interval for livestock – other cattle}$$

$$S^{sh} = \text{Subsidy interval for livestock – sheep}$$

$$S^{sm} = \text{Subsidy interval for SME}$$

The sets are defined first, and they include sets of livestock categories, cattle shed places, land usage and subsidy intervals. Livestock categories are divided into three categories according to the hierarchical distribution shown in Figure 3.1. Cattle shed places are places for adult cows, places for calves under six months, places for cattle over six months, and places for sheep. Land usage include roughage, grain, and potatoes. The sets for subsidy intervals define the intervals that subsidy amounts depend upon, and these sets are defined according to the Agriculture Agreement.

4.2.2 Subscripts

o	Tiny animal type, $o \in O^a$
m	Minor animal type, $m \in O_k^{mi}, \forall k \in O^{ma}$
k	Major animal type, $k \in O^{ma}$
p	Stall space type, $p \in P$
l	Farmland type, $l \in L$
i	Subsidy interval, $i \in S^{dc} \cup S^{sc} \cup S^{oc} \cup S^{sh} \cup S^{sm}$

4.2.3 Margin Parameters

A_o^a	Margin for animals, $o \in O^a$
A_l^l	Margin for farmland, $l \in L$
A_i^{dc}	Margin for livestock subsidies for dairy cows, $i \in S^{dc}$
A_i^{sc}	Margin for livestock subsidies for suckler cows, $i \in S^{sc}$
A_i^{oc}	Margin for livestock subsidies for other cattle, $i \in S^{oc}$
A_i^{sh}	Margin for livestock subsidies for sheep, $i \in S^{sh}$
A^{mi}	Margin for milk production subsidies
A^{be}	Margin for beef production subsidies
A_i^{sm}	Margin for SME subsidies, $i \in S^{sm}$
A^{cl}	Margin for cultural landscape subsidies
A_l^{al}	Margin for acreage landscape subsidies, $l \in L$

The margin parameters include margins for animals, farmland, and subsidies. The margin for animals includes the margin from sale of beef and milk less costs of forage and other variable costs. Where applicable, the animal margins also include price subsidies for milk production, price subsidies for beef production, and production subsidies for animals on pasture. Animal margins depend on several factors: realised milk price, realised beef price, roughage cost, concentrate cost, fodder consumption, and which region the farm is located in. Thus, the margin of animals will be different from farm to farm.

4.2.4 Other Parameters

L^p	Pasture land
L^c	Cultivated land
Q	Milk quota
D	Maximal subsidy amount
N	Maximal number of animals
d^a_p	Stall space available, $p \in P$
$d^r_{o,p}$	Stall space requirement, $o \in O^a$, $p \in P$
c_o	Forage need, $o \in O^a$
b	Crop yield
g_o	Milk deliveries, $o \in O^a$
h_o	Productive life, $o \in O^a$
r	Birth rate dairy cows
f^p_k	Factor for animals on pasture, $k \in O^{ma}$
f^r_k	Factor for animals on roughage, $k \in O^{ma}$

Other parameters include all non-margin parameters. Most of these parameters come from user inputs, e.g. milk quota, pasture land, and cultivated land. The parameters for maximal subsidy amount and factors for pasture and roughage are gathered from the subsidy regulations in the Agriculture Agreement.

4.2.5 Variables

x_o^a	Number of animals, $o \in O^a$
x_i^{dc}	Number of dairy cows with livestock subsidies, $i \in S^{dc}$
x_i^{sc}	Number of suckler cows with livestock subsidies, $i \in S^{sc}$
x_i^{oc}	Number of other cattle with livestock subsidies, $i \in S^{oc}$
x_i^{sh}	Number of sheep with livestock subsidies, $i \in S^{sh}$
x^{mi}	Number of dairy cows with milk production subsidies
x^{be}	Number of suckler cows with beef production subsidies
x_i^{sm}	Number of dairy cows with SME subsidies, $i \in S^{sm}$
x_k^p	Number of animals with pasture subsidies, $k \in O^{ma}$
x_k^r	Number of animals with roughhage subsidies, $k \in O^{ma}$
y_l	Decare of l grown, $l \in L$
z^{be}	Binary variable: 1 if six or more suckler cows
z^{sm}	Binary variable: 1 if 52 or more dairy cows

The decision variables include variables for the total number of animals, variables for the number of animals eligible for the different subsidies, decares of land grown, and two binary variables to indicate whether the farm is eligible for beef production subsidies and SME subsidies. Apart from the two binary variables, all other variables are continuous. This includes the number of animals, which are allowed to be fractional numbers. The variables for number of animals are defined as the average annual number of animals. Since the variables represent average numbers, it makes sense to state that "we have 0.5 cows and 0.5 calves". This would be the case if the farm has one cow and one calf for half a year and zero cows and zero calves for the other half a year. The alternative to continuous variables is to use integer variables which forces the number of animals to take integer values. However, the way the program is formulated here, it is necessary to allow fractional numbers for at least some of the animal classes. This is because the logical relationships between animal classes must allow for some fractionality to avoid making the program too restrictive. This is elaborated further in constraints 4.6–4.10.

4.2.6 Objective Function

$$\max \sum_{o\in O^a} A_o^A x_o^a + \sum_{l\in L} A_l^l y_l + \sum_{i\in S^{dc}} A_i^{dc} x_i^{dc} + \sum_{i\in S^{sc}} A_i^{sc} x_i^{sc} + \sum_{i\in S^{oc}} A_i^{oc} x_i^{oc} + \sum_{i\in S^{sh}} A_i^{sh} x_i^{sh}$$
$$+ A^{mi} x^{mi} + A^{be} x^{be} + \sum_{i\in O^{sm}} A_i^{sm} x_i^{sm} + (A^{cl} + A^{al}_{\text{roughage}}) \sum_{k\in O^{ma}} f_k^r x_k^r \quad (4.1)$$
$$+ \sum_{l\in L\setminus \text{roughage}} (A^{cl} + A_l^{al}) y_l$$

The objective function is total gross margin of the farm, which includes livestock margins and subsidy margins. Maximising total gross margin is a common objective in whole-farm linear programming models (Morrison et al., 1986; Conway and Killen, 1987; Veysset et al., 2005; Crosson et al., 2006; Hansen, 2009; Flaten et al., 2012; Reidsma et al., 2018).

4.2.7 Constraints

4.2.7.1 Farm Constraints

Milk production cannot exceed the milk quota

$$\sum_{o\in O^a} g_o x_o \leq Q \quad (4.2)$$

Constraint 4.2 makes sure that the total milk production does not exceed the milk quota. Each farm has a milk quota that limits how much milk they can produce each year. In reality, milk quotas can also be sold or purchased on a designated market, but in this model the milk quota is taken as fixed, similar to the models of Hansen (2009) and Flaten et al. (2012). The total milk production cannot exceed the milk quota, which means that it's not possible to keep inventory of milk. Some of the milk production is fed to calves, wasted due to quality issues, and consumed by the household, and this "loss of milk" is already calculated into the variable g_o. For example, a cow that produces 5 000 kg of milk annually (equal to 4 854 litres) will only deliver 4 466 litres of milk annually.

Number of animals cannot exceed the number of stalls

$$\sum_{o \in O^a} d^r_{o,p} x_o \leq d^a_p \quad \forall p \in P \tag{4.3}$$

Growth acreage cannot exceed the available farmland

$$\sum_{l \in L} y_l \leq L^c \tag{4.4}$$

Total forage need cannot exceed the available forage on the farm

$$\sum_{o \in O^a} c_o x^a_o - b(0.6L^p + y_{\text{roughage}}) \leq 0 \tag{4.5}$$

Constraint 4.3 ensures that the farm has enough stall spaces in the cattle shed to house all animals. Constraint 4.4 makes sure that the decares of grown crops cannot exceed the decares of cultivated farmland. The decision of which crops to grow is determined in the optimisation procedure, similar to several other farming optimisation models, e.g. Doole and Romera (2015) and Veysset et al. (2005). Cultivated farmland can be used to grow crops and for animal grazing. Pasture farmland is restricted to animal grazing and thus can only be used to fulfil the fodder needs of livestock. Constraint 4.5 ensures that enough farmland is used to grow fodder for the farm's livestock. There is no opportunity to purchase or sell fodder, implying that the fodder demand by the livestock has to be satisfied with own production. A similar restriction is made in Flaten et al. (2012) and Veysset et al. (2005). The utilisation of farmland is given by the crop yield parameter b. The utilisation of pasture land is 60% of the utilisation of cultivated farmland, and thus the parameter for pasture land is multiplied by 0.6.

Balance between number of heifers calving and number of dairy cows slaughtered

$$\sum_{o \in O^{ti}_{\text{oc,h}}} \frac{12}{h_o} x^a_o - \sum_{o \in O^{ti}_{\text{dc,dc}}} \frac{12}{h_o} x^a_o = 0 \tag{4.6}$$

Balance between number of calves turning into heifers and number of heifers calving

$$\frac{12}{h_{\text{hc6mo}}} x^a_{\text{hc6mo}} - \sum_{o \in O^{ti}_{\text{oc,h}}} \frac{12}{h_o} x^a_o = 0 \tag{4.7}$$

Balance between number of calves born and number of calves turning into heifers or being sold

$$0.5r \sum_{o \in O^{ti}_{\text{dc,dc}}} x^a_o - \sum_{o \in O^{ti}_{\text{oc,hc}}} \frac{12}{h_o} x^a_o = 0 \tag{4.8}$$

Balance between number of calves turning into bulls and number of bulls slaughtered

$$\frac{12}{h_{\text{bc6mo}}} x^a_{\text{bc6mo}} - \sum_{o \in O^{ti}_{\text{oc,b}}} \frac{12}{h_o} x^a_o = 0 \tag{4.9}$$

Balance between number of calves born and number of calves turning into bulls or being sold

$$0.5r \sum_{o \in O^{ti}_{\text{dc,dc}}} x^a_o - \sum_{o \in O^{ti}_{\text{oc,bc}}} \frac{12}{h_o} x^a_o = 0 \tag{4.10}$$

Constraints 4.6 to 4.10 handles the logical relationships between animals. Together, the constraints make sure that the number of animals on the farm is in a steady state. The number of cows slaughtered each year have to be replaced by new-born calves. The constraints can be seen as the movement of animals through different stages in the life cycle. Similar constraints were modelled in Crosson et al. (2006).

Constraint 4.6 balances the number of dairy cows being slaughtered and the number of heifers calving and turning into a dairy cow. The rate of replacement of the different animals are given by the fraction $\frac{12}{h_o}$. Here, h_o represent the productive life of each animal type. E.g. the productive life of calves is six months and the productive life of heifers is 18 months. A lower productive life results in a higher replacement rate. If the productive life of dairy cows is 24 months, 50% of the dairy cow population should be replaced each year. Constraints 4.7 and 4.9 are made using the same logic as 4.6 – the number of animals going out of a given animal category should be equal to the number of animals entering that same category.

Constraints 4.8 and 4.10 represent the number of calves born each year. The number of calves that each dairy cow give birth to each year is given by the birth rate parameter r. A birth rate of one means that one calf is born per dairy cow, every year. For simplicity, we assume that 50% of born calves are bull (male) calves and 50% are heifer (female) calves. This is approximately true for large data samples, but deviations will occur on farm level, especially on farms with few cows. However, since there exist a market for the purchase of calves, it is possible for the farmers to correct such deviations by purchasing calves of the gender that they are missing and by selling calves of the other gender. Another way the farmer can correct the problem of too few or too many female calves, is by changing the productive life of dairy cows. The productive life of dairy cows determines how many cows have to be replaced annually. A longer productive life results in fewer cows having to be replaced. As such, if too few female calves are born (meaning that there are less females to replace dairy cows), the productive life of cows can be increased sufficiently such that the two numbers balance each other.

Previously, we stated that the decision variables for number of animals are continuous variables, allowing fractional number of animals. Constraints 4.6–4.10 shows why it is necessary to allow some fractionality. The rate of replacement is given by $\frac{12}{h_o}$. This is the proportion of each animal class that is replaced annually. E.g. constraint 4.6 was defined as follows:

$$\sum_{o \in O^{ti}_{\mathrm{oc,h}}} \frac{12}{h_o} x^a_o - \sum_{o \in O^{ti}_{\mathrm{dc,dc}}} \frac{12}{h_o} x^a_o = 0$$

where $\sum_{o \in O^{ti}_{\mathrm{oc,h}}} \frac{12}{h_o} x^a_o$ is the number of heifers calving each year (and turning into dairy cows), and $\sum_{o \in O^{ti}_{\mathrm{dc,dc}}} \frac{12}{h_o} x^a_o$ is the number of dairy cows slaughtered each year (and having to be replaced). Now, assume that $h_{o \in O^{ti}_{\mathrm{oc,h}}} = 18$ and $h_{o \in O^{ti}_{\mathrm{dc,dc}}} = 24$, i.e. the productive life of heifers is 18 months, and the productive life of dairy cows is 24 months, which gives the following expression:

$$\sum_{o \in O^{ti}_{\mathrm{oc,h}}} \frac{12}{18} x^a_o - \sum_{o \in O^{ti}_{\mathrm{dc,dc}}} \frac{12}{24} x^a_o = 0$$

which can be rewritten as

$$\sum_{o \in O^{ti}_{\mathrm{oc,h}}} x^a_o = \frac{18}{24} \sum_{o \in O^{ti}_{\mathrm{dc,dc}}} x^a_o = \frac{3}{4} \sum_{o \in O^{ti}_{\mathrm{dc,dc}}} x^a_o$$

The expression above states that the average annual number of heifers should be $\frac{3}{4}$ of the average annual number of dairy cows. This will often result in a fractional number of heifers. By imposing an integer restriction on the number of heifers, it would only be possible to have number of dairy cows in a multiple of four – 4, 8, 12, etc. And this is before taking into consideration the relationship between calves and heifers, which imposes further restrictions. Thus, imposing integer restrictions on all animal variables would make the model too restrictive and the optimal solutions would not be very valuable.

The case is different for the animal variables denoting number of dairy cows. The number of dairy cows, their birth rate, and their productive life directly determine how many calves will be born each year and how many heifers are needed to replace slaughtered cows. As such, the dairy cow variables are dominating calf, bull, and heifer variables. The numbers of these animals are directly dependent on the number of dairy cows. The implication is that it will be possible to define the number of dairy cows as an integer variable without imposing unnecessary restriction to the model. It is not certain that imposing integer restrictions to the number of dairy cows will affect the optimal solution. Constraint 4.3 limits the number of dairy cows to the number of stall places in the cattle shed. The number of stall places, given by d^a_p, will be integer, meaning that the number of dairy cows will be integer, given that no other constraints are more restrictive and limiting the number of dairy cows before the stall places does. From the runs of the model with continuous dairy cows, we observe that the optimal number of dairy cows is integer in most cases. An instance where the number of dairy cows is not integer is discussed further in section 6.3.1.

The variables for suckler cows are created differently than the variables for dairy cows. The suckler cows variables aggregate all types of suckler cows into one variable. Calves, heifers, bulls, and adult suckler cows are all put together and the margin variable is an average value of all these animals. Thus, there is no need for any constraints to control the relationship for suckler cows, this is already taken care of in the definition of the variables.

4.2.7.2 Subsidies Constraints

The following constraints are related to the farming subsidies. Several authors have incorporated the Norwegian agriculture subsidy scheme into optimisation models (Hansen, 2009; Flaten et al., 2012). The agriculture subsidies are negotiated every year resulting in changes to the subsidy scheme and subsidy rates from year to year. The subsidy scheme has changed since Hansen (2009) and Flaten et al. (2012) developed their models. Therefore, the subsidy constraints in this optimisation model are formulated directly from the Agriculture Agreement for 2018–2019.

Upper limit for subsidies for livestock and small and medium sized dairy farms

$$\begin{aligned}\sum_{i \in S^{dc}} A_i^{dc} x_i^{dc} + \sum_{i \in S^{sc}} A_i^{sc} x_i^{sc} + \sum_{i \in S^{oc}} A_i^{oc} x_i^{oc} + \sum_{i \in S^{sh}} A_i^{sh} x_i^{sh} \\ + \sum_{i \in O^{sm}} A_i^{sm} x_i^{sm} \leq D\end{aligned} \tag{4.11}$$

Production subsidy for livestock – dairy cows

Number of dairy cows in the first subsidy interval

$$x_{1-14}^{dc} \leq 14 \tag{4.12}$$

Number of dairy cows in the second subsidy interval

$$x_{1-14}^{dc} + x_{15-30}^{dc} \leq 30 \tag{4.13}$$

Number of dairy cows in the third subsidy interval

$$x_{1-14}^{dc} + x_{15-30}^{dc} + x_{31-50}^{dc} \leq 50 \tag{4.14}$$

Number of dairy cows with subsidies cannot exceed the actual number of dairy cows

$$\sum_{i \in S^{dc}} x_i^{dc} - \sum_{m \in O_{\mathrm{dc}}^{mi}} \sum_{o \in O_{\mathrm{dc},m}^{ti}} x_o^a \leq 0 \tag{4.15}$$

Production subsidy for livestock – suckler cows

Number of suckler cows in the first subsidy interval

$$x^{sc}_{1-50} \leq 50 \tag{4.16}$$

Number of suckler cows with subsidies cannot exceed the actual number of suckler cows

$$\sum_{i \in S^{sc}} x^{sc}_i - \sum_{m \in O^{mi}_{\mathrm{sc}}} \sum_{o \in O^{ti}_{\mathrm{sc},m}} x^a_o \leq 0 \tag{4.17}$$

Production subsidy for livestock – other cattle

Number of other cattle with subsidies cannot exceed the actual number of heifers and bulls

$$\sum_{i \in S^{oc}} x^{oc}_i - \sum_{m \in O^{mi}_{\mathrm{oc}}} \sum_{o \in O^{ti}_{\mathrm{oc},m}} x^a_o \leq 0 \tag{4.18}$$

Production subsidy for livestock – sheep

First subsidy interval

$$x^{sh}_{1-126} \leq 126 \tag{4.19}$$

Number of sheep with subsidy cannot exceed the actual number of sheep

$$\sum_{i \in S^{sh}} x^{sh}_i - \sum_{m \in O^{mi}_{\mathrm{sh}}} \sum_{o \in O^{ti}_{\mathrm{sh},m}} x^a_o \leq 0 \tag{4.20}$$

Livestock subsidies are given by constraints 4.12 to 4.20. Livestock subsidies are separated into livestock category and number of animals. The four livestock categories are dairy cow, suckler cow, other cattle, and sheep. For each livestock category, intervals are defined for the number of animals and the corresponding subsidy amount. The subsidy amount is degressive over the number of animals, i.e. the first interval of animals is given a higher subsidy amount than the second interval. In the livestock subsidy formulations, we are utilising the fact that the subsidies are degressive. E.g. the variable x^{dc}_i is the number of dairy cows in interval i. Since the subsidy amount is degressive, the first interval will always be maximised before the second interval is maximised, and so on. We are also creating two kinds of constraints for each livestock category. The first type makes sure that the number of animals within a given interval cannot exceed the actual interval size, e.g. $x^{dc}_{1-14} \leq 14$

states that the number of dairy cows in the interval from 1 to 14 cannot exceed 14. The second type of constraint ensures that the number of animals with livestock subsidies cannot exceed the actual number of animals. E.g. $\sum_{i \in S^{dc}} x_i^{dc} - \sum_{m \in O_{\text{dc}}^{mi}} \sum_{o \in O_{\text{dc},m}^{ti}} x_o^a \leq 0$ states that the total number of dairy cows with livestock subsidies cannot exceed the actual number of dairy cows on the farm. It is worth noting that if the livestock subsidy structure would change such that the subsidy amounts are no longer degressive, the current model formulation would not work. However, the use of degressive amounts is in line with the purpose of the government subsidies favouring small farms over big farms and we do not expect the degressive structure to be removed.

Production subsidy for dairy production

Maximum five dairy cows can be given subsidies

$$x^{mi} \leq 5 \tag{4.21}$$

Number of dairy cows with subsidies cannot exceed the actual number of dairy cows

$$x^{mi} - \sum_{m \in O_{\text{dc}}^{mi}} \sum_{o \in O_{\text{dc},m}^{ti}} x_o^a \leq 0 \tag{4.22}$$

Constraint 4.21 and 4.22 are related to the operational subsidies for milk production. Here, a constant subsidy rate is given per number of dairy cow for a maximum of five dairy cows. Constraint 4.21 states that maximum five dairy cows are eligible for subsidies and 4.22 makes sure that the number of animals eligible for subsidies does not exceed the actual number of dairy cows.

Production subsidy for cattle beef production

Maximum 40 suckler cows can be given subsidies

$$x^{be} \leq 40 z^{be} \tag{4.23}$$

Number of suckler cows with subsidies cannot exceed the actual number of suckler cows

$$x^{be} - \sum_{m \in O_{\text{sc}}^{mi}} \sum_{o \in O_{\text{sc},m}^{ti}} x_o^a \leq 0 \tag{4.24}$$

Programming the binary variable z^{be} to take the value 1 if and only if there are six or more suckler cows on the farm

$$z^{be} - \sum_{m \in O^{mi}_{\text{sc}}} \sum_{o \in O^{ti}_{\text{sc},m}} x^a_o + 5 - N(1 - z^{be}) \leq 0 \tag{4.25}$$

Constraints 4.23 to 4.25 are related to the operational subsidies for specialised beef production. This subsidy is awarded based on the number of suckler cows the farm has in its possession. The structure of this subsidy is different from the previous. Here, a maximum of 40 suckler cows can be given subsidies. However, farms with less than six suckler cows are not given anything. Thus, the subsidy is only given for farms with six or more suckler cows. The subsidy amount is calculated based on the total number of suckler cows and not the number of suckler cows above six (see Figure 2.9b). The constraints 4.23 and 4.24 are similar in structure to the constraints for operational subsidies for milk production. We also need a binary variable, z^{be}, that takes the value one if there are at least six suckler cows on the farm. Constraint 4.25 makes sure that the binary variable takes the appropriate value.

$$z^{be} - \sum_{m \in O^{mi}_{\text{sc}}} \sum_{o \in O^{ti}_{\text{sc},m}} x^a_o + 5 - N(1 - z^{be}) \leq 0$$

In the expression, N can be any number that is always greater than the maximum number of suckler cows the farm can potentionally have. When $z^{be} = 1$, we get the following expression:

$$\sum_{m \in O^{mi}_{\text{sc}}} \sum_{o \in O^{ti}_{\text{sc},m}} x^a_o \geq 6$$

Thus, the number of suckler cows should be greater or equal to six in order for the binary variable z^{be} to be 1. The subsidy is only awarded when $z^{be} = 1$ as can be seen from constraint 4.23. Whenever $\sum_{m \in O^{mi}_{\text{sc}}} \sum_{o \in O^{ti}_{\text{sc},m}} x^a_o < 6$, i.e. the number of suckler cows is lower than 6, z^{be} will be zero and no subsidies are awarded.

Production subsidy for small and medium sized dairy farms

Number of animals in first interval cannot exceed 23

$$x^{sm}_{1-23} \leq 23(1 - z^{sm}) \tag{4.26}$$

Number of animals in first interval cannot exceed actual number of animals

$$x^{sm}_{1-23} - \sum_{m \in O^{mi}_{\mathrm{dc}}} \sum_{o \in O^{ti}_{\mathrm{dc},m}} x^a_o \leq 0 \tag{4.27}$$

Number of animals in second interval cannot exceed 28

$$x^{sm}_{24-51} \leq 28(1 - z^{sm}) \tag{4.28}$$

Setting the binary variable z^{sm}

$$\sum_{i \in S^{sm}} x^{sm}_i - \sum_{m \in O^{mi}_{\mathrm{dc}}} \sum_{o \in O^{ti}_{\mathrm{dc},m}} x^a_o + N z^{sm} \geq 0 \tag{4.29}$$

SME subsidies are structured in two intervals, where the subsidy amount is positive in the first interval and negative in the second interval. SME subsidies are only given to farms with a total of 51 dairy cows or less (as of 2019). The mathematical formulation of SME subsidies differs from other subsidies because the subsidy amount is negative in the second interval. We cannot use "less than or equal to" operators to determine the number of dairy cows in the second interval, as the optimisation procedure will then choose to not accept the negative subsidies. We include a binary variable which determines if there are more than 51 dairy cows on the farm. The four constraints operate together. When $z^{sm} = 0$, i.e. the number of dairy cows is 51 or less, we get the following constraint:

$$\sum_{i \in S^{sm}} x^{sm}_i \geq \sum_{m \in O^{mi}_{\mathrm{dc}}} \sum_{o \in O^{ti}_{\mathrm{dc},m}} x^a_o$$

From 4.26 and 4.28 we know that $\sum_{i \in S^{sm}} x^{sm}_i$ can at maximum be equal to 51. Thus, the inequality expression displayed above does not hold when $\sum_{m \in O^{mi}_{\mathrm{dc}}} \sum_{o \in O^{ti}_{\mathrm{dc},m}} x^a_o > 51$, i.e. when the number of dairy cows is greater than 51. When this happens, z^{sm} must be equal to 1, such that the inequality expression changes to:

$$\sum_{i \in S^{sm}} x^{sm}_i \geq \sum_{m \in O^{mi}_{\mathrm{dc}}} \sum_{o \in O^{ti}_{\mathrm{dc},m}} x^a_o - N$$

$z^{sm} = 1$ implies that $\sum_{i \in S^{sm}} x_i^{sm} = 0$ leading to

$$N \geq \sum_{m \in O_{\text{dc}}^{mi}} \sum_{o \in O_{\text{dc},m}^{ti}} x_o^a$$

which always hold.

Production subsidy for acreage and cultural landscape

Number of animals with subsidies for pasture cannot exceed the actual number of animals

$$x_k^p - \sum_{m \in O_k^{mi}} \sum_{o \in O_{k,m}^{ti}} x_o^a \leq 0 \quad \forall k \in O^{ma} \tag{4.30}$$

Pasture farmland with subsides cannot exceed actual pasture farmland

$$\sum_{k \in O^{ma}} f_k^p x_k^p - L^p \leq 0 \tag{4.31}$$

Number of animals with subsidies for forage cannot exceed the actual number of animals

$$x_k^r - \sum_{m \in O_k^{mi}} \sum_{o \in O_{k,m}^{ti}} x_o^a \leq 0 \quad \forall k \in O^{ma} \tag{4.32}$$

Forage farmland with subsidies cannot exceed the sum of actual forage farmland and 60 percent of pasture farmland eligible for subsidies

$$\sum_{k \in O^{ma}} f_k^r x_k^r - 0.6 \sum_{k \in O^{ma}} f_k^p x_k^p - y_{\text{roughage}} \leq 0 \tag{4.33}$$

The last group of constraints, from 4.30 to 4.33 are all related to the subsidies for acreage and cultural landscape. These subsidies are mainly dependent on the crops grown on the farmland. However, for roughage crops the number of animals on the farm will also influence the subsidy amount. Therefore, the subsidies for acreage and cultural landscape are calculated differently from all other subsidies. The four constraints here are only related to roughage subsidies. Subsidies for other crops are calculated directly in the objective function. For other crops the farmland eligible for subsidies equals the actual farmland. For roughage, the farmland eligible for subsidies may be lower than the actual farmland used to grow roughage. The farmland eligible for roughage subsidies cannot exceed a calculated farmland acreage, which is given by a factor multiplied by the number of animals. Pasture farmland eligible for subsidies are given by the expression $\sum_{k \in O^{ma}} f_k^p x_k^p$, and constraint 4.31 ensures that the eligible pasture land does not exceed the actual pasture land on the farm. Roughage farmland eligible for subsidies are given by the expression $\sum_{k \in O^{ma}} f_k^r x_k^r$. Eligible roughage farmland cannot exceed the sum of actual roughage farmland and 60% of the pasture land eligible for subsidies, and this is expressed in constraint 4.33.

The full mathematical formulation of the optimisation model is reprinted in appendix A1.

5 Data

The purpose of the optimisation model is to assist dairy farmers' decision making by finding the optimal allocation of their resources. The model assists in deciding the number and kind of animals to have on the farm, and how to allocate the farmland between different activities. The model optimises each farm individually. Because of this, the model relies on input data from the farmer. Every farm has different characteristics which make the input-data unique for each farm that uses the optimisation model. There will be differences in arable farmland, crop yield, number of cattle shed spaces, number of calves born per dairy cow, productive life of dairy cows, and milk quota. There can also be structural differences in which some farms can grow grains or potatoes or house sheep, while other farms cannot. Additionally, farms usually receive different prices for the milk and beef they deliver. This price difference can be due to differences in geographical location of the farm and differences in the quality of the milk/beef delivered. There will also be differences in the forage and concentrate consumption per animal for different farms, and differences in prices paid for concentrate and to produce forage.

Each farm is unique, and the optimisation model has to take into account these unique characteristics of the farms. As such, a large portion of the input data should be set specifically according to which farm is using the model. Still, another portion of the data will be constant for all farms. Data related to subsidies are constant for all farms, with the exception of the region which the farm is located in (this is entered by the user of the model). In addition to the subsidies data, the following data is fixed for all farms:

- Shed stalls requirement, for all animal types
- Productive life of animals, except for dairy cows
- Milk yield per dairy cow

Shed stalls requirement defines what kind of stalls each animal type uses in the shed. E.g. dairy cows use stalls for cows, while calves use stalls for calves. This usage of stalls is given solely by the type of animal and is thus equal for all farms. As such, the data for this parameter is calculated. The productive life of animals is also given by the type of animal. E.g. calves have productive life of six months, while a heifer calving at 24 months of age have a productive life of 18 months. However, the productive life of dairy cows can be different for different farms, and the data value should be set by the individual farm. Milk yield is also only dependent on the animal type. Dairy cows are divided into 11 categories depending on their annual milk yield, and the milk yield parameter is set appropriate for each dairy cow category (see Figure 3.1).

Livestock margin parameters are calculated based on both farm specific inputs and fixed inputs. E.g. the fodder consumption and fodder prices can be set by the user of the model, while the quantity of milk delivered is fixed for all instances.

Fodder consumption is the amount of food that each animal type consumes annually. This measurement is given in food units (FEm). Fodder consumption depends on the nutrition demand by each animal type. Generally, farms develop a fodder plan which determines how much fodder each animal consumes annually. The fodder plans can be different between farms. However, reference fodder plans exist. In Norway, these reference plans are developed by TINE and other industry players. Since the fodder consumption relies on the farm-specific fodder plans, data for fodder consumption should be set specific to which farm is using the optimisation model. Entering fodder consumption for all animal types may however require significant work by each farmer, and in some cases the farmers might not know how much fodder each animal consumes, especially if it's an animal type that the farm currently doesn't have in its possession. Therefore, standard values for fodder consumption are provided in the model. These standard fodder consumption values are gathered from the industry reference values, which for this thesis is provided by TINE and Nortura.

5.1 Description of the Data

This section will outline the different data types that are used optimisation model.

5.1.1 Farm Data

Data that are required to be entered by the user include:

- Region for price subsidies for milk

- Region for price subsidies for beef
- Region for production subsidies for acreage and cultural landscape
- Realised milk price (in kr/l)
- Milk quota (in l)
- Realised beef price (in kr/kg)
- Realised sheep wool price (in kr/kg)
- Forage price (in kr/FEm)
- Concentrate price (in kr/FEm)
- Cultivated farmland (in daa)
- Pasture farmland (in daa)
- Crop yield (in FEm/daa)
- Number of stalls for all animal types
- Birth rate dairy cows (in calves/year)
- Productive life dairy cows (in months)
- Forage consumption for all animal types (in FEm)
- Concentrate consumption for all animal types (in FEm)

The three regions, region for price subsidies for milk, region for price subsidies for beef, and region for production subsidies for acreage and cultural landscape, are used in determining the subsidy rate for price subsidies and some production subsidies. The three regions are set according to the geographical location of the farm. The parts of the country which are included in each region are determined by the Norwegian Agriculture Agency, and can be found online and in the Agricultural Agreement (LMD, 2018a).

TINE dairy farms receive a price per litre milk delivered set by TINE. The price is calculated from a base price and adjusted according to seasonal variations, quality parameters, organic production, amongst others (TINE, 2019). The realised milk price is set individually by each farm equal to the price they receive from TINE. Beef prices are set by Nortura. Similar to milk prices, the beef prices vary according to different parameters, most notably quality characteristics of the meat. Each farm should input the price they receive per kilogram of beef delivered, and for each animal type.

The milk quota determines the upper limit on how much milk a farm can sell commercially, each year. The input data for the milk quota is set equal to the current quota the farm possesses. In the model, the milk quota is a constant parameter and the model is not optimising the size of the quota. However, in the analysis part of the thesis, the effect of a decrease in the milk quota is investigated.

Price of fodder include both price of forage and price of concentrate. The prices have to be entered by the user of the optimisation model and set equal to the realised production cost of forage and purchase cost of concentrate. Cultivated farmland refers to the farmland where it is possible to grow and harvest crops. Pasture farmland is the farmland that can solely be used for grazing livestock. The two farmland parameters should be entered by the farm using the optimisation model, where both parameters are measured in decare. Crop yield is the production performance of the farmland, measured in fodder units per decare of farmland. The crop yield measure represents how effectively farmland is converted into forage for the livestock. The measure is used to calculate how much fodder the farm produces.

Number of stall places has to be entered by the user of the model and it is a measure of how many animals the farm can fit into its shed. The birth rate of dairy cows is a measure of the average annual number of calves born from dairy cows. In Norway, the average birth rate is approximately equal to 1 (Nortura, nd) but some farms may have more rapid calving resulting in a higher birth rate while some farms may have less frequent calving. Productive life of dairy cows is a time parameter which measures the time from first calving until slaughtering. This is the period in which the cow produces milk. The productive life can be altered by the individual user to represent the real slaughtering age on the farm.

Forage and concentrate consumption are measures of how much fodder each animal consumes each year. Forage consumption is assumed to be covered by production on the farmland while concentrate consumption is supplied from purchases.

5.1.2 Subsidy Data

Data on subsidies affecting the Norwegian agriculture sector is gathered from NAA (2018). NAA publishes the Agriculture Agreement, which is the document outlining the agriculture subsidy schemes and all subsidy rates. The Agriculture Agreement is updated annually. The subsidy data is integrated into the model and there is no need for the individual user to adjust any of the subsidy data. The subsidy rates used in this model are from the Agriculture Agreement 2018–2019, retrieved from regjeringen.no.

5.2 Numerical Experiments

For the purpose of our analysis in this thesis, we are using experimental data to approximate three different Norwegian dairy farms located in the south-west. The numerical data sets we develop are based on standard reference values for fodder consumption and input prices, and statistical data from the Norwegian dairy farming industry.

Farm sizes are determined based on statistics from the Norwegian dairy farming industry. The data on farm sizes are provided by the Norwegian Agriculture Agency. This data shows the number of dairy cows, suckler cows, and other animals on all Norwegian farms. The data also provides information on cultivated farmland and pasture farmland, as well as data on milk and meat deliveries from each farm (NAA, 2019a; NAA, 2018). In the numerical runs of the model, we use average numbers for milk deliveries per cow and farmland area per cow. This way the farms will be representative for the real-life situations of Norwegian dairy farms. We use three different farm sizes to study the marginal effects of subsidies on farm decision making, especially focusing on changes in the livestock structure for bigger farms.

The location of the farms is in Jæren in Rogaland county, which is in region A for milk subsidies, region 1 for beef subsidies, and region 2 for production subsidies. Jæren is an area with good conditions for cattle and crop farming. Thus, Jæren has some of the lowest subsidy rates in the country. In fact, the farms do not receive any price subsidies for milk or beef. As such, the income from milk production is equal to the price the farm receives from TINE.

We are using standard reference values for milk prices, beef prices, fodder prices, and fodder consumption. These values are provided by TINE and Nortura. For the numerical runs of the model, the milk price is set equal to the base price received from TINE, currently at NOK/l 4.39 (TINE, 2019). Beef prices vary from NOK/kg 41.42 to NOK/kg 47.95 depending on which animal the beef comes from. Fodder prices are NOK/FEm 0.90 for forage and NOK/FEm 3.00 for concentrate. The fodder consumption per animal type is gathered from standard reference values for a farm with high quality forage production and the values are listed in the appendix. The milk quota is equal to 9 000 l/cow. The milk quota is set fairly high, above the average milk delivery per cow of Norwegian dairy farms, which for 2018 was 6 950 l/cow (NAA, 2018; NAA, 2019a). The milk quota is set this high to ensure that the farm can achieve the maximum milk production of their dairy cows. Having a lower milk quota may affect the optimal solution. In the following chapter we will study a scenario where the milk quota is lower to analyse the effect the milk quota has on the optimal allocation of farm resources. The total farmland area is set equal to the average farmland area per adult cow in Norwegian dairy farms multiplied by the number of stall places for adult cows. The average cultivated farmland per cow is 12 daa, and the average pasture farmland per cow is 3 daa, as of 2018 (NAA, 2018). The yield on the cultivated land is 700 FEm/daa (4 953 MJ/daa), which is a measure of how effectively the farm produces fodder for its livestock, and the climatic conditions in the area. Dairy cows have a productive life of 24 months, close to the industry average (Nortura, nd). This results in a cull rate of 50%, meaning that every year, half of the dairy cow population is slaughtered and have to be replaced. The livestock on the farm does not spend enough time on pasture land or on rangeland to be eligible for grazing subsidies. This assumption only has minor practical concerns. The subsidies for pasture and rangeland are added directly to the margin of the livestock and is not dependent on the total farmland area or the size of the livestock. In this analysis, we focus on subsidies that are non-linear.

The number of stall places for animals for the three farms is determined based on industry statistics. This is to make the farm sizes representative of real Norwegian dairy farms. The number of stall places for the small dairy farm is equal to the 25$^{\text{th}}$ percentile of number of cows in dairy farms in Norway, stall places for the medium sized dairy farm is equal to the median, and stall places for the large farm is equal to the 75$^{\text{th}}$ percentile. The small farm can house 15 cows, the medium farm can house 23 cows, and the large farm can house 38 cows. The number of stall places for other cattle and calves are set equal to the number of places for cows. There are no places for sheep, as this is the situation for most Norwegian dairy farms. Also, since we are focusing our study on dairy farms, the inclusion of sheep is not very relevant. Sheep do not compete with cattle for stalls in the shed.

6 Results

In the analysis part of the thesis, the farming optimisation model is used to analyse three dairy farms of different sizes. The analysis will focus on the optimal livestock structure, profit distribution, and the relative contribution of subsidies on different farm sizes. Further in the analysis, two scenarios are studied. First, the effect of a lower milk quota. Second, the effect of lower milk prices. We are using the numerical data described in the preceding chapter.

6.1 Analysis of a Small Dairy Farm

The first part of the analysis will study the optimal solution for a small dairy farm. The input data is summarised in the table below.

Table 6.1: The input parameters used to define a small farm. The parameters are based on reference values and industry statistics.

Region:		
Region price subsidies milk	A	
Region price subsidies beef	1	
Region production subsidies	2	
Cattle shed places:		
Adult cows	15	
Cattle > 6 mo.	15	
Calves < 6 mo.	15	
Sheep	0	
Cultivated land	180	daa
Pasture land	45	daa
Crop yield	700	FEm/daa
Milk quota	135 000	l
Productive life dairy cow	24	months

The model is run in AMPL using the CPLEX solver. The optimal objective function is 713 110, which represent the total gross margin for the farm. For a farm running the model, it is of interest to see the optimal structure of the livestock, how the subsidies affect the total gross margin, which factors are constraining further expansion, and how the farmland is utilised.

We will first obtain an overview of the livestock structure. The second level of the livestock hierarchy shown in Figure 3.1 is used to visualise the livestock composition, i.e. livestock are grouped into dairy cows, suckler cows, heifers, heifer calves, bull, bull calves, and sheep. In Figure 6.1, the number of animals in each group is summarised and visualised.

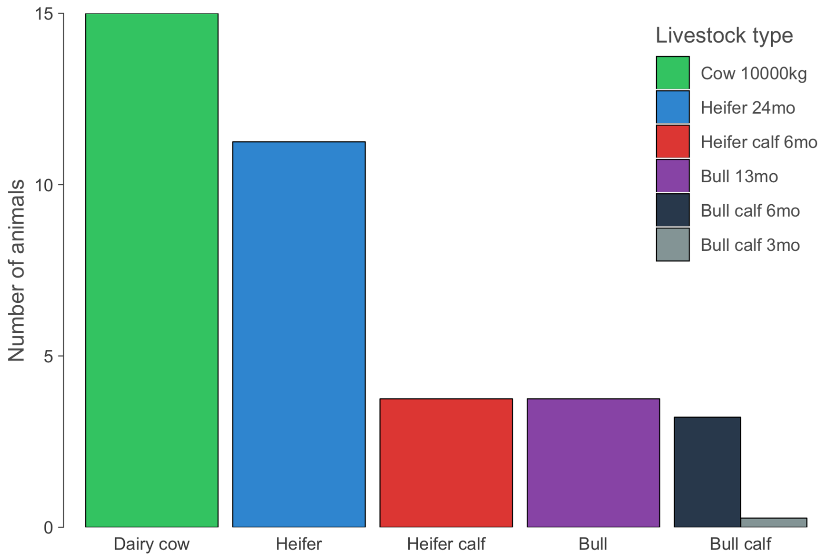


Figure 6.1: Small farm livestock structure obtained from the optimisation model. The resulting farm is a pure dairy farm; no suckler cows are present in the optimal solution. All heifers are recruited to replace culled dairy cows. Most bulls are slaughtered at 13 months age. All dairy cows produce 10 000 kg of milk annually.

In the optimal solution, the farm has 15 dairy cows, which is equal to the number of stalls for cows. Naturally, suckler cows are competing for the same stalls as dairy cows, but in the optimal solution there are no suckler cows. This illustrates that, under the current assumptions, dairy cows are more profitable, and it is more profitable to allocate stall places to dairy cows, at the expense of suckler cows. Having dairy cows inevitably results in other animals on the farm, as the cows must produce offspring to stay in lactation. Since we have set dairy cows to give birth once a year on average in the input of the model, the farm will obtain calves equal to the number of dairy cows. When counting up the number of calves (and other animals) in Figure 6.1, we are counting the average annual number of animals. Calves are classified as calves for the first 6 months of their lives. From the 6$^{\text{th}}$ month, calves are classified as heifers or bulls depending on their gender. Thus, having one calf for six months will result in an average annual number of calves of 0.5. This has implications for the use of stalls. A calf only needs a stall assigned to calves for half a year, meaning that it's possible to have two calves per stall place, each year.

The optimal solution provides information about which decisions to make regarding the offspring of dairy cows. From Figure 6.1, we can see that all heifer calves present in the optimal solution are of type heifer calf 6mo. These heifer calves are reared to become milk producing dairy cows. The optimisation model can choose between these heifer calves and heifer calves that are sold after three months. In the optimal solution, the farm does not have any heifer calves that are sold at three months age. Given the numerical data used in this analysis, it is actually impossible to have heifer calves sold at three months old. This is because the number of heifer calves born is *exactly* large enough to replace the dairy cows that are culled. This fact can be described by the productive life of dairy cows and heifers. Productive life is the number of months that an animal spends in its current category. For the numerical runs, we assumed that the productive life of dairy cows is 24 months, which means that, on average, 50% of the dairy cows have to be replaced annually. Heifer calves have a productive life of six months and heifers have a productive life of 18 months. Thus, the age of the heifer at first calving is 24 months. The productive life of dairy cows therefore equals the age of heifers at first calving. Dairy cows give birth to one calf annually, of which 50% are heifer calves (female calves). Since the productive life of dairy cows equals the age of heifers at first calving (24 months), and the cull rate (replacement rate) of dairy cows is equal to the birth rate of heifer calves (50%), the number of heifers calving for the first time will be exactly equal to the number of dairy cows slaughtered every year. If the productive life of dairy cows was longer, meaning that dairy cows are slaughtered at an older age, we would have more heifer calves than what is needed to replace slaughtered cows. In such a scenario, it would be possible for the optimisation model to choose heifer calves that are sold at three months age.

For bull calves (male calves) born, the farm is faced with two possible decisions. The farm can either feed the bull calf to adult age (13-19 months) before slaughtering them, or the farm can sell the calf at three months of age. In the optimal solution of our model, 3.75 bulls are fed up and slaughtered at 13 months of age. It is also possible to keep bulls until 15, 17, or 19 months age, but the optimisation model suggests keeping bulls only until they are 13 months of age. The key takeaway is that the optimisation model suggests that it is more profitable to slaughter bulls at 13 months of age than at an older age, given the current conditions on the farm. It doesn't necessarily mean that 13 months slaughter age is the optimal strategy for all farms. Number of stall places, forage production and crop yield will influence the optimal slaughter age and these parameters are different between farms.

Subsidies are a significant contribution to farmers' finances and the complex structure of the Norwegian agricultural subsidies creates challenges of optimal decision making. This is one of the benefits by using a farm management tool such as this optimisation model. Since this optimisation model is maximising gross margin while taking into account subsidy payments, it is interesting to investigate how the subsidies affect the overall financial performance of the farm, and not just look at the overall gross margin in itself. In Figure 6.2, the total income is decomposed into income sources. A farm's total income stems from the direct margin of the livestock, the direct margin of the farmland, and subsidies. For the small dairy farm studied here, income from subsidies represent 38.9% of total income. Direct livestock margin equals 57.4% of total income, and the remaining 3.7% is from farmland margin. Amounting to 38.9%, subsidies are a major factor affecting the profitability of the farm we are analysing here. In this model, livestock margin does not include labour costs, costs of rent, electricity, administration or any fixed costs. After deducting all costs, subsidies can be crucial for farms to stay profitable.

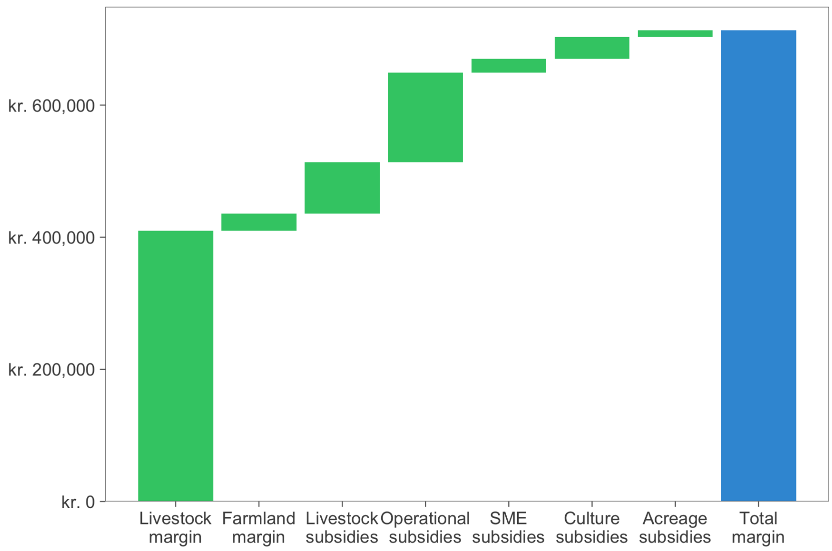


Figure 6.2: Small farm income distribution in the optimal solution. The direct livestock margin is mainly the income from sale of milk. Subsidies have a significant impact of income, with operational subsidies being the biggest contributor.

The total subsidy payment comes from four different subsidy schemes. These are livestock subsidies, operational subsidies, SME subsidies, and subsidies for acreage and cultural landscape (here separated into subsidies for acreage and subsidies for cultural landscape). Of all subsidy schemes, the operational subsidy is the largest contributor, equalling 48.7% of total subsidies – or 18.9% of total income. Operational subsidies for milk production are given to a maximum of five dairy cows per farm. However, the size of the subsidy is much larger than other subsidy schemes. As such, the operational subsidies are a large contribution for small farms. Livestock subsidy is the second largest contributor at 28.1% of total subsidies. The livestock subsidies are degressive, meaning that farms with fewer animals receives a higher rate per head than farms with more animals. However, in contrast to operational subsidies, the livestock subsidies do not have a maximal limit on the number of heads that are eligible for subsidies. For example – in the case of dairy cows – the farmer receives NOK/head 770 when surpassing 51 dairy cows, regardless of how many he is in possession of.

Table 6.2: Small farm livestock margin in the optimal solution, for dairy cows and other cattle. Other cattle includes heifers, bulls and calves. The margin for other cattle is negative because of feeding costs.

	Dairy cow		**Other cattle**	
Number	15.0		22.2	
Livestock margin	NOK	458 246	NOK	−48 713
Livestock subsidies	NOK	60 914	NOK	17 119
Operational subsidies	NOK	135 050	NOK	0
SME subsidies	NOK	21 000	NOK	0
Total subsidies	**NOK**	**216 964**	**NOK**	**17 119**
Total livestock profit	**NOK**	**675 210**	**NOK**	**−31 594**

Decision making on a dairy farm consists of allocating resources under constraints. First, the number of animals on a farm cannot exceed the stalls available in the shed. Second, the total milk production cannot exceed the milk quota that the farm possesses. Third, the total fodder consumption of the farm's livestock cannot exceed the total forage production on the farm. The optimisation model is maximising the gross margin of the dairy farm while respecting these constraints, amongst others. It is of interest to investigate which constraints are imposing restrictions in the optimal solution. A constraint that often limits the farm's potential to expand is the number of stalls in the cattle shed, which represents a physical limit on how many animals a farm can have. There are also other constraints. Having a milk quota that is not sufficiently large will limit the farm from utilising its full production potential and may be harmful for profits. Not producing enough fodder is also harmful as it might limit how many animals the farm can have or how much milk the cows can produce. The factors that impose limitations or constrains a farm's expansion is displayed in Figure 6.3, where the utilisation rate of each constraining factor is included. The utilisation rate shows how large a proportion of the constraining factor is used in the optimal solution. E.g. if the utilisation rate of the milk quota is 100%, then the total milk production is exactly equal to the available milk quota. Usually, this indicates that the milk quota is not sufficiently large. However, the model also returns a theoretical maximum utilisation rate of milk production. The theoretical maximum milk production is calculated based on how much milk the highest yielding cows can produce and how many dairy cows the farm can fit. In the model we assume that the maximum milk yield of dairy cows is to 10 000 kg. Converting 10 000 kg to litres and deducting milk fed to calves, waste due to quality issues, and household consumption, this results in an annual production slightly below 8 900 litres milk. By multiplying this amount with the number of cow stalls, we obtain the theoretically maximum total milk production for the farm. The theoretical milk utilisation rate is a good measure to see if a farm is producing as much as possible.

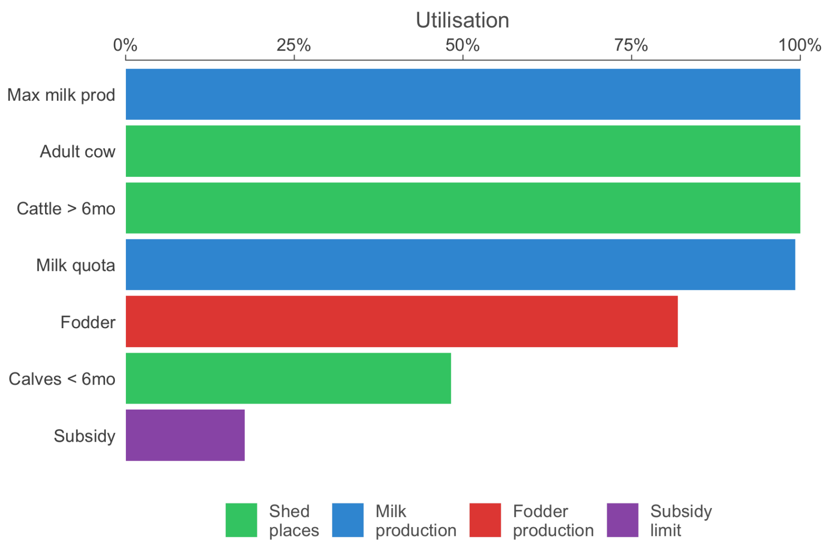


Figure 6.3: Small farm constraining factors in the optimal solution. Shed places for adult cows and cattle is fully utilised and limits further expansion. Milk production is at its theoretical maximum while a small portion of the milk quota is unused.

As we see from the plot above, the utilisation rate is 100% for maximum milk production, cow stalls and adult cattle stalls. This indicates that the milk production equals the theoretical maximum, meaning the farm has 15 dairy cows each producing 10 000 kg of milk annually. Both heifers and bulls are competing for cattle stalls, and from Figure 6.1 we see that the farm has both heifers and bulls. On average, 11.25 heifers are filling these stalls, while the remaining 3.75 stalls are used by bulls. 99.2% of the milk quota is used, leaving a small portion of the quota unused. From a pure economic perspective, it is costly to have quota that is unused and, in these cases, where some of the quota is unused it's recommended to either rent out the unused quota or to sell it. Our optimisation model does not allow for the trade of milk quotas as it is difficult to precisely model milk quota prices. As such, any unused quota will remain as it is. The fodder production has a utilisation rate of 81.9%, meaning that 81.9% of the potential fodder production is consumed by the livestock on the farm. The potential fodder production is given by the total acreage of the farmland and the crop yield rate. It represents how much fodder can be produced at the farm. Since the utilisation rate is lower than 100%, the livestock on the farm requires less fodder than what can be produced. This creates an opportunity for remaining farmland to be used to grow other crops, such as grain. Alternatively, the farmer can invest in more stall places to have more animals on the farm.

In the shed, only 48% of the places for calves are used. Having unused calf shed places is generally not recommended as it is a poor use of resources. In this optimisation model, however, the number of calves is limited by the number of calves born. Thus, it is not possible to have more calves with the current model formulation. In reality, a farmer can see this as an opportunity to purchase recently born calves and feed them up to six months age for resell. It is worth noting that purchasing calves increases the demand for fodder, and the farmland area should be sufficiently large to enable this increase in fodder demand. Including the possibility for farmers to purchase calves would be an interesting extension to the optimisation model.

We have also included the utilisation of subsidies. This is because there is an upper limit on the amount of subsidies that a single farm can receive. For 2019, this maximum subsidy amount is NOK 560 000 (LMD, 2018a). The subsidy schemes that count towards this upper limit are production subsidy for livestock and production subsidy for small and medium sized dairy farms. The maximum subsidy amount does not impose a restriction for further expansion of the farm, such as the number of stalls does. However, this subsidy limit will, when reached, reduce the marginal profitability of expanding the farm as the farm does not receive any additional subsidies for having more animals. In the optimal solution, only 17.7% of the subsidy limit is reached, meaning that for this small farm, the maximum limit does not impose any restrictions.

To end the analysis of the small farm, we will investigate how the farmland is used. As previously noted, in the optimal solution, the farm utilises 81.9% of its fodder production potential. Thus, not all farmland needs to be used for fodder production. Forage is grown for feed, but it is also possible to grow grains or potatoes for commercial sale. The structure of the farmland is visualised below, in Figure 6.4.

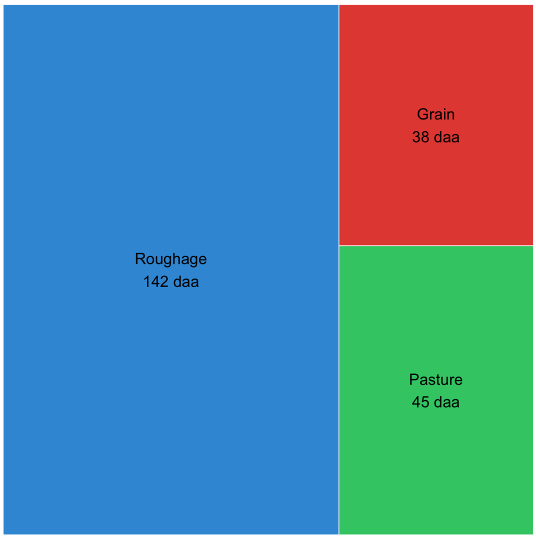


Figure 6.4: Small farm farmland usage in the optimal solution. Most of the farmland (142 daa) is used to grow roughage (forage) to feed the livestock. 38 daa is used to grow grains for commercial sale.

The farmland consists of 180 daa of cultivated land and 45 daa of pasture land. Of the 180 daa of cultivated land, 142 daa is used to grow forage used to feed the livestock. The remaining 38 daa is used to grow grains for commercial sale. Pasture land can only be used to produce fodder for livestock and it is thus not possible to grow cash crops on this land.

Table 6.3: Small farm farmland margin in the optimal solution. Pasture land and forage is used to produce fodder for the livestock and does not generate any income, but are awarded subsidies. Grain is sold commercially for profit.

	Pasture land		Cultivated land			
			Forage		Grain	
Area	45 daa		142 daa		38 daa	
Farmland margin	NOK	0	NOK	0	NOK	26 276
AK subsidies	NOK	0	NOK	27 453	NOK	15 766
Total margin	**NOK**	**0**	**NOK**	**27 453**	**NOK**	**42 042**

AK subsidies for forage is calculated based on both forage grown on pasture land and forage grown on cultivated land. In this table, all subsidies for forage are displayed in the forage column.

6.2 Comparison of Three Dairy Farms

In this section, two more dairy farms are analysed. The first is a medium sized dairy farm and the second is a large dairy farm. The data used to create these farms were given in chapter 5. A brief overview of the input data for the three farms are displayed in Table 6.4.

Table 6.4: Input parameters for farms of three sizes; small, medium, and large. The parameters are based on industry statistics and replicate average Norwegian dairy farms.

	Small dairy farm		**Medium dairy farm**		**Large dairy farm**	
Cow stalls	15		23		38	
Cattle stalls	15		23		38	
Calf stalls	15		23		38	
Milk quota	135 000	l	207 000	l	342 000	l
Cultivated farmland	180	daa	276	daa	456	daa
Pasture farmland	45	daa	69	daa	114	daa

The first thing to study is if the composition of the livestock changes for bigger farms. The small farm had only dairy cows and bulls, meaning that is was more profitable for the small farm to have dairy cows than to have suckler cows. As explained in chapter 2, dairy cows and suckler cows are affected by several subsidy schemes. Based purely on the livestock subsidy payment, it's more profitable for a farm to obtain their first suckler cow than their 15^{th} dairy cow because the marginal subsidy payment for the first suckler cow is greater than the marginal subsidy amount for the 15^{th} dairy cow. Operational subsidies for dairy cows reach its maximum at five dairy cows, meaning that bigger farms receive no extra subsidy. For suckler cows, the operational subsidies increase up to the 40th suckler cow. However, at their respective maximal values, the operational subsidy for dairy cow is larger than the operational subsidy for suckler cow, totalling 144 050 for dairy cows, versus 131 280 for suckler cows in this region. Again, based purely on subsidy payments, for farms with five dairy cows, it is more profitable to obtain suckler cows. Lastly, the subsidies for small and medium sized enterprises reaches its maximum value at 23 dairy cows. From the 24th dairy cows and upwards to the 51th, the subsidy amount is negative, thus reducing the total subsidy payments that the farm receives. From the perspective of subsidies, there are clear incentives for bigger farms to change their livestock structure from only dairy cows to a mix of dairy cows and suckler cows, in accordance to the government objective to have a varied farm structure. The optimal livestock composition found by the model for the small, medium and large farms are displayed in Figure 6.5.

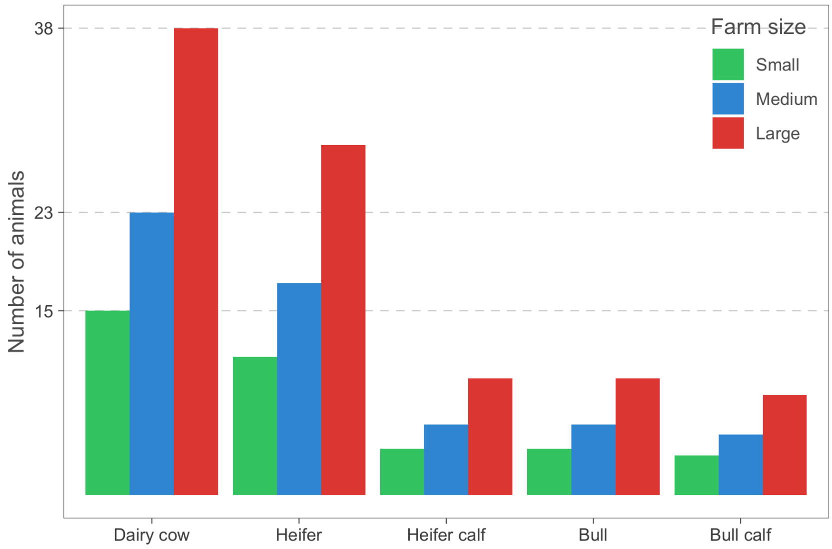


Figure 6.5: Three farms livestock structure in the optimal solutions. The relation between all livestock categories are similar in all three solutions.

In Figure 6.5 we can see that the livestock structure remains unchanged in the optimal solution of the small, medium, and large farms. The number of animals in each category increases proportional to the size of the farms. The ratio between the livestock classes are constant for all three farms. All farms are pure dairy farms in the optimal solutions. The results indicate that the optimal allocation of livestock does not depend on the size of the farm. Given the assumptions made, our model suggests that large farms will – similar to medium and small farms – have a livestock structure of pure dairy cows, with bulls slaughtered at 13 months age. In the previous paragraph, it was argued that the subsidy schemes provide large farms with incentives to obtain additional suckler cows instead of dairy cow. For the three farms studied here, the results from our optimisation model suggests that these incentives are not great enough to change the livestock structure in favour of suckler cows. It is more profitable to obtain the 15th, 23rd, and 38th dairy cow than it is to obtain the first suckler cow when taking the whole margin of these animals into account. One of the political objectives of the Norwegian agriculture subsidy system is to obtain a "diverse agriculture with a *varied farm structure*". Based on our findings and given the assumptions in the model, the subsidy system does not lead to a varied farm structure in terms of promoting suckler cows, neither for small, medium, or large farms.

Data on Norwegian dairy farms gathered by the Norwegian Agriculture Agency in 2018 shows that out of 7 776 dairy farms, 941 farms also have suckler cows – leaving 6 835 dairy farms without any suckler cows. Only 12% of dairy farms have at least one suckler cow. The proportion of dairy farms with suckler cows increase with size of the farm's livestock. Of the 1 697 dairy farms with cultivated farmland of 140–220 decares, 6.3% have suckler cows. Of the dairy farms with cultivated farmland of 236–316 daa, 11.5% have suckler cows, and of farms with cultivated land of 416–496 daa, 16.9% have suckler cows.

Table 6.5: Proportion of dairy farms with suckler cows. These data show that the proportion of dairy farms with suckler cows is greater for large farms than for small farms. The majority of dairy farms have no suckler cows at all.

	Cultivated farmland		
	140–220 daa	236–316 daa	416–496 daa
Number of dairy farms	1 697	1 283	544
with suckler cows (%)	107 (6.3%)	148 (11.5%)	92 (16.9%)
without suckler cows (%)	1 590 (93.7%)	1 135 (88.5%)	452 (83.1%)

Data is based on Norwegian farms that had dairy cows and delivered milk in 2018. Note that the intervals displayed here are not including all farms. The intervals are based on the three farm sizes analysed in this section, which had cultivated land of 180 daa, 276 daa, and 456 daa. The intervals are set to include ± 40 daa.

Even though the proportion is increasing, the data on Norwegian dairy farms shows that most farms do not have any suckler cows in their herds. With the experimental data used to create the three farms studied in this thesis, the optimisation model suggests that it's most profitable to keep a pure dairy cow herd, and this reflects the real situation for most Norwegian dairy farms. However, this does not mean that the optimisation model simply confirms the real-life situation in Norwegian dairy farming without providing farmers with any decision support. A decision support tool is very useful for the 12% of dairy farms with suckler cows to study if the choice of having suckler cows is the optimal allocation of the farm's resources. Similarly, for the 88% of farms without suckler cows, the optimisation model is useful to analyse if it is profitable to replace some dairy cows with suckler cows.

Another comparison between model results and actual data that is interesting to study is the number of dairy cows and other cattle. In the optimal solution of the three farms, the number of other cattle were 22.2, 34.1, and 56.3 for the small, medium, and large farm, respectively. The ratio of other cattle to dairy cows was constant at 1.48 for all farms. Industry data on Norwegian dairy farms without suckler cows reveal that the ratio of other cattle to dairy cows is 1.59 on average. The ratio is greater for larger farms, increasing from 1.50 for farms with cultivated farmland of 140–220 daa, to 1.60 for 236–316 daa, to 1.72 for 416–496 daa. The results obtained from the farming decision model suggest a constant ratio between other cattle and dairy cows while the industry data shows a higher average ratio and a ratio that is increasing by farm size. The difference might be due to the fact that the model suggests to slaughter bulls at 13 months of age, while the average slaughter age for bulls was closer to 18 months in 2018 (NDHRS, 2019).

With the degressive structure of the subsidies, it is interesting to analyse the change in subsidy's share of total income for bigger farms. The share of total income from livestock subsidies, operational subsidies, SME subsidies, and AK subsidies is 33.8% for the medium sized farm and 27.1% for the large farm. To compare, the share was 38.9% for the small farm. The diminishing contribution from subsidies is clear, subsidies' share of total income is 30.2% lower for the large farm compared to the small farm. For the small farm, operational subsidies were the largest subsidy class, contributing to 48.7% of total subsidies. This share drops to 39.6% for the medium sized farm and 32.9% for the large farm. This is due to the fact that the operational subsidy awarded to dairy cows is not scalable above five dairy cows. As such, a small farm with 15 cows, a medium farm with 23 cows, and a large farm with 38 farms, all receive the same operational subsidy amount. Subsidies for small and medium sized dairy farms reaches its maximum level at 23 dairy cows. The medium sized farm gets the most out of this subsidy class, while the large farm receives less than both the small and medium sized farms.

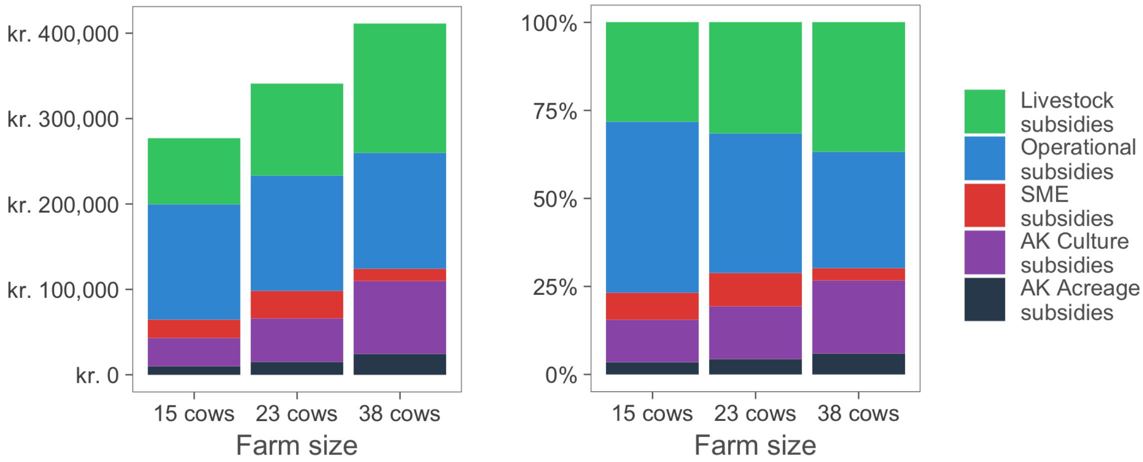


Figure 6.6: Subsidy structure of three farms. On the left, total subsidy payment received by a small, medium sized and large farm. On the right, the relative proportion of each subsidy class of total subsidy payment received by a small, medium and large farm.

For the large farm, the biggest subsidy contribution comes from livestock subsidies, equalling 36.9% of total subsidies (31.5% for medium and 28.1% for small farms). From Figure 6.6 it can be seen that AK subsidies are also an important contributor to total subsidies, especially for larger dairy farms. The AK subsidies are linearly scalable and not degressive as the other subsidy classes. AK subsidies are dependent on both the total number of animals and the total farmland area, as described in the review of the subsides. Thus, as farms expand, they need to have sufficient farmland area per number of animals for the AK subsidies to increase. In the farms studied in this thesis, the ratio between farmland area and number of stalls for cows is constant, which explains why the AK subsidies are increasing in magnitude for the bigger farms.

From the previous discussion, it's apparent that the relative contribution of the subsidies diminish as the dairy farm grows in size. In deciding whether or not to expand a dairy farm, the measure of key interest is the marginal subsidy amount. The marginal subsidy amount is the subsidy payment the farm receives for having one more dairy cow. Since the subsidy schemes are degressive, the marginal subsidy amount decreases as the number of animals increases. Generally, if the sum of the marginal subsidy amount and the marginal direct income is greater than the marginal cost, having one more dairy cow is profitable, at least in the short term. We will continue using the three farms and analyse the average marginal subsidy amount that the small farm obtained, and how it changes with an expansion from small to medium, and from medium to large.

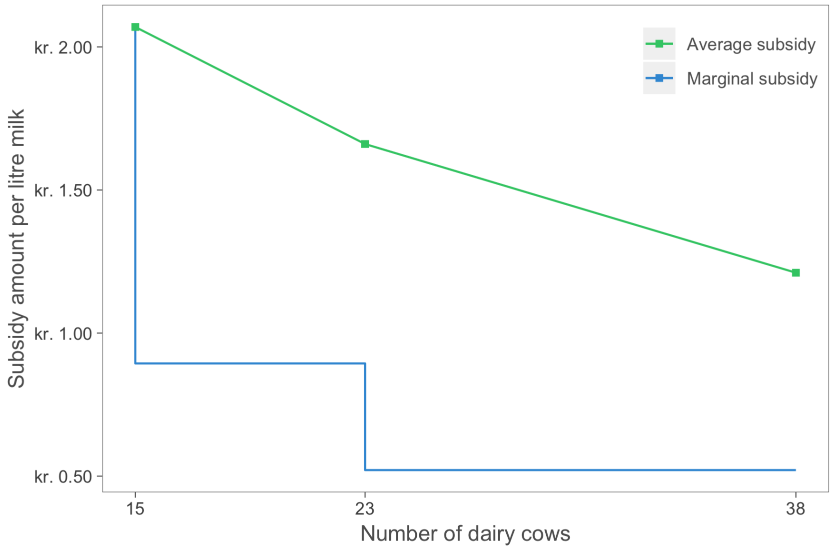


Figure 6.7: Average and marginal subsidy of three farms in the optimal solutions. The marginal subsidy is decreasing as farms increase in size.

The small farm with 15 dairy cows received on average NOK 2.07 in subsidy payment per litre of milk delivered. This includes all subsidy classes; livestock subsidies, operational subsidies, SME subsidies, and AK subsidies. The price a dairy farm receives for its milk (excluding subsidies) was set to NOK/l 4.39. Thus, for the small farm, the subsides received per litre of milk delivered is almost half of the milk price (2.07/4.39=0.47). Eight dairy cows are needed to expand the farm from a small sized to a medium sized dairy farm. On average, these eight cows receive NOK 0.89 in subsidies per litre milk delivered, resulting in a decrease of 56.8%. It is clear that as farms increase their dairy cow herd, it puts more pressure on their cost structure. The small farm receives NOK 6.46 per litre milk (including subsidies), meaning that the farm is still profitable with variable costs per litre milk produced equal to 147% of the milk price. The medium sized farm is still profitable with variable costs equal 120% of the milk price. The marginal subsidy income drops even further for the large farm. The large farm has 38 dairy cows, 15 more than the medium sized farm. The marginal subsidy income for these 15 cows is NOK 0.52 per litre milk delivered, 41.7% lower than the medium sized farm and 74.8% lower than the small farm. The large dairy farm can have variable costs equal to 112% of the milk price and still stay profitable. As farms increase their dairy cow herd, the marginal subsidy amount decreases and the amount by which the variable cost can exceed milk income diminishes, especially for farms increasing beyond the median size of Norwegian dairy farms. In general, large farms are much more dependent on having an efficient cost structure than smaller farms which to a larger extent can rely on subsidy payments.

Table 6.6: Margin comparison of three farms in the optimal solutions. The table shows livestock margin, farmland margin, and subsidy margin per number of dairy cow, for all three farms. Total margin is the sum of the three margins multiplied by number of dairy cows.

	Small dairy farm	**Medium dairy farm**	**Large dairy farm**
Number of dairy cows	15	23	38
Livestock margin per dairy cow	27 302	27 302	27 302
Farmland margin per dairy cow	1 752	1 752	1 752
Subsidy margin per dairy cow	18 487	14 834	10 816
Total margin	**713 110**	**1 009 418**	**1 515 048**

6.3 Scenario Analyses

In the following section, we will analyse the effects on the optimal solution from changes in two key dairy farm inputs – milk quota and milk price. First, we analyse how the optimal solution changes when the size of the milk quota is set equal to the average milk delivery for Norwegian dairy farms. Second, we investigate the effects of reduced milk prices on the optimal livestock structure.

6.3.1 Milk Quota Decrease

The three farms studied in the preceding section all had a milk quota that was sufficiently large for the farms to produce at the theoretical maximum defined in the optimisation model. In reality, not all dairy farms have such a large milk quota. Data from the Norwegian Agriculture Agency show that dairy farms on average deliver 6 950 litre of milk per dairy cow. A farm with a quota lower than production potential is interested to know how much income increases with a larger quota. With this knowledge, the farmer can obtain the maximum price he is willing to pay for extra quota. The opposite is also true. A farmer with a large quota wants to know how much he can sell portions of the quota for. In the first scenario analysis, we will study the optimal solution for a small farm that has a quota approximately equal to the average milk delivery of Norwegian farms. The numerical data used for the farm we will study here is the same data as we used to represent the small farm analysed previously in the section, with the exception of the milk quota that is set to 7 000 litres per cow. The farm can house 15 cows and thus has a milk quota of 105 000 litres, 30 000 litres less than the previously analysed small farm.

Solving the optimisation model for a small farm with 105 000 litres milk quota results in a change in the livestock structure. In the optimal solution, the farm has 11.8 dairy cows producing 10 000 kg of milk each. These 11.8 cows produce enough milk to fill the quota. However, the dairy cows do not fill all the cow stalls in the shed. The free stall places are filled by suckler cows. There are 3.2 suckler cows in the optimal solution. All suckler cows are heavy breeds with slaughter age at 14 months of age. These results tell us that, for this small dairy farm, it is more profitable to only have 10 000 kg dairy cows producing enough milk to cover the quota, and then use the remaining cow stalls to house suckler cows. The alternative would have been to produce less milk per cow and fill the stall places purely with dairy cows. Apart from the addition of suckler cows, the other animals present on the farm is similar to the solution of a small farm with a larger milk quota. The number of heifers and bulls is reduced accordingly to the reduction in calves born from fewer dairy cows.

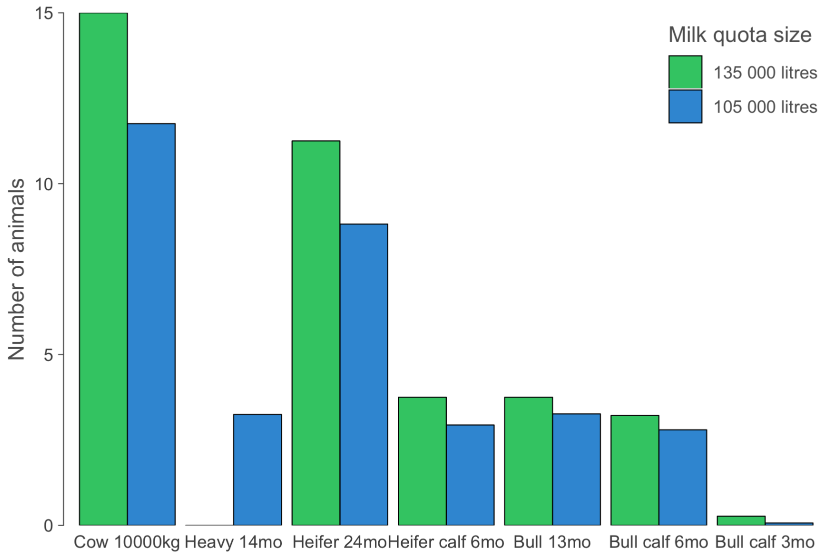


Figure 6.8: Livestock structure after milk quota decrease. The number of dairy cows (cow 10 000kg) is exactly large enough to fill the milk quota. Remaining shed places are filled by suckler cows (heavy 14mo).

The total gross margin of the small farm with a milk quota of 135 000 litres was 713 110. The farm analysed in this section, with a quota of 105 000 litres has a gross margin of 650 862 in the optimal solution. The difference in gross margin is 62 249. Thus, the small dairy farm with space for 15 adult cows and with a current quota of 105 000 litres should rationally pay a maximum of 62 249 for an additional 30 000 litres of milk quota. This results in a price of NOK/l 2.07. However, as we saw in the analysis of the small farm with 135 000 litres quota, the entire quota was not used. With a maximum production per cow of 10 000 kg, the farm produced 133 981 litres of milk annually. This means that a milk quota of 133 981 litres is sufficient to obtain the same results as we obtained for the small farm with 135 000 litres quota. A quota increase of 28 981 litres is necessary to obtain the same milk production as the small farm with a large quota. This translates to an increase in total margin of NOK 2.15 per litre of increased milk quota. From an economic perspective, it's rational to acquire the additional quota for an annual cost not exceeding NOK/l 2.15. As of 7 April 2019, the annual rental costs of milk quotas lay below NOK/l 2.15 (Melkebørs, nd; Melkekvoter, nd) for Rogaland county, concluding that the small farm studied here is better off increasing its quota and utilise its full milk production potential, than to leave the quota low and to keep suckler cows.

In the optimal solution of the small farm with 105 000 litres milk quota, the number of dairy cows is 11.8 and the number of suckler cows is 3.2. Previously in the analysis, when the milk quota was sufficiently large, the optimal solution always returned integer number of cows. Now, with a smaller quota, the optimal variables are no longer integer. The number of cows was previously limited by the stall places in the shed (which is an integer number), but the number of cows is now limited by the milk quota. The optimisation model sets the number of cows such that the milk quota is exactly filled, and this results in fractional number of cows. The issue of fractional numbers was discussed in chapter 4, where we argued that since the decision variables for number of animals are defined as annual average number of animals, the use of fractional numbers is appropriate. Having an integer farming optimisation model could however be an interesting addition to our current model, and this topic is discussed further in chapter 7.

6.3.2 Sensitivity to Milk Price Decreases

From the previous analyses, we found that the dairy farms were more profitable having a pure dairy cow herd than a combined dairy cow and suckler cow herd. Three farms replicating a small, a medium, and a large Norwegian dairy farm all had a pure dairy cow herd, when the milk quota was sufficiently large. The small farm with a lower quota is better off increasing its quota to the maximum potential given the current milk quota prices. The choice of being a pure dairy cow farm depends of several factors such as location, subsidies, fodder costs, milk prices, and beef prices. Since milk is the main output of dairy cows, the price of milk is crucial to determine the profitability of dairy cows. In farming decision making, the choice between dairy cows and suckler cows represents an allocation of competing resources (dairy cows vs suckler cows) under physical constraints (cowshed stalls). Given constant beef prices, an increase in the milk price will make dairy cows relatively more competitive to suckler cows. On the other hand, a decrease in the milk price will make dairy cows less competitive relative to suckler cows. The ratio between milk and beef prices will affect the trade-off between having dairy cows who produce milk and suckler cows who produce beef.

In this section we study a reduction in the milk price of 10%, 20%, and 30%. The numerical data is adjusted for all three farms creating nine new data sets. Nine optimisation models are run in AMPL and the results are compared. The milk quota is set to its original size, i.e. the quota is sufficiently large to produce at the full potential. When studying the optimal livestock structure, a reduction of the milk price with 10%, 20%, or 30% can be interpreted equally to a reduction in the price ratio between milk and beef with 10%, 20% or 30% respectively. It's the relative profitability that affects the choice between dairy cows and suckler cows, and when milk prices decrease the relative profitability of suckler cows increase compared to dairy cows. We are interested in seeing if and how the optimal livestock structure is affected by decreasing milk price.

The effects of a milk price reduction are similar for both the small and the medium sized farm. A decrease in the milk price up to 30% does not affect the optimal livestock structure. Thus, small and medium sized farms are better off with a pure dairy cow herd even when milk prices drop by as much as 30%. This illustrates the fact that dairy cows are significantly more profitable than suckler cows, especially for small and medium sized farm. For the large farm, however, there was a change in livestock structure at 30% reduction. For a milk price reduction of 10% and 20%, the optimal solution with a pure dairy cow herd remained in place. But, when the milk price dropped by as much as 30%, it became more profitable to keep some suckler cows on the farm. The livestock structure of the large farm can be seen in Figure 6.9 below.

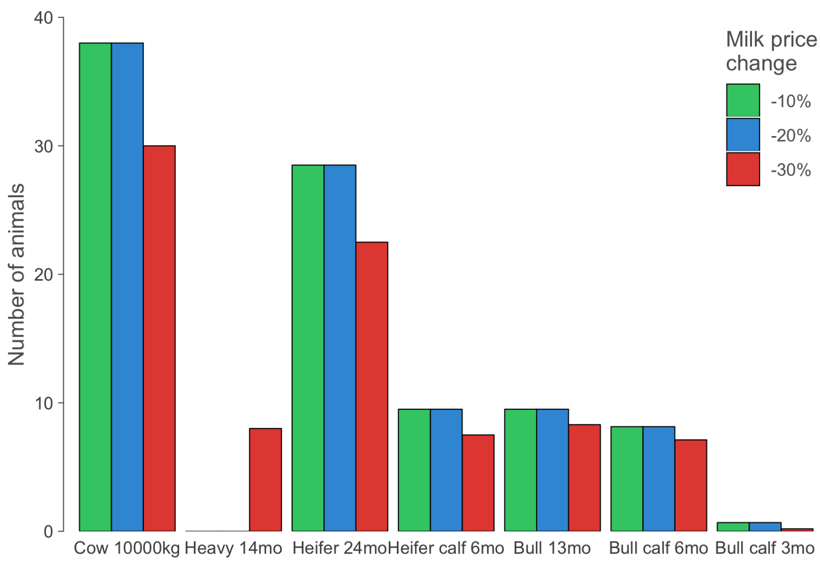


Figure 6.9: Livestock structure on the large farm after milk price decrease. Eight dairy cows are replaced by suckler cows when the milk price drops by 30%.

For the large farm, when milk prices drop by up to 20%, the optimal number of dairy cows is 38, equal to the number of stalls. When the milk price drops by 30%, the optimal number of dairy cows is 30. The remaining 8 stalls are filled by suckler cows. We see that the decrease in milk price does not lead to a change in livestock structure from pure dairy cow to pure suckler cow. It is still profitable to keep a majority of dairy cows on the farm. However, when the large farm has 30 dairy cows, it is more profitable to obtain the 1st suckler cow than the 31st dairy cow. It is also more profitable to obtain the 8th suckler cow than the 31st dairy cow. This also means that the first 30 dairy cows are more profitable than the first 8 suckler cows. The fact that suckler cows become relatively more profitable than dairy cows after the 30th dairy cow is due to the degressive structure of the agriculture subsidies. The marginal subsidy amount given to dairy cows reduces at specific points, as defined in the Agriculture Agreement. By studying the agreement, we find that the marginal production subsidies for livestock decreases from NOK/head 2 562 to NOK/head 1 100 after the 30th dairy cow (LMD, 2018a). This drop in marginal subsidies for dairy cows is sufficiently large to make a new suckler cow more profitable than a new dairy cow, given that the milk to beef price ratio has decreased by 30%.

In our model, we do not take into account the buying and selling of milk quotas. It's possible for farmers to sell or rent out unused quota. For the large farm, it was profitable to not use some of the quota when milk prices decreased by 30%. This unused quota has an opportunity cost since it can be sold or rented out, instead of staying with the farm unused. Incorporating a possibility to sell or rent out the unused quota in the model, would increase the profitability of the farm analysed here. When a farm is using its entire quota, it is simultaneously losing out on the potential income of selling or renting out the quota. This is because the farm is having an asset that can be sold in the market. The annual income from selling or renting out the quota can be interpreted as an additional cost of producing milk. For every litre of milk produced, this cost represents the lost income of no longer being able to sell or rent out a quota of one litre. Thus, incorporating milk quota trade into the model will increase the relative competitiveness of suckler cows to dairy cows. This will ultimately make the farm more sensitive to milk price changes. Thus, the percentage decrease in milk price that is required to alter the livestock structure in the favour of suckler cows is in reality lower than depicted here.

However, incorporating milk quota trade adds more complexity to the model. A major challenge is determining the correct price that can be received from selling or renting out quota. A decrease in milk prices will affect not only a single farm, but all farms in the industry. A decrease in milk prices makes it less profitable to produce milk and reduces the attractiveness of milk production. Since the profitability decreases, it is rational to assume that also the demand for milk quota decreases. Furthermore, decreased demand of milk quotas will decrease the market price for milk quotas. As such, a milk price decrease will also decrease the market price for milk quotas. In the situation analysed here, with a 30% decrease in milk price, it might be difficult for the farmer to find potential buyers of his excess quota. The farm must at least expect to obtain a lower price for his quota than what he would have obtained before the milk price decrease. In the previous paragraph, we argued that incorporating milk quota trade in the model will increase the sensitivity to milk price decreases. However, as argued here, the market price for milk quotas will most likely decrease following a milk price decrease. It is expected that the milk quota market price dynamics will offset some of the increased sensitivity to milk price decreases. If the milk quota market collapses and nobody is willing to buy quotas, effectively valuing milk quotas at zero, the sensitivity to a 30% milk price decrease will not change at all by incorporating milk quota trade into the model. The topic of milk quota valuation is further discussed in section 7.3.

6.4 Summary

In this analysis, we studied dairy farms of three sizes with animals representing the 25% percentile, the median, and the 75% percentile of the number of cows on Norwegian dairy farms. Farmland area, birth rates, and productive lives were set approximately equal to industry average. The milk quota was sufficiently large to allow for a maximal production of 10 000 kg of milk per cow. The farms were located in Jæren in Rogaland county.

The results from this farming optimisation model suggests that keeping a pure dairy cow herd is the most profitable choice for all three farm sizes. It also suggests that it's profitable to fill all cow stalls. We found that total subsidies equal 38.9%, 33.8% and 27.1% of total margin for the small, medium and large farm, respectively. This results in a decrease of 30.2% in the relative contribution of subsidies for a large farm compared to a small farm. Operational subsidies are the biggest subsidy contributor for both small and medium sized farms, while livestock subsidies are the biggest contribution for large farms. For the first 15 dairy cows, the marginal subsidy payment per litre milk produced is equal to NOK/l 2.07. For the next 8 cows, the marginal subsidy is NOK/l 0.89, and for the next 15 it is NOK/l 0.52. The marginal subsidy is 74.8% less for cow number 24 to 38 than for cow number 1 to 15, meaning that larger farms can rely less on subsidy payments and are required to have lower variable production costs than small farms. The small farm is profitable with variable costs up to 147% of the milk price, while the large farm is profitable with variable costs up to 112% of the milk price.

For the small farm, when the milk quota was reduced to the industry average of 7 000 litres per cow, total margin decreased by NOK 62 249. The margin reduction equals NOK 2.15 per litre of milk production lost compared to the small farm with a high quota. The quota valuation of NOK/l 2.15 for the small farm is higher than annual rental prices and suggests that it's more profitable to rent additional quota than to keep suckler cows for a small farm.

Reducing the price of milk by 10%, 20% and 30% had no effect on the optimal variables for both the small and the medium sized farm. However, the large farm experienced changes in the optimal solution when the price of milk reduced by 30%. With a 30% reduction, the farm had 8 fewer dairy cows and 8 more suckler cows than the optimal solution under the base milk price.

7 Discussion and Extensions

In this chapter, we will discuss our findings from the analysis and what these results implies for the end-user. Further, several extensions of the scope of research is suggested, both as research questions but also to develop the proposed optimisation model to a fully integrated decision support system.

7.1 Validity of the Results

The solutions obtained in the analysis are only representative for the three specific farms we have investigated, and may differ for farms with other characteristics. The validity of the results depends greatly on the assumptions we have made when deciding the input parameters of the farms as well as the realised price achieved on output products. Although the three farms are representing small, medium and large farms in Norway, no farm is similar. The geographical location of a farm not only has implications for the subsidy rates, but also for the crop yield, prices of milk, beef and milk quota, as well as prices for fodder, grains and potatoes. Even if regional deficiency schemes are in place to make up for unfavourable production environments, the crop yield will depend on the soil, so that some locations might consequently experience lower yields than other, more favourable locations. The three farms analysed are located in one of the most favourable regions of the country, namely Jæren. Not only does this result in zero regional subsidies, but it is also why the farms have a fairly high crop yield.

Further, the shed and farmland capacity represents physical limitations to the farm activities. We have assumed the shed to have no stalls for sheep, as well as having an equal amount of stalls for cows, cattle and calves within each farm. In reality, the number and combination of stalls in the shed varies from farm to farm, and represents a great investment to change. When it comes to the farmland, we decided that every farm must be self-sufficient in regards of producing enough forage to feed their own livestock. Although the crop yield sets the premises of how many food units (FEm) a decare of farmland produces, the farmland can be a constraining factor if you have more animals than you are able to feed. Feeding is also an important factor when it comes to milk yield per cow.

Higher yielding cows require more food, and thus more land per cow. In our base case, the farmland and crop yield is more than large enough to feed the highest yielding cows, which in turn allows the farms to fill their milk quota. Nevertheless, the quota on the three farms is set to be attainable by construction, as we multiplied the number of dairy cow stalls with the theoretical yield of the highest producing cows. Still, we could end up in a situation where there is not enough land to feed as many high yielding cows as we could fit in the shed. With the current feeding plans provided to the model, this was possible nonetheless. Additionally, farmers often have their own feeding plans, which are likely to give different results if they were loaded into this optimisation model.

The last crucial parameter is the productive life of dairy cows. This impacts the flow of animals and their stages throughout the model, and other values would produce different solutions. This parameter is discussed in detail in section 7.4. Finally, we have made assumptions about the productive lives of calves and heifers as well as the birth rate of calves. As these are not programmed as input parameters, they will affect every farm similarly, and should thus not be responsible for changes between solutions.

Even though our model proposes pure dairy farms in all cases except for the scenario analysis with reduced milk quota for the small farm and when the milk price drops with 30% for the large farm, this does not mean suckler cows are always disregarded in favor of dairy cows. It would therefore be interesting to analyse the livestock structure of farms with similar characteristics as the three farms we have analysed, only located in other parts of the country with a crop yield corresponding to those locations. For the farmer it can also be useful to run the optimisation model with other values for stall places than he actually has to analyse the marginal income of an extra stall. This will constitute the willingness to invest in a larger shed, and may be highly relevant as Norwegian dairy farms are getting larger for every year. An increase in the shed capacity could involve solutions where extra suckler cows are favoured instead of extra dairy cows. Farmland and milk quota are important parameters when analysing if you should expand the shed capacity. The decares of farmland can restrict the model if the farmer has more stalls than animals he is able to feed with his current total crop yield and feeding plan. Hansen (2009) emphasises the need for analysing the utilisation of milk quota before deciding to make changes in the herd, and it may be that it is more profitable to increase the milk yield of the cows you have – or to buy or rent more quota – than it is to invest in a larger shed. Trade and valuation of milk quota is discussed further in section 7.3.

The results obtained from the analyses in the preceding chapter should be interpreted as valid only for the specific farms studied and under the assumption we made. The main purpose of this thesis was to develop a tool to assist dairy farmers' decision making and we achieved this by creating a farming optimisation model. The analyses in the preceding chapter illustrated how the optimisation model can be used by farmers to make decisions regarding the number of animals to have, the milk yield per cow, slaughtering age of bulls and suckler cows, and how to allocate farmland between different crops. The model also provides the farmers with insights into subsidy's contributions to total margin which can be useful if the farmer expects the subsidy scheme to change in the future. Lastly, as we saw from the analysis of a milk quota decrease, the farming optimisation model can be used to estimate the change in profitability resulting from a change in quota size which helps to estimate the price a farmer is willing to pay for quota, or the price a farmer is willing to sell the quota for.

7.2 Continuous vs Integer Variables

The optimisation model presented in this thesis is defined with continuous variables for the number of animals. The use of continuous variables is appropriate for quantities that are dividable. Animals cannot be divided and it might therefore seem inappropriate to use continuous variables for the number of animals. However, as stated in chapter 4, the variables for number of animals are defined as the annual average number of animals. Quantities that represent average values can be formulated as continuous variables, even when the original quantity (here number of animals) is indivisible. To define the number of animals as an annual average is in accordance with the industry standard NDHRS (2019). From a technical perspective, the use of continuous variables is fine. However, this optimisation model is meant to assist farmers in their decision making. For farmers, it might be difficult to interpret fractional numbers of animals. If the optimisation model suggests the farmer to keep 11.5 dairy cows, this can cause some confusion. Should the farmer keep 11 or 12 cows? Making the optimisation model intuitive for the farmers is an important priority in developing a farming decision support tool, a point also mentioned by Rose et al. (2016) and Lundström and Lindblom (2018).

In chapter 4, we discussed the use of continuous versus integer variables for the number of animals. In the model, the relationships between animals are defined in a way that results in fractionally for some of the animal classes. For example, the number of calves, number of heifers, and number of bulls will be equal to a fixed proportion of the number of dairy cows. Since the model is defined in a way that requires the use of fractional numbers, imposing a integer restriction for the number of animals would be inappropriate for our model. Using continuous variables, we have to make sure that the proper interpretation of the fractional numbers are given to the user of the farming decision model.

In order to develop a model that only returns integer numbers for the number of animals, it is necessary to expand the model into a dynamic model. A dynamic model takes into the effect of time. In our model, we are looking at the annual average number of animals. A dynamic model is able to state how many animals the farm should have at a specific point of time. Where a static model can state that the farm should have 11.5 dairy cows on annual average, a dynamic model can state that the farm should have 11 dairy cows during the first half of the year and 12 dairy cows in the second half of the year. For further research, it would be interesting to expand the farming optimisation model to also take time into account. It would be a helpful tool for Norwegian dairy farmers in planning their farm several years ahead.

7.3 Valuation of Milk Quota

In the current model formulation, the milk quota is taken as a constant parameter and the model is not optimising the size of the quota. However, there are markets for buying, selling, and renting milk quotas. This creates an opportunity for farmers to either increase their quota by purchasing or renting quotas or to reduce their quota by selling or renting out existing quotas.

We analysed two situations for a small farm; one situation with a quota below the milk production potential; and another situation with a quota above the milk production potential. We valued the difference between the quotas at NOK 2.15 per litre, above the rental prices on the market, thus suggesting that it's profitable to rent additional quota to obtain a higher milk production. By adding the possibility to rent quota to the model, decisions such as this one would be computed by the optimisation model. Ultimately, including a milk quota valuation in the model will create a more realistic picture of the decisions facing a dairy farm.

As argued in section 6.3.2, since milk quotas are the farm's assets, the holding cost of milk quotas can be interpreted as a cost of producing milk. Currently, this cost is not incorporated into the model. By including a milk quota valuation the cost of producing milk increases and the overall margin of dairy cows decreases. Thus, including milk quota valuation leads to an increase in the relative profitability of suckler cows. This change in relative profitability may affect the optimal solution the model obtains.

From the analysis, we found that the optimal livestock structure changed for the large dairy farm when the price of milk drops by 30%. Since a drop in milk prices decreases the profitability of milk production, the effects of such a drop can be compared to the effect of including milk quota valuations. Since the livestock structure changed with a drop in milk prices, we could also expect to observe something similar by including milk quota valuations. However, it all depends on the current price quotas are traded for. The traded volume in the milk quota market is fairly low which makes it difficult to model the correct quota price. At any given time, it's possible to find the price at which milk quotas are traded, but it's more difficult to understand how the milk quota prices are affected by changes in the dairy market, e.g. changes in milk prices. Hennessy et al. (2012) studied the functioning of the Irish milk quota market. They estimated the economic value of milk quotas using an optimisation model and compared the estimated economic value to the actual trading value of milk quotas. The authors found that milk quotas were undervalued in some regions, and overvalued in other regions. This study was done for the Irish milk quota market. We have not found any similar studies for the Norwegian milk quota market. The implication of these undervaluations and overvaluations of milk quotas is that we cannot easily obtain the correct milk quota price using an economic framework. Hence, if the Norwegian milk quota market cannot be modelled using economic framework, it would be difficult to estimate the valuation of milk quotas in different scenarios. This would ultimately make it difficult to incorporate milk quota valuations into a whole-farm decision support system. For further research in the dairy agricultural sector, it would be interesting to study the Norwegian milk quota market, and to develop an economic framework to estimate the valuation of milk quotas in different regions in Norway.

7.4 Variable Productive Life of Dairy Cows – Find Optimal Cull Rate

For the farms analysed in the preceding chapter, we set the productive life of dairy cows equal to 24 months, arguing that this is approximately equal to the average productive life of dairy cows in Norway. However, dairy cows can live a much longer life than four years. It would be interesting to investigate the effects that a prolonged productive life will have on the optimal solution.

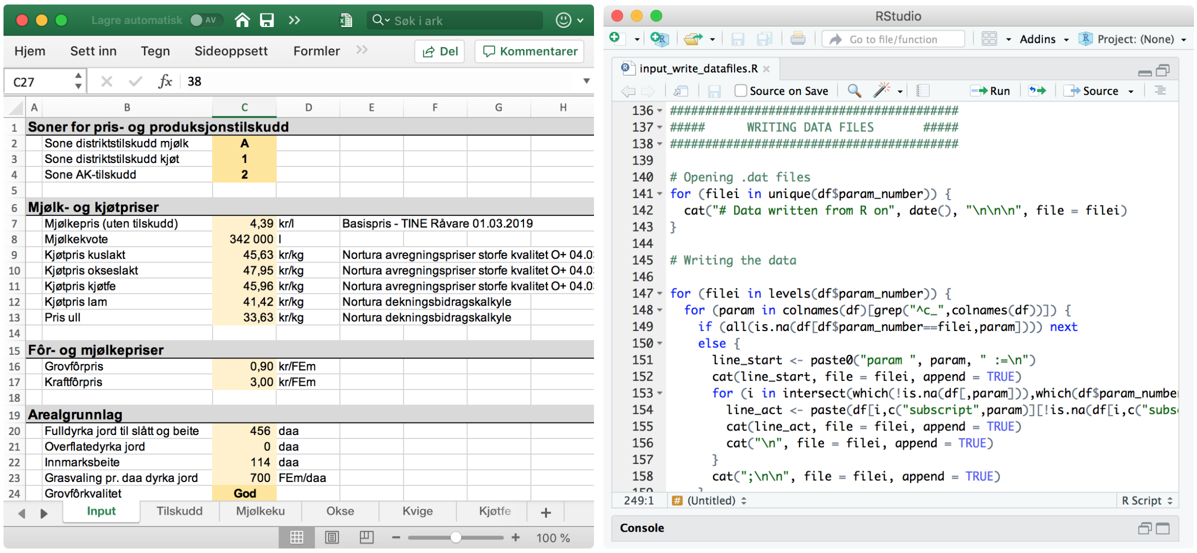
The number of dairy cows that a farm can have is limited by the cowshed capacity, meaning that the number of dairy cows in the optimal solution cannot change by increasing the productive life. However, by increasing the productive life, the number of dairy cows that have to be replaced each year is reduced. In turn, the so-called cull rate will be reduced. This means that the demand for heifers is reduced, and since heifers are costly to have, the overall margin will improve. Also, since the cull rate is lower, less heifer calves needs to be used as replacement cows. This creates an opportunity to sell the heifer calves (or heifers) that are not needed to replace the farm's own dairy cows. Isolated, this will increase the profitability of the farm.

It is necessary to also take into account the change in dairy cow's variable costs when the productive life is increased. Increased productive life increases the variable veterinary costs and might influence the milk yield as the dairy cow ages. In order to incorporate a variable productive life for dairy cows, it is necessary to correctly model how the costs and production output changes as the productive life increases. Being able to estimate the economic value of a dairy cow's productive life is very relevant for farm management as farmers can make more informed decisions about culling rate.

Finding the optimal cull rate for dairy cows is important to obtain the optimal dairy farm structure. Animals is a scarce resource on a farm, and being able to increase the slaughter age will create an opportunity to keep more cows for other meat production and not to replace milk producing dairy cows. Heikkilä et al. (2008) studied optimal replacement policy for cows of Ayrshire and Holstein-Friesian Finnish herds. The authors found that the optimal mean parity (number of calves) were 3.8 and 3.7 for the two breeds, respectively. These results were higher than the average mean parity of 2.3, suggesting that the cows were slaughtered earlier than optimal. We haven't found similar studies for the Norwegian Red breed – the dairy cow found on Norwegian farms. The average mean parity for Norwegian Red is 2.7 (NDHRS, 2019). If Norwegian Red cows are slaughtered too early, this is an ineffective use of dairy cows. As also stated by Sommerseth (2018), it would be interesting to study the optimal replacement policy for Norwegian Red. The optimal policy could be integrated into the optimisation model, which would provide farmers with highly relevant information.

7.5 Developing a Whole-Farm Decision Support System

In this thesis, it has been our objective to propose an optimisation model designed to assist farmers in making decisions related to milk and beef production. The whole model is in fact structured in two parts. The first part is an input system that requires the farmer to enter certain parameters that is specific for his farm. Such an input system is incorporated into farming decision systems developed by other authors (e.g. Veysset et al., 2005; Ashfield et al., 2014; Rupnik et al., 2019). The inputs from the farmers are used to calculate the parameters used in the optimisation model. The actual optimisation model is the second part of our model. It takes the parameters given by the farmer as input, before it maximises the gross margin of the farm. Finally, an output file is created that gives the farmer relevant information on how to optimally organise his farm.



(a) Input system in Excel (b) R-script to write data files

Figure 7.1: Input data handling. The Excel file on the left (a) is used by the user of the model to enter data about the farm. The R-script on the right (b) is used to write AMPL-style data files.

The Excel-file shown in Figure 7.1a generates a CSV-file with all the parameters necessary to run the optimisation model in AMPL. The experimental data described in chapter 5 were all entered into the Excel sheet. The data files that AMPL can read needs to be formatted in a specific way. This is done with R through the integrated development environment RStudio (R Core Team, 2018). A script in R is created to process the CSV-file from Excel. The data from this CSV-file is formatted in such a way that the file can be read by AMPL. Using Excel and R to create AMPL data files makes it easy to make changes to input parameters without needing to edit the AMPL data files directly. AMPL handles the optimisation procedure using the solver CPLEX. Then, AMPL writes the results of the optimisation model to a CSV-file that is further processed in R to create output files to summarise and visualise the key output values from the optimisation model. The analyses done in the preceding chapter were all done in R based on the CSV-files written by AMPL.

Using different software for different tasks is a first step towards a fully integrated system to assist dairy farming decision making. The procedure of data processing, optimisation and visualisation of the results is currently a bit tedious in our model. Thus, we cannot claim it is a fully integrated decision support system. Wolfert et al. (2017); Jones et al. (2017) and Higgins et al. (2019) provides inspiring reviews of recent smart farming DSSs with emerging technologies such as Cloud Computing, Internet of Things, Artificial Intelligence and Big Data. Fornés (2019) and Rupnik et al. (2019) have developed examples cloud-based systems embedding data collection with mathematical models to present real-time information in the cloud in an automated process. Although the proposed model in this thesis is not fully integrated in an automated process from back-end to front-end, we argue it can be perceived as a DSS prototype. More importantly, the model is still entirely capable of assisting farmers in their decision making. In the following are two proposals for turning our farm management tool into a decision support system.

A complete, interactive decision support system of the optimisation model can for example be made in R by utilising the Shiny (Chang et al., 2019) package together with the AMPL API (AMPL Optimization Inc., nd). Shiny is a framework for building interactive web applications with R, binding inputs and outputs together. It contains prebuilt widgets, with the possibility to customise your own, enabling to build responsive applications with a satisfactory user-interface. To start with, this application can be put online by using RStudio's hosting service (RStudio, nd), so that farmers can access it online. A long term goal can be to get a website domain to host the application on a standalone server. To integrate the Shiny application with the optimisation model built in this thesis, AMPL Optimization Inc. (nd) provides an API which gives access to AMPL models and run AMPL commands from external programs. The API is available in R through the rAMPL library (AMPL Optimization Inc., 2018). Figure 7.2 shows a possible architecture and a sequence of interactions in the proposed decision support system.

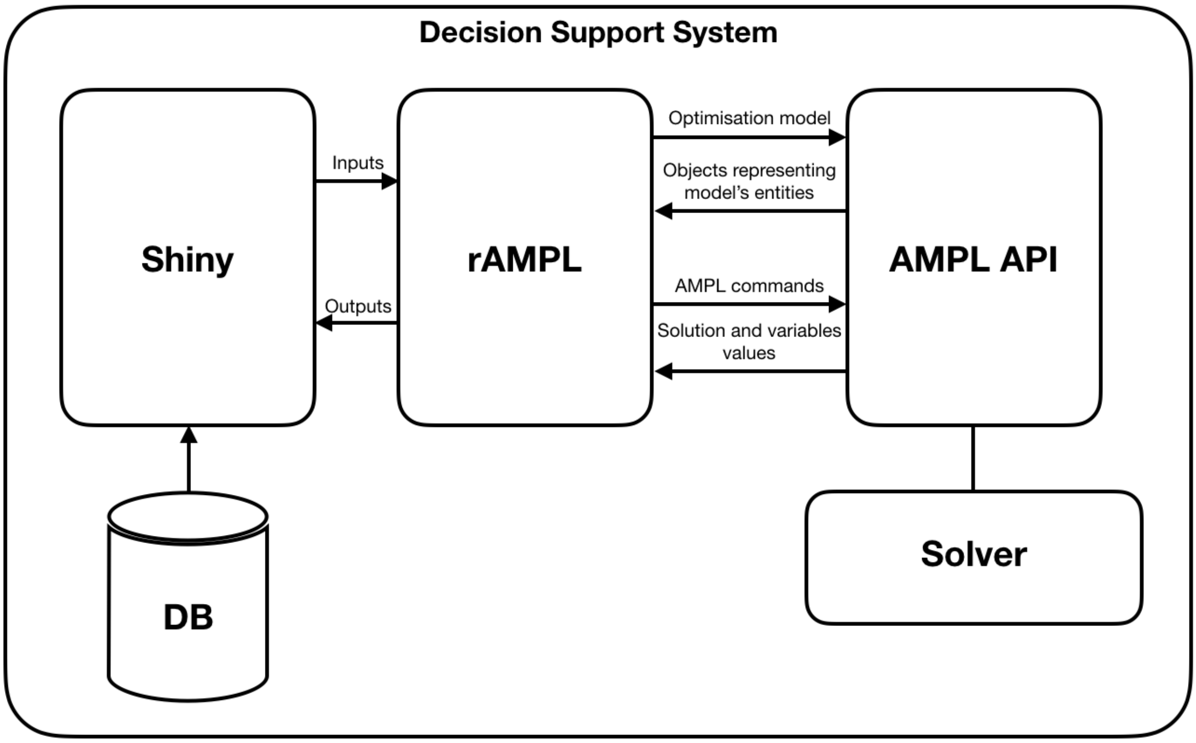


Figure 7.2: A decision support system. The proposed optimisation model is integrated with Shiny (Chang et al., 2019) and rAMPL (AMPL Optimization Inc., 2018) through AMPL API. This illustration is adapted from (AMPL Optimization Inc., nd)

In this proposed system, the farmer should go to the web application (Shiny) where he can upload his data according to the Excel-template presented earlier, or enter his farm characteristics as inputs (DB). Shiny communicates the inputs to the AMPL API via the functions of rAMPL – where the R-commands we have already developed for writing AMPL data is included – and runs the optimisation model. Instead of writing the results of the optimisation as CSV-files which then is further processed in R to create output files for visualisation, the results are directly retrieved from the AMPL API through rAMPL. The visualisation procedure is still conducted in R, but are now sent back to Shiny and visualised online in the web application, conveniently accessible to the farmer.

A disadvantage of using AMPL and the AMPL API in a DSS is that the user is required to have the AMPL executable and rAMPL library installed locally. This may be challenging, not only as it becomes more tedious for the farmers, but also because AMPL requires a valid license to be operated, which is costly. Most likely this license is needed when operated through the API too. This might call for changing to a different software for running the optimisation. Perhaps open source is a valid option if sufficient solvers exists.

However, an advantage of using AMPL API is not only its access to great solvers, but all model generation and solver interaction is handled directly by AMPL. Thus, the API library acts only as an intermediary, leading to great stability and speed. The extra memory and CPU usage depends mostly on how much data is read back from AMPL, making the size of the optimisation irrelevant (AMPL Optimization Inc., nd).

A second and more idealistic way forward is to facilitate for automated data collection on Norwegian farms, and put the model "out in the cloud", so the farmer can operate the model from anywhere. Today, new farm-machinery and infrastructure are equipped with sensors, enhancing the possibilities of collecting big on-site data and thus developing advanced farm-specific DSSs (Capalbo et al., 2017). Access to real time on-site data could represent a great improvement of the model. Jones et al. (2017) points out that the value of on-site data for improved farm management tools should motivate farmers to facilitate for accurate data collection. In Norway, it might be reasonable that TINE, as a farmer owned cooperative, could facilitate such data collection on the farms, and make the data available through an API. Then, from a developers perspective, extending the model to fetch farm-specific on-site data may only require to access this API. The model will then take use of the on-site data together with public data it already is provided with to find improved solutions for a specific farm, increasing its relevance for the farmer. Note that sharing of private data calls for appropriate use when it comes to confidentiality and security. Measures such as only the farmer who the data belongs to can access the data, or anonymising the data when shared outside of TINE can be taken. For research purpose, anonymous data works just fine as long as you know which data belongs to the same farm.

Making the model available in the cloud as it is now can be difficult due to the possible license requirement of AMPL. Anyhow, as long as the inputs and results of what would be a DSS is automatically presented in an appropriate user-interface once it has been supplied with input data, the farmers can access the system locally if not in the cloud if they are provided with the DSS as a software. Although this is more old fashioned these days, and would not be as convenient for the farmers, this could be made possible as for example an R script like the DSS of Rupnik et al. (2019). For a thorough description of cloud modelling, the reader is directed to Fornés (2019, p. 5-9).

8 Conclusion

The purpose of this thesis was to develop a farm management tool to assist Norwegian dairy farmers' decision making. To achieve this, we constructed a mixed integer linear programming model which maximises the total gross margin under the constraints of farming activities and subsidies.

We created three dairy farms using experimental data and the optimisation model was applied to the farms. The main conclusions that can be drawn from this thesis are:

- Given the assumptions and experimental data used in the model, we found that farms of small, medium, and large sizes located in Jæren, Norway are best off having pure dairy cow herds. The farms had an alternative to keep suckler cows, but dairy cows were found to be more profitable, irrespective of the farm's size. Furthermore, we found that total subsidy payments account for 38.9%, 33.8%, and 27.1% of total gross margin for a farm of small, medium, and large size, respectively. Also, the marginal subsidy amount per litre milk delivered was NOK 2.07 for the first 15 dairy cows, NOK 0.89 for the next eight dairy cows, and NOK 0.52 for the next 15 dairy cows.

- For the small farm studied here, we found that reducing the milk quota from 135 000 litres to 105 000 litres resulted in a varied farm structure, with both dairy cows and suckler cows. However, we also found that the lost income following the quota reduction was greater than the market prices of quotas, ultimately suggesting that the farm is better off renting quota to obtain maximal milk production.

- Lastly, we found that the optimal solutions obtained for the three farms studied here were little sensitive to decreases in the milk prices. For both the small and medium sized farms, the optimal solutions remained unchanged when the price of milk decreased by up to 30%. For the large farm, we found that the optimal livestock structure changes when milk prices drop by 30%. In the new optimal solution, the farm had eight suckler cows, compared to zero in the original solution.

The results obtained from the model runs using experimental data should only be interpreted for the specific farms studied and under the assumptions made in generating the input data. The main contribution of this thesis was the development an optimisation model to assist dairy farmers' decision making. We identified some weaknesses of the LP model that would be valuable to improve in further extensions of the model: the use of continuous variables, milk quota valuations, and variable cull rate. We used continuous variables to express the number of animals on the farm, arguing that the variables were defined as annual average numbers. Reformulating the mathematical model into a dynamic model would allow for the use of integer variables, which is an improvement from the current formulation as it allows the farmer to see exactly how many animals to have at any given time and it would clarify the natural dynamic relationship between cows and its offspring. Milk quotas are traded on open markets but the farming LP model did not open for this possibility. We argued that correctly modelling milk quota prices is a challenging task which is outside the scope of this thesis. However, incorporating milk quota valuations in the LP model would further improve the usefulness of the model and represent an interesting future extension. The cull rate of dairy cows was fixed at 50% representing a productive life of 24 months. The choice of cull rate is an economically motivated decision that farmers make and being able to incorporate this decision into an optimisation model framework could be valuable to farmers. Including a variable cull rate requires detailed modelling of the cost dynamics as cows become older and we were currently not able to correctly model this. Further research into the optimal culling rate of Norwegian Red dairy cows would be valuable for Norwegian farmers.

For this thesis, we created a two-stage system. First, an Excel file for input handling, and second, an AMPL file to handle the actual optimisation. We also used R-scripts to process both the input data and the output data. This is a first step in developing a fully integrated whole-farm decision support system. For further work, we proposed some solutions of making the decision support system available online using Shiny and AMPL API, or by utilising open-source software. Making a full DSS in a natural continuation of the work done in this thesis and would be a powerful tool for Norwegian dairy farmers.

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Appendix

A1 Optimisation Model

maximise

$$\sum_{o\in O^a} A_o^A x_o^a + \sum_{l\in L} A_l^l y_l + \sum_{i\in S^{dc}} A_i^{dc} x_i^{dc} + \sum_{i\in S^{sc}} A_i^{sc} x_i^{sc} + \sum_{i\in S^{oc}} A_i^{oc} x_i^{oc} + \sum_{i\in S^{sh}} A_i^{sh} x_i^{sh}$$
$$+A^{mi} x^{mi} + A^{be} x^{be} + \sum_{i\in O^{sm}} A_i^{sm} x_i^{sm} + (A^{cl} + A^{al}_{\text{roughage}}) \sum_{k\in O^{ma}} f_k^r x_k^r$$
$$+ \sum_{l\in L\setminus\text{roughage}} (A^{cl} + A_l^{al}) y_l$$

subject to

$$\sum_{o\in O^a} g_o x_o \leq Q$$

$$\sum_{o\in O^a} d^r_{o,p} x_o \leq d^a_p \quad \forall p \in P$$

$$\sum_{l\in L} y_l \leq L^c$$

$$\sum_{o\in O^a} c_o x_o^a - b(0.6L^p + y_{\text{roughage}}) \leq 0$$

$$\sum_{o\in O^{ti}_{\text{oc,h}}} \frac{12}{h_o} x_o^a - \sum_{o\in O^{ti}_{\text{dc,dc}}} \frac{12}{h_o} x_o^a = 0$$

$$\frac{12}{h_{\text{hc6mo}}} x^a_{\text{hc6mo}} - \sum_{o\in O^{ti}_{\text{oc,h}}} \frac{12}{h_o} x_o^a = 0$$

$$0.5r \sum_{o \in O^{ti}_{\mathrm{dc,dc}}} x^a_o - \sum_{o \in O^{ti}_{\mathrm{oc,hc}}} \frac{12}{h_o} x^a_o = 0$$

$$\frac{12}{h_{\mathrm{bc6mo}}} x^a_{\mathrm{bc6mo}} - \sum_{o \in O^{ti}_{\mathrm{oc,b}}} \frac{12}{h_o} x^a_o = 0$$

$$0.5r \sum_{o \in O^{ti}_{\mathrm{dc,dc}}} x^a_o - \sum_{o \in O^{ti}_{\mathrm{oc,bc}}} \frac{12}{h_o} x^a_o = 0$$

$$\sum_{i \in S^{dc}} A^{dc}_i x^{dc}_i + \sum_{i \in S^{sc}} A^{sc}_i x^{sc}_i + \sum_{i \in S^{oc}} A^{oc}_i x^{oc}_i + \sum_{i \in S^{sh}} A^{sh}_i x^{sh}_i + \sum_{i \in O^{sm}} A^{sm}_i x^{sm}_i \leq D$$

$$x^{dc}_{1-14} \leq 14$$

$$x^{dc}_{1-14} + x^{dc}_{15-30} \leq 30$$

$$x^{dc}_{1-14} + x^{dc}_{15-30} + x^{dc}_{31-50} \leq 50$$

$$\sum_{i \in S^{dc}} x^{dc}_i - \sum_{m \in O^{mi}_{\mathrm{dc}}} \sum_{o \in O^{ti}_{\mathrm{dc},m}} x^a_o \leq 0$$

$$x^{sc}_{1-50} \leq 50$$

$$\sum_{i \in S^{sc}} x^{sc}_i - \sum_{m \in O^{mi}_{\mathrm{sc}}} \sum_{o \in O^{ti}_{\mathrm{sc},m}} x^a_o \leq 0$$

$$\sum_{i \in S^{oc}} x_i^{oc} - \sum_{m \in O_{\text{oc}}^{mi}} \sum_{o \in O_{\text{oc},m}^{ti}} x_o^a \leq 0$$

$$x_{1-126}^{sh} \leq 126$$

$$\sum_{i \in S^{sh}} x_i^{sh} - \sum_{m \in O_{\text{sh}}^{mi}} \sum_{o \in O_{\text{sh},m}^{ti}} x_o^a \leq 0$$

$$x^{mi} \leq 5$$

$$x^{mi} - \sum_{m \in O_{\text{dc}}^{mi}} \sum_{o \in O_{\text{dc},m}^{ti}} x_o^a \leq 0$$

$$x^{be} \leq 40 z^{be}$$

$$x^{be} - \sum_{m \in O_{\text{sc}}^{mi}} \sum_{o \in O_{\text{sc},m}^{ti}} x_o^a \leq 0$$

$$z^{be} - \sum_{m \in O_{\text{sc}}^{mi}} \sum_{o \in O_{\text{sc},m}^{ti}} x_o^a + 5 - N(1 - z^{be}) \leq 0$$

$$x_{1-23}^{sm} \leq 23(1 - z^{sm})$$

$$x_{1-23}^{sm} - \sum_{m \in O_{\text{dc}}^{mi}} \sum_{o \in O_{\text{dc},m}^{ti}} x_o^a \leq 0$$

$$x_{24-51}^{sm} \leq 28(1 - z^{sm})$$

$$\sum_{i \in S^{sm}} x_i^{sm} - \sum_{m \in O^{mi}_{\text{dc}}} \sum_{o \in O^{ti}_{\text{dc},m}} x_o^a + N z^{sm} \geq 0$$

$$x_k^p - \sum_{m \in O^{mi}_k} \sum_{o \in O^{ti}_{k,m}} x_o^a \leq 0 \quad \forall k \in O^{ma}$$

$$\sum_{k \in O^{ma}} f_k^p x_k^p - L^p \leq 0$$

$$x_k^r - \sum_{m \in O^{mi}_k} \sum_{o \in O^{ti}_{k,m}} x_o^a \leq 0 \quad \forall k \in O^{ma}$$

$$\sum_{k \in O^{ma}} f_k^r x_k^r - 0.6 \sum_{k \in O^{ma}} f_k^p x_k^p - y_{\text{roughage}} \leq 0$$

A2 Input Parameters

Table A2.1: Input parameters used in the numeric runs of the optimisation model. Small farm/medium farm/large farm.

Region, price subsidies milk	A	
Region, price subsidies beef	1	
Region, production subsidies	2	
Milk price	4.39	NOK/l
Milk quota	135/207/342	thousand l
Meat price, cow	45.63	NOK/kg
Meat price, bull	47.95	NOK/kg
Meat price, suckler cow	45.96	NOK/kg
Meat price, lamb	41.42	NOK/kg
Wool price	33.63	NOK/kg
Price of forage	0.90	NOK/FEm
Price of concentrate	3.00	NOK/FEm
Cultivated land	180/276/456	daa

Pasture land	45/69/114	daa
Crop yield	700	FEm/daa
Stall places, adult cow	15/23/38	
Stall places, cattle > 6 mo.	15/23/38	
Stall places, calves < 6 mo.	15/23/38	
Stall places, sheep	0/0/0	
Birth rate cows	1	
Productive life dairy cows	24	mo
Forage/concentrate consumption, dairy cow 5 000 kg	4 295/533	FEm
Forage/concentrate consumption, dairy cow 5 500 kg	4 334/669	FEm
Forage/concentrate consumption, dairy cow 6 000 kg	4 399/755	FEm
Forage/concentrate consumption, dairy cow 6 500 kg	4 399/944	FEm
Forage/concentrate consumption, dairy cow 7 000 kg	4 391/1 140	FEm
Forage/concentrate consumption, dairy cow 7 500 kg	4 418/1 292	FEm
Forage/concentrate consumption, dairy cow 8 000 kg	4 423/1 490	FEm
Forage/concentrate consumption, dairy cow 8 500 kg	4 423/1 692	FEm
Forage/concentrate consumption, dairy cow 9 000 kg	4 428/1 898	FEm
Forage/concentrate consumption, dairy cow 9 500 kg	4 428/2 118	FEm
Forage/concentrate consumption, dairy cow 10 000 kg	4 419/2 349	FEm
Forage/concentrate consumption, bull calf 3 mo	1 001/137	FEm
Forage/concentrate consumption, bull calf 6 mo	1 001/137	FEm
Forage/concentrate consumption, bull 13 mo	1 874/1 058	FEm
Forage/concentrate consumption, bull 15 mo	2 329/827	FEm
Forage/concentrate consumption, bull 17 mo	2 707/713	FEm
Forage/concentrate consumption, bull 19 mo	3 019/698	FEm
Forage/concentrate consumption, heifer calf 3 mo	2 788/451	FEm
Forage/concentrate consumption, heifer calf 6 mo	2 788/451	FEm
Forage/concentrate consumption, heifer 24 mo	2 788/451	FEm
Forage/concentrate consumption, light suckler cow 14 mo	4 300/575	FEm
Forage/concentrate consumption, light suckler cow 17 mo	4 800/700	FEm
Forage/concentrate consumption, light suckler cow 20 mo	5 613/888	FEm
Forage/concentrate consumption, heavy suckler cow 14 mo	4 213/1 168	FEm

Forage/concentrate consumption, heavy suckler cow 17 mo	5 150/1 168	FEm
Forage/concentrate consumption, heavy suckler cow 20 mo	5 900/1 293	FEm
Forage/concentrate consumption, sheep	400/700	FEm

A3 Optimisation Output

Table A3.1: AMPL optimal solution

grossMargin	713110.42	small
v_animals[cow_5000]	0	small
v_animals[cow_5500]	0	small
v_animals[cow_6000]	0	small
v_animals[cow_6500]	0	small
v_animals[cow_7000]	0	small
v_animals[cow_7500]	0	small
v_animals[cow_8000]	0	small
v_animals[cow_8500]	0	small
v_animals[cow_9000]	0	small
v_animals[cow_9500]	0	small
v_animals[cow_10000]	15	small
v_animals[heifer_24mo]	11.25	small
v_animals[heifercalf_6mo]	3.75	small
v_animals[heifercalf_3mo]	0	small
v_animals[bull_13mo]	3.75	small
v_animals[bull_15mo]	0	small
v_animals[bull_17mo]	0	small
v_animals[bull_19mo]	0	small
v_animals[bullcalf_6mo]	3.21	small
v_animals[bullcalf_3mo]	0.27	small
v_animals[sheep]	0	small
v_animals[light_14mo]	0	small
v_animals[light_17mo]	0	small
v_animals[light_20mo]	0	small

v_animals[heavy_14mo]	0	small
v_animals[heavy_17mo]	0	small
v_animals[heavy_20mo]	0	small
v_acreageGrow[roughage]	142.46	small
v_acreageGrow[grain]	37.54	small
v_acreageGrow[potato]	0	small
v_prodLiveDairycow[dairycow1_14]	14	small
v_prodLiveDairycow[dairycow15_30]	1	small
v_prodLiveDairycow[dairycow31_50]	0	small
v_prodLiveDairycow[dairycow51_Inf]	0	small
v_prodLiveSucklercow[sucklercow1_50]	0	small
v_prodLiveSucklercow[sucklercow51_Inf]	0	small
v_prodLiveOthercattle[othercattle1_Inf]	22.23	small
v_prodLiveSheep[sheep1_126]	0	small
v_prodLiveSheep[sheep127_Inf]	0	small
v_prodMilk	5	small
v_prodBeef	0	small
v_prodSMEDairycow[dairycow1_23]	15	small
v_prodSMEDairycow[dairycow24_51]	0	small
v_prodAKPasture[dairycow]	6.43	small
v_prodAKPasture[othercattle]	0	small
v_prodAKPasture[sheep]	0	small
v_prodAKPasture[sucklercow]	0	small
v_prodAKForage[dairycow]	12.1	small
v_prodAKForage[othercattle]	0	small
v_prodAKForage[sheep]	0	small
v_prodAKForage[sucklercow]	0	small
grossMargin	1009417.85	medium
v_animals[cow_5000]	0	medium
v_animals[cow_5500]	0	medium
v_animals[cow_6000]	0	medium
v_animals[cow_6500]	0	medium

v_animals[cow_7000]	0	medium
v_animals[cow_7500]	0	medium
v_animals[cow_8000]	0	medium
v_animals[cow_8500]	0	medium
v_animals[cow_9000]	0	medium
v_animals[cow_9500]	0	medium
v_animals[cow_10000]	23	medium
v_animals[heifer_24mo]	17.25	medium
v_animals[heifercalf_6mo]	5.75	medium
v_animals[heifercalf_3mo]	0	medium
v_animals[bull_13mo]	5.75	medium
v_animals[bull_15mo]	0	medium
v_animals[bull_17mo]	0	medium
v_animals[bull_19mo]	0	medium
v_animals[bullcalf_6mo]	4.93	medium
v_animals[bullcalf_3mo]	0.41	medium
v_animals[sheep]	0	medium
v_animals[light_14mo]	0	medium
v_animals[light_17mo]	0	medium
v_animals[light_20mo]	0	medium
v_animals[heavy_14mo]	0	medium
v_animals[heavy_17mo]	0	medium
v_animals[heavy_20mo]	0	medium
v_acreageGrow[roughage]	218.44	medium
v_acreageGrow[grain]	57.56	medium
v_acreageGrow[potato]	0	medium
v_prodLiveDairycow[dairycow1_14]	14	medium
v_prodLiveDairycow[dairycow15_30]	9	medium
v_prodLiveDairycow[dairycow31_50]	0	medium
v_prodLiveDairycow[dairycow51_Inf]	0	medium
v_prodLiveSucklercow[sucklercow1_50]	0	medium
v_prodLiveSucklercow[sucklercow51_Inf]	0	medium

v_prodLiveOthercattle[othercattle1_Inf]	34.09	medium
v_prodLiveSheep[sheep1_126]	0	medium
v_prodLiveSheep[sheep127_Inf]	0	medium
v_prodMilk	5	medium
v_prodBeef	0	medium
v_prodSMEDairycow[dairycow1_23]	23	medium
v_prodSMEDairycow[dairycow24_51]	0	medium
v_prodAKPasture[dairycow]	9.86	medium
v_prodAKPasture[othercattle]	0	medium
v_prodAKPasture[sheep]	0	medium
v_prodAKPasture[sucklercow]	0	medium
v_prodAKForage[dairycow]	18.56	medium
v_prodAKForage[othercattle]	0	medium
v_prodAKForage[sheep]	0	medium
v_prodAKForage[sucklercow]	0	medium
grossMargin	1515048.27	large
v_animals[cow_5000]	0	large
v_animals[cow_5500]	0	large
v_animals[cow_6000]	0	large
v_animals[cow_6500]	0	large
v_animals[cow_7000]	0	large
v_animals[cow_7500]	0	large
v_animals[cow_8000]	0	large
v_animals[cow_8500]	0	large
v_animals[cow_9000]	0	large
v_animals[cow_9500]	0	large
v_animals[cow_10000]	38	large
v_animals[heifer_24mo]	28.5	large
v_animals[heifercalf_6mo]	9.5	large
v_animals[heifercalf_3mo]	0	large
v_animals[bull_13mo]	9.5	large
v_animals[bull_15mo]	0	large

v_animals[bull_17mo]	0	large
v_animals[bull_19mo]	0	large
v_animals[bullcalf_6mo]	8.14	large
v_animals[bullcalf_3mo]	0.68	large
v_animals[sheep]	0	large
v_animals[light_14mo]	0	large
v_animals[light_17mo]	0	large
v_animals[light_20mo]	0	large
v_animals[heavy_14mo]	0	large
v_animals[heavy_17mo]	0	large
v_animals[heavy_20mo]	0	large
v_acreageGrow[roughage]	360.91	large
v_acreageGrow[grain]	95.09	large
v_acreageGrow[potato]	0	large
v_prodLiveDairycow[dairycow1_14]	14	large
v_prodLiveDairycow[dairycow15_30]	16	large
v_prodLiveDairycow[dairycow31_50]	8	large
v_prodLiveDairycow[dairycow51_Inf]	0	large
v_prodLiveSucklercow[sucklercow1_50]	0	large
v_prodLiveSucklercow[sucklercow51_Inf]	0	large
v_prodLiveOthercattle[othercattle1_Inf]	56.32	large
v_prodLiveSheep[sheep1_126]	0	large
v_prodLiveSheep[sheep127_Inf]	0	large
v_prodMilk	5	large
v_prodBeef	0	large
v_prodSMEDairycow[dairycow1_23]	23	large
v_prodSMEDairycow[dairycow24_51]	15	large
v_prodAKPasture[dairycow]	16.29	large
v_prodAKPasture[othercattle]	0	large
v_prodAKPasture[sheep]	0	large
v_prodAKPasture[sucklercow]	0	large
v_prodAKForage[dairycow]	30.66	large

v_prodAKForage[othercattle]	0	large
v_prodAKForage[sheep]	0	large
v_prodAKForage[sucklercow]	0	large
grossMargin	650861.6	small_lowquota
v_animals[cow_5000]	0	small_lowquota
v_animals[cow_5500]	0	small_lowquota
v_animals[cow_6000]	0	small_lowquota
v_animals[cow_6500]	0	small_lowquota
v_animals[cow_7000]	0	small_lowquota
v_animals[cow_7500]	0	small_lowquota
v_animals[cow_8000]	0	small_lowquota
v_animals[cow_8500]	0	small_lowquota
v_animals[cow_9000]	0	small_lowquota
v_animals[cow_9500]	0	small_lowquota
v_animals[cow_10000]	11.76	small_lowquota
v_animals[heifer_24mo]	8.82	small_lowquota
v_animals[heifercalf_6mo]	2.94	small_lowquota
v_animals[heifercalf_3mo]	0	small_lowquota
v_animals[bull_13mo]	3.26	small_lowquota
v_animals[bull_15mo]	0	small_lowquota
v_animals[bull_17mo]	0	small_lowquota
v_animals[bull_19mo]	0	small_lowquota
v_animals[bullcalf_6mo]	2.8	small_lowquota
v_animals[bullcalf_3mo]	0.07	small_lowquota
v_animals[sheep]	0	small_lowquota
v_animals[light_14mo]	0	small_lowquota
v_animals[light_17mo]	0	small_lowquota
v_animals[light_20mo]	0	small_lowquota
v_animals[heavy_14mo]	3.24	small_lowquota
v_animals[heavy_17mo]	0	small_lowquota
v_animals[heavy_20mo]	0	small_lowquota
v_acreageGrow[roughage]	126.4	small_lowquota

v_acreageGrow[grain]	53.6	small_lowquota
v_acreageGrow[potato]	0	small_lowquota
v_prodLiveDairycow[dairycow1_14]	11.76	small_lowquota
v_prodLiveDairycow[dairycow15_30]	0	small_lowquota
v_prodLiveDairycow[dairycow31_50]	0	small_lowquota
v_prodLiveDairycow[dairycow51_Inf]	0	small_lowquota
v_prodLiveSucklercow[sucklercow1_50]	3.24	small_lowquota
v_prodLiveSucklercow[sucklercow51_Inf]	0	small_lowquota
v_prodLiveOthercattle[othercattle1_Inf]	17.89	small_lowquota
v_prodLiveSheep[sheep1_126]	0	small_lowquota
v_prodLiveSheep[sheep127_Inf]	0	small_lowquota
v_prodMilk	5	small_lowquota
v_prodBeef	0	small_lowquota
v_prodSMEDairycow[dairycow1_23]	11.76	small_lowquota
v_prodSMEDairycow[dairycow24_51]	0	small_lowquota
v_prodAKPasture[dairycow]	6.43	small_lowquota
v_prodAKPasture[othercattle]	0	small_lowquota
v_prodAKPasture[sheep]	0	small_lowquota
v_prodAKPasture[sucklercow]	0	small_lowquota
v_prodAKForage[dairycow]	10.96	small_lowquota
v_prodAKForage[othercattle]	0	small_lowquota
v_prodAKForage[sheep]	0	small_lowquota
v_prodAKForage[sucklercow]	0	small_lowquota