Essays on fishery management

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General introduction

Fishing activities have accompanied human civilizations for thousands of years, supporting some of our ancestors through a hard life. As the technology advances, we have obtained more and more tools to detect what is out there in the mysterious ocean. Now we are able to explore species, collect data, measure samples and estimate quantities for various stocks. However, at the same time, the efficiency and capacity of removing large amounts of biomass out of the sea is rocketing. Without a long-term vision and proper decision-making mechanisms, it is easy to act myopically and overuse the power of modern technology. Today, it is a globally recognized issue that many fish populations, which were productive and sound before, are becoming overly exploited and poorer.

To pursue better management and more sustainable resources, an important instrument in the hands of researchers is modelling. Population dynamics of fish is one of the oldest realms in quantitative population ecology (Shertzer, Williams, Prager, & Vaughan, 2014). Such mathematical models focus on the biological processes such as growth, recruitment, natural mortality and fishing mortality. As the literature evolves, many studies address more of the other aspects, for example climate uncertainty, predation relationship, ecosystem analysis, market interaction and other economic elements.

All three chapters in the thesis share the common research field of fishery modelling and management. The goal is to propose modelling approaches, innovative recipes and management guidelines that potentially improve our understanding of the topic. In addition, all chapters employ bioeconomic models that incorporate both biological and economic elements. Numerical examples are applied in every chapter to demonstrate the model and to offer relevant discussions for real populations.

The readers will find a wide spectrum of methods and techniques through the chapters, such as non-linear programming, dynamic optimization in discrete or continuous time and simulation. When the research question focuses on the first-best policy, optimization analysis is weighted more. When the study is mainly about effects of specified policies, simulation results are more emphasized.

The structure of the models develops progressively in the thesis. In Chapter 1, to avoid possible oversimplification of a biomass model, we build an age-structured single species model. The time increment is the same as a typical yearly model. In Chapter 2, we still study a single species,

but instead of the numerous age classes we categorize the stock into two groups of immature and mature fish. The time frame here is more complicated and unconventional: seasonal and uneven time increment are introduced in discrete dynamic optimization. In Chapter 3, instead of single species models, we work with a predator-prey relationship in an ecosystem. Both optimization and simulation are conducted in a continuous time framework.

Any fishery model is inevitably an abstract description of the world and contains multiple assumptions and distortions of reality. The essence of modelling is the balance between where to simplify and where to elaborate. For three chapters that have rather diversified biological structure, time increment and mathematical methods, a unified goal is to present a matching approach for each problem. For example, to look into seasonality in a year, Chapter 2 employs a two-dimensional size-structured model that not only simplifies the age classes but also systematically captures the season-dependent behaviour of each group.

Stochasticity is an intrinsic feature of real-world fisheries, caused by mechanisms that are often tangled. While modelling the bioeconomic processes, it is a common practice to build a stochastic population growth. In Chapter 1, randomness from the environment is linked to both individual weight gain and recruitment of the cohorts, allowing us to study them separately. In Chapter 2, the core of the study is the innovative approach of the seasonal optimization model itself, therefore, stochasticity is not modelled. In Chapter 3, environmental stochasticity is summed up into one volatility term in the dynamic equation. Stochasticity may not be the centre of the research question in every chapter but is indeed a crucial aspect for reasonable modelling.

In Chapter 1, the effects of recruitment patterns and environmental impacts on the optimal exploitation of a fish population are investigated. Based on a discrete-time age-structured bioeconomic model of Northeast Atlantic mackerel, we introduce the mechanisms that generate 6 scenarios of the problem. Using the simplest scenario, optimizations are conducted under 8 different parameter combinations. Then, the problem is solved for each scenario and simulations are conducted with constant fishing mortalities. We find that any parameter combination that favours the older fish tends to lend itself to pulse fishing pattern. The simulations indicate that a constant fishing mortality around 0.06 performs the best. A comparison between the optimal and the historical harvest shows that for most of the time, the optimal exploitation precedes the historical one, leading to much higher net profit and lower fishing cost.

Chapter 2 is an attempt to treat seasonality in a systematic way while most of the extant literature tends to neglect periodicity in fishery modelling. We present a multi-season multi-state bioeconomic model and apply a periodic Bellman approach using dynamic programming to obtain the optimal feedback policy of each season. Our numerical illustration demonstrates that a seasonal dynamic optimization model allows for naturally occurring seasonal moratorium or potentially a Marine Protected Area (MPA). It shows that there exist optimal dynamic paths that develop into a permanent equilibrium cycle, which consists of one harvesting season followed by a moratorium period. This indicates an optimal closure of the fishery that a yearly model would overlook. Fishing pressure on the mature stock elicits even heavier harvesting in the next season on the same group. A protective moratorium of the immature could hinder the value of the whole stock.

Chapter 3 demonstrates a predator-prey system of cod and capelin that confronts a possible scenario of prey extinction under the first-best policy in a stochastic world. We discover a novel 'super-harvest' phenomenon that the optimal harvest of the predator is even higher than the myopic policy on part of the state space. Following the idea of harvesting the predator to protect the prey, we ban prey harvest and increase predator catch in a designated state space area based on the optimal policy. When we scale up the predator harvest by 1.5, the prey recovery rate escalates for as much as 28% at a cost of 5% value loss. We establish two strategies: modest deviation from the optimal on a large area or intense measure on a small area. It seems more cost-effective to target the stock space with accuracy than to simply boost predator harvest when the aim is to achieve remarkable improvement of prey recovery probability.

Shertzer, K. W., Williams, E. H., Prager, M. H., & Vaughan, D. S. (2014). Fishery Models ☆.
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Chapter 1

1. Optimization of age-structured bioeconomic model: recruitment, weight gain and environmental effects

Yuanming Ni [a]

Abstract

More and more fishery researchers begin to acknowledge that one-dimensional biomass models may omit key information when generating management guidelines. For the more complicated age-structured models, numerous parameters require a proper estimation or a reasonable assumption.

In this paper, the effects of recruitment patterns and environmental impacts on the optimal exploitation of a fish population are investigated. Based on a discrete-time age-structured bioeconomic model of Northeast Atlantic mackerel, we introduce the mechanisms that generate 6 scenarios of the problem. Using the simplest scenario, optimizations are conducted under 8 different parameter combinations. Then, the problem is solved for each scenario and simulations are conducted with constant fishing mortalities.

It is found that a higher environmental volatility leads to more net profits but with a lower probability of achieving the mean values. Any parameter combination that favours the older fish tends to lend itself to pulse fishing pattern. The simulations indicate that a constant fishing mortality around 0.06 performs the best. A comparison between the optimal and the historical harvest shows that for more than 70% of the time, the optimal exploitation precedes the historical one, leading to 43% higher net profit and 34% lower fishing cost.

Keywords

Age-structured, bioeconomic, recruitment, optimization

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1.1 Introduction

Many economic fishery studies have described the state of a population using biomass as the only variable (Schaefer, 1954). Such surplus production models use lumped parameters to describe the stock dynamics. In recent years, fishery biologists and economists widely begin to recognize that such one-dimensional models are too simple for developing realistic management guidelines (Tahvonen, 2008). One concern is about the dangerous tendency to catch small and immature fish (Pauly, Christensen, Dalsgaard, Froese, & Torres, 1998). Another is that ever increasing fishing pressure may cause various systematic changes in the internal structure and evolution of fish populations (Anderson et al., 2008), which may have crucial economic consequences that may not be captured by the biomass approach.

The age-structured framework is pioneered by Baranov (1945), Beverton & Holt (1957), and Leslie (1945) among others. Many extensions have been explored since and have dominated fishery management (Hilborn & Walters, 1991). However, analysing the problem technically is still challenging. Clark (2010) has pointed out that an analytical solution for the general age-structured problem is unattainable. Many models lend themselves to Mathematical Programming (Operational Research) and simulations, but not much to analytical studies. Still, it is possible to formulate a proper model and simulate how different factors influence the optimal fishing mortalities numerically. Our model is inspired by, among others, Walters (1969), Hannesson (1975), Getz & Haight (1989), Horwood (1987) and Tahvonen (2009).

NEA (Northeast Atlantic) Mackerel (*Scomber scombrus*) is a fish species with high commercial values. In the 1960s and 1970s, the annual catches of mackerel in the Northeast Atlantic, mainly North Sea, rose steeply, resulting in an extreme drop in the 1980s. Consequently, the mackerel stock has been at low level for many years with poor recruitment. ICES (International Council for the Exploration of the Sea) advices have been proposing limits on the fishing mortality or the size of the catch to improve the situation of the unsustainable harvest. But still in 2014, as in all years since 2008, unilateral quotas have been set higher than the TAC indicated by the ICES Management Plan (ICES, 2014). It is thus of high interest to investigate the management of NEA mackerel.

Among many environmental factors, food availability can be crucial and representative for all species. As typical plankton feeders, mackerel is affected by the abundance, distribution and composition of zooplankton to a large extent (Reid, Walsh, & Turrell, 2001). Most zooplankton species have a life span shorter than one year, therefore no age structure is applied for the

zooplankton population. The characteristics such as density, average size and distribution of zooplankton that interact with the mackerel stock are many. To simplify the problem reasonably, a single zooplankton index is used to describe prey density or the food availability for mackerel. Note that due to the position of zooplankton as a primary producer in the low trophic level, several assumptions are implied, which may or may not fall into the category of standard predator-prey models (Yodzis, 1994). First, the consumption of zooplankton by mackerel this year will not influence the prey density next year. Due to the many predators that zooplankton has simultaneously, the sole impact from mackerel is difficult to quantify. Second, the food supply of mackerel depends entirely on zooplankton abundance. Other prey species are not included.

Random variations in the environment affect the dynamics of populations through changes in individual life histories (Benton, Lapsley, & Beckerman, 2002). In this paper, two interactions between the environmental factor (zooplankton index) and the fish population (mackerel) are considered. The first interaction is the influence by the zooplankton abundance on the mackerel recruitment. Mackerel spawns between May and July, which coincides with the zooplankton boom (Lockwood, 1988). It is known that higher food availability can bring down the natural mortality especially of the small fish larva by allowing them to spend more time in the deeper, darker and safer area of the sea. Since this interaction mainly applies for the first-year juveniles, it can be translated to a constant natural mortality plus a varying recruitment influenced by the zooplankton abundance. The argument for the second interaction is the strong and positive connection between weight and price. On the final product market, a mackerel can be called 'large size' if the average weight is 400 to 600 grams (4-6 category); 'small size' if the average weight is 200 to 400 grams (2-4 category). The price differences between categories can be significant. Assuming one price for all age classes is not realistic.

There have been different approaches addressing the two interactions: recruitment and weight. To deal with recruitment, some chose an exogenous and constant recruitment such as (Beverton & Holt, 1957) and (Clark, 2010). While some assumed that recruitment can be endogenous and stock-dependent, such as (Walters, 1969) and (Getz & Haight, 1989). In this paper, both approaches will be applied. To deal with weight, some studies such as (Walters, 1969) referred to the von Bertalanffy function as the rule that determines the individual fish weight. This study links mackerel's annual weight gain to the exogenous zooplankton index, which can be either deterministic or stochastic under different scenarios.

Due to its complexity, the age-structured model can be sensitive to many parameters (Horwood, 1987). It is thus necessary to be cautious when it comes to combinations of parameters. In the literature, one issue has been connected to the parameter sensitivity: the choice between smooth, stable harvesting over time and periodic (oscillatory or pulse) fishing (Tahvonen, 2010). Clark (2010) argued that when there exists perfect selectivity, it is optimal to harvest each cohort at its maximum biomass, creating a stable harvest strategy. Hannesson (1975) pointed out that non-selective gear leads to pulse fishing. He also showed that discounting shortens the intervals between fishing periods. Tahvonen (2009) proves that when there are 2 age classes with endogenous recruitment, optimal harvest is pulse fishing under specific conditions such as non-selective gear. Steinshamn (2011) showed that pulse fishing becomes less attractive as the distribution of the species moves from uniform to schooling. Rocha, Gutiérrez, & Antelo (2012) concluded that imperfect selectivity increases the optimal lifespan and the optimal pulse length.

The approaches employed in this paper are innovative in several aspects. First, while many predator-prey models have been focusing on higher trophic levels (Major, 1978; Von Westernhagen, Westernhagen, & Rosenthal, 1976), this paper investigates the lower trophic level including the primary production. Second, a series of 8 parameter combinations has been examined before introducing the random environmental interactions, which offers some thought-provoking results. Third, based on a common framework of the model, 6 different scenarios have been applied in order to fully investigate the problem. Last but not least, based on the same population dynamics, both simulation and optimization are conducted, offering deeper understanding of the model.

1.2 Model

1.2.1 Model formulation

The population dynamics is:

$$x_{i+1,t+1} = x_{i,t}e^{-(m_i+s_i f_t)} \ (i = 0,1,\dots,n-1; t = 1,2,\dots,T-1) \qquad (1.1)$$

where $x_{i,t}$ is the number of fish individuals of age i at time t measured in millions; $x_{i,1}$ is given by historical data as the initial status of the stock; m_i is the natural mortality of age i; s_i is the selectivity parameter of age i and f_t indicates the fishing mortality at time t and is also the decision variable of the model. The dynamics indicates that every year part of the cohort dies out of natural causes and another part is being harvested. Both events are assumed to happen instantaneously. The rest of the cohort survives the year and continue to grow and reproduce.

The maximum age of the fish in the model is denoted by n and T indicates the end period. It is assumed that all fish with age above n will naturally die.

In order to describe the recruitment, the Spawning Stock Biomass (SSB) is calculated as following:

$$S_t = \sum_i u_i x_{i,t} w_{i,t} \qquad (1.2)$$

where S_t is the SSB at time t measured in million tonnes; u_i is the maturity ogive (proportion of sexually matured individuals in that age class) and $w_{i,t}$ is the individual weight of the average fish of age i at time t.

The endogenous recruitment can be generalized as a recruitment function: $x_{0,t+1} = \varphi(S_t)$.

Harvest can be obtained from the well-known Beverton-Holt model:

$$H_t = \sum_i h_{i,t} = \sum_i \frac{s_i f_t}{s_i f_t + m_i} [1 - e^{-(m_i + s_i f_t)}] w_{i,t} x_{i,t} \qquad (1.3)$$

where H_t is the harvest biomass measured in million tonnes at time t and $h_{i,t}$ is the harvest biomass measured in million tonnes of age i at time t.

The problem's objective is to maximize the following:

$$Z = \max_{f_t} \sum_{t=1}^{T} \sum_{i=0}^{n} (1 + r)^{-t} (p_{i,t} h_{i,t} - c f_t) \qquad (1.4)$$

where Z is the present value of the total net profit, r is the discrete discount rate, $p_{i,t}$ indicates the unit weight price for age i at time t and c is the calibrated cost parameter.

The objective function is subject to the following constraints:

1. Population dynamics: Eq. (1.1)
2. Sustainability constraint: SSB does not fall below a proposed biomass reference point in the end: $S_T \geq B_{lim}$
3. Non-negativity for variables $x_{i,t}$, $w_{i,t}$ and f_t
4. Effort restriction (admissible controls defined by harvest capacity): $f_t \in F$

The problem is solved in GAMS as a nonlinear programming problem using solver NLP.

According to Steinshamn (2011), the stock elasticity parameters for different fish species vary, resulting in various population dynamics and production functions. The model is concise and easy to analyse in the extreme cases where stock elasticity equals either zero or one. Zero stock elasticity lend itself to pure schooling fishery where production function is independent of the stock. Mackerel, which has a certain schooling behaviour, has a stock elasticity between zero and one. A larger stock level, even for schooling species, naturally leads to higher probability of finding the fish schools given the same level of searching effort. Thus, we believe the production function is not completely stock independent. In addition, another study of a schooling species Norwegian Spring-Spawning Herring uses a similar model (Bjørndal, Gordon, Lindroos, & Kaitala, 2000). Therefore, a stock elasticity of one is applied in the model.

Optimizing with respect to fishing mortality is equivalent to finding the optimal effort if we follow $f_t = qE_t$, where q is the catchability parameter. The cost parameter c in the model can be reckoned as the unit cost of effort multiplied by the catchability parameter. Both paradigms are interchangeable common practices. This paper focuses on optimizing the fishing mortality.

1.2.2 Two interactions

We introduce the zooplankton index ρ_t which is assumed to take the form of a Bounded Random Walk (BRW) (Nicolau, 2002; Vandromme et al., 2011). This specified process[1] has a mean reverting property around 1 but behaves like a random walk in the range of [0.6,1.4]. Another constraint of $\rho_t \in [0.5,1.5]$ is forced in order to avoid extreme outliers.

The first interaction of the zooplankton influencing mackerel's average weight gain goes as follows:

$$w_{i+1,t+1} = w_{i,t} + [0.036(\rho_t - 1) + 0.055] \qquad w_{0,t} \approx 0 \qquad (1.5)$$

Usually growth rates are difficult to determine from catches because schools are sorted by size and their mobility prevents representative sampling (Skagen, 1989). A small difference between weight of catch and weight of stock exists in the data. It is ignored in the model for simplicity. It is assumed that the weight for the first age class is virtually zero. As the same cohort

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[1] $\rho_{t+1} = 0.01[100\rho_t + e^{-120}(e^{-3(100\rho_t - 100)} - e^{3(100\rho_t - 100)}) + \sigma_\rho \varepsilon_t]$ where ε_t is a sequence of independent and identically distributed random variables with $E[\varepsilon_t] = 0$ and $Var[\varepsilon_t] = 1$ and the volatility $\sigma_\rho = 2$.

accumulates its weight over time, the zooplankton index ρ_t decides how much weight is gained each year for all cohorts.

Fig. 1.1 displays the historical average individual weight of the fish. When it is assumed that $\rho_t = 1$ for all t, the weight gain is constant every year, creating a linear weight development pattern that can be written as:

$$w_{i+1,t+1} = w_{i,t} + 0.055 \tag{1.6}$$
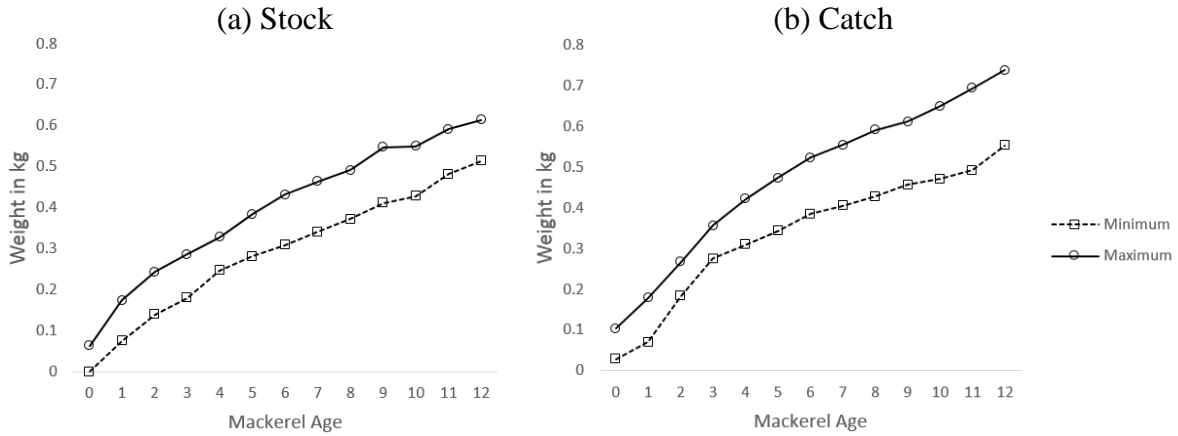
Fig. 1.1 Minimum and maximum weight of mackerel by age class from year 1980 to 2014

The possible maximum and minimum individual weights respectively are 0.83 kg and 0.49 kg at the age of 12 in the model. There exist cases where maximum weight is reported to be 3.5 kg[2]. Such extreme values will not be considered in the model.

The second interaction is about zooplankton affecting mackerel's recruitment. Under different scenarios, recruitment can be: first, exogenously given and fixed as 4500 million, which is the historical mean recruitment from 1980 to 2014; second, governed by a recruitment function; third, exogenous and random from a normal distribution $N(4500,2000)$ based on historical data (ICES, 2014). For the first and third case, the behaviour of zooplankton index does not alter the recruitment. For the second case, we choose the Ricker formula (Ricker, 1954) as an estimation of the recruitment function and include the zooplankton index in a multiplicative form:

$$\varphi(S_t, \rho_t) = \rho_t \alpha S_t e^{-\beta S_t} \tag{1.7}$$

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[2] http://www.imr.no/temasider/fisk/makrell/makrell/en

where $\alpha = 6.37$ and $\beta = 0.52$ for NEA mackerel (Simmonds, Campbell, Skagen, Roel, & Kelly, 2011).
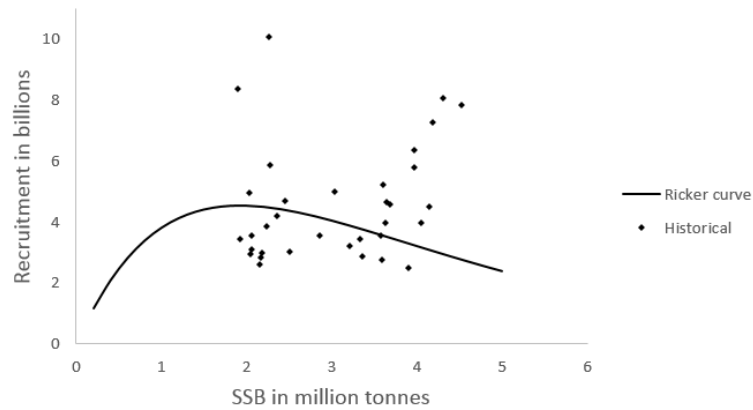
Fig. 1.2 Ricker recruitment function and the historical recruitment data

We see from Fig. 1.2 that the historical recruitment data (square dots) exhibits rather high volatility and spreads widely around the estimated curve. The recruitment curve seems to have limited explanatory power about the relationship between the SSB and the recruitment next year.

1.2.3 Scenarios illustration

The zooplankton index can either be deterministic and fixed as 1 (denoted as D) or stochastic as a BRW process (denoted as S). Recruitment can be fixed as 4500 million (denoted as F) or governed by recruitment curve as in Eq. (1.7) (denoted as C) or random from a normal distribution N (4500,2000) (denoted as R). All scenario combinations are listed in Table 1.1. In scenarios DF, DC and DR, weight gain is governed by Eq. (1.6) while in scenarios SF, SC and SR by Eq. (1.5).

Table 1.1. Scenario illustrations

	Zooplankton index	
Recruitment	Deterministic	Stochastic
Fixed	DF	SF
Curve	DC	SC
Random	DR	SR

1.3 Parameters

This section applies the simplest scenario DF where the zooplankton index is deterministic as 1 throughout the entire period. Recruitment is fixed as the historical mean. Individual weight is assumed to have a constant yearly gain as in Eq. (1.6).

1.3.1 Parameter specifications

It is a common practice to set the age classes of mackerel from 0 to 12, where the 0 age class is the recruitment of that year. So $i \in [0,1,2, \dots 12]$ and $n = 12$. The whole modelling period is set to be 100 years in order to gain long term insights of the problem. Thus $t \in [1,2, \dots T]$ and $T = 100$. The original status of the stock for each age class in the model comes from the historical data in year 1980. Discount rate r is set to 5%. The effort constraint can be chosen as $f_t \in [0,10]$, which has a high enough upper bound for fishing mortality (Horwood, 1987).

Fishing costs c is calibrated to 23000 in order to obtain a cost-revenue ratio around 70%: a number that has been observed for pelagic fisheries such as mackerel, herring, blue whiting and capelin (Fiskeridirektoratet). Minimum SSB B_{lim} is 1.84 million tonnes (ICES, 2014). Maturity ogives of mackerel u_i are presented in Table. 1.2.

The three varying parameters are price, natural mortality and selectivity. For such parameters, we either apply a constant number, which is what usually has been done, or utilize the age structure of the model by assigning age-specific or weight-dependent parameter values.

Price of mackerel in Norwegian kroner per kilogram (NOK/kg) is either constant as the mean price $p = 8.46$ NOK/kg or linear as a function of weight: $p_{i,t} = 19.87 w_{i,t}$ (Zimmermann & Heino, 2013). Natural mortality of mackerel is assumed to be a constant $m = 0.15$ for all age classes (ICES, 2014) or age-specific: $m_i = 0.32 - 0.02i$.

Selectivity s_i is assumed to be either knife-edge selective (denoted s_i') where only classes above a certain age are harvest or non-selective (denoted s_i^*) where every age class lend itself to some fishing mortality. Both cases are shown in Table. 1.2. Note that age-class zero is of no interest for harvesting. The non-selective case can be calculated as the age-specific fishing mortality divided by maximum fishing mortality in the same year. According to ICES (2014), there has been a slow shift from selecting older classes to younger classes over time. To embody this shift, we apply the average historical values on the first 35 years and use the values in year 35 for the remaining periods.

**Table 1.2** Parameter values for maturity ogive and selectivity

i	u_i	s_i'	s_i^*	
		$t \in [1,T]$	$t \in [1,35]$	$t \in [36,T]$
0	0	0	0.03	0.01
1	0.106	1	0.1	0.04
2	0.539	1	0.18	0.18
3	0.913	1	0.37	0.43
4	0.998	1	0.64	0.72
5	0.999	1	0.73	0.82
6	0.999	1	0.9	0.83
7	1	1	1	1
8	1	1	1	1
9	1	1	1	1
10	1	1	1	1
11	1	1	1	1
12	1	1	1	1

1.3.2 Parameter combinations

The three varying parameters are combined and explored: constant vs. weight-dependent price; knife-edge selective vs. non-selective; constant vs. age-specific natural mortality. This gives a total of 8 different combinations. It is found that the optimization results are highly sensitive to parameter assumptions. In other words, a small change in parameter combination may lead to rather distinct results.

It seems that weight-dependent price $p_{i,t}$, knife-edge selectivity s_i' and constant natural mortality m lend themselves to pulse fishing. A possible explanation is that the weight-specific price structure puts higher value on older classes, justifying the waiting period before harvesting. With selectivity s_i', younger age classes are more vulnerable towards harvesting and this creates a relatively lower fishing pressure for the older age classes. Selectivity parameters applied have in fact very limited difference: both s_i' and s_i^* are the same above age 7. However, it induces obvious changes of the results. This paper numerically illustrates the scale of the issue, which should never be underestimated. In many studies, natural mortality is assumed to be constant

for all. When the bigger fish has a higher probability to survive, this also gives incentive to wait
for the stock to accumulate.
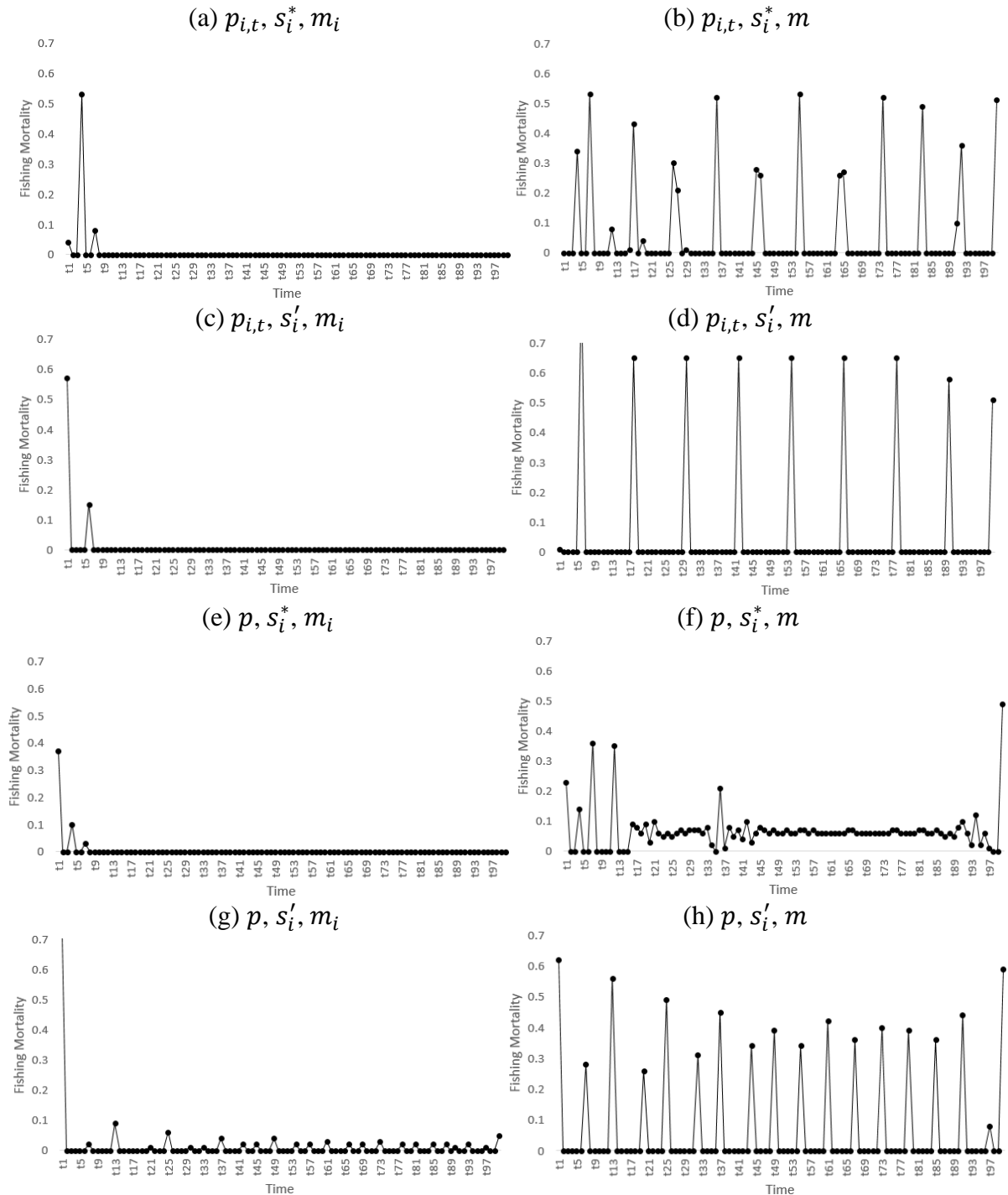
Fig. 1.3 Optimal fishing mortalities for 8 parameter combinations

To sum up, any parameter choice that favours the older age classes, for example by assigning
higher value or decreasing the chance of death of older classes, tends to lend itself to pulse
fishing pattern.

1.4 Optimization

In this section, we choose the parameter combination: price $p_{i,t}$ is weight-dependent, selectivity is s_i^* and natural mortality is a constant m. This combination has a modest tendency towards favouring the pulse fishing pattern. All other parameter values are specified as in Section 1.3.1. All scenarios in Table. 1.1 will be explored and summarized in this section.

1.4.1 Mean results

For each scenario of SF, SC and SR, 1000 realizations of the zooplankton index are drawn randomly. Each represents a possible outcome of the environmental development path during the 100 years. The model is treated as a deterministic nonlinear programming problem under each path. Optimization is conducted for each realization and mean results are obtained by taking the average.

Several indicators of the results are used for interpretation. Net profits Z and fishing costs C are measured in million dollars and calculated as the mean from the 1000 scenarios. The fishing cost C for the whole period is calculated as: $C = \sum_t (1 + r)^{-t} c f_t$.

The average time series of harvest $\bar{H}_t$ and stock biomass $\bar{B}_t$ are measured in million tonnes and are obtained as the average from the 1000 scenarios. $\bar{H}$, $\bar{B}$ and $\bar{x}_{0,t}$ are the average harvest, stock biomass and recruitment respectively. Only periods from $t18$ to $t90$ are used to calculate the mean results in order to avoid the adjusting phases in the beginning and at the end of the model, which have extremely high volatility. σ_Z^* denotes the standard deviation of the sample for net profits and σ_C^* denotes the standard deviation of the sample for fishing costs.

As illustrated in Table. 1.3 the only modelling difference between scenario DF and DR is the randomness of recruitment. Scenario DR has a higher profit margin on average but with a lower probability of actually reaching the mean. Note that random recruitment in scenario DR is from a symmetric probability distribution around the same mean as DF. The model seems to be able to efficiently capture and utilize the extremely high recruitment to reach higher average profits. It is also noticeable that σ_C^* is generally larger than σ_Z^*. This may be explained by the way fishing costs and sales revenues are calculated. On one hand, many elements are involved in determining the sales revenue such as individual weight and stock size, thus smoothing out potential variances. On the other hand, the calculation of total fishing cost is purely linked to fishing mortality, which may have high volatility.

Scenario DC results in the lowest net profit, lowest cost and poorest harvest and stock biomass. This is mainly due to the poor recruitments. If the recruitments generated by the model strictly follow the estimated Ricker curve, it ends up with only 74% of what has been the historical average. With the fact that recruitment data usually is rather volatile, letting a single recruitment function to take over seems an unreliable approach leading to a pessimistic scenario.

Table 1.3 Mean results

(Z and C are net profit and fishing cost for the whole period. σ^* represents the standard deviation of the sample. $\overline{H}$ is the average harvest in million tonnes. $\overline{B}$ is the average stock biomass in million tonnes. $\bar{x}_{0,t}$ is the average recruitment in millions. Mean values are obtained from $t18$ to $t90$.)

Scenario Indicators	DF	DC	DR	SF	SC	SR
Z	12504	8761	14222	12955	9491	14596
C	29817	23733	30843	29954	24442	30821
σ_Z^*	/	/	1497	2228	1851	2811
σ_C^*	/	/	1884	2860	4199	3164
$\overline{H}$	0.2	0.1	0.22	0.19	0.13	0.23
$\overline{B}$	5.17	4.15	5.16	5.19	4.14	5.16
$\bar{x}_{0,t}$	4500	3308	4509	4500	3290	4498

Compared to DF, scenario SF lends itself to slightly higher values of average net profit and fishing cost. The improvements are rather trivial. It seems that a nonlinear individual weight development path may not create huge differences in the results. However, note that the increases in σ_Z^* and σ_C^* are much higher between scenario SF and DF than between DR and DF, which is mainly caused by varying annual weight gains. This implies that when recruitment is fixed, the randomness of weight gain, which may be presumably small, is transferred to the volatility of the value as well as cost of harvesting.

Similar to DC, scenario SC has a poor performance: in more than 70% of the time, scenario SC leads to lower profits than SR. However, scenario SC has a higher net profit than DC on average due to the introduction of the random environmental factor. Scenario SR, similar to DR, has the highest profit and cost on average all the scenarios. The varying annual weight gain almost doubles σ_Z^* and σ_C^* by switching from DR to SR, which is not a surprise given two sources of randomness in the model.

To sum up, higher volatility of the zooplankton index, implying either varying recruitment or volatile weight gains, leads to higher net profits on average but together with a lower probability of actually hitting the mean values. It can be interpreted as the risk upon the fishing industry brought by nature. When recruitment is fixed, volatile weight gains cause considerable increase of σ_Z^* and σ_C^*. When weight gain is constant, random recruitment also lends itself to larger volatility of profits and costs. Strong and extremely good recruitment can be utilized by the model to reach better profits. Net profits usually have smaller variances than fishing costs mainly due to the structure of the model. Recruitment governed by a recruitment function tends to lead to the weakest zero age-class, thus the poorest overall performance.

1.4.2 Time series results

Fig. 1.4 displays the mean estimated optimal fishing mortality time series of 1000 realizations under each scenario. Scenarios DF, DC, SC and SF exhibit various scales of pulse fishing pattern while DR and SR have a more stable harvest. For DR and SR, the normally distributed recruitments are generated with a rather large standard deviation, leading to widely distinctive optimization results. With all the realizations, they simply cancel out and smoothed out when the mean time series are presented.

The volatility of simulated recruitments in scenario SC is roughly 600 while this number is about 2000 in DR and SR. When the volatility of recruitment is zero (scenario SF) or relatively small (scenario SC), the pulse fishing patter clearly remains even after taking the average. With a fixed recruitment as in SF, the suggested fishing activity is periodic and reaches a peak about every 9 years. Yet the fishing mortality peaks are of moderate levels. For scenario SC, the recruitment is comparatively weaker, so the waiting time or moratorium period is longer. About every 14 years, the time series of scenario SC suggest quite a strong harvesting action.

A similar conclusion could be drawn when comparing scenario DF and DC. A guaranteed recruitment offers the possibility for more frequent fishing activities and lower fishing mortality in general.

While the time series results for scenario DC and SC are extremely alike, more distinctions can be found between DF and SF. When the weight gain is stochastic, the 'waiting period' is not strictly moratorium and the strong harvest years becomes more conservative.
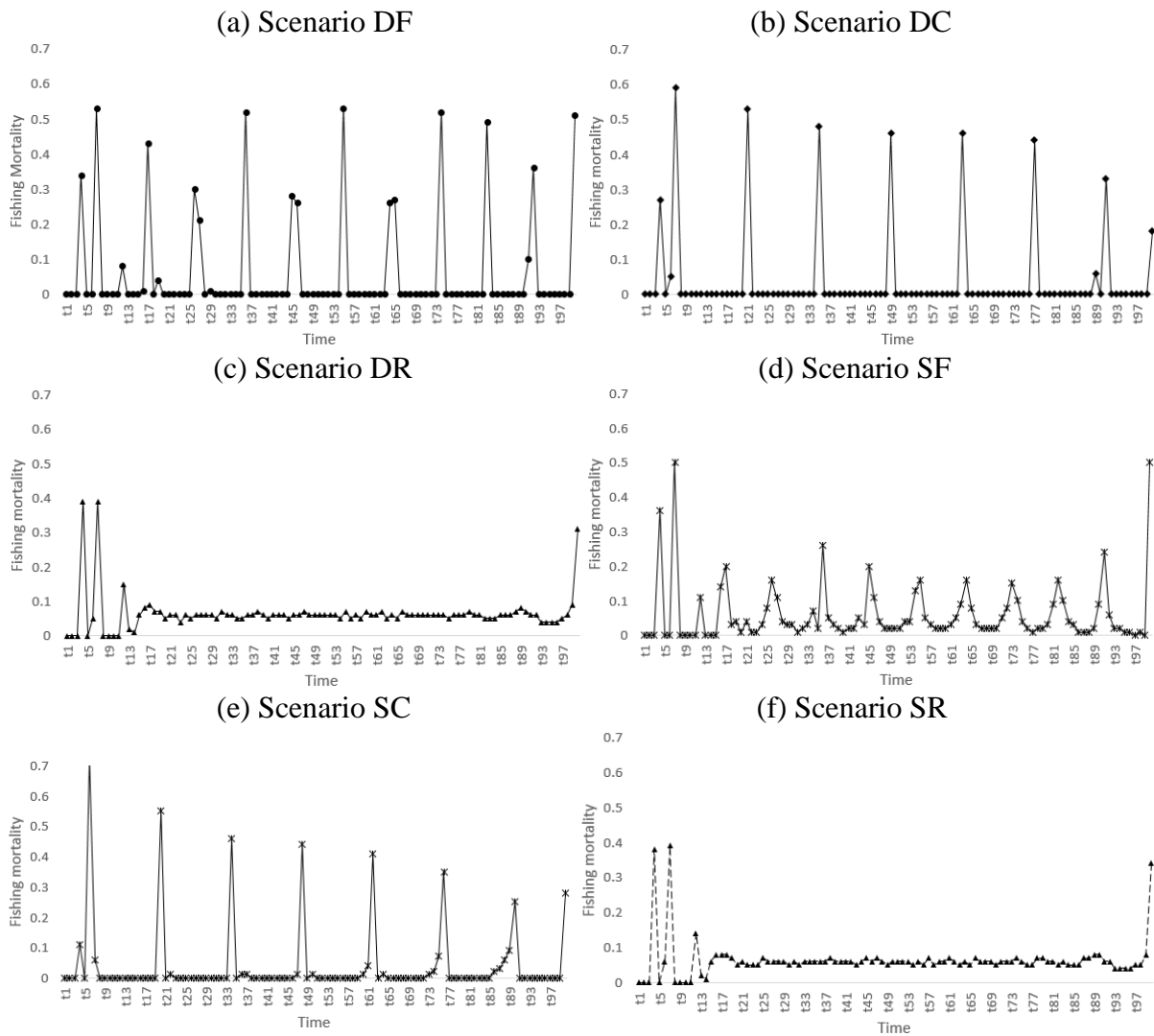
**Fig. 1.4** Mean estimated optimal fishing mortalities

1.5 Simulation

A straightforward policy that is easy to implement in reality is a constant fishing mortality. In this section, fishing mortalities from 0.02 to 0.1 are assessed through simulations of the stock in a period of 100 years under the 6 scenarios. All parameters are the same as in Section 1.4.

Simulating with a constant fishing mortality has been a tool for analysing such bioeconomic problems. Bjørndal, Ussif, & Sumaila (2003) investigated the Norwegian spring spawning herring (NSSH) and found that with a time horizon of 20 years, a constant fishing mortality of 0.15 is economically optimal. They assumed that the price of the fish is a constant and applied 20 years as the simulation period, which is much shorter compared with this study.

As shown in Fig. 1.5, regardless of the choice of scenarios, net profit is maximized when fishing mortality is around 0.06. In the management plan simulations of ICES advice 2015, the NEA

mackerel stock is simulated with different target fishing mortalities from 0.2 to 0.35. No economic elements are accounted for in these simulations. This paper promotes a much lower fishing mortality than what is being evaluated in the ICES advice. One aspect that potentially contributes to this difference is the weight-specific price structure. This assumption includes crucial economic aspects that could be neglected in pure biological studies. When the fishing mortality is high and constant, the composition of the entire population may shift towards smaller individuals, leaving the cohorts limited time to accumulate body weight in time and therefore lose its value.
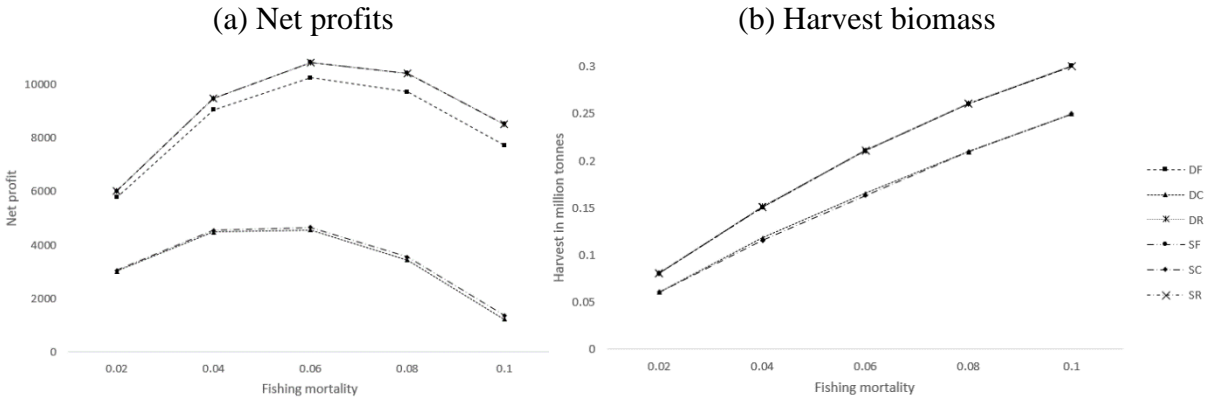

Fig. 1.5 Simulated net profits and harvest biomass under different fishing mortalities for each scenario

The level of net profits is largely influenced by the overall recruitment and the random environmental factor. In scenario DC and SC where recruitment is determined by the Ricker function, the recruitments are systematically lower, leading to smaller profits and smaller optimal fishing mortality. In the other scenarios, DF has the lowest profits, which goes in line with the findings from Section 1.4.1. Compared to Table. 1.3, simulations with a fixed fishing mortality lead to average net profits that are at least 1 σ_Z^* lower than the optimization results.

The curve of total harvest biomass against fishing mortality in Fig. 1.5(b) is slightly concave. Since the fishing mortality is kept constant for years in the simulation, a heavier harvesting gives rise to a smaller stock biomass and smaller individuals. When the stock is harvested to a poor level, even large fishing effort will still have little return. This may explain what can be observed in Fig. 1.5(a) and (b): the more we manage to harvest, the worse it becomes regarding net profits once the fishing mortality goes beyond the optimal level.

1.6 Historical vs. optimal harvest

It is of interest to apply some real data to the model and make comparison between historical harvest and optimal harvest offered by the optimization model. Similar to scenario SC, this section assumes that the zooplankton index is stochastic and recruitment is governed by a recruitment curve as in Eq. (1.7), in order to fully capture the two interactions of recruitment and weight gain. Parameter T is changed to 40 years in order to cover the available data from 1980 to 2014. Fishing cost c is adjusted to 11000.
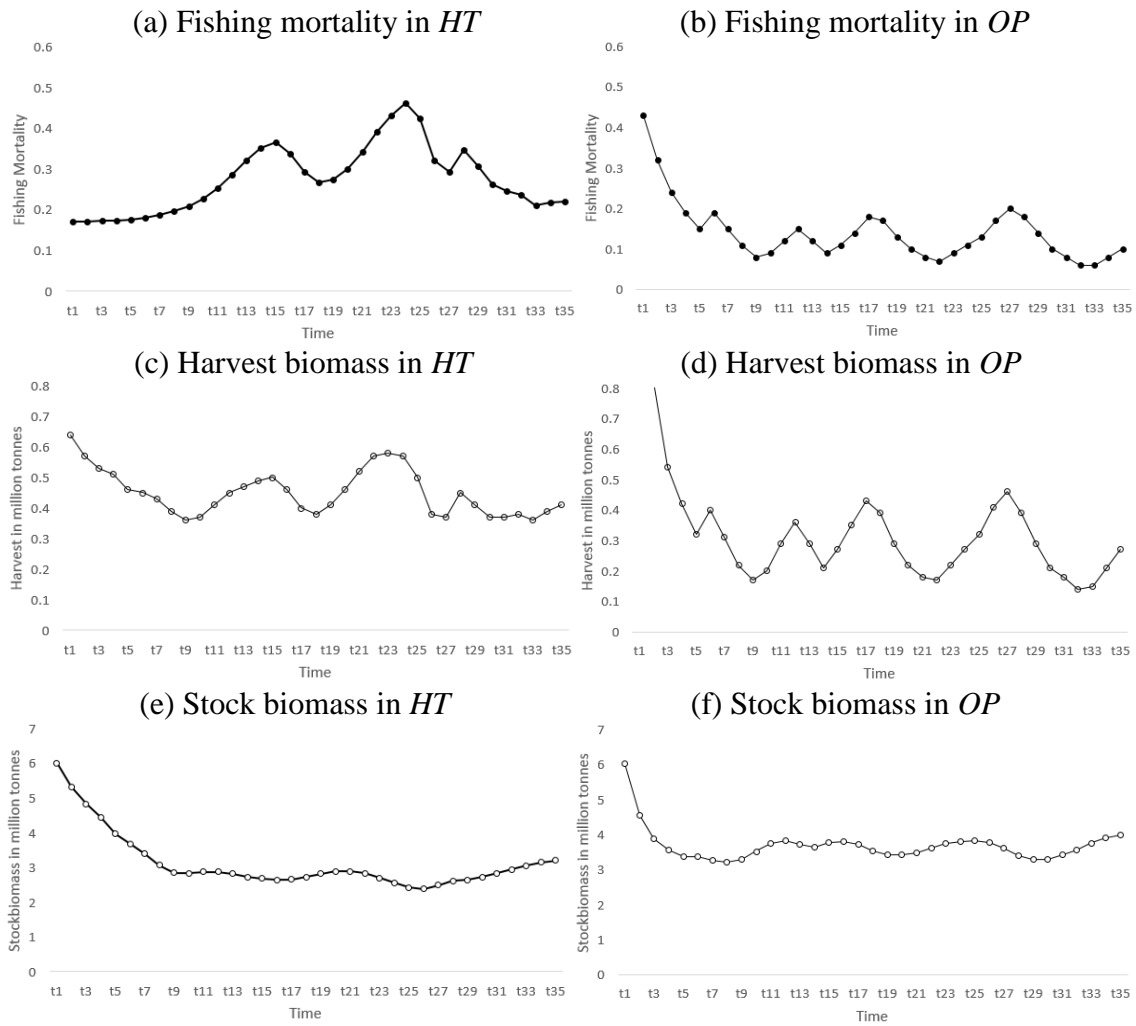
Fig. 1.6 Results for scenarios HT and OP

We use $'HT'$ to indicate the results of historically applied harvest and $'OP'$ for the optimal harvest. Both have the same random number generator seed. Since the stock has never been managed under a pulse fishing regime with consecutive years of strict moratorium, the optimal exploitation requires certain constraint in order to remain practical and comparable. To offer relevant proposals, an extra constraint of $k_1 \leq \frac{f_{t+1}}{f_t} \leq k_2$ is added in the optimization, where

$k_1 = 0.75$ and $k_2 = 1.25$ are the minimum and maximum annual change rate respectively from historical data.

As shown in Table. 1.4, *HT* leads to 43% lower profit, 34% higher cost and a larger cost-revenue ratio on average. In addition, with more than twice the fishing mortality and 1.6 times the harvest biomass of *OP*, *HT* maintains 32% lower stock biomass level. The historical exploitation is economically inefficient and biologically unsustainable.

As presented in Fig. 1.6, it is no surprise that the results of *HT* show that NEA mackerel stock had been harvested unsustainably. The stock biomass kept decreasing to a minimum level around 2.4 million tonnes in year 2005. In year 1994 and 2003, fishing mortality peaked to 0.37 and 0.46 respectively. After the second peak, fishing mortality came down to around 0.22, leading to a slight recover in stock biomass.

The fishing mortalities in Fig. 1.6(b) can be the proposal for management plan of the NEA mackerel from our model. It not only leads to a higher net profit but also a higher and more stable stock biomass, which is crucial for a healthy stock structure and sustainable fishery resource management. The proposed harvest policy secures both biological and economical potential of the stock, diminishing the possibility of potential population collapse.

Table 1.4 Results for scenarios *HT* and *OP*

(Z and C are net profit and fishing cost for the whole period. σ^* represents the standard deviation of the sample. $\bar{H}$ is the average harvest in million tonnes. $\bar{B}$ is the average stock biomass in million tonnes. $\bar{x}_{0,t}$ is the average recruitment in millions. $\bar{f}$ is the average fishing mortality. Mean values are obtained from $t15$ to $t35$.)

Scenario Indicators	HT	OP
Z	15944	28216
C	47309	31067
σ_Z^*	4080	2769
σ_C^*	396	388
$\bar{H}$	0.44	0.28
$\bar{B}$	2.76	3.63
$\bar{x}_{0,t}$	4369	3778
$\bar{f}$	0.312	0.118

As Hannesson (2011) pointed out: 'What pulse fishing means is that a stock of fish is fished down heavily for a short period of time and then left to replenish itself for a longer period. But what does the industry do in the meantime?' The added constraint in this section successfully limits the variations of fishing mortality over time.

It is shown in Table. 1.5 that as the constraint gets tighter, both profit and cost tend to decrease; the shadow cost for having the constraint rises. Still, the scale of the constraint shadow cost is low: within 10%. At a relatively low cost, the constraint has made the proposal more realistic since stable quotas are favoured by fishers as they enable the decision making to be more predictable and correct. When the annual change rate of fishing mortality is within 2% ($k_1 = 0.98$ and $k_2 = 1.02$), it is still valid that in more than 70% of the time, OP results in higher average net profits than HT. It seems possible to obtain better results than HT if the harvest strategy is to apply a constant fishing mortality level. Such a simple strategy can have limited risk, higher mean profits and more straightforward implications for the sector.

Table 1.5 Results for OP under various sets of k_1 and k_2

(Z and C are net profit and fishing cost for the whole period. Shadow cost is calculated as the percentage difference of the objective value Z with (set 1-4) and without (set 5) the underlying constraint. σ^* represents the standard deviation of the sample.)

Parameter sets	1	2	3	4	5
k_1	0.75	0.9	0.95	0.98	$+\infty$
k_2	1.25	1.1	1.05	1.02	$-\infty$
Z	28216	27934	27512	26894	29332
C	31067	30451	30547	30209	31770
Shadow cost	3.8%	4.8%	6.2%	8.3%	/
σ_Z^*	2769	2748	2825	2942	2903

1.7 Conclusion

This paper sets up an age-structured bioeconomic model of NEA mackerel and introduces an environmental factor that affects the weight gain process and the yearly recruitment of the population. To deal with such a complex model, certain assumptions are required. All the simplifications are based on existing studies. Several methods have been implemented to analyse the problem: simple optimization of nonlinear programming problem; extracting

average results from multiple optimizations; simulations and comparison between historical and optimal policy.

It is found that weight-dependent price, knife-edge selectivity and constant natural mortality promote pulse fishing as the optimal exploitation pattern. Any parameter combination that advocates the older age classes encourages the policy to give time for the population to accumulate biomass before removing a large quantity in a short period. The behaviour of the optimal policy is quite sensitive to the combination of parameters. The challenge for such age-structured models lies not only in estimating the numerous age-dependent parameters, but also in choosing the assumptions that sway the results to the minimum extent.

The mean results from multiple optimizations suggest that a higher volatility of the environmental factor leads to higher net profits on average but with a smaller likelihood of achieving the mean values. When recruitment is constant, the seemingly small stochasticity in the weight gaining process causes huge volatility in net profits and fishing costs. Although the Ricker recruitment function is estimated from the historical data in a mathematically sound manner, still it seems oversimplified for generating reasonable recruitments. In our numerical example, even with a certain amount of randomness involved, the recruitment curve constantly produces the most pessimistic scenarios of all.

When a constant policy is applied for simulations, it is found that a fishing mortality around 0.06 produces the highest net profits. This result promotes a much more conservative policy for NEA mackerel than what ICES reports have examined. The simulations yield at least one σ_Z^* lower net profits than optimizations regardless of the scenario, which is an expected situation. In reality, a stable and predictable fishing pattern without intense harvesting peaks is practical and valuable to the industry and management of the resource.

Comparisons between the historical and the optimal harvest prove that the past exploitation has been economically inefficient and biologically unsustainable. A proper and constant fishing mortality policy has the possibility to exceed the performance of the historical harvest. When conducting the optimization in this section of the paper, an extra constraint on the annual change rate of the decision variable is attached in order to smooth out the policy. The cost of having such a constraint is inexpensive: within 10% of the total profit. The comparison manifests that in more than 70% of the time, the optimal harvest offered by the model leads to 43% higher net profit and 34% lower fishing cost.

Appendix

Table A. Notations and parameter values

Subscripts		Definitions
i		Age
t		Time/Period
Variables		
$x_{i,t}$		Number of individuals of age i at time t (millions)
f_t		Fishing mortality at time t (decision variable)
s_i		Selectivity at age i
$w_{i,t}$		Average individual weight of age i at time t (tons)
$h_{i,t}$		Harvest biomass at time t of age i (million tons)
H_t		Harvest biomass at time t (million tons)
S_t		Spawning Stock Biomass (SSB) at time t (million tons)
$p_{i,t}$		Unit price of age i at time t (1000NOK/ton)
ρ_t		Zooplankton index
Parameters	Values	
m	0.15	Natural mortality (same for all age classes)
m_i	$0.32 - 0.02i$	Natural mortality at age i
n	12	Maximum age class
T	100	End period
u_i	Table 1.2	Proportion of sexually matured individuals at age i
s_i'	Table 1.2	Knife-edge selectivity
s_i^*	Table 1.2	Non-selective selectivity
α, β	6.37; 0.52	Parameters for the Ricker's recruitment function
p	8.46	Constant unit price (NOK/kilo)
r	0.05	Discount rate
c	23000	Calibrated cost parameter for pure schooling fisheries
B_{lim}	1.84	Biomass reference limit (million tons)
F	[0,10]	Fishing mortality range

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Chapter 2

2. Seasonality matters: a multi-season, multi-state dynamic optimization in fisheries

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Abstract

Many biological and economic processes in fisheries occur seasonally though most of the extant literature tends to neglect periodicity. This work is an attempt to treat seasonality in a systematic way. We present a multi-season multi-state bioeconomic model and apply a periodic Bellman approach using dynamic programming to obtain the optimal feedback policy of each season. Our approach has rich potentials. It could deal with seasonal patterns of arbitrary uneven lengths: some may span years and some may occur within a year.

Our numerical illustration demonstrates that a seasonal dynamic optimization model allows for naturally occurring seasonal moratorium or potentially a Marine Protected Area (MPA). It shows that there exists optimal dynamic paths that develop into a permanent equilibrium cycle, which consists of one harvesting season followed by a moratorium period. This indicates an optimal closure of the fishery that a yearly model would overlook. Fishing pressure on the mature stock elicits even heavier harvesting in the next season on the same group. A protective moratorium of the immature hinders the value of the whole stock.

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2.1 Introduction

2.1.1 Motivation behind seasonality

Numerous commercial fisheries exhibit periodic patterns that undergo periods of feeding, reproduction, migration and harvesting (Clark, 2010). However, most commercial fisheries are managed quintessentially on a yearly basis. For example, agencies like the International Council for the Exploration of the Sea (ICES) collect annual data and provide annual advice regarding stock status, reference points and Total Allowable Catches (TACs) (Fisheries - European Commission, 2016). Often, annual TACs are allotted to different vessel groups that target the resource in different seasons (Kvamsdal et al., 2017). Therefore, we conduct this work believing that research and policymaking should incorporate seasonality in fisheries to a larger and deeper extent.

Our approach is an initial attempt that applies Dynamic Programming (DP) to obtain the optimal feedback control policy with both seasonality and multidimensional states in a bioeconomic model under an infinite time horizon. It has rich potential because this approach makes it possible to take into consideration arbitrary season lengths including longer seasonality that spans more than a single year. For instance, we may model El Nino conditions that happen at intervals spanning several years combined with seasonal features that take place annually.

Many fisheries with notable seasonal variations have been studied using diverse methodology. Some ecological studies focus on the biological aspects of the seasonal stock such as population growth (Durant et al., 2008). Some stand from an economic and empirical point of view: Hannesson, Salvanes, & Squires (2010) analysed the effect of technological change on labour and productivity in the seasonal Lofoten fishery using data over 130 years. Hermansen & Dreyer (2010) investigated how the regime aiming to shift the harvesting pattern towards the low season performed for the Lofoten fishery. Seasonal changes were considered an existing negative impact to the processing industry and data about the regime was collected to test the hypotheses empirically. Some investigate the consequences for fisheries exposed to seasonal environmental disturbances. Huang & Smith (2011) employed the optimal control theory on an annual fishery with a seasonal hypoxia shock and found that the pollution shifts the harvest opening earlier in the year. Various nonlinear programming optimization models establish multiple periods with time dependent parameters and variables to capture seasonality. Önal et al. (1991) developed a nonlinear mathematical programming model with multi-periods and

size-structure to determine the optimal harvesting pattern for the Texas shrimp fishery. The optimal policy during 1000 days were compared with the actual observation and substantial overfishing was found especially in spring and summer. There are also highly detailed simulation models that assess fishery management policies (Pelletier et al., 2009). They can incorporate complex seasonality in the dynamics without much restriction since the 'curse of dimensionality' in itself does not impose challenge on such what-if analyses.

Although many researchers have approached seasonality in the literature, most have built intra-seasonal bioeconomic models to analyse within-season harvesting in regulated open access fisheries (Smith, 2012). Larkin & Sylvia (1999) set up a nonlinear programming model to determine the time path of harvest by allocating effort across months (intra-season harvest pattern) and annual quotas between competing industry sectors (catch rights) and found that intra-season quality generated higher net benefits than the allocation of catch rights. Holland (2011) simulated an intra-annual numerical model to optimize effort and catch over a one-year period in the Maine lobster fishery under seasonal variability in catchability. Such intra-season studies tend to assume that fishing takes place over a single season, with a frequently fixed season length (Kvamsdal et al., 2017).

Both the profound role of seasonality in fisheries and the inadequate studies in the literature call for further research in this direction. Seasonality in fisheries is a common phenomenon: both biological processes, such as spawning, and human activities, such as harvesting, are seasonal rather than smoothly distributed over time (Bjørndal & Munro, 2012; Clark, 2010). At the same time, seasonality is complex and difficult to address: when considering one-year time increments, discrete-time models tend to neglect seasonality; when considering autonomous optimal feedback policies, continuous-time models neglect it as well (Kvamsdal et al., 2017). Moreover, when seasonality is investigated instead of being disregarded, those intra-season studies that imply a single harvesting season can be an oversimplification of many real world fisheries.

We hereby argue that it is appropriate to turn to discrete-time models (Bjørndal & Munro, 2012) and we propose uneven time increments that are less than one year in order to incorporate seasonality. To fill the gap in the current literature, we introduce multiple harvesting seasons within each year. Each season length can be adjusted according to specific fisheries. The stock develops alternately according to seasonal dynamics and the cycle goes on in an infinite time horizon.

2.1.2 Remarks on the approach

A rich literature on fisheries management includes implementation of the DP approach (Clark, 1985; Lane, 1988; Mangel & Clark, 1983; Poudel et al., 2012; Walters, 2002) but few address the issue of seasonality. With the recent finding on the periodic Bellman approach (Kvamsdal et al., 2016) , we are able to simplify the problems that consist of multiple periods in one cycle and solve them with a common DP technique. Nevertheless, it is widely recognized that discrete DP suffers from the curse of dimensionality (Anderson et al., 1981). The escalating computing time brought about by additional dimensions remains. Therefore, we begin with the simplest case of two-dimensional states, seeking to preserve both solution efficiency and theoretic insights.

With multidimensional states, a bioeconomic model can describe either interacting species that often includes predator-prey relationships (Poudel et al., 2012; Sandal & Steinshamn, 2010) or a single stock that is categorized into age or size groups (Tahvonen, 2014). Age-structured models, which are widely used to assign catch quotas, have gained more popularity among economists as a consequence of the critics on the 'oversimplified' traditional biomass approach (Tahvonen, 2010; Townsend, 1986). Due to generally large cohort numbers, few of the age-structured studies employ the DP approach.

To better understand the inner structure of a stock, we adopt a commonly accepted practice and divide a population into mature (adult) and immature (juvenile) individuals (Holden & Conrad, 2015; Sandal & Steinshamn, 2003). Though simple, this practice is intuitive and allows inclusion of critical biological processes such as recruitment, as well as other potential model expansions and specifications such as size-dependent prices.

We achieve the optimal feedback solution with the control variable (seasonal harvests) as a direct function of the state variable (mature and immature group biomasses). The feedback models take prevailing stocks as inputs and automatically respond to the changes in the states, and thus adapt to new situations instantaneously (Sandal & Steinshamn, 1997). In addition, the feedback approach can easily be extended with stochasticity since it is superior to the alternative time paths approach when faced with uncertainty (Agnarsson et al., 2008).
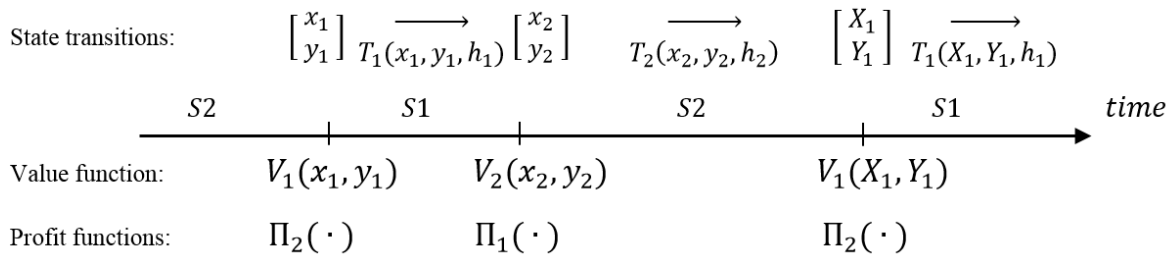
In Section 2, we first demonstrate the basic model structure with seasonality and two-dimensional states; we then specify the biological state transitions and economic rent functions; lastly, we establish the credibility of our method and results by referring to the recent theoretic

findings on the periodic Bellman approach. In Section 3, a simple depiction of the solution procedure is provided. We then elaborate on parameter specifications and introduce two cases. Section 4 manifests the numerical results, analyses the effects and provides possible explanations. Conclusions and discussions follow in Section 5.

2.2 The model

2.2.1 Model structure

As elaborated in Section 1, we choose a simple but representative model structure aiming to access both conceptual insights and solution feasibility. We look at an infinite horizon, discounted, discrete-time dynamic optimization problem with two-dimensional states and two seasons. The states are mature biomass x and immature biomass y and the two seasons are denoted as S1 and S2. It is straightforward to depict the seasonal state transitions as follows:

State transitions: $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \xrightarrow{T_1(x_1,y_1,h_1)} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \xrightarrow{T_2(x_2,y_2,h_2)} \begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} \xrightarrow{T_1(X_1,Y_1,h_1)}$

S2 ⊢———— S1 ————⊢———— S2 ————⊢———— S1 ————→ *time*

Value function: $V_1(x_1,y_1)$ $\quad V_2(x_2,y_2)$ $\qquad\qquad V_1(X_1,Y_1)$

Profit functions: $\Pi_2(\cdot)$ $\qquad \Pi_1(\cdot)$ $\qquad\qquad \Pi_2(\cdot)$

Fig. 2.1 Two-dimensional two-season dynamic cycle in an infinite time horizon: state transitions, value function and profit functions

The population at the start of S1 is represented by the biomass of two age groups, namely mature (x_1) and immature (y_1). Seasonal harvest policy or control of the system is h_1 for S1 and h_2 for S2 respectively. There are two dynamic operators during a cycle noted as $T_1(x_1,y_1,h_1)$ and $T_2(x_2,y_2,h_2)$, driven by its seasonal transitions and policy. States x_2 and y_2 are the new states at the beginning of S2 after dynamic operator T_1. States X_1 and Y_1 at the end of S2 are the states at the beginning of the next S1. The periodic cycle goes on infinitely as demonstrated in Fig. 2.1.

As a practice of tradition (Gordon, 1954; Scott, 1955), we believe it is legitimate to set our objective as to maximize the economic rent from the fishery resource. The core purpose of fisheries management is to take into account both the conservation of the resource base, as well as the exploitation of it (Bjørndal et al., 2004). Profit maximization achieves many of the prevalent fishery objectives, for example, it implies conservation of fish stocks and marine

environment to the extent that this conservation contributes to harvesting profits and conservation in itself is valuable (Arnason, 2009).

Seasonal net economic gains are denoted as Π_1 and Π_2. In each season, we assume that the stock first undergoes biological processes, such as growth and spawning, and then becomes available for the human harvesting activities at the end of the season. Therefore, Π_1 is obtained at the end of S1 while Π_2 is realized at the end of S2.

The value function manifests the best possible value of the objective and is non-autonomous, depending on which season the present time is. V_1 indicates the value at the beginning of S1 while V_2 represents the value when standing at the end of S1 or at the beginning of S2. Note that V_1 and V_2 are not the value of each separate season but rather of the entire infinite time horizon and together they constitute the value function of our problem:

$$
V(t,x,y) = \left\{ \begin{array}{l} V_1(x,y), \quad t = S1 \\ V_2(x,y), \quad t = S2 \end{array} \right. = \left\{ \begin{array}{l} \max_{h_1} \{\beta_1 \Pi_1(h_1, x, y) + \beta_1 V_2(T_1(x, y, h_1))\} \\ \max_{h_2} \{\beta_2 \Pi_2(h_2, x, y) + \beta_2 V_1(T_2(x, y, h_2))\} \end{array} \right. \tag{2.1}
$$

Where t indicates the season, $\beta_1 = \beta^\theta$ and $\beta_2 = \beta^{1-\theta}$. The parameter θ indicates the length proportion of S1 in a whole cycle and β is the cycle discount factor. A varying θ generates flexibility and allows the model to deal with species or ecological systems of various kinds.

2.2.2 A simplified fishery

Our bioeconomic model describes a single species stock in which the parameter for seasonal harvesting selectivity can be modified. This simplified fishery model serves as a starting point to probe the implications of modelling seasonality in such problems and to stimulate and support further research in this direction. Certain seasonal features are in reference to the Northeast Arctic (NEA) cod stock. The adults of the NEA cod migrate from the Barents Sea to the Norwegian coast to spawn in the spring, with a remarkable amount aggregated in Lofoten. The larvae then drifts north and reaches the Barents Sea by summer.

We assume that spawning or recruitment takes place during S1, setting S1 as the shorter season in this context. Harvesting is assumed to take place smoothly within the season. We introduce a major difference between seasons about harvesting: in S1, only the mature group lends itself to harvesting while in S2 the selection between x and y remains unfixed. This protection of the young in the spawning season can be advocated by multiple arguments: modelling-wise, it simplifies the decision variables; economically, it may be easier to take the mature group in S1

when they agglomerate in the spawning ground; biologically, sparing the immature at this critical time of the year is likely to advantage future prosperity of the stock. If the season is short enough, then one can choose to ignore the growth during the season (Mangel, 1984). Thus, we assume for simplicity that the mature group has little change except for biomass loss due to harvesting as shown in Eq. (2.2).

$$x_2 = x_1 - h_1 \tag{2.2}$$

$$y_2 = F(x_1, y_1) = min\,[\,y_{max},\ (b_1 + b_2 x_1)y_1 + b_3 x_1\,] \tag{2.3}$$

Function $F(x_1, y_1)$ in Eq. (2.3) is linear in y_1 with upper bound y_{max} and it effectively grasps the spawning dynamics: both the slope $(b_1 + b_2 x_1)$ and the intercept $(b_3 x_1)$ are dependent on x_1. As x_1 escalates, $F(x_1, y_1)$ rises via both slope and intercept until it reaches the maximum biomass y_{max}. This structure implies that a stronger mature group produces more offspring creating higher population density for the immature and thus induce better fitness and faster growth rate of the immature according to 'the Allee effect' (Allee, 1931). When y_1 becomes zero, x_1 still contributes as much as $(b_3 x_1)$ to y_1 through spawning, which maintains the immature group even without any to start with. When x_1 is zero, y_1 will grow to $b_1 y_1$ after S1. Here the 'biomass growth' counts for both individual weight gain and biomass loss caused by natural mortality, so b_1 is not necessarily larger than one.

$$X_1 = D_x(x_2, y_2) - sh_2 \tag{2.4}$$
$$Y_1 = D_y(x_2, y_2) - (1 - s)h_2 \tag{2.5}$$
$$D_x(x_2, y_2) = mx_2 + ay_2 \tag{2.6}$$
$$D_y(x_2, y_2) = G(y_2) - ay_2 - cx_2 y_2 \tag{2.7}$$
$$G(y_2) = \frac{g_1 y_2}{1 + g_2 y_2} \tag{2.8}$$

During S2, both groups can lend themselves to harvesting where a gear selectivity parameter s indicates the proportion of h_2 that comes from the mature group. Having a parameter s enables the model to adjust according to fishing gear or policy. For example, $s = 1$ may represent a policy to spare all the immature individuals and $s = 0$ can signify a fishing gear type that perfectly selects all the young fish. $D_x(x_2, y_2)$ and $D_y(x_2, y_2)$ are the dynamics of the mature and immature group respectively before harvest in S2.

On top of the natural growth and mortality within the mature group itself, which is assumed linear, every year, a part of the immature matures and transforms to join the mature group. Thus,

the dynamics of the mature group during S2 consists of three parts: linear growth (mx_2), maturity transformation (ay_2) and harvest (sh_2).

The function $G(y_2)$ has a form resembling that of the discrete-time Beverton-Holt recruitment function (Beverton & Holt, 1957). It provides a non-linear growth for the immature group during the longer season. In the function, smaller states correspond to higher growth rates and as the state increases, the immature biomass after S2 asymptotically approaches a fixed level. Note that the original Beverton-Holt function gives the number of population but we are working with stock biomass instead. Cannibalistic behaviour is expressed in the model as cx_2y_2. The multiplicative form means that only when both groups coexist, will there be biomass loss due to cannibalism. The dynamics of the immature group include the following items: non-linear growth $G(y_2)$, maturity transformation (ay_2), cannibalistic behaviour (cx_2y_2) and harvest $(1-s)h_2$.

The size and value of the fish stock as a renewable source can grow and change over time due to biological factors, regulatory decisions as well as market conditions (Clark & Munro, 1975). After elaborating on the biological facets, we now focus on the economic conditions, which can be decisive in determining the value functions.

Inspired from the NEA cod stock as one of the most important fisheries worldwide with minimum prices set through negotiations (Pettersen & Myrland, 2016), we assume that the unit price is a typical downward sloping function of the seasonal harvest with a price floor. Both seasons share the same price function structure as in Eq. (2.9): when harvest is big, the unit price of the fish goes towards the minimum p_{min} and when harvest is small, the price approaches the maximum p_{max}. We can easily adjust parameter values to enable various price ranges for each season. In this work, we employ the same price function for both seasons leaving costs as the only element contributing to the economic difference between seasons. Numerical details are elaborated in Section 3.3.

$$P(h) = p_{min} + (p_{max} - p_{min})e^{-\alpha h} \tag{2.9}$$

$$\Pi_1(h_1, x_1) = P(h_1)h_1 - k_1 h_1 \tag{2.10}$$

$$\Pi_2(h_2, x_2, y_2) = P(h_2)h_2 - \frac{k_2 h_2}{D_x(x_2, y_2) + D_y(x_2, y_2)} \tag{2.11}$$

For the seasonal cost functions, there is an important concept in fisheries management: the sensitivity of harvesting cost to the biomass density, which is referred to as the marginal stock

effect (MSE)[3] (Clark & Munro, 1975). This establishes an inverse relationship between stock size and harvesting cost, making it possible to account for density dependence and thereby analyse anything from completely uniformly distributed stock to perfectly schooling fish (Steinshamn, 2011).

A stock-cost parameter reflects the level of stock dependency in the harvesting cost. One critical value is zero, signifying that the unit harvest cost is completely independent of the aggregated stock size and the total harvesting cost is simply proportional to the harvest amount. This can be a valid assumption when the stock is highly densified and it may elicit overfishing because taking the last school of fish can remain profitable given a constant unit cost (Bjørndal & Conrad, 1987). For positive values of the stock-cost parameter, the unit harvest cost is dependent on the stock size inversely: less densified stocks are difficult to search and have higher unit harvest cost; more densified stocks are easy to find and have lower unit cost. Such stock dependency is quintessentially an economic protection of the weaker stocks.

In S1, when the population migrates into a compact enough spawning ground, the biomass density increases as a result of the gathering. We infer that this aggregation or agglomeration leads to salient population densification in S1 and this effectively sets the value of the implied stock-cost parameter to be zero[4] as in Eq. (2.10). In S2, the stock is assumed to be dispersed in a vast enough area with an implicit stock-cost parameter of one: the seasonal unit harvest cost is dependent on the total stock size as in Eq. (2.11). We choose the state right before harvesting as the reference for the stock size, which may underestimate the costs when the harvest amount is significant. Having such season-specific cost structure makes it possible to capture more of the diverse implications of seasonality.

2.2.3 The Periodic Bellman approach

Kvamsdal et al. (2016) have shown that the classical Bellman approach can be extended to periodic problems. Instead of formulating the problem using one high-dimensional single equation with the annual contraction factor, we can work directly with the coupled dynamic programming equations and legitimately obtain the same unique solution.

---

3 A parameter usually referred to as the stock elasticity, or the schooling parameter in some studies, embodies the MSE either in the production function or in the unit harvest cost function.

4 Though empirically zero is hardly a realistic number, a stock independent cost structure can provide profound implications from our seasonal model.

If the optimal control $\{h_1^*, h_2^*\}$ is known, the value functions in Eq. (2.12) are the fixed points of the Bellman operators, for which β_1 and β_2 are the seasonal contraction factors respectively.

$$V_1(x_1, y_1) = \beta_1 \Pi_1(x_1, y_1, h_1^*) + \beta_1 V_2(T_1(x_1, y_1, h_1^*))$$
$$V_2(x_2, y_2) = \beta_2 \Pi_2(x_2, y_2, h_2^*) + \beta_2 V_1(T_2(x_2, y_2, h_2^*)) \qquad (2.12)$$

In our two-season setting, the coupled dynamic programming equations are displayed in Eq. (2.1). We establish our numerical work based on the theoretical findings about the periodic Bellman approach summarized above. The procedure here is to utilize both of the coupled value function equations at the same time and solve for control h_1 from the first contraction process and h_2 from the second. We demonstrate further numerical algorithm details in Section 3.1.

2.3 Numerical approach

2.3.1 Solution procedure

We discretize the two-dimensional state space evenly with N1 grid points along x and N2 grid points along y. All the state transitions before control are $F(x_1, y_1)$, $D_x(x_2, y_2)$ and $D_y(x_2, y_2)$, which can be calculated once for all. For any state ending up outside the grid, it will be set to the upper bounds of the grid: x_{max} and y_{max}. Lower bounds for both dimensions are zero.

In the numerical scheme, we use the escapements after each season as the decision variable of that season. The policy is bounded so that the states remain non-negative at all times.

Starting with a rough estimate of the value functions, we seek to utilize them to arrive at the optimal policies. For each grid point, we find the corresponding upper and lower bounds for the policy and then discretize this feasible part of the escapement space evenly using a number of points that are proportional to the length of the feasible range. Hence, the step length of the discretized escapement is comparable for all grid points. For each grid point, we interpolate within the grid and obtain all the feasible values according to Eq. (2.12). We then pick the maximum value and update the current value function on this grid point, completing Eq. (2.1). Subsequently, we locate the escapement level that provides the highest value and update it as the optimal escapement on this grid point, which in turn gives us the optimal harvest. One policy iteration is accomplished when all the grid points are updated with the newest values and policies for both S1 and S2.

Then a number of coupled value iterations follow, contracting both value functions without updating the policies. When the value functions have moved closer to the true fixed points, they

are put into another round of policy iteration. This iterating process continues until the change in the value functions and policies are negligible.

2.3.2 Phenomenological parameter specifications

We construct a reasonable and stylized population to work with through theoretical parameterization. The focus and purpose of the numerical illustration is to model seasonality. The first season, S1, lasts from January to April and represents the shorter and more intense harvest season (Hermansen & Dreyer, 2010). In this work, we conduct the numerical approximation with an annual discount factor of 0.97 and accordingly, the seasonal discount factors are $\beta_1 = 0.97^{\frac{1}{3}}$ and $\beta_2 = 0.97^{\frac{2}{3}}$. Referring to several stock assessments from ICES, the state space is set to be 4000 (x_{max}) by 4000 (y_{max}) with 200 (N1 and N2) points distributed evenly along each dimension. All parameters introduced earlier are positive numbers. The unit of the states as well as other state-related concepts such as harvest and biomass difference is kiloton (10^6kg). The unit of the profits and costs is million Norwegian kroner (10^6NOK).

The biological parameters mimicking the scale of the population and its dynamics without harvest are specified below:

$$y_2 = F(x_1, y_1) = min[\, 4000, (0.92 + 0.00005x_1)y_1 + 0.2x_1 \,] \tag{2.13}$$

$$D_x(x_2, y_2) = 0.75x_2 + 0.2y_2 \tag{2.14}$$

$$D_y(x_2, y_2) = \frac{1.2y_2}{1+0.00005y_2} - 0.2y_2 - 0.00001x_2y_2 \tag{2.15}$$

The inherent equilibrium cycle of the stock lies well inside the stock space: see the white diamond and the white hexagon in Fig. 2.2(a). If left alone, the stock will not collapse with a low biomass level so it never exhibits critical depensation (Clark, 2010). Yet, the biomass recovery in such situations is very slow as shown in Fig. 2.2(b). If either group is wiped out every year consecutively in whichever season, the population eventually collapses to zero. This phenomenon goes in line with the single species setting here that neither group can be sustained by itself.
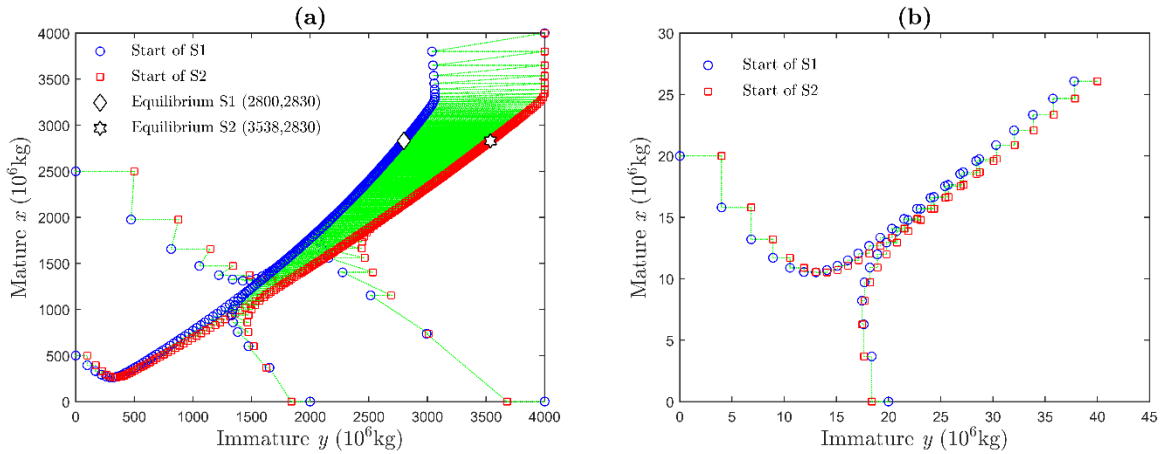
Fig. 2.2 Dynamic paths without harvesting:

(a) Full state space in 200 years with initial states (0, 2000), (0, 4000), (500, 0), (2500, 0) and (4000, 4000); (b) Partial state space in 20 years with initial states (0, 20) and (20, 0)

For state transitions, there exists more nonlinearity and complexity for the immature than the mature, so we demonstrate the immature group visually. A message from Fig. 2.3 is the different level of sensitivity and susceptibility brought by the states between seasons. When spawning activities occur in S1, the new immature state depends heavily on the mature group while the link is not as strong in the other case in S2. This is intuitive in the sense that the spawning season may display the uttermost direct impact from the mature to the immature group.
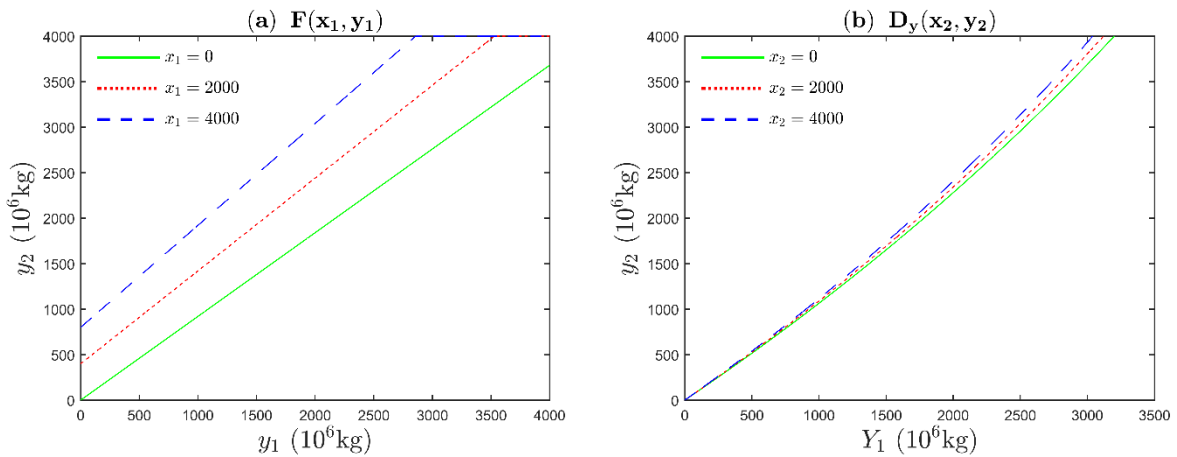
Fig. 2.3 Immature group state transitions without harvesting:

(a) $F(x_1, y_1)$ in S1; (b) $D_y(x_2, y_2)$ in S2

Having determined the biological parameters, we proceed to elaborate on the parameters that address the economic aspects.

Regarding the price, we argue that the longer period S2 may elicit less intensive harvesting, lower average harvesting rate and limited supply to the market. These consequences in turn increase prices especially when the demand is not necessarily dropping in S2. Following this argument, we keep the minimum price 10 and raise the maximum price from 18 to 25, roughly doubling the min-max price difference, arriving at a shifted and higher price function in S2. This can reflect the unmet demand when harvesting happens less intensively during S2.

$$P_1(h_1) = 10 + (18 - 10)e^{-0.002h_1} \qquad (2.16)$$

$$P_2(h_2) = 10 + (25 - 10)e^{-0.002h_2} \qquad (2.17)$$

In order to achieve reasonable results and comparable scales, we calibrate the cost parameters and employ the following numerical expressions for the seasonal economic gains:

$$\Pi_1(h_1) = P_1(h_1)h_1 - 6h_1 \qquad (2.18)$$

$$\Pi_2(h_2, x_2, y_2) = P_2(h_2)h_2 - \frac{24000h_2}{D_x(x_2,y_2)+D_y(x_2,y_2)} \qquad (2.19)$$

We display Π_1 and Π_2 in Fig. 2.4 to offer a brief visualization of the economic drive behind the optimization. The profit Π_1 only depends on seasonal harvest while Π_2 is also state dependent. For most states, Π_2 is monotonically increasing, while for some states it is concave e.g. when $D_x + D_y = 2000$.
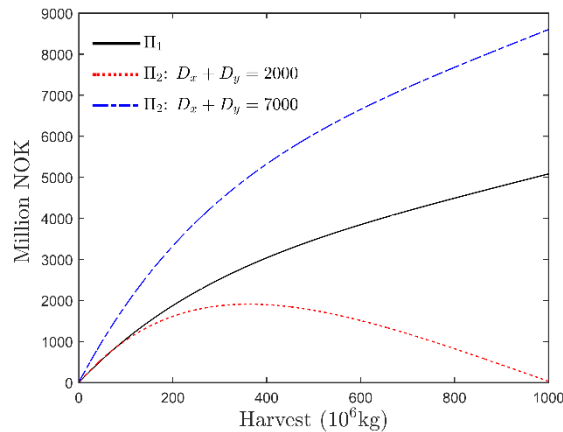
Fig. 2.4 Seasonal net economic gains (million NOK)

The graphical comparison of the unit harvest cost is illustrated in Fig. 2.5. The red surface manifesting the costs in S2 writes as $\frac{24000}{D_x(x_2,y_2)+D_y(x_2,y_2)}$. The blue surface represents the unit

harvest cost of 6 NOK/kg in S1. When the stocks are abundant, S2 costs are lower than S1. When either group is getting scarce, it becomes much more costly to harvest in S2.
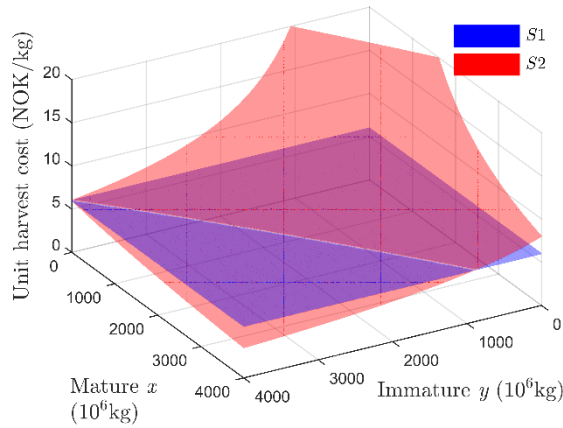
Fig. 2.5 Seasonal unit harvest costs (NOK/kg) on the state space

2.3.3 Two cases

We look at two special values of selectivity parameter s: zero and one. For any in between values, the optimization results are analysed but not included here because they lie between the results when s equals to zero and one. When $s = 0$, the whole stock is harvested separately timewise: in S1 only the mature is harvested and in S2 only the immature is harvested. Therefore, we denote this case as MIH (Mature Immature Harvest). When $s = 1$, only the mature group lends itself to harvesting during both S1 and S2. This case is referred to as MMH (Mature Mature Harvest).

For case MIH, the harvest range in S2 is $h_2 \in [0, D_y(x_2, y_2)]$; for case MMH, it is $h_2 \in [0, D_x(x_2, y_2)]$. The escapement policy constraints are $u_1 \in [0, x_1]$ and $u_2 \in \begin{cases} [0, D_y(x_2, y_2)], s = 0 \\ [0, D_x(x_2, y_2)], s = 1 \end{cases}$ respectively for each season.

We present here what we believe is most representative and informative. The cases are independent and can potentially represent specific fisheries policies. We argue that it is legitimate to compare them thanks to properly established numerical scales.

Despite a certain level of parameter sensitivity, the general features of the numerical results remain robust. For example, a 10% increase or decrease of the maturity parameter a leads to roughly 3% change in h_1 and 5% in h_2 under case MIH. A 10% increase or decrease of the price parameter α leads to an average change of 3% in h_1 and 4% in h_2 on the state space.

2.4 Results

The most interesting and relevant findings of our results are summarized in Table 2.1. We elaborate and provide more detailed observation and analysis for each finding.

Table 2.1 Highlights of findings

No.	Findings	Explanation & Implications
1	Seasonal moratorium in S2 at the equilibrium cycle.	Equilibrium cycle acts as a target escapement policy. Emerges naturally as the optimal policy.
2	Some states begin with positive year round harvests and later evolve into harvesting (S1)-closure (S2).	Typically, moratorium is employed to recover the stock. We find novel optimal trajectories.
3	Overshooting potentially caused by contraction process that is stronger in one direction.	A rising group biomass could be the overshooting effect instead of a signal to increase harvest.
4	A 'valley' effect (declining harvest with rising states) exists for harvest in S1	We find it closely related to the unit profit difference between seasons.
5	Fishing pressure on the mature in S2 arouses even more aggressive harvesting on the mature in S1.	It is more efficient to gain biomass by having a large amount of immature rather than mature in the beginning of S1.
6	The mature is pressed to lower levels along its optimal path towards the equilibrium cycle in Case MMH.	When the immature is protected and can fully utilize the growth, the mature can be harvested to a lower level.
7	Case MMH results in a lower value of the resource in the long term.	Higher efficiency of transferring the mature to the immature via spawning. We should avoid high pressure on the mature in S2 so that they reproduce better in S1.

2.4.1 Dynamic paths under optimal harvest

Starting from various initial states, we follow the seasonal optimal harvesting policies and reach the permanent 'equilibrium' that consists of two seasonal states in a yearly cycle as in Fig. 2.6.

One novel observation is that the equilibrium h_2 in both cases are zero[5], signifying that the optimal harvesting includes a seasonal closure of the fishery, which is not forced by regulations but emerge naturally instead (Finding No.1). In both cases, it is optimal to harvest 57 kilotons of the mature fish during S1 and leave the whole stock to itself during S2. The cycle acts as a target escapement policy. In the optimal equilibrium cycle, the location of the stock during S2 can also be treated as a Marine Protected Area (MPA). A typical yearly model may simply overlook such important implications provided by our model with uneven season lengths and multiple harvesting seasons.
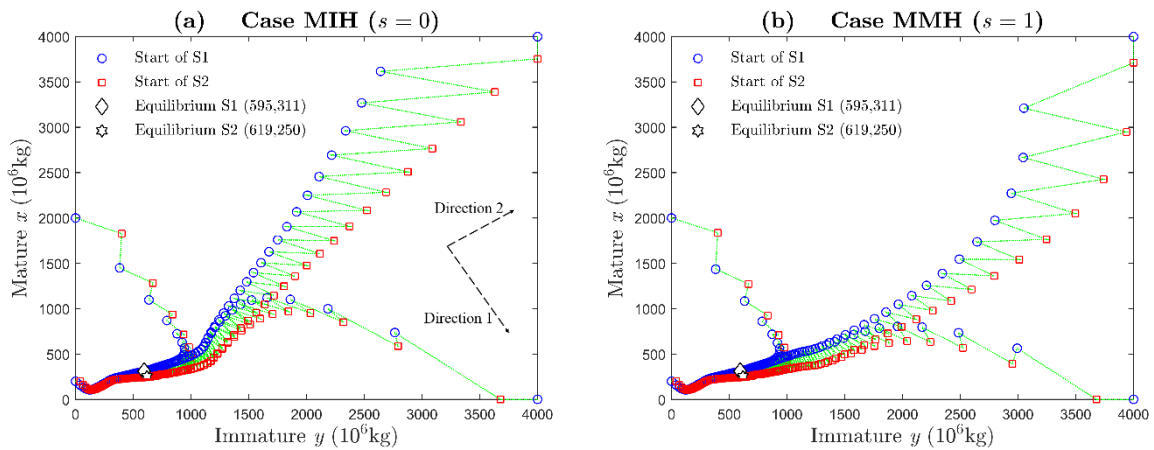
Fig. 2.6 Dynamic paths with optimal policy in 200 years with initial states: (0, 4000), (200, 0), (2000, 0) and (4000, 4000). Contraction directions illustrated in (a).

Harvest moratorium is not a new topic or finding from other fisheries models (Clark, Clarke, & Munro, 1979; Kasperski & Wieland, 2009; Kennedy, 1992). Nonetheless, it is novel to bring seasonality into such models and reach a moratorium during a part of the annual cycle as a long-term optimal steady situation. This coincides with practices in real fisheries: harvesting is often assumed to take place during the whole year but many fleets only harvest seasonally.

Typically, the stock first goes through moratorium to recover the biomass and then lends itself to harvesting when it becomes abundant enough. Our results contradict this pattern and indicate that many initial states begin with positive harvests in both seasons but later evolve and settle down in an annual equilibrium cycle of harvesting (S1)-closure (S2) (Finding No.2). Visual examples of such trajectories include initial states of (4000, 4000) and (0, 4000) in Fig. 6.

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5 A consistency check for the validity of our numerical results is that both cases end up in the same equilibrium cycle with the same equilibrium values since selectivity in S2 makes no difference when we do not harvest in S2 at all.

Moreover, even when the stock undergoes moratorium first and harvesting later, our results exhibit diversified paths towards the equilibrium cycle. When the initial state is 200 kilotons of the mature, an annual moratorium first applies, and then h_1 becomes positive but S2 remains a moratorium period. When the initial state enlarges to 2000 kilotons of the mature, S1 is the only harvesting season and this situation is sustained for the entire trajectory.

In both cases, various initial states seem to join a certain path converging to the equilibrium cycle. The hallmarks of the trajectories imply that the contraction can be strong in one direction so that the states move quickly towards the path and weak in another so that they progress slowly along the path. We find that the contraction is much faster along direction 1^6 than direction 2^7 (see Fig. 2.6(a)) for both cases in the neighbourhood of the equilibrium cycle. Though our calculation is valid closely around the equilibrium states, the features seem rather global in the state space.

Overshooting could come as a result of the speed and direction of the contraction process (Finding No.3). Given an initial state far from the equilibrium states, it takes a long time to approach the steady state. Together with a faster speed along a certain direction, overshooting can emerge. For example, starting with the maximum immature state in Case MIH (see Fig. 2.6(a)), the mature stock quickly rises above the equilibrium then slowly drops towards the steady state. Therefore, observing an increase of group biomass could potentially be the overshooting effect instead of a clear indication to harvest more.

2.4.2 Seasonal optimal harvest policies

We identify decreasing harvest with increasing states ('valley') on h_1 surfaces as shown in Fig. 2.8 (Finding No.4). To investigate closely, we demonstrate seasonal unit profit of harvest and seasonal harvest along the diagonal of the state space. For example, according to the solid blue line in Fig. 2.7(a), it is optimal to harvest 174 kilotons of the mature on top of the 'valley' (diagonal states 960) and only 152 in the bottom (diagonal states 1500) even though both states are more prolific in the latter case.

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6 The eigenvalue is 0.58 for direction 1 with eigenvector of (-0.82, 0.58) for both cases. A smaller eigenvalue indicates a stronger and faster contraction. The direction of contraction is specified by the eigenvector.

7 The eigenvalue is 0.97 for direction 2 with eigenvector of (0.48, 0.88) for both cases.
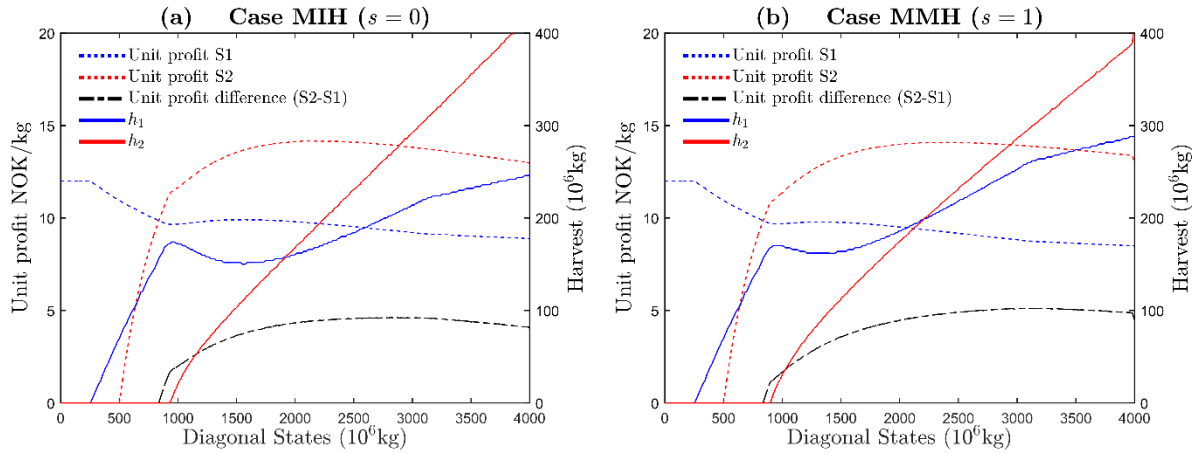
Fig. 2.7 Seasonal unit profit in NOK/kg (blue and red dotted lines), unit profit difference in NOK/kg (black dotted lines) and harvest policy in S1 and S2 (blue and red solid lines) along the diagonal of the state space

It is counterintuitive to diminish harvest given stocks that are more abundant. Since Π_1 is monotonically increasing with harvest, the key must lie in the benefits from later periods. In Fig. 2.7, when h_2 (red solid line) remains zero, h_1 (blue solid line) seems to be rising with the states at a fast rate. When h_2 becomes positive, the fishing pressure is diverted partly to S2. Consequently, h_1 increases less drastically. Moreover, when the economic advantage of S2 is prevailing (dotted black line in Fig. 2.7), h_1 decreases and forms the dent in order to exploit higher profits from S2. The 'valley' is an interesting and robust effect because of the trade-off, or competition between harvesting seasons.

We observe that part of the optimal h_1 surface with a weak mature stock reaches the myopic solution, in which merely the present season's net revenue is valued. The immature group is not harvested in S1 and a considerable part of it will transit to join the mature in the next season S2, maintaining the stock biomass level. Therefore, in the myopic part, it is economically justified to wipe out the mature group in S1 when the immature is abundant. The results of h_2 in Fig. 2.8 evince that the natural moratorium area on the stock space in S2 tends to maintain its shape and size under different parameter s.
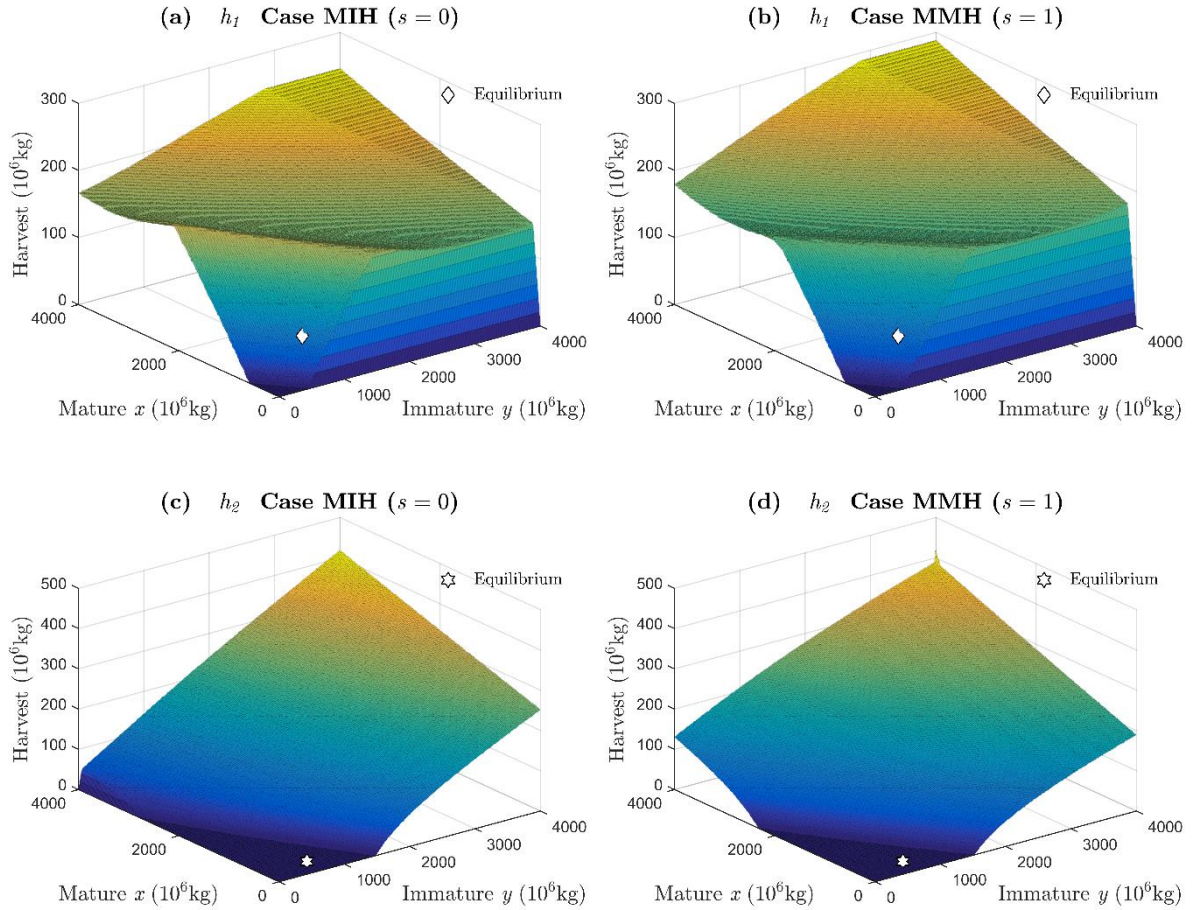
Fig. 2.8 Optimal harvest policies

(a): h_1 of Case MIH (equilibrium state indicated by white diamond) (b): h_1 of Case MMH (white diamond) (c): h_2 of Case MIH (white hexagram) (d): h_2 of Case MMH (white hexagram)

2.4.3 Selectivity s in S2

The key divergences of h_2 between the two cases, where s is the only varying parameter, lie along the two zero axes as shown in Fig. 2.8. On the one hand, when the mature group is zero i.e. $x_2 = 0$, what is left available for harvesting in S2 is $D_x(0, y_2) = a y_2$ of the mature and $D_y(0, y_2) = G(y_2) - a y_2$ of the immature. Referring to Fig. 2.3(b), we deduce that there is less harvesting potential when we select the mature group D_x with $s = 1$. Consequently, h_2 along this axis exhibits stronger conservation when $s = 1$. On the other hand, when the immature group is zero i.e. $y_2 = 0$, what is left available to take is $D_x(y_2, 0) = m x_2$ of the mature and $D_y(x_2, 0) = 0$ of the immature. Hence, for Case MIH when $s = 0$, h_2 along this axis is all zero while for Case MMH when $s = 1$, h_2 becomes positive as the mature group prospers.

We probe into the trade-off between harvesting in S1 and in S2. Fig. 2.9 manifests the proportion of h_1 to the total annual harvest if we follow the optimal policy during the year. The grey surface depicts this proportion with each state in the stock space as the starting state of S1. The blue part at the bottom displays the natural moratorium zone where it is optimal to leave the stock to itself throughout the year. The red plain highlights the area where the proportion is smaller than 50%, meaning that h_2 is more dominant in the annual harvest. We find that they cover locations in the state space that correspond to the area where the unit harvest cost in S2 (red surface) lies beneath the unit cost in S1 (blue surface) in Fig. 2.5. It is straightforward that when it is cheaper in S2, more is harvested during S2. However, due to higher prices in S2, the red planes cover some additional state space area: areas with abundant immature and little mature in Case MIH and areas with little immature and abundant mature in Case MMH.
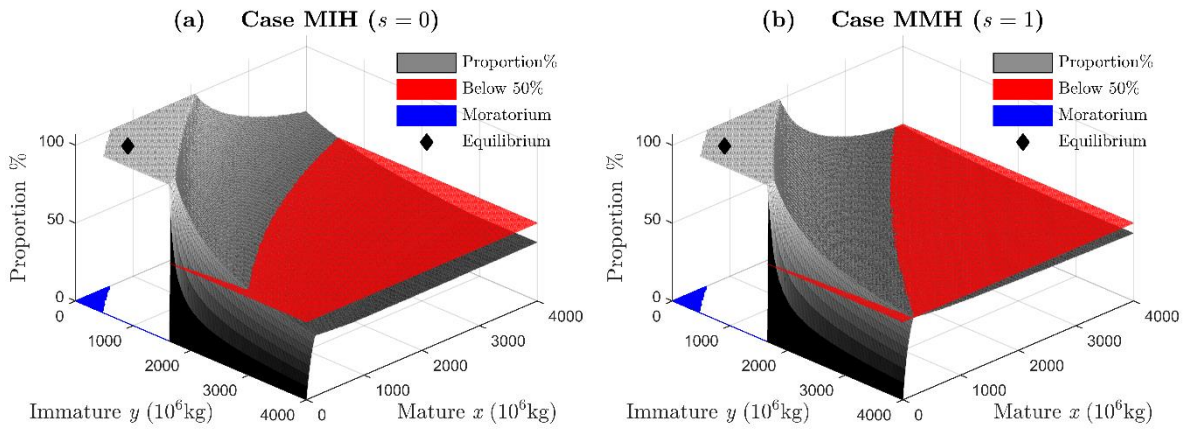
Fig. 2.9 Proportion of h_1 to the annual harvest following the optimal policy. Grey surface is the percentage. Blue area is the annual moratorium zone. Red plane represents 50%. Black diamond is the equilibrium state in S2.

Selecting only the mature in S2 ($s = 1$) enhances h_1 on most part of the stock space not only with regard to the harvest biomass level in Fig. 2.8 but also the relative proportion in Fig. 2.9. It may seem counterintuitive that the fishing pressure on the mature in S2 arouses even more aggressive harvesting in S1 on the same group (Finding No.5). This thought-provoking phenomenon may be explained from the biological perspective: the strongest growth of the stock is the stepwise linear transition of the immature during S1, which is dependent on both states. The marginal biomass increment incurred by one more state unit is $(0.00005y + 0.2)$ of the mature and $(0.00005x + 0.92)$ of the immature. Apparently, it is more efficient to gain biomass by having a copious amount of the immature rather than the mature in the beginning of S1. When $s = 0$, we are forced to diminish the immature in the beginning of S1; hence it is

crucial to retain the mature at a higher level in order to exploit the growth. When $s = 1$, the immature group is free from being harvested and can fully utilize the growth, so the mature group can lend itself to harvesting towards a lower level. One piece of evidence is the optimal path of the maximum states as the initial point in Fig. 2.6: Case MMH ($s = 1$) tends to press the mature group to lower levels along its path towards the equilibrium cycle (Finding No.6).

The surfaces of both value functions are almost identical between cases. Therefore, we calculate the average value in the state space as an indicator for its general level. The difference between the two cases is 1274 million NOK for V_1 and 972 for V_2. Both are lower for Case MMH ($s = 1$), suggesting that the compulsory selection targeting the mature group in S2 results in a lower value of the resource in the long term (Finding No.7). This may relate to the high efficiency of transferring mature biomass to the immature through spawning and the relatively low efficiency of transferring immature to the mature via natural maturation. Hence, when determining the selectivity s, it can be worthy to avoid putting all the fishing pressure on the mature group in S2 and to allow it to produce more offspring in the spawning season S1. A complete ban on harvesting a certain part of a stock may stem from conservative purposes but can also hinder the value and potential of the whole stock.

2.5 Conclusions and discussions

We employ a DP technique in a periodic Bellman approach to present a seasonal model numerically. We obtain the optimal feedback harvesting policies given multiple uneven seasons and two-dimensional states. Our method provides a useful tool for seasonal regulation measures, such as periodic moratorium, MPA, seasonal TACs, fleet-specific TACs and seasonal target escapement policies. The trade-off of harvesting between seasons is driven by multi-fold mechanisms, which leads to profound implications for bioeconomic modelling and policymaking.

Key findings are listed in Table 2.1 and the most intriguing ones are rather counterintuitive. Our approach allows a naturally occurring seasonal closure of the fishery in the equilibrium cycle. This cannot occur in a non-seasonal model. The permanent equilibrium cycle in Case MIH and Case MMH resembles a target escapement policy (Reed, 1979). Instead of an annual target, which is a common practice, our results propose alternating optimal seasonal target stock levels. A moratorium period or a MPA is usually considered a specified policy scheme to be evaluated (Pelletier et al., 2009). Unlike a typical policy that imposes a moratorium to recover the stock, we find that many states first undergo harvesting all year round and then develop into

the long-term seasonal moratorium. In a standard single state bioeconomic model, higher levels of the stock often indicate bigger harvest potentials (Clark, 2010). According to our results, a growing group biomass could be the overshooting effect instead of a clear signal to increase harvest. On part of the state space, the optimal seasonal harvest declines with rising states, creating a 'valley'. This may originate from the unit profit difference between seasons. Similar discoveries exist in some multi-species studies (Leif K. Sandal & Steinshamn, 2010). Fishing pressure on the mature in one season leads to even heavier harvesting on the same group in the next season. Protecting the immature may derive from conservative purposes but in fact diminish the value of the whole stock. Therefore, it can be beneficial to avoid only harvesting the mature group in order to let it reproduce better in the spawning season. This finding goes in line with the general concern regarding the Spawning Stock Biomass (SSB). For example, ICES often employs SSB reference limits to ensure a strong spawning stock.

Our approach can potentially be adapted for varying biological and economic seasonality in fisheries. Some forage fish, such as herring, make vast migrations between spawning, feeding and nursery grounds. In this case, the season number and season lengths can be adjusted according to behaviours that are shaped by ocean currents, plankton abundance, trade-off between predator avoidance and growth (Brönmark et al., 2008). Although the curse of dimensionality is inevitable, the multidimensional states may instead represent predator-prey multispecies relationships, such as cod and capelin in the Barents Sea. This allows for more focus on ecological analysis and food web studies, which are gaining more attention as a research topic. For single species models, the biological dynamics could include depensation so that the model becomes more realistic for species that are particularly vulnerable at lower states. The decision variables, i.e. harvesting policies, may be extended to season-state-specific instead of simplified to season-specific as in our presentation.

Possible modelling extensions exist in various directions. It is interesting to employ size(group)-dependent price functions (Zimmermann et al., 2011). The model can be in continuous time and apply the periodic Bellman approach to investigate seasonality. Stochasticity can be added to the biological transitions via different fluctuations in such as the ocean temperature, food availability, cannibalistic behaviour, predator distribution etc. Randomness may also be incorporated in the price and cost functions. We are currently working on establishing stochastic models with seasonality.

Furthermore, the big potential and rich implications of our approach go beyond the model we present in this work. This approach enables us to take into consideration longer seasonality that spans more than a single year. For example, the El Nino conditions affect the ocean temperature drastically at intervals spanning several years, influencing numerous ocean lives as well as commodity prices in different countries. In addition, the flexibility to employ arbitrary uneven season lengths makes it possible to combine more than a single seasonal pattern in the model. For instance, we can model El Nino as a condition that happens once every several years together with a seasonal feature, such as upwelling in the spring, which takes place annually. Except for natural phenomenon, such combined seasonal patterns may potentially be extended to market prices, technological breakthroughs, economic trends and countless other aspects in natural resource modelling.

Seasonal models require season-specific data for real fishery management. An approach with sophisticated seasonality surely puts a burden on the managerial costs due to more frequent monitoring of the stock. Assuming that the required data is available, the techniques of inferring input parameter values from the observations are conceptually the same as for a classical yearly model.

Appendix

Table A1. Notations in the model.

Variables	Definitions	Unit
x	Mature group biomass	kiloton (10^6 kg)
y	Immature group biomass	kiloton (10^6 kg)
x_1	Mature group biomass in the beginning of S1	kiloton (10^6 kg)
y_1	Immature group biomass in the beginning of S1	kiloton (10^6 kg)
x_2	Mature group biomass in the beginning of S2	kiloton (10^6 kg)
y_2	Immature group biomass in the beginning of S2	kiloton (10^6 kg)
X_1	Mature group biomass in the end of S2	kiloton (10^6 kg)
Y_1	Immature group biomass in the end of S2	kiloton (10^6 kg)
h_1	Harvest biomass in S1 from the mature group	kiloton (10^6 kg)
h_2	Harvest biomass in S2 from either group	kiloton (10^6 kg)
h	Either h_1 or h_2	kiloton (10^6 kg)
$F(x_1, y_1)$	Immature group biomass in the beginning of S2	kiloton (10^6 kg)
$D_x(x_2, y_2)$	Mature group biomass before harvest in S2	kiloton (10^6 kg)
$D_y(x_2, y_2)$	Immature group biomass before harvest in S2	kiloton (10^6 kg)
$G(y_2)$	Nonlinear biomass growth for the immature group in S2	kiloton (10^6 kg)
$P_1(h_1), P_2(h_2)$	Unit price of the fish	NOK/kg
Π_1	Net economic gain in S1	Million NOK
Π_2	Net economic gain in S2	Million NOK

Parameters	Definitions	Values
b_1, b_2, b_3	Parameters of the nonlinear growth $F(x, y)$ for the immature in S1	0.92, 0.00005, 0.2
x_{max}, y_{max}	Biomass upper bound for both states	4000, 4000
s	Selectivity of harvesting in S2	0 or 1
m	Linear growth for the mature in S2	0.75
a	Linear biomass transition from immature to mature in S2	0.2
c	Cannibalistic behavior in S2	0.00001
g_1, g_2	Nonlinear growth for the immature in S2	1.2, 0.00005
p_{min}, p_{max}, α	Parameters for the pricing function (P_L or P_H)	10, 18, -0.002 or 10, 25, -0.002
k_1	Constant unit cost in S1	6
k_2	Cost parameter in S2	24000

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Chapter 3

3. Greed is good: from super-harvest to recovery in a stochastic predator-prey system

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Abstract

This paper demonstrates a predator-prey system of cod and capelin that confronts a possible scenario of prey extinction under the first-best policy in a stochastic world. We discover a novel 'super-harvest' phenomenon that the optimal harvest of the predator is even higher than the myopic policy, or the 'greedy solution', on part of the state space. This intrinsic attempt to harvest more predator to protect the prey is a critical evidence supporting the idea behind 'greed is good'.

We ban prey harvest and increase predator harvest in a designated state space area based on the optimal policy. Three heuristic recovery plans are generated following this principle. We employ stochastic simulations to analyse the probability of prey recovery and evaluate corresponding costs in terms of value loss percentage.

We find that the alternative policies enhance prey recovery rates mostly around the area of 50% recovery probability under the optimal policy. When we scale up the predator harvest by 1.5, the prey recovery rate escalates for as much as 28% at a cost of 5% value loss. We establish two strategies: modest deviation from the optimal on a large area or intense measure on a small area. It seems more cost-effective to target the stock space with accuracy than to simply boost predator harvest when the aim is to achieve remarkable improvement of prey recovery probability.

Keywords: stock recovery, resilience, predator-prey, ecosystem, stochastic

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3.1 Introduction

Marine fisheries are vital resources for human society and ecosystem, especially with growing world population and increasing food demands (FAO, 2008). The current progress towards sustainable fisheries is at an insufficient rate and the stock recovery is generally overwhelmed by unsustainable fishing practices (Teh, Cheung, Christensen, & Sumaila, 2017). To improve the current fisheries management requires an effort to address both the direct economic gains and the indirect ecological values of the resource. To assess the existence value and the risk of extinction of any species, it is more realistic to include dynamic stock interactions from an ecosystem perspective. Feasible management practices that aim to rebuild a weak stock in a system should be evaluated with regards to its effects and costs.

With widely recognized depletion of various global fisheries and increasing climate uncertainty, many researchers and policy makers have prioritized their focus to sustainability, stock recovery and collapse. Incorporating sustainability considerations adds additional layers of complexity to conventional models (Howarth, 1995). Woodward & Bishop (1999) included a sustainability constraint in their model to suggest long-term sustainable management in a deterministic setting. Kama & Schubert (2004) chose to derive decision rules of a sustainable development under a special case of preference uncertainty. Britten, Dowd, Kanary, & Worm (2017) revealed how a changing environmental context can reform the recovery timeline and delay the rebuilding of depleted fish stocks. Rosa, Vaz, Mota, & Silva (2018) developed an age-structured model where the objective function incorporates the risk of fishery collapse, in addition to profit maximization and fishers' preference for stable landings. They managed to illustrate that their framework assists the analysis and design of harvest control rules. Diwakar Poudel, Sandal, & Kvamsdal (2015) discovered that the risk of stock collapse due to stochastically induced critical depensation increases with stochasticity in a single species model.

Healthy and diverse marine ecosystems are essential in order to ensure they are resilient to inevitable shocks and stresses. It has become clear that ecosystem-based fisheries management (EBFM) is a desired approach towards resilient fisheries (Link et al., 2012). In contrast to treating different species individually and separately, an ecosystem-based approach deals with the interacting components in a systematic and dynamic way. The most common models of single species ignores the ecological as well as the technological and economic interactions among species (Kasperski, 2010). This may lead to misleading results and incorrect policy decisions causing over or under exploitation of the stocks (Fleming & Alexander, 2002; Maravelias, Damalas, Ulrich, Katsanevakis, & Hoff, 2011). Usually the economic interactions

play an important role in generating the overall harvesting pressure on the commercially valuable species.

Multispecies models in the literature have been attempts to account for ecosystem concerns. Earlier multispecies studies focused mainly on a predator-prey relationship from different trophic levels (May, Beddington, Clark, Holt, & Laws, 1979; Yodzis, 1994). However, they merely addressed the biological yields without considering the economic aspects of harvesting. Later, some suggested deterministic bioeconomic models with an optimal equilibrium solution (Fleming & Alexander, 2002; Kar & Chaudhuri, 2004). They found it difficult to solve for the optimal paths even with linear objective functions. Some concluded that multispecies management provides distinct advantages allowing for more realistic modelling of growth rates and better understanding of fish population dynamics (Hollowed et al., 2000). Nonetheless, multispecies bioeconomic models are limited due to unavailability of the analytical solutions (Posch & Trimborn, 2010) and computational difficulties (Singh, Weninger, & Doyle, 2006).

Most multispecies bioeconomic studies propose optimal harvesting in a deterministic setting (Clark, 2010; Woodward & Bishop, 1999; Sandal & Steinshamn, 2010). However, most of the economic and biological processes take place in an uncertain environment in reality (Charles & Munro, 1985). Uncertainties in fishery include stock measurement error, parameter estimation errors, environmental variability influencing the growth of fish stocks, structural uncertainty and model error (Charles, 1998; Sethi, Costello, Fisher, Hanemann, & Karp, 2005; Nøstbakken & Conrad, 2007; Roughgarden & Smith, 1996; Poudel, Sandal, & Kvamsdal, 2015; Kvamsdal, Poudel, & Sandal, 2016). Most of the extant literature that evaluates long-term stock management does not consider such uncertainties sufficiently. Stochastic models with a single species have gained popularity over the years (Clark & Kirkwood, 1986; Hannesson, 1987; Sandal & Steinshamn, 2017; Sethi et al., 2005; Singh et al., 2006; Kugarajh, Sandal, & Berge, 2006; Bruce, J, & Christopher, 2009), but stochastic multispecies models are still uncommon in the literature (Agnarsson et al., 2008).

We employ a feedback policy approach where the optimal control (harvest) is a direct function of the state variable (stock). Instead of the commonly used time paths approach, the feedback approach is superior when faced with uncertainty (Agnarsson et al., 2008). We also apply the DP (Dynamic Programming) technique, conducting value and policy iterations to solve for the optimal policy and value (Judd, 1998). The DP approach is a useful method when considering the multispecies management model under stochasticity (Sanchirico & Springborn, 2011).

This study is inspired by previous work of Sandal & Steinshamn (2010). In this paper, we work with a continuous-time stochastic multispecies predator-prey bioeconomic model. Based on the optimal policy derived from the numerical solution of a predator-prey system, we generate alternative harvesting policies in search for recovery of the less valuable prey stock. We conduct simulations to investigate the probability of prey recovery in a certain period of time, which also mirrors the risk of prey collapse. We progressively refine the recovery plans using three heuristics to explore the consequential benefits and costs. Using the DP technique, we evaluate the costs of implementing the alternative policies, providing references for the existence value of the prey species. We introduce the concept of value elasticity of recovery, which sheds light on a possible state-dependent recovery approach for further research.

3.2 Predator-prey system

3.2.1 Model

We employ a continuous-time predator-prey bioeconomic model. The general interdependent deterministic biological growth model is similar to those of Clark (1990), Agnarsson et al. (2008) and Poudel et al. (2012). Letting x be the prey species state and y be the predator stock state, the continuous-time deterministic growth increments of the system are:

$$dx = [f(x,y) - u_x]dt$$
$$dy = [g(x,y) - u_y]dt \tag{3.1}$$

Functions $f(x,y)$ and $g(x,y)$ are the biological growth functions of the prey and predator respectively, while u_i stands for the harvest rate of species ($i = x, y$). The term dt is the time increment. Furthermore, a two-species interaction model with stochastic dynamics is generated by adding volatility terms in Eq. (3.1) in the following way:

$$\begin{pmatrix} dx \\ dy \end{pmatrix} = F(x, y, u_x, u_y)dt + \sigma(x,y) \begin{pmatrix} dB_x \\ dB_y \end{pmatrix} \tag{3.2}$$

where $F(x, y, u_x, u_y) = \begin{pmatrix} f(x,y) - u_x \\ g(x,y) - u_y \end{pmatrix}$ and $\sigma(x,y) = \begin{pmatrix} \sigma_{11}x & \sigma_{12}y \\ \sigma_{21}x & \sigma_{22}y \end{pmatrix}$.

It can formally be considered as the two-dimensional controlled *Ito-process*: $dZ = F(Z, u) + \sigma(Z)dB$. In Eq. (3.2), term $\sigma(x,y)$ is the diffusion matrix, and dB_x and dB_y denote the incremental basic Brownian motion, which is independent and identically distributed (*i.i.d.*) with mean zero and variance dt. The additive noise formulation is a general Wiener process and contains the multiplicative case (Poudel et al., 2015; Kvamsdal et al., 2016). We assume

the stock biomass states and harvests to be nonnegative.

The economic part of the model consists of the net revenues from harvesting both species, which can be obtained by adding revenue from each stock.[8] Let $\pi\left(x, y, u_x, u_y\right)$ be the total net revenue, where $\pi_x\left(x, u_x\right)$ and $\pi_y\left(y, u_y\right)$ are the revenues from x and y respectively:

$$\pi\left(x, y, u_x, u_y\right) = \pi_x\left(x, u_x\right) + \pi_y\left(y, u_y\right) \tag{3.3}$$

$$= p_x(u_x)u_x - c_x(x, u_x) + p_y(u_y)u_y - c_y(y, u_y)$$

where $p_i(\cdot)$ and $c_i(\cdot)$ are inverse demand functions and cost functions respectively. We assume that the objective of the fisheries management authority (such as a regional fisheries management organization that acts as the sole owner of the resource) is to maximize the expected net present value (NPV) of harvesting activities of the fishery over an infinite time horizon. This can be achieved by maximizing the following functional with respect to the policy or control variable u_i.

$$J\left(x, y, u_x, u_y\right) = E\left[\int_0^\infty e^{-\delta t}\, \pi\left(x, y, u_x, u_y\right)dt\right] \tag{3.4}$$

The nonnegative parameter δ is the discount rate, and E is the expectation operator. The value function and the optimal policy can be obtained by solving the Hamilton-Jacobi-Bellman (HJB) equation:

$$\delta V(x, y) = \max_{u \in U \subset R_+^2}\{\pi(x, y, u_x, u_y) + DV^T(x, y)F\left(x, y, u_x, u_y\right) \tag{3.5}$$

$$+ \frac{1}{2}tr[\sigma(x, y)\sigma^T(x, y)D^2V(x, y)]\}$$

where $DV(x, y) = \begin{pmatrix} \frac{\partial}{\partial x}V(x, y) \\ \frac{\partial}{\partial y}V(x, y) \end{pmatrix}$ and $D^2V(x, y) = \begin{pmatrix} \frac{\partial^2}{\partial x^2}V(x, y) & \frac{\partial^2}{\partial x \partial y}V(x, y) \\ \frac{\partial^2}{\partial y \partial x}V(x, y) & \frac{\partial^2}{\partial y^2}V(x, y) \end{pmatrix}$.

Closed-form solutions are usually rare because of the difficulty in solving the HJB equation given nonlinearity and boundary conditions. The Markov chain approximation approach is one of the most effective numerical methods for such problems with nonlinear control. The numerical optimization results will be presented in section 2.3. The solution procedure will not be emphasized with details in this paper.

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[8] We assume that there is no market interactions between the demand for and prices of the two species. Therefore, the revenues from both species are added together.

3.2.2 Numerical specifications

The diversified ecosystem in the Barents Sea harbours two key fish species, namely capelin (*Mallotus villosus*), a plankton feeder, and Northeast Arctic cod (*Gadus morhua*), the main predator of capelin. Cod is considered the main resource of the Norwegian commercial white fish industry (Kugarajh et al., 2006), while capelin is the largest pelagic stock in the Barents Sea and potentially the most abundant in the world. The relationship between cod and capelin is highly dynamic in the Barents Sea ecosystem (Bogstad et al. 1997). As the prey, capelin is crucial for the growth of juvenile cod (Dalpadado & Bogstad, 2004). The cod recruitment and survival rate are directly affected by climatic conditions and availability of food. Higher temperature during spawning and more capelin have a positive effect on cod recruitment (Hjermann et al., 2007). Given various kinds of uncertainties in the Barents Sea ecosystem (Flaaten et al., 1998), we apply a stochastic multispecies model consisting of cod and capelin as the interacting predator and prey species.

Functional forms of the biological and economic components of the model, as well as the specifications of parameter values are based on the works of Agnarsson et al. (2008) and Sandal & Steinshamn (2010). Built on empirical data and analysis from existing work, our model ensures that the functional forms are relevant and the parameter values occupy a realistic part of the parameter space. We assume that a single authority who seeks to maximize the joint benefit of the predator-prey system manages both stocks. The upper bounds for the state space is $xmax = 10000$ and $ymax = 12000$. We set 100 grid points along each dimension of the state space to discretize the problem numerically. The biological growth functions of capelin (prey *x*) and cod (predator *y*) in Eq. (3.1) are specified as:

$$\begin{aligned} f(x,y) &= a_1 x^2 - a_2 x^3 - a_3 xy \\ g(x,y) &= b_1 y^2 - b_2 y^4 + b_3 xy \end{aligned} \tag{3.6}$$

where a_1, a_2, a_3, b_1, b_2, and b_3 are parameters. The first two terms in Eq. (3.6) for each species represent the biomass growth in the absence of the other species and hence stand for the aggregated effects of the rest of the ecosystem. The xy-term represents the interactions between the two stocks. The numerical specification goes as follows[9]:

$$\begin{aligned} f(x,y) &= 1.8 \cdot 10^{-4} x^2 - 1.19 \cdot 10^{-8} x^3 - 2.1 \cdot 10^{-4} xy \quad (10^6 kg/year) \\ g(x,y) &= 2.2 \cdot 10^{-4} y^2 - 3.49 \cdot 10^{-11} y^4 + 1.82 \cdot 10^{-5} xy \quad (10^6 kg/year) \end{aligned} \tag{3.7}$$

---

9 The value of parameter a_1 is $1.8 \cdot 10^{-3}$ on the referred papers, which is supposedly a typo.

The volatility of each species is assumed to be a linear function of its own stock level. It is also assumed that there is no correlation between the stochastic terms. The diffusion matrix is thus specified as: $\sigma_{11} = 0.2$; $\sigma_{12} = 0$; $\sigma_{21} = 0$ and $\sigma_{22} = 0.2$.

The functional forms of the economic part in Eq. (3.3) are specified as follows:

$$
\begin{aligned}
p_x(u_x) &= p_1 \\
c_x(x, u_x) &= q_1 u_x^{\alpha_1} \\
p_y(u_y) &= p_2 - p_3 u_y \\
c_y(y, u_y) &= \frac{q_2 u_y^{\alpha_2}}{y}
\end{aligned}
\tag{3.8}
$$

where $p_1, q_1, \alpha_1, p_2, q_2, \alpha_1, \alpha_2$, and p_3 are price, cost, and elasticity parameters. We assume that capelin is an unevenly distributed schooling species and the unit cost of harvesting is independent of its stock level. The simplified revenue function can be rewritten as:

$$
\pi\left(y, u_x, u_y\right) = p_1 u_x - q_1 u_x^{\alpha_1} + p_2 u_y - p_3 u_y^2 - \frac{q_2 u_y^{\alpha_2}}{y}
\tag{3.9}
$$

The corresponding numerical specification is:

$$
\pi\left(y, u_x, u_y\right) = u_x - 0.07 u_x^{1.4} + 12.65 u_y - 0.00893 u_y^2 - 5848.1 \frac{u_y}{y} \quad (10^6 NOK)
\tag{3.10}
$$

It is also worth mentioning that both stocks have commercial value but the predator is much more worthy in the market. The unit price of cod is 12.65 NOK/kg while that of capelin is 1NOK/kg. The optimal feedback solutions are calculated with a 5% discount rate $(\delta = 0.05)$.

3.2.3 Evidences for heuristics from the optimal policy

We interpret the optimal harvests from the point of view of how the first-best policy determines the development of the prey species capelin. As shown by the blue dashed lines in Fig. 3.1, in a deterministic world, some initial states end up with capelin extinction while some others go to the other extreme of capelin prosperity. It seems that a slight change in the starting point could give rise to drastic differences of capelin stock development. In a single species setting, both stocks are sustainable on its own. The coexistence of two stocks in a predator-prey relationship gives rise to a possibility that the prey may disappear.
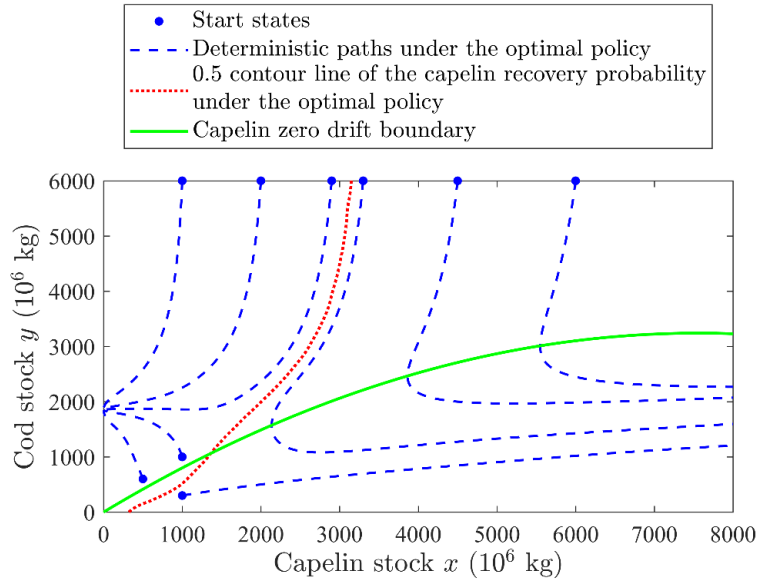
Fig 3.1 Development paths from various initial states

If capelin disappears, we lose the direct revenues from harvesting capelin as a commercial species as well as the indirect revenues due to a weaker cod stock. The risk of capelin collapse is embedded in the optimization model and the way that the optimal policy deals with this possibility is reflected in itself. We observe that the optimal harvest of both species expresses some level of intentional 'prey protection and recovery' strategy.
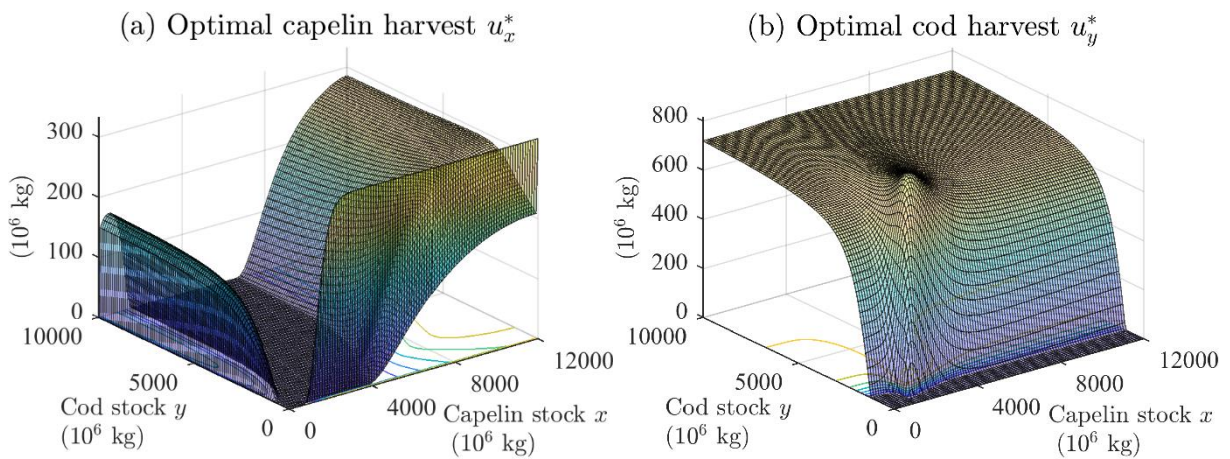

Fig 3.2 Optimal harvest for (a) capelin u_x^* and (b) cod u_y^*

The first evidence is the moratorium region or the 'valley' phenomenon in the optimal capelin harvest as shown in Fig. 3.2(a) with a surface plot of the harvest and in Fig. 3.3(a) with a contour plot of the harvest. When capelin stock is low, for example below 2000·10^6kg, it will most likely go extinct due to predation no matter how much human harvests. Therefore, the optimal capelin harvest is positive in this region in order to take advantage of whatever value that can still be acquired. Inside the moratorium area, i.e. bottom of the 'valley', the optimal policy

equals zero, seeking to avoid capelin from disappearing or to slow down the extinction process. The conservation of capelin is stronger in the presence of higher volatility because a more stochastic cod stock intuitively requires more abundant food resource (Poudel et al., 2014).

The second evidence is that on part of the state space the optimal cod harvest exceeds the myopic cod harvest, forming what we call a 'super-harvest' phenomenon. The myopic or 'greedy' solution to the optimization model only takes into account a single period when calculating the profit function. We solve for the myopic harvests by maximizing $e^{-\delta}\pi\left(x, y, u_x, u_y\right)$ with respect to u_x and u_y. The results are displayed in Appendix Fig. B. Usually the greedy harvest is, as the name implies, larger than the optimal policy. Therefore, it is very novel and counterintuitive to observe an obvious bump in Fig. 3.2(b) and a significant state space area of the super-harvest in Fig. 3.3(b). Inside the 0 contour line in Fig. 3.3(b), the positive difference numbers together with the purple colours indicate how much the optimal harvest is even greedier than the greedy solution.
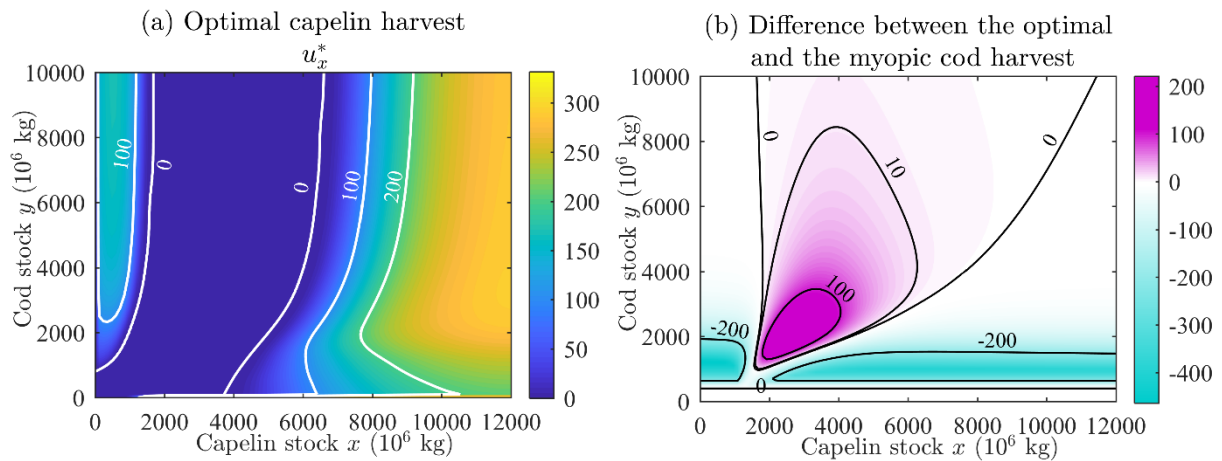
Fig 3.3 (a) Contour plot of the optimal capelin harvest u_x^*; (b) Contour plot of the difference between the optimal cod harvest u_y^* and the myopic cod harvest

Referring to the green line of capelin zero drift boundary as shown in Fig. 3.1, one possible explanation is that it is optimal to harvest a bit more cod so that the cod state shrinks at a faster speed. This enhances the probability that the states enter the area of positive capelin growth and land on the safe side for capelin. When cod is abundant, the speed-up effect could accumulate for a long time as cod transits from high to low. Thus, a low level of super-harvest applies to most of the cod abundant region. The optimal cod harvest bump (dark purple region in Fig. 3.3(b)) is probably where the system would extract the most potential. Given that the super-harvest emerges naturally and intrinsically with the optimal policy, the message is that it is worthy to give up some short-term revenues from the predator if the prey has a higher

probability of long-term recovery. There is no additional existence value of the prey in the objective function and super-harvest does not appear when there is only one species.

The strategy of prey protection and recovery manifested in the optimal policy could be amplified when the non-economic values of the system are accommodated as well. In this work, we follow the two evidences analysed above, establish alternative polices according to various heuristics and evaluate the recovery plans concerning effects and costs.

We define A on the state space as the area in which the recovery plan replaces the optimal policy. The first evidence, i.e. moratorium of the optimal capelin harvest, is an intuitive and straightforward strategy. Following this, we set all capelin harvest to zero in A for all recovery plans. Due to the large area of moratorium in Fig. 3.3(a), some alternative harvests may result in no change for capelin policy. The second evidence, i.e. super-harvest of the optimal cod policy, is an active and more aggressive approach where the system chooses to be 'merciful' to the prey by being 'greedy' to the predator. Parameter θ $(\theta > 1)$ describes the degree of deviation of the alternative cod harvest from the optimal cod harvest in A. The higher θ gets, the bigger existence value we bestow implicitly to the prey species.

3.2.4 Simulation settings

In order to evaluate the effect of a recovery plan, we look at the probability of capelin recovery and the improvement achieved by implementing the alternative policy instead of the optimal. We conduct Monte Carlo simulations with a feedback policy. To imitate a continuous-time Markov process, we apply a time unit of one year and a time step dt of 0.01 with 2000 periods, which leads to a simulated time horizon of 20 years. For each initial state, we simulate 2000 realizations and then calculate the corresponding probabilities. A trajectory is counted as capelin recovery if x exceeds or equals to $4000 \cdot 10^6$kg in the end of the simulation period. Similarly, a trajectory is considered capelin collapse if x ends up smaller than $120 \cdot 10^6$kg. The sum of capelin recovery and collapse probabilities equals one on most of the state space. We therefore conclude that 20 years is a long enough simulation period for most states to settle down either as capelin recovery or collapse.
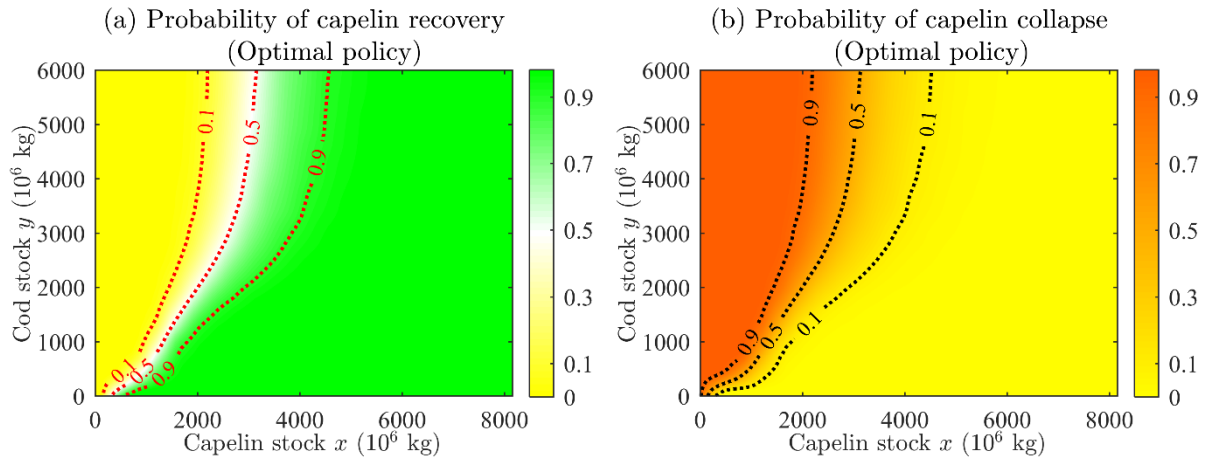
Fig 3.4 Contour plots of the probability of (a) capelin recovery and (b) capelin collapse

As shown in Fig. 3.4, the contour lines for both figures extend more and more vertically as cod stock increases, offering limited new information. In addition, the investigation interest shrinks as capelin becomes more and more plentiful. Thus, when presenting the results, we focus on the state space area where cod stock y is smaller than $6000 \cdot 10^6$kg and capelin stock x is smaller than $8000 \cdot 10^6$kg. A total of 100 points from a 10-by-10 even grid are chosen as the initial states for the simulations. The results are then transferred onto the fine grid of the state space using cubic interpolation.

In Fig. 3.4(a), less than 10% of realizations end up as recovered for states within the area to the left of the 0.1 contour line. More than 90% of realizations are considered as capelin recovery for states within the area to the right of the 0.9 contour line. Similar conclusions could be drawn from the probability map of capelin collapse in Fig. 3.4(b). The risk of prey extinction is highly mirrored to the probability of prey recovery. Therefore, we focus on presenting and interpreting the results of capelin recovery in the rest of the paper.
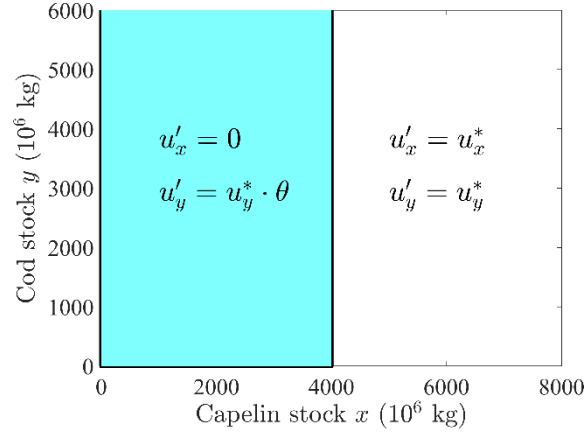
3.3 The Simple Heuristic (SH)

3.3.1 Recovery plan of SH

The objective of all the recovery plans is to decrease the risk of capelin extinction and to promote the sustainability and resilience of the predator-prey system while considering the cost and practicality of the alternative policy. Following the two evidences in section 2.3, the recovery plan of Simple Heuristic (SH) is generated out of plain intuitions. We ban the prey harvest and increase the predator harvest on area A^{SH} where both stocks coexist and the prey is considered weak. For capelin stocks lager than $4000 \cdot 10^6$kg, the likelihood of capelin recovery is already very high. Deviation from the optimal policy is deemed unnecessary in this case.

Therefore, the area, as shown in Fig. 3.5, is defined as $A^{SH} = x \in (0,4000] \cap y \in (0, ymax]$. The alternative harvest is calculated as follows:

$$u'_x(x,y) = 0$$
$$u'_y(x,y) = u^*_y(x,y) \cdot \theta$$

(3.11)

Fig 3.5 State space area A^{SH} (blue colored) and corresponding policy of the Simple Heuristic (SH) recovery plan for capelin and cod

Harvesting the predator in a greedy manner has its practicality when it comes to implementation of the recovery policy. Taking more predator not only helps to recover the prey but also imposes no pressure regarding extra monetary investment. Putting more cod on the market lowers the total profit according to the objective function, but it can be positive for the local labour market as well such as the processing companies. In addition, raising the quota of a valuable commercial species is unlikely to confront strong opposition from fishers. Sandal & Steinshamn (2010) have investigated the possibility of rescuing the prey by harvesting the predator. In this paper, we focus on the probabilistic evaluation of the recovery plan together with the corresponding cost.

3.3.2. Evaluation of SH

We employ two values of parameter θ, i.e. 1.2 and 1.5, and demonstrate on the state space the probability of capelin recovery, the improvement of the probability by the recovery plan and the value loss for diverging from the optimal policy.

We plot the contour lines of capelin recovery probability under the optimal policy (see Fig. 3.4(a)) as a reference using red dotted lines. As illustrated in Fig. 3.6(a,b), harvesting more predator than the optimal cod policy increases the likelihood of capelin recovery and pushes the blue dashed contour lines to the left. When θ takes the higher value of 1.5, the shift towards

left is more obvious. The two styles of contour lines merge where cod state lies within the moratorium region. In the moratorium area, cod harvest remains zero no matter what value θ takes.
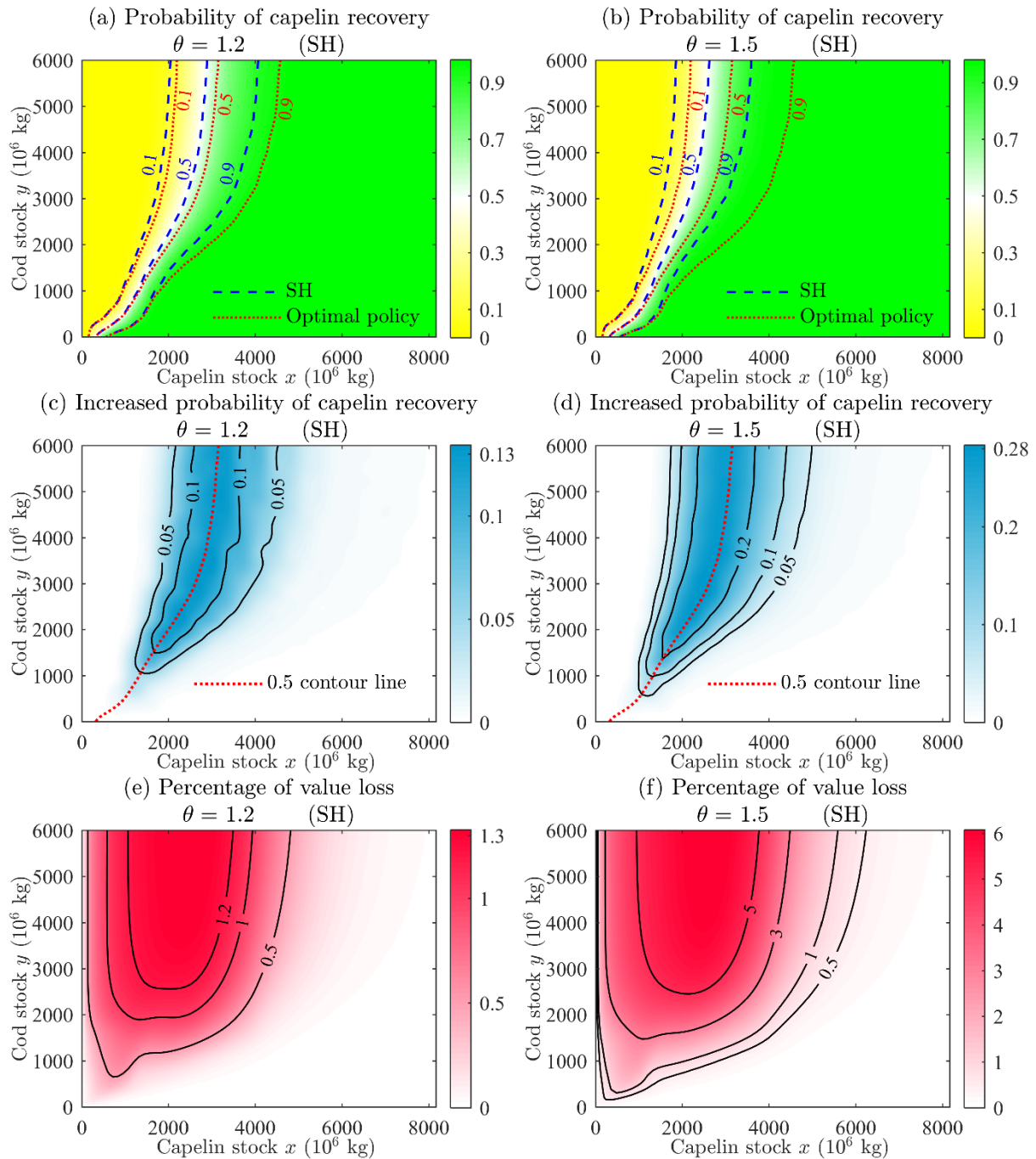
Fig 3.6 Recovery plan of Simple Heuristic (SH) with $\theta = 1.2$ and $\theta = 1.5$:

(a,b) Probability of capelin recovery; (c,d) Increased probability of capelin recovery;

(e,f) Percentage of value loss compared with the optimal value

In order to describe the performance of the recovery plan, we present the difference of capelin recovery probability between the alternative and the optimal policy. From Fig. 3.6(c,d), we

observe that the enhancement is most evident on the narrow blue area within the 0.05 contour line, which wraps around the red dotted 0.5 contour line of capelin recovery rate under the optimal policy. Comparing with the marked areas in Fig. 3.3, where the optimal policy intends to rescue the prey in search for extracting the most value, the blue areas fall inside of the moratorium region in Fig. 3.3(a) and coincides with the super-harvest area to a large extent.

When $\theta = 1.2$, the most effective case is for the recovery rate to rise from around 50% to 63%, leading to a maximum improvement of 13% in the darkest blue part. When $\theta = 1.5$, the most fruitful case is for the recovery probability to escalate from around 50% to 78%, which is rather prominent. The improvement looks rather trivial and negligible outside of the 0.05 contour lines. It is either because capelin stock is doomed to go extinct when the predator is strong and the prey is weak or because capelin species is quite safe already even without the extra harvest on cod.

Note that the approach of calculating recovery probabilities automatically leaves out some special situations. For example, a state may begin by developing into capelin collapse and then shift towards the safe equilibrium thanks to a weakening cod stock. But 20 years of simulation period is not long enough for it to be considered as recovered. Therefore, the boost in capelin recovery rate exposes only a part of all the effects produced by the recovery plan. For some states that are outside of and close to area A^{SH}, the likelihood of ending up with a recovered capelin stock also grows. Although the alternative harvest has a sharp change on the boundary, the impact distributes more progressively.

Corresponding to the benefit of the recovery plan, the other side of the coin is the incurred cost related to implementing the alternative instead of the optimal policy. By applying the value iteration DP technique, we are able to solve for the value function of a given policy. The amount of value difference between the alternative and the optimal harvest is the value loss in absolute terms.

The percentage of value loss, manifested in Fig. 3.6(e,f), is the percentage number of the value loss compared to the optimal value function. When $\theta = 1.2$, the worst case is that the recovery plan costs 1.3% of the optimal value. When $\theta = 1.5$, the alternative harvest could result in a value loss of as much as 6% of the optimal value. While the capelin recovery probability increment is approximately linear to the excessive cod harvest, the percentage of value loss is more sensitive to the change in alternative policy.

All the states that suffer from value losses spread fairly widely on the stock space. While it is definitive that the states inside A^{SH}, except for the moratorium region, are subjected to value losses due to deviation from the optimal policy, it is less obvious for the states outside. Keep in mind that the value of each state is the sum of the expected and discounted profits in an infinite time horizon. In a stochastic world, as long as there exists a possibility for a state outside A^{SH} to enter the area of inevitable value losses at some point, the total value will be lower than the optimal no matter how long the state stays inside of A^{SH}. Therefore, as cod stock enlarges, the predator is able to bring down capelin faster and drag the state deeper inside of A^{SH}, which provokes a heavier value loss. As a result, we can clearly observe that the contour lines from Fig. 3.6(f) form an asymmetric 'U' shape leaning towards the right.

The approach of solving the value function for a recovery plan and then obtaining the value loss percentage from it is a novel and interesting method. We construct the recovery plan intending to preserve the capelin stock as a prey for the cod, but other benefits such as development for the processing companies and environmental significance for other related species are also concomitant. Thus, the value loss we acquire here can be considered as a reference for the upper bound of the existence value of capelin as a food source for cod.

3.4 The Refined Heuristic (RH)

3.4.1 Recovery plan of RH

Proceeding from the Simple Heuristic, we continue to refine the area of the alternative policy in pursuit of more promising capelin recovery and less value loss. From the previous results, we notice that certain states already have little risk of capelin extinction under the optimal policy and appears to have limited improvement when we switch to the alternative policy. Hence, we could alleviate the value loss by avoiding to carry out a non-optimal policy on the area that holds insubstantial need to rebuild capelin.

As a result, for the recovery plan of the Refined Heuristic (RH), we establish A^{RH} using the 0.9 contour line of capelin recovery probability under the optimal policy. As displayed in Fig. 3.7, the blue area A^{RH} includes the states that have a capelin recovery probability less than 90%. It is an attempt to distribute the efforts in a smarter way that they can be put to better use.
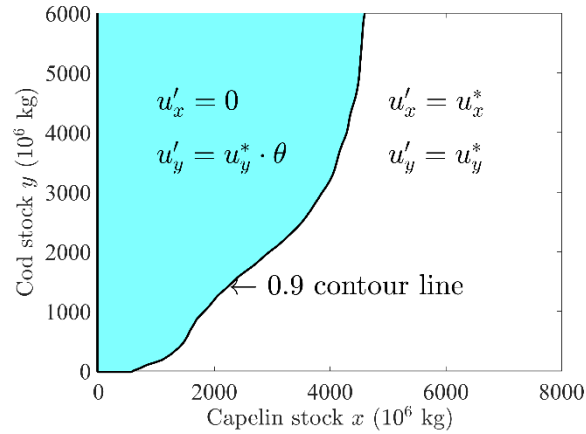
Fig 3.7 State space area A^{RH} (blue coloured) and corresponding policy of the Refined Heuristic (RH) recovery plan for capelin and cod

3.4.2. Evaluation of RH

As illustrated in Fig. 3.8, the capelin recovery rate (figures a,b) and the increased probability of capelin recovery (figures c,d) resemble very much the counterparts from SH under the same value of θ. Shrinking the unnecessary policy deviation does not contribute to notable capelin recovery enhancement but mainly to sparing the value loss.

Evidently, the contour lines of the same values distribute densely and narrowly under RH instead of widely and dispersedly with SH. This has disparate implications depending on where the state stands on the stock space. For the state of $2000 \cdot 10^6$kg capelin and $5000 \cdot 10^6$kg cod, the percentage of value loss is bound to reach the worst case of around 1.3% with $\theta = 1.2$ whichever recovery plan there is. For the state of $4000 \cdot 10^6$kg capelin and $2000 \cdot 10^6$kg cod with $\theta = 1.5$, the percentage of value loss is merely 0.2% for RH but is 1.2% for SH. The recovery plan of RH may not deliver a pronounced improvement regarding value loss in the former case but certainly performs better in the latter case. In addition, the maximum percentage of value loss is lower under RH for either choice of θ and the number of states involved in any definite value losses are much smaller under RH.
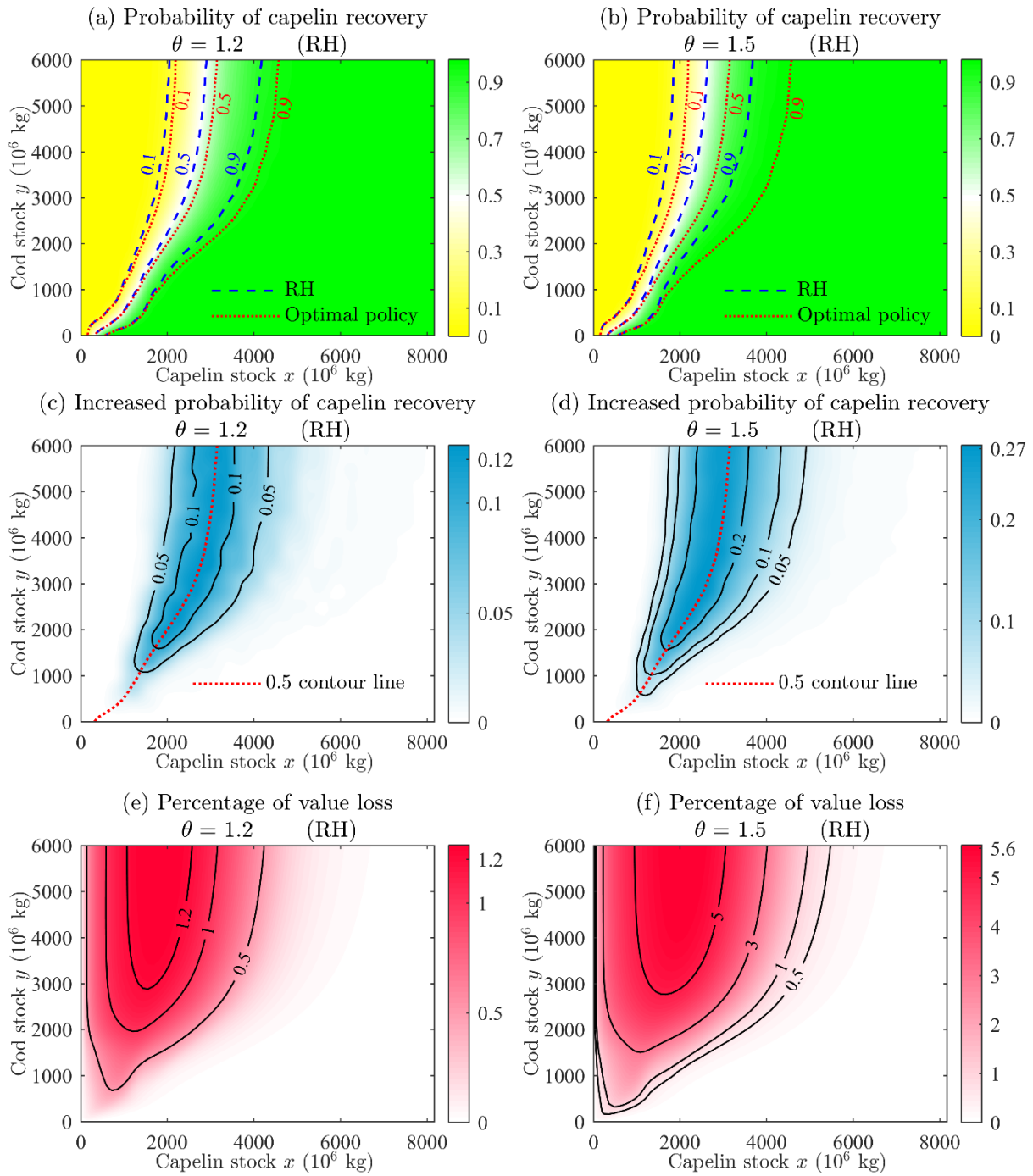
Fig 3.8 Recovery plan of Refined Heuristic (RH) with $\theta = 1.2$ and $\theta = 1.5$:
(a,b) Probability of capelin recovery; (c,d) Increased probability of capelin recovery;
(e,f) Percentage of value loss compared with the optimal value

Inspired by the concept of price elasticity of demand in economics, we divide the change of
capelin recovery probability by the percentage of value loss and refer to the quotient as the
'value elasticity of recovery'. For various states, the value elasticity of recovery can be very

distinct. The following Table 3.1 lists a comparison of the elasticity for the exact same state under SH and RH. This state lies within the alternative policy region in both recovery plans.

Table 3.1 Value elasticity of recovery for the state of $2500 \cdot 10^6$kg capelin and $3000 \cdot 10^6$kg cod under recovery plans of SH and RH

Recovery plan	Increased capelin recovery probability	Percentage of value loss	Value elasticity of recovery
SH $(\theta = 1.2)$	12.87%	1.25%	10.3
SH $(\theta = 1.5)$	28.79%	5.56%	5.2
RH $(\theta = 1.2)$	12.30%	0.99%	12.4
RH $(\theta = 1.5)$	27.76%	4.58%	6.1

While the price elasticity of demand in economics measures the responsiveness of the demanded quantity to a change in the price, the value elasticity of recovery estimates the sensitivity of capelin recovery probability increase to a unit of value loss. Under the recovery plan of SH with $\theta = 1.2$, for each percentage of value loss, the alternative policy is able to achieve an average of 10.3% capelin recovery rate increase for the chosen state. As θ rises, the value loss escalates at a higher speed making the value elasticity of recovery generally lower for both SH and RH.

The value elasticity of recovery goes up by roughly 20% from SH to RH under the same θ. This finding reinforces the argument behind RH that the recovery plan is deliberately refined to be more efficient at promoting capelin stock at the same amount of value cost. However, there are apparently some states with a low capelin stock that suffer from value losses but do not enjoy much privilege of capelin recovery. The value elasticity of recovery is zero for such states. Does it imply that it is useless to implement any recovery plan on such an area? We continue to another heuristic that utilizes the innate information from the optimal policy and avoids large area of zero elasticity when capelin stock is poor.

3.5 The Target Heuristic (TH)

3.5.1 Recovery plan of TH

After exploring the recovery potentials and value costs under the recovery plans of SH and RH, we pursue to target the relevant stock space area with higher levels of precision and sophistication. In the recovery plan of TH, the variable becomes the area of the alternative policy instead of the value of θ.

The inspiration and justification that lead to constructing A^{TH} are threefold. First, the novel phenomenon of 'super-harvest' remains a major drive behind restricting the alternative policy within the super-harvest boundary. It follows the innate feature from the first-best policy. Second, RH has proven to be more efficient and beneficial in comparison with SH, therefore we continue to utilize the contour lines of capelin recovery probability from the optimal policy. Third, we can observe clearly from Fig. 3.6(c,d) and 3.8(c,d) that the increased capelin recovery rates concentrate on a part of the stock space with a capelin stock roughly above $1500 \cdot 10^6$kg. The question follows spontaneously: is it still rewarding to deviate from the optimal policy at a cost of value loss when capelin is weak? It is intriguing to create cases where we switch back to the first-best policy for small capelin stocks.
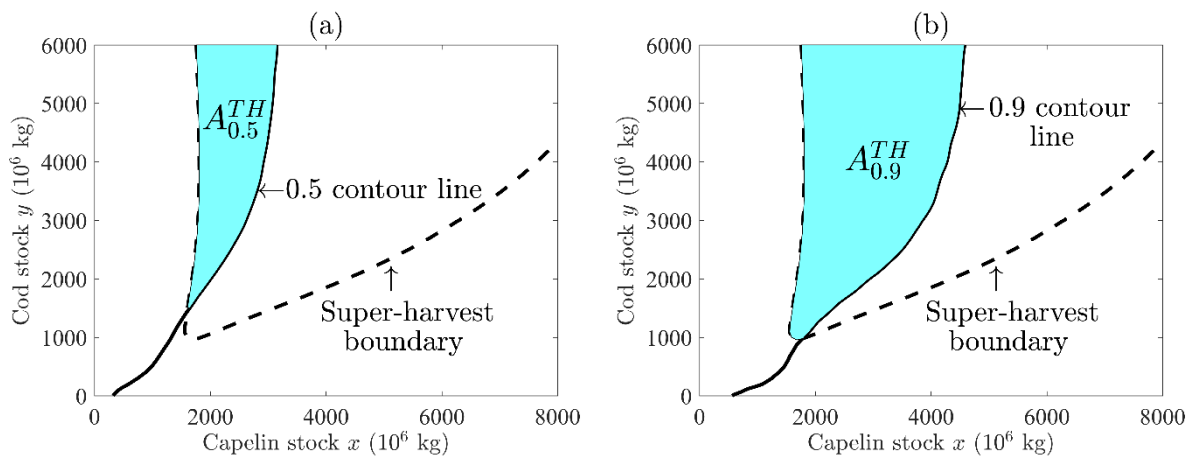
Fig 3.9 State space areas of the Target Heuristic (TH) recovery plan for capelin and cod (a) $A_{0.5}^{TH}$ and (b) $A_{0.9}^{TH}$

As displayed in Fig. 3.9, the narrow blue regions on the stock space lie in between the super-harvest boundary (dashed line) and the 0.5 or 0.9 contour line (solid line). The region of $A_{0.5}^{TH}$ covers the super-harvest area that lend itself to a capelin recovery probability of less than 50%. The area of $A_{0.9}^{TH}$ includes super-harvest states that have a capelin recovery rate of less than 90%. The latter is a stronger tool to preserve capelin and to boost the resilience of the predator-prey system. The recovery plan of TH takes advantage of the information extracted from the optimal policy in order to target very specific states on the stock space. With fewer states being affected by the recovery plan, we escalate the value of θ to 1.8 to produce an intense recovery policy on a concentrated region on the stock space, which also leads to comparable results to the previous SH and RH.

3.5.2. Evaluation of TH

We discover that the case of $A_{0.5}^{TH}$ with $\theta = 1.8$ produces similar results as SH and RH with $\theta = 1.2$ and the case of $A_{0.9}^{TH}$ with $\theta = 1.8$ generates resembling results to SH and RH with $\theta = 1.5$. For SH and RH, a large part of the state space is involved in some level of recovery rate increase but for many states it is merely neglectable. However, for TH the states with enhancements are more gathered within a outlined area. If we reckon the outermost contour line as a threshold for any noteworthy improvement, then the areas inside the 0.05 contour lines in Fig. 3.10(c,d) are marginally smaller than their counterparts in SH and RH. In addition, the maximum of capelin recovery rate increase is the highest under TH. The recovery plan of TH successfully achieves rather adequate capelin recovery effects and comparable results to those from SH and RH.

Compared to SH and RH, the recovery plan of TH employs a higher θ on a smaller targeted area, which is a trade-off between the number of states that lend themselves to alternative policies and the degree of deviation from the optimal policy. From Fig. 3.10(a), we notice that the 0.5 contour line shifts towards left to the utmost extent among the three contour lines, which is not the case for SH and RH. This is also reflected in Fig. 3.10(c) that the dark blue area mainly gathers to the left of 0.5 contour line. Moreover, the spaces between the 0.05 and 0.1 contour line are much tighter than that of SH and RH, indicating a sharper rise of improvement with $A_{0.5}^{TH}$. For the case of $A_{0.9}^{TH}$, the main differences emerge around the lower left corner of the state space. This region is left out by the super-harvest and is therefore not targeted under TH. As a result, the dark blue area in Fig. 3.10(d) distributes alongside the red dotted 0.5 contour line and ends where cod stock is above $1000 \cdot 10^6$kg while for SH and TH, it elongates and spreads until where cod is about $500 \cdot 10^6$kg.

For SH and RH, we seek to rescue inadequate capelin stocks even though they are much likely to develop towards extinction. The recovery plan manages to prolong the process of capelin collapse, which endows the value of capelin existing for a longer period of time in the system. For TH, the implicit argument is that the efforts to sustain the weak capelin states are not worthy and therefore we decide to extract the remaining commercial values from the system. Instead of trying to keep the poor capelin stocks, doomed sooner or later, present for more years, we focus on altering the ending for capelin on the selected parts of the state space, i.e. $A_{0.5}^{TH}$ and $A_{0.9}^{TH}$. Such distinctions consequently spawn considerable differences with regards to the percentage of value loss.
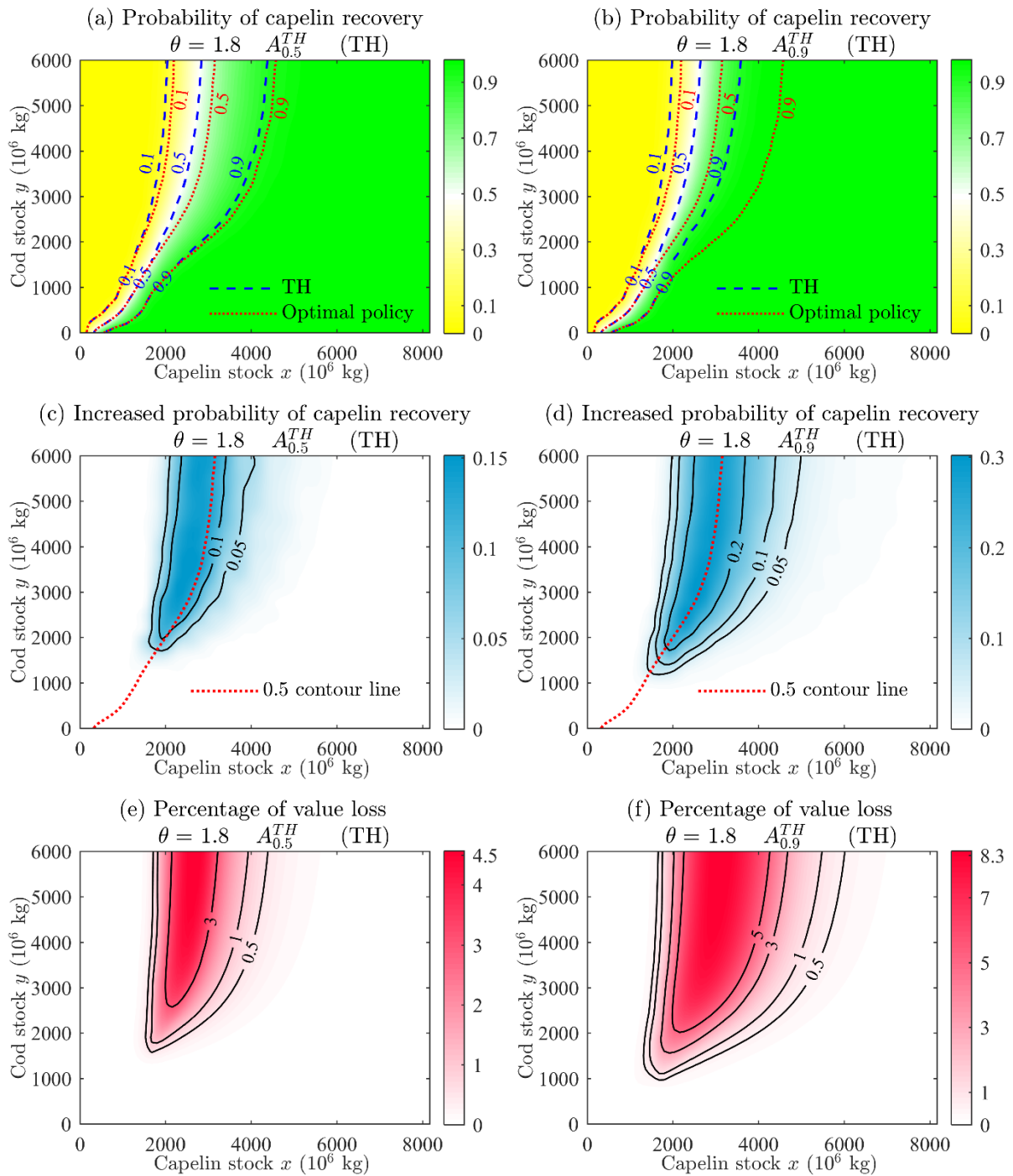
Fig 3.10 Recovery plan of Target Heuristic (TH) with $A_{0.5}^{TH}$ and $A_{0.9}^{TH}$:

(a,b) Probability of capelin recovery; (c,d) Increased probability of capelin recovery;

(e,f) Percentage of value loss compared with the optimal value

For most of the states with a capelin stock less than $2000 \cdot 10^6$kg, implementing the recovery plan of TH leads to zero value loss. But a weak capelin stock could experience a percentage of value loss up to 5% using SH and RH. If we put the case of $A_{0.5}^{TH}$ in Fig. 3.10(e) together with

the case of SH when $\theta = 1.2$ in Fig. 3.8(e), we see that the worst of value loss is 4.5% for the former and 1.2% for the later. The 0.5 contour lines signify an extensive shrinkage of the dark red area. For the majority of states, the value under the recovery plan of $A_{0.5}^{TH}$ does not deviate from the optimal value function. But for the ones that do deviate, bigger sacrifices in value are made in order to acknowledge the ecological importance of the prey. Similar characteristics hold for the case of $A_{0.9}^{TH}$ in Fig. 3.10(f) and the case of SH when $\theta = 1.5$ in Fig. 3.8(f). The highest percentage of value loss is 8.3% for the former and 6% for the latter, with a smaller gap between the two. Again, the outermost 0.5 contour lines manifest an obvious reduction of the 'suffering' area. And the innermost 5 contour lines reveal that the percentage of value loss rockets drastically towards the centre under TH.

The value elasticity of recovery for the same state ($2500 \cdot 10^6$kg capelin and $3000 \cdot 10^6$kg cod) is 4.36 for the case of $A_{0.5}^{TH}$ and 4.03 for the case of $A_{0.9}^{TH}$. Since the value of θ remains unchanged, it is expected that the two elasticities are close. Similar to previous results, the case of a stronger deviation from the optimal policy yields a lower value elasticity of recovery. It is also anticipated that the elasticity numbers from SH and RH are higher than those from TH. The distinctive design about TH is to let a smaller number of states carry the gains and losses of a more intensive approach. For this specified state, one percentage of value loss exchanges approximately four percentage of increase of capelin recovery probability. To determine whether this number can be considered sufficient would be another potential research direction.

3.6 Conclusion and discussion

In our stochastic predator-prey setting, a capelin stock develops towards two opposite endings: a prosperous ecosystem or a devasted one. We discover that the optimal policy inherently makes an effort to promote the prey, sometimes going so far as to be even greedier than the 'greedy harvest' for the predator. The super-harvest is an unconventional and thought-provoking discovery. A certain amount of economic benefit is sacrificed through excessive predator removal in order to drive the states faster into the region of capelin growth so that the risk of prey collapse becomes lessened. Note that the optimization objective does not put extra value on having an ecosystem instead of a single stock. Therefore, the super-harvest phenomenon implies that even though no existence value of the prey is deliberately counted, the optimal solution somehow calls out for maintaining the food source in the system for a longer period of time. The value loss could be reckoned as a reference for the upper bound of the existence value of the prey.

It has been shown that implementation of adequate policies to reduce fishing mortality is crucial for overexploited stocks to recover, underlining the positive impacts of science-based management (Zimmermann & Werner, 2019). The idea of acting greedily at harvesting the predator in order to spare the prey guides our heuristics that seek to rebuild the prey stock. To elevate the resilience of the system, we need to alter the probability of capelin extinction and recovery intentionally by deviating from the first-best policy.

One could potentially produce alternative management plans that are instantly effective but come with an unacceptable cost. Thus, we propose a succession of heuristics that modify the optimal policy in a way that both promote capelin recovery and limit the value loss. We generate three recovery plans with various degrees of complexity. The Simple Heuristic (SH) follows a straightforward rule that all states with a capelin stock less than a certain level apply the alternative policy. The Refined Heuristic (RH) selects the states with a capelin recovery rate less than 90%. The Target Heuristic (TH) focuses on the states of super-harvest that at the same time has a recovery rate less than 50% or 90%. Within the active area of the recovery plan, capelin harvest is zero and cod harvest is scaled up with parameter θ.

Our results show that all of the recovery plans manage to lift capelin recovery probabilities and shift the contour lines. The improvement in recovery rate mainly takes place around the contour line of 50% under the optimal policy. It is not a surprise that the approach contributes most to the area of states where future development is most obscure. The similarities dominate the results between SH and RH. When $\theta = 1.2$, the capelin recovery probability can grow by as much as around 13%. When $\theta = 1.5$, the maximum capelin recovery rate increase is roughly 28%.

While RH performs slightly better than SH regarding recovery, it delivers a much more visible difference regarding value loss percentage. The refined plan under RH successfully avoids unnecessary value losses on numerous states. With a specific example state, we demonstrate that this leads to at least 20% increase in value elasticity of recovery for RH. The value elasticity of recovery reflects the efficiency of transferring one percent of value loss into recovery probability growth and the exquisiteness in the design of the policy. The choice of θ is certainly crucial for the elasticity and should depend on the urgency degree of rebuilding the endangered stock.

The TH selects part of the super-harvest area and sets the cod harvest to 1.8 times of the first-best policy. It resembles a sharp knife with intense effort on a tiny state space area, creating

comparable recovery results and concentrated value losses. Very poor capelin stocks are directly harvested in the same resolute way as the optimal policy. The maximum value loss is 4.5% while this number is only about 1.3% for SH and RH under $\theta = 1.2$. The worst value loss is 8.3% while it is around 5.8 % for SH and RH under $\theta = 1.5$.

Let us assume the first 'jump' is from zero capelin recovery probability increase to the mild cases with a maximum increment below 15% and the second 'jump' is from the mild to the best improvements. An intriguing question is: what is smarter to do? To make the first jump, SH and RH lend themselves to a minor loss in value while TH chooses to sacrifice a small group of states. It is a judgement call to determine which strategy is more suitable depending on specific constraints. To make the second jump, both the area and level of value loss roughly double under TH. But for SH and RH, in addition to the area expansion the level of value loss has rocketed much more. It seems smarter to work with the area refinement than to simply increase θ when we seek to move from modest to pronounced recovery improvement. We emphasize the importance of the super-harvest finding and hope for better processes to target the accurate state space area for stock recovery with future research.
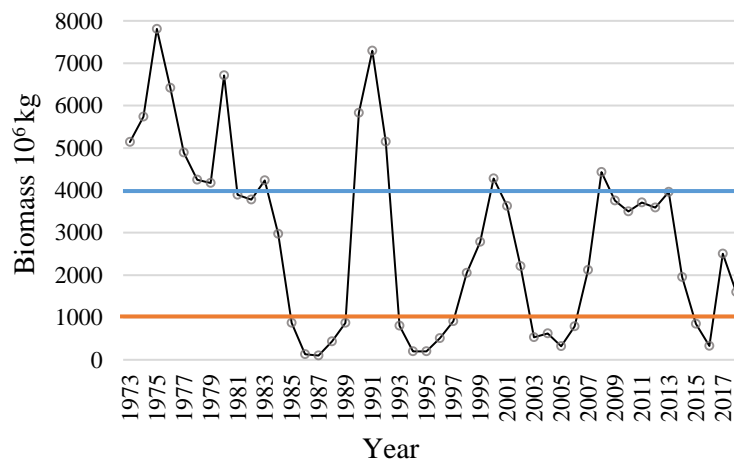
Fig 3.11 Total biomass of capelin stock in the Barents Sea
from 1973 to 2017 with unit of 10^6 kg

The capelin stock in the Barents Sea has a history of repeated collapses during the recent 50 years (Gjøsæter, Bogstad, & Tjelmeland, 2009). As displayed in Fig. 3.11, we highlight the most relevant range of states between $1000 \cdot 10^6$kg and $4000 \cdot 10^6$kg using colored solid lines. Yearly state transitions take place in a drastic and sharp manner inside this range. The overall trend is decreasing over the years despite the volatile ups and downs. Better stock management is called for in order to maintain a healthy ecosystem.

For the three times where capelin stock hit bottom and bounced above the blue line of $4000 \cdot 10^6$kg afterwards, it took 4-5 years to rise above the yellow line of $1000 \cdot 10^6$kg. According to our study, even with increased predator removal and banned capelin harvest, not much can be improved when capelin is extremely weak. A period of 5 consecutive years with a collapsed capelin stock signifies huge economic losses. Precautionary measures should be taken to avoid capelin state dropping below the lower yellow line. With the recovery plans proposed in our paper, we establish a buffer area on the state space with enhanced growth for capelin. It helps not only to rebuild the stock faster but more importantly to escape being trapped in a poor state for years.
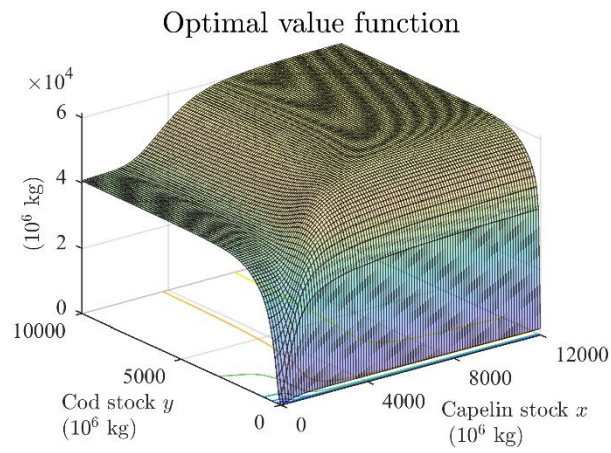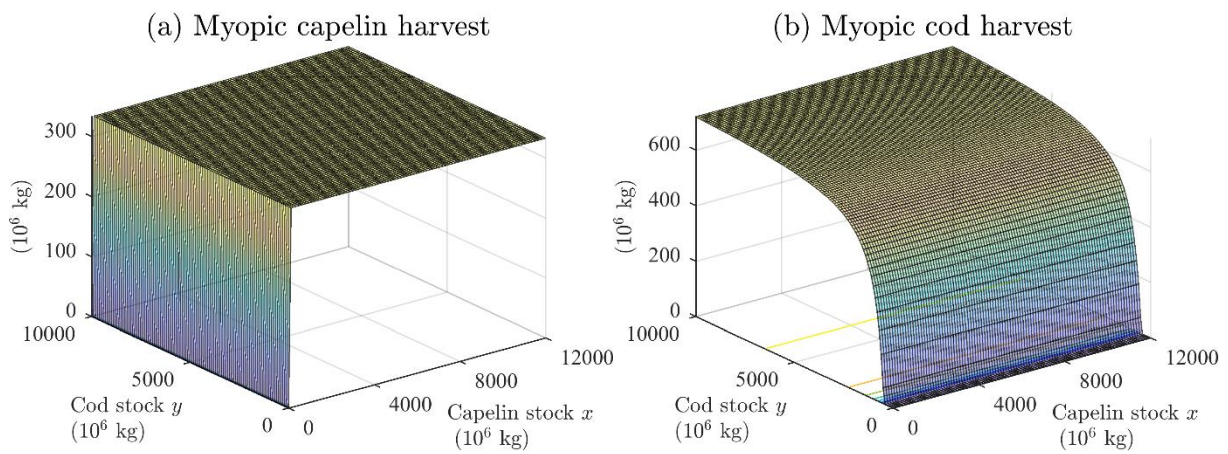
Appendix

Fig A. Optimal value function

Fig B. Myopic harvest policy for (a) capelin and (b) cod

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