ESSAYS ON INVESTMENTS IN EMERGING TECHNOLOGIES: A REAL OPTIONS APPROACH

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Introduction

Investments in emerging technologies are particularly risky, since, apart from economic uncertainty, firms must consider not only uncertainty in the arrival of innovations but also the presence of potential rivals. Moreover, embedded sources of risk may be transient, further complicating the investment decision. Thus, in order to develop efficient investment and operational policies, firms must account for the evolution and interaction of various types of uncertainties and also for the likely presence of a rival. For example, Netflix started out as a DVD-by-mail business, but gained a substantial first-mover advantage by being an early adaptor of the opportunities presented by online content delivery (Financial Times, 2018). Also, note that, although emerging technologies presented favourable opportunities for Netflix, they may be detrimental for those that are not making a timely technology transition. This occurred to Kodak, where its traditional market evaporated within a short time frame (The Economist, 2012), whereas its closest competitor, Fuji, aggressively explored other business avenues, such as copying and videotapes, and is consequently still a profitable company (Harvard Business Review, 2016). Although technological innovations present both opportunities and pitfalls for firms, innovations are often considered beneficial for the society as a whole. More specifically, emerging technologies such as renewable energy technologies have improved tremendously the last decade, thereby alleviating society's dependence on fossil fuels, yet the development of emerging technologies have been partly driven by government subsidies (Duffy *et al.*, 2015). In turn, this introduces political risk which became evident in Spain after the gradual removal of promised subsidies (The Economist, 2013). Hence, this thesis seeks to better understand how political uncertainty affects technology adoption, but also how risk aversion in a competitive environment impacts the incentive to invest or abandon technologies.

Chapter 1 discusses how the interaction between policy risk and technological uncertainty may impact investment decisions. In this context, politicians with the best of intentions seek to reduce emissions, encourage technology adoption and ensure energy security. However, firms must also consider the likelihood of a sudden change in the political climate. Further complicating the investment decision is the plethora of possible policy instruments with different risk characteristics, such as fixed or premium feed-in tariffs and renewable energy certificate trading (Boomsma *et al.*, 2012; Schallenberg-Rodriguez & Haas, 2012). Chapter 1 investigates a support scheme that takes the form of a fixed premium on top of the output price, and among other results, the thesis shows that greater likelihood of subsidy retraction lowers the incentive to invest, yet greater likelihood of subsidy provision facilitates investment. This is in line with, Boomsma & Linnerud (2015) when a subsidy retraction impacts existing as well as planned projects. In fact, there are two opposing forces: **i.** a likely retraction creates an incentive for early adoption to take advantage of the available subsidy and **ii.** the extra profit from operating with the subsidy is also believed to be short lived, and the latter force dominates the former. In addition, sequential technology improvements with embedded options complicate the effect of subsidies, since embedded options to invest in improved technology versions increase the investment incentive, and, as a consequence, the impact of subsidy retraction is less pronounced.

The presence of a rival and attitudes towards risk further complicate the problem of sequential technology adoption under uncertainty, which is the topic considered in Chapter 2. More specifically, duopolistic competition to adopt improved technology versions necessitates a more nuanced analysis of the rival's best response. Also, since investment opportunities typically involve technical risk that cannot be diversified, firms are likely to exhibit risk aversion. Indeed, risk neutral valuation may no longer be possible, since markets for technical risk are likely to be underdeveloped, thus preventing the construction of a replicating portfolio. Within the context of duopolistic competition, risk aversion typically increases the incentive to postpone investment, and the delayed entry by the follower is beneficial for the first-mover, who gets to enjoy monopoly profits for a longer time. However, the incentive to invest first and pre-empt a rival hastens technology adoption, particularly when this entails embedded options to adopt improved technology versions in the future. Although early market entry might secure monopoly profits, a potential rival can adopt an improved technology, thus leaving an incumbent worse off in the future. Consequently, we might encounter a war of attrition scenario, where neither firm wants to be the first to enter. By contrast, a firm that controls the innovation process does not face the threat of pre-emption. Nevertheless, technology spillover can take place when technologies are difficult to patent, which allows a rival to enter the market shortly after a proprietary leader. In the same line of work as Siddiqui & Takashima (2012), this thesis investigates how competition impacts sequential technology adoption strategies. For example, a firm may choose to adopt either every technology sequentially or wait for a new technology to become available before deciding which one to adapt. Results indicate that a firm would choose the latter strategy only when a more productive technology is likely to arrive. Furthermore, technological uncertainty may turn a pre-emption game into a game where the second-mover gets the higher payoff, and, thus, both firms will postpone technology adoption.

In order to shed further light on the investment decision under technological uncertainty, we also analyse how disruptive technologies create incentives to abandon existing technologies in Chapter 3. For example, Kodak developed digital cameras and a photo sharing webpage, but used the webpage primarily to promote printing of digital photos. Thus, Kodak's failure is also due to its reluctance abandon an old technology and embrace emerging technologies (Harvard Business Review, 2016). In this context, attitudes towards risk raises the incentive to abandon an existing project, yet the impact of risk aversion becomes more complex when a firm can also choose production capacity. In essence, the firm can reduce its exposure to price risk either through a smaller project or by investing later at a higher price. Results indicate that increasing risk aversion and technological uncertainty hasten investment by decreasing the amount of installed capacity. Furthermore, technological uncertainty may in fact reduce the loss in project value in the absence of managerial discretion over project scale.

Chapter 4 also focuses on disruptive innovations, but rather from the perspective of business cycles. There is a vast literature on business cycles pioneered by Schumpeter, who discusses how entrepreneurship and innovation are initiated under a harsh economic climate, which, in turn, create a fertile ground for economic expansions (Schumpeter, 1942). This is especially relevant for investments in emerging technologies that depend on innovations. In addition, such investments often rely on subsidies, such as those discussed in Chapter 1. Hence, indicators for political risk and leading economic indicators could potentially predict the likelihood of a regime switch (Filardo, 1994). Furthermore, these indicators are likely to be time-varying causing the likelihood for an economic expansion or recession to change over time. Thus, this thesis presents a technique to approximate the option value with time-varying transition probabilities that are determined by an indicator of future economic conditions. Results indicate that when the probability of a regime switch is low, the option value is greater (less) in the good (bad) regime under time-varying transition probabilities compared to fixed transition probabilities.

Although each chapter emphasizes on different sources of uncertainty, they all seek to account for the value of managerial flexibility. Most prominent, is the flexibility to wait for more information before committing to an irreversible investment. Other types of managerial discretion include the ability the scale the size of a project or the flexibility to abandon an existing market regime in order to enter a new one. In all cases, the value of managerial discretion is affected by underlying uncertainties, such as economic and technological uncertainty (Dixit & Pindyck, 1994). The former is typically reflected in output price fluctuations. More specifically, the output price is governed by a geometric Brownian motion (GBM), which implies that over an infinitesimal time period the firm expects the price change to be normally distributed. Another source of uncertainty stems from regime switching, which may be caused either by technological innovations or by a change in business climate. This can be formally introduced via a Poisson process, when the likelihood of a regime switch is constant for all time periods. However, Chapter 4 relaxes the assumption of constant regime-switching probabilities, and allows the likelihood of a sudden regime switch to depend on an economic indicator.

In order to value investment opportunities under several sources of uncertainty, a dynamic programming approach is employed. In fact, with infinite time horizon the investment problem gets a recursive structure, which facilitates theoretical analysis. Although, dynamic programming relies on a subjective discount rate, it can be used even when markets are incomplete and a replicating portfolio cannot be created (Dixit & Pindyck, 1994). One particular class of dynamic programming problems is called optimal stopping, where stopping corresponds to making decisions such as investment or abandonment, and the optimal timing of the decision is found by comparing the value of waiting for one more time period to the expected value from investment or abandonment. Hence, by employing this technique, we are able solve a broad spectrum of investment problems and shed light on interactions between several sources of uncertainty and optimal decisions.

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Chapter 1

Sequential Investment in Emerging Technologies under Policy Uncertainty

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Abstract

Investment in emerging technologies, such as renewable energy, is particularly challenging, since, apart from uncertainty in revenue streams, firms must also take into account both policy uncertainty and the random arrival of innovations. We assume that the former is reflected in the sudden provision and retraction of a support scheme, which takes the form of a fixed premium on top of the output price. Thus, we analyse how price, technological, and policy uncertainty interact to affect the decision to invest sequentially in successively improved versions of an emerging technology. We show that greater likelihood of subsidy retraction lowers the incentive to invest, whereas greater likelihood of subsidy provision facilitates investment. However, embedded options to invest in improved technology versions raise the value of the investment opportunity, thereby mitigating the impact of subsidy retraction and making the impact of subsidy provision more pronounced. Additionally, by allowing for sequential policy interventions, we find that the impact of policy uncertainty becomes less pronounced as the number of policy interventions increases.

Keywords: investment analysis, real options, renewable energy, policy uncertainty

1.1 Introduction

Promoting investment in alternative energy technologies may not only rely on the successful implementation of support schemes, but may also require investment in clean technology research and development (R&D) (Lomborg, 2001; The Economist, 2015a and 2015b). However, from the perspective of private firms, investment in emerging technologies is considerably risky since it is typically made in the light of economic and technological uncertainty, where the latter is often reflected in the random arrival of innovations. Consequently, within an environment of increasing economic uncertainty, the viability of private firms depends crucially on the timely adoption of technological innovations. For example, subsidies for renewable energy (RE) technologies fuelled a boom in solar panel manufacturing in China and allowed solar production capacity to increase significantly. Combined with the decrease in the price of silicon, the main component of traditional solar panels, this reduced the competitive advantage of US companies, many of which either went bankrupt or were purchased by Chinese companies (The New York Times, 2013). Also, Germany's biggest utilities, initially invested heavily in coal- and gas-fired power stations, yet are now transitioning into low-carbon emission technologies (Financial Times, 2016a).

While various papers analyse how investment in technological innovations is affected by price and technological uncertainty (Grenadier & Weiss, 1997; Huisman & Kort, 2002; Chronopoulos & Siddiqui, 2015), insights on the interaction of these features with policy uncertainty are not equally developed. In fact, in most cases, insights are based on numerical or simulation methods, which are crucial for studying more complex settings, but do not allow for analytical tractability. However, the latter is necessary for understanding the implications of policy uncertainty for investment, for example, why the incentive to either accelerate or postpone investment increases as the likelihood of subsidy retraction increases depending on the specifications of a model (Boomsma & Linnerud, 2015; Adkins & Paxson, 2016). In turn, this will also enable a better understanding of any implications resulting from the potential to invest sequentially in successively improved versions of an emerging technology, which is particularly crucial for sectors characterised by intense R&D activity.

Indeed, although emerging technologies often enjoy government support, the absence of a clear policy framework, which is frequently reflected in the sudden provision or retraction of a support scheme, discourages investment decisions. For example, although promises of 10%

annual returns boosted the Spanish solar industry in 2008, the subsequent reduction of subsidies at different points in time increased producers' reluctance to commit to future investments (The Economist, 2013). More recently, political uncertainty regarding the UK's future within the European Union prompted Siemens to re-evaluate its long-term investment strategy in RE (Financial Times, 2016b). Furthermore, empirical research based on small hydropower projects indicates that uncertainty regarding future subsidy provision increases the incentive to postpone investment. In fact, even promises to include existing projects in a prospective support scheme may not be as successful in promoting investment decisions as policymakers may expect (Linnerud *et al.*, 2014).

Despite recent attempts to incorporate policy uncertainty within real options models (Fleten *et al.*, 2016), insights involving the combined impact of price, technological, and policy uncertainty are limited, as these features are frequently analysed in isolation. We address this disconnect by incorporating these features in a real options framework for sequential investment in technological innovations. Thus, we provide insights not only on how price, policy, and technological uncertainty interact to affect the optimal investment policy, but also on how policymakers can devise more efficient policy mechanisms in order to incentivise investment in emerging technologies. The scope of our model does not include the option to choose between alternative projects (Grenadier & Weiss, 1997; Chronopoulos & Siddiqui, 2015), but emphasises on how price, policy, and technological uncertainty interact to affect sequential investment decisions. Thus, we assume that the firm adopts each technology that becomes available (compulsive strategy) and ignores the possibility to wait for both technologies to arrive in order to have the option to adopt either the old one (laggard strategy) or the new one directly (leapfrog strategy).

We show that greater likelihood of subsidy retraction postpones investment by decreasing the expected value of a project, yet the likely provision of a subsidy raises the investment incentive. Interestingly, we also find that the option to invest sequentially in improved versions of a technology raises the value of an investment opportunity, and, thus, may either mitigate the impact of policy uncertainty or make it more pronounced. These results have important implications for the current policymaking process in many countries that seek to stimulate investment in RE power plants. Indeed, many countries implement a variety of policy interventions and selective support schemes, without taking into account particular features of investment projects or considering that private firms may act more cautiously in the light of the uncertainties emerging from frequent switches between policy regimes. Additionally, our results deviate from those of earlier literature (Chronopoulos *et al.*, 2016; Adkins & Paxson, 2016), thereby indicating that the impact of policy uncertainty on the optimal investment policy depends on model specifications. Consequently, by deriving analytical results, where possible, regarding the impact of policy uncertainty on the optimal investment policy, we offer a direction for further research on the appropriate model specification that aims at capturing features of low-carbon investments, e.g., irreversibility, delay, and embedded options. These features are crucial as they impinge upon the radical policy imperatives for structural change in electricity markets to meet ambitious sustainability targets.

We proceed by discussing some related work in Section 1.2 and introduce assumptions and notation in Section 1.3. In Section 1.4.1, we address the problem of optimal investment timing taking into account only price and technological uncertainty. We introduce policy uncertainty in Section 1.4.2 and 1.4.3 in the form of sudden retraction and provision of a subsidy, respectively. In Section 1.4.4, we allow for the sudden provision of a retractable subsidy, and, in Section 1.4.5, we allow for infinite provisions and retractions. Section 1.5 presents numerical results for each case, while Section 1.6 concludes the paper and offers directions for further research.

1.2 Related Work

The seminal work of McDonald & Siegel (1985) and Dixit & Pindyck (1994) has spawned a substantial literature in the area of investment under uncertainty. A strand of this literature illustrates the amenability of real options theory to emerging technologies, R&D, telecommunications, and the energy sector (Bastian-Pinto *et al.*, 2010; Koussis *et al.* 2007; Rothwell, 2006; Siddiqui & Fleten, 2010; Lemoine, 2010; Farzan *et al.*, 2015; Franklin, 2015). Nevertheless, analytical formulations of problems that address investment in emerging technologies typically do not combine crucial features such as price, policy, and technological uncertainty. Indeed, most of this literature either addresses the impact of technological uncertainty on investment decisions ignoring the implications of policy uncertainty (Alvarez & Stenbacka, 2001; Schwartz & Zozaya-Gorostiza, 2003) or allows for policy uncertainty without taking into account the sequential nature of investment in emerging technologies (Boomsma *et al.*, 2012; Adkins & Paxson, 2016). Consequently, models that incorporate price, technological, and policy uncertainty in analytical frameworks for sequential investment remain somewhat underdeveloped.

In the area of investment under policy uncertainty, Boomsma *et al.* (2012) develop a real options model in order to investigate how investment behaviour is affected by regulatory uncertainty as well as changes of support scheme. They show that the value of an investment opportunity under policy uncertainty is greater than under RE certificate trading, which is higher than under a premium feed-in tariffs. In the same line of work, Boomsma & Linnerud (2015) find that the prospect of subsidy retraction increases the rate of investment if it is applied to new projects, while it slows down investment if it has a retroactive effect. Adkins & Paxson (2016) develop an analytical model for investment under price, quantity, and policy uncertainty. The latter is reflected in the random provision and retraction of a subsidy, which takes the form of a fixed premium on quantity. Their results indicate that the prospect of a permanent subsidy retraction (provision) facilitates (postpones) investment. Additionally, they find that the value of the option to invest increases as the correlation between the price of electricity and quantity of electricity produced increases, since this raises the aggregate volatility. Chronopoulos *et al.* (2016) ignore quantity uncertainty, yet allow for discretion over capacity and sequential policy interventions. They find that greater likelihood of a subsidy retraction may facilitate investment, yet results in smaller projects. Although these papers address the impact of policy uncertainty on investment timing and capacity sizing decisions, they ignore the implications of technological uncertainty and how sequential investment opportunities may impact the optimal investment policy.

Examples of frameworks for sequential investment under uncertainty include Majd & Pindyck (1987), who show how traditional valuation methods understate the value of a project by ignoring the flexibility embedded in the time to build. Dixit & Pindyck (1994) develop a model for sequential investment assuming that the value of the project depreciates exponentially and that the investor has an infinite number of investment option. In the same line of work, Gollier *et al.* (2005) compare a sequence of small nuclear power plants with a single nuclear power plant of large capacity. Their results indicate that the value of modularity may even trigger investment in the initial module at an electricity price level below the now-or-never net present value (NPV) threshold. Wu *et al.* (2009) analyse investment in enterprise resource planning systems, which can be installed either in full or sequentially, and the authors are able to solve a complex compounded real options problem by utilising stochastic programming. By comparing a lumpy to a stepwise investment strategy, Kort *et al.* (2010) show that higher price uncertainty raises the attractiveness of the former strategy by increasing the reluctance to make costly switches between different stages.

Allowing for technological uncertainty, Balcer & Lippman (1984) find that the optimal timing of technology adoption under infinite switching options is influenced by expectations about future technological changes and that increasing technological uncertainty tends to delay adoption. Grenadier & Weiss (1997) develop a model for sequential investment in order to study how the innovation rate and technological growth impact the optimal technology adoption strategy, and find that a firm may adopt an available technology even though more valuable innovations may occur in the future. Farzin *et al.* (1998) assume that technological innovations follow a Poisson process and find that the NPV rule can be used as an investment criterion in most cases. By contrast, Doraszelski (2001) identifies an error in Farzin *et al.* (1998) and shows that a firm will always defer investment when it takes the value of waiting into account. Mehrez *et al.* (2000) develop a discrete-time model for maintenance and replacement of a technology with either a more efficient one that is available or with a technology that will arrive at a random point in time. Huisman & Kort (2004) analyse how technological uncertainty impacts the competitive equilibrium and find that, when technological uncertainty becomes sufficiently large, the competition changes from a pre-emption game into a war of attrition game. While these papers present a comprehensive modelling of investment in technological innovations, they ignore the implications of policy and technological uncertainty for sequential investment.

More pertinent to our analysis is Chronopoulos & Siddiqui (2015), who develop a real options framework for sequential investment in technological innovations and analyse how the endogenous relationship between economic and technological uncertainty impacts both the optimal technology adoption strategy and the associated investment rule. Their results indicate that, although economic uncertainty postpones investment, uncertainty in the arrival of innovations may accelerate the adoption of an existing technology. We extend Chronopoulos & Siddiqui (2015) by introducing policy uncertainty in the form of sudden provision and retraction of a support scheme. Since technological uncertainty and increased intervention of government policy in trading arrangements may affect the optimal investment policy of private firms, significantly, and explore their combined impact in this paper. We assume that the output price fluctuates stochastically according to a geometric Brownian motion (GBM) and that policy interventions and technological innovations follow independent Poisson processes. Thus, we show that greater likelihood of subsidy retraction lowers the incentive to invest by decreasing the expected value of the project, whereas, greater likelihood of subsidy provision facilitates investment. Interestingly, results also indicate that an embedded option to invest in a more efficient technology raises the value of the investment opportunity. This implies that sequential investment opportunities mitigate the impact of policy uncertainty in the case of sudden subsidy retraction, and make the impact of policy uncertainty more pronounced in the case of subsidy provision. Also, we illustrate how the impact of policy uncertainty becomes less pronounced, when the rate of policy interventions increases, and diminishes under infinite provisions and retractions.

1.3 Assumptions and Notation

We consider a price-taking firm with a perpetual option to invest in $n = 1, 2$ successively improved versions of a technology, each with infinite lifetime, facing price, technological, and policy uncertainty. Given a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, we assume that technological and policy uncertainty follow independent Poisson processes, $\left\{M_t^i, t \geq 0\right\}$, where t is continuous and denotes time, $\lambda_i \geq 0$ denotes the intensity of the Poisson process, and $i = \{\tau, p\}$ (denoting technological and policy uncertainty, respectively). Intuitively, M_t^i counts the number of random times $h_m^i, m \in \mathbb{N}$ that occur between 0 and t, and $T_m^i = h_m^i - h_{m-1}^i$ is the time interval between subsequent Poisson events. Also, we assume that there is no operating cost associated with each technology and that the output price at time t, E_t, is independent of M_t^i (Boomsma & Linnerud, 2015). Note that the independence between price and technological uncertainty facilitates the analysis when firms have no information about the decisions made by R&D companies (Chronopoulos & Siddiqui, 2015).

The output price follows a GBM (Boomsma *et al.*, 2012), which is described in (1). We denote by μ the annual growth rate, by σ the annual volatility, by dZ_t the increment of the standard Brownian motion, and by $\rho \geq \mu$ the subjective discount rate. With respect to our motivating examples, although energy prices are mean reverting, empirical evidence based on 127 years of data indicates that the rate of mean reversion is low enough, and, therefore, assuming a GBM for investment analysis is unlikely to lead to large errors (Pindyck, 1999).

$$dE_t = \mu E_t dt + \sigma E_t dZ_t, \quad E_0 \equiv E > 0 \tag{1}$$

We denote the output of technology n by D_n ($D_2 \geq D_1$) and the corresponding investment cost by I_n. We let $a = 0, 1$ denote the presence ($a = 1$) or absence ($a = 0$) of a subsidy that can be provided and retracted b and c times, respectively, and assume that the subsidy takes the form of a fixed premium, y, on top of the output price, E_t. Thus, the time of investment in technology n is denoted by $\tau_{n,a}^{b,c}$, while $\varepsilon_{n,a}^{b,c}$ is the corresponding optimal investment threshold. For example, the subsidy is not available initially, then, under sudden provision of a permanent subsidy, the optimal time to invest in the second technology is $\tau_{2,0}^{1,0}$, while the corresponding optimal investment threshold is $\varepsilon_{2,0}^{1,0}$. Finally, $F_{n,a}^{b,c}(\cdot)$ is the maximised expected NPV from investing in technology n, while $\Phi_{n,a}^{b,c}(\cdot)$ is the expected value (NPV) of the active project inclusive of embedded options.

In line with Chronopoulos & Siddiqui (2015), we assume that a new technology is more efficient in that it can produce a greater output compared to an older one, yet its adoption entails a greater capital expenditure. This implies that at the point α where the expected NPVs of the profits of the two technologies are equal, i.e. $\Phi_{1,a}^{b,c}(\alpha) = \Phi_{2,a}^{b,c}(\alpha)$, we have $\Phi_{2,a}^{b,c}(\alpha) > 0$. Otherwise, if $\Phi_{2,a}^{b,c}(\alpha) < 0$, then no trade-off exits between the two technologies and only the new technology presents a viable investment opportunity, because its expected value is always greater than that of the old technology for all the positive values of its range (Décamps *et al.*, 2006). If we ignore technological and policy uncertainty, then this condition simplifies to $\frac{D_2}{I_2} < \frac{D_1}{I_1}$. In terms of context, a firm may have a plot of land that will be used to build a wind farm and that the investment decision is divided in two steps. In step one, the firm develops this property with an embedded option to increase its utilisation via the adoption of a new technology if prices increase. However, this requires not only an additional investment cost, since the new technology covers greater demand, but also a cost for decommissioning the old technology.

1.4 Analytical Results

Problem Formulation: The firm's value function at different states of operation is indicated in Figure 1.1 and is determined via backward induction, by following the steps outlined below:

1. Initially, we assume that the firm is operating the second technology, and, thus, holds the value function $\Phi_{2,a}^{b,c}(E)$, following the adoption of the second technology at $\tau_{2,a}^{b,c}$.

2. Prior to the adoption of the second technology, the firm holds the value function $F_{2,a}^{b,c}(E)$, consisting of the value from operating the first technology and a single embedded option to invest in the second one. The latter will be exercised at time $\tau_{2,a}^{b,c}$, and, thus, the firm

will obtain the value function $\Phi_{2,a}^{b,c}(E)$.

3. Before the arrival of the second technology, the firm holds the value function $\Phi_{1,a}^{b,c}(E)$, which consists of the expected value from operating the first technology with the embedded option to invest in the second one, which has yet to become available.

4. Finally, before time $\tau_{1,a}^{b,c}$ the firm holds the value function $F_{1,a}^{b,c}(E)$, i.e., the option to invest in the first technology with a single embedded option to invest in the second one, that has yet to become available.

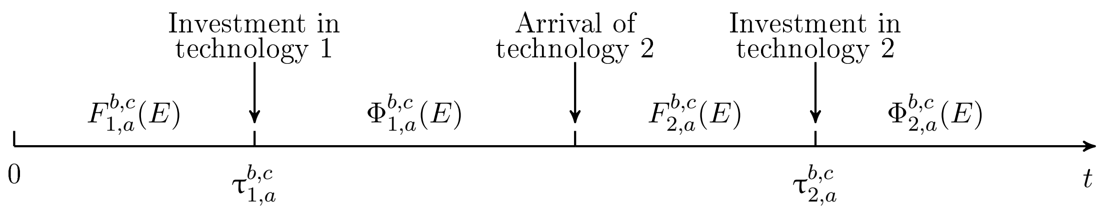


Figure 1.1: State transition diagram

1.4.1 Benchmark Case: Investment without Policy Uncertainty

We assume that a firm has the option to invest in each technology facing only price and technological uncertainty. The expected value of the revenues from operating the second technology in the presence ($a = 1$) or absence ($a = 0$) of a subsidy is indicated in (2).

$$\Phi_{2,a}^{0,0}(E) \quad = \quad \frac{D_2 E\,(1 + ay)}{\rho - \mu} \tag{2}$$

Next, the value of the option to invest in the second technology is indicated in (3). The first term on the top part of (3) reflects the expected present value of the revenues from operating the first technology, while the second term represents the option to invest in the second one. Note that $\beta_1 > 1$ is the positive root of the quadratic $\frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - \rho = 0$. The first term in the bottom part of (3) reflects the expected value from operating the second technology, while, the second term is the investment cost (all proofs can be found in the appendix).

$$F_{2,a}^{0,0}(E) \quad = \quad \begin{cases} \frac{D_1 E(1+ay)}{\rho-\mu} + A_{2,a}^{0,0} E^{\beta_1} & , E < \varepsilon_{2,a}^{0,0} \\ \\ \Phi_{2,a}^{0,0}(E) - I_2 & , E \geq \varepsilon_{2,a}^{0,0} \end{cases} \tag{3}$$

The optimal investment threshold, $\varepsilon_{2,a}^{0,0}$, and the endogenous constant, $A_{2,a}^{0,0}$, are obtained analytically by applying value-matching and smooth-pasting conditions to the two branches of (3).

These conditions are indicated in (A-3) and (A-4), respectively, and the resulting expressions for $\varepsilon_{2,a}^{0,0}$ and $A_{2,a}^{0,0}$ are indicated in (4).

$$\varepsilon_{2,a}^{0,0} = \frac{\beta_1 I_2 (\rho - \mu)}{(\beta_1 - 1)(D_2 - D_1)(1 + ay)} \quad \text{and} \quad A_{2,a}^{0,0} = \left(\frac{1}{\varepsilon_{2,a}^{0,0}}\right)^{\beta_1} \left(\frac{(D_2 - D_1)(1 + ay)\,\varepsilon_{2,a}^{0,0}}{\rho - \mu} - I_2\right) \tag{4}$$

Next, we assume that the firm is operating the first technology holding an embedded option to adopt the second one, which has yet to become available. The dynamics of the value function $\Phi_{1,a}^{0,0}(E)$ are described in (5), where $\mathbb{E}_E$ denotes the expectation operator that is conditional on the initial output price, E. The first term on the right-hand side of (5) is the immediate profit from operating the first technology. As the second term indicates, with probability $\lambda_\tau dt$ the second technology will arrive and the firm will receive the value function $F_{2,a}^{0,0}(E)$, whereas, with probability $1 - \lambda_\tau dt$, no innovation will occur and the firm will continue to hold the value function $\Phi_{1,a}^{0,0}(E)$.

$$\Phi_{1,a}^{0,0}(E) = D_1 E (1 + ay)\, dt + (1 - \rho dt)\, \mathbb{E}_E \left\{ \lambda_\tau dt F_{2,a}^{0,0}(E + dE) + (1 - \lambda_\tau dt)\, \Phi_{1,a}^{0,0}(E + dE) \right\} \tag{5}$$

By expanding the right-hand side of (5) using Itô's lemma and solving the resulting ordinary differential equation, we can rewrite (5) as in (6), where $A_{1,a}^{0,0} \leq 0$ and $B_{1,a}^{0,0} \geq 0$ are determined analytically via value-matching and smooth-pasting conditions between the two branches and $\delta_1 > 1, \delta_2 < 0$ are the roots of the quadratic $\frac{1}{2}\sigma^2 \delta(\delta - 1) + \mu\delta - (\rho + \lambda_\tau) = 0$. The first term on the top part of (6) represents the expected present value of the revenues from operating the first technology, while the second term is the option to invest in the second one, adjusted via the third term because the second technology has yet to become available. The first two terms on the bottom part of (6) represent the expected profit from the two technologies. Notice that both the output and investment cost in the second technology are adjusted by λ_τ, since the second technology is not available yet (Alvarez & Stenbacka, 2001; Huisman & Kort, 2004). The third term reflects the likelihood of the price dropping in the waiting region prior to the arrival of an innovation.

$$\Phi_{1,a}^{0,0}(E) = \begin{cases} \frac{D_1 E (1+ay)}{\rho - \mu} + A_{2,a}^{0,0} E^{\beta_1} + A_{1,a}^{0,0} E^{\delta_1} & , E < \varepsilon_{2,a}^{0,0} \\ \frac{(\lambda_\tau D_2 + (\rho - \mu) D_1) E (1+ay)}{(\rho - \mu)(\rho - \mu + \lambda_\tau)} - \frac{\lambda_\tau I_2}{\rho + \lambda_\tau} + B_{1,a}^{0,0} E^{\delta_2} & , E \geq \varepsilon_{2,a}^{0,0} \end{cases} \tag{6}$$

Finally, the value of the option to invest in the first technology is indicated in (7), where the optimal investment threshold, $\varepsilon_{1,a}^{0,0}$, and the endogenous constant, $C_{1,a}^{0,0} \geq 0$, are determined numerically via value-matching and smooth-pasting conditions between the two branches. The top part on the right-hand side of (7) is the value of the option to invest in the first technology, while the bottom part reflects the expected value of the active project. The latter consists of the expected value from operating the first technology inclusive of the embedded option to invest in the second one, and is indicated in the top part of (6), reduced by the investment cost.

$$F_{1,a}^{0,0}(E) = \begin{cases} C_{1,a}^{0,0} E^{\beta_1} & , E < \varepsilon_{1,a}^{0,0} \\ \\ \Phi_{1,a}^{0,0}(E) - I_1 & , E \geq \varepsilon_{1,a}^{0,0} \end{cases} \tag{7}$$

1.4.2 Permanent Subsidy Retraction

We extend the previous framework by assuming that a subsidy is available and that it may be retracted permanently at a random point in time, T_1^p. Consequently, the expected value of the revenues from operating the second technology is indicated in (8). The first term on the right-hand side is the expected present value of the revenues in the absence of the subsidy, while, the second term, is the expected extra value due to the presence of a subsidy, that has an exponential lifetime and will be retracted at T_1^p.

$$\mathbb{E}_E \left[\int_0^\infty e^{-\rho t} D_2 E_t dt + \int_0^{T_1^p} e^{-\rho t} D_2 E_t y dt \right] = \frac{D_2 E}{\rho - \mu} + \mathbb{E} \left\{ \frac{D_2 E y \left[1 - e^{-(\rho-\mu) T_1^p} \right]}{\rho - \mu} \right\} \tag{8}$$

Since $T_1^p \sim \exp(\lambda_p)$, by evaluating the expectation of this expression with respect to T_1^p we obtain (9). Notice that the subsidy will never be retracted if $\lambda_p = 0$, whereas a greater λ_p raises the likelihood of subsidy retraction and lowers the expected NPV of the project.

$$\begin{aligned} \Phi_{2,1}^{0,1}(E) &= \frac{D_2 E}{\rho - \mu} + \int_0^\infty \lambda_p e^{-\lambda_p T_1^p} \frac{D_2 E y \left[1 - e^{-(\rho-\mu) T_1^p} \right]}{\rho - \mu} dT_1 \\ &= \frac{D_2 E}{\rho - \mu} + \frac{D_2 E y}{\rho - \mu + \lambda_p} \end{aligned} \tag{9}$$

Next, we assume that the firm is operating the first technology and holds a single embedded option to invest in the second one. The dynamics of the firm's value function are described in (10), where the first term on the right-hand side reflects the immediate profit from operating the first technology. As the second term indicates, the option to invest in the second technology will be exercised in the permanent absence of a subsidy with probability $\lambda_p dt$. By contrast, with probability $1 - \lambda_p dt$, no policy intervention will take place and the firm will continue to hold the option to invest in the second technology in the presence of a retractable subsidy.

$$F_{2,1}^{0,1}(E) = D_1 E(1+y)dt + (1-\rho dt)\,\mathbb{E}_E\left\{\lambda_p dt F_{2,0}^{0,0}(E+dE) + (1-\lambda_p dt)\, F_{2,1}^{0,1}(E+dE)\right\} \quad (10)$$

By expanding the right-hand side of (10) using Itô's lemma and solving the resulting ordinary differential equation, we obtain (11), where $\varepsilon_{2,1}^{0,1}$ and $A_{2,1}^{0,1} \geq 0$ are determined via value-matching and smooth-pasting conditions, while $\eta_1 > 1, \eta_2 < 0$ are the roots of the quadratic $\frac{1}{2}\sigma^2\eta(\eta - 1) + \mu\eta - (\rho + \lambda_p) = 0$. The first two terms in the top part of (11) represent the expected value of the revenues from operating the first technology, while the third term is the option to invest in the second one in the absence of a subsidy, adjusted via the fourth term since the subsidy is currently available.

$$F_{2,1}^{0,1}(E) \quad = \quad \begin{cases} \frac{D_1 E}{\rho-\mu} + \frac{D_1 E y}{\rho-\mu+\lambda_p} + A_{2,0}^{0,0} E^{\beta_1} + A_{2,1}^{0,1} E^{\eta_1} & , E < \varepsilon_{2,1}^{0,1} \\ \Phi_{2,1}^{0,1}(E) - I_2 & , E \geq \varepsilon_{2,1}^{0,1} \end{cases} \quad (11)$$

Next, we step back and assume that an innovation has not taken place yet, but may occur over an infinitesimal time interval dt with probability $\lambda_\tau dt$. The dynamics of the value function $\Phi_{1,1}^{0,1}(E)$ are described in (12), where the first term on the right-hand side represents the immediate profit from operating the first technology and the second term reflects the expected value in the continuation region. Notice that if the subsidy is retracted with probability $\lambda_p dt$, then either an innovation will take place with probability $\lambda_\tau dt$ and the firm will receive the value function $F_{2,0}^{0,0}(E)$, or no innovation will take place with probability $1 - \lambda_\tau dt$ and the firm will continue to operate the first technology in the absence of a subsidy. Similarly, if no policy intervention occurs with probability $1 - \lambda_p dt$, then the firm will either receive the value function $F_{2,1}^{0,1}(E)$ with probability $\lambda_\tau dt$, or it will continue to hold the value function $\Phi_{1,1}^{0,1}(E)$ with probability $1 - \lambda_\tau dt$.

$$\begin{aligned} \Phi_{1,1}^{0,1}(E) \quad = \quad & D_1 E(1+y)dt + (1-\rho dt)\,\mathbb{E}_E\left\{\lambda_p dt\left[\lambda_\tau dt F_{2,0}^{0,0}(E+dE) + (1-\lambda_\tau dt)\,\Phi_{1,0}^{0,0}(E+dE)\right]\right. \\ & \left. + (1-\lambda_p dt)\left[\lambda_\tau dt F_{2,1}^{0,1}(E+dE) + (1-\lambda_\tau dt)\,\Phi_{1,1}^{0,1}(E+dE)\right]\right\} \end{aligned} \quad (12)$$

The expression of $\Phi_{1,1}^{0,1}(E)$ is indicated in (13), where $A_{1,1}^{0,1} \leq 0$ and $B_{1,1}^{0,1} \leq 0$ are determined numerically via value-matching and smooth-pasting conditions, while $\theta_1 > 1, \theta_2 < 0$ are the roots of the quadratic $\frac{1}{2}\sigma^2\theta(\theta-1) + \mu\theta - (\rho + \lambda_p + \lambda_\tau) = 0$. The first two terms in the top part of (13) represent the expected revenues from operating the first technology, while the third term is the option to invest in the second one, adjusted via the fourth term due to policy uncertainty. The fifth term reflects the loss in option value due to the absence of the second technology, and is adjusted via the last term due to policy uncertainty. The first three terms in the bottom part of (13) represent the expected revenues from investing the second technology, while the last two terms reflect the likelihood of the price dropping in the waiting region before the arrival of the second technology, adjusted for policy uncertainty.

$$\Phi_{1,1}^{0,1}(E) = \begin{cases} \frac{D_1 E}{\rho-\mu} + \frac{D_1 E y}{\rho-\mu+\lambda_p} + A_{2,0}^{0,0} E^{\beta_1} + A_{2,1}^{0,1} E^{\eta_1} + A_{1,0}^{0,0} E^{\delta_1} + A_{1,1}^{0,1} E^{\theta_1} & , E < \varepsilon_{2,1}^{0,1} \\ \frac{\lambda_\tau D_2 E + (\rho-\mu) D_1 E}{(\rho-\mu)(\rho-\mu+\lambda_\tau)} + \frac{[\lambda_\tau D_2 + (\rho-\mu+\lambda_p) D_1] E y}{(\rho-\mu+\lambda_p)(\rho-\mu+\lambda_p+\lambda_\tau)} - \frac{\lambda_\tau I_2}{\rho+\lambda_\tau} & \\ + B_{1,0}^{0,0} E^{\delta_2} + B_{1,1}^{0,1} E^{\theta_2} & , E \geq \varepsilon_{2,1}^{0,1} \end{cases} \tag{13}$$

The dynamics of the option to invest in the first technology are described in (14). Notice that, over an infinitesimal time interval dt, either the subsidy will be retracted with probability $\lambda_p dt$ and the firm will receive the option to invest in the absence of a subsidy, or no policy intervention will take place with probability $1 - \lambda_p dt$ and the firm will continue to hold the value function $F_{1,1}^{0,1}(E)$.

$$F_{1,1}^{0,1}(E) = (1-\rho dt)\, \mathbb{E}_E \Big\{ \lambda_p dt F_{1,0}^{0,0}(E+dE) + (1-\lambda_p dt)\, F_{1,1}^{0,1}(E+dE) \Big\} \tag{14}$$

The expression of $F_{1,1}^{0,1}(E)$ is indicated in (15), where $\varepsilon_{1,1}^{0,1}$ and $C_{1,1}^{0,1} \geq 0$ are obtained numerically via value-matching and smooth-pasting conditions. The first term in the top part of (15) is the option to invest in the absence of a subsidy, adjusted via the second term since the subsidy is currently available. The bottom part represents the expected value from operating the first technology inclusive of the embedded option to invest in the second one, which is obtained by paying the investment cost I_1.

$$F_{1,1}^{0,1}(E) = \begin{cases} C_{1,0}^{0,0} E^{\beta_1} + C_{1,1}^{0,1} E^{\eta_1} & , E < \varepsilon_{1,1}^{0,1} \\ \Phi_{1,1}^{0,1}(E) - I_1 & , E \geq \varepsilon_{1,1}^{0,1} \end{cases} \tag{15}$$

We can investigate the impact of λ_p and λ_τ on the optimal investment rule by expressing $F_{1,1}^{0,1}(E)$ as in (16). The optimal investment rule is obtained by applying the first-order necessary condition (FONC) to (16) and is indicated in (17), where we equate the marginal benefit (MB) of delaying investment to the marginal cost (MC). Note that the second-order sufficiency condition (SOSC) requires the value function, $F_{1,1}^{0,1}(E)$, to be concave at the investment threshold, which is shown in Chronopoulos *et al.* (2011) and Chronopoulos & Lumbreras (2017) for the more general case of a risk-averse decisionmaker.

$$F_{1,1}^{0,1}(E) = \left(\frac{E}{\varepsilon_{1,1}^{0,1}}\right)^{\beta_1} \left[\Phi_{1,1}^{0,1}\left(\varepsilon_{1,1}^{0,1}\right) - I_1 - C_{1,1}^{0,1}\varepsilon_{1,1}^{0,1\,\eta_1}\right], \quad E < \varepsilon_{2,1}^{0,1} \tag{16}$$

The first two terms on the left-hand side consist of the stochastic discount factor multiplied by the incremental project value created by waiting until the price is higher. These terms are positive, decreasing functions of the output price, as waiting longer allows the project to start at a higher initial price, yet the rate at which this benefit accrues diminishes due to the effect of discounting. The third term represents the reduction in the MC of waiting due to saved investment cost. Similarly, the first two terms on the right-hand side reflect the opportunity cost of forgone cash flows discounted appropriately. The fourth and third term on the left- and right-hand side, respectively, reflect the loss in option value, since the second technology is not available yet. Specifically, the fourth term on the left-hand side is the MB from postponing the loss in value, whereas the third term on the right-hand side is the MC from a potentially greater impact of the loss from waiting for a higher threshold price. The last two terms on the left- and the right-hand side reflect the necessary adjustments of MB and MC of waiting due to policy uncertainty.

$$\begin{aligned}
&\left(\frac{E}{\varepsilon_{1,1}^{0,1}}\right)^{\beta_1}\left[\frac{D_1}{\rho-\mu} + \frac{D_1 y}{\rho-\mu+\lambda_p} + \frac{\beta_1 I_1}{\varepsilon_{1,1}^{0,1}} - \beta_1 A_{1,0}^{0,0}\varepsilon_{1,1}^{0,1\,\delta_1-1} - \beta_1 A_{1,1}^{0,1}\varepsilon_{1,1}^{0,1\,\theta_1-1} + \left[\beta_1 C_{1,1}^{0,1} + \eta_1 A_{2,1}^{0,1}\right]\varepsilon_{1,1}^{0,1\,\eta_1-1}\right] \\
&= \left(\frac{E}{\varepsilon_{1,1}^{0,1}}\right)^{\beta_1}\left[\frac{\beta_1 D_1}{\rho-\mu} + \frac{\beta_1 D_1 y}{\rho-\mu+\lambda_p} - \delta_1 A_{1,0}^{0,0}\varepsilon_{1,1}^{0,1\,\delta_1-1} - \theta_1 A_{1,1}^{0,1}\varepsilon_{1,1}^{0,1\,\theta_1-1} + \left[\eta_1 C_{1,1}^{0,1} + \beta_1 A_{2,1}^{0,1}\right]\varepsilon_{1,1}^{0,1\,\eta_1-1}\right]
\end{aligned} \tag{17}$$

As shown in Proposition 1, greater likelihood of subsidy retraction lowers the MB by more than the MC, thereby raising the incentive to postpone investment. Intuitively, the incentive to invest early in order to take advantage of the subsidy for a longer period is mitigated completely by the rapid reduction in the value of the active project due to subsidy retraction.

Proposition 1. *Greater likelihood of subsidy retraction raises the optimal investment threshold.*

The relative loss in option value due to subsidy retraction is $\frac{F_{1,1}^{0,0}(E)-F_{1,1}^{0,1}(E)}{F_{1,1}^{0,0}(E)}$. If $\lambda_p = 0$, then the subsidy will never be retracted and the relative loss in option value is zero. By contrast, as λ_p

increases, the relative loss increases. Indeed, greater likelihood of subsidy retraction lowers the expected value of the available subsidy. This implies that $C_{1,1}^{0,1}E^{\eta_1} \to 0 \Rightarrow F_{1,1}^{0,1}(E) \to F_{1,1}^{0,0}(E)$, as shown in Proposition 2. Also, the relative loss in option value is always below one, since the firm can invest even in the absence of a subsidy, albeit at a higher price threshold.

Proposition 2. $\frac{F_{1,1}^{0,0}(E)-F_{1,1}^{0,1}(E)}{F_{1,1}^{0,0}(E)} \in \left[0, 1-\frac{1}{(1+y)^{\beta_1}}\right]$.

1.4.3 Provision of a Permanent Subsidy

As the increasing replacement of fossil-fuel with RE facilities may deteriorate the financial risk-return performance of incremental investments (Muñoz & Bunn, 2013), subsidies may be required to support green investments. Like in Section 1.4.2, we assume that there is a single policy intervention, and, therefore, we denote the random time at which it takes place by T_1^p. The expected present value of the revenues from operating the second technology is indicated in (18), and, according to the right-hand side, it consists of the expected value of the project in the absence of the subsidy (first term) and the extra value of the subsidy (second term) that will be provided at time T_1^p.

$$\mathbb{E}_E\left[\int_0^\infty e^{-\rho t}D_2E_t dt + \int_{T_1^p}^\infty e^{-\rho t}D_2E_t y dt\right] = \frac{D_2E}{\rho-\mu} + \mathbb{E}\left\{\frac{D_2Eye^{-(\rho-\mu)T_1^p}}{\rho-\mu}\right\} \tag{18}$$

Since $T_1^p \sim \exp(\lambda_p)$, taking the expectation of this expression with respect to T_1^p yields (19).

$$\Phi_{2,0}^{1,0}(E) \quad = \quad \frac{D_2E}{\rho-\mu} + \frac{\lambda_p D_2 E y}{(\rho-\mu+\lambda_p)(\rho-\mu)} \tag{19}$$

The dynamics of the option to invest in the second technology are described in (20), where the first term on the right-hand side represents the instantaneous profit from operating the first technology. The second term indicates that, depending on the provision of a subsidy, the firm will receive either $F_{2,1}^{0,0}(E)$ with probability $\lambda_p dt$ or $F_{2,0}^{1,0}(E)$ with probability $1-\lambda_p dt$.

$$F_{2,0}^{1,0}(E) = D_1Edt + (1-\rho dt)\,\mathbb{E}_E\left\{\lambda_p dt F_{2,1}^{0,0}(E+dE) + (1-\lambda_p dt)\,F_{2,0}^{1,0}(E+dE)\right\} \tag{20}$$

The expression of $F_{2,0}^{1,0}(E)$ is indicated in (21), where $\varepsilon_{2,0}^{1,0}$, $A_{2,0}^{1,0} \leq 0$, $B_{2,0}^{2,0} \geq 0$, and $C_{2,0}^{1,0} \geq 0$, are determined numerically via value-matching and smooth-pasting conditions between the three branches. Note that, unlike the case of sudden subsidy retraction, $F_{2,0}^{1,0}(E)$ is now defined over three different regions of E: (i) if $E < \varepsilon_{2,1}^{0,0}$, then the firm would not invest even in the presence of a subsidy, (ii) if $\varepsilon_{2,1}^{0,0} \leq E < \varepsilon_{2,0}^{1,0}$, then the firm would invest immediately if the subsidy is provided, and (iii) if $E \geq \varepsilon_{2,0}^{1,0}$, then investment will take place immediately even in the absence of the subsidy.

$$F_{2,0}^{1,0}(E) = \begin{cases} \frac{D_1 E}{\rho-\mu} + \frac{\lambda_p y D_1 E}{(\rho-\mu)(\rho-\mu+\lambda_p)} + A_{2,1}^{0,0} E^{\beta_1} + A_{2,0}^{1,0} E^{\eta_1} & , E < \varepsilon_{2,1}^{0,0} \\ \frac{\lambda_p D_2 E(1+y)+(\rho-\mu) D_1 E}{(\rho-\mu)(\rho-\mu+\lambda_p)} - \frac{\lambda_p I_2}{\rho+\lambda_p} + B_{2,0}^{1,0} E^{\eta_2} + C_{2,0}^{1,0} E^{\eta_1} & , \varepsilon_{2,1}^{0,0} \leq E < \varepsilon_{2,0}^{1,0} \\ \Phi_{2,0}^{1,0}(E) - I_2 & , E \geq \varepsilon_{2,0}^{1,0} \end{cases} \tag{21}$$

Next, the dynamics of the value of the active project prior to the arrival of the second technology are described in (22), where the first term on the right-hand side reflects the instantaneous profit from operating the first technology. As the second term indicates, within an infinitesimal time interval dt a subsidy will be provided with probability $\lambda_p dt$ and then the firm will receive either the value function $F_{2,1}^{0,0}(E)$ or $\Phi_{1,1}^{0,0}(E)$ depending on the arrival of an innovation. By contrast, a subsidy will not be provided with probability $1 - \lambda_p dt$, and, depending on the arrival of an innovation, the firm will receive either the value function $F_{2,0}^{1,0}(E)$ or $\Phi_{1,0}^{1,0}(E)$.

$$\begin{aligned} \Phi_{1,0}^{1,0}(E) = D_1 E dt \quad &+ \quad (1-\rho dt)\, \mathbb{E}_E \Bigg\{ \lambda_p dt \Big[\lambda_\tau dt F_{2,1}^{0,0}(E+dE) + (1-\lambda_\tau dt)\, \Phi_{1,1}^{0,0}(E+dE) \Big] \\ &+ \quad (1-\lambda_p dt) \Big[\lambda_\tau dt F_{2,0}^{1,0}(E+dE) + (1-\lambda_\tau dt)\, \Phi_{1,0}^{1,0}(E+dE) \Big] \Bigg\} \end{aligned} \tag{22}$$

Notice that (22) must be solved separately for each of the expressions of $F_{2,1}^{0,0}(E)$, $\Phi_{1,1}^{0,0}(E)$, and $F_{2,0}^{1,0}(E)$ that are indicated in (3), (6), and (21), respectively. Thus, $\Phi_{1,0}^{1,0}(E)$ is also defined over three different regions of E. Indeed, following the same approach as in Section 1.4.2, we obtain the expression for $\Phi_{1,0}^{1,0}(E)$ that is described in (23), where $A_{1,0}^{1,0}$, $B_{1,0}^{1,0}$, $C_{1,0}^{1,0}$ and $D_{1,0}^{1,0}$ are determined via value-matching and smooth-pasting conditions between the three branches. Each branch reflects the expected value of the first technology with an embedded option to invest in the second one. The second technology is not available yet and the corresponding investment option will not be exercised if the output price is low, i.e. $E < \varepsilon_{2,1}^{0,0}$ (top branch), however it will be exercised instantly if the subsidy is provided (middle branch) or immediately regardless of the subsidy (bottom branch).

$$\Phi_{1,0}^{1,0}(E)=\begin{cases}\frac{D_1E}{\rho-\mu}+\frac{\lambda_pD_1Ey}{(\rho-\mu)(\rho-\mu+\lambda_p)}+A_{2,1}^{0,0}E^{\beta_1}+A_{2,0}^{1,0}E^{\eta_1} & \\ +A_{1,1}^{0,0}E^{\delta_1}+A_{1,0}^{1,0}E^{\theta_1} & ,\ E<\varepsilon_{2,1}^{0,0}\\ \left[\frac{[\lambda_\tau D_2+(\rho-\mu)D_1]}{\rho-\mu+\lambda_\tau}+\frac{\lambda_\tau D_2}{\rho-\mu+\lambda_p}\right]\frac{\lambda_pE(1+y)}{(\rho-\mu)^2\left(1+\frac{\lambda_p+\lambda_\tau}{\rho-\mu}\right)} & \\ +\frac{D_1E}{\rho-\mu+\lambda_p}-\left(\frac{1}{\rho+\lambda_\tau}+\frac{1}{\rho+\lambda_p}\right)\frac{\lambda_\tau\lambda_pI_2}{\rho+\lambda_p+\lambda_\tau}+B_{2,0}^{1,0}E^{\eta_2} & \\ +C_{2,0}^{1,0}E^{\eta_1}+B_{1,1}^{0,0}E^{\delta_2}+B_{1,0}^{1,0}E^{\theta_2}+C_{1,0}^{1,0}E^{\theta_1} & ,\ \varepsilon_{2,1}^{0,0}\le E<\varepsilon_{2,0}^{1,0}\\ \left[\frac{\lambda_p(1+y)}{\rho-\mu+\lambda_\tau}+\frac{\lambda_py}{\rho-\mu+\lambda_p}+1\right]\frac{\lambda_\tau D_2E}{(\rho-\mu)^2\left(1+\frac{\lambda_p+\lambda_\tau}{\rho-\mu}\right)}+\frac{D_1E}{\rho-\mu+\lambda_\tau} & \\ \times\left[\frac{\lambda_py}{(\rho-\mu)\left(1+\frac{\lambda_p+\lambda_\tau}{\rho-\mu}\right)}+1\right]-\frac{\lambda_\tau I_2}{\rho+\lambda_\tau}+B_{1,1}^{0,0}E^{\delta_2}+D_{1,0}^{1,0}E^{\theta_2} & ,\ E\ge\varepsilon_{2,0}^{1,0}\end{cases}\tag{23}$$

Like in (14), the dynamics of the option to invest in the first technology with a single embedded option to upgrade to the second one are described in (24).

$$F_{1,0}^{1,0}(E)=(1-\rho dt)\,\mathbb{E}_E\left\{\lambda_pdtF_{1,1}^{0,0}(E+dE)+(1-\lambda_pdt)\,F_{1,0}^{1,0}(E+dE)\right\}\tag{24}$$

The expression of $F_{1,0}^{1,0}(E)$ is indicated in (25), where $\varepsilon_{1,0}^{1,0}$, $G_{1,0}^{1,0}$, $H_{1,0}^{1,0}$, and $J_{1,0}^{1,0}$, are determined numerically via value-matching and smooth-pasting conditions between the three branches. The first term in the top branch of (25) reflects the value of the option to invest in the presence of a subsidy, adjusted via the second term due to policy uncertainty. The first two terms in the second branch represent the expected value of the project if the subsidy is provided, while the third term is the option to invest in the second technology, adjusted for technological uncertainty via the fourth term. The last two terms reflect the likelihood of the price either dropping below $\varepsilon_{1,1}^{0,0}$ or increasing beyond $\varepsilon_{1,0}^{1,0}$.

$$F_{1,0}^{1,0}(E)\quad=\quad\begin{cases}C_{1,1}^{0,0}E^{\beta_1}+G_{1,0}^{1,0}E^{\eta_1} & ,E<\varepsilon_{1,1}^{0,0}\\ \frac{\lambda_pD_1E(1+y)}{(\rho-\mu)(\rho-\mu+\lambda_p)}-\frac{\lambda_pI_1}{\rho+\lambda_p}+A_{2,1}^{0,0}E^{\beta_1}+\frac{\lambda_p}{\lambda_p-\lambda_\tau}A_{1,1}^{0,0}E^{\delta_1} & \\ +H_{1,0}^{1,0}E^{\eta_2}+J_{1,0}^{1,0}E^{\eta_1} & ,\varepsilon_{1,1}^{0,0}\le E<\varepsilon_{1,0}^{1,0}\\ \Phi_{1,0}^{1,0}(E)-I_1 & ,E\ge\varepsilon_{1,0}^{1,0}\end{cases}\tag{25}$$

Although it is not possible to express the value of the option to invest as in (16), we can analyse the impact of λ_p on $\varepsilon_{1,0}^{1,0}$ by applying the FONC to the value-matching condition between the bottom two branches of (25), and, thus, obtain (26). The first term on the left-hand side represents the extra benefit from allowing the project to start at a higher output price, the second term reflects the reduction in the MC due to saved investment cost, and the third term is the MB from being able to delay investment should the output price drop below $\varepsilon_{1,1}^{0,0}$. The first term on the right-hand side is the MC of the forgone cash flows, while the second term is positive and represents the MC associated with the absence of the second technology. The fourth term on the left-hand side reflects the increase in the MB of waiting due to the likelihood of a subsidy, whereas the third term on the right-hand is the corresponding MC of waiting because the subsidy is not available yet.

$$\begin{aligned}&\left(\frac{E}{\varepsilon_{1,0}^{1,0}}\right)^{\eta_1}\left[\frac{D_1}{\rho-\mu+\lambda_p}+\frac{\eta_1\rho I_1}{(\rho+\lambda_p)\,\varepsilon_{1,0}^{1,0}}+\theta_1 A_{1,0}^{1,0}\varepsilon_{1,0}^{1,0\theta_1-1}+(\eta_1-\eta_2)\,H_{1,0}^{1,0}\varepsilon_{1,0}^{1,0\eta_2-1}\right]\\&=\left(\frac{E}{\varepsilon_{1,0}^{1,0}}\right)^{\eta_1}\left[\frac{\eta_1 D_1}{\rho-\mu+\lambda_p}-\frac{(\delta_1-\eta_1)\,\lambda_\tau}{\lambda_\tau-\lambda_p}A_{1,1}^{0,0}\varepsilon_{1,0}^{1,0\delta_1-1}+\eta_1 A_{1,0}^{1,0}\varepsilon_{1,0}^{1,0\theta_1-1}\right]\end{aligned}\tag{26}$$

As shown in Proposition 3, greater likelihood of subsidy provision lowers the MB by more than the MC, thereby decreasing the optimal investment threshold.

Proposition 3. *Greater likelihood of subsidy provision lowers the optimal investment threshold.*

The relative loss in option value due to policy uncertainty is $\frac{F_{1,1}^{0,0}(E)-F_{1,0}^{1,0}(E)}{F_{1,1}^{0,0}(E)}$, and, unlike the case of sudden subsidy retraction, it decreases with greater λ_p. Indeed, as shown in Proposition 4, for $\lambda_p = 0$ the subsidy will never be provided and the relative loss in option value is maximised. By contrast, the relative loss in option value decreases with greater λ_p, since the subsidy is more likely to be available permanently.

Proposition 4. $\frac{F_{1,1}^{0,0}(E)-F_{1,0}^{1,0}(E)}{F_{1,1}^{0,0}(E)} \in \left[1-\frac{1}{(1+y)^{\beta_1}}, 0\right]$.

1.4.4 Provision of a Retractable Subsidy

Here, we assume that the subsidy that was provided at time T_1^p may be retracted at time T_2^p. Consequently, once the subsidy is provided at T_1^p, the firm receives the value of a retractable subsidy. The expected value of the project can be calculated as indicated in (27). Unlike (18), the second term on the left-hand side of (27) indicates that the subsidy is only available until time T_2^p. Using the properties of the Erlang distribution regarding the joint density of T_1^p and

T_2^p, we can express the expected value of the active project as in the bottom line of (27).

$$\begin{aligned}\mathbb{E}_E\left[\int_0^\infty e^{-\rho t}D_2E_tdt+\int_{T_1^p}^{T_2^p}e^{-\rho t}D_2E_tydt\right]&=\frac{D_2E}{\rho-\mu}+\mathbb{E}\left\{\frac{D_2Ey\left[e^{-(\rho-\mu)T_1^p}-e^{-(\rho-\mu)T_2^p}\right]}{\rho-\mu}\right\}\\ =\frac{D_2E}{\rho-\mu}+\frac{D_2Ey}{\rho-\mu}&\left[\int_0^\infty\lambda_pe^{-\lambda_pT_1^p}e^{-(\rho-\mu)T_1^p}dT_1^p-\int_0^\infty\lambda_p^2T_2^pe^{-\lambda_pT_2^p}e^{-(\rho-\mu)T_2^p}dT_2^p\right]\end{aligned}\tag{27}$$

The analytical expression of (27) is indicated in (28). Note that, unlike (19), the subsidy will be available for a smaller time period, and, therefore, its expected value is reduced, since $\frac{\lambda_p}{(\rho-\mu+\lambda_p)^2}\leq\frac{\lambda_p}{(\rho-\mu)(\rho-\mu+\lambda_p)}$.

$$\Phi_{2,0}^{1,1}(E)\quad=\quad\frac{D_2E}{\rho-\mu}+\frac{\lambda_pD_2Ey}{(\rho-\mu+\lambda_p)^2}\tag{28}$$

Next, we assume that the firm operates the first technology and holds a single embedded investment option. The latter, will either be exercised in the presence of a retractable subsidy with probability λ_pdt, or in the absence of a subsidy that has yet to be provided with probability $1-\lambda_pdt$. Thus, the dynamics of the value function $F_{2,0}^{1,1}(E)$ are described in (29). Notice that the ordinary differential equation that is obtained by expanding the right-hand side of (29) using Itô's lemma must be be solved for each expression of $F_{2,1}^{0,1}(E)$, that is indicated in (11). Thus, the expression for $F_{2,0}^{1,1}(E)$ is indicated in (D-1).

$$F_{2,0}^{1,1}(E)\quad=\quad D_1Edt+(1-\rho dt)\,\mathbb{E}_E\left\{\lambda_pdtF_{2,1}^{0,1}(E+dE)+(1-\lambda_pdt)\,F_{2,0}^{1,1}(E+dE)\right\}\tag{29}$$

The dynamics of the value function $\Phi_{1,0}^{1,1}(E)$ are indicated in (30), where the first term on the right-hand side reflects the immediate profit from operating the first technology. The second term indicates that if a subsidy is provided with probability λ_pdt, then the firm will receive either the value function $F_{2,1}^{0,1}(E)$ or $\Phi_{1,1}^{0,1}(E)$ conditional on the arrival of an innovation. Alternatively, a subsidy will not be provided with probability $1-\lambda_pdt$, and, contingent on the arrival of the second technology, the firm will receive either the value function $F_{2,0}^{1,1}(E)$ or $\Phi_{1,0}^{1,1}(E)$. Solving the differential equation that is obtained by expanding the right-hand side of (30) using Itô's lemma, we obtain the expression that is indicated in (D-2).

$$\begin{aligned}\Phi_{1,0}^{1,1}(E)=D_1Edt\quad&+\quad(1-\rho dt)\,\mathbb{E}_E\left\{\lambda_pdt\left[\lambda_\tau dtF_{2,1}^{0,1}(E+dE)+(1-\lambda_\tau dt)\,\Phi_{1,1}^{0,1}(E+dE)\right]\right.\\ &+\quad\left.(1-\lambda_pdt)\left[\lambda_\tau dtF_{2,0}^{1,1}(E+dE)+(1-\lambda_\tau dt)\,\Phi_{1,0}^{1,1}(E+dE)\right]\right\}\end{aligned}\tag{30}$$

Similarly, the dynamics of the value of the option to invest in the first technology are described in (31). Notice that, over an infinitesimal time interval dt, either the subsidy will become available and the option will be exercised in the presence of a retractable subsidy, or no policy intervention will take place and the firm will continue to hold the value function $F_{1,0}^{1,1}(E)$. Solving (31) for each expression of $F_{1,1}^{0,1}(E)$ that is indicated in (15), we obtain (D-3).

$$F_{1,0}^{1,1}(E) = (1 - \rho dt)\,\mathbb{E}_E \left\{ \lambda_p dt F_{1,1}^{0,1}\,(E + dE) + (1 - \lambda_p dt)\, F_{1,0}^{1,1}\,(E + dE) \right\} \tag{31}$$

As will be shown numerically, the likely retraction of the subsidy after its initial provision decreases the incentive to invest compared to the case of permanent subsidy provision. This happens because the reduction in the lifetime of the subsidy renders it less valuable, thereby increasing the incentive to postpone investment.

1.4.5 Infinite Provisions and Retractions

Here, we assume that a subsidy is subject to infinite provisions and retractions. Taking into account that $\frac{\lambda_p}{(\rho-\mu+\lambda_p)^2} + \frac{\lambda_p^3}{(\rho-\mu+\lambda_p)^4} + \frac{\lambda_p^5}{(\rho-\mu+\lambda_p)^6} + \ldots = \frac{\lambda_p}{(\rho-\mu+\lambda_p)^2} \Big/ \left(1 - \frac{\lambda_p^2}{(\rho-\mu+\lambda_p)^2}\right) = \frac{\lambda_p}{(\rho-\mu)(\rho-\mu+2\lambda_p)}$, the expected value of the profits from operating the second technology when the subsidy is initially absent, i.e., $a = 0$, is indicated in (32).

$$\Phi_{2,0}^{\infty,\infty}(E) = \frac{D_2 E}{\rho - \mu} + \frac{\lambda_p D_2 E y}{(\rho - \mu)\,(\rho - \mu + 2\lambda_p)} \tag{32}$$

By contrast, if the subsidy is initially available, i.e., $a = 1$, then the subsidy will be retracted and provided infinitely many times. Consequently, the expected value of the subsidy is $\frac{(\rho-\mu+\lambda_p)D_2 E y}{(\rho-\mu)(\rho-\mu+2\lambda_p)}$, and the expected NPV of the second technology is indicated in (33).

$$\Phi_{2,1}^{\infty,\infty}(E) = \frac{D_2 E}{\rho - \mu} + \frac{(\rho - \mu + \lambda_p)\, D_2 E y}{(\rho - \mu)\,(\rho - \mu + 2\lambda_p)} \tag{33}$$

The dynamics of the option to invest in the second technology are described in (34) for $a = 0, 1$. The first term on the right-hand side is the instantaneous profit from operating the first technology, while the subsequent terms represent the expected value of the project in the continuation region, that depends on whether the subsidy is provided or retracted.

$$\begin{aligned} F_{2,a}^{\infty,\infty}(E) = D_1 E\,(1 + ya)\,dt + (1 - \rho dt)\,\mathbb{E}_E \Big\{ & \lambda_p dt F_{2,1-a}^{\infty,\infty}(E + dE) \\ & + (1 - \lambda_p dt)\, F_{2,a}^{\infty,\infty}(E + dE) \Big\} \end{aligned} \tag{34}$$

Similarly, the dynamics of the firm's value function when it operates the first technology holding a single embedded option to invest in the second one, are described in (35). The first term on the right-hand side indicates the immediate profit from operating the first technology, while the remaining terms represent the expected value in the continuation region, that, unlike (34) depends on both policy uncertainty and the arrival of the second technology.

$$\Phi_{1,a}^{\infty,\infty}(E) = D_1 E\,(1+ya)\,dt + (1-\rho dt)\,\mathbb{E}_E\Bigg\{\lambda_p dt\Big[\lambda_\tau dt F_{2,1-a}^{\infty,\infty}(E+dE) + (1-\lambda_\tau dt) \\ \times \Phi_{1,1-a}^{\infty,\infty}(E+dE)\Big] + (1-\lambda_p dt)\,\Big[\lambda_\tau dt F_{2,a}^{\infty,\infty}(E+dE) + (1-\lambda_\tau dt)\,\Phi_{1,a}^{\infty,\infty}(E+dE)\Big]\Bigg\} \tag{35}$$

Finally, the dynamics of the firm's value function in the initial state are indicated in (36). The expression of $F_{1,a}^{\infty,\infty}(E)$ for each value of a is indicated in (E-8) and (E-9), and is obtained by first expanding the right-hand side of (36) using Itô's lemma and then solving the set of coupled ordinary differential equation corresponding to $a = 0, 1$.

$$F_{1,a}^{\infty,\infty}(E) = (1-\rho dt)\,\mathbb{E}_E\Big\{\lambda_p dt F_{1,1-a}^{\infty,\infty}(E+dE) + (1-\lambda_p dt)\,F_{1,a}^{\infty,\infty}(E+dE)\Big\} \tag{36}$$

1.5 Numerical Results

For the numerical results we assume that $\rho = 0.1$, $\mu = 0.01$, $\sigma \in [0.2, 0.3]$, $y = 0.1$, $I_1 = 500$, $I_2 = 1500$, $D_1 = 8$, $D_2 = 16$, and $\lambda_p, \lambda_\tau \in [0, 1]$. In line with Section 1.3, we assume that a portion of I_2, e.g. 500 is used for decommissioning the existing technology and the remaining amount (1000) for investing in the new one. Thus, the assumption $\frac{D_2}{I_2} < \frac{D_1}{I_1}$ is satisfied. In turn, this creates a trade-off between the two technologies in that their corresponding NPVs become equal at a positive NPV. Therefore, the NPV of the second (first) technology is greater than that of the first (second) one at high (low) output prices. Figure 1.2 illustrates the project and option value as well as the optimal investment threshold in the second technology in the case of permanent subsidy retraction (left panel) and permanent subsidy provision (right panel). As the left panel illustrates, greater likelihood of subsidy retraction lowers the value of the investment opportunity and the expected value of the active project. In turn, this increases the incentive to delay investment and raises the required investment threshold. By contrast, greater likelihood of subsidy provision raises the value of the option to invest and lowers the required investment threshold.

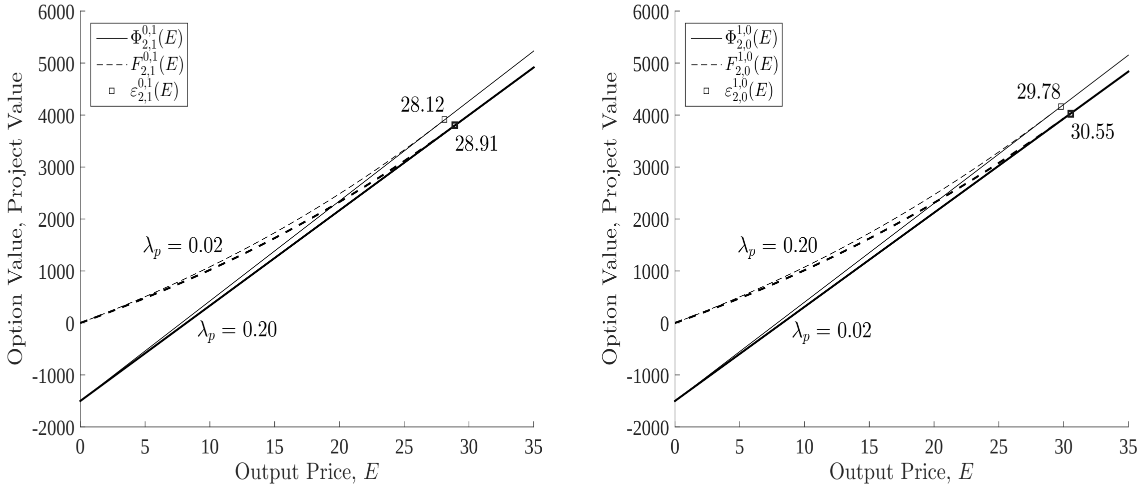
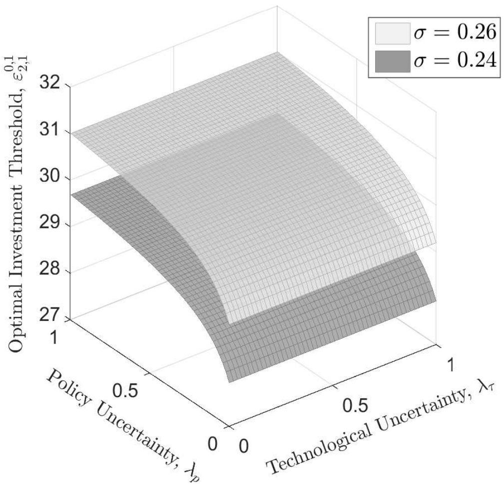


Figure 1.2: Option and project value for investment in the second technology under permanent subsidy retraction (left panel) and permanent subsidy provision (right panel) for $\lambda_p = 0.02, 0.2$ and $\sigma = 0.24$. Greater likelihood of subsidy retraction (provision) lowers (raises) the expected value of the project and decreases (increases) the investment incentive.

Similarly, Figure 1.3 illustrates the impact of technological and policy uncertainty on the optimal investment threshold in the second (left panel) and the first technology (right panel) under sudden subsidy retraction. Notice that the threat of permanent subsidy retraction decreases the firm's incentive to invest and raises the optimal investment threshold, as shown in Proposition 1. Interestingly, this is in contrast to Chronopoulos *et al.* (2016) and Adkins & Paxson (2016), who show that greater likelihood of subsidy retraction accelerates investment. Note, however, that, in these models, the value of the active project is a linear function of λ_p, yet, in our model, the impact of policy uncertainty on the value of the active project is exponential. Intuitively, this implies that the incentive to invest early in order to take advantage of the subsidy for a longer period does not compensate the loss in value due to subsidy retraction. Consequently, assumptions regarding the impact of policy uncertainty on the value of the active project may have crucial implications regarding a firm's optimal investment policy.

As the right panel illustrates, the possibility to invest in a more efficient technology, mitigates the impact of policy uncertainty. This happens because the prospect of sequential investment increases the value of the initial investment opportunity and mitigates the loss in option value due to subsidy retraction. Additionally, greater price uncertainty raises the opportunity cost of investment, and, in turn, the value of waiting, thereby increasing the incentive to postpone investment (Dixit & Pindyck, 1994). These results have important implications for both private firms and policymakers. Indeed, the former can take into account the impact of policy uncertainty on the value of the project and the option to invest, and, thus, make more informed investment and operational decisions. Similarly, the latter can devise more effective policy mechanisms by taking into account how firms may respond to policy uncertainty in the light of sequential investment opportunities which particularly relevant to emerging technologies and the RE sector.



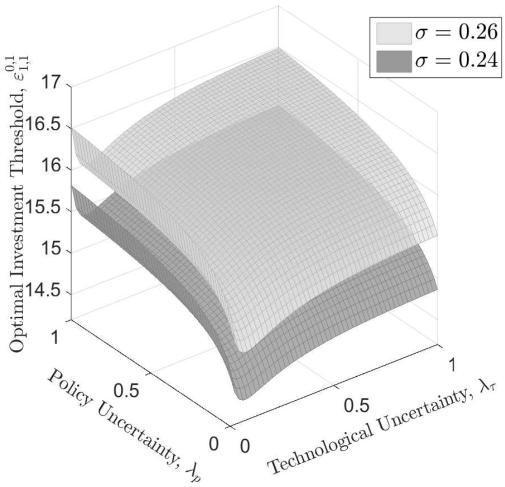


Figure 1.3: Impact of λ_p and λ_τ on the optimal investment threshold in the second (left) and the first technology (right) under sudden subsidy retraction. Greater likelihood of technological innovation raises the value of the project and increases the incentive to invest in the existing technology, thereby mitigating the impact of subsidy retraction.

Unlike the case of sudden subsidy retraction, the left panel in Figure 1.4 indicates that if a firm holds a single investment option, then greater likelihood of subsidy provision accelerates investment, as shown in Proposition 3. As the right panel illustrates, this result becomes more pronounced when investing in the first technology and the likelihood of the second technology increases. This happens because an increase in λ_τ raises the value of the embedded option to invest in the second technology, and, in turn, the value of the option to invest in the first one. Intuitively, a greater likelihood of subsidy provision raises the value of the investment opportunity, and, in turn, the firm's incentive to invest. This implies that the rapid increase in project value due to subsidy provision mitigates the firm's incentive to postpone investment due to the temporary absence of the subsidy.

Figure 1.5 illustrates how the impact of policy uncertainty on the optimal investment threshold can be decomposed with respect to the MB and MC of delaying investment. Notice that greater likelihood of subsidy retraction lowers both the MB and the MC curve, yet

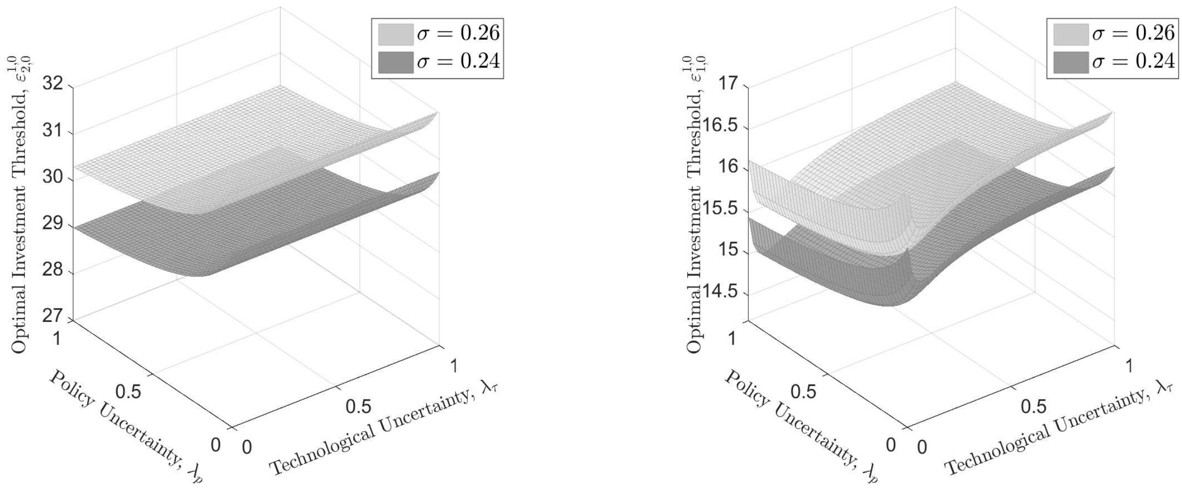


Figure 1.4: Impact of λ_p and λ_τ on the optimal investment threshold in the second (left) and the first technology (right) under sudden subsidy provision. The likely arrival of a new technology raises the incentive to invest in the existing one and makes the impact of subsidy provision more pronounced.

the latter shifts down by more than the former, and, as a result, the two curves intersect at a higher threshold (left panel). Intuitively, the extra cost from delaying investment is reflected partly in the loss in value due to the absence of the second technology. This loss becomes more pronounced as both the output price and the likelihood of subsidy retraction increase. By contrast, as the right panel illustrates, greater likelihood of subsidy provision decreases both the MB and MC of delaying investment, yet the MB decreases by more, thereby decreasing the marginal value of delaying investment, and, in turn, the optimal investment threshold.

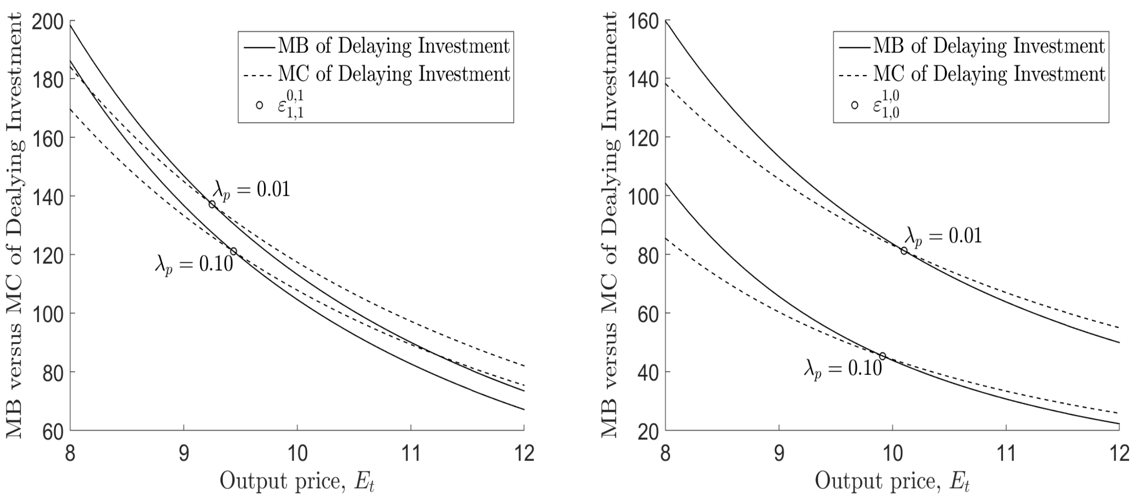


Figure 1.5: Impact of λ_p on the MB and MC of delaying investment for a permanent sudden retraction (left) and a permanent provision (right), for $\lambda_\tau = 0.02$ and $\sigma = 0.24$.

The relative loss in option value due to sudden subsidy retraction and provision is illustrated in the left and right panel of Figure 1.6, respectively. According to the left panel, greater likelihood of subsidy retraction raises the relative loss in option value, as shown in Proposition 2, and this result becomes more pronounced as the rate of innovation increases. By contrast, the right panel illustrates that, in the case of sudden subsidy provision, the relative loss in option value decreases with greater λ_p, as shown in Proposition 4. Again, this result becomes more pronounced as the rate of innovation increases. This is in line with Propositions 1 and 3, as it implies that the incentive to postpone (accelerate) investment increases with greater likelihood of subsidy retraction (provision), and this becomes more pronounced in the presence of embedded investment options. Notice also that the relative loss in option value is never zero because the firm can always exercise an investment option whether a subsidy is present or not, albeit at a higher price threshold.

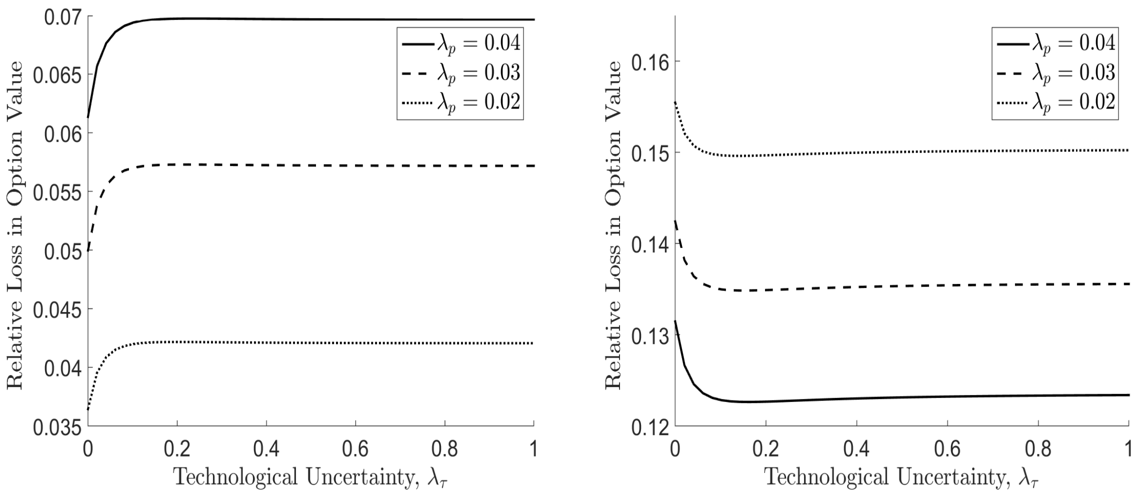


Figure 1.6: Impact of λ_p and λ_τ on the relative loss in options value under permanent subsidy retraction (left) and permanent subsidy provision (right), for $\sigma = 0.24$. The likely arrival of a more efficient technology raises (lowers) the relative loss in option value due to subsidy retraction (provision).

Figure 1.7 illustrates how the likely retraction of a subsidy following its initial provision impacts the optimal investment policy as well as the relative loss in option value. As the left panel illustrates, the retraction of the subsidy lowers the expected value of the project, and, in turn, the expected value of the investment opportunity, thereby increasing the incentive to postpone investment and raising the required investment threshold. In the right panel, the arrows indicate the direction of increasing policy interventions. Notice that the relative loss in option value under the sudden provision of a retractable subsidy (thick curves) is greater than the relative loss in option value in the case of sudden provision of a permanent subsidy (thin curves).

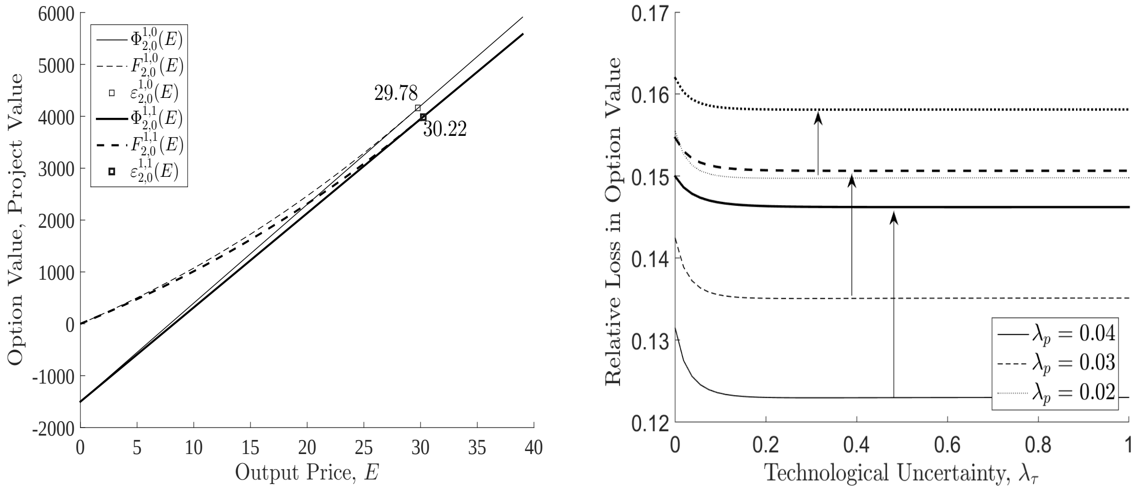


Figure 1.7: Option and project value for investment in the second technology under the provision of a permanent and a retractable subsidy for $\lambda_p = 0.2$ and $\sigma = 0.24$ (left panel) and relative loss in option value (right panel). The likely retraction of a subsidy following its initial provision increases the incentive to postpone investment and raises the relative loss in option value.

Figure 1.8 illustrates the impact of λ_p and λ_τ on the optimal investment threshold in the case of provision of a permanent and a retractable subsidy. As both panels illustrate, $\lambda_p = 0$ implies that the subsidy will never be provided, and, therefore, $\epsilon_{2,0}^{1,1} = \epsilon_{2,0}^{1,0}$. However, an increase in λ_p implies that the extra value due the provision of the subsidy is reduced due to the likelihood of a subsequent subsidy retraction. Consequently, relative to the case of permanent subsidy provision, the likelihood that the subsidy will be available temporarily decreases the investment incentive and raises the optimal investment threshold, i.e., $\epsilon_{2,0}^{1,1} > \epsilon_{2,0}^{1,0}$. Nevertheless, as the right panel illustrates, the possibility to upgrade an existing technology by adopting a more efficient version creates an opposing force that mitigates the impact of subsidy retraction. This result reveals an important feature of the interaction between technological and policy uncertainty, that is crucial from the perspective of both policymakers and private firms due to the increasing R&D activity in many industries. Indeed, support schemes that aim to stimulate investment in emerging technologies are likely to be revised frequently within a volatile economic environment. Consequently, taking into account both the particular nature of investment in emerging technologies and how private firms are likely to respond to frequent revisions of a

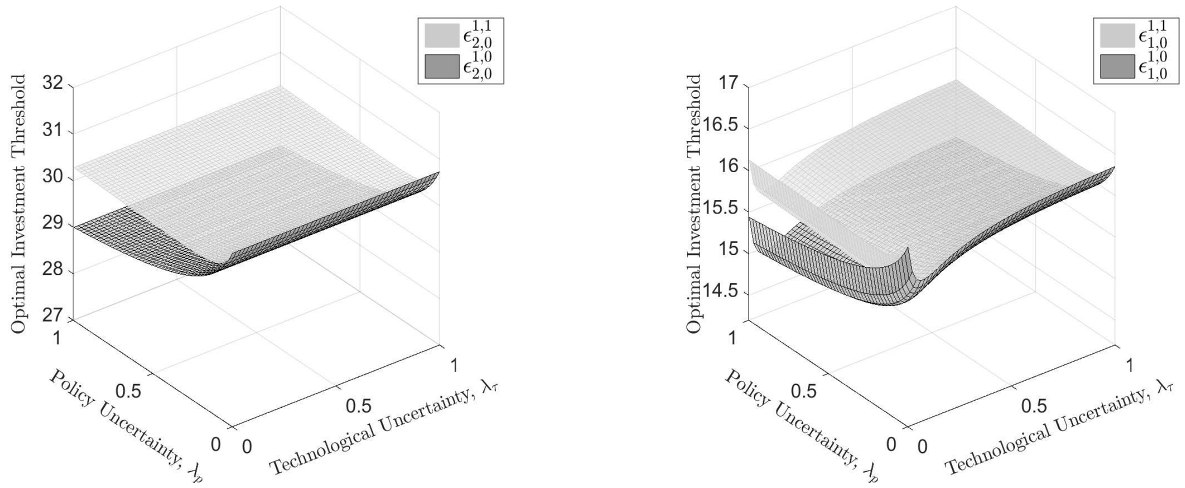


Figure 1.8: Impact of λ_p and λ_τ on the optimal investment threshold in the second (left) and the first technology version (right) under sudden provision of a permanent and a retractable subsidy, for $\sigma = 0.24$. The likely retraction of a subsidy following its initial provision decreases the expected value of the project and increases the incentive to postpone investment. However, the option to adopt a more efficient technology mitigates the impact of subsidy retraction.

support scheme when a project entails embedded investment options, will enable more informed policymaking decisions.

The impact of λ_p and λ_τ on the optimal investment threshold under infinite provisions and retractions is illustrated in Figure 1.9. Even though the optimal investment thresholds present a similar behaviour as in the case of permanent subsidy provision and retraction, increasing number of policy interventions make the impact of policy uncertainty less pronounced. Indeed, the optimal investment threshold in the case of infinite provisions and retractions is lower (higher) compared to the case of temporary (permanent) subsidy provision, i.e., $\epsilon_{2,0}^{1,0} < \epsilon_{2,0}^{\infty,\infty} < \epsilon_{2,0}^{1,1}$. Intuitively, subsequent subsidy provisions (retractions) raise (lower) the expected option and project value, and, as a result, the optimal investment threshold is in between the initial scenarios of permanent and temporary subsidy provision. Notice also that the investment incentive is greater if the subsidy is currently available, i.e., $\epsilon_{2,1}^{\infty,\infty} < \epsilon_{2,0}^{\infty,\infty}$. This happens because, due to policy uncertainty and the effect of discounting, the first policy intervention has a greater impact on the expected value of the project, and, in turn, the investment decision. Additionally, the investment thresholds for $a = 0$ (the subsidy is initially absent) and $a = 1$ (the subsidy is initially present) converge to each other when the rate of policy interventions increases. In fact, as the right panel illustrates, this convergence becomes more pronounced in the presence of embedded investment options.

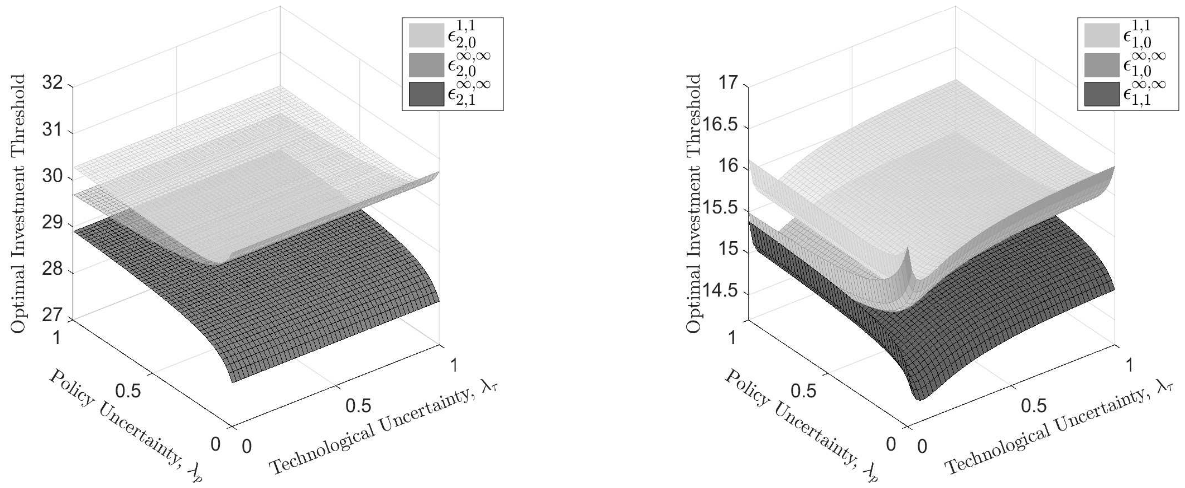


Figure 1.9: Impact of λ_p and λ_τ on the optimal investment threshold in the second (left) and the first technology (right) under infinite provisions and retractions, for $\sigma = 0.24$. Increasing number of policy interventions make the impact of policy uncertainty on the optimal investment threshold less pronounced.

1.6 Conclusions

In an era of increasing economic uncertainty, firms in sectors such as energy, manufacturing, and telecommunications require managerial strategies that are responsive to market conditions. For example, the implications of the structural transformation of the power sector for both market participants and policymakers are considered to be crucial as they are expected to change substantially the wholesale market dynamics (Sensfuß *et al.*, 2008). Within this environment, private firms are required to make accurate investment decisions, while policymakers must take into account how private firms respond to different sources of uncertainty in order to incentivise investment. In this paper, we develop a real options framework in order to investigate how price, policy, and technological uncertainty interact to affect the decision to invest sequentially in an emerging technology. Therefore, we consider a private firm with an option to invest sequentially in successively improved versions of a technology, facing technological and policy uncertainty. The latter is implemented by analysing the case of sudden subsidy retraction, sudden provision of a permanent and retractable subsidy, as well as infinite provisions and retractions.

We show that greater likelihood of subsidy retraction decreases the investment incentive and postpones investment, whereas, greater likelihood of subsidy provision increases the expected value of the project and accelerates investment. Additionally, we find that increasing the number of policy interventions mitigates the impact of policy uncertainty on the optimal investment rule by reducing the expected value of the subsidy. Interestingly, however, allowing for sequential investment opportunities raises the expected value of the project and increases the incentive to invest. This implies that the possibility to invest in a more efficient technology mitigates the impact of policy uncertainty in the case of subsidy retraction, but can make the impact of policy uncertainty more pronounced in the case of sudden subsidy provision. These results are particularly relevant to the energy sector, where frequent revisions of support schemes create uncertain responses to incentives while technological innovations create sequential investment opportunities. Thus, these results provide complementary insights to the well-established energy systems models by addressing the impact of incentives upon market agents. Indeed, understanding how an increasing rate of policy interventions and technological innovations may influence the propensity to invest is particularly crucial for the design of policies that aim to promote long-term investment decisions. Additionally, by deriving rigorous results regarding the impact of policy uncertainty on sequential investment decisions, we provide a direction for a better understanding of the different results observed in the literature (Boomsma *et al.*, 2015; Adkins & Paxson, 2016), and, in turn, insights for identifying the appropriate model specification that aims at capturing policy uncertainty.

Although the application of real options is a selective process, the implications can be strong, and alongside other aspects of behavioural economics, would appear to be essential in understanding how support schemes may be implemented in order to create investment incentives. A limitation of the current framework is reflected in the independence between price and policy uncertainty. This limitation can be relaxed by developing a two-factor model in order to investigate how the correlation between price and policy uncertainty impacts the optimal investment policy. Additionally, empirical research regarding the rate of policy interventions would provide crucial insights not only on the appropriate model specification, but also on how to configure model parameters in order to model realistic situations in various industries. In order to relax the assumption of a GBM, a mean-reverting process could be implemented within the same framework, while allowing for different technology adoption strategies, e.g. leapfrog and laggard, would enable further investigation of how the dominant strategy is affected by technological and policy uncertainty. Also, allowing for strategic interactions will provide insights on how policy measures may enhance or reduce the competitive advantage of power plants depending on their asymmetries, related to cost and operational flexibility. For example, a carbon-price floor can influence the value of operational flexibility, thereby inducing investment in a RE facility by decreasing the value of operational flexibility embedded in a commodity-based facility (Chronopoulos *et al.*, 2014), while capacity sizing could potentially partly offset political risk (Chronopoulos *et al.*, 2017).

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1.7 Appendix

A Benchmark Case

The dynamics of the value function $F_{2,a}^{0,0}(E)$ are described in (A-1).

$$F_{2,a}^{0,0}(E) = D_1 E\,(1+ya)\,dt + (1-\rho dt)\,\mathbb{E}_E\left\{F_{2,a}^{0,0}(E+dE)\right\} \tag{A-1}$$

Using Itô's lemma, we expand the right-hand side of (A-1), and, thus, we obtain (A-2).

$$\frac{1}{2}\sigma^2 E^2 F_{2,a}^{0,0''}(E) + \mu E F_{2,a}^{0,0'}(E) - \rho F_{2,a}^{0,0}(E) + D_1 E\,(1+ya) = 0 \tag{A-2}$$

Notice that the solution of the homogeneous part of (A-2) is $F_{2,a}^{0,0}(E) = A_{2,a}^{0,0}E^{\beta_1} + B_{2,a}^{0,0}E^{\beta_2}$. However, $E \to 0 \Rightarrow B_{2,a}^{0,0}E^{\beta_2} \to \infty$, and, therefore, $B_{2,a}^{0,0} = 0$. The expression for $F_{2,a}^{0,0}(E)$ is indicated in (3). Also, $\varepsilon_{2,a}^{0,0}$ and $A_{2,a}^{0,0}$ are indicated in (4) and are determined analytically via the value-matching and smooth-pasting conditions indicated in (A-3) and (A-4), respectively

$$\frac{D_2\varepsilon_{2,a}^{0,0}\,(1+ay)}{\rho-\mu} - I_2 = \frac{D_1\varepsilon_{2,a}^{0,0}(1+ay)}{\rho-\mu} + A_{2,a}^{0,0}\varepsilon_{2,a}^{0,0^{\beta_1}} \tag{A-3}$$

$$\frac{D_2\,(1+ay)}{\rho-\mu} = \frac{D_1(1+ay)}{\rho-\mu} + \beta_1 A_{2,a}^{0,0}\varepsilon_{2,a}^{0,0^{\beta_1-1}} \tag{A-4}$$

Next, the firm is operating the first technology version and holds an option to invest in the second one. The dynamics of the value function are described in (5), and by expanding the right-hand side of (5) using Itô's lemma, we obtain (A-5).

$$\frac{1}{2}\sigma^2 E^2 \Phi_{1,a}^{0,0''}(E) + \mu E \Phi_{1,a}^{0,0'}(E) - (\rho+\lambda_\tau)\Phi_{1,a}^{0,0}(E) + \lambda_\tau F_{1,a}^{0,0}(E) + D_1 E\,(1+ya) = 0 \tag{A-5}$$

The endogenous constants $A_{1,a}^{0,0}$ and $B_{1,a}^{0,0}$ are indicated in (A-6) and (A-7) and are determined by the value-matching and smooth-pasting conditions between the two branches of (6).

$$A_{1,a}^{0,0} = \frac{\varepsilon_{2,a}^{0,0^{-\delta_1}}}{\delta_2 - \delta_1} \left[\frac{\lambda_\tau (\delta_2 - 1)(D_2 - D_1)(1 + ya)\varepsilon_{2,a}^{0,0}}{(\rho - \mu)(\rho - \mu + \lambda_\tau)} - \frac{\delta_2 \lambda_\tau I_2}{\lambda_\tau + \rho} + (\beta_1 - \delta_2) A_{2,a}^{0,0} \varepsilon_{2,a}^{0,0^{\beta_1}} \right] \leq 0 \quad \text{(A-6)}$$

$$B_{1,a}^{0,0} = \frac{\varepsilon_{2,a}^{0,0^{-\delta_2}}}{\delta_1 - \delta_2} \left[\frac{\lambda_\tau (1 - \delta_1)(D_2 - D_1)(1 + ya)\varepsilon_{2,a}^{0,0}}{(\rho - \mu)(\rho - \mu + \lambda_\tau)} + \frac{\delta_1 \lambda_\tau I_2}{\lambda_\tau + \rho} - (\beta_1 - \delta_1) A_{2,a}^{0,0} \varepsilon_{2,a}^{0,0^{\beta_1}} \right] \geq 0 \quad \text{(A-7)}$$

B Permanent Subsidy Retraction

Proposition 1 *Greater likelihood of subsidy retraction raises the optimal investment threshold.*
Proof: Notice that greater λ_p lowers the expected value of the project, thereby reducing both the MB and the MC of delaying investment. Also, notice that the first five terms on the left-hand side of (17) are less sensitive to changes in λ_p than the first four terms on the right-hand side, since $\theta_1 \geq \eta_1 \geq \beta_1 \geq 1$ and $\theta_1 \geq \delta_1 \geq \beta_1 \geq 1$.

We start by assuming that $\lambda_\tau = 0$ and denote the last terms on the left- and right-hand side of (17) by $G = \beta_1 C_{1,1}^{0,1} + \eta_1 A_{2,1}^{0,1}$ and $H = \eta_1 C_{1,1}^{0,1} + \beta_1 A_{2,1}^{0,1}$, respectively. Notice that the ratio between $C_{1,1}^{0,1}$ and $A_{2,1}^{0,1}$ is equal to $\left[\frac{(D_2 - D_1) I_1}{D_1 I_2}\right]^{\eta_1} \frac{I_2}{I_1} < 1$, and, thus, G and H can be expressed as in (B-1) and (B-2), respectively.

$$G = \beta_1 C_{1,1}^{0,1} + \eta_1 A_{2,1}^{0,1} = \left[\beta_1 + \eta_1 \left(\frac{(D_2 - D_1) I_1}{D_1 I_2} \right)^{\eta_1} \frac{I_2}{I_1} \right] C_{1,1}^{0,1} \quad \text{(B-1)}$$

$$H = \eta_1 C_{1,1}^{0,1} + \beta_1 A_{2,1}^{0,1} = \left[\eta_1 + \beta_1 \left(\frac{(D_2 - D_1) I_1}{D_1 I_2} \right)^{\eta_1} \frac{I_2}{I_1} \right] C_{1,1}^{0,1} \quad \text{(B-2)}$$

Since $C_{1,1}^{0,1}$ impacts G and H in the same way, the effect of λ_p on the optimal investment threshold depends on how it impacts the expressions within the brackets. According to (B-3), the impact of λ_p of H is more pronounced, which implies that the MC of delaying investment decreases by more than the MB, thereby increasing the marginal value of delaying investment.

$$\frac{\partial G}{\partial \lambda_p} < \frac{\partial H}{\partial \lambda_p} \Leftrightarrow \left[\left(\frac{(D_2 - D_1) I_1}{D_1 I_2} \right)^{\eta_1} \frac{I_2}{I_1} - 1 \right] \frac{\partial \eta_1}{\partial \lambda_p} + (\eta_1 - \beta_1) \frac{\partial}{\partial \lambda_p} \left[\left(\frac{(D_2 - D_1) I_1}{D_1 I_2} \right)^{\eta_1} \frac{I_2}{I_1} \right] < 0 \quad \text{(B-3)}$$

If $\lambda_\tau > 0$, then both the MB and the MC of delaying investment increase due to the embedded investment option, yet λ_τ only impacts $C_{1,1}^{0,1}$, and, therefore, the overall effect remains unchanged. □

Proposition 2 $\frac{F_{1,1}^{0,0}(E)-F_{1,1}^{0,1}(E)}{F_{1,1}^{0,0}(E)} \in \left[0, 1-\frac{1}{(1+y)^{\beta_1}}\right]$.

Proof: The relative loss in option value due to subsidy retraction is outlined in (B-4).

$$\frac{F_{1,1}^{0,0}(E)-F_{1,1}^{0,1}(E)}{F_{1,1}^{0,0}(E)} = \frac{\left(C_{1,1}^{0,0}-C_{1,0}^{0,0}\right)E^{\beta_1}-C_{1,1}^{0,1}E^{\eta_1}}{C_{1,1}^{0,0}E^{\beta_1}} \tag{B-4}$$

Notice that $\lambda_p = 0 \Rightarrow F_{1,1}^{0,0}(E) = F_{1,1}^{0,1}(E) \Rightarrow \frac{F_{1,1}^{0,0}(E)-F_{1,1}^{0,1}(E)}{F_{1,1}^{0,0}(E)} = 0$. By contrast, as λ_p increases, the relative loss increases since $C_{1,1}^{0,1} \to 0$. Also, notice that $\varepsilon_{2,1}^{0,0} = \frac{\varepsilon_{2,0}^{0,0}}{1+y}$, $A_{2,1}^{0,0} = A_{2,0}^{0,0}(1+y)^{\beta_1}$, and, $\varepsilon_{1,1}^{0,0} = \frac{\varepsilon_{1,0}^{0,0}}{1+y}$. Thus, $A_{1,1}^{0,0} = (1+y)^{\delta_1} A_{1,0}^{0,0}$, and by substituting $\varepsilon_{1,1}^{0,0}$, $A_{1,1}^{0,0}$ and $A_{2,1}^{0,0}$ in the expression for $C_{1,1}^{0,0}$, we obtain (B-5).

$$C_{1,1}^{0,0} = (1+y)^{\beta_1}\frac{1}{\varepsilon_{1,1}^{0,0\beta_1}}\left(\frac{D_1\varepsilon_{1,0}^{0,0}}{\rho-\mu} - I_1 + A_{2,0}^{0,0}\varepsilon_{1,0}^{0,0\beta_1} + A_{1,0}^{0,0}\varepsilon_{1,0}^{0,0\delta_1}\right) = (1+y)^{\beta_1} C_{1,0}^{0,0} \tag{B-5}$$

Hence, $\frac{C_{1,1}^{0,0}}{C_{1,0}^{0,0}} = (1+y)^{\beta_1}$, and, thus, $\frac{F_{1,1}^{0,0}(E)-F_{1,1}^{0,1}(E)}{F_{1,1}^{0,0}(E)} = 1-\frac{1}{(1+y)^{\beta_1}}$ □

C Provision of a Permanent Subsidy

Proposition 3 *Greater likelihood of subsidy provision lowers the optimal investment threshold.*

Proof: If $\lambda_\tau = 0$, then (26) can be rewritten as in (C-1), since $\theta_1 = \eta_1$.

$$\left(\frac{E}{\varepsilon_{1,0}^{1,0}}\right)^{\eta_1}\left[\frac{D_1}{\rho-\mu+\lambda_p} + \frac{\eta_1\rho I_1}{(\rho+\lambda_p)\varepsilon_{1,0}^{1,0}} + (\eta_1-\eta_2)H_{1,0}^{1,0}\varepsilon_{1,0}^{1,0\eta_2-1}\right] = \left(\frac{E}{\varepsilon_{1,0}^{1,0}}\right)^{\eta_1}\left[\frac{\eta_1 D_1}{\rho-\mu+\lambda_p}\right] \tag{C-1}$$

By inserting the expression for $H_{1,0}^{1,0} = \frac{1}{(\eta_1-\eta_2)\varepsilon_{1,1}^{0,0\eta_2}}\left((\eta_1-\beta_1)C_{1,1}^{0,0}\varepsilon_{1,1}^{0,0\beta_1} - (\eta_1-1)\frac{\lambda_p D_1\varepsilon_{1,1}^{0,0}(1+y)}{(\rho-\mu)(\rho-\mu+\lambda_p)} + \eta_1\frac{\lambda_p I_1}{\rho+\lambda_p}\right)$ in (C-1), subtracting the left from the right-hand side, and taking the derivative with respect to λ_p we obtain (C-2).

$$\frac{\partial}{\partial\lambda_p}\left[\frac{(\eta_1 D_1(\rho-\mu)+\lambda_p D_1)\left[\varepsilon_{1,1}^{0,0^{1-\eta_2}}(1+y) - \varepsilon_{1,0}^{1,0^{1-\eta_2}}\right]}{(\rho-\mu)(\rho-\mu+\lambda_p)} + \frac{\eta_1\rho I_1\left[\varepsilon_{1,0}^{1,0^{-\eta_2}} - \varepsilon_{1,1}^{0,0^{-\eta_2}}\right]}{\rho+\lambda_p}\right]$$
$$+(\eta_1-\eta_2)H_{1,0}^{1,0}\log\left(\frac{\varepsilon_{1,0}^{1,0}}{\varepsilon_{1,1}^{0,0}}\right)\frac{\partial\eta_2}{\partial\lambda_p} < 0 \tag{C-2}$$

Starting with the second term on the left-hand side of (C-2), we notice that:

$$\frac{\partial}{\partial\lambda_p}\frac{\eta_1}{\rho+\lambda_p} = \frac{\frac{\partial\eta_1}{\partial\lambda_p}(\rho+\lambda_p)-\eta_1}{(\rho+\lambda_p)^2} < \frac{\frac{\partial\eta_1}{\partial\lambda_p}(\rho+\lambda_p)-\sqrt{\frac{\rho+\lambda_p}{\sigma^2}}}{(\rho+\lambda_p)^2} < 0 \tag{C-3}$$

And by inserting $\frac{\partial \eta_1}{\partial \lambda_p} = \frac{1}{\sigma^2 \sqrt{\left(\frac{\mu}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2(\rho+\lambda_p)}{\sigma^2}}}$ into the inequality $\frac{\partial \eta_1}{\partial \lambda_p}(\rho+\lambda_p) - \sqrt{\frac{\rho+\lambda_p}{\sigma^2}} < 0$ we obtain $0 < \left(\frac{\mu}{\sigma^2}-\frac{1}{2}\right)^2 + \frac{(\rho+\lambda_p)}{\sigma^2}$, which cannot be negative. Next, we take the partial derivative of the first term on the left-hand side of (C-2) with respect to λ_p and we obtain (C-4).

$$\frac{\partial}{\partial \lambda_p} \frac{\eta_1 D_1(\rho-\mu) + \lambda_p D_1}{(\rho-\mu)(\rho-\mu+\lambda_p)} = \frac{D_1\left[\frac{\partial \eta_1}{\partial \lambda_p}(\rho+\lambda_p-\mu) - \eta_1 + 1\right]}{(\rho-\mu)(\rho-\mu+\lambda_p)^2} \tag{C-4}$$

Similarly, we can show that $\frac{\partial \eta_1}{\partial \lambda_p}(\rho+\lambda_p-\mu)-\eta_1+1<0$, and, that, ${\varepsilon_{1,1}^{0,0}}^{1-\eta_2}(1+y) - {\varepsilon_{1,0}^{1,0}}^{1-\eta_2} < 0$, and $\varepsilon_{1,1}^{0,0}(1+y) = \varepsilon_{1,0}^{0,0}$. The final term in (C-2) is negative because $\frac{\partial \eta_2}{\partial \lambda_p} < 0$, while the other terms are positive. Consequently, the MB of delaying investment decreases by more than the MC. If $\lambda_\tau > 0$, then both the MB and the MC of delaying investment increase due to the embedded investment option, however, since policy uncertainty impacts the embedded investment option in the same way, the overall effect is maintained. □

Proposition 4 $\frac{F_{1,1}^{0,0}(E) - F_{1,0}^{1,0}(E)}{F_{1,1}^{0,0}(E)} \in \left[1 - \frac{1}{(1+y)^{\beta_1}}, 0\right]$

Proof: Notice that the relative loss in option value when $\lambda_p = 0$ is $\frac{F_{1,1}^{0,0}(E) - F_{1,0}^{1,0}(E)}{F_{1,1}^{0,0}(E)} = \frac{C_{1,1}^{0,0} - C_{1,0}^{0,0}}{C_{1,1}^{0,0}}$, which, from Proposition 2, is $1 - \frac{1}{(1+y)^{\beta_1}}$. Furthermore, when λ_p increases the value of the adjustment term, $G_{1,0}^{1,0}$, approaches zero, and the relative loss is zero. Since $\frac{\partial}{\partial \lambda_\tau} G_{1,0}^{1,0} > 0$ and $G_{1,0}^{1,0} \leq 0$, the relative loss is decreasing when we increase λ_τ. □

D Provision of a Retractable Subsidy

By expanding the right-hand side of (29) using Itô's lemma and solving the resulting ordinary differential equation, we obtain (D-1). The first two terms in the top part of (D-1) reflect the expected profit from operating the first technology. The third term represents the option to invest in the second technology in the permanent absence of a subsidy, adjusted via the last term, since the subsidy will be provided and subsequently retracted. Similarly, the first three terms in the middle part represent the expected profit from operating the second technology, while the last two terms represent the likelihood of the price either dropping in the waiting region or rising above $\varepsilon_{2,0}^{1,1}$.

$$F_{2,0}^{1,1}(E) = \begin{cases} \frac{D_1 E}{\rho-\mu} + \frac{\lambda_p D_1 E y}{(\rho-\mu+\lambda_p)^2} + A_{2,0}^{0,0} E^{\beta_1} + \left[\frac{\lambda_p A_{2,1}^{0,1}}{\frac{1}{2}\sigma^2 - \eta_1\sigma^2 - \mu} \ln E + A_{2,0}^{1,1}\right] E^{\eta_1} & , E < \varepsilon_{2,1}^{0,1} \\ \frac{\lambda_p D_2 E + (\rho-\mu) D_1 E}{(\rho-\mu)(\rho-\mu+\lambda_p)} + \frac{\lambda_p D_2 E y}{(\rho-\mu+\lambda_p)^2} - \frac{\lambda_p}{\rho+\lambda_p} I_2 + B_{2,0}^{1,1} E^{\eta_2} + C_{2,0}^{1,1} E^{\eta_1} & , \varepsilon_{2,1}^{0,1} \leq E < \varepsilon_{2,0}^{1,1} \\ \Phi_{2,0}^{1,1}(E) - I_2 & , E \geq \varepsilon_{2,0}^{1,1} \end{cases} \tag{D-1}$$

Similarly, by expanding the right-hand side of (30) using Itô's lemma and solving the resulting ordinary differential equation for each expression of $F_{1,0}^{1,1}(E)$ that is indicated in (D-1), we obtain (D-2). Note that $A_{1,0}^{1,1}$, $B_{1,0}^{1,1}$, $C_{1,0}^{1,1}$ and $D_{1,0}^{1,1}$ are determined via value-matching and smooth-pasting conditions between the three branches.

$$\Phi_{1,0}^{1,1}(E) = \begin{cases} \frac{D_1 E}{\rho-\mu} + \frac{\lambda_p D_1 E y}{(\rho-\mu+\lambda_p)^2} + A_{2,0}^{0,0} E^{\beta_1} + A_{1,0}^{0,0} E^{\delta_1} + A_{1,0}^{1,1} E^{\theta_1} \\ + \frac{\lambda_p A_{1,1}^{0,1} \ln E}{\frac{1}{2}\sigma^2 - \theta_1\sigma^2 - \mu} E^{\theta_1} + \left(\frac{\lambda_p}{\lambda_\tau} A_{2,1}^{0,1} + A_{2,0}^{1,1}\right) E^{\eta_1} \\ + \frac{\lambda_\tau \lambda_p A_{2,1}^{0,1} E^{\eta_1}}{(\theta_2-\theta_1)\left(\frac{1}{2}\sigma^2 - \eta_1\sigma^2 - \mu\right)\frac{1}{2}\sigma^2} \left[\frac{(\eta_1-\theta_1)\ln E - 1}{(\eta_1-\theta_1)^2} - \frac{(\eta_1-\theta_2)\ln E - 1}{(\eta_1-\theta_2)^2}\right] & , E < \varepsilon_{2,1}^{0,1} \\ \\ \left[\frac{\lambda_\tau D_2 + (\rho-\mu) D_1}{(\rho-\mu)(\rho-\mu+\lambda_\tau)} + \frac{[\lambda_\tau D_2 + (\rho-\mu+\lambda_p) D_1] y}{(\rho-\mu+\lambda_p)(\rho-\mu+\lambda_p+\lambda_\tau)} + \frac{\lambda_\tau D_2}{(\rho-\mu)(\rho-\mu+\lambda_p)}\right. \\ \left. + \frac{\lambda_\tau D_2 y}{(\rho-\mu+\lambda_p)^2}\right] \frac{\lambda_p E}{(\rho-\mu+\lambda_p+\lambda_\tau)} + \frac{D_1 E}{(\rho-\mu+\lambda_p)} + B_{1,0}^{0,0} E^{\delta_2} + \frac{\lambda_p B_{1,1}^{0,1} \ln E}{\frac{1}{2}\sigma^2 - \theta_2\sigma^2 - \mu} E^{\theta_2} \\ - \frac{(2\rho+\lambda_p+\lambda_\tau)\lambda_\tau\lambda_p I_2}{(\rho+\lambda_p+\lambda_\tau)(\rho+\lambda_p)(\rho+\lambda_\tau)} + B_{2,0}^{1,1} E^{\eta_2} + C_{2,0}^{1,1} E^{\eta_1} + B_{1,0}^{1,1} E^{\theta_2} + C_{1,0}^{1,1} E^{\theta_1} & , \varepsilon_{2,1}^{0,1} \leq E < \varepsilon_{2,0}^{1,1} \\ \\ \frac{\lambda_\tau D_2 E + (\rho-\mu) D_1 E}{(\rho-\mu)(\rho-\mu+\lambda_\tau)} + \frac{\lambda_p [\lambda_\tau D_2 + (\rho-\mu+\lambda_p) D_1] E y}{(\rho-\mu+\lambda_p)(\rho-\mu+\lambda_p+\lambda_\tau)^2} + \frac{\lambda_p \lambda_\tau D_2 E y}{(\rho-\mu+\lambda_p)^2(\rho-\mu+\lambda_p+\lambda_\tau)} \\ + B_{1,0}^{0,0} E^{\delta_2} + \frac{\lambda_p B_{1,1}^{0,1} \ln E}{\frac{1}{2}\sigma^2 - \theta_1\sigma^2 - \mu} E^{\theta_2} - \frac{\lambda_\tau I_2}{\rho+\lambda_\tau} + D_{1,0}^{1,1} E^{\theta_2} & , E \geq \varepsilon_{2,0}^{1,1} \end{cases} \quad \text{(D-2)}$$

Finally, the expression of $F_{1,0}^{1,1}(E)$ is indicated in (D-3), where $\varepsilon_{1,0}^{1,1}$, $G_{1,0}^{1,1}$, $H_{1,0}^{1,1}$, and $J_{1,0}^{1,1}$ are determined via value-matching and smooth-pasting conditions between the three branches. The first term on the top branch of (D-3) is the option to invest in the permanent presence of a subsidy, adjusted via the second term due to policy uncertainty. The second branch reflects the expected project value if the subsidy becomes available, and the bottom branch is expected project value when the price is high enough so that investment is optimal even in the absence of a subsidy.

$$F_{1,0}^{1,1}(E) = \begin{cases} C_{1,1}^{0,0} E^{\beta_1} + \left(\frac{\lambda_p C_{1,1}^{0,1}}{\frac{1}{2}\sigma^2 - \eta_1\sigma^2 - \mu} \ln E + G_{1,0}^{1,1}\right) E^{\eta_1} & , E < \varepsilon_{1,1}^{0,1} \\ \\ \frac{\lambda_p D_1 E}{(\rho-\mu)(\rho-\mu+\lambda_p)} + \frac{\lambda_p D_1 E y}{(\rho-\mu+\lambda_p)^2} - \frac{\lambda_p I_1}{\rho+\lambda_p} + A_{2,0}^{0,0} E^{\beta_1} - \frac{\lambda_p}{\lambda_\tau} A_{1,1}^{0,1} E^{\theta_1} \\ + \frac{\lambda_p A_{2,1}^{0,1} \ln E}{\frac{1}{2}\sigma^2 - \theta_1\sigma^2 - \mu} E^{\eta_1} + \frac{\lambda_p}{\lambda_p - \lambda_\tau} A_{1,0}^{0,0} E^{\delta_1} + H_{1,0}^{1,1} E^{\eta_2} + J_{1,0}^{1,1} E^{\eta_1} & , \varepsilon_{1,1}^{0,1} \leq E < \varepsilon_{1,0}^{1,1} \\ \\ \Phi_{1,0}^{1,1}(E) - I_1 & , E \geq \varepsilon_{1,0}^{1,1} \end{cases} \quad \text{(D-3)}$$

E Infinite Provisions and Retractions

By expanding the right-hand side of (34) using Itô's lemma and adding the differential equations corresponding to $a = 0, 1$ we obtain $\overline{f}(E) = F_{2,1}^{\infty,\infty}(E) + F_{2,0}^{\infty,\infty}(E)$, whereas by subtracting them we obtain $\underline{f}(E) = F_{2,1}^{\infty,\infty}(E) - F_{2,0}^{\infty,\infty}(E)$. The dynamics of $\overline{f}(E)$ and $\underline{f}(E)$ are described in

(E-1) and (E-2), respectively.

$$\frac{1}{2}\sigma^2 E^2 \overline{f}^{''}(E) + \mu E \overline{f}^{'}(E) - \rho \overline{f}(E) + 2D_1E + D_1Ey = 0 \tag{E-1}$$

$$\frac{1}{2}\sigma^2 E^2 \underline{f}^{''}(E) + \mu E \underline{f}^{'}(E) - \rho \underline{f}(E) - 2\lambda_p \underline{f}(E) + D_1Ey = 0 \tag{E-2}$$

If $a = 1$, then the expression of $F_{2,1}^{\infty,\infty}(E)$ is indicated in (E-3), where ξ is the solution to the quadratic $\frac{1}{2}\sigma^2\xi(\xi-1)+\mu\xi-(\rho+2\lambda_p)=0$. The first two terms on the top part represent the expected profit from operating the first technology, while the last two terms reflect the adjusted value of the option to invest in the second one.

$$F_{2,1}^{\infty,\infty}(E) = \begin{cases} \frac{D_1E}{\rho-\mu} + \frac{(\rho-\mu+\lambda_p)D_1Ey}{(\rho-\mu)(\rho-\mu+2\lambda_p)} + A_{2,1}^{\infty,\infty}E^{\beta_1} + B_{2,0}^{\infty,\infty}E^{\xi_1} & , E < \varepsilon_{2,1}^{\infty,\infty} \\ \Phi_{2,1}^{\infty,\infty}(E) - I_2 & , E \geq \varepsilon_{2,1}^{\infty,\infty} \end{cases} \tag{E-3}$$

Similarly, if $a = 0$, then the expression of $F_{2,0}^{\infty,\infty}(E)$ is indicated in (E-4), where $A_{2,1}^{\infty,\infty}$, $B_{2,0}^{\infty,\infty}$, $C_{2,0}^{\infty,\infty}$, $D_{2,0}^{\infty,\infty}, \varepsilon_{2,1}^{\infty,\infty}, \varepsilon_{2,0}^{\infty,\infty}$ are obtained numerically via value-matching and smooth-pasting conditions between the branches of (E-3) and (E-4). The first two terms in the top branch of (E-4) represent the expected profit from operating the first technology, while the third term is the expected value of the option to invest in the second technology, adjusted via the fourth term due to policy uncertainty. The first three terms in the second branch, represent the expected value of the project, while the last two terms represent the likelihood of the price either dropping below $\varepsilon_{2,1}^{\infty,\infty}$ or rising above $\varepsilon_{2,0}^{\infty,\infty}$.

$$F_{2,0}^{\infty,\infty}(E) = \begin{cases} \frac{D_1E}{\rho-\mu} + \frac{\lambda_p D_1 Ey}{(\rho-\mu)(\rho-\mu+2\lambda_p)} + A_{2,1}^{\infty,\infty}E^{\beta_1} - B_{2,0}^{\infty,\infty}E^{\xi_1} & , E < \varepsilon_{2,1}^{\infty,\infty} \\ \frac{\lambda_p D_2 E + (\rho-\mu)D_1E}{(\rho-\mu)(\rho-\mu+\lambda_p)} + \frac{\lambda_p D_2 Ey}{(\rho-\mu)(\rho-\mu+2\lambda_p)} - \frac{\lambda_p I_2}{(\rho+\lambda_p)} & \\ +C_{2,0}^{\infty,\infty}E^{\eta_2} + D_{2,0}^{\infty,\infty}E^{\eta_1} & , \varepsilon_{2,1}^{\infty,\infty} \leq E < \varepsilon_{2,0}^{\infty,\infty} \\ \Phi_{2,0}^{\infty,\infty}(E) - I_2 & , E \geq \varepsilon_{2,0}^{\infty,\infty} \end{cases} \tag{E-4}$$

Next, the dynamics of the firm's value function before the arrival of the second technology are described in (35) for $a = 0, 1$. By expanding the right-hand side of (35) using Itô's lemma we obtain (E-5).

$$\begin{aligned} &\frac{1}{2}\sigma^2 E^2 \Phi_{1,a}^{\infty,\infty''}(E) + \mu E \Phi_{1,a}^{\infty,\infty'}(E) - (\rho+\lambda_p+\lambda_\tau)\Phi_{1,a}^{\infty,\infty}(E) + \lambda_p \Phi_{1,1-a}^{\infty,\infty}(E) + \lambda_\tau F_{2,a}^{\infty,\infty}(E) \\ &+D_1E\,(1+ya) = 0 \end{aligned} \tag{E-5}$$

Following a similar approach, we let $\overline{\phi}(E) = \Phi_{1,1}^{\infty,\infty}(E) + \Phi_{1,0}^{\infty,\infty}(E)$. Notice that (E-5) has to be solved for each expression of $F_{2,1}^{\infty,\infty}(E)$ that is indicated in (E-3), and, thus, the expression of $\overline{\phi}(E)$ is indicated in (E-6).

$$\overline{\phi}(E) = \begin{cases} \frac{(2+y)D_1E}{\rho-\mu} + 2A_{2,1}^{\infty,\infty}E^{\beta_1} + A_{1,1}^{\infty,\infty}E^{\delta_1} & ,E < \varepsilon_{2,1}^{\infty,\infty} \\ \frac{\lambda_\tau(\rho-\mu+2\lambda_p)D_2E}{(\rho-\mu)(\rho-\mu+\lambda_p)(\rho-\mu+\lambda_\tau)} + \frac{[\lambda_\tau D_2+(\rho-\mu)D_1]Ey}{(\rho-\mu)(\rho-\mu+\lambda_\tau)} & \\ +\frac{[\lambda_\tau+2(\rho-\mu+\lambda_p)]D_1E}{(\rho-\mu+\lambda_p)(\rho-\mu+\lambda_\tau)} - \frac{\lambda_\tau(\rho+2\lambda_p)}{(\rho+\lambda_p)(\rho+\lambda_\tau)}I_2 & \\ -\frac{\lambda_\tau\left(C_{2,0}^{\infty,\infty}E^{\eta_2}+D_{2,0}^{\infty,\infty}E^{\eta_1}\right)}{\lambda_p-\lambda_\tau} + B_{1,1}^{\infty,\infty}E^{\delta_2} + C_{1,1}^{\infty,\infty}E^{\delta_1} & ,\varepsilon_{2,1}^{\infty,\infty} \leq E < \varepsilon_{2,0}^{\infty,\infty} \\ \frac{(2+y)D_1E}{(\rho-\mu+\lambda_\tau)} + \frac{\lambda_\tau(2+y)D_2E}{(\rho-\mu)(\rho-\mu+\lambda_\tau)} - -\frac{2\lambda_\tau I_2}{\rho+\lambda_\tau} + D_{1,1}^{\infty,\infty}E^{\delta_2} & ,E \geq \varepsilon_{2,0}^{\infty,\infty} \end{cases} \tag{E-6}$$

Similarly, we set $\underline{\phi}(E) = \Phi_{1,1}^{\infty,\infty}(E) - \Phi_{1,0}^{\infty,\infty}(E)$, and the expression of $\underline{\phi}(E)$ is indicated in (E-7), where κ_1 is the positive root of $\frac{1}{2}\sigma^2\kappa(\kappa-1)+\mu\kappa-(\rho+\lambda_\tau+2\lambda_p) = 0$. Consequently, $\Phi_{1,1}^{\infty,\infty}(E)$ and $\Phi_{1,0}^{\infty,\infty}(E)$ can be expressed as a linear combination of (E-6) and (E-7).

$$\underline{\phi}(E) = \begin{cases} \frac{D_1Ey}{\rho-\mu+2\lambda_p} + 2B_{2,0}^{\infty,\infty}E^{\xi_1} + G_{1,1}^{\infty,\infty}E^{\kappa_1} & ,E < \varepsilon_{2,1}^{\infty,\infty} \\ \left[D_1y + \frac{\lambda_\tau[D_2-D_1]}{(\rho-\mu+\lambda_p)} + \frac{\lambda_\tau D_2y}{(\rho-\mu+2\lambda_p)}\right]\frac{E}{\rho-\mu+\lambda_\tau+2\lambda_p} & \\ -\frac{\lambda_\tau\rho I_2}{(\rho+\lambda_p)(\rho+\lambda_\tau+2\lambda_p)} - \frac{\lambda_\tau\left[C_{2,0}^{\infty,\infty}E^{\eta_2}+D_{2,0}^{\infty,\infty}E^{\eta_1}\right]}{\lambda_\tau+\lambda_p} & \\ +H_{1,1}^{\infty,\infty}E^{\kappa_2} + J_{1,1}^{\infty,\infty}E^{\kappa_1} & ,\varepsilon_{2,1}^{\infty,\infty} \leq E < \varepsilon_{2,0}^{\infty,\infty} \\ \frac{D_1Ey}{\rho-\mu+\lambda_\tau+2\lambda_p} + \frac{\lambda_\tau D_2Ey}{(\rho-\mu+2\lambda_p)(\rho-\mu+\lambda_\tau+2\lambda_p)} + K_{1,1}^{\infty,\infty}E^{\kappa_2} & ,E \geq \varepsilon_{2,0}^{\infty,\infty} \end{cases} \tag{E-7}$$

Finally, the option to invest in the first technology is indicated in (E-8) for $a = 1$. The first term in the top branch of (E-8) reflects the value of the investment opportunity, adjusted via the second term for policy uncertainty, since the subsidy is currently available, while the bottom branch is the value of the active project.

$$F_{1,1}^{\infty,\infty}(E) = \begin{cases} L_{1,1}^{\infty,\infty}E^{\beta_1} + M_{1,0}^{\infty,\infty}E^{\xi_1} & ,E < \varepsilon_{1,1}^{\infty,\infty} \\ \Phi_{1,1}^{\infty,\infty}(E) - I_1 & ,E \geq \varepsilon_{1,1}^{\infty,\infty} \end{cases} \tag{E-8}$$

If $a = 0$, then the option to invest in the first technology is described in (E-9), and, like in

Section 1.4.3 and 1.4.4, it is defined over three different regions of E. The first term on the top branch of (E-9) is the option to invest, adjusted via the second term due to policy uncertainty. The second branch reflects the value of the project provided that the subsidy becomes available, and the bottom branch is the expected value of the project if the price is sufficiently high so that investment would take place even in the absence of a subsidy.

$$F_{1,0}^{\infty,\infty}(E) = \begin{cases} L_{1,1}^{\infty,\infty} E^{\beta_1} - M_{1,0}^{\infty,\infty} E^{\xi_1} & , E < \varepsilon_{1,1}^{\infty,\infty} \\ \left[\frac{1+\frac{y}{2}}{\rho-\mu} + \frac{y}{2(\rho-\mu+2\lambda_p)}\right] \frac{\lambda_p D_1 E}{(\rho-\mu+\lambda_p)} - \frac{\lambda_p I_1}{\rho+\lambda_p} & \\ +\frac{1}{2} A_{2,1}^{\infty,\infty} E^{\beta_1} + \frac{\lambda_p A_{1,1}^{\infty,\infty} E^{\delta_1}}{2(\lambda_p-\lambda_\tau)} - \frac{B_{2,0}^{\infty,\infty} E^{\xi_1}}{2} & \\ -\frac{\lambda_p G_{1,1}^{\infty,\infty} E^{\kappa_1}}{2(\lambda_p+\lambda_\tau)} + N_{1,0}^{\infty,\infty} E^{\eta_2} + O_{1,0}^{\infty,\infty} E^{\eta_1} & , \varepsilon_{1,1}^{\infty,\infty} < E < \varepsilon_{1,0}^{\infty,\infty} \\ \Phi_{1,0}^{\infty,\infty}(E) - I_1 & , E \geq \varepsilon_{1,0}^{\infty,\infty} \end{cases} \tag{E-9}$$

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Chapter 2

Strategic Technology Switching under Risk Aversion and Uncertainty

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Abstract

Sequential investment opportunities or the presence of a rival typically hasten investment under risk neutrality. By contrast, increasing economic uncertainty or risk aversion raise the incentive to postpone investment in the absence of competition. We analyse how economic and technological uncertainty, reflected in the random arrival of innovations, interact with attitudes towards risk to impact both the optimal technology adoption strategy and the optimal investment policy within each strategy, under a proprietary and a non-proprietary duopoly. Results indicate that technological uncertainty increases the follower's investment incentive yet delays the entry of the non-proprietary leader. Also, we show that the proprietary leader's optimal investment policy is not affected by the likely arrival of an innovation, yet competition induces the proprietary leader to adopt a new technology at a price threshold lower than in the case of monopoly. Additionally, we show that the likely arrival of innovations decreases the relative loss in the value of the leader due to the follower's entry, while the corresponding impact of risk aversion is ambiguous. Interestingly, we also find that a higher first-mover advantage with respect to a new technology does not affect the leader's entry, and that technological uncertainty may turn a pre-emption game into a war of attrition, where the second-mover gets the higher payoff.

Keywords: investment analysis, real options, competition, risk aversion

2.1 Introduction

Emerging technologies are subject to frequent upgrades that become available at random points in time and the firm that adopts them first can capture a greater market share (Lieberman & Montgomery, 1988; Zachary *et al.*, 2015). Hence, firms investing in emerging technologies must take into account both strategic interactions and the sequential nature of such investments. Furthermore, emerging technologies typically entail technical risk that cannot be diversified, and, therefore, firms are likely to exhibit risk aversion. Indeed, the underlying commodities of such projects are typically not freely traded, thus preventing the construction of a replicating portfolio. Consequently, risk-neutral valuation may not be possible as the assumption of hedging via spanning assets breaks down. Although various models have been developed in order to analyse sequential investment under price and technological uncertainty, most of these either ignore strategic interactions (Grenadier & Weiss, 1997; Doraszelski, 2001; Chronopoulos & Siddiqui, 2015) or assume risk neutrality (Huisman & Kort, 2003, 2004; Weeds, 2002). Consequently, how strategic interactions impact sequential investment decisions and how price and technological uncertainty interact with risk aversion to impact the optimal investment policy remain important open research questions.

Incorporating such features in an analytical framework for sequential investment is crucial as these are pertinent to various industries, e.g., computer software, telecommunications, pharmaceutical, etc. For example, firms producing brand-name drugs enjoy high revenues so long as their patents are protected. In the early 1980s, a drug which soothes both pain and inflammation was a costly patented product. Today, Boots, a British chemist, sells generic tablets for just 2.5 pence per pill (Wall Street Journal, 2013b). In the area of telecommunications, Apple's iPhone sales declined prior to the introduction of iPhone 4s in 2012, while, at the same time, Samsung's Galaxy S_3, the closest rival to Apple's market leading iPhone, took close to 18% of the market (Financial Times, 2012). The legal debate between Apple and Samsung reflects a highly competitive environment in which firms can potentially profit from adopting other firms' patented technologies. Of course, there are various other competitive advantages that a firm may have, that may not be related to the adoption of patented technologies, however, their analysis is beyond the scope of this paper. For example, Samsung is more vertically integrated than Apple, and, thus, can bring products to the market more quickly (Wall Street Journal, 2013a).

We consider the case of duopolistic competition, where two identical firms invest sequentially in technological innovations facing price and technological uncertainty. Within this context, we analyse the case of proprietary and non-proprietary duopoly. The former occurs when a firm controls the innovation process, and, therefore, does not face the threat of pre-emption. By contrast, the latter occurs when the innovation process is exogenous to both firms, and, therefore, they fight for the leader's position. Hence, we contribute to the existing literature by first developing a utility-based framework for sequential investment in order to analyse how price and technological uncertainty interact with risk aversion to impact investment under duopolistic competition. Second, we derive analytical expressions, where possible, for the optimal entry threshold of the leader and the follower. Thus, for each firm, we determine both the optimal technology adoption strategy, and, within each strategy, the optimal investment rule. Finally, we provide managerial insights for investment decisions based on analytical and numerical results.

We proceed by discussing some related work in Section 2.2 and introduce assumptions and notation in Section 2.3. We begin the analysis with the benchmark case of monopoly in Section 2.4. In Section 2.5, we assume that firms adopt each technology that becomes available (compulsive strategy) and analyse the case of proprietary and non-proprietary duopoly in Sections 2.5.1 and 2.5.2, respectively. In Section 2.5.3, we also consider how pre-emption may lead to a war of attrition. In Section 2.6, we assume that a firm may wait for a new technology to become available before deciding to either skip an old technology and invest directly in the new one (leapfrog strategy) or to adopt the old technology first and then the new one (laggard strategy). In Section 2.7, we provide numerical results for each case and illustrate how attitudes towards risk interact with price and technological uncertainty to impact the optimal technology adoption strategy and the associated investment rule. Section 2.8 concludes the paper and offers directions for further research.

2.2 Related Work

Real options models often address the problem of optimal investment timing without considering strategic interactions (McDonald & Siegel, 1985 and 1986), while the ones that do, either ignore the sequential nature of investment opportunities (Pawlina & Kort, 2006; Siddiqui & Takashima, 2012) or attitudes towards risk (Huisman & Kort, 2015). In the area of competition, Spatt &

Sterbenz (1985) analyse how the degree of rivalry impacts the learning process and the decision to invest, and find that increasing the number of players hastens investment and that the investment decision resembles the standard NPV rule. Via a deterministic model, Fudenberg & Tirole (1985) show that a high first-mover advantage results in a pre-emption equilibrium with dispersed adoption timings, as it increases a firm's incentive to pre-empt investment by its rivals. Smets (1993) first developed a continuous-time model of strategic real options under product market competition, stochastic demand and irreversibility. Extending the framework of Fudenberg & Tirole (1985), Huisman & Kort (1999) find that uncertainty creates a positive option value of waiting that raises the required investment threshold. Specifically, they find that, in deterministic models, a high first-mover advantage leads to a pre-emption equilibrium, yet, in stochastic models, higher uncertainty may turn a pre-emption into a simultaneous investment equilibrium.

In the same line of work, Lambrecht & Perraudin (2003) incorporate incomplete information into an equilibrium model in which firms invest strategically. Also, Pennings (2004) develops a monopoly and duopoly model for irreversible investment and examines quality choice and entry timing under demand uncertainty. He finds that if the leader produces a low quality product, then the follower faces a large irreversible investment, and, therefore, will postpone investment until demand is sufficiently high. In turn, this increases the period of monopoly profits for the leader. Paxson & Pinto (2005) develop a rivalry model that allows for price and quantity uncertainty, and, among other results, they find that an increase in the correlation between profits per unit and quantity of units produced raises their aggregate volatility, and, in turn, the investment trigger of both the leader and the follower. Takashima *et al.* (2008) assess the effect of competition on the investment decision of firms with asymmetric technologies under price uncertainty. They show how mothballing options facilitate investment, thereby offering a competitive advantage to a thermal power plant over a nuclear power plant. By contrast, lower variable and construction costs favour coal- and oil-thermal power plants. Bouis *et al.* (2009) analyse investment in markets with more than two identical competitors. In the setting including three firms, they find that, if the entry of the third firm is delayed, then the second firm has an incentive to invest earlier so that it can enjoy the duopoly market structure for a longer time. This increases the incentive for the first firm to delay investment, as it faces a shorter period in which it can enjoy monopoly profits. In the same line of work, Armada *et al.* (2011) introduce a setting with several competitors who arrive according to a Poisson process.

Also, Graham (2011) finds that an equilibrium may not exist when allowing for asymmetric information over revenues, while Thijssen *et al.* (2012) present an analytical model that deals with the coordination problem in pre-emptive competition. Allowing for capacity sizing as well as entry and exit decisions under duopolistic competition, Lavrutich (2017) finds that the follower can set capacity strategically, so that the leader has an incentive to exit.

Examples of early work in the area of investment under technological uncertainty include Balcer & Lippman (1984), who analyse the optimal timing of technology adoption taking into account the expected flow of technological progress. A model for sequential investment in technological innovations is developed by Grenadier & Weiss (1997), who assume that a firm may either adopt each technology that becomes available (compulsive), or wait for a new technology to arrive before adopting either the new (leapfrog) or the old technology (laggard), or purchase only an early innovation (buy and hold). Their results indicate that a firm may adopt an available technology even though more valuable innovations may occur in the future, while decisions on technology adoption are path dependent. Assuming that innovations follow a Poisson process, Farzin *et al.* (1998) investigate the impact of technological uncertainty on the optimal timing of technology adoption, yet ignore price uncertainty. Doraszelski (2001) revisits the analytical framework of Farzin *et al.* (1998) and shows that, compared to the net present value (NPV) approach, a firm will defer technology adoption when it takes the option value of waiting into account. Weeds (1999) analyses the decision to invest in a research project and finds that increasing technological and economic uncertainty postpone investment, while technological uncertainty may accelerate abandonment when the profitability of the project declines. Also, Chronopoulos & Siddiqui (2015) find that uncertainty over the arrival of innovations facilitates the adoption of an existing technology, while Lukas *et al.* (2017) show how optimal capacity is related to a product's life-cycle when technological lifetime is uncertain. Although the aforementioned papers offer a comprehensive analysis of investment under technological uncertainty, they assume risk-neutrality and ignore the implications of strategic interactions.

Allowing for economic and technological uncertainty, Weeds (2002) analyses strategic investment in competing research projects and identifies the existence of non-cooperative and cooperative games. The former involve **i.** a pre-emptive competition where firms invest sequentially and **ii.** a symmetric outcome in which investment is more delayed than in the case of monopoly. The latter involves sequential investment, yet compared to the non-cooperative (pre-emptive leader-follower) game, the investment triggers are higher. Also, compared to the optimal cooperative investment pattern, investment is found to be more delayed when firms act non-cooperatively as each refrains from investing in the fear of starting a patent race. Miltersen & Schwartz (2004) analyse how competition in the development and marketing of a product impacts investment in R&D. They find that competition not only increases production and reduces prices, but also shortens the development stage and raises the probability of a successful outcome. Huisman & Kort (2004) study a dynamic duopoly in which firms compete in the adoption of new technologies under price and technological uncertainty. Their results indicate that taking into account the likely arrival of a new technology could turn a pre-emption game into one where the second mover gets the highest payoff. Leippold & Stromberg (2017) extend Huisman & Kort (2004) by allowing for market incompleteness and find that undiversifiable risk may accelerate technology adoption.

Examples of analytical models for investment under uncertainty that allow for risk aversion include Henderson & Hobson (2002), who introduce market incompleteness in the framework of Merton (1969) by allowing for a second, non-tradable asset and address the question of how to price and hedge this random payoff. Alvarez & Stenbacka (2004) implement attitudes towards risk via a hyperbolic absolute risk aversion (HARA) utility function and develop an analytical framework for optimal regime-switching. They show that if the decision-maker is risk seeking, then increasing price uncertainty does not necessarily decelerate investment. A similar result is indicated in Henderson (2007), who shows that idiosyncratic risk raises the incentive to accelerates investment and lock in the investment payoff. Hugonnier & Morellec (2013) use the framework of Karatzas & Shreve (1999) in order to determine the analytical expression for the expected utility of a perpetual stream of cash flows that follows a geometric Brownian motion (GBM). Thus, they express the investment policy as the solution to an optimal stopping-time problem and find that greater risk aversion lowers the expected utility of the project and reduces the probability of investment. By contrast, Chronopoulos *et al.* (2011) show that operational flexibility in the form of suspension and resumption options mitigates the impact of risk aversion and increases the incentive to invest. Also, a utility-based framework that allows for Markov-regime switching is described in Chronopoulos & Lumbreras (2017). Although these papers address the impact of risk aversion on investment and operational decisions under price uncertainty, they ignore the implications of both technological uncertainty and competition.

More pertinent to our analysis is Siddiqui & Takashima (2012), who analyse the extent to which sequential decision making offsets the impact of competition under risk neutrality. They find that a duopoly firm's value relative to a monopolist's decreases (increases) with uncertainty as long as the loss in market share is high (low). Also, they show that this loss in value decreases if a firm adopts a sequential investment approach. Similar to Siddiqui & Takashima (2012), we consider a spillover-knowledge duopoly in which both firms invest sequentially in technological innovations and the follower can enter the market after the leader. Within this context, we analyse how attitudes towards risk interact with price and technological uncertainty to affect the technology adoption strategy (compulsive, leapfrog, and laggard) of each firm. We assume that technological innovations arrive according to a Poisson process, while price uncertainty is modelled via a geometric Brownian motion (GBM). Results indicate that technological uncertainty has a non-monotonic impact on the required investment threshold of the follower and the non-proprietary leader, yet it does not impact the proprietary leader's optimal investment policy. Furthermore, we find that the likely arrival of a new technology decreases the leader's relative loss in value due to the presence of a rival, and that increasing risk aversion raises the incentive to delay investment, yet it has an ambiguous impact on the relative loss in the value of the leader. Surprisingly, by comparing a compulsive with a leapfrog/laggard strategy under proprietary duopoly, we find that the latter strategy may dominate even under risk aversion, provided that the rate of innovation and the output price are sufficiently high. Finally, we find that a higher first-mover advantage with respect to a new technology does not affect the leader's entry, and that technological uncertainty may turn a pre-emption game into a war of attrition.

2.3 Assumptions and Notation

Given a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, we introduce technological uncertainty by assuming that innovations follow a Poisson process $\{M_t, t \geq 0\}$, where t is continuous and denotes time. We assume that the output price $\{E_t, t \geq 0\}$ is independent of the process $\{M_t, t \geq 0\}$, and evolves according to a GBM, as in (1), where μ is the annual growth rate, σ is the annual volatility, and dZ_t is the increment of the standard Brownian motion. Also, $\rho > \mu$ denotes the subjective discount rate and r is the risk-free rate.

$$dE_t = \mu E_t dt + \sigma E_t dZ_t, \quad E_0 \equiv E > 0 \tag{1}$$

We assume that the firms' risk preferences are described by power-function with constant relative risk aversion, which is indicated in (2). Note that standard economic theory assumes that decision-makers are typically risk averse and that risk-seeking behaviour is less plausible (Pratt, 1964). Nevertheless, we assume that $\gamma \in [0.7, 1.3]$, and, thus, examine the implications of both risk-averse $\gamma \in [0.7, 1]$ and risk-seeking behaviour $\gamma \in [1, 1.3]$ to enable comparisons with both Hugonnier & Morellec (2013) and Chronopoulos & Lumbreras (2017).

$$U(E) \quad = \quad \frac{E^{\gamma}}{\gamma}, \quad \gamma > 0 \tag{2}$$

We let $a = p, n$ denote proprietary and non-proprietary duopoly, respectively, and $b = m, \ell, f$ denote the monopolist, the leader and the follower, where the leader is the first firm to enter the market in the case of competition. Also, we assume that each firm holds perpetual options to invest in two technologies, each with an infinite lifetime. There is no operating cost associated with each technology, while the investment cost is I_i, $i = 1, 2$ $(I_1 \leq I_2)$ and the corresponding output is D_i, where $D_{\underline{i}}$ or $D_{\bar{i}}$ indicates that there is either one $(\underline{i})$ or two $(\bar{i})$ firms in the market, respectively. Hence, D_i is decreasing in the number of active firms and increasing in i. Thus, depending on the number of firms in the industry, a firm's option to invest in technology i while operating technology $i-1$ is denoted by $F^{ab}_{i-1,i}(\cdot)$, and the expected utility from operating technology i inclusive of embedded options is denoted by $\Phi^{ab}_{i}(\cdot)$. Also, the time and output price at investment are denoted by $\tau^{ab}_{i-1,i}$ and $E^{ab}_{i-1,i}$, respectively, while the optimal investment threshold is denoted by $\varepsilon^{ab}_{i-1,i}$. For example, $F^{n\ell}_{0,1}(\cdot)$ is the non-proprietary leader's option to invest in the first technology with a single embedded option to upgrade it by adopting the second one, $\tau^{n\ell}_{0,1}$ is the time of investment and $\varepsilon^{n\ell}_{0,1}$ is the optimal investment threshold.

As in Chronopoulos & Siddiqui (2015), we assume that each technological version has greater output than the older one yet it is more costly. In essence, this implies that there is a trade-off between the two technologies, i.e. the first technology is more lucrative for low output prices while the second technology is preferred when output prices are high. This condition implies that there is a α where the expected NPVs of the profits of the two technologies are equal, i.e. $\Phi^{b,c}_{1,a}(\alpha) = \Phi^{b,c}_{2,a}(\alpha)$, we have $\Phi^{b,c}_{2,a}(\alpha) > 0$. In terms of context, a firm may possess an investment opportunity to develop a production facility, and the investment decision is divided in two steps. In step one, the firm develops the energy production facility with an embedded option to increase its utilisation or retrofit it with new technology later. For example, oil production facilities have been converted to utilise gas reserves, but at a substantial cost in order to implement export facilities and retrofitting (Støre *et al.*, 2018).

2.4 Benchmark Case: Monopoly

First, we consider the benchmark case where a monopolist holds a single investment opportunity. This has already been analysed in Hugonnier & Morellec (2013) and Conejo *et al.* (2016), but we present the analysis here for ease of exposition and to allow for comparisons. Since $U(\cdot)$ is not separable, the key insight is to decompose all the cash flows of the project into disjoint time intervals. Hence, we assume that the monopolist has initially placed the amount of capital required for investment in a certificate of deposit and earns a risk-free rate, r. Thus, until time $\tau_{0,1}^m$, the monopolist earns the instantaneous utility $U(rI_1)$. At time $\tau_{0,1}^m$, the monopolist swaps this risk-free cash flow in return for the instantaneous utility $U(ED_{\underline{1}})$, as shown in Figure 2.1.

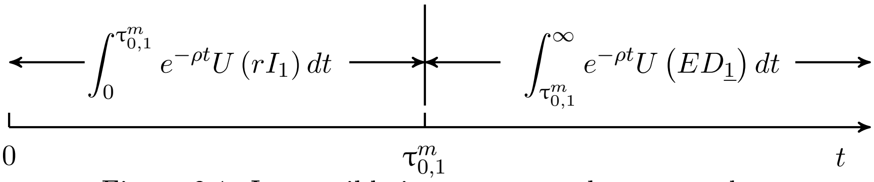


Figure 2.1: Irreversible investment under monopoly

The time-zero expected discounted utility of all the cash flows of the project is described in (3), where $\mathbb{E}_E[\cdot]$ denotes the expectation operator that is conditional on the initial output price, E.

$$\mathbb{E}_E\left[\int_0^{\tau_{0,1}^m} e^{-\rho t} U(rI_1)\, dt + \int_{\tau_{0,1}^m}^{\infty} e^{-\rho t} U\left(ED_{\underline{1}}\right) dt\right] \tag{3}$$

By decomposing the first integral, we can rewrite (3) as in (4).

$$\int_0^{\infty} e^{-\rho t} U(rI_1)\, dt + \mathbb{E}_E\left[\int_{\tau_{0,1}^m}^{\infty} e^{-\rho t}\left[U\left(ED_{\underline{1}}\right) - U(rI_1)\right] dt\right] \tag{4}$$

Notice that the first term in (4) is deterministic, as it does not depend on the investment threshold. Therefore, the optimisation objective is reflected in the second term and can be written as in (5) using the law of iterated expectations and the strong Markov property of the GBM. The latter states that the values of the process $\{E_t, t \geq 0\}$ after time $\tau_{0,1}^m$ are independent of the values of the process before time $\tau_{0,1}^m$ and depend only on the value of the process at time $\tau_{0,1}^m$. Note that the stochastic discount factor is $\mathbb{E}_E\left[e^{-\rho\tau}\right] = \left(\frac{E}{E_\tau}\right)^{\beta_1}$ (Dixit & Pindyck, 1994), $\beta_1 > 0, \beta_2 < 0$ are the roots of the quadratic $\frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - \rho = 0$, and $\mathcal{S}$ is the set of stopping times generated by the filtration of the process $\{E_t, t \geq 0\}$.

$$F_{0,1}^m(E) \;=\; \sup_{\tau_{0,1}^m \in \mathcal{S}} \mathbb{E}_E\left[e^{-\rho\tau_{0,1}^m}\right] \mathbb{E}_{E_{0,1}^m}\left[\int_0^{\infty} e^{-\rho t}\left[U\left(ED_{\underline{1}}\right) - U(rI_1)\right] dt\right] \tag{5}$$

Using Theorem 9.18 of Karatzas & Shreve (1999), the maximised expected value of the option to invest can be expressed as in (6)

$$F_{0,1}^{m}(E) = \max_{E_{0,1}^{m}>E} \left(\frac{E}{E_{0,1}^{m}}\right)^{\beta_1} \Phi_1^{m}\left(E_{0,1}^{m}\right) \tag{6}$$

where $\Phi_1^m(\cdot)$ is the expected utility of the active project and is described in (7).

$$\Phi_1^{m}(E) = \Upsilon U\left(ED_{\underline{1}}\right) - \frac{U(rI_1)}{\rho}, \quad \Upsilon = \frac{\beta_1\beta_2}{\rho(\beta_1-\gamma)(\beta_2-\gamma)} \tag{7}$$

Solving the unconstrained optimisation problem (6), we obtain the optimal investment threshold that is indicated in (8). Note that, although the investment threshold is commonly expressed in terms of β_1, it is more expedient to use β_2 in our case, due to the relationship $\beta_1\beta_2 = -2\rho \,/\, \sigma^2$. Also, the second-order sufficiency condition (SOSC) requires the objective function to be concave at $\varepsilon_{0,1}^m$, which is shown in Chronopoulos & Lumbreras (2017).

$$\varepsilon_{0,1}^{m} = rI_1 \left[\frac{\beta_2-\gamma}{\beta_2 D_{\underline{1}}^{\gamma}}\right]^{\frac{1}{\gamma}} \tag{8}$$

Note that the analysis of sequential technology adoption for the monopolist is identical to the follower's (see Section 2.5.1), except for replacing $D_{\bar{i}}$ by $D_{\underline{i}}$. Therefore, we omit this for ease of exposition.

2.5 Compulsive strategy

2.5.1 Proprietary Duopoly

Follower

We extend the benchmark case of Section 2.4 by assuming that there are two firms in the market competing in the adoption of technological innovations. First, we consider the optimal investment policy of the follower. As illustrated in Figure 2.2, the follower is initially in state $(0,1)$ and holds the option to invest in the first technology, and, thus, move to state 1. Once an innovation takes place, the follower moves to state $(1,2)$, where she has the option to invest in the second technology and move to state 2. We denote a transition due to an innovation (investment) by a dashed (solid) line. Note that the follower will always adopt each technology after the leader. Hence, to alleviate notation, we will indicate the presence of two firms via $\overline{1}$ and $\overline{2}$ only when it is necessary to avoid confusion, i.e., when it is not implied by the superscript.

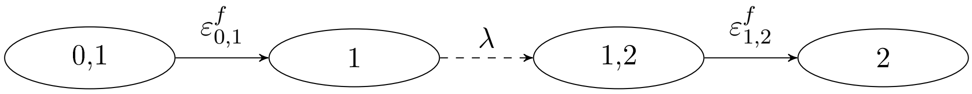


Figure 2.2: State-transition diagram for the proprietary follower under a compulsive strategy

Similar to the benchmark case, we assume that the amount of capital required for the adoption of each technology is exchanged at investment for the risky cash flows of the project. For example, at time $\tau^f_{0,1}$ the follower exchanges the capital required for investing in the first technology for the risky cash flows of the project. Analogously to (4) and (5), this results in the instantaneous utility $U\left(ED_{\overline{1}}\right) - U\left(rI_1\right)$, which accrues from $\tau^f_{0,1}$ until $\tau^f_{1,2}$, as indicated in Figure 2.3. Similarly, at $\tau^f_{1,2}$ the follower exchanges the capital required for investing in the second technology for the risky cash flows it generates.

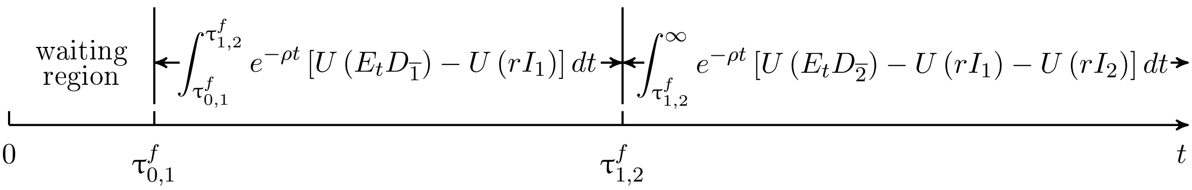


Figure 2.3: Sequential investment under a compulsive strategy

The follower's objective is to maximise the time-zero discounted expected utility of all the cash flows of the project, which is described in (9). The first (second) integral in (9) indicates the expected utility of the cash flows from operating the first (second) technology.

$$\mathbb{E}_E\left[\int_{\tau^f_{0,1}}^{\tau^f_{1,2}} e^{-\rho t}\left[U\left(E_t D_{\overline{1}}\right) - U\left(rI_1\right)\right] dt + \int_{\tau^f_{1,2}}^{\infty} e^{-\rho t}\left[U\left(E_t D_{\overline{2}}\right) - U\left(rI_1\right) - U\left(rI_2\right)\right] dt\right] \tag{9}$$

By decomposing the first integral, we can rewrite (9) as in (10).

$$\begin{aligned}\mathbb{E}_E\left[e^{-\rho \tau^f_{0,1}}\right]\Bigg[\mathbb{E}_{E^f_{0,1}}\int_0^{\infty} e^{-\rho t}\left[U\left(E_t D_{\overline{1}}\right) - U\left(rI_1\right)\right] dt + \mathbb{E}_{E^f_{0,1}}\left[e^{-\rho\left(\tau^f_{1,2}-\tau^f_{0,1}\right)}\right]\\ \times \mathbb{E}_{E^f_{1,2}}\int_0^{\infty} e^{-\rho t}\left[\left(D^{\gamma}_{\overline{2}} - D^{\gamma}_{\overline{1}}\right) U\left(E_t\right) - U\left(rI_2\right)\right] dt\Bigg]\end{aligned} \tag{10}$$

Next, we determine the follower's value function in each state using backward induction. Therefore, we first assume that the follower has already adopted and operates the first technology. The expected utility of the project's cash flows is indicated in (11), where the first term is the expected utility from operating the first technology and the second term is the maximised expected value of the embedded option to adopt the second one.

$$\mathbb{E}_E\left[\int_0^\infty e^{-\rho t}\left[U\left(E_t D_{\overline{1}}\right)-U\left(rI_1\right)\right]dt\right]+A_{1,2}^f E^{\beta_1} \quad ,E<\varepsilon_{1,2}^f \tag{11}$$

Like in (4), the first term in (11) does not depend on the investment threshold, and, therefore, the optimisation objective is reflected in the second term. The latter, is expressed in (12) as the maximised discounted expected utility from adopting the second technology.

$$\begin{aligned} A_{1,2}^f E^{\beta_1} &= \max_{E_{1,2}^f>E}\left(\frac{E}{E_{1,2}^f}\right)^{\beta_1}\mathbb{E}_{E_{1,2}^f}\int_0^\infty e^{-\rho t}\left[\left(D_{\overline{2}}^\gamma-D_{\overline{1}}^\gamma\right)U\left(E_t\right)-U\left(rI_2\right)\right]dt \\ &= \max_{E_{1,2}^f>E}\left(\frac{E}{E_{1,2}^f}\right)^{\beta_1}\left[\Upsilon\left(D_{\overline{2}}^\gamma-D_{\overline{1}}^\gamma\right)U\left(E_{1,2}^f\right)-U\left(rI_2\right)\right]dt \end{aligned} \tag{12}$$

Solving this unconstrained optimisation problem, we obtain the expression of the optimal investment threshold that is indicated in (13) (all proofs can be found in the appendix).

$$\varepsilon_{1,2}^f = rI_2\left[\frac{\beta_2-\gamma}{\beta_2\left(D_{\overline{2}}^\gamma-D_{\overline{1}}^\gamma\right)}\right]^{\frac{1}{\gamma}} \tag{13}$$

Equivalently, we can express the follower's value function in state $(1,2)$ as in (14). The first two terms on the top part reflect the expected utility of the cash flows from operating the first technology, while the third term represents the option to adopt the second one. The bottom part represents the expected utility of the profits from operating the second technology, $\Phi_2^f(E)=\Upsilon U\left(ED_{\overline{2}}\right)-\frac{U(rI_1)+U(rI_2)}{\rho}$.

$$F_{1,2}^f(E) = \begin{cases}\Upsilon U\left(ED_{\overline{1}}\right)-\frac{U(rI_1)}{\rho}+A_{1,2}^f E^{\beta_1} & ,E<\varepsilon_{1,2}^f \\ \Phi_2^f(E) & ,E\geq\varepsilon_{1,2}^f\end{cases} \tag{14}$$

Next, we step back to state 1, where the follower is operating the first technology and holds an embedded option to invest in the second one, that has yet to become available. The dynamics of the follower's value function are described in (15), where the first term on the right-hand side represents the instantaneous utility of the profits from operating the first technology and the second term is the expected utility of the project in the continuation region. As the second term indicates, with probability λdt the second technology will arrive and the follower will receive the value function, $F_{1,2}^f(E)$, whereas, with probability $1-\lambda dt$, no innovation will occur and the follower will continue to hold the value function, $\Phi_1^f(E)$.

$$\Phi_1^f(E) = [U(ED_{\overline{1}}) - U(rI_1)]\,dt + (1-\rho dt)\,\mathbb{E}_E\left[\lambda dt F_{1,2}^f(E+dE) + (1-\lambda dt)\,\Phi_1^f(E+dE)\right] \tag{15}$$

By expanding the right-hand side of (15) using Itô's lemma, we can rewrite (15) as in (16), where $\Lambda = \frac{\Upsilon}{\lambda\Upsilon+1}$ and $\delta_1 > 0, \delta_2 < 0$ are the roots of the quadratic $\frac{1}{2}\sigma^2\delta(\delta-1)+\mu\delta-(\rho+\lambda)=0$. Also, $A_1^f > 0$ and $B_1^f < 0$ are determined analytically by applying value-matching and smooth-pasting conditions to the two branches of (16). The first two terms on the top part represent the expected utility of the revenues and cost, respectively. The third term is the option to invest in the second technology, adjusted via the last term since the second technology is not available yet. The first three terms on the bottom part, represent the expected utility of operating the second technology, while the fourth term represents the likelihood of the price dropping into the waiting region prior to the arrival of an innovation.

$$\Phi_1^f(E) = \begin{cases} \Upsilon U(ED_{\overline{1}}) - \frac{U(rI_1)}{\rho} + A_{1,2}^f E^{\beta_1} + A_1^f E^{\delta_1} & , E < \varepsilon_{1,2}^f \\ \Lambda\left[\lambda\Upsilon U(ED_{\overline{2}}) + U(ED_{\overline{1}})\right] - \frac{\lambda U(rI_2)}{(\lambda+\rho)\rho} - \frac{U(rI_1)}{\rho} + B_1^f E^{\delta_2} & , E \geq \varepsilon_{1,2}^f \end{cases} \tag{16}$$

Finally, the follower's value function in state $(0,1)$ is indicated in (17). By applying value-matching and smooth-pasting conditions to the two branches of (17), we can solve for the optimal investment threshold, $\varepsilon_{0,1}^f$, and the endogenous constant, $A_{0,1}^f$, numerically.

$$F_{0,1}^f(E) = \begin{cases} A_{0,1}^f E^{\beta_1} & , E < \varepsilon_{0,1}^f \\ \Phi_1^f(E) & , E \geq \varepsilon_{0,1}^f \end{cases} \tag{17}$$

Note that by setting $\gamma = 1$, we can retrieve the same value functions and investment thresholds as Chronopoulos & Siddiqui (2015), who analyse sequential investment under risk neutrality.

Leader

Next, we consider the optimal investment policy of the proprietary leader. Notice that once the leader invests in the first technology, thereby moving from state $(0,\underline{1})$ to state $\underline{1}$, she receives monopoly profits until the follower enters. This could be an industry with weak patent protection, where knowledge spillover allows the follower to enter immediately after the leader. Once the follower adopts the first technology, both firms share the market in state $\overline{1}$. The same process is then repeated with respect to the second technology.

$$0,\underline{1} \xrightarrow{\varepsilon^{p\ell}_{0,\underline{1}}} \underline{1} \xrightarrow{\varepsilon^{f}_{0,1}} \overline{1} \overset{\lambda}{\dashrightarrow} \overline{1},\underline{2} \xrightarrow{\varepsilon^{p\ell}_{\overline{1},\underline{2}}} \underline{2} \xrightarrow{\varepsilon^{f}_{1,2}} \overline{2}$$

Figure 2.4: State-transition diagram for the proprietary leader under a compulsive strategy

Assuming that the follower chooses the optimal investment policy, the value function of the proprietary leader in state $\underline{2}$ is described in (18). The first two terms on the right-hand side reflect the monopoly profits from operating the second technology and the third term is the expected reduction in the proprietary leader's profits due to the follower's entry. The endogenous constant $A^{p\ell}_{\underline{2}}$ is obtained by value-matching (18) with the bottom part of (14), i.e., the follower's value function $\Phi^{f}_{2}(E)$, at $\varepsilon^{f}_{1,2}$, and is indicated in (A–5).

$$\Phi^{p\ell}_{\underline{2}}(E) \quad = \quad \Upsilon U\left(ED_{\underline{2}}\right) - \frac{U\left(rI_1\right) + U\left(rI_2\right)}{\rho} + A^{p\ell}_{\underline{2}} E^{\beta_1}, \quad E < \varepsilon^{f}_{1,2} \tag{18}$$

Next, the value function of the proprietary leader in state $\left(\overline{1},\underline{2}\right)$ is described in (19). The first two terms on the top part reflect the expected utility of the cash flows from operating the first technology, and the third term is the embedded option to invest in the second one.

$$F^{p\ell}_{\overline{1},\underline{2}}(E) \quad = \quad \begin{cases} \Upsilon U\left(ED_{\overline{1}}\right) - \frac{U(rI_1)}{\rho} + A^{p\ell}_{\overline{1},\underline{2}} E^{\beta_1} & , E < \varepsilon^{p\ell}_{\overline{1},\underline{2}} \\ \Phi^{p\ell}_{\underline{2}}(E) & , E \geq \varepsilon^{p\ell}_{\overline{1},\underline{2}} \end{cases} \tag{19}$$

The endogenous constant, $A^{p\ell}_{\overline{1},\underline{2}}$, and the optimal investment threshold, $\varepsilon^{p\ell}_{\overline{1},\underline{2}}$, can be obtained analytically via value-matching and smooth-pasting conditions and are indicated in (20).

$$\varepsilon^{p\ell}_{\overline{1},\underline{2}} = rI_2 \left[\frac{\beta_2 - \gamma}{\beta_2\left(D^{\gamma}_{\underline{2}} - D^{\gamma}_{\overline{1}}\right)}\right]^{\frac{1}{\gamma}} \text{ and } \quad A^{p\ell}_{\overline{1},\underline{2}} = \left(\frac{1}{\varepsilon^{p\ell}_{\overline{1},\underline{2}}}\right)^{\beta_1} \left[\Phi^{p\ell}_{\underline{2}}\left(\varepsilon^{p\ell}_{\overline{1},\underline{2}}\right) - \Upsilon U\left(\varepsilon^{p\ell}_{\overline{1},\underline{2}} D_{\overline{1}}\right) + \frac{U(rI_1)}{\rho}\right] \tag{20}$$

Corollary 1 indicates the necessary condition for a trade-off to exist between the two technologies, and is a consequence of the assumption about the second technology being more efficiency yet also more costly in Section 2.3. Note that, by setting $\gamma = 1$, we can retrieve the condition under risk neutrality, as in Chronopoulos & Siddiqui (2015),

Corollary 1. *A trade-off between the two technologies exists iff* $\frac{D^{\gamma}_{1}}{I^{\gamma}_{1}} > \frac{D^{\gamma}_{2}}{I^{\gamma}_{1}+I^{\gamma}_{2}}$.

Using Corollary 1, we can show that the proprietary leader will not invest in the second technology before the follower adopts the first one, as indicated in Proposition 1. Intuitively, the second technology is considerably more costly and wont be adopted when the output price is below the follower's required investment threshold for the first technology.

Proposition 1. $\varepsilon^{p\ell}_{\overline{1},\underline{2}} > \varepsilon^{f}_{0,1} \Leftrightarrow \frac{D_{\overline{1}}^{\gamma}}{I_1^{\gamma}} > \frac{D_{\underline{2}}^{\gamma}}{I_1^{\gamma}+I_2^{\gamma}}$.

Interestingly, unlike Chronopoulos *et al.* (2014), the leader's required investment threshold in the second technology is lower than that of the monopolist, as shown in Proposition 2. Intuitively, the entry of the follower reduces the monopoly profits of the leader with respect to the first technology. In turn, this raises the value of the leader's option to invest in the second technology and lowers the required adoption threshold, thereby extending the corresponding period of monopoly profits.

Proposition 2. $\varepsilon^{p\ell}_{\overline{1},\underline{2}} < rI_2\left[\frac{\beta_2-\gamma}{\beta_2\left(D_{\underline{2}}^{\gamma}-D_{\underline{1}}^{\gamma}\right)}\right]^{\frac{1}{\gamma}} = \varepsilon^{m}_{1,2}$.

In state $\overline{1}$, the leader shares the market with the follower waiting for the arrival of the second technology. The dynamics of the value function of the leader are described in (21), where the first term on the right-hand side reflects the instantaneous profit from operating the first technology. The second term is the discounted expected value in the continuation region, where the proprietary leader gets either $F^{p\ell}_{\overline{1},\underline{2}}(E)$ or $\Phi^{p\ell}_{\overline{1}}(E)$, depending on whether an innovation occurs or not.

$$\Phi^{p\ell}_{\overline{1}}(E) = \left[U\left(ED_{\overline{1}}\right) - U\left(rI_1\right)\right]dt + (1-\rho dt)\,\mathbb{E}_E\left[\lambda dt F^{p\ell}_{\overline{1},\underline{2}}(E+dE) + (1-\lambda dt)\,\Phi^{p\ell}_{\overline{1}}(E+dE)\right] \quad (21)$$

The proprietary leader's value function in state $\overline{1}$ is indicated in (22), where $A^{p\ell}_{\overline{1}}$ and $C^{p\ell}_{\overline{1}}$ are determined by value matching and smooth pasting the two branches, while $B^{p\ell}_{\overline{1}}$ is obtained by value matching (22) with the top branch of (19) at $\varepsilon^{f}_{1,2}$. The first two (three) terms in the top (bottom) part of (22) reflect the expected utility of the profits under a low (high) output price. The third term on the top part is the option to invest in the second technology adjusted via the fourth term due to technological uncertainty. The fourth term on the bottom part reflects the reduction in the expected utility of the leader's profits due to the follower's entry adjusted for technological uncertainty via the fifth term. The last term reflects the likelihood of the price dropping in the waiting region.

$$\Phi^{p\ell}_{\overline{1}}(E) = \begin{cases} \Upsilon U\left(ED_{\overline{1}}\right) - \frac{U(rI_1)}{\rho} + A^{p\ell}_{\overline{1},\underline{2}}E^{\beta_1} + A^{p\ell}_{\overline{1}}E^{\delta_1} & , E < \varepsilon^{p\ell}_{\overline{1},\underline{2}} \\ \Lambda\left[\lambda\Upsilon U\left(ED_{\underline{2}}\right) + U\left(ED_{\overline{1}}\right)\right] - \frac{U(rI_1)}{\rho} - \frac{\lambda U(rI_2)}{\rho(\rho+\lambda)} & \\ +A^{p\ell}_{\underline{2}}E^{\beta_1} + B^{p\ell}_{\overline{1}}E^{\delta_1} + C^{p\ell}_{\overline{1}}E^{\delta_2} & , E \geq \varepsilon^{p\ell}_{\overline{1},\underline{2}} \end{cases} \quad (22)$$

The value function of the proprietary leader in state $\underline{1}$ is indicated in (23), where $A_{\underline{1}}^{p\ell} < 0$ is obtained by value matching (23) with the top branch in (22) at $\varepsilon_{0,1}^{f}$ and is described in (A–6). The first two terms in (23) reflect the expected utility from operating the first technology, and, the third term, is the expected reduction in the proprietary leader's profits due to the follower's entry.

$$\Phi_{\underline{1}}^{p\ell}(E) \quad = \quad \Upsilon U\left(ED_{\underline{1}}\right) - \frac{U\left(rI_1\right)}{\rho} + A_{\underline{1}}^{p\ell}E^{\beta_1}, \quad E < \varepsilon_{0,1}^{f} \tag{23}$$

In state $(0,\underline{1})$, the proprietary leader holds the option to invest in the first technology with an embedded option to invest in the second one, that has yet to become available. The expression of $F_{0,\underline{1}}^{p\ell}(E)$ is described in (24), where the top part is the value of the option to invest and the bottom part is the expected utility of the active project inclusive of the embedded option to invest in the second technology.

$$F_{0,\underline{1}}^{p\ell}(E) \quad = \quad \begin{cases} A_{0,\underline{1}}^{p\ell}E^{\beta_1} & , E < \varepsilon_{0,\underline{1}}^{p\ell} \\ \Phi_{\underline{1}}^{p\ell}(E) & , E \geq \varepsilon_{0,\underline{1}}^{p\ell} \end{cases} \tag{24}$$

The expression of $\varepsilon_{0,\underline{1}}^{p\ell}$ and $A_{0,\underline{1}}^{p\ell}$ is indicated in (25). Notice that, as shown in Proposition 3, the leader's decision to adopt the first technology is independent of technological uncertainty.

$$\varepsilon_{0,\underline{1}}^{p\ell} = \frac{rI_1}{D_{\underline{1}}}\left[\frac{\beta_2-\gamma}{\beta_2}\right]^{\frac{1}{\gamma}} \quad \text{and} \quad A_{0,\underline{1}}^{p\ell} = \left(\frac{1}{\varepsilon_{0,\underline{1}}^{p\ell}}\right)^{\beta_1}\Phi_{\underline{1}}^{p\ell}\left(\varepsilon_{0,\underline{1}}^{p\ell}\right) \tag{25}$$

Proposition 3. *The proprietary leader's required investment threshold for the first technology is independent of λ.*

2.5.2 Non-Proprietary Duopoly

With two firms in the market fighting for the leader's position, each one of them faces the risk of pre-emption. Note that, under a compulsive strategy, the follower will invest in each technology after the leader has already adopted it. Consequently, the value function of the follower in each state is the same as in Section 2.5.1. However, to determine the non-proprietary leader's optimal investment policy, starting with the second technology, we must consider the strategic interactions between the leader and the follower. Note that the leader's value function in state $\underline{2}$ is described in (18), i.e., $\Phi_{\underline{2}}^{n\ell}(E) \equiv \Phi_{\underline{2}}^{p\ell}(E)$. We let $\varepsilon_{\overline{1},\underline{2}}^{n\ell}$ denote the point of intersection between the value function of the leader and the follower. If $E < \varepsilon_{\overline{1},\underline{2}}^{n\ell}$, then a firm is better off being the follower, since $F^f_{1,2}(E) > \Phi^{n\ell}_{\underline{2}}(E)$. By contrast, if $E > \varepsilon^{n\ell}_{\overline{1},\underline{2}}$, then a firm is better off being a leader, since $F^f_{1,2}(E) < \Phi^{n\ell}_{\underline{2}}(E)$. Consequently, the point of indifference between being a leader and a follower is $\varepsilon^{n\ell}_{\overline{1},\underline{2}}$ and is determined numerically by solving (26).

$$F^f_{1,2}(E) \quad = \quad \Phi^{n\ell}_{\underline{2}}(E) \tag{26}$$

Note that there are two possible scenarios: **i.** $\varepsilon^f_{0,1} > \varepsilon^{n\ell}_{\overline{1},\underline{2}}$ and **ii.** $\varepsilon^f_{0,1} < \varepsilon^{n\ell}_{\overline{1},\underline{2}}$. In the former scenario, the follower invests in the first technology after the leader can pre-empt the second one. implies that the leader does not face the risk of pre-emption, since the follower is assumed here to adopt a compulsive strategy, and, therefore, will not skip the first technology. In the latter scenario, the follower adopts the first technology before the leader can pre-empt the second one. This implies that the leader faces the threat of pre-emption. Consequently, the leader's optimal investment threshold in the second technology is $\max\left\{\varepsilon^f_{0,1}, \varepsilon^{n\ell}_{\overline{1},\underline{2}}\right\}$, as shown in Proposition 4. Intuitively, although the leader can pre-empt the second technology at $\varepsilon^{n\ell}_{\overline{1},\underline{2}}$, she may choose to delay adoption until the follower's entry at $\varepsilon^f_{0,1}$, provided that $\varepsilon^f_{0,1} > \varepsilon^{n\ell}_{\overline{1},\underline{2}}$. Doing so, the leader captures the same value function, albeit at a higher threshold, closer to the utility-maximising one.

Proposition 4. *The optimal investment threshold of the non-proprietary leader for the second technology is* $\max\left\{\varepsilon^f_{0,1}, \varepsilon^{n\ell}_{\overline{1},\underline{2}}\right\}$, *where* $\varepsilon^{n\ell}_{\overline{1},\underline{2}}$ *satisfies the condition* $F^f_{1,2}(E) = \Phi^{n\ell}_{\underline{2}}(E)$.

Next, we step back prior to the arrival of the second technology and assume that the firm that pre-empts the first technology is better placed to also adopt the second technology, if the follower has not entered the market. The non-proprietary leader's value function is indicated in (27). The first two terms reflect the expected utility of the monopoly profits from operating the first technology and the third term is the reduction in expected utility due to the entry of the follower. Note that the latter depends on whether $\varepsilon^f_{0,1} > \varepsilon^{n\ell}_{\overline{1},\underline{2}}$ or $\varepsilon^f_{0,1} < \varepsilon^{n\ell}_{\overline{1},\underline{2}}$.

$$\Phi^{n\ell}_{\underline{1}}(E) \quad = \quad \Upsilon U\left(ED_{\underline{1}}\right) - \frac{U\left(rI_1\right)}{\rho} + A^{n\ell}_{\underline{1}} E^{\beta_1} \tag{27}$$

Note that the last term on the right-hand side of (27) depends on whether the follower invests in the first technology before, $\varepsilon^f_{0,1} < \varepsilon^{n\ell}_{\overline{1},\underline{2}}$, or after, $\varepsilon^f_{0,1} > \varepsilon^{n\ell}_{\overline{1},\underline{2}}$, the leader can pre-empt the second one. In each case, the amount of reduction in the value of the leader due to the entry of the follower is different. In the former case, i.e. $\varepsilon^f_{0,1} < \varepsilon^{n\ell}_{\overline{1},\underline{2}}$, the leader does not have an advantage regarding the second technology, since the technology will be adopted at the indifference point between being a leader and a follower. Consequently, $A^{n\ell}_{\underline{1}}$ is determined by value matching (27) with (16). In the latter case, i.e. $\varepsilon^{f}_{0,1} \geq \varepsilon^{n\ell}_{\overline{1},\underline{2}}$, upon the follower's entry, the leader will receive the reduced value from operating the first technology with the expected value from pre-empting the second one. In this case, $A^{n\ell}_{\underline{1}}$ is obtained by value-matching (27) with the value function whose dynamics are described in (28). The first term indicates the instantaneous utility of leader's reduced profits due to follower's entry and the second term is the expected value of the profits in the continuation region. Note that with probability λdt the second technology will become available and the leader will get to pre-empt it, whereas with probability $1-\lambda dt$ the leader will continue to operate the first technology. Expanding (28) using Itô's lemma and solving the resulting ordinary differential equation yields (A–7).

$$\Phi^{n\ell}_{\overline{1}}(E) = \left[U\left(ED_{\overline{1}}\right) - U\left(rI_1\right)\right]dt + \left(1-\rho dt\right)\mathbb{E}_E\left[\lambda dt\Phi^{p\ell}_{\underline{2}}(E+dE) + \left(1-\lambda dt\right)\Phi^{n\ell}_{\overline{1}}(E+dE)\right] \quad (28)$$

Following the same reasoning as in (26), the leader's pre-emption threshold in the first technology, $\varepsilon^{n\ell}_{0,\underline{1}}$, is determined by solving (29).

$$F^{f}_{0,1}(E) \quad = \quad \Phi^{n\ell}_{\underline{1}}(E) \quad (29)$$

2.5.3 War of Attrition

Due to the competitive advantage created by ignoring the first technology, and, therefore not incurring the associated cost, a firm may choose to invest in the second one directly. Here, we consider how pre-emption of the first technology by a rival motivates a firm to adopt the second technology directly and ignore a compulsive strategy. Note that the difference in investment strategies prevents a comparison between the two firms in a way similar to that of Section 2.5.2. Like Takashima *et al.* (2008), we take the perspective of each firm separately and analyse their value functions assuming initially that it is possible for each firm to assume both roles, i.e., leader and follower. Then, we conclude which role is feasible for each firm. Since we have already determined the pre-emption threshold for the second technology under a compulsive strategy in (26), we only need to determine the pre-emption threshold when the first technology is ignored.

We denote as follower the firm that gets pre-empted in the adoption of the first technology, and, therefore, may have a greater incentive to adopt the second one directly. The follower's value function when investing in the second technology directly is described in (30). The top part is the value of the option to invest and the bottom part is the expected utility of the active project.

$$F_{0,2}^{f}(E) = \begin{cases} A_{0,2}^{f} E^{\beta_1} & , E < \varepsilon_{0,2}^{f} \\ \Upsilon U\left(ED_{\overline{2}}\right) - \frac{U(rI_2)}{\rho} & , E \geq \varepsilon_{0,2}^{f} \end{cases} \tag{30}$$

The expression of $A_{0,2}^{f}$ and $\varepsilon_{0,2}^{f}$ is obtained through value-matching and smooth-pasting conditions and is indicated in (31).

$$\varepsilon_{0,2}^{f} = \frac{rI_2}{D_{\overline{2}}}\left[\frac{\beta_2 - \gamma}{\beta_2}\right]^{\frac{1}{\gamma}} \quad \text{and} \quad A_{0,2}^{f} = \left(\frac{1}{\varepsilon_{0,2}^{f}}\right)^{\beta_1}\left[\Upsilon U\left(\varepsilon_{0,2}^{f} D_{\overline{2}}\right) - \frac{U\left(rI_2\right)}{\rho}\right] \tag{31}$$

The corresponding leader's value function is denoted by $\widetilde{\Phi}_{\underline{2}}^{n\ell}(\cdot)$ and is described in (32). Note that $\widetilde{A}_{\underline{2}}^{n\ell}$ is determined by value matching (32) with the bottom part of (30) at $\varepsilon_{0,2}^{f}$. The pre-emptive leader's threshold, $\widetilde{\varepsilon}_{0,\underline{2}}^{n\ell}$, satisfies the condition $F_{0,2}^{f}\left(\widetilde{\varepsilon}_{0,\underline{2}}^{n\ell}\right) = \widetilde{\Phi}_{\underline{2}}^{n\ell}\left(\widetilde{\varepsilon}_{0,\underline{2}}^{n\ell}\right)$.

$$\widetilde{\Phi}_{\underline{2}}^{n\ell}(E) = \Upsilon U\left(ED_{\underline{2}}\right) - \frac{U\left(rI_2\right)}{\rho} + \widetilde{A}_{\underline{2}}^{n\ell} E^{\beta_1} \quad , \widetilde{\varepsilon}_{0,\underline{2}}^{n\ell} < E \leq \varepsilon_{0,2}^{f} \tag{32}$$

Consequently, skipping the first technology is a feasible strategy provided that $\widetilde{\varepsilon}_{0,\underline{2}}^{n\ell} < \varepsilon_{\overline{1},\underline{2}}^{n\ell}$. Intuitively, the follower in the first technology can invest in the second technology first provided the pre-emption threshold of the compulsive leader is greater than the threshold of directly adopting the second technology. Note that the feasibility of skipping the first technology and adopting the second one directly can be analysed by comparing the relative value of the two strategies, i.e., $\widetilde{\Phi}_{\underline{2}}^{n\ell}(E)/F_{0,1}^{f}(E)$.

2.6 Leapfrog and Laggard Strategy

Assuming that the leader has proprietary rights on each technology, she may decide to ignore a technology temporarily in order to wait for a new one to arrive before deciding which one to invest in. If the leader ignores the first technology, then only the second one will be commercialised, and, therefore, the follower's value function is indicated in (30). Given the follower's optimal response, the proprietary leader can choose whether to adopt a leapfrog or a laggard strategy as illustrated in Figure 2.5. Instead of moving from $(0, \underline{1})$ to $\underline{1}$, the leader moves to state $(0, \underline{1} \vee \underline{2})$, and then, either invests in the first technology, holding the option to switch to the second one, i.e., state $(\underline{1}, \underline{2})$, or $(\vee)$ invests directly in the second technology, thereby moving to state $\underline{2}$.

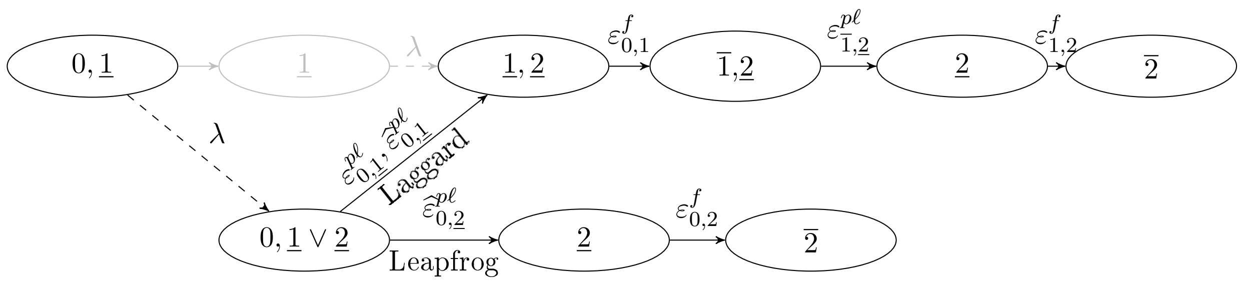


Figure 2.5: Proprietary duopoly under a leapfrog/laggard strategy

Notice that the value function of the proprietary leader in states $\left(\overline{1},\underline{2}\right)$, $\underline{2}$, and $\overline{2}$ following a laggard strategy is the same as in Section 2.5.1, while her value function in state $\underline{2}$ following a leapfrog strategy is indicated in (32). Hence, we proceed directly to state $(\underline{1},\underline{2})$, where the leader operates the first technology and earns monopoly profits until the follower enters at $\varepsilon^{f}_{0,1}$. The leader's value function is described in (33), where the third term on the right-hand side reflects the expected reduction in the leader's profits due to the follower's entry. The endogenous constant, $A^{p\ell}_{\underline{1},\underline{2}}$, is obtained by value matching (33) with the bottom part of (19) at $\varepsilon^{f}_{0,1}$.

$$F^{p\ell}_{\underline{1},\underline{2}}(E) \quad = \quad \Upsilon U\left(ED_{\underline{1}}\right) - \frac{U\left(rI_1\right)}{\rho} + A^{p\ell}_{\underline{1},\underline{2}}E^{\beta_1} \tag{33}$$

Due to the presence of the second technology, there exist two waiting regions: **i.** $E \leq \varepsilon^{p\ell}_{0,\underline{1}}$ and **ii.** $\widehat{\varepsilon}^{p\ell}_{0,\underline{1}} \leq E \leq \widehat{\varepsilon}^{p\ell}_{0,\underline{2}}$ (Décamps *et al.*, 2006). Hence, the value function in state $(0,\underline{1}\vee\underline{2})$ is described in (34), where $B^{p\ell}_{0,\underline{1}\vee\underline{2}}$, $C^{p\ell}_{0,\underline{1}\vee\underline{2}}$, $\widehat{\varepsilon}^{p\ell}_{0,\underline{1}}$, and $\widehat{\varepsilon}^{p\ell}_{0,\underline{2}}$ are obtained numerically via value-matching and smooth-pasting conditions between the bottom three branches, and $\widetilde{\Phi}^{n\ell}_{\underline{2}}(E)$ is indicated in (32). Notice that, if $E < \varepsilon^{p\ell}_{0,\underline{1}}$, then the firm will wait until $E = \varepsilon^{p\ell}_{0,\underline{1}}$ and then adopt the first technology. By contrast, if $\widehat{\varepsilon}^{p\ell}_{0,\underline{1}} \leq E \leq \widehat{\varepsilon}^{p\ell}_{0,\underline{2}}$, then the firm will either invest directly in the second technology if $E \uparrow \widehat{\varepsilon}^{p\ell}_{0,\underline{2}}$, or it will invest in the first one and hold the option to switch to the second if $E \downarrow \widehat{\varepsilon}^{p\ell}_{0,\underline{1}}$.

$$F^{p\ell}_{0,\underline{1}\vee\underline{2}}(E) \quad = \quad \begin{cases} A^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_1} & , E < \varepsilon^{p\ell}_{0,\underline{1}} \\ F^{p\ell}_{\underline{1},\underline{2}}(E) & , \varepsilon^{p\ell}_{0,\underline{1}} \leq E < \widehat{\varepsilon}^{p\ell}_{0,\underline{1}} \\ B^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_2} + C^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_1} & , \widehat{\varepsilon}^{p\ell}_{0,\underline{1}} \leq E < \widehat{\varepsilon}^{p\ell}_{0,\underline{2}} \\ \widetilde{\Phi}^{n\ell}_{\underline{2}}(E) & , \widehat{\varepsilon}^{p\ell}_{0,\underline{2}} \leq E \end{cases} \tag{34}$$

Finally, in state $(0,\underline{1})$ either the second technology will become available with probability λdt and the proprietary leader will receive the value function $F^{p\ell}_{0,\underline{1}\vee\underline{2}}(E)$, or no innovation will take place with probability $1-\lambda dt$ and the leader will continue to hold the value function $\widehat{F}^{p\ell}_{0,\underline{1}}(E)$.

$$\widehat{F}^{p\ell}_{0,\underline{1}}(E) \quad = \quad (1-\rho dt)\,\mathbb{E}_E\left[\lambda dt F^{p\ell}_{0,\underline{1}\vee\underline{2}}(E+dE)+(1-\lambda dt)\,\widehat{F}^{p\ell}_{0,\underline{1}}(E+dE)\right] \tag{35}$$

The expression of the value function in state $(0,\underline{1})$ is indicated in (36), where $D^{p\ell}_{\underline{1}}$, $G^{p\ell}_{\underline{1}}$, $H^{p\ell}_{\underline{1}}$, $J^{p\ell}_{\underline{1}}$, $K^{p\ell}_{\underline{1}}$, $L^{p\ell}_{\underline{1}}$, and $M^{p\ell}_{\underline{1}}$ are determined numerically via the value-matching and smooth-pasting conditions between the branches of (36). Notice that (36) has five branches, which is a consequence of the value function $\widetilde{\Phi}^{n\ell}_{\underline{2}}(E)$ changing to the bottom branch of (30) when the follower enters the market.

$$\widehat{F}^{p\ell}_{0,\underline{1}}(E)=\begin{cases} A^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_1}+D^{p\ell}_{\underline{1}}E^{\delta_1} & ,E<\varepsilon^{p\ell}_{0,\underline{1}} \\ \lambda\Lambda\Upsilon U\left(D_{\underline{1}}E\right)-\frac{\lambda U(rI_1)}{\rho(\rho+\lambda)}+A^{p\ell}_{\underline{1},\underline{2}}E^{\beta_1}+G^{p\ell}_{\underline{1}}E^{\delta_1}+H^{p\ell}_{\underline{1}}E^{\delta_2} & ,\varepsilon^{p\ell}_{0,\underline{1}}\leq E<\widehat{\varepsilon}^{p\ell}_{0,\underline{1}} \\ B^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_2}+C^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_1}+J^{p\ell}_{\underline{1}}E^{\delta_1}+K^{p\ell}_{\underline{1}}E^{\delta_2} & ,\widehat{\varepsilon}^{p\ell}_{0,\underline{1}}\leq E<\widehat{\varepsilon}^{p\ell}_{0,\underline{2}} \\ \lambda\Lambda\Upsilon U\left(D_{\underline{2}}E\right)-\frac{\lambda U(rI_2)}{\rho(\rho+\lambda)}+\widetilde{A}^{n\ell}_{\underline{2}}E^{\beta_1}+L^{p\ell}_{\underline{1}}E^{\delta_1}+M^{p\ell}_{\underline{1}}E^{\delta_2} & ,\widehat{\varepsilon}^{p\ell}_{0,\underline{2}}\leq E<\varepsilon^{f}_{0,\overline{2}} \\ \lambda\Lambda\Upsilon U\left(D_{\overline{2}}E\right)-\frac{\lambda U(rI_2)}{\rho(\rho+\lambda)} & ,\varepsilon^{f}_{0,\overline{2}}\leq E \end{cases} \tag{36}$$

2.7 Numerical Results

Proprietary duopoly with compulsive firms

For the numerical results, the parameter values are $\mu=0.01$, $\rho=r=0.08$, $\sigma\in[0.1,0.25]$, $I_1=500$, $I_2=1500$, $D_{\overline{1}}=8$, $D_{\overline{2}}=15$, $D_{\underline{1}}=11$, $D_{\underline{2}}=19$, and $\lambda\in\mathbb{R}^+$. These values ensure that there is a trade-off between the two technologies, as shown in Corollary 1. Figure 2.6 illustrates the value function of the leader and the follower for the case of investment in the first (left panel) and the second (right panel) technology under a compulsive strategy. According to the right panel, the proprietary leader has the option to delay investment, and, therefore, adopts the second technology at $E=19.76$. By contrast, the non-proprietary leader faces the risk of pre-emption and adopts the second technology at $E=14.58$. Indeed, for $E<14.58$ the option value of the follower is greater than the project value of the leader, while for $E>14.58$ the opposite is observed. Consequently, $E=14.58$ indicates the point of indifference between being the leader or the follower. For $14.58<E\leq 32.12$, the leader enjoys monopoly profits, however, once the follower invests in the second technology at 32.12, then both firms share the market. The left panel illustrates the value function of the leader and the follower when

contemplating investment in the first technology, while holding an embedded option to invest in the second one, that has yet to become available. Notice that, upon adoption of the first technology at $E = 7.88$, the value function of the proprietary leader (thin curve) is not the same as that of the follower (thick curve), since the leader holds the option to invest in the second technology first. Consequently, unlike state $\overline{2}$, the value function of the proprietary leader value matches with her own value function in state $\overline{1}$ at $E = 7.88$ and not with the follower's value function.

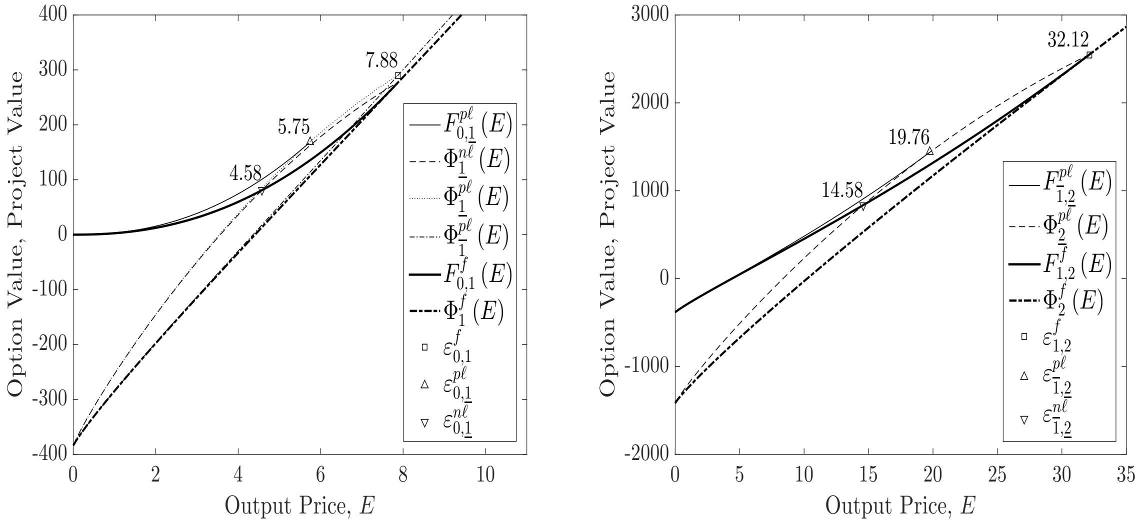


Figure 2.6: Option and project value of the leader and the follower in the first (left panel) and the second technology (right panel) under a proprietary and a non-proprietary duopoly for $\lambda = 0.1$, $\gamma = 0.9$, and $\sigma = 0.2$.

Figure 2.7 illustrates the impact of λ and γ on the required investment threshold of the proprietary leader (left panel) and the follower (right panel) for $\sigma = 0.18, 0.20$. Notice that price uncertainty increases the required investment threshold of both the leader and the follower by raising the opportunity cost of investing, thereby increasing the value of waiting. This effect has also been confirmed empirically (Dunne & Mu, 2010). Interestingly, while the impact of technological uncertainty on the required investment threshold of the follower is non-monotonic, the proprietary leader's decision to invest is not affected by technological uncertainty. The former result is in line with Chronopoulos & Siddiqui (2015), who show that greater λ increases a firm's incentive to adopt the currently available technology in order to have a shot at the yet unreleased version. The latter result happens because the the follower invests in the first technology before the leader invests in the second one, as shown in Proposition 1. Consequently, the adoption of the first technology does not affect the leader's prospective monopoly profits from the second one, thus resulting in a myopic strategy, as indicated in Proposition 3.

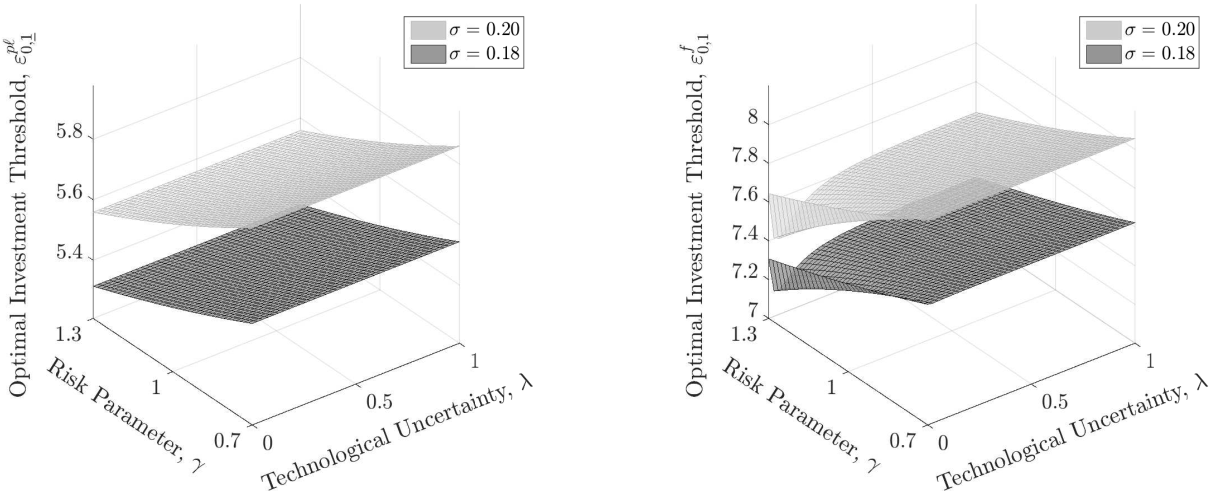


Figure 2.7: Impact of λ and γ on the optimal investment threshold of the proprietary leader (left panel) and the follower (right panel).

The right panel of Figure 2.8 illustrates the impact of λ and γ on the required investment threshold of the non-proprietary leader for $\sigma = 0.18, 0.20$. Notice that, although the impact of γ and σ is the same as in Figure 2.7, greater λ induces later adoption for the leader. Intuitively, this happens because earlier entry of the follower, as illustrated in the right panel of Figure 2.7, reduces the period of monopoly profits for the non-proprietary leader, thereby decreasing the attractiveness of the first technology. Also, as the left panel illustrates, increasing the first-mover advantage raises the investment incentive and lowers the required entry threshold of the

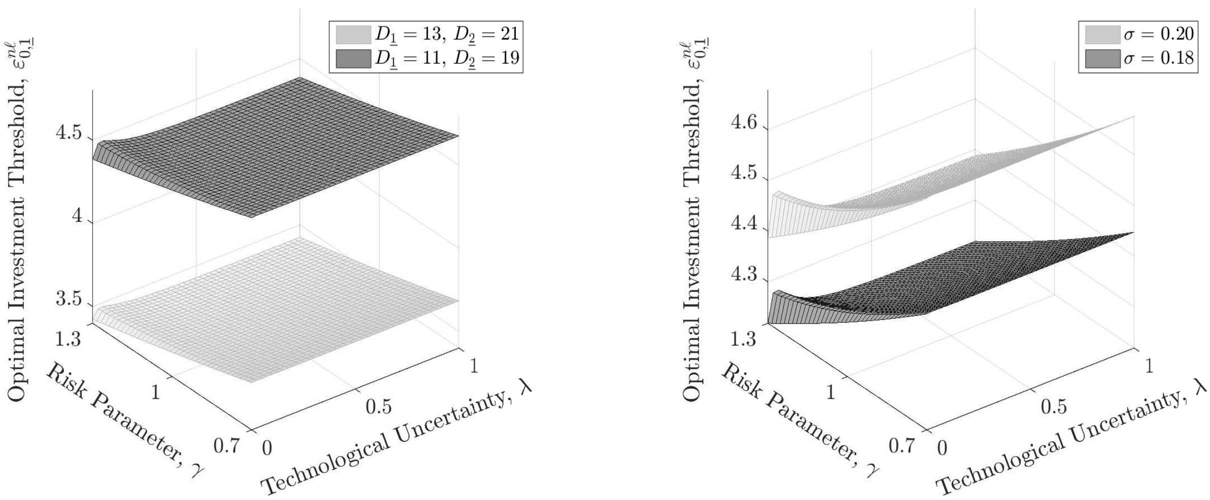
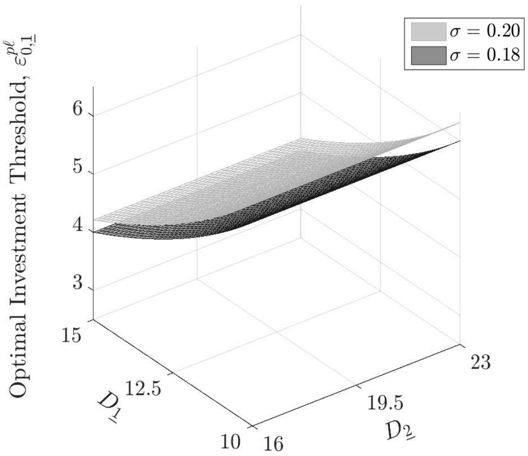


Figure 2.8: Impact of first-mover advantage on $\varepsilon^{n\ell}_{0,\underline{1}}$ (left panel) and impact of λ and γ on $\varepsilon^{n\ell}_{0,\underline{1}}$ for $\sigma = 0.18, 0.2$ (right panel).

Figure 2.9 illustrates the impact of greater first-mover advantage on the required investment threshold of the proprietary (left panel) and non-proprietary compulsive leader (right panel). As both panels illustrate, the leader's required investment threshold in the first technology is not affected by the first-mover advantage in the second one. More specifically, since the follower's entry threshold is not affected by changes in $D_{\underline{1}}$, the period of monopoly profits for the leader in the first technology is unchanged and so is the leader's optimal adoption threshold. By contrast, a greater first-mover advantage in the first technology accelerates investment, while the threat of pre-emption increases the investment incentive, as illustrated in the right panel.



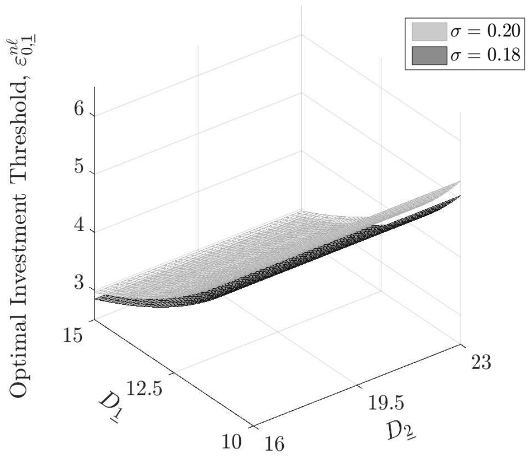


Figure 2.9: Impact of $D_{\underline{1}}$ and $D_{\underline{2}}$ on the optimal investment threshold of the proprietary (left panel) and non-proprietary leader (right panel).

In order to calculate the leader's relative loss in value, we use the follower's analysis from Section 2.5.1 to find the monopolist's option value under sequential investment. The impact of γ and σ on the relative loss in the value of the proprietary and non-proprietary leader is indicated in the left- and the right-hand side expression of (37), respectively, and is illustrated in Figure 2.10.

$$\frac{A^m_{0,1}{\varepsilon^{n\ell}_{0,\underline{1}}}^{\beta_1} - A^{p\ell}_{0,1}{\varepsilon^{n\ell}_{0,\underline{1}}}^{\beta_1}}{A^m_{0,1}{\varepsilon^{n\ell}_{0,\underline{1}}}^{\beta_1}} \quad \text{and} \quad \frac{A^m_{0,1}{\varepsilon^{n\ell}_{0,\underline{1}}}^{\beta_1} - \Phi^{n\ell}_{\underline{1}}(\varepsilon^{n\ell}_{0,\underline{1}})}{A^m_{0,1}{\varepsilon^{n\ell}_{0,\underline{1}}}^{\beta_1}} \tag{37}$$

In line with Siddiqui & Takashima (2012) and Chronopoulos *et al.* (2014), the left panel in Figure 2.10 indicates that the relative loss in the value of the proprietary leader increases (decreases) with greater price uncertainty when the first-mover advantage is high (low). Intuitively, this happens because, under low discrepancy in market share, the increase in the proprietary leader's value of investment opportunity due to the follower's late investment is greater than the expected loss due to the entry of the follower. However, when the discrepancy is high the period of time with monopoly profits in the second technology is more pronounced causing the relative loss to increase. Also, as the right panel illustrates, greater price uncertainty and a lower first-mover advantage decreases the relative loss in value for the non-proprietary leader.

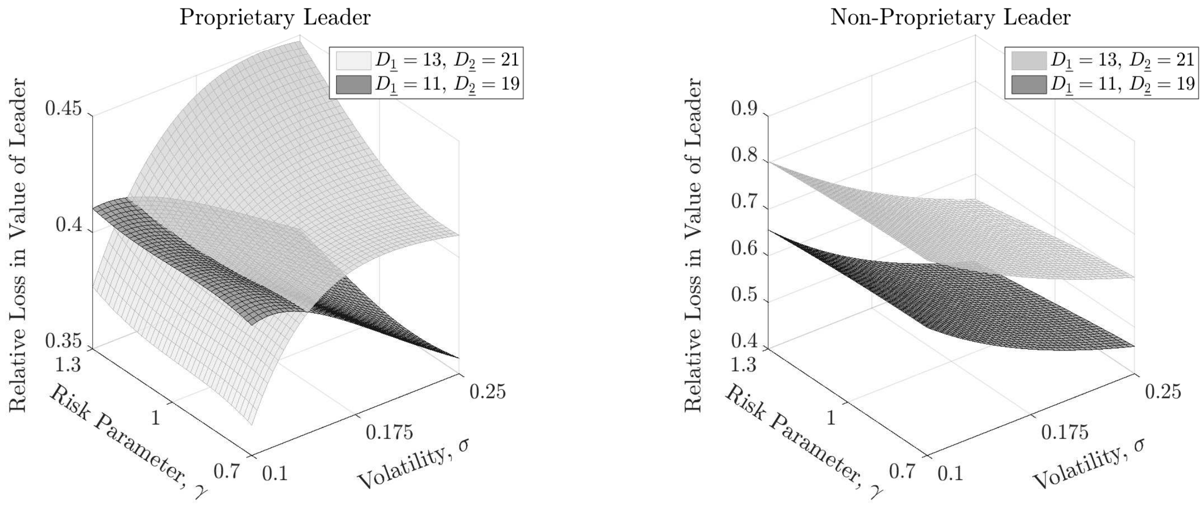


Figure 2.10: Relative loss in the value of the proprietary (left panel) and non-proprietary leader (right panel) versus γ and σ for $\lambda = 0.1$.

The impact of γ and λ on the relative loss in value for the proprietary (left panel) and non-proprietary leader (right panel) is illustrated in Figure 2.11. As both panels illustrate, a higher innovation rate lowers the relative loss in the value of the leader by raising the expected utility of the embedded option to adopt a more efficient technology. Interestingly, risk aversion has an ambiguous effect on the relative loss in the value of the proprietary leader (left panel). More specifically, under a low (high) rate of technological innovation, greater risk aversion decreases (increases) the relative loss in the value of the leader. This happens because greater risk aversion postpones the entry of the follower, thereby allowing the leader to enjoy monopoly profits for a longer time. However, when λ is high, the second technology is more likely to become available, which in turn, gives the leader greater incentive to invest than the monopolist, as shown in Proposition 2. Consequently, the impact of greater risk aversion is mitigated by higher technological uncertainty.

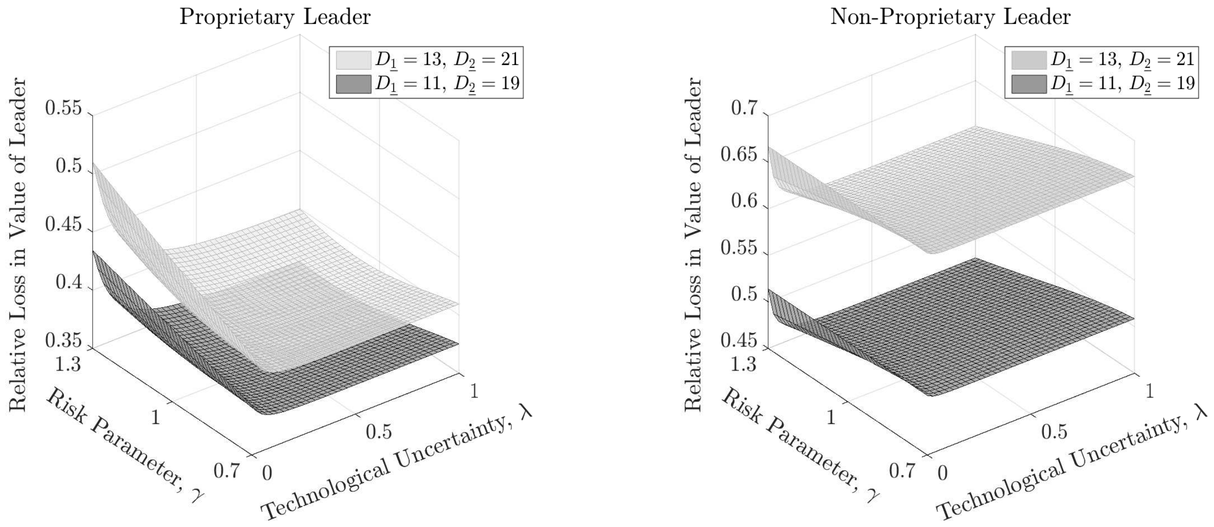


Figure 2.11: Relative loss in the value of the proprietary (left panel) and non-proprietary leader (right panel) versus γ and λ.

War of attrition.

The top panel in Figure 2.12 indicates that, for $\lambda = 0.1$, $\gamma = 0.9$ and $\sigma = 0.2$, the pre-emption threshold for the second technology when the first one has already been adopted is 14.58. Yet, the bottom panel illustrates that direct pre-emption of the second technology, without adopting the first one, requires a threshold of 8.31. Consequently, the competitive advantage from ignoring the first technology can facilitate the direct pre-emption of the second one.

However, although skipping the first technology in order to pre-empt the second may be feasible, it may still be optimal to proceed with a compulsive strategy. Figure 2.13 illustrates the relative value of **i.** pre-empting the second technology directly and **ii.** adopting compulsive strategy under a low (left panel) and a high (right panel) output price. The relative value of these two strategies is described in (38). In this comparison, we ignore technological uncertainty by assuming that both technologies are available.

$$RV_1 = \frac{\widetilde{\Phi}_{\underline{2}}^{n\ell}(E)}{F_{0,1}^{f}(E)} \tag{38}$$

Note that if the output price is low, then it is always better to be a compulsive follower as the left panel of illustrates. However, under a high output price (right panel), increasing price uncertainty makes it optimal to skip the first technology in order to pre-empt the second one. Interestingly, lower risk aversion also increases the relative value of pre-empting the second technology directly. In fact, even under risk neutrality ($\gamma = 1$), it is optimal to ignore the first

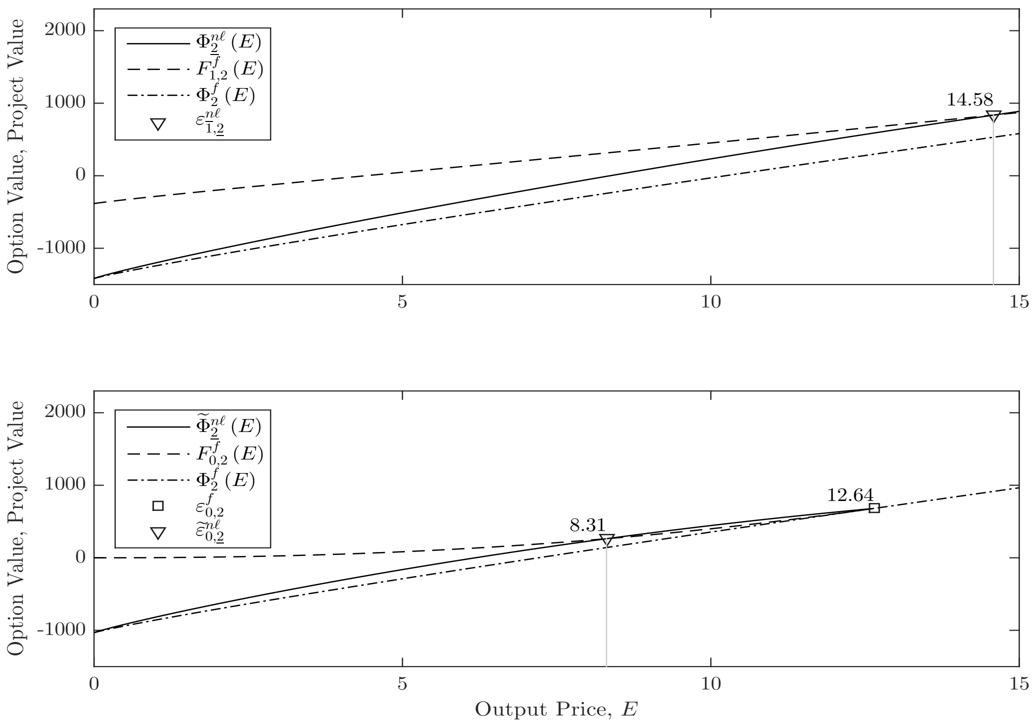


Figure 2.12: Value function of the leader and the follower, when the follower adopts a leapfrog strategy under a non-proprietary duopoly with $\lambda = 0.1$, $\gamma = 0.9$ and $\sigma = 0.2$.

technology and pre-empt the second one directly, provided that price uncertainty is adequately high.

Proprietary duopoly under a leapfrog/laggard strategy.

The left panel in Figure 2.14 illustrates the value function of the proprietary leader under a leapfrog/ laggard strategy for $\gamma = 0.9, 1.0$. Notice that if $\gamma = 0.9$ and $E < 5.75$, then the leader will wait until $E = 5.75$ to adopt the first technology and enjoy monopoly profits until the follower enters at $E = 7.90$. By contrast, if $E \in [13.27, 15.13]$, then the leader will either adopt a laggard strategy if $E \downarrow 13.27$ or a leapfrog strategy if $E \uparrow 15.13$. Also, lower risk aversion raises the expected utility of the project and lowers the investment thresholds. As the right panel illustrates, greater price uncertainty raises all investment threshold, yet decreases the likelihood of a laggard strategy by narrowing the intermediate waiting region.

Figure 2.15 illustrates the relative value of the compulsive and leapfrog/laggard strategy for the proprietary leader. More specifically, the left panel illustrates the top branch in (39), which compares the first branch of (36) with (23) when the output price is low, i.e., $E < \varepsilon_{0,\underline{1}}^{p\ell}$. Similarly, the right panel illustrates the expression in the bottom-branch, which compares the

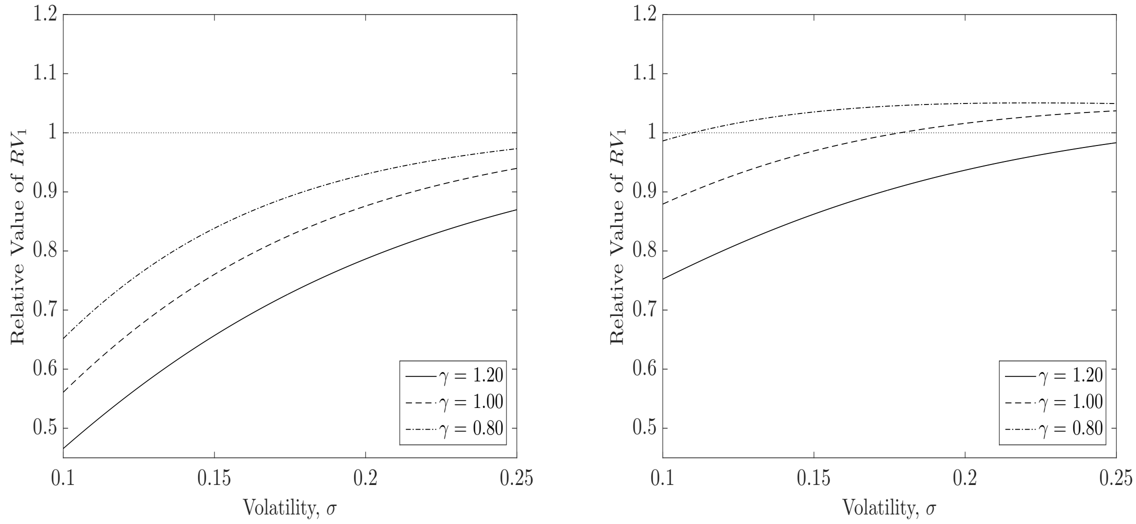


Figure 2.13: Relative value of the war of attrition strategy compared to the compulsive strategy for the follower evaluated at $E = \widehat{\varepsilon}^{n\ell}_{0,\underline{2}}$ (left panel) and $E = \varepsilon^{f}_{0,2}$ (right panel).

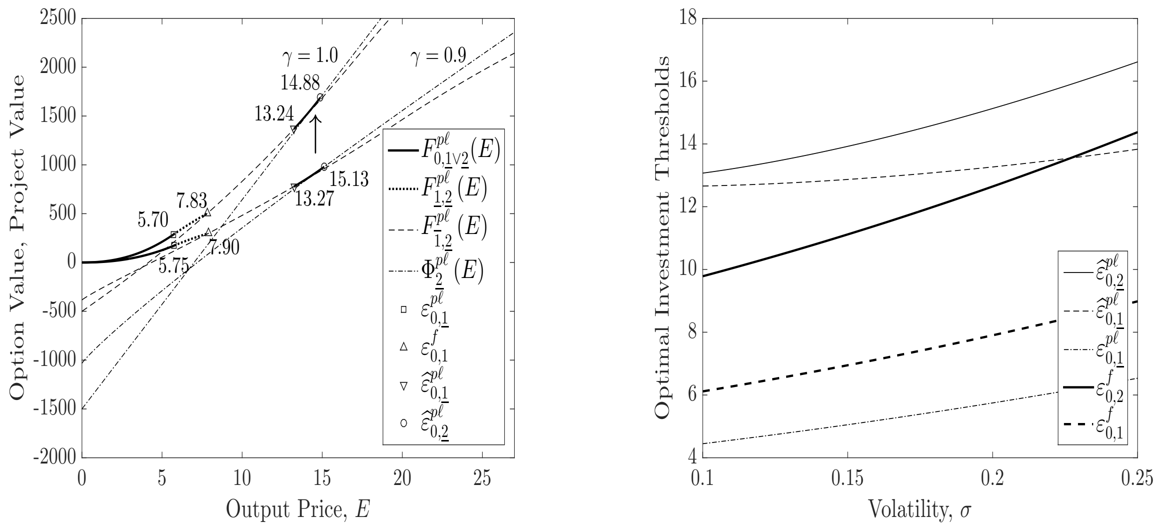


Figure 2.14: Value function of the proprietary leader for $\sigma = 0.2$ (left panel) and optimal investment thresholds for $\gamma = 0.9$ (right panel), under proprietary duopoly with a leapfrog/laggard strategy.

bottom part of (36) with the top part of (22) when the output price is high, i.e., $E = \varepsilon^{p\ell}_{0,\underline{2}}$.

$$RV_2 \quad = \quad \begin{cases} \dfrac{A^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_1} + D^{p\ell}_{\underline{1}}E^{\delta_1}}{A^{p\ell}_{0,\underline{1}}E^{\beta_1}} & , E < \varepsilon^{p\ell}_{0,\underline{1}} \\[2ex] \dfrac{B^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_2} + C^{p\ell}_{0,\underline{1}\vee\underline{2}}E^{\beta_1} + J^{p\ell}_{\underline{1}}E^{\delta_1} + K^{p\ell}_{\underline{1}}E^{\delta_2}}{\Upsilon U\left(ED_{\overline{1}}\right) - \frac{U(rI_1)}{\rho} + A^{p\ell}_{\overline{1},\underline{2}}E^{\beta_1} + A^{p\ell}_{\overline{1}}E^{\delta_1}} & , \widehat{\varepsilon}^{p\ell}_{0,\underline{1}} < E < \widehat{\varepsilon}^{p\ell}_{0,\underline{2}} \end{cases} \tag{39}$$

As the right panel illustrates, the compulsive strategy dominates when the output price is low. This happens because a firm must wait longer to invest in the more capital intensive technology and the associated payoff does not offset the foregone revenues from ignoring the existing one.

In fact, greater risk aversion promotes the adoption of a compulsive strategy and makes the leapfrog/laggard strategy relative less attractive. Interestingly, unlike Chronopoulos & Siddiqui (2015), the same result holds even at a high output price, as long as the discrepancy in market share is large. However, as the right panel illustrates, a leapfrog strategy may dominate, under a high output price and a low discrepancy in market share.

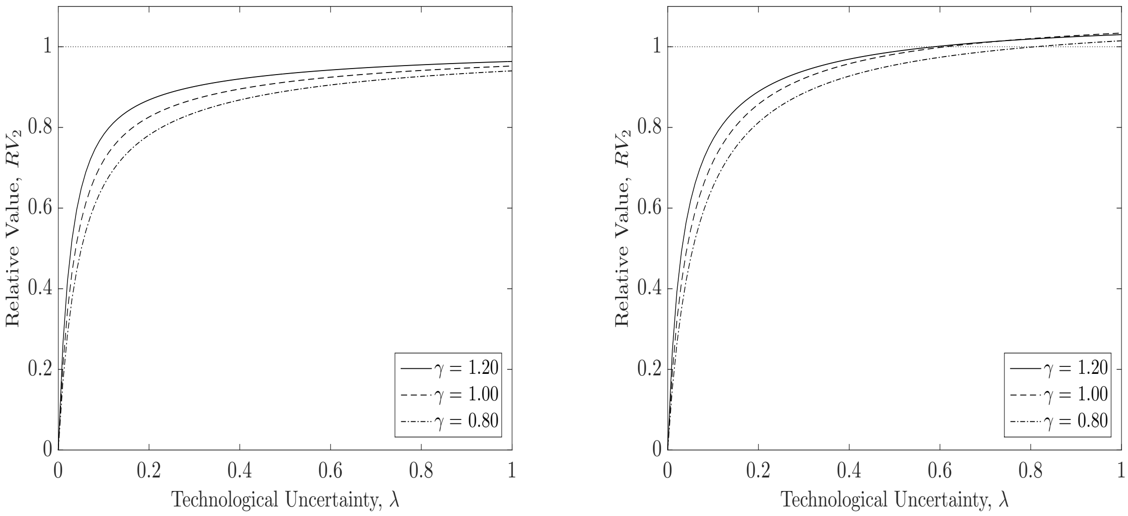


Figure 2.15: Relative value of compulsive and leapfrog/laggard strategy for the proprietary leader at $E = \varepsilon^{p\ell}_{0,\underline{1}}$ (left panel) and $E = \widehat{\varepsilon}^{p\ell}_{0,\underline{2}}$ where $D_{\underline{1}} = 9$, $D_{\underline{2}} = 17$ (right panel) for $\sigma = 0.2$.

2.8 Conclusions

We analyse how attitudes towards risk interact with price and technological uncertainty to impact sequential investment decisions of firms within the context of duopolistic competition. The analysis is motivated by four main features of the modern economic environment: **i.** increasing competition due to the deregulation of many sectors of the economy, such as energy and telecommunications; **ii.** market incompleteness and associated attitudes towards idiosyncratic risk; **iii.** the sequential nature of investment decisions in emerging technologies, e.g., energy, and the R&D-based sector of the economy; and **iv.** technological uncertainty. We incorporate these features into a utility-based framework for investment under uncertainty by assuming that two identical firms compete in the sequential adoption of technological innovations. More specifically, we assume that the firms compete in the adoption of two technologies, of which the first one is available, while the arrival of the second, more efficient one is subject to technological uncertainty.

Results indicate that insights from traditional real options models do not extend naturally to a competitive setting with various interacting uncertainties. Indeed, we find that technological uncertainty increases the follower's incentive to adopt the existing technology. This is in line with Chronopoulos & Siddiqui (2015), who address the problem of sequential investment in technological innovations ignoring, however, both the implications of strategic interactions and attitudes towards risk. Interestingly, under a proprietary duopoly, the leader's required investment threshold for both technologies is independent of technological uncertainty. In addition, we show that competition induces earlier technology adoption in the case of proprietary duopoly relative to the case of monopolist. Furthermore, we find that although greater price uncertainty lowers the relative loss in the value of the non-proprietary leader, in the case of proprietary duopoly, the impact of price uncertainty on the relative loss in the leader's option value depends on the discrepancy in market share. Also, a higher innovation rate lowers the relative loss in the value of both the proprietary and the non-proprietary leader. With respect to the technology adoption strategy, we show how the threat of pre-emption creates an incentive to ignore the existing technology in order to adopt the new one directly, and identify when this strategy dominates under different rates of technological innovation and levels of risk aversion. Similarly, by comparing a compulsive to a leapfrog/laggard strategy under a proprietary duopoly we find that the former strategy always dominates under a low output price. However, the latter strategy may dominate when the discrepancy in market share is small, provided that both the rate of innovation and the output price is high.

Extensions in the same line of work may include the flexibility to choose not only the time of investment but also the size of the project. In line with Huisman & Kort (2015), this will also enable the analysis of how different types of strategic interactions impact social welfare in terms of the time of investment and the amount of installed capacity. Additionally, other types of uncertainties may also be relevant within the context of strategic interactions. For example, regulatory risk regarding the availability of subsidies for specific technologies may impact strategic interactions, significantly. Other strategies may also be analysed as in Grenadier and Weiss (1997), or asymmetries can be included to analyse proprietary duopoly as in Takashima *et al.* (2008). Furthermore, we can investigate how technological uncertainty impacts the propensity to form joint ventures by reinterpreting γ as the elasticity of profit with respect to demand (De Hek & Mukherjee, 2011). Finally, to determine the robustness of the analytical and numerical results, it may be interesting to apply an alternative stochastic process such as a mean reverting GBM as well as other utility functions, e.g., Epstein-Zin utility or preferences in accordance with Prospect theory.

2.9 Appendix

A Compulsive Strategy

Follower

The expected utility from operating the second technology is given in (A–1)

$$\Phi_2^f(E) = \Upsilon U\left(ED_{\overline{2}}\right) - \frac{U\left(rI_1\right) + U\left(rI_2\right)}{\rho} \tag{A–1}$$

and the value function of the follower in state $(1,2)$ is indicated in (A–2).

$$F_{1,2}^f\left(E\right) = \begin{cases} \left[U\left(ED_{\overline{2}}\right) - U\left(rI_1\right)\right]dt + (1-\rho dt)\mathbb{E}_E\left[F_{1,2}^f(E+dE)\right] & , E < \varepsilon_{1,2}^f \\ \Phi_2^f(E) & , E \geq \varepsilon_{1,2}^f \end{cases} \tag{A–2}$$

By expanding the top branch on the right-hand side of (A–2) using Itô's lemma and solving the resulting ordinary differential equation, we obtain (A–3).

$$F_{1,2}^f(E) = \Upsilon U\left(ED_{\overline{2}}\right) - \frac{U\left(rI_1\right)}{\rho} + A_{1,2}^f E^{\beta_1} + C_{1,2}^f E^{\beta_2} \tag{A–3}$$

Notice that $\beta_2 < 0 \Rightarrow C_{1,2}^f E^{\beta_2} \to \infty$ as $E \to 0$. Hence, we must have $C_{1,2}^f = 0$. The endogenous constant, $A_{1,2}^f$, and the required investment threshold, $\varepsilon_{1,2}^f$, are obtained via the value-matching and smooth-pasting conditions. Thus, the value function in state $(1,2)$ is described in (14).

Next, by expanding the right-hand side of (15) using Itô's lemma we obtain the differential equation (A–4), where $\mathcal{L} = \frac{1}{2}\sigma^2 E^2 \frac{\mathbf{d}^2}{\mathbf{d}E^2} + \mu E \frac{\mathbf{d}}{\mathbf{d}E}$ denotes the differential generator. By solving (A–4) for each expression of $F_{1,2}^f(E)$ that is indicated in (14) we obtain (16).

$$\left[\mathcal{L} - (\rho+\lambda)\right]\Phi_1^f(E) + \lambda F_{1,2}^f(E) + U\left(D_{\overline{1}}E\right) - U\left(\rho I_1\right) = 0 \tag{A–4}$$

Leader

In a state $\underline{2}$, the value function of the leader described in (18) will value match with the bottom part of the leader's value function (14), because for $E \geq \varepsilon_{1,2}^f$ the two firms will share the market.

Hence, $A_{\underline{2}}^{p\ell}$ is described in (A–5).

$$A_{\underline{2}}^{p\ell} = \left(\frac{1}{\varepsilon_{1,2}^{f}}\right)^{\beta_1} \Upsilon U\left(\varepsilon_{1,2}^{f}\right)\left[D_{\bar{2}}^{\gamma} - D_{\underline{2}}^{\gamma}\right] \tag{A–5}$$

By contrast, in state $\underline{1}$, $A_{\underline{1}}^{p\ell}$ is obtained by value matching (23) with the top branch in (22) at $\varepsilon_{0,1}^{f}$. Hence, the endogenous constant $A_{\underline{1}}^{p\ell}$ is indicated in (A–6).

$$A_{\underline{1}}^{p\ell} = \left(\frac{1}{\varepsilon_{0,1}^{f}}\right)^{\beta_1} \left[\Upsilon U\left(\varepsilon_{0,1}^{f}\right)\left[D_{\bar{1}}^{\gamma} - D_{\underline{1}}^{\gamma}\right] + A_{\bar{1},\underline{2}}^{p\ell}\varepsilon_{0,1}^{f\,\beta_1} + A_{\bar{1}}^{p\ell}\varepsilon_{0,1}^{f\,\delta_1}\right] \tag{A–6}$$

If the follower in the first technology chooses to ignore it in order to adopt the second one directly, then her value function is obtained by expanding (28) using Itô's lemma and solving the resulting ordinary differential equation. The solution is indicated in (A–7). The first three terms in the top part reflect the expected utility of the monopoly profits from operating the second technology, the fourth term reflects the loss in value due to the follower's entry and the final term adjusts for technological uncertainty. In the bottom part, both would invest immediately if the second technology becomes available, and both players get the expected NPV. The endogenous constant, $A_{\bar{1}}^{n\ell}$ is obtained by value-matching the two branches at $\varepsilon_{1,2}^{f}$.

$$\Phi_{\bar{1}}^{n\ell}(E) = \begin{cases} \Lambda\left[\lambda\Upsilon U\left(ED_{\underline{2}}\right) + U\left(ED_{\bar{1}}\right)\right] - \frac{U(rI_1)}{\rho} - \frac{\lambda U(rI_2)}{\rho(\rho+\lambda)} + A_{\underline{2}}^{p\ell}E^{\beta_1} + A_{\bar{1}}^{n\ell}E^{\delta_1} & , E < \varepsilon_{1,2}^{f} \\ \Lambda\left[\lambda\Upsilon U\left(ED_{\bar{2}}\right) + U\left(ED_{\bar{1}}\right)\right] - \frac{U(rI_1)}{\rho} - \frac{\lambda U(rI_2)}{\rho(\rho+\lambda)} & , E \geq \varepsilon_{1,2}^{f} \end{cases} \tag{A–7}$$

□

Corollary 1 *There is a trade-off between the two technologies iff* $\frac{D_1^{\gamma}}{I_1^{\gamma}} > \frac{D_2^{\gamma}}{I_1^{\gamma}+I_2^{\gamma}}$.

Proof: Let ε denote the indifference point between the two projects, i.e., $\Phi_1^{ab}(\varepsilon) = \Phi_2^{ab}(\varepsilon)$.

$$\begin{aligned} \Phi_1^{ab}(\varepsilon) = \Phi_2^{ab}(\varepsilon) \quad &\Leftrightarrow \quad \Upsilon U\left(D_2\varepsilon\right) - \frac{U(rI_2) + U(rI_1)}{\rho} = \Upsilon U\left(D_1\varepsilon\right) - \frac{U\left(rI_1\right)}{\rho} \\ &\Leftrightarrow \quad \varepsilon = \left(\frac{\gamma U(rI_2)}{\Upsilon\rho\left(D_2^{\gamma} - D_1^{\gamma}\right)}\right)^{\frac{1}{\gamma}} \end{aligned} \tag{A–8}$$

A trade-off between the technologies requires that $\Phi_1^{ab}(\varepsilon) > 0$.

$$\Phi_1^{ab}(\varepsilon) > 0 \Rightarrow \Upsilon U\left(D_1\varepsilon\right) - \frac{U\left(rI_1\right)}{\rho} > 0 \Rightarrow \frac{D_1^{\gamma}}{I_1^{\gamma}} > \frac{D_2^{\gamma}}{I_1^{\gamma} + I_2^{\gamma}} \tag{A–9}$$

□

Proposition 1 $\varepsilon_{\bar{1},\underline{2}}^{p\ell} > \varepsilon_{0,1}^{f} \Leftrightarrow \frac{D_1^{\gamma}}{I_1^{\gamma}} > \frac{D_2^{\gamma}}{I_1^{\gamma}+I_2^{\gamma}}$.

Proof: From Chronopoulos & Siddiqui (2014), we know that $\varepsilon^{f}_{0,1}$ is described in (A–10).

$$\varepsilon^{f}_{0,1} \quad = \quad \frac{rI_1}{D_{\bar{1}}}\left[\frac{\beta_2-\gamma}{\beta_2}\right]^{\frac{1}{\gamma}} \tag{A–10}$$

Also, the expression for $\varepsilon^{p\ell}_{\bar{1},\underline{2}}$ is indicated in in (A–11).

$$\varepsilon^{p\ell}_{\bar{1},\underline{2}} \quad = \quad rI_2\left[\frac{\beta_2-\gamma}{\beta_2\left(D^{\gamma}_{\underline{2}}-D^{\gamma}_{\bar{1}}\right)}\right]^{\frac{1}{\gamma}} \tag{A–11}$$

Consequently

$$\begin{aligned} \varepsilon^{p\ell}_{\bar{1},\underline{2}} > \varepsilon^{f}_{0,1} \quad & \Leftrightarrow \quad rI_2\left(\frac{\beta_2-\gamma}{\beta_2}\right)^{\frac{1}{\gamma}}\left(\frac{1}{D^{\gamma}_{\underline{2}}-D^{\gamma}_{\underline{1}}}\right)^{\frac{1}{\gamma}} > \frac{rI_1}{D_{\bar{1}}}\left(\frac{\beta_2-\gamma}{\beta_2}\right)^{\frac{1}{\gamma}} \\ & \Leftrightarrow \quad D^{\gamma}_{\bar{1}}I^{\gamma}_2 > I^{\gamma}_1\left(D^{\gamma}_{\underline{2}}-D^{\gamma}_{\underline{1}}\right) \\ & \Leftrightarrow \quad \frac{D^{\gamma}_{\bar{1}}}{I^{\gamma}_1} > \frac{D^{\gamma}_{\underline{2}}}{I^{\gamma}_1+I^{\gamma}_2} \end{aligned} \tag{A–12}$$

which holds according to Corollary 1. □

Proposition 2 $\varepsilon^{p\ell}_{\bar{1},\underline{2}} < rI_2\left[\frac{\beta_2-\gamma}{\beta_2\left(D^{\gamma}_{\underline{2}}-D^{\gamma}_{\underline{1}}\right)}\right]^{\frac{1}{\gamma}} = \varepsilon^{m}_{1,2}$.

Proof: Since the only difference between a monopolist and a follower is the demand coefficient (Dixit & Pindyck, 1994), we can use (4) to determine $\varepsilon^{m}_{1,2}$ by replacing $D_{\bar{i}}$ with $D_{\underline{i}}$, $i = 1, 2$. Based on the analytical expression of $\varepsilon^{p\ell}_{\bar{1},\underline{2}}$ and $\varepsilon^{m}_{1,2}$, we have:

$$\varepsilon^{p\ell}_{\bar{1},\underline{2}} = rI_2\left[\frac{\beta_2-\gamma}{\beta_2\left(D^{\gamma}_{\underline{2}}-D^{\gamma}_{\bar{1}}\right)}\right]^{\frac{1}{\gamma}} < rI_2\left[\frac{\beta_2-\gamma}{\beta_2\left(D^{\gamma}_{\underline{2}}-D^{\gamma}_{\underline{1}}\right)}\right]^{\frac{1}{\gamma}} = \varepsilon^{m}_{1,2} \quad \Leftrightarrow \quad D_{\underline{1}} > D_{\bar{1}} \tag{A–13}$$

□

Proposition 3 *The proprietary leader's required investment threshold for the first technology is independent of λ.*

Proof: We can alternatively express the first branch in (24) as in (A–14) to investigate the impact of λ.

$$F^{p\ell}_{0,\underline{1}}(E) \quad = \quad \max_{E^{p\ell}_{0,\underline{1}}>E}\left(\frac{E}{E^{p\ell}_{0,\underline{1}}}\right)^{\beta_1}\Phi^{p\ell}_{\underline{1}}\left(E^{p\ell}_{0,\underline{1}}\right) \tag{A–14}$$

The optimal investment rule is found by applying the first-order necessary conditions to (A–14) with respect to $E_{0,\underline{1}}^{p\ell}$ and is outlined in (A–15), where the marginal benefit (MB) of delaying the investment is equal to the marginal cost (MC).

$$\gamma \Upsilon U\left(D_{\underline{1}}\right) \varepsilon_{0,\underline{1}}^{p\ell\ \gamma-1}+\frac{\beta_1 U\left(rI_1\right)}{\varepsilon_{0,\underline{1}}^{p\ell}\rho}-\beta_1 A_{\underline{1}}^{p\ell}\varepsilon_{0,\underline{1}}^{p\ell\ \beta_1-1}=\beta_1 \Upsilon U\left(D_{\underline{1}}\right) \varepsilon_{0,\underline{1}}^{p\ell\ \gamma-1}-\beta_1 A_{\underline{1}}^{p\ell}\varepsilon_{0,\underline{1}}^{p\ell\ \beta_1-1} \quad \text{(A–15)}$$

The first term on the left-hand side reflects the extra benefit from allowing the project to start at a higher price threshold and the second term is the increase in MB form postponing the investment cost. Similarly, the first term on the right-hand side represents the opportunity cost of forgone cash flows. The third term on the left-hand side represents the MB of postponing the loss in value due to the follower's entry, while the second term on the right-hand side is the MC from waiting, and, thus, incurring a greater loss in value when the follower enters. These opposing forces cancel each other, because the follower will enter before the second technology arrives. Consequently, the leader's investment threshold in the first technology does not impact her possible monopoly profits in the second technology, and, thus, the leader adopts a myopic investment strategy. □

Proposition 4 *The optimal investment threshold of the non-proprietary leader for the first technology is* $\max\left\{\varepsilon_{0,1}^{f},\varepsilon_{\overline{1},\underline{2}}^{n\ell}\right\}$, *where* $\varepsilon_{\overline{1},\underline{2}}^{n\ell}$ *satisfies the condition* $F_{1,2}^{f}(E)=\Phi_{\underline{2}}^{n\ell}(E)$.

Proof: Ideally, the leader would invest at the threshold that maximises her expected utility, i.e., at $\varepsilon_{\overline{1},\underline{2}}^{p\ell}$. However, the threat of pre-emption lowers the adoption threshold to $\varepsilon_{\overline{1},\underline{2}}^{n\ell}$. The price threshold at which the firm is indifferent between being the leader or the follower is defined implicitly through the equality $F_{1,2}^{f}(E)=\Phi_{\underline{2}}^{n\ell}(E)$. Given that the follower adopts a compulsive strategy, there are two possible scenarios:

i. $\varepsilon_{0,1}^{f}>\varepsilon_{\overline{1},\underline{2}}^{n\ell}$

ii. $\varepsilon_{0,1}^{f}<\varepsilon_{\overline{1},\underline{2}}^{n\ell}$

In the former scenario, the threat of pre-emption is eliminated, however, in the latter the threat still exists. If $\varepsilon_{0,1}^{f}>\varepsilon_{\overline{1},\underline{2}}^{n\ell}$, then the leader will invest at $\varepsilon_{0,1}^{f}$, since $F_{\overline{1},\underline{2}}^{n\ell}\left(\varepsilon_{0,1}^{f}\right)>F_{\overline{1},\underline{2}}^{n\ell}\left(\varepsilon_{\overline{1},\underline{2}}^{n\ell}\right)$. By contrast, if $\varepsilon_{0,1}^{f}<\varepsilon_{\overline{1},\underline{2}}^{n\ell}$, then the leader will have to pre-empt the first technology at $\varepsilon_{\overline{1},\underline{2}}^{n\ell}$. Consequently, the optimal investment threshold is $\max\left\{\varepsilon_{0,1}^{f},\varepsilon_{\overline{1},\underline{2}}^{n\ell}\right\}$. □

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Chapter 3

Optimal Risk Adoption and Capacity Investment in Disruptive Innovations

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Abstract

Disruptive innovations often formulate new market regimes and create incentives to abandon existing, less attractive ones. However, the decision to abandon an established market regime depends not only on market factors, such as economic and technological uncertainty, but also on attitudes towards risk. Although risk aversion typically raises the incentive to postpone (accelerate) investment (abandonment) by decreasing the expected utility of a project, the impact of risk aversion becomes more complex when a firm has discretion over both the time of investment and the size of a project within a regime-switching economic environment. Taking into account attitudes towards risk and the random arrival of innovations, we develop a utility-based, regime-switching framework for analysing how a private firm may choose to initially invest in an existing market-regime and subsequently abandon it in order to enter a new one, when it has discretion over investment timing and project scale. Results indicate that increasing risk aversion and technological uncertainty hasten investment by decreasing the amount of installed capacity, and that the likely arrival of innovations may in fact reduce the relative loss in project value in the absence of managerial discretion over project scale. Also, we show how the incentive to abandon an existing (new) regime may increase (decrease) depending on the sensitivity of a demand function to capacity expansion.

Keywords: investment analysis, regime switching, risk aversion

3.1 Introduction

The rapid pace of innovation and intense research and development (R&D) activities in many industries, such as renewable energy (RE), transportation and IT, have resulted in an increasing number of disruptive technologies. For example, electric cars were considerably more expensive eight years ago, yet are now selling at an exponential rate (The Guardian, 2017). This is expected to drive down the price of batteries that hold the key to unleashing new levels of green growth. However, at the same time, this will also limit the demand for products designed for cars running on fossil fuel (The Financial Times, 2017). The electricity production process has also been revolutionized. For example, 25 years ago, a wind turbine could supply a handful of normal homes, but such turbines have now been supplanted by others, with superior technology and size, which each supplies hundreds of homes (The Financial Times, 2018a). Consequently, timely technology switch is key for corporate strategy and may in fact determine the success or failure of a company as a whole. Yet, risk aversion often raises the reluctance to abandon an existing market regime in order to enter a new one. In fact, this decision becomes particularly complex when it entails irreversible capacity investment in the light of technological and demand uncertainty (The Economist, 2012).

Indeed, a particular challenge associated with disruptive technologies is that they often entail idiosyncratic risk that cannot be diversified. Consequently, the assumption of hedging via spanning assets breaks down as the underlying commodities are not likely to be freely traded, and, thus, enable the construction of a replicating portfolio. Hence, risk-neutral valuation may not be possible, and analytical methods for capital budgeting and risk assessment must allow for utility functions to capture risk-averse agents. This is particularly crucial, since market-based approaches to electricity decarbonization rely upon incentives, and, therefore, their effectiveness is as much a function of behavioural as it is of fundamental economics.

Despite the growing literature in the area of investment under uncertainty and risk aversion (Alvarez & Stenbacka, 2004; Chronopoulos & Lumbreras 2017), models that address investment and capacity sizing decisions within a regime-switching economic environment typically ignore attitudes towards risk. Therefore, we consider a risk-averse firm that, under the risk of a technological disruption, must decide to abandon an existing market regime in order to enter a new one. An early transition to the new regime allows the firm to limit exposure to downside risk, while, on the other hand, waiting enables the firm to observe the market and make a more informed irreversible capacity investment decision. By analysing this trade-off, our contribution to the existing literature is threefold: **i.** we develop a regime-switching, utility-based framework for sequential investment under uncertainty and operational flexibility in order to derive optimal investment and operational thresholds; **ii.** we show how attitudes towards risk interact with economic and technological uncertainty to affect not only the optimal regime-switching strategy, but also optimal investment and capacity-sizing decisions; and **iii.** we provide managerial insights for investment and operational decisions based on analytical and numerical results.

We proceed by discussing some related work in Section 3.2 and introduce assumptions and notation in Section 3.3. The problem of investment in a new regime is addressed in Section 3.4.1. In Section 3.4.2, we tackle the problem of abandoning an old regime in order to invest in a new one, and, in Section 3.4.3, we analyse the problem of investment under regime switching. Section 3.5 provides numerical examples for each case and Section 3.6 concludes, offering directions for future research.

3.2 Literature Review

Examples of early work in the area of investment under uncertainty that allow for discretion over project scale include Dangl (1999) and Pindyck (1988). The former, tackles the problem of investing in a project with continuously scalable capacity, and, the latter, considers a firm that expands its capital stock incrementally with operational flexibility. Dangl (1999) finds that demand uncertainty raises the optimal capacity and makes waiting the optimal strategy even when demand is high. Allowing for discrete capacity sizing, Dixit (1993) develops a model for choosing among mutually exclusive projects under uncertainty. The decision rule involves **i.** ranking the projects by capacity; **ii.** finding the investment threshold for each project; and **iii.** selecting the largest project for which the optimal threshold exceeds the current price. Décamps *et al.* (2006) extend Dixit (1993) by identifying a second waiting region around the indifference point between the net present values (NPVs) of two projects. Within this region, a firm will select the smaller (larger) project if the price drops (increases) sufficiently. Applications of these models to RE investment are presented in (Bøckman *et al.*, 2008; Fleten *et al.*, 2007), while policy-oriented applications are described in (Boomsma *et al.*, 2012; Siddiqui & Fleten, 2010). However, these models are based on the assumption of risk neutrality, and, therefore, attitudes towards risk are not taken into account.

Within the context of investment in disruptive technologies, Hagspiel *et al.* (2013) consider a risk-neutral, price-setting firm that faces both technological change and a declining profit stream. The firm can abandon an established technology by either exiting the corresponding market regime permanently or by investing in a new one. However, the new regime is assumed to be available, and, therefore, technological uncertainty is not considered. Their results indicate that with (without) discretion over capacity, the relationship between the optimal investment threshold and price uncertainty is monotonic (non-monotonic). In addition, the firm abandons the current regime more easily when economic uncertainty is high and when the market for the innovative product is very attractive. Considering the case of electric vehicles, Lukas *et al.* (2013) study the impact of uncertainty over technological life cycle on the decision to invest and scale the capacity of a project under risk neutrality. Results indicate that the investment threshold follows an S-curve, segmented with respect to the optimal capacity choice, which depends on the degree of product life-cycle uncertainty. Filomena *et al.* (2014) analyse the problem of technology selection and capacity investment for electricity generation in a competitive environment under uncertainty, considering: **i.** the portfolio of technologies; **ii.** each technology's capacity; and **iii.** the technology's production level for every scenario. Results indicate that portfolio diversification arises even with risk-neutral firms and technologies with different cost expectations.

Examples of recent work that attempts to reconcile risk aversion with real options models include Henderson & Hobson (2002), who extend Merton (1969) by introducing market incompleteness via a second risky asset on which no trading is allowed. In the same line of work, Henderson (2007) assumes that part of the uncertainty associated with the investment payoff can be hedged via a risky asset that is correlated with the investment payoff, yet the remaining risk is idiosyncratic. Results indicate that higher risk aversion or lower correlation between the project value and the hedging asset lowers both the option value and the investment threshold. Using the analytical framework of Karatzas & Shreve (1999), Hugonnier & Morellec (2013) account for a decision-maker's risk aversion via a constant relative risk aversion (CRRA) utility function. Thus, they show that risk aversion erodes the value of a project and raises the required investment threshold. Extensions of Hugonnier & Morellec (2013) that allow for operational flexibility and discretion over project scale are presented in Chronopoulos *et al.* (2011) and Chronopoulos *et al.* (2013).

Recent examples of regime-switching frameworks for investment under uncertainty that allow for attitudes towards risk include Alvarez & Stenbacka (2001, 2004). These frameworks assume that the structure of the underlying stochastic process may be affected by either a change in volatility while holding the drift fixed (Alvarez & Stenbacka, 2004) or a change in drift while holding the volatility constant (Alvarez & Stenbacka, 2001). Their results challenge those from traditional real options literature by showing how increasing uncertainty does not necessarily decelerate investment. An extention of (Alvarez & Stenbacka, 2001, 2004) that considers a structural change of the underlying stochastic process in terms of both the drift and the volatility is presented in Matomäki (2013). More recently, Chronopoulos & Lumbreras (2017) develop a regime-switching model for investment under uncertainty, yet assume an exogenous price process and ignore discretion over project scale. They find that, depending on market-regime asymmetry, greater risk aversion and price uncertainty in a new regime may accelerate regime switching.

Since attitudes towards risk and discretion over both investment timing and project scale may impact the optimal investment policy significantly, we explore their interaction and combined impact in this paper. The scope of our model does not include the option to choose between alternative market regimes (technologies) (Grenadier & Weiss, 1997), but emphasises on how demand and technological uncertainty interact to affect sequential investment decisions. Thus, we assume that the firm enters (invests in) each regime that becomes available (compulsive strategy) and ignores the possibility to wait for both regimes to arrive in order to have the option to adopt either the old one (laggard strategy) or the new one directly (leapfrog strategy).

By incorporating attitudes towards risk via a hyperbolic absolute risk aversion (HARA) utility function, we find that the interaction between demand and technological uncertainty is rather strong and that market regime asymmetry can impact the decision to abandon an existing regime in order to switch to a new one, considerably. Specifically, we find that greater (lower) economic uncertainty in a new market regime raises (reduces) the value of the option to invest in it and increases (decreases) the incentive to abandon a mature market regime. However, greater economic uncertainty in the mature regime raises the value of waiting, thereby increasing the incentive to postpone abandonment and delaying entry in the new regime. Furthermore, we show how greater likelihood of technological innovations lowers the relative loss in value due to an incorrect capacity choice.

3.3 Assumptions and Notation

We consider a firm with a perpetual option to invest in a project of infinite lifetime facing demand and technological uncertainty. The demand shock process $\left\{\Theta_t^{(k)}, t \geq 0\right\}$, where t denotes time and $k = 1, 2, 3$ denotes the different market regimes, follows a Markov-modulated geometric Brownian motion (GBM) that is described in (1). We denote by μ_k the annual growth rate, by σ_k the annual volatility and by dZ_t the increment of the standard Brownian motion. Also, $\rho > \mu_k$ denotes the subjective discount while r is the risk-free rate.

$$d\Theta_t^{(k)} = \mu_k \Theta_t^{(k)} dt + \sigma_k \Theta_t^{(k)} dZ_t, \quad \Theta_0^{(k)} \equiv \Theta > 0 \tag{1}$$

The firm has discretion over both the time of investment and the size of the project and faces either a multiplicative (m) or an iso-elastic (iso) inverse demand function. Consequently, the price process $\left\{P_{t,d}^{(k)}, t \geq 0\right\}$, where $d = \{iso, m\}$, depends on not only the demand shock process but also on the market output, $Q_{t,d}^{(k)}$. This relationship is expressed in (2), where $\mathcal{H}(\cdot)$ is a continuous, decreasing function of the capacity.

$$P_{t,d}^{(k)}\left(\Theta_t^{(k)}, Q_{t,d}^{(k)}\right) = \Theta_t^{(k)} \mathcal{H}\left(Q_{t,d}^{(k)}\right) \tag{2}$$

As indicated in (3), η and ξ are positive constants determining how responsive the price is to capacity changes. For example, $1/\xi$ is the demand elasticity, while η impacts prices linearly.

$$\mathcal{H}\left(Q_{t,d}^{(k)}\right) = \begin{cases} 1 - \eta Q_{t,m}^{(k)} & , d = m \\ Q_{t,iso}^{(k)^{-\xi}} & , d = iso \end{cases} \tag{3}$$

Technological innovations follow a Poisson process $\{N_t, t \geq 0\}$ with intensity ν. Hence, the probability of an innovation occurring within an infinitesimal time interval dt is νdt. Once an innovation takes place, two things happen: **i.** the market parameters for the existing technology switch from regime 1 to regime 2; and **ii.** market regime 3 emerges. We assume that the emergence of a new market regime reduces the attractiveness of the existing one so that $\mu_3 > \mu_1 > \mu_2$. Furthermore, the multiplicative inverse demand function has a fixed price intercept, i.e. the market has limited size (Boonman & Hagspiel, 2014). A fixed market size is more pertinent to a declining market (regime 2), whereas the iso-elastic demand function has no capacity limit and is consequently more suitable for an expanding market (regimes 1 and 3). For example, the demand for RE, such as wind and solar, has been expanding in tandem with efficiency improvements. By contrast, nuclear energy capacity in Germany is capped and set to be phased out because RE is now a viable alternative (The Financial Times, 2018b). Also, the firm's preferences are described by a specific utility function taken from the HARA class of utility functions, namely a power-function with constant relative risk aversion indicated in (4). We assume that γ can be below 1, and, thus, examine the implications of risk-aversion but also $\gamma > 1$, i.e. risk-seeking behaviour. Note that standard economic theory deems risk-seeking behaviour unlikely (Pratt, 1964), yet we allow for risk-seeking behaviour to enable comparisons with Chronopoulos & Lumbreras (2017) and also to explore implications of such preferences.

$$U\left(P_{t,d}^{(k)}\right) = \frac{1}{\gamma} P_{t,d}^{(k)^{\gamma}} \tag{4}$$

The operating cost is constant and equal to c (\$/unit) for all regimes and the cost of abandoning the incumbent regime is fixed and denoted by E (\$). By contrast, the investment cost $I(\cdot)$ (\$) behaves as in (5), where $b^{(k)}$, $\lambda > 1$ are constants. Hence, $I(\cdot)$ is a convex function of the capacity. This implies that the marginal investment cost is increasing, and, as a result, this model is more suitable for describing projects that exhibit diseconomies of scale, e.g. alternative energy technologies (Bøckman *et al.*, 2008; Chronopoulos *et al.*, 2017; Iyer *et al.*, 2014; Siddiqui & Takashima, 2012). In line with Huisman & Kort (2015), we assume that the firm always produces up to capacity, $Q_d^{(k)}$, this is often called the "market clearance assumption" and arise when it is costly to ramp up and down capacity or commitments to workers and suppliers hinders temporary adjustments (Hagspiel *et al.*, 2013). For ease of exposition we set $I_d^{(k)} \equiv I\left(Q_d^{(k)}\right)$.

$$I\left(Q_d^{(k)}\right) = b^{(k)} Q_d^{(k)^{\lambda}} \tag{5}$$

Also, we let $i = 0, 1$ denote the state of a project (technology). For example, a technology is either active if $i = 1$ or abandoned if $i = 0$. We let $\tau_{i,d}^{(k)}$ denote the time of investment ($i = 1$) or abandonment ($i = 0$) of a technology in regime k, $\Theta_{i,d}^{(k)^*}$ denote the corresponding optimal investment threshold and $Q_d^{(k)^*}$ the optimal capacity. Also, $F_{i,d}^{(k)}(\cdot)$ is the maximised expected value of the option to invest in or abandon a regime and $\Phi_{i,d}^{(k)}(\cdot)$ is the maximised expected utility of the project. For example, the time of investment in regime 1 is denoted by $\tau_{1,d}^{(1)}$, the state variables by $\Theta_{1,d}^{(1)}$ and $Q_d^{(1)}$ and the corresponding optimal investment threshold and capacity by $\Theta_{1,d}^{(1)^*}$ and $Q_d^{(1)^*}$, respectively.

3.4 Model

3.4.1 Regime 3

The value function within each market regime is determined via backward induction, as in Chronopoulos & Lumbreras (2017). Therefore, we begin by assuming that, after having just exited the second regime, the firm is in an inactive state and considers investing in the third one. Following the same approach as Hugonnier & Morellec (2013) and Conejo *et al.* (2016), we decompose the cash flows of the project into disjoint time intervals, as in Figure 3.1. We assume that the capital required for realisation of the project is initially invested in a certificate of deposit and earns a risk-free rate (r) up to time $\tau_{1,d}^{(3)}$. At time $\tau_{1,d}^{(3)}$ the firm swaps the risk-free cash flow for the risky cash flow that the project generates and fixes the capacity of the project.

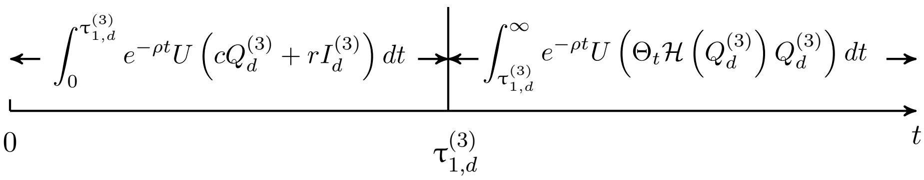

Figure 3.1: Irreversible investment in regime 3

The objective is to determine the investment policy that maximises the time-zero expected discounted utility of all the cash flows of the project. This is indicated in (6), where $\mathbb{E}_\Theta[\cdot]$ is the expectation operator conditional on the initial value of the demand shock parameter, Θ.

$$\mathbb{E}_\Theta\left[\int_0^{\tau_{1,d}^{(3)}} e^{-\rho t} U\left(cQ_d^{(3)} + rI_d^{(3)}\right) dt + \int_{\tau_{1,d}^{(3)}}^{\infty} e^{-\rho t} U\left(\Theta_t \mathcal{H}\left(Q_d^{(3)}\right) Q_d^{(3)}\right) dt\right] \tag{6}$$

Next, we decompose the first integral in (6) and rewrite it as in (7). Note that the first term in (7) is deterministic. Hence, the optimisation objective is reflected in the second term.

$$\int_0^{\infty} e^{-\rho t} U\left(cQ_d^{(3)} + rI_d^{(3)}\right) dt + \mathbb{E}_\Theta\left[\int_{\tau_{1,d}^{(3)}}^{\infty} e^{-\rho t}\left[U\left(\Theta_t \mathcal{H}\left(Q_d^{(3)}\right) Q_d^{(3)}\right) - U\left(cQ_d^{(3)} + rI_d^{(3)}\right)\right] dt\right] \tag{7}$$

Using the law of iterated expectations and the strong Markov property of the GBM, we can express the optimisation objective, i.e. the second term in (7), as in (8). The first term is the stochastic discount factor, $\mathbb{E}_\Theta\left[e^{-\rho\tau}\right] = \left(\frac{\Theta}{\Theta_\tau}\right)^\beta$ (Dixit & Pindyck, 1994), and the second term is the expected utility of the project's cash flows

$$\mathbb{E}_\Theta\left[e^{-\rho \tau_{1,d}^{(3)}}\right]\left[V_{1,d}^{(3)}\left(\Theta_{1,d}^{(3)}, Q_d^{(3)}\right) - \int_0^{\infty} e^{-\rho t} U\left(cQ_d^{(k)} + rI_d^{(k)}\right) dt\right] \tag{8}$$

where:

$$V_{1,d}^{(k)}(\Theta, Q_d) = \mathbb{E}_\Theta\left[\int_0^\infty e^{-\rho t}\left[U\left(\Theta_t \mathcal{H}\left(Q_d^{(k)}\right)Q_d^{(k)}\right)\right]dt\right] \tag{9}$$

Proposition 1 extends Theorem 9.18 of Karatzas & Shreve (1999) to allow for an inverse demand function in the analytical expression for the expected utility of a perpetual stream of cash flows when the demand shock parameter follows a Markov-modulated GBM. Note that, the contribution of Proposition 1 is two-fold: **i.** it facilitates insights on how attitudes towards risk impact the expected utility of a project within a regime-switching environment and **ii.** it enables the analysis of the feedback effect of capacity expansion on price under risk aversion.

Proposition 1. *The expected utility of a perpetual stream of cash flows* $P_{t,d}^{(k)}\left(\Theta_t^{(k)}, Q_d^{(k)}\right)Q_{t,d}^{(k)}$, *where* $\Theta_t^{(k)}$ *follows a Markov-modulated GBM is*

$$\mathbb{E}_\Theta\left[\int_0^\infty e^{-\rho t}U\left(\Theta_t^{(k)}\mathcal{H}\left(Q_d^{(k)}\right)Q_d^{(k)}\right)dt\right] = \mathcal{A}^{(k)}U\left(\Theta\mathcal{H}\left(Q_d^{(k)}\right)Q_d^{(k)}\right).$$

where

$$\mathcal{A}^{(k)} = \frac{\beta_{1k}\beta_{2k}}{\left(\rho+\nu\mathbb{1}_{k=1}\right)\left(\gamma-\beta_{1k}\right)\left(\gamma-\beta_{2k}\right)}$$

$\mathbb{1}_{k=1}$ *is an indicator function,* β_{ik} $i = 1, 2$ *and* $k = 1, 2, 3$, *are the roots of the quadratic* $\frac{1}{2}\sigma_k^2\beta\left(\beta-1\right)+\mu_k\beta-\left(\rho+\nu\mathbb{1}_{k=1}\right)=0$.

Using Proposition 1, we can write $V_{1,d}^{(3)}\left(\Theta, Q_d^{(3)}\right)$ as in (10).

$$V_{1,d}^{(3)}\left(\Theta, Q_d^{(3)}\right) = \mathcal{A}^{(3)}U\left[\Theta\mathcal{H}\left(Q_d^{(3)}\right)Q_d^{(3)}\right] \tag{10}$$

Thus, the firm's maximised value of investment opportunity can be expressed as in (11)

$$F_{1,d}^{(3)}(\Theta) = \max_{\Theta_{1,d}^{(3)}>\Theta}\left(\frac{\Theta}{\Theta_{1,d}^{(3)}}\right)^{\beta_{13}}\left[\Phi_{1,d}^{(3)}\left(\Theta_{1,d}^{(3)}\right)-\frac{1}{\rho}U\left(cQ_d^{(3)^*}+rI_d^{(3)^*}\right)\right] \tag{11}$$

where $\Phi_{1,d}^{(k)}\left(\Theta_{1,d}^{(k)}\right) \equiv V_{1,d}^{(k)}\left(\Theta_{1,d}^{(k)}, Q_d^{(k)^*}\right)$ is the expected utility of the revenues when the capacity is chosen optimally, i.e. $Q_d^{(3)^*}$ satisfies the condition for optimal capacity at investment, which is indicated in (12).

$$\frac{\partial}{\partial Q_d^{(3)}}\left[V_{1,d}^{(3)}\left(\Theta_{1,d}^{(3)}, Q_d^{(3)}\right)-\frac{1}{\rho}U\left(cQ_d^{(3)}+rI_d^{(3)}\right)\right]=0 \tag{12}$$

Equivalently, the value of the option to invest in regime 3 can be expressed as in (13). The top part on the right-hand side of (13) is the value of the option to invest and the bottom part is the maximised expected utility of the active project.

$$F_{1,d}^{(3)}(\Theta) = \begin{cases} A_{1,d}^{(3)}\Theta^{\beta_{11}} & ,\Theta < \Theta_{1,d}^{(3)^*} \\ \Phi_{1,d}^{(3)}(\Theta) - \frac{1}{\rho}U\left(cQ_d^{(3)^*} + rI_d^{(3)^*}\right) & ,\Theta \geq \Theta_{1,d}^{(3)^*} \end{cases} \tag{13}$$

By applying value-matching and smooth-pasting conditions to (13) together with the condition for optimal capacity choice at investment in (12) (see Hagspiel *et al.*, 2013), we obtain the analytical expression for $A_{1,d}^{(3)}$, $\Theta_{1,d}^{(3)^*}$ and $Q_d^{(3)^*}$. These are indicated in (14), (15) and (16), respectively. Note that the second-order sufficiency condition (SOSC) requires the objective function to be concave at $\Theta_{1,d}^{(3)^*}$, which is shown in Chronopoulos & Lumbreras (2017).

$$A_{1,d}^{(3)} = \left(\frac{1}{\Theta_1^{(3)^*}}\right)^{\beta_{11}} \left[\Phi_{1,d}^{(3)}\left(\Theta_1^{(3)^*}\right) - \frac{1}{\rho}U\left(cQ_d^{(3)^*} + rI_d^{(3)^*}\right)\right] \tag{14}$$

$$Q_d^{(3)^*} = \left[\frac{c}{\rho b}\frac{\gamma - \beta_{13}\xi}{\beta_{13}(\lambda+\xi-1) - \lambda\gamma}\right]^{\frac{1}{\lambda-1}} \tag{15}$$

$$\Theta_1^{(3)^*} = \left(cQ_d^{(3)^*\xi} + rbQ_d^{(3)^*\lambda+\xi-1}\right)\left(\frac{\beta_{23}-\gamma}{\beta_{23}}\right)^{\frac{1}{\gamma}} \tag{16}$$

Proposition 2 indicates that, due to the inverse demand function, the optimal capacity under immediate investment is always finite. This result is in contrast to earlier literature that assumes an exogenous price process. For example, Chronopoulos *et al.* (2013) assume an exogenous price and find that the amount of installed capacity becomes infinitely large as $\lambda \downarrow 1$.

Proposition 2. *The optimal capacity under a "now-or-never" investment opportunity is finite for all* $\lambda \geq 1$.

Also, Proposition 3 indicates how price sensitivity to capacity expansion may impact the optimal investment threshold and optimal capacity depending on the relationship between market parameters. Intuitively, a lower ξ leads to greater installed capacity, since the market is more attractive, i.e. an additional unit of capacity impacts prices less when ξ is low. This is in line with Boonman & Hagspiel (2014) who find that ξ has to be small, otherwise the first market entrant will not install any capacity.

Proposition 3. *The optimal capacity and optimal investment threshold decrease in* ξ *iff* $\beta_{13} > \frac{\gamma(\lambda+1)}{\lambda+2\xi-1}$.

Proposition 4 indicates the condition under which the optimal capacity is smaller under an iso-elastic inverse demand function than a multiplicative inverse demand function. In order to ensure that it is more lucrative to operate under an iso-elastic inverse demand function than a multiplicative, which we assumed in Section 3.3, it is reasonable to assume that Proposition 4 holds.

Proposition 4. $Q_{iso}^{(3)^*} > Q_m^{(3)^*}$ *iff* $\frac{1-\eta Q_m^{(3)^*}}{1-2\eta Q_m^{(3)^*}} > \frac{1}{1-\xi}$.

3.4.2 Regime 2

Here, we assume that the firm is active in regime 2 and that it holds an embedded option to abandon it in order to invest in regime 3. As the first term in the top part of (17) indicates, prior to abandonment the firm receives the cash flows of the active project in regime 2 holding the option to abandon it following a sufficient decrease of the output price. As the first term of the bottom branch indicates, by abandoning regime 2 the firm salvages the operating cost reduced by the incremental cost of abandonment and obtains the option to invest in the third regime, as indicated in the second term. Notice that, since the project is active, the capacity has already been chosen optimally either in regime 1 or regime 2. Here, we assume that the capacity was fixed upon investment in regime 2, and is therefore denoted as $Q_d^{(2)^*}$.

$$\Phi_{1,d}^{(2)}(\Theta) = \begin{cases} \mathcal{A}^{(2)} U\left(\Theta \mathcal{H}\left(Q_d^{(2)^*}\right) Q_d^{(2)^*}\right) + B_{0,d}^{(2)} \Theta^{\beta_{22}} & ,\Theta > \Theta_{0,d}^{(2)^*} \\ \frac{U\left(cQ_d^{(2)^*} - rE\right)}{\rho} + F_{1,d}^{(3)}(\Theta) & ,\Theta \leq \Theta_{0,d}^{(2)^*} \end{cases} \tag{17}$$

Next, we step back and consider the problem of optimal investment in regime 2 with a single embedded abandonment option. The maximised value of the option to invest in regime 2 is described in the top part of (18). After investment, the firm receives the value of the active project, which is indicated in the bottom part. The first term reflects the expected profits obtained by operating in the second regime, while the second term reflects the operating and investment cost that the firm has to pay at investment.

$$F_{1,d}^{(2)}(\Theta) = \begin{cases} A_{1,d}^{(2)} \Theta^{\beta_{12}} & ,\Theta < \Theta_{1,d}^{(2)^*} \\ \Phi_{1,d}^{(2)}(\Theta) - U\left(cQ_d^{(2)^*} + rI_d^{(2)^*}\right) & ,\Theta \geq \Theta_{1,d}^{(2)^*} \end{cases} \tag{18}$$

Again, we apply the first-order necessary condition (FONC) with regards to capacity at the investment threshold as indicated in (19), together with the smooth-pasting and value-matching conditions between the branches of (18) and (17), we obtain a set of equations that can be solved numerically for $A_{1,d}^{(2)}$, $B_{0,d}^{(2)}$, $\Theta_{1,d}^{(2)^*}$, $\Theta_{0,d}^{(2)^*}$ and $Q_d^{(2)^*}$.

$$\frac{\partial}{\partial Q_d^{(2)}} \left[V_{1,d}^{(2)} \left(\Theta_{1,d}^{(2)^*}, Q_d^{(2)} \right) - U \left(c Q_d^{(2)} + r I_d^{(2)} \right) \right] = 0 \tag{19}$$

To emphasise on the value of the option to invest in the third regime, we also determine the value of the option to abandon the second regime permanently, which is indicated in (20). Notice that the absence of the option to switch to the third regime lowers the value in the bottom branch, and, in turn, the value of the option to invest in the first place.

$$\widetilde{\Phi}_{1,d}^{(2)}(\Theta) = \begin{cases} \mathcal{A}^{(2)} U \left(\Theta \mathcal{H} \left(\widetilde{Q}_d^{(2)^*} \right) \widetilde{Q}_d^{(2)^*} \right) + B_{0,d}^{(2)} \Theta^{\beta_{22}} & , \Theta > \widetilde{\Theta}_{0,d}^{(2)^*} \\ \frac{U\left(c\widetilde{Q}_d^{(2)^*} - rE \right)}{\rho} & , \Theta \leq \widetilde{\Theta}_{0,d}^{(2)^*} \end{cases} \tag{20}$$

Note that in the absence of the option to invest in regime 3 we can obtain an analytical expression for the optimal abandonment threshold. And for a given capacity, lower abandonment cost hastens abandonment, since it is less costly to exit the market. In contrast, lower operating costs delays abandonment, because it makes the current regime more lucrative to operate in. Furthermore, the derivative of the optimal abandonment threshold with respect to η under a multiplicative inverse demand function is always greater than zero. This occurs, since increasing η lowers the output price for a given capacity, which in turn reduces the attractiveness of the second regime and induces earlier abandonment.

$$\widetilde{\Theta}_{0,d}^{(2)^*} = \left(\frac{\beta_{12} - \gamma}{\beta_{12}} \right)^{\frac{1}{\gamma}} \frac{c\widetilde{Q}_d^{(2)^*} - rE}{\mathcal{H}\left(\widetilde{Q}_d^{(2)^*} \right) \widetilde{Q}_d^{(2)^*}}, \quad and, \quad \frac{\partial \widetilde{\Theta}_{0,m}^{(2)^*}}{\partial \eta} = \left(\frac{\beta_{12} - \gamma}{\beta_{12}} \right)^{\frac{1}{\gamma}} \frac{\left(c\widetilde{Q}_m^{(2)^*} - rE \right)}{\left(1 - \eta \widetilde{Q}_m^{(2)^*} \right)^2} > 0 \tag{21}$$

3.4.3 Regime 1

The expected utility of the active project in the first regime is indicated in (22), where the first term on the right-hand side is the utility of the immediate profits and the second term is the expected utility in the continuation region. As the second term indicates, within an infinitesimal time interval dt, a regime switch may take place with probability νdt and the firm will receive the value function $\Phi_{1,d}^{(2)}(\Theta)$. By contrast, no innovation will occur with probability $1 - \nu dt$ and the firm will continue to hold the value function $\Phi_{1,d}^{(1)}(\Theta)$.

$$\begin{aligned} \Phi_{1,d}^{(1)}(\Theta) = \; & U\left(\Theta \mathcal{H}\left(Q_d^{(1)^*} \right) Q_d^{(1)^*} \right) dt + (1 - \rho dt) \Big\{ \nu dt \mathbb{E}_\Theta \left[\Phi_{1,d}^{(2)}(\Theta + d\Theta) \right] \\ & + (1 - \nu dt) \mathbb{E}_\Theta \left[\Phi_{1,d}^{(1)}(\Theta + d\Theta) \right] \Big\} \end{aligned} \tag{22}$$

Notice that (22) has to be solved for each expression of $\Phi_{1,d}^{(2)}(\Theta)$ indicated in (17). By expanding the right-hand side of (22) using Itô's lemma and solving the resulting ordinary differential equation, we obtain (23), where β_{11}, β_{21} are the roots of $\frac{1}{2}\sigma_1^2\beta(\beta-1)+\mu_1\beta-(\rho+\nu)=0$ and $\mathcal{C}_{ik}=-\nu/\left[\frac{1}{2}\sigma_1^2\beta_{ik}(\beta_{ik}-1)+\mu_1\beta_{ik}-(\rho+\nu)\right]$. The first term on the top part of (23) represents the expected utility of the profits from operating in the first regime, while the second term is the expected utility of the cash flows after abandonment of regime 2, adjusted for technological uncertainty. The third term is the option to invest in regime 3, adjusted via the fourth term because the second regime has yet to become available. The first term in the bottom part of (23) represents the expected profit from operating in the first regime that might suddenly switch to the second. The second term reflects abandonment option from regime two, that is adjusted by the final term in order to account for technological uncertainty. If $\nu=0$, then the second, third and fourth term in the upper branch are zero. Intuitively, $\nu=0$ implies that no regime switching will take place, and, as a result, the first technology will continue to operate in the first regime. By contrast, $\lim_{\nu\to\infty}\Phi_{1,d}^{(1)}(\Theta)=\Phi_{1,d}^{(2)}(\Theta)$ since $\lim_{\nu\to\infty}\nu\mathcal{A}^{(1)}=1$ and $\lim_{\nu\to\infty}\mathcal{A}^{(1)}=0$.

$$\Phi_{1,d}^{(1)}(\Theta)=\begin{cases}\mathcal{A}^{(1)}U\left(\Theta\mathcal{H}\left(Q_d^{(1)^*}\right)Q_d^{(1)^*}\right)+\frac{\nu U\left(cQ^{(1)^*}-rE\right)}{\rho(\nu+\rho)} & \\ +\mathcal{C}_{13}F_{1,d}^{(3)}(\Theta)+A_{1,d}^{(1)}\Theta^{\beta_{11}} & ,\Theta\leq\Theta_{0,d}^{(1)^*}\\ \left[1+\nu\mathcal{A}^{(2)}\right]\mathcal{A}^{(1)}U\left(\Theta\mathcal{H}\left(Q_d^{(1)^*}\right)Q_d^{(1)^*}\right) & \\ +\mathcal{C}_{22}B_{0,d}^{(2)}\Theta^{\beta_{22}}+B_{0,d}^{(1)}\Theta^{\beta_{21}} & ,\Theta>\Theta_{0,d}^{(1)^*}\end{cases}\tag{23}$$

Next, the dynamics of the option to invest in the first regime are described in (24). The first term on the right-hand side of (24) indicates that, while waiting to invest in the first regime, an innovation may take place with probability νdt and the firm will receive the value function $F_{1,d}^{(2)}(\Theta)$. By contrast, with probability $1-\nu dt$ no innovation will take place and the firm will continue to hold the value function $F_{1,d}^{(1)}(\Theta)$.

$$F_{1,d}^{(1)}(\Theta)\quad=\quad(1-\rho dt)\left\{\nu dt\mathbb{E}_\Theta\left[F_{1,d}^{(2)}(\Theta+d\Theta)\right]+(1-\nu dt)\,\mathbb{E}\left[F_{1,d}^{(1)}(\Theta+d\Theta)\right]\right\}\tag{24}$$

By expanding the right-hand side of (24) using Itô's lemma we obtain (25), which must be solved together with (26), i.e. the differential equation for the value of the option to abandon in the second regime, where $\mathcal{L}$ is the differential operator, i.e. $\mathcal{L} = \frac{1}{2}\sigma^2\Theta^2\frac{d^2}{d\Theta^2} + \mu\Theta\frac{d}{d\Theta}$.

$$\begin{aligned} \left[\mathcal{L} - (\rho + \nu)\right] F_{1,d}^{(1)}(\Theta) + \nu F_{1,d}^{(2)}(\Theta) &= 0 \qquad (25) \\ \left[\mathcal{L} - \rho\right] F_{1,d}^{(2)}(\theta) &= 0 \qquad (26) \end{aligned}$$

Hence, the value of the option to invest in regime 1 is obtained by solving (25)-(26) and is described in (27)

$$F_{1,d}^{(1)}(\Theta) = \begin{cases} A_1^{(1)}\Theta^{\beta_{11}} + \mathcal{C}_{12}A_{1,d}^{(2)}\Theta^{\beta_{12}} & ,\Theta < \Theta_{1,d}^{(1)^*} \\ \Phi_{1,d}^{(1)}(\Theta) - U\left(cQ_d^{(1)^*} + rI_d^{(1)^*}\right) & ,\Theta \geq \Theta_{1,d}^{(1)^*} \end{cases} \qquad (27)$$

Next, $A_{1,d}^{(1)}$, $B_{0,d}^{(1)}$, $A_{1,d}^{(1)}$, $B_{0,d}^{(2)}$, $Q_d^{(1)^*}$, $\Theta_{1,d}^{(1)^*}$ and $\Theta_{0,d}^{(1)^*}$ are obtained numerically via value-matching and smooth-pasting conditions between the two branches of (17), (23) and (27) together with the FONC as indicated in (28)

$$\frac{\partial}{\partial Q_d^{(1)}}\left[V_{1,d}^{(1)}\left(\Theta_{1,d}^{(1)^*}, Q_d^{(1)}\right) - U\left(cQ_d^{(1)} + rI_d^{(1)}\right)\right] = 0 \qquad (28)$$

3.5 Numerical Examples

3.5.1 Regime 3

For the numerical examples, we assume the following parameter values: $\eta = 0.1$, $\xi = 0.1$, $\lambda = 3$, $b_1 = b_2 = 250$, $b_3 = 500$, $r = \rho = 7\%$, $\sigma_k \in [0.15, 0.2]$, $\gamma \in [0.9, 1.1]$, $\mu_1 = 0.5\%$, $\mu_2 = -0.5\%$, $\mu_3 = 1\%$, $\nu = 1\%$, $\Theta = 30$, $E = 200$ and $c = 50$. In line with Chronopoulos & Siddiqui (2015), we assume that a new regime (technology) is more attractive in that it exhibits a higher growth rate compared to the incumbent one, yet entry entails a greater capital expenditure. In terms of context, a firm may have a plot of land that will be used to develop a power generation facility and that the investment decision is divided in two steps. In step one, the firm develops this property using an existing technology holding an embedded option to adopt a new technology if the price increases. However, this requires not only an additional investment cost, since the new technology is more capital intensive, but also a cost for decommissioning the old technology. Furthermore, we consider an iso-elastic inverse demand function in regime 1 and 3, but a multiplicative one in regime 2. This allows us to not only investigate the impact of lower growth rate and greater economic uncertainty in a declining market (regime 2), but also how a market with less potential impacts the decision to abandon.

The left panel in Figure 3.2 illustrates the option and project value as well as the maximised NPV for $\mu_3 = 1\%, 2\%$, while the right panel illustrates the impact of γ on $\Theta_{1,iso}^{(3)^*}$. Notice that an increase in μ_3 raises the attractiveness of the third regime. In turn, this raises the incentive to install a bigger project, thereby increasing the required investment threshold. In the right panel, we investigate the impact of different risk preferences, where $\gamma < 1$ implies risk aversion, $\gamma = 1$ risk-neutrality and $\gamma > 1$ risk-seeking behaviour. As the right panel illustrates, greater risk-aversion, reflected in smaller values of γ, accelerates investment by increasing the firm's incentive to build a smaller project and reduce exposure to downside risk. Also, increasing uncertainty postpones investment by raising the opportunity cost of investment, and, in turn, the value of waiting.

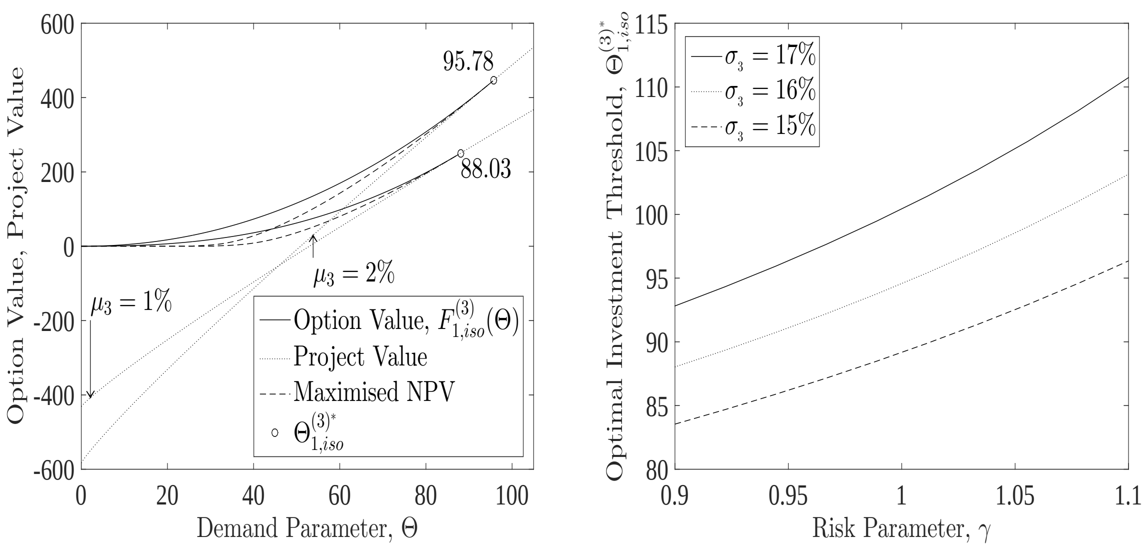

Figure 3.2: Impact of γ on option and project value for $\sigma_3 = 0.16$ (left panel) and optimal investment threshold versus γ (right panel).

In line with Figure 3.2, the left panel of Figure 3.3 indicates that it is optimal to invests in greater capacity when risk aversion decreases. Also, greater demand uncertainty increases the optimal capacity by raising the required investment threshold. By contrast, as the right panel illustrates, a higher ξ increases the price sensitivity to capacity expansion, and leads to lower installed capacity (right panel). Although the firm reduces forgone revenue by investing earlier and thereby increases the project value, it does not offset the feedback effect that capacity expansion has on the output price.

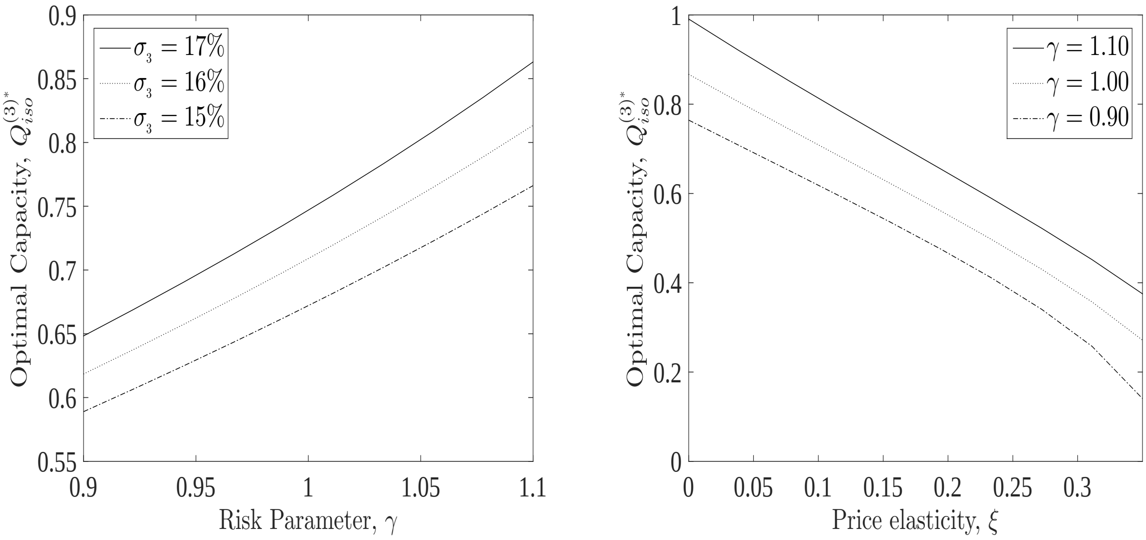

Figure 3.3: Optimal capacity versus γ (left panel) for $\xi = 0.1$ and versus ξ with $\sigma_3 = 0.16$ (right panel).

3.5.2 Regime 2

In the left panel of Figure 3.4, we hold uncertainty in regime 2 fixed and change uncertainty in regime 3 and vice versa. Doing so, we identify two effects: **i.** Greater (lower) uncertainty in regime 3 raises (reduces) the value of the embedded option to switch regimes and invest in the new technology and increases (decreases) the incentive to abandon regime 2; and **ii.** Greater uncertainty in regime 2 raises the value of waiting, thereby increasing the incentive to postpone abandonment and delaying entry in regime 3. Regarding the latter effect, the firm would not want to make an irreversibly decision and abandon regime 2 permanently due to a temporary downturn, which is more likely to happen when uncertainty is high. Interestingly, the left panel also indicates that increasing risk aversion postpones abandonment by lowering the required abandonment threshold. This seemingly counter-intuitive result happens because greater risk aversion lowers the amount of installed capacity, and, in turn, the expected utility of the operating cost. In effect, this reduces exposure to downside risk and increases the incentive to abandon the project less easily. The right panel indicates that increasing ξ hastens abandonment, this happens because the firm invests in less capacity but earlier in the third regime, which necessitates an earlier abandonment of the second regime. Similarly, η induces the firm to abandon the second regime earlier. This implies that the more responsive the inverse demand function in regime 2 is to capacity expansion, the greater is the incentive to abandon regime 2.

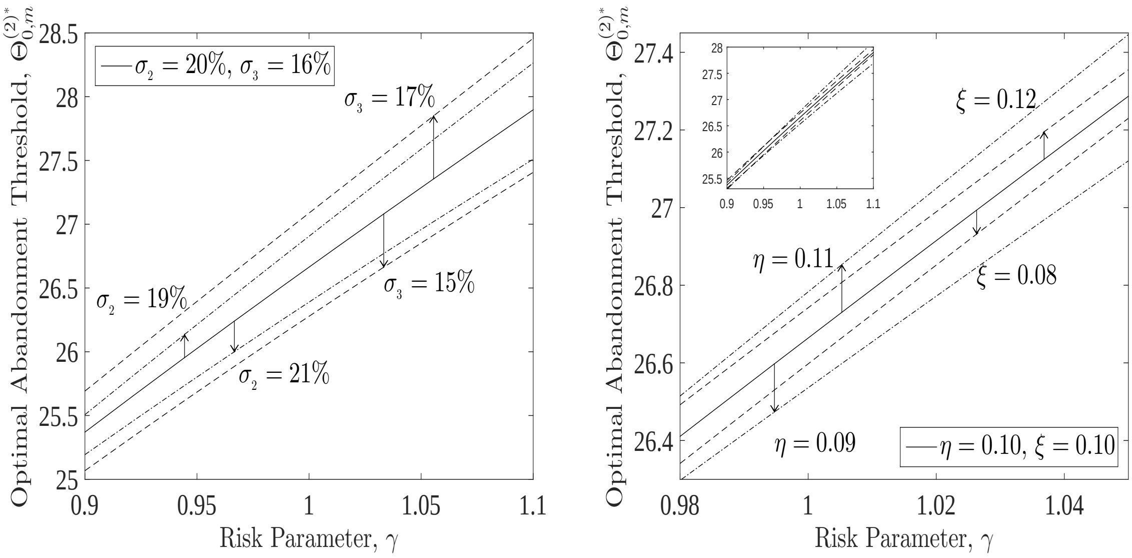

Figure 3.4: Optimal abandonment threshold versus σ_2 and σ_3 (left panel) and η and ξ (right panel) for $\sigma_2 = 0.20$ and $\sigma_3 = 0.16$.

3.5.3 Regime 1

Figure 3.5 illustrates the impact of ν and γ on the optimal investment threshold (left panel) and the optimal capacity (right panel). Interestingly, as the left panel indicates, greater likelihood of regime switching raises a firm's incentive to invest in the first regime. However, according to the right panel, earlier investment results in the installation of a smaller project. This happens because the emergence of a new market regime reduces the attractiveness of the incumbent one (regime 1), thereby raising the incentive to invest sooner in a smaller project in regime 1 while market conditions are still favourable. Intuitively, the firm would not want to commit to a project of large capacity that is based on a technology that will soon become obsolete.

Figure 3.6 illustrates the impact of ν and σ_1 on the relative loss in option value due to fixed capacity, which is indicated in (29).

$$\frac{F_{1,d}^{(1)}\left(\Theta, Q_d^{(1)^*}\right) - F_{1,d}^{(1)}\left(\Theta, Q_d^{(1)}\right)}{F_{1,d}^{(1)}\left(\Theta, Q_d^{(1)^*}\right)} \tag{29}$$

The left panel in Figure 3.6 indicates under an iso-elastic inverse demand function that, as $Q_{iso}^{(1)}$ increases, the relative loss in option value diminishes when $Q_{iso}^{(1)} < Q_{iso}^{(1)^*}$ becomes zero for

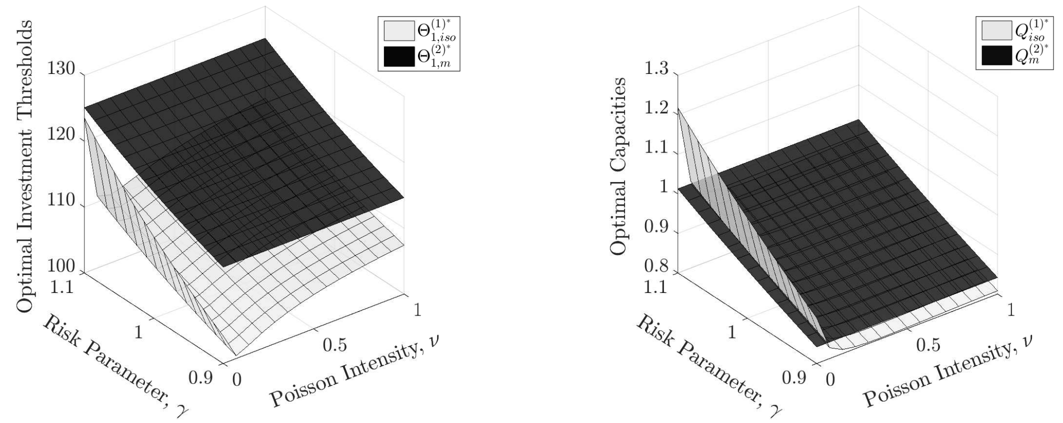

Figure 3.5: Impact of ν, γ and ξ on optimal investment threshold (left panel) and optimal capacity (right panel) in regime 1 for $\sigma_1 = 0.19, \sigma_2 = 0.20$ and $\sigma_3 = 0.16$.

$Q_{iso}^{(1)} = Q_{iso}^{(1)^*}$ and increases if $Q_{iso}^{(1)} > Q_{iso}^{(1)^*}$. This implies that increasing demand uncertainty raises (lowers) the relative loss in option value when the amount of installed capacity is lower (greater) than the optimal one. By contrast, greater likelihood of innovation lowers the amount of installed capacity and reduces (increases) the relative loss in option value when the size of the project is lower (greater) than the optimal level. This result reveals an important feature of capacity choice, e.g. industries which are likely to be disrupted may benefit from a more conservative investment strategy to avoid overinvestment in capacity.

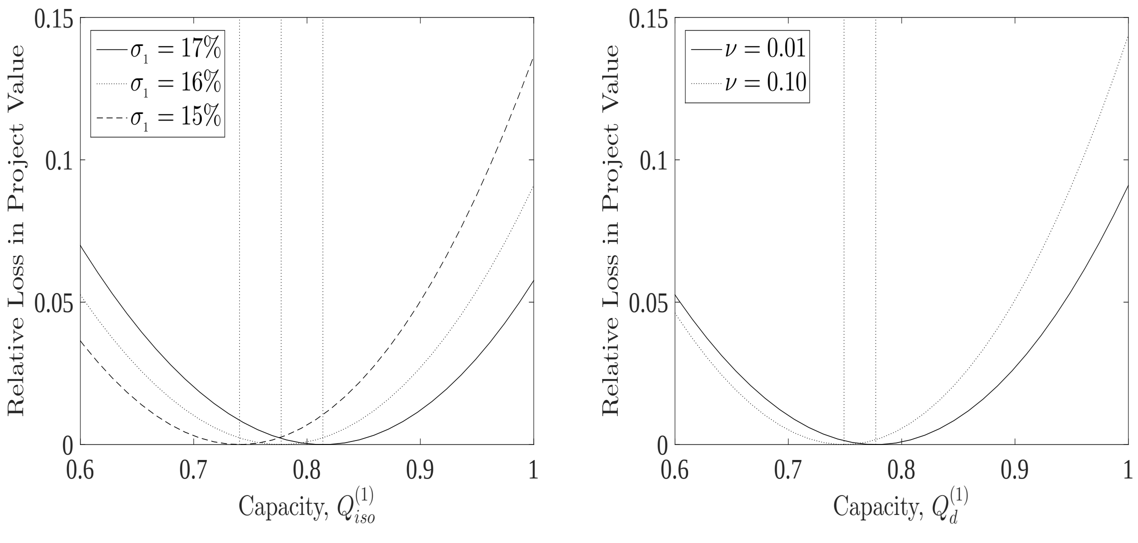

Figure 3.6: Relative loss in option value due to fixed capacity in regime 1 for $\nu = 0.01$ (left panel) and $\sigma = 16\%$ (right panel).

3.6 Conclusions

The increasing frequency of disruptive innovations indicates that the developing world reflects a rapid-growing market rather than just a low-cost manufacturing base. Within this context, the viability of private firms relies crucially on investment strategies that are responsive to market conditions. Therefore, we develop a utility-based, regime switching framework in order to analyse how technological and demand uncertainty interact with attitudes towards risk to impact the decision to abandon an existing market regime and entry into a new one. Results indicate that managerial discretion, risk aversion and market regime asymmetry, in terms of demand structure and economic uncertainty, can have a crucial impact on the decision to give up a mature technology in order to enter a new, possibly riskier, yet more profitable regime.

We provide analytical indications of the effect of managerial discretion in the light of interacting uncertainties that tend to be absent from the long-term economic models that support policy initiatives. Failure to understand these properly implies an increasing likelihood of cycles of under- or over-investment. Hence, understanding the behavioural inclinations to invest is crucial and real options analysis can provide important insights. In terms of future research, it would be interesting to apply an alternative stochastic process, such as a mean-reverting process, or a different class of utility functions, and, thus, assess the robustness of the numerical and theoretical results. Also, other aspects of the real options literature, such as competition, time to build and alternative technology adoption strategies may be included within this framework in line with Bar-Ilan & Strange (1996), Chronopoulos & Siddiqui (2015) and Goto *et al.* (2017).

3.7 Appendix

Proposition 1. *The expected utility of a perpetual stream of cash flows* $P_{t,d}^{(k)}\left(\Theta_t^{(k)}, Q_d^{(k)}\right) Q_{t,d}^{(k)}$, *where* $\Theta_t^{(k)}$ *follows a Markov-modulated GBM is*

$$\mathbb{E}_\Theta\left[\int_0^\infty e^{-\rho t} U\left(\Theta_t^{(k)} \mathcal{H}\left(Q_d^{(k)}\right) Q_d^{(k)}\right) dt\right] \;=\; \mathcal{A}^{(k)} U\left(\Theta \mathcal{H}\left(Q_d^{(k)}\right) Q_d^{(k)}\right).$$

where

$$\mathcal{A}^{(k)} \;=\; \frac{\beta_{1k}\beta_{2k}}{\left(\rho+\nu\mathbb{1}_{k=1}\right)\left(\gamma-\beta_{1k}\right)\left(\gamma-\beta_{2k}\right)}$$

$\mathbb{1}_{k=1}$ *is an indicator function,* β_{ik} $i=1,2$ *and* $k=1,2,3$*, are the roots of the quadratic* $\frac{1}{2}\sigma_k^2\beta\left(\beta-1\right)+\mu_k\beta-\left(\rho+\nu\mathbb{1}_{k=1}\right)=0$.

Proof: The differential equation governing the value process is indicated in (A–1), where $V_1^{(k)}\left(\Theta, Q_d^{(k)}\right)$ is an unknown value function. The first term on the right-hand side is the instantaneous profit for an arbitrary level of Θ, and capacity, $Q_d^{(k)}$, and the second term is the discounted future value. Furthermore, in order to assess the value of a profit stream in the current regime we let $V_1^{(k+1)}\left(\Theta, Q_d^{(k+1)}\right)=0$.

$$\begin{aligned} V_{1,d}^{(k)}\left(\Theta, Q_d^{(k)}\right) = U\left(\Theta\mathcal{H}\left(Q_d^{(k)}\right) Q_d^{(k)}\right) dt \quad &+ \left(1-\rho dt\right)\left\{\nu dt \mathbb{1}_{k=1}\mathbb{E}_\Theta\left[V_{1,d}^{(k+1)}\left(\Theta+d\Theta, Q_d^{(k+1)}\right)\right]\right.\\ &\left.+\left(1-\nu dt\mathbb{1}_{k=1}\right)\mathbb{E}_\Theta\left[V_{1,d}^{(k)}\left(\Theta+d\Theta, Q_d^{(k)}\right)\right]\right\} \end{aligned} \quad \text{(A–1)}$$

Using Itô's lemma, we can expand the right-hand side of (A–1) to obtain (A–2), and expressing the first term on the right-hand side of (A–1) as $U\left(\mathcal{H}\left(Q\right)Q\right)\Theta^\gamma$.

$$\begin{aligned} \frac{\sigma_k^2}{2}\Theta^{(k)2}\frac{d^2}{d\Theta^2}V_{1,d}^{(k)}\left(\Theta, Q_d^{(k)}\right)+\mu_k\Theta^{(k)}\frac{d}{d\Theta}V_{1,d}^{(k)}\left(\Theta, Q_d^{(k)}\right)-\left(\rho+\nu\mathbb{1}_{k=1}\right)V_{1,d}^{(k)}\left(\Theta, Q_d^{(k)}\right)&\\ +U\left(\mathcal{H}\left(Q_d^{(k)}\right)Q\right)\Theta^{(k)^\gamma}=0& \end{aligned} \quad \text{(A–2)}$$

Notice that, the firm produces up to capacity and thus the inverse demand function $\mathcal{H}\left(Q_d^{(k)}\right)$ and capacity $Q_d^{(k)}$ are constants. We conjecture that $V_{1,d}^{(k)}\left(\cdot\right)$ can be expressed as $V_{1,d}^{(k)}\left(\Theta, Q\right)= A\Theta^\gamma$. Substituting this expression into (A–2) yields (A–3).

$$\left[\frac{1}{2}\sigma_k^2\left(\gamma-1\right)\gamma+\mu_k\gamma-\left(\rho+\nu\mathbb{1}_{k=1}\right)\right]A\Theta^\gamma=-U\left(\mathcal{H}\left(Q\right)Q\right)\Theta^\gamma \quad \text{(A–3)}$$

and we find that $A=\frac{-U(\mathcal{H}(Q)Q)}{\frac{1}{2}\sigma^2(\gamma-1)\gamma+\mu\gamma-(\rho+\nu\mathbb{1}_{k=1})}$. In Theorem 9.18, Karatzas & Shreve (1999) find a closed-form expression for the expected utility of stochastic cash flows that follow an exogenous GBM, without demand uncertainty. Specifically, they find that $\mathbb{E}_\Theta\left[\int_0^\infty e^{-\rho t} U\left(P_t^{(k)}\right) dt\right] = \mathcal{A}^{(k)} U(P)$ where $\mathcal{A}^{(k)} = \frac{\beta_{1k}\beta_{2k}}{(\rho+\nu\mathbb{1}_k)\left(\gamma-\beta_{1k}\right)\left(\gamma-\beta_{2k}\right)}$. It can be shown that $\mathcal{A}^{(k)} = \frac{-1}{\frac{1}{2}\sigma^2(\gamma-1)\gamma+\mu\gamma-(\rho+\nu\mathbb{1}_{k=1})}$, which coincides with the denominator we obtained from (A–3), thus we can express the value function with demand uncertainty as in (A–4).

$$G_1^{(3)}(\Theta, Q) = \mathcal{A}^{(k)} U(PQ) \tag{A–4}$$

□

Proposition 2. *The optimal capacity under a "now-or-never" investment opportunity is finite for all $\lambda \geq 1$.*

Proof: The optimal capacity under a *now-or-never* investment decision is obtained by using the bottom branch of $F_{1,d}^{(3)}(\Theta)$ and solving $\frac{\mathrm{d}F_{1,d}^{(3)}(\Theta)}{\mathrm{d}Q_d^{(3)}} = 0$. If we let $\lambda \to 1$, then, under a multiplicative demand function, the condition for optimal capacity choice becomes $f(x) = 0$, where:

$$f(x) = \rho\mathcal{A}_3(1-2\eta x) - (1-\eta x)^{1-\gamma}\left(\frac{c+rb}{\Theta}\right)^\gamma \tag{A–5}$$

We can show that a solution exists via the intermediate value theorem. If we can find a constant u such that $f(a) < u < f(b)$, then $\exists x_0 \in (a,b)$ s.t. $f(x_0) = u$. First, we set $x = \frac{1}{\eta}$, and the second term in (A–5) is then zero and $f(x) < 0$. On the other hand, if $x \to 0$, the first term is positive, and the second term can be made arbitrarily small by setting a greater Θ, thus $f(x) > 0$. Consequently, there exists a $Q_m^{(3)^*}$ the satisfies (A–5). Also, by letting $\lambda \to 1$ under an iso-elastic demand function, the expression for the optimal capacity is described in (A–6) for $\xi \neq 1$.

$$Q_d^{(3)^*} = \left[\frac{(c+rb)^\gamma}{\rho\mathcal{A}_3(1-\xi)\,\theta^{(3)\gamma}}\right]^{-\frac{1}{\xi\gamma}} \tag{A–6}$$

□

Proposition 3. *The optimal capacity and optimal investment threshold decrease in ξ iff $\beta_{13} > \frac{\gamma(\lambda+1)}{\lambda+2\xi-1}$.*

Proof: We start by calculating the partial derivative of $\partial Q^{(3)^*}$ with respect to ξ.

$$\begin{aligned}\frac{\partial Q_{iso}^{(3)^*}}{\partial\xi} &= \frac{1}{\lambda-1}\left[\frac{c}{\rho b}\frac{\gamma-\beta_{13}\xi}{\beta_{13}(\lambda+\xi-1)-\lambda\gamma}\right]^{\frac{2-\lambda}{\lambda-1}} \times \\ &\frac{c}{\rho b}\frac{-\beta_{13}\left[\beta_{13}(\lambda+\xi-1)-\lambda\gamma-(\gamma-\beta_{13}\xi)\right]}{\left[\beta_{13}(\lambda+\xi-1)-\lambda\gamma\right]^2}\end{aligned} \tag{A–7}$$

Hence:

$$\begin{aligned}\frac{\partial Q_{iso}^{(3)^*}}{\partial \xi} > 0 \quad &\Leftrightarrow \quad \beta_{13}(\lambda + 2\xi - 1) - \gamma(\lambda + 1) > 0 \\ &\Leftrightarrow \quad \beta_{13} > \frac{\gamma(\lambda + 1)}{\lambda + 2\xi - 1}\end{aligned} \tag{A–8}$$

Since the optimal investment threshold is a monotonically increasing function of $Q_{iso}^{(3)^*}$, we have $\frac{\partial \Theta_1^{(3)^*}}{\partial \xi} > 0$. □

Proposition 4. $Q_{iso}^{(3)^*} > Q_m^{(3)^*}$ *iff* $\frac{1-\eta Q_m^{(3)^*}}{1-2\eta Q_m^{(3)^*}} > \frac{1}{1-\xi}$.

Proof: The optimal capacity conditions for iso-elastic inverse demand function (top branch) and an multiplicative (lower branch) are given in (A–9).

$$\begin{cases} \rho \mathcal{A}_3 \frac{(\beta_{23}-\gamma)}{\beta_{23}} - \frac{1}{1-\xi} \frac{\left(c + rb\lambda Q_{iso}^{(3)^*\,\lambda-1}\right)}{\left(c + rbQ_{iso}^{(3)^*\,\lambda-1}\right)} = 0 \\ \rho \mathcal{A}_3 \frac{(\beta_{23}-\gamma)}{\beta_{23}} - \frac{1-\eta Q_m^{(3)^*}}{1-2\eta Q_m^{(3)^*}} \frac{\left(c + rb\lambda Q_m^{(3)^*\,\lambda-1}\right)}{\left(c + rbQ_m^{(3)^*\,\lambda-1}\right)} = 0 \end{cases} \tag{A–9}$$

Notice that, the first terms are identical for both branches, and also the second part of the second term, denoted by $C\left(Q_d^{(3)^*}\right) = \frac{\left(c + rb\lambda Q_d^{(3)^*\,\lambda-1}\right)}{\left(c + rbQ_d^{(3)^*\,\lambda-1}\right)}$. Since $\lambda > 1 \Rightarrow \frac{\partial C\left(Q^{(3)^*}\right)}{\partial Q^{(3)^*}} > 0$ and $\frac{\partial}{\partial Q_m^{(3)^*}} \frac{1-\eta Q_m^{(3)^*}}{1-2\eta Q_m^{(3)^*}} < 0$, we have that $Q_{iso}^{(3)^*} > Q_m^{(3)^*} \Leftrightarrow \frac{1-\eta Q_m^{(3)^*}}{1-2\eta Q_m^{(3)^*}} > \frac{1}{1-\xi}$. □

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Chapter 4

The value of turning-point detection for optimal investment

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Abstract

Understanding the dynamic evolution of business cycles is key for investment in emerging technologies, especially, since these technologies are associated with periods of economic growth whose duration depends on disruptive innovations, market saturation and economic uncertainty. Furthermore, recessions are often accompanied by greater economic uncertainty, which further incentivises firms to postpone investment. We develop a regime-switching, real options model for investment under uncertainty that facilitates time-varying transition probabilities in order to capture the dynamic evolution of an economic indicator. Specifically, we consider a private firm with a perpetual option to invest in a production facility within a dynamically evolving regime-switching economic environment, and develop a numerical approach to approximate the value of the investment opportunity. Results indicate that, ignoring the dynamic evolution of transition probabilities can result in severe valuation errors. Indeed, we find that when the probability of a regime switch is initially low, the option value is greater (less) in the good (bad) regime under time-varying transition probabilities than under fixed transition probabilities. In contrast, when the probability is initially high, we find that the impact of the initial state is reduced, and also that incorrectly assuming fixed-transition probabilities leads to overvalued investment opportunities.

Keywords: investment analysis, time-varying transition probabilities, real options

4.1 Introduction

Although firms recognise the cyclical nature of economic growth, predicting when economic conditions will turn is notoriously hard. Empirical evidence suggests that firms attempt to time the market by issuing equity at high market values compared to book and past market values, with the intent to exploit temporary fluctuations in the business climate (Baker & Wurgler, 2002). However, market timing complicates the investment process, since firms need to weigh the value of waiting for more information against the probability of a recession. Consequently, theoretical models for investment under uncertainty have tried to incorporate both price uncertainty and regime switching (Driffill *et al.*, 2013; Goto *et al.*, 2017). Although these models give a good theoretical foundation to evaluate potential investments, they assume that transition probabilities are fixed.

Fixed transition probabilities may be a reasonable starting point, however, firms often have additional information about the likelihood of an economic turning point which causes a new market regime to emerge. For instance, leading indicators, such as building permits and interest rates (Stock & Watson, 1998) or the Purchasing Managers' Index (PMI) can predict future economic conditions (Koenig, 2002). In fact, The PMI aims to capture executive's sentiment regarding future economic growth by asking questions about new orders, production and employment, and is an important indicator of the economy's health. Such indicators give firms reasons to adjust their expectations regarding future expansions and contractions. Nevertheless, traditional models often assume a constant expected growth rate and price uncertainty, or, at best, assume fixed transition probabilities (FTP), thus disregarding important information that firms possess. In this paper, we relax this assumption by allowing for time-varying transition probabilities (TVTP) and we use a least square approach to approximate the value of an investment opportunity. Our model provides a decision-support tool which can evaluate investment opportunities in the presence of a looming recession.

We proceed by discussing some related work in section 4.2 and introduce assumptions and notation in Section 4.3. In Section 4.4, we present the benchmark model of investment under regime switching following the approach Goto *et al.* (2017). Next, we proceed in Section 4.5 by presenting a numerical approach for option pricing under regime switching. Finally, Section 4.7 summarises the paper and provides suggestions for future research.

4.2 Literature Review

Examples of early work that incorporates policy uncertainty in the form of regime switching include Dixit & Pindyck (1994), who consider a tax credit on the investment cost which might be present or absent depending on the current regime. Like Dixit & Pindyck (1994), Hassett & Metcalf (1999) investigate a tax credit subject to regime switching, yet allows the tax credit to be correlated with output price, and find that the policy uncertainty delays capital investments. Driffill *et al.* (2003) study how business cycle conditions may impact a monopolist's entry and exit decisions, and find that, firms mainly invest during economic expansions and abandon in recessions. Guo *et al.* (2005) analyse the option to incrementally increase capacity in a regime switching economy, and find that capacity expansions are subject to lumpy investment when a regime suddenly switches from a recession to an economic expansion, whereas firms will adjust capacity incrementally within a regime. Driffill *et al.* (2013) develop a model for analysing the state of the economy with discrete shifts between booms and busts. The value in each regime is found via a stochastic discount factor which is greater during recessions than expansions, since investors marginal rate of substitution is greater during recessions. They find that Markov regime switching results in delayed investment. Grenadier *et al.* (2014) study how economic busts impact an agent's propensity to abandon real estate projects. Results indicate that after a bust, agents strategically decide to abandon unsuccessful projects, in order to hide their abilities and thus blend in with the crowd. In the same line of work as Driffill *et al.* (2013), Goto *et al.* (2017) allow for competition in a regime-switching model, where two asymmetric firms compete to secure a period of monopoly profits. They show that even a disadvantaged firm can have an incentive to invest first, which occurs after a switch to the good regime where suddenly both firms find it lucrative to invest. Although regime switching is a crucial feature affecting the viability of capital projects, the aforementioned papers ignore the time-varying nature of transition probabilities and the presence of unobservable regimes. Consequently, models that investigate the impact of the dynamic evolution of business cycles on a firm's propensity to invest remain somewhat underdeveloped.

In parallel, but separate from the the real options literature, statisticians and economists have developed analytical techniques to estimate models with TVTP. In their pioneering model, Diebold *et al.* (1994) extend Hamilton (1989) by illustrating how transition probabilities can depend on other economic variables by employing an expectation-maximisation algorithm to estimate transition probabilities. Filardo (1994) uses maximum likelihood to investigate if US output data alternates between periods of expansion and contraction of varying durations, and their results indicate the existence of two states with different duration. Creal *et al.* (2013) introduce a framework to estimate time-varying parameters, which utilises the complete density of the dependent variable to calculate a score, which is used to find the time-varying parameters. Bazzi *et al.* (2016) adopt the framework of Creal *et al.* (2013) to study TVTP and demonstrate its applicability to US industrial production. Although these papers develop the necessary tools to estimate and forecast TVTP, they do not consider how forecasting TVTP may impact the value of an investment opportunity.

Since the option to invest depends on the dynamic evolution of transition probabilities, which hinders analytical tractability, numerical methods must be adopted. Longstaff & Schwartz (2001) introduce a simulation method for valuing American options. Their main insight is to use ordinary least squares to estimate the conditional expected payoff. The firm then decides whether to exercise the option by comparing the current state with the future expected value, and Stentoft (2004) shows that the approximated option value converges to the true option value in a multiperiod, multidimensional setting, yet convergence rates are uncertain in this general case. The flexibility of the approach lends itself well to a wide range of applications. For example, Boomsma *et al.* (2012) use this method to price real options under different support scheme, while Hsu & Schwartz (2008) investigate multistage R&D projects. Cortazar *et al.* (2008) extend Brennan & Schwartz's (1985) model by including three correlated stochastic processes which represent relevant commodity prices.

In this paper we develop a simulation method for investments under TVTP in which we investigate the tradeoff between waiting for more information against the possibility of a worsening business climate. Thus, we first implement an analytical benchmark case and solve it both analytically and numerically. Next, we allow for TVTP to capture changing expectations through time. Consequently, we derive new insights regarding how expectations impact optimal investment decisions. Results indicate that, low regime uncertainty creates value in an expansionary period i.e. high growth. Although, small errors in the estimated parameters are amplified in the valuation calculation, the contribution from TVTP on option value is even greater in a period of economic expansion when a sudden regime switch is unlikely initially.

4.3 Assumptions and Notation

Uncertainty is modelled on a probability space $(\Omega, \mathcal{F}, P)$, where Ω is the set of all possible realisations of the economy, which is endowed with a filtration $\{\mathcal{F}_t; t \in [0, \infty)\}$ generated by relevant state variables as in Longstaff & Schwartz (2001). We consider a firm with a perpetual option to invest in a project of infinite lifetime facing price uncertainty. The output price process $\left\{P_t^{(\epsilon)}, t \geq 0\right\}$, where t denotes time and $\epsilon \in \{1, 2\}$ denotes the current regime, follows a Markov-modulated geometric Brownian motion (GBM) that is described in (1). We denote by μ_ϵ the annual growth rate, by σ_ϵ the annual volatility and by dZ_t the increment of the standard Brownian motion.

$$dP_t^{(\epsilon)} = \mu_\epsilon P_t^{(\epsilon)} dt + \sigma_\epsilon P_t^{(\epsilon)} dZ_t, \quad P_0 \equiv P > 0 \tag{1}$$

In order to facilitate a discrete approximation of the continuous-time stochastic process, we let $m \in M$ be the scenario, $n \in N$ be the discrete time step, and T be duration of the time horizon. We set T sufficiently high so as not to impact the option value at $t = 0$, significantly. Equivalently to dt in continuous time, we have $\Delta t = T/N$ in discrete time. Also, for clarity, we introduce extra indices in the simulation part, for example $\epsilon_{n,m}$ to indicate the time step (n) and the current scenario (m). The analytical solution to (1) is indicated for the continuous-time case in (2) between time t and $t + dt$, and the discrete approximation for a scenario is outlined in (3), where $\omega_{P,n} \sim \mathcal{N}(0, 1)$ (Miranda & Fackler, 2002).

$$\begin{aligned} P_{t+dt}^{(\epsilon)} &= P_t^{(\epsilon)} \exp\left(\left[\mu_\epsilon - 0.5\sigma_\epsilon^2\right] dt + \sigma_\epsilon dZ_t\right) && (2) \\ P_n^{(\epsilon)} &= P_{n-1}^{(\epsilon)} \exp\left(\left[\mu_\epsilon - 0.5\sigma_\epsilon^2\right] \Delta t + \sigma_\epsilon \sqrt{\Delta t}\, \omega_{P,n}\right) && (3) \end{aligned}$$

When transition probabilities are fixed, regime switching follows a Poisson process $\{J_t, t \geq 0\}$ and is independent of the process $\left\{P_t^{(\epsilon)}, t \geq 0\right\}$. Consequently, the time τ between two subsequent regime switches is exponentially distributed, i.e. $\tau \sim \exp\left(\lambda^{(\epsilon)}\right)$. The parameter $\lambda^{(\epsilon)}$ is the intensity of the Poisson process, and, thus, the probability of a regime switch within an infinitesimal time interval dt, is $\lambda^{(\epsilon)} dt$. In contrast, under TVTP we assume that there is a state variable determining the current transition probability $\{W_t, t \geq 0\}$. Normally, W_t is an observed time series with unknown data generating process and mapping to transition probabilities (Diebold *et al.*, 1994; Filardo, 1994). But, to facilitate numerical examples, and to avoid probabilities drifting towards an absorbing barrier (1 or 0), we assume that W_t is stationary and is generated by the autoregressive process, as shown in Bazzi *et al.* (2016). The process is indicated in (4), where φ_1, $|\varphi_2| < 1$ and $\omega_{W,n} \sim \mathcal{N}(0, \sigma_{W,n})$.

$$W_n = \varphi_1 + \varphi_2 W_{n-1} + \omega_{W,n}, \quad W_0 \equiv W \tag{4}$$

Furthermore, the conditional expectation $\mathbb{E}[\cdot|\mathcal{F}_t]$ of the process $\{W_t\}$ at $t+n$ given the information set $\mathcal{F}_t$ is given in (5). The second term inside the expectation operator converges to zero as the number of time steps tends to infinity, while the first term is a convergent geometric series. Thus, the unconditional mean is $\lim_{t\to\infty} \mathbb{E}[W_t|\mathcal{F}_t] = \frac{\varphi_1}{1-\varphi_2}$ (Enders, 2008).

$$\mathbb{E}\left[W_{t+n}|\mathcal{F}_t\right] = \mathbb{E}\left[\varphi_1 \sum_{i=0}^{n-1} \varphi_2^i + \varphi_2^n W_t|\mathcal{F}_t\right] \tag{5}$$

The state variable, W_t, has an associated probability generating process $p_n^{(\epsilon,\hat{\epsilon})} = \Phi\left(\alpha^{(\epsilon)} W_t\right)$, where $p_n^{(\epsilon,\hat{\epsilon})}$ is the probability of going from state ϵ to state $\hat{\epsilon}$ at time step n. Also, the probability of no event occurring is $p_n^{(\epsilon,\epsilon)} = 1 - p_n^{(\epsilon,\hat{\epsilon})}$, $\alpha^{(\epsilon)}$ and $\alpha^{(\hat{\epsilon})}$ are constants, and $\Phi(\cdot)$ is the cumulative normal density function. Although there are several candidate functions which maps a time series process to probabilities, such as the logistic transfomration (Bazzi *et al.*, 2016), any specification that maps W_t into a unit interval is a valid candidate (Filardo, 1994), thus we adopt $\Phi(\cdot)$ which is already implemented in Matlab by Ding (2012).

Furthermore, we assume that $\mu_1 > \mu_2$, which implies that the first regime can be interpreted as a period of expansion, while the second regime is characterized by lower growth or retraction. Additionally, economic retractions usually exhibit greater volatility, and, therefore, we assume that $\sigma_2 > \sigma_1$ (Driffill & Sola, 1998), and that $\lambda^{(2)} > \lambda^{(1)}$ since recessions often have a shorter duration than expansions (Harding & Pagan, 2002). In line with Goto *et al.* (2017) we assume that the discount rate, r is constant across states since it produces no qualitative difference. We denote by $V_{b,c}^{(\epsilon)}(P)$ the value of the option to invest and by $G_{b,c}^{(\epsilon)}(P)$ the expected value of the profits from operating in the current regime, where b denotes if the solution is found analytically (an) or numerically (nu) and c denotes whether the value function is subject to FTP (f) or TVTP (v). Finally, we denote the project's output by D, and the investment cost by K.

4.4 Benchmark Model

Here, we assume that regimes are observable and that transition probabilities are fixed. Although this case has already been analysed in Goto *et al.* (2017), we present the analysis here for ease of exposition and to allow for comparisons. The optimal investment threshold in each regime is denoted by $P^{(\epsilon)^*}$. Since the first regime exhibits a lower volatility and a higher growth rate, we have that $P^{(1)^*} < P^{(2)^*}$. Consequently, there are three different investment regions depending on the current price: **i.** if the price is high, i.e. $P^{(2)^*} \leq P$, then the firm invests independently of the state of the economy; **ii.** if $P^{(1)^*} \leq P < P^{(2)^*}$, then the firm will invest only if the economy is in the good regime; and **iii.** if $P < P^{(1)^*} < P^{(2)^*}$, then the firm will postpone investment.

The firm's value function is described in (6), where the first term on the right-hand side of (6) is the immediate profit from operating in the current regime. As the second term indicates, with probability $\lambda^{(\epsilon)}dt$ another regime will appear and the firm will receive the value function $G_{an,f}^{(\hat{\epsilon})}(P)$, whereas, with probability $1 - \lambda^{(\epsilon)}dt$, no regime switch will occur and the firm will continue to hold the value function $G_{an,f}^{(\epsilon)}(P)$.

$$G_{an,f}^{(\epsilon)}(P) = DPdt + (1 - \rho dt)\,\mathbb{E}\left[\lambda^{(\epsilon)}dtG_{an,f}^{(\hat{\epsilon})}(P + dP) + \left(1 - \lambda^{(\epsilon)}dt\right)G_{an,f}^{(\epsilon)}(P + dP)\,|\mathcal{F}_0\right] \quad (6)$$

By expanding the right-hand side of (6) using Itô's lemma, we obtain the ordinary differential equation indicated in (7).

$$\frac{1}{2}\sigma_\epsilon^2 P^2 \frac{dG_{an,f}^{(\epsilon)}(P)}{dP^2} + \mu_\epsilon P \frac{dG_{an,f}^{(\epsilon)}(P)}{dP} - rG_{an,f}^{(\epsilon)}(P) + \lambda^{(\epsilon)}\left(G_{an,f}^{(\hat{\epsilon})}(P) - G_{an,f}^{(\epsilon)}(P)\right) + DP = 0 \quad (7)$$

The expression for $G_{an,f}^{(\epsilon)}(P)$ can be determined analytically and is indicated in (8). Notice that, if $\lambda^{(\epsilon)} = 0$ a regime switch will never occur, and the firm obtains the value of operating in the current regime indefinitely.

$$G_{an,f}^{(\epsilon)}(P) = \frac{\left(r + \lambda^{(\epsilon)} + \lambda^{(\hat{\epsilon})} - \mu_{\hat{\epsilon}}\right)DP}{\left(r + \lambda^{(\epsilon)} - \mu_\epsilon\right)\left(r + \lambda^{(\hat{\epsilon})} - \mu_{\hat{\epsilon}}\right) - \lambda^{(\epsilon)}\lambda^{(\hat{\epsilon})}} \quad (8)$$

Next, we consider the case where the firm is currently in the first regime. The top branch reflects the value of the option to invest and is determined by solving (A–3). As the bottom part in (9) indicates, if $P \geq P^{(1)^*}$, then the firm will invest and obtain the expected value from operating in the first regime (first term) by paying the investment cost in the second term.

$$V_{an,f}^{(1)}(P) = \begin{cases} c_1^{(1)} P^{\gamma_1} + c_2^{(1)} P^{\gamma_2} & , P < P^{(1)^*} \\ G_{an,f}^{(1)}(P) - K & , P \geq P^{(1)^*} \end{cases} \tag{9}$$

The option value is indicated in the top branch of (10), which takes into account the likelihood of investing in regime one. The first two terms in the second branch represent the option value to wait, while the subsequent two terms are the expected cash flows, and are found by solving (A–1) in the appendix. The expected value of the active firm in the second regime is indicated in bottom branch, and the endogenous constants and investment threshold in (9) and (10) are determined through value-matching and smooth-pasting conditions between the different branches in both regimes, numerically.

$$V_{an,f}^{(2)}(P) = \begin{cases} c_1^{(2)} P^{\gamma_1} + c_2^{(2)} P^{\gamma_2} & , P < P^{(1)^*} \\ b_1 P^{\delta_1} + b_2 P^{\delta_2} + \frac{\lambda^{(2)} G^{(1)}(P)}{r + \lambda^{(2)} - \mu_2} - \frac{\lambda^{(2)} K}{r + \lambda^{(2)}} & , P^{(1)^*} < P < P^{(2)^*} \\ G_{an,f}^{(2)}(P) - K & , P \geq P^{(1)^*} \end{cases} \tag{10}$$

4.5 Numerical Solution Procedure

4.5.1 TVTP Estimation

In practice, the true probability of a regime switch is unobserved and has to be estimated. This is commonly done through a regime-switching model as described in Hamilton (2008), but, in our case, with a slight modification to capture TVTP (Bazzi *et al.*, 2016; Ding, 2012). Here, we denote by $\theta = (\mu_\epsilon, \sigma_\epsilon, \alpha^{(\epsilon)}, \varphi_1, \varphi_2)$ the vector containing all relevant parameters. Also, the probability of being in a given state at time n is defined as in (11).

$$\xi_n^{(\epsilon)} = Pr\left(\epsilon | \mathcal{F}_n; \theta\right) \tag{11}$$

The probability density function of observing the current realisation of $\log\left(\Delta P_n^{(\epsilon)}\right)$ conditional on a state and the distribution's parameters, θ, is given in (12). In other words, since the log-difference is approximately equal to the percentage change, this is the probability of observing the percentage price change at n, given the state and the its mean and variance. Notice that the exponential function is maximised when the numerator is zero, i.e. the log-differenced price, is equal to $\left[\mu_\epsilon - 0.5\sigma_\epsilon^2\right] \Delta t$. Consequently, since the expectation of the log-differenced price is

$\left[\mu_\epsilon - 0.5\sigma_\epsilon^2\right]\Delta t$, the probability density function is maximised under the true, unobserved state.

$$\eta_n^{(\epsilon)} = f\left(\log\left(\Delta P_n^{(\epsilon)}\right)\Big|\epsilon, \mathcal{F}_{t-1};\theta\right) = \frac{1}{\sqrt{2\pi}\sigma_\epsilon}\exp\left[-\frac{\left[\log\left(\Delta P_n^{(\epsilon)}\right) - \left(\mu_\epsilon - 0.5\sigma_\epsilon^2\right)\Delta t\right]^2}{2\sigma_\epsilon^2}\right] \tag{12}$$

Next, we can calculate the conditional probability density function of the n^{th} observation according to (13). The first term on the right-hand side, $p_n^{(i,j)}$, is the probability of going from state i to state j. This is obtained at each time step by observing W_n and calculating the resulting probability, using the formula, $p_n^{(i,j)} = \Phi\left(\alpha^{(i)}W_n\right)$. Assuming that we have obtained the filtered probabilities from the previous time step, $\xi_{n-1}^{(\epsilon)}$, we can calculate the predictive probabilities for state ϵ as $\sum_{i=1}^{2} p_n^{(i,\epsilon)}\xi_{n-1}^{(i)}$, i.e. the probability for being in state ϵ. Finally, this probability has to be multiplied by $\eta_n^{(\epsilon)}$, reflecting the probability of observing the current realisation of $\log\left(\Delta P_n^{(\epsilon)}\right)$.

$$f\left(\log\left(\Delta P_n^{(\epsilon)}\right)\Big|\mathcal{F}_{t-1};\theta\right) = \sum_{i=1}^{2}\sum_{j=1}^{2} p_n^{(i,j)}\xi_{n-1}^{(i)}\eta_n^{(j)} \tag{13}$$

The update rule for the probability in each state is indicated in (14). Notice that there are two ways to enter state ϵ: **i.** the process has stayed in the same state or **ii.** the process switched from $\hat{\epsilon}$. Thus, we sum the densities for both these options and scale them by the denominator to get the probability of the state at time n.

$$\xi_n^{(\epsilon)} = \frac{\sum_{i=1}^{2} p_n^{(i,\epsilon)}\xi_{n-1}^{(i)}\eta_n^{(\epsilon)}}{f\left(\log\left(\Delta P_n^{(\epsilon)}\right)\Big|\mathcal{F}_{n-1};\theta\right)} \tag{14}$$

To start the algorithm, we set the probabilities at time step $n = 1$ to the unconditional probability, which implies that $W_1 = \frac{\varphi_1}{1-\varphi_2}$ and $\xi_1^{(\epsilon)} = \left(1 - p_1^{(\hat{\epsilon},\hat{\epsilon})}\right) / \left(2 - p_1^{(\epsilon,\epsilon)} - p_1^{(\hat{\epsilon},\hat{\epsilon})}\right)$. Next, we iterate through the time series from $n = 1$ to $n = N$ to obtain the sample conditional log likelihood which is indicated in (15). In essence, this is the likelihood of observing all the percentage price changes given our set of parameters θ.

$$\sum_{t=1}^{T}\log f\left(\log\left(\Delta P_n^{(\epsilon)}\right)\Big|\mathcal{F}_{n-1};\theta\right) \tag{15}$$

Finally, by optimising (15), we can find an estimate of θ, which is denoted $\hat{\theta}$. This can be done via a standard non-linear optimiser in Matlab, see for example the implementation of Ding (2012) and Perlin (2015).

4.5.2 Monte Carlo Simulation of Price Scenarios

Based on the estimation procedure in Section 4.5.1, we can simulate paths of the modulated GBM, where we start by simulating M scenarios. This is illustrated in the top panel of Figure 4.1 for $M = 7$. Notice that a regime switch may occur between any two points in time, and, also, that the first regime has greater growth rate (blue) compared to the second regime (red) on average, in accordance with the assumptions in Section 4.3.

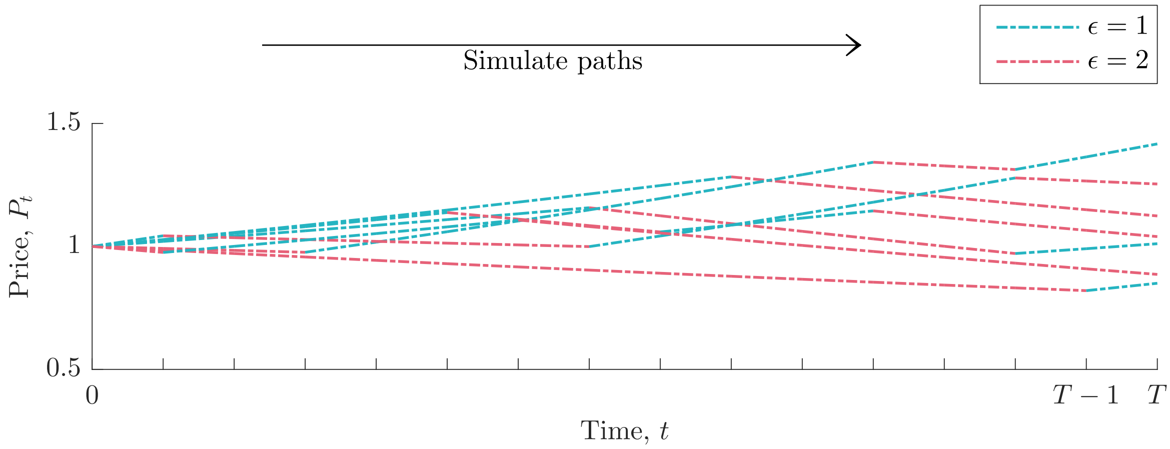

Figure 4.1: Illustrative example of simulation procedure

More specifically, the simulation is carried out by using the algorithm outlined below and yields a discrete approximation of the price process. Furthermore, since we observe W at time zero, we can forecast future transition probabilities by utilising (5) under TVTP for the entire time frame. For example, if W is high it will return to its long term mean after some time, but the transition probabilities in the beginning will be either high or low compared to long term average, depending on the sign of $\alpha^{(\epsilon)}$. Having established the transitions probabilities we follow the algorithm outlined below to simulate price scenarios.

1. We adopt the approach of Hamilton (1989) and assign an initial state $\epsilon_{1,m}$ to each scenario $m \in M$ in accordance with the unconditional probability as in Section 4.5.1.

2. Update $n := n + 1$, and generate $u_{n,m} \sim U(0,1), \forall m \in M$. Then we can check if $\lambda^{(\epsilon_{n-1,m})}\Delta t > u_{n,m}$ under FTP or $p_{n-1}^{(\epsilon_{n-1,m})} > u_{n,m}$ under TVTP. Since $u_{n,m}$ is uniform, the probability that $u_{n,m}$ is less than the transition probability, is exactly equal to the transition probability. Thus, if this condition is true, change the current regime, i.e. set $\epsilon_{n,m} = 3 - \epsilon_{n-1,m}$, otherwise keep the previous state $\epsilon_{n-1,m}$.

3. Simulate M standard normally distributed variables, i.e. $(\omega_{P,1}, \omega_{P,2}, \ldots, \omega_{P,M})$ and cal-

culate the value of the modulated GBM for all $m \in M$ according to (3).

4. Repeat steps 2 and 3 until $n = N$.

4.5.3 Option-Pricing Algorithm

Having simulated the paths of the price process, we work backwards to evaluate the investment option as illustrated in Table 4.1, where each bracket contains the current state, the price observed in Figure 4.1 and the project value. For example, at time T for path 1, the firm is currently in the first regime, and the output price is 1.42, thus, the NPV at expiration date is $G^{(1)}_{an,f}(1.42) - K = 6.78$, and in path 6 the firm chooses not to invest since the expected NPV is negative in regime 2. Prior to the final date, the firm compares the value of immediate exercise with the expected cash flows form continuing to wait, and will only exercise if immediate exercise is more valuable. Hence, the firm needs to identify the conditional expectations of continuing. In order to establish this expectation we use cross-sectional information in the simulated paths, which is done by regressing the subsequent realised cash flows from continuation, on a set of basis functions with the current realisations as input for both states, which will be discussed in the next section. Notice that, when M is big some paths will have switched regime, while others have remained unchanged, and the future expectation reflects the regime uncertainty. Next, the fitted value of this regression is an efficient unbiased estimate of the conditional expectation (Longstaff & Schwartz, 2001). For example, in the first path at $T-1$, it was not optimal to exercise the option and thus the new project value is the discounted future value, i.e. $e^{-r\Delta t}6.78 = 6.74$, where $\Delta t = 2/30$ and $r = 0.1$.

Table 4.1: Illustrative table for the pricing algorithm, where each bracket contains the state, price and project value.

Path	T-2	T-1	T
1	[1, 1.31, 6.69]	[1, 1.36, 6.74]	[1, 1.42, 6.78]
2	[2, 1.28, 3.69]	[2, 1.27, 3.55]	[2, 1.25, 3.42]
3	[2, 1.17, 2.58]	[2, 1.15, 2.30]	[2, 1.12, 2.03]
4	[2, 1.09, 1.68]	[2, 1.07, 1.41]	[2, 1.04, 1.13]
5	[1, 0.97, 1.95]	[1, 0.99, 1.96]	[1, 1.01, 1.98]
6	[2, 0.93, 0.00]	[2, 0.91, 0.00]	[2, 0.89, 0.00]
7	[2, 0.83, 0.07]	[1, 0.82, 0.07]	[1, 0.85, 0.07]

Hence, we extend Longstaff & Schwartz (2001) to value an American option when the price process follows a modulated GBM, and for each regime we approximate the option value by a linear combination of basis functions $L_j(\cdot)$ with coefficients $\beta_j^{(\epsilon)}$. Thus, $F^{(\epsilon)}(\cdot)$ is the approximated value of continuation as indicated in (16).

$$F^{(\epsilon)}\left(P_n^{(\epsilon)}\right) = \sum_{j=0}^{k} \beta_j^{(\epsilon)} L_j\left(P_n^{(\epsilon)}\right) \tag{16}$$

According to Longstaff & Schwartz (2001) there are several types of possible basis functions $L(\cdot)$ that can be used, including Laguerre, Hermite, Legendre, Chebyshev, Gegenbauer and Jacobi polynomials. Longstaff & Schwartz (2001) find that their numerical results are robust to the different possible basis functions, thus we follow their approach and use Laguerre polynomials with $k = 3$ as in (A–8). The option-pricing algorithm can be summarised in the following steps.

1. Simulate $M \times N$ outcomes of the price process as described in Section 4.5.2, and calculate the expected value of immediate exercise. This is done, either by using the bottom branch in (9) and (10) for FTP, or by simulating the expected value of a *now-or-never* investment opportunity for TVTP, i.e. calculate $\mathbb{E}\left[V_{b,c}^{(\epsilon)}\left(P_{n,m}^{(\epsilon)}\right)|\mathcal{F}_n\right]$, $\forall m \in M$ and $\forall n \in N$.

2. Start at $n \coloneqq N$, where the firm will exercise the option only if it is in the money. Consequently, for all m find the pay-off vector, $H_n = \max\left(0, V_{b,c}^{(\epsilon)}\left(P_{n,m}^{(\epsilon)}\right)\right)$.

3. Step back to $n \coloneqq n - 1$ and approximate the option value define $X_n^{(\epsilon)} = F^{(\epsilon)}\left(P_{n,m}^{(\epsilon)}\right)$, $\forall m \in M$. To improve efficiency, we omit outcomes which are not in the money at the current n, i.e. exercising today would lead to a loss. Also, in order to estimate the values of $\beta_j^{(\epsilon)}$ we divide the the value of waiting, H_{n+1}, into two subsets depending on the state at time n and discount it, i.e. $Y_n^{(\epsilon)} = e^{-r\Delta t} H_{n+1}|\epsilon$. As stated in Perlin (2015) and Turner *et al.* (1989) when the market regime is known, we can use linear regression as specified in (17) to estimate the conditional expectation for the two states.

$$\begin{aligned} Y_n^{(\epsilon)} &= X_n^{(\epsilon)} + u_\epsilon \\ &\text{where} \\ &u_\epsilon \sim N(0, \sigma_\epsilon^2) \end{aligned} \tag{17}$$

4. Calculate the conditional expectation $\mathbb{E}\left[Y_n^{(\epsilon)}|X_n^{(\epsilon)}\right]$ for each scenario (m), and exercise the option only if immediate exercise yields a higher return. Consequently, update the current value function, by $H_n = \max\left(\mathbb{E}\left[Y_n^{(\epsilon)}|X_n^{(\epsilon)}\right], V_{b,c}^{(\epsilon)}\left(P_{n,m}^{(\epsilon)}\right)\right)$.

5. Repeat step 3-4 until $n = 1$. At $n = 1$, calculate an average value for each regime, which is then the option value for each state.

4.6 Numerical Examples

For the numerical examples, the values of different parameters are given in Table 4.2, and we set $T = 100$ for the simulation. Furthermore, we set the values for $\alpha^{(\epsilon)}$ such that the unconditional probability is the same as under FTP, i.e. $\lambda^{(\epsilon)}\Delta t = \Phi\left(\alpha^{(\epsilon)}\frac{\varphi_1}{1-\varphi_2}\right)$. Figure 4.2

Table 4.2: True parameters used for numerical examples

μ_1	μ_2	σ_1	σ_2	$\lambda^{(1)}$	$\lambda^{(2)}$	φ_1	φ_2	σ_W	$\alpha^{(1)}$	$\alpha^{(2)}$	r	P	K	D
3%	-2%	10%	20%	0.15	0.25	0.1	0.996	0.1	-0.1	-0.09	0.1	1	10	1

illustrates the option value in regime one (left panel) and two (right panel) with FTP. Here we use the analytical solution from Section 4.4 as a benchmark (green solid line) to the simulation approach (red dotted line). Notice that the confidence interval (CI) diminishes as M increases, yet at a lower rate when M is greater than 1500. Also, the mean is very close to the analytical solution for $M > 1500$.

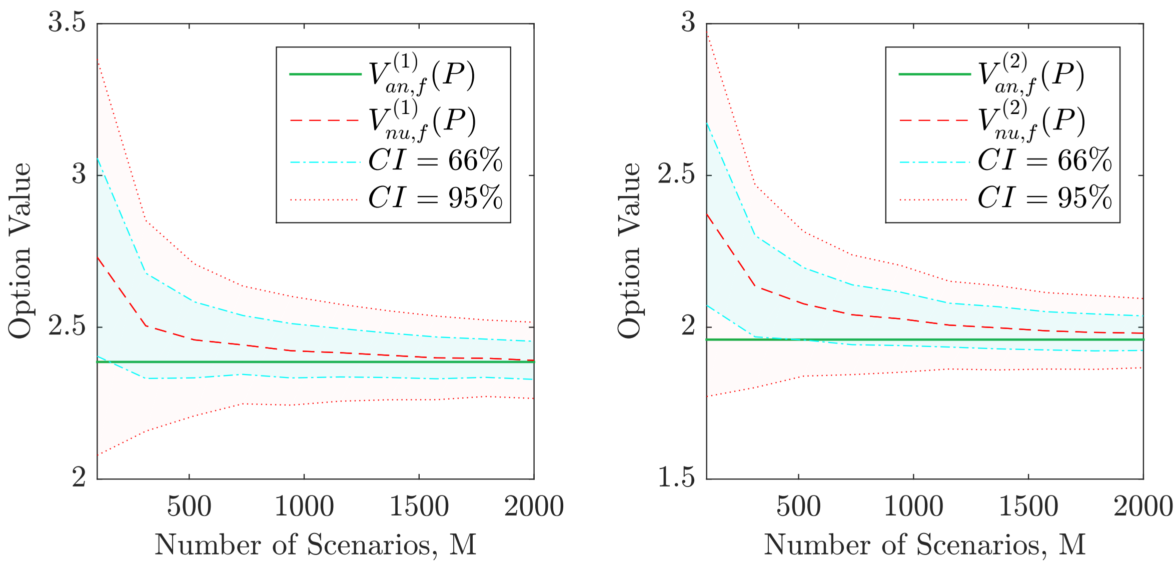

Figure 4.2: Impact of M on option value and estimation uncertainty for the first regime (left panel) and the second regime (right panel). The model is estimated one thousand times for each M in order to find the CIs, and $N = 2000$

To investigate the performance of our pricing procedure with TVTP, we consider two simulated paths which represent an observed process driving probabilities and a modulated GBM. The firm observes these time series and estimates the model parameters $(\hat{\theta})$. The resulting parameters are indicated in Table 4.3. Notice that both FTP and TVTP method are able to efficiently capture the first two moments in both regimes. Although, the true data generating process here is time-varying, the expected long-term transition intensities are $\lambda^{(1)} = 0.15$ and $\lambda^{(2)} = 0.3$ which the FTP model seems to capture well.

Table 4.3: True parameters and estimated parameters under FTP and TVTP, with $M = 1500$ and $N = 2000$

	μ_1	μ_2	σ_1	σ_2	$\lambda^{(1)}$	$\lambda^{(2)}$	$\alpha^{(1)}$	$\alpha^{(2)}$
True	3%	-2%	10%	20%	Na	Na	-0.097	-0.09
FTP	2.5%	-2.3%	10%	20%	0.16	0.26	Na	Na
TVTP	2.5%	-2.3%	10%	20%	Na	Na	-0.099	-0.091

In order to illustrate the importance of TVTP we let the initial value be $W = 1.5\frac{\varphi_1}{1-\varphi_2}$. This implies that W is 50% above it long term average, which in turn implies that the probability of a regime switch is small at the beginning, before returning to its long-term average, as illustrated in Figure 4.3. Notice that, the true transition probabilities (blue solid line), are similar to the estimated transition probabilities (red dotted line), especially for the first twenty years, which are more important due to discounting.

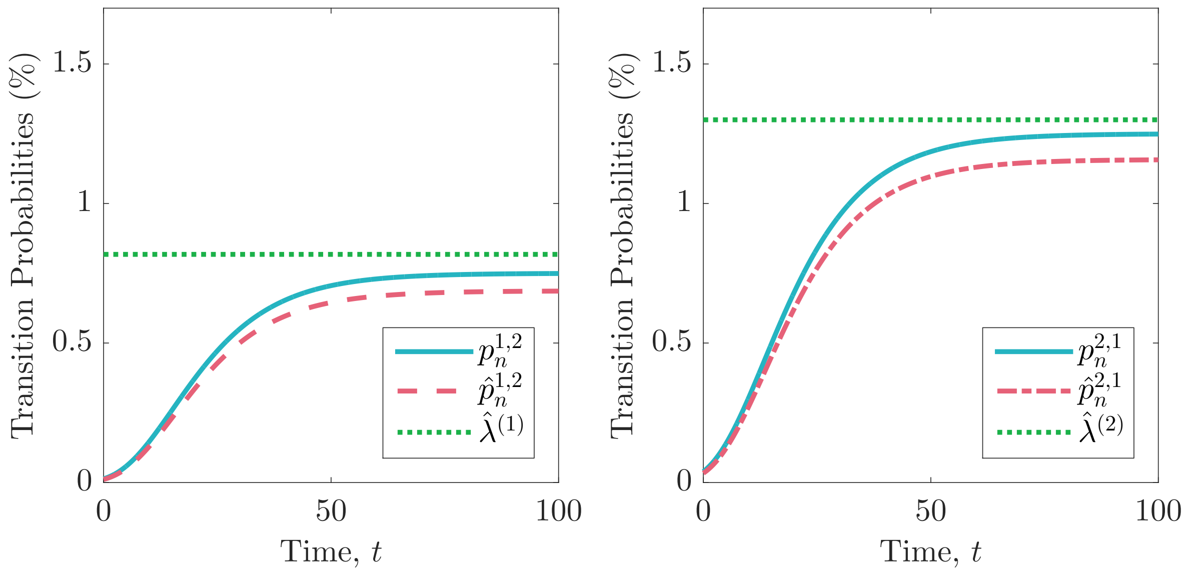

Figure 4.3: Evolution of forecasted transition probabilities for regime one (left panel) and regime two (right panel)

By using the forecasted transition probabilities, we can price the option to invest. This is repeated one thousand times both assuming FTP and TVTP, and is illustrated in Figure 4.4, where each dot is an outcome of a simulation, and the value in regime 1 is on the x-axis and the value in regime 2 on the y-axis. The left panel uses the true parameters while the right panel uses the estimated parameters. Recall that the transition probabilities are low in the beginning in this example since W is high and $\alpha^{(\epsilon)}$ is negative, which makes the initial state more important. As a result, the value increases in regime one while it decreases in regime two for TVTP compared to FTP. Although, the valuation assuming FTP with true parameters (left panel) or estimated parameters (right panel) is less affected by estimation errors than TVTP, FTP undervalues the good regime and overvalues the bad regime.

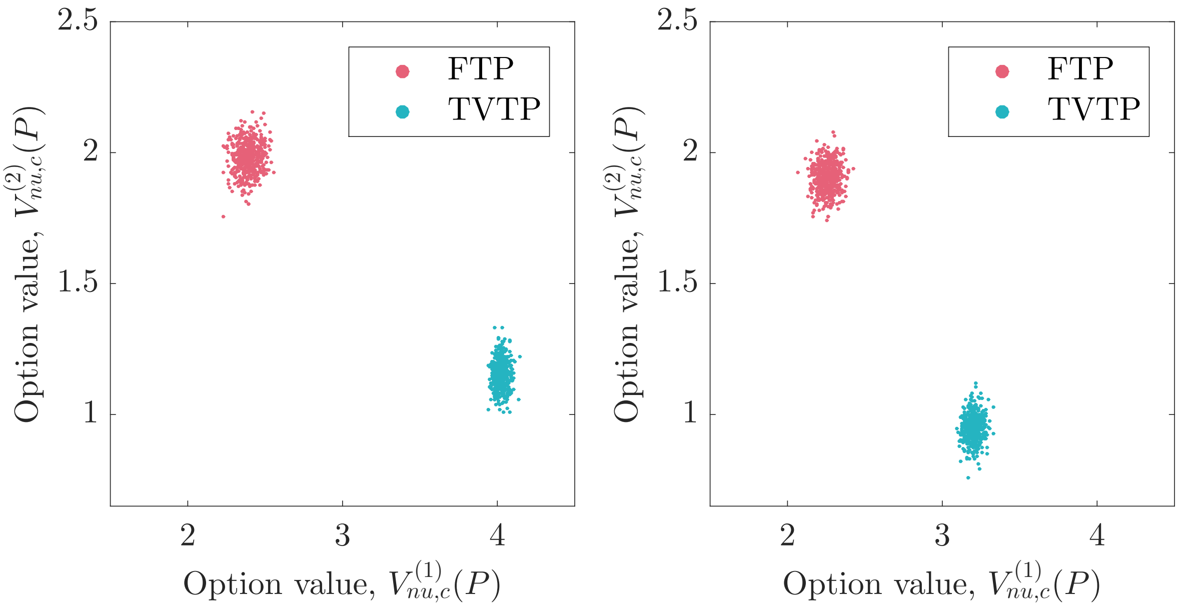

Figure 4.4: Option value using true model parameters (left panel) and estimated (right panel) with FTP and TVTP repeated one thousand times, $W = 1.5\frac{\varphi_1}{1-\varphi_2}$, $M = 1500$, $N = 2000$

Figure 4.5 illustrates the case where TVTP are high in the beginning, which implies a high likelihood for a regime switch. For example, it could be the case where leading indicators predict a likely regime switch. Notice that, the values in regime 1 under TVTP is reduced compared to Figure 4.4, because the firm is unlikely to stay in the good regime for an extended period. Furthermore, the option values for both regimes are greater under FTP than under TVTP. This occurs because the project values under TVTP are subject to a greater likelihood of regime switching than under FTP, which in turn reduces the values of a *now-or-never* investment in the first regime more than it increases the project values in the second regime. The non-symmetric impact on the two regimes is due to the convex nature of the exponential function, where a decreased expected growth rate due to more frequent regime switches in the good regime, has a greater impact on the project values than an equivalent increase in growth rates in the bad regime. In addition, since wrongly assuming FTP leads to greater values of a *now-or-never* investment in the first regime but lower *now-or-never* values in the second regime compared to TVTP, the alternation between states creates additional volatility which increases the option value. Driffill *et al.* (2003) find a similar result, where imposing a one-state model when the true data-generating process is a two-state model induces volatility. This is because the time-varying mean is captured as additional volatility in a one-state model.

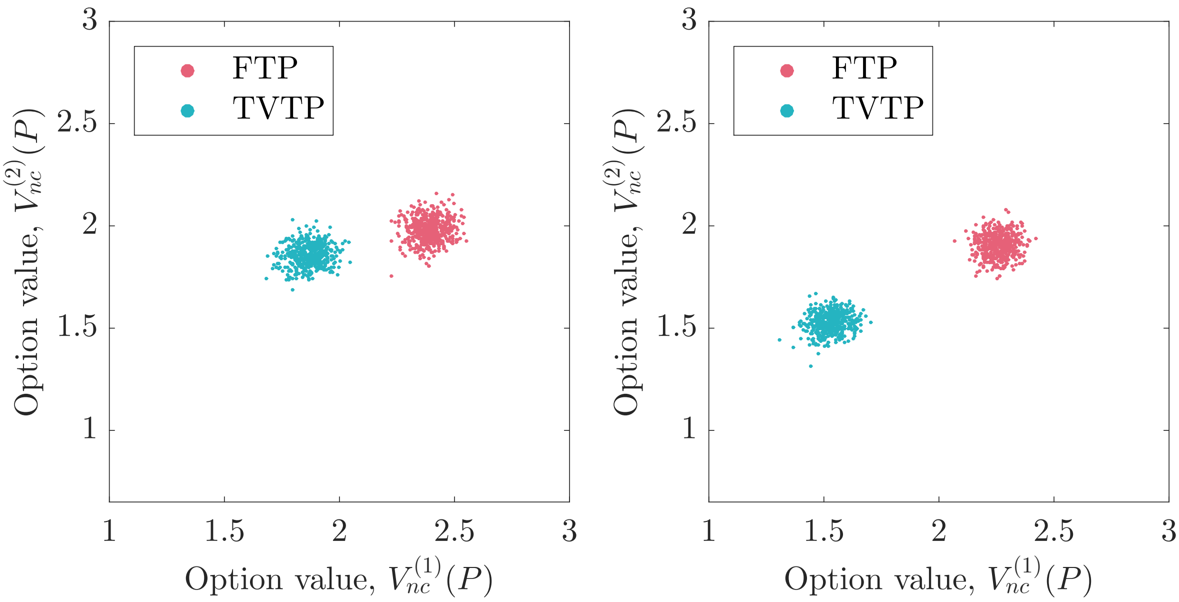

Figure 4.5: Option value using true model parameters (left panel) and estimated (right panel) with FTP and TVTP repeated one thousand times, $W = 0.5\frac{\varphi_1}{1-\varphi_2}$, $M = 1500$, $N = 2000$

4.7 Conclusions

We develop a real options framework in order to address the problem of optimal investment under time-varying regime uncertainty and how future expectations impacts the option value. Although empirical evidence suggests that transition probabilities are time-varying (Filardo, 1994; Aloui & Jammazi, 2009), valuation frameworks that address TVTP are limited. Hence, our simulation approach extends the real options literature by developing a technique to price options under TVTP. More specifically, we capture the TVTP through an exogenous process which determines the transition probabilities, and thus impacts the option value.

Our results indicate that, when there is a low forecasted likelihood of a regime switch, the initial state is more important, which causes an increase in project value for the first regime, while the second regime becomes less valuable. This is crucial for firms contemplating to invest in RE markets, where firms might have additional information indicating that markets are likely remain unchanged. In contrast, we find that increasing the likelihood of regime switching, reduces the impact of the initial state, and also that incorrectly assuming FTP can lead overvalued projects.

Apart from empirically tests of the model, further analysis for different processes governing the transition probabilities and the price process are interesting extension. For example, a mean-reverting process might be more suitable than a GBM for firms connected to commodity markets, or transition probabilities could be cyclical (Bazzi *et al.*, 2016). Also, the impact of the specific functional form for calculating the transition probabilities remains an interesting question for future work.

4.8 Appendix

A Benchmark Model

In scenario **ii.** ($p^{(1)} \leq P < p^{(2)}$), where the firm would wait if $\epsilon = 2$, but invest immediately if $\epsilon = 1$. Note that, we have already found the solution in the first regime in (8), while the ordinary differential equation (ODE) indicated in (A–1) describes the second regime.

$$\frac{1}{2}\sigma_2^2 P^2 \frac{dV^{(2)}(P)}{dP^2} + \mu_2 P \frac{dV^{(2)}(P)}{dP} - rV^{(2)}(P) + \lambda^{(2)} \left(G^{(1)}(P) - K - V^{(2)}(P)\right) = 0 \quad \text{(A–1)}$$

We adopt the same candidate function as Goto *et al.* (2017) for the second regime and the solution is indicated in (A–2). The roots of the quadratic $\frac{\sigma_2}{2}\delta\left(\delta - 1\right) + \mu_2\delta - \left(r + \lambda^{(2)}\right) = 0$ are δ_1 and δ_2.

$$V^{(2)}(P) = b_1 P^{\delta_1} + b_2 P^{\delta_2} + \frac{\lambda^{(2)} G^{(1)}(P)}{r + \lambda^{(2)} - \mu_2} - \frac{\lambda^{(2)} K}{r + \lambda^{(2)}} \quad \text{(A–2)}$$

Finally, we consider the third scenario, where the price is not high enough to warrant an investment independent of the current regime. The value function must then satisfy the ODEs indicated in (A–3).

$$\begin{aligned}
&\frac{1}{2}\sigma_2^2 P^2 \frac{dV^{(1)}(P)}{dP^2} + \mu_2 P \frac{dV^{(1)}(P)}{dP} - rV^{(1)}(P) + \lambda^{(1)}\left(V^{(2)}(P) - V^{(1)}(P)\right) = 0 \\
&\frac{1}{2}\sigma_2^2 P^2 \frac{dV^{(2)}(P)}{dP^2} + \mu_2 P \frac{dV^{(2)}(P)}{dP} - rV^{(2)}(P) + \lambda^{(2)}\left(V^{(1)}(P) - V^{(2)}(P)\right) = 0
\end{aligned} \tag{A–3}$$

The candidate function $V^{(\epsilon)}(P)$ is conjectured to be as outlined in (A–4),

$$\begin{aligned}
V^{(1)}(P) &= AP^{\gamma} \\
V^{(2)}(P) &= BP^{\gamma}
\end{aligned} \tag{A–4}$$

where A and B are constants to be determined. Substituting in the candidate solution (A–4) into (A–3), we obtain (A–5)

$$\begin{aligned}
&\left[-\frac{\sigma_1}{2}\gamma(\gamma-1) - \mu_1\gamma + \left(r + \lambda^{(1)}\right)\right] A = \lambda^{(1)} B \\
&\left[-\frac{\sigma_2}{2}\gamma(\gamma-1) - \mu_2\gamma + \left(r + \lambda^{(2)}\right)\right] A = \lambda^{(2)} B
\end{aligned} \tag{A–5}$$

where we can eliminate A and B from the two equations in (A–5), and obtain a fourth order polynomial as indicated in (A–6)

$$\left[\frac{\sigma_1}{2}\gamma(\gamma-1) + \mu_1\gamma - (r + \lambda_1)\right]\left[\frac{\sigma_2}{2}\gamma(\gamma-1) + \mu_2\gamma - (r + \lambda_2)\right] = \lambda^{(1)}\lambda^{(2)} \tag{A–6}$$

Since $\lim_{P\to 0} V^{(\epsilon)}(P) = 0$, we use only the two positive solutions, γ_1 and γ_2 from (A–6), and the solution is outlined in (A–7). Due to (A–5), the constants in (A–7) has to satisfy the following relationship $c_k^{(1)} = c_k^{(2)}\left[-\frac{\sigma_1}{2}\gamma_k(\gamma_k - 1) - \mu_1\gamma_k + \left(r + \lambda^{(1)}\right)\right]/\lambda^{(1)}$.

$$V^{(\epsilon)}(P) = c_1^{(\epsilon)} P^{\gamma_1} + c_2^{(\epsilon)} P^{\gamma_2} \tag{A–7}$$

Laguerre Polynomials

The Laguerre polynomials are given in (A–8) which are used to approximate the option value.

$$\begin{aligned}
L_0(X) &= 1 \\
L_1(X) &= -X + 1 \\
L_2(X) &= \frac{1}{2}\left(X^2 - 4X + 2\right) \\
L_3(X) &= \frac{1}{6}\left(-X^3 + 9X^2 - 18X + 6\right)
\end{aligned} \tag{A–8}$$

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