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Slow but Steady Wins the Race

An inquiry into the fundamental relation between systematic risk and stock returns with empirical evidence from the Oslo Stock Exchange in the period of 1990 – 2018

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Abstract

A value-weighted (equal-weighted) portfolio comprised of the twenty percent of the stocks on the Oslo Stock Exchange with the lowest beta each month produced cumulative excess returns of 1241% (692%) from 1990 to 2018. A value-weighted (equal-weighted) portfolio comprised of the twenty percent of the stocks on the Oslo Stock Exchange with the highest beta only generated cumulative excess returns of 6% (22%) over the same period.

The beta anomaly refers to the low (high) abnormal returns of stocks with a high (low) beta. In this thesis, we examine the presence of a beta anomaly on the Oslo Stock Exchange in the period of 1990 to 2018, and perform a replicating study of *A Lottery Demand-Based Explanation of the Beta Anomaly*, by Bali, Brown, Murray, and Tang, to investigate whether the notion of lottery demand - investors' disproportionately high demand for lottery-like stocks - can explain the beta anomaly. Our results demonstrate an economically large and statistically significant beta anomaly on the Oslo Stock Exchange relative to conventional asset pricing models. We also find that our proxy for lottery demand, a variable MAX, correlates negatively with future stock returns. However, our results do not support the postulation that lottery demand plays an important role in generating the beta anomaly on the Oslo Stock Exchange, and our conclusions thus deviate from those of the paper we replicate.

Preface

This thesis marks the end of our time at NHH, and it has certainly been a worthy last obstacle. The process of writing this thesis has been a humbling exercise in persistence, and we have developed a profound respect for the effort it takes to produce presentable and reliable results.

The thesis is an inquiry into the fundamental relation between risk and return on the Oslo Stock Exchange. As such, we believe (and hope) our work is of value and interest to both academics and market practitioners.

We would like to extend our gratitude to our supervisor, Jørgen Haug, for his apt comments and general guidance throughout the process of writing this thesis.

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Contents

1 Introduction

The high (low) abnormal returns of stocks with low (high) beta – commonly referred to as *the beta anomaly* – is the oldest and one of the most robust stock market anomalies documented in empirical asset pricing research. The anomaly has piqued the interest of many a researcher since the 1970's, when early empirical research by Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), and Haugen and Heinz (1975) revealed that the security market line was, in reality, flatter than predicted by the acclaimed CAPM. The finding is still considered anomalous as the positive beta-return relation predicted by the CAPM is embedded in most modern asset pricing models.

There is a plethora of research documenting the anomaly across geographies, time periods, and asset classes. Although the cause of the anomaly is highly debated in the international scientific community, there appears to be a broad consensus regarding its existence. Interestingly, the three studies examining the beta anomaly on the Oslo Stock Exchange (Frazzini & Pedersen, 2014; Juneja & Bordvik, 2017; Christensen, 2019) present conflicting results regarding its existence.

With this backdrop, we attempt to kill two birds with one stone in this thesis; to thoroughly probe into the existence of the beta anomaly on the Oslo Stock Exchange, and to test the proposed explanation for the anomaly which we find the most intuitively appealing – the lottery demand-based explanation of the beta anomaly.

As such, this thesis is both an inquiry into the existence of the beta anomaly on the Oslo Stock Exchange, and a replicating study of *A Lottery Demand-Based Explanation of the Beta Anomaly*, by Bali, Brown, Murray, and Tang (2017). The central postulation of their paper is that investors' demand for lottery-like stocks plays an important role in generating the beta anomaly. Their logical reasoning is as follows. Investors have a disproportionately high demand for stocks with a payoff structure resembling that of real lotteries, and such lottery-stocks are, for the most part, also high-beta stocks. Lottery investors should therefore exert a disproportionately high price pressure on high-beta stocks relative to low-beta stocks and thus contribute to generating the beta anomaly.

The postulation of Bali et al. (2017) is underpinned by three principal hypotheses, which we use in this thesis to test the lottery demand-based explanation of the beta anomaly. The three hypotheses we test are: (I) the beta anomaly is present in the Norwegian stock market, (II) there is a lottery demand phenomenon in the Norwegian stock market, and (III) lottery demand plays an important role in generating the beta anomaly. We replicate the study on a sample of all listed companies on the Oslo Stock Exchange in the period of 1985 to 2018.

We test **The Beta Anomaly (I)** by performing univariate portfolio analyses on monthly quintile portfolios sorted on an ascending ordering of the stocks' market beta. We demonstrate that a zero-cost portfolio with a long position in the low-beta quintile portfolio and a short position in the high-beta quintile portfolio ("low-high beta portfolio" hereafter) generates economically large and statistically significant positive abnormal returns relative to the CAPM. We also demonstrate that a value-weighted low-high beta portfolio generates economically large and statistically significant abnormal returns relative to the Fama and French (1993) and Carhart (1997) four-factor (FFC4) model augmented with the liquidity factor of Næs, Skjeltorp, and Ødegaard (2009) (FFC4 + LIQ). The abnormal returns are, however, statistically insignificant for the corresponding equal-weighted portfolio, and our robustness tests illustrate that the statistical significance of the beta anomaly relative to the FFC4 + LIQ model is sensitive to our choice of time period and data filters. Nevertheless, our combined results strongly indicate that low-beta stocks outperform high-beta stocks on a risk-adjusted basis in our sample, underpinning the existence of a beta anomaly on the Oslo Stock Exchange.

The Lottery Demand Phenomenon (II) refers to the high (low) abnormal returns of stocks that experience a low (high) amount of lottery demand-price pressure. We follow Bali et al. (2017) and use a variable MAX as a proxy for lottery demand. MAX is defined as the average of the five highest daily returns in the previous month. To test the lottery demand phenomenon on the Oslo Stock Exchange, we first evaluate whether MAX is an accurate proxy of lottery demand, and subsequently analyze the relation between MAX and one-month-ahead abnormal returns.

To assess whether MAX is a good proxy for lottery demand on the Oslo Stock Exchange, we measure the idiosyncratic volatility (IVOL), idiosyncratic skewness (ISKEW), and the stock price (PRICE) for each MAX-sorted quintile portfolio. We find that quintile portfolios constructed to be monotonically increasing in MAX are also monotonically increasing in IVOL and ISKEW, and monotonically decreasing in PRICE. We conclude that MAX effectively captures the lottery-stock characteristics put forth by Kumar (2009).

As with the beta anomaly, we test the relation between MAX and future abnormal returns by constructing monthly quintile portfolios based on an ascending ordering of MAX. We demonstrate that a zero-cost portfolio with a long position in the low-MAX quintile portfolio and a short position in the high-MAX quintile portfolio ("low-high MAX portfolio" hereafter) generates statistically significant abnormal returns relative to the FFC4 + LIQ model. The results are generally robust to variations in data filters but not across different time periods. Our results thus strongly indicate that there is a statistically significant negative relation between MAX and future abnormal returns in our sample. Still, the ambiguous results from our robustness tests prevent us from concluding with great certainty.

We test the **Lottery Demand-Based Explanation of the Beta Anomaly (III)** by analyzing the returns of the beta-sorted portfolios controlling for MAX using three different methodologies. In general, we find limited evidence suggesting that lottery demand, as measured by MAX, plays an important role in generating the beta anomaly in our sample. A bivariate portfolio analysis demonstrates that controlling for MAX has a limited impact on the abnormal returns of the low-high beta portfolio, and a univariate portfolio analysis sorting on the portion of beta that is orthogonal to MAX yields similar results. We find that in three out of the four conducted tests in the univariate and bivariate portfolio analyses, the abnormal returns of the low-high beta portfolio remain statistically significant despite the portfolio being neutralized to MAX.

By augmenting the FFC4 + LIQ factor model with a lottery demand factor FMAX, we find that the abnormal returns of the low-high beta portfolio are no longer statistically significant. However, the abnormal returns remain economically large, and we demonstrate that an IVOL factor constructed analogously to the FMAX factor is equally capable of explaining the abnormal returns associated with the beta anomaly as FMAX. Seen in conjunction with the results from the bivariate portfolio analysis and the univariate portfolio analysis sorting on the component of beta that is orthogonal to MAX, we find that our analyses do not provide any conclusive evidence in favor of the lottery-demand explanation of the beta anomaly.

When we reverse the roles of MAX and beta, we find that the low-high MAX portfolio no longer generates statistically significant abnormal returns when the portfolio is constructed to have a neutral exposure to beta. The results demonstrate that the documented negative relation between MAX and abnormal returns in our sample cannot necessarily be attributed to investor demand for lottery-like assets. Consequently, the statistically significant abnormal returns of the low-high MAX portfolio in our sample cannot be interpreted to illustrate a statistically significant lottery demand phenomenon on the Oslo Stock Exchange.

We contribute to the existing literature in two principal ways. Firstly, by replicating the study of Bali et al. (2017) on a Norwegian sample, we provide an out-of-sample robustness test of their results, which we believe could prove important in generalizing the findings in the paper. Secondly, we expand the study of Bali et al. (2017), most notably by analyzing the long-term cumulative returns of portfolios sorted on beta and MAX and assessing the trading costs associated with investing in the portfolios. However, we also deviate by providing additional robustness tests, particularly by assessing the strong observed correlation between IVOL and the proxy for lottery demand, MAX.

From a personal standpoint, we find the topic interesting as the studies on the beta anomaly in Norway offer conflicting conclusions, and a proof of its existence could potentially alter investors' perception of the beta-return relation in Norway. Furthermore, we believe that shedding light on the performance of lottery-like stocks could be a wake-up call for many Norwegian retail investors. Stock discussions among retail investors, for instance, in the school cafeteria or in Norwegian online forums, often revolve around stocks with lottery traits.

The remainder of the thesis is organized as follows. Section 2 lays out relevant asset pricing theory and presents existing literature on the beta anomaly. Section 3 describes our data and the adjustments we have made. Section 4 introduces the variables and presents the methodology used to estimate them. Section 5 describes our empirical methodology, and section 6 presents our results from testing hypothesis I-III. Section 7 discusses limitations to our paper and brings suggestions for further research. Section 8 concludes.

2 Theory and Literature Review

The purpose of this section is to provide the reader with the necessary theoretical foundation to interpret our results and give an overview of the existing literature on the topic. The section encompasses relevant asset pricing theory, literature on the existence of the beta anomaly and the proposed explanations for it, as well as an introduction to the lottery demand-based explanation of the beta anomaly.

2.1 A Brief Introduction to Asset Pricing

In general, the word *anomaly* means a deviation from the common rule. As such, an asset pricing anomaly refers to an observed deviation from conventional asset pricing models and theory. Since this paper is on the beta anomaly and its explanations, we begin this section by giving a brief introduction to the history and theory[1] behind the asset pricing models discussed in this thesis.

Modern Portfolio Theory and Tobin's Separation Theorem

All neoclassical equilibrium asset pricing models build upon the seminal work of Markowitz and Tobin. Markowitz (1952) recognized how cross-sectional correlation in stock returns affects the variance of a portfolio and was the first to entertain the notion of a mean-variance efficient frontier of stock portfolios. His *Modern Portfolio Theory* proposes that all portfolios but those on the frontier are inferior, and that all risk-averse, mean-variance optimizing investors select portfolios along the frontier corresponding to their risk preferences.

Tobin (1958), through his *Separation Theorem*, introduced a risk-free asset to the investable universe of Markowitz. The inclusion of a risk-free asset implies that investors can scale the risk-return relation of any portfolio to suit their risk preferences. As such, all investors select the risky portfolio on Markowitz's efficient frontier with the highest return per unit of risk – the tangency portfolio – and borrow or lend at the risk-free rate to achieve the desired portfolio risk-return relation. The resulting risk-return relation faced by investors in the market can be expressed by the acclaimed *Capital Market Line* (CML), illustrated in figure 1 and given by

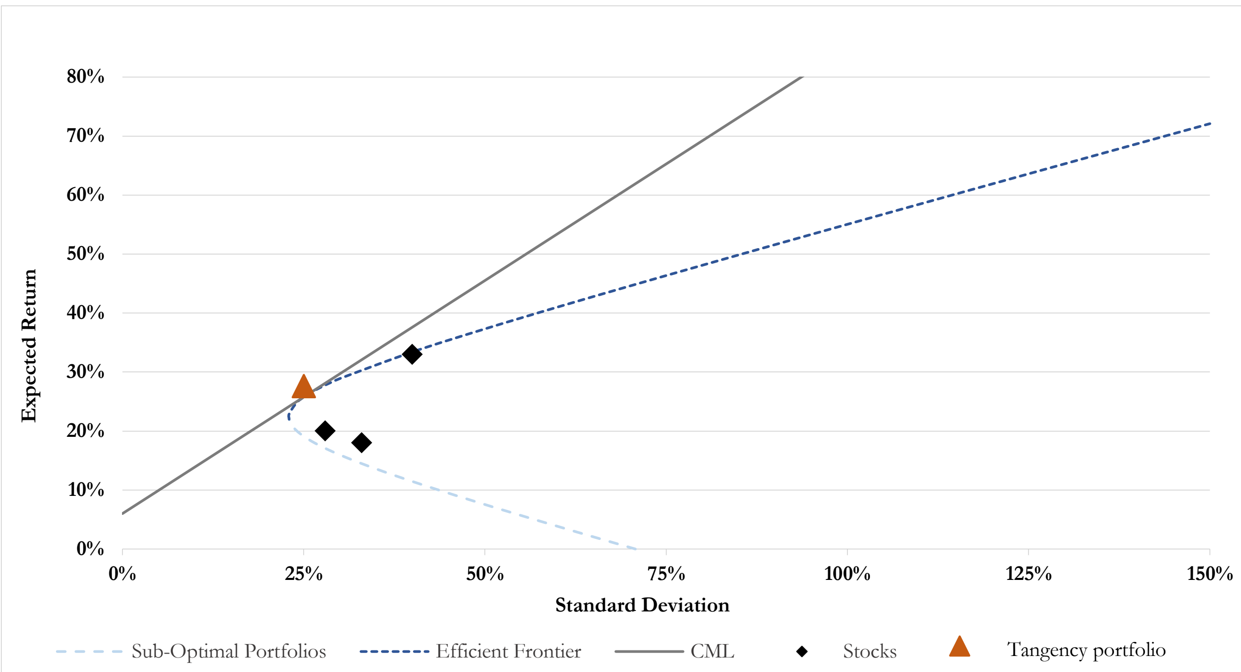
$$r_p - r_f = \left(r_{p*} - r_f\right) * \frac{\sigma_p}{\sigma_{p*}}$$

[1] The general theory on asset pricing models is primarily based on Body, Kane and Marcus (2014)

where subscript p refers to any efficient, investable portfolio (i.e., any combination of the tangency portfolio and the risk-free asset), and subscript $p*$ refers to the tangency portfolio. Hence, the risk premium of an efficient portfolio is equal to the risk premium of the tangency portfolio, multiplied by the variance ratio of the efficient portfolio to the tangency portfolio. The CML depicts the expected return and standard deviation of all combinations of the tangency portfolio and the risk-free asset, and its slope represents the Sharpe ratio of the tangency portfolio.

Figure 1: Illustration of the Efficient Frontier, the CML and the Tangency Portfolio

The figure illustrates Markowitz' efficient frontier of stock portfolios, the Capital Market Line (CML) and the tangency portfolio in a fictive market consisting of three stocks.



The Capital Asset Pricing Model

The combined work of Markowitz and Tobin constituted a paradigm shift in finance. However, despite its theoretical elegance, the CML can only be used to price efficient portfolios. It is thus of little use in explaining cross-sectional differences in individual stock returns. In this regard, the CAPM was the pioneer (Sharpe, 1964; Lintner 1965; Mossin 1966). The CAPM builds on the theoretical foundation of Markowitz and Tobin and further impose the assumptions that all investors share the same investment universe and have a homogeneous market view. If these assumptions hold, the CAPM predicts that all investors would generate the same efficient frontier and face the same optimal risky portfolio. When faced with the same optimal risky portfolio, all investors would hold the same portfolio weights in risky assets. The efficient portfolio must then be the value-weighted portfolio of all the assets in the investable universe – the market portfolio. Thus, in pricing a single stock, the appropriate risk to consider for an investor is the additional risk the security contributes to the market portfolio if included. This incremental increase in portfolio risk is measured by a stock's beta. Ultimately, the risk premium, $E[R_i]$, of an asset i can be expressed by

$$E[R_i] = \beta_i E[R_m]$$

where,

$$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)}$$

The equation above states the CAPM on its unconditional form.[2] It implies that the risk premium of a single asset should be a positive, linear function of its sensitivity towards the market risk premium as measured by beta.

The core strength of the CAPM is that it is theoretically consistent under its assumptions. It provides an intuitive framework on how assets should be priced in market equilibrium and offers an explanation to why market risk should be the only priced factor. However, as demonstrated in numerous empirical studies since the middle of the 1970s, the risk-return relation observed empirically is not in line with the CAPM's predictions.[3]

Arbitrage Pricing Theory (APT)

Interestingly, the CAPM's empirical inadequacies sparked an ever-growing body of research dedicated to exploring effects in cross-sectional stock returns that violate the CAPM's basic tenets. One of these efforts was Ross' (1976) Arbitrage Pricing Theory (APT), developed as an alternative to the empirically flawed CAPM. The central intuition behind APT is that several systematic risk factors influence long-term stock returns, and that expected stock returns can be described by a linear combination of the stocks' sensitivity towards these risk factors if all arbitrage opportunities are fully exploited. In general, the expected excess return of stock i under APT can be expressed by

$$E[R_i] = \sum_{k=1}^{n} \beta_{i,k} F_k$$

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[2] The graph representation of the unconditional CAPM is often referred to as the security market line (SML).

[3] The early empirical evidence against the CAPM is presented in section 2.2.

where F_k represents the risk premium of systematic risk factor k, and $\beta_{i,k}$ represents stock, i's sensitivity towards risk factor k. At first glance, the general APT model appears very similar to the CAPM. The only noticable difference is that APT allows for several systematic risk factors. However, the models differ significantly in their theoretical foundation as the CAPM is an equilibrium model, while the APT is underpinned by a no-arbitrage assumption.

The CAPM assumes all market participants to be risk-averse, mean-variance optimizers. Consequently, in the event of a mispricing, all market participants slightly tilt their portfolios to alter their exposure to the mispriced security and the market returns to equilibrium. APT, on the other hand, is based on the law of one price, and as Ross (1976) puts it, "*is much more an arbitrage relation than an equilibrium condition […]*" (p. 355). APT solely relies on three postulations (Bodie, Kane, & Marcus, 2014): (i) stock returns can be described by a factor model, (ii) the investable universe is vast enough to diversify away idiosyncratic risk, and (iii) all arbitrage-opportunities are fully exploited. Underpinned by these assumptions, the APT ensures fair pricing through the following logic. If all investors are well-diversified, there is no exposure to the idiosyncratic risk of any single security, and the only relevant risk exposure of an investor is his exposure to the systematic risk factors. Consequently, provided that the relevant systematic risk factors in explaining future stock returns are known in the market and all arbitrage opportunities are exploited, it follows from the law of one price that two stocks with the same risk factor sensitivity must be priced equally. If not, it is possible to construct a replicating portfolio with the same factor exposure, but at a different price than the mispriced asset, and by constructing a zero-cost, long-short portfolio, it is possible to generate arbitrage profits on the mispricing.

Although APT is useful in the sense that it is light in the assumptions, it does not provide any guidance to which systematic risk factors that are relevant in explaining expected returns. The dominant approach in determining these risk factors in practice is through empirical analysis of company-specific characteristics as proxies for systematic risk factors. The renowned Fama and French 3-Factor model (FF3) (Fama & French, 1993) is one example of such an APT-based model. The model gained its wide popularity by demonstrating that by adding the two factors *small minus big (SMB)*[4] and *high minus low*

[4] The SMB factor is based the size effect (Banz, 1981) - that small firms offer higher risk-adjusted returns than large firms.

(HML)[5] to the unconditional CAPM, the model's ability to predict cross-sectional variation in stock returns increased substantially (Dimson & Mussavian, 1998).

The strength of the APT-based models is thus that they offer increased precision in predicting cross-sectional differences in stock returns compared to equilibrium models, such as the CAPM (Dimson & Mussavian, 1999). However, what the APT-based models make up in predictive power, they lack in theoretical foundation. There is no underlying theoretical foundation explaining why any of the conventional factors should be related to returns.

A stock market anomaly is merely a reflection of an asset pricing model's inability to explain cross-sectional differences in stock returns. As such, when we evaluate our results against commonly accepted asset pricing models, we are, in a broader sense, solely testing the validity of the applied models. At this point in time, it appears to be no common agreement on which factors are the "true" factors in the APT-based models and, as such, we must make do with what is common in the asset pricing literature. However, the continuous discoveries of deviations from conventional models and theory prove that there is still headroom in better understanding what drives stock returns. This paper aims to investigate one of the oldest, most disputed anomalies – the beta anomaly.

2.2 An Overview of Low-Risk Anomalies

Although the focus of this paper is on the beta anomaly, we deem it important to clarify the position of the beta anomaly within the vast universe of stock market anomalies. The beta anomaly falls under the broader category of low-risk anomalies – a body of anomalies directly related to the risk-return relation in stock returns. The low-risk anomaly (or equivalently, the low-risk effect) comprises the empirical finding that an investor's increased exposure to volatility does not command a greater risk-adjusted return as predicted by conventional asset pricing models.

However, the volatility of a stock can be defined as the sum of the stock's systematic risk and idiosyncratic risk. As such, the research streams on the topic of low risk anomalies has developed into three separate, yet interrelated anomalies: the total volatility anomaly, the idiosyncratic volatility anomaly, and the beta anomaly. In the following, we briefly introduce the total volatility anomaly and the idiosyncratic volatility anomaly, but the primary focus of the next section lies on the beta anomaly and its explanations.

[5] The HML factor is based on the finding of Fama and French (1992) - that value stocks (stocks with a high book-to-market ratio) outperform growth stocks (stocks with a low book-market ratio).

The Total Volatility Anomaly

The total volatility anomaly refers to the positive (negative) abnormal returns of stocks with low (high) total volatility. The initiation of the research stream on the topic was highly motivated by the empirical findings of Clarke, de Silva & Thorley (2006) which revealed that the returns of minimum variance portfolios constructed on US data in the period of 1968-2005 generated comparable returns as the market portfolio, but with a standard deviation 25% lower than that of the market. With this backdrop, Blitz and van Vliet (2007) examined the performance of decile portfolios sorted by three-month historical return volatility on a global data sample, spanning from 1986 to 2006. They document a positive, economically large and statistically significant alpha spread between the two extreme volatility-sorted decile portfolios (low-risk portfolio minus high risk-portfolio) relative to the CAPM and the FF3 model in the US, Europe, and Japan. The findings are supported by several more recent studies; Baker and Haugen (2012) document statistically significant differences in returns and Sharpe ratios between quintile portfolios sorted by total volatility across 33 different markets from 1990 to 2011, and Blitz, Pang and van Vliet (2013) document the total volatility anomaly in a sample comprised of observations from 30 emerging equity markets.

The Idiosyncratic Volatility (IVOL) Anomaly

The IVOL anomaly refers to the high (low) risk-adjusted returns of stocks with low (high) idiosyncratic volatility. This empirical finding is considered anomalous as it contradicts the fundamental assumption of the CAPM that all market participants hold the optimal, well-diversified portfolio and should thus not be compensated (or penalized) for exposure to idiosyncratic risk.

The finding was formalized by Ang, Hodrick, Xing, and Zhang (2006) in the highly influential[6] paper *The Cross-Section of Volatility and Expected Returns*. The main sample in the paper comprises all US stocks listed on NYSE, NASDAQ, and AMEX from 1963 to 2000. To demonstrate the anomaly, the stocks are sorted into quintile portfolios by IVOL each month, and the difference in one-month-ahead abnormal returns between the high-IVOL portfolio and the low-IVOL portfolio is calculated. Evidently, the long-short portfolio with a long position in the high-IVOL portfolio and a short position in the low-IVOL portfolio generates a statistically significant negative monthly alpha of 1.19% relative to the FF3 model. The results alone constitute a powerful manifestation of the IVOL anomaly, and the paper marked the beginning of an entire stream of research within IVOL anomalies.

[6] The paper ranks 20th on the list of the most cited articles published in *The Journal of Finance* of all time.

The Beta Anomaly

The beta anomaly refers to the high (low) abnormal returns of stocks with low (high) beta. Not only is the beta anomaly the oldest stock market anomaly, it is also widely considered as one of the greatest anomalies in finance as it challenges the very core of the CAPM. The anomaly was discovered during the first empirical tests of the CAPM in the 1970's when Black, Jensen, and Scholes (1972), Fama and MacBeth (1973) and Haugen and Heinz (1975) found that the relation between beta and returns in the stock market was flatter than the CAPM's predictions.[7] Nevertheless, the CAPM's notion that higher systematic risk commands a greater return stayed conventional knowledge until the nineties when Fama and French (1992) discovered that beta was largely unpriced in the market when controlling for size. Following this finding is an extensive body of literature documenting the beta anomaly across geographies, time periods and asset classes (Rouwenhosrst, 1999; Blitz & van Vliet, 2007; Baker, Bradley & Wurgler, 2011; Blitz, Pang & van Vliet, 2012; Baker, Bradley & Talifeiro, 2014; Frazzini & Pedersen, 2014; Bali, Brown, Murray & Tang, 2017).

To the best of our knowledge, there are only three studies commenting on the existence of the beta anomaly in Norway, of which two are master theses. Frazzini and Pedersen (2014) document the abnormal returns of their betting-against-beta (BAB) strategy across 20 countries in the period of 1984 to 2009 and find no evidence of a statistically significant beta anomaly in Norway. Juneja and Bordvik (2017) investigate the beta anomaly in the Norwegian market in the period 1986-2014 and argue that there is no beta anomaly in Norway relative to the unconditional CAPM. Christensen (2019), in a master thesis investigating the suitability of mispricing models on the Oslo Stock Exchange, finds a statistically significant beta anomaly relative to various pricing models in the period 1998-2018. However, he does not make further comments on the finding, as this was not the purpose of the paper.

2.3 Explanations of the Beta Anomaly

In this thesis, we test the lottery-demand based explanation of the beta anomaly in the Norwegian stock market. However, ever since the discovery of the anomaly in the mid-1970s, the anomaly has been attributed to numerous other explanations than that of investors' demand for lottery-like stocks.

[7] More specifically, when examining the empirical fit of the security market line, Black, Jensen, and Scholes (1972) found that the return of zero-beta stocks were higher than the risk-free rate (i.e., the intercept of the CAPM-implied SML was too low), and that a higher beta commanded a lower increase in return than predicted by the CAPM (i.e., the CAPM-implied SML was too steep).

This section first introduces the reader to what we believe are the most recognized suggested explanations for the anomaly, followed by a review of the key findings in the paper we replicate.

Leverage Constraints

Black (1972, 1993), who co-wrote the paper first documenting the anomaly, was also the first to hypothesize that the beta anomaly may be due to leverage constraints among market participants. Most notably, he theoretically showed that in the presence of leverage constraints, the linear relation between systematic risk and return would be flatter than predicted by the unconditional CAPM.

According to the CAPM, all market participants hold the efficient portfolio and lever (or de-lever) this portfolio in accordance with their risk preferences. In the presence of leverage constraints, however, an investor requiring expected returns in excess of the efficient portfolio returns has no other option but to deviate from the efficient portfolio weights and disproportionately allocate capital to high-beta stocks. Frazzini and Pedersen (2014) argue that several types of large institutional investors are subject to inflexible investment mandates, and consequently constrained in the amount of leverage they can use. This results in a disproportionately high demand price-pressure being exerted on high-beta stocks, which subsequently decrease (increase) the future return of high-beta (low-beta) stocks. Frazzini and Pedersen (2014) attribute the observed inverted beta-return relation in the stock market to the abovementioned effect.

Constraints on Short Selling

Given the long-standing empirical evidence of the beta anomaly, one would expect capable investors to exploit the mispricing and drive the risk-return relation closer to an equilibrium. After all, why would unconstrained arbitrageurs let such an opportunity pass?

Baker, Bradley, and Wurgler (2011) show that the stocks comprising the most volatile portfolios tend to be small and illiquid. Such stocks are expensive to trade, and particularly expensive to short-sell, due to substantial borrowing costs and a lack of stocks available for borrowing. Furthermore, Novy-Marx and Velikov (2018) show that Frazzini and Pedersen's (2014) BAB strategy – which is designed to capitalize on the beta anomaly – allocates close to 40 cents of every dollar invested in the short portfolio to stocks in the smallest decile in terms of market capitalization.[8] Even though the strategy produces impressive profits on paper, exploiting the anomaly is far from trivial in the real world.

[8] Listed stocks on AMEX, NASDAQ, and NYSE grouped into deciles by NYSE decile size breaks.

Considerable trading costs might deter investors from *Betting Against Beta*, and the distorted risk-return relation may prevail in the markets.

Benchmarking

Although constraints on short selling is an intuitive explanation for the dismal return of high-beta portfolios, the explanation falls short in explaining why market participants refrain from overweighting low-beta stocks given their superior performance. In this regard, Baker et al. (2011) note that large institutional investors are predominantly evaluated based on relative performance measures[9], as opposed to absolute returns. One example of such a relative performance measure is the information ratio (IR), which is defined as

$$IR = \frac{Portfolio\ return - Benchmark\ return}{Tracking\ error}$$

where *benchmark return* refers to the return of the benchmark the fund manager is evaluated against, and *tracking error* is the standard deviation of the return difference between the portfolio and the benchmark. In short, if an investor seeking to maximize IR is unable to enter levered positions, which is the case for many mutual funds and pension funds, allocating capital to low-beta stocks often lead to a greater increase in tracking error than benchmark outperformance. This discourages fund managers to disproportionately allocate capital to low-beta stocks as this would lead to a lower IR even though portfolio alpha would increase.

Coskewness as a Priced Factor

The CAPM assumes that investors only care about the mean and variance of the return distribution, which implies that investors deem higher-order distribution moments irrelevant in explaining equity returns. However, Schneider, Wagner, and Zechner (2017) offer a conflicting view. They assume that investors require compensation for exposure to negative coskewness and demonstrate that CAPM alphas of beta-sorted portfolios are directly related to residual coskewness risk. Consequently, in their view, factor model alphas of volatility sorted portfolios – the beta anomaly – is merely a reflection of compensation for coskewness risk. As expected residual coskewness is impossible to observe ex-ante for market participants, the authors use option-implied ex-ante skewness to proxy for expected coskewness. The proxy is rather successful, and they demonstrate empirically that factor models

[9] According to Sensoy (2009), 61.3% of US mutual funds are benchmarked to the S&P500 while 94.6% are benchmarked to a popular US index.

accounting for ex-ante skewness erode the CAPM alphas of volatility-sorted portfolios and the *BAB* factor. However, the validity of their postulation depends entirely on the assumption that coskewness is priced in the market.

IVOL

Analogous to the beta anomaly, the IVOL anomaly has withstood numerous empirical tests across geographies and time periods. Liu, Stambaugh, and Yuan (2018) suggest that the two anomalies are intertwined in the form that the IVOL anomaly is the driver behind the beta anomaly. The empirical foundation for their view is that the beta anomaly is non-existent when controlling for IVOL, while the IVOL anomaly persists after controlling for beta. Additionally, they analyze this finding further by segmenting this effect into over- and underpriced stocks (equivalent to Stambaugh, Yu & Yuan, 2015) and find that the relation between IVOL and returns is positive for underpriced stocks, while the opposite is documented for overpriced stocks. This leads to the conclusion that the beta anomaly is only present in periods of high cross-sectional correlation between beta and IVOL, and in periods when the market is overpriced. Furthermore, they attribute the general prevalence of the IVOL anomaly to investors being relatively less able (or willing) to short overpriced stocks than they are able to enter long positions in underpriced stocks, leading to a negative IVOL-return relation in the stock market as a whole.

Bruno and Haug (2018) offer a more theoretically sophisticated explanation for the IVOL anomaly. They prove mathematically that equity IVOL should be negatively correlated with expected equity returns, both in the cross-section and in the time series. The technical reason is that equity returns and equity IVOL have opposite responses to increases in asset IVOL. The proof is rooted in the law of one price and the portfolio view of equity. According to the law of one price, excess equity returns should be proportional to excess asset returns, with the constant of proportionality being the elasticity of equity with regards to assets. The elasticity is a function of several factors, of which the most notable are leverage and asset IVOL. As elasticity decreases in asset IVOL, the relation between excess equity returns and asset IVOL is negative. On the other hand, equity IVOL is proved to be positively correlated with asset IVOL. Consequently, variation in asset IVOL across stocks induces a negative relation between equity IVOL and equity returns in the cross-section.

It is important to note that Bruno and Haug (2018) do not suggest that their finding on the IVOL-return relation is an explanation for the beta anomaly. However, in line with the postulation of Liu et al. (2018), if IVOL is highly correlated with beta in the cross-section, factor model alphas generated by beta-sorted portfolios could exist due to the IVOL-return relation described by Bruno and Haug (2018).

Lottery Demand

An intuitive and appealing explanation of the beta anomaly is that of investors' demand for lottery-like stocks. Lottery behavior is a classic example of human contradiction of rationality - the expected value of buying a lottery ticket is never positive. Yet however, the international gambling industry generates a gross gaming yield[10] in excess of USD 450bn annually (Statista, 2018).

In 2002, Daniel Kahneman received the Nobel Prize for his research on behavioral economics, and in particular, for his work on the *Cumulative Prospect Theory* (CPT) (Kahneman & Tversky, 1992). Simply put, the CPT is a synthesis of several observed human biases transformed into a decision-making model under risk. Consequently, it can be considered as an alternative to, and a disproof of, the use of expected value as the rule in human decision making. To avoid diving into the CPT in its entirety, we only consider the observed human bias that people tend to overweight small probabilities and thus have a predisposed inclination to engage in activities with a small probability of a large payoff, regardless of if the expected value is negative. Examples of such behavior are the purchase of excess insurance or lottery tickets.

Several researchers have proposed that this inclination affects investor behavior in the stock markets to the extent that it influences asset prices. Barbaris and Huang (2007) argue that this inclination is inconsistent with the notion that investors are strict mean-variance optimizers. They argue that under CPT, investors also consider positive skewness a desirable trait, and argue that this can cause overpricing in positively skewed stocks. Similarly, Mitton and Vorkink (2007) attribute portfolio underdiversification – which itself is a contradiction of mean-variance optimization - to preferences for skewness among investors.

Kumar (2009) argues that lottery-stocks must have a payoff structure resembling that of real lotteries. He infers that lotteries are cheap to enter, exhibit high variance in the distribution of the payoffs, and involve a small probability of a large payoff. As such, Kumar (2009) defines lottery-stocks as stocks with high IVOL (volatile payoff distribution), high skewness (possibility of a large payoff) and a low stock price (cheap entry). He underpins his postulation by showing that that people who exhibit

[10] Gross gaming yield is defined as the amount retained by operators after the payment of winnings but before the deduction of the costs of the operation (Statista, 2019).

gambling behavior outside of the stock market invest disproportionately in lottery-stocks compared to other individual investors and institutional investors. Kumar's (2009) definition of lottery-stocks has generally been accepted by academics further exploring the lottery effect, such as Bali, Cakci, and Whitelaw (2011) and Han and Kumar (2013).

2.4 Dissecting the Paper – The Key Findings of Bali et al. (2017)

Although there are several efforts to link lottery-behavior to asset prices, there is, to our knowledge, only one paper directly relating it to the beta anomaly. The paper, which we base a significant part of our thesis on, *A Lottery Demand-Based Explanation of the Beta Anomaly*, by Bali et al. (2017), postulates that lottery demand plays an important role in generating the anomalous beta-return relation observed in the markets since the 1970s. The study is conducted using data on all publicly listed stocks in the US from August 1963 through December 2012.[11] The central logical reasoning underpinning their paper can best be expressed in the words of the authors:

"[…] lottery investors generate demand for stocks with high probabilities of large short-term up moves in the stock price. Such up moves are partially generated by a stock's sensitivity to the overall market – market beta. A disproportionately high (low) amount of lottery demand-based price pressure is therefore exerted on high-beta (low-beta) stocks, pushing the prices of such stocks up (down) and therefore decreasing (increasing) future returns." (p. 1)

To test this notion, they formulate and test three different hypotheses. Below follows a summary of their key findings under each hypothesis and an explanation as to why the hypothesis is relevant for testing their main problem statement.

1. The beta anomaly is prevalent in the US stock market. In order to recommend lottery demand as the preferred explanation for the beta anomaly, it is imperative to first prove the existence of the anomaly. By constructing beta-sorted decile portfolios and generating a zero-cost portfolio with a long position in the high-beta decile portfolio and a short position in the low-beta decile portfolio, the authors successfully generate an economically large and statistically significant beta anomaly relative to conventional asset pricing models. This result is hardly surprising. The anomaly has been documented in the US market across numerous studies since the 1970's.

2. There is a "lottery demand phenomenon" in the US stock market. The lottery demand phenomenon refers to the low (high) abnormal returns of stocks that experience a high (low) amount

[11] To evaluate the robustness of their results, Bali et al. (2017) also conduct additional analyses on an extended sample from 1931 through 2012.

of lottery demand-price pressure. The testability of this hypothesis thus relies on two critical assumptions: (i) an accurate proxy for lottery demand exists, and (ii) future abnormal returns and lottery demand have opposite responses to an increase in the proxy (i.e. lottery demand is negatively related to future stock returns).

The authors follow Bali et al. (2011) and use a variable MAX as a proxy for lottery demand. MAX is defined as the average of the five highest daily returns in the previous month. A zero-cost portfolio with a long position in a high-MAX decile portfolio and a short position in a low-MAX decile portfolio yields economically large and statistically significant negative abnormal returns relative to conventional asset pricing models. In other words, MAX is negatively correlated with future stock returns. The results are robust to various robustness tests, such as variations in how MAX is measured. The statistically significant alpha produced from the long-short MAX portfolio is hereafter referred to as the "lottery demand phenomenon."

The finding that MAX can neutralize the beta anomaly is not itself a proof of validity for the lottery demand-based explanation. The missing piece is that MAX must be an accurate proxy of lottery demand. The authors argue that since MAX is positively correlated with IVOL and idiosyncratic skewness, and negatively correlated with stock price - the three lottery-traits put forth by Kumar (2009) – MAX is an accurate measure of lottery demand. As an additional argument to support their conclusion to the hypothesis, the authors also examine the beta anomaly and the lottery demand phenomenon while controlling for the degree of institutional ownership. They find that the abnormal returns of zero-cost beta-sorted decile portfolios and zero-cost MAX-sorted decile portfolios are statistically insignificant in stocks with a high degree of institutional ownership and highly statistically significant in stocks with a low degree of institutional ownership. This test is based on the notion that behavioral biases (such as lottery behavior) are primarily concentrated among individual investors, as suggested by Kumar (2009).

3. Lottery demand, as measured by MAX, neutralizes the beta anomaly. This hypothesis tests the very heart of the paper and can thus be regarded as the most crucial hypothesis. Without digging into the details of their methodology, the authors use various tests to demonstrate that MAX neutralizes the beta anomaly. More specifically, they show that the anomaly is no longer present when beta-sorted portfolios are neutralized to the MAX variable, regression specifications control for MAX, or factor models include a MAX-factor. To demonstrate the explanatory power of MAX relative to other factors, the authors show that the beta anomaly is robust to controlling for other factors known to be correlated with future stock returns, such as the three Fama and French (1993) factors, IVOL and idiosyncratic skewness.

3 Data

This section presents our main stock sample, our sources of data, and the adjustments made.

3.1 Stock Sample

Our main source of data is *Børsprosjektet* at NHH. *Børsprosjektet* contains stock data on all publicly listed companies in Norway from January 1980 through December 2018. However, the data sample used in this paper begins on January 1st, 1985. The shortening of the dataset is a consequence of unsatisfactory data quality in the database prior to 1985; observations on returns and the number of shares outstanding were missing for a significant part of the sample.

We retrieve both daily and monthly stock data. The resulting dataset contains observations for 885 different stocks over a period of 34 years. Each stock is assigned a unique identifying code, which corresponds to the variable *SecurityId* in the database. We have used the variable *Generic* for data on stock prices and *ReturnAdjGeneric* for stock returns. *Generic* is a collective variable equal to the latest available daily *last price*. The variable *last price* is only available on days the stock has been traded, and the *Generic* variable thus reflects the last available daily closing price. *ReturnAdjGeneric* computes the simple nominal returns adjusted for dividends, stock splits and reverse splits. Furthermore, the variables *SharesIssued* and *OffShareTurnover* are used to obtain the number of shares outstanding and the number of officially traded shares for the period, respectively. It is worth noting that observations with no official turnover will have no *last price*, and *ReturnAdjGeneric* will therefore equal zero in these instances. Twenty-five percent of the daily return observations in the dataset are equal to zero due to no official turnover. Summary statistics for the mentioned variables in the pre-filtered dataset are presented in Table 1.

Table 1: Summary Statistics of Unfiltered Data

The table presents summary statistics for the variables in our unfiltered dataset. The data sample was retrieved from NHH's *Børsprosjektet* and covers the period from Jan. 1985 through Dec. 2018. Panel A presents summary statistics for the daily data while Panel B presents summary statistics for the monthly data. *Generic* is a collective variable equal to the last available daily closing price. *ReturnAdjGeneric* computes the nominal simple returns adjusted for dividends, stock splits and reverse splits. *MCAP* equals the securities' market capitalization computed as the product of *SharesIssued* and *Generic*. *ShareIssued* represents the number of outstanding shares, while *OffShareTurnover* equals the number of officially traded shares for the given day (month) in the daily (monthly) dataset.

Panel A: Daily Data

Variable	N	Mean	SD	Min	Max
Generic	1 669 695	108.87	365.14	0.02	24 000.00
ReturnAdjGeneric	1 669 695	0.00	0.04	-0.97	14.00
SharesIssued	1 669 694	101 402 586	364 085 777	0.00	20 640 180 097
MCAP	1 669 694	4 699 603 509	22 583 178 786	0.00	682 689 344 752
OffShareTurnover	1 246 260	634 391	4 832 068	1	1 576 555 064

Panel B: Monthly Data

Variable	N	Mean	SD	Min	Max
Generic	82 221	110.29	385.44	0.02	23 000.00
ReturnAdjGeneric	81 515	0.01	0.17	-0.97	8.24
SharesIssued	82 428	99 501 088	358 422 356	0.00	20 272 457 825
MCAP	82 221	4 616 019 303	22 346 483 092	0.00	631 352 126 394
OffShareTurnover	79 082	9 683 428	57 133 033	-1 946 722 005[I]	1 989 745 188

[I] There are four monthly stock observations in the monthly dataset with a negative value of *OffShareTurnover*. We do not, however, rely on the monthly values of *OffShareTurnover* for any calculations, as all turnover calculations in this paper are based on the daily observations of *OffShareTurnover* which we find to be correct. As the corresponding monthly observations of *Generic*, *ReturnAdjGeneric* and *SharesIssued* are also correct for the stocks with the negative monthly values of *OffShareTurnover*, we do not remove the observations.

3.1.1 Data Filters

In line with Bali et al. (2017), we limit our analysis to only include common shares. In *Børsprosjektet*, this corresponds to *A shares*, *B shares*, and *ordinary shares*. We do, however, deviate by also including *Primary Capital Certificates* in our study. Although not technically a common stock, *Primary Capital Certificates* are listed on the exchange and trade correspondingly. In our view, to omit *Primary Capital Certificates* would lead to an inaccurate representation of the Norwegian investment universe as Norwegian savings banks make up a significant share of the investment opportunities within the Norwegian financial sector. Lastly, we limit our analysis to stocks listed on the Oslo Stock Exchange (OSE hereafter) and exclude stocks listed on Oslo Axess.

According to professor Bernt Arne Ødegaard (2019, Ødegaard hereafter), not all stocks should necessarily be included when conducting empirical asset pricing analyses on the OSE. Low valued stocks (penny stocks) and illiquid stocks can be particularly problematic. It is common in the asset pricing literature to remove penny stocks as they may have microstructure-issues related to illiquidity and highly exaggerated returns. With regards to a Norwegian stock sample, Ødegaard recommends removing all observations for a given stock in years where its share price has been observed at a level below NOK 10 or its market capitalization has been observed at a level below NOK 1mn. In our sample, 389 out of the 885 stocks have at some point in time traded below NOK 10, and removing them would have a significant impact on our sample size. In general, we believe it is important to be cautious when filtering based on share price and market capitalization as these variables are directly linked to stock returns. In addition, we find that filtering on a yearly interval may be too strict for our sample as it would induce undesirable biases to our analyses. One example of such a bias is that the poor performance of several oil service companies following the oil price crash in 2014 would be omitted from the analyses while their strong performance in the years leading up to the crisis would be included (e.g. DOF Subsea or Odfjell Drilling).

To better preserve our sample size and avoid unwanted biases, we lower Ødegaard's stock price restriction to NOK 1 and only remove stocks in months where its share price has been observed below NOK 1 or its market capitalization has been observed below NOK 1mn.[12] By removing months of observations rather than years, we aim to reduce the potential bias stemming from poorly performing stocks falling out of our sample. Filtering on a monthly interval is also in line with Bali et al. (2017).

To reduce the impact of illiquid stocks, Ødegaard removes stocks with less than 20 trading days in a given year. We generally follow his example, but we also impose an additional restriction targeting the trading volume: each year, we remove the 2.5 percent of the stocks in the sample with the lowest average daily turnover[13] in NOK. The trading-volume restriction is primarily imposed to reduce the impact of potential microstructure-issues in our sample due to our more lenient stock price restriction.[14] The liquidity filters are enforced on a yearly interval in line with Ødegaard as liquidity is not directly linked to stock performance and should thus not bias our results. We have illustrated the

[12] We do not find it constructive to use relative filters based on market capitalization and share price, although it is common in the literature. Relative measures would result in the removal of stocks with high market capitalization and high share prices form our sample in the 1980s and 1990s.

[13] Defined as the average trading volume in days the stock has been traded over the course of a year.

[14] Increasing the liquidity of our sample should also make the results of our analysis more representative for what an investor could expect to achieve in the market by replicating our methodology.

impact of our filters on the number of stocks in our sample in Table 2, and summary statistics for our filtered sample are presented in Table A.1 in the appendix.

3.2 Factors and Risk-free rate

We retrieve historical daily data and monthly data on the Fama and French factors, the Carhart (1997) momentum factor, the liquidity factor, and estimates of the historical risk-free rate from professor Bernt Arne Ødegaard's website[15] for the period of 1985 to 2018. The Fama and French factors are calculated according to Fama and French (1998) and the momentum factor is calculated according to Carhart (1997) on Norwegian data. The liquidity factor for the OSE is constructed following the methodology of Næs, Skjeltorp, and Ødegaard (2009), equal to the monthly difference in returns between the return of the least liquid portfolio and the most liquid portfolio out of three portfolios sorted on relative bid-ask spread in the previous month. The estimates of the risk-free rate are the forward-looking interest rates of borrowing at the given date of the stated period. Data on Frazzini and Pedersen's (2014) betting-against-beta (BAB) factor on Norwegian data is retrieved from AQRs website[16] for the period of 1985 to 2018.

3.3 Index Data

We download daily and monthly historical market returns for the OBX and the Oslo Børs All-share index (OSEAX) from Ødegaard's website. The OBX is adjusted for dividends and consists of the 25 most liquid stocks on the OSE ranked by 6-month trailing turnover. The OSEAX is also adjusted for dividends and consists of all stocks listed on the OSE. The OSEAX has return observations for our entire sample period (Jan. 1985 – Dec. 2018), while the OBX returns start from January 1987. We use the MSCI World index returns as our proxy for the global market portfolio and download monthly MSCI index returns for the period 1987-2018 from Compustat. The index has 1,651 constituents representing 23 developed markets and covers approximately 85% of the free float-adjusted market capitalization in each country (MSCI, 2019). The MSCI returns are converted to NOK using daily NOK/USD exchange rates collected from Norges Bank.

[15] Link to website: http://finance.bi.no/~bernt/financial_data/ose_asset_pricing_data/index.html
[16] Link to website: https://www.aqr.com/Insights/Datasets/Betting-Against-Beta-Equity-Factors-Monthly

Table 2: Evolution in the Number of Stocks in the Data Sample per Year after Imposing Filters

The table presents the number of unique stocks in our data sample for each year after imposing filters.

Explanation of columns: "Total Stocks" refers to the total number of stocks in the data sample for a given year before filters. The requirement for a stock to be included in a given year is one valid return observation. "Security Type" is the number of stocks left in the sample after free-float shares are removed. "Market" is the number of stocks left in the sample after stocks listed on Oslo Axess are removed. "MCAP" represents the number of stocks left in the sample after stocks are removed for the months their market capitalization is observed below NOK 1mn. "Share Price" represents the number of stocks left in the sample after stocks are removed for the months their stock price is observed below NOK 1. "Trading Days" represents the number of stocks left in the sample after all stocks that have been traded less than 20 days for a given year is removed. "Turnover" represents the filtered stocks in the 2.5 percentile in terms of average daily turnover over the course of a year.

Year	Total Stocks	Security Type	Market (OSE)	MCAP >NOK 1M	Share Price > NOK 1	Trading Days > 20	Turnover > 2.5 percentile
1985	164	164	164	142	142	138	136
1986	173	173	173	153	153	145	142
1987	168	168	168	152	152	141	137
1988	153	152	152	139	139	121	118
1989	165	155	155	148	148	136	134
1990	179	167	167	159	159	146	145
1991	166	153	153	148	148	136	136
1992	164	153	153	148	148	120	120
1993	175	165	165	162	161	135	135
1994	185	175	175	173	173	153	152
1995	185	185	185	185	183	165	163
1996	203	203	203	203	203	189	187
1997	248	248	248	246	244	226	222
1998	268	268	268	265	262	243	243
1999	261	261	261	261	258	239	239
2000	257	257	257	257	253	236	235
2001	243	243	243	243	238	219	217
2002	224	224	224	224	214	200	200
2003	216	216	216	215	197	180	180
2004	207	207	207	205	197	191	186
2005	238	238	238	238	237	225	221
2006	257	257	257	257	257	248	243
2007	290	290	264	264	264	261	256
2008	283	283	248	246	246	241	238
2009	265	265	228	227	221	214	211
2010	257	257	220	216	211	209	207
2011	251	251	211	207	204	201	198
2012	240	240	205	200	196	196	195
2013	240	240	202	200	197	194	191
2014	235	235	197	195	194	191	190
2015	229	229	194	194	191	191	189
2016	220	220	192	192	188	187	183
2017	227	227	202	202	200	198	194
2018	220	220	202	202	200	199	194

4 Variables

This section presents the variables in our dataset and the methodology used to estimate them.

4.1 Market Returns

We construct our own market indexes using our filtered data sample to ensure that the investment opportunities in our constructed portfolios and the market index are consistent. We construct both a value-weighted (VW hereafter) and an equal-weighted (EW hereafter) market index. The EW index returns for month t +1 are computed as the sum of the monthly returns for the individual stocks in month t + 1, divided by the total number of stocks in month t + 1. The VW index returns for month t +1 are computed based on

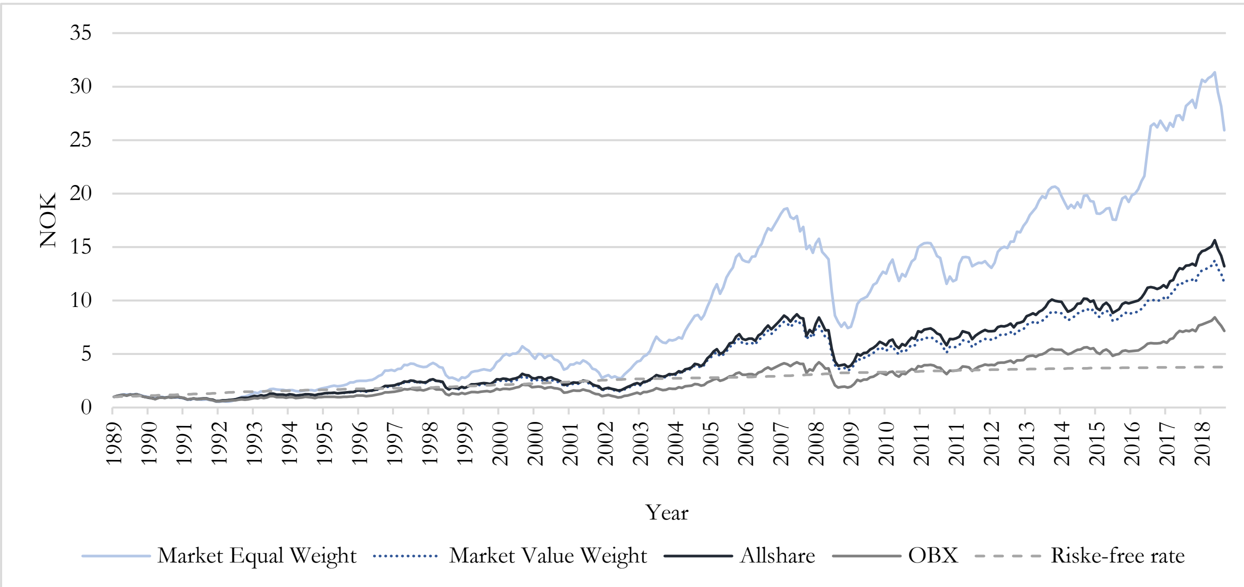
$$r_{M,t+1} = \sum_{i=1}^{n} \frac{r_{i,t+1} * MC_{i,t}}{MC_{M,t}}$$

where $r_{M,t+1}$ is the VW market return in month t +1, $r_{i,t+1}$ is the return of stock i in month $t + 1$, $MC_{i,t}$ is the market capitalization of stock i in month t and $MC_{M,t}$ equals the sum of the individual stocks' market capitalization in month t.

We have plotted our index returns "Market Equal Weight" and "Market Value Weight" in Figure 2 against the returns of the OBX and the OSEAX from 1990 to 2018. Our VW market index tracks the OSEAX well, and we attribute the minor observable deviation to the filters we have applied to our dataset and our use of a monthly rebalancing frequency. The OSEAX is rebalanced on a semi-annual basis.

Figure 2: Cumulative Index Returns

The figure presents the cumulative performance (in NOK) of a NOK 1 investment in the different market indices from Jan. 1990 through Dec. 2018. The EW and VW market index are based on our filtered data sample. Allshare, OBX and the risk-free rate are retrieved from Ødegaard's website.



4.2 Beta Estimation

In this thesis, we evaluate the performance of quintile portfolios sorted on ex-ante estimates of market beta and the amount of lottery demand for a stock, and we thus estimate each stock's beta rather than estimating betas on a portfolio level. We use three different approaches to estimate a stock's market beta. Our main model estimates the market beta (β_{5Y}) at the end of month *t* to be the slope coefficient from a regression of excess stock returns on excess VW market returns using monthly returns from the 60-month period prior to and including month *t*. We require a stock to have a minimum of 36 valid monthly returns for beta computations and define valid returns as monthly returns with official share turnover greater than 0. Furthermore, only valid returns are included in the regression. By conducting a simulated computer, quasi-experiment, Serra and Montelac (2013) find that including return periods without share turnover in beta estimation results in beta estimates that are statistically different from the underlying betas. They conclude that the best alternative for stocks with low liquidity is to use the trade-to-trade method and only include valid stock returns and the corresponding market returns in the beta regression.

Bali et al. (2017) use betas calculated on a one-year period of daily returns in their focal analyses. This is consistent with Daves, Erhardt, and Kunkel (2000), who argue that daily return frequency produces lower standard errors. Furthermore, Daves et al. (2000) prefer a return history shorter than three years to reduce the chance of significant changes to companies' beta. On the other hand, Dimson (1979) argues that the beta of companies that are not traded every day is biased downwards and, according to Scholes and Williams (1997), the problems with non-synchronism between stock returns and market returns are increasingly serious when daily data is used. Since lower frequencies can result in biased beta estimates for illiquid stocks, Damodaran (1994) and Koller, Goedhart, and Wessels (2002) prefer the use of monthly data. As discussed in the data section (section 3.1.1), the OSE is relatively illiquid, and our filtered daily sample contains 324 364 daily zero-return observations with no official turnover, which amounts to 22% of our total observations. As we study the performance of portfolios sorted on beta with a small sample of Norwegian stocks, biased estimates of beta can be especially impactful. We therefore select monthly frequency as our preferred beta-estimation method in our focal analyses throughout the thesis.

To evaluate the robustness of our results and further explore the relation between beta and one-month-ahead excess returns, we also estimate a stock's market beta (β_{1Y}) using one year of daily returns, and by regressing the monthly stock returns on the MSCI World index (β_{MSCI}). We estimate β_{1Y} for month *t* following the methodology of Bali et al. (2017) using daily excess returns from the 12-month period up to and including month *t*. We require a minimum of 200 valid daily return observations in the calculation period, and in line with our estimation of β_{5Y}, only valid returns are included in the regression. β_{MSCI} is calculated following the same methodology as β_{5Y}, with the exception that we regress the monthly excess stock returns on the MSCI World index returns in NOK in excess of the risk-free rate in Norway.

4.3 Lottery Demand

We measure the amount of lottery demand for a stock following Bali et al. (2011) and Bali et al. (2017) using the variable MAX. MAX is calculated, for each month *t*, as the average of the five highest daily returns of the stock in month *t*. In line with Bali et al. (2017), we require a minimum of 15 valid daily return observations in the given month to estimate MAX.

4.4 Other Variables

In addition to estimating beta and the amount of lottery demand for each stock, we also estimate each stock's idiosyncratic volatility (IVOL) and idiosyncratic skewness (ISKEW). We calculate the **IVOL** of a stock in month *t* following the methodology of Bali et al. (2017) as the standard deviation of the residuals from a regression of valid excess stock returns on the Fama and French 3-factor model.[17] We use one month of daily return data and require a minimum of 15 valid daily returns within the given month. The Fama and French (1993) three-factor regression specification is

$$r_{i,d} = a + b_1 MKT_d + b_2 SMB_d + b_3 HML_d + e_{i,d}$$

where MKT_d represents the VW excess market returns on day *d*, and SMB_d and HML_d represent the return of the size factor and book-to-market factor as calculated by Ødegaard, respectively, on day *d*. **ISKEW** is calculated in a similar manner following Boyer, Mitton, and Vorkink (2010) as the skewness[18] of the residuals from a regression of valid excess stock returns on the Fama and French 3-factor model. We use one month of daily return data and require a minimum of 15 valid daily returns. Summary statistics of the variables are presented in Table 3 below.

Table 3: Summary Statistics of Calculated Variables

The table presents summary statistics for the estimated variables in our dataset. See section 4.2 for calculations on β_{5Y}, β_{1Y} and β_{MSCI}. See section 4.3 for calculations on MAX. See section 4.4 for calculations on IVOL and ISKEW.

Variable	N	Mean	SD	Min	Max
β_{5Y}	44354	0.895	0.581	-2.277	6.322
β_{1Y}	35731	0.866	0.504	-2.177	4.902
β_{MSCI}	44344	0.775	0.732	-4.947	5.497
MAX	43528	0.037	0.028	-0.003	1.147
IVOL	43528	0.025	0.019	0.000	0.812
ISKEW	43528	0.218	0.726	-3.736	4.310

[17] We use HML and SMB factors estimated on Norwegian data.

[18] Skewness is calculated using the *skewness* function in R where skewness is defined by $\gamma_1 = \frac{u_3}{u_2^{\frac{3}{2}}}$, where u_3 and u_2 are the third and second moments.

5 Methodology

The methodology section is divided into three subsections. The first subsection (5.1) presents the methodology used to examine the beta anomaly and the lottery demand phenomenon in the Norwegian stock market. The second subsection (5.2) presents the methodology used to assess the ability of lottery demand to explain the beta anomaly in Norway. The third subsection (5.3) outlines how we present and evaluate the generated results.

5.1 The Beta Anomaly and the Lottery Demand Phenomenon

5.1.1 Univariate Portfolio Analysis

To assess the beta anomaly and the lottery demand phenomenon on the OSE, we perform univariate portfolio analyses as is common in the literature. Bali et al. (2017) conduct univariate portfolio analyses by dividing their sample into decile portfolios. However, constructing decile portfolios based on our Norwegian sample would result in an insufficient number of stocks per portfolio as the sample is too narrow in the cross-section. According to Ødegaard, a minimum of ten Norwegian stocks are needed for a diversified portfolio, and by using quintile breakpoints, our portfolios meet this requirement in most years. Using quartile breakpoints would be preferable from the perspective of diversification; however, it would also reduce the difference in average beta and MAX values across our constructed portfolios and thus make the relation between our sorting variables and returns increasingly difficult to test. We believe that the use of quintile breakpoints best balances the two considerations.

We follow the same portfolio formation methodology for all measures of beta and MAX throughout the thesis. Starting in December 1989, we sort all stocks into quintile portfolios at the end of month *t* based on an ascending order of the sorting variable. We then calculate the corresponding excess portfolio returns in month *t+1*. December 1989 (at market close) is the natural starting point for our analysis as we use a 60-month return window to calculate β_{5Y}.[19] Furthermore, we use the same starting point for all portfolio formations and return calculations, regardless of the sorting variable. This is to ensure consistency in the analysis between portfolios sorted by beta and portfolios sorted by MAX. The portfolios are rebalanced monthly, and the procedure is repeated through December 2018, resulting in a total of 348 monthly observations of portfolio excess returns.

[19] Data from 1985 to 1990 (60 monthly return observations) is used to estimate β_{5Y} and the first portfolio sort is therefore conducted at market close December 1989. This implies that our analysis covers portfolio returns from January 1990 through December 2018.

We calculate both VW and EW portfolio excess returns to increase the robustness of our results. On the one hand, VW returns are preferable to EW returns as capital will be allocated according to the individual stocks' market capitalization. We do not control for transaction costs in our return calculations, and large allocations relative to a stock's market capitalization can yield impracticable "paper profits" as the illiquidity of the stock relative to the size of the allocation could result in considerable transaction costs. Furthermore, EW portfolios tend to have a higher turnover than VW portfolios as they require continuous rebalancing to maintain equal weights. EW portfolio returns may thus exhibit a minor value bias as the portfolio (by construction) tends to sell winning stocks and buy losing stocks. On the other hand, we find that the EW portfolios arguably provide a better representation of how the average stock in the portfolio performed. As the OSE is dominated by a few large stocks, VW portfolio returns could be heavily tilted towards the performance of only a handful of stocks. This could make portfolio returns undiversified and potentially result in a distorted relation between our sorting variables and VW returns. However, as introduced in the data section (section 3), the OSE is also greatly influenced by illiquid stocks, which may make EW returns artificially high. Hence, we deem it necessary to consider both VW and EW returns in our analyses.

5.2 The Lottery Demand Phenomenon as an Explanation of the Beta Anomaly

To investigate the role lottery demand plays in generating the beta anomaly on the OSE, we follow the methodology of Bali et al. (2017)[20] and explore the relation between market beta and one-month-ahead excess returns after controlling for MAX. To ensure that our results are sufficiently robust, we control for MAX using three different methods. First, we conduct a bivariate portfolio analysis, which entails a conditional double sort on MAX, then on β_{5Y}. We subsequently conduct a univariate portfolio analysis on portfolios sorted on the component of β_{5Y} that is orthogonal to MAX. Lastly, we construct a lottery demand factor and assess the factor's ability to explain the abnormal returns associated with the beta anomaly.

5.2.1 Bivariate Portfolio Analysis

We conduct the bivariate portfolio analysis following Bali et al. (2017) by performing a conditional double sort. At the end of each month *t*, we sort all stocks into quintile portfolios based on ascending values of MAX. For each MAX quintile, we then sort the stocks into quintile portfolios based on an

[20] We only deviate by not performing Fama and MacBeth (1973) regressions as we do not have the data to estimate the control variables used by Bali et al. (2017) on our Norwegian data sample.

ascending ordering of β_{5Y}. We only include monthly stock observations with estimates of both β_{5Y} and MAX for the given month.

Bali et al. (2017) divide their stock sample into decile portfolios and end up with a matrix of 10x10 portfolios. However, as our Norwegian stock sample is far too narrow in the cross-section to generate one hundred well-diversified portfolios, we are forced to deviate from the paper we replicate. Henceforward, we use a 5x5 sort in our bivariate portfolio analyses. A 5x5 sort resulting in 25 portfolios will arguably result in undiversified portfolios given our sample.[21] However, we find that a 5x5 sort is preferred as it makes the generated results directly comparable with our univariate portfolio analyses. To see why, consider a 4x4 sort whose results indicate that controlling for MAX neutralizes the beta anomaly. In isolation, such a result would constitute evidence in favor of the lottery-demand based explanation of the beta anomaly. However, since we have not documented the beta anomaly with a quartile portfolio sort, such a result could either indicate that MAX neutralizes the anomaly, or that the beta anomaly is not detected when performing the univariate analysis with quartile portfolios. This ambiguity would make it impossible to draw precise conclusions from our results. The discussion on why we use quintile portfolios in the univariate analyses can be found in section 5.1.1.

To account for our narrow cross-section of stocks and ensure comparable results with the univariate portfolio analysis, we focus our analysis on the average MAX quintile portfolio, withing each beta quintile. We create the average MAX quintile by first performing the conditional double sort on MAX, then on β_{5Y} to generate a 5x5 portfolio matrix. We subsequently sum the stocks across the MAX quintiles for each beta quintile to create five beta sorted portfolios that should be neutralized to MAX by construction. Solely focusing on the averages will not allow us to explore potential non-linear relations between beta and returns when controlling for MAX, but we argue that the individual portfolios resulting from the double-sort are not sufficiently diversified to provide valuable information regarding potential non-linear relations.[22] As such, we find it to be most constructive to limit our analysis to the averages.

[21] Dividing our sample into 25 portfolios would result in portfolios containing as few as 4 stocks during the early 1990s.
[22] We argue that undiversified portfolios cannot be used to assess the relation between beta and returns while controlling for MAX as we believe the idiosyncratic risk of the portfolios would distort the observed relation.

5.2.2 Univariate Portfolio Analysis of Beta Orthogonal to MAX

To test the robustness of our results, we also construct beta-sorted portfolios that are neutralized to MAX by sorting stocks on the component of β_{5Y} that is orthogonal to MAX ($\beta_{5Y\perp M}$). We estimate $\beta_{5Y\perp M}$ for each stock at the end of month *t* as the sum of the residual and the intercept from a cross-sectional regression of β_{5Y} on MAX. Only stocks with both an estimate of β_{5Y} and MAX for the given month are included in the regression. At the end of each month *t,* we sort stocks into quintile portfolios based on an ascending order of $\beta_{5Y\perp M}$.

5.2.3 FMAX Factor

Following Bali et al. (2017), we construct a MAX factor (FMAX) using the methodology of Fama and French (1993). At the end of each month *t,* we sort all stocks into two groups based on market capitalization[23] and independently sort all stocks into three groups based on an ascending ordering of MAX using the 30th and 70th percentile as breakpoints. The intersections create six portfolios, and the return of the FMAX factor in month *t +1* equals the average return of the two VW low-MAX portfolios less the average return of the two VW high-MAX portfolios.[24]

5.3 Portfolio Evaluation

The focus of our analyses lies on the one-month-ahead (month *t*+1) portfolio excess returns. To test the relation between one-month-ahead returns and our sorting characteristics we calculate the average excess returns (R), Sharpe Ratios (SR) and alphas for each of our quintile portfolios and the zero-cost portfolio (low-high portfolio)[25] with a long position in quintile portfolio 1 and a short position in quintile portfolio 5. Alphas are estimated relative to four different asset pricing models: The CAPM (CAPM), The Fama and French (1993) three-factor model (FF3), the FF3 model including the momentum factor by Carhart (1997) (FFC4), and lastly, the FFC4 model augmented with the liquidity factor by Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ).[26] We use the VW market index as the market factor when regressing the VW portfolio returns, and we use the EW market index as the market factor when regressing the EW portfolio returns.

To formally test for the presence of the beta anomaly, the lottery demand phenomenon, and test whether the beta anomaly persists after controlling for MAX, we follow the methodology of Bali et al.

[23] Using median market capitalization as the breakpoint.
[24] The factor is constructed to have a neutral exposure to market capitalization.
[25] Sharpe ratios and its components are not reported for bivariate sorts or univariate sorts on orthogonal variables.
[26] Only CAPM alphas and FFC4 + LIQ alphas are reported for bivariate sorts and univariate sorts on orthogonal variables.

(2017) and test the null hypothesis of portfolio alphas equal to zero using t-statistics adjusted following Newey and West (1987) using four lags.[27] We are particularly interested in the alpha of the low-high portfolio, which should be statistically significant[28] and positive in the presence of a beta anomaly or a lottery demand phenomenon. Lastly, to assess the FMAX factor's ability to explain the returns associated with the beta anomaly, we augment the FFC4 + LIQ model with the FMAX factor and test the null hypothesis of portfolio alpha for the low-high β_{5Y}-sorted portfolio equal to zero.

6 Analysis

This section presents the results of our analyses. The structure of this section closely resembles the logic put forth in Bali et al. (2017) to make the replication as accurate as possible. Specifically, the structure follows the hypotheses as presented in section 2.4 in a way that each section is dedicated to one hypothesis. The section begins with an assessment of whether the beta anomaly is present in our Norwegian sample. Then we assess whether MAX is a good proxy of the amount of lottery demand for a stock before we test the relation between MAX and one-month-ahead returns. Ultimately, we test whether controlling for lottery demand, as measured by MAX, neutralizes the beta anomaly. Each section first presents the results obtained relative to the hypotheses, followed by a brief review of the findings of Bali et al. (2017) and a comparison between our results and those of Bali et al. (2017) from the US market.

6.1 Hypothesis 1 – The Existence of a Beta Anomaly in Norway

6.1.1 Cumulative Portfolio Returns

We begin or analysis on the relation between beta and excess stock returns by examining the historical performance of quintile portfolios sorted by beta. Figure 3 presents the historical cumulative excess return on an investment of NOK 1 in each of the β_{5Y}-sorted quintile portfolios, starting in January 1990.[29] The results are striking. The VW low-beta portfolio (portfolio 1) generated cumulative excess returns of 1241% from January 1990 through December 2018, while the VW high-beta portfolio (portfolio 5) generated cumulative excess returns of 6.3% over the same time period. The returns of

[27] We use the function *NeweyWest()* from the *sandwich* package in R for the HAC variance-covariance estimator proposed by Newey and West (1987). We do not use a prewhitened estimation function and set the argument *prewhite = FALSE* and use finite sample adjustments by setting *adjust = TRUE*. Bali et al. (2017) use six lags, but as our sample covers a shorter time period, the same number of lags is not necessarily optimal. Following Hanck, Arnold, Gerber and Schmelzer (2019) *Introduction to Econometrics in R*, we estimate the number of lags following:

$lags = (0.75 * T^{\frac{1}{3}}) - 1.$

[28] Statistical significance refers to a p-value below 5% throughout the rest of the thesis.

[29] The cumulative excess returns of quintile portfolios sorted by β_{1Y} are presented in Figure B.1 in the appendix.

the EW portfolios exhibit the same pattern; the cumulative excess returns were 692% and 22.3% for the EW low-beta portfolio and the EW high-beta portfolio, respectively.[30] In terms of performance rankings among the five portfolios, we note that the low-beta portfolio has been the best performer across both portfolio weighting schemes, while the opposite is true for the high-beta portfolio. The results are thus in stark contrast to the beta-return relation suggested by the CAPM.

6.1.2 Univariate Portfolio Analyses

To formally test for the presence of the beta anomaly on the OSE, we conduct a univariate portfolio analysis on the quintile portfolios sorted on β_{5Y}. Table 4 presents portfolio characteristics, Sharpe ratios, and monthly factor model alphas for VW and EW β_{5Y}-sorted quintile portfolios. First, we note that both ex-ante and ex-post portfolio betas increase monotonically from the low-beta portfolio (quintile 1) to the high-beta portfolio (quintile 5) for both portfolio formation schemes. The ex-ante VW (EW) portfolio betas increase (by construction) from 0.284 (0.226) for quintile 1 to 1.631 (1.707) for quintile 5. Ex-post VW (EW) portfolio betas represent the slope coefficient from a regression of VW (EW) excess portfolio returns on VW (EW) excess market returns. Monotonically increasing ex-post betas hence validate our model by illustrating that the low-beta (high-beta) portfolio formed using ex-ante betas tends to have a low (high) beta ex-post.

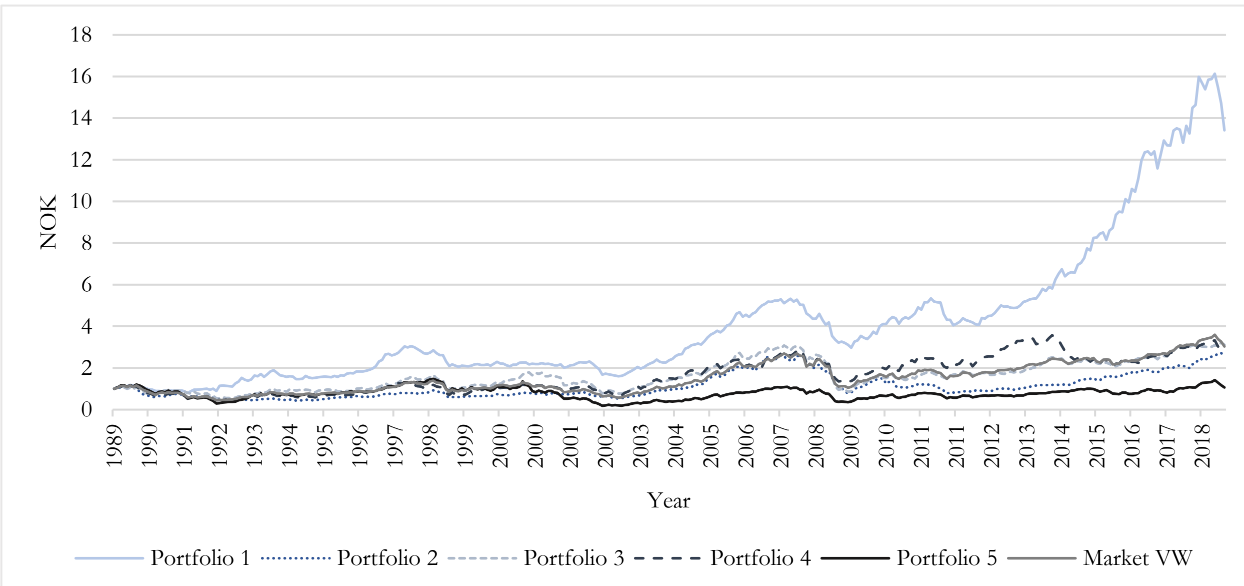
The average one-month-ahead VW (EW) excess returns decrease from 0.85% (0.69%) for the low-beta portfolio to 0.38% (0.22%) for the high-beta portfolio, albeit not monotonically. The VW (EW) low-beta portfolio achieved an annualized Sharpe ratio of 0.645 (0.544), while the VW (EW) high-beta portfolio generated an annualized Sharpe ratio of 0.158 (0.181). The portfolio Sharpe ratios illustrate that the low-beta quintile portfolio has been the superior investment in our sample period for an undiversified investor focused on maximizing excess returns relative to total volatility.

[30] The trading costs associated with replicating the beta-sorted quintile portfolios are assessed in section F in the appendix.

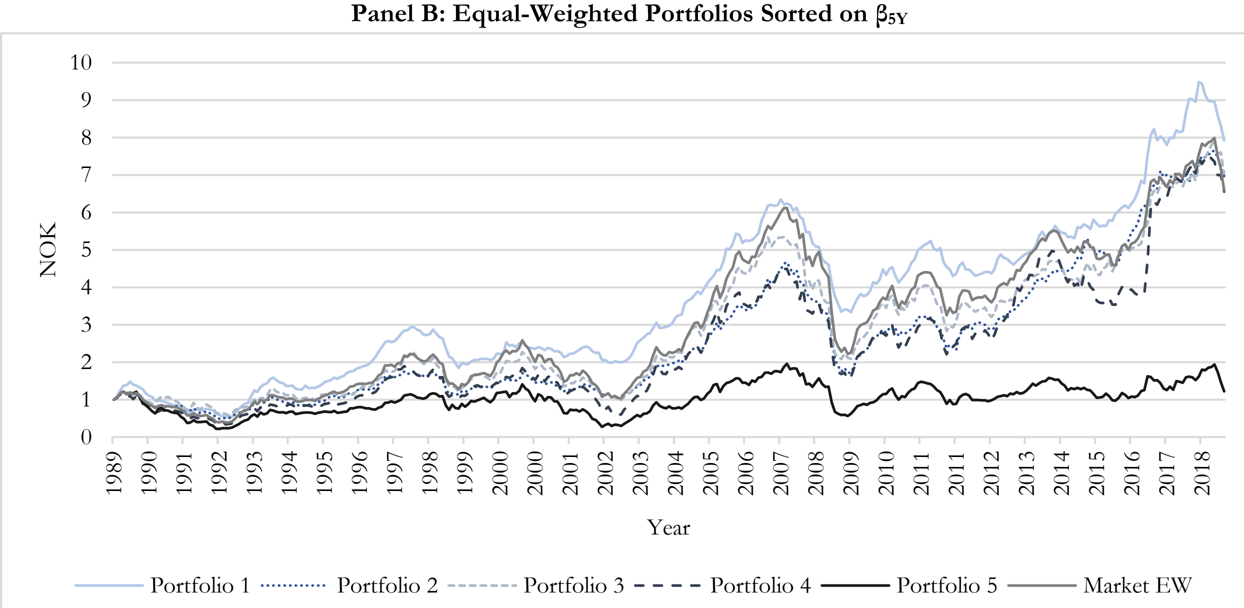
Figure 3: Cumulative Excess Returns of Quintile Portfolios Sorted on β_{5Y}

At the end of each month *t*, all stocks are sorted into quintile portfolios based on an ascending ordering of β_{5Y} and the portfolio excess returns are calculated for month *t +1*. The figure presents the historical cumulative excess returns (in NOK) of a NOK 1 investment in each of the quintile portfolios from Jan. 1990 through Dec. 2018. The quintile portfolios are rebalanced monthly, all dividends and cash payouts are assumed to be reinvested and the return calculation assumes no transaction costs. The Portfolio *i* in the figure correspond to the β_{5Y}-sorted quintile portfolio *i*, while Market VW (EW) illustrates the value-weighted (equal-weighted) excess return of a portfolio consisting of all the stocks in our filtered dataset. Panel A presents the cumulative excess returns of the value-weighted β_{5Y} quintile portfolios and panel B presents the cumulative excess returns of the equal-weighted β_{5Y} quintile portfolios.

Panel A: Value-Weighted Portfolios Sorted on β_{5Y}



Panel B: Equal-Weighted Portfolios Sorted on β_{5Y}



To formally test for the presence of the beta anomaly in our sample, we evaluate the beta-sorted portfolio alphas relative to four different factor models, presented in Table 4. Our discussion will primarily focus on the alphas relative to the CAPM and the FFC4 + LIQ model, and we are particularly interested in the abnormal returns of the zero-cost, low minus high beta portfolio ("low-high portfolio" hereafter"). We find that CAPM alphas decline monotonically from quintile 1 to quintile 5 and that the CAPM alpha for the VW (EW) low-high portfolio is both economically large and statistically significant with a magnitude of 0.87% (0.79%). The low-beta portfolio has thus outperformed the high-beta portfolio in our model when systematic risk is measured according to the CAPM.

The abnormal returns of the VW (EW) quintile portfolios relative to the FFC4 + LIQ model decline from 0.37% (0.11%) for quintile 1 to -0.28% (-0.24%) for quintile 5, albeit not monotonically. Neither of the quintile portfolios generates statistically significant alphas relative to the FFC4 + LIQ model, but the VW (EW) low-high portfolio achieves a FFC4 + LIQ alpha of 0.65% (0.35%) with a corresponding t-statistic of 2.245 (1.401). The FFC4 + LIQ alpha for the VW low-high portfolio is both economically large and statistically significant, and we find that both the long side (quintile 1) and the short side (quintile 5) of the trade contributes to the alpha. Although not statistically significant, we note that the reported FFC4 + LIQ alpha of the EW low-high portfolio implies a 4.3% annual alpha and should still be considered economically large. Hence, the results from our main model indicate that the low-beta portfolio has outperformed the high-beta portfolio on a risk-adjusted basis when risk is measured relative to conventional asset pricing models. However, the lack of statistical significance for the FFC4 + LIQ alpha of the EW low-high portfolio moderates our confidence in our conclusion.

Table 4: Univariate Portfolio Analysis Sorting on β_{5Y}

The table presents the results of a univariate portfolio analysis on the relation between excess and abnormal returns in month t+1 and the variable β_{5Y} in month t. At the end of each month t, all stocks are sorted into quintile portfolios based on an ascending ordering of β_{5Y}. The table consists of two panels (A and B) and each panel is divided into three sections.
Explanation of sections: "Portfolio Characteristics" presents the time-series mean number of stocks in each quintile portfolio (Portfolio length), the time-series mean portfolio market share (calculated as the sum of the market capitalization within each portfolio divided by total market capitalization) for each month t (Market share), the time-series mean of the monthly average β_{5Y} for the stocks in each quintile portfolio (β ex-ante), and the slope coefficient from a regression of VW (EW) portfolio excess returns on the VW (EW) market excess returns (β ex-post). "Portfolio Sharpe Ratio" presents the time-series mean monthly portfolio excess returns in month $t+1$ (R), the standard deviation of monthly portfolio excess returns in month $t+1$ (SD), and an annualized portfolio Sharpe ratio (SR). "Factor Model Alphas" presents monthly portfolio alphas relative to the CAPM (CAPM), the Fama-French 3-Factor model (FF3), the Fama-French-Carhart 4-Factor model (FFC4) and the FFC4 model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags.
Explanation of panels: All reported numbers in panel A are calculated using value-weighted portfolios, while all numbers in panel B are calculated using equal-weighted portfolios.
General: The column labeled Low-High β_{5Y} refers to a zero-cost, long-short portfolio with a long position in quintile portfolio 1 and a short position in quintile portfolio 5. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.

Panel A: Value-Weighted Portfolios

Value	β_{5Y} 1 (Low)	β_{5Y} 2	β_{5Y} 3	β_{5Y} 4	β_{5Y} 5 (High)	Low-High β_{5Y}
			Portfolio Characteristics			
Portfolio length	25	25	26	25	25	
Market share	3 %	14 %	28 %	32 %	23 %	
β ex-ante	0.284	0.602	0.848	1.135	1.631	-1.347
β ex-post	0.493	0.844	0.966	1.003	1.297	-0.804
			Portfolio Sharpe Ratio			
R	0.85 %	0.46 %	0.54 %	0.51 %	0.38 %	0.47 %
SD	4.58 %	6.07 %	6.37 %	6.51 %	8.34 %	7.03 %
SR	0.645	0.263	0.293	0.271	0.158	0.233
			Factor Model Alphas			
CAPM	0.61 %	0.05 %	0.07 %	0.02 %	-0.25 %	0.87 %
	(3.214)	(0.214)	(0.383)	(0.109)	-(1.187)	(3.027)
FF3	0.39 %	0.08 %	0.07 %	0.06 %	-0.35 %	0.74 %
	(1.979)	(0.337)	(0.370)	(0.319)	-(1.544)	(2.405)
FFC4	0.37 %	0.02 %	0.11 %	0.10 %	-0.24 %	0.61 %
	(1.838)	(0.083)	(0.631)	(0.603)	-(1.071)	(1.995)
FFC4 + LIQ	0.37 %	0.01 %	0.11 %	0.09 %	-0.28 %	0.65 %
	(1.813)	(0.028)	(0.631)	(0.541)	-(1.363)	(2.245)

Panel B: Equal-Weighted Portfolios

Value	β_{5Y} 1 (Low)	β_{5Y} 2	β_{5Y} 3	β_{5Y} 4	β_{5Y} 5 (High)	Low-High β_{5Y}
			Portfolio Characteristics			
Portfolio length	25	25	26	25	25	
Market share	3 %	14 %	28 %	32 %	23 %	
β ex-ante	0.226	0.586	0.846	1.134	1.707	-1.481
β ex-post	0.572	0.762	0.872	1.102	1.365	-0.793
			Portfolio Sharpe Ratio			
R	0.69 %	0.70 %	0.73 %	0.81 %	0.47 %	0.22 %
SD	4.40 %	5.22 %	5.81 %	7.25 %	8.97 %	7.06 %
SR	0.544	0.461	0.437	0.386	0.181	0.109
			Factor Model Alphas			
CAPM	0.28 %	0.15 %	0.10 %	0.01 %	-0.52 %	0.79 %
	(1.696)	(1.094)	(0.715)	(0.068)	-(2.520)	(2.655)
FF3	0.12 %	0.16 %	0.15 %	0.07 %	-0.29 %	0.41 %
	(0.835)	(1.184)	(1.028)	(0.436)	-(1.376)	(1.410)
FFC4	0.07 %	0.10 %	0.11 %	0.07 %	-0.16 %	0.23 %
	(0.451)	(0.748)	(0.671)	(0.442)	-(0.755)	(0.776)
FFC4 + LIQ	0.11 %	0.12 %	0.10 %	0.03 %	-0.24 %	0.35 %
	(0.777)	(0.895)	(0.606)	(0.219)	-(1.300)	(1.401)

6.1.3. Comparison of Results with the US Market

The results from our univariate portfolio analysis sorting on β_{5Y} are generally in line with the results Bali et al. (2017) obtain when they perform a similar analysis on the US market. However, our methods differ slightly as they use decile portfolios to account for the larger cross-section of stocks in their sample, and as they sort stocks on β_{1Y} in their primary model. In their US sample, the ex-ante EW portfolio betas increase from -0.00 for decile 1 to 2.02 for decile 10. The spread in their EW ex-ante portfolio betas is somewhat larger than what we find on Norwegian data, but this is to be expected as they use decile breakpoints while we use quintile breakpoints. In line with our results, they find that the FFC4 + PS[31] alphas decrease from decile 1 to decile 10, albeit not monotonically. However, in contrast to our results, the FFC4 + PS alphas of decile portfolio 1 and 10 are statistically significant in their sample, with a magnitude of 0.23% and -0.26%, respectively. Furthermore, Bali et al. (2017) report the performance of a high-low decile portfolio (in contrast to our low-high quintile portfolio) and present a FFC4 + PS alpha equal to -0.49% with a t-statistic of -2.26 for the high-low decile portfolio. Similar to our results, they find that both the long side and the short side of their high-low portfolio has contributed to the abnormal returns. The paper also reports results using VW decile portfolios

[31] "PS" refers to Pastor and Stambaugh's (2003) liquidity factor.

sorted on β_{5Y} for an extended sample period. They find that in the period 1931-2012, the high-low β_{5Y} portfolio has generated an economically large and statistically significant FFC4 alpha of -0.48% per month.

The main difference between our results and those of Bali et al. (2017) is the increased statistical power of their estimates. We largely attribute this discrepancy to their use of a data sample with a longer time series (1963-2012) and a larger cross-section of stocks. The larger cross-section increases the precision of their estimates, and it also allows for the use of decile portfolios. The use of decile portfolios results in a larger spread between the portfolio averages of the sorting variable and, given the existence of a relation between the sorting variable and returns, the use of decile portfolios should contribute to increased statistical significance in the results.

6.1.4 Robustness Tests of the Beta Anomaly

As the beta anomaly is not as well documented in Norway as it is in the US, and as the results from our univariate analysis on β_{5Y} are not unambiguous, we conduct several robustness tests to validate our results. Table 5 reports the CAPM alphas and the FFC4 + LIQ alphas for the low-high beta portfolios over five different time periods. The VW portfolio delivers a statistically significant CAPM alpha across all tested periods, while the FFC4 + LIQ alpha is statistically significant at the 5% level for all periods except the period spanning from 1990 to 2013. The EW portfolios produce statistically significant CAPM alphas in all tested periods, and the FFC4 + LIQ alpha becomes statistically significant at the 5% level in the period 2000-2018.

We argue that a lower alpha for the VW low-high portfolio in the period 1990 to 2013 may be a result of the period ending near the peak of the oil price cycle. As presented in Table A.4 in the appendix, a significant portion of the companies in the top 20 list of the most frequent companies in quintile portfolio 5 has direct exposure to the oil price. Quintile portfolio 1, on the other hand, is in most periods comprised of stocks with no direct oil-price exposure. Ending the period near the peak of the oil price cycle will, therefore, result in a biased estimate of the relative performance between quintile portfolios 1 and 5. In general, the OSE is heavily tilted towards stocks with significant exposure to the oil price, and we argue it is essential to be aware of the cyclical nature of the oil price when evaluating portfolio performance on the OSE. In our full sample from 1990 to 2018, oil-price-sensitive companies have been through several cycles, and with 2018 not representing a clear cyclical peak or trough, we argue that our full sample period should provide fair estimates of the long-term performance of stocks with a high degree of oil-price exposure. Therefore, we do not consider the reduced alpha of the VW low-high portfolio from 1990-2013 as a major point of concern for the robustness of our results.

Table 5: Low-High β_{5Y} Portfolio Alphas for Different Sample Time Periods

The table presents the results from several univariate portfolio analyses on β_{5Y} using different time periods. At the end of each month t, all stocks are sorted into quintile portfolios based on an ascending ordering of β_{5Y}. The table presents the monthly alphas of a zero-cost, long-short portfolio with a long position in quintile 1 (low-beta) and short position in quintile 5 (high-beta) relative to the CAPM (CAPM) and the Fama-French-Carhart 4-Factor model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags. The columns refer to the use of different time periods of our data sample. Panel A reports the results for value-weighted portfolios, while panel B reports the results for equal-weighted portfolios.

Panel A: Value-Weighted Portfolios

Value	1990-2018	1995-2018	2000-2018	1990-2013	1990-2007
			Factor Model Alphas		
CAPM	0.87 %	0.88 %	1.03 %	0.74 %	0.97 %
	(3.027)	(2.920)	(3.059)	(2.341)	(2.474)
FFC4 + LIQ	0.65 %	0.60 %	0.76 %	0.59 %	0.84 %
	(2.245)	(1.959)	(2.190)	(1.873)	(2.122)

Panel B: Equal-Weighted Portfolios

Value	1990-2018	1995-2018	2000-2018	1990-2013	1990-2007
			Factor Model Alphas		
CAPM	0.79 %	0.83 %	0.88 %	0.69 %	0.99 %
	(2.655)	(2.560)	(2.764)	(2.024)	(2.319)
FFC4 + LIQ	0.35 %	0.42 %	0.53 %	0.30 %	0.39 %
	(1.401)	(1.526)	(1.951)	(1.078)	(1.082)

We also examine the sensitivity of our results with regard to our data filters. Table 6 presents CAPM alphas and FFC4 + LIQ alphas for the low-high β_{5Y} portfolio given some select variations in our data filters. We find that the CAPM alphas of the low-high VW and EW portfolios remains statistically significant at the 5% level for all tested variations in data filters. For the VW portfolios, the FFC4 + LIQ alpha is statistically significant for all variations in data filters, except when we impose a stricter filter on market capitalization and remove stocks in months when their market capitalization has been observed below NOK 1bn. Interestingly, we find that the FFC4 + LIQ alpha is statistically significant for EW portfolios when including the minimum NOK 1bn filter on market capitalization or removing the minimum NOK 1 filter on share price.

Table 6: Low-High β_{5Y} Portfolio Alphas for Variations in Data Filters

The table presents the results from several univariate portfolio analyses on β_{5Y} for varying data filters. At the end of each month t, all stocks are sorted into quintile portfolios based on an ascending ordering of β_{5Y}. The table presents the monthly alphas of a zero-cost, long-short portfolio with a long position in quintile 1 (low-beta) and short position in quintile 5 (high-beta) relative to the CAPM (CAPM) and the Fama-French-Carhart 4-Factor model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags. The columns refer to different variations in data filters. "Primary Model" refers to the data filters applied to the main model as discussed in section 3.1.1. "Primary Capital Certificates" reports the zero-cost portfolio alphas when primary capital certificates are excluded from the sample. "Share Price" presents the zero-cost portfolio alphas when there is no restriction on stock price in the sample. "Large Stocks" presents the zero-cost portfolio alphas when stocks are removed for the months their market capitalization is observed below NOK 1bn. "Turnover" presents the zero-cost portfolio alphas when there is no restriction on stock turnover in the data sample. All results reported in panel A are calculated using value-weighted portfolios, while all results in panel B are calculated using equal-weighted portfolios. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018.

Panel A: Value-Weighted Portfolios

Value	Primary Model	Primary Capital Certificates	Share Price (0)	Large Stocks (1bn)	Turnover (0%)
		Factor Model Alphas			
CAPM	0.87 %	0.97 %	0.95 %	0.69 %	0.84 %
	(3.027)	(3.088)	(3.226)	(1.983)	(2.873)
FFC4 + LIQ	0.65 %	0.71 %	0.75 %	0.56 %	0.62 %
	(2.245)	(2.266)	(2.510)	(1.549)	(2.123)

Panel B: Equal-Weighted Portfolios

Value	Primary Model	Primary Capital Certificates	Share Price (0)	Large Stocks (1bn)	Turnover (0%)
		Factor Model Alphas			
CAPM	0.79 %	0.82 %	1.11 %	1.04 %	0.82 %
	(2.655)	(2.599)	(3.627)	(3.372)	(2.713)
FFC4 + LIQ	0.35 %	0.37 %	0.69 %	0.74 %	0.39 %
	(1.401)	(1.382)	(2.684)	(2.353)	(1.564)

Lastly, we report the results from using β_{1Y} as the sorting variable in a univariate portfolio analysis in Table B.1 in the appendix.[32] The results from sorting on β_{1Y} are generally in line with the results from sorting on β_{5Y} as the VW (EW) low-high β_{1Y} portfolio generates an economically large CAPM alpha of 0.47% (0.92%) and FFC4 + LIQ alpha of 0.33% (0.76%). The results do, however, differ in that the source of the FFC4 + LIQ alpha is mostly generated by shorting quintile portfolio 5 in the β_{1Y} model, while it is more evenly distributed between the long side and the short side of the trade in the β_{5Y} model. We also note that the CAPM alpha and the FFC4 + LIQ alpha for the low-high portfolio is statistically significant for the EW portfolios, but not for the VW portfolios, which is opposite of the

[32] We also report the results from univariate portfolio analysis sorting on β_{MSCI} in table B.2 in the appendix.

results from the β_{5Y} model. Despite differences in statistical significance between the two models, we find the results from sorting on β_{1Y} to be supportive of the existence of a beta anomaly as all tests indicate that the low-beta portfolio has outperformed the high-beta portfolio on a risk-adjusted basis.

6.1.5 Comparison of Results with Other Studies in the Norwegian Market

As mentioned in the literature review (section 2.2), there are, to the best of our knowledge, three other studies commenting on the beta anomaly in a Norwegian sample. Frazzini and Pedersen (2014) report the abnormal returns of their Betting-Against-Beta (BAB) factor in Norway in the period of 1984 to 2009. Surprisingly, the factor, which is long low-beta stocks and short high-beta stocks, generates a statistically insignificant, negative alpha of -0.06% relative to the FFC4 model. The finding is not commented further in the paper.

The updated performance of the BAB factor constructed by Frazzini and Pedersen (2014) for the period 1990-2018 is reported on AQR's website. In Table B.3 in the appendix, we report the CAPM alpha and the FFC4 + LIQ alpha for the Norwegian BAB factor in the period 1990-2018. We find that the BAB factor has generated a statistically significant CAPM alpha of 1.06% and FFC4 + LIQ alpha of 0.63%. The results are arguably in line with the documented abnormal returns of our low-high β_{5Y} portfolios. As such, we do not find the results of Frazzini and Pedersen (2014) to contradict our findings.

Christensen (2019) documents a beta anomaly on the OSE as a part of his master thesis investigating the suitability of various mispricing models on the OSE in a sample from 1998 to 2018, which excludes financial firms. He presents statistically significant monthly abnormal returns of 1.26% relative to the FF3 model of a VW zero-cost portfolio with a long position in the low-beta quintile portfolio and a short position in the high-beta quintile portfolio. As can be seen from Table 5 and Table 6, we obtain a statistically significant monthly VW FFC4 + LIQ alpha of 0.76% in the period 2000-2018 and a statistically significant monthly VW FFC4 + LIQ alpha of 0.71% in the full sample period when we exclude primary capital certificates. Our results are thus fairly in line with those of Christensen (2019), given the differing sample.

In their master thesis, Juneja and Bordvik (2017) report of a statistically insignificant monthly VW (EW) FF3 model alpha of -0.27% (-0.57%) for a zero-cost portfolio with a long position in the high-beta quintile portfolio and a short-position in the low-beta quintile portfolio from July 1986 to June 2014. Based on these results, they argue that the beta anomaly is not present in the Oslo Stock

Exchange. However, as their documented alphas are both negative and large in magnitude, we argue that their findings do not contradict our results. Moreover, their study is not directly comparable to ours as they rebalance the portfolios on a yearly basis while we use a monthly interval. Consequently, we find no critical deviations from our results in previously conducted research on the beta anomaly with a Norwegian sample.

6.1.6 Conclusion on the Beta Anomaly

All tests considered, we find strong evidence of a beta anomaly in our sample. When systematic risk is measured according to the CAPM, we document that the low-high β_{5Y} portfolios generate both economically large and statistically significant alpha across all tested variations in model specifications and data filters. However, when we expand our factor model and measure portfolio returns relative to the FFC4 + LIQ model, our results are not unambiguous. We find that the FFC4 + LIQ alpha is both economically large and statistically significant for the VW low-high β_{5Y} portfolio, but only economically large for the EW low-high portfolio. Although the lack of statistical significance makes our results more uncertain, we find it to be excessively strict to require statistical significance across all model specifications in order to infer the presence of a beta anomaly when using Norwegian data. As discussed in the comparison of our results with those of Bali et al. (2017), the small Norwegian sample and the use of quintile portfolios reduces the precision of our results. Even though our portfolios fulfill the diversification requirements of Ødegaard, we do not believe that our portfolios are adequately diversified to provide robust results across all variations of model specifications. At the beginning of the 1990s, our portfolios comprise close to 10 stocks, and minor variations in portfolio composition due to changes in model specifications or data filters could alter portfolio returns in non-trivial ways. Hence, we argue that the results from several models and robustness tests must be considered collectively to evaluate the presence of a beta anomaly on the OSE.

Considering that we find positive and economically large FFC4 + LIQ alphas for the low-high beta portfolio across all tested model specifications, and that the FFC4 + LIQ alpha is statistically significant for the VW portfolio from 1990-2018 and the EW portfolio from 2000-2018, we argue that our results suggest that we can disregard the null hypothesis of FFC4 + LIQ alpha equal to zero for the low-high β_{5Y} portfolio with a reasonably high degree of certainty. We hence infer that there has been a beta anomaly on the Oslo Stock Exchange in the period 1990-2018, which cannot be explained by the FFC4 + LIQ factor model.[33]

6.2 Hypothesis 2 – The Existence of a Lottery Demand Phenomenon in Norway

Having demonstrated the beta anomaly on the OSE, we proceed to examine the existence of the lottery demand phenomenon. In short, the lottery demand phenomenon refers to the low (high) abnormal returns of stocks that experience a disproportionately high (low) demand from investors with lottery-preferences. Kumar (2009) argues that stocks with high IVOL, high ISKEW, and a low share price are particularly attractive to lottery-investors, and Bali et al. (2011) find that the variable MAX successfully captures these lottery traits. Consequently, to assess the presence of a lottery demand phenomenon on the OSE, we first evaluate whether MAX effectively captures the lottery characteristics defined by Kumar (2009) on the OSE, and subsequently analyze the relation between MAX and one-month-ahead returns.

6.2.1 MAX as a Proxy for Lottery Demand

To evaluate whether MAX is a good proxy for lottery demand, we follow the methodology of Bali et al. (2017) and examine the relation between MAX and the three lottery traits put forth by Kumar (2009) in our sample. Table 7 reports the time-series average MAX, stock price, IVOL, and ISKEW for the MAX-sorted EW quintile portfolios. The average MAX values increase monotonically (by construction) from quintile 1 to quintile 5, and the difference between the average MAX value of the high portfolio and the average MAX value of the low portfolio of 5.7% is highly statistically significant with a corresponding t-statistic of -55.4. Average IVOL and average ISKEW exhibit a similar pattern across the portfolios; both variables increase monotonically across the MAX-sorted portfolios and the difference in the average values between the two extreme portfolios is highly statistically significant for both variables. The average stock price decreases monotonically from portfolio 1 (111) to portfolio 5 (41), and the average difference of NOK 71 is both economically large and statistically significant. The only discrepancy between our results and those of Bali et al. (2017) is that we observe a somewhat larger difference in the average IVOL and ISKEW between the low and high portfolios, while they find a larger spread in average stock prices and values of MAX. Nevertheless, the results presented in

[33] It is important to note that we document the anomaly relative to asset pricing models. The estimated factor model alphas can hence either be interpreted to be "true" in the sense that they illustrate the potential for riskless returns, or to be the result of a model error where the market correctly prices risk that is not reflected in the factor model.

Table 7 firmly indicate that MAX is successful in sorting stocks along the lottery-traits identified by Kumar (2009) in our sample.

Table 7: Lottery Characteristics of MAX-Sorted Quintile Portfolios

All stocks in our sample are sorted into quintile portfolios based on an ascending ordering of MAX at the end of each month *t*. The table presents the time-series means of the monthly average values of MAX, Share Price, idiosyncratic volatility (IVOL) and idiosyncratic skewness (ISKEW) for the stocks in each of the MAX-sorted quintile portfolios. The column labeled Low-High MAX presents the time-series mean difference in average characteristics between the low-MAX portfolio (quintile 1) and the high-MAX portfolio (quintile 5). The numbers in parentheses are t-statistics testing the null hypothesis of the average difference equal to zero. Our sample contains monthly observations of stock characteristics from Jan. 1990 through Dec. 2018.

	MAX 1 (Low)	MAX 2	MAX 3	MAX 4	MAX 5 (High)	Low-High MAX
			Stock Characteristics			
MAX	0.015	0.024	0.031	0.042	0.072	-0.057
						-(55.4)
Stock Price	111	97	80	62	41	71
						(39.6)
IVOL	0.013	0.017	0.021	0.027	0.046	-0.033
						-(53.6)
ISKEW	-0.048	0.080	0.183	0.273	0.546	-0.594
						-(59.4)

6.2.2 Cumulative Portfolio Returns

Having confirmed that MAX captures the lottery characteristics defined by Kumar (2009), we proceed to examine the relation between the variable MAX and one-month-ahead excess stock returns. Parallel to our approach on the beta anomaly, we begin our analysis by examining the historical performance of quintile portfolios sorted on MAX. Figure 4 presents the historical cumulative excess returns of a NOK 1 initial investment in each MAX-sorted quintile portfolio from January 1990 through December 2018. The figure contains two charts to illustrate the performance of both VW and EW portfolios.

Figure 4 illustrates that an investment of NOK 1 in the low-MAX portfolio has generated substantially higher absolute returns than a corresponding investment in the high-MAX portfolio. The VW (EW) low-MAX portfolio has generated a cumulative excess return of 207% (530%) compared to the -15.4% (79%) cumulative excess return of the VW (EW) high-MAX portfolio.[34] However, in contrast to our

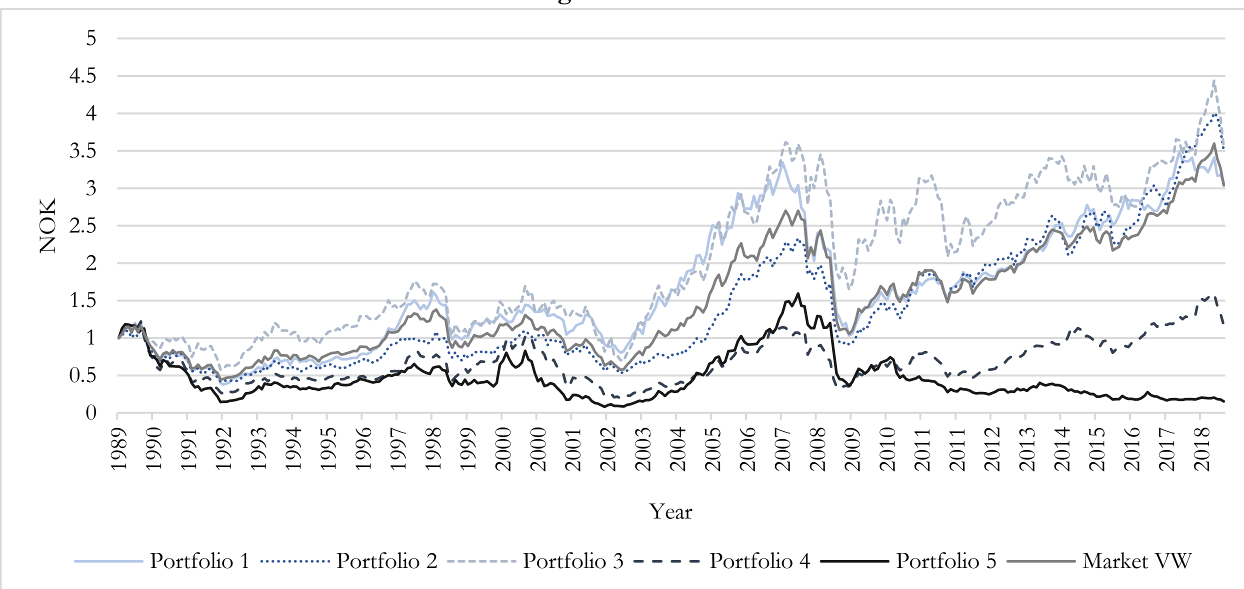
[34] The trading costs associated with replicating the MAX-sorted quintile portfolios are assessed in section F in the appendix.

analysis of the β_{5Y}-sorted quintile portfolios, we note that the low-MAX (high-MAX) portfolio does not distinguish itself as the best (worst) performer.

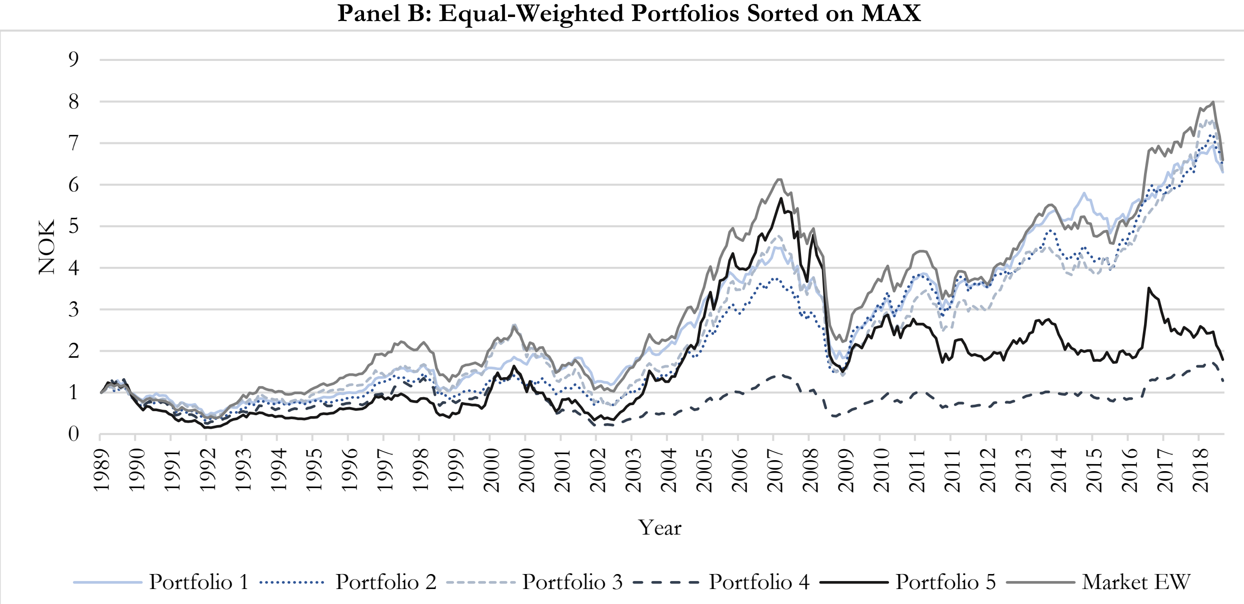
Figure 4: Cumulative Excess Returns of Quintile Portfolios Sorted on MAX

At the end of each month *t*, all stocks are sorted into quintile portfolios based on an ascending ordering of MAX and the portfolio excess returns are calculated for month *t +1*. The figure presents the historical cumulative excess returns (in NOK) of a NOK 1 investment in each of the quintile portfolios from Jan. 1990 through Dec. 2018. The quintile portfolios are rebalanced monthly, all dividends and cash payouts are assumed to be reinvested and the return calculation assumes no transaction costs. The portfolio *i* in the figure correspond to the MAX-sorted quintile portfolio *i*, while Market VW (EW) illustrates the value-weighted (equal-weighted) excess returns of a portfolio consisting of all the stocks in our filtered dataset. Panel A presents the cumulative excess returns of the value-weighted MAX quintile portfolios and panel B presents the cumulative excess returns of the equal-weighted MAX quintile portfolios.

Panel A: Value-Weighted Portfolios Sorted on MAX



Panel B: Equal-Weighted Portfolios Sorted on MAX



6.2.3 Univariate Portfolio Analyses

Analogous to our analysis on the beta anomaly, we formally analyze the relation between MAX and one-month-ahead returns by conducting a univariate portfolio analysis, this time on portfolios sorted by MAX instead of beta. Table 8 reports portfolio characteristics, Sharpe ratios, and monthly factor model alphas for VW and EW quintile portfolios sorted by MAX. The results in Table 8 show that the average MAX values for the VW (EW) quintile portfolios increase monotonically (by construction) from 1.6% (1.5%) for quintile 1 to 6.6% (7.2%) for quintile 5. Interestingly, we do not find a strong negative relation between MAX and one-moth-ahead excess returns, as documented by Bali et al. (2017). There is no indication of a monotonically decreasing relation between one-month-ahead excess returns and MAX, and the mean excess return of the EW low-high portfolio is both negative and very small in magnitude (-0.03%) in our sample. However, we note that ex-ante and ex-post portfolio betas increase monotonically from quintile 1 to quintile 5, consistent with the positive MAX-beta relation observed by Bali et al. (2017).

The CAPM alphas of the MAX-sorted quintile portfolios exhibit an overall negative relation to MAX, although they are not monotonically decreasing for either EW or VW portfolios. The low-high MAX portfolio generates a CAPM alpha of 0.71% for the VW portfolio and 0.53% for the EW portfolio. The alphas are economically large, but only statistically significant at the 10% level for both portfolio formation schemes.

Interestingly, we document stronger abnormal returns for the low-high MAX portfolio relative to the FFC4 + LIQ model than the CAPM.[35] The FFC4 + LIQ alphas for the VW (EW) portfolios decrease from 0.05% (0.28%) for quintile 1 to -0.90% (-0.38%) for quintile 5. In contrast to our results on the beta anomaly, we observe that several of the quintile portfolios generate statistically significant abnormal returns relative to the FFC4 + LIQ model. The VW (EW) low-high MAX portfolio generates a FFC4 + LIQ alpha of 0.94% (0.66%) with corresponding t-statistics of 2.431 (2.110), which is both

[35] Factor model alphas in multifactor models equals the intercept term from a regression of excess portfolio returns on factor returns and is estimated using $\hat{\alpha} = \bar{y} - \sum_{i=1}^{n} \hat{\beta}_i \bar{X}_i$, where $\hat{\alpha}$ equals the estimated intercept, $\bar{y}$ equals the average excess returns of the portfolio, $\hat{\beta}_i$ equals the estimated slope coefficient for factor i, and $\bar{X}_i$ equals the average returns associated with factor i. The inclusion of additional factors that the portfolio loads negatively in can thus result in estimates of alpha with a greater magnitude, despite the additional factors increasing the explanatory power of the model (R^2). We note that the increased factor model alpha for the low-high MAX portfolio in the FFC4 + LIQ factor model relative to the CAPM can largely be attributed to the low-high MAX portfolio's negative loading in the HML factor.

economically large and statistically significant. We note that the FFC4 + LIQ alpha for the low-high MAX portfolio stems mostly from the short side of the trade (shorting portfolio 5).

6.2.4 Comparison of the Results with the US Market

The results from our univariate portfolio analysis sorting on MAX are, for the most part, in line with the results of Bali et al. (2017). We have constructed MAX according to the methodology of Bali et al. (2017) and our analysis only differs with regards to our use of quintile breakpoints instead of decile breakpoints. On the US sample, Bali et al. (2017) report ex-ante EW portfolio MAX values increasing from 0.66% for decile 1 to 7.62% for decile 10. Parallel to our analysis of the beta-sorted portfolios, we find that Bali et al. (2017) report a somewhat larger spread in the sorting variable between their low and high portfolio, which we again attribute to their use of decile breakpoints.

In line with our results, they find that the FFC4 + PS alphas decrease from decile 1 (0.24%) to decile 10 (-1.15%). We note that the FFC4 + PS alphas experience a large drop from decile 9 (-0.37%) to decile 10 (-1.15%) in the US sample. This is similar to what we find for our VW portfolios, but the FFC4 + LIQ alpha actually increases from quintile 4 (-0.46%) to quintile 5 (-0.38%) for our EW portfolios. Bali et al. (2017) report that the high-low portfolio achieved a FFC4 + PS alpha of -1.38% with a corresponding t-statistic of (-8.09), which is both economically large and highly statistically significant.[36]

Although our results are generally in line with the results of Bali et al. (2017), we note that, in addition to increased statistical significance, they also find a significantly higher FFC4 + PS alpha in absolute terms for their high-low MAX portfolio. This stands in contrast to the findings in our comparison of the beta anomaly, where the magnitude of our FFC4 + PS alpha for the low-high β_{5Y} portfolio lined up closely with the corresponding results of Bali et al. (2017).[37]

[36] Bali et al. (2017) find that the FFC4 + PS alpha of the high-low portfolio is mostly attributable to shorting decile 10

Table 8: Univariate Portfolio Analysis Sorting on MAX

The table presents the results of a univariate portfolio analysis on the relation between excess and abnormal returns in month *t*+1 and the variable MAX in month *t*. At the end of each month *t*, all stocks are sorted into quintile portfolios based on an ascending ordering of MAX. The table consists of two panels (A and B) and each panel is divided into three sections.

Explanation of sections: "Portfolio Characteristics" presents the time-series mean number of stocks in each quintile portfolio (Portfolio length), the time-series mean portfolio market share (calculated as the sum of the market capitalization within each portfolio divided by the total market capitalization) for each month *t* (Market share), the time-series mean of the monthly average value of MAX for the stocks in each quintile portfolio (MAX ex-ante), the time-series mean of the monthly average β_{5Y} for the stocks in each quintile portfolio (β ex-ante), and the slope coefficient from a regression of VW (EW) portfolio excess returns on the VW (EW) market excess returns (β ex-post). "Portfolio Sharpe Ratio" presents the time-series mean monthly portfolio excess returns in month *t+1* (R), the standard deviation of monthly portfolio excess returns in month *t+1* (SD), and the annualized portfolio Sharpe ratio (SR). "Factor Model Alphas" presents monthly portfolio alphas relative to the CAPM (CAPM), the Fama-French 3-Factor model (FF3), the Fama-French-Carhart 4-Factor model (FFC4) and the FFC4 model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags.

Explanation of panels: All reported numbers in panel A are calculated using value-weighted portfolios, while all numbers in panel B are calculated using equal-weighted portfolios.

General: The column labeled Low-High MAX refers to a zero-cost, long-short portfolio with a long position in quintile portfolio 1 and a short position in quintile portfolio 5. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.

Panel A: Value-Weighted Portfolios

Value	MAX 1 (Low)	MAX 2	MAX 3	MAX 4	MAX 5 (High)	Low-High MAX
			Portfolio Characteristics			
Portfolio length	25	25	25	25	25	
Market share	31%	30%	21%	13%	6%	
MAX ex-ante	0.016	0.024	0.031	0.041	0.066	-0.050
β ex-ante	0.841	0.894	0.928	0.942	0.951	-0.109
β ex-post	0.922	0.954	1.055	1.280	1.463	-0.541
			Portfolio Sharpe Ratio			
R	0.51 %	0.55 %	0.60 %	0.43 %	0.07 %	0.44 %
SD	5.98 %	6.11 %	6.85 %	8.55 %	10.98 %	8.73 %
SR	0.294	0.313	0.305	0.174	0.021	0.175
			Factor Model Alphas			
CAPM	0.06 %	0.09 %	0.09 %	-0.20 %	-0.65 %	0.71 %
	(0.390)	(0.606)	(0.540)	-(0.873)	-(1.796)	(1.673)
FF3	0.06 %	0.12 %	0.08 %	-0.31 %	-1.08 %	1.14 %
	(0.385)	(0.820)	(0.460)	-(1.285)	-(3.051)	(2.760)
FFC4	0.04 %	0.09 %	0.20 %	-0.27 %	-0.86 %	0.89 %
	(0.240)	(0.584)	(1.188)	-(1.146)	-(2.471)	(2.169)
FFC4 + LIQ	0.05 %	0.08 %	0.18 %	-0.29 %	-0.90 %	0.94 %
	(0.310)	(0.536)	(1.118)	-(1.334)	-(2.719)	(2.431)

Panel B: Equal-Weighted Portfolios

Value	MAX 1 (Low)	MAX 2	MAX 3	MAX 4	MAX 5 (High)	Low-High MAX
			Portfolio Characteristics			
Portfolio length	25	25	25	25	25	
Market share	31 %	30 %	21 %	12 %	6 %	
MAX ex-ante	0.015	0.024	0.031	0.042	0.072	-0.057
β ex-ante	0.849	0.992	1.084	1.156	1.205	-0.356
β ex-post	0.782	0.966	1.094	1.286	1.552	-0.771
			Portfolio Sharpe Ratio			
R	0.68 %	0.74 %	0.79 %	0.43 %	0.71 %	-0.03 %
SD	5.40 %	6.43 %	7.08 %	8.39 %	10.48 %	7.77 %
SR	0.437	0.401	0.384	0.179	0.234	-0.012
			Factor Model Alphas			
CAPM	0.12 %	0.05 %	0.00 %	-0.49 %	-0.41 %	0.53 %
	(0.769)	(0.303)	-(0.023)	-(2.496)	-(1.672)	(1.608)
FF3	0.33 %	0.31 %	0.17 %	-0.39 %	-0.44 %	0.77 %
	(2.545)	(2.206)	(1.260)	-(2.060)	-(1.742)	(2.490)
FFC4	0.29 %	0.33 %	0.21 %	-0.39 %	-0.35 %	0.64 %
	(2.246)	(2.235)	(1.512)	-(2.038)	-(1.324)	(2.029)
FFC4 + LIQ	0.28 %	0.29 %	0.15 %	-0.46 %	-0.38 %	0.66 %
	(2.108)	(2.036)	(1.181)	-(2.709)	-(1.479)	(2.110)

6.2.5 Robustness Tests of the Lottery Demand Phenomenon

We conduct several additional tests to examine whether our results are robust. Table C.1 in the appendix reports the abnormal returns of the low-high MAX portfolio relative to the CAPM and the FFC4 + LIQ model over five different time periods, while Table C.2 in the appendix reports the corresponding measures for different variations in our data filters. We find that the abnormal returns of the low-high MAX portfolio relative to both the CAPM and FFC4 + LIQ are large in magnitude across all time periods for both EW and VW portfolios. However, the CAPM alphas are not statistically significant for any of the tested periods, and the FFC4 + LIQ alpha is only statistically significant for the period 1990-2018. Table C.2 shows that the results are robust to all variations in data filters except for the exclusion of small stocks (market cap below NOK 1bn). Our robustness tests indicate that the results from our main model are generally robust to the data filters we have applied, but not for varying time periods. All test considered, we find that the low-high MAX portfolio generates economically large abnormal returns relative to CAPM and the FFC4 + LIQ factor model for all tested variations in model specifications. The CAPM alpha of the low-high MAX portfolio is, however, in most cases not statistically significant, and while the FFC4 + LIQ alpha is statistically significant for both EW and

VW portfolios in our main model, the results are not robust to varying time periods. Consequently, we find our results to strongly indicate that there is a negative relation between MAX and future abnormal returns in our sample, but the ambiguous results from our robustness tests prevent us from concluding with great certainty.

Our analysis demonstrates that MAX effectively captures the lottery-traits defined by Kumar (2009) on the Oslo Stock Exchange, and that there is a negative relation between MAX and returns in our sample. However, it is important to note that although these results are in line with those of Bali et al. (2017) and Bali et al. (2011), the results do not, in isolation, prove that the observed negative relation between MAX and abnormal returns can solely be attributed to investor demand for lottery-like assets.[38]

6.3 Hypothesis 3 – Lottery Demand as an Explanation of the Beta Anomaly

In the previous section of the report, we demonstrated the presence of the beta anomaly and that there is a negative correlation between MAX and abnormal returns on the OSE. In this section, we proceed to examine whether lottery demand, as measured by MAX, can explain the documented beta anomaly. The rationale of Bali et al. (2017) is that MAX should correlate positively with market beta as up-moves in stock prices are partially explained by the stock's sensitivity to the market. Lottery investors should therefore exert a disproportionately high price pressure on high-beta stocks relative to low-beta stocks and thus contribute to generating the beta anomaly.

6.3.1 The Cross-Sectional Relation between MAX and Beta

The lottery demand-based explanation of the beta anomaly is contingent on MAX being correlated with beta in the cross-section. As such, we begin our analysis by assessing the cross-sectional relation between MAX and beta in our sample. Table 9 presents the time-series average of the monthly cross-sectional Pearson product-moment correlation ("Pearson correlation" hereafter) between MAX and our estimates of beta. We find that that the Pearson correlation between MAX and β_{5Y} (β_{1Y}) is 0.20 (0.28) on average in our sample, illustrating that in most months, lottery stocks are also high-beta stocks. In the US sample, Bali et al. (2017) report that the Person moment correlation between MAX and β_{1Y} is 0.30 on average. Although our estimates of the correlation between β_{1Y} and MAX are remarkably similar, we note that we estimate a weaker correlation between MAX and β_{5Y} compared to

[38] There could be other factors correlated with both MAX and future stock returns explaining the observed negative relation.

Bali et al. (2017). However, the methodology used to estimate MAX more closely resembles the methodology used to estimate β_{1Y} than β_{5Y}, and we hence argue that β_{1Y} by construction should exhibit a stronger correlation with MAX. As such, we find our results to be in line with what Bali et al. (2017) find in the US.

Table 9: Average Monthly Cross-Sectional Correlation between Variables

The table presents the time-series average of the monthly cross-sectional Pearson product-moment correlation between the variables. The data sample runs from Jan. 1990 through Dec. 2018.

	β_{5Y}	β_{1Y}	β_{MSCI}	MAX	IVOL	ISKEW
β_{5Y}	1.00	0.56	0.70	0.20	0.13	0.04
β_{1Y}	0.56	1.00	0.37	0.28	0.17	0.03
β_{MSCI}	0.70	0.37	1.00	0.11	0.05	0.02
MAX	0.20	0.28	0.11	1.00	0.87	0.30
IVOL	0.13	0.17	0.05	0.87	1.00	0.19
ISKEW	0.04	0.03	0.02	0.30	0.19	1.00

6.3.2 Clarification of the Data Sample

Having demonstrated that MAX is positively correlated with beta in our sample, we proceed to formally test whether the beta anomaly persists after controlling for MAX. However, before we embark on the analyses, we must make a clarifying comment on the differing data samples used in the following analysis.

In the previous sections (6.1 and 6.2) of the report, we demonstrated the relation between our sorting variables and one-month-ahead excess returns. To maximize the precision of our results given our limited dataset, we utilized all observations of the sorting variable of interest in the univariate portfolio analysis. As discussed in section 4 of the report, the data requirements for estimating the different sorting variables vary, which implies that we have used different data samples for the univariate portfolio analysis depending on the sorting variable of interest.

In the following section of the report, we examine whether the low-high beta (MAX) portfolio generates statistically abnormal returns when the portfolio is constructed to be neutral to MAX (beta). This form of bivariate portfolio analysis requires a data sample with stock observations that contain estimates of both MAX and beta, and the dataset used will therefore differ from the ones originally employed to document the beta anomaly and the lottery demand phenomenon. The results from the bivariate portfolio analysis will therefore include both the effect from adding a control variable and the use of a differing sample. As the Norwegian dataset is small, the use of a differing sample could alter our results in non-trivial ways. To isolate the effect of adding a control variable, we therefore report the results for both the beta anomaly and lottery demand phenomenon using a common sample in Table 14 and Table D.1 and D.2 in the appendix.[39]

6.3.3 The Beta Anomaly When Controlling for MAX

Bivariate Portfolio Analysis

To assess the relation between market beta and future excess returns after controlling for MAX, we first explore the performance of the quintile portfolios generated by performing a conditional double sort on MAX, then on β_{5Y}. Table 10 reports portfolio characteristics and factor model alphas for the average MAX quintile portfolios, within each β_{5Y} quintile. We first note that our conditional double sort is successful in creating β_{5Y} -sorted portfolios that are neutralized to MAX. The ex-ante average MAX values are almost identical across the β_{5Y} quintiles and the ex-ante and ex-post portfolio betas increase monotonically.

The results in Table 10 illustrate that the zero-cost portfolio with a long position in the average MAX portfolio in the low β_{5Y} quintile and a short position in the average MAX portfolio in the high β_{5Y} quintile (low-high portfolio), generates economically large and statistically significant abnormal returns for both VW and EW portfolios relative to the CAPM and the FFC4 + LIQ factor model. Interestingly, we also note that both the CAPM alpha and the FFC4 + LIQ alpha for the EW low-high portfolio are greater in magnitude when controlling for MAX. The bivariate portfolio analysis thus indicates that the beta anomaly remains statistically significant after controlling for MAX.

[39] Using a common sample, we find that both the low-high β_{5Y} portfolio and the low-high MAX portfolio generates statistically significant CAPM alpha and a FFC4 + PS alpha for both the EW and VW portfolios.

Table 10: Bivariate Portfolio Analysis Sorting on MAX, Then on β_{5Y}

The table presents the results from a bivariate portfolio analysis on the relation between the abnormal returns in month *t*+1 and the variable β_{5Y} in month *t* while controlling for MAX. The table presents portfolio characteristics and factor model alphas for the average MAX quintile portfolio within each β_{5Y} quintile. We create the average MAX quintile by performing a conditional double sort first on MAX, then on β_{5Y} to generate a 5x5 portfolio matrix at the end of each month *t*. We subsequently sum the stocks across the MAX quintiles for each β_{5Y} quintile to create five β_{5Y}-sorted portfolios that by construction should be neutralized to MAX. We only include monthly stock observations with estimates of both β_{5Y} and MAX for the given month. The table consists of two panels (A and B) and each panel is divided into two sections.

Explanation of sections: "Portfolio Characteristics" presents the time-series mean of the average values of MAX for the stocks in each of the β_{5Y}-sorted portfolios (MAX ex-ante), the time-series mean of the average values of β_{5Y} for the stocks in each of the β_{5Y}-sorted portfolios (β ex-ante), and the slope coefficient from a regression of VW (EW) portfolio excess returns on the VW (EW) market excess returns (β ex-post). "Factor Model Alphas" presents monthly portfolio alphas relative to the CAPM (CAPM) and the Fama-French-Carhart 4-Factor model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for the VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.

Explanation of panels: All reported numbers in Panel A are calculated using value-weighted portfolios, while all numbers in Panel B are calculated using equal-weighted portfolios.

General: The column labeled Low-High β_{5Y} refers to a zero-cost, long-short portfolio with a long position in the average MAX portfolio in β_{5Y} quintile 1 and a short position in the average MAX portfolio in β_{5Y} quintile 5.

Panel A: Value-Weighted Portfolios

Value	β_{5Y} 1 (Low)	β_{5Y} 2	β_{5Y} 3	β_{5Y} 4	β_{5Y} 5 (High)	Low-High β_{5Y}
			Portfolio Characteristics			
MAX ex-ante	0.029	0.028	0.027	0.026	0.028	0.001
β ex-ante	0.520	0.794	0.973	1.169	1.559	-1.039
β ex-post	0.819	1.004	1.036	1.083	1.227	-0.408
			Factor Model Alphas			
CAPM	0.23 %	0.35 %	0.06 %	-0.14 %	-0.37 %	0.59 %
	(0.989)	(1.481)	(0.311)	-(0.802)	-(1.823)	(1.993)
FFC4 + LIQ	0.17 %	0.42 %	0.18 %	-0.02 %	-0.42 %	0.59 %
	(0.768)	(1.894)	(0.834)	-(0.132)	-(2.045)	(2.034)

Panel B: Equal-Weighted Portfolios

Value	β_{5Y} 1 (Low)	β_{5Y} 2	β_{5Y} 3	β_{5Y} 4	β_{5Y} 5 (High)	Low-High β_{5Y}
			Portfolio Characteristics			
MAX ex-ante	0.035	0.035	0.035	0.035	0.036	-0.001
β ex-ante	0.461	0.782	1.009	1.261	1.717	-1.256
β ex-post	0.841	0.944	1.025	1.184	1.276	-0.434
			Factor Model Alphas			
CAPM	0.42 %	0.09 %	-0.07 %	-0.31 %	-0.79 %	1.21 %
	(2.238)	(0.446)	-(0.350)	-(1.453)	-(3.471)	(4.007)
FFC4 + LIQ	0.47 %	0.16 %	0.09 %	-0.07 %	-0.48 %	0.95 %
	(2.355)	(0.725)	(0.551)	-(0.400)	-(2.304)	(3.090)

Univariate Portfolio Analysis Sorting on $\beta_{5Y\perp M}$

To test the robustness of our results, we also construct beta-sorted portfolios that are neutralized to MAX by sorting stocks into quintile portfolios based on the portion of β_{5Y} that is orthogonal to MAX ($\beta_{5Y\perp M}$). Table 11 reports the portfolio characteristics and the factor model alphas for the $\beta_{5Y\perp M}$-sorted portfolios. We note that the methodology successfully sorts stocks into quintile portfolios with low variation in average MAX values and monotonically increasing betas, similar to our bivariate portfolio analysis.

The EW low-high $\beta_{5Y\perp M}$ portfolio generates a statistically significant CAPM alpha and FFC4 + LIQ alpha, but the CAPM alpha and the FFC4 + LIQ alpha is only statistically significant at the 10% level for the VW low-high $\beta_{5Y\perp M}$ portfolio. Comparing the results with the abnormal returns of the VW (EW) low-high β_{5Y} portfolio constructed using the same sample (Table 14), we find that controlling for MAX resulted in a 21% (19%) reduction in CAPM alpha and a 24% (20%) reduction in FFC4 + LIQ alpha. Although controlling for MAX resulted in reduced alphas for the low-high portfolio, we note that the abnormal returns of the VW (EW) low-high $\beta_{5Y\perp M}$ remains statistically significant at the 10% (5%) level and is not statistically different from the abnormal returns of the VW (EW) low-high β_{5Y} portfolio. As such, we find the results to be consistent with the findings of the bivariate portfolio analysis.

Evaluating the results from our univariate portfolio analysis on $\beta_{5Y\perp M}$-sorted portfolios in conjunction with the results from the bivariate portfolio analysis, we find that the beta anomaly remains statistically significant after controlling for MAX in three out of the four tests we have conducted. We therefore conclude that our analysis does not provide strong evidence in support of the theory that lottery demand measured by MAX, plays an important role in generating the beta anomaly in our sample.

Our results deviate substantially from what Bali et al. (2017) find when they conduct the same analyses on their US sample. They find no evidence of a beta anomaly after controlling for MAX and report a FFC4 + PS alpha for the EW high-low $\beta_{1Y\perp M}$ portfolio of 0.08% percent, which is both economically small and statistically indistinguishable from zero.

Table 11: Univariate Portfolio Analysis Sorting on the Orthogonal of β_{5Y} on MAX ($\beta_{5Y\perp M}$)

The table presents the results from a univariate portfolio analysis on the relation between the excess and abnormal returns in month *t*+1 and the portion of β_{5Y} that is orthogonal to MAX in month *t*.
The table consists of two panels (A and B) and each panel is divided into two sections.
Explanation of sections: "Portfolio Characteristics" presents the time-series mean of the average values of MAX for the tocks in each of the $\beta_{5Y\perp M}$ -sorted portfolios (MAX ex-ante), the time-series mean of the average values of β_{5Y} for the stocks in each of the $\beta_{5Y\perp M}$ -sorted portfolios (β ex-ante), and the slope coefficient from a regression of VW (EW) portfolio excess returns on the VW (EW) market excess returns (β ex-post). "Factor Model Alphas" presents monthly portfolio alphas relative to the CAPM (CAPM) and the FFC4 model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for the VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags.
Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.
Explanation of panels: All reported numbers in Panel A are calculated using value-weighted portfolios, while all numbers in Panel B are calculated using equal-weighted portfolios.
General: The column labeled Low-High $\beta_{5Y\perp M}$ refers to a zero-cost, long-short portfolio with a long position in quintile portfolio 1 and a short position in quintile portfolio 5.

Panel A: Value-Weighted Portfolios

Value	$\beta_{5Y\perp M}$ 1 (Low)	$\beta_{5Y\perp M}$ 2	$\beta_{5Y\perp M}$ 3	$\beta_{5Y\perp M}$ 4	$\beta_{5Y\perp M}$ 5 (High)	Low-High $\beta_{5Y\perp M}$
			Portfolio Characteristics			
MAX ex-ante	0.030	0.027	0.025	0.027	0.032	-0.002
β ex-ante	0.466	0.751	0.947	1.215	1.690	-1.223
β ex-post	0.756	1.044	1.000	1.118	1.320	-0.564
			Factor Model Alphas			
CAPM	0.26 %	0.28 %	0.09 %	-0.04 %	-0.37 %	0.63 %
	(1.096)	(1.215)	(0.513)	-(0.181)	-(1.604)	(1.865)
FFC4 + LIQ	0.23 %	0.40 %	0.09 %	-0.05 %	-0.34 %	0.57 %
	(1.042)	(1.678)	(0.484)	-(0.202)	-(1.402)	(1.692)

Panel B: Equal-Weighted Portfolios

Value	$\beta_{5Y\perp M}$ 1 (Low)	$\beta_{5Y\perp M}$ 2	$\beta_{5Y\perp M}$ 3	$\beta_{5Y\perp M}$ 4	$\beta_{5Y\perp M}$ 5 (High)	Low-High $\beta_{5Y\perp M}$
			Portfolio Characteristics			
MAX ex-ante	0.038	0.034	0.032	0.034	0.037	0.001
β ex-ante	0.446	0.763	0.982	1.257	1.782	-1.336
β ex-post	0.825	0.944	1.023	1.137	1.355	-0.530
			Factor Model Alphas			
CAPM	0.22 %	0.13 %	-0.03 %	-0.44 %	-0.64 %	0.86 %
	(1.271)	(0.764)	-(0.154)	-(1.962)	-(2.790)	(2.839)
FFC4 + LIQ	0.27 %	0.17 %	0.12 %	-0.19 %	-0.34 %	0.61 %
	(1.523)	(0.981)	(0.681)	-(0.926)	-(1.578)	(1.970)

Alternative Measures of Beta

Bali et al. (2017) use β_{1Y} as their estimate of market beta in their focal analyses. In section 6.3.1, we report results demonstrating a weaker correlation between MAX and β_{5Y} (0.20) in our sample than what Bali et al. (2017) document between MAX and β_{1Y} (0.30) in their US sample.[40] The correlation between the variable of interest and the control variable is one of the factors which will determine the portfolio composition resulting from a conditional double-sort or a univariate portfolio sort using

[40] The monthly Pearson moment correlation between β_{1Y} and MAX has been 0.28 on average in our sample.

orthogonal components. To examine whether our deviating findings can be attributed to our use of $\beta_{5Y,}$ we redo our analysis with β_{1Y} as the estimate of beta.

We report the results of a univariate portfolio analysis on β_{1Y} using only observations that contain estimates of both β_{1Y} and MAX in Table D.1 in the appendix. We find that the VW (EW) low-high β_{1Y} portfolio generates a FFC4 + LIQ alpha of 0.24% (0.73%) with a corresponding t-statistic of 0.658 (1.979) and note that the beta anomaly is only statistically significant for the EW portfolio. The results of a bivariate analysis on portfolios sorted first on MAX, then on β_{1Y}, are reported in Table D.3 in the appendix. We find that after controlling for MAX, the FFC4 + LIQ alpha of the EW low-high portfolio is reduced to 0.56% and is only statistically significant at the 10% level. In line with our previous results, we find that controlling for MAX does not result in a significant reduction in FFC4 + LIQ alpha for the EW low-high portfolio. When also considering that the beta anomaly remains economically large and statistically significant at the 10% level for the EW low-high portfolio after controlling for MAX, we find the results from the robustness test to be in line with the results from our main model. As such, we conclude that our deviating findings are not likely to be a result of differing methodologies used to estimate beta.[41][42]

6.3.4 The Lottery Demand Phenomenon When Controlling for β_{5Y}

Bivariate Portfolio Analysis

In the previous section of the report, we demonstrated that the beta anomaly persists after controlling for MAX. To further explore the relation between the two variables, we reverse the roles, and proceed to assess the relation between MAX and one-month-ahead excess returns, while controlling for beta.

We begin by performing a bivariate portfolio analysis sorting first on β_{5Y}, then on MAX, and we report the results in Table 12 below. A zero-cost portfolio with a long position in the average β_{5Y} portfolio in MAX quintile 1 and short position in the average β_{5Y} portfolio in MAX quintile 5 (low-high portfolio) does not generate a statistically significant CAPM alpha or FFC4 + LIQ alpha for either the EW or VW portfolios. Our results indicate that there is not a statistically significant negative relation between MAX and one-month ahead abnormal returns when we control for $\beta_{5Y.}$

[41] Bali et al. (2017) also conduct a robustness test using an extended sample and find that the high-low β_{5Y} sorted portfolio no longer generates statistically significant FFC4 + PS alpha when the portfolio is neutralized to MAX.
[42] In table D.4 in the appendix we report the results from a univariate portfolio analysis sorting on the component of β_{1Y} that is orthogonal to MAX. We find the results to be in line with the results from the bivariate portfolio analysis on β_{1Y} controlling for MAX.

Table 12: Bivariate Portfolio Analysis Sorting on β_{5Y}, Then on MAX

The table presents the results from a bivariate portfolio analysis on the relation between the abnormal returns in month t+1 and the variable MAX in month t while controlling for β_{5Y}. The table presents portfolio characteristics and factor model alphas for the average β_{5Y} quintile portfolio within each MAX quintile. We create the average β_{5Y} quintile by performing a conditional double sort first on β_{5Y}, then on MAX to generate a 5x5 portfolio matrix at the end of each month t. We subsequently sum the stocks across the β_{5Y} quintiles for each MAX quintile to create five MAX-sorted portfolios that by construction should be neutralized to β_{5Y}. We only include monthly stock observations with estimates of both β_{5Y} and MAX for the given month. The table consists of two panels (A and B) and each panel is divided into two sections.

Explanation of sections: "Portfolio Characteristics" presents the time-series mean of the average values of MAX for the stocks in each of the MAX-sorted portfolios (MAX ex-ante), the time-series mean of the average values of β_{5Y} for the stocks in each of the MAX-sorted portfolios (β ex-ante), and the slope coefficient from a regression of VW (EW) portfolio excess returns on the VW (EW) market excess returns (β ex-post). "Factor Model Alphas" presents monthly portfolio alphas relative to the CAPM (CAPM) and the Fama-French-Carhart 4-Factor model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for the VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.

Explanation of panels: All reported numbers in Panel A are calculated using value-weighted portfolios, while all numbers in Panel B are calculated using equal-weighted portfolios.

General: The column labeled Low-High MAX refers to a zero-cost, long-short portfolio with a long position in the average β_{5Y} portfolio in MAX quintile 1 and a short position in the average β_{5Y} portfolio in MAX quintile 5.

Panel A: Value-Weighted Portfolios

Value	MAX 1 (Low)	MAX 2	MAX 3	MAX 4	MAX 5 (High)	Low-High MAX
			Portfolio Characteristics			
MAX ex-ante	0.017	0.023	0.030	0.038	0.058	-0.042
β ex-ante	1.046	1.050	1.076	1.078	1.192	-0.146
β ex-post	0.937	1.011	1.083	1.084	1.296	-0.359
			Factor Model Alphas			
CAPM	-0.02 %	0.07 %	0.26 %	-0.26 %	-0.51 %	0.49 %
	-(0.137)	(0.358)	(1.374)	-(1.236)	-(1.675)	(1.370)
FFC4 + LIQ	0.00 %	0.22 %	0.27 %	-0.29 %	-0.69 %	0.69 %
	(0.016)	(1.140)	(1.309)	-(1.359)	-(2.145)	(1.769)

Panel B: Equal-Weighted Portfolios

Value	MAX 1 (Low)	MAX 2	MAX 3	MAX 4	MAX 5 (High)	Low-High MAX
			Portfolio Characteristics			
MAX ex-ante	0.016	0.024	0.030	0.039	0.066	-0.050
β ex-ante	0.999	1.030	1.049	1.064	1.091	-0.092
β ex-post	0.805	0.942	1.030	1.149	1.362	-0.557
			Factor Model Alphas			
CAPM	-0.07 %	0.07 %	-0.09 %	-0.06 %	-0.60 %	0.53 %
	-(0.442)	(0.422)	-(0.460)	-(0.311)	-(2.430)	(1.716)
FFC4 + LIQ	0.10 %	0.29 %	0.16 %	0.00 %	-0.49 %	0.60 %
	(0.725)	(2.096)	(0.941)	-(0.022)	-(1.960)	(1.907)

Univariate Portfolio Analysis Sorting on $MAX_{\perp\beta}$

Analogous to our bivariate portfolio analysis controlling for MAX, we test the robustness of our results by also performing a univariate portfolio analysis on the portion of MAX that is orthogonal to β_{5Y} ($MAX_{\perp\beta}$). The results are presented in Table 13 below. In line with the results from the bivariate portfolio analysis, we find that the low-high $MAX_{\perp\beta}$ portfolio does not generate a statistically significant CAPM alpha or FFC4 + LIQ alpha for VW or EW portfolios. The univariate portfolio analysis on $MAX_{\perp\beta}$ hence provides additional evidence suggesting that the lottery demand phenomenon as measured by MAX is not statistically significant in our sample after controlling for beta.

The results from our analyses of the relation between MAX and abnormal returns when controlling for beta also deviate significantly from the results obtained by Bali et al. (2017) as they document that the abnormal returns of the high-low MAX portfolio remain economically large and highly statistically significant after controlling for beta.

6.3.5 Interpretation of the Results from Univariate and Bivariate Portfolio Sorts

The results from the bivariate portfolio analysis and univariate portfolio analysis sorting on the portion of beta that is orthogonal to MAX do not provide evidence in support of the theory that lottery demand as measured by MAX plays an important role in generating the documented beta anomaly on the OSE for the period 1990-2018. Our results demonstrate that a portfolio that has been long low-beta stocks and short high-beta stocks, while maintaining a neutral exposure to MAX, has generated statistically significant abnormal returns in three out of the four tests conducted using our main model sorting on β_{5Y}.

Furthermore, we demonstrate that the low-high MAX portfolio no longer generates statistically significant abnormal returns when the portfolio is constructed to be neutral to beta. We argue that the results illustrate that the negative relation observed between MAX and abnormal returns in our sample, cannot necessarily solely be attributed to investor demand for lottery-like assets. Moreover, since the abnormal returns of the low-high MAX portfolio are not statistically significant after controlling for beta, we find that the negative relation between MAX and abnormal returns documented in section 6.2 of the thesis, cannot be interpreted to illustrate the presence of a statistically significant lottery demand phenomenon on the Oslo Stock Exchange.

Table 13: Univariate Portfolio Analysis Sorting on the Orthogonal of MAX on β_{5Y} ($MAX_{\perp\beta}$)

The table presents the results from a univariate portfolio analysis on the relation between the excess and abnormal returns in month *t*+1 and the portion of MAX that is orthogonal to β_{5Y} in month *t*.
The table consists of two panels (A and B) and each panel is divided into two sections.
Explanation of sections: "Portfolio Characteristics" presents the time-series mean of the average values of MAX for the stocks in each of the $MAX_{\perp\beta}$ -sorted portfolios (MAX ex-ante), the time-series mean of the average values of β_{5Y} for the stocks in each of the $MAX_{\perp\beta}$ -sorted portfolios (β ex-ante), and the slope coefficient from a regression of VW (EW) portfolio excess returns on the VW (EW) market excess returns (β ex-post). "Factor Model Alphas" presents monthly portfolio alphas relative to the CAPM (CAPM) and the FFC4 model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for the VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.
Explanation of panels: All reported numbers in Panel A are calculated using value-weighted portfolios, while all numbers in Panel B are calculated using equal-weighted portfolios.
General: The column labeled Low-High $MAX_{\perp\beta}$ refers to a zero-cost, long-short portfolio with a long position in quintile portfolio 1 and a short position in quintile portfolio 5.

Panel A: Value-Weighted Portfolios

Value	$MAX_{\perp\beta}$ 1 (Low)	$MAX_{\perp\beta}$ 2	$MAX_{\perp\beta}$ 3	$MAX_{\perp\beta}$ 4	$MAX_{\perp\beta}$ 5 (High)	Low-High $MAX_{\perp\beta}$
			Portfolio Characteristics			
MAX ex-ante	0.017	0.023	0.029	0.038	0.062	-0.045
β ex-ante	1.088	1.030	1.042	1.057	1.190	-0.102
β ex-post	0.957	1.029	1.037	1.146	1.298	-0.341
			Factor Model Alphas			
CAPM	-0.04 %	0.18 %	0.13 %	-0.39 %	-0.58 %	0.53 %
	-(0.290)	(1.009)	(0.666)	-(1.521)	-(1.691)	(1.339)
FFC4 + LIQ	-0.02 %	0.22 %	0.22 %	-0.44 %	-0.77 %	0.75 %
	-(0.140)	(1.126)	(1.097)	-(1.701)	-(2.127)	(1.777)

Panel B: Equal-Weighted Portfolios

Value	$MAX_{\perp\beta}$ 1 (Low)	$MAX_{\perp\beta}$ 2	$MAX_{\perp\beta}$ 3	$MAX_{\perp\beta}$ 4	$MAX_{\perp\beta}$ 5 (High)	Low-High $MAX_{\perp\beta}$
			Portfolio Characteristics			
MAX ex-ante	0.017	0.023	0.030	0.038	0.068	-0.051
β ex-ante	1.095	1.010	1.024	1.033	1.068	0.027
β ex-post	0.871	0.930	0.972	1.155	1.354	-0.483
			Factor Model Alphas			
CAPM	-0.08 %	0.19 %	0.09 %	-0.40 %	-0.55 %	0.47 %
	-(0.497)	(1.058)	(0.486)	-(1.820)	-(2.146)	(1.387)
FFC4 + LIQ	0.12 %	0.37 %	0.34 %	-0.29 %	-0.51 %	0.64 %
	(0.812)	(2.333)	(1.993)	-(1.264)	-(1.962)	(1.922)

Our results deviate substantially from what Bali et al. (2017) document in the US, but we still find them to be reasonable. As discussed in section 2.3, there are many competing explanations for the beta anomaly. Given that some other factor than lottery demand measured by MAX generates the anomaly in Norway, we would expect the same factor to impact the relation between MAX and returns, given that MAX by construction should exhibit some correlation with β_{5Y}.[43]

[43] The results from a bivariate portfolio analysis using a conditional double sort, or a univariate portfolio analysis on orthogonal components cannot be used to determine the "true" causal relation between the sorting variables and returns.

6.3.6 FMAX Factor

The results from section 6.3 illustrate that the abnormal returns of the low-high beta portfolio remain statistically significant when portfolios are constructed to have a neutral exposure to MAX. In this section, we continue our analysis on the beta anomaly controlling for MAX, but instead controlling for portfolio exposure to MAX, we include a MAX factor (FMAX) designed to capture the returns associated with lottery demand in our factor models. We proceed to assess to what degree the FMAX factor can explain the abnormal returns of the low-high beta portfolio. In Table 14 we present the abnormal returns of the β_{5Y}-sorted quintile portfolios, relative to the CAPM, the FFC4 + LIQ, the FFC4 + LIQ + FMAX, and the FFC4 + LIQ + FIVOL factor models.

We find that when we augment the FFC4 + LIQ factor model with the FMAX factor, the abnormal returns for the VW and EW low-high β_{5Y} portfolios are no longer statistically significant. The VW (EW) low-high β_{5Y} portfolio generates a statistically significant FFC4 + LIQ alpha of 0.76% (0.73%) with a corresponding t-statistic of 2.045 (2.416). When we add the FMAX factor to the model, we find that the VW (EW) low-high β_{5Y} portfolio generates a FFC4 + LIQ + FMAX alpha of 0.48% (0.45%) with a corresponding t-statistic of 1.160 (1.374), which is not statistically significant.

We evaluate the robustness of our results and report the abnormal returns of the β_{1Y}-sorted quintile portfolios in Table D.1 in the appendix. In line with our main model, we find that the EW low-high β_{1Y} portfolio has not generated statistically significant alpha when returns are evaluated using the FFC4 + LIQ + FMAX model.

Bali et al. (2017) find that the returns associated with the beta anomaly are no longer economically large or statistically significant when measured against the FFC4 + PS model augmented with the FMAX factor. They report FFC4 + PS + FMAX alpha of 0.06% for the EW high-low β_{1Y} portfolio. Although our results are in line with regards to reduced statistical significance, we note the abnormal returns of the low-high portfolio remain economically large in our sample. Augmenting the FFC4 + LIQ model with the FMAX factor, we find that alphas decline from 0.73% to 0.49% for our EW low-high β_{5Y} portfolio, which illustrates a much weaker effect than what Bali et al. (2017) report. In their sample, augmenting the FFC4 + PS model with the FMAX factor results in alphas increasing from -0.49% to 0.06% for the high-low β_{1Y} portfolio.

Our model only controls for MAX and our measure of beta, and there could be omitted variables that would alter the observed relation if included in the analysis.

Table 14: Univariate Portfolio Analysis Sorting on β_{5Y} (Common Sample)

The table presents the results of a univariate portfolio analysis on the relation between excess and abnormal returns in month *t*+1 and the variable β_{5Y} in month *t*. At the end of each month *t*, all stocks with an estimate of both β_{5Y} and MAX are sorted into quintile portfolios based on an ascending ordering of β_{5Y}. The table consists of two panels (A and B) and each panel is divided into three sections. **Explanation of sections:** "Portfolio Characteristics" presents the time-series mean of the monthly average MAX for the stocks in each quintile portfolio (MAX ex-ante), the time-series mean of the monthly average β_{5Y} for the stocks in each quintile portfolio (β ex-ante), and the slope coefficient from a regression of VW (EW) portfolio excess returns on the VW (EW) market excess returns (β ex-post). Factor Model Alphas" presents portfolio alphas relative to the CAPM (CAPM), the Fama-French-Carhart four-factor model (FFC4) model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ), the FFC4 + LIQ model augmented with a lottery demand factor FMAX and the FFC4 + LIQ model augmented with an IVOL factor FIVOL. We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags. See section 5.2.4 for the construction of the FMAX and FIVOL factors. **Explanation of panels:** All reported numbers in Panel A are calculated using value-weighted portfolios, while all numbers in Panel B are calculated using equal-weighted portfolios. **General:** The column labeled Low-High β_{5Y} refers to a zero-cost, long-short portfolio with a long position in quintile portfolio 1 and a short position in quintile portfolio 5. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.

Panel A: Value-Weighted Portfolios

Value	β_{5Y} 1 (Low)	β_{5Y} 2	β_{5Y} 3	β_{5Y} 4	β_{5Y} 5 (High)	Low-High β_{5Y}
			Portfolio Characteristics			
MAX ex-ante	0.025	0.025	0.025	0.029	0.036	-0.011
β ex-ante	0.460	0.773	0.979	1.269	1.740	-1.280
β ex-post	0.728	1.030	0.975	1.136	1.430	-0.702
			Factor Model Alphas			
CAPM	0.40 %	-0.02 %	0.23 %	-0.18 %	-0.40 %	0.80 %
	(1.848)	-(0.090)	(1.284)	-(0.803)	-(1.509)	(2.187)
FFC4 + LIQ	0.33 %	0.08 %	0.36 %	-0.15 %	-0.43 %	0.76 %
	(1.634)	(0.368)	(1.962)	-(0.716)	-(1.524)	(2.045)
FFC4 + LIQ + FMAX	0.25 %	0.06 %	0.44 %	-0.05 %	-0.22 %	0.48 %
	(1.248)	(0.257)	(2.154)	-(0.239)	-(0.719)	(1.160)
FFC4 + LIQ + FIVOL	0.21 %	0.16 %	0.38 %	-0.04 %	-0.13 %	0.34 %
	(1.098)	(0.701)	(1.715)	-(0.200)	-(0.497)	(0.976)

Panel B: Equal-Weighted Portfolios

Value	β_{5Y} 1 (Low)	β_{5Y} 2	β_{5Y} 3	β_{5Y} 4	β_{5Y} 5 (High)	Low-High β_{5Y}
			Portfolio Characteristics			
MAX ex-ante	0.031	0.033	0.033	0.037	0.043	-0.013
β ex-ante	0.412	0.760	0.986	1.267	1.804	-1.392
β ex-post	0.714	0.936	1.014	1.216	1.405	-0.691
			Factor Model Alphas			
CAPM	0.30 %	-0.04 %	0.14 %	-0.41 %	-0.76 %	1.07 %
	(1.941)	-(0.243)	(0.778)	-(1.619)	-(3.175)	(3.544)
FFC4 + LIQ	0.30 %	0.07 %	0.32 %	-0.23 %	-0.43 %	0.73 %
	(1.943)	(0.431)	(1.812)	-(0.979)	-(1.930)	(2.416)
FFC4 + LIQ + FMAX	0.22 %	0.03 %	0.33 %	-0.11 %	-0.23 %	0.45 %
	(1.423)	(0.155)	(1.802)	-(0.459)	-(0.893)	(1.374)
FFC4 + LIQ + FIVOL	0.28 %	-0.04 %	0.31 %	-0.04 %	-0.21 %	0.49 %
	(1.731)	-(0.256)	(1.663)	-(0.198)	-(0.845)	(1.516)

Bali et al. (2017) use the explanatory power of the FMAX factor on the abnormal returns of the beta-sorted portfolios as evidence in support of their theory that lottery demand for stocks generates the beta anomaly. Although we also find that the low-high beta portfolio has not generated statistically significant abnormal returns when we augment the FFC4 + LIQ model with the FMAX factor, we do not interpret the results as illustrating a causal relation between lottery demand and the beta anomaly. Firstly, we note that the observed effect of including the FMAX factor in the model has a much smaller effect on the magnitude of the abnormal returns in our sample, relative to their study. Furthermore, augmenting factor models with a FMAX factor provides information regarding the correlation of returns associated with lottery demand and returns associated with the beta anomaly. As we, in contrast to Bali et al. (2017), find no evidence in support of the theory that lottery demand generates the beta anomaly when we analyze the performance of beta-sorted portfolios with neutral exposure to MAX, we argue that we do not have a basis for interpreting the observed correlation in returns as illustrating a causal relation.

By construction, MAX should correlate positively with risk measures such as total volatility, beta and idiosyncratic volatility as it measures extreme movements in stock prices (although only positive). Risky stocks tend to be risky on several measures, and we would hence expect some correlation in the returns of low-high portfolios constructed using different risk measures. To test our hypothesis, we construct an IVOL factor (FIVOL) analogous to the FMAX factor. We report the results from augmenting the FFC4 + LIQ factor model with the FIVOL factor in Table 14, above. We find that neither the VW nor the EW low-high β_{5Y} portfolio generates statistically significant abnormal returns when returns are measured against the FFC4 + LIQ + FIVOL model, and we find that the FIVOL factor is equally capable of explaining the abnormal returns associated with the beta anomaly as the FMAX factor.[44][45] As such, the documented correlation between FMAX and the abnormal returns associated with the beta anomaly cannot, by itself, be used to infer that lottery demand plays an important role in generating the beta anomaly. Hence, we find that our conclusion from the bivariate portfolio analysis remains valid, and we argue that our analyses do not provide any conclusive evidence in support of

[44] The monthly Person correlation between MAX and IVOL has been 0.87 on average in our sample. This implies that the portfolios constructed using IVOL and MAX will be very similar with regards to stock composition, and portfolio returns will consequently exhibit a strong correlation.

[45] Due to the explanatory power of the FIVOL factor on the abnormal returns associated with the beta anomaly, we also conduct a bivariate portfolio analysis sorting first on IVOL then β_{5Y} and a univariate portfolio analysis on the portion of β_{5Y} that is orthogonal to IVOL. The results are presented in appendix E. In short, the results indicate that the beta anomaly persists after controlling for IVOL.

the theory that lottery demand as measured by MAX plays an important role in generating the beta anomaly on the Oslo Stock Exchange

7 Limitations and Further Research

In this section, we briefly review the most apparent limitations of our thesis and suggest areas for further research. In our view, the most obvious candidate for interesting further research is an analysis of the trading costs associated with exploiting the beta anomaly on the OSE. In Appendix F, we provide a short discussion on the topic and assess the portfolio turnover for the beta-sorted and MAX-sorted quintile portfolios. We believe that the portfolio turnover can serve as a very rough estimate of the trading costs, but a more comprehensive analysis is needed to reveal the actual trading costs associated with such a strategy.

Controlling for Additional Factors

An important limitation of our thesis is that we have not replicated all the analyses presented in Bali et al. (2017). Most notably, Bali et al. (2017) conduct multiple bivariate portfolio sorts to control for other variables documented to have explanatory power in predicting future stock returns. They use these to strengthen their lottery demand-based explanation of the beta anomaly as the beta anomaly remains statistically significant when controlling all variables, except for MAX. They also document that the inclusion of MAX in a Fama-MacBeth (1973) regression controlling for the other variables results in a significant positive relation between beta and stock returns. Performing a similar analysis on the Norwegian data sample would be interesting. However, we have not had the capacity, nor the required data on the Norwegian stock market to estimate the numerous variables documented to predict future stock returns in the literature. As such, we emphasize that our results should be considered in conjunction with the work of Bali et al. (2017) as an out-of-sample robustness test evaluating the applicability of their findings to the Oslo Stock Exchange.

Degree of Institutional Ownership

Another limitation of our paper is that we do not examine the beta anomaly or the lottery demand phenomenon among stocks with a varying degree of institutional ownership.[46] Bali et al. (2017) find that both the beta anomaly and the lottery demand phenomenon is strong (weak) among stocks with a low (high) degree of institutional ownership and argue that this is due to individuals being more

[46] The analysis was omitted due to the lack of available data on the degree of institutional ownership of the stocks listed on the OSE.

prone to exhibit lottery-behavior than institutions. In that regard, it would be interesting to examine whether the same is true on the OSE as it perhaps could provide increased clarity as to why MAX fails to explain the beta anomaly in our sample. Even though it would not solve the challenge of MAX being correlated with other measures known to predict future returns, a result indicating that the lottery demand phenomenon is stronger among stocks with a low degree of institutional ownership on the OSE would support the theory that MAX is a good proxy of investors demand for lottery-like assets.

New Proxies for Lottery Demand

As we have discovered throughout the process of writing this thesis, interpreting our results in a statistically correct manner when MAX is used as a proxy for lottery demand is challenging as MAX correlates with other factors which themselves correlate with future stock returns. For example, Bruno and Haug (2018) prove mathematically that equity IVOL should be negatively correlated with expected equity returns in the cross-section. In our sample, the Pearson correlation between MAX and IVOL has been 0.87 on average, illustrating that the variables are highly correlated. Consequently, an observed negative relation between MAX and future returns cannot conclusively be attributed to lottery demand from investors. However, we still find the lottery demand-based explanation of the beta anomaly to be intriguing, and further research on the subject using new and possibly creative measures of lottery demand would, in our view, be very interesting. In particular, measures that do not exhibit a strong correlation with variables known to have explanatory power in predicting future stock returns would be of high interest.

8 Conclusion

The high (low) abnormal returns of stocks with low (high) beta – the beta anomaly – is the oldest and one of the most robust stock market anomalies documented in empirical asset pricing research. From the 1970s to present, there have been numerous efforts to explain the phenomenon. In this thesis, we test the lottery demand-based explanation proposed by Bali et al. (2017) by replicating their paper on a Norwegian data sample. We examine the presence of both the beta anomaly and the lottery demand phenomenon before we assess whether lottery demand plays an important role in generating the beta anomaly in our sample.

By examining the performance of monthly quintile portfolios sorted on an ascending ordering of beta, we find strong evidence suggesting that there has been a beta anomaly on the Oslo Stock Exchange in the period 1990-2018. A value-weighted (equal-weighted) portfolio comprised of the twenty percent of the stocks on the OSE with the lowest beta each month would have produced cumulative excess returns of 1241% (692%) from 1990 to 2018. A corresponding portfolio comprised of the twenty percent of the stocks on the Oslo Stock Exchange with the highest beta would only have generated cumulative excess returns of 6% (22%) over the same period.

To examine the beta anomaly, we sort the stocks in our sample into quintile portfolios based on an ascending ordering of market beta. We find that a zero-cost portfolio with a long position in the low-beta quintile portfolio and a short position in the high-beta quintile portfolio (low-high beta portfolio) generates economically large and statistically significant positive abnormal returns relative to the CAPM. The results are robust across various time periods, data filter variations, and portfolio weighting schemes. We also demonstrate that a value-weighted, low-high beta portfolio generates economically large and statistically significant abnormal returns relative to the Fama and French (1993) and Carhart (1997) four-factor model (FFC4) augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). The abnormal returns are, however, statistically insignificant for the corresponding equal-weighted portfolio, and our robustness tests illustrate that the statistical significance of the beta anomaly relative to the FFC4 + LIQ factor model is sensitive to our choice of time period and data filters. Even though we are not able to demonstrate the beta anomaly relative to the FFC4 + LIQ model in an entirely uncontestable manner, our combined results strongly indicate that there is a beta anomaly on the Oslo Stock Exchange.

Following Bali et al. (2017), we use a variable MAX, defined as the average of the five highest daily returns over the past month, as a proxy for a stock's lottery demand. We validate that MAX is an accurate proxy of lottery demand on the Oslo Stock Exchange relative to Kumar's (2009) definition by demonstrating that quintile portfolios constructed to be monotonically increasing in MAX are also monotonically increasing in idiosyncratic volatility and idiosyncratic skewness, and monotonically decreasing in average stock price.

The lottery demand phenomenon refers to the high (low) abnormal returns of stocks that experience a low (high) amount of lottery demand-price pressure. To analyze the relation between MAX and one-month-ahead excess returns, we sort the stocks in our sample into quintile portfolios based on an ascending ordering of MAX. We find that a zero-cost portfolio with a long position in the low-MAX quintile portfolio and a short position in the high-MAX quintile portfolio (low-high MAX portfolio) generates positive and economically large abnormal returns relative to the CAPM. The abnormal returns are only statistically significant at the 10% level. However, we find that both a value-weighted and an equal-weighted low-high MAX portfolio generates economically large and statistically significant alphas relative to the FFC4 + LIQ model. The results are generally robust to variations in data filters, but not across time periods. We therefore argue that our results strongly indicate that there is a negative relation between MAX and future abnormal returns in our sample, but the ambiguous results from our robustness tests prevent us from concluding with great certainty.

We test the lottery demand-based explanation of the beta anomaly by analyzing the returns of the beta sorted portfolios while controlling for MAX using three different methodologies. In short, we find very limited evidence suggesting that lottery demand measured by MAX plays an important role in generating the beta anomaly in our sample. A bivariate portfolio analysis demonstrates that controlling for MAX has a limited impact on the abnormal returns of the low-high beta portfolio, and a univariate portfolio analysis on the portion of beta that is orthogonal to MAX yields similar results. We find that in three out of the four tests conducted in the univariate and bivariate portfolio analyses, the abnormal returns of the low-high beta portfolio remain statistically significant despite the portfolio being neutralized to MAX.

When we augment the FFC4 + LIQ factor model with a lottery demand factor FMAX, we find that the abnormal returns of the low-high beta portfolio are no longer statistically significant. However, the abnormal returns remain economically large, and we demonstrate that an IVOL factor constructed analogously to the FMAX factor is equally capable of explaining the abnormal returns associated with the beta anomaly as FMAX. Seen in conjunction with the results from the bivariate portfolio analysis and the univariate portfolio analysis sorting on the component of beta that is orthogonal to MAX, we find that our analyses do not provide any conclusive evidence in favor of the lottery-demand explanation of the beta anomaly.

When we reverse the roles of MAX and beta, we find that the low-high MAX portfolio no longer generates statistically significant abnormal returns when the portfolio is constructed to be neutral to beta. The results hold for all four conducted tests and suggest that the negative relation between MAX and abnormal returns documented in our sample not necessarily can be attributed to investor demand for lottery-like assets. As such, we believe our analyses illustrate a potential challenge concerning MAX as a measure of lottery demand. Moreover, since the abnormal returns of the low-high MAX portfolio are statistically insignificant after controlling for beta, we argue that the negative relation documented between MAX and abnormal returns in our thesis cannot be interpreted to demonstrate a statistically significant lottery demand phenomenon on the Oslo Stock Exchange.

In conclusion, our thesis documents the presence of a statistically significant beta anomaly on the Oslo Stock Exchange. However, we do not find conclusive evidence of a lottery demand phenomenon in Norway, and the documented beta anomaly in our sample does not appear to be a manifestation of investors' demand for lottery-like assets as measured by MAX. As such, the anomaly remains largely unexplained, and the most interesting finding in this thesis is, perhaps, that the moral of the legendary fable, *The Tortoise and the Hare*, appears to also apply to investments on the Oslo Stock Exchange:

Slow but steady wins the race

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Appendices

Appendix A: Descriptive Statistics

Table A.1: Summary Statistics of Filtered Data

The table presents summary statistics for the observations in our filtered dataset. The data sample was retrieved from NHH's *Børsprosjektet* and covers the period from Jan. 1985 through Dec. 2018. Panel A presents summary statistics for the daily data while Panel B presents summary statistics for the monthly data. *Generic* is a collective variable equal to the last available daily closing price. *ReturnAdjGeneric* computes the nominal simple returns adjusted for dividends, stock splits and reverse splits. *MCAP* equals the securities' market capitalization computed as the product of *SharesIssued* and *Generic*. *ShareIssued* represents the number of outstanding shares, while *OffShareTurnover* equals the number of officially traded shares for the given day (month) in the daily (monthly) dataset.

Panel A: Daily Data

Variable	N	Mean	SD	Min	Max
Generic	1,447,685	97.44	181.47	1.00	4,900.00
ReturnAdjGeneric	1,447,685	0.00	0.04	-0.97	4.48
SharesIssued	1,447,685	96,145,093	273,865,647	40,500	8,899,016,805
MCAP	1,447,685	5,361,859,792	24,181,700,485	1,238,328	682,689,344,752
OffShareTurnover	1,123,321	515,893	3,262,448	1	469,322,211

Panel B: Monthly Data

Variable	N	Mean	SD	Min	Max
Generic	70,071	97.56	182.41	1.00	3,960.00
ReturnAdjGeneric	70,071	0.01	0.16	-0.97	8.24
SharesIssued	70,071	95,533,950	271,484,096	40,500	8,899,016,805
MCAP	70,071	5,334,997,041	24,118,812,261	1,793,082	631,352,126,394
OffShareTurnover	69,260	8,129,068	47,955,585	-1,946,722,005 [J]	1,989,745,188

[J] There are four monthly stock observations in the monthly dataset with a negative value of *OffShareTurnover*. We do not, however, rely on the monthly values of *OffShareTurnover* for any calculations, as all turnover calculations in this paper are based on the daily observations of *OffShareTurnover* which we find to be correct. As the corresponding monthly observations of *Generic*, *ReturnAdjGeneric* and *SharesIssued* are also correct for the stocks with the negative monthly values of *OffShareTurnover*, we do not remove the observations.

Table A.2: Summary Statistics of Variables

The table presents summary statistics for all the estimated variables used in this thesis. The data sample runs from Jan. 1990 through Dec. 2018. $\beta_{5Y,}$ β_{1Y} and β_{MSCI} are estimates of beta, MAX is a proxy for lottery demand, IVOL refers to idiosyncratic volatility and ISKEW refers to idiosyncratic skewness. $\beta_{5Y\perp M}$ is the component of β_{5Y} orthogonal to MAX, $\beta_{1Y\perp M}$ is the component of β_{1Y} orthogonal to MAX, $\beta_{5Y\perp IVOL}$ is the component of β_{5Y} orthogonal to IVOL and $MAX_{\perp\beta}$ is the component of MAX orthogonal to $\beta_{5Y.}$

Variable	N	Mean	SD	Min	Max
β_{5Y}	44354	0.895	0.581	-2.277	6.322
β_{1Y}	35731	0.866	0.504	-2.177	4.902
β_{MSCI}	44344	0.775	0.732	-4.947	5.497
MAX	43528	0.037	0.028	-0.003	1.147
IVOL	43528	0.025	0.019	0.000	0.812
ISKEW	43528	0.218	0.726	-3.736	4.310
$\beta_{5Y\perp M}$	32132	0.839	0.574	-2.677	5.945
$\beta_{1Y\perp M}$	34675	0.639	0.487	-2.842	4.229
$\beta_{5Y\perp IVOL}$	32132	0.832	0.590	-3.249	6.105
$MAX_{\perp\beta 5}$	32132	0.026	0.027	-0.069	1.138

Table A.3: Average Monthly Cross-Sectional Correlation Between Variables

The table presents the time-series average of the monthly cross-sectional Person product-moment correlation between the variables. The data sample runs from Jan. 1990 through Dec. 2018.
$\beta_{5Y,}$ β_{1Y} and β_{MSCI} are estimates of market beta, MAX is a proxy for lottery demand, IVOL refers to idiosyncratic volatility and ISKEW refers to idiosyncratic skewness. $\beta_{5Y\perp M}$ is the component of β_{5Y} orthogonal to MAX, $\beta_{1Y\perp M}$ is the component of β_{1Y} orthogonal to MAX, $\beta_{5Y\perp IVOL}$ is the component of β_{5Y} orthogonal to IVOL and $MAX_{\perp\beta}$ is the component of MAX orthogonal to $\beta_{5Y.}$

	β_{5Y}	β_{1Y}	β_{MSCI}	MAX	IVOL	ISKEW	$\beta_{5Y\perp M}$	$\beta_{1Y\perp M}$	$\beta_{5Y\perp IVOL}$	$MAX_{\perp\beta}$
β_{5Y}	1.00	0.56	0.70	0.20	0.13	0.04	0.96	0.51	0.95	0.00
β_{1Y}	0.56	1.00	0.37	0.28	0.17	0.03	0.51	0.94	0.50	0.18
β_{MSCI}	0.70	0.37	1.00	0.11	0.05	0.02	0.63	0.35	0.62	-0.03
MAX	0.20	0.28	0.11	1.00	0.87	0.30	0.00	0.00	0.00	0.96
IVOL	0.13	0.17	0.05	0.87	1.00	0.19	-0.04	-0.08	-0.04	0.85
ISKEW	0.04	0.03	0.02	0.30	0.19	1.00	-0.02	-0.06	-0.02	0.29
$\beta_{5Y\perp M}$	0.96	0.51	0.63	0.00	-0.04	-0.02	1.00	0.53	0.99	-0.20
$\beta_{1Y\perp M}$	0.51	0.94	0.35	0.00	-0.08	-0.06	0.53	1.00	0.52	-0.09
$\beta_{5Y\perp IVOL}$	0.95	0.50	0.62	0.00	-0.04	-0.02	0.99	0.52	1.00	-0.20
$MAX_{\perp\beta}$	0.00	0.18	-0.03	0.96	0.85	0.29	-0.20	-0.09	-0.20	1.00

Table A.4: Top 20 most frequent companies in portfolios by sort variable

The table presents the company names of the top 20 most frequent stocks in quintile portfolio 1 (Panel A) and quintile portfolio 5 (Panel B) in the different univariate analyses. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. The columns refer to the different sorting variables used in the univariate portfolio sorts.

Panel A: Quintile Portfolio 1

	β_{5Y}	β_{1Y}	β_{MSCI}	MAX
1	Arendals Fossekompani	Sparebanken Møre	Sparebanken Øst	Sparebanken Møre
2	Voss Veksel- og Landmandsbank	Hafslund ser. B	Arendals Fossekompani	Orkla
3	Indre Sogn Sparebank	Hafslund ser. A	SpareBank 1 BV	SpareBank 1 SMN
4	Sparebanken Sør	SpareBank 1 Nord-Norge	Sandsvær Sparebank	SpareBank 1 Nord-Norge
5	SpareBank 1 BV	SpareBank 1 SR-Bank	Totens Sparebank	Hafslund ser. B
6	SpareBank 1 Ringerike Hadeland	Sparebanken Øst	Aurskog Sparebank	SpareBank 1 SR-Bank
7	Byggma	SpareBank 1 SMN	Norwegian Car Carriers	Sparebanken Vest
8	Aurskog Sparebank	Sandnes Sparebank	TTS Group	Norsk Hydro
9	Sandsvær Sparebank	Sparebanken Vest	Voss Veksel- og Landmandsbank	Sparebanken Øst
10	Melhus Sparebank	Olav Thon Eiendomsselskap	Indre Sogn Sparebank	Hafslund ser. A
11	Sparebanken Øst	Ekornes	Byggma	DNB
12	Rieber & Søn	Totens Sparebank	SpareBank 1 Ringerike Hadeland	Olav Thon Eiendomsselskap
13	Gyldendal	Gjensidige NOR Sparebank	Skue Sparebank	Sandnes Sparebank
14	Skue Sparebank	Bolig- og Næringsbanken	Gyldendal	Kongsberg Gruppen
15	Sparebanken Vest	Kverneland	Hol Sparebank	Veidekke
16	Hol Sparebank	AF Gruppen	NRC Group	Odfjell ser. A
17	Totens Sparebank	Odfjell ser. A	Rieber & Søn	Totens Sparebank
18	AF Gruppen	Kongsberg Gruppen	DNO	Bonheur
19	Tide	NextGenTel Holding	AF Gruppen	Bolig- og Næringsbanken
20	SpareBank 1 Østfold Akershus	Goodtech	Sparebanken Sør	Telenor

Panel B: Quintile Portfolio 5

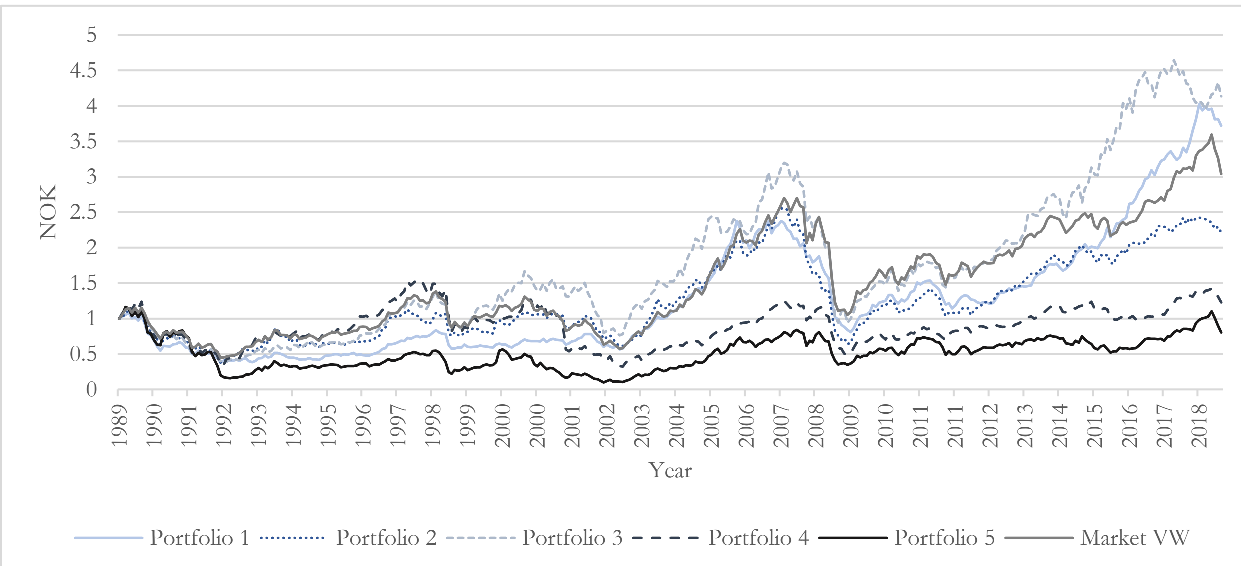
	β_{5Y}	β_{1Y}	β_{MSCI}	MAX
1	Petroleum Geo-Services	Petroleum Geo-Services	Norske Skogindustrier	DNO
2	Subsea 7	Subsea 7	Petroleum Geo-Services	Jinhui Shipping and Transportation
3	Storebrand	TGS-NOPEC Geophysical Company	Atea	Techstep
4	DNO	Norsk Hydro	Eltek	Solon Eiendom
5	Jinhui Shipping and Transportation	DNO	Storebrand	NRC Group
6	Eltek	Dolphin Drilling	Schibsted ser. A	Apptix
7	NCL Holding	Akastor	SAS Norge B	Hexagon Composites
8	Akastor	Frontline	Royal Caribbean Cruises	Reach Subsea
9	Tandberg Data	Storebrand	Nordic Semiconductor	Petroleum Geo-Services
10	Kongsberg Automotive	Prosafe	Jinhui Shipping and Transportation	Hiddn Solutions
11	Questerre Energy Corporation	Tandberg	Subsea 7	Frontline
12	Atea	DNB	Elkem	Navamedic
13	TGS-NOPEC Geophysical Company	Equinor	SAS AB	Funcom
14	Norsk Hydro	Questerre Energy Corporation	Kongsberg Automotive	Petrolia
15	Yara International	Atea	Frontline	REC Silicon
16	SAS Norge B	Seadrill	Norsk Hydro	Rocksource
17	Elkem	NCL Holding	Hiddn Solutions	Opticom
18	Dolphin Drilling	REC Silicon	Tandberg	EMS Seven Seas
19	Ocean Rig	Golden Ocean Group	Tandberg Data	Questerre Energy Corporation
20	Songa Offshore	Royal Caribbean Cruises	BW Offshore Limited	Incus Investor

Appendix B: The Beta Anomaly

Figure B.1: Cumulative Excess Returns of Quintile Portfolios Sorted on β_{1Y}

At the end of each month *t,* all stocks are sorted into quintile portfolios based on an ascending ordering of β_{1Y} and the portfolio excess returns are calculated for month *t +1*. The figure presents the historical cumulative excess returns (in NOK) of a NOK 1 investment in each of the quintile portfolios from Jan. 1990 through Dec. 2018. The quintile portfolios are rebalanced monthly, all dividends and cash payouts are assumed to be reinvested and the return calculation assumes no transaction costs. The Portfolio *i* in the figure correspond to the β_{1Y}-sorted quintile portfolio *i*, while Market VW (EW) illustrates the value-weighted (equal-weighted) excess return of a portfolio consisting of all the stocks in our filtered dataset. Panel A presents the cumulative excess returns of the value-weighted β_{1Y} quintile portfolios and panel B presents the cumulative excess returns of the equal-weighted β_{1Y} quintile portfolios.

Panel A: VW Portfolios sorted on β_{1Y}



Panel B: EW Portfolios sorted on β_{1Y}

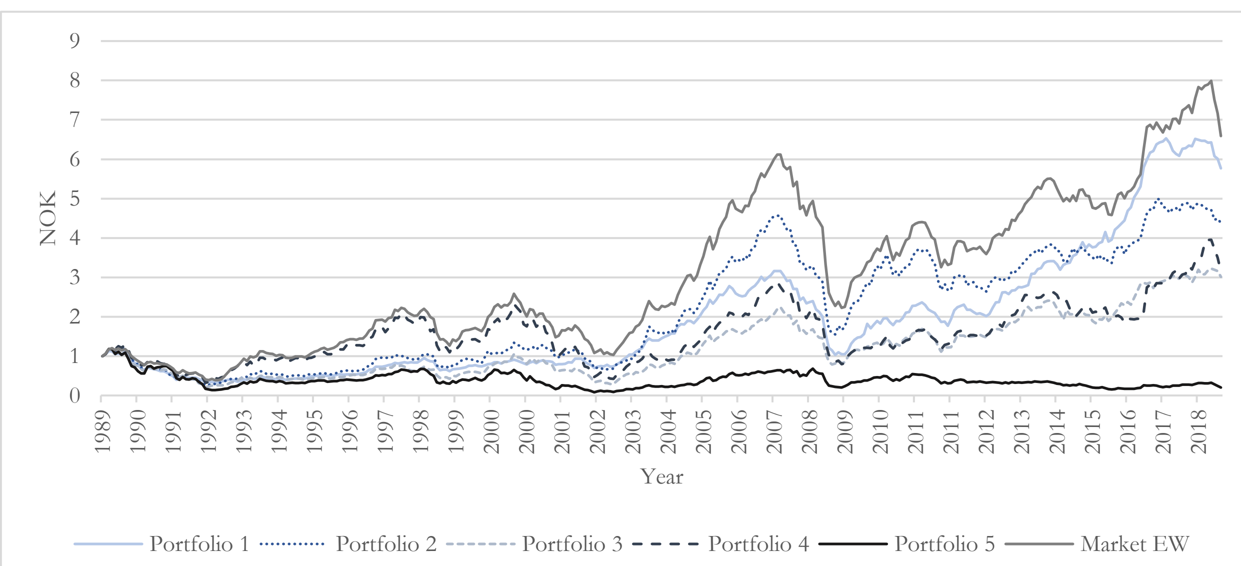


Table B.1: Univariate Portfolio Analysis Sorting on β_{1Y}

The table presents the results of a univariate portfolio analysis on the relation between excess and abnormal returns in month *t*+1 and the variable β_{1Y} in month *t*. At the end of each month *t*, all stocks are sorted into quintile portfolios based on an ascending ordering of β_{1Y}. The table consists of two panels (A and B) and each panel is divided into three sections.

Explanation of sections: "Portfolio Characteristics" presents the time-series mean number of stocks in each quintile portfolio (Portfolio length), the time-series mean portfolio market share (calculated as the sum of the market capitalization within each portfolio divided by total market capitalization) for each month *t* (Market share), the time-series mean of the monthly average β_{1Y} for the stocks in each quintile portfolio (β ex-ante), and the slope coefficient from a regression of VW (EW) portfolio excess returns on the VW (EW) market excess returns (β ex-post). "Portfolio Sharpe Ratio" presents the time-series mean monthly portfolio excess returns in month *t+1* (R), the standard deviation of monthly portfolio excess returns in month *t+1* (SD), and an annualized portfolio Sharpe ratio (SR). "Factor Model Alphas" presents monthly portfolio alphas relative to the CAPM (CAPM), the Fama-French 3-Factor model (FF3), the Fama-French-Carhart 4-Factor model (FFC4) and the FFC4 model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags.

Explanation of panels: All reported numbers in panel A are calculated using value-weighted portfolios, while all numbers in panel B are calculated using equal-weighted portfolios.

General: The column labeled Low-High β_{1Y} refers to a zero-cost, long-short portfolio with a long position in quintile portfolio 1 and a short position in quintile portfolio 5. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.

Panel A: Value-Weighted Portfolios

Value	β_{1Y} 1 (Low)	β_{1Y} 2	β_{1Y} 3	β_{1Y} 4	β_{1Y} 5 (High)	Low-High β_{1Y}
			Portfolio Characteristics			
Portfolio length	21	21	20	21	21	
Market share	5%	10%	17%	32%	36%	
β ex-ante	0.360	0.660	0.873	1.117	1.548	-1.188
β ex-post	0.666	0.901	0.954	1.061	1.433	-0.768
			Portfolio Sharpe Ratio			
R	0.50%	0.43%	0.63%	0.33%	0.40%	0.10%
SD	4.91%	6.26%	6.51%	7.16%	9.52%	7.47%
SR	0.354	0.238	0.335	0.161	0.147	0.046
			Factor Model Alphas			
CAPM	0.18%	-0.01%	0.16%	-0.19%	-0.30%	0.47%
	(0.922)	-(0.057)	(0.846)	-(1.013)	-(1.013)	(1.258)
FF3	0.08%	-0.11%	0.18%	-0.18%	-0.36%	0.44%
	(0.441)	-(0.588)	(0.907)	-(0.893)	-(1.120)	(1.096)
FFC4	0.08%	-0.10%	0.15%	-0.07%	-0.20%	0.29%
	(0.444)	-(0.508)	(0.773)	-(0.373)	-(0.607)	(0.705)
FFC4 + LIQ	0.08%	-0.10%	0.13%	-0.07%	-0.25%	0.33%
	(0.431)	-(0.502)	(0.674)	-(0.367)	-(0.807)	(0.879)

Panel B: Equal-Weighted Portfolios

Value	β_{1Y} 1 (Low)	β_{1Y} 2	β_{1Y} 3	β_{1Y} 4	β_{1Y} 5 (High)	Low-High β_{1Y}
			Portfolio Characteristics			
Portfolio length	21	21	20	21	21	
Market share	5%	10%	17%	32%	36%	
β ex-ante	0.343	0.649	0.869	1.117	1.607	-1.263
β ex-post	0.732	0.960	1.072	1.206	1.579	-0.847
			Portfolio Sharpe Ratio			
R	0.65%	0.64%	0.59%	0.67%	0.14%	0.51%
SD	5.30%	6.54%	7.27%	8.13%	10.83%	8.22%
SR	0.424	0.340	0.280	0.286	0.044	0.215
			Factor Model Alphas			
CAPM	0.12%	-0.05%	-0.19%	-0.20%	-1.00%	1.12%
	(0.603)	-(0.284)	-(0.981)	-(0.973)	-(3.296)	(2.831)
FF3	0.23%	0.09%	0.03%	0.01%	-0.69%	0.92%
	(1.346)	(0.536)	(0.185)	(0.065)	-(2.296)	(2.312)
FFC4	0.16%	0.05%	0.03%	0.07%	-0.49%	0.65%
	(0.934)	(0.307)	(0.141)	(0.344)	-(1.530)	(1.597)
FFC4 + LIQ	0.16%	0.03%	-0.04%	0.00%	-0.60%	0.76%
	(0.936)	(0.158)	-(0.210)	(0.020)	-(2.100)	(2.063)

Univariate Portfolio Analysis Sorting on β_{MSCI}

In Table B.2 we report the results from a univariate portfolio analysis sorting on β_{MSCI} and using the MSCI World Index in NOK as the market factor in the factor models. We do not require the low-high β_{MSCI} portfolio to generate statistically significant abnormal returns to establish the presence of a beta anomaly in Norway, as we are mostly interested in confirming the anomaly relative to a country specific index, which is common practice in the literature. We report the results as we find it interesting to assess beta-sorted portfolios from an international perspective. With the development of online trading, even retail investors now have easy access to international capital markets and according to the Norwegian Government, 39.3% of company shares listed on the Oslo Stock Exchange were owned by foreign investors (Oslo Børs, 2019). Without diving into a comprehensive discussion on benchmarking, we argue that the choice of benchmark to use as a proxy for the market portfolio is not obvious.

We find that when sorting our stocks into quintile portfolios based on β_{MSCI}, we find no evidence of a beta anomaly. The CAPM alpha of the VW (EW) low-high β_{MSCI} portfolio is economically large but not statistically significant, while the FFC4 + LIQ alpha is neither economically large nor statistically significant. Although we find the results to be interesting, a more comprehensive analysis on the potential causes of the results lies beyond the scope of this thesis.

Table B.2: Univariate Portfolio Analysis Sorting on β_{MSCI}

The table presents the results of a univariate portfolio analysis on the relation between excess and abnormal returns in month *t*+1 and the variable β_{MSCI} in month *t*. At the end of each month *t*, all stocks are sorted into quintile portfolios based on an ascending ordering of β_{MSCI}. The table consists of two panels (A and B) and each panel is divided into three sections.

Explanation of sections: "Portfolio Characteristics" presents the time-series mean number of stocks in each quintile portfolio (Portfolio length), the time-series mean portfolio market share (calculated as the sum of the market capitalization within each portfolio divided by total market capitalization) for each month *t* (Market share), the time-series mean of the monthly average β_{MSCI} for the stocks in each quintile portfolio (β ex-ante MSCI), the time-series mean of the monthly average β_{5Y} for the stocks in each quintile portfolio (β ex-ante OSE), and the slope coefficient from a regression of portfolio excess returns on the excess returns of the MSCI World Index in NOK in excess of the Norwegian risk-free rate (β ex-post). "Portfolio Sharpe Ratio" presents the time-series mean monthly portfolio excess returns in month *t+1* (R), the standard deviation of monthly portfolio excess returns in month *t+1* (SD), and an annualized portfolio Sharpe ratio (SR). "Factor Model Alphas" presents monthly portfolio alphas relative to the CAPM (CAPM), the Fama-French 3-Factor model (FF3), the Fama-French-Carhart 4-Factor model (FFC4) and the FFC4 model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the MSCI World Index as the market factor in the factor models, while the remaining factors are estimated on the Oslo Stock Exchange. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags.

Explanation of panels: All reported numbers in Panel A are calculated using value-weighted portfolios, while all numbers in Panel B are calculated using equal-weighted portfolios.

General: The column labeled Low-High β_{MSCI} refers to a zero-cost, long-short portfolio with a long position in quintile portfolio 1 and a short position in quintile portfolio 5. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.

Panel A: Value-Weighted Portfolios

Value	β_{MSCI} 1 (Low)	β_{MSCI} 2	β_{MSCI} 3	β_{MSCI} 4	β_{MSCI} 5 (High)	Low-High β_{MSCI}
			Portfolio Characteristics			
Portfolio length	25	25	26	25	25	
Market share	0.079	0.238	0.198	0.268	0.217	
β ex-ante MSCI	0.069	0.436	0.727	1.052	1.605	-1.536
β ex-ante OSE	0.591	0.772	0.909	1.114	1.451	-0.860
β ex-post MSCI	0.716	0.753	0.980	1.027	1.236	-0.519
			Portfolio Sharpe Ratio			
R	0.66%	0.62%	0.45%	0.61%	0.46%	0.20%
SD	6.29%	5.31%	6.39%	6.64%	8.18%	7.40%
SR	0.362	0.405	0.244	0.317	0.193	0.094
			Factor Model Alpha			
CAPM	0.31%	0.25%	-0.03%	0.11%	-0.15%	0.46%
	(1.216)	(1.569)	-(0.172)	(0.693)	-(0.681)	(1.276)
FF3	-0.11%	0.22%	0.05%	0.18%	-0.27%	0.16%
	-(0.463)	(1.330)	(0.292)	(1.089)	-(1.123)	(0.429)
FFC4	-0.10%	0.13%	0.01%	0.27%	-0.12%	0.02%
	-(0.390)	(0.730)	(0.064)	(1.774)	-(0.531)	(0.068)
FFC4 + LIQ	-0.08%	0.13%	0.01%	0.26%	-0.16%	0.09%
	-(0.312)	(0.724)	(0.041)	(1.682)	-(0.762)	(0.255)

Panel B: Equal-Weighted Portfolios

Value	β_{MSCI} 1 (Low)	β_{MSCI} 2	β_{MSCI} 3	β_{MSCI} 4	β_{MSCI} 5 (High)	Low-High β_{MSCI}
			Portfolio Characteristics			
Portfolio length	25	25	26	25	25	
Market Share	8%	24%	20%	27%	22%	
β ex-ante MSCI	-0.018	0.439	0.713	1.045	1.684	-1.702
β ex-ante OSE	0.417	0.663	0.867	1.088	1.463	-1.045
β ex-post MSCI	0.718	0.796	0.874	1.022	1.258	-0.540
			Portfolio Sharpe Ratio			
R	0.82%	0.79%	0.69%	0.40%	0.68%	0.14%
SD	5.72%	5.47%	5.91%	6.74%	8.38%	7.01%
SR	0.497	0.499	0.403	0.207	0.280	0.071
			Factor Model Alphas			
CAPM	0.30%	0.21%	0.06%	-0.33%	-0.23%	0.53%
	(1.637)	(1.469)	(0.416)	-(2.387)	-(1.123)	(1.653)
FF3	0.04%	0.21%	0.13%	-0.20%	0.02%	0.02%
	(0.269)	(1.375)	(1.022)	-(1.390)	(0.086)	(0.085)
FFC4	0.01%	0.15%	0.03%	-0.18%	0.15%	-0.14%
	(0.076)	(0.987)	(0.282)	-(1.202)	(0.795)	-(0.502)
FFC4 + LIQ	0.06%	0.15%	0.04%	-0.22%	0.08%	-0.02%
	(0.350)	(0.993)	(0.322)	-(1.499)	(0.436)	-(0.082)

Table B.3: Factor Model Alphas for the Norwegian BAB Factor in the Period 1990-2018

The table presents monthly alphas and corresponding t-statistics for the returns associated with Norwegian BAB factor relative to the CAPM (CAPM), the Fama-French-Carhart four-factor model (FFC4) model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ), the FFC4 + LIQ model augmented with a lottery demand factor FMAX and the FFC4 + LIQ model augmented with an IVOL factor FIVOL. We use the value-weighted market portfolio as the market factor in the factor models. The t-statistics are adjusted following Newey and West (1987) using four lags. The sample contains monthly data on the BAB factor in Norway for the period 1990-2018. Data on the BAB factor is downloaded from AQR's website, and the factor is constructed following the methodology of Frazzini and Pedersen (2014).

Factor Model	Alpha	t-statistics
CAPM	1.06 %	2.92
FFC4 + LIQ	0.63 %	1.78
FFC4 + LIQ + FMAX	0.52 %	1.43
FFC4 + LIQ + FIVOL	0.48 %	1.33

Appendix C: Lottery Demand Phenomenon

Table C.1: Low-High MAX Portfolio Alphas for Different Sample Time Periods

The table presents the results from several univariate portfolio analyses on MAX using different time periods. At the end of each month t, all stocks are sorted into quintile portfolios based on an ascending ordering of MAX. The table presents the monthly alphas of a zero-cost, long-short portfolio with a long position in quintile 1 (low-MAX) and short position in quintile 5 (high-MAX) relative to the CAPM (CAPM) and the Fama-French-Carhart 4-Factor model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags. The columns refer to the use of different time periods of our data sample. Panel A reports the results for value-weighted portfolios, while Panel B reports the results for equal-weighted portfolios.

Panel A: Value-Weighted Portfolios

Value	1990-2018	1995-2018	2000-2018	1990-2013	1990-2007
			Factor Model Alphas		
CAPM	0.71%	0.75%	0.92%	0.43%	0.19%
	(1.673)	(1.543)	(1.770)	(0.930)	(0.343)
FFC4 + LIQ	0.94%	0.85%	0.96%	0.81%	0.60%
	(2.431)	(1.897)	(1.921)	(1.892)	(1.128)

Panel B: Equal-Weighted Portfolios

Value	1990-2018	1995-2018	2000-2018	1990-2013	1990-2007
			Factor Model Alphas		
CAPM	0.53%	0.53%	0.53%	0.38%	0.27%
	(1.608)	(1.366)	(1.194)	(1.088)	(0.664)
FFC4 + LIQ	0.66%	0.51%	0.42%	0.59%	0.47%
	(2.110)	(1.428)	(1.003)	(1.757)	(1.125)

Table C.2: Low-High MAX Portfolio Alphas for Variations in Data Filters

The table presents the results from several univariate portfolio analyses on MAX for varying data filters. At the end of each month t, all stocks are sorted into quintile portfolios based on an ascending ordering of MAX. The table presents the monthly alphas of a zero-cost, long-short portfolio with a long position in quintile 1 (low-MAX) and short position in quintile 5 (high-MAX) relative to the CAPM (CAPM) and the Fama-French-Carhart 4-Factor model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags. The columns refer to different variations in data filters. "Primary Model" refers to the data filters applied to the main model as discussed in section 3.1.1. "Primary Capital Certificates" reports the zero-cost portfolio alphas when primary capital certificates are excluded from the sample. "Share Price" presents the zero-cost portfolio alphas when there is no restriction on stock price in the sample. "Large Stocks" presents the zero-cost portfolio alphas when stocks are removed for the months their market capitalization is observed below NOK 1bn. "Turnover" presents the zero-cost portfolio alphas when there is no restriction on stock turnover in the data sample. All results reported in panel A are calculated using value-weighted portfolios, while all results in Panel B are calculated using equal-weighted portfolios. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018.

Panel A: Value-Weighted Portfolios

Value	Primary Model	Primary Capital Certificates	Share Price	Large Stocks	Turnover
		Factor Model Alphas			
CAPM	0.71%	0.81%	0.98%	0.43%	0.72%
	(1.673)	(1.964)	(2.249)	(1.289)	(1.699)
FFC4 + LIQ	0.94%	1.07%	1.23%	0.50%	0.96%
	(2.431)	(2.845)	(3.076)	(1.577)	(2.464)

Panel B: Equal-Weighted Portfolios

Value	Primary Model	Primary Capital Certificates	Share Price	Large Stocks	Turnover
		Factor Model Alphas			
CAPM	0.53%	0.48%	0.96%	0.74%	0.54%
	(1.608)	(1.407)	(2.705)	(2.538)	(1.639)
FFC4 + LIQ	0.66%	0.62%	1.14%	0.80%	0.67%
	(2.110)	(1.941)	(3.462)	(2.945)	(2.145)

Appendix D: Lottery-Demand Based Explanation of the Beta Anomaly

Table D.1: Univariate Portfolio Analysis Sorting on β_{1Y} (Common Data Sample)

The table presents the results of a univariate portfolio analysis on the relation between excess and abnormal returns in month *t*+1 and the variable β_{1Y} in month *t*. At the end of each month *t*, all stocks with an estimate of both β_{1Y} and MAX are sorted into quintile portfolios based on an ascending ordering of β_{1Y}. The table consists of two panels (A and B) and each panel is divided into three sections.

Explanation of sections: "Portfolio Characteristics" presents the time-series mean of the monthly average MAX for the stocks in each quintile portfolio (MAX ex-ante), the time-series mean of the monthly average β_{1Y} for the stocks in each quintile portfolio (β ex-ante), and the slope coefficient from a regression of VW (EW) portfolio excess returns on the VW (EW) market excess returns (β ex-post). Factor Model Alphas" presents portfolio alphas relative to the CAPM (CAPM), the Fama-French-Carhart four-factor model (FFC4) model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ), the FFC4 + LIQ model augmented with a lottery demand factor FMAX and the FFC4 + LIQ model augmented with an IVOL factor FIVOL. We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags. See section 5.2.4 for the construction of the FMAX and FIVOL factors.

Explanation of panels: All reported numbers in Panel A are calculated using value-weighted portfolios, while all numbers in Panel B are calculated using equal-weighted portfolios.

General: The column labeled Low-High β_{1Y} refers to a zero-cost, long-short portfolio with a long position in quintile portfolio 1 and a short position in quintile portfolio 5. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.

Panel A: Value-Weighted Portfolios

Value	β_{1Y} 1 (Low)	β_{1Y} 2	β_{1Y} 3	β_{1Y} 4	β_{1Y} 5 (High)	Low-High β_{1Y}
			Portfolio Characteristics			
MAX ex-ante	0.024	0.024	0.026	0.028	0.035	-0.012
β ex-ante	0.370	0.670	0.883	1.127	1.556	-1.185
β ex-post	0.695	0.904	0.979	1.074	1.449	-0.754
			Factor Model Alphas			
CAPM	0.06%	0.06%	0.11%	0.01%	-0.35%	0.41%
	(0.309)	(0.338)	(0.580)	(0.053)	-(1.160)	(1.103)
FFC4 + LIQ	-0.06%	0.01%	0.12%	0.14%	-0.30%	0.24%
	-(0.309)	(0.069)	(0.620)	(0.658)	-(0.955)	(0.658)
FFC4 + LIQ + FMAX	-0.09%	-0.13%	0.08%	0.17%	0.01%	-0.10%
	-(0.486)	-(0.631)	(0.389)	(0.762)	(0.018)	-(0.270)
FFC4 + LIQ + FIVOL	0.02%	-0.13%	0.10%	0.21%	0.08%	-0.06%
	(0.109)	-(0.674)	(0.494)	(0.947)	(0.283)	-(0.174)

Panel B: Equal-Weighted Portfolios

Value	β_{1Y} 1 (Low)	β_{1Y} 2	β_{1Y} 3	β_{1Y} 4	β_{1Y} 5 (High)	Low-High β_{1Y}
			Portfolio Characteristics			
MAX ex-ante	0.028	0.031	0.034	0.036	0.045	-0.017
β ex-ante	0.352	0.659	0.878	1.127	1.614	-1.262
β ex-post	0.752	0.966	1.070	1.208	1.591	-0.839
			Factor Model Alphas			
CAPM	0.06%	0.00%	-0.20%	-0.11%	-1.04%	1.10%
	(0.299)	-(0.003)	-(1.015)	-(0.522)	-(3.295)	(2.738)
FFC4 + LIQ	0.09%	0.10%	-0.05%	0.11%	-0.64%	0.73%
	(0.511)	(0.524)	-(0.244)	(0.509)	-(2.139)	(1.979)
FFC4 + LIQ + FMAX	-0.03%	0.06%	-0.06%	0.13%	-0.29%	0.27%
	-(0.153)	(0.290)	-(0.289)	(0.635)	-(0.937)	(0.717)
FFC4 + LIQ + FIVOL	0.03%	0.01%	-0.09%	0.12%	-0.39%	0.42%
	(0.193)	(0.037)	-(0.464)	(0.592)	-(1.291)	(1.216)

Table D.2: Univariate Portfolio Analysis Sorting on MAX (Common Data Sample)

The table presents the results of a univariate portfolio analysis on the relation between excess and abnormal returns in month *t*+1 and the variable MAX in month *t*. At the end of each month *t*, all stocks with an estimate of both β_{5Y} and MAX are sorted into quintile portfolios based on an ascending ordering of MAX. The table consists of two panels (A and B) and each panel is divided into three sections.

Explanation of sections: "Portfolio Characteristics" presents the time-series mean of the monthly average MAX for the stocks in each quintile portfolio (MAX ex-ante), the time-series mean of the monthly average β_{5Y} for the stocks in each quintile portfolio (β ex-ante), and the slope coefficient from a regression of VW (EW) portfolio excess returns on the VW (EW) market excess returns (β ex-post). Factor Model Alphas" presents portfolio alphas relative to the CAPM (CAPM), the Fama-French-Carhart four-factor model (FFC4) model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ), the FFC4 + LIQ model augmented with a lottery demand factor FMAX and the FFC4 + LIQ model augmented with an IVOL factor FIVOL. We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags. See section 5.2.4 for the construction of the FMAX and FIVOL factors.

Explanation of panels: All reported numbers in Panel A are calculated using value-weighted portfolios, while all numbers in Panel B are calculated using equal-weighted portfolios.

General: The column labeled Low-High MAX refers to a zero-cost, long-short portfolio with a long position in quintile portfolio 1 and a short position in quintile portfolio 5. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.

Panel A: Value-Weighted Portfolios

Value	MAX 1 (Low)	MAX 2	MAX 3	MAX 4	MAX 5 (High)	Low-High MAX
			Portfolio Characteristics			
MAX ex-ante	0.015	0.023	0.030	0.039	0.062	-0.047
β ex-ante	0.945	1.025	1.110	1.180	1.370	-0.425
β ex-post	0.905	0.977	1.064	1.182	1.434	-0.529
			Factor Model Alpha			
CAPM	-0.04%	0.32%	0.12%	-0.29%	-0.84%	0.80%
	-(0.232)	(1.945)	(0.702)	-(1.288)	-(2.507)	(2.075)
FFC4 + LIQ	-0.06%	0.38%	0.23%	-0.35%	-1.06%	1.00%
	-(0.345)	(2.072)	(1.292)	-(1.585)	-(3.111)	(2.529)

Panel B: Equal-Weighted Portfolios

Value	MAX 1 (Low)	MAX 2	MAX 3	MAX 4	MAX 5 (High)	Low-High MAX
			Portfolio Characteristics			
MAX ex-ante	0.015	0.023	0.030	0.039	0.069	-0.054
β ex-ante	0.835	0.976	1.070	1.135	1.213	-0.378
β ex-post	0.713	0.916	1.019	1.196	1.446	-0.733
			Factor Model Alphas			
CAPM	0.03%	0.20%	0.10%	-0.44%	-0.66%	0.70%
	(0.195)	(1.243)	(0.612)	-(2.195)	-(2.620)	(2.148)
FFC4 + LIQ	0.18%	0.44%	0.33%	-0.34%	-0.58%	0.76%
	(1.188)	(3.002)	(2.213)	-(1.843)	-(2.126)	(2.204)

Table D.3: Bivariate Portfolio Analysis Sorting on MAX, Then on β_{1Y}

The table presents the results of a bivariate portfolio analysis on the relation between the abnormal returns in month *t*+1 and the variable β_{1Y} in month *t* while controlling for MAX. The table presents portfolio characteristics and factor model alphas for the average MAX quintile portfolio within each β_{1Y} quintile. We create the average MAX quintile by performing a conditional double sort first on MAX, then on β_{1Y} to generate a 5x5 portfolio matrix at the end of each month *t*. We subsequently sum the stocks across the MAX quintiles for each β_{1Y} quintile to create five β_{1Y} -sorted portfolios that by construction should be neutralized to MAX. We only include monthly stock observations with estimates of both β_{1Y} and MAX for the given month. The table consists of two panels (A and B) and each panel is divided into two sections.

Explanation of sections: "Portfolio Characteristics" presents the time-series mean of the average values of MAX for stocks in each of the β_{1Y}-sorted portfolios (MAX ex-ante), the time-series mean of the average values of β_{1Y} for stocks in each of the β_{1Y} -sorted portfolios (β ex-ante), and the slope coefficient from a regression of VW (EW) portfolio excess returns on the VW (EW) market excess returns (β ex-post). "Factor Model Alphas" presents monthly portfolio alphas relative to the CAPM (CAPM) and the Fama-French-Carhart 4-Factor model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for the VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.

Explanation of panels: All reported numbers in panel A are calculated using value-weighted portfolios, while all numbers in panel B are calculated using equal-weighted portfolios.

General: The column labeled Low-High β_{1Y} refers to a zero-cost, long-short portfolio with a long position in the average MAX portfolio in β_{1Y} quintile 1 and a short position in the average MAX portfolio in β_{1Y} quintile 5.

Panel A: Value-Weighted Portfolios

Value	β_{1Y} 1 (Low)	β_{1Y} 2	β_{1Y} 3	β_{1Y} 4	β_{1Y} 5 (High)	Low-High β_{1Y}
			Portfolio Characteristics			
MAX ex-ante	0.029	0.029	0.028	0.027	0.027	0.001
β ex-ante	0.418	0.689	0.875	1.072	1.388	-0.970
β ex-post	0.747	0.929	0.964	1.045	1.289	-0.542
			Factor Model Alphas			
CAPM	-0.03%	-0.16%	0.09%	0.29%	-0.35%	0.33%
	-(0.135)	-(0.858)	(0.360)	(1.615)	-(1.860)	(1.133)
FFC4 + LIQ	-0.17%	-0.18%	0.05%	0.44%	-0.30%	0.14%
	-(0.823)	-(1.001)	(0.208)	(2.331)	-(1.538)	(0.473)

Panel B: Equal-Weighted Portfolios

Value	β_{1Y} 1 (Low)	β_{1Y} 2	β_{1Y} 3	β_{1Y} 4	β_{1Y} 5 (High)	Low-High β_{1Y}
			Portfolio Characteristics			
MAX ex-ante	0.035	0.034	0.035	0.035	0.036	-0.001
β ex-ante	0.410	0.687	0.895	1.124	1.514	-1.104
β ex-post	0.853	0.961	1.146	1.206	1.404	-0.551
			Factor Model Alphas			
CAPM	0.12%	-0.27%	-0.09%	-0.31%	-0.81%	0.93%
	(0.675)	-(1.338)	-(0.509)	-(1.498)	-(3.264)	(3.014)
FFC4 + LIQ	0.16%	-0.17%	0.04%	-0.09%	-0.40%	0.56%
	(0.897)	-(0.854)	(0.268)	-(0.450)	-(1.841)	(1.840)

Table D.4: Univariate Portfolio Analysis Sorting on the Orthogonal of β_{1Y} on MAX ($\beta_{1Y\perp M}$)

The table presents the results of a bivariate portfolio analysis on the relation between the abnormal returns in month *t*+1 and the variable β_{1Y} in month *t* while controlling for MAX. The table presents portfolio characteristics and factor model alphas for the average MAX quintile portfolio within each β_{1Y} quintile. We create the average MAX quintile by performing a conditional double sort first on MAX, then on β_{1Y} to generate a 5x5 portfolio matrix at the end of each month *t*. We subsequently sum the stocks across the MAX quintiles for each β_{1Y} quintile to create five β_{1Y} -sorted portfolios that by construction should be neutralized to MAX. We only include monthly stock observations with estimates of both β_{1Y} and MAX for the given month. The table consists of two panels (A and B) and each panel is divided into two sections.

Explanation of sections: "Portfolio Characteristics" presents the time-series mean of the average values of MAX for stocks in each of the β_{1Y}-sorted portfolios (MAX ex-ante), the time-series mean of the average values of β_{1Y} for stocks in each of the β_{1Y} -sorted portfolios (β ex-ante), and the slope coefficient from a regression of VW (EW) portfolio excess returns on the VW (EW) market excess returns (β ex-post). "Factor Model Alphas" presents monthly portfolio alphas relative to the CAPM (CAPM) and the Fama-French-Carhart 4-Factor model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for the VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.

Explanation of panels: All reported numbers in panel A are calculated using value-weighted portfolios, while all numbers in panel B are calculated using equal-weighted portfolios.

General: The column labeled Low-High β_{1Y} refers to a zero-cost, long-short portfolio with a long position in the average MAX portfolio in β_{1Y} quintile 1 and a short position in the average MAX portfolio in β_{1Y} quintile 5.

Panel A: Value-Weighted Portfolios

Value	$\beta_{1Y\perp M}$ 1 (Low)	$\beta_{1Y\perp M}$ 2	$\beta_{1Y\perp M}$ 3	$\beta_{1Y\perp M}$ 4	$\beta_{1Y\perp M}$ 5 (High)	Low-High $\beta_{1Y\perp M}$
			Portfolio Characteristics			
MAX ex-ante	0.030	0.027	0.026	0.026	0.031	-0.001
β ex-ante	0.382	0.644	0.835	1.067	1.497	-1.115
β ex-post	0.711	0.896	0.978	1.027	1.382	-0.671
			Factor Model Alphas			
CAPM	-0.12%	0.00%	0.08%	-0.05%	-0.29%	0.16%
	-(0.650)	(0.010)	(0.457)	-(0.286)	-(1.169)	(0.484)
FFC4 + LIQ	-0.27%	-0.10%	0.01%	0.09%	-0.23%	-0.04%
	-(1.381)	-(0.548)	(0.037)	(0.495)	-(0.898)	-(0.129)

Panel B: Equal-Weighted Portfolios

Value	$\beta_{1Y\perp M}$ 1 (Low)	$\beta_{1Y\perp M}$ 2	$\beta_{1Y\perp M}$ 3	$\beta_{1Y\perp M}$ 4	$\beta_{1Y\perp M}$ 5 (High)	Low-High $\beta_{1Y\perp M}$
			Portfolio Characteristics			
MAX ex-ante	0.038	0.033	0.033	0.033	0.038	0.000
β ex-ante	0.400	0.665	0.872	1.110	1.583	-1.183
β ex-post	0.851	0.991	1.089	1.149	1.506	-0.655
			Factor Model Alphas			
CAPM	-0.09%	0.11%	-0.11%	-0.20%	-1.02%	0.93%
	-(0.464)	(0.547)	-(0.640)	-(0.897)	-(3.739)	(2.714)
FFC4 + LIQ	-0.07%	0.19%	0.06%	0.06%	-0.65%	0.58%
	-(0.398)	(0.867)	(0.412)	(0.314)	-(2.615)	(1.767)

Appendix E: The Beta Anomaly while Controlling for IVOL

Table E.1: Bivariate Portfolio Analysis Sorting on IVOL, Then on β_{5Y}

The table presents the results of a bivariate portfolio analysis on the relation between abnormal returns in month *t*+1 and the variable β_{5Y} in month *t* while controlling for IVOL. The table presents portfolio characteristics and factor model alphas of the average IVOL quintile portfolio within each β_{5Y} quintile. We create the average IVOL quintile by performing a conditional double sort first on IVOL, then on β_{5Y} to generate a 5x5 portfolio matrix at the end of each month *t*. We subsequently sum the stocks across the IVOL quintiles for each β_{5Y} quintile to create five β_{5Y}-sorted portfolios that by construction should be neutralized to IVOL We only include monthly stock observations with estimates of both β_{5Y} and IVOL for the given month. The table consists of two panels (A and B) and each panel is divided into two sections.

Explanation of sections: "Portfolio Characteristics" presents the time-series mean of the average values of MAX for the stocks in each of the β_{5Y}-sorted portfolios (MAX ex-ante), the time-series mean of the average values of β_{5Y} for the stocks in each of the β_{5Y}-sorted portfolios (β ex-ante), and the slope coefficient from a regression of VW (EW) portfolio excess returns on the VW (EW) market excess returns (β ex-post). "Factor Model Alphas" presents monthly portfolio alphas relative to the CAPM (CAPM) and the Fama-French-Carhart 4-Factor model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags.

Explanation of panels: All reported numbers in panel A are calculated using value-weighted portfolios, while all numbers in panel B are calculated using equal-weighted portfolios.

General: The column labeled Low-High β_{5Y} refers to a zero-cost, long-short portfolio with a long position in the average IVOL portfolio in β_{5Y} quintile 1 and a short position in the average IVOL portfolio in β_{5Y} quintile 5. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.

Panel A: Value-Weighted Portfolios

Value	β_{5Y} 1 (Low)	β_{5Y} 2	β_{5Y} 3	β_{5Y} 4	β_{5Y} 5 (High)	Low-High β_{5Y}
			Portfolio Characteristics			
MAX ex ante	0.027	0.026	0.027	0.027	0.029	-0.003
β ex ante	0.514	0.778	0.948	1.144	1.530	-1.017
β ex post	0.765	0.988	1.027	1.102	1.203	-0.438
			Factor Model Alpha			
CAPM	0.46 %	0.17 %	0.29 %	-0.19 %	-0.16 %	0.62 %
	(2.122)	(0.659)	(1.613)	-(1.019)	-(0.864)	(2.252)
FFC4 + LIQ	0.42 %	0.23 %	0.32 %	-0.13 %	-0.09 %	0.51 %
	(2.171)	(0.920)	(1.529)	-(0.719)	-(0.466)	(1.854)

Panel B: Equal-Weighted Portfolios

Value	β_{5Y} 1 (Low)	β_{5Y} 2	β_{5Y} 3	β_{5Y} 4	β_{5Y} 5 (High)	Low-High β_{5Y}
			Portfolio Characteristics			
MAX ex ante	0.033	0.034	0.035	0.036	0.038	-0.006
β ex ante	0.453	0.775	1.003	1.264	1.733	-1.280
β ex post	0.776	1.013	1.041	1.201	1.241	-0.465
			Factor Model Alpha			
CAPM	0.25 %	0.20 %	-0.04 %	-0.47 %	-0.67 %	0.91 %
	(1.682)	(0.925)	-(0.251)	-(2.316)	-(2.919)	(3.600)
FFC4 + LIQ	0.29 %	0.26 %	0.06 %	-0.27 %	-0.32 %	0.61 %
	(1.963)	(1.292)	(0.360)	-(1.351)	-(1.618)	(2.296)

Table E.2: Univariate Portfolio Analysis Sorting on the Orthogonal of β_{5Y} on IVOL ($\beta_{5Y\perp IVOL}$)

The table presents the results of a univariate portfolio analysis on the relation between excess and abnormal returns in month *t*+1 and the portion of β_{5Y} that is orthogonal to IVOL in month *t*. The table consists of two panels (A and B) and each panel is divided into two sections.

Explanation of sections: "Portfolio Characteristics" presents the time-series mean of the average values of MAX for stocks in each of the $\beta_{5Y\perp IVOL}$ -sorted portfolios (MAX ex-ante), the time-series mean of the average values of β_{5Y} for stocks in each of the $\beta_{5Y\perp IVOL}$ -sorted portfolios (β ex-ante), and the slope coefficient from a regression of VW (EW) portfolio excess returns on the VW (EW) market excess returns (β ex-post). "Factor Model Alphas" presents monthly portfolio alphas relative to the CAPM (CAPM) and the FFC4 model augmented with the liquidity factor of Næs, Skjeltorp and Ødegaard (2009) (FFC4 + LIQ). We use the VW (EW) market portfolio as the market factor in the factor models when estimating alphas for VW (EW) portfolios. The numbers in parentheses are t-statistics adjusted following Newey and West (1987) using four lags.

Explanation of panels: All reported numbers in panel A are calculated using value-weighted portfolios, while all numbers in panel B are calculated using equal-weighted portfolios.

General: The column labeled Low-High $\beta_{5Y\perp IVOL}$ refers to a zero-cost, long-short portfolio with a long position in quintile portfolio 1 and a short position in quintile portfolio 5. Our sample contains portfolio returns from Jan. 1990 through Dec. 2018. Portfolios are rebalanced monthly.

Panel A: Value-Weighted Portfolios

Value	$\beta_{5Y\perp IVOL}$ 1 (Low)	$\beta_{5Y\perp IVOL}$ 2	$\beta_{5Y\perp IVOL}$ 3	$\beta_{5Y\perp IVOL}$ 4	$\beta_{5Y\perp IVOL}$ 5 (High)	Low-High $\beta_{5Y\perp IVOL}$
			Portfolio Characteristics			
MAX ex ante	0.031	0.027	0.025	0.027	0.032	-0.001
β ex ante	0.477	0.751	0.948	1.213	1.682	-1.205
β ex post	0.768	1.025	0.979	1.147	1.315	-0.547
			Factor Model Alpha			
CAPM	0.55 %	0.36 %	-0.04 %	-0.08 %	-0.49 %	1.03 %
	(2.273)	(1.564)	-(0.221)	-(0.373)	-(2.179)	(2.951)
FFC4 + LIQ	0.56 %	0.42 %	-0.03 %	-0.06 %	-0.49 %	1.05 %
	(2.363)	(1.760)	-(0.155)	-(0.293)	-(2.116)	(2.918)

Panel B: Equal weighted portfolios

Value	$\beta_{5Y\perp IVOL}$ 1 (Low)	$\beta_{5Y\perp IVOL}$ 2	$\beta_{5Y\perp IVOL}$ 3	$\beta_{5Y\perp IVOL}$ 4	$\beta_{5Y\perp IVOL}$ 5 (High)	Low-High $\beta_{5Y\perp IVOL}$
			Portfolio Characteristics			
MAX ex ante	0.038	0.033	0.032	0.034	0.038	0.001
β ex ante	0.462	0.764	0.978	1.252	1.774	-1.312
β ex post	0.860	0.941	0.981	1.143	1.356	-0.496
			Factor Model Alpha			
CAPM	0.53 %	0.02 %	-0.15 %	-0.47 %	-0.68 %	1.21 %
	(2.523)	(0.101)	-(0.757)	-(2.067)	-(3.005)	(3.653)
FFC4 + LIQ	0.60 %	0.02 %	0.09 %	-0.26 %	-0.40 %	1.00 %
	(2.806)	(0.112)	(0.507)	-(1.312)	-(2.014)	(3.001)

Appendix F: Returns and Transaction Costs

A Word of Warning to Market Practitioners

In this thesis we find that a portfolio long stocks with low beta and short stocks with high beta has generated statistically significant abnormal returns relative to the CAPM. Furthermore, we document that the low-beta portfolio has generated higher absolute cumulative excess returns than both the high-beta portfolio and the market portfolio over the sample period. An important caveat to such analyses is that the documentation of an anomaly does not necessarily imply that it can be converted into a profitable trading strategy.

Firstly, the results of our analyses are based entirely on historical observations and there is no guarantee that the documented relation between market beta and abnormal returns will hold in the future. Secondly, our analysis does not account for trading costs or the costs associated with borrowing stocks for short-selling, and as such, our results cannot be used to evaluate the historical profitability of "betting-against-beta". As discussed in section 2.3 of the thesis Baker, Bradly and Wurgler (2011) show that the stocks comprising the most volatile portfolios tend to be small and illiquid, which are typically expensive to trade and especially short sell. Given that that trading costs prevents investors from *betting against beta*, the documented anomaly can hardly be considered irrational.

Estimating the trading costs associated with betting against beta on the Oslo Stock Exchange lies beyond the scope of this paper, and further research on the topic would be interesting. We have nevertheless computed a few simple metrics which (we hope) will allow investors to from a more informed opinion regarding the trading costs associated with replicating the quintile portfolios constructed in our analysis. Generally, we find that the trading costs will largely be a function of the liquidity of the stocks in the portfolio and the portfolio turnover. We have not estimated the liquidity of the individual stocks in our sample, but we argue that market share of the portfolios in our analyses will function as a reasonable proxy for the trading costs associated with illiquidity. We report the time series average market share for our β_{5Y} sorted portfolios and MAX-sorted portfolios in Table 4 and Table 8, respectively. We note that the stocks in the β_{5Y} (MAX) sorted quintile portfolio 1 (5) on average have constituted 3% (6%) of the total market. Combined with a qualitative assessment of the most frequent stocks in the portfolios reported in Table A.4 in the appendix, we argue it is reasonable to assume that the costs of trading stocks in the β_{5Y} quintile 1 portfolio and MAX quintile 5, will be relatively large.

We have not formally assessed the turnover of the quintile portfolios in our analysis, but in Table F.1 below we report relevant metrics on the monthly rebalancing process of our portfolio formation methodology. Not surprisingly, we find that the average turnover measured by the number of stocks that enter and leave the portfolios, is lowest for the β_{5Y}-sorted portfolios and highest for the MAX sorted portfolios[47]. In figure F.3, F.4 and F.5 in the appendix we also illustrate the development in portfolio turnover relative to portfolio size for our beta and MAX sorted portfolios over our sample period. All else equal, we note that the portfolio turnover suggests that the trading costs associated with replicating the β_{5Y}-sorted portfolios should be significantly lower than the trading costs of replicating the MAX sorted portfolios.

Lastly, we also want to specify that although our analysis has been focused on the performance of low-high portfolios, we would by no means recommend shorting the high-beta or high-MAX portfolio based on the results of this thesis alone. Firstly, our analysis does not include borrowing costs, which could be significant. Furthermore, it is important to note that the high-beta and high-MAX portfolios are constructed to experience high volatility in monthly returns. This is illustrated by the standard deviations of the portfolio returns reported in Table 4 and Table 8 for the high-β_{5Y} and high-MAX portfolios. We have also plotted the monthly excess returns of the high-β_{5Y} and high-MAX portfolios in figure F.1 and F.2 in the appendix and we note that over the course of our sample period the high-portfolios have experience several large-up moves in portfolio returns. As such, portfolios with large short positions in the high portfolios will have a substantial risk of experiencing margin calls or having their equity wiped, which would be detrimental to the long-term cumulative returns.

[47] Turnover measures the number of stocks that enter and leave the portfolio. Hence, it does not measure the number of trades. We rebalance our portfolios each month to construct equal- and value-weighted portfolios, and an accurate replication of our methodology would require significantly more trades than the reported turnover.

Table F.1: Portfolio Turnover

At the end of each month *t,* all stocks are sorted into quintile portfolios based on an ascending ordering of the sorting variable. The table presents the time-series mean of the number of stocks in the portfolio prior to rebalancing each month (Stocks Prior to Rebalancing), number of stocks that leave the portfolio each month (Stocks Sold), number of stocks that enter the portfolio each month (Stocks Bought), the number of stocks in the portfolio post rebalancing (Stocks Post Rebalancing) and the monthly portfolio turnover equal to the sum of Stocks Bough and Stocks Sold (Turnover). The results are reported for portfolios sorted on β_{5Y}, β_{1Y} and MAX. Panel A reports the results for Quintile Portfolio 1, while Panel B reports the results for Quintile Portfolio 5. The sample contains monthly observations from Jan. 1990 through Dec. 2018 and contains 348 monthly observations of portfolio rebalancing.

Panel A: Quintile Portfolio 1 (Low)

Sorting Variable	Stocks Prior to Rebalancing	Stocks Sold	Stocks Bought	Stocks Post Rebalancing	Turnover
β_{5Y}	25.5	1.5	1.6	25.5	3.1
β_{1Y}	20.6	3.1	3.1	20.6	6.1
MAX	25.0	15.6	15.6	25.1	31.2

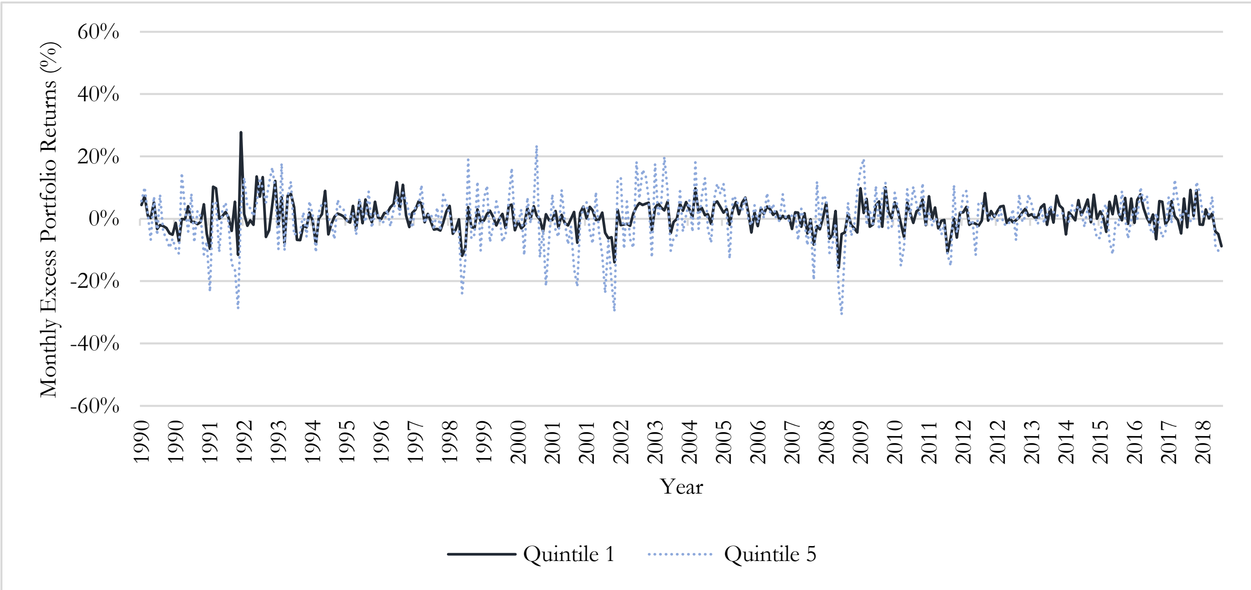
Panel B: Quintile Portfolio 5 (High)

Sorting Variable	Stocks Prior to Rebalancing	Stocks Sold	Stocks Bought	Stocks Post Rebalancing	Turnover
β_{5Y}	25.5	1.6	1.6	25.5	3.1
β_{1Y}	20.6	2.2	2.2	20.6	4.4
MAX	25.1	15.3	15.3	25.1	30.6

Figure F.1: Monthly Excess Returns of Quintile 1 and Quintile 5 Sorted on β_{5Y}

At the end of each month *t*, all stocks are sorted into quintile portfolios based on an ascending ordering of β_{5Y}. The figure plots the monthly excess returns for quintile portfolio 1 and quintile portfolio 5, from Jan. 1990 through Dec. 2018. Panel A plots the monthly excess returns for the value-weighted portfolios, while Panel B plots monthly excess returns for the equal-weighted portfolios.

Panel A: Value-Weighted Portfolios



Panel B: Equal-Weighted Portfolios

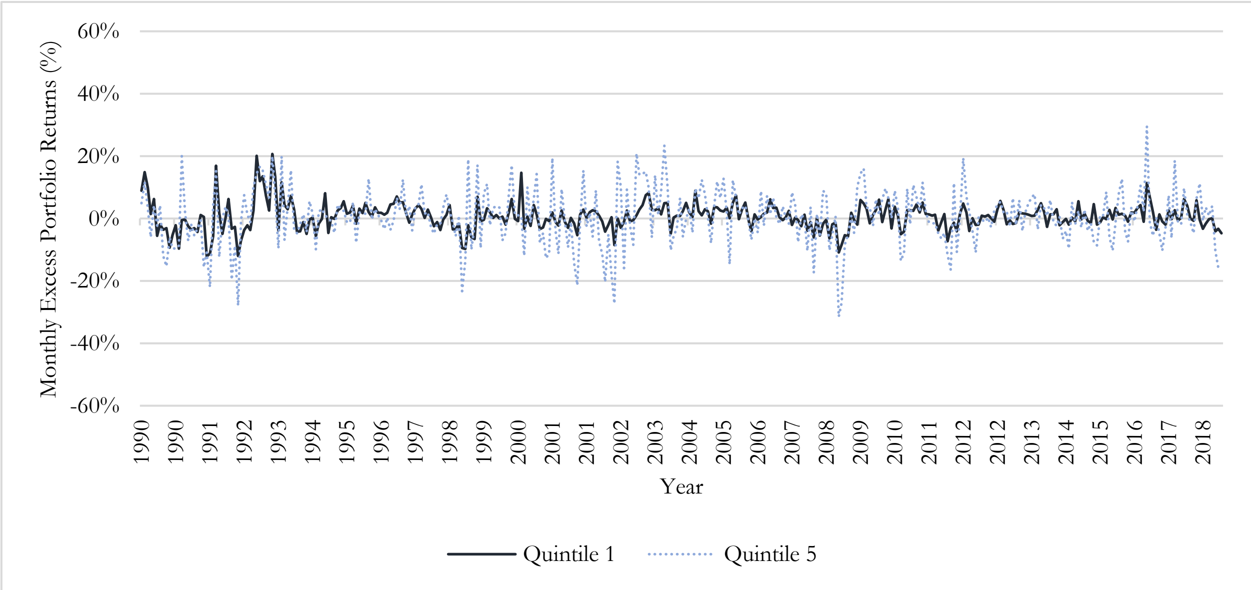
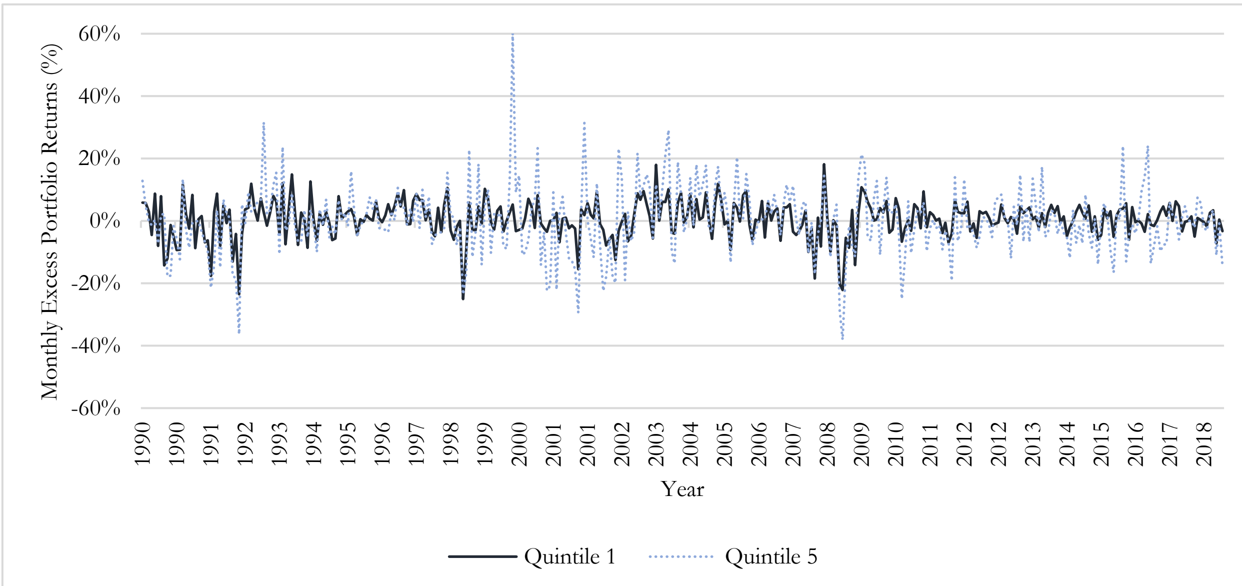


Figure F.2: Monthly Excess Returns of Quintile 1 and Quintile 5 Sorted on MAX

At the end of each month *t*, all stocks are sorted into quintile portfolios based on an ascending ordering of MAX. The figure plots the monthly excess returns for quintile portfolio 1 and quintile portfolio 5, from Jan. 1990 through Dec. 2018. Panel A plots the monthly excess returns for the value-weighted portfolios, while Panel B plots monthly excess returns for the equal-weighted portfolios.

Panel A: Value-Weighted Portfolios



Panel B: Equal-Weighted Portfolios

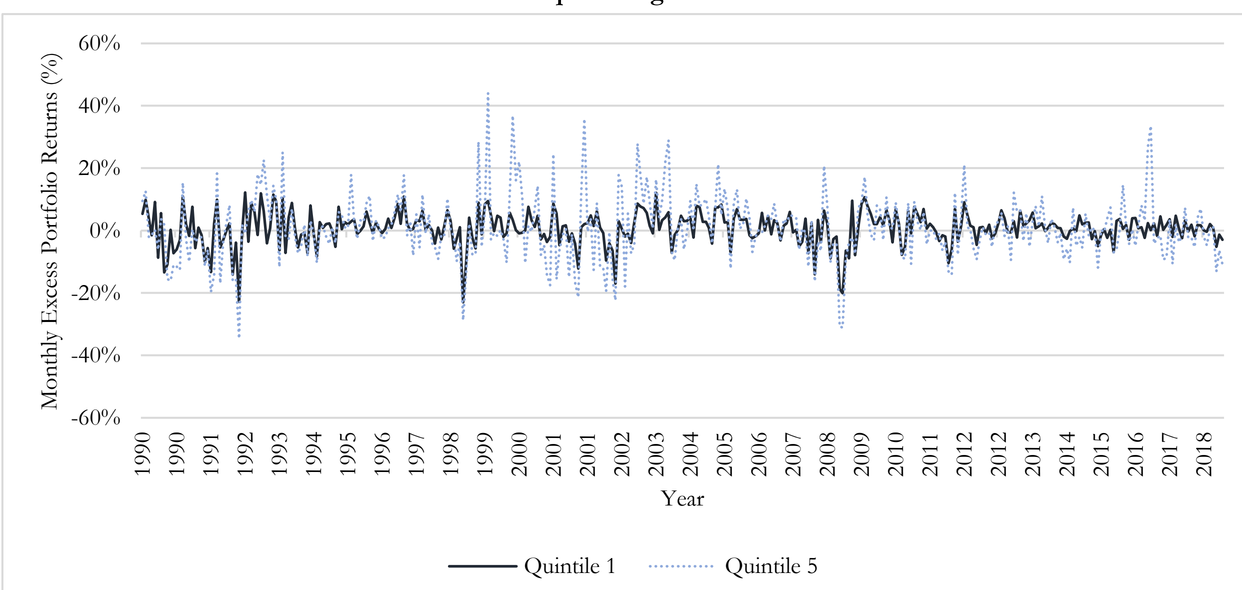
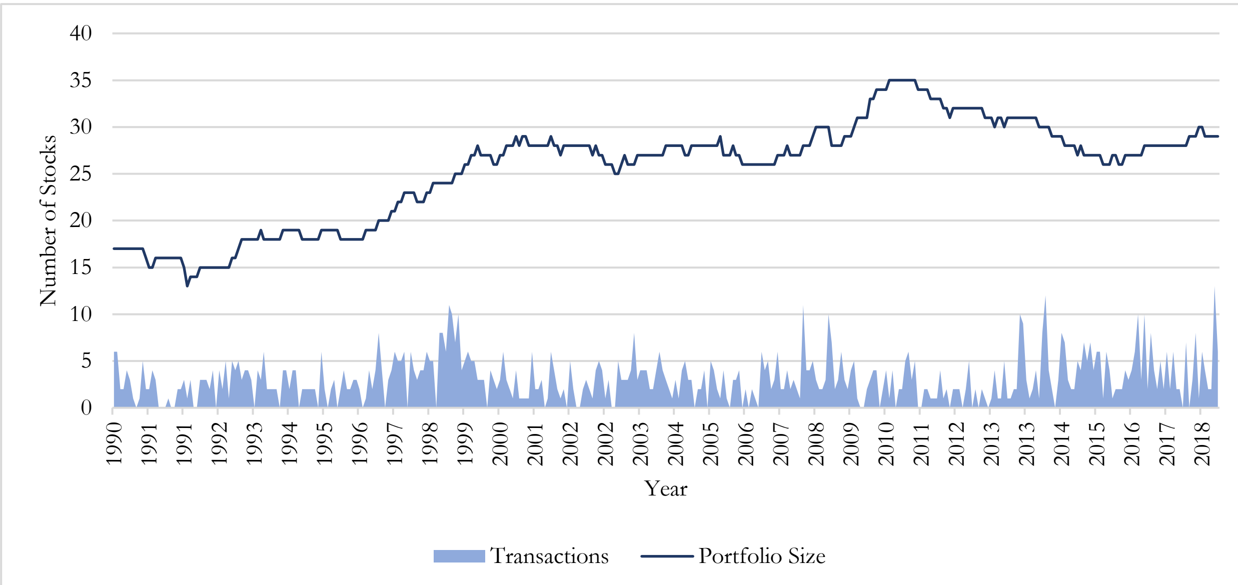


Figure F.3: Transactions vs. Portfolio Size for β_{5Y}-Sorted Quintile Portfolios

At the end of each month *t*, all stocks are sorted into quintile portfolios based on an ascending ordering of β_{5Y}. The figure illustrates the number of stocks in the quintile portfolios for each month *t*, and the sum of the number of stocks that enter and leave the quintile portfolios for each month *t* (Transactions). The dataset used contains monthly observations of the quintile portfolios from Jan. 1990 through Dec. 2018, resulting in a total of 348 monthly observations. Panel A reports the results for Quintile Portfolio 1 (low-beta), while Panel B reports the results for Quintile Portfolio 5 (high-beta).

Panel A: Quintile Portfolio 1



Panel B: Quintile Portfolio 5

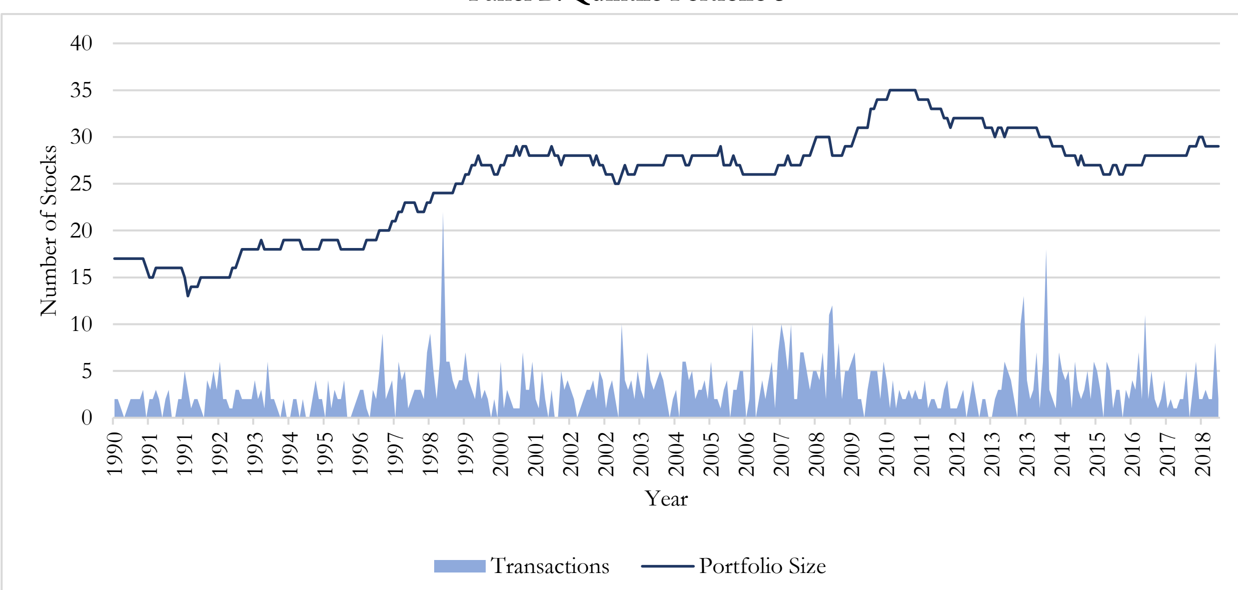
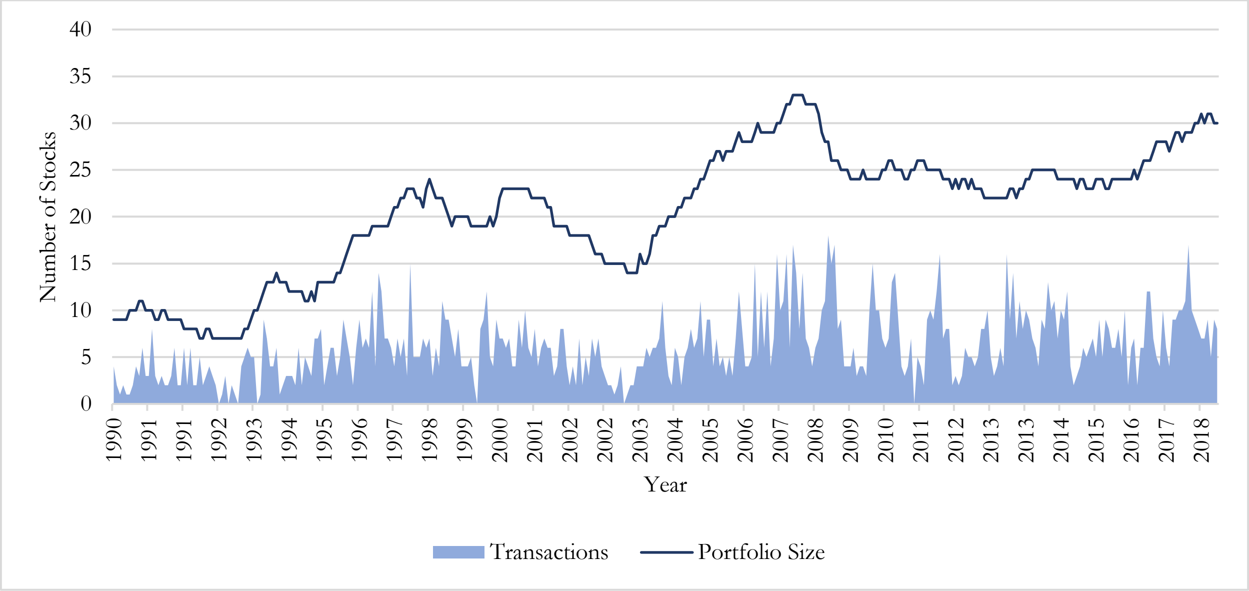


Figure F.4: Transactions vs. Portfolio Size for β_{1Y}-Sorted Quintile Portfolios

At the end of each month *t*, all stocks are sorted into quintile portfolios based on an ascending ordering of β_{1Y}. The figure illustrates the number of stocks in the quintile portfolios for each month *t*, and the sum of the number of stocks that enter and leave the quintile portfolios for each month *t* (Transactions). The dataset used contains monthly observations of the quintile portfolios from Jan. 1990 through Dec. 2018, resulting in a total of 348 monthly observations. Panel A reports the results for Quintile Portfolio 1 (low-beta), while Panel B reports the results for Quintile Portfolio 5 (high-beta).

Panel A: Quintile Portfolio 1



Panel B: Quintile Portfolio 5

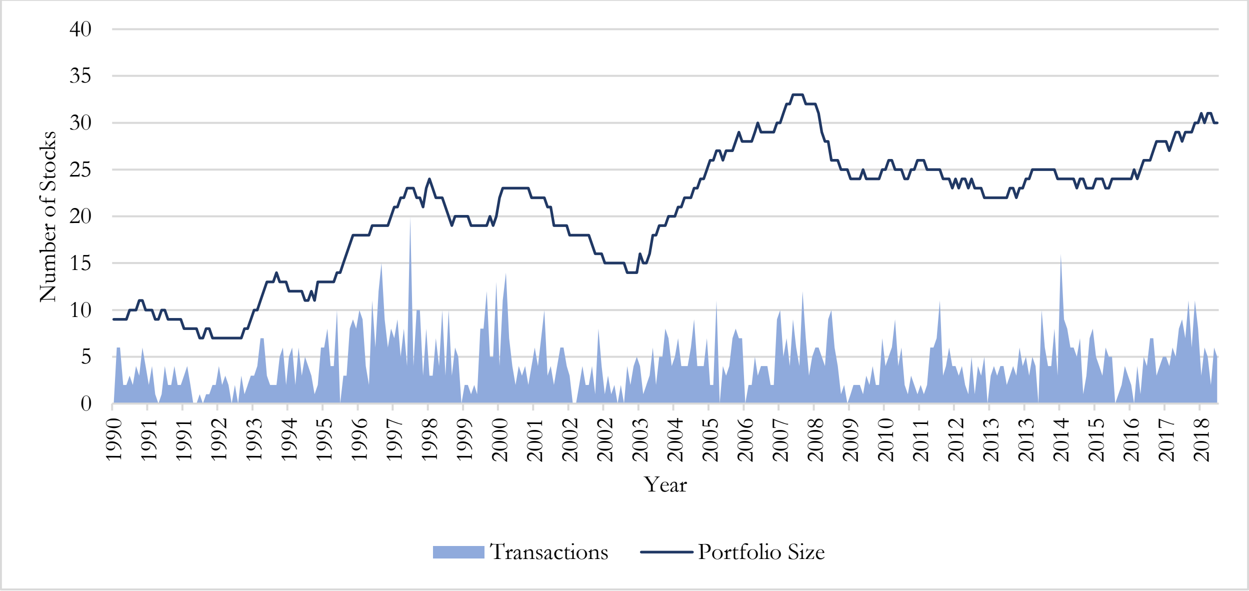
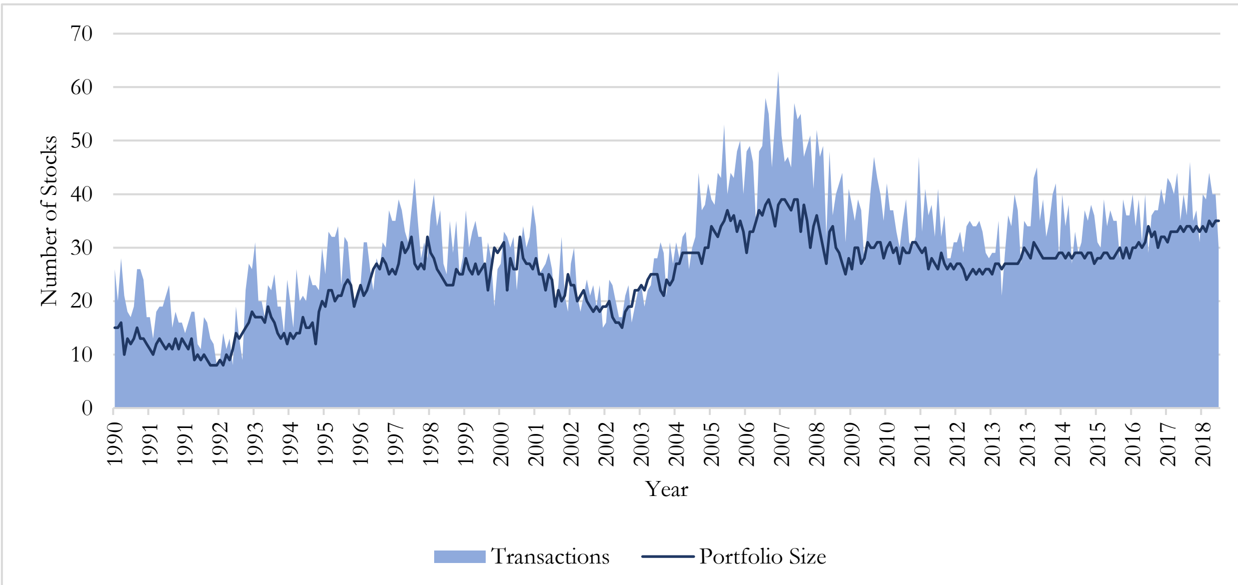


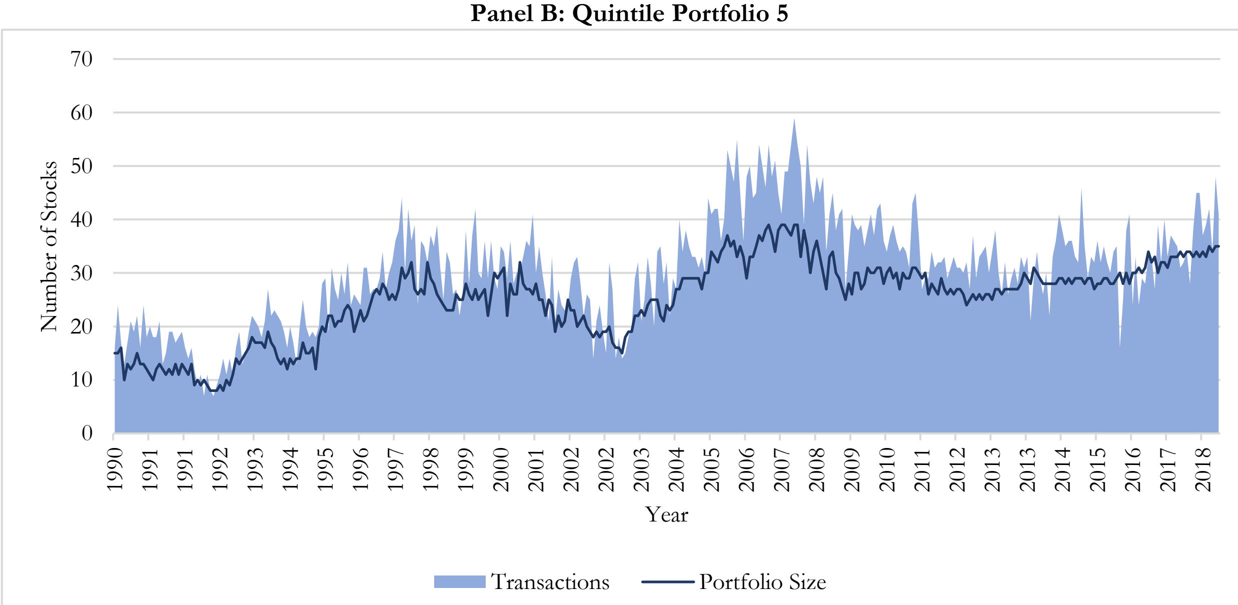
Figure F.5: Transactions vs. Portfolio Size for MAX Sorted Quintile Portfolios

At the end of each month *t*, all stocks are sorted into quintile portfolios based on an ascending ordering of MAX. The figure illustrates the number of stocks in the quintile portfolios for each month *t*, and the sum of the number of stocks that enter and leave the quintile portfolios for each month *t* (Transactions). The dataset used contains monthly observations of the quintile portfolios from Jan. 1990 through Dec. 2018, resulting in a total of 348 monthly observations. Panel A reports the results for Quintile Portfolio 1 (low-MAX), while Panel B reports the results for Quintile Portfolio 5 (high-MAX).

Panel A: Quintile Portfolio 1



Panel B: Quintile Portfolio 5



R Script

This section presents the R-code used to generate the focal results of this thesis. Some calculations have been performed in excel, and we have not included the code used to generate all supportive calculations. The full code and the dataset will be granted upon request.

```
########## Explanation of most important variables ##########


### General Variables ###

# MCAP = Market Capitalization
# LMCAP = Lagged Market Capitalization
# M.VW = Estimated returns of the value-weighted market portfolio
# M.EW = Estimated returns of the equal-weighted market portfolio
# MSCI = Returns of the MSCI World index in NOK
# E.M.VW = Estimated excess returns of the value-weighted market portfolio
# E.M.EW = Estimated excess returns of the equal-weighted market portfolio
# E.MSCI = Excess returns of the MSCI World index in NOK
# iskew = Idosyncratic skewness
# ReturnAdjGeneric = Stock returns (Monthly and daily)
# E.ReturnAdjGeneric = Excess stock returns (Monthly and daily)


### Sorting variables ###

# beta.monthly = Refers to B_5Y in the main paper
# beta.daily = Refers to B_1Y in the main paper
# beta.msci = Refers to B_MSCI in the main paper
# max = Refers to MAX in the main paper
# ivol = Idosyncratic volatility
# obeta.monthly = Component of beta.monthly orthogonal to max
# obeta.daily = Component of beta.daily orthogonal to max
# obmax = Component of max orthogonal to beta.monthly
# ibeta.monthly = Component of beta.monthly orthogonal to ivol
```

```
##########  Section 1: Install and load packages ##########


# Install packages
install.packages("dplyr")
install.packages("readxl")
install.packages("openxlsx")
install.packages("tidyr")
install.packages("readr")
install.packages("lubridate")
install.packages("read.xlsx")
install.packages("ggplot2")
install.packages("stargazer")
install.packages("sandwich")
install.packages("lmtest")
install.packages("moments")
install.packages("psych")
install.packages("markovchain")

# Load packages
library(dplyr)
library(readxl)
library(openxlsx)
library(tidyr)
library(readr)
library(lubridate)
library(ggplot2)
library(stargazer)
library(sandwich)
library(lmtest)
library(zoo)
library(moments)
library(Hmisc)
library(psych)
library(markovchain)



########## Section 2: Import data ##########



### Daily data
# Set Working Directory to import daily data
setwd(XXX)

## Import data on stocks from Børsprosjektet at NHH

# Create empty dataframe with colnames
d <- read.csv("1980.csv", header = TRUE, sep = ";")
df_daily <- d[0,]

# Loop inn stock data for all time periods (could not download more than 50 000 rows of data at a
time from Børsprosjektet)
data.list <- list.files( pattern = '*.csv')

for (i in 1:length(data.list)) {
  d <- read.csv(data.list[i], header = TRUE, sep = ";")
  df_daily <- rbind(df_daily,d)}

# Delete excess variables
rm(d, data.list)

## Import remaining daily data

# Import Data on Market Returns (Professor Bernt Arne Ødegaard)
dfm_daily <- read.xlsx("market_returns.xlsx")
# Import data on Factors (Professor Bernt Arne Ødegaard)
df_factors_daily <- read.xlsx("Daily factors.xlsx")
# Import data on Risk-free rate (Professor Bernt Arne Ødegaard)
df_rf_daily <- read.xlsx("Daily rf.xlsx")
```

```r
# Import data on USD/NOK Exchange Rate
df_exchange_daily <- read.xlsx("USDNOK.xlsx")

### Monthly data
# Set Working Drectory to import monthly data
setwd(XXX)

## Import Stock Data for three different time periods
df_1999 <- read.csv("Monthly data 1980-1999.csv", header = TRUE, sep =";")
df_2009 <- read.csv("Monthly data 2000-2009.csv", header = TRUE, sep =";")
df_2019 <- read.csv("Monthly data 2010-2019.csv", header = TRUE, sep =";")
df_monthly <- rbind(df_1999, df_2009, df_2019)

# Delete unnecessary variables
rm(df_1999, df_2009, df_2019)

## Import remaining monthly data

# Import Data on Market Returns (Professor Bernt Arne Ødegaard)
dfm_monthly <- read.xlsx("Monthly market returns.xlsx")
# Import data on MSCI World Index Returns
df_msci_monthly <- read.xlsx("MSCI World 1985-2019.xlsx")
# Import data on Factors (Professor Bernt Arne Ødegaard)
df_factors_monthly <- read.xlsx("Factors.xlsx")
# Import data on Risk-free Rate (Professor Bernt Arne Ødegaard)
df_rf_monthly <- read.xlsx("Risk free rate.xlsx")
# Import data on the BAB factor from AQR
df_bab_monthly <- read.xlsx("BAB factor clean.xlsx")

### New Environment to save output
setwd(XXX)



########## Section 3: Change date format for imported data ##########



# Daily stock data
df_daily <- df_daily %>%
  rename( TradeDate = ï..ï..ï..TradeDate)
df_daily$TradeDate <- gsub(x = df_daily$TradeDate, pattern = "00:00:00", replacement = "", fixed
= T )
df_daily$TradeDate <- as.Date(df_daily$TradeDate)

# Daily market data
dfm_daily$date <- as.Date(as.character(dfm_daily$date), "%Y%m%d")
dfm_daily <- dfm_daily %>%
  rename(TradeDate = date)

# Daily factor data
df_factors_daily$date <- as.Date(as.character(df_factors_daily$date), "%Y%m%d")
df_factors_daily <- df_factors_daily %>%
  rename(TradeDate = date)

# Daily rf data
 df_rf_daily$date <- as.Date(as.character(df_rf_daily$date), "%Y%m%d")
df_rf_daily <- df_rf_daily %>%
  rename(TradeDate = date)

# Daily exchange rate data
df_exchange_daily$Date <- as.Date(as.character(df_exchange_daily$Date), "%Y-%m-%d")
df_exchange_daily <- df_exchange_daily %>%
  rename(TradeDate = Date)

# Monthly stock data
df_monthly <- df_monthly %>%
  rename( TradeDate = ï..ï..ï..TradeDate)
df_monthly$TradeDate <- gsub(x = df_monthly$TradeDate, pattern = "00:00:00", replacement = "",
fixed = T )
df_monthly$TradeDate <- as.Date(df_monthly$TradeDate)
```

```
# Set all dates first day of the month
df_monthly$TradeDate <- floor_date(df_monthly$TradeDate, "month")

# Monthly market data
dfm_monthly$date <- as.Date(as.character(dfm_monthly$date), "%Y%m%d")
dfm_monthly <- dfm_monthly %>%
  rename(TradeDate = date)
# Set all dates to first day of the month
dfm_monthly$TradeDate <- floor_date(dfm_monthly$TradeDate, "month")

# Monthly factor
df_factors_monthly$date <- as.Date(as.character(df_factors_monthly$date), "%Y%m%d")
df_factors_monthly <- df_factors_monthly %>%
  rename(TradeDate = date)
# Set all dates to first day of the month
df_factors_monthly$TradeDate <- floor_date(df_factors_monthly$TradeDate, "month")

# Montly rf
df_rf_monthly$date <- as.Date(as.character(df_rf_monthly$date), "%Y%m%d")
df_rf_monthly <- df_rf_monthly %>%
  rename(TradeDate = date)
# Set all dates to first day of the month
df_rf_monthly$TradeDate <- floor_date(df_rf_monthly$TradeDate, "month")

# Monthly bab factor
df_bab_monthly$Date <- as.Date(as.character(df_bab_monthly$Date), "%m/%d/%Y")
df_bab_monthly <- df_bab_monthly %>%
  rename(TradeDate = Date)
# Set all dates to first day of the month
df_bab_monthly$TradeDate <- floor_date(df_bab_monthly$TradeDate, "month")

# MSCI monthly data
df_msci_monthly$Date <- as.Date(as.character(df_msci_monthly$Date), "%Y%m%d")
df_msci_monthly <- df_msci_monthly %>%
  rename(TradeDate = Date)
# Set all dates to first day of the month
df_msci_monthly$TradeDate <- floor_date(df_msci_monthly$TradeDate, "month")



########## Section 4: Data filtering ##########



### Daily data

## Preparations

# Compute year, month, MCAP (Market Capitalization) and LMCAP (Lagged Market Capitalization)
variables
df_daily <- df_daily %>% mutate(Year = year(TradeDate)) %>% mutate(Month =
floor_date(df_daily$TradeDate, "month")) %>%
  mutate(Turnover_adj = Generic * OffShareTurnover) %>% mutate(MCAP = Generic * SharesIssued) %>%
group_by(SecurityId) %>%
  arrange(TradeDate) %>% mutate(LMCAP = lag(MCAP)) %>% ungroup()

# Compute yearly variables needed for filtering
df_daily <- df_daily %>% group_by(SecurityId, Year) %>% mutate(Trading_days_year =
sum(!is.na(OffShareTurnover))) %>%
  mutate(Turnover_adj_year = mean(Turnover_adj, na.rm = TRUE)) %>% ungroup()

# Compute monthly variables needed for filtering
df_daily <- df_daily %>%  group_by(SecurityId, Month) %>%  mutate(Trading_days_month =
sum(!is.na(OffShareTurnover))) %>%
  mutate(MCAP_month_low = min(MCAP, na.rm = TRUE)) %>% mutate(Generic_month_low = min(Generic,
na.rm = TRUE)) %>% ungroup()

# Create relative liquidity limit
k <- df_daily %>% select("Year", "SecurityId", "Turnover_adj_year") %>%
  distinct() %>% group_by(Year) %>% mutate(Turnover_adj_limit = quantile(Turnover_adj_year,
0.025, type = 3, na.rm = TRUE)) %>%
```

```
  ungroup() %>% select("Year", "Turnover_adj_limit") %>% distinct()
df_daily <- merge(df_daily, k, by = "Year", all.x = TRUE)

# Remove excess variable
rm(k)

## Filtering

# Use data post 1985 with observations of ReturnAdjGeneric
filter(!is.na(ReturnAdjGeneric))
df_daily <- df_daily %>% filter(TradeDate >= as.Date("1985-01-01") & TradeDate <= as.Date("2018-
12-31")) %>%

# Create summary statistics of daily data prior to filters
summary_statistics_prefilter_daily <- df_daily %>% select("Generic",
"ReturnAdjGeneric","SharesIssued" ,"MCAP", "OffShareTurnover") %>%
  describe( ., na.rm = TRUE, skew = FALSE, quant = c(0.25,0.75))

# Apply filters
df_daily <- df_daily %>%
  filter(SecurityTypeId != 7) %>%                # Remove free float shares from sample
  filter(Market == "OSE") %>%                    # Remove stocks listed not listed on OSE from
sample
  filter(MCAP_month_low >= 1000000) %>%          # Remove stocks for months with less than NOK 1m
MCAP
  filter(Generic_month_low >= 1) %>%             # Remove stocks for months with less than NOK 1
Generic
  filter(Trading_days_year >= 20) %>%            # Remove stocks for years with less than 20
trading days
  filter(Turnover_adj_year > Turnover_adj_limit) # Remove the 2.5% of stocks for years with the
lowest average daily turnover in NOK

## Create summary statistics of daily data post filters
summary_statistics_postfilter_daily <- df_daily %>% select("Generic",
"ReturnAdjGeneric","SharesIssued" ,"MCAP", "OffShareTurnover") %>%
  describe( ., na.rm = TRUE, skew = FALSE, quant = c(0.25,0.75))

### Monthly Data

## Preparations

# Calculate Year, Month, MCAP, and LMCAP variable
df_monthly <- df_monthly %>% mutate(Year = year(TradeDate)) %>% mutate(MCAP =
Generic*SharesIssued) %>%
  group_by(SecurityId) %>% arrange(TradeDate) %>% mutate(LMCAP = lag(MCAP)) %>% ungroup()

## Filtering

# Use data post 1985
df_monthly <- df_monthly %>%
  filter(TradeDate >= as.Date("1985-01-01") & TradeDate <= as.Date("2018-12-31") )

# Create summary statistics of monthly data prior to filters
summary_statistics_prefilter_monthly <- df_monthly %>%
  select("Generic", "ReturnAdjGeneric","SharesIssued" ,"MCAP", "OffShareTurnover") %>%
  describe( ., na.rm = TRUE, skew = FALSE, quant = c(0.25,0.75))

# Remove NA values for ReturnAdjGeneric. Results in only one return observation per month (Have
checked this)
df_monthly <- df_monthly %>% filter(!is.na(ReturnAdjGeneric))

# Extract unique combinations of SecurityID and Month from daily data
merge <- df_daily %>% select("Month","SecurityId","Trading_days_month") %>% distinct() %>%
rename(TradeDate = Month)

# Merge the unique combinations of SecurityID and Month from the daily data with the monthly
dataset
# This procedure effectively applies the filters applied to the daily data, to the monthly data.
df_monthly <- merge(merge, df_monthly, by = c("TradeDate","SecurityId"), all.x = FALSE, all.y =
FALSE) # 87 453 - 72 783
rm(merge) # Remove excess dataframe
```

```r
# Create summary statistics of monthly data post filters
summary_statistics_postfilter_monthly <- df_monthly %>%
  select("Generic", "ReturnAdjGeneric","SharesIssued" ,"MCAP", "OffShareTurnover") %>%
  describe( ., na.rm = TRUE, skew = FALSE, quant = c(0.25,0.75))



########## Section 5: Compute Market Returns ##########

# Note that VW returns are calculated using lagged market capitalization

## 1: Calculate daily market returns using filtered data

# Create LMCAP_return variable
df_daily <- df_daily %>% mutate(LMCAP_return = ReturnAdjGeneric * LMCAP)

# Create EW and VW Market Returns
dfma_daily <- df_daily %>% filter(!is.na(LMCAP)) %>% filter(LMCAP > 0) %>%
  group_by(TradeDate) %>% summarise(SecurityIds = length(ReturnAdjGeneric),
  M.EW = sum(ReturnAdjGeneric)/length(ReturnAdjGeneric),M.VW = sum(LMCAP_return)/sum(LMCAP))

## 2: Calculate monthly market returns using filtered data

# Create LMCAP_Return variable
df_monthly <- df_monthly %>% mutate(LMCAP_return = ReturnAdjGeneric * LMCAP)

# Compute EW and VW Market Returns
dfma_monthly <- df_monthly %>% filter(!is.na(LMCAP_return)) %>%
  filter(LMCAP > 0) %>% group_by(TradeDate) %>% summarise(
  M.EW = sum(ReturnAdjGeneric)/length(ReturnAdjGeneric), M.VW = sum(LMCAP_return)/sum(LMCAP))

## 3: Calculate MSCI Index returns in NOK

# Extract last daily exchange rate each month
df_exchange_monthly <- df_exchange_daily %>% mutate(month =
floor_date(df_exchange_daily$TradeDate, "month")) %>%
  group_by(month) %>% arrange(TradeDate) %>%
  mutate(RateLast = tail(Rate, 1)) %>% ungroup() %>%
  select("month", "RateLast") %>% distinct() %>%
  rename(TradeDate = month)

# Merge exchange rate with MSCI price and compute simple MSCI World Index returns in NOK
df_msci_monthly <- merge(df_msci_monthly, df_exchange_monthly,  by = "TradeDate", all.x = TRUE,
all.y = FALSE)
df_msci_monthly <- df_msci_monthly %>% mutate(Close.NOK = Close.USD*RateLast) %>%
  arrange(TradeDate) %>% mutate(Close.NOK.L = lag(Close.NOK)) %>%
  mutate(MSCI = Close.NOK/Close.NOK.L -1) %>% select("TradeDate", "MSCI")

# Remove excess variables
rm(df_exchange_monthly, df_exchange_daily)



########## Section 6: Combine dataframes ##########



# Create df_market_daily
df_market_daily <- merge(dfma_daily, dfm_daily, by = "TradeDate", all.x = TRUE, all.y = FALSE)
df_market_daily <- merge(df_market_daily, df_rf_daily, by = "TradeDate", all.x = TRUE, all.y =
FALSE)
df_market_daily <- merge(df_market_daily, df_factors_daily, by = "TradeDate", all.x = TRUE, all.y
= FALSE)
df_market_daily$Rf <- na.locf(df_market_daily$Rf, fromLast = TRUE ) # Replace NA rf values with
next value

# Create df_market_monthly
df_market_monthly <- merge(dfma_monthly, dfm_monthly, by = "TradeDate", all.x = TRUE, all.y =
FALSE)
```

```
df_market_monthly <- merge(df_market_monthly, df_rf_monthly, by = "TradeDate", all.x = TRUE,
all.y = FALSE)
df_market_monthly <- merge(df_market_monthly, df_factors_monthly, by = "TradeDate", all.x = TRUE,
all.y = FALSE)
df_market_monthly <- merge(df_market_monthly, df_bab_monthly, by = "TradeDate", all.x = TRUE,
all.y = FALSE)
df_market_monthly <- merge(df_market_monthly, df_msci_monthly, by = "TradeDate", all.x = TRUE,
all.y = FALSE)
df_market_monthly <- merge(df_market_monthly, df_brent_monthly, by = "TradeDate", all.x = TRUE,
all.y = FALSE)

# Merge daily stock data with daily market data
df_daily <- merge(df_daily, df_rf_daily, by = "TradeDate", all.x = TRUE, all.y = FALSE)
df_daily <- merge(df_daily, dfma_daily, by = "TradeDate", all.x = TRUE, all.y = FALSE)
df_daily <- merge(df_daily, df_factors_daily, by = "TradeDate", all.x = TRUE, all.y = FALSE)
df_daily$Rf <- na.locf(df_daily$Rf, fromLast = TRUE ) # Replace NA rf values with next value

# Merge monthly stock data with monthly market data
df_monthly <- merge(df_monthly, df_rf_monthly, by = "TradeDate", all.x = TRUE, all.y = FALSE)
df_monthly <- merge(df_monthly, dfma_monthly, by = "TradeDate", all.x = TRUE, all.y = FALSE)
df_monthly <- merge(df_monthly, df_msci_monthly, by = "TradeDate", all.x = TRUE, all.y = FALSE)
df_monthly <- merge(df_monthly, df_brent_monthly, by = "TradeDate", all.x = TRUE, all.y = FALSE)

# Remove excess dataframes
rm(dfma_daily , dfm_daily, df_rf_daily, df_factors_daily)
rm(df_factors_monthly, df_rf_monthly, dfm_monthly, dfma_monthly, df_bab_monthly, df_msci_monthly)

# Extract needed variables from df_daily
df_daily  <- df_daily %>% select("TradeDate", "Year", "Month", "SecurityId", "SecurityName",
"SecurityTypeId",
  "ReturnAdjGeneric", "LogReturnAdjGeneric" , "Generic" ,
  "MCAP", "LMCAP", "OffShareTurnover" , "M.EW" , "M.VW" , "Rf", "SMB", "HML")

# Extract needed variables from df_monthly
df_monthly  <- df_monthly %>% select("TradeDate", "Year","SecurityId", "SecurityName",
"SecurityTypeId",
  "ReturnAdjGeneric", "LogReturnAdjGeneric" ,"Generic" ,
  "MCAP", "LMCAP", "OffShareTurnover" , "Trading_days_month", "M.EW" , "M.VW", "MSCI", "Rf",
"Brent")

# Calculate Market Returns in excess of the risk-free rate
df_daily <- df_daily %>% mutate(E.ReturnAdjGeneric = ReturnAdjGeneric - Rf) %>%
  mutate(E.M.EW = M.EW - Rf) %>% mutate(E.M.VW = M.VW - Rf)
df_market_daily <- df_market_daily %>% mutate(E.M.EW = M.EW - Rf) %>%
  mutate(E.M.VW = M.VW - Rf)
df_monthly <- df_monthly %>% mutate(E.ReturnAdjGeneric = ReturnAdjGeneric - Rf) %>%
  mutate(E.M.EW = M.EW - Rf) %>% mutate(E.M.VW = M.VW - Rf) %>% mutate(E.MSCI = MSCI - Rf)
df_market_monthly <- df_market_monthly %>% mutate(E.M.EW = M.EW - Rf) %>%
  mutate(E.M.VW = M.VW - Rf) %>% mutate(E.MSCI = MSCI - Rf)

# Create master dataframe
master <- df_monthly



##########  Section 7: Estimate Variables ##########



# Note that all variables are estimated at the end of month t and merged with dates for the month
t+1, hence in the master dataframe variable estimates for month t will have corresponding excess
returns for month t+1

# Create date vector (Will decide when first variables are calculated and must be specified)
date <- seq(as.Date("1985-01-01"), as.Date("2018-12-01"), by = "months")


### 1: beta.monthly: At the end of each month t, for each stock i, estimate market beta based on
60 months of monthly return observations

# Create empty dataframe
```

```r
beta_monthly <- data.frame(SecurityId = integer(), beta.monthly = numeric(), Tradedate =
as.Date(character()), stringsAsFactors = FALSE)

## Model
v <- 60 # Number of months with monthly returns used to calculate beta
w <- 36 # Number of valid return observations needed in the estimation period for a stock to be
included

for (i in 1:(length(date)-v)) {
  beta.tab <- df_monthly %>% filter(TradeDate >= date[i] & TradeDate < date[i+v]) %>%
    filter(Trading_days_month > 0) %>% group_by(SecurityId) %>% filter(n() >= w) %>%
    do(ols.model = lm(data = ., formula = E.ReturnAdjGeneric ~ E.M.VW)) %>% mutate(beta.monthly =
coef(ols.model)[2]) %>%
    select("SecurityId", "beta.monthly") %>% mutate(TradeDate = date[v+i])
  beta_monthly <- rbind(beta_monthly,beta.tab)}


### 2: beta.daily: At the end of each month t, for each stock i, estimate market beta based on 12
months of daily return observations

# Create empty dataframe
beta_daily <- data.frame(SecurityId = integer(), beta.daily = numeric(), Tradedate =
as.Date(character()), stringsAsFactors = FALSE)

## Model
n <- 12  # Number of months with daily returns used to calculate beta
m <- 200 # Number of valid return observations (observations with official turnover>0) needed in
the estimation period for stock to be included

for (i in 1:(length(date)-n)) {
  beta.tab <- df_daily %>% filter(TradeDate >= date[i] & TradeDate < date[i+n]) %>%
    filter(!is.na(OffShareTurnover)) %>% group_by(SecurityId) %>% filter(n() >= m) %>%
    do(ols.model = lm(data = ., formula = E.ReturnAdjGeneric ~ E.M.VW)) %>% mutate(beta.daily =
coef(ols.model)[2]) %>%
    select("SecurityId", "beta.daily") %>% mutate(TradeDate = date[n+i])
  beta_daily <- rbind(beta_daily,beta.tab)}


### 3: beta.msci: At the end of each month t, for each stock i, estimate market beta based on 60
months of monthly return observations using the MSCI World Index as market factor

# Create empty dataframe
beta_msci <- data.frame(SecurityId = integer(), beta.msci = numeric(), Tradedate =
as.Date(character()), stringsAsFactors = FALSE)

## Model
v <- 60 # Number of months with monthly returns used to calculate beta
w <- 36 # Number of valid return observations needed in the calculation period for stock to be
included

for (i in 1:(length(date)-v)) {
  beta.tab <- df_monthly %>% filter(TradeDate >= date[i] & TradeDate < date[i+v]) %>%
    filter(Trading_days_month > 0) %>% filter(!is.na(E.MSCI)) %>% group_by(SecurityId) %>%
    filter(n() >= w) %>%do(ols.model = lm(data = ., formula = E.ReturnAdjGeneric ~ E.MSCI)) %>%
    mutate(beta.msci = coef(ols.model)[2]) %>% select("SecurityId", "beta.msci") %>%
    mutate(TradeDate = date[v+i])
  beta_msci <- rbind(beta_msci,beta.tab)}


### 4: max: At the end of each month t, for each stock i, estimate max based on 1 month of daily
return observations

# Create empty  dataframe
max_daily <- data.frame(SecurityId = integer(), max = numeric(), Tradedate =
as.Date(character()), stringsAsFactors = FALSE)

## Model
v <- 1  # Number of months with daily returns used to calculate max
w <- 15 # Number of valid return observations needed in the estimation period for stock to be
included
```

```
for (i in 1:(length(date)-v)) {
  max.tab <- df_daily %>% filter(TradeDate >= date[i] & TradeDate < date[i+v]) %>%
    group_by(SecurityId) %>% filter(sum(!is.na(OffShareTurnover)) >= w) %>%
    summarise(max = mean(tail(sort(ReturnAdjGeneric),5))) %>% select("SecurityId", "max") %>%
    mutate(TradeDate = date[v+i])
  max_daily <- rbind(max_daily,max.tab)}


### 5: ivol and iskew: At the end of each month t, for each stock i, estimate ivol and iskew
based on 1 month of daily return observations

# Create empty dataframe
ivol_daily <- data.frame(SecurityId = integer(), ivol = numeric(),
  iskew = numeric(), Tradedate = as.Date(character()), stringsAsFactors = FALSE)

## Model
v <- 1 # Number of months with daily returns used to calculate ivol and iskew
w <- 15 # Number of valid return observations needed in the calculation period for stock to be
included

for (i in 1:(length(date)-v)) {
  ivol.tab <- df_daily %>% filter(TradeDate >= date[i] & TradeDate < date[i+v]) %>%
    filter(!is.na(OffShareTurnover)) %>% group_by(SecurityId) %>% filter(n() >= w) %>%
    do(ols.model = lm(data = ., formula = E.ReturnAdjGeneric ~ E.M.VW + SMB + HML)) %>%
    mutate(ivol = sd(residuals(ols.model))) %>% mutate(iskew = skewness(residuals(ols.model)))
%>%
    select("SecurityId", "ivol", "iskew") %>% mutate(TradeDate = date[v+i])
  ivol_daily <- rbind(ivol_daily,ivol.tab)}


### Merge variables with master dataframe
master <- merge(master, beta_daily, by = c("TradeDate", "SecurityId"), all.x = TRUE, all.y =
FALSE)
master <- merge(master, beta_monthly, by = c("TradeDate", "SecurityId"), all.x = TRUE, all.y =
FALSE)
master <- merge(master, beta_msci, by = c("TradeDate", "SecurityId"), all.x = TRUE, all.y =
FALSE)
master <- merge(master, ivol_daily, by = c("TradeDate", "SecurityId"), all.x = TRUE, all.y =
FALSE)
master <- merge(master, max_daily, by = c("TradeDate", "SecurityId"), all.x = TRUE, all.y =
FALSE)


### 6: obeta.monthly: At the end of each month t, for each stock i, estimate the portion of
beta.monthly orthogonal to max

# Calculate monthly o.alfa and o.beta from crossectional regressions
obeta.monthly.tab <- master %>% group_by(TradeDate) %>% filter(!is.na(beta.monthly)) %>%
  filter(!is.na(max)) %>% do(ols.model = lm(data = ., formula = beta.monthly ~ max)) %>%
  mutate(obeta.monthly.alfa = coef(ols.model)[1]) %>% mutate(obeta.monthly.beta =
coef(ols.model)[2]) %>%
  select("TradeDate", "obeta.monthly.alfa", "obeta.monthly.beta")

# Merge with master dataframe  and calculate obeta
master <- merge(master, obeta.monthly.tab, by = "TradeDate", all.x = TRUE)
master <- master %>% mutate(obeta.monthly = beta.monthly - (obeta.monthly.beta * max))


### 7: obeta.daily: At the end of each month t, for each stock i, estimate the portion of
beta.daily orthogonal to max

# Calculate monthly o.alfa and o.beta from crossectional regressions
obeta.daily.tab <- master %>% group_by(TradeDate) %>% filter(!is.na(beta.daily)) %>%
  filter(!is.na(max)) %>% do(ols.model = lm(data = ., formula = beta.daily ~ max)) %>%
  mutate(obeta.daily.alfa = coef(ols.model)[1]) %>% mutate(obeta.daily.beta = coef(ols.model)[2])
%>%
  select("TradeDate", "obeta.daily.alfa", "obeta.daily.beta")

# Merge with master dataframe and calculate obeta
master <- merge(master, obeta.daily.tab, by = "TradeDate", all.x = TRUE)
master <- master %>% mutate(obeta.daily = beta.daily - (obeta.daily.beta * max))
```

```
### 8: obmax: At the end of each month t, for each stock i, estimate the portion of max
orthogonal to beta.monthly

# Calculate monthly o.alfa and o.beta from crossectional regressions
obmax.tab <- master %>% group_by(TradeDate) %>% filter(!is.na(max)) %>%
  filter(!is.na(beta.monthly)) %>% do(ols.model = lm(data = ., formula = max ~ beta.monthly)) %>%
  mutate(obmax.alfa = coef(ols.model)[1]) %>% mutate(obmax.beta = coef(ols.model)[2]) %>%
  select("TradeDate", "obmax.alfa", "obmax.beta")

# Merge with master dataframe and calculate OBMAX
master <- merge(master, obmax.tab, by = "TradeDate", all.x = TRUE)
master <- master %>% mutate(obmax = max - (obmax.beta * beta.monthly))


### 9: Ibeta.monthly: At the end of each month t, for each stock i, estimate the portion of
beta.monthly orthogonal to ivol

# Calculate monthly o.alfa and o.beta from crossectional regressions
ibeta.monthly.tab <- master %>% group_by(TradeDate) %>% filter(!is.na(beta.monthly)) %>%
  filter(!is.na(ivol)) %>% do(ols.model = lm(data = ., formula = beta.monthly ~ ivol)) %>%
  mutate(ibeta.monthly.alfa = coef(ols.model)[1]) %>%mutate(ibeta.monthly.beta =
coef(ols.model)[2]) %>%
  select("TradeDate", "ibeta.monthly.alfa", "ibeta.monthly.beta")

# Merge with master dataframe  and calculate obeta
master <- merge(master, ibeta.monthly.tab, by = "TradeDate", all.x = TRUE)
master <- master %>% mutate(ibeta.monthly = beta.monthly - (ibeta.monthly.beta * max))

# Remove excess dataframes
rm(beta_daily, beta_monthly, beta_msci, max_daily, ivol_daily, beta_brent, beta.tab, max.tab,
ivol.tab ,brent.tab,
   omax.tab, obeta.daily.tab, obeta.monthly.tab, ibeta.monthly.tab, obmax.tab)

### Filter to use have estimates of all variables for same time period
master <- master %>%
  filter(TradeDate >= as.Date("1990-01-01") & TradeDate <= as.Date("2018-12-31") )



########## Section 8: Summary Statistics and Correlation: Variables ##########



## Create summary statistics for estimated variables
summary_statistics_variables <- master %>%
  select("beta.monthly", "beta.daily", "beta.msci" , "max", "ivol", "iskew",
  "obeta.monthly", "obeta.daily", "ibeta.monthly" ,"omax", "obmax") %>%
  describe( ., na.rm = TRUE, skew = FALSE, quant = c(0.25,0.75))

## Create correlation matrix with averages of monthly cross-sectional correlations(not same as
average correlation)
variable_cor <- master %>%
  select("TradeDate", "beta.monthly", "beta.daily", "beta.msci" , "max",
  "ivol", "iskew", "obeta.monthly", "obeta.daily", "ibeta.monthly" ,"omax", "obmax")
variable_cor <- variable_cor %>% group_by(TradeDate) %>%
  do(cormat = cor(select(., - matches("TradeDate")), use = "pairwise.complete.obs"))
variable_cor <- Reduce("+", variable_cor$cormat)/length(variable_cor$cormat)



########## Section 9: Sort stocks into portfolios ##########

# At the end of each month t, we sort all stocks into portfolios based on and ascending ordering
of the sorting variables
# In this section it is important to note what filters are applied to the different sorts as they
have implications for the double-sorts and factor constructions in the following sections


### Calculate additional variables needed to create portfolios
```

```r
# Note that all measures using mcap are computed using LMCAP.
master <- master %>% mutate(mcap_return = E.ReturnAdjGeneric * LMCAP) %>%
  mutate(mcap_beta_monthly = LMCAP * beta.monthly) %>% mutate(mcap_beta_daily = LMCAP *
beta.daily) %>%
  mutate(mcap_beta_msci = LMCAP * beta.msci) %>% mutate(mcap_max = LMCAP * max) %>%
  mutate(mcap_ivol = LMCAP * ivol)


#### 1: Sort into quintile portfolios based on sorting variables to be used in univariate
portfolio analysis

### Construct quintile portfolios sorted on an ascending ordering of beta.monthly
# Only observations with an estimate of beta.monthly are included in the sort
sort_bm <- master %>% group_by(TradeDate) %>% filter(!is.na(beta.monthly)) %>%
  mutate(Portfolio_b_m = ifelse(beta.monthly <= quantile(beta.monthly, 0.20, type = 3), 1,
  ifelse(beta.monthly > quantile(beta.monthly, 0.20, type = 3)
  & beta.monthly <= quantile(beta.monthly, 0.40, type = 3), 2,
  ifelse(beta.monthly > quantile(beta.monthly, 0.40, type = 3)
  & beta.monthly <= quantile(beta.monthly, 0.60, type = 3), 3,
  ifelse(beta.monthly > quantile(beta.monthly, 0.60, type = 3)
  & beta.monthly <= quantile(beta.monthly, 0.80, type = 3), 4,
  5))))) %>% ungroup() %>%   select(SecurityId, TradeDate, Portfolio_b_m)

### Construct quintile portfolios sorted on an ascending ordering of beta.daily
# Only observations with an estimate of beta.daily are included in the sort
sort_bd <- master %>% group_by(TradeDate) %>% filter(!is.na(beta.daily)) %>%
  mutate(Portfolio_b_d = ifelse(beta.daily <= quantile(beta.daily, 0.20, type = 3), 1,
  ifelse(beta.daily > quantile(beta.daily, 0.20, type = 3) & beta.daily <= quantile(beta.daily,
0.40, type = 3), 2,
  ifelse(beta.daily > quantile(beta.daily, 0.40, type = 3) & beta.daily <= quantile(beta.daily,
0.60, type = 3), 3,
  ifelse(beta.daily > quantile(beta.daily, 0.60, type = 3) & beta.daily <= quantile(beta.daily,
0.80, type = 3), 4,
  5))))) %>% ungroup() %>% select(SecurityId, TradeDate, Portfolio_b_d)

### Construct quintile portfolios sorted on an ascending ordering of beta.msci
# Only observations with an estimate of beta.msci are included in the sort
sort_msci <- master %>% group_by(TradeDate) %>% filter(!is.na(beta.msci)) %>%
  mutate(Portfolio_b_msci = ifelse(beta.msci <= quantile(beta.msci, 0.20, type = 3), 1,
  ifelse(beta.msci > quantile(beta.msci, 0.20, type = 3) & beta.msci <= quantile(beta.msci, 0.40,
type = 3), 2,
  ifelse(beta.msci > quantile(beta.msci, 0.40, type = 3) & beta.msci <= quantile(beta.msci, 0.60,
type = 3), 3,
  ifelse(beta.msci > quantile(beta.msci, 0.60, type = 3) & beta.msci <= quantile(beta.msci, 0.80,
type = 3), 4,
  5))))) %>% ungroup() %>% select(SecurityId, TradeDate, Portfolio_b_msci)

### Construct quintile portfolios sorted on an ascending ordering of max
# Only observations with an estimate of max are included in the sort
sort_max <- master %>% group_by(TradeDate) %>% filter(!is.na(max)) %>%
  mutate(Portfolio_max = ifelse(max <= quantile(max, 0.20, type = 3), 1,
  ifelse(max > quantile(max, 0.20, type = 3) & max <= quantile(max, 0.40, type = 3), 2,
  ifelse(max > quantile(max, 0.40, type = 3) & max <= quantile(max, 0.60, type = 3), 3,
  ifelse(max > quantile(max, 0.60, type = 3) & max <= quantile(max, 0.80, type = 3), 4,
  5))))) %>% ungroup() %>% select(SecurityId, TradeDate, Portfolio_max)

### Construct quintile portfolios sorted on an ascending ordering of ivol
# Only observations with an estimate of ivol are included in the sort
sort_ivol <- master %>% group_by(TradeDate) %>% filter(!is.na(ivol)) %>%
  mutate(Portfolio_ivol = ifelse(ivol <= quantile(ivol, 0.20, type = 3), 1,
  ifelse(ivol > quantile(ivol, 0.20, type = 3) & ivol <= quantile(ivol, 0.40, type = 3), 2,
  ifelse(ivol > quantile(ivol, 0.40, type = 3) & ivol <= quantile(ivol, 0.60, type = 3), 3,
  ifelse(ivol > quantile(ivol, 0.60, type = 3) & ivol <= quantile(ivol, 0.80, type = 3), 4,
  5))))) %>% ungroup() %>% select(SecurityId, TradeDate, Portfolio_ivol)


#### 2: Sort stocks into quintile portfolios based on orthogonal components to be used in
univariate portfolio analysis

### Construct quintile portfolios sorted on an ascending ordering of obeta.monthly
# Only observations with an estimate of obeta.monthly are included in the sort
```

```
sort_obeta_monthly <- master %>% group_by(TradeDate) %>% filter(!is.na(obeta.monthly)) %>%
  mutate(Portfolio_obeta_monthly = ifelse(obeta.monthly <= quantile(obeta.monthly, 0.20, type =
3), 1,
  ifelse(obeta.monthly > quantile(obeta.monthly, 0.20, type = 3) & obeta.monthly <=
quantile(obeta.monthly, 0.40, type = 3), 2,
  ifelse(obeta.monthly > quantile(obeta.monthly, 0.40, type = 3) & obeta.monthly <=
quantile(obeta.monthly, 0.60, type = 3), 3,
  ifelse(obeta.monthly > quantile(obeta.monthly, 0.60, type = 3) & obeta.monthly <=
quantile(obeta.monthly, 0.80, type = 3), 4,
  5))))) %>% ungroup() %>% select(SecurityId, TradeDate, Portfolio_obeta_monthly)

### Construct quintile portfolios sorted on an ascending ordering of obeta.daily
# Only observations with an estimate of obeta.daily are included in the sort
sort_obeta_daily <- master %>% group_by(TradeDate) %>% filter(!is.na(obeta.daily)) %>%
  mutate(Portfolio_obeta_daily = ifelse(obeta.daily <= quantile(obeta.daily, 0.20, type = 3), 1,
  ifelse(obeta.daily > quantile(obeta.daily, 0.20, type = 3) & obeta.daily <=
quantile(obeta.daily, 0.40, type = 3), 2,
  ifelse(obeta.daily > quantile(obeta.daily, 0.40, type = 3) & obeta.daily <=
quantile(obeta.daily, 0.60, type = 3), 3,
  ifelse(obeta.daily > quantile(obeta.daily, 0.60, type = 3) & obeta.daily <=
quantile(obeta.daily, 0.80, type = 3), 4,
  5))))) %>% ungroup() %>% select(SecurityId, TradeDate, Portfolio_obeta_daily)

### Construct quintile portfolios sorted on an ascending ordering of obmax
# Only observations with an estimate of obmax are included in the sort
sort_obmax <- master %>% group_by(TradeDate) %>% filter(!is.na(obmax)) %>%
  mutate(Portfolio_obmax = ifelse(obmax <= quantile(obmax, 0.20, type = 3), 1,
  ifelse(obmax > quantile(obmax, 0.20, type = 3) & obmax <= quantile(obmax, 0.40, type = 3), 2,
  ifelse(obmax > quantile(obmax, 0.40, type = 3) & obmax <= quantile(obmax, 0.60, type = 3), 3,
  ifelse(obmax > quantile(obmax, 0.60, type = 3) & obmax <= quantile(obmax, 0.80, type = 3), 4,
  5))))) %>% ungroup() %>% select(SecurityId, TradeDate, Portfolio_obmax)


### Construct quintile portfolios sorted on an ascending ordering of ibeta.monthly
# Only observations with an estimate of ibeta.monthly are included in the sort
sort_ibeta_monthly <- master %>% group_by(TradeDate) %>% filter(!is.na(ibeta.monthly)) %>%
  mutate(Portfolio_ibeta_monthly = ifelse(ibeta.monthly <= quantile(ibeta.monthly, 0.20, type =
3), 1,
  ifelse(ibeta.monthly > quantile(ibeta.monthly, 0.20, type = 3) & ibeta.monthly <=
quantile(ibeta.monthly, 0.40, type = 3), 2,
  ifelse(ibeta.monthly > quantile(ibeta.monthly, 0.40, type = 3) & ibeta.monthly <=
quantile(ibeta.monthly, 0.60, type = 3), 3,
  ifelse(ibeta.monthly > quantile(ibeta.monthly, 0.60, type = 3) & ibeta.monthly <=
quantile(ibeta.monthly, 0.80, type = 3), 4,
  5))))) %>% ungroup() %>% select(SecurityId, TradeDate, Portfolio_ibeta_monthly)


#### 3: Sorts into portfolios to be part of a conditional double-sort used for bivariate
portfolio analysis

### Construct quintile portfolios sorted on an ascending ordering of max to be used in a
conditional double sort on max then beta.monthly)
# Only observations with an estimate of both max and beta.monthly are included in the sort
sort_max2 <- master %>% group_by(TradeDate) %>% filter(!is.na(max)) %>%
  filter(!is.na(beta.monthly)) %>%
  mutate(Portfolio_max2 = ifelse(max <= quantile(max, 0.20, type = 3), 1,
  ifelse(max > quantile(max, 0.20, type = 3) & max <= quantile(max, 0.40, type = 3), 2,
  ifelse(max > quantile(max, 0.40, type = 3) & max <= quantile(max, 0.60, type = 3), 3,
  ifelse(max > quantile(max, 0.60, type = 3) & max <= quantile(max, 0.80, type = 3), 4,
  5))))) %>% ungroup() %>% select(SecurityId, TradeDate, Portfolio_max2)

### Construct quintile portfolios sorted on an ascending ordering of max to be used a in
conditional double sort on max then beta.daily)
# Only observations with an estimate of both max and beta.daily are included in the sort
sort_max3 <- master %>% group_by(TradeDate) %>% filter(!is.na(max)) %>%filter(!is.na(beta.daily))
%>%
  mutate(Portfolio_max3 = ifelse(max <= quantile(max, 0.20, type = 3), 1,
  ifelse(max > quantile(max, 0.20, type = 3) & max <= quantile(max, 0.40, type = 3), 2,
  ifelse(max > quantile(max, 0.40, type = 3) & max <= quantile(max, 0.60, type = 3), 3,
  ifelse(max > quantile(max, 0.60, type = 3) & max <= quantile(max, 0.80, type = 3), 4,
  5))))) %>% ungroup() %>% select(SecurityId, TradeDate, Portfolio_max3)
```

```r
### Construct quintile portfolios sorted on an ascending ordering of beta.monthly to be used in
conditional double-sort on beta.monthly then max
# Only observations with an estimate of both max and beta.monthly are included in the sort
sort_mb <- master %>% group_by(TradeDate) %>% filter(!is.na(beta.monthly)) %>%filter(!is.na(max))
%>%
  mutate(Portfolio_mb = ifelse(beta.monthly <= quantile(beta.monthly, 0.20, type = 3), 1,
  ifelse(beta.monthly > quantile(beta.monthly, 0.20, type = 3) & beta.monthly <=
quantile(beta.monthly, 0.40, type = 3), 2,
  ifelse(beta.monthly > quantile(beta.monthly, 0.40, type = 3) & beta.monthly <=
quantile(beta.monthly, 0.60, type = 3), 3,
  ifelse(beta.monthly > quantile(beta.monthly, 0.60, type = 3) & beta.monthly <= quantile(max,
0.80, type = 3), 4,
  5))))) %>% ungroup() %>% select(SecurityId, TradeDate, Portfolio_mb)

### Construct quintile portfolios sorted on an ascending ordering of ivol to be used in
conditional double-sort on ivol then beta monthly
# Only observations with an estimate of both ivol and beta.monthly are included in the sort
sort_ivol2 <- master %>% group_by(TradeDate) %>% filter(!is.na(ivol)) %>%
filter(!is.na(beta.monthly)) %>%
  mutate(Portfolio_ivol2 = ifelse(ivol <= quantile(ivol, 0.20, type = 3), 1,
  ifelse(ivol > quantile(ivol, 0.20, type = 3) & ivol <= quantile(ivol, 0.40, type = 3), 2,
  ifelse(ivol > quantile(ivol, 0.40, type = 3) & ivol <= quantile(ivol, 0.60, type = 3), 3,
  ifelse(ivol > quantile(ivol, 0.60, type = 3) & ivol <= quantile(ivol, 0.80, type = 3), 4,
  5))))) %>% ungroup() %>% select(SecurityId, TradeDate, Portfolio_ivol2)


#### 4: Sorts into portfolios to be used to be used in factor construction

### Construct two equal-sized portfolios sorted on an ascending ordering of LMCAP for max and
ivol factor calculations
# Same data filters applied for estimates of max and ivol, implies that all estimates of max will
have a corresponding estimate of ivol and the same sort on LMCAP can be used for both factor
constructions
# Only observations with an estimate of max/ivol and LMCAP are included in the sort
sort_LMCAP <- master %>% group_by(TradeDate) %>% filter(!is.na(LMCAP)) %>% filter(LMCAP > 0) %>%
  filter(!is.na(max)) %>% mutate(Portfolio_LMCAP = ifelse(LMCAP <= quantile(LMCAP, 0.50, type =
3), 1,2)) %>%
  ungroup() %>% select(SecurityId, TradeDate, Portfolio_LMCAP)

### Construct two equal-sized portfolios sorted on an ascending ordering of LMCAP for
beta.monthly factor calculations
# Only observations with an estimate of beta.monthly and LMCAP are included in the sort
sort_LMCAP2 <- master %>% group_by(TradeDate) %>% filter(!is.na(LMCAP)) %>% filter(LMCAP > 0) %>%
  filter(!is.na(beta.monthly)) %>% mutate(Portfolio_LMCAP2 = ifelse(LMCAP <= quantile(LMCAP,
0.50, type = 3), 1,2)) %>%
  ungroup() %>% select(SecurityId, TradeDate, Portfolio_LMCAP2)


### Merge portoflio sorts with master dataframe.
master <- merge(master, sort_bm, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_bd, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_msci, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_max, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_ivol, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_obeta_daily, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_obeta_monthly, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_ibeta_monthly, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_LMCAP, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_LMCAP2, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_max2, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_max3, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_ivol2, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_omax, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_mb, by = c("TradeDate", "SecurityId"), all.x = TRUE)
master <- merge(master, sort_obmax, by = c("TradeDate", "SecurityId"), all.x = TRUE)


# Remove excess dataframes
rm(sort_bm, sort_bd, sort_msci, sort_max, sort_ivol, sort_obeta_daily, sort_obeta_monthly,
sort_ibeta_monthly,
```

```
  sort_LMCAP, sort_max2, sort_ivol2, sort_omax, sort_LMCAP2, sort_max3, sort_mb, sort_obmax)

########## Section 9: Construct Factor Returns ##########

#### 1: Construct FMAX factor

# Sort stocks into three portfolios based on an ascending ordering of MAX
# Only observations that have been previously sorted into an LMCAP portfolio are included
sort_fmax_0 <- master %>% group_by(TradeDate) %>% filter(!is.na(Portfolio_LMCAP)) %>%
  mutate(Portfolio_fmax_0 = ifelse(max <= quantile(max, 0.30, type = 3), 1,
  ifelse(max > quantile(max, 0.30, type = 3) & max <= quantile(max, 0.70, type = 3), 2,3))) %>%
  ungroup() %>% select(SecurityId, TradeDate, Portfolio_fmax_0)
master <- merge(master, sort_fmax_0, by = c("TradeDate", "SecurityId"), all.x = TRUE)
rm(sort_fmax_0)

# Create the four portfolios formed by the intersections between the low and high max portfolios
and the two LMCAP portfolios
sort_fmax_1 <- master %>%
  mutate(Portfolio_fmax = ifelse(Portfolio_LMCAP == 1 & Portfolio_fmax_0 == 1, 11,
                           ifelse(Portfolio_LMCAP == 2 & Portfolio_fmax_0 == 1, 21,
                           ifelse(Portfolio_LMCAP == 1 & Portfolio_fmax_0 == 3, 13,
                           ifelse(Portfolio_LMCAP == 2 & Portfolio_fmax_0 == 3, 23,
                           NA))))) %>% ungroup() %>% select(SecurityId, TradeDate,
Portfolio_fmax)
master <- merge(master, sort_fmax_1, by = c("TradeDate", "SecurityId"), all.x = TRUE)


# Calculate the VW monthly returns for the 4 portfolios created by the intersections
fmax_factor <- master %>% filter(!is.na(Portfolio_fmax)) %>% group_by(TradeDate, Portfolio_fmax)
%>%
  summarise(return_VW = sum(mcap_return)/sum(LMCAP)) %>% rename(portfolio = Portfolio_fmax)

# Compute the monthly FMAX factor returns equal to the average returns of the two low-max
portfolios less the average returns of the two high-max portfolios
fmax_factor <- fmax_factor %>% spread(key = portfolio, value = return_VW) %>%
  mutate(FMAX = (`11`+`21`)/2 - (`13`+`23`)/2 ) %>% select(TradeDate, FMAX)

# Remove excess dataframes
rm(sort_fmax_1)

##### 2: Construct FIVOL factor

# Sort stocks into three portfolios based on an ascending ordering of ivol
# Only observations that have been previously sorted into an LMCAP portfolio are included
sort_fivol_0 <- master %>% group_by(TradeDate) %>% filter(!is.na(Portfolio_LMCAP)) %>%
  mutate(Portfolio_fivol_0 = ifelse(ivol <= quantile(ivol, 0.30, type = 3), 1,
                             ifelse(ivol > quantile(ivol, 0.30, type = 3) & ivol <=
quantile(ivol, 0.70, type = 3), 2,
                             3))) %>% ungroup() %>% select(SecurityId, TradeDate,
Portfolio_fivol_0)
master <- merge(master, sort_fivol_0, by = c("TradeDate", "SecurityId"), all.x = TRUE)
rm(sort_fivol_0)

# Create the four portfolios formed by intersections between the low and high ivol portfolios and
the two LMCAP portfolios
sort_fivol_1 <- master %>%
  mutate(Portfolio_fivol = ifelse(Portfolio_LMCAP == 1 & Portfolio_fivol_0 == 1, 11,
                           ifelse(Portfolio_LMCAP == 2 & Portfolio_fivol_0 == 1, 21,
                           ifelse(Portfolio_LMCAP == 1 & Portfolio_fivol_0 == 3, 13,
                           ifelse(Portfolio_LMCAP == 2 & Portfolio_fivol_0 == 3, 23,
                           NA))))) %>% ungroup() %>% select(SecurityId, TradeDate,
Portfolio_fivol)
master <- merge(master, sort_fivol_1, by = c("TradeDate", "SecurityId"), all.x = TRUE)

# Calculate the VW monthly returns for the 4 portfolios created by the intersections
fivol_factor <- master %>% filter(!is.na(Portfolio_fivol)) %>% group_by(TradeDate,
Portfolio_fivol) %>%
  summarise(return_VW = sum(mcap_return)/sum(LMCAP)) %>% rename(portfolio = Portfolio_fivol)
```

```
# Compute the monthly FIVOL factor returns equal to the average returns of the two low-ivol
portfolios less the average returns of the two high-ivol portfolios
fivol_factor <- fivol_factor %>% spread(key = portfolio, value = return_VW) %>%
  mutate(FIVOL = (`11`+`21`)/2 - (`13`+`23`)/2 ) %>% select(TradeDate, FIVOL)

# Remove excess dataframes
rm(sort_fivol_1)


##### 3: Create FBETA factor

# Sort the stocks into three portfolios based on an ascending ordering of beta.monthly
# Only observations that have been previously sorted into an LMCAP2 portfolio are included
sort_fbeta_0 <- master %>% group_by(TradeDate) %>%
  filter(!is.na(Portfolio_LMCAP2)) %>%
  mutate(Portfolio_fbeta_0 = ifelse(beta.monthly <= quantile(beta.monthly, 0.30, type = 3), 1,
                             ifelse(beta.monthly > quantile(beta.monthly, 0.30, type = 3) &
beta.monthly <= quantile(beta.monthly, 0.70, type = 3), 2,
                             3))) %>% ungroup() %>% select(SecurityId, TradeDate,
Portfolio_fbeta_0)
master <- merge(master, sort_fbeta_0, by = c("TradeDate", "SecurityId"), all.x = TRUE)
rm(sort_fbeta_0)

# Create the four portfolios formed by the intersections between the low and high beta.monthly
portfolios and the two LMCAP2 portfolios
sort_fbeta_1 <- master %>%
  mutate(Portfolio_fbeta = ifelse(Portfolio_LMCAP2 == 1 & Portfolio_fbeta_0 == 1, 11,
                           ifelse(Portfolio_LMCAP2 == 2 & Portfolio_fbeta_0 == 1, 21,
                           ifelse(Portfolio_LMCAP2 == 1 & Portfolio_fbeta_0 == 3, 13,
                           ifelse(Portfolio_LMCAP2 == 2 & Portfolio_fbeta_0 == 3, 23,
                           NA))))) %>% ungroup() %>% select(SecurityId, TradeDate,
Portfolio_fbeta)
master <- merge(master, sort_fbeta_1, by = c("TradeDate", "SecurityId"), all.x = TRUE)


# Calculate the VW returns for the 4 portfolios created by the intersections
fbeta_factor <- master %>% filter(!is.na(Portfolio_fbeta)) %>% group_by(TradeDate,
Portfolio_fbeta) %>%
  summarise(return_VW = sum(mcap_return)/sum(LMCAP)) %>% rename(portfolio = Portfolio_fbeta)

# Compute the monthly FBETA factor returns equal to the average returns of the two low-
beta.monthly portfolios less the average returns of the two high-beta.monthly portfolios
fbeta_factor <- fbeta_factor %>% spread(key = portfolio, value = return_VW) %>%
  mutate(FBETA = (`11`+`21`)/2 - (`13`+`23`)/2 ) %>% select(TradeDate, FBETA)

# Remove excess dataframes
rm(sort_fbeta_1)



##########  Section 10: Perform Conditional Double-sorts ##########



# Note that the conditional double sorts depend on correct filtering of observations in the
primary sorts performed in section 8.
# In this section we create 5 portfolios representing the sum of stocks in the control variable
quintile portfolios within each quintile of the sorting variable of interest
# When we later compute the portfolio returns, we will estimate the performance of the average
control variable quintile portfolio within each quintile of the variable of interest


### 1: Perform a conditional double-sort on max then beta.monthly
## Create 5 portfolios representing the sum of stocks in the max quintile portfolios within each
beta.monthly quintile

# Create BNM portfolios
portfolio_bnm <- master %>%filter(!is.na(Portfolio_max2)) %>%
  group_by(Portfolio_max2, TradeDate) %>%
    mutate(Portfolio_BNM = ifelse(beta.monthly <= quantile(beta.monthly, 0.20, type = 3), 1,
```

```
    ifelse(beta.monthly > quantile(beta.monthly, 0.20, type = 3) & beta.monthly <=
quantile(beta.monthly, 0.40, type = 3), 2,
    ifelse(beta.monthly > quantile(beta.monthly, 0.40, type = 3) & beta.monthly <=
quantile(beta.monthly, 0.60, type = 3), 3,
    ifelse(beta.monthly > quantile(beta.monthly, 0.60, type = 3) & beta.monthly <=
quantile(beta.monthly, 0.80, type = 3), 4,
    5))))) %>% ungroup() %>%select(SecurityId, TradeDate, Portfolio_BNM)

# Merge with master dataframe
master <- merge(master, portfolio_bnm, by = c("TradeDate", "SecurityId"), all.x = TRUE)

# Remove excess dataframes
rm(portfolio_bnm)


### 2: Perform a conditional double-sort on max then beta.daily
## Create 5 portfolios representing the sum of stocks in the max quintile portfolios within each
beta.daily quintile

# Create BNMD portfolios
portfolio_bnmd <- master %>%
  filter(!is.na(Portfolio_max3)) %>%
  group_by(Portfolio_max3, TradeDate) %>%
  mutate(Portfolio_BNMD = ifelse(beta.daily <= quantile(beta.daily, 0.20, type = 3), 1,
  ifelse(beta.daily > quantile(beta.daily, 0.20, type = 3) & beta.daily <= quantile(beta.daily,
0.40, type = 3), 2,
  ifelse(beta.daily > quantile(beta.daily, 0.40, type = 3) & beta.daily <= quantile(beta.daily,
0.60, type = 3), 3,
  ifelse(beta.daily > quantile(beta.daily, 0.60, type = 3) & beta.daily <= quantile(beta.daily,
0.80, type = 3), 4,
  5))))) %>% ungroup() %>% select(SecurityId, TradeDate, Portfolio_BNMD)

# Merge with master dataframe
master <- merge(master, portfolio_bnmd, by = c("TradeDate", "SecurityId"), all.x = TRUE)

# Remove excess dataframes
rm(portfolio_bnmd)


### 3: Perform a conditional double-sort on beta.monthly then max
## Create 5 portfolios representing the sum of stocks in the beta.monthly quintile portfolios
within each max quintile

# Create MNB portfolios
portfolio_mnb <- master %>%
  filter(!is.na(Portfolio_mb)) %>%
  group_by(Portfolio_mb, TradeDate) %>%
  mutate(Portfolio_MNB = ifelse(max <= quantile(max, 0.20, type = 3), 1,
  ifelse(max > quantile(max, 0.20, type = 3) & max <= quantile(max, 0.40, type = 3), 2,
  ifelse(max > quantile(max, 0.40, type = 3) & max <= quantile(max, 0.60, type = 3), 3,
  ifelse(max > quantile(max, 0.60, type = 3) & max <= quantile(max, 0.80, type = 3), 4,
  5))))) %>% ungroup() %>% select(SecurityId, TradeDate, Portfolio_MNB)

# Merge with master dataframe
master <- merge(master, portfolio_mnb, by = c("TradeDate", "SecurityId"), all.x = TRUE)

# Remove excess dataframes
rm(portfolio_mnb)


### 4: Perform a conditional double-sort on ivol then beta.monthly
## Create 5 portfolios representing the sum of stocks in the ivol quintile portfolios withing
each beta.monthly quintile

# Create BNI portfolios
portfolio_bni <- master %>%
  filter(!is.na(Portfolio_ivol2 )) %>%
  group_by(Portfolio_ivol2, TradeDate) %>%
  mutate(Portfolio_BNI = ifelse(beta.monthly <= quantile(beta.monthly, 0.20, type = 3), 1,
  ifelse(beta.monthly > quantile(beta.monthly, 0.20, type = 3) & beta.monthly <=
quantile(beta.monthly, 0.40, type = 3), 2,
```

```
  ifelse(beta.monthly > quantile(beta.monthly, 0.40, type = 3) & beta.monthly <=
quantile(beta.monthly, 0.60, type = 3), 3,
  ifelse(beta.monthly > quantile(beta.monthly, 0.60, type = 3) & beta.monthly <=
quantile(beta.monthly, 0.80, type = 3), 4,
  5))))) %>% ungroup() %>%select(SecurityId, TradeDate, Portfolio_BNI)

# Merge with master dataframe
master <- merge(master, portfolio_bni, by = c("TradeDate", "SecurityId"), all.x = TRUE)

# Remove excess dataframes
rm(portfolio_bni)



########## Section 11: Compute monthly and timer-series average portfolio excess returns, sharpe
ratios and portfolio characteristics ##########



#### 1: Quintile portfolios sorted on beta.monthly

### Calculate monthly VW (EW) portfolio excess returns and portfolio characteristics
portfolio_month <- master %>% filter(!is.na(Portfolio_b_m)) %>% group_by(TradeDate,
Portfolio_b_m) %>%
  summarise(length = length(E.ReturnAdjGeneric), mcap = sum(LMCAP, na.rm = TRUE),
  er_month_ew = sum(E.ReturnAdjGeneric)/length(E.ReturnAdjGeneric), beta_monthly_ew =
mean(beta.monthly, na.rm = TRUE),
  beta_daily_ew = mean(beta.daily, na.rm = TRUE), max_ew = mean(max, na.rm = TRUE), er_month_vw =
sum(mcap_return)/sum(LMCAP),
  beta_monthly_vw = sum(mcap_beta_monthly, na.rm = TRUE)/sum(LMCAP), beta_daily_vw =
sum(mcap_beta_daily, na.rm = TRUE)/sum(LMCAP),
  max_vw = sum(mcap_max, na.rm = TRUE)/sum(LMCAP))

# Compute the metrics for the low-high portfolio (portfolio 6)
portfolio_month <- portfolio_month %>% group_by(TradeDate) %>%arrange(Portfolio_b_m) %>%
  summarise(Portfolio_b_m = 6, length = 0, mcap = first(mcap)-last(mcap), er_month_ew =
first(er_month_ew)-last(er_month_ew),
  beta_monthly_ew = first(beta_monthly_ew)-last(beta_monthly_ew), beta_daily_ew =
first(beta_daily_ew)-last(beta_daily_ew),
  max_ew = first(max_ew)-last(max_ew), er_month_vw = first(er_month_vw)-last(er_month_vw),
  beta_monthly_vw = first(beta_monthly_vw)-last(beta_monthly_vw), beta_daily_vw =
first(beta_daily_vw)-last(beta_daily_vw),
  max_vw = first(max_vw)-last(max_vw)) %>% bind_rows(portfolio_month, .) %>% arrange(TradeDate)
%>% rename(portfolio = Portfolio_b_m)

# Compute time-series averages
portfolio_month_c <- portfolio_month %>% group_by(portfolio) %>% summarise(length =
round(mean(length)),
  mcap = mean(mcap), beta_monthly_ew = mean(beta_monthly_ew), beta_daily_ew = mean(beta_daily_ew,
na.rm = TRUE),
  max_ew = mean(max_ew, na.rm = TRUE), sd_month_ew = sd(er_month_ew),
  er_month_ew = mean(er_month_ew), sr_month_ew = er_month_ew/sd_month_ew, beta_monthly_vw =
mean(beta_monthly_vw, na.rm = TRUE),
  beta_daily_vw = mean(beta_daily_vw, na.rm = TRUE), max_vw = mean(max_vw), sd_month_vw =
sd(er_month_vw, na.rm = TRUE),
  er_month_vw = mean(er_month_vw), sr_month_vw = er_month_vw/sd_month_vw) %>% t()


#### 2: Quintile portfolios sorted on beta.daily

### Calculate monthly VW (EW) portfolio excess returns and portfolio characteristics
portfolio_day <- master %>% filter(!is.na(Portfolio_b_d)) %>% group_by(TradeDate, Portfolio_b_d)
%>%
  summarise(length = length(E.ReturnAdjGeneric),mcap = sum(LMCAP, na.rm = TRUE),
  er_day_ew = sum(E.ReturnAdjGeneric)/length(E.ReturnAdjGeneric),beta_monthly_ew =
mean(beta.monthly, na.rm = TRUE),
  beta_daily_ew = mean(beta.daily, na.rm = TRUE),max_ew = mean(max, na.rm = TRUE),
  er_day_vw = sum(mcap_return)/sum(LMCAP),beta_monthly_vw = sum(mcap_beta_monthly, na.rm =
TRUE)/sum(LMCAP),
  beta_daily_vw = sum(mcap_beta_daily, na.rm = TRUE)/sum(LMCAP),max_vw = sum(mcap_max, na.rm =
TRUE)/sum(LMCAP))
```

```
# Compute the metrics for the low-high portfolio (portfolio 6)
portfolio_day <- portfolio_day %>% group_by(TradeDate) %>% arrange(Portfolio_b_d) %>%
summarise(Portfolio_b_d = 6,
  length = 0,mcap = first(mcap)-last(mcap),er_day_ew = first(er_day_ew)-last(er_day_ew),
  beta_monthly_ew = first(beta_monthly_ew)-last(beta_monthly_ew),beta_daily_ew =
first(beta_daily_ew)-last(beta_daily_ew),
  max_ew = first(max_ew)-last(max_ew),er_day_vw = first(er_day_vw)-
last(er_day_vw),beta_monthly_vw = first(beta_monthly_vw)-last(beta_monthly_vw),
  beta_daily_vw = first(beta_daily_vw)-last(beta_daily_vw), max_vw = first(max_vw)-last(max_vw))
%>%
  bind_rows(portfolio_day, .) %>% arrange(TradeDate) %>% rename(portfolio = Portfolio_b_d)

# Compute time-series averages
portfolio_day_c <- portfolio_day %>% group_by(portfolio) %>% summarise(length =
round(mean(length)),
  mcap = mean(mcap),beta_monthly_ew = mean(beta_monthly_ew),beta_daily_ew = mean(beta_daily_ew),
  max_ew = mean(max_ew), sd_day_ew = sd(er_day_ew),er_day_ew = mean(er_day_ew),sr_day_ew =
er_day_ew/sd_day_ew,
  beta_monthly_vw = mean(beta_monthly_vw),beta_daily_vw = mean(beta_daily_vw),max_vw =
mean(max_vw),
  sd_day_vw = sd(er_day_vw),er_day_vw = mean(er_day_vw),sr_day_vw = er_day_vw/sd_day_vw) %>% t()


#### 3: Quintile portfolios sorted on beta.msci

### Calculate monthly VW (EW) portfolio excess returns and portfolio characteristics
portfolio_msci <- master %>% filter(!is.na(Portfolio_b_msci)) %>% group_by(TradeDate,
Portfolio_b_msci) %>%
  summarise(length = length(E.ReturnAdjGeneric),mcap = sum(LMCAP, na.rm = TRUE),
  er_msci_ew = sum(E.ReturnAdjGeneric)/length(E.ReturnAdjGeneric), beta_monthly_ew =
mean(beta.monthly, na.rm = TRUE),
  beta_msci_ew = mean(beta.msci, na.rm = TRUE), max_ew = mean(max, na.rm = TRUE),
  er_msci_vw = sum(mcap_return)/sum(LMCAP), beta_monthly_vw = sum(mcap_beta_monthly, na.rm =
TRUE)/sum(LMCAP),
  beta_msci_vw = sum(mcap_beta_msci, na.rm = TRUE)/sum(LMCAP), max_vw = sum(mcap_max, na.rm =
TRUE)/sum(LMCAP))

# Compute the metrics for the low-high portfolio (portfolio 6)
portfolio_msci <- portfolio_msci %>% group_by(TradeDate) %>% arrange(Portfolio_b_msci) %>%
  summarise(Portfolio_b_msci = 6, length = 0, mcap = first(mcap)-last(mcap),er_msci_ew =
first(er_msci_ew)-last(er_msci_ew),
  beta_monthly_ew = first(beta_monthly_ew)-last(beta_monthly_ew), beta_msci_ew =
first(beta_msci_ew)-last(beta_msci_ew),
  max_ew = first(max_ew)-last(max_ew), er_msci_vw = first(er_msci_vw)-last(er_msci_vw),
  beta_monthly_vw = first(beta_monthly_vw)-last(beta_monthly_vw),beta_msci_vw =
first(beta_msci_vw)-last(beta_msci_vw),
  max_vw = first(max_vw)-last(max_vw)) %>% bind_rows(portfolio_msci, .) %>% arrange(TradeDate)
%>% rename(portfolio = Portfolio_b_msci)

# Compute time-series averages
portfolio_msci_c <- portfolio_msci %>% group_by(portfolio) %>%
  summarise(length = round(mean(length)), mcap = mean(mcap), beta_monthly_ew =
mean(beta_monthly_ew),
  beta_msci_ew = mean(beta_msci_ew), max_ew = mean(max_ew, na.rm = TRUE),
  sd_msci_ew = sd(er_msci_ew), er_msci_ew = mean(er_msci_ew), sr_msci_ew = er_msci_ew/sd_msci_ew,
  beta_monthly_vw = mean(beta_monthly_vw), beta_msci_vw = mean(beta_msci_vw),
  max_vw = mean(max_vw, na.rm = TRUE), sd_msci_vw = sd(er_msci_vw),
  er_msci_vw = mean(er_msci_vw), sr_msci_vw = er_msci_vw/sd_msci_vw) %>% t()


#### 4: Quintile portfolios sorted on max

### Calculate monthly VW (EW) portfolio excess returns and portfolio characteristics
portfolio_max <- master %>% filter(!is.na(Portfolio_max)) %>%
  group_by(TradeDate, Portfolio_max) %>% summarise(length = length(E.ReturnAdjGeneric),
  mcap = sum(LMCAP, na.rm = TRUE), er_max_ew =
sum(E.ReturnAdjGeneric)/length(E.ReturnAdjGeneric),
  beta_monthly_ew = mean(beta.monthly, na.rm = TRUE), max_ew = mean(max, na.rm = TRUE),
  er_max_vw = sum(mcap_return)/sum(LMCAP), beta_monthly_vw = sum(mcap_beta_monthly, na.rm =
TRUE)/sum(LMCAP),
```

```r
  max_vw = sum(mcap_max, na.rm = TRUE)/sum(LMCAP), price = mean(Generic, na.rm = TRUE),
  ivol = mean(ivol, na.rm = TRUE), iskew = mean(iskew, na.rm = TRUE))

# Compute the metrics for the low-high portfolio (portfolio 6)
portfolio_max <- portfolio_max %>% group_by(TradeDate) %>% arrange(Portfolio_max) %>%
  summarise(Portfolio_max = 6, length = 0, mcap = first(mcap)-last(mcap),
  er_max_ew = first(er_max_ew)-last(er_max_ew), beta_monthly_ew = first(beta_monthly_ew)-
last(beta_monthly_ew),
  max_ew = first(max_ew)-last(max_ew), er_max_vw = first(er_max_vw)-last(er_max_vw),
  beta_monthly_vw = first(beta_monthly_vw)-last(beta_monthly_vw), max_vw = first(max_vw)-
last(max_vw),
  price = first(price) - last(price), ivol = first(ivol)-last(ivol), iskew = first(iskew)-
last(iskew)) %>%
  bind_rows(portfolio_max, .) %>% arrange(TradeDate) %>% rename(portfolio = Portfolio_max)

# Compute time-series averages
portfolio_max_c <- portfolio_max %>% group_by(portfolio) %>%
  summarise(length = round(mean(length)),mcap = mean(mcap),
  beta_monthly_ew = mean(beta_monthly_ew), max_ew = mean(max_ew), sd_max_ew = sd(er_max_ew),
  er_max_ew = mean(er_max_ew), sr_max_ew = er_max_ew/sd_max_ew, beta_monthly_vw =
mean(beta_monthly_vw),
  max_vw = mean(max_vw), sd_max_vw = sd(er_max_vw), er_max_vw = mean(er_max_vw),
  sr_max_vw = er_max_vw/sd_max_vw, price = mean(price),ivol = mean(ivol), iskew= mean(iskew)) %>%
t()


#### 5. Quintile portfolios representing the sum of stocks in the quintile max portfolios within
each beta.monthly quintile portfolio

### Calculate monthly VW (EW) portfolio excess returns and portfolio characteristics
portfolio_BNM <- master %>% filter(!is.na(Portfolio_BNM)) %>% group_by(TradeDate, Portfolio_BNM)
%>%
  summarise(length = length(E.ReturnAdjGeneric), mcap = sum(LMCAP, na.rm = TRUE),
  er_BNM_monthly_ew = sum(E.ReturnAdjGeneric)/length(E.ReturnAdjGeneric),beta_monthly_ew =
mean(beta.monthly, na.rm = TRUE),
  max_ew = mean(max, na.rm = TRUE), er_BNM_monthly_vw = sum(mcap_return)/sum(LMCAP),
  beta_monthly_vw = sum(mcap_beta_monthly, na.rm = TRUE)/sum(LMCAP), max_vw = sum(mcap_max, na.rm
= TRUE)/sum(LMCAP))

# Compute the metrics for the low-high portfolio (portfolio 6)
portfolio_BNM <- portfolio_BNM %>% group_by(TradeDate) %>% arrange(Portfolio_BNM) %>%
  summarise(Portfolio_BNM = 6,
  length = 0, mcap = first(mcap)-last(mcap), er_BNM_monthly_ew = first(er_BNM_monthly_ew)-
last(er_BNM_monthly_ew),
  beta_monthly_ew = first(beta_monthly_ew)-last(beta_monthly_ew),max_ew = first(max_ew)-
last(max_ew),
  er_BNM_monthly_vw = first(er_BNM_monthly_vw)-last(er_BNM_monthly_vw), beta_monthly_vw =
first(beta_monthly_vw)-last(beta_monthly_vw),
  max_vw = first(max_vw)-last(max_vw)) %>% bind_rows(portfolio_BNM, .) %>% arrange(TradeDate) %>%
rename(portfolio = Portfolio_BNM)

# Compute time-series averages
portfolio_BNM_c <- portfolio_BNM %>% group_by(portfolio) %>% summarise(length = mean(length),
  mcap = mean(mcap),beta_monthly_ew = mean(beta_monthly_ew),max_ew = mean(max_ew, na.rm = TRUE),
  sd_BNM_monthly_ew = sd(er_BNM_monthly_ew), er_BNM_monthly_ew = mean(er_BNM_monthly_ew),
  sr_BNM_monthly_ew = er_BNM_monthly_ew/sd_BNM_monthly_ew, beta_monthly_vw =
mean(beta_monthly_vw, na.rm = TRUE),
  max_vw = mean(max_vw),sd_BNM_monthly_vw = sd(er_BNM_monthly_vw),
  er_BNM_monthly_vw = mean(er_BNM_monthly_vw), sr_BNM_monthly_vw =
er_BNM_monthly_vw/sd_BNM_monthly_vw) %>% t()


#### 6: Quintile portfolios sorted on the component of beta.monthly orthogonal to max

### Calculate monthly VW (EW) portfolio excess returns and portfolio characteristics
portfolio_obeta_monthly <- master %>% filter(!is.na(Portfolio_obeta_monthly)) %>%
  group_by(TradeDate, Portfolio_obeta_monthly) %>% summarise(length = length(E.ReturnAdjGeneric),
  mcap = sum(LMCAP, na.rm = TRUE), er_obeta_monthly_ew =
sum(E.ReturnAdjGeneric)/length(E.ReturnAdjGeneric),
  beta_monthly_ew = mean(beta.monthly, na.rm = TRUE), max_ew = mean(max, na.rm = TRUE),
```

```r
  er_obeta_monthly_vw = sum(mcap_return)/sum(LMCAP), beta_monthly_vw = sum(mcap_beta_monthly,
na.rm = TRUE)/sum(LMCAP),
  max_vw = sum(mcap_max, na.rm = TRUE)/sum(LMCAP))

# Compute the metrics for the low-high portfolio (portfolio 6)
portfolio_obeta_monthly <- portfolio_obeta_monthly %>% group_by(TradeDate)
%>%arrange(Portfolio_obeta_monthly) %>%
  summarise(Portfolio_obeta_monthly = 6,length = 0,
  mcap = first(mcap)-last(mcap),er_obeta_monthly_ew = first(er_obeta_monthly_ew)-
last(er_obeta_monthly_ew),
  beta_monthly_ew = first(beta_monthly_ew)-last(beta_monthly_ew),max_ew = first(max_ew)-
last(max_ew),
  er_obeta_monthly_vw = first(er_obeta_monthly_vw)-last(er_obeta_monthly_vw),
  beta_monthly_vw = first(beta_monthly_vw)-last(beta_monthly_vw), max_vw = first(max_vw)-
last(max_vw)) %>%
  bind_rows(portfolio_obeta_monthly, .) %>% arrange(TradeDate) %>% rename(portfolio =
Portfolio_obeta_monthly)

# Compute time-series averages
portfolio_obeta_monthly_c <- portfolio_obeta_monthly %>% group_by(portfolio) %>%
  summarise(length = round(mean(length)),mcap = mean(mcap),beta_monthly_ew =
mean(beta_monthly_ew),
  max_ew = mean(max_ew, na.rm = TRUE), sd_obeta_monthly_ew = sd(er_obeta_monthly_ew),
  er_obeta_monthly_ew = mean(er_obeta_monthly_ew),sr_obeta_monthly_ew =
er_obeta_monthly_ew/sd_obeta_monthly_ew,
  beta_monthly_vw = mean(beta_monthly_vw, na.rm = TRUE), max_vw = mean(max_vw),
  sd_obeta_monthly_vw = sd(er_obeta_monthly_vw), er_obeta_monthly_vw = mean(er_obeta_monthly_vw),
  sr_obeta_monthly_vw = er_obeta_monthly_vw/sd_obeta_monthly_vw) %>% t()


#### 7. Quintile portfolios representing the sum of stocks in the quintile max portfolios within
each beta.daily quintile portfolio

### Calculate monthly VW (EW) portfolio excess returns and portfolio characteristics
portfolio_BNMD <- master %>%filter(!is.na(Portfolio_BNMD)) %>% group_by(TradeDate,
Portfolio_BNMD) %>%
  summarise(length = length(E.ReturnAdjGeneric),mcap = sum(LMCAP, na.rm = TRUE),
  er_BNMD_monthly_ew = sum(E.ReturnAdjGeneric)/length(E.ReturnAdjGeneric),beta_daily_ew =
mean(beta.daily, na.rm = TRUE),
  max_ew = mean(max, na.rm = TRUE),er_BNMD_monthly_vw = sum(mcap_return)/sum(LMCAP),
  beta_daily_vw = sum(mcap_beta_daily, na.rm = TRUE)/sum(LMCAP),max_vw = sum(mcap_max, na.rm =
TRUE)/sum(LMCAP))

# Compute the metrics for the low-high portfolio (portfolio 6)
portfolio_BNMD <- portfolio_BNMD %>% group_by(TradeDate) %>% arrange(Portfolio_BNMD) %>%
summarise(Portfolio_BNMD = 6,
  length = 0, mcap = first(mcap)-last(mcap),er_BNMD_monthly_ew = first(er_BNMD_monthly_ew)-
last(er_BNMD_monthly_ew),
  beta_daily_ew = first(beta_daily_ew)-last(beta_daily_ew),max_ew = first(max_ew)-last(max_ew),
  er_BNMD_monthly_vw = first(er_BNMD_monthly_vw)-last(er_BNMD_monthly_vw), beta_daily_vw =
first(beta_daily_vw)-last(beta_daily_vw),
  max_vw = first(max_vw)-last(max_vw)) %>% bind_rows(portfolio_BNMD, .) %>% arrange(TradeDate)
%>% rename(portfolio = Portfolio_BNMD)

# Compute time-series averages
portfolio_BNMD_c <- portfolio_BNMD %>% group_by(portfolio) %>% summarise(length = mean(length),
  mcap = mean(mcap), beta_daily_ew = mean(beta_daily_ew),max_ew = mean(max_ew, na.rm = TRUE),
  sd_BNMD_monthly_ew = sd(er_BNMD_monthly_ew), er_BNMD_monthly_ew = mean(er_BNMD_monthly_ew),
  sr_BNMD_monthly_ew = er_BNMD_monthly_ew/sd_BNMD_monthly_ew, beta_daily_vw = mean(beta_daily_vw,
na.rm = TRUE),
  max_vw = mean(max_vw), sd_BNMD_monthly_vw = sd(er_BNMD_monthly_vw),er_BNMD_monthly_vw =
mean(er_BNMD_monthly_vw),
  sr_BNMD_monthly_vw = er_BNMD_monthly_vw/sd_BNMD_monthly_vw) %>% t()


#### 8: Quintile portfolios sorted on the component of beta.daily orthogonal to max

### Calculate monthly VW (EW) portfolio excess returns and portfolio characteristics
portfolio_obeta_daily <- master %>% filter(!is.na(Portfolio_obeta_daily)) %>% group_by(TradeDate,
Portfolio_obeta_daily) %>%
  summarise(length = length(E.ReturnAdjGeneric),mcap = sum(LMCAP, na.rm = TRUE),
```

```
  er_obeta_daily_ew = sum(E.ReturnAdjGeneric)/length(E.ReturnAdjGeneric), beta_daily_ew =
mean(beta.daily, na.rm = TRUE),
  max_ew = mean(max, na.rm = TRUE), er_obeta_daily_vw = sum(mcap_return)/sum(LMCAP),
  beta_daily_vw = sum(mcap_beta_daily, na.rm = TRUE)/sum(LMCAP), max_vw = sum(mcap_max, na.rm =
TRUE)/sum(LMCAP))

# Compute the metrics for the low-high portfolio
portfolio_obeta_daily <- portfolio_obeta_daily %>% group_by(TradeDate) %>%
arrange(Portfolio_obeta_daily) %>%
  summarise(Portfolio_obeta_daily = 6, length = 0, mcap = first(mcap)-last(mcap),
  er_obeta_daily_ew = first(er_obeta_daily_ew)-last(er_obeta_daily_ew), beta_daily_ew =
first(beta_daily_ew)-last(beta_daily_ew),
  max_ew = first(max_ew)-last(max_ew), er_obeta_daily_vw = first(er_obeta_daily_vw)-
last(er_obeta_daily_vw),
  beta_daily_vw = first(beta_daily_vw)-last(beta_daily_vw), max_vw = first(max_vw)-last(max_vw))
%>%
  bind_rows(portfolio_obeta_daily, .) %>% arrange(TradeDate) %>% rename(portfolio =
Portfolio_obeta_daily)

# Compute time-series averages
portfolio_obeta_daily_c <- portfolio_obeta_daily %>% group_by(portfolio) %>%
  summarise(length = round(mean(length)), mcap = mean(mcap),beta_daily_ew = mean(beta_daily_ew),
  max_ew = mean(max_ew, na.rm = TRUE), sd_obeta_daily_ew = sd(er_obeta_daily_ew),
  er_obeta_daily_ew = mean(er_obeta_daily_ew), sr_obeta_daily_ew =
er_obeta_daily_ew/sd_obeta_daily_ew,
  beta_daily_vw = mean(beta_daily_vw, na.rm = TRUE), max_vw = mean(max_vw), sd_obeta_daily_vw =
sd(er_obeta_daily_vw),
  er_obeta_daily_vw = mean(er_obeta_daily_vw), sr_obeta_daily_vw =
er_obeta_daily_vw/sd_obeta_daily_vw) %>% t()


#### 9. Quintile portfolios representing the sum of stocks in the quintile beta.monthly
portfolios within each max quintile portfolio

### Calculate monthly VW (EW) poirtfolio excess returns and portfolio characteristics
portfolio_MNB <- master %>% filter(!is.na(Portfolio_MNB)) %>% group_by(TradeDate, Portfolio_MNB)
%>%
  summarise(length = length(E.ReturnAdjGeneric), mcap = sum(LMCAP, na.rm = TRUE),
  er_MNB_monthly_ew = sum(E.ReturnAdjGeneric)/length(E.ReturnAdjGeneric),beta_monthly_ew =
mean(beta.monthly, na.rm = TRUE),
  max_ew = mean(max, na.rm = TRUE), er_MNB_monthly_vw = sum(mcap_return)/sum(LMCAP),
  beta_monthly_vw = sum(mcap_beta_monthly, na.rm = TRUE)/sum(LMCAP), max_vw = sum(mcap_max, na.rm
= TRUE)/sum(LMCAP))

# Compute the metrics for the low-high portfolio (portfolio 6)
portfolio_MNB <- portfolio_MNB %>% group_by(TradeDate) %>%
  arrange(Portfolio_MNB) %>% summarise(Portfolio_MNB = 6, length = 0,mcap = first(mcap)-
last(mcap),
  er_MNB_monthly_ew = first(er_MNB_monthly_ew)-last(er_MNB_monthly_ew),beta_monthly_ew =
first(beta_monthly_ew)-last(beta_monthly_ew),
  max_ew = first(max_ew)-last(max_ew), er_MNB_monthly_vw = first(er_MNB_monthly_vw)-
last(er_MNB_monthly_vw),
  beta_monthly_vw = first(beta_monthly_vw)-last(beta_monthly_vw), max_vw = first(max_vw)-
last(max_vw)) %>%
  bind_rows(portfolio_MNB, .) %>% arrange(TradeDate) %>% rename(portfolio = Portfolio_MNB)

# Compute time-series averages
portfolio_MNB_c <- portfolio_MNB %>% group_by(portfolio) %>%
  summarise(length = mean(length), mcap = mean(mcap), beta_monthly_ew = mean(beta_monthly_ew),
  max_ew = mean(max_ew, na.rm = TRUE), sd_MNB_monthly_ew = sd(er_MNB_monthly_ew),
er_MNB_monthly_ew = mean(er_MNB_monthly_ew),
  sr_MNB_monthly_ew = er_MNB_monthly_ew/sd_MNB_monthly_ew, beta_monthly_vw =
mean(beta_monthly_vw, na.rm = TRUE),
  max_vw = mean(max_vw), sd_MNB_monthly_vw = sd(er_MNB_monthly_vw), er_MNB_monthly_vw =
mean(er_MNB_monthly_vw),
  sr_MNB_monthly_vw = er_MNB_monthly_vw/sd_MNB_monthly_vw) %>% t()


#### 10: Quintile portfolios sorted on the component of max orthogonal to beta.monthly

### Calculate monthly VW (EW) portfolio excess returns and portfolio characteristics
```

```
portfolio_obmax <- master %>% filter(!is.na(Portfolio_obmax)) %>% group_by(TradeDate,
Portfolio_obmax) %>%
  summarise(length = length(E.ReturnAdjGeneric),mcap = sum(LMCAP, na.rm = TRUE),
  er_obmax_ew = sum(E.ReturnAdjGeneric)/length(E.ReturnAdjGeneric), beta_monthly_ew =
mean(beta.monthly, na.rm = TRUE),
  max_ew = mean(max, na.rm = TRUE), er_obmax_vw = sum(mcap_return)/sum(LMCAP),
  beta_monthly_vw = sum(mcap_beta_monthly, na.rm = TRUE)/sum(LMCAP), max_vw = sum(mcap_max, na.rm
= TRUE)/sum(LMCAP))

# Compute the metrics for the low-high portfolio (portfolio 6)
portfolio_obmax <- portfolio_obmax %>% group_by(TradeDate) %>% arrange(Portfolio_obmax) %>%
  summarise(Portfolio_obmax = 6, length = 0, mcap = first(mcap)-last(mcap), er_obmax_ew =
first(er_obmax_ew)-last(er_obmax_ew),
  beta_monthly_ew = first(beta_monthly_ew)-last(beta_monthly_ew), max_ew = first(max_ew)-
last(max_ew),
  er_obmax_vw = first(er_obmax_vw)-last(er_obmax_vw), beta_monthly_vw = first(beta_monthly_vw)-
last(beta_monthly_vw),
  max_vw = first(max_vw)-last(max_vw)) %>% bind_rows(portfolio_obmax, .) %>% arrange(TradeDate)
%>% rename(portfolio = Portfolio_obmax)

# Compute time-series averages
portfolio_obmax_c <- portfolio_obmax %>% group_by(portfolio) %>%
  summarise(length = mean(length), mcap = mean(mcap), beta_monthly_ew = mean(beta_monthly_ew),
  max_ew = mean(max_ew, na.rm = TRUE), sd_obmax_ew = sd(er_obmax_ew), er_obmax_ew =
mean(er_obmax_ew),
  sr_obmax_ew = er_obmax_ew/sd_obmax_ew, beta_monthly_vw = mean(beta_monthly_vw, na.rm = TRUE),
  max_vw = mean(max_vw), sd_obmax_vw = sd(er_obmax_vw), er_obmax_vw =
mean(er_obmax_vw),sr_obmax_vw = er_obmax_vw/sd_obmax_vw) %>% t()


#### 11. Quintile portfolios representing the sum of stocks in the quintile ivol portfolios
within each beta.monthly quintile portfolio

### Calculate monthly VW (EW) poirtfolio excess returns and portfolio characteristics
portfolio_BNI <- master %>% filter(!is.na(Portfolio_BNI)) %>% group_by(TradeDate, Portfolio_BNI)
%>%
  summarise(length = length(E.ReturnAdjGeneric), mcap = sum(LMCAP, na.rm = TRUE),
  er_BNI_monthly_ew = sum(E.ReturnAdjGeneric)/length(E.ReturnAdjGeneric),beta_monthly_ew =
mean(beta.monthly, na.rm = TRUE),
  max_ew = mean(max, na.rm = TRUE),er_BNI_monthly_vw = sum(mcap_return)/sum(LMCAP),
  beta_monthly_vw = sum(mcap_beta_monthly, na.rm = TRUE)/sum(LMCAP),max_vw = sum(mcap_max, na.rm
= TRUE)/sum(LMCAP))

# Compute the metrics for the low-high portfolio (portfolio 6)
portfolio_BNI <- portfolio_BNI %>% group_by(TradeDate) %>%arrange(Portfolio_BNI) %>%
  summarise(Portfolio_BNI = 6,length = 0, mcap = first(mcap)-last(mcap),
  er_BNI_monthly_ew = first(er_BNI_monthly_ew)-last(er_BNI_monthly_ew),
  beta_monthly_ew = first(beta_monthly_ew)-last(beta_monthly_ew), max_ew = first(max_ew)-
last(max_ew),
  er_BNI_monthly_vw = first(er_BNI_monthly_vw)-last(er_BNI_monthly_vw), beta_monthly_vw =
first(beta_monthly_vw)-last(beta_monthly_vw),
  max_vw = first(max_vw)-last(max_vw)) %>% bind_rows(portfolio_BNI, .) %>%
  arrange(TradeDate) %>% rename(portfolio = Portfolio_BNI)

# Compute time-series averages
portfolio_BNI_c <- portfolio_BNI %>% group_by(portfolio) %>%
  summarise(length = mean(length),mcap = mean(mcap),beta_monthly_ew = mean(beta_monthly_ew),
  max_ew = mean(max_ew, na.rm = TRUE), sd_BNI_monthly_ew = sd(er_BNI_monthly_ew),
  er_BNI_monthly_ew = mean(er_BNI_monthly_ew), sr_BNI_monthly_ew =
er_BNI_monthly_ew/sd_BNI_monthly_ew,
  beta_monthly_vw = mean(beta_monthly_vw, na.rm = TRUE), max_vw = mean(max_vw),
  sd_BNI_monthly_vw = sd(er_BNI_monthly_vw), er_BNI_monthly_vw = mean(er_BNI_monthly_vw),
  sr_BNI_monthly_vw = er_BNI_monthly_vw/sd_BNI_monthly_vw) %>% t()


#### 12: Quintile portfolios sorted on the component of beta.monthly orthogonal to ivol

### Calculate monthly VW (EW) portfolio excess returns and portfolio characteristics
portfolio_ibeta_monthly <- master %>% filter(!is.na(Portfolio_ibeta_monthly)) %>%
  group_by(TradeDate, Portfolio_ibeta_monthly) %>% summarise(length = length(E.ReturnAdjGeneric),
```

```r
  mcap = sum(LMCAP, na.rm = TRUE), er_ibeta_monthly_ew =
sum(E.ReturnAdjGeneric)/length(E.ReturnAdjGeneric),
  beta_monthly_ew = mean(beta.monthly, na.rm = TRUE), max_ew = mean(max, na.rm = TRUE),
  er_ibeta_monthly_vw = sum(mcap_return)/sum(LMCAP), beta_monthly_vw = sum(mcap_beta_monthly,
na.rm = TRUE)/sum(LMCAP),
  max_vw = sum(mcap_max, na.rm = TRUE)/sum(LMCAP))

# Compute the metrics for the low-high portfolio (portfolio 6)
portfolio_ibeta_monthly <- portfolio_ibeta_monthly %>%
  group_by(TradeDate) %>% arrange(Portfolio_ibeta_monthly) %>%
  summarise(Portfolio_ibeta_monthly = 6, length = 0, mcap = first(mcap)-last(mcap),
  er_ibeta_monthly_ew = first(er_ibeta_monthly_ew)-last(er_ibeta_monthly_ew),
  beta_monthly_ew = first(beta_monthly_ew)-last(beta_monthly_ew),max_ew = first(max_ew)-
last(max_ew),
  er_ibeta_monthly_vw = first(er_ibeta_monthly_vw)-last(er_ibeta_monthly_vw),
  beta_monthly_vw = first(beta_monthly_vw)-last(beta_monthly_vw),max_vw = first(max_vw)-
last(max_vw)) %>%
  bind_rows(portfolio_ibeta_monthly, .) %>% arrange(TradeDate) %>% rename(portfolio =
Portfolio_ibeta_monthly)

# Compute time-series averages
portfolio_ibeta_monthly_c <- portfolio_ibeta_monthly %>% group_by(portfolio) %>%
  summarise(length = mean(length),mcap = mean(mcap),beta_monthly_ew = mean(beta_monthly_ew),
  max_ew = mean(max_ew, na.rm = TRUE), sd_ibeta_monthly_ew = sd(er_ibeta_monthly_ew),
  er_ibeta_monthly_ew = mean(er_ibeta_monthly_ew),sr_ibeta_monthly_ew =
er_ibeta_monthly_ew/sd_ibeta_monthly_ew,
  beta_monthly_vw = mean(beta_monthly_vw, na.rm = TRUE), max_vw = mean(max_vw),
  sd_ibeta_monthly_vw = sd(er_ibeta_monthly_vw), er_ibeta_monthly_vw = mean(er_ibeta_monthly_vw),
  sr_ibeta_monthly_vw = er_ibeta_monthly_vw/sd_ibeta_monthly_vw) %>% t()




########## Section 12: Construct dataframe with all monthly portfolio excess returns ##########



portfolio_returns <- merge(portfolio_month[,c("TradeDate", "portfolio", "er_month_ew",
"er_month_vw")],
  portfolio_day[,c("TradeDate", "portfolio", "er_day_ew", "er_day_vw")], by =
c("TradeDate","portfolio"), all.x = TRUE, all.y = TRUE)
portfolio_returns <- merge(portfolio_returns,
  portfolio_msci[,c("TradeDate", "portfolio", "er_msci_ew", "er_msci_vw")],
  by = c("TradeDate","portfolio"), all.x = TRUE, all.y = TRUE)
portfolio_returns <- merge(portfolio_returns,
  portfolio_max[,c("TradeDate", "portfolio", "er_max_ew", "er_max_vw")],by =
c("TradeDate","portfolio"), all.x = TRUE, all.y = TRUE)
portfolio_returns <- merge(portfolio_returns,
  portfolio_obeta_daily[,c("TradeDate", "portfolio", "er_obeta_daily_ew", "er_obeta_daily_vw")],
  by = c("TradeDate","portfolio"), all.x = TRUE, all.y = TRUE)
portfolio_returns <- merge(portfolio_returns,
  portfolio_obeta_monthly[,c("TradeDate", "portfolio", "er_obeta_monthly_ew",
"er_obeta_monthly_vw")],
  by = c("TradeDate","portfolio"), all.x = TRUE, all.y = TRUE)
portfolio_returns <- merge(portfolio_returns,
  portfolio_ibeta_monthly[,c("TradeDate", "portfolio", "er_ibeta_monthly_ew",
"er_ibeta_monthly_vw")],
  by = c("TradeDate","portfolio"), all.x = TRUE, all.y = TRUE)
portfolio_returns <- merge(portfolio_returns,
 portfolio_BNM[,c("TradeDate", "portfolio", "er_BNM_monthly_ew", "er_BNM_monthly_vw")],
 by = c("TradeDate","portfolio"), all.x = TRUE, all.y = TRUE)
portfolio_returns <- merge(portfolio_returns,
 portfolio_BNMD[,c("TradeDate", "portfolio", "er_BNMD_monthly_ew", "er_BNMD_monthly_vw")],
 by = c("TradeDate","portfolio"), all.x = TRUE, all.y = TRUE)
portfolio_returns <- merge(portfolio_returns,
 portfolio_BNI[,c("TradeDate", "portfolio", "er_BNI_monthly_ew", "er_BNI_monthly_vw")],
 by = c("TradeDate","portfolio"), all.x = TRUE, all.y = TRUE)
portfolio_returns <- merge(portfolio_returns,
 portfolio_BNO[,c("TradeDate", "portfolio", "er_BNO_monthly_ew", "er_BNO_monthly_vw")],
 by = c("TradeDate","portfolio"), all.x = TRUE, all.y = TRUE)
portfolio_returns <- merge(portfolio_returns,
```

```
 portfolio_MNB[,c("TradeDate", "portfolio", "er_MNB_monthly_ew", "er_MNB_monthly_vw")],
 by = c("TradeDate","portfolio"), all.x = TRUE, all.y = TRUE)
portfolio_returns <- merge(portfolio_returns,
 portfolio_obmax[,c("TradeDate", "portfolio", "er_obmax_ew", "er_obmax_vw")],
 by = c("TradeDate","portfolio"), all.x = TRUE, all.y = TRUE)

# Merge dataframe containing portfolio excess returns with market returns and factor returns to
prepare for factor model regressions
portfolio_returns <- merge(portfolio_returns, df_market_monthly, by = "TradeDate")
portfolio_returns <- merge(portfolio_returns, fmax_factor, by = "TradeDate", all.x = TRUE)
portfolio_returns <- merge(portfolio_returns, fivol_factor, by = "TradeDate", all.x = TRUE)
portfolio_returns <- merge(portfolio_returns, fbeta_factor, by = "TradeDate", all.x = TRUE)



########## Section 14: Identify most frequent companies in each portfolio ##########



# Create dataframe with SecurityID and Company Names
company_lookup <- df_daily %>% select(SecurityId, SecurityName)
company_lookup <- distinct(company_lookup)

### 1: Create overview of top 20 companies in portfolio 5

# Extract top 20 companies in beta.monthly portfolio 5
company.bm <- master %>% group_by(SecurityId, Portfolio_b_m) %>% summarise(n())
company.bm <- merge(company.bm, company_lookup, by = "SecurityId", all.x = TRUE)
company.bm <- company.bm %>% filter(Portfolio_b_m == 5) %>% rename(BetaMonthly = SecurityName)
%>%
  arrange(-`n()`) %>% select(BetaMonthly) %>% slice(1:20)

# Extract top 20 companies in beta.monthly portfolio 5
company.bd <- master %>% group_by(SecurityId, Portfolio_b_d) %>% summarise(n())
company.bd <- merge(company.bd, company_lookup, by = "SecurityId", all.x = TRUE)
company.bd <- company.bd %>% filter(Portfolio_b_d == 5) %>% rename(BetaDaily = SecurityName) %>%
  arrange(-`n()`) %>% select(BetaDaily) %>% slice(1:20)

# Extract top 20 companies in beta.msci portfolio 5
company.msci <- master %>% group_by(SecurityId, Portfolio_b_msci) %>% summarise(n())
company.msci <- merge(company.msci, company_lookup, by = "SecurityId", all.x = TRUE)
company.msci <- company.msci %>% filter(Portfolio_b_msci == 5) %>% rename(BetaMsci =
SecurityName) %>%
  arrange(-`n()`) %>% select(BetaMsci) %>% slice(1:20)

# Extract top 20 companies in max portfolio 5
company.max <- master %>% group_by(SecurityId, Portfolio_max) %>% summarise(n())
company.max <- merge(company.max, company_lookup, by = "SecurityId", all.x = TRUE)
company.max <- company.max %>% filter(Portfolio_max == 5) %>% rename(Max = SecurityName) %>%
  arrange(-`n()`) %>% select(Max) %>% slice(1:20)

# Create combined dataframe with top 20 companies in portfolio 5 for the different sorting
variables
Port5_companies <- cbind(company.bm, company.bd, company.msci, company.max)


### 2: Create overview of top 20 companies in portfolio 1

## Extract top 20 companies in beta.monthly portfolio 1
company.bm <- master %>% group_by(SecurityId, Portfolio_b_m) %>% summarise(n())
company.bm <- merge(company.bm, company_lookup, by = "SecurityId", all.x = TRUE)
company.bm <- company.bm %>% filter(Portfolio_b_m == 1) %>% rename(BetaMonthly = SecurityName)
%>%
  arrange(-`n()`) %>% select(BetaMonthly) %>% slice(1:20)

## Extract top 20 companies in beta.daily portfolio 1
company.bd <- master %>% group_by(SecurityId, Portfolio_b_d) %>% summarise(n())
company.bd <- merge(company.bd, company_lookup, by = "SecurityId", all.x = TRUE)
company.bd <- company.bd %>% filter(Portfolio_b_d == 1) %>% rename(BetaDaily = SecurityName) %>%
  arrange(-`n()`) %>% select(BetaDaily) %>% slice(1:20)
```

```
## Extract top 20 companies in beta.msci portfolio 1
company.msci <- master %>% group_by(SecurityId, Portfolio_b_msci) %>% summarise(n())
company.msci <- merge(company.msci, company_lookup, by = "SecurityId", all.x = TRUE)
company.msci <- company.msci %>% filter(Portfolio_b_msci == 1) %>% rename(BetaMsci =
SecurityName) %>%
  arrange(-`n()`) %>% select(BetaMsci) %>% slice(1:20)

## Extract top 20 companies in max portfolio 1
company.max <- master %>% group_by(SecurityId, Portfolio_max) %>% summarise(n())
company.max <- merge(company.max, company_lookup, by = "SecurityId", all.x = TRUE)
company.max <- company.max %>% filter(Portfolio_max == 1) %>% rename(Max = SecurityName) %>%
  arrange(-`n()`) %>% select(Max) %>% slice(1:20)

# Create combined dataframe with top 20 companies in portfolio 1 for the different sorting
variables
Port1_companies <- cbind(company.bm, company.bd, company.msci, company.max)

## Delete excess dataframes
rm(company.bm, company.bd, company.msci, company.max)



########## Section 15: Examine portfolio turnover ##########



### 1: Estimate turnover for portfolios sorted on beta.monthly

## Compute the number of stocks that enter the portfolio each month
buy <- master %>% select(TradeDate, SecurityId, Portfolio_b_m) %>%
  arrange(SecurityId, TradeDate) %>% mutate(Lag_Portfolio_b_m = lag(Portfolio_b_m)) %>%
  mutate(Trade = ifelse(is.na(Lag_Portfolio_b_m) & is.na(Portfolio_b_m), 0,
  ifelse(is.na(Lag_Portfolio_b_m) & !is.na(Portfolio_b_m), 1,
  ifelse(!is.na(Lag_Portfolio_b_m) & is.na(Portfolio_b_m), 1,
  ifelse(Lag_Portfolio_b_m == Portfolio_b_m, 0, 1))))) %>%
  group_by(TradeDate, Portfolio_b_m) %>% summarise(Buy = sum(Trade, na.rm = TRUE),
  Stocks_Post = sum(!is.na(Portfolio_b_m))) %>% filter(!is.na(Portfolio_b_m)) %>%
  rename(Portfolio = Portfolio_b_m)

## Compute the number of stocks that leave the portfolio each month
sell <- master %>% select(TradeDate, SecurityId, Portfolio_b_m) %>%
  arrange(SecurityId, TradeDate) %>% mutate(Lag_Portfolio_b_m = lag(Portfolio_b_m)) %>%
  mutate(Trade = ifelse(is.na(Lag_Portfolio_b_m) & is.na(Portfolio_b_m), 0,
  ifelse(is.na(Lag_Portfolio_b_m) & !is.na(Portfolio_b_m), 1,
  ifelse(!is.na(Lag_Portfolio_b_m) & is.na(Portfolio_b_m), 1,
  ifelse(Lag_Portfolio_b_m == Portfolio_b_m, 0, 1))))) %>%
  group_by(TradeDate, Lag_Portfolio_b_m) %>% summarise(Sell = sum(Trade, na.rm = TRUE),
  Stocks_Prior = sum(!is.na(Lag_Portfolio_b_m))) %>% filter(!is.na(Lag_Portfolio_b_m)) %>%
  rename(Portfolio = Lag_Portfolio_b_m)

## Compute the portfolio turnover (enter + leave)
Turnover_B5Y <- merge(buy, sell, by = c("TradeDate", "Portfolio")) %>%
  mutate(Transactions = Buy + Sell) %>% mutate(Turnover = Sell/Stocks_Prior) %>%
  mutate(Turnover_Buy = Buy/Stocks_Post)


### 2: Estimate turnover for portfolios sorted on beta.daily

## Compute the number of stocks that enter the portfolio each month
buy <- master %>% select(TradeDate, SecurityId, Portfolio_b_d) %>%
  arrange(SecurityId, TradeDate) %>% mutate(Lag_Portfolio_b_d = lag(Portfolio_b_d)) %>%
  mutate(Trade = ifelse(is.na(Lag_Portfolio_b_d) & is.na(Portfolio_b_d), 0,
  ifelse(is.na(Lag_Portfolio_b_d) & !is.na(Portfolio_b_d), 1,
  ifelse(!is.na(Lag_Portfolio_b_d) & is.na(Portfolio_b_d), 1,
  ifelse(Lag_Portfolio_b_d == Portfolio_b_d, 0, 1))))) %>%
  group_by(TradeDate, Portfolio_b_d) %>% summarise(Buy = sum(Trade, na.rm = TRUE),
  Stocks_Post = sum(!is.na(Portfolio_b_d))) %>% filter(!is.na(Portfolio_b_d)) %>%
  rename(Portfolio = Portfolio_b_d)

## Compute the number of stcoks that leave the portfolio each month
sell <- master %>% select(TradeDate, SecurityId, Portfolio_b_d) %>%
```

```
  arrange(SecurityId, TradeDate) %>% mutate(Lag_Portfolio_b_d = lag(Portfolio_b_d)) %>%
  mutate(Trade = ifelse(is.na(Lag_Portfolio_b_d) & is.na(Portfolio_b_d), 0,
  ifelse(is.na(Lag_Portfolio_b_d) & !is.na(Portfolio_b_d), 1,
  ifelse(!is.na(Lag_Portfolio_b_d) & is.na(Portfolio_b_d), 1,
  ifelse(Lag_Portfolio_b_d == Portfolio_b_d, 0,1))))) %>%
  group_by(TradeDate, Lag_Portfolio_b_d) %>% summarise(Sell = sum(Trade, na.rm = TRUE),
  Stocks_Prior = sum(!is.na(Lag_Portfolio_b_d))) %>% filter(!is.na(Lag_Portfolio_b_d)) %>%
  rename(Portfolio = Lag_Portfolio_b_d)

## Compute the portfolio turnover (enter + leave)
Turnover_B1Y <- merge(buy, sell, by = c("TradeDate", "Portfolio")) %>%
  mutate(Transactions = Buy + Sell) %>% mutate(Turnover_Sell = Sell/Stocks_Prior) %>%
  mutate(Turnover_Buy = Buy/Stocks_Post)


### 3: Estimate turnover for portfolios sorted on max

## Compute the number of stocks that enter the portfolio each month
buy <- master %>% select(TradeDate, SecurityId, Portfolio_max) %>%
  arrange(SecurityId, TradeDate) %>% mutate(Lag_Portfolio_max = lag(Portfolio_max)) %>%
  mutate(Trade = ifelse(is.na(Lag_Portfolio_max) & is.na(Portfolio_max), 0,
  ifelse(is.na(Lag_Portfolio_max) & !is.na(Portfolio_max), 1,
  ifelse(!is.na(Lag_Portfolio_max) & is.na(Portfolio_max), 1,
  ifelse(Lag_Portfolio_max == Portfolio_max, 0,1))))) %>%
  group_by(TradeDate, Portfolio_max) %>% summarise(Buy = sum(Trade, na.rm = TRUE),
  Stocks_Post = sum(!is.na(Portfolio_max))) %>% filter(!is.na(Portfolio_max)) %>%
  rename(Portfolio = Portfolio_max)

## Compute the number of stocks that leave the portfolio each month
sell <- master %>% select(TradeDate, SecurityId, Portfolio_max) %>%
  arrange(SecurityId, TradeDate) %>% mutate(Lag_Portfolio_max = lag(Portfolio_max)) %>%
  mutate(Trade = ifelse(is.na(Lag_Portfolio_max) & is.na(Portfolio_max), 0,
  ifelse(is.na(Lag_Portfolio_max) & !is.na(Portfolio_max), 1,
  ifelse(!is.na(Lag_Portfolio_max) & is.na(Portfolio_max), 1,
  ifelse(Lag_Portfolio_max == Portfolio_max, 0,1))))) %>%
  group_by(TradeDate, Lag_Portfolio_max) %>% summarise(Sell = sum(Trade, na.rm = TRUE),
  Stocks_Prior = sum(!is.na(Lag_Portfolio_max))) %>% filter(!is.na(Lag_Portfolio_max)) %>%
  rename(Portfolio = Lag_Portfolio_max)

## Compute the portfolio turnover (enter + leave)
Turnover_MAX <- merge(buy, sell, by = c("TradeDate", "Portfolio")) %>%
  mutate(Transactions = Buy + Sell) %>% mutate(Turnover = Sell/Stocks_Prior) %>%
  mutate(Turnover_Buy = Buy/Stocks_Post)



########## Section 16: Factor model regressions ##########



# Note, we have not included the regression models reporting the factor loadings.
# The following presents the code used to generate the ex-post betas, portfolio alphas and
corresponding t-statistics reported in the main paper

# Create vector equal to the number of portfolios to be used in loop. Portfolio 6 equals the low-
high portfolio
k = c(1,2,3,4,5,6)

# Create empty dataframe to store market beta from CAPM regressions(ex-post portfolio betas),
factor model alphas and corresponding t-statistics
alfa <- data.frame(P1.a = numeric(), P2.a = numeric(), P3.a = numeric(), P4.a = numeric(), P5.a =
numeric(), P6.a = numeric(),
                   P1.t = numeric(), P2.t = numeric(), P3.t = numeric(), P4.t = numeric(), P5.t =
numeric(), P6.t = numeric())

### Estimate the number of lags to use in Newey West Adjustment (lag=4)
m <- portfolio_returns %>% filter(portfolio == 6)
m <- count(m)
m <- round(0.75*(m^(1/3)))
lag <- m-1
```

```
#### 1: Portfolios sorted on beta.monthly

## beta.monthly sorted VW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_month_vw ~ E.M.VW, data = p) # CAPM
  reg2 <- lm(er_month_vw ~ E.M.VW + SMB + HML, data = p) # FF3
  reg3 <- lm(er_month_vw ~ E.M.VW + SMB + HML + MOM, data = p) # FF3 + MOM
  reg4 <- lm(er_month_vw ~ E.M.VW + SMB + HML + MOM + LIQ, data = p) # FF3 + MOM + LIQ
  reg5 <- lm(er_month_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FMAX, data = p) # FF3 + MOM + LIQ +
FMAX
  reg6 <- lm(er_month_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FIVOL, data = p) # FF3 + MOM + LIQ +
FIVOL

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # The following code was used to store data in a manner which was easy to combine with
portfolio characteristics in excel to create the tables used in univariate and bivariate
portfolio analysis in the main paper
  # The code used to extract market betas, alphas and corresponding t-statistics is similar for
all regressions, and as such, we will only report the code for beta.monthly VW portfolios

  # Extract portfolio Ex-post betas and Aplhas
  alfa[1,i] = Coef1[2,1] # Ex-post beta (Beta from CAPM regression) adj. newey west
  alfa[2,i] = Coef1[1,1] # Alpha CAPM reg adj. newey west
  alfa[3,i] = Coef2[1,1] # Alpha FF3 reg adj. newey west
  alfa[4,i] = Coef3[1,1] # Alpha FF3 + MOM reg adj. newey west
  alfa[5,i] = Coef4[1,1] # Aplha FF3 + MOM + LIQ reg adj. wewey west
  alfa[6,i] = Coef5[1,1] # Alpha FF3 + MOM + LIQ + FMAX reg adj. newey west
  alfa[7,i] = Coef6[1,1] # Alpha FF3 + MOM + LIQ + FIVOL reg adj. newey west

  alfa[1,6+i] = Coef1[2,1] # Ex-post beta (Beta from CAPM regression) adj. newey west
  alfa[2,6+i] = Coef1[1,3] # t-stat alpha CAPM reg adj. newey west
  alfa[3,6+i] = Coef2[1,3] # t-stat alpha FF3 reg adj. newey west
  alfa[4,6+i] = Coef3[1,3] # t-stat alpha FF3 + MOM reg adj. newey west
  alfa[5,6+i] = Coef4[1,3] # t-stat aplha FF3 + MOM + LIQ reg adj. wewey west
  alfa[6,6+i] = Coef5[1,3] # t-stat alpha FF3 + MOM + LIQ + FMAX reg adj. newey west
  alfa[7,6+i] = Coef6[1,3] # t-stat alpha FF3 + MOM + LIQ + FIVOL reg adj. newey west
}

rownames(alfa) <- c( "Beta" ,"MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                     "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_b_m_VW <- alfa


## beta.monthly sorted EW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
```

```
  reg1 <- lm(er_month_ew ~ E.M.EW, data = p)
  reg2 <- lm(er_month_ew ~ E.M.EW + SMB + HML, data = p)
  reg3 <- lm(er_month_ew ~ E.M.EW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_month_ew ~ E.M.EW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_month_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_month_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
  }

rownames(alfa) <- c("Beta" ,"MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_b_m_EW <- alfa


#### 2: Portfolios sorted on beta.daily

## beta.daily sorted VW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_day_vw ~ E.M.VW, data = p)
  reg2 <- lm(er_day_vw ~ E.M.VW + SMB + HML, data = p)
  reg3 <- lm(er_day_vw ~ E.M.VW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_day_vw ~ E.M.VW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_day_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_day_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
  }

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_b_d_VW <- alfa
```

```
## beta.daily sorted EW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_day_ew ~ E.M.EW, data = p)
  reg2 <- lm(er_day_ew ~ E.M.EW + SMB + HML, data = p)
  reg3 <- lm(er_day_ew ~ E.M.EW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_day_ew ~ E.M.EW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_day_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_day_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
  }

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_b_d_EW <- alfa


#### 3: Portfolios sorted on beta.msci

## beta.msci sorted VW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_msci_vw ~ E.M.VW, data = p)
  reg2 <- lm(er_msci_vw ~ E.M.VW + SMB + HML, data = p)
  reg3 <- lm(er_msci_vw ~ E.M.VW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_msci_vw ~ E.M.VW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_msci_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_msci_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
```

```r
  }

rownames(alfa) <- c("Beta" ,"MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_b_msci_VW <- alfa


## beta.msci sorted EW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_msci_ew ~ E.M.EW, data = p)
  reg2 <- lm(er_msci_ew ~ E.M.EW + SMB + HML, data = p)
  reg3 <- lm(er_msci_ew ~ E.M.EW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_msci_ew ~ E.M.EW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_msci_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_msci_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
  }

rownames(alfa) <- c("Beta" ,"MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_b_msci_EW <- alfa


#### 4: Portfolios sorted on max

## max sorted VW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_max_vw ~ E.M.VW, data = p)
  reg2 <- lm(er_max_vw ~ E.M.VW + SMB + HML, data = p)
  reg3 <- lm(er_max_vw ~ E.M.VW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_max_vw ~ E.M.VW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_max_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_max_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
```

```r
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
  }

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_max_VW <- alfa


## max sorted EW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_max_ew ~ E.M.EW, data = p)
  reg2 <- lm(er_max_ew ~ E.M.EW + SMB + HML, data = p)
  reg3 <- lm(er_max_ew ~ E.M.EW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_max_ew ~ E.M.EW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_max_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_max_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
  }

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_max_EW <- alfa


#### 5: Portfolios representing the average max quintile portfolio within each beta.monthly
quintile

## BNM VW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_BNM_monthly_vw ~ E.M.VW, data = p)
  reg2 <- lm(er_BNM_monthly_vw ~ E.M.VW + SMB + HML, data = p)
  reg3 <- lm(er_BNM_monthly_vw ~ E.M.VW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_BNM_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_BNM_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_BNM_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
```

```
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
  }

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_BNM_VW <- alfa

## BNM EW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_BNM_monthly_ew ~ E.M.EW, data = p)
  reg2 <- lm(er_BNM_monthly_ew ~ E.M.EW + SMB + HML, data = p)
  reg3 <- lm(er_BNM_monthly_ew ~ E.M.EW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_BNM_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_BNM_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_BNM_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics as used for beta.monthly VW
  }

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_BNM_EW <- alfa


#### 6: Portfolios sorted on the component of beta.monthly orthogonal to max

## obeta_monthly VW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_obeta_monthly_vw ~ E.M.VW, data = p)
  reg2 <- lm(er_obeta_monthly_vw ~ E.M.VW + SMB + HML, data = p)
  reg3 <- lm(er_obeta_monthly_vw ~ E.M.VW + SMB + HML + MOM, data = p)
```

```
  reg4 <- lm(er_obeta_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_obeta_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_obeta_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
  }

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_obeta_monthly_VW <- alfa


## obeta_monthly EW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_obeta_monthly_ew ~ E.M.EW, data = p)
  reg2 <- lm(er_obeta_monthly_ew ~ E.M.EW + SMB + HML, data = p)
  reg3 <- lm(er_obeta_monthly_ew ~ E.M.EW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_obeta_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_obeta_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_obeta_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
  }

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_obeta_monthly_EW <- alfa


#### 7: Portfolios representing the average max quintile portfolio within each beta.daily
quintile

## BNMD VW portfolios
for (i in 1:length(k)) {
```

```r
  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_BNMD_monthly_vw ~ E.M.VW, data = p)
  reg2 <- lm(er_BNMD_monthly_vw ~ E.M.VW + SMB + HML, data = p)
  reg3 <- lm(er_BNMD_monthly_vw ~ E.M.VW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_BNMD_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_BNMD_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_BNMD_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
}

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_BNMD_VW <- alfa


## BNMD EW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_BNMD_monthly_ew ~ E.M.EW, data = p)
  reg2 <- lm(er_BNMD_monthly_ew ~ E.M.EW + SMB + HML, data = p)
  reg3 <- lm(er_BNMD_monthly_ew ~ E.M.EW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_BNMD_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_BNMD_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_BNMD_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
}

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
```

```
alfa_BNMD_EW <- alfa


#### 8: Portfolios sorted on the component of beta.daily orthogonal to max

## obeta.daily VW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_obeta_daily_vw ~ E.M.VW, data = p)
  reg2 <- lm(er_obeta_daily_vw ~ E.M.VW + SMB + HML, data = p)
  reg3 <- lm(er_obeta_daily_vw ~ E.M.VW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_obeta_daily_vw ~ E.M.VW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_obeta_daily_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_obeta_daily_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
}

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_obeta_daily_VW <- alfa


## obeta.daily EW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_obeta_daily_ew ~ E.M.EW, data = p)
  reg2 <- lm(er_obeta_daily_ew ~ E.M.EW + SMB + HML, data = p)
  reg3 <- lm(er_obeta_daily_ew ~ E.M.EW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_obeta_daily_ew ~ E.M.EW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_obeta_daily_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_obeta_daily_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
```

```r
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
}

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_obeta_daily_EW <- alfa


#### 9: Portfolios representing the average beta.monthly quintile portfolio within each max
quintile

## MNB VW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_MNB_monthly_vw ~ E.M.VW, data = p)
  reg2 <- lm(er_MNB_monthly_vw ~ E.M.VW + SMB + HML, data = p)
  reg3 <- lm(er_MNB_monthly_vw ~ E.M.VW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_MNB_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_MNB_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_MNB_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
}

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_MNB_VW <- alfa


### MNB EW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_MNB_monthly_ew ~ E.M.EW, data = p)
  reg2 <- lm(er_MNB_monthly_ew ~ E.M.EW + SMB + HML, data = p)
  reg3 <- lm(er_MNB_monthly_ew ~ E.M.EW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_MNB_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_MNB_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_MNB_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
```

```r
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
}

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_MNB_EW <- alfa


#### 10: Portfolios sorted on the component of max orthogonal to beta.monthly

### OBMAX VW
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_obmax_vw ~ E.M.VW, data = p)
  reg2 <- lm(er_obmax_vw ~ E.M.VW + SMB + HML, data = p)
  reg3 <- lm(er_obmax_vw ~ E.M.VW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_obmax_vw ~ E.M.VW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_obmax_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_obmax_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
}

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_obmax_VW <- alfa


### OBMAX EW
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_obmax_ew ~ E.M.EW, data = p)
  reg2 <- lm(er_obmax_ew ~ E.M.EW + SMB + HML, data = p)
  reg3 <- lm(er_obmax_ew ~ E.M.EW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_obmax_ew ~ E.M.EW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_obmax_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_obmax_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FIVOL, data = p)
```

```
  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
}

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_obmax_EW <- alfa


#### 11: Portfolios representing the average ivol quintile portfolio within each beta.monthly
quintile

## BNI VW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_BNI_monthly_vw ~ E.M.VW, data = p)
  reg2 <- lm(er_BNI_monthly_vw ~ E.M.VW + SMB + HML, data = p)
  reg3 <- lm(er_BNI_monthly_vw ~ E.M.VW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_BNI_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_BNI_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_BNI_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
  }

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_BNI_VW <- alfa

## BNI EW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])
```

```r
  # Perform factor model regressions
  reg1 <- lm(er_BNI_monthly_ew ~ E.M.EW, data = p)
  reg2 <- lm(er_BNI_monthly_ew ~ E.M.EW + SMB + HML, data = p)
  reg3 <- lm(er_BNI_monthly_ew ~ E.M.EW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_BNI_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_BNI_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_BNI_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
  }

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_BNI_EW <- alfa


#### 12: Portoflios sorted on the component of beta.monthly orthogonal to ivol

## ibeta VW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_ibeta_monthly_vw ~ E.M.VW, data = p)
  reg2 <- lm(er_ibeta_monthly_vw ~ E.M.VW + SMB + HML, data = p)
  reg3 <- lm(er_ibeta_monthly_vw ~ E.M.VW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_ibeta_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_ibeta_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_ibeta_monthly_vw ~ E.M.VW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
  }

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_ibeta_monthly_VW <- alfa
```

```r
### Ibeta EW portfolios
for (i in 1:length(k)) {

  p <- portfolio_returns %>%
    filter(portfolio == k[i])

  # Perform factor model regressions
  reg1 <- lm(er_ibeta_monthly_ew ~ E.M.EW, data = p)
  reg2 <- lm(er_ibeta_monthly_ew ~ E.M.EW + SMB + HML, data = p)
  reg3 <- lm(er_ibeta_monthly_ew ~ E.M.EW + SMB + HML + MOM, data = p)
  reg4 <- lm(er_ibeta_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ, data = p)
  reg5 <- lm(er_ibeta_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FMAX, data = p)
  reg6 <- lm(er_ibeta_monthly_ew ~ E.M.EW + SMB + HML + MOM + LIQ + FIVOL, data = p)

  # Estimate Newey West correlation Matrix
  NW1 <- NeweyWest(reg1, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW2 <- NeweyWest(reg2, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW3 <- NeweyWest(reg3, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW4 <- NeweyWest(reg4, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW5 <- NeweyWest(reg5, lag = 4, prewhite = FALSE, adjust = TRUE)
  NW6 <- NeweyWest(reg6, lag = 4, prewhite = FALSE, adjust = TRUE)

  # Estimate coefficients using Newey West correlation matrix
  Coef1 <- coeftest(reg1, vcov. = NW1)
  Coef2 <- coeftest(reg2, vcov. = NW2)
  Coef3 <- coeftest(reg3, vcov. = NW3)
  Coef4 <- coeftest(reg4, vcov. = NW4)
  Coef5 <- coeftest(reg5, vcov. = NW5)
  Coef6 <- coeftest(reg6, vcov. = NW6)

  # Same code used to extract ex-post betas, alphas and t-statistics
  }

rownames(alfa) <- c("Beta", "MKT", "FF3", "FF3 + MOM", "FF3 + MOM + LIQ",
                    "FF3 + MOM + LIQ + FMAX", "FF3 + MOM + LIQ + FIVOL")
alfa_ibeta_monthly_EW <- alfa
```