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Comparing structural credit models

and their applicability to banks

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Acknowledgements

We have with great enthusiasm enjoyed diving into the peculiarities of modeling banks through the approach of option pricing theory. In a world where banks are increasingly important to the global financial system and its stability, we have been inspired by joining the research field concerning both shareholders and other stakeholders of banks. With our thesis, we therefore hope to further improve the understanding of the implications derived from different approaches to bank modeling.

We wish to thank our supervisor Svein-Arne Persson for his qualitative and technical assistance, in addition to his patience with us as newcomers to this intriguing research field. We would also like to thank Atreya, Mjøs and Persson togheter with Nagel and Purnanandam for their research contributions, providing the foundation for this thesis.

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Executive summary

Throughout this thesis we have presented a narrow overview of the research field of structural credit models and their applicability to banks. We have focused on two of the newer contributions to the field by Nagel and Purnanandam (2019)(NP) and Atreya, Mjøs and Persson (2019)(AMP), and provided a thorough, but not exhaustive, comparison and evaluation of these models.

We have found that the different approaches of the two models provide logical results for both risk-neutral probability of default (RNPD)[1] and credit spreads[2], each displaying strengths and weaknesses compared to the banking industry. Both models account for the crucial characteristic of banks in that the value of their loans, and therefore their assets, have a naturally capped upside. Accordingly, both models rely on the use of a standard Brownian motion to describe the uncertainty of borrower asset values, and then value the banks claim on these through their respective loans.

In our comparison we find that the NP model provides somewhat higher estimates for both RNPD and credit spread relative to the AMP model for different borrower risk parameters. We then discuss various characteristics and assumptions of both models as explanatory for the observed deviation between the models. We also discuss whether each of these characteristics appear realistic in light of the banking industry.

Lastly, we touch upon additional common deviations from the banking industry of structural credit models like the ones we compare. Here we point to the complexity of loan types, debt structure, bank income sources and bank's borrowers as difficult elements to incorporate in detail. Nonetheless, we argue that the models in focus presents reasonable simplifications of the complex banking industry.

[1] See introduction for definition.
[2] See introduction for definition.

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1 Introduction

1.1 Introduction

In the aftermath of the great financial crisis of the 21st century, economists and regulators across the globe have directed great emphasis on bank risks and their impact on financial stability. On one hand, this has lead to meaningful critique of highly levered bank structures and accompanying increased bank regulations such as the third Basel accords (BIS, 2017). On the other hand, business executives have argued that leverage, as an element of bank risk, is a natural part of the banking industry and claim that stricter capital requirements will adversely affect economic growth (Gornall and Strebulaev, 2018).

The debate on bank risks and their impact on financial stability has also become an emphasised subject in the academic field of risk measuring and its application to banks. One such example is Keeley (1990), explaining high bank leverage as a result of moral hazards. Another is DeAngelo and Stulz (2015) arguing that the bank's role as a liquidity provider explains the high bank leverage observed around the world. Unlike both of these, Admati et al. (2013) concludes that high leverage is not necessary for banks in order to perform their functions or operate efficiently, since bank equity is not socially expensive. However, this paper represents a purely qualitative discussion of typical bank fallacies, and does not specifically account for each of the banks stakeholders. Accordingly, Admati et al. (2013) argued for a need for more quantitative models that may substantiate empirical data.

At the same time, researchers have demonstrated meaningful progress in the field of measuring credit risk and its application to banks. This research field consists of both 1) model-based approaches, and 2) traditional approaches using historical data of defaults (Toto, 2016). Looking at the model based approach, a line is drawn between structural[3]- and reduced-form[4] models. These approaches differ on their application of empirical observations and their determination of default probabilities and the time of default (Jarrow and Protter, 2004). In the past decades there has been a debate on which of these

[3] Structural models use the evolution of firms' structural variables, such as asset and debt values, to determine the time of default (Toto, 2016).

[4] In reduced-form models, default is treated as an unexpected event whose probability is governed by a default-intensity process (Toto, 2016).

model categories are better. However, as discussed by Wang (2009), both categories hold certain pros and cons, implying that they may be appropriate for different applications, supported by Toto (2016). Accordingly, increased accuracy of both these categories of credit risk models have provided valuable information on various aspects regarding banks such as capital structure, bank default probabilities, implications of financial regulations, and much more.

In our thesis, we tackle the field of credit models for banks by looking exclusively at structural credit models. We do so by comparing two of the more recent papers by Nagel and Purnanandam (2019) and Atreya, Mjøs and Persson (2019). Despite significant differences in their approach, these papers build on the evolution of the research field going back to the option pricing scheme model of Merton (1974). Along the way, multiple researchers have provided crucial insights to the applicability of such models to banks, including Leland (1994), Dermine and Lajeri (2001) and Gornall and Strebulaev (2018) amongst others.

In our comparison, we introduce estimates for the given parameters, based on empirical data form the Norwegian financial sector. In Norway, banks and mortgage companies play a crucial part in the economy, accounting for nearly 80% of the total credit to Norwegian households and companies (Norges Bank, 2019a). This translates to nearly four times the annual Norwegian state budget (Finansdepartementet, 2019). Furthermore, we find that Norwegian banks are highly levered relative to average firms, with equity accounting for only around 10% of the banks' total balance sheet values (Finans Norge, 2019). This is similar to what (Gornall and Strebulaev, 2018) found for US banks.

Our approach in comparing the two models includes their application to the modeling of risk-neutral probability of default (RNPD)[5] of a bank and corresponding credit spread[6] under a given set of parameters. Furthermore, we structure our analysis to discuss the differences displayed by the models, and compare these to what may appear reasonable in the financial industry. Accordingly, the main issue of this thesis is to answer the following:

[5]The risk-neutral probability of default is the calculated probability of default under the assumption that prices are calculated as their discounted expected values using risk-adjusted probabilities (Davis, 2017).

[6]Credit spreads are defined as the difference in yield between a corporate bond and a Treasury bond (or similar estimate for risk-free rate) of the same maturity (Romo, 2014).

What are the differences, and corresponding major strengths and weaknesses of the model presented by Atreya,Mjøs and Persson (2019) compared to that of Nagel and Purnanandam (2019)?

As our application of the models in comparison is rather narrow, we recognize that our thesis paves the way for further use and evaluation of the models. Due to the models being rather general, they are both applicable to multiple additional interesting topics such as bank regulation and its consequences.

The rest of the paper is organized as follows. Section 2 presents the theory behind the models of comparison in addition to two of their key predecessors. In section 3, our choice of parameters is discussed, while section 4 provides the methodology we have applied. In section 5, our findings are illustrated and further discussed in section 6. Section 7 concludes.

1.2 Limitations

Due to the vast extent of literature on the subject of banks and their characteristics, we have made certain limitations to our thesis to adequately answer the issue of topic introduced. First, we have only taken into account structural credit models, and specifically focused on two models in addition to their theoretical background.

Secondly, we have focus on their implications for banks RNPD and the accompanying credit spread. As both models are general enough to be applied to a vast number of bank elements, our analysis is therefore far from exhaustive in its evaluation of the models.

2 Theory

In this section we introduce the theoretical foundation of our thesis. It includes 4 models in their respective historical order as the latter two models are structured as modifications of the prior ones. The following parts include a qualitative and technical introduction to each of the models, illustrating why some of the adjustments have been critical to evaluating banks in light of option pricing theory.

The first model, by Merton (1974), lay much of the foundation for this field of research. Secondly, we present the adjustments made by Dermine and Lajeri (2001) which tackled the issue of banks' assets being constrained differently than other firms. The last two models are far more recent to the research field, and provide the basis for this thesis' findings and analysis, in which they are further compared and analyzed.

2.1 The Merton model (1974)

In May 1974, Robert C. Merton published a paper on the pricing of corporate debt, focusing on the risk structure of interest rate. The paper introduces a model as an extension of the Black and Scholes formula (1973), utilizing the insights from pricing options to value the debt and equity of a firm. By holding the term structure in the model given for most of his paper, Merton primarily focused on the impact of changes to the firm's probability of default on the price of debt and hence equity. The paper is structured as a thorough mathematical derivation of his findings, including multiple examples of application. However, as we merely utilize the conceptual insights of his findings, we will in this section focus on the explanation, rather than the mathematical derivation of Merton's (1974) findings.

The Merton model includes a variety of assumptions and simplifications. It starts by including the efficient markets hypothesis by Fama (1970) and Samuelson (1965) and the Miller-Modigliani (1958) theorem of capital structure invariance. Furthermore, Merton (1974) defines the asset value (V_t) of a firm as a "diffusion-type stochastic process", implying the firm asset value may drift in either direction at any point of time. This is given by

$$dV_t = (rV_t - C)dt + \sigma V_t dW_t. \quad (2.1)$$

Here, r is the continuous risk-free interest rate, C is the total payout by the firm to either shareholders or creditors, dt represents the increment of time t, σ is the instantaneous standard deviation of the return on the firm (volatility of firms asset value), and dW_t is a standard Gauss-Wiener process as a risk-neutral probability measure.

The fundamental insights of Merton (1974) then revolves around pricing the debt and equity instruments of a firm with the asset process described above. This can be illustrated with a simple balance sheet model of a firm as displayed below, where the firm is financed with equity and one instrument of zero-coupon debt.

Assets	Debt + Equity
A_t	$D_t = \min(\bar{D}, A_t)$
	$E_t = \max(A_t - \bar{D}, 0)$

Here, the debt- (D_t) and equity (E_t) time t values are determined by the asset value (A_t) at the time of debt maturity and the face value of the debt given by $\bar{D}$. If the asset value is above the face value of debt at the time of debt maturity, the creditor of the firm receive its respective face value $\bar{D}$ and equity holders capture the remaining value of A_t. However, if $A_t < \bar{D}$, the creditor takes over the firm, and hence receives the remaining value A_t, while equity holders receive zero.

In his paper, Merton (1974) discovered that the option theory provided by Black and Scholes (1973) could be used to value the firm's debt and equity at any point of time t prior to debt maturity. First, looking at the firm's equity, the relation described in the table above represents the cash flow of a call option with the strike price equal to the face value of debt. Accordingly, the shareholders may "exercise" their respective option on the remaining value of the firm's assets in cases where its value surpasses the face value of debt at time of maturity. The price of this option then equals the equity value at any point of time t.

Secondly, the firm's debt value can be rewritten as a function of the firm's equity so that

$$D_t = \bar{D}e^{-rT} - \max(\bar{D} - A_t, 0). \tag{2.2}$$

The first part of the equation is simply the discounted face value of debt to time t, where r is defined earlier and T is the remaining time to maturity of the debt. However, the latter part represents a put option on the firm's value with the strike price equal to the face value of the debt. Hence, the creditor receives its face value of debt less any potential difference of $\bar{D} - A_t$ in the case that $A_t < \bar{D}$.

Merton (1974) assumed the options applied to be European of type, implying that they may only be exercised at time of maturity. The equation for equity value is then given by

$$E_t = A_t\Phi(d_1) - De^{-rT}\Phi(d_2), \tag{2.3}$$

where

$$d_1 = \frac{\ln\frac{A_t}{D} + (r + \frac{\sigma_v^2}{2})T}{\sigma_v\sqrt{T}}, \tag{2.4}$$

and

$$d_2 = d_1 - \sigma_v\sqrt{T}. \tag{2.5}$$

Here, E_t, A_t, D, T and r are already defined. Furthermore, σ_v represents the standard deviation of the equity (can be calculated from stock returns) and $\Phi()$ represents the cumulative standard normal distribution. Applying the put-call-parity (Stoll, 1969), we can calculate the put option value, and hence the debt value at time t.

The findings of Merton (1974) presented above allows for a pricing of equity and corporate debt in a simplistic model of limited and observable variables at any given time t. The paper goes on to further develop and apply the insight to risk structure of interest rates as well as pricing of both preferred stock and callable bonds.

2.2 Dermine and Lajeri (2001)

The Merton model was created without any specific company or industry as foundation. Naturally, as different firms in different industries vary considerably in the types of assets they hold, the model does not fit equally well across the board. In this sense, some industries are in need of modifications to the Merton model of varying degrees to make more sense. Banks in specific are part of this group, with a key issue being that the upside potential of a bank's assets is naturally capped. In 2001, Dermine and Lajeri published a research note which explicitly looks at the lending risk of banks' assets, effectively accounting for this characteristic (Dermine and Lajeri, 2001).

The findings of Dermine and Lajeri are supported by simulation-based research from Lucas (1995), McAllister and Mingo (1996) and CreditMetrics (1997) on loan portfolios. In these studies, the authors find evidence of highly left-skewed distributions of the value of loan portfolios due to correlation across defaults. The distributions are categorized by a high probability for minimal changes in the value of the loan portfolio at the same time as the tail is longer to the left with lower values. The left tail is explained by credit losses during for instance recessions, where the loan losses can be considerable, while under normal circumstances the interest and principal are reimbursed due to few loan defaults (Dermine and Lajeri, 2001). Hence, their findings substantiate the modeling of a capped upside for a bank.

Looking at traditional banks, the asset side is usually comprised of a majority of lending to households and/or corporate borrowers. Taking Norwegian banks as an example, we have looked at 10 years of empirical data on their balance sheet structure. Here, we find that on average more than 72% of the banks' assets comprised of loans to customers and other credit institutions (SSB, 2019a). With assets primarily comprised of loans, the value of a bank's assets can not surpass the sum of the discounted face value of the loans and their respective interest payments, in effect capping the upside valuation of the assets. Similar to valuing the equity as a call option in the Merton model, Dermine and Lajeri (2001) applies the option scheme in their own model, with the twist of introducing the capped upside into the call option valuation. Their research note is centered around the pricing of deposit insurance, but is nonetheless equally relevant for the evaluation of a bank's assets, debt and equity.

The model of Dermine and Lajeri centers around one bank with one corporate borrower. The borrower funds its assets (A) through a loan (L) from the bank in addition to equity (E_f), while the bank funds its assets (comprising of one loan and a deposit insurance (P)) through deposits (D) and equity (E_b). Below the balance sheets of the borrower and the bank are displayed:

Borrower:

Assets	Debt + Equity
A	L
	E_f

Bank:

Assets	Debt + Equity
L	D
P	E_b

Then, the research note goes on to present the market value of the bank's equity in a standard option style (Black and Scholes, 1973), as a call option on the bank's assets:

$MV_e = Call(Value\ of\ loan, \bar{D}) = Call(\bar{L} - Put(A, \bar{L}), \bar{D})$

Here, the $Call$ and Put are defined by Black and Scholes (1973), while A is given by the table above, and $\bar{D}$ and $\bar{L}$ represents the face values of the bank debt (D) and the loan (L) respectively. As the assets consists of a loan and an insurance on deposits, the call option representing the equity may be rewritten as a function of both parts. Here the loan can be represented by the value of the loan at maturity less a put option due to the fact that the borrower's limited liability allows it to sell its assets A at maturity at the price of $\bar{L}$, in effect representing the bank taking over the borrowers' assets in the case of default. Applying the put-call-parity theorem (Stoll, 1969), the research note formulates the value of the loan as

$$L = \mathrm{e}^{-rT}\bar{L} - Put(A, \bar{L}), \tag{2.6}$$

where r represents the instantaneous risk-free rate and T the time to debt maturity. Here, the bank's assets equal the equity (call option) plus the discounted value of the exercise price ($\bar{D}$) less the liability of the deposit insurer (put option). Hence, we can rewrite the market value of the bank equity as

$$\begin{aligned} MV_e &= Call(\bar{L} - Put(A, \bar{L}), \bar{D}) \\ &= \mathrm{e}^{-rT}\bar{L} - Put(A, \bar{L}) - \mathrm{e}^{-rT}\bar{D} + Put(\bar{L} - Put(A, \bar{L}), \bar{D}) \\ &= L - D + Put(A, \bar{D}). \end{aligned} \tag{2.7}$$

Here, we see that the bank's equity is bounded upwards by the value of the loan, less the liability to the depositors in the case of a solvent borrower, while bounded downward by the put received from the deposit insurer in the case of borrower default. Applying the risk-neutral valuation methodology, the final valuation formula of equity value is given by

$$\begin{aligned} MV_e &= Call(A, \bar{D}) - Call(A, \bar{L}) \\ &= A\mathrm{N}(\frac{\ln\frac{A}{\bar{D}} + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}) - \mathrm{e}^{-rt}\bar{D}\mathrm{N}(\frac{\ln\frac{A}{\bar{D}} + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}) \\ &- A\mathrm{N}(\frac{\ln\frac{A}{\bar{L}} + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}) + \mathrm{e}^{-rt}\bar{L}\mathrm{N}(\frac{\ln\frac{A}{\bar{L}} + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}). \end{aligned} \tag{2.8}$$

Here, N(.) represents the cumulative normal distribution and σ the instantaneous volatility of the borrower asset value. This equation may be interpreted as a call on the asset value of the borrower at the exercise price $\bar{D}$, net of a call given to the borrower on the same asset at the exercise price $\bar{L}$. The latter two parts of the equation depicts the value loss resulting from the capped upside. This is a decreasing value of the the loan repayments, approaching zero as L goes to infinity.

2.3 Two more recent approaches

In recent years, multiple approaches to structural credit models of a bank has been proposed with a variety of purposes. These include estimating the bank's probability of default, pricing deposit insurance, modeling the effect of the deposit insurance on bank shareholders, optimizing the bank's capital structure, and much more. In this section we introduce two such structural models; the first exploring banks' risk dynamics and distance to default by Stefan Nagel and Amiyatosh Purnanandam (2019), and the second seeking to optimize banks' capital structure with regards to the shareholders interest by

Nikhil Atreya, Aksel Mjøs and Svein-Arne Persson (2019).

2.3.1 Nagel and Purnanandam (2019)

In 2019, Nagel and Purnanandam (NP) provided their contribution to the field of evaluating banks from an option perspective. Their paper presents a structural model for banks, and focuses on the implications of specific bank assets characteristics to their default risk and distance to default. Additionally, it provides quarterly empirical bank panel data from 1987 to 2016, and discusses the pitfalls of the standard Merton model on bank risk dynamics, government deposit guarantees and more (Nagel and Purnanandam, 2019).

The model presented in the paper is a modification of the Merton model, distinguished by three central characteristics. First, the model assumes a log-normal distribution for the borrowers assets over time, not the bank's. Hence, the capped upside of the bank is represented by its assets consisting of a pool of zero-coupon loans in which the borrower assets comprise the loans' respective collateral. Secondly, the loans are modeled with staggered maturities such that a fraction of the loans mature each period. Concurrently, the bank redistributes the payoff from the repaid loans to new loans under equal characteristics. This implies that the loan-to-value ratio is reset with each maturing loan as the new loan will be given at the same fixed initial loan-to-value ratio. Lastly, the bank's asset is modeled as a senior claim on the borrower assets.

With these characteristics, the model assumes a bank with a pool of loans constructed in N cohorts (denoted by τ) in which the asset values (A_t) of each borrower (denoted i) follows a log-normal process presented by the stochastic differential equation

$$\frac{dA_t^{\tau,i}}{A_t^{\tau,i}} = (r-\delta)dt + \sigma(\sqrt{\rho}dW_t + \sqrt{1-\rho}dZ_t^{\tau,i}). \tag{2.9}$$

Here, W_t and Z_t are independent Brownian motions, δ is the depreciation rate, r is the risk-free rate and σ is the instantaneous borrowers asset volatility. Moreover, t introduces the time element, and dt represents the increment of time. The Z_t process introduces the idiosyncratic risk parameters, while ρ is included as the correlation of asset values due to their common exposure to W_t.

Furthermore, the model introduces a fixed initial loan-to-value level l for all loans and an accompanying promised yield on the loans μ. Here, μ is endogenously solved for within the model together with F_1 being the face value of the first round of loans provided by the bank. Then, a time to maturity for the loans is set in order to evaluate the assets of the bank at a certain point of time $t = T$. At this point, the model first solves for the aggregate borrower asset value of cohort τ given by

$$A^{\tau}_{T-\tau} = \frac{1}{N}\exp\{(r-\delta)T - \frac{1}{2}\rho\sigma^2 T + \sigma\sqrt{\rho}(W_{T-\tau} - W_{-\tau})\}. \tag{2.10}$$

Here, exp{x} represents the notation e^x. Furthermore, NP defines the aggregate log asset value as

$$a^{\tau}_{T-\tau} = \frac{1}{N}[(r-\delta)T - \frac{1}{2}\sigma^2 T + \sigma\sqrt{\rho}(W_{T-\tau} - W_{-\tau)}]. \tag{2.11}$$

Here, the idiosyncratic risk is completely diversified away when assuming a continuum of borrowers in each cohort. The stochastic component is therefore solely dependent on the Brownian motions represented by $W_{T-\tau} - W_{-\tau}$. This is used to calculate the banks payoff (L) from the loans of cohort τ at the given time, obtained by

$$L^{\tau}_{T-\tau}(\mu) = \frac{1}{N}[A^{\tau}_{T-\tau}\Phi\{d_1(\mu)\} + F_1(\mu)\Phi\{d_2(\mu)\}], \tag{2.12}$$

where

$$d_1(\mu) = \frac{\ln F_1(\mu) - a^{\tau}_{T-\tau}}{\sqrt{1-\rho}\sqrt{T}\sigma} - \sqrt{1-\rho}\sqrt{T}\sigma, \tag{2.13}$$

and

$$d_2(\mu) = -\frac{\ln F_1(\mu) - a^{\tau}_{T-\tau}}{\sqrt{1-\rho}\sqrt{T}\sigma}. \tag{2.14}$$

In this equation, Φ represents the truncated log-normal distribution, with the standard normal cumulative distribution functions d_1 and d_2. This implies that the idiosyncratic risk of borrowers assets are present in the calculation of the banks payoff from individual borrowers, given by the expression $\sqrt{1-\rho}\sqrt{T}\sigma$ in d_1 and d_2, despite being diversified away in the aggregated borrower asset values.

The model is then constructed so that the bank debt, also presented as a zero-coupon loan debt, matures on a given date H with face value D. As some loans may have matured by this date, the model introduces a recalibration effect in which the payoff from the matured loans is immediately used for new loans to similar borrowers within each cohorts at the same fixed loan-to-value ratio. Consequently, the model provides a new loan face value (F_2) from the initially found μ (same μ for all loans as they are given on equal terms of borrower risk), and then calculates similar aggregate borrower asset values and corresponding bank payoffs from the cohorts. Here, the model specifies the importance of utilizing the new time horizons for the new loans, and its implied changes to the equations presented above (for further explanation, see appendix A1).

From the set of equations presented above, we can calculate the bank's asset value at a given point of time $t = H$. This is simply done by discounting the bank's payoffs from the loans within each cohort to the specified time H so that

$$V_H = \sum_{\tau<H} \mathrm{e}^{-r(\tau+T-H)} E_H^{\mathbb{Q}}[L_{2T-\tau}^{\tau}] + \sum_{\tau\geq H} \mathrm{e}^{-r(\tau-H)} E_H^{\mathbb{Q}}[L_{T-\tau}^{\tau}], \tag{2.15}$$

where $E_H^{\mathbb{Q}}[.]$ denotes a conditional expectation under the risk-neutral measure at the time of bank debt maturity. In the equation above, the only source of stochastic variation is given by the Brownian motion W_t. Hence, applying a reasonable set of parameters (r, σ, δ, ρ, T, τ, N and H), NP (2019) provides a set of 10,000 simulations of W_t. These are then applied to illustrate the distribution of V_H under the risk-neutral measure.

Now, we may introduce different capital structures and illustrate both the banks ability to repay its debt, and the corresponding equity values. In order to make the model more realistic, NP (2019) further introduces single payment dividends (Y_H) to the banks equity holders, given by

$$Y_H = V_H(1 - e^{-\gamma H}). \tag{2.16}$$

Here, γ is defined as the payout level, and the payments are modeled to be paid out just before maturity. Furthermore, the model presents its equity (S_H) and debt (B_H) values by

$$S_H = \max[V_H - Y_H - D, 0], \tag{2.17}$$

and

$$B_H = V_H - Y_H - S_H. \tag{2.18}$$

We have adapted the NP (2019) model in Excel to illustrate the distribution of the balance sheet values in the following figures. We start by running a set of 10 000 simulations of a Brownian motion for each period the bank provides loans. We then calculate the value of the promised payment on loans (μ) and its corresponding face value of loans (F_1) (see Methodology section for further explanation). Then, we calculate the aggregate borrower asset values at loan maturity for each cohort (see equation 2.10) with their respective aggregate log asset values (see equation 2.11), some of which have been rolled over from their first round of loans. Furthermore, we calculate the bank's respective payoffs from each cohort (see equation 2.12), and discount these values back to the time of maturity for the banks debt (see equation 2.15). At this point, we can illustrate the distributions of the balance sheet values for a given capital structure, as displayed in the following figures.

Here we have applied the parameters in accordance with the original paper by NP (2019). Hence, the parameters are set at $N = 10$, $H = 5$, $T = 10$, $\sigma = 0.2$, $\rho = 0.5$, $r = 0.01$, $\delta = 0.005$, $l = 0.66$, $\gamma = 0.002$ and $D = 0.70$. Correspondingly, the first cohort of loans is assumed to be given at $t = -9$, implying that the loans in some cohorts are rolled over into new loans by the time of the banks' debt maturity. Furthermore, we have made a model adjustment compared to NP(2019)(see the first part of the Methodology section for further explanation).

Figure 2.1: Simulated bank asset values

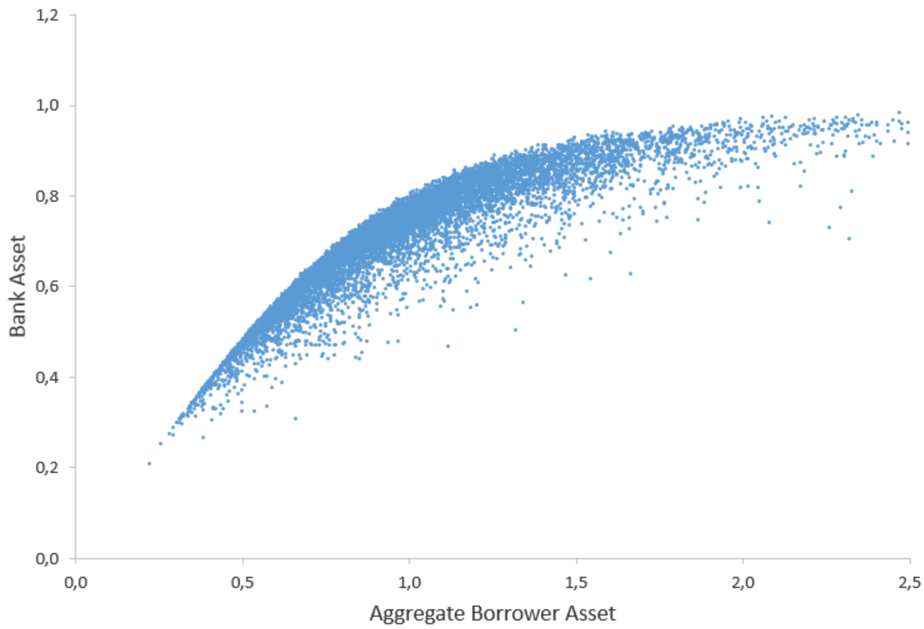

Figure 2.2: Simulated bank equity values

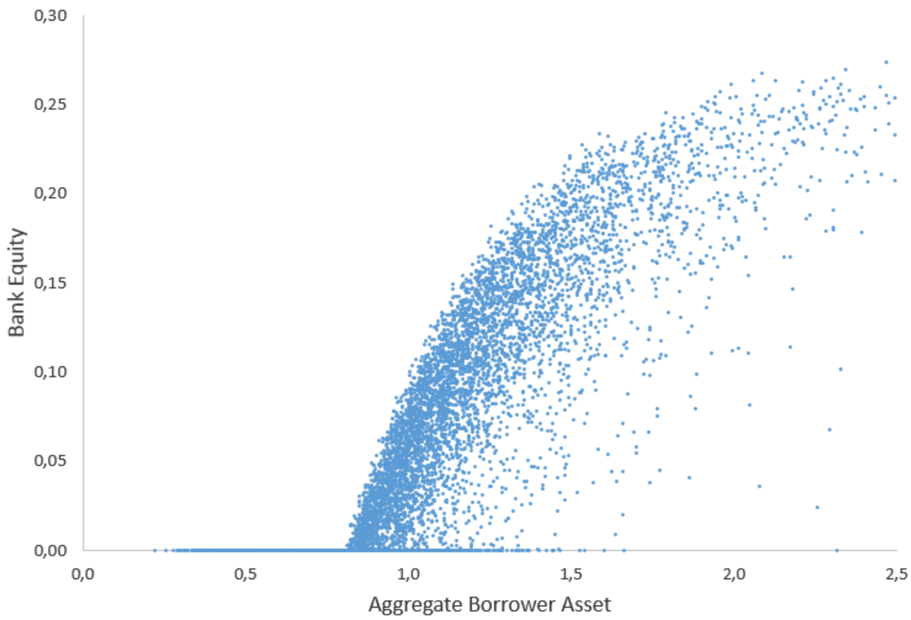

Figure 2.3: Simulated bank debt values

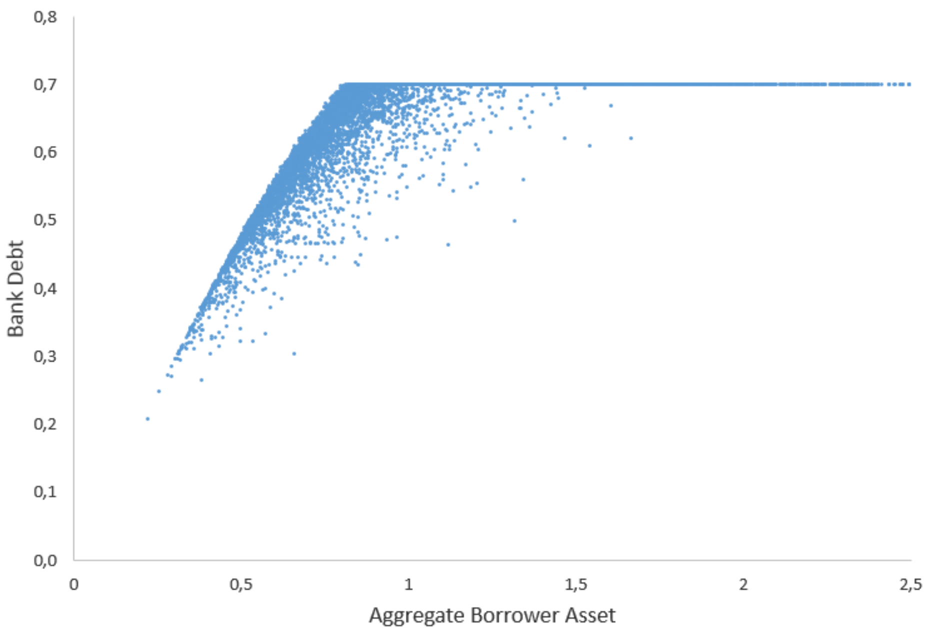

Figure 2.1, 2.2 and 2.3: *Illustrates the simulated 1) bank asset, 2) equity and 3) debt values at bank maturity ($H = 5$) as a function of aggregate borrower asset values [7]. The figures are based on 10 000 simulations. Each simulated value corresponds to a dot in the figures.*

From the graphs above we can point out some important findings of the models nature. First, there is a clear concavity to both the bank's asset- and equity values. This is mainly driven by the staggered maturities of the loans implying that many loans are not matured at the time $t = H$. Secondly, this may also be driven by the idiosyncratic risk of the individual loans, increasing the borrowers' default option value and thereby reducing the bank's value of the loans.

Another point of notice is the clear dispersion in the bank's asset value despite the aggregate borrower asset value. This may also be driven by the staggered maturities of the loans. In cases where borrower asset values perform poorly until the maturity of the first few cohorts, the number of defaults may be significant and the corresponding aggregate payoffs from the loans will be low. Thereby the value of the new loans of the bank will be low. As a result, the banks asset value at time $t = H$ will be reduced, despite a scenario

[7] Aggregate borrower asset value is the total collateral value of the borrowers within all cohorts at their respective time of loan maturity, discounted to the time of the banks debt maturity.

with strong improvement in borrower asset values up until the time of loan maturity, due to the loans capped upside.

In the remaining part of NP's (2019) paper, they compare and visualize the difference of their model and that of Merton (1974), in addition to including an empirical point of view as mentioned earlier. Here, the paper illustrates the pitfall of Merton (1974) in underestimating the asset volatility of a bank in situations of shocks to borrower asset volatility due to the fixed asset volatility of the Merton model. For further elaboration, see appendix A1 or the paper included in the reference list.

2.3.2 Atreya, Mjøs and Persson (2019)

In the Fall of 2019, Atreya, Mjøs and Persson (AMP) provided their working paper on banks' capital structure in a shareholder perspective. The paper presents a structural model illustrating why shareholders are better of with close to 100% leverage in a bank in cases of reasonable parameter assumptions. The paper further provides illustrative examples of the effect of interest rate shocks to optimal bank leverage amongst other elements (Atreya et al., 2019).

The model presented by AMP (2019) represents a set of modifications of the Leland (1994) and Merton (1974) models. It starts by defining a bank which only provides asset-backed loans to a single borrower at a time. It then explicitly defines the loans to customers and the bank's debt as perpetual coupon paying instruments, excluding the characteristics of a fixed maturity. Then the paper strategically provides the model's structure, starting with the borrower.

At the borrower level, taxes and bankruptcy costs are disregarded, due to the focus on the bank's optimal capital structure. The model then specifies the borrower asset value (A_t) so that

$$\frac{dA_t}{A_t} = rdt + \sigma dW_t, \tag{2.19}$$

where σ is the constant borrower asset volatility and W_t is a standard Brownian motion. dt represents the increment of time. Due to the assumption of continuous coupon paying loans, r represents the constant continuously compounded risk-free rate of return.

In the AMP model, the bank's borrowers finance their assets at a given fixed loan-to-value ratio $0 < L < 1$. Hence, given a constant A_0, the borrower selects a loan value $\hat{B}$ so that its initial leverage at loan origination is given by

$$L = \frac{\hat{B}}{A_0}. \tag{2.20}$$

For the given loan size, the borrower defaults on its loan at the time when its asset value (A_t) reaches the threshold value $\bar{A}$, also representing the value the bank recovers in the event of borrower default. The value of the loan is expressed as a function of both the borrower's asset value and the threshold value

$$B(A_t) = \frac{c\hat{B}}{r} - (\frac{c\hat{B}}{r} - \bar{A})(\frac{A_t}{\bar{A}})^{-\gamma}, \tag{2.21}$$

where

$$\gamma = \frac{2r}{\sigma^2} > 0, \tag{2.22}$$

and c represents the continuous coupon payment on the borrower loan. The last element of the loan value expression above represents a security yielding 1 in the event of borrower default. Denoted by its time t price, we have that

$$G_t = (\frac{A_t}{\bar{A}})^{-\gamma}. \tag{2.23}$$

Here, the process takes values in the interval $(0, 1]$ for values of $A_t > \bar{A}$. Applying Ito's lemma, it can be shown that the process of G_t, like the process A_t, is a geometric Brownian motion. Using G_t, we find that the initial borrower defaults at time t when $G_t = 1$ and $A_t = \bar{A}$. The threshold value is here determined endogenously in the model for a given value of debt $(\hat{B})$ and coupon rate (c). This determination is based on Black and Cox (1976) so that

$$\bar{A} = \Psi\hat{B}, \tag{2.24}$$

where

$$\Psi = \frac{c}{r}\frac{\gamma}{1+\gamma} < 1 \tag{2.25}$$

is the factor multiplied by the face value of borrower debt to determine the borrower's optimal default threshold. Utilizing the findings above, we can now calculate the initial value of G and the corresponding coupon rate c from

$$G = G_0 = (L\Psi)^{\gamma} \tag{2.26}$$

and

$$\frac{c}{r}(1 - \frac{G}{\gamma+1}) = 1. \tag{2.27}$$

Next, the model provides the characteristics of the bank. At this point, the paper postpones the introduction of capital market frictions until the final part of optimization. The bank is here presented with only one borrower at a time, so that when the borrower defaults, the bank issues its recovered amount into a new loan at equal borrower terms such as an equal and constant asset volatility (σ) and initial leverage (L). With the borrowers denoted by j, the recovered amount of $\bar{A}^j$ equals the face value amount of the next borrower, so that

$$\hat{B}^{j+1} = \bar{A}^j = B\Psi^j, \tag{2.28}$$

for all $j \geq 1$ and where $\hat{B}^1 = B$. The relation between the bank and borrower can be illustrated by their respective balance sheets:

Borrower j balance sheet	
A_t^j	$D_t^j = B(A_t^j)$
	$E_t^j = A_t^j - D_t^j$

Bank balance sheet	
$B_t = D_t^j$	$D_t(B)$
	$E_t(B) = B_t - D_t(B)$

Furthermore, the model proceeds to generalize the process of the bank's asset values when the first borrower defaults. This is done by returning to the defined process G_t in a rewritten manner

$$G_t = Ge^{\sigma\gamma Y_t}, \tag{2.29}$$

where Y_t represents G_t expressed as a Brownian motion (for elaborated calculations, please see appendix A2). Drawing on the findings of Merton (1974), AMP (2019) further defines

$$d = \frac{1}{\sigma\gamma}\ln(\frac{1}{G}) \tag{2.30}$$

as the borrowers' normalized distance to default. This is by assumption the same at loan origination for all the bank's future borrowers. Extending these findings to a case where a borrower default results in a new loan to a new borrower, we have that when the default time of the bank's borrower number $n = 1, 2, \ldots$

$$\tau(n) = \inf\{t \geq 0 : G_t = \frac{1}{G^{n-1}}\} = \inf\{t \geq 0 : Y_t = n \cdot d\}. \tag{2.31}$$

Here, Y_t counts the number of normalized distances to default and inf represents the abbreviation of infimum [8]. By defining the number of defaults up to time t by N_t as

$$N_t = \lfloor \frac{\eta_t}{d} \rfloor, \tag{2.32}$$

where

$$\eta_t = \sup_{0<s<t} Y_s. \tag{2.33}$$

Here, the notation $\lfloor x \rfloor$ represents the greatest integer less than or equal to x (Graham et al., 1994), while sup represents the abbreviation of supremum [9]. We can further calculate the state price of all the bank's future borrowers at time t in terms of N_t and Y_t, given by

[8] Infimum is the largest quantity that is less than or equal to each of a given set or subset of quantities (Lexico, 2019).

[9] Supremum is the smallest quantity that is greater than or equal to each of a given set or subset of quantities (Lexico, 2019).

$$\Pi_t = Ge^{\sigma\gamma(Y_t - N_t d)}. \tag{2.34}$$

It can be graphically shown that whenever Π_t reaches 1, this indicates a borrower default and will automatically reset Π_t to the next borrower of the bank. By now, we can solve for the bank's asset value (B_t) in a frictionless scenario, given by

$$B_t = \frac{c}{r} B\Psi^{N_t}(1 - \frac{\Pi_t}{\gamma + 1}). \tag{2.35}$$

Introducing the element of capital structure, the model defines F as the face value of debt and i as the continuous interest rate paid on the bank's debt. The problem of solving for the debt and equity components in such a scenario has been studied extensively, and bases its solution on the number of borrower defaults. This is solved by

$$n^* = \lceil \frac{\ln(iF) - \ln(cB)}{\ln\Psi} \rceil, \tag{2.36}$$

in which the notation $\lceil x \rceil$ represents the least integer greater than or equal to x (Graham et al., 1994), and i is simultaneously solved for by its definition

$$i = r(\frac{1 - \frac{(G\Psi)^{n^*}}{L_B}}{1 - G^{n^*}}), \tag{2.37}$$

where L_B is the initial leverage ratio of the bank. The equity of the bank can then be calculated as

$$E(B) = \sup_{\tau} E[\int_0^{\tau} (cB\Psi^{N_t} - iF)\mathrm{e}^{-rt} dt], \tag{2.38}$$

and by defining τ as $\tau(n^*)$ using the definiton of τ from equation 2.31, we solve the equity

$$E(B) = B - \{\frac{iF}{r} - (\frac{iF}{r} - \bar{B}^*)G^{n*}\}, \tag{2.39}$$

where

$$\bar{B}^* = B\Psi^{n^*}. \tag{2.40}$$

The paper goes on introducing standard capital market frictions of taxation (θ) and bankruptcy cost (α) at the bank level, holding the borrowers clear of the bankruptcy cost. Accordingly, the bank's after-tax income is given by $cB\Psi^{N_t}$ and the bank's interest payments on debt are now θiF. Additionally, the cost of bankruptcy is given by $\alpha\bar{B}$, where $\bar{B}$ represents the bank's default threshold. For the optimization in the model, the bank's enterprise value is the sum of its assets and the trade-off between the frictions introduced, where the latter is maximized. This is defined as

$$V(\bar{n}) = B + X(\bar{n}), \tag{2.41}$$

where the trade-off function is given by

$$X(\bar{n}) = T(\bar{n}) - C(\bar{n}) = \frac{\theta iF}{r}(1 - G^{\bar{n}}) - \alpha B(\Psi G)^{\bar{n}}. \tag{2.42}$$

Combining the findings above, AMP presents the optimal value of equity for the shareholders as

$$E_f(B) = B - \{\frac{(1-\theta)iF}{r} - (\frac{(1-\theta)iF}{r} - \bar{B}_f^*)G^{n_f^*}\}, \tag{2.43}$$

where

$$n_f^* = \lceil \frac{\ln[(1-\theta)iF] - \ln(cB)}{\ln\Psi} \rceil, \tag{2.44}$$

and

$$\bar{B}_f^* = B\Psi^{n_f^*}. \tag{2.45}$$

The last part of the model is the optimization of the trade-off value to maximize the bank's value. As the trade-off benefit increases with iF due to the tax advantage, we set it to its maximum value for a given n_f^*. We then have to account for the fact that the optimal number of borrower defaults is limited to the natural numbers, denoted $n^* \in \mathbb{N}$. Hence, the optimization is a discrete problem. This is approached by solving the trade-off function for the integers below and above the real positive number of borrower defaults, denoted $t \in \mathbb{R}^+$. This gives us the trade-off function

$$X(t) = \frac{\theta}{r}\frac{cB\Psi^{t-1}}{(1-\theta)}(1-G^t) - \alpha B(\Psi G)^t = B\Psi^t(\theta K(1-G^t) - \alpha G^t). \tag{2.46}$$

The optimal number of borrower defaults is then set at the integer found above, which provides the greatest value of the trade-off function, given by

$$n^* = \lfloor t^* \rfloor + 1\{X\}, \tag{2.47}$$

where

$$t^* = \frac{\frac{\theta K \ln\Psi}{(\theta K + \alpha)\ln(\Psi G)}}{\ln G}, \tag{2.48}$$

and

$$1\{X\} = \begin{cases} 1 \text{ if } X(\lfloor t^* \rfloor) > X\lceil t^* \rceil) \\ 0 \text{ otherwise.} \end{cases} \tag{2.49}$$

Finally, the paper defines the bank's enterprise value ($V(B)^*$) including capital market frictions as a function of the equations presented. This is given by

$$V(B)^* = B + \frac{\theta(iF)^*}{r}(1-G^{n^*}) - \alpha B(\Psi G)^{n^*} = B + X(n^*). \tag{2.50}$$

The model can then be visualized on multiple parameters. In the paper, AMP (2019) focus on the impact of variation in the borrowers leverage and borrower asset volatility as risk parameters for the bank. In Figure 2.4, the optimal bank leverage as a function of these borrower risk parameters is illustrated.

Figure 2.4: Bank leverage vs. borrower risk parameters

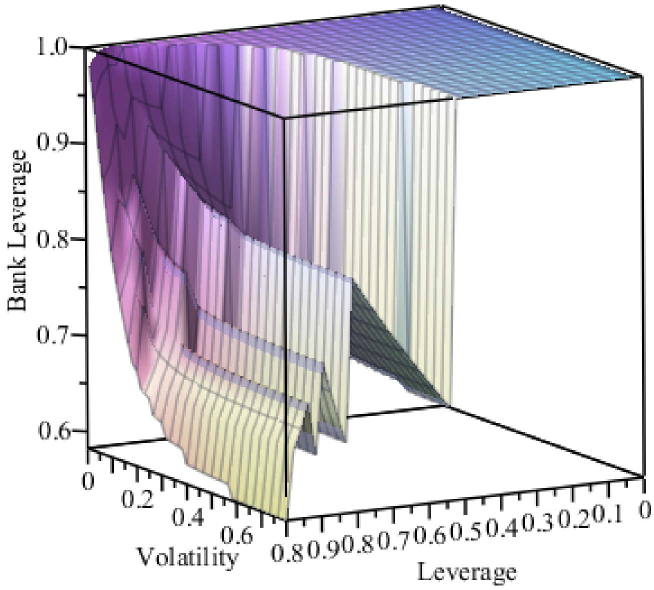

Figure 2.4: *The optimal bank leverage for different borrower risk parameters (σ and L). $r = 2\%$, $\theta = 27\%$, $\alpha = 22\%$ and $B = 100$.*

Figure 2.4 depicts bank leverage of nearly 100% as optimal for shareholders in a large part of what can be considered as reasonable levels of both borrower leverage and asset volatility. Once the borrower leverage grows, the figure becomes discontinuous with stepwise moves in the optimal level of bank leverage. This is explained by the relation between the bank interest rate (i) on its debt and the borrower risk parameters. At certain points of borrower leverage, the bank will adapt its debt structure and i will change, due to the calculation of n^* (equation 2.37). The paper goes on to discuss the following implications of the figure above, such as for regulatory interventions, coupon payments, optimal number of borrower defaults and more.

3 Choice of parameters

In this section we introduce the parameters applied in the following sections of the thesis to the models of NP and AMP. Some parameters will be treated as variables to analyse their impact on the models, while those introduced here are based on empirical observations and rational discussion in light of the current situation in the Norwegian financial sector. We note that the parameters regarding time horizon are fixed rather arbitrarily in accordance with NP (2019), something we discuss further in the analysis section.

3.1 Common input parameters in both models

Besides the parameters treated as variables in the approach in the following sections, there are two parameters that are treated as given constants in both models; risk-free rate (r) and the bank leverage ratio.

Looking at the risk-free rate, we find 10-year Norwegian government bond yields to be a decent indicator for Norwegian banks. During the course of preparing this thesis, we have observed that these yields have been ranging mostly between 1-1.5% by late 2019 (Norges Bank, 2019b). However, due to the current state of a demographic shift in the Norwegian population, combined with slightly lower growth expectations, the yields are expected to remain low (Carvalho et al., 2017)(IMF, 2019). We therefore argue that $r = 0.01$ is a conservatively fair level.

Considering the parameter of bank leverage, we have looked at the historical leverage ratio [10] of Norwegian banks (Finans Norge, 2019). However, this ratio includes some off-balance sheet items in the denominator, implying that the ratio is artificially low for a pure balance sheet driven model. Hence, we have also looked at historical balance sheet values for Norwegian banks [11], and found that the banks on average operated with approximately 90% book leverage (Finans Norge, 2019).

[10] The leverage ratio consists of core capital, and the exposure target includes all capitalized items and off-balance sheet items calculated without risk weighting (Finanstilsynet, 2017).

[11] We have excluded foreign branches here, as they may be affected by deviating capital requirements in the respective home countries.

3.2 Specific parameters of the NP model

In addition to the parameters included in both models, the NP model incorporates a borrower asset correlation (ρ), a bank asset depreciation rate (δ) and a bank payout level (γ). Due to lack of relevant Norwegian figures, we have continued with the parameters provided in the NP paper of $\rho = 0.5$, $\delta = 0.005$ and $\gamma = 0.02$.

3.3 Specific parameters of the AMP model

Due to the inclusion of financial market frictions, the AMP model also incorporates two new parameters: income tax (θ) and bankruptcy cost (α). In their paper they base these figures on empirical observations from US banks of $\theta = 0.27$ and $\alpha = 0.22$. With regards to the tax rate, we have looked at 10 years of empirical data on Norwegian banks, providing an average rate of $\theta = 0.24$ (SSB, 2019b). However, concerning the bankruptcy cost, there is little to no relevant recent empirical data from the Norwegian banking sector. Hence, we have continued with the estimate provided by AMP (2019).

4 Methodology

In this section we explain our application of the NP and AMP models presented in the theory section as well as the steps to our findings.

4.1 Simulations and adjustments of the NP model

In this section we first present some minor adjustments to the NP model used in our application before discussing our approach to simulations and further calculations within the model. For this model we have utilized Excel to provide the figures in the following Findings section.

4.1.1 Model adjustments

In our application of the NP model we have made an adjustment to certain expressions due to our understanding of a potential error in the original version. The adjustment regards the use of $(1/N)$ in equations 2.10 and 2.12 in the theory section. To our understanding, applying this part of the expression both when calculating the aggregate borrower asset and the payoff from a cohort will double the normalization needed when having multiple cohorts. Hence, we have disregarded this effect up until the calculation of the banks asset value. This implies that displaying the aggregate borrower asset value or payoff from a specific cohort independently would be misleading as these are not yet adjusted for the total number of cohorts within the bank.

4.1.2 Time to maturity and number of cohorts

Similar to NP (2019), our results are based on maturities staggered across 10 cohorts of borrowers, bank debt maturity of 5 years and the bank issuance of zero-coupon loans with maturity of 10 years. In figure 4.1, we have illustrated the 10 cohorts and their respective maturities used to estimate the bank asset value at t = H. Each line represent the 10-year loans of a cohort, while the dotted line illustrates the time of the bank's debt maturity. Furthermore, the time frame begins at t = -9 due to an assumption of the first cohort's initial loan maturity at time 1.

Figure 4.1: Staggered cohorts

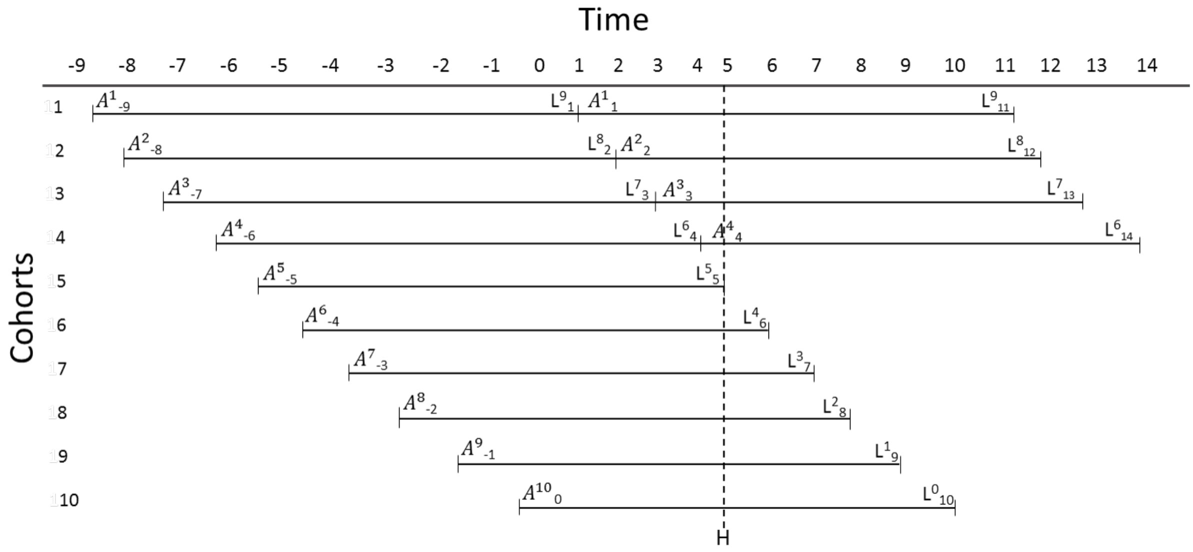

Figure 4.1: *Illustration of the 10 cohorts and their respective maturities used to estimate the bank asset value at t* $= H$*. Parameters:* $N = 10, T = 10, H = 5$.

As we can see from figure 4.1, the cohorts mature at different times. At the maturity date of the bank debt ($t = 5$), cohort 1, 2, 3 and 4 have already been rolled over into new loans, illustrated with an additional line in the figure. Cohort 5 matures at the same time as the banks' debt, while cohort 6, 7, 8, 9 and 10 matures in the periods following.

4.1.3 Simulating the standard Brownian motion of the model

In the NP model, the Brownian motion W_t depicts the only stochastic variation to the determination of the aggregate borrower asset values of the respective cohorts. Hence, this is also the only stochastic variation to the final calculation of the bank asset value. To provide the distribution of both the balance sheet values and risk parameters of the bank, we have therefore calculated a set of 10 000 simulations of the process W_t, consistent with the approach of NP.

Approaching the simulations, we have applied the Excel command of NORMSINV(RAND()), providing the inverse of a standard cumulative distribution with $E(x) = 0$ and $\sigma = 1$. The RAND() function then returns a random number $0 < x < 1$. We then set $W_0 = 0$. Furtermore, the approach is adjusted for the time intervals so that

W_{t+1} = NORMSINV(RAND()) + W_t, implying that the process evolves over time as a standard Brownian motion. Running a set of 10 000 simulations, we find the distribution of 10 000 bank asset values, and may thereby evaluate parameters such as asset volatility, RNPD and credit spreads. In order to assign a RNPD and credit spread to different combinations of borrower leverage and volatility, we have run 10 000 simulations for each pair of different borrower leverage and borrower volatilities (yielding a total of 1 000 000 simulations) and estimated the bank asset value and volatility, borrower asset value, RNPD and credit spread based on the arithmetic average for values from each set of simulations.

4.1.4 Further calculations

4.1.4.1 Endogenously solving for μ and F

As introduced in the presentation of NP (2019) in the theory section, the promised yield on loans (μ) and the face value of initial loans (F_1) provided by the bank are solved for endogenously in the NP model. The approach here relies on the insight that the initial borrower leverage can be modeled as the present face value $F_1(\mu)$ less a Black-Scholes put option. As the initial borrower leverage is a defined constant within the model, and F_1 is a function of μ, this can be utilized to solve for μ and F_1.

In our approach to the NP model, we incorporate the findings above into the Goal Seek function of Excel. Filling in a standard Black-Scholes put, F_1 as a function of μ and the equation of borrower leverage (see Appendix A.1) as a function of $F_1(\mu)$ and the put, we then Goal Seek the latter cell to the given value of borrower leverage we want, by changing μ only. This returns the promised yield on loans (μ) and its respective face value (F_1).

When executing this exercise, we noted that the starting value set in the cell in which borrower leverage was calculated had a slight impact on the final value of μ. Though the practical impact of the effect was nearly unnoticeable, it should be recognized as a minor weakness of utilizing the Excel function of Goal Seek in this application.

4.1.4.2 RNPD- and credit spread calculations

In our calculations of RNPD and credit spread we have assumed a fixed bank leverage at time 0, allowing for the variation of the borrower risk parameters. This fixation is done under the assumption that the bank's assets consist solely of the loans provided to 10 cohorts at time 0, the first being provided at time t =-9. We then discount the respective face values of the loans to time 0. Furthermore, we allow the size of bank debt to fluctuate under various combinations of borrower risk parameters so that its discounted time 0 value, divided by the time 0 asset value, remains a constant (leverage) ratio. This is done by the use of Excel's Goal Seek command for each set of borrower risk parameters.

In the process of calculating the RNPD we turn to the appendix of the NP (2019) paper. Here, they introduce the function

$$RNPD = \Phi(\frac{-\ln(V_t) + \ln(D) - (r - \gamma - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}). \tag{4.1}$$

Here, $\Phi()$ represents the standard cumulative distribution function, V_t the bank's asset value and D the bank's face value of debt. σ is the bank asset volatility and is found by calculating the standard deviation of the distribution of bank asset values given by the set of simulations. Lastly, the remaining parameters are as presented in the theory section.

In the following sections we also evaluate the credit spread on bank debt in the NP model. The credit spread is here defined in accordance with the paper as RNPD multiplied by the loss given default (LGD). Hence, we can write the credit spread as

$$Credit\ spread = RNPD \cdot LGD, \tag{4.2}$$

where LGD is given by

$$LGD = 1 - \min[\frac{V_H}{F}, 1]. \tag{4.3}$$

In accordance with NP (Nagel and Purnanandam, 2019), $RNPD$ is provided by equation 4.1, while LGD is calculated as 1 less the recovery rate. We define the recovery rate as the minimum of the discounted asset value (V_H) divided by the face value of debt and 1, utilizing the distributions of the bank's asset values from simulations. We then calculate the average credit spread within each set of 10 000 simulations, obtaining a single credit spread estimate for each set of the chosen parameters in the model.

4.2 Application of the AMP model

In this section we discuss our application of the AMP model to evaluate bank RNPD and the credit spread of its debt. For this model we have utilized Maple to provide the figures in the following Findings section.

4.2.1 RNPD calculations

In the sections below we introduce further calculations on the basis of the AMP model. Due to AMP (2019) including the element of financial market frictions (referred to as frictions) within their model, we have provided separate illustrations to account for the effect of such frictions in comparison with the NP model.

4.2.1.1 RNPD - without frictions

In the calculations of AMP's RNPD without frictions, we utilize the models frictionless definition of the optimal number of borrower defaults (n^* from 2.36) and bank default threshold ($\bar{B}^*$ from 2.40). In this case, n^* is dependent on both the face value of the bank's debt (F) and the interest (i) on the respective debt. Here, i is again dependent on n^*, implying the need for simultaneously solving for both parameters. This is done through a process which start with an arbitrary value for i, and then gradually adjusts the value of i until it simultaneously solves both equations. This can be done both in Excel through the use of the Goal Seek function, or in Maple (which we utilize) by writing the process as a procedure. Here, the bank leverage ratio is given by $L_B = \frac{F}{B}$. Hence, fixing both F and B allows us to implicitly fix the initial bank leverage under the assumption that the bank debt is issued at par.

Furthermore, we apply the standard results (see Harrison (1985) or Lando (2004)) for Brownian motions in the process of calculating RNPD. Defining m_t as the minimum value of the bank's asset upon time t, the probability $Q(m_t < \bar{B})$ can be written as $Q(\eta_t > \bar{n}^* d)$. Both the distribution of η_t and d are known from the theory section. $\bar{n}$ is however defined by

$$\bar{n} = n(\bar{B}) = \bar{m} + 1 - \frac{\ln(\bar{G})}{\ln(G)}, \tag{4.4}$$

where

$$\bar{G} = (\gamma + 1)(1 - \frac{r}{c}\frac{\bar{B}}{B\Psi^{\bar{m}}}), \tag{4.5}$$

and

$$\bar{m} = m(\bar{B}) = \lfloor \frac{\ln(\bar{B}) - \ln(B)}{\ln(\Psi)} \rfloor. \tag{4.6}$$

From Harrison (1985) or Lando (2004), the function of Q() gives us the RNPD from the AMP model, where Q() is defined by

$$Q(\tau > t) = \Phi(\frac{x_0 - \mu t}{\sqrt{t}}) - e^{2\mu x_0}\Phi(\frac{-x_0 - \mu t}{\sqrt{t}}), \tag{4.7}$$

where

$$x_0 = \bar{n}^* d, \tag{4.8}$$

and

$$\mu = (\frac{r}{\sigma\gamma}) - \frac{\sigma\gamma}{2}. \tag{4.9}$$

4.2.1.2 RNPD - with frictions

Similar steps as described above are applied in the calculation including capital market frictions. However, including capital market frictions slightly changes the calculation of the bank debt interest rate (i) and face value (F). Due to the inclusion of frictions, we apply the n^* (equation 2.47) given by t^* and $X(t)$ (equation 2.46). These results are then applied in equation 2.50, optimizing the bank enterprise value. As the effect of frictions allow for a greater enterprise value of the bank, driven by the tax advantage less bankruptcy cost in the $X(t)$ equation, the bank leverage ratio is calculated as

$$L_B = \frac{F}{V_B}, \tag{4.10}$$

where V_B is given by equation 2.50. A fixed initial bank leverage L_B and n^* is then applied to calculate the bank debt interest rate given by equation 2.37. We then solve for the optimal number of borrower defaults including frictions and the bank debt interest rate, given by equation 2.44. The remaining process of calculating the RNPD of the bank under various borrower risk parameters (borrower asset volatility and borrower leverage) then follows the same process as described for the case without capital market frictions.

4.2.2 Credit spread calculations

In the calculation of credit spreads from the AMP model we define the credit spread as the continuous interest i paid on the bank debt less the risk-free rate r. Hence, we have utilized the same line of equations presented in the section on RNPD above. Here we define the interest rate i along the way, both with and without capital market frictions. The last steps in order to calculate the credit spread is then simply to subtract the risk-free interest rate, and then adjust the rate from its continuous characteristics to an annual rate, so that it is comparable with the results from the NP model. The latter is here done by

$$credit\ spread_{annualized} = e^{(i_{continuous} - r)} - 1. \tag{4.11}$$

5 Findings

In this section we have applied the models of NP and AMP presented in the theory section, accompanied with the adjustments, extensions and additions presented in the methodology section. We visualize both models' evaluation of a bank's RNPD, and the accompanying credit spread under various borrower risk parameters. In this section we only provide a brief introduction to the findings, while in the following section of analysis we elaborate on their explanations.

A key element to note in this section is that the AMP model appears to display elements of discontinuity in the following figures. This is explained by the relation between the bank interest rate (i) on its debt and the borrower risk parameters. At certain points of borrower leverage, the bank will adapt its debt structure and i will change, due to the calculation of n^* (equation 2.37). However, as we have fixed the initial level of bank leverage in this section, the entire effect will materialize in a lower i, for certain levels of borrower leverage. As a result, increased borrower leverage may in some regions contribute to a decline in both RNPD and credit spreads, despite being considered a risk parameter of the bank, due to a decline in i.

Another point of notice is that the figures illustrating the NP model are based on a 10x10 grid of point observations, implying that some sharp edges within the illustrations may be due to a limited set of observations. In comparison, those of the AMP model are based on a 49x49 grid. This difference is due to the use of Excel for the NP model, while Maple is used for the AMP model.

5.1 Risk-neutral probability of default

In figure 5.1 below, we have illustrated the RNPD of the NP model. Here, the parameters are set to $N = 10$, $H = 5$, $T = 10$, $\rho = 0.5$, $r = 0.01$, $\delta = 0.005$, $\gamma = 0.002$, while we let borrower asset volatility (σ) and borrower leverage (l) vary. The model excludes the effect of capital market frictions, and is fixed at an initial bank leverage ratio of 0.9.

Figure 5.1: NP - RNPD

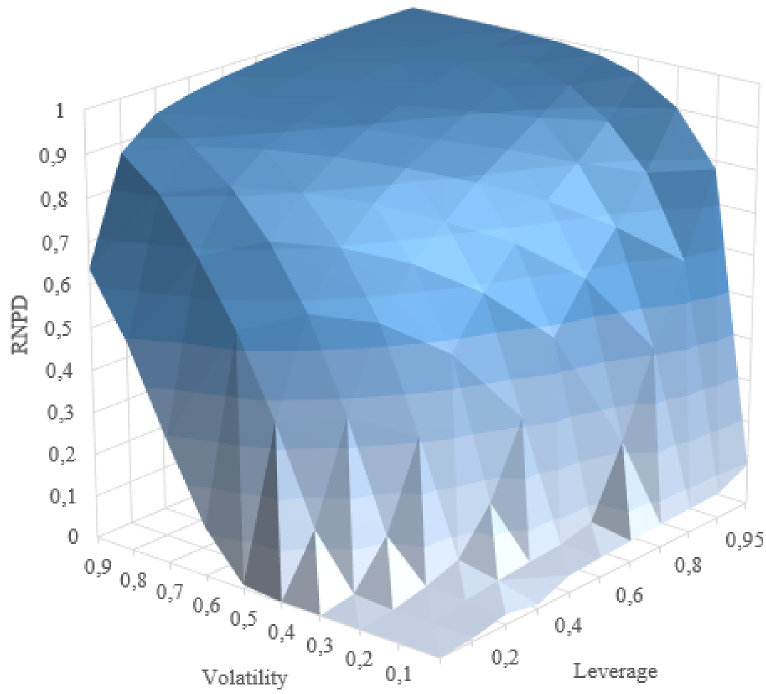

Figure 5.1: *NP model with RNPD for different borrower risk parameters (σ,l).*

From the figure 5.1, we see that both parameters of borrower risk have extensive impact on the default probability of the bank. Especially for cases in which one of the parameters approach 1, even incremental increases from the starting point of zero for the other risk parameter leads to RNPD jumping towards 1.

In figure 5.2, we have illustrated the RNPD of the AMP model excluding frictions. Here, the parameters are set to $r = 0.01$, $B = 100$, $T = 5$ and $F = 90$, while we let borrower asset volatility (σ) and borrower leverage (L) vary. The fixture of B and F implies an initial bank leverage of 0.9, corresponding to the same value as the NP figure above.

Figure 5.2: AMP - RNPD (no frictions)

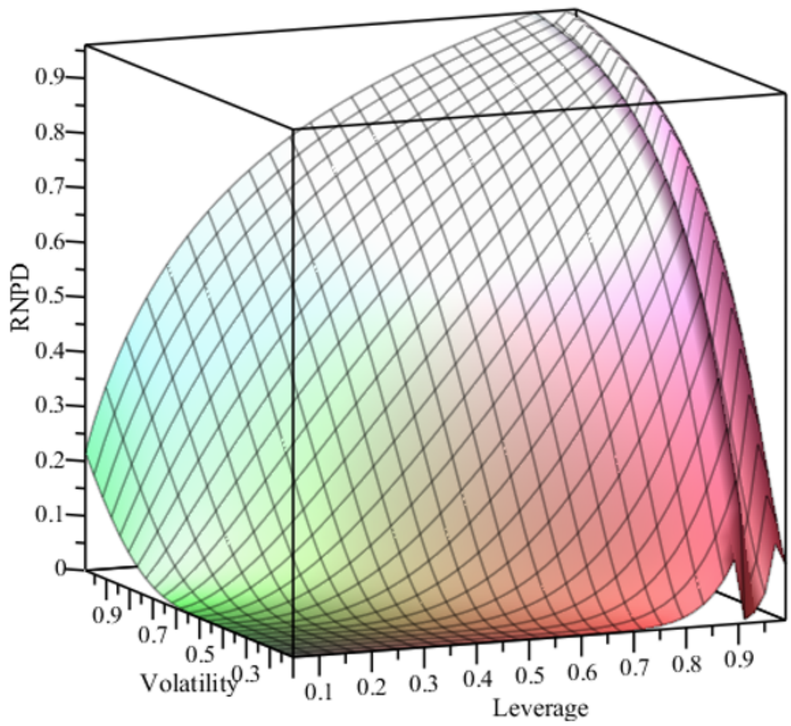

Figure 5.2: *The AMP model without frictions' estimates of RNPD as a function of borrower risk (σ and L).*

In figure 5.2 above, we find similar results as the NP model in which large values of σ and L yields a RNPD moving towards 1. However, the impact of increasing borrower risk parameters appear to have a slightly less adverse effect on the RNPD of the bank, compared to the NP model. Also, the figure displays an element of discontinuity in the upper interval of borrower leverage, as discussed in the beginning of this section.

Lastly, we have included an illustration of the AMP model including frictions in figure 5.3. The figure shows clear similarities to the model of no frictions, though it displays a somewhat lower upper limit to RNPD. Also here we find elements of discontinuity as discussed earlier in this section.

Figure 5.3: AMP - RNPD (frictions)

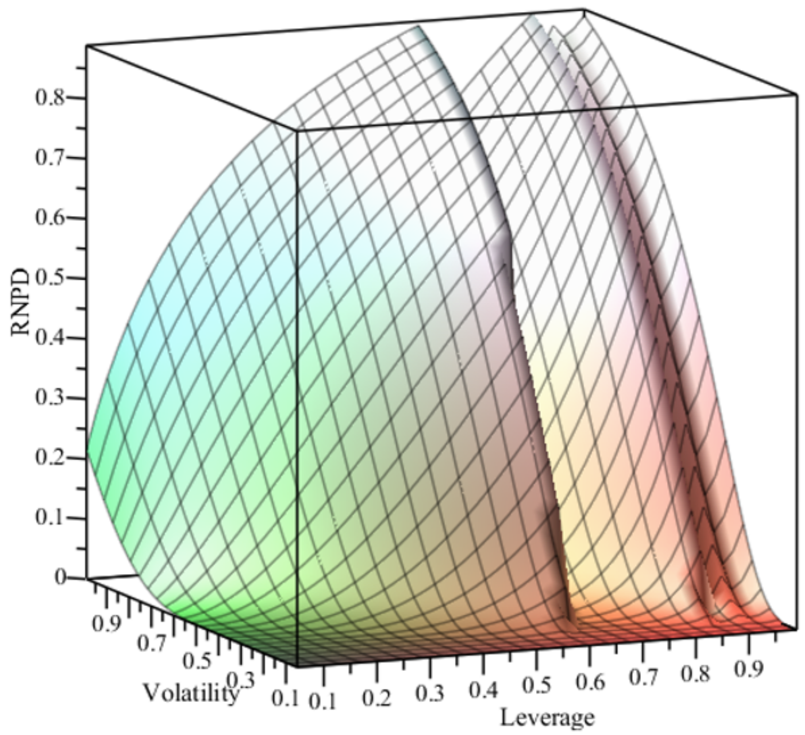

Figure 5.3: *The AMP model with frictions' estimates of RNPD as a function of borrower risk (σ and L).*

Below we have summarized the findings on RNPD in a table of results for different values of borrower risk parameters. The table illustrates that the NP model provides remarkably higher values of RNPD in the mid region of borrower risk parameters, while converges in the end points. This is in line with what we observe from the previous figures. Note that the discussion on discontinuity for the AMP model (see beginning of the Findings section) affects the results here.

Table 5.1: RNPD estimates

Input	NP	AMP - no frictions	AMP - frictions
$\sigma = 0.1$, leverage $= 0.3$	0.00	0.00	0.00
$\sigma = 0.2$, leverage $= 0.4$	0.15	0.00	0.00
$\sigma = 0.4$, leverage $= 0.6$	0.79	0.25	0.25
$\sigma = 0.6$, leverage $= 0.8$	0.97	0.75	0.44
$\sigma = 0.7$, leverage $= 0.9$	1.00	0.89	0.75

Table 5.1: *Displays estimates for RNPD under the two models for various set of borrower risk parameter inputs.*

5.2 Credit spreads on bank debt

In figure 5.4 below, we have illustrated the bank's credit spread under the NP model, and we have applied the exact same input parameters as under the RNPD section above.

Figure 5.4: NP - Credit Spread

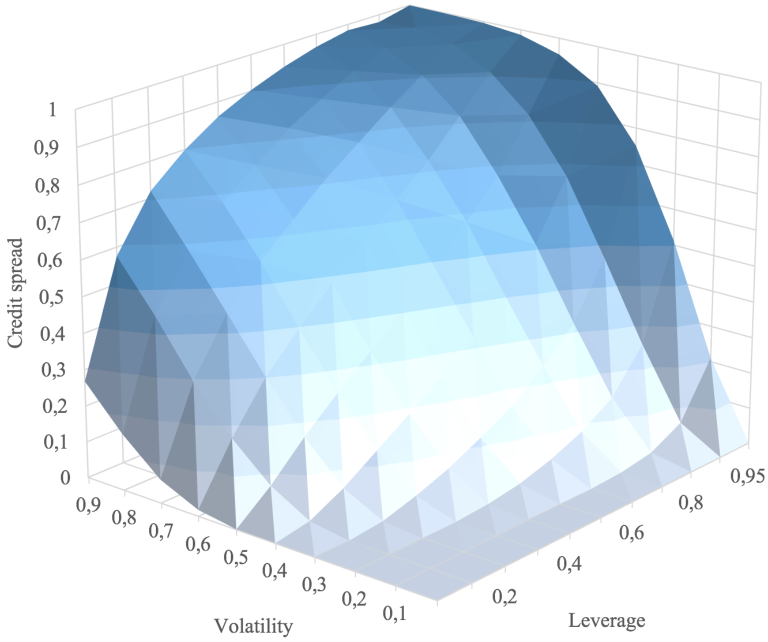

Figure 5.4: *Credit spreads on bank debt interests estimated with the NP model for different borrower risk parameters (σ and l).*

In figure 5.4, we find a clear pattern of increasing credit spread for higher borrower risk parameters as expected. However, the steepness of the surface is substantial, yielding a credit spread of almost 1 (100%) for the maximum displayed values of the risk parameters. Also, it appears that increases to borrower asset volatility have a greater impact to credit spreads at low levels, compared to that of borrower leverage.

In figure 5.5, we have illustrated the bank's credit spread under the AMP model without frictions, and we have applied the exact same input parameters as under the RNPD section.

Figure 5.5: AMP - Credit spread (no frictions)

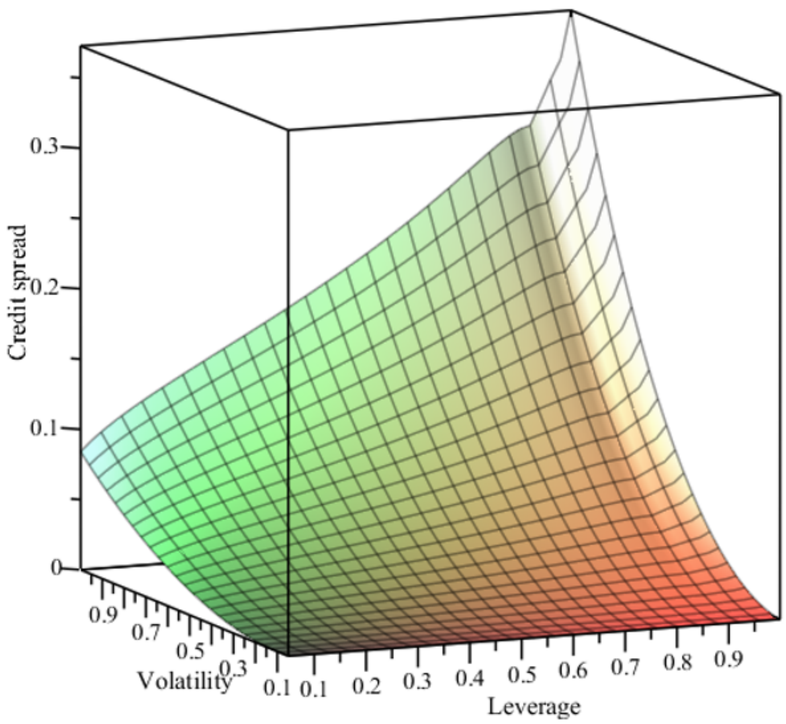

Figure 5.5: *Credit spreads on bank debt interests estimated with the AMP model without frictions for different borrower risk parameters (σ and L).*

Interestingly, figure 5.5 displays a credit spread that merely reaches slightly below 0.4, compared to nearly 1 in the NP model. Nonetheless, we find similar trends in development along the axis, despite borrower volatility appearing to induce a more convex impact on the credit spread in this model, compared to prior figures.

Lastly, we have also exhibited a version of the AMP model including capital market frictions in figure 5.6. Here, we find that the credit spread evolves to a slightly lower upper limit, as expected. The discontinuity is clearly more present when including frictions, though the overall trend appears fairly similar to the model without frictions.

Figure 5.6: AMP - Credit spread (frictions)

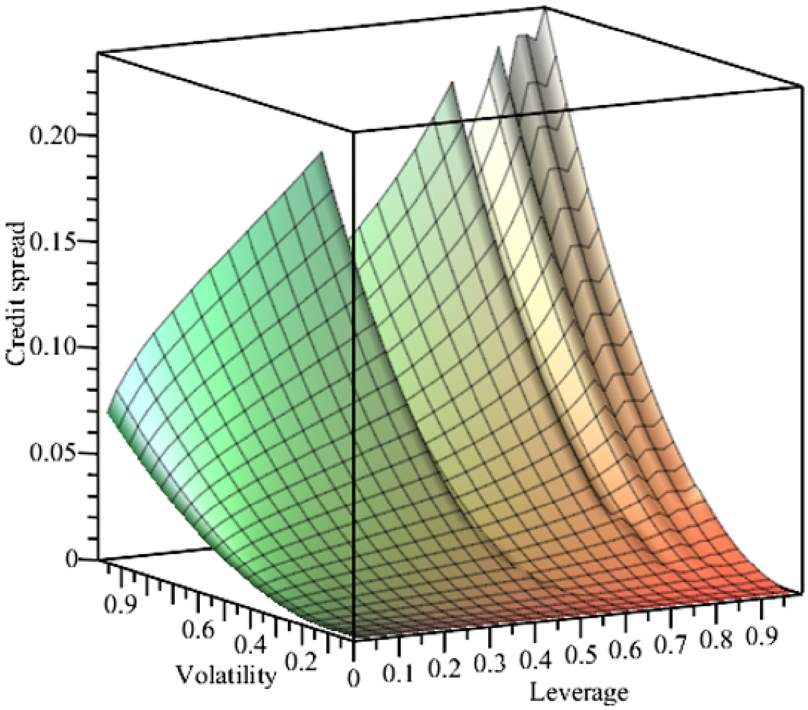

Figure 5.6: *Credit spreads on bank debt interests estimated with the AMP model with frictions for different borrower risk parameters (σ and L).*

Below we have summarized the findings on credit spreads in a table of results for different values of borrower risk parameters. The table illustrates that the NP model provides vastly higher values of credit spreads as the borrower risk parameters increases, as the previous figures illustrate. Note that the discussion on discontinuity for the AMP model (see beginning of the Findings section) affects the results here.

Table 5.2: Credit spread estimates

Input	**NP**	**AMP - no frictions**	**AMP - frictions**
$\sigma = 0.1$, leverage $= 0.3$	0.00	0.00	0.00
$\sigma = 0.2$, leverage $= 0.4$	0.01	0.00	0.00
$\sigma = 0.4$, leverage $= 0.6$	0.32	0.03	0.02
$\sigma = 0.6$, leverage $= 0.8$	0.82	0.09	0.07
$\sigma = 0.7$, leverage $= 0.9$	0.96	0.13	0.11

Table 5.2: *Displays estimates (in %) for credit spreads under the two models for various set of borrower risk parameter inputs.*

6 Analysis

In this section we discuss the differences between the two models presented by Nagel and Purnanandam (2019) and Atreya, Mjøs and Persson (2019). The discussion is comprised of two parts. First, we analyse the results shown in the previous section, emphasising the models output on RNPD and credit spreads. Secondly, we will discuss potential disconnections of the models to the banking industry that are not covered in the first part.

6.1 Analyzing RNPD and credit spreads

6.1.1 Simulation versus optimization

As presented in the theory section, the models incorporate two fundamentally different mathematical methods in order to estimate the RNPD and credit spread. Our application of the AMP's model includes an element of optimization in its estimate of the RNPD and credit spread as explained in the theory section, while the NP model's estimates are based on 10 000 simulation runs. As these methods have different effects on the results presented in the findings section, we briefly discuss their implications below.

The figures in the findings section indicates that there exists meaningful difference in the estimates of the models, especially the credit spread estimates at higher borrower risk (and the RNPD calculations at borrower leverage and volatility in the mid-section). An explanation of the significantly lower results from the AMP model compared to NP may be that AMP is based on optimizing the number of borrower defaults under certain constraints, hence we receive the RNPD and credit spread only for optimal solutions. On the other hand, simulations take a greater spectre of values into account in order to look at the performance of a system. Hence, it is reasonable to expect some deviance in the results to be driven by the use of these different methods.

Furthermore, we expect the variation in values under simulation to be highest in the mid-section compared to values estimated close to the endpoint of borrower asset and borrower volatility. This is due to to the capped volatility of the simulated values near the endpoints as the RNPD cannot become greater than one or less than zero. This means that the distance between the estimates from the simulation and optimization should converge close to the endpoints of borrower leverage and volatility compared to the middle layer. This is exactly the case for the RNPD figures.

For the credit spreads the estimates from both models converges towards zero based on the assumption that the risk-free rate is always lower than or equal to the rate of risky bonds. Nonetheless, in practice, the credit spreads can become negative under very special scenarios like during the financial crisis were Bhanot and Guo (2011) found evidence of negative credit spreads on three occasions on an American Express bond.

Further, another question is whether 10 000 simulations in the NP model are enough in accordance with the central limit theorem (CLT), which is necessary in order to provide reasonable estimates of the model. The CLT states that the distribution of a sample converges towards the true population parameter with increasing sample size. Often a sample size of 30 or more is sufficient in order for the CLT to hold (Ganti, 2019). In figure 6.1, the probability density function for various numbers of simulations are illustrated.

Figure 6.1: Simulation distributions

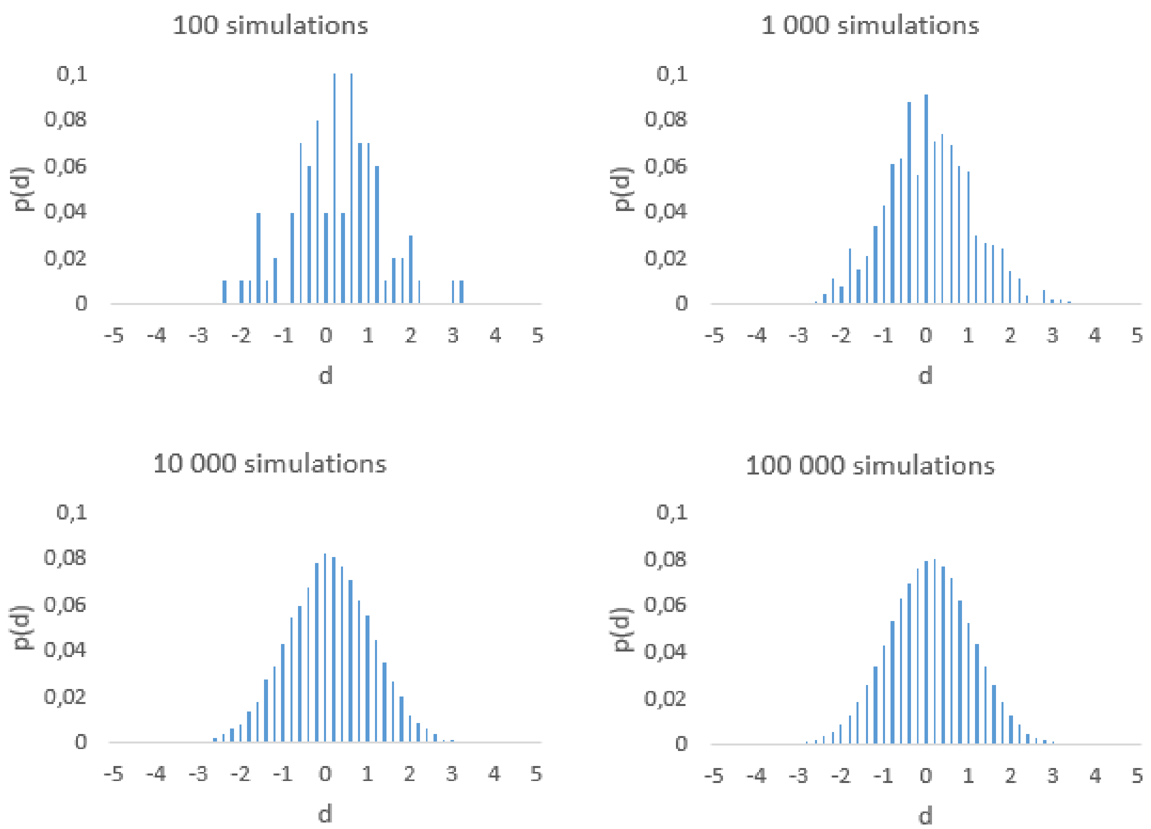

Figure 6.1: *Illustrates the distributions from 100, 1 000, 10 000 and 100 000 simulation runs. Like NP, we use 10 000 simulation runs.*

As we can see from the figures, the probability density function (PDF) based on 100 simulations is far from symmetrical. However, the PDF becomes increasingly symmetrical with a greater number of simulations. With 1000 simulations, the PDF is not perfectly symmetrical, but has improved meaningfully compared to 100 simulations. In accordance with the NP (2019), the use of 10 000 simulations appears to be a sufficient number of simulations based on the PDF, which is not far from perfectly symmetrical. As we can see from figure 6.1, the symmetry does not change much between 10 000 and 100 000 simulations, hence 10 000 simulations appears to be an appropriate number for precision. This is substantiated by Ritter et al. (2001), who argues that when iterations are inexpensive, running 10 000 simulations is satisfactory due to stable estimates.

6.1.2 Differences in time horizon

The two models presented in the findings section have an inherently different way of incorporating the element of time. While NP (2019) defines the lending and debt of the bank as bonds with a given year of maturity, AMP (2019) assumes that both the lending and debt of the bank are perpetual contracts.

Banks, like other firms tend to operate with a given time to maturity for both their debt, and the loans they provide. However, banks also typically prefers a stable capital structure with the intention of providing business for an indefinite time. Hence, the NP model is only appropriate in the evaluation of a given limited time horizon, while the AMP model is also suited for an indefinite time horizon. This might seem rather negligible at first sight, but for models that are intended to represent banks in general, this implies that an argument must be made for what is a reasonable event horizon in the NP model. As this may be difficult to define for general banks, an arbitrary time t in the model may severely impact its results.

For illustrative purposes we have exemplified the effect in figure 6.2 for RNPD, showing that different time horizons have significant impact on RNPD for equal risk parameters. Here, the parameters are set equal to those used in the theory section. From the figures we can see that increasing the time to loan maturity (T) leads to increased RNPD, all else equal.

Figure 6.2: Effect of time on RNPD (NP model)

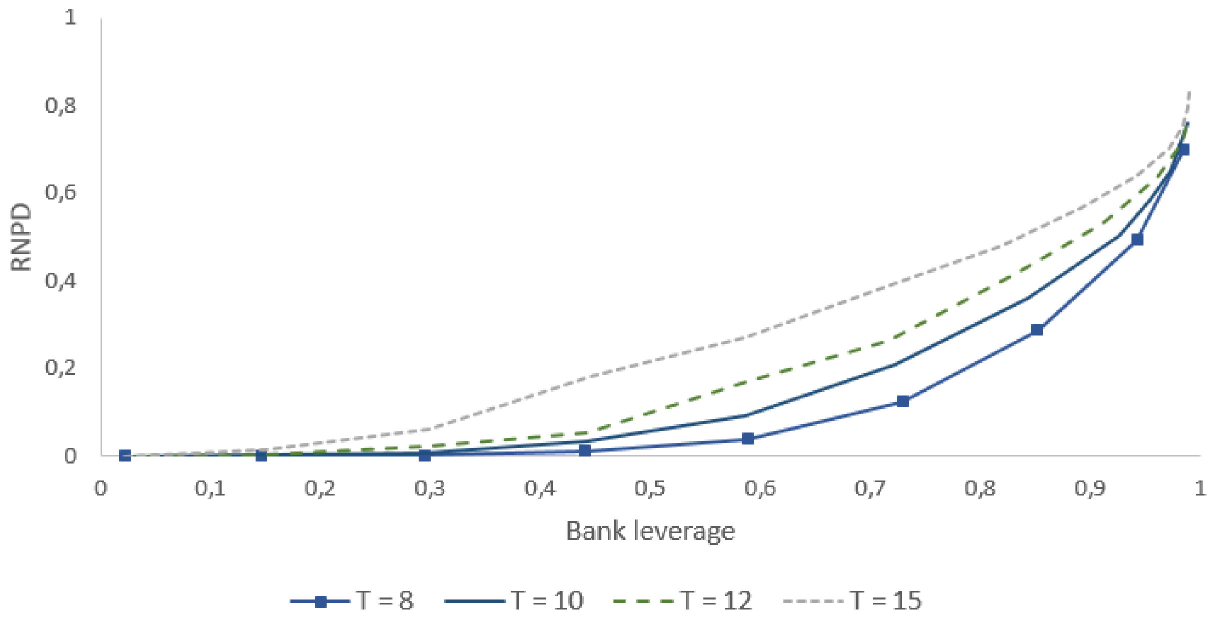

Figure 6.2: *Illustrates the effect of different borrower loan maturities on RNPD. $N = 10$ and $H = 5$ is held constant, while T varies among the various lines.*

In the figures of RNPD in the findings section the issue of time is solved endogenously (see Harrison (1985) or Lando (2004)), allowing for the specification of time in the AMP model. Hence, the calculations of RNPD are done under the same time horizon for both models.

The case is however different for the computation of credit spreads. Here, the AMP model is fundamentally based on a perpetual time horizon for all loans and debt, implying that fixing a time of maturity for bank debt is a rather complicated matter. Our approach, as described in the method section is to convert the continuous credit spread provided by the AMP model to an annualized rate. However, as this rate is calculated without a specific point of maturity for the bank debt, it is not entirely comparable to the results from the NP model. Hence, some of the deviation between the two models' respective credits spreads may be due to the issue of time horizon.

Another important issue is related to the effect of a limited time horizon in the NP model. Despite the downside of an arbitrary time to maturity as discussed above, it has an important impact on the collateral of a loan held by the bank. In the AMP model, the bank only issues new loans in the event of a borrower default. Hence, a long period of positive borrower asset evolution would imply that the collateral related to the bank's loan would accumulate accordingly. This is depicted in figure 6.3 in which we look at a case of a meaningful borrower asset value growth over a period of 30 years, found by trial and error for $\sigma = 0.2$.

In a typical bank, loans will at some point of time be repaid and the bank will reissue their respective payoff. Hence, accumulated collateral of such a loan will be reset to that of the initial borrower leverage ratio. For a long sustained period of borrower asset value growth as in figure 6.3, this will not be accounted for in the AMP model. However, the fixed time to maturity in the NP model captures this effect, as depicted in figure 6.3.

Figure 6.3: Borrower asset value as collateral

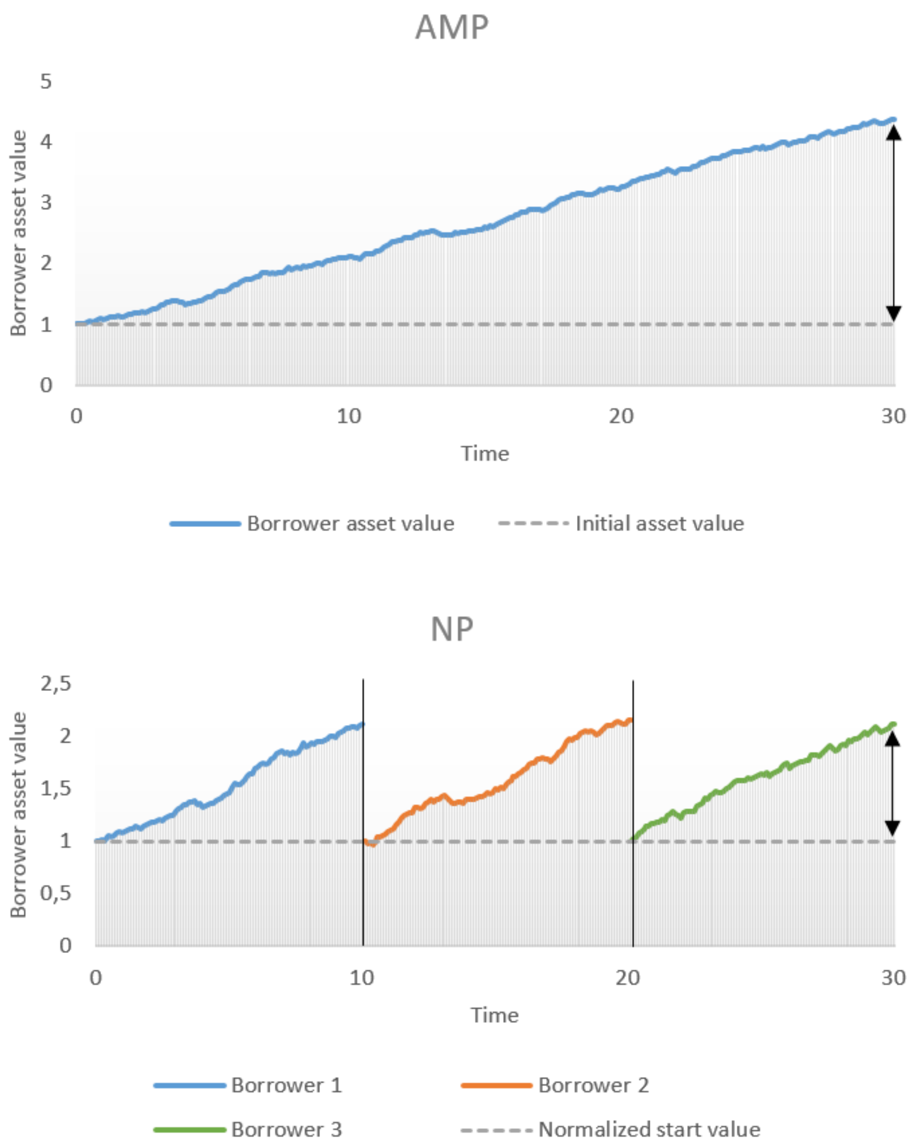

Figure 6.3: *Illustrates the borrower asset value and its role as collateral for a loan in a period of positive borrower asset value evolution. The blue line displays the development of borrower 1 for the AMP model, while the orange and green lines display two additional borrowers for the NP model. The doted line at value 1 is the defined initial value of borrower assets in both models. The solid black lines demonstrate that the loans in the NP model are repaid and redistributed to new loans every 10 years as we set* $T = 10$. *The arrows on the right hand side illustrates the collateral (above initial level) in period 30.*

Here, we see that as the loan reaches its maturity (set to $T = 10$), the bank captures its payoff, and then reissues a loan at equal initial parameters (here we have set the initial borrower asset value to 1[12]). In effect, the accumulated collateral may become unrealistically large in the AMP model in cases of sustained borrower asset value growth, as depicted by the arrows on the right hand side of the graphs in figure 6.3 (in addition to the collateral from the initial borrower leverage), which are considerably larger for AMP. As a result, the AMP model may underestimate the realistic risk of loan losses in such periods due to the artificially high collateral that follows. Accordingly, this characteristic contributes to explaining the lower observations of both RNPD and credit spreads for the AMP model in the findings section.

6.1.3 Zero-coupon- versus continuous coupon bonds

Another element of difference to the models concerns their assumptions on coupon payments. The loans and debt of the bank in the NP model are structured as zero-coupon bonds, implying that all the income and costs regarding the bonds are present in their respective face values. In the AMP model, they are structured with a continuous coupon or interest accompanying their lack of maturity. However, in the banking industry, lending and debt primarily consists of loans with coupon or interest payments (Lindquist et al., 2016). Taking the Norwegian financial sector as an example, banks' debt mainly consists of customer deposits and covered bonds, both heavily dependent on their interest/coupon payments (SSB, 2019a). Hence, the AMP approach appears the more realistic one.

One major implication of the different assumptions regarding the coupon payments above is that the bank's asset value becomes the only driver of default in the NP model. As the model excludes periodical coupon payments, the bank may only default on its debt at the time of maturity. This is a rather improbable implication of the model's coupon assumptions in light of the financial sector. Fluctuations in the ability to pay interest on debt may in fact be a crucial indicator of bank defaults, regardless of the time to maturity of its debt. This element is at least to some degree captured in the AMP model as it endogenously calculates the number of borrower defaults that optimizes the bank's enterprise value on the basis of the calculated coupon payments from borrowers, affecting the interest paid on the bank's debt and the bank's optimal time of default.

[12]This is a simplification as the payoff from borrower 1 would imply a larger loan to borrower 2 (and the payoff from both 1 and 2 leads to a larger loan to borrower 3) and hence a greater initial borrower asset value for borrower 2 and 3 for a fixed initial borrower leverage ratio. Nonetheless, the insight from the example holds.

The same argument holds for the borrower asset process. In the NP model, borrowers may only default at the time of their respective maturity, compared to borrowers in the AMP model defaulting as their asset value reaches their calculated threshold value, which depends directly on their coupon payments. The implications of this observation are severe, and may be illustrated by a simplified example as in figure 6.4. Here, we have mapped a standard Brownian motion with annual volatility of 20%, representing the evolution of borrower asset value, starting at the value 1. Setting the NP time of maturity to $T = 10$, we look at the bank's loss given a weak borrower asset evolution. The key takeaway is that the bank under the NP model will lose the entire difference between the borrower asset value at the time of maturity and the face value of the loan (set to 0.7), given by the arrow at the right hand side. However, under the AMP model, the borrower will default the first time its asset value breaches the threshold value (also set to 0.7). The bank will then redistribute the recaptured amount into a new loan to a borrower at the same initial borrower leverage ratio. Hence, the bank will recapture the safety from collateral at the fixed initial leverage ratio, ensuring that it loses no more than that of the first default during the given time period for this example. As a result, the loss of the bank under the AMP model becomes significantly lower than under the NP model, which is reflected in a lower RNPD and credit spread in the findings section.

However, the argument also goes the other way, as the case may be made where the AMP model would reflect a loss from a borrower default, while the NP model does not, should i.e. the borrowers asset value increase substantially in the last few years of the example. Nonetheless, the example illustrates how the loss potential for a given time period is significantly larger under the NP model.

Figure 6.4: Borrower asset value and default

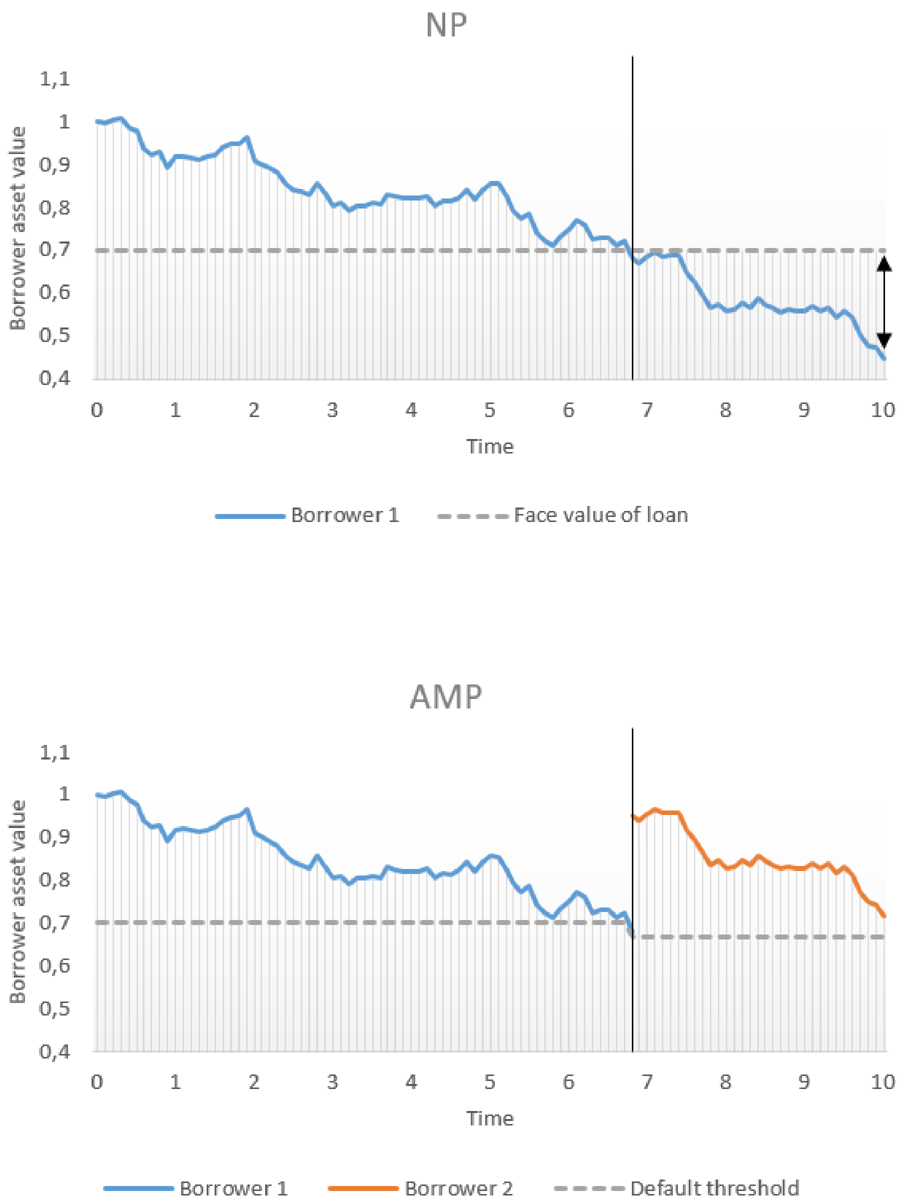

Figure 6.4: *Illustrates the borrower asset value and default threshold in NP and AMP over time. The blue line displays the development of borrower 1. The doted line at value 0.7 is the default threshold in AMP and face value of loan in NP. The solid line demonstrates the time of default in AMP, while the orange line is the asset value of the new borrower after default. In NP, the borrowers default first at $T = 10$, hence the bank loss is higher compared to AMP due to the negative development in borrower asset value.*

Looking at the figures on credit spread for AMP and NP in the findings section, this argument appears to bode well with the observed differences. Especially, as borrower asset volatility increases, the NP model displays signs of concavity for credit spread, compared to convexity for the AMP model. This may partly be explained by the characteristics described in the example above, greatly increasing the bank's downside risk under the NP model compared to the AMP model, especially as borrower asset volatility increases.

6.1.4 Capital market frictions in the AMP model

The AMP model in its complete form includes parameters for capital market frictions of income tax and bankruptcy cost, as opposed to the NP model assuming perfect capital markets. From a practical point of view, this makes the AMP model more realistic as there are hardly any banks escaping such frictions in the real world. However, as NP lacks such parameters, our findings section provides two versions of the AMP model; with and without frictions.

A key implication of introducing capital market frictions to the AMP model is that the bank's initial enterprise value may surpass its initial asset value denoted by its initial loan to a borrower. This is due to the trade-off function given by X(t) (equation 2.46) which optimizes the difference between increased enterprise value from tax benefits less the bankruptcy cost. Hence, the bank may withstand a greater number of borrower defaults before defaulting on its own debt for a given set of risk parameters. This is in fact what the findings appear to illustrate, as the RNPD and credit spread for a given set of parameters mostly trends lower for the AMP model including financial market frictions compared to the one that does not. It is here important to note that the model's technicalities responsible for the areas of discontinuity changes somewhat when including frictions, largely explaining what appears to be areas of deviation from this argument.

6.1.5 Number of borrowers

The two models provide quite different borrower structures for a bank. In the NP model, the bank provides loans to an unspecified number of borrowers divided into a set of cohorts, capturing the effects of diversification in the banking industry. In contrast, the AMP model operates with a more simplified model with only one borrower at a time. From an industry perspective, the approach of NP (2019) is here more realistic.

One of the main implications of a multiple borrower structure is that the asset values of the borrowers are typically not perfectly correlated. Hence, if a group of borrowers default due to weak asset performance, others may perform better, reducing the negative impact on the bank's payoff from defaults. Despite this effect being lost in the AMP model with only one borrower at a time, this may partly be captured by applying a somewhat lower estimate of borrower asset volatility in the model.

6.2 Further deviations from the banking industry

As discussed in the prior analysis, both the models of NP (2019) and AMP (2019) display strengths and weaknesses in comparison with each other and the banking industry. However, there are some points of criticism left out of the discussion so far. These are not unique for the models analysed in this thesis, but rather general for this field of research, due to the complexity of accounting for an increasing number of bank characteristics in a structural model.

One such point of notice is the vast number of types of loans in the financial sector. In practice, loans can be provided with and without collateral, with varying maturities, with floating interest rates pegged to external factors, to vastly different borrowers, etc. These characteristics are extremely complicated to model specifically, hence explaining the need for simplifying assumptions such as a common volatility estimate and exclusively modeling loans with collateral.

Another point of notice is the variation in debt structures found in banks across the world. As discussed previously in the analysis, Norwegian banks utilize deposits and covered bonds as their primary source of debt funding (SSB, 2019a). The models could here have adapted a branched debt structure to incorporate the complexity of the banking industry such as Sundaresan and Wang (2014). However, it may be argued that similarities of these debt securities should limit the potential disadvantage of simplifying the debt as a single type of instrument such as the models analyzed in this thesis.

A potentially larger deviance from the banking industry is that the income stream tends to stem from far more than one source. Taking Norwegian banks as an example, the last 10-years of empirical data depicts almost 25% of income as derived from other sources than interest income (SSB, 2019b). Other income sources may here include provisional income, financial market activities, dividend income from subsidiaries, insurance business, etc. Hence, the simplification of a bank where its only business regards asset backed loans may disregard important elements of typical banks, which could have meaningful impact on aspects of bank evaluation.

Last, but not least, geographical presence of a bank may have considerable implications for what are deemed realistic input parameters. I.e. a large number of smaller Norwegian banks are only narrowly present in a geographical sense, implying that regional economic factors may dictate vastly different values of borrower asset correlation (ρ) and asset volatility (σ), compared to larger banks (Cook, 2019). However, as the models analyzed above are focused on general banks, this effect is left unaccounted for.

7 Conclusion

Throughout our thesis we have presented a narrow overview of the research field of structural credit models and their applicability to banks. We have focused on two of the newer contributions to the field by Nagel and Purnanandam (2019) and Atreya, Mjøs and Persson (2019), and provided a thorough, but not exhaustive, comparison and evaluation of these models.

In our work, we have put meaningful emphasis on the objectivity of our evaluation, especially due to our supervisor being among the authors of the AMP model. We have therefore approached our analysis from strictly theoretical and empirical view, based on the results we have found in combination with our accumulated knowledge of banks.

We have found that the different approaches of the two models provide similarly logical results for both RNPD and credit spreads, each displaying strengths and weaknesses compared to the banking industry. Both models account for the crucial characteristic of banks in that the value of their loans, and therefore their assets, have a naturally capped upside. Accordingly, both models rely on the use of a standard Brownian motion to describe the uncertainty of borrower asset values, before valuing the banks claim on these through their respective loans.

The NP model adapts the bank characteristics of multiple borrowers of which their assets are not perfectly correlated. It also provides an arguably sufficient number of simulations of its borrower asset processes, providing a reasonable estimate for the asset volatility of the bank. As the model operates with a given time to maturity for both loans to borrowers and bank debt, it appears well suited to evaluate any single bank in which duration for loans and debt maturity are given. Nonetheless, its assumption of loans and bank debt as zero-coupon bonds is a clear deviation from typical bank practice. Combined with a fixed time horizon, this has meaningful impact on the modeled bank loss from borrower defaults due to the bank lacking the ability to capture its claim on collateral at the moment the borrower first becomes insolvent. However, the fixed time horizon also implies a periodical reset of collateral, which corresponds to limited time horizon of any single loan.

The AMP model structures the respective loans and debt of a bank as continuously paying bonds without a given maturity, hence avoiding the issues of fixed time, but disregarding the effect of periodically reset collateral. It also incorporates capital market frictions, which are both realistic and relevant to banks across the world. As the model utilizes optimization, it disregards the need for a bank asset volatility estimate, implying that the bank actively chooses its optimal point of default. Hence, the model is well suited in its application to a generalized bank, and may therefore be argued to fit regulatory purposes that are encompassing the majority of the banking industry. Nonetheless, also the AMP model displays some weakness, such as relying on a single borrower at a time. This choice of model structure may to some degree be defended by adjusting the volatility estimate for borrower asset values, but still represents a clear disconnection to the banking industry.

We find the deviations of the models illustrated in our findings section to be closely linked to the discussion above. I.e. the NP model tends to display both greater RNPD and credit spread for equal values of borrower risk parameters. Here we point to the effect of an arbitrary time horizon, and the structure of zero-coupon bonds as driving the bank risk artificially high in the NP model. This is especially supported by the appearance of concavity displayed when borrower asset volatility increases, compared to convexity for the AMP model. We also point to the effect of simulations compared to optimization, in which a greater number of scenarios are accounted for in the NP model. We also discuss the impact of capital market frictions within the AMP model, allowing the bank to reduce both its RNPD and credit spread for a given set of borrower risk parameters, due to the maximization of its trade-off function.

Lastly, we touch upon some common disconnections of structural credit models to the present banking industry. We find that their lack of loan diversity, bank debt diversity, bank income diversity and various borrower characteristics contributes to somewhat distancing the models from today's banks. However, we believe the models present reasonable simplifications of such banks, and recognize the rather extreme complications and impracticality of accurately accounting for the vast amount of details regarding the banking industry.

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Appendix

In this section we present each and every equation provided in the papers of NP and AMP. We begin by introducing the parameters utilized in the models, followed by the respective series of equations chronologically as in the paper.

A1 Nagel and Purnanandam model

A1.1 Parameter definitions

r -> risk-free rate

l -> borrower leverage ratio

σ -> borrower asset volatility

ρ -> borrower asset correlation

δ -> depreciation rate

γ -> payout level

A1.2 Model equations

Borrower asset value process

$$\frac{dA_t^{\tau,i}}{A_t^{\tau,i}} = (r-\delta)dt + \sigma(\sqrt{\rho}dW_t + \sqrt{1-\rho}dZ_t^{\tau,i}). \tag{.1}$$

Face value of borrower loan

$$F_1(\mu) = le^{\mu T}. \tag{.2}$$

Payoff at maturity $t = T - \tau$

$$L_{T-\tau}^{\tau,i}(\mu) = \min[A_{T-\tau}^{\tau,i}, F_1(\mu)]. \tag{.3}$$

Borrower leverage for competitively priced loans

$$l = e^{-rT} E^{\mathbb{Q}}_{-\tau}[L^{\tau,i}_{T-\tau}(\mu)]. \tag{.4}$$

Aggregate value of collateral in cohort τ

$$A^{\tau}_{T-\tau} = \frac{1}{N}\exp\{(r-\delta)T - \frac{1}{2}\rho\sigma^2 T + \sigma\sqrt{\rho}(W_{T-\tau} - W_{-\tau})\}, \tag{.5}$$

and its aggregate log asset value

$$a^{\tau}_{T-\tau} = \frac{1}{N}[(r-\delta)T - \frac{1}{2}\sigma^2 T + \sigma\sqrt{\rho}(W_{T-\tau} - W_{-\tau})]. \tag{.6}$$

Payoff at maturity received by bank

$$L^{\tau}_{T-\tau}(\mu) = \frac{1}{N}[A^{\tau}_{T-\tau}\Phi\{d_1(\mu)\} + F_1(\mu)\Phi\{d_2(\mu)\}], \tag{.7}$$

where

$$d_1(\mu) = \frac{\ln F_1(\mu) - a^{\tau}_{T-\tau}}{\sqrt{1-\rho}\sqrt{T}\sigma} - \sqrt{1-\rho}\sqrt{T}\sigma, \tag{.8}$$

and

$$d_2(\mu) = -\frac{\ln F_1(\mu) - a^{\tau}_{T-\tau}}{\sqrt{1-\rho}\sqrt{T}\sigma}. \tag{.9}$$

New loans issued to borrowers at maturity

$$F_2(\mu) = L^{\tau}_{T-\tau} e^{\mu T}. \tag{.10}$$

Reset borrower asset value for new loans issued

$$A^{\tau,i}_{(T-\tau)+} = \frac{L^{\tau}_{T-\tau}}{l}. \tag{.11}$$

Payoff at maturity for new loans received by bank

$$L^{\tau}_{2T-\tau}(\mu) = \frac{1}{N}[A^{\tau}_{2T-\tau}\Phi(d_3) + F_2(\mu)\Phi(d_4)], \tag{.12}$$

where

$$d_3 = \frac{\ln F_2(\mu) - a^{\tau}_{2T-\tau}}{\sqrt{1-\rho}\sqrt{T}\sigma} - \sqrt{1-\rho}\sqrt{T}\sigma, \tag{.13}$$

and

$$d_4 = -\frac{\ln F_2(\mu) - a^{\tau}_{2T-\tau}}{\sqrt{1-\rho}\sqrt{T}\sigma}. \tag{.14}$$

Aggregate value of banks loan portfolio at time $t = H$

$$V_H = \sum_{\tau<H} e^{-r(\tau+T-H)} E^{\mathbb{Q}}_H[L^{\tau}_{2T-\tau}] + \sum_{\tau\geq H} e^{-r(\tau-H)} E^{\mathbb{Q}}_H[L^{\tau}_{T-\tau}]. \tag{.15}$$

Bank payout at time $t = H$

$$Y_H = V_H(1 - e^{-\gamma H}). \tag{.16}$$

Ex-dividend bank equity value at time $t = H$

$$S_H = \max[0, V_H - Y_H - D]. \tag{.17}$$

Bank debt value at time $t = H$

$$B_H = V_H - Y_H - S_H. \tag{.18}$$

A2 Atreya, Mjøs and Persson model

A2.1 Parameter definitions

r -> continuos risk-free rate

L -> borrower leverage ratio

σ -> borrower asset volatility

θ -> bank income tax rate

α -> bankruptcy cost of the bank

A2.2 Model equations

Borrower asset value process

$$\frac{dA_t}{A_t} = rdt + \sigma dW_t. \tag{.19}$$

Borrower leverage at loan origination

$$L = \frac{\hat{B}}{A_0}. \tag{.20}$$

Value of the borrower loan (Black and Cox, 1976)

$$B(A_t) = \frac{c\hat{B}}{r} - (\frac{c\hat{B}}{r} - \bar{A})(\frac{A_t}{\bar{A}})^{-\gamma}, \tag{.21}$$

where

$$\gamma = \frac{2r}{\sigma^2} > 0. \tag{.22}$$

Price of security paying 1 in the case of borrower default

$$G_t = (\frac{A_t}{\bar{A}})^{-\gamma}. \tag{.23}$$

The process G_t using Itô's lemma

$$\frac{dG_t}{G_t} = rdt - \sigma\gamma dW_t. \tag{.24}$$

The borrower defaults at time $\tau(1)$, where

$$\tau(1) = \inf\{t \geq 0 : A_t = \bar{A}\} = \inf\{t \geq 0 : G_t = 1\}. \tag{.25}$$

Borrower threshold value (Black and Cox, 1976)

$$\bar{A} = \Psi\hat{B}, \tag{.26}$$

where

$$\Psi = \frac{c}{r}\frac{\gamma}{\gamma+1} < 1. \tag{.27}$$

Initial value of state price of the borrower's default

$$G = G_0 = (L\Psi)^{\gamma}. \tag{.28}$$

Assuming the loans are granted at par, the coupon rate (c) can be found from

$$\frac{c}{r}(1 - \frac{G}{\gamma+1}) = 1. \tag{.29}$$

Expressing deterministic sequence of loan amounts to borrower $j+1$

$$\hat{B}^{j+1} = \bar{A}^{j} = B\Psi^{j}. \tag{.30}$$

Bank asset value when the first borrower is solvent

$$B_t = B(A_t) = \frac{c\hat{B}^1}{r} - (\frac{c\hat{B}^1}{r} - \bar{A})(\frac{A_t}{\bar{A}})^{-\gamma} = \frac{c\hat{B}^1}{r}(1 - \frac{G_t}{\gamma+1}). \tag{.31}$$

G_t can be expressed by an arithmetic Brownian motion Y_t with dynamics

$$dY_t = vdt - dW_t, \tag{.32}$$

where

$$v = \frac{r}{\sigma\gamma} - \frac{\sigma\gamma}{2}. \tag{.33}$$

G_t can then be expressed by Y_t as

$$G_t = Ge^{\sigma\gamma Y_t}, \tag{.34}$$

where

$$d = \frac{1}{\sigma\gamma}\ln(\frac{1}{G}). \tag{.35}$$

Default time of first borrower expressed by Y_t

$$\tau(1) = \inf\{t \geq 0 : G_t = 1\} = \inf\{t \geq 0 : Y_t = d\}. \tag{.36}$$

Default time of the banks borrower number $n = 1, 2, ...$

$$\tau(n) = \inf\{t \geq 0 : G_t = \frac{1}{G^{n-1}}\} = \inf\{t \geq 0 : Y_t = n \cdot d\}. \tag{.37}$$

By defining

$$\eta_t = \sup_{0 \leq s \leq t} Y_s, \tag{.38}$$

the number of borrower defaults up to time t is

$$N_t = \lfloor \eta_t / d \rfloor. \tag{.39}$$

State price of default of the borrower at time t

$$\Pi_t = Ge^{\sigma\gamma(Y_t - N_t d)}. \tag{.40}$$

Face value of borrower loan at time t

$$\hat{B}_t = B\Psi^{N_t}. \tag{.41}$$

Time t value of the bank's asset

$$B_t = \frac{c}{r} B\Psi^{N_t}(1 - \frac{\Pi_t}{\gamma + 1}). \tag{.42}$$

The bank defaults at time

$$\tau_{\bar{B}} = \inf\{t \geq 0 : B_t = \bar{B}\}. \tag{.43}$$

Minimum of the bank's asset value up to time t

$$m_t = \inf_{0 \leq s \leq t} B_s. \tag{.44}$$

Distribution of the bank's default time

$$Q(\tau_{\bar{B}} < t) = Q(m_t < \bar{B}). \tag{.45}$$

The bank's default probability (see Harrison,1985)

$$Q(m_t < \bar{B}) = Q(\eta_t > \bar{n} \cdot d), \tag{.46}$$

where

$$\bar{n} = n(\bar{B}) = \bar{m} + 1 - \frac{\ln(\bar{G})}{\ln(G)}, \tag{.47}$$

and

$$\bar{m} = m(\bar{B}) = \max\{n : B\Psi^n > \bar{B}\} = \lfloor \frac{\ln(\bar{B}) - \ln(B)}{\ln(\Psi)} \rfloor, \tag{.48}$$

and

$$\bar{G} = (\gamma + 1)(1 - \frac{r}{c}\frac{\bar{B}}{B\Psi\bar{m}}). \tag{.49}$$

$\tau_{\bar{B}}$ has an inverse Gaussian (IG) distribution

$$\tau_{\bar{B}} \sim \mathrm{IG}(\frac{n_{\bar{B}} \cdot d}{v}, (\bar{n} \cdot d)^2). \tag{.50}$$

Time 0 value of the state price of the bank's default

$$\Pi_{\bar{B}} = G^{\bar{n}}. \tag{.51}$$

State price of the bank's default when L approaches 100%

$$\lim_{L \nearrow 1} \Pi_{\bar{B}} = (\frac{B}{\bar{B}})^{-\gamma}. \tag{.52}$$

Shareholders maximize time 0 value by solving

$$E(B) = \sup_{\tau} \mathbb{E}[\int_0^{\tau} (cB\Psi^{N_t} - iF)e^{-rt}dt], \tag{.53}$$

which is solved by the following

$$n^* = \lceil \frac{\ln(iF) - \ln(cB)}{\ln\Psi} \rceil, \tag{.54}$$

and

$$\bar{\tau}^* = \tau(n^*), \tag{.55}$$

and

$$E(B) = B - \{\frac{iF}{r} - (\frac{iF}{r} - \bar{B}^*)G^{n^*}\}. \tag{.56}$$

The bank's optimal default threshold

$$\bar{B}^* = B\Psi^{n^*}. \tag{.57}$$

Time value 0 of the bank's own debt

$$D(B) = \frac{iF}{r} - (\frac{iF}{r} - \bar{B}^*)G^{n^*}. \tag{.58}$$

Interest rate paid on the bank's debt

$$i = r(\frac{1 - \frac{(G\Psi)^{n^*}}{L_B}}{1 - G^{n^*}}), \tag{.59}$$

where

$$L_B = \frac{F}{B}. \tag{.60}$$

Time 0 value of the tax benefit of the bank from debt financing

$$T(\bar{n}) = \frac{\theta iF}{r}(1 - G^{\bar{n}}). \tag{.61}$$

Time 0 value of the bankruptcy cost of the bank from debt financing

$$C(\bar{n}) = \alpha\bar{B}G^{\bar{n}} = \alpha B(\Psi G)^{\bar{n}}. \tag{.62}$$

Time 0 sum of the bank's asset value and trade-off value

$$V(\bar{n}) = B + X(\bar{n}), \tag{.63}$$

where

$$X(\bar{n}) = T(\bar{n}) - C(\bar{n}) = \frac{\theta iF}{r}(1 - G^{\bar{n}}) - \alpha B(\Psi G)^{\bar{n}}. \quad (.64)$$

Shareholders optimize the time 0 equity value for a given amount of bank leverage by solving

$$E_f(B) = \sup_{\tau}\mathbb{E}[\int_0^{\tau}(cB\Psi^{N_t} - (1-\theta)iF)e^{-rt}dt], \quad (.65)$$

Which is solved by the following

$$n_f^* = \lceil \frac{\ln[(1-\theta)iF] - \ln(cB)}{\ln\Psi} \rceil, \quad (.66)$$

and

$$\bar{\tau}_f^* = \tau(n_f^*), \quad (.67)$$

and

$$E_f(B) = B - \{\frac{(1-\theta)iF}{r} - (\frac{(1-\theta)iF}{r} - \bar{B}_f^*)G^{n_f^*}\}, \quad (.68)$$

where

$$\bar{B}_f^* = B\Psi^{n_f^*}. \quad (.69)$$

The time 0 value of the bank's payment to creditors before tax

$$\frac{iF}{r} - (\frac{iF}{r} - \bar{B}_f^*)G^{n_f^*}. \quad (.70)$$

The time 0 value of the bank's tax deductions on interest payments

$$T(n_f^*) = \frac{\theta iF}{r}(1 - G^{n_f^*}). \tag{.71}$$

Net time 0 value of the bank's debt liability

$$D_f(B) = \frac{(1-\theta)iF}{r} - (\frac{(1-\theta)iF}{r} - \bar{B}_f^*)G^{n_f^*}. \tag{.72}$$

Net time 0 value of the bank's debt liability in the perspective of creditors

$$D_f^c(B) = \frac{iF}{r} - (\frac{iF}{r} - (1-\alpha)\bar{B}_f^*)G^{n_f^*}. \tag{.73}$$

Setting iF to maximum value for optimal enterprise value

$$(iF)^* = \frac{cB\Psi^{n_f^*-1}}{1-\theta}. \tag{.74}$$

Trade off as a function of the number of borrower defaults with repect to t $\in \mathbb{R}^+$

$$X(t) = \frac{\theta}{r}\frac{cB\Psi^{(t-1)}}{(1-\theta)}(1 - G_t) - \alpha B(\Psi G)^t = B\Psi^t(\theta K(1 - G^t) - \alpha G^t), \tag{.75}$$

where the constant

$$K = \frac{\gamma + 1}{\gamma(1-\theta)} > 1. \tag{.76}$$

Optimal number of borrower defaults

$$n^* = \lfloor t^* \rfloor + 1\{X\}, \tag{.77}$$

where

$$t^* = \frac{\frac{\theta K \ln\Psi}{(\theta K + \alpha)\ln(\Psi G)}}{\ln G}, \tag{.78}$$

and

$$1\{X\} = \begin{cases} 1 \text{ if } X(\lfloor t^* \rfloor) > X\lceil t^* \rceil) \\ 0 \text{ otherwise.} \end{cases} \tag{.79}$$

The bank's optimal cash flow dedicated to debt service

$$(iF)^* = \frac{cB\Psi^{n^*-1}}{1-\theta} = rK\bar{B}^*. \tag{.80}$$

The value of the bank's optimal debt

$$D(B)^x = \frac{(iF)^*}{r} - (\frac{(iF)^*}{r} - (1-\alpha)B\Psi^{n^*})G^{n^*}. \tag{.81}$$

The corresponding interest rate on bank debt

$$i = \frac{r}{1-(1-\frac{1-\alpha}{K})G^{n^*}}. \tag{.82}$$

The optimum enterprise value of the bank

$$V(B)^* = B + \frac{\theta(iF)^*}{r}(1-G^{n^*}) - \alpha B(\Psi G)^{n^*} = B + X(n^*). \tag{.83}$$