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Private Real Estate Investing: Expectations vs. Reality

An empirical analysis on the characteristics of private real estate in Oslo between 2005-2018

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Abstract

In this thesis, we analyze the characteristics of private real estate (PRE) as an asset. The analysis is done in several increments, starting from the one-year holding period index risk and return of PRE. Each subsequent model adjusts the risk and return measure of PRE to either account for a market imperfection, or for a retail investor perspective. After all adjustments are implemented, we are left with a final model adjusted for individual house risk and liquidity risk, imputed rent, leverage, owner costs, taxation and a five-year holding period. We find that the imputed rent, leverage and holding period have the largest impact on the return, while the individual house risk, leverage and holding period substantially affect the risk. Furthermore, we find that these adjustments increase the annual Sharpe-ratio of PRE from 0.71 in our initial model, to 1.82 in our final model.

For each increment we also analyze the Sharpe ratio of the tangent mixed-asset portfolio consisting of the PRE asset, a broad stock fund and broad a bond fund. As a result of the increase in the PRE asset Sharpe ratio, the allocation to this asset in the tangent mixed-asset portfolio increases from 32.05% to 50.82% in the final model. The Sharpe ratio of the tangent portfolio therefore increases from 0.86 to 1.85, and unlike in the initial model, including PRE in the mixed-asset portfolio significantly improves portfolio efficiency in the final model. By comparing the efficient frontiers with and without PRE, we find that while the dominating characteristic of PRE in initial mixed-asset portfolio is to be risk reducing, the dominating characteristic in the final model is to be return enhancing. The PRE asset in all models exhibits both characteristics to some extent, and the size of the effects is the largest in the final model.

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1 Introduction

In 2013 the Norwegian Fund and Asset Management Association (VFF) conducted a nationwide survey to map Norwegians' savings in mutual funds. Several of the questions in the survey were on the five-year expectations of the risk and return of different assets such as stocks, stock funds, money market funds and real estate. Out of all the assets, respondents ranked the primary home as the asset with the highest expected five-year return, and believed this return could be achieved at a risk below that of a money market fund. In commenting the results, the administrative director of VFF stated that "these results contradict one of the main pillars of financial theory, being the correlation between risk and return". In this thesis we analyze the characteristics of PRE as an asset, both individually, but also in conjunction with traditional financial assets to examine whether there exists a free lunch in the Norwegian housing market.

To derive the PRE risk and return we follow the methodology of Case & Shiller (1987) and create a weighted repeat sales (WRS) house price index (HPI) using data on apartment sales in Oslo in the period 2003-2018. We find that the average annual index return is 7.23% and risk is 6.02%, yielding a Sharpe ratio of 0.71. As a comparison, investing in a broad global stock fund over the period yields a Sharpe ratio of 0.61 and investing in a broad global bond fund yields a Sharpe ratio of 0.47. We therefore find that the PRE asset has a higher Sharpe ratio than the common investment alternatives over the period we are examining.

As the PRE asset has the highest Sharpe ratio, the allocation to this asset in a mixed-asset portfolio with the stock fund and bond fund is 32.05%. The allocation to this asset is limited by the positive correlations between PRE and both of the financial assets. The Sharpe ratio of the tangent mixed-asset portfolio is 0.86 and is higher than the Sharpe ratio of the tangent financial asset portfolio being 0.68. The tangent financial asset portfolio consists of the broad global stock and bond funds. Despite the increase in the Sharpe ratio, the alpha from running a regression of the tangent mixed-asset portfolio on the tangent financial asset portfolio is insignificant. This implies that for a one-year holding period, including the index risk and return of PRE in the investment universe does not significantly improve portfolio efficiency for the well-diversified investor.

To examine what drives to improvement in the portfolio efficiency, we follow Lee & Stevenson (2004) and compare the mixed-asset efficient frontier to the financial asset efficient frontier. The first way of doing this is to compare the risks of the frontiers for given levels of return (evaluating the risk reduction ability of PRE). The second is to compare the returns of the frontiers for given levels of risk (evaluating the return enhancement ability of PRE). In doing this, we find that the dominating characteristic of PRE is to be risk reducing, though the asset exhibits both risk reducing and return enhancing abilities. This is in line with the related literature, as Lee & Stevenson find that direct real estate should be considered a risk reducer rather than a return enhancer in the mixed-asset portfolio.

The analysis above constitutes the first out of seven such analyses conducted in this thesis. Each analysis adds a layer of complexity through adjusting the PRE asset, either for market imperfections, or for a retail investor perspective. We make these adjustments because we argue that the index risk and return does not reflect what an investor can realistically obtain in the market. While several of the adjustments are made in related literature, no paper to our knowledge includes all. In addition to this, each incremental adjustment is analyzed individually. We therefore argue that we derive one of the most holistic measures of PRE risk and return, and we do so using high quality data which we gather manually from primary sources.

Following Case & Shiller (1990) we adjust our index return for imputed rent, leverage and taxation.[1] Following Goetzmann (1993) we adjust our risk for individual house risk and our risk and return for a five-year holding period. In line with Jordà et al. (2017) we adjust our return for operating costs, and analogously to Cheng et al. (2013) we adjust our risk for liquidity risk, and return for transaction costs. After these adjustments we reach our final model; the five-year holding period tax model. This is the model we argue most accurately represents the risk and return of PRE over the period we examine.

In this final model, the average annual return of the PRE asset increases to 17.40%, mainly driven by leverage, the imputed rent and the five-year holding period. The risk of this asset also increases compared to the initial model, and is now 8.36%. While the leverage and individual house risk are the main drivers of risk-increases, the increase in holding

[1] Taxation is the only market imperfection that also affects the financial assets.

period from one to five years mitigates the effect of these. The PRE asset now has the highest return out of the assets we compare, yet the risk is slightly below that of the stock fund. The Sharpe ratio of the PRE asset is now 1.82. This is considerably higher than for the one-year holding period index risk and return, and for the stock and bond fund, being 0.80 and 0.48, respectively. Similarly to the initial model, the PRE asset has the highest Sharpe ratio, but this time, the out-performance compared to the financial assets is larger.

Accompanying the large increase in the PRE asset Sharpe ratio, is an increase in the allocation to this asset to 50.82% in the tangent mixed-asset portfolio. This is in line with the vastly spread literature on optimal/suggested allocations to PRE ranging from 0.00% to 10.00% in Ziobrowski et al. (1999) to from below 50.00% to above 80.00% in Webb & Rubens (1986). The Sharpe ratio of this tangent mixed-asset portfolio is 1.85 and similarly to the one-year index risk and return model, this is above the tangent financial asset portfolio Sharpe ratio which is now 0.69. As with the PRE asset and the financial assets, the out-performance of the tangent mixed-asset portfolio compared to the financial asset portfolio is larger. Therefore, unlike the initial model, in our final model the regression of the tangent mixed-asset portfolio on the tangent financial asset portfolio yields a significantly positive annual alpha of 5.69%. This indicates that expanding the investment universe to include PRE now yields significant benefits to the well-diversified investor.

In this final model, the significant benefits from including PRE in the investment universe are mainly driven by the return enhancing qualities of this asset. This contradicts the findings of Lee & Stevenson (2004), and our findings in the initial model. Furthermore, compared to the one-year index risk and return model, the increase in portfolio efficiency from both risk reduction and return enhancement increases. This is in line with Lee & Stevenson who find that direct real estate should be considered a strategic asset in the mixed-asset portfolio, especially for those investors with longer holding periods.

The change in the conclusion implies that the benefits of PRE are dependent on investment horizon, with longer horizons yielding higher benefits. We conduct several additional robustness tests, both to the inputs, and the assumptions. For the inputs we exchange the type of apartment and the number of apartments, while for the assumptions we change the wealth level, apartment value and leverage of the PRE asset. We find that our five-year holding period results are robust to changes in type of apartment and number of apartments. Likewise, we find that our results are not sensitive to wealth or apartment value, but that they are sensitive to leverage. This is in line with the findings of Boyd et al. (1998) who state that leveraged real estate improves the efficiency of a mixed-asset portfolio for taxable investors.

In summary, we provide strong evidence that over a five-year holding period, real estate yields a risk and return superior to that of the common investment alternatives, even in the most comprehensive model. The allocation to this asset is robustly significant, and including PRE in a mixed-asset portfolio provides improved investment opportunities through its mainly return enhancing abilities. Though superior, the extent of this out-performance is nowhere near the expectations of the asset, as the risk of PRE is close to that of the global stock fund, not to that of a money market fund.

An outline of the thesis is as follows; Section II reviews literature on the adjustments made to the PRE risk and return. Section III describes the data. Section IV presents the methodology used to create the WRS HPI. Section V describes the portfolio construction and empirical results. In section VI we conduct several robustness tests to verify our results. Section VII concludes and discusses topics for further research and the implication of our results.

2 Literature Review

In this section, we present literature that is related to both our contributions, and include our results when appropriate. We begin with presenting literature on our first contribution, the HPI. Then, we present a short literature review of the challenges with using the HPI index risk and return in the Modern Portfolio Theory (MPT) framework. Following this, we present literature on our second contribution, the improvements we make to the accuracy of the PRE index risk and return measures. These are either targeted towards reducing the challenges with using the HPI risk and return in the MPT framework (the market imperfections), or a result of our retail investor perspective.

We begin with constructing a HPI for Oslo for the period 2003-2018 to derive the index returns. We follow Case & Shiller (1987) and create a WRS HPI. From the index, we find that the average annual nominal return of PRE is 7.23%, with a standard deviation of 6.02%. This return is high, and risk is low, compared to the 135-year nominal Norway average return from Jordà et al. (2017) of 4.62% and risk of 8.08%, and the 200-year nominal return from Døskeland (2014) of 3.40%. It therefore appears that we may be looking at an extraordinary period were PRE has outperformed its historical average, with a lower degree of volatility. The out-performance can also be a result of this paper analyzing the Oslo market, which has behaved differently than the national housing market. Now that index risk and return is derived, we turn to the challenges of using this directly in the MTP framework.

Academic research has been addressing the benefit of adding PRE to a portfolio mainly invested in listed equities and bonds since the early 1980s. The early approaches used predominantly mean-variance optimization based directly on index returns, e.g. Fogler (1984), Irwin & Landa (1987), Webb & Rubens (1987) and Firstenberg et al. (1988). Later studies acknowledge that direct application of mean-variance optimization can be problematic, as PRE breaches a number of assumptions behind the standard MPT framework. In particular, investments tend to be illiquid according to Liu et al. (1990a). In addition to this, Chua (1999) postulates that there are significant costs associated with investing in PRE. Lastly, Young & Graff, (1995), Young et al. (2006) and Young (2008) find that returns are not normally distributed. In this paper, we discuss all of these weaknesses, and adjust for most of them, in addition to the individual investor adjustments.

The first adjustment we make is to the HPI risk. We follow Goetzmann (1993) and calculate a measure of individual house risk. The individual house risk not a result of a breach in the MPT assumptions, but rather a result of our individual investor perspective. Unlike stocks and bonds where there is one return series for a single stock and one for the market, there is solely one return series for housing. Therefore, the market risk must be adjusted upwards to account for the risk of one single house. We find that when including individual risk, the standard deviation of the PRE asset almost doubles, from 6.02% to 10.60%. This is in line with Goetzmann who finds that on average in the four cities studied, the total risk is 108% higher than the market risk. Similarly, Jordà et al. (2017) find that U.S. local housing return volatility is about twice as large as aggregate volatility, and recognize that it is much more difficult to invest in a diversified housing portfolio than a well-diversified equity portfolio.

The second adjustment to the risk is to account for liquidity risk, which is a market imperfection of the PRE asset. Jordà et al. (2017) state that "differences in the liquidity and the financial structures of the investment claim" are some of the reasons why PRE differs from equity. Several papers attempt to adjust for this risk, either by subtracting an arbitrary illiquidity premium, such as Hoesli & Lizieri (2007) or using more complex search-based approaches like those of Fisher et al. (2003), Bond et al. (2006) and Cheng et al. (2013). We follow the procedure of Cheng et al. and find that for the apartments in Oslo, the average time on market (TOM) is 25 days. This is much lower than in Cheng et al. where the average TOM is between 6 and 15 months. For this reason, they find that the conventional return volatility should be adjusted upward by as much as 97% in order to fully reflect liquidity risk. In our paper, we find that for the Oslo market, the upward adjustment in return volatility is much lower, only 3.65%. The small increase from liquidity risk can be a result of the period examined as Cheng et al. finds that the magnitude of liquidity risk is too large to be ignored, especially in down markets when liquidity risk is a great concern. As no such down markets are represented in our data, our TOM is most likely downwards biased.

The first return adjustment, the imputed rent, is made due to the individual investor perspective, similarly to the individual house risk. This rent is included as we argue that one must consume housing services somehow, either through renting, or though owning the home one lives in. We follow Case & Shiller (1990) and add this rent to the PRE investment for simplicity. They describe the imputed rent as “the value of housing services that accrues to the owner”, and argue that the imputed rent is a critical component of the return to investment in owner occupied housing. When adding the imputed rent, our PRE return increases to 11.53%. In comparing this to the long term Jordà et al. (2017) total PRE return of 11.34%, the Oslo PRE returns seem to be in line with their historical national average[2]

We continue to follow Case & Shiller (1990), and adjust our PRE return for leverage. This is also done due to the individual perspective, but can be argued to be a market imperfection as well, because leverage leads to a significant cost of PRE, the mortgage. Jordà et al. (2017) mention adjusting for leverage but choose not to, based on calculations using 20% leverage and a mortgage rate of 2.5%, as this only increases the return from 7% to 8.1%. They therefore conclude that this adjustment is not consequential for the main conclusions they present in the paper. This statement is true, because they do not adjust for taxation. When we leverage the PRE asset with 54.11% debt, and an average mortgage rate (MR) of 3.93%, the return on equity increases to 20.49%. Case & Shiller use a higher level of leverage (80%) while we use the average level of leverage in Oslo in 2017 and 2018.

The first market imperfection adjustment that is made to the return of PRE, is to include the operating cost. Though Case & Shiller (1990) include the imputed rent, they do not disclose whether or not it is net of operating costs. These costs are accounted for in Jordà et al. (2017) where the authors state that any homeowner incurs costs for maintenance and repairs which lower the rental yield and thus the effective return on housing. We deal with this issue similarly to these authors and subtract all costs that must be paid regardless of ownership status from the PRE asset return. In Jordà et al. the average nominal net yield for Norway in the long run is 6.72%, while our nominal annual average is 3.51%. This difference can be a result of the yield in Oslo falling substantially in recent years, as the rent level has lagged the house prices according to Grytten (2009).

[2]It does not matter whether Jordà et al. (2017) recognize that the rent is "imputed" or not, both papers use the total return.

The next market imperfection that affects the return of PRE is the transaction cost. While this cost is typically negligible for equities and bonds, it can be significant for properties. This cost is mentioned, but not adjusted for in Case & Shiller (1990). Jordà et al. (2017) conduct some calculations and conclude that these costs do not significantly affect the return of the PRE asset, and therefore do not adjust their returns. They use a higher level of transaction costs, 7.7% against our average of 5.39%, yet divide these over a long holding period of ten years. Therefore, their estimated transaction costs lower the PRE return by less than one percent per year. Because we leverage our PRE returns and initially utilize a one-year holding period, including transaction costs in our paper lowers the return by 11.70%.

The last market imperfection adjusted for is taxation. This is done in Case & Shiller (1990) and acknowledged in Jordà et al. (2017), yet not adjusted for, even though the authors state that from an investor‘s perspective, adjusting for taxes is clearly important. This is especially crucial when examining Norway because taxes are often stated as the main reason for why the home-ownership rate is high[3]. This market imperfection is the first to affect all assets, not only PRE. As taxation negatively affects the financial assets and positively affects the PRE asset, this last step increases the relative attractiveness of the real estate asset.

The last individual investor adjustment made is to increase the holding period, and is done following Goetzmann (1993), with some alterations. While Goetzmann utilizes the initial average annual index return for the five-year holding period, we expand the holding period using the most comprehensive model. We therefore have variables that are dependent on realization, and variables whose assumptions are altered for longer holding periods. As a result of this, we cannot use the same average annual return for a longer holding period, and our PRE return increases from 7.79% to 17.40%, mostly due to the decreased effect of transaction costs. We continue to follow Goetzmann and re-calculate the holding period risk, accounting for our new returns, the autocorrelation in the returns and the reduced price risk. We also reduce our liquidity risk in the same way as the price risk, based on the findings of Lin & Vandell (2007) and Bond et al. (2007), who show that liquidity risk is an issue mainly for short-term investors. We find that all of these components lead to a significantly reduced risk, from 15.83% to 8.36%, while in Goetzmann the risk increases

[3]http://www.oecd.org/economy/surveys/norway-2018-OECD-economic-survey-overview.pdf

for two of the cities studied, and decreases in the remaining two when moving from a one- to five-year holding period.

The final weakness of including PRE in the MPT framework is that the HPI returns are not normally distributed. Young (2008) argues that real estate return distributions are non-normal. Goetzmann (1993) also expresses concerns about this, as homeowners may have an aversion to taking a loss on their home sale. According to Young, the non-normality of the returns implies that asset diversification is far less effective at reducing the impact of non-systematic investment risk on real estate portfolios than in the case of assets with normally distributed investment risk. Though we find that our HPI returns exhibit some evidence of aversion to realize losses, the non-normality is not problematic for our conclusion. We are looking at PRE investments from the perspective of an individual investor. Hence, no non-systematic risk are diversified away.

As one can infer from the literature review, the topic of PRE as an asset is extensively researched. Our paper depends on the insights from multiple sources, many applying an institutional perspective. The main sources of the thesis are the Case & Shiller (1987) and Goetzmann (1993) papers, as the replication of these using Norwegian data yields our HPI. These papers are therefore presented in detail in the methodology section. Now, we move on to the data section where the gathering and adaption of the data series related to the adjustments made in this section, are presented.

3 Data

In this section, we provide a detailed description of the data we use for our analyses. First, we describe the financial asset data needed to create the benchmark portfolios. Then, we briefly describe the main types of house price indices and the data needed to construct these. Following this, we describe the time series needed for the adjustments to the PRE index risk and return. We also briefly explain the difference between a cooperative (co-op) and a freehold apartment and present the data needed to adjust our model to account for these. Lastly, we comment on the data quality and suitability. A list of the data used for the models is provided in the appendix.

3.1 Financial Assets

We use mutual funds as proxies for the traditional financial assets. This simplifies the calculation of taxation in our final one-year holding period model. We choose some of the broadest and most popular funds in Norway as these best represent realistic investment alternatives to PRE for the retail investor. We require the funds to be active for the entire period from 2004 to 2018. Moreover, the funds have to be large (assets under management), and broad (exposure to geography and sectors). For stocks we use the Global fund from SKAGEN Fondene which manages 28.9 billion NOK. For bonds we use the average of SKAGEN Avkastning which manages 1.6 billion NOK and DNB Obligasjon which manages 1.8 billion NOK. The indexes are provided using reinvestment of dividends (total return) and are net of management fees. We also calculate a market capitalization-weighted average of the ten most popular mutual stock funds in Norway, and compare this to the Global fund. We find that the chosen fund is representative. We also exchange the Global fund with a Norwegian fund and note that the results do not change.

To compute the Sharpe ratio and tangent portfolios we use the average of ten-year government bond rates from 2005-2018, provided by Norges Bank.

Before we describe the data used to construct the house price index, we present summary statistics for the financial asset.

Table 3.1: Summary Statistics Financial Assets

Annual summary statistics for the stock and bond funds used to create the benchmark, and the risk-free rate used to calculate the Sharpe ratio and tangent portfolios. The number of observations varies, yet all data is annualized. The time period analyzed is 2005-2018

Summary Statistics	Stock Fund	Bond Fund	10-year Government Bond
Average Return	11.52%	3.88%	2.95%
Standard Deviation	14.13%	2.00%	1.12%
Maximum Return	31.07%	7.40%	4.78%
Minimum Return	-19.15%	0.51%	1.33%
Number of Observations	5110	5110	14
Acronym	SF	BF	R_{F_1}

3.2 House Price Index

There are two widely applied methodologies that can be replicated to create the HPI, the hedonic HPI and the repeat-sales (RS) HPI. In the words of Silverstein (2014) the hedonic approach " allows the index to accurately measure the changes in the value of a home over time based on a single sale through inference using the typical value associated with the changes in house attributes over time". The advantage of this method is that it utilizes all observations, while the disadvantage is that there is no accurate measure of the house quality variable in Norway. The second main methodology is the RS HPI. This method controls for housing quality through comparing the sales of the same house across time, and was introduced by Bailey et al. (1963). This method entails comparing only paired sales prices for the same house, and hence all observations of a house sold only once are lost. Another drawback of this method is that sales prices are not entered into the index until they are paired with a subsequent sale, causing the index to be revised to reflect the addition of each new sales pair. There also exist combinations of these two main methods, yet no such hybrid indices are widely available in Norway.

In this paper, we use a modified version of Bailey et al. (1963) RS HPI, namely the WRS HPI. This version is based on Case & Shiller (1987) and argues that paired sales with longer time intervals between them should be given less weight than those with shorter time intervals. This can be due to random differences in level of renovation and neighbourhood quality. In the original RS method, homes sold after long time intervals have a large influence on the index relative to homes sold over short intervals. Therefore, Case & Shiller give less weight to long time interval observations. Our thesis is dependent on using a RS methodology due to the individual house risk adjustment. We follow Case & Shiller as we agree with the modification made to Bailey et al., and are interested in the impact of this modification on the Oslo market data.

Our WRS index is created with data from Eiendomsverdi, and we receive data on nearly 250 000 homes sold in Oslo between 2003 and 2019. The data contains information on sales date, sales price, common debt, living area, estate type, bedrooms and geographic area for each property sold in the period. In addition, Eiendomsverdi creates a unique ID for each unique property.

We start with filtering the data to only look at either freehold or co-op apartments. Out of 242,013 initial observations, 100,757 are freehold apartments, and 90,839 are co-op apartments. By using the Unique ID field created by Eiendomsverdi we remove observations that appear to be data entry errors; if two sales prices with the same ID are different by a factor of ten, they are removed. Subsequently, observations are excluded if there is evidence that the apartment has been refurbished. This is done by checking the number of bedrooms and the size of the living area of the apartment. If this changes during the period, the observations are dropped. After these adjustments, we are left with 100,655 freehold apartments and 61,442 co-op apartments. We see that there appears to be more data-entry mistakes for the co-ops.

Finally, we use the Unique ID field to create a new data set with the remaining observations, containing prices and dates for purchase and sale. Each pair contains two observations, and because not all apartments are sold twice or more during our time period, we lose some observations in this step. After also removing pairs containing missing values, we are left with a total of 38,636 clean freehold apartment pairs, and 14,763 clean co-op apartment pairs. Given these criteria, we end up using a total of 106,798 of our initial observations. Now that the data needed to construct the index is presented, we move on to the data used to account for the adjustments discussed in the previous section. Summary statistics for the WRS HPI are presented together with those of the adjustments.

3.3 Adjustments

The first adjustment we make is to include individual house risk. The data needed to execute this, is the same as the data needed to construct the HPI, and this procedure is described in the methodology section. The second adjustment to the total PRE risk is to add the liquidity risk. To calculate the liquidity risk, we receive data on the average time on market for different areas of Oslo from Real Estate Norway. We combine this data with our data from Eiendomsverdi (the same data used to create the index) and create an annual weighted average time on market for Oslo. This is done by weighting days on market with number of apartments sold in the different Oslo regions.

The next adjustment is the imputed rent. Unlike the individual house- and the liquidity risk, it affects the PRE return. This variable is included in Case & Shiller (1990), but we adjust the measure slightly. Rental units are on average smaller and of lower quality than owner-occupied units. To correct for this, Case & Shiller scale up the rent by the average number of rooms in an owner occupied unit over the average number of rooms in a rental unit. As we do not have Norwegian data on the average number of rooms in rental units, we instead compare the rent and price of apartments with two bedrooms, which is the median number of bedrooms in our data set. In this way we are certain to compare the rent with the price of similar sized apartments.

The rent data is gathered from The Municipality of Oslo and we use the average monthly market rent for Oslo annually. We use this data, rather than Statistic Norway‘s estimate of imputed rent because Statistics Norway uses effective values. Though effective values are superior, we have a nominal HPI. For our HPI and rent index to be comparable, they must both use nominal numbers. The HPI cannot be constructed effectively as there is no available variable measuring house quality (renovations and improvements in housing standards) in Norway. According to Case & Shiller (1990), for purposes of constructing an index of returns to homeowners, the unadjusted index that fails include depreciation is the appropriate one.

Following the imputed rent we also incorporate leverage. To do this, we need two data series. The first is the average loan-to-value (LTV) in Oslo, which is the value of the mortgage to the value of the property. There is no national register for this historical data in Norway. We therefore use data from the Financial Supervisory Authority of Norway‘s annual bank survey. The data for Oslo reaches back only two years, and the mid-year LTV is on average 54.11%. The second data series needed for leverage is the mortgage rate. We gather this data series from Statistics Norway. Data on mortgage interest rates for households reaches back to 2008. We therefore use the average interest rate on loans to households for the period from 2005-2008. Based on the years where both interest rates are recorded, the difference is minor, most likely because the mortgage is the largest component of household debt. For simplicity, we assume that no instalments are paid on the mortgage over the holding period though recognizing that this creates an upward bias in the PRE return.

We divide the next adjustment, owner cost, into two main parts. The first is the operating cost and the second is the transaction cost of PRE. The operating cost consists of municipal costs, insurance of the building, administrative costs and maintenance. We obtain these from Huseiernes Bokostnadsindeks for Oslo in the period from 2010 to 2017. For the years 2005-2010 and 2017-2018 we adjust the values with their corresponding consumer price index (CPI) component from Statistics Norway. For the municipal costs we use "water supply and various costs related to housing", for the insurance we use "insurance" and for the maintenance we use "maintenance of housing". As we do not have a value for the remaining various costs, we compare the share of maintenance, insurance and municipal costs to total costs in the income statement of several freehold buildings and find that these typically account for 90% of the total costs. We assume that this relationship is constant over our period and scale up our operating costs by 10%.

The second owner cost is the transaction cost. The transaction cost is divided into purchase costs and sales costs. The purchase costs mainly consist of the document duty, but also the registration fees, mortgage certificate, house-purchase insurance and owner-change fee. The sales costs mainly consist of the broker-fee, but also the registration fee, mortgage certificate, house-sales insurance, appraisal, styling, photography, internet ads, and open house. Most of these costs are either capped by, or set by the government, such as the document duty, registration fees and mortgage certificate. These time series are provided by Kartverket. The broker fee is provided by The Financial Supervisory Authority of Norway and is calculated by dividing the total broker compensation by the total value conveyed in Oslo annually. The house-purchase insurance, owner-change fee, house-sale insurance and appraisals are provided by major market actors and are hence considered representative. The final sales costs, styling, photography, internet ads and open house are CPI adjusted for the period using the appropriate discount rate. To do this we use "acquisition costs" from Eurodata for the period 2010-2018 and "other products and services" from Statistics Norway from 2005-2010. Through discussions with several realtors it is evident that styling and photography was not common until 2010, hence these costs are zero from 2005-2010.

The last market imperfection is the tax. The data for the general income tax rate (GITR), property tax rates (PTR) and wealth tax rates (WTR) from 2007–2018 are provided by The Norwegian Tax Administration. From 2005-2007 the GITR is gathered from the National Budget, provided online by the Ministry of Finance while the WTR is gathered from Lovdata. The wealth tax valuation of primary homes (WVH), secondary homes (WVSH), stocks (WVS) and bonds (WVB) are found in the "lignings-ABC", later called "skatte-ABC" which is provided by The Norwegian Tax Administration for the years from 2010-2018. In 2010, there was a change in the calculation of housing valuation. According to the Ministry of Finance, both before and after this change, the maximum valuation of primary homes was 30% of the market value. We therefore extrapolate the valuation of 25% used in the period after 2010, to the years between 2005 and 2010 as our results are not sensitive to this simplification. The maximum valuation also applies for secondary housing before 2010. This is because the new law in 2010 was implemented to distinguish the taxation of primary and secondary housing. In the years following, the valuation of secondary housing increased in increments to 90% of market value in 2018. However, the valuation for homes is only a guideline for new homes, in reality the wealth tax valuation of housing in percent of market value is often less than the advised percentage valuation.

The bond and stock fund profits are taxed with the GITR. The stock profit is net of a risk free rate (R_{F_2}), and is adjusted with a factor for upwards adjustment (FUA) in 2017 and 2018. The data on these variables is also provided by the Norwegian Tax Administration. In 2006 there was a tax reform, hence there was no deductible risk free interest rate in 2005.

Before describing the dated used to adapt the analysis to co-op apartments, we present summary statistics for the WRS HPI and most of the variables used for the adjustments of this.

Table 3.2: Summary Statistics Freehold WRS HPI and Market Imperfections

Annual summary statistics for the freehold WRS HPI (capital appreciation model) and the additional variables needed to account for market imperfections. The number of observations varies, yet all data is annualized. Summary statistics of tax rates are uninterpretable, and are therefore excluded. The period analyzed is 2005-2018

Summary Statistics	Capital Appreciation	Days on Market	Imputed Rent Yield	Loan to Value	Private Debt Mortgage Rate	Operating Cost	Purchase Cost	Sales Cost
Mean	7.23%	28.27	4.30%	54.11%	3.93%	0.78%	2.91%	2.48%
Standard Deviation	6.02%	7.37	0.43%	1.89%	1.25%	0.10%	0.11%	0.18%
Maximum	17.61%	42.22	5.05%	56.00%	6.88%	0.95%	3.10%	2.77%
Minimum	-6.11%	20.49	3.57%	52.22%	2.51%	0.57%	2.76%	2.05%
Number of Observations	38636	168	14	2	56	56	56	56
Acronym	CAP	TOM	Y	LTV	MR, PMR	OC	TC	

3.4 Cooperatives

Co-ops are the second main type of apartments in Oslo. The key difference between co-op and freehold apartments relates to the rights of the unit owner. For a freehold property, one has unrestricted ownership of a piece of real estate, while for a co-op, the real estate is owned by all of the members, but one has the right to one housing unit within the co-op. In addition to the rights of the owner, the transaction costs are lower for the co-op. For this type of apartment, you do not have to pay the document fee upon purchase, the registration fees are lower, and the owner-change fee is paid by the seller instead of the buyer of the unit. Furthermore, the purchase of a co-op is often done with a combination of common debt, personal debt and equity unlike freeholds where the entire purchase is financed with personal debt and equity.

As the differences between the two apartment types indicate that a home owner may not be indifferent in a choice between them, we split our data set and calculate an index for each type separately. To do this, we begin with repeating the procedure of the WRS index using the data for co-ops instead of freehold units. To account for the difference in transaction costs, we gather new data on the registration fees. The registration fees from 2006-2018 are provided by Kartverket. It was not until 2006 that co-ops were included in the land registration, hence there was no registration fee before this time. Lastly, we account for the difference in financing by gathering data on the mortgage rate for the common debt of co-ops. We receive data from BORI, a Norwegian building society. Their data on floating mortgage rates reaches back to 2007, and we therefore extrapolate the average of the rate in 2007 and 2008 back to 2005 upon recommendation of a BORI employee.

Before describing the data quality and suitability, we present summary statistics for the co-op WRS HPI and the adjustments made to this that differ from the freehold adjustments.

Table 3.3: Summary Statistics Co-Op WRS HPI and Market Imperfections

Annual summary statistics for the Co-op WRS HPI (capital appreciation model) and the additional variables needed to account for market imperfections for this type of apartment. Variables not included in this table are similar to the freehold. The number of observations varies, yet all data is annualized. The period analyzed is 2005-2018

Summary Statistics	Captial Appreciation	Common Debt to Value	Common Debt Morgage Rate	Purchase Cost	Sales Cost
Mean	7.48%	8.87%	3.81%	0.29%	2.94%
Standard Deviation	6.69%	13.96%	1.29%	0.04%	0.19%
Maximum	19.45%	100.00%	7.00%	0.35%	3.21%
Minimum	-7.03%	0.00%	2.15%	0.22%	2.52%
Number of Observations	14763	29526	14	56	56
Acronym	CAP	CDTV	CDMR	TC	

3.5 Data Quality and Suitability

The data presented in this section is retrieved from reliable sources and industry experts. Multiple data series are cross-checked, both by using several sources for the same data, but also through comparisons with results from existing literature. The use of extrapolation, CPI adjustments and other assumptions is minimized and only used as a final resolution if data cannot be found. Furthermore, this solution is only applied to variables that do not affect the result to a large extent. In addition to this, these adjustments are not done without consulting industry experts, or the providers of the data.

Although the numbers gathered in this section represent a faithful attempt to estimate the different times series of investor returns, they are constructed from a number of different data sources. Consequently, they have a much higher degree of uncertainty associated with them, and their summary statistics. Several authors such as Michaud (1989) and Jorion (1985) point out that estimation errors affect the risky end of the efficient frontier to a much larger extent. In this thesis, we are mostly interested in the tangent portfolios, which are seldom found at the risky end of the efficient frontier. Therefore, this uncertainly is not the main data concern in this thesis.

An issue with our data is the length of our time series. According to Boyd et al. (1998), longer time periods are superior because they encompass many of the major economic and political occurrences and trends that can contribute to variability in investment performance. The main data foundation of this thesis is the individual-level house sales data, and this data limits the analysis to 16 years.[4] The issue of the time series is not necessarily that 16 years is too short, as Case & Shiller (1987) use 16 years in their analysis, but rather that the current housing cycle in Norway has been long. While the Case & Shiller paper encompasses an entire housing cycle, the last house price crash in Norway was in 1987-1992, approximately 30 years ago[5]. As a result of this, one needs to be cautious of generalizing the results of this paper, as the PRE returns utilized may be overstated and therefore do not accurately predict the future. This effect is demonstrated in the table below.

Table 3.4: Annual Real Rate of Return in Different Time Periods

Norwegian data from an analysis by Bjørn Erik Sættem, savings economist in Nordnet Bank demonstrating that the relative out-performance of PRE versus the financial assets is dependent on the time horizon analyzed

	1871-2018	1950-2018	1983-2018	1993-2018	2000-2018	2010-2018
Primary Home Without Rental Income	1.20%	2.20%	3.10%	4.90%	3.70%	2.60%
Secondary Home With 3% Real Rental Income	4.10%	5.10%	6.00%	7.70%	6.70%	5.60%
Stocks USA in NOK	6.40%	6.70%	8.00%	6.70%	2.80%	12.10%
Stocks Norway	N/A	N/A	8.30%	6.50%	5.30%	6.70%
Interest Rate	2.10%	1.60%	3.20%	2.10%	1.50%	0.10%
Inflation	3.00%	4.50%	3.10%	2.00%	2.10%	2.10%

NB! returns of international stocks are affected by changes in exchange rates.
Sources: Nordnet, Norges Bank, Macrobond, Oslo Stock Exchange, Statistics Norway

Another challenge with using historical data, that is not an issue in this thesis, is that when a specific time period is chosen, it can bias the result because the sample can be chosen a priori to give specific results. In our model, the time period examined is limited by the data used, and we have therefore not chosen a specific year to start the analysis.

Now that the data is presented, and the quality and suitability of it discussed, we move on to the methodology section where the WRS HPI is described in more detail.

[4]The analysis is limited to 14 years as the time series on the adjustments only goes back to 2005.
[5]https://forskning.no/okonomi/her-er-verdens-storste-boligbobler/1358124

4 Methodology

In this section, we introduce the methodology used to create the WRS index returns, individual house risk, correlations and the five-year holding period risk. These are some of the most important inputs needed to include PRE in the MPT framework, and provide the basis for the following models to build on to. In the next section, Portfolio Construction and Empirical Results, we present the simple risk and return calculations used to include the remaining adjustments of PRE. The numerical results of the methodology section are presented with their respective models in the next section.

4.1 WRS Index

The index risk and return calculations are based on Case & Shiller (1987) and we use their WRS method to estimate the quarterly returns to residential properties in Oslo with our created sales pairs. The WRS method is based on the assumption that the log price P_i, of the ith house at time t can be viewed to have three components;

$$P_{i,t} = m_t + d_{i,t} + e_i \tag{4.1}$$

Where m is the log of the city-wide level of housing prices at a particular time and captures the location or market effect on the price. d captures the impact of individual property drift and is assumed to follow the Gaussian random walk. e is an identically distributed normal noise term with a mean of zero and variance σ_e^2. This is some random variable that is named price risk to distinguish it from the components of the true value. It is further assumed that m, d and e are uncorrelated. The components of value is the difference in the market term which is estimated by; $\mu_t = m_t - m_{t-1}$ and the difference in the individual property noise for which $\delta_{i,t} = d_{i,t} - d_{i,t-1}$ for the log return series y_i :

$$y_i = \sum_{t=b_i}^{s_t} (\mu_t + \delta_{i,t}) + e_{i,s_i} - e_{i,b_i} \tag{4.2}$$

Where b_i and s_i are the purchase date and sales date for the ith property, respectively.

Under the assumption that the three effects are uncorrelated, that d follows a random walk and the e reflects a trading noise; the variance of the paired log price change from time s to b, can be derived by:

$$Var(y_{t,s}) = \sigma_{\mu}^2 + (s-b)\sigma_d^2 + 2\sigma_e^2 \tag{4.3}$$

In the first step of creating the WRS index, the original RS method is followed diligently. This involves running a regression where the ith observation of the dependent variable is the log of the price of the ith house at its second sales date, minus the log of its price on its first sales date. The independent variables only consist of dummy variables, one for each time period except from the first. For each sales pair, the dummy variables are zero, except for the dummy corresponding to the first sale, where it is -1, and for the second sale where it is $+1$. The estimated coefficients are then taken as the log price index.

In the second step, the squared residuals $(y_i - \hat{y}_i)^2$ from the first step regression are regressed on a constant and the time interval between sales. Under the assumption that the variance of δ_{t-1} grows linearly in time with an expectation of zero, we may estimate the variance of this term by the regressed squared residuals on the holding period for each property, $s_i - b_i$. The slope from the regression in formula 4.4 estimates the variance of the drift term ($\beta = \sigma_d^2$), while the intercept from the regression estimates the variance of the price risk term ($\alpha = 2\sigma_e^2$).

$$(y_i - \hat{y}_i)^2 = \alpha + \beta(s_i - b_i) + e_i \tag{4.4}$$

In the third step, a generalized least squares regression is run by first dividing each of the observations in the step-one regression by the square root of the fitted value of the step-two regression, and re-running the regression. This supplies us with the final return series. The final return series can be used to estimate the expected return and volatility of a home investment.

However, according to Goetzmann (1992) the log transformation results in a downward bias of the arithmetic mean at each point in time. A geometric index underestimates the percentage change in the average value of a portfolio. The problem persists when we extend the estimation to a number of periods. Shiller (1991) also follows up on the Case & Shiller (1987) paper, and argues that the arithmetic index is in fact superior if one wishes to study the covariance between housing prices and prices of other assets for the purpose of constructing a well-diversified portfolio. Lewis (2016) on the other hand, argues that the geometric mean is better as the arithmetic mean is sensitive to extreme property values, and as a result, the prices can be skewed upwards by high property values.

We follow Goetzmann (1992) and apply a simple modification to the geometric WRS index that approximates the appreciation rates obtained from the arithmetic index.

$$\tilde{I} = 100 * e^{\mu_t + \frac{1}{2}\sigma_d^2 t} \tag{4.5}$$

This means that a simple arithmetic average of price relatives (the house price in the last quarter divided by house price in the first quarter of the sample) should be, assuming this model and log normally distributed prices, at the end of a sample t quarters long, higher by a factor of $exp(\frac{1}{2}\sigma_d^2 t)$ than the corresponding geometric average.

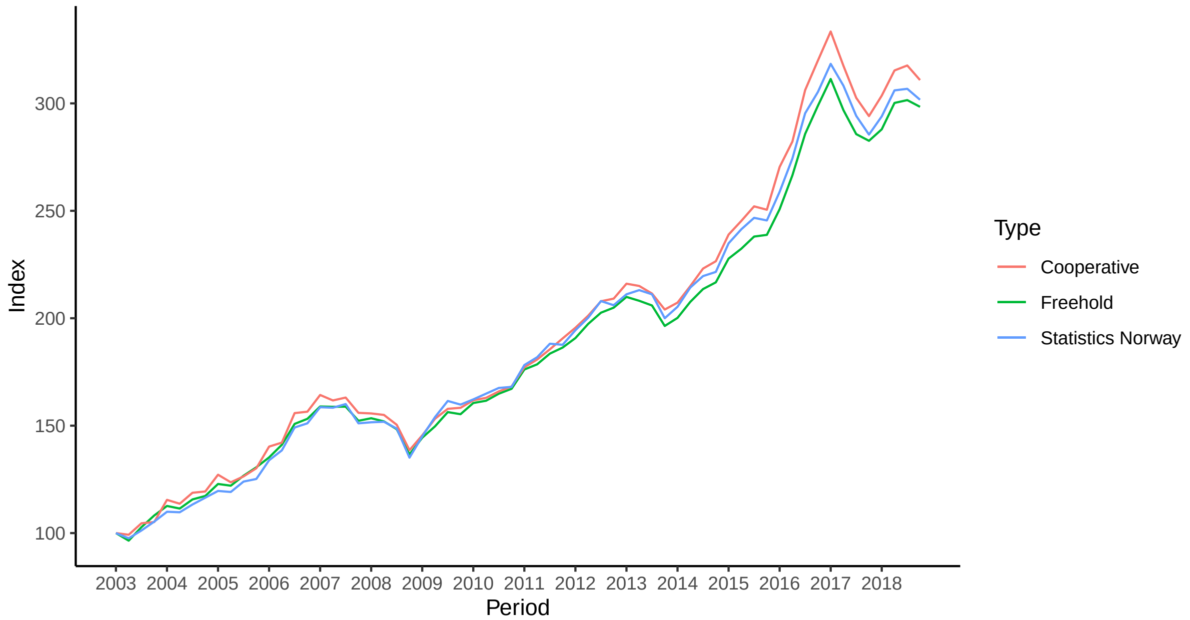

Figure 4.1: House Price Indices

WRS house price indices for cooperative and freehold apartments, created using data on homes sold in Oslo from 2003 until 2018. The indices are plotted together with the hedonic house price index from Statistics Norway, in which both apartments types are merged into one index, showing a comparison of the three.

Figure 4.1 plots the WRS indexes for the co-op and freehold apartments. The performance variance is small across the red and blue line. The nominal annual average returns from 2005-2018 are 7.23% and 7.48% for freeholds and co-ops respectively, and we observe that prices have nearly tripled over the period. Figure 4.1 also shows a comparison of the WRS indexes with the hedonic sales index published by Statistics Norway for apartments sold in Oslo over the same period. We observe little variance in the performance, indicating that the WRS indexes do not suffer from the loss of data due to the requirement of sales pairs. We also observe that the Statistics Norway index lies close to our WRS indexes with a nominal annual average of 7.50% for the period 2003-2018. As the market return and single home return is the same, we measure the individual house risk and add this to the variance of the index presented above.

4.2 Individual House Risk

The WRS method estimates μ_t, and the residuals from the second stage regression capture the variation due to the price risk e_i and the individual property specific drift term $\delta_{i,t}$. As shown in formula 4.3 and 4.4, the constant term of the second stage regression is an estimate of $2\sigma_e^2$, and the slope term is the estimate of σ_d^2. As we assume that price risk at the time of purchase and sale are independent of each other, we follow Case & Shiller (1987) and divide this quantity in half to estimate the error associated with a single sale. The estimate of these factors alongside with the estimated risk of the market factor are used to provide an approximation of the average risk associated with a one-year investment in a single home. As the three components are assumed independent of each other, their variance may be summed. We estimate β using quarterly observations, and this must therefore be multiplied with four to obtain an annual estimate.

$$\sigma_h = \sqrt{\frac{\hat{\alpha}}{2} + 4\beta + \sigma_\mu^2} \tag{4.6}$$

As these measures affect the risk, yet not the return of the real estate asset, we now comment on the effect of these in the calculation of the correlations between PRE and the financial assets.

4.3 Correlations

In addition to the risk and return estimates, we also need an estimate of the correlation between the returns of the PRE asset and the returns of the financial assets. We use the three components of PRE risk identified previously, under the assumption that $\delta_{i,t}$ and $e_{i,t}$ are uncorrelated with other asset classes. Given these assumptions, the covariance between μ_t, and other asset classes is the only relevant measure to calculate the correlation. We follow Goetzmann (1993) and find the desired correlation between the returns to PRE and the returns to asset A for a one-year holding period with the following formula;

$$\rho_{PRE,A} = \frac{\sigma_{\mu,A}}{\sigma_A\sqrt{\frac{\hat{\alpha}}{2} + 4\beta + \sigma_\mu^2}} \tag{4.7}$$

A full correlation matrix for all models can be found in the appendix. Now, we move on to the final risk alteration made in the thesis.

4.4 Five-Year Risk

In the next step we need to estimate the risk associated with a five-year investment in PRE. According to Goetzmann (1993), the volatility is affected by two additional factors. The first factor is the autocorrelation of the return. We must account for the annual autocorrelation in each of the four lags when estimating the five-year variance. This factor also applies for the stock and bond fund. A positive autocorrelation implies that the five-year variance of the series is greater than five times the annual variance, while a negative autocorrelation implies that the five-year variance is less than five times the annual variance. The annualized five-year standard deviation $(\sigma^2_{\mu,5Y})$ is calculated as:

$$\sqrt{\frac{s'Ps}{5}} where: P \equiv \begin{bmatrix} \rho_0 & \rho_1 & \rho_2 & \rho_3 & \rho_4 \\ \rho_1 & \rho_0 & \rho_1 & \rho_2 & \rho_3 \\ \rho_2 & \rho_1 & \rho_0 & \rho_1 & \rho_2 \\ \rho_3 & \rho_2 & \rho_1 & \rho_0 & \rho_1 \\ \rho_4 & \rho_3 & \rho_2 & \rho_1 & \rho_0 \end{bmatrix} \begin{matrix} \rho_i \equiv ith\ order\ correlation\ coefficient \\ s = column\ vector\ of\ five\ \sigma_h \end{matrix} \tag{4.8}$$

The second factor influencing the long-term risk for PRE is the price risk. As it is not time related, this is a fixed quantity. Hence, it has a diminishing effect on the variance for longer holding periods. Based on these two factors, we calculate the standard deviation of a five-year investment in PRE by the following formula:

$$\sigma_{h,5y} = \sqrt{\frac{\frac{\hat{\alpha}}{2}}{5} + 4\beta + \sigma^2_{\mu,5y}} \tag{4.9}$$

Now that the methodology used to derive the index risk and return of PRE, and some of the more technical alterations are accounted for, we move on to presenting the models individually. Here, the numerical results from this section, and the formulas used to include the remaining alterations are deducted and analyzed.

5 Portfolio Construction and Empirical Results

In this section we present the risk, return and correlation calculations utilized in the efficient frontiers for each individual model. None of the models in this thesis allow for short selling. The first section provides an overview of our investment universe before including PRE. The next section introduces PRE, utilizing a freehold primary apartment and a one-year holding period. This section is divided into several sub-sections where each adds a level of complexity to the return and risk calculations. Starting from a capital appreciation model, we include individual house risk, liquidity risk, imputed rent, leverage, owner costs and finally taxation. This is done to derive the most comprehensive comparison of the assets in our investment universe. The last section increases the holding period for the primary freehold apartment from one to five years. First, we examine the benchmark portfolio.

5.1 Benchmark

Before including PRE in our portfolio, we examine an investment universe comprised solely of the stock and bond funds. The returns of the funds are calculated using the following formula[6]:

$$R_{SF/BF} = \left(\frac{P_{t+1} + Div - fees}{P_t}\right) \tag{5.1}$$

The market risk is calculated using the formula for standard deviation;

$$\sigma_{SF/BF} = \sqrt{\frac{\sum(R_i - R_{avg})^2}{n-1}} \tag{5.2}$$

The average annual one-year stock fund return is 11.52% and the annual standard deviation is 14.12%. For the bond fund, these values are 3.88% and 1.99%, respectively. For any risky asset or portfolio, the Sharpe ratio is defined as the ratio of the excess return to the standard deviation of that return. We define the excess return on a risky asset as the

[6]Notice that because our fund data is provided with reinvestment of dividends and after management fees, the comparison with PRE is biased until these variables have been accounted for. It is unclear in which way this will bias the results of the first models as this is dependent on the relative size of the dividends and management fees.

difference between the one-year holding period return of the risky asset, and the return on 10-year Norwegian government bonds. The average return on a 10-year Norwegian government bond in our period is 2.95%. Hence, the annual Sharpe ratios for the stock and bond funds are 0.61 and 0.47, respectively.

The correlation between our stock and bond fund is positive, at 0.27[7]. This moderately positive correlation can be a result of the use of annual correlations instead of for example monthly or daily. We use annual correlations because for some variables we include later, only annual data is available, and we need the results to be comparable.

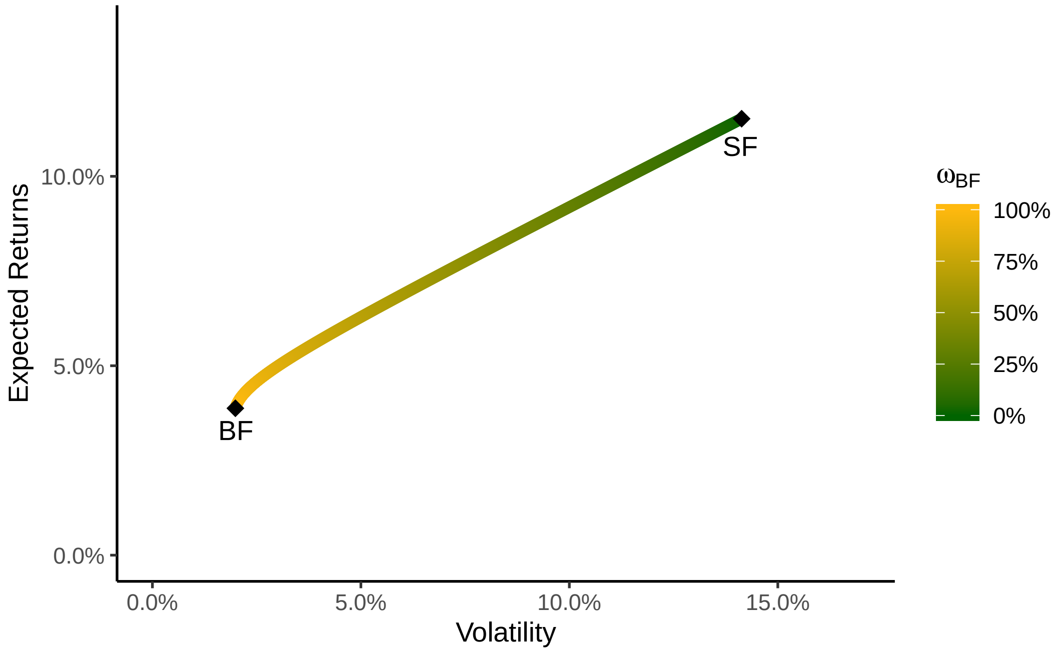

Figure 5.1: Possible Benchmark Portfolios

Portfolios comprised of positive mixtures of the stock and bond funds, plotted in a mean-variance diagram. The portfolios are created using data over the time period 2005 through 2018, and they are based upon 0.1% increments.

The efficient frontier in figure 5.1 is the one we use as the benchmark to which we compare all of the succeeding efficient frontiers, up until the tax model. The high correlation between the assets generates small diversification benefits. This can be observed in the figure, as the efficient frontier is relatively straight.

[7]This appears to be high, especially in light of the Norwegian Pension fund‘s proposition to increase its share allocation among other things based on the negative correlation between stocks and bonds. They argue that in 1990, the historical correlation was 0.4, but it had (in 2016) been -0,3 if using the return from the last eighteen years.

The tangent portfolio of the financial assets allocates 18.42% of the wealth to the stock fund and 81.58% to the bond fund. This high bond allocation can be a result of the period in question and the positive correlation between the assets. In the period examined, the stock fund incurs draw-downs in the 2007 financial crisis, the 2010 European debt crises, the plunge in oil prices in 2016 and the global uncertainty surrounding the trade war in 2018. This results in a high stock fund risk, and therefore small allocations to this asset. This is illustrated in figure 5.2 below.

Figure 5.2: Stock Fund and Bond Fund Returns

Plot showing the annual returns of the stock and bond fund. The time period covered is 2005-2018.

Furthermore, the bond fund is not to be mistaken for a money market fund, as it has a higher weighted average maturity, and some allocations to corporate debt, yielding a higher risk and return. The return of the tangent portfolio of the stock and bond fund is 5.28% and the standard deviation is 3.42%. The Sharpe ratio of this portfolio is 0.68 and is therefore higher than the Sharpe ratios of the financial assets individually. This is as expected, because the tangent portfolio allocates wealth to both assets. Later, this tangent portfolio is considered the benchmark portfolio to which we compare the tangent mixed-asset portfolio of the stock fond, bond fund and PRE, or only PRE. Once again, this is up until the tax model. Taxation is the first market imperfection to impact the financial assets, and new risks and returns are calculated, leading to a new benchmark efficient frontier and a new tangent benchmark portfolio.

Now that the investment universe comprised of stock and bond funds is presented, we are ready to introduce PRE into our benchmark portfolio, thereby creating the mixed-asset portfolio.

5.2 One-Year Holding Period for the Freehold Unit

While realizing that this is a highly stylized example, we deduct our final and most comprehensive model using a one-year holding period. Each improvement to the PRE risk and return measure is presented individually, and subsequently, their effect on a mixed-asset portfolio is analyzed. When we deduct the more realistic five-year holding period model in the next section, it is based on the calculations and intuition of the final one-year holding period model, being the tax model. To arrive at this model, we begin with analyzing the index risk and return of PRE.

5.2.1 Capital Appreciation Model

In this first model, we examine how the PRE WRS index risk and return fares in the MPT framework. Below is the return formula used for the PRE return in the capital appreciation model; P_{t+1}[8] is the sales price, and P_t is the purchase price. For the PRE market risk, we utilize the same formula as the one presented for the stock and bond fund, being the formula for standard deviation.

$$R_A = (\frac{P_{t+1} - P_t}{P_t}) \tag{5.3}$$

Using the return formula above, the average annual WRS index return of a one-year investment in a freehold apartment in Oslo is 7.23%. This is higher than the bond fund return and below the global stock fund return. Likewise, the risk of PRE is 6.02% and lies between that of the stock and bond funds. Rated by Sharpe ratios, PRE has the highest ratio of 0.71, the stock fund places second with 0.61 and the bond fund has the lowest with 0.47. Following the analysis of the risk and return of PRE, we now add this asset to the MPT framework to analyze the characteristics of real estate in a mixed-asset portfolio. To derive the mixed-asset efficient frontier, we first need to calculate the correlations between PRE and the financial assets.

[8] $P_{t+1} = exp(WRS_{t+1}) * P_t$

According to Norges Bank Investment Management, low correlations of PRE returns with returns of other asset classes are typically the key argument for introducing PRE into a mixed-asset portfolio. While we find a weak, but positive correlation between PRE and the bond fund of 0.04, and a moderate positive correlation with the stock fund of 0.47, existing literature finds mixed results for different markets. While Hoesli & Lizieri (2007) find correlations close to zero for PRE in the US and UK and a negative correlation with bonds in Australia, Hoesli et al. (2004) find moderate positive levels of correlation with equities and moderately negative correlations with bonds. Also literature on the same markets is inconclusive as Boyd et al. (1998) find that PRE is uncorrelated or negatively correlated with stocks and bonds in the US. In general, it appears that these assets are stronger positively correlated in Norway than other markets, at least for the period being examined[9].

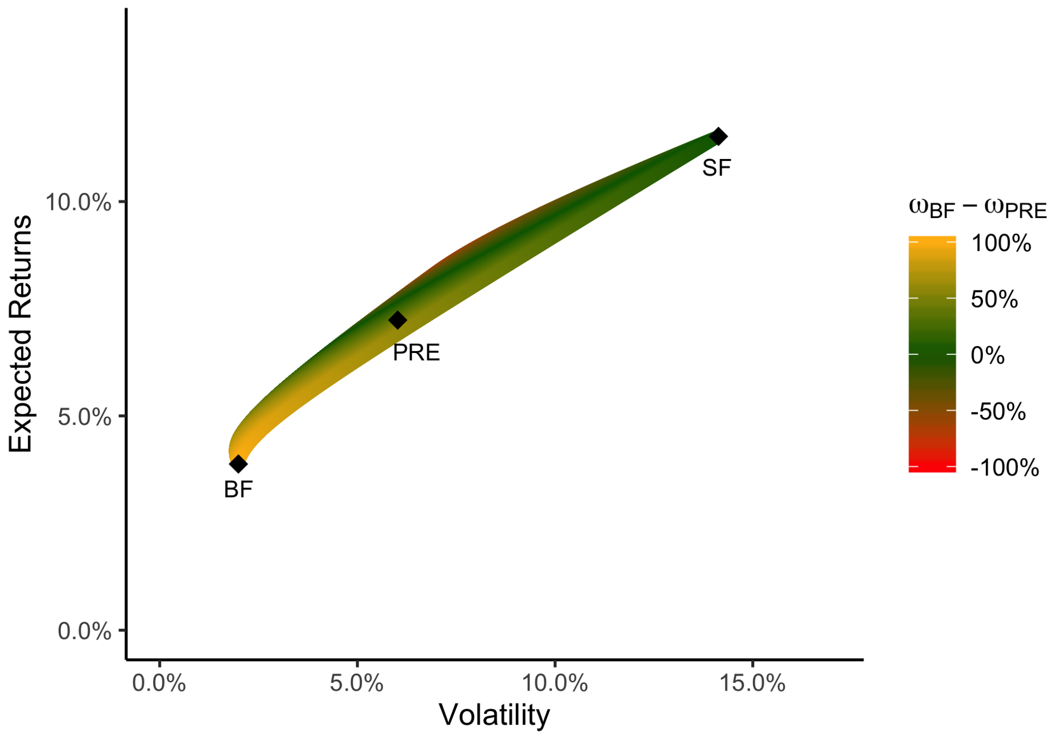

Figure 5.3: Possible Capital Appreciation Model Portfolios

Portfolios comprised of positive mixtures of the stock fund, bond fund and PRE asset from the capital appreciation model, plotted in a mean-variance diagram. The portfolios are created using data over the time period 2005 through 2018, and they are based upon 0.1% increments. The efficient frontier is displayed in 3-D.

Looking at the mean-variance diagram in figure 5.3, PRE is placed in between the stock

[9]As with the correlations between the financial assets, this can be a result of using annual returns.

and bond fund. The efficient frontier is once again fairly straight. This is due to the correlations between all of the assets being positive, indicating small diversification benefits. As it is challenging to conduct a visual comparison of this mixed-asset efficient frontier and the benchmark efficient frontier, we utilize two methods for systematic comparison of the models.

The first method is to compare the financial ratios of the tangent portfolios. The advantage of this, is that it is simple and calculates a few ratios that are easy to follow when moving from one model to the next. The second method is presented in Lee & Stevenson (2004) and examines the incremental impact of including PRE by comparing the risk of efficient portfolios with and without property for the same level of return, and the return of portfolios with and without property for the same level of risk. The advantage of this procedure is that one can analyze whether the PRE asset is mostly return enhancing or risk reducing, and whether this changes when moving along the efficient frontier.

In this paper we utilize both methods, yet focus on the financial ratio method. Here, we compare the tangent benchmark portfolio with the tangent mixed-asset portfolio, or simply PRE. The financial ratios we use are the Sharpe ratio, Jensen‘s alpha, appraisal ratio (AR), and the information ratio (IR). The focus is on the Sharpe ratio, due to our retail investor perspective. A retail investor most likely cares about the total risk of the investment, and not solely the systematic risk. Furthermore, the alpha, AR and IR are not able to account for any of our adjustments we make directly on the risk measure, as they are based on the return series of the assets. We choose to still include these measures because as the liquidity risk is small, they are relevant for the aggregate of our individual investors.

To compare the financial measures of the tangent portfolios, we begin with deriving the tangent mixed-asset portfolio. The allocation to the PRE asset in this portfolio is 32.05%. This allocation is positive, and a result of the PRE asset having the highest Sharpe ratio out of the assets we compare. The reason for why the allocation to this asset is not larger, is due to the positive correlation between this asset and both of the financial assets. The return of this portfolio is 5.35% and risk is 2.80%, compared to the tangent benchmark portfolio where the return is 5.28% and the risk is 3.42%. As expected due to the positive allocation to the PRE asset, the Sharpe ratio of the tangent portfolio is

0.86 and considerably higher than that of the benchmark portfolio, being 0.68. As the return of the tangent mixed-asset portfolio is similar to that of the benchmark, it is the reduction in tangent portfolio risk that drives the increase in the Sharpe ratio.

To examine whether the increased portfolio efficiency is significant for a well-diversified investor, we calculate the alpha of the PRE asset, and the mixed-asset portfolio. In doing this, we first run a regression of the PRE return on the tangent benchmark portfolio return. This yields a positive, yet insignificant annual alpha of 2.30%. This alpha is Jensens' alpha and takes into consideration the capital asset pricing model (CAPM) market theory and includes a risk-adjusted component in its calculation. For this first model, the alpha can be interpreted correctly. Given the return predicted based on the exposure to market risk, the PRE asset yields 2.30% excess return, yet due to variation in its return series, this excess return is not significant.[10]

We repeat this procedure, but instead of using the PRE return, we use the tangent mixed-asset portfolio return and run a regression of this on the same benchmark. The alpha from this regression is 0.71%, and is also insignificant. The alpha from this second regression is always "less extreme" than that of the PRE return. This is because the tangent mixed-asset portfolio incorporates both the return and risk of the PRE asset when deriving the allocation to this asset. This extenuating effect becomes especially clear in the leverage model.

Following the Sharpe ratio and alphas, we calculate the AR and IR of the tangent mixed-asset portfolio. The AR [11] compares the portfolio's alpha to the portfolio's unsystematic risk, showing how many units of active return including PRE in the tangent mixed-asset portfolio produces per unit of risk. The AR of the tangent mixed-asset portfolio is 0.37. The IR[12] is a measurement of the tangent mixed-asset portfolio's returns beyond the returns of the benchmark, compared to the volatility of those returns. The IR is 0.03. A high AR and low IR is a result of the tangent mixed-asset portfolio producing high levels of return per unit of unsystematic risk, yet not compared to its benchmark.

Moving on to the second method for systematic comparison, we find that the overall

[10] Alternatively, the alpha can be interpreted as the model being wrong.

[11] $AR = \frac{\alpha_{RE,S,B}}{\sqrt{\sigma^2_{\epsilon_{RE,S,B}}}}$

[12] $IR = \frac{R_{RE,S,B} - R_{S,b}}{\sqrt{\sigma^2_{R(RE,S,B)-R(S,B)}}}$

dominating quality for PRE is to be risk reducing. This is due to the relative improvement in risk for given return levels is larger than the relative improvement in return for given risk levels. Though the dominating quality of real estate is to be risk-reducing, the asset has both risk reducing and return enhancing qualities. This is consistent with the findings of Lee & Stevenson (2004) who find that direct real estate should be considered a risk reducer rather than a return enhancer in the mixed-asset portfolio.

In summary, the PRE asset has the highest Sharpe out of the assets compared, and receives a positive allocation in the tangent mixed-asset portfolio. Though the Sharpe ratio of this portfolio is superior to that of the benchmark, the alfa from this regression is not significant. This indicates that including PRE in the investment universe does not produce significant gains, at least not for a well-diversified investor. The PRE asset is mainly risk reducing, and as both its risk reducing and return enhancing abilities are moderate, the shift in the efficient frontier in this model is therefore small. We now move on to the next model, where we expect the shift from including PRE in the mixed-asset portfolio to be even smaller.

5.2.2 Individual House- and Liquidity Risk Model

In this section we adjust the PRE market risk to account for two additional sources of risk; individual house risk and liquidity risk. We begin with the individual house risk. This risk occurs because unlike with stock and bond funds, an individual cannot invest in a market index of homes and thereby diversify away unsystematic risk. Therefore, we use our measure of individual house risk presented in the methodology section, and add this to the market risk in the formula below[13].

$$\sigma_h = \sqrt{\frac{\hat{\alpha}}{2} + 4\beta + \sigma_\mu^2} \tag{5.4}$$

After adding the individual risk to the PRE asset market risk, the standard deviation almost doubles, increasing from 6.02% to 10.60%. Compared to Case & Shiller (1987), our risk regression has a larger loading on the price risk, and less on the drift term[14], though the total individual house risk is similar. The drift term is included because Case

[13] This is the same formula that was presented in the methodology section

[14] Our price risk= 0.00738, our drift term=0.00006, Case & Shiller price risk= 0.0042, and drift term=0.001125

& Shiller assume that there is a drift through time of individual house values, due to for example random differences in the amount of upkeep expended across houses, or random changes in neighbourhood quality.

A potential explanation for the small loading on this factor in our model can be that according to the deputy rector of the OsloMet, Norwegians are “world champions of refurbishments”. He argues that this is because many Norwegians own the housing they live in, and it makes economic sense to keep the housing of a high standard. Furthermore, Norwegians in general have a good private economy, and due to the climate they spend a lot of our time indoors[15]. These factors can result in the housing being constantly refurbished and kept of a high quality, thereby reducing the impact of "random differences in the amount of upkeep expended across houses".

Moreover, according to the sociologists Ljunggren and Andersen, over time there has only been minor changes in the “attractiveness” of the different areas of Oslo. Furthermore, there are clear divisions between the west and east side of the city, and the segregation does not seem to be changing[16]. This can reduce the impact of "random changes in neighborhood quality". Hence, most of the individual risk in the Oslo market is price risk, which is a random error. The large increase in total risk from including individual risk decreases the Sharpe ratio of the PRE asset from 0.71 to 0.40, a considerable reduction.

Following this adjustment, we also include the liquidity risk. This is the risk that stems from the “unplanned” extra time you must spend in the PRE market because it takes time to sell an apartment. In some markets, this risk is substantial, but in Oslo the market is highly liquid and this risk is insignificant. This is most likely a result of the period being examined as it is especially in down markets that liquidity becomes an issue.

We use this new market variance that accounts for liquidity risk[17], σ^2_{IL}, and replace the

[15] https://forskning.no/hus-og-hjem-okonomi/derfor-er-nordmenn-verdensmestere-i-a-pusse-opp/1573237

[16] https://forskning.no/sosiale-relasjoner-demografi-etnisitet/oslos-gylne-ghettoer/562169

[17] $\sigma^2_{IL} = \frac{(T_H + t_{TOM})^2 * \sigma^2_\mu + (\sigma^2_\mu + R_A) * \sigma^2_{TOM}}{T_H + t_{TOM}}$, where T_u is the time on the market for the financial assets, t_{TOM} is the extra time on market for the PRE asset (25 days) and R_A is the return on PRE from our first model. Based on Cheng et al. (2013). We use the procedure without assuming i.i.d returns as Case & Shiller (1987) find that housing returns are serially correlated and partially predictable.

"old" market variance, σ_μ^2, in the total standard deviation formula;

$$\sigma_h = \sqrt{\frac{\hat{\alpha}}{2} + 4\beta + \sigma_{IL}^2} \tag{5.5}$$

This risk measure includes the individual house risk, market risk and liquidity risk. With the new risk measure, the standard deviation of PRE increases by 0.12 percentage points from 10.60% to 10.72%. With such a trivial effect on the total risk, and an unchanged return of 7.23%, the Sharpe ratio of PRE remains unchanged at 0.40. After including the effect of the two additional risk measures, the Sharpe ratio of the PRE asset is now the lowest out of the assets we compare.

As seen in the methodology section, the correlations between the assets are not impacted by including independent risk variables. Therefore, though the correlations between the assets change when including individual and liquidity risk, the covariance is unchanged. As it is the covariance that determines the curvature of the efficient frontier, there are no diversification benefits from including individual house- or liquidity risk.

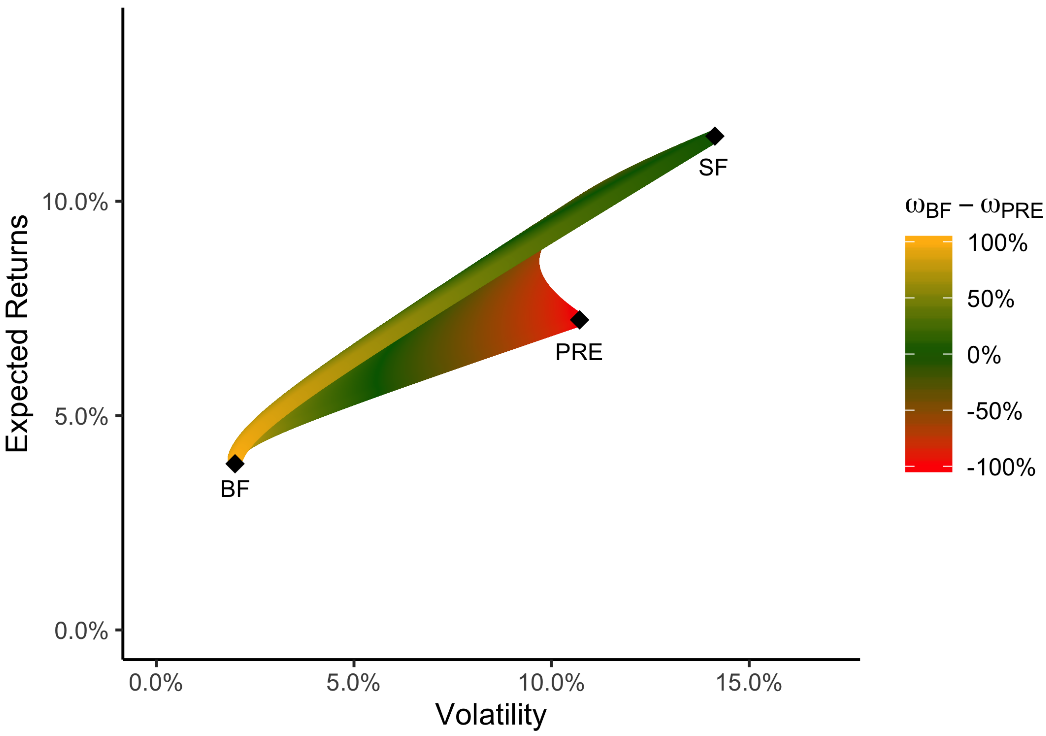

Figure 5.4: Possible Individual House- and Liquidity Risk Model Portfolios

Portfolios comprised of positive mixtures of the stock fund, bond fund and PRE asset from the individual house- and liquidity risk model, plotted in a mean-variance diagram. The portfolios are created using data over the time period 2005 through 2018, and they are based upon 0.1% increments. The efficient frontier is displayed in 3-D.

From the mean-variance diagram in figure 5.4, it is evident that PRE is less attractively placed. The allocation to this asset in the tangent portfolio is now 11.32%, down from 32.05%. This reduction is a result of the added risk measures reducing the Sharpe ratio of PRE from 0.71 to 0.40 while the Sharpe ratios of the stock and bond funds remain unchanged. The return of the tangent mixed-asset portfolio is now 5.31%, down from 5.35% in the capital appreciation model, and the risk is now 3.22%, up from 2.80%. With a decrease in the return and increase in the risk compared to the previous model, it is natural that the Sharpe ratio of the tangent mixed-asset portfolio decreases, and is now 0.73, down from 0.86. As a result of PRE still receiving an allocation in the tangent portfolio, the Sharpe ratio of this portfolio is still above that of the benchmark, being 0.68. The now reduced out-performance of the PRE asset, and therefore the tangent mixed-asset portfolio are also reflected in the alphas.

In the capital appreciation model we run a regression of the PRE return on the benchmark. We do not repeat the procedure for this model, because none of these return variables are adjusted. The alpha from the regression of the new tangent mixed-asset portfolio return on the tangent benchmark return is 0.26% and insignificant[18], down from 0.71% (which is also insignificant) in the previous model. Both the AR and IR are unchanged at a two decimal place from including the individual house- and liquidity risk. This is as mentioned a result of the inability of these ratios to incorporate changes made directly to the risk measure. Though unchanged at a two decimal level, both ratios experience a slight decrease due to the reduced allocation to PRE in the tangent mixed-asset portfolio. Ideally, if the individual house- and liquidity risk were captured in the return series, both ratios would have been reduced considerably. The decrease in all financial ratios is a result of a reduction in the efficiency gain from including PRE in the mixed-asset portfolio.

A counterintuitive result in this model is that despite the negative impact of the increased total risk, the dominating characteristic of real estate is still to be risk reducing. This is a result of the allocation to PRE being reduced for low levels of risk while it remains unchanged for high levels of risk compared to the previous model. Even with the adverse effect of our two additional risk measures, using the PRE asset to reduce risk in the high-return portfolios is still superior to using the bond fund. Though the allocation to real estate is mostly unchanged for the high-return portfolios, the increased risk of this asset has a negative impact on the efficiency gain from both risk reduction and return enhancement.

In summary, as expected, the inclusion of individual house- and liquidity risk reduces the Sharpe ratio of the PRE asset, and therefore the allocation to this in the tangent mixed-asset portfolio. Hence, the Sharpe ratio of this portfolio is also reduced and the alpha is insignificant. Surprisingly, the dominating characteristic of PRE is still to be risk reducing, yet this effect is reduced compared to the previous model.

With the three risk components presented above, we have reached our most comprehensive risk measure and we now move on to variables that affect our return estimate. For all of the following one-year holding period models, this is the risk formula we use. The only difference is that we re-calculate the market risk component (and hence the liquidity risk) based on the new return series in each model. An entirely new risk formula is derived when we move on to the five-year holding period.

[18]Most likely, if the alpha could include the risk measures added in this section, it would have been negative

5.2.3 Imputed Rent Model

Now, we move on to adjustments that affect the return series, while keeping the last model for risk. Before adding the first variable, we emphasize that all of these models “add on” to the previous model, as with the risk measures. This is clear from appendix A, where the data needed to construct each model always consists of all the data needed for previous models as well. The first return adjustment that is added to the capital appreciation return, is the imputed rent yield (Y);

$$R_B = \frac{P_{t+1} - P_t}{P_t} + Y \tag{5.6}$$

As a result of adding the imputed rent, the average annual return of PRE increases from the index return of 7.23%, to 11.53%. The return of PRE is now slightly above that of the stock fund, which is unchanged at 11.52%. The standard deviation of PRE decreases from 10.72% to 10.61%. The reduction indicates that the imputed rent is lower in periods where the housing prices increase. This is a result of the imputed rent being more stable than housing prices, as rents are often contract-based and inflation-adjusted. Since the yield is calculated by dividing the imputed rent by median home price, the yield decreases when house prices increase. This effect slightly reduces the market variance of the return. The Sharpe ratio of housing is now 0.81, up from 0.40. This large effect demonstrates the importance of including imputed rent when comparing PRE to other assets. The Sharpe ratio of the PRE asset is now once again the highest out of the assets we compare.

Because of our new return series, and the fact that we add different yields every year, we now have new covariances between PRE and the financial assets. The correlation between PRE and the stock fund remains unchanged while the correlation between PRE and the bond fund increases from 0.02 to 0.04. This implies that the diversification benefits from adding PRE to the stock fund are unchanged while the diversification benefits of adding PRE to a bond fund slightly decrease. Adding the imputed rent, a steady and continuing cash flow, gives the PRE asset qualities that make it more similar to the bond fund. Hence, the direction of the changes in the correlations seems to be justified.

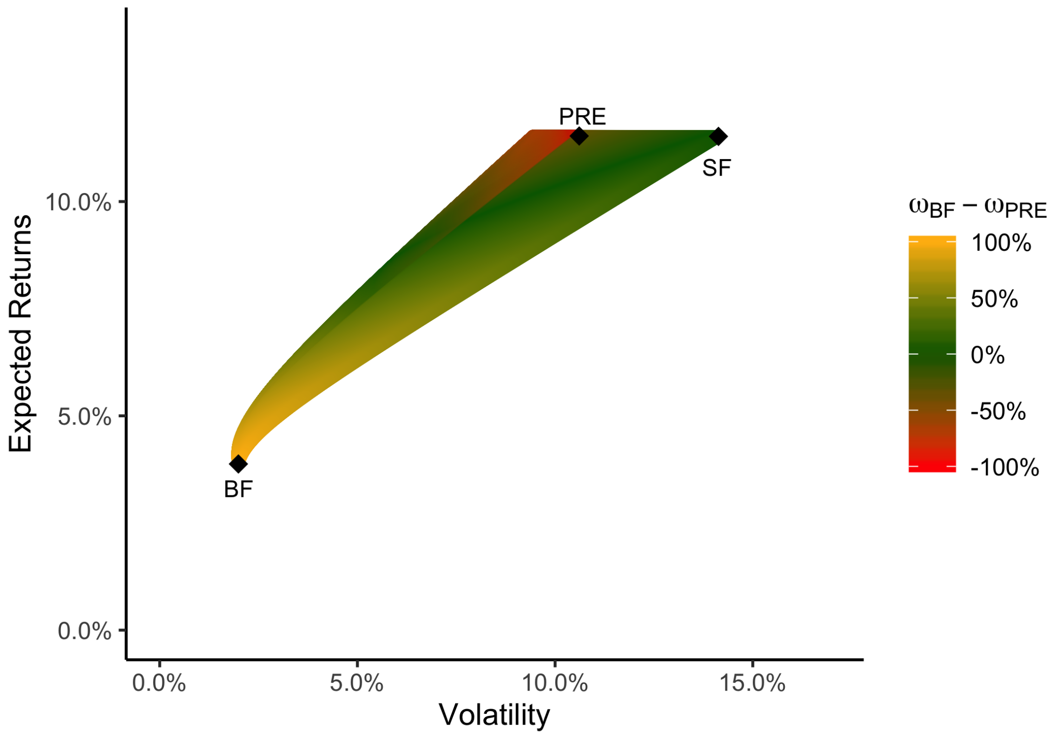

Figure 5.5: Possible Imputed Rent Model Portfolios

Portfolios comprised of positive mixtures of the stock fund, bond fund and PRE asset from the imputed rent model, plotted in a mean-variance diagram. The portfolios are created using data over the time period 2005 through 2018, and they are based upon 0.1% increments. The efficient frontier is displayed in 3-D.

Looking at the mean-variance diagram in figure 5.5 we see that the efficient frontier has shifted outwards. The shape of the frontier is a result of the almost identical return of the stock fund and PRE. As a result of the more attractive placement of PRE in the mean-variance diagram, the allocation to PRE in the tangent mixed-asset portfolio increases from 11.32% to 68.44%. The large allocation to PRE in this model is a result of the real estate having more bond-like characteristics, and it therefore replaces the bond fund in the tangent mixed-asset portfolio. As this portfolio is typically found for low levels or risk and return, the allocation to the bond fund is large. When real estate now completely replaces the bond fund, the allocation to PRE is large instead. Though the standard deviation of PRE itself is slightly reduced, the large allocation to this (still risky) asset increases the risk of the tangent mixed-asset portfolio from 3.22% to 9.44%. The total return of this portfolio increases from 5.31% to 11.52%. With a larger allocation to PRE, the Sharpe ratio of the tangent mixed-asset portfolio increases to 0.91, up from 0.73. Similarly to the Sharpe ratio, we find that the alphas increase compared to the previous models.

Now that we have included the imputed rent, the PRE return series is adjusted and we once again run a regression of the PRE return on the tangent benchmark return. This produces an alpha of 6.65%, up from 2.30% in the previous model, and is now significantly positive. The alpha from the regression of the tangent mixed-asset portfolio return on the benchmark is 4.65%, up from 0.26% and is also significantly positive. For similar reasons to the Sharpe ratio (improved asset risk-return), the AR increases from 0.37 in the previous model to 0.99. The IR increases more, from 0.03 to 1.19. The large increase in the IR is a result of the imputed rent increasing the difference in return between the tangent mixed-asset portfolio and the benchmark, while decreasing the difference in the variability of these returns, and therefore the standard deviation of this. The increase in the financial ratios is a result of the increased portfolio efficiency from including PRE in the mixed-asset portfolio.

The large increase in return from adding the imputed rent changes the dominating characteristic of PRE compared to existing literature and our previous models, as it is now to be return enhancing[19]. Compared to the previous model, the improved portfolio efficiency from both return enhancement and risk reduction increases.

In summary, including the imputed rent leads to a considerable increase in the PRE asset Sharpe ratio as it both increases the return and decreases the risk of this asset. This leads to a larger allocation to this asset in the tangent mixed-asset portfolio and an increase in the Sharpe ratio of this. In this model, including PRE in the investment universe leads to significantly improved risk-adjusted returns for a well-diversified investor, and the dominating characteristic of PRE is now to be return enhancing. Before presenting the next return adjustment, we comment on the issue of reinvestment of the imputed rent.

We do not adjust for reinvestment of imputed rent as this does not reflect reality.[20] First of all, it is not obvious what this imputed rent should be invested in. It could be the stock fund, the bond fund, a money market fund, the primary home, a secondary home or it could be held in cash. Assuming that it should be invested in our WRS index; on the one

[19]As we can only compare the portfolios for the interval of risk and return included in the benchmark portfolio, models where the PRE asset has the highest risk, return or both can be biased.

[20]Most likely some of the imputed rent would be netted out against the instalments on the mortgage, but in this paper we assume no such payments are made on the loan over the holding period.

hand, investing it in the primary home is problematic because 14 years' worth of imputed rent cannot all be spent on refurbishments that increase the apartment value comparably[21]. On the other hand, 14 years' worth of imputed rent is also not enough to save up and purchase a secondary home, and you cannot purchase 0.3% of a secondary home each month. Therefore, for now, we continue to build on the model without reinvestment of the imputed rent, though we recognize the small bias this creates, being an advantage to the financial assets.

Now, we move on to the next variable which will continue to increase the return of PRE, but this time, most likely also the risk.

5.2.4 Leverage Model

To create a more realistic comparison of our assets, we leverage the imputed rent model. Once again, this model builds on to the three previous models. We do not lever the stock and bond funds, as this is rarely done in reality for a retail investor. Furthermore, the public companies on Oslo Stock Exchange are typically already leveraged at around 60.98%[22]. The leveraging affects the return on equity (ROE) of the PRE asset through two different channels.

First, adding debt leverages the ROE due to a "gearing effect". Secondly, the mortgage payment reduces the total return, and hence the ROE of the asset. Including debt can add a third component, instalments, but we assume that no instalments are paid over the holding period. The new return formula accounting for these two effects is;

$$R_C = \left(\frac{P_{t+1} - P_t}{P_t} + Y - (LTV * MR) \right) * \frac{1}{(1 - LTV)} \tag{5.7}$$

We leverage the capital appreciation, imputed rent and mortgage cost. This is because we want to calculate the return to the investor's equity, not the total return. When adding debt, the ROE increases because the total return of the PRE asset (before including the mortgage) is larger than the mortgage rate. The return of the PRE asset is now 20.49%, up from 11.53% in the imputed rent model. The downside of leverage is that both gains

[21]At some point home buyers are not willing to spend more on an apartment of a certain size, no matter how much it has been refurbished.

[22]Estimate derived from Statistics Norway per 31.12.2016

and losses are enhanced, and the standard deviation of PRE therefore increases from 10.61% to 16.30%. This increase is moderate compared to the accompanying increase in return, and as a result, the efficient frontier shifts outwards. This contradicts the Modigliani and Miller (MM) theorem (1958, 1963), but is most likely a result of the period 2005-2018 not encompassing an entire housing cycle (we return to this discussion in the robustness section). The Sharpe ratio of the PRE asset increases to 1.08, up from 0.81. The out-performance of the Sharpe ratio of the PRE asset compared to the financial assets therefore increases compared to the previous model.

The correlation between PRE and the financial assets also increases. The correlation between the stock fund and PRE increases from 0.26 to 0.37 while the correlation between the bond fund and PRE increases from 0.04 to 0.06. This is because these two correlations are already positive, and when we leverage the PRE asset, its variance increases, thereby increasing its covariance with the other assets. This leads to decreased diversification benefits of PRE, making our efficient frontier less curved. In Boyd et al. (1998) the effect of leverage is the opposite, as the correlations between the financial assets and PRE are initially negative and then become stronger negatively correlated as a result of the leverage.

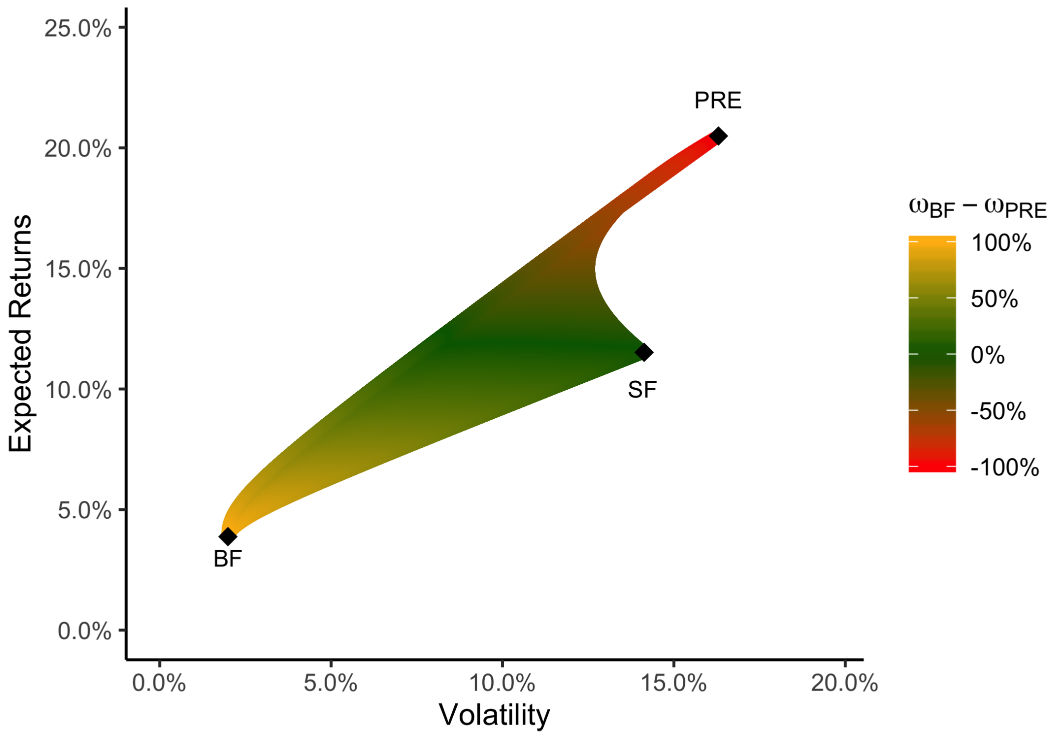

Figure 5.6: Possible Leverage Model Portfolios

Portfolios comprised of positive mixtures of the stock fund, bond fund and PRE asset from the leverage model, plotted in a mean-variance diagram. The portfolios are created using data over the time period 2005 through 2018, and they are based upon 0.1% increments. The efficient frontier is displayed in 3-D.

From the mean-variance diagram in figure 5.6 we observe that PRE is located to the far right of the efficient frontier. Despite the net improvement in the risk-return of PRE, the allocation to this asset in the tangent portfolio decreases from 68.44% to 24.21%. This is due to similar reasons as the large PRE allocation in the previous model. The new risk and return characteristics of PRE make this asset more similar to the stock fund, and this is therefore the asset that PRE mainly replaces. As the tangent portfolio is typically found for low levels of risk and return, the allocation to the stock fund is small, and the allocation to PRE when it replaces most of the stock fund is therefore also small. As a result of the decreased allocation to PRE, the return of the tangent mixed-asset portfolio decreases from 11.52% to 8.12% and risk from 9.44% to 4.50%. The Sharpe ratio of the tangent mixed-asset portfolio however, increases from 0.91 to 1.15. This increase occurs because the Sharpe ratio of PRE is higher than in the previous model, and this asset still receives a positive allocation in the tangent mixed-asset portfolio. The increase in Sharpe ratio is once again accompanied by an increase in the alphas from the regression of the PRE asset and the tangent mixed-asset portfolio on the tangent benchmark portfolio.

We repeat the procedure of running a regression of the PRE return on the benchmark portfolio. This exercise yields a significantly positive alpha of 13.08%, up from 6.65% in the previous model. This is the highest alpha found in any of the models (including the five-year holding period), and is a result of this being the last model after only upward adjustments to the PRE return. In the next models, we account for the costs from investing in PRE. Hence, this alpha is unrealistic, and not to be expected in the market. Following this regression, we add the new PRE asset to the tangent mixed-asset portfolio and run a regression of this on the tangent benchmark portfolio. This alpha is now reduced to 3.13%, down from 4.65%, yet is still significantly positive. The PRE return alpha increases while the tangent mixed-asset portfolio alpha decreases compared to the previous model as a result of the tangent mixed-asset portfolio incorporating both the risk and return of the PRE asset. While the PRE asset alpha solely measures the increased return as a result of leverage, the mixed-asset tangent portfolio incorporates the increased risk that often accompanies increased return through the allocation to the PRE asset in the tangent portfolio.

Following the development in the Sharpe ratio and PRE asset alpha, the AR increases slightly from 0.99 to 1.03. The IR on the other hand, is reduced from 1.19 to 0.93. This indicates that the leverage effect on the tracking error is relatively larger than the effect on the return difference between the tangent portfolio and the benchmark. The increase in all but one financial ratio is the result of increased portfolio efficiency from including PRE in the mixed-asset portfolio.

Similarly to the previous model, the dominating characteristic of PRE in the mixed-asset portfolio is to be return enhancing. Compared to the imputed rent model, both the return enhancement and risk reduction ability of PRE are larger.

In summary, leveraging the PRE asset leads to an increase in the Sharpe ratio of this asset, implying that the increase in return is not compensated for with an equivalent increase in risk. Though the allocation to this asset in the tangent mixed-asset portfolio is reduced, the Sharpe ratio of this portfolio increases. As with the imputed rent model, the gains for a well-diversified investor from including PRE are significant, and the dominating characteristic of this asset is to be return enhancing. Now, we move from the adjustments that increase the return of PRE, to adjustments that decrease it.

5.2.5 Owner Cost Model

In this model we add two additional variables, the operating costs and transaction costs, together denoted owner costs. These adjustments add on to the previous models, and we begin with the operating costs. When we include the imputed rent yield, it is because we argue that in our investment universe, one must live somewhere, either through renting or owning. Yet, adding the entire imputed rent to the PRE asset is unjustified, because some costs need to be paid, regardless of ownership status. We therefore adjust the average annual return to take into account the operating costs (OC) incurred regardless of housing situation, and use net yield (NY) instead of Y-OC;

$$R_D = \left(\frac{P_{t+1} - P_t}{P_t} + NY - (LTV * MR) \right) * \frac{1}{(1 - LTV)} \tag{5.8}$$

These operating costs mainly consist of insurance of the building, municipal costs and maintenance. Before moving on to the effect of these on the PRE risk and return, there is another cost that must be deducted from the PRE return. This is the transaction cost, and is related to both purchase and sale of the housing unit. Building on to the return formula with operating costs, the PRE return with owner costs is calculated by;

$$R_E = \left(\frac{P_{t+1} - P_t - TC}{P_t} + NY - (LTV * MR) \right) * \frac{1}{(1 - LTV)} \tag{5.9}$$

As expected, including owner cost reduces the average annual return of the PRE asset. The average annual return is now 6.53%, down from 20.49% in the leverage model. This decrease appears to be large considering that the owner costs are approximately 6%. Yet, the average annual return decreases by almost 14%. This is an effect of the leverage. If we deduct all the costs before leveraging the PRE asset, the total return is now 3%. If we then add leverage of 2.17, the return is 6.53%.[23] The risk is slightly reduced, from 16.30% to 15.96%. This reduction is caused by a similar effect as with the imputed rent. If the house value increases and the owner costs remain the same, the costs in percent of median house value will decrease. One could argue that this is realistic as several of the operating and transaction costs are government-determined and only change annually by a small pre-determined interval. The new annual Sharpe ratio of PRE is reduced from

[23] Another way of looking at this is taking the owner costs of 6.17% and leveraging this, which equals the 14% that are deducted from the return.

1.08 to 0.22. This significant drop is a result of the owner costs reducing the return, yet not greatly impacting the risk of the asset, due to their low variance. The PRE asset now once again has the lowest Sharpe ratio out of the assets we compare.

As we subtract different owner costs each year, our return series is affected and we have new covariances between our assets. The correlations between PRE and the financial assets are slightly reduced. These reduced correlations imply a small increase in the diversification benefits from including PRE in the mixed-asset portfolio.

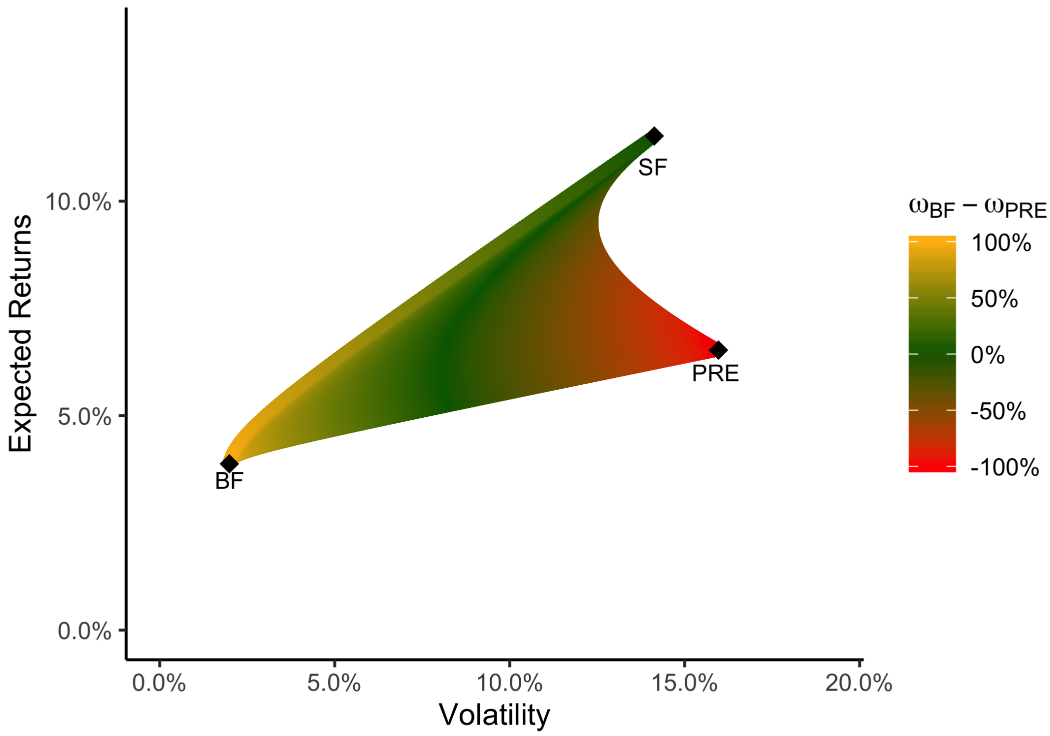

Figure 5.7: Possible Owner Cost Model Portfolios

Portfolios comprised of positive mixtures of the stock fund, bond fund and PRE asset from the owner cost model, plotted in a mean-variance diagram. The portfolios are created using data over the time period 2005 through 2018, and they are based upon 0.1% increments. The efficient frontier is displayed in 3-D.

From the efficient frontier in figure 5.7 it is clear that PRE is the riskiest asset, but does not have the highest expected return. As all assets are now presented annually and after costs, this is the first unbiased comparison of PRE, the stock fund and the bond fund (based on our adjustments). The efficient frontier in this model is almost identical to that of the benchmark. This implies that the allocation to PRE along the efficient frontier is minimal. This is seen in the tangent mixed-asset portfolio, as the allocation to this asset decreases from 24.21% in the previous model, to 0.64% in this model. The return of this tangent portfolio is 5.27%, down from 8.12%, and the portfolio risk is reduced to 3.40% from 4.50% in the previous model. As a result of the negative development in the PRE asset Sharpe ratio, the tangent mixed-asset portfolio Sharpe ratio is reduced from 1.15 to 0.68. The small allocation to PRE in this model is obvious from the fact that the Sharpe ratio of the tangent portfolio with PRE, is now the same as the Sharpe ratio of the tangent portfolio without PRE. The same development is reflected in the alphas.

With owner costs, the regression of the PRE return on the tangent benchmark portfolio return yields an insignificantly negative alpha of -0.71%, down from 13.08% and significant in the previous model. Similarly, the alpha from the regression of the tangent mixed-asset portfolio on the benchmark yields an insignificant alpha of 0.01%, down from 3.13% and significant in the previous model. Once again, the tangent portfolio has a mitigating effect on the change in the PRE asset characteristics. The low level of this alpha is a result of the low allocation to PRE in the tangent mixed-asset portfolio. The regression above has largely the same portfolio as both the dependent and independent variable and the constant is therefore zero. With an insignificant alpha, the AR is also insignificant at -0.07, down from 1.03 in the leverage model. The IR is negative at -0.17, down from 0.93, a result of the benchmark return being 5.28%, which is higher than the tangent mixed-asset portfolio return, being 5.27%, while the tracking error is mainly unchanged from the previous model.

Due to the minimal allocation to PRE along the entire frontier, real estate neither has return enhancing nor risk reducing qualities the mixed-asset portfolios.

In summary, as expected, the owner costs lead to a reduction in the PRE asset Sharpe and therefore the allocation to this asset in the tangent mixed-asset portfolio. The Sharpe ratio of this portfolio is also reduced and a well-diversified investor no longer obtains a significant alpha from including PRE in the investment universe. As the allocation to PRE is minimal, it neither has return enhancing or risk reduction abilities. Now, we move on to the final model, which introduces the only adjustment that also affects the financial assets.

5.2.6 Tax Model

In the final one-year holding period model, we do not add any risk or return variables to the owner cost model. The only adjustment that remains is to calculate the after-tax values for the assets, including the risk-free asset. With a one-year holding period we assume that the owner lives in the home exactly one whole year and hence is exempt from the capital gains tax. The imputed rent is not taxed in Norway. Though we have property and wealth taxes, the wealth tax does not affect the median Oslo citizen and the property tax does not affect the median apartment.

The new return for PRE is;

$$R_G = \left(\frac{P_{t+1} - P_t - TC}{P_t} + NY - (1 - GITR) * LTV * MR - PTR \right.$$
$$\left. -(WVH - LTV) * WTR \right) * \frac{1}{(1 - LTV)} \tag{5.10}$$

This formula includes the GITR but also the PTR and WTR[24] though in the base-case these two last values are zero.

In this last one-year model, for the first time, the stock fund, bond fund and risk-free asset returns are affected. For all the financial assets, both capital gains and dividends are taxed. As we are using mutual funds, only the taxation of capital gains is relevant. The profit from the sale of stocks is reduced by R_{F_2} and then increased by the FUA before being multiplied with the GITR.

The new return for the stock fund is;

$$R_{SF} = \left(\frac{P_{t+1} + Div - fees - P_t}{P_t} \right) - (R_{SF} - R_{F_2}) * FUA * GITR - WTR * WVS \tag{5.11}$$

The bond fund and risk-free profits are taxed with the GITR (no FUA). The new return for the bond fund (and the risk-free rate used for the tangent portfolio) is;

$$R_{BF} = \left(\frac{P_{t+1} + Div - fees - P_t}{P_t} \right) - (1 - GITR) - WTR * WVB \tag{5.12}$$

The return of PRE increases from 6.53% to 7.79% when including taxes. This appears counterintuitive, but is a result of the only tax effect for PRE being the deductible

[24] We deduct taxes based on their income year for comparability. In reality, the rates from the income year determine the taxes paid in the following year, which is the tax assessment year.

mortgage rate. As the mortgage is a cost, reducing it increases the return. The effect of taxation on the stock and bond funds on the other hand, is negative, especially for the stock fund. The average annual stock fund return decreases from 11.52% to 8.67% and the bond fund return decreases from 3.88% to 2.82%. The return of the risk-free asset used in our Sharpe ratio calculations decreases from 2.95% to 2.14% after taxation.

The standard deviation of the PRE asset decreases from 15.96% to 15.83%, and the standard deviation of the stock fund decreases from 14.12% to 10.08%. This is due to the GITR having a stronger impact on the returns in years where they are high, especially because of the FUA. The standard deviation of the bond fund is reduced from 1.99% to 1.42%, for the same reasons as with the stock fund (except the FUA). The annual Sharpe ratio of the PRE asset is now 0.36, up from 0.22. The Sharpe ratio of the stock fund increases from 0.61 to 0.65 and the bond fund Sharpe ratio increases from 0.47 to 0.48 as a result of taxation. The Sharpe ratio of the PRE asset is therefore still the lowest out of the assets we compare, yet the under-performance compared to the previous model is reduced.

The covariance between the assets also changes due to the effect of the different tax laws, and the change in these over the period. The new correlation is weaker between PRE and the stock fund, and slightly strengthened between PRE and the bond fund. Now that we adjust the returns of the stock and bond funds as well, their correlations change for the first time, yet insignificantly due to the similar taxation of the two assets.[25]

[25] The FUA separates the taxation of the assets, this was introduced in 2016 and has therefore been in effect for two out of the fourteen years analyzed

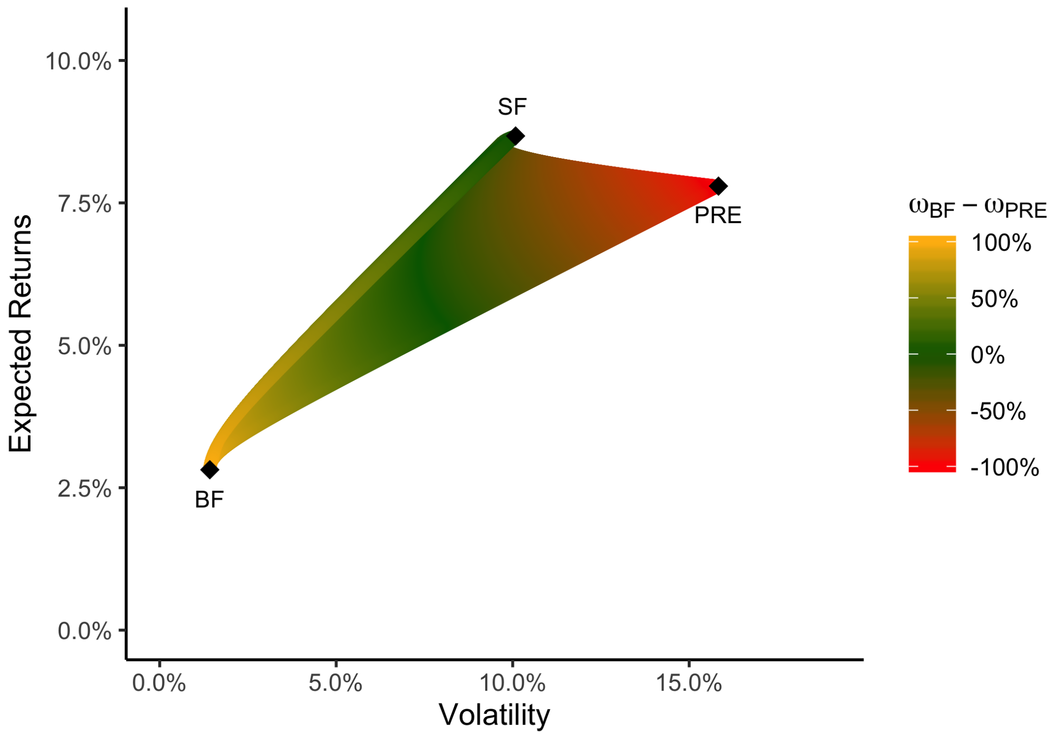

Figure 5.8: Possible Tax Model Portfolios

Portfolios comprised of positive mixtures of the stock fund, bond fund and PRE asset from the tax model, plotted in a mean-variance diagram. The portfolios are created using data over the time period 2005 through 2018, and they are based upon 0.1% increments. The efficient frontier is displayed in 3-D.

Looking at the mean-variance diagram in figure 5.8, PRE is located to the far right and does not appear to be represented along the efficient frontier. The high risk of the PRE asset does not seem to be adequately compensated for, as the stock fund has lower risk yet yields higher returns. At this point, a new benchmark efficient frontier and tangent benchmark portfolio are calculated using the after-tax stock and bond fund values.

The allocation to the stock fund in the after tax benchmark is 19.60%, up from 18.42%, and the allocation to the bond fund is 80.40%, down from 81.58% in the before-tax benchmark. The new return is 3.96%, down from 5.28%. This is a result of the taxation affecting both assets in the portfolio adversely. The new benchmark risk is 2.53%, down from 3.42%. This because taxation reduces the risk of the financial assets by reducing the variability of their returns. The Sharpe ratio of the tangent benchmark portfolio is 0.72 and has therefore increased as a result of taxation. This development is also seen for the tangent mixed-asset portfolio Sharpe ratio.

As taxation increases the relative attractiveness of PRE compared to the financial assets, the allocation to real estate in the tangent mixed-asset portfolio increases to 3.50% from 0.64% in the owner cost model. The return of this portfolio is 3.97% down from 5.27% and the risk is 2.50%, down from 3.40% in the owner cost model. Due to the small allocation to PRE, the Sharpe ratio of the tangent mixed-asset portfolio is 0.73 and is therefore slightly above that of the tangent benchmark portfolio. This slight out-performance is reflected in the alphas as well.

The alpha from regressing the after-tax PRE return on the after-tax tangent benchmark return is positive, but insignificant at 1.33%, up from -0.71% in the owner cost model. Similarly, the alpha from running a regression of the tangent mixed-asset portfolio on the benchmark is 0.04%, up from 0.01%, and is also insignificant. These results can be driven by the inclusion of taxes bringing the tangent portfolio down to even lower levels of risk and return. Therefore, while the risk and return characteristics of PRE improve compared to the previous model, the allocation to this asset is still small, as the asset has a high risk and return. The AR of this portfolio is 0.09, up from -0.07 while the IR is 0.03 up from -0.17. These ratios indicate a slight improvement in the risk-return characteristics of the PRE asset, both individually, and relatively to the benchmark. The low financial ratios are a result of the small increase in portfolio efficiency from including the PRE asset in the mixed-asset portfolio.

As there are some allocations to PRE along the efficient frontier, we find that the dominating characteristic of this asset is still to be return enhancing. The result may be surprising as the allocation to PRE in this model most likely is a result of diversification benefits. This can be a result of the weakness of the comparison and the bias that occurs when the risk or return of the PRE asset is found for higher levels than the risk or return of the benchmark. Furthermore, as both the risk reduction and return enhancement effects are small, the return enhancement effect is marginally above that of the risk reduction. Compared to the previous model, the risk reduction and return enhancement gains are larger, yet the impact of these effects on the efficient frontier is still small.

In summary, the Sharpe ratio of PRE in the most comprehensive one-year holding period model is 0.36 and is the lowest out of the assets we compare. The allocation to this asset in the tangent mixed-asset portfolio is small, and is most likely a result of diversification benefits. As a result of the small allocation to real estate in the tangent mixed-asset portfolio, the Sharpe ratio of this portfolio is only slightly above that of the benchmark portfolio. There are therefore no significant gains for a well-diversified investor from including PRE in the investment universe. The dominating characteristic of PRE is to be return enhancing, yet this effect is small and only marginally larger than the risk reduction effect. We therefore find that the risk-return trade-off from investing in PRE for a one-year holding period is inferior to that of the stock and bond funds, yet some real estate may be included in the portfolio for diversification.

We now move on to a more realistic holding period and expect the relative risk-return trade-off from investing in real estate to improve. Before doing this, we present a short recap of the models analyzed in this section.

Table 5.1: One-Year Holding Period for Freehold Units

Panel A consists of the summary statistics for the freehold PRE and financial assets. Panel B provides summary statistics for the tangent portfolios. The benchmark is the tangent portfolio consisting of the stock and bond funds, and the tangent mixed-asset portfolios consist of the different measures of PRE risk and return, combined with the stock and bond funds. Panel C consists of financial measures obtained when comparing the tangent mixed-asset portfolio to the tangent benchmark portfolio. All values are annual and from the time period 2005-2018.

Panel A: Summary Statistics Assets	Stock Fund	Bond Fund	Capital Appreciation	Individual and Liquidity Risk	Imputed Rent	Leverage	Owner Cost	Stock Fund after Tax	Bond Fund after tax	Tax
Average Return	11.52%	3.88%	7.23%	7.23%	11.53%	20.49%	6.53%	8.67%	2.82%	7.79%
Standard Deviation	14.12%	1.99%	6.02%	10.72%	10.61%	16.30%	15.96%	10.08%	1.42%	15.83%
Sharpe Ratio	0.61	0.47	0.71	0.40	0.81	1.08	0.22	0.65	0.48	0.36
Maximum Return	31.07%	7.39%	17.61%	17.61%	21.31%	43.49%	29.63%	22.37%	5.33%	30.37%
Minimum Return	-19.15%	0.51%	-6.10%	-6.10%	-1.10%	-10.41%	-23.94%	-12.72%	0.37%	-21.66%
Skewness	-0.61	0.06	-0.35	-0.35	-0.34	-0.46	0.45	-0.57	0.04	-0.42
Kurtosis	2.61	2.02	2.81	2.81	2.62	3.08	3.24	2.47	2.06	3.10

Panel B: Summary Statistics Portfolios	Benchmark	Capital Appreciation	Individual and Liquidity Risk	Imputed Rent	Leverage	Owner Cost	Benchmark after Tax	Tax
Stock Fund Weight	18.42%	5.22%	13.75%	31.56%	3.32%	18.02%	19.60%	16.80%
Bond Fund Weight	81.58%	62.73%	74.93%	0.00%	72.46%	81.34%	80.40%	79.60%
Private Real Estate Weight	0.00%	32.05%	11.32%	68.44%	24.21%	0.64%	0.00%	3.50%
Average Return	5.28%	5.35%	5.31%	11.52%	8.12%	5.27%	3.96%	3.97%
Standard Deviation	3.42%	2.80%	3.22%	9.44%	4.50%	3.40%	2.53%	2.50%
Sharpe Ratio	0.68	0.86	0.73	0.91	1.15	0.68	0.72	0.73
Maximum Return	9.47%	7.95%	9.10%	19.68%	11.75%	9.68%	7.24%	7.05%
Minumum return	-3.11%	-2.63%	2.94%	-6.76%	-2.87%	-3.18%	-2.20%	-2.61%
Skewness	-0.98	-1.77	-1.40	-1.03	-1.59	-1.03	-0.95	-1.32
Kurtosis	3.61	5.58	4.84	3.97	5.1	3.75	3.5	4.6

Panel C: Financial Measures Portfolios	Capital Appreciation	Individual and Liquidity Risk	Imputed Rent	Leverage	Owner Cost	Tax
Alpha Private Real Estate	2.30%	2.30%	6.65%**	13.08%**	-0.71%	1.33%
Beta Private Real Estate	0.85	0.85	0.83	1.91	1.84	2.37
Alpha All Assets	0.71%	0.26%	4.65%*	3.13%*	0.01%	0.04%
Beta All Assets	0.73***	0.90***	1.68***	0.89**	1.00***	1.00***
Appraisal Ratio All Assets	0.37	0.37	0.99	1.03	-0.07	0.09
Information Ratio All Assets	0.03	0.03	1.19	0.93	-0.17	0.03

*Signif. codes: 0'***' 0.001 '**' 0.01 '*' 0.05*

5.3 Five-Year Holding Period for the Freehold Unit

Utilizing a one-year holding period creates a simple example to follow, but is far from reality for the median Oslo citizen. According to "Prognosesenteret" young people on average live in their housing for three years, families with young children live in the same unit on average for four years, while families with older children live in the same unit on average eleven years. Adults without children live in the same unit on average fourteen years[26]. In order to better reflect reality, we increase our holding period to five years. We argue that this is fair, given the assumption that there is a majority of young people, families with young children and adults living in apartments in Oslo.

Changing the holding period adds several layers of complexity to the PRE asset. In this section we present the adjustments made when moving from a one- to five-year holding period in our tax model. We are still solely analyzing primary freehold apartments in Oslo, and we start with the return adjustments. As we are using average annual returns, the changes to the returns are not a direct result of changing the holding period, but more changes in our assumptions as a result of the change in holding period. These adjustments are presented in the same order as they appear in our one-year holding period model.

The first return variable we adjust is the leverage. The leverage used for a one-year holding period is not realistic to use over a five-year holding period. This is because even though we assume that no instalments are paid, the capital appreciation of the housing decreases the LTV each year. To account for this, we decrease the LTV proportionally with the capital appreciation of housing over five years and average these LTVs. The LTV for the five-year holding period is 47.29%, down from 54.11% in the one-year holding period. Hence, the five-year investment is on average less leveraged, even without instalments. This effect reduces the risk and return of the PRE asset. The "no instalments" assumption is less applicable for a longer time period, but we argue that it is better to keep it for comparability with the one-year holding period model.

In addition to the leverage, some of the owner costs are also affected by the holding period. While the operating costs are independent of realization and paid annually, the transaction costs are only paid at the time of purchase and sale. These costs are now paid

[26]https://www.aftenposten.no/bolig/i/QogG6J/slik-bor-vi-i-norge-i-dag

once every five years and annualized. This effect increases the risk and return of the PRE asset.

A challenge with the longer holding period is that what is considered maintenance and repair, and what is considered refurbishment, depends on the time horizon one is examining. Over one year, painting a wall can be considered maintenance, while looking at twenty years, shifting the entire bathroom or kitchen can be considered maintenance. We choose to use the same measure of maintenance costs as in the one-year holding period for consistency. As these are operating costs, they are paid annually and are therefore not adjusted in our five-year holding period[27].

The new return formula is;

$$R_{G,5y} = \left(\sqrt[5]{\frac{P_{t+5} - P_t - TC}{P_t}} + NY - (1 - GITR) * LTV_{5y} * MR - PTR \right. \\ \left. -(WVH - LTV5y) * WTR \right) * \frac{1}{(1 - LTV_{5y})} \tag{5.13}$$

The average annual return of PRE increases from 7.79% to 17.40% when moving from the one-year to the five-year holding period. For the stock and bond fund, the capital gains tax is paid every five years and therefore their risk and return change slightly as well[28] (due to compounding). This is the only return effect from the longer holding period for the financial assets.

New return stock fund;

$$R_{SF,5y} = \sqrt[5]{\frac{P_{t+5} + Div - fees - P_t}{P_t} - (R_{SF} - R_{F_2}) * FUA * GITR} - WTR * WVS \tag{5.14}$$

New return bond fund;

$$R_{BF,5y} = \sqrt[5]{\frac{P_{t+5} + Div - fees - P_t}{P_t} * (1 - GITR)} - WTR * WVB \tag{5.15}$$

The average annual return of the stock fund increases from 8.67% to 9.38% and the average annual return of the bond fund increases from 2.82% to 2.88%. The return of the risk-free asset incurs the same taxation as the bond fund, and the return of this asset therefore increases from 2.14% to 2.18% Though the return measures of all the assets are

[27] $P_{t+5} = exp(WRS_{t+5}) * P_t$

[28] For the five-year holding period, the question surrounding the treatment of the time value of money arises. In this paper we decide to not include this aspect as it complicates the analysis, and there is no consensus on how it should be incorporated in this model.

altered, the most important adjustment made when moving from a one-year to a five-year holding period relates to the measure of risk.

To adjust the risk for the longer holding period, we calculate a new market variance to account for the new return series and the autocorrelation of this. Following this, we add liquidity risk to the new market variance and include this into our total risk calculation[29]. The new risk formula for our five-year holding period is;

$$\sigma_{h,5y} = \sqrt{\frac{\frac{\hat{\alpha}}{2}}{5} + 4\beta + \frac{\sigma^2_{IL,5y}}{5}} \tag{5.16}$$

In moving from the one-year holding period to the five-year holding period, three variables therefore affect the new standard deviation. The first is the new return series based on the changes made to the transaction costs and LTV. The second is the calculated autocorrelation, and the third is the treatment of the individual price risk and the liquidity risk. In total, the standard deviation of PRE decreases from 15.83% in the one-year holding period, to 8.36% in the five-year holding period, a considerable reduction.

In our analysis we find that reducing transaction costs and LTV positively impacts the market variance of the new return series, as expected. The new return series reduces the standard deviation from 15.83% to 14.82%. Also, because the autocorrelation is mostly negative, this effect reduces the standard deviation further from 14.82% to 11.98%. The last effect of the increased holding period is that the individual price risk and liquidity risk are divided by the holding period. The reason behind this adjustment is that according to Goetzman (1993) "while the variance of the market risk and drift grow linearly in time, the price risk does not". The liquidity risk is treated similarly, as Cheng et al. (2013) state that "This result is in fact consistent with our common perception that liquidity risk would be less of an issue to investors who are able to hold private assets for a longer period of time". If we increase the holding period enough, these two risk components become insignificant. This effect decreases the standard deviation of PRE from 11.98% to 8.36%. All three effects reduce the standard deviation of PRE when going from the one- to five-year holding period, making the five-year holding period less risky.

Following the two first steps of the PRE risk adjustments, the average annual standard

[29] We calculate the liquidity risk and the autocorrelation based on the new return series and add these risk measures together in $\sigma^2_{IL,5y}$

deviation for the stock fund increases from 10.08% to 10.65% after the tax adjustment, and is then reduced to 9.11% due to mostly negative autocorrelations. While Mackinnon & Al Zaman (2009) find that return volatility in the long run is reduced due to mean reversion, they find that the effect is stronger for listed equities than for PRE. We find the opposite, as the negative autocorrelation leads to a 19.16% reduction in the PRE volatility and 14.55% reduction in the stock fund volatility. The bond fund risk increases from 1.42% to 1.48% due to the tax adjustment, and then slightly increases further to 1.49% because of the mainly positive autocorrelations.

Therefore, while the increase in holding period leads to increased returns for all three assets, the risk of the financial assets increase, while the risk of PRE decreases. This is because for PRE, the positive effect on the return from reduced transaction costs is larger than the negative effect of reduced leverage, resulting in an increased return. The positive effect of reduced leverage on the risk however, is larger than the negative effect of reduced transaction costs, resulting in a reduced risk. For the financial assets, the effect of reduced taxes on the return is positive, while the effect of reduced taxes on the risk is negative.

The Sharpe ratio of PRE increases to 1.82, up from 0.36 in the one-year holding period as the risk effect discussed above is reinforced by the negative autocorrelation and decreased price- and liquidity risk. The Sharpe ratio of the stock fund increases from 0.65 to 0.80, indicating that the positive effect of the negative autocorrelation on the risk is larger than the negative effect of reduced taxes. The Sharpe ratio of the bond fund is unchanged at 0.48, indicating that the two effects on the risk cancel out against the increased return. The Sharpe ratio of the PRE asset is therefore once again the highest out of the assets we compare, and this out-performance is the largest found in any of the models so far.

The five-year holding period also results in significant changes in the correlations of the assets. The correlation between PRE and the stock fund increases from 0.36 to 0.71 and the correlation between PRE and the bond fund increases from 0.05 to 0.09. The correlation between the stock and bond fund changes less, from 0.27 to 0.28. Also these results contrast those of MacKinnon & Al Zaman (2009) and Rehring (2012) who find that correlations between real estate and other asset classes tend to decrease with an increasing investment horizon.

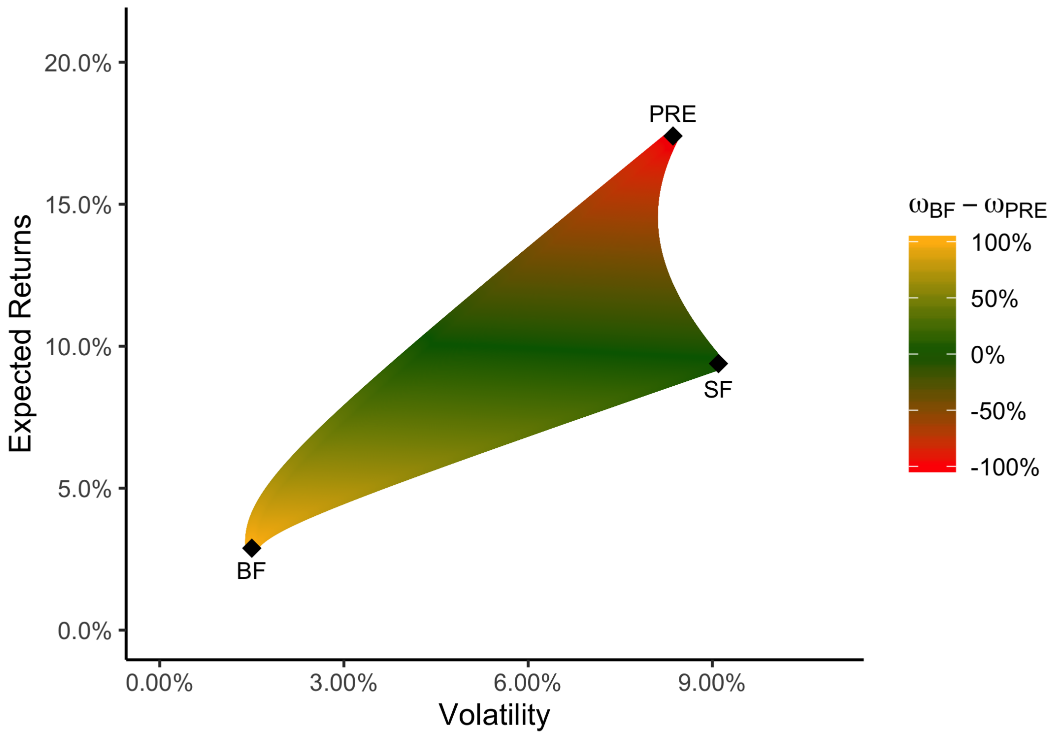

Figure 5.9: Possible Five-Year Holding Period Model Portfolios

Portfolios comprised of positive mixtures of the stock fund, bond fund and PRE asset from the five-year holding period tax model, plotted in a mean-variance diagram. The portfolios are created using data over the time period 2005 through 2018, and they are based upon 0.1% increments. The efficient frontier is displayed in 3-D.

Looking at the mean-variance diagram with the five-year holding period in figure 5.9 the PRE asset now has a higher return, yet lower risk than the stock fond. This is a slight diversion from financial theory on the correlation between risk and return, and is most likely a result of time period examined. As with the tax model, a new benchmark efficient frontier and tangent benchmark portfolio are calculated with the five-year risk and return of the stock and bond fund.

The new return of the tangent benchmark portfolio is 4.81%, up from 3.96% in the one-year holding period. The risk of this portfolio also increases, and is now 3.83%, up from 2.53%. The allocation to the bond fund is now 71.01% and the allocation to the stock fund is 28.99%, implying a small shift towards the stock fund compared to the one-year holding period model. While the Sharpe ratio of the stock fund increases and the ratio of the bond fund is unchanged, the Sharpe ratio of the benchmark portfolio decreases from 0.72 to 0.69 compared to the one-year holding period. This is a result of the increase in correlations between the financial assets when moving from the one-year to five-year holding period. Unlike the Sharpe ratio of the tangent benchmark portfolio, the Sharpe ratio of the tangent mixed-asset portfolio increases when we increase the holding period.

As can one can infer from the mean-variance diagram, the allocation to the PRE asset increases for the five-year holding period. The allocation to this asset is now 50.82%, up from 3.50% in the one-year holding period. The return of the tangent mixed-asset portfolio is now 10.27%, up from 3.97%, and the risk is 4.38%, up from 2.50% in the one-year holding period. The Sharpe ratio of the tangent mixed-asset portfolio is 1.85. This is considerably higher than the Sharpe ratio of the tangent benchmark portfolio and the one-year holding period tangent mixed-asset portfolio being 0.69 and 0.73, respectively. As the Sharpe ratio of the benchmark portfolio decreases, while the Sharpe ratio of the tangent mixed-asset portfolio increases compared to the one-year holding period model, the relative out-performance from including PRE in the portfolio increases. This result is reflected in the alpha regressions as well.

The regression of the PRE return on the tangent benchmark portfolio yields an alpha of 11.12% which is significantly positive, up from 1.33% which is insignificant for the one-year holding period. Similarly, the alpha from the regression of the tangent mixed-asset portfolio on the tangent benchmark portfolio is 5.69% and is significantly positive, up from 0.04% which is insignificant for the one-year holding period. The AR of the tangent mixed-asset portfolio is 1.02, up from 0.09 in the one-year holding period, reflecting the attractive development in the risk-return characteristics also seen in the Sharpe ratio. The IR also increases significantly, from 0.03 to 0.98. This implies that the risk-return characteristics of PRE improve relatively to the change in the risk-return characteristics of the benchmark. It is evident from the financial ratios that including PRE in the mixed-asset portfolio creates an outward shift in the efficient frontier.

The driver of this shift is the return enhancement characteristic of PRE, similarly to the final one-year holding period model. The increased efficiency from the return enhancement on the other hand, is larger than for the one-year holding period, as is the effect from risk reduction. This is consistent with the results of Lee & Stevenson (2004) being that direct real estate should be considered as a strategic asset in the mixed-asset portfolio, especially for investors with longer holding periods.

In summary, increasing the holding period leads to an improvement in both the risk and return characteristics of the PRE asset. The outperformance of the Sharpe ratio of this asset compared to the financial assets is larger than for the one-year holding period. This results in a large allocation to this asset in the tangent mixed-asset portfolio, and an increase in the Sharpe ratio of this. The increased efficiency from both risk reduction and return enhancement results in a significant alpha for the well-diversified investor who includes PRE in the investment universe. The improvement in portfolio efficiency is mostly driven by return enhancement, similarly to the one-year holding period. The conclusion from the previous section is therefore changed, and we find that the risk-return characteristics of PRE are superior when looking a five-year holding period. For a longer holding period than five years, the out-performance of the tangent portfolio with PRE compared to the benchmark increases. The incremental increase is expected to be much smaller, as the effect of these variables going from one to five years is larger than going from five to for example ten years.

With this being our most comprehensive model, we are interested in how much of the return is generated by characteristics of PRE itself, and how much is a result of the "external factors" being leverage and taxation. We therefore remove the effect of these two factors on our final model and analyze the impact on our results.

As the effect of both leverage and taxation on the return of PRE is positive, the return of this asset is reduced from 17.40% to 10.52% when removing these variables. The largest detrimental impact is from the leverage. This variable also has the largest effect on the risk of the asset, which is reduced from 8.36% to 5.44%, yielding a new Sharpe ratio of 1.39. The effect on the PRE asset Sharpe ratio is therefore moderate, as a result of the leverage impacting both the risk and return of an asset in the same direction. The initial Sharpe ratios of the stock and bond fund are 0.72 and 0.45, respectively. The Sharpe ratio of the PRE asset is therefore below its leveraged and taxed value, yet still the highest out of the assets we compare.

As removing the leverage and tax affects the return series of all three assets, new correlations are calculated. We find that the correlation between PRE and the financial assets decreases, and the correlation between the financial assets increases. The change in relative attractiveness of the assets and their correlations results in an increased allocation to PRE in the tangent mixed-asset portfolio of 59.06%. The return of the portfolio is

7.80%, risk is 3.38% and Sharpe ratio is 1.44. Compared to the tangent mixed-asset portfolio with leverage and taxation, the risk, return and Sharpe ratio decrease. The alpha from the regression of the tangent mixed-asset portfolio on the tangent benchmark portfolio is 3.35% and significantly positive. This implies that the inclusion of PRE leads to a significant improvement in portfolio efficiency for a well-diversified investor.

In comparing the efficient frontiers, we find that the improvement in portfolio efficiency is driven by the risk reducing ability of PRE, rather than the return enhancing such as in the original tax model. As the five-year impact of taxation on a primary home (with the low mortgage rates in recent years) is small, this implies that it is mainly the leverage that makes PRE return enhancing in the mixed-asset portfolio.

In summary, while the "external factors" have a large impact on the risk and return that can be expected from a five-year holding period PRE investment, they are not the sole cause of the relative out-performance of this asset. Without these variables, PRE still has the highest Sharpe ratio, and including this asset in the investment universe yields significant gains for a well-diversified investor. Unlike the original five-year holding period model, the dominating characteristic of PRE without taxation and leverage in the mixed-asset portfolio is to be risk reducing. This has implications for an investor as it implies that it is the level of leverage that determines the impact of including PRE on portfolio efficiency.

Before moving on to the robustness section where several of the assumptions used in this section are adjusted, we present the same table as for the one-year holding period. All of the models are included in the table, but we only comment on the tax model as it is based on the same adjustments as for the one-year holding period model. If a model is different from the one-year holding period, we comment on it in this section.

Table 5.2: Five-Year Holding Period for Freehold Units

Panel A consists of the summary statistics for the freehold PRE and financial assets. Panel B provides summary statistics for the tangent portfolios. The benchmark is the tangent portfolio consisting of the stock and bond funds, and the tangent mixed-asset portfolios consist of the different measures of PRE risk and return, combined with the stock and bond funds. Panel C consists of financial measures obtained when comparing the tangent mixed-asset portfolio to the tangent benchmark portfolio. All values are annual and from the time period 2005-2018.

Panel A: Summary Statistics Assets	Stock Fund	Bond Fund	Capital Appreciation	Individual and Liquidity Risk	Imputed Rent	Leverage	Owner Cost	Stock Fund after Tax	Bond Fund after Tax	Tax
Average Return	11.52%	3.88%	7.23%	7.23%	11.53%	18.35%	16.44%	9.38%	2.88%	17.40%
Standard Deviation	11.84%	2.05%	3.66%	5.61%	5.40%	8.47%	8.60%	9.11%	1.49%	8.36%
Sharpe Ratio	0.72	0.45	1.17	0.76	1.59	1.82	1.57	0.80	0.48	1.82
Maximum Return	31.07%	7.40%	17.61%	17.61%	21.31%	38.18%	36.57%	25.11%	5.52%	37.14%
Minimum Return	-19.15%	0.51%	-6.11%	-6.11%	-1.06%	-8.17%	-10.60%	-11.61%	0.40%	-8.87%
Skewness	-0.61	0.06	-0.35	-0.35	-0.34	-0.44	-0.45	-0.38	0.07	-0.42
Kurtosis	2.61	2.02	2.81	2.81	2.62	3.02	3.06	2.20	2.06	2.93

Panel B: Summary Statistics Portfolios	Benchmark	Capital Appreciation	Individual and Liquidity Risk	Imputed Rent	Leverage	Owner Cost	Benchmark	Tax
Stock Fund Weight	30.78%	0.00%	7.25%	0.00%	0.00%	0.00%	28.99%	0.00%
Bond Fund Weight	69.22%	37.35 %	57.24%	36.64%	38.65%	45.18%	71.01%	49.18%
Private Real Estate Weight	0.00%	62.35%	35.50%	63.36%	61.35%	54.82%	0.00%	50.82%
Average Return	6.23%	5.97%	5.62%	8.73%	12.76%	10.77%	4.81%	10.27%
Standard Deviation	4.31%	2.45%	2.99%	3.55%	5.33%	4.98%	3.83%	4.38%
Sharpe Ratio	0.76	1.23	0.89	1.63	1.84	1.57	0.69	1.85
Maximum Return	12.98%	11.87%	8.92%	14.39%	24.35%	21.14%	9.86%	19.77%
Minimum Return	-5.54%	-3.61%	-3.26%	-0.48%	-4.82%	-5.58%	-3.10%	-4.32%
Skewness	-0.85	-0.84	-1.70	-0.81	-0.72	-0.82	-64.40%	-0.74
Kurtosis	3.34	3.53	5.49	3.26	3.42	5.60	2.87%	3.40

Panel C: Financial Measures Portfolios	Capital Appreciation	Individual and Liquidity Risk	Imputed Rent	Leverage	Owner Cost	Tax
Alpha Private Real Estate	2.41%	2.24%	6.60%**	11.47%**	9.51%*	11.12%*
Beta Private Real Estate	0.92	0.62	0.60	1.20	1.21	1.59
Alpha All Assets	1.46%	0.86%	4.24%**	7.1%**	5.28%*	5.69%**
Beta All Assets	0.48*	0.55***	0.47*	0.82	0.77*	0.92
Appraisal Ratio All Assets	0.39	0.39	1.17	1.07	0.87	1.02
Information Ratio All Assets	-0.06	-0.19	0.55	0.97	0.74	0.98

*Signif. codes: 0'***' 0.001 '**' 0.01 '*' 0.05*

6 Robustness and Sensitivities

In this section, we adjust some of the assumptions used in the analysis. The two first adjustments relate to the robustness of the allocation to PRE, while the two last adjustments are sensitivity analyses related to assumptions behind the PRE asset. To evaluate the robustness of the allocation to PRE, the freehold apartment is exchanged with a co-op apartment, both for the one- and five-year holding period. We also evaluate the robustness of the allocation to PRE within the freehold unit, by exchanging the primary home for a primary and secondary home. To evaluate the sensitivity of the taxation assumption, the wealth and property values are varied, and similarly the one-year holding period LTV is varied, both before and after taxation. We begin with exchanging the freehold with the co-op apartment type.

6.1 Cooperative Unit

In this section we examine the after-tax one- and five-year holding period models for the second main type of apartment that exists in Oslo, namely the co-op. This type of apartment stands out from the freehold apartment that we have focused on up until now, and we adjust our models accordingly. This is to see whether our results from the previous section are sensitive to changing the apartment type.

6.1.1 One-Year Holding Period for the Cooperative Unit

The co-op model builds on a second WRS index created using only the co-op observations of our data set. Because the tax model is the most comprehensive, we utilize this for the co-op apartment. We only comment on the steps leading up to this final model if there is an adjustment compared to the freehold apartment models.

The first potential adjustment relates to the imputed rent. Unfortunately, the data on rent levels in Oslo is not split between freehold and co-op units. This is most likely due to the difficulty of renting out co-ops and the low level of rent observations as a result of this. A possible solution is to use the same rent levels as for the freehold and divide these by the median co-op value. The problem with this solution is that it would create a higher rent yield as the median co-op is less expensive than the median freehold. Such an increase in rent yield is not justified. For comparability between the freehold and co-op total returns, we therefore use the same rent yield for the co-op as in the freehold models.

The next adjustment is the leverage. For the co-op units in the data set, the average common debt to value (CDTV) is 8.87%. Under the assumption that the total LTV is unchanged, the private debt of the co-op model is 45.24% and is financed with the same MR as the private debt in the freehold model, now denoted PMR (private mortgage rate). The mortgage rate on the common debt is denoted CDMR (common debt mortgage rate) and has historically been below the PMR, yet the difference has been diminishing in recent years[30].

The second potential adjustment relates to the operating costs. The level of these costs for the co-op relative to the freehold model is not evident in our data, or among our sources. On the one hand, it can be argued that the operating costs are higher for co-ops because more common space has to be maintained. In addition to this, the co-ops often have more administrative costs as they use professional accounting and cleaning services. On the other hand, one can also infer that these costs are divided by more inhabitants as co-op buildings are often larger than freehold buildings. Furthermore, the scale of large co-ops can able them to bargain and receive superior terms on for example internet and TV providers. The net effect of these variables is unclear and we therefore leave the operating costs unchanged.

The remaining owner costs, being the transaction costs, are adjusted. The document fee is removed, the registration fees are reduced and the owner-change fee is moved from the purchase price to the sales price. This substantially reduces the transaction costs and increases the return of co-ops.

The risk of this asset is calculated following the same procedure as the individual house- and liquidity risk freehold model, while the new return formula is shown below;

$$R_H = \frac{1}{(1-LTV)} * \left(\frac{P_{t+1} - P_t - TC}{P_t} + NY - (1-GITR) * \Big(((LTV - CDTV) * PMR) + CDTV * CDMR \Big) - PTR - (WVH - LTV) * WTR \right) \tag{6.1}$$

The after-tax one-year holding period return of co-ops is 13.08%. This is considerably

[30] These are average values and can vary substantially between cooperatives.

higher than the return for freeholds being 7.79%, and is a result of the reduction in transaction costs and slightly lower funding costs (and the leverage effect on this difference). The risk of the co-op model is also higher, at 16.90%, while the risk of the freehold is 15.83%. The Sharpe ratio of the co-op is 0.65, almost twice the size of the Sharpe ratio of the freehold which is 0.36. This increase in Sharpe ratio implies that the increase in the return when moving from the freehold to co-op model is not countered by an equivalent increase in the risk. The Sharpe ratio of the co-op apartment is therefore the same as the Sharpe ratio of the stock fund, and these are the highest out of the assets we compare.

As a result of the entirely new return series, the correlations between the co-op and both of the financial assets are lower than that of the freehold and the financial assets, indicating slightly improved diversification benefits. The effect is fairly small, as can be deducted from the relatively straight efficient frontier figure 4.1 and the correlations found in the Appendix.

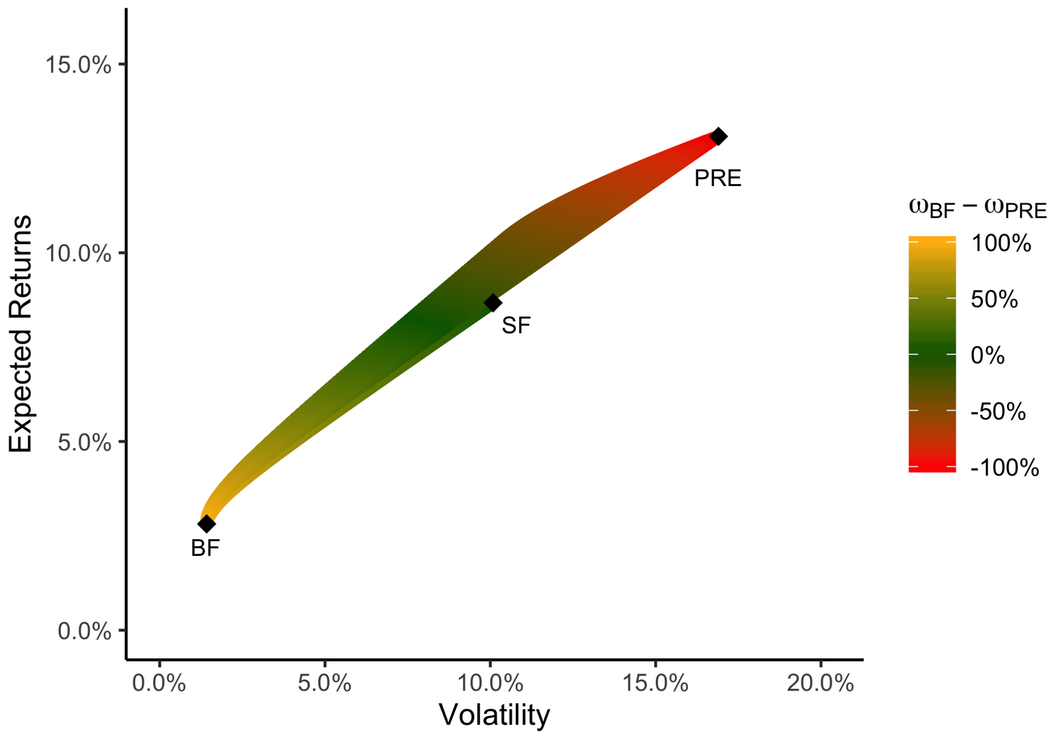

Figure 6.1: Possible One-year Holding Period Cooperative Model Portfolios

Portfolios comprised of positive mixtures of the stock fund, bond fund and cooperative apartment from the tax model, plotted in a mean-variance diagram. The portfolios are created using data over the time period 2005 through 2018, and they are based upon 0.1% increments. The efficient frontier is displayed in 3-D.

In the mean-variance diagram in figure 6.1, the co-op is located to the far right and receives allocations along the efficient frontier for high levels of risk. For simplicity, in the succeeding analysis of the financial ratios, the "one-year co-op portfolio" is referring to the one-year holding period tangent mixed-asset portfolio with the co-op apartment as the PRE asset. The same explanation holds for the "one-year freehold portfolio".

We find that despite the out-performance of the Sharpe ratio of the co-op asset, the allocation to this asset in the one-year co-op portfolio is small, at 9.40%. This is most likely a result of the tangent portfolio typically being found for low levels of risk and return, while the co-op asset has a high risk and return. Though small, the allocation to the co-op is larger than that of the freehold in the one-year freehold portfolio, being 3.50%. The return of the one-year co-op portfolio is 4.48%, while the return of the one-year freehold portfolio is 3.97%. Similarly, we find that the risk of the one-year co-op portfolio is 2.73% which is also higher than that of the one-year freehold portfolio, being 2.50%.

As the relative increase in the return is larger than the relative increase in the risk when moving from the one-year freehold portfolio to the one-year co-op portfolio, the Sharpe ratio of the one-year co-op portfolio is the highest. This Sharpe ratio is 0.86, and is therefore considerably above that of the one-year freehold portfolio, being 0.73. As the one-year freehold portfolio Sharpe is above the benchmark Sharpe ratio, being 0.72, both one-year PRE portfolios outperform the benchmark, though the relative out-performance of the portfolio with the co-op is larger. Similarly to the Sharpe ratio, the alpha of the co-op is superior to that of the freehold apartment.

As with the previous models, we run a regression of the co-op return on the tangent benchmark portfolio return. This produces an alpha of 6.40%. Though this is considerably higher than the alpha of 1.33% for the one-year holding period freehold asset return, it is still not significant. The alpha from running the regression of the one-year co-op portfolio on the tangent benchmark portfolio is 0.58% and is, as expected, not significant though it is considerably above the alpha of the one-year freehold portfolio being 0.04%. The AR and IR follow the same development as the Sharpe ratio and alphas when moving from the one-year freehold portfolio to the one-year co-op portfolio.

The AR of the one-year co-op portfolio is 0.45, which is higher than the one-year freehold portfolio AR, being 0.09. This measure indicates that the one-year co-op portfolio creates more return per unit of risk than the one-year freehold portfolio. The IR of the one-year co-op portfolio is 0.40, and is also substantially higher than that of the one-year freehold portfolio, being 0.03. This implies that the risk-return trade-off of the co-op is also superior relative to the benchmark (which is the same for both models). Another result that is altered when exchanging the freehold apartment with a co-op, is the effect of this PRE asset on the mixed-asset portfolio.

The dominating characteristic of the co-op is to be risk reducing, contradicting the result from the freehold model, yet in line with existing literature. This result is surprising given the placement of the asset in the efficient frontier, and is most likely due to the largest gains from PRE existing for higher levels of risk and return than the benchmark portfolio. Furthermore, the difference between the two effects is moderate, as the increase in portfolio efficiency from risk reduction and return enhancement is similar.

In summary, the decision between a co-op and a freehold apartment for a one-year holding period has a large impact on the risk and return an investor can expect. The co-op apartment yields a higher return, yet is riskier. The Sharpe ratio of the co-op is higher than that of the freehold, and this asset therefore receives a larger allocation in the tangent mixed-asset portfolio. Similarly, the Sharpe ratio of this portfolio is superior to that of the freehold tangent mixed-asset portfolio. Yet, the inclusion of the PRE asset in the investment universe does still not provide significant gains for a well-diversified investor. Unlike the freehold model, the dominating characteristic of PRE in the mixed-asset portfolio is to be risk reducing, yet this result is most likely due to a weakness in the method for comparing efficient frontiers. We now move on to the more realistic holding period also for this type of apartment, and expect the difference between the co-op and freehold apartment to diminish.

6.1.2 Five-Year Holding Period for the Cooperative Unit

Now, the same procedure is repeated for the co-op using a five-year holding period. The PRE return adjustments for this holding period are similar to those of the five-year holding period for the freehold apartment; a reduction in the LTV, and that the transaction costs are paid once every five years and annualized. For simplicity, the common debt is assumed to be unchanged, implying that the private debt is reduced from 45.24% to 38.42% to account for the reduction in total LTV from 54.11% to 47.29%. The returns of the stock and bond funds are adjusted in the same way as in the five-year holding period for the freehold model.

The new PRE return is;

$$R_{H,5y} = \frac{1}{(1 - LTV_{5y})} * \left(\sqrt[5]{\frac{P_{t+5} - P_t - TC}{P_t}} + NY - (1 - GITR) * \Big(((LTV_{5y} - CDTV) \right. \\ \left. *PMR) + CDTV * CDMR \Big) - PTR - (WVH - LTV_{5y}) * WTR \right) \tag{6.2}$$

As a result of the adjustments made to the PRE return measurement, the return of the co-op increases to 18.48%, from 13.08% in the one-year holding period. This return is also above that of the five-year holding period freehold apartment, being 17.40%. The difference between the five-year holding period returns of these apartment types is therefore smaller than the difference between the one-year holding period returns. This diminishing difference between the apartment types is reflected in the return, yet not in the riskiness of these assets.

The risk of the co-op, stock fund and bond fund are adjusted to take into account the new return series and autocorrelation. The risk of the co-op is also adjusted for the decrease in price- and liquidity risk. For the co-op we therefore follow the same three steps when moving from the one-year to the five-year holding period as for the freehold model. The first change stems from the new return series and reduces the standard deviation from 16.90% in the one-year holding period to 15.52% in the five-year holding period. This decrease indicates that the new return series is less volatile than the one-year holding period returns, which is as expected due to the reduction in leverage. The second risk adjustment comes from the net negative autocorrelation, and decreases the risk further, from 15.52% to 12.69%. The net negative autocorrelation indicates that also the co-op asset exhibits mean reversion, yet slightly less than the freehold unit as it reduces the risk by 18.24% while the risk is reduced with 19.16% in the freehold model. Finally, the price- and liquidity risk are divided by the holding period; this decreases the standard deviation from 12.69% to 9.35%. This is higher than the five-year holding period freehold risk, being 8.36%, yet lower than the one-year holding period co-op risk, being 16.90%. The volatility of the stock and bond funds is identical to the five-year holding period with the freehold model.

The Sharpe ratio of the co-op therefore increases from 0.65 in the one-year holding period, to 1.75. Though this Sharpe ratio is higher than that of the stock and bond fund, being 0.80 and 0.48, respectively, it below that of the comparable freehold apartment, being 1.82. As the difference between the returns of the two apartment types diminishes, and the difference between their risks remains mainly unchanged, the Sharpe ratios of the apartments types converge towards a common value. For a long enough holding period, the Sharpe ratios of these two apartments type will therefore be equal.

As expected from the conversion of the co-op and freehold returns, the evolution of the correlations when moving from the one-year to the five-year holding period is similar to that of the freehold model, with an increase in all correlations. The correlation between the co-op and stock fund increases from 0.35 to 0.63, while the correlation between the co-op and bond fund increases from 0.04 to 0.08. Compared to the freehold model, the correlation between co-ops and the financial assets is slightly lower, indicating diversification benefits.

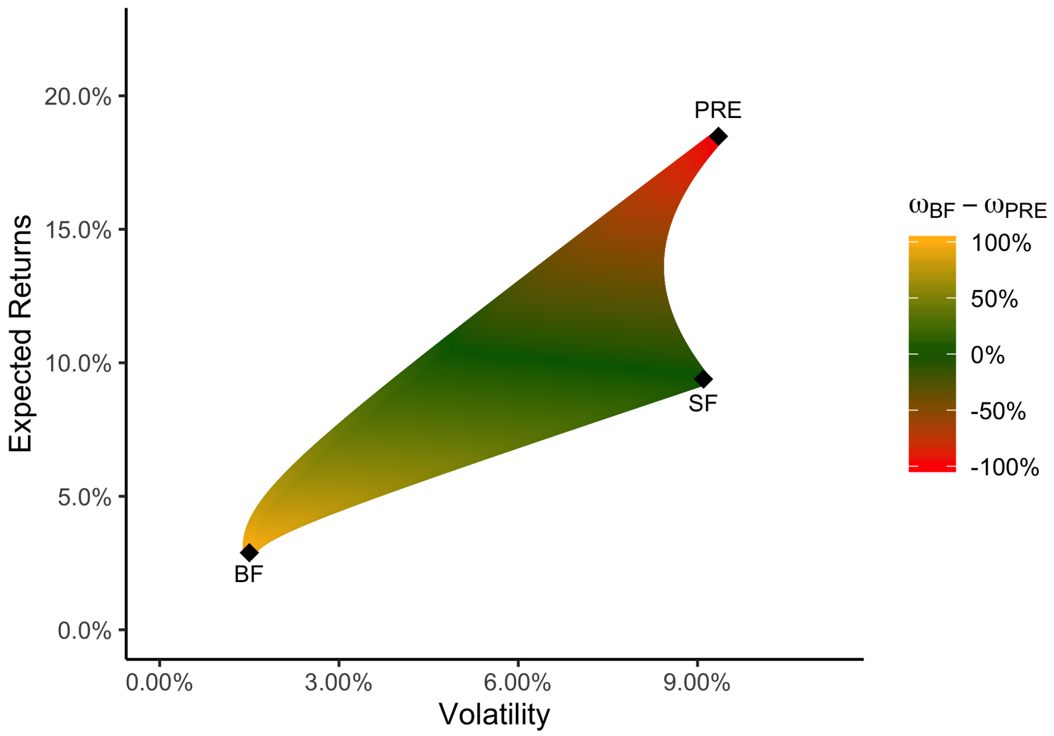

Figure 6.2: Possible Five-Year Holding Period Cooperative Model Portfolios

Portfolios comprised of positive mixtures of the stock fund, bond fund and cooperative apartment from the five-year holding period tax model, plotted in a mean-variance diagram. The portfolios are created using data over the time period 2005 through 2018, and they are based upon 0.1% increments. The efficient frontier is displayed in 3-D.

The efficient frontier in figure 6.2 clearly resembles that of the five-year holding period freehold model. In this model, we continue to use the same notations as for the one-year holding period. In addition to these, we add the "five-year co-op portfolio" for the five-year holding period tangent mixed-asset portfolio with the co-op apartment as the PRE asset. The explanation is similar for the "five-year freehold portfolio".

The allocation to the co-op in five-year co-op portfolio increases to 44.40%, from 9.40% in the one-year co-op portfolio. This is still slightly below that of the five-year freehold portfolio, with the allocation to the freehold apartment being 50.82%. The return of the five-year co-op portfolio is 9.82%, and risk is 4.30%. This return and risk is higher than the one-year co-op portfolio return and risk, being 4.48% and 2.73%, yet lower than the five-year freehold portfolio return and risk, being 10.27% and 4.38%. Similarly to the relative risk and return characteristics, the Sharpe ratio of the five-year co-op tangent portfolio is 1.78, and is thereby in between that of the one-year co-op portfolio and five-year freehold portfolio Sharpe ratios, being 0.86 and 1.85, respectively. Unlike the co-op portfolio Sharpe ratio, the alpha of the co-op asset is the highest for the five-year holding period as well.

The alpha from the regression of the five-year holding period co-op return on the tangent benchmark portfolio is 12.11% and significant. This annual alpha is marginally above that of the freehold model, being 11.12%, and hence supports the assumption that the difference between the apartment types is diminishing for longer holding periods. Similarly, the alpha from running a regression of the five-year co-op portfolio on the benchmark is 5.42% and significantly positive. This alpha is below that of the five-year freehold portfolio which is 5.69%. The lower alpha is a result of the reduced allocation to the PRE asset in the five-year co-op portfolio compared to the five-year freehold portfolio. This is because the tangent portfolios take into account both the risk and return characteristics of the assets. Compared to the one-year holding period, both co-op alphas increase significantly. This development is also reflected in the AR and IR.

The AR of the five-year co-op portfolio is 1.00 and is therefore over twice size of the one-year co-op portfolio AR which is 0.45. The increase is expected, and is a result of the improvement in the risk and return characteristics of the co-op when moving from a one-year to a five-year holding period. The IR also increases, from 0.40 in the one-year co-op portfolio to 0.92 in the five-year co-op portfolio. This result is also expected as the return of the co-op (and therefore the tangent mixed-asset portfolio) increases relatively more than that of the benchmark whiles the volatility of both return series increases simultaneously. Therefore, the difference between the return series (and standard deviation of this) is mainly unchanged. Compared to the five-year freehold portfolio, both the five-year co-op portfolio AR and IR are slightly lower. This is because the co-op market risk is higher than the freehold market risk, and the higher co-op return does not counter this. As with the risk and return characteristics of the PRE assets, and the financial ratios of their tangent mixed-asset portfolios, the dominating characteristic of the PRE assets in the five-year holding period is similar.

The dominating characteristic of both five-year holding period models is to be return enhancing. Furthermore, the effect of the return enhancement and the risk reduction is also larger for both five-year models than for their comparable one-year models.

In summary, as expected, the decision between a co-op and freehold apartment for a five-year holding period is less critical when it comes to the expectations to the PRE asset‘s risk and return. The Sharpe ratios of both of the PRE assets are higher than those of the financial assets, yet the freehold asset now has a higher Sharpe ratio than the co-op asset. Therefore, the allocation to the freehold apartment in the tangent mixed-asset portfolio is higher than the allocation to the co-op apartment. As a result of this, the Sharpe ratio of the tangent mixed-asset portfolio with the freehold is the highest. The alpha from including both types of PRE in the tangent mixed-asset portfolio is significant, implying an increased portfolio efficiency for the well-diversified investor. The dominating characteristic of both PRE assets is now also the same, to be return enhancing. Lastly, the strength of both the return enhancement and risk reduction effects is much larger for the five-year models, than the one-year models. Therefore, as expected, the difference between the PRE assets is diminishing for longer holding periods.

Before we analyze the allocation to PRE within the freehold apartment type rather than for different apartment types, we present a table summarizing the one- and five-year holding periods for the freehold and co-op tax models.

Table 6.1: One- and Five-Year Holding Period for Freehold and Cooperative Units

Panel A consists of the summary statistics for the freehold PRE, cooperative PRE and financial assets. The one-year holding period PRE models for the two apartment types build on the tax model. Panel B provides summary statistics for the three tangent portfolios. The benchmark is the tangent portfolio consisting of the stock and bond funds, and both PRE tangent portfolios consist of the tax model PRE and the stock and bond funds. Panel C consists of financial measures obtained when comparing the tangent mixed-asset portfolio to the tangent benchmark portfolio. All values are annual and from the time period 2005-2018.

	One-Year Holding Period				**Five-Year Holding Period**			
Panel A: Summary Statistics Assets	Stock Fund	Bond Fund	Freehold	Cooperative	Stock Fund	Bond Fund	Freehold	Cooperative
Average Return	8.67%	2.82%	7.79%	13.08%	9.38%	2.88%	17.40%	18.48%
Standard Deviation	10.08%	1.42%	15.83%	16.90%	9.11%	1.49%	8.36%	9.35%
Sharpe Ratio	0.65	0.48	0.36	0.65	0.80	0.48	1.82	1.75
Maximum Return	22.37%	5.33%	30.37%	39.09%	25.11%	5.52%	37.13%	40.95%
Minimum Return	-12.70%	0.37%	-21.66%	-18.70%	-11.61%	0.40%	-8.87%	-9.44%
Skewness	-0.57	0.04	-0.42	-0.23	-0.38	0.07	-0.42	-0.26
Kurtosis	2.47	2.06	3.10	3.00	2.20	2.06	2.93	2.92
Panel B: Summary Statistics Portfolios	Benchmark		Freehold	Cooperative	Benchmark		Freehold	Cooperative
Weight Stock Fund	19.60%		16.80%	11.94%	28.99%		0.00%	0.00%
Weight Bond Fund	80.40%		79.60%	78.65%	71.01%		49.18%	55.60%
Weight Private Real Estate	0.00%		3.50%	9.40%	0.00%		50.82%	44.40%
Average return	3.96%		3.97%	4.48%	4.81%		10.27%	9.82%
Standard Deviation	2.53%		2.50%	2.73%	3.83%		4.38%	4.30%
Sharpe Ratio	0.72		0.73	0.86	0.69		1.85	1.78
Maximum return	7.24%		7.05%	6.90%	9.86%		19.77%	19.20%
Minimum return	-2.20%		-2.61%	-2.98%	-3.10%		-4.32%	-3.98%
Skewness	-0.95		-1.32	-1.86	-0.64		-0.74	-0.66
Kurtosis	3.50		4.60	5.94	2.87		3.40	3.42
Panel C: Financial Measures Portfolios			Freehold	Cooperative			Freehold	Cooperative
Alpha Private Real Estate			1.33%	6.40%			11.12%*	12.11%*
Beta Private Real Estate			2.37	2.49			1.59	1.62
Alpha All Assets			0.04%	0.58%			5.69%**	5.42%*
Beta All Assets			1.00***	0.96***			0.92	0.85
Appraisal Ratio All Assets			0.09	0.45			1.02	1.00
Information Ratio All Assets			0.03	0.40			0.98	0.92

*Signif. codes: 0'***' 0.001 '**' 0.01 '*' 0.05*

6.2 Secondary Home

In this section, we include a secondary freehold apartment in the MPT framework. This is to examine the robustness of the tangent allocation to real estate in the five-year freehold mixed-asset portfolio. In doing this, we investigate whether the allocation to PRE should be in one freehold apartment or in a combination of primary and secondary freehold apartments.

We include the secondary housing in our MPT framework in two steps. The first step creates a new return series for the secondary apartment by adjusting the taxation. The second step finds the tangent allocation to the primary and secondary home, and adds this combined PRE asset to the mixed-asset portfolio. The second step is included because the secondary apartment investment cannot be considered independently from the primary apartment.

The first step therefore builds on the five-year freehold model and adjusts the taxation of the PRE return series. For secondary homes, both the capital gains and rent income is taxed by the GITR. Similarly to the stock and bond funds, the capital gains tax is incurred every five years and is annualized. In the calculation of the capital gains tax the apartment price plus purchase costs is considered the "entry" value, and the apartment price less sales costs is considered the "exit value, implying that the tax is paid of the net profit. The tax on rent income is similarly based on the net yield, and all of the operating costs are therefore tax deductible[31]. As with the primary home, the mortgage rate is still tax deductible. We continue using the same LTV as for the primary home, being 47.29% for the five-year holding period.

The new return formula is;

$$R_{SH,5y} = \left(\sqrt[5]{\frac{(P_{t+5} - P_t - TC) * (1 - GITR)}{P_t}} + (1 - GITR) * \Big(NY - (LTV_{5y} * MR)\Big) - PTR - (WVSH - LTV_{5y}) * WTR \right) * \frac{1}{(1 - LTV_{5y})} \tag{6.3}$$

The return of the secondary home is 11.42%, down from 17.40% for the primary home. This is a result of the PRE asset incurring more taxation as the secondary home is

[31] skatteetaten.no/person/skatt/hjelp-til-riktig-skatt/bolig-og-eiendeler/bolig-eiendom-tomt/utleie/utleie-av-del-av-egen-bolig/fradrag/

considered an investment similarly to stocks and bonds. As expected, increased taxation also reduces the volatility of the returns, and the risk[32] of this PRE asset is lower; at 7.20% compared to the primary home return volatility being 8.36%. The Sharpe ratio of the secondary home is 1.28 and is therefore below that of the primary home being 1.82. This indicates that the negative effect of the taxation on the PRE return is larger than the positive effect of taxation on PRE risk. This is as expected because the opposite effect is observed when we lever the return in the leverage model. Despite the increased taxation of the secondary home, it nevertheless has a higher Sharpe ratio than the financial assets, as the Sharpe ratios of the stock and bond fund are 0.80 and 0.48, respectively. Compared to the primary home, the correlation between the secondary home and the financial assets is lower, indicating improved diversification benefits.

The second step creates a combined PRE asset with the tangent portfolio of the primary and secondary freehold apartments. The allocation to these two assets is sensitive to the assumptions made about the relative leverage of these. In this paper it is assumed that the secondary home is purchased to increase the mortgage and hence decrease net wealth, because the mortgage on the primary home is paid off. We therefore lever the secondary apartment with 47.29% debt and assume that the primary home is purchased with 100% equity. While recognizing that these assumptions do not universally hold, this model demonstrates the return adjustments that can be used to include secondary housing in the MPT framework. With the five-year holding period, both apartments are assumed to be purchased and sold simultaneously. After deriving the allocation to the primary apartment, (W_{PH}) and secondary apartment, (W_{SH}), through finding the tangent portfolio of these two assets, we insert these weights into the new return formula;

$$\begin{aligned} R_{PH+SH,5y} = W_{PH}\left(\sqrt[5]{\frac{P_{t+5}-P_t-TC}{P_t}} + NY - PTR - (WVH * WTR)\right) \\ +W_{SH}\left(\sqrt[5]{\frac{(P_{t+5}-P_t-TC)*(1-GITR)}{P_t}} + (1-GITR)*(NY-(LTV_{5y}*MR)) \right. \\ \left. -PTR-(WVSH-LTV_{5y})*WTR\right) * \frac{1}{(1-LTV_{5y})} \end{aligned} \tag{6.4}$$

The new risk formula for $\sigma_{PH+SH,5y}$ is;

[32] We assume no change in the individual risk or liquidity risk as the incremental change from moving from one to two homes is assumed to be insignificant.

$$\sqrt{W_{PH}^2 * (\frac{\frac{\hat{\alpha}}{2}}{5} + 4\beta + \frac{\sigma_{PH,IL,5y}^2}{5}) + W_{SH}^2 * (\frac{\frac{\hat{\alpha}}{2}}{5} + 4\beta + \frac{\sigma_{SH,IL,5y}^2}{5}) + 2 * W_{PH} * W_{SH} * COV(PH, SH)} \tag{6.5}$$

The tangent PRE portfolio allocates 100% to the primary home, and hence there are no benefits from moving from a primary apartment to the combination of a primary and secondary apartment, given our assumptions. The reason for this is that the return series for the secondary apartment created in the first step, build on the same WRS HPI as the primary home, and they are therefore almost perfectly correlated. The sole difference between the return series relates to the leverage and taxation, and as seen previously, leveraging an investment that positively covaries with another, increases the covariance. With two assets that are nearly perfectly correlated, the tangent portfolio allocates 100% to the asset with the highest Sharpe ratio. In our paper, this is the primary home. Therefore, the tangent portfolio allocation to PRE consists only of the primary home. Further research should analyze the effect of purchasing a secondary home in a different market, as this return series would build on a different house price index. This secondary home can potentially provide diversification benefits if the house market in the second city is driven by different economic factors.

The analysis above indicates that there are no benefits for the median home buyer from dividing the tangent PRE allocation between a primary and secondary apartment in Oslo. In our model, the secondary apartment still has a higher Sharpe ratio than the financial assets. This implies that if there is a restriction on the amount of wealth that can go into the primary home, there will be allocations to the secondary apartment. We do not attempt to incorporate this in our model as this restriction needs to be connected to a certain apartment price or wealth level, while we solely utilize returns. In reality, if the wealth level of an individual is sufficiently high, allocating 50.82% of this wealth to one single primary home is unreasonable. The value of this apartment will be "so high", that the investor cannot expect the risk and return calculated based on median apartments in Oslo. In such a situation, it would be beneficial to allocate wealth to a secondary apartment instead. As the focus in this paper is on the median Oslo citizen, wealth levels that would lead to overly-expensive primary homes are not a concern.

In summary, our results are robust to the amount of homes that the investor should purchase. This is because the tangent portfolio of a primary and secondary home allocates 100% to the primary home. Moving on from the robustness of the tangent allocation we now examine sensitivities to some of our key assumptions.

6.3 Tax Sensitivities

In Case & Shiller (1990), both the mortgage rate and property tax are deductible by the marginal individual income tax. This implies that for higher marginal tax rates, more of the mortgage is tax deductible. This creates a stronger incentive to purchase PRE over other assets for individuals with higher income levels. As seen in the portfolio construction section, in Norway, the property tax is not deductible, and the mortgage rate is deductible by the GITR, which is independent of your income level. It is therefore evident that the incentive to purchase real estate over the financial assets does not increase with income level. Yet, Norway is one of only six countries in the world that levies the wealth tax. This tax rate is progressive in the way that the activation threshold has a relatively larger impact on wealth levels that are marginally above it, compared to higher wealth levels.

We therefore conduct a sensitivity analysis where we assume different values of net wealth (excluding housing wealth), and examine whether investing the sum that corresponds to the median freehold apartment value each year in PRE, rather than the financial assets, yields tax savings. The tax savings are calculated as the total wealth tax paid in percent of the median apartment value.

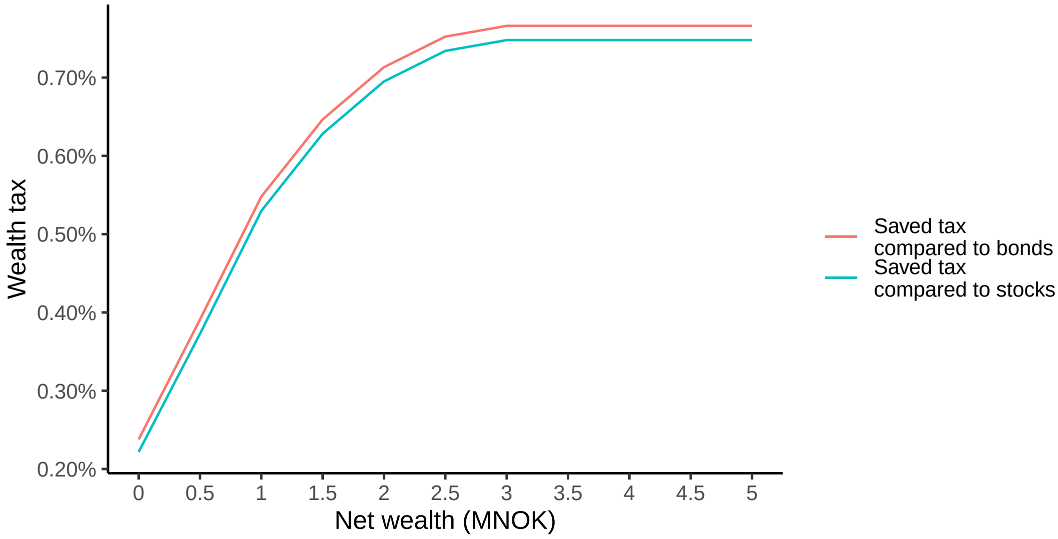

Figure 6.3: Tax Sensitivities

Tax savings with different values of net wealth, calculated as the total wealth tax paid in % of median home value. The saved tax compared to bonds is the difference in the tax paid in % of median home value from investing in the bond fund, rather than the primary home. The saved tax compared to stocks can be interpreted similarily

From figure 6.3 it is evident that the incentive to purchase PRE over the financial assets is only increasing for low levels of wealth. After a certain point, the tax advantage is constant. This is because for higher wealth levels, the net negative wealth value of the primary home does not reduce the total net wealth below the activation threshold. A weakness of this sensitivity analysis is that for higher levels of wealth, the assumption that the individual would purchase a home for the median apartment value becomes less realistic. Most likely, there is a positive correlation between the value of the primary home and the wealth for some wealth levels, while after a certain wealth level, further increases in the apartment value are unrealistic (as mentioned in the secondary home analysis). We do not attempt to model this.

For housing values over a certain level, the property tax is incurred as well, and will begin to decrease the incentive to purchase housing over the financial assets. The total incentive to purchase PRE over financial assets may therefore be curved. It increases for low levels of wealth and then decreases as the wealthier investor purchases a more expensive home, thereby incurring the property tax.

In summary, our results are not sensitive to changes in the wealth or property value as our scope does not include high wealth individuals. Furthermore, the activation threshold for both types of taxation are high, and in the event of incurring these, the tax rates are low and do not have a significant impact on the analysis. We find that for low levels of wealth, the incentive to purchase PRE over the stock and bond fund increases, while for higher levels of wealth this incentive becomes constant. At this point it is likely that the investor purchases a more expensive primary home, and the property tax will therefore reduce the incentive to purchase PRE over the stock and bond fund. While our results are not directly sensitive to income level as in Case & Shiller (1990), we revisit this question in the next section while including the effect of leverage.

6.4 Leverage Sensitivities

In our base case scenario, we use a LTV of 54.11% for a one-year holding period. This is the average mid-year leverage on new loans in Oslo in 2017 and 2018. We use the same LTV for the full period being analyzed, from 2005 to 2018, due to the lack of reliable historical data. We recognize that over time, the average LTV varies. The average annual LTV can depend on many factors, such as location in the business cycle (economic expansion or contraction) and the interest rate level. In addition to this, other factors such as the maximum level of permitted debt (which has varied over the period) can determine the average LTV. Due to the possible bias from using average debt based on solely two years, we construct allocation tables for three levels of debt in addition to the LTV utilized for the one-year holding period.

Table 6.2: Leverage Sensitivities

The first model "Before Tax" is the owner cost freehold model, while the second model "After Tax" is the tax freehold model. For different levels of target return the allocation table finds the minimal standard deviation and allocation to the different assets. All values are in percent, and based on the time period 2005-2018

		Before Tax				After Tax			
Leverage	Return	St.Dev	W.BF	W.SF	W.PRE	St.Dev	W.BF	W.SF	W.PRE
0	5.00	2.99	82.98	14.20	2.80	4.09	62.66	37.20	0.10
	6.50	5.34	63.36	33.83	2.80	6.50	37.03	62.86	0.10
	8.00	7.91	43.10	53.30	3.50	8.97	11.51	88.49	0.00
	10.00	11.43	17.20	79.60	3.20				
	11.00	13.20	4.40	92.70	2.90				
20.00	5.00	3.00	83.48	14.20	2.30	4.09	62.50	37.00	0.40
	6.50	5.35	63.46	33.72	2.80	6.50	36.60	62.70	0.70
	8.00	7.92	45.05	53.57	1.30	8.97	11.10	88.20	0.60
	10.00	11.43	19.52	79.98	0.50				
	11.00	13.21	4.80	92.70	2.50				
54.11	5.00	3.01	84.50	14.20	1.30	4.00	62.00	32.80	5.20
	6.50	5.35	65.60	34.20	0.20	6.38	35.70	54.90	9.40
	8.00	7.92	45.70	53.60	0.50	8.79	9.70	78.10	12.20
	10.00	11.43	19.70	80.00	0.30				
	11.00	13.21	6.30	92.90	0.80				
85.00	5.00	3.02	85.28	14.71	0.00	3.72	73.40	22.50	5.00
	6.50	5.36	65.67	34.33	0.00	5.88	54.30	6.60%	6.50
	8.00	7.92	46.00	54.00	0.00	8.10	35.60	55.00	9.30
	10.00	11.44	19.92	79.88	0.20	11.08	10.30	77.00	12.70
	11.00	13.21	7.40	3.09	0.10	12.59	0.50	84.00	15.50

In our owner cost (before-tax) model we find that increasing the leverage does not provide significant diversification benefits for any level of return. This is in line with economic theory and the MM (1958, 1963) theorem. Unlike the effect of leverage on the imputed rent model, leverage does not create a shift in the efficient frontier in the owner cost model. This is most likely because including the owner costs reduces the upward bias in the returns caused by only analyzing half of a housing cycle. Our results are now more comparable with those in Boyd et al. (1998) who state that there are no significant portfolio benefits for the non-taxable investor who leverages real estate. Furthermore, the more debt one adds to the PRE asset, the less desirable it becomes for non-taxable investors at virtually all levels of risk preference.

For the taxable investor, leveraging the PRE asset provides diversification gains for all levels of risk preference. The higher the return, the greater the gains from the leverage are. Compared to Boyd et al. (1998) we find overall smaller gains from leverage. This is because as mentioned previously, in the US both the property tax and mortgage rate are tax deductible and they are tax deductible by the marginal income tax rate. Furthermore, Boyd et al. use the maximum marginal income tax rate while we use the GITR in Norway, and only the mortgage is tax deductible.

While Boyd et al. (1998) find that there exists an optimal amount of debt for real estate investors, we find that increasing the leverage improves portfolio efficiency for all levels of LTV. This can be a result of the time period used in the Boyd et al. paper, where the average mortgage rate is higher than the total return excluding the mortgage rate. The time period examined in this thesis is characterized by historically low mortgage rates and high PRE returns. Therefore, in the period from 2005-2018 home owners have experienced solely the positive effect of leverage, and our model concludes that the maximum level of LTV is optimal. As not all investors can obtain the maximum level of LTV, our results are indirectly sensitive to income level.

While in Case & Shiller (1990), the effect of higher income levels is direct through the tax deductibility of the property tax and mortgage rate, the effect is less noticeable in Norway. Here, it comes in connection with the leverage, as the size of the mortgage an individual can obtain, is restricted. The first main restriction relates to the total LTV permitted, while the second limits the amount of times the annual income an individual can lend. According to the Financial Supervisory Authority of Norway, the younger home buyers are often restricted by this last rule, rather than the maximum LTV. Therefore, as leveraging the PRE investment yields higher portfolio efficiency, the incentive to purchase PRE over the financial assets increases for higher levels of income.

In summary, we find that while our before-tax results are not sensitive to leverage, our after-tax results are. As the efficiency gain from leverage increases for higher levels of LTV, and these levels are restricted by income level, our results are indirectly sensitive to income level. Now that the results have been presented and analyzed, we move on to the conclusion where the most central results are analyzed in more detail. Furthermore, the relevance and implications of the results are discussed.

7 Conclusions

In this thesis, we examine the characteristics of PRE, both individually, and in a mixed-asset portfolio together with traditional financial assets. We find that in our most comprehensive five-year model, the average annual return of the PRE asset is 17.40% and the volatility of this return is 8.36%. Out of the assets we compare, PRE has the highest return, yet the stock fund has the highest risk. This small divergence between the risk and return correlation can be a result of several factors. It could be the consequence of a market imperfection, the period we examine, or that our most comprehensive model does not include all relevant variables. We argue that it is most likely a result of the period we examine. From 2005-2018 PRE has outperformed its historical average with a low degree of risk, as a result of this time period not encompassing an entire housing cycle. The stock fund, on the other hand, incurs several draw-downs over the period, resulting in a low average annual return and high risk. The results should therefore not be generalized, or used to extrapolate expectations into the future.

In examining the effect of including PRE in a mixed-asset portfolio, we find that the allocation to the PRE asset is 50.82%, and that the PRE asset completely replaces the stock fund, and some of the bond fund allocation. This portfolio yields a Sharpe ratio of 1.85, thereby strongly out-performing the tangent benchmark portfolio Sharpe ratio, being 0.69. Furthermore, we find that the alpha from the regression of the tangent mixed-asset portfolio on the tangent benchmark portfolio is 5.69% and significant. The interpretation of this, is that including our final measure of PRE risk and return in the investment universe yields significantly improved portfolio efficiency for a well-diversified investor. The improved portfolio efficiency is the largest for high levels of risk and return and is mainly driven by the return enhancing ability of PRE.

In addition to this, we find that even without the effect of the "external factors", being leverage and taxation, an allocation to PRE in the mixed-asset portfolio yields superior portfolio efficiency. The Sharpe ratio is of this portfolio is reduced to 1.44, yet is still significantly above the benchmark Sharpe ratio of 0.69. The removal of these factors alters the effect of PRE on the mixed-asset portfolio as the dominating quality of this asset without leverage and taxation is to be risk reducing. The implication of this, is that the desired effect of PRE on the investor's portfolio can be adapted through the leverage of this asset.

In our robustness section we find that while the five-year results are robust to apartment type, the one-year results are not. The decision between a freehold and co-op apartment affects the risk and return that can be expected from investing in PRE for an investor with a short investment horizon. While the one-year freehold average annual return and risk is 7.79% and 15.85% the co-op values are 13.08% and 16.80%, respectively. In reality, the decision of what type of apartment to purchase is driven by many factors that cannot be incorporated in the risk-and return measures of PRE, as the ownership rights, location and the standard of these apartment types differ.

Furthermore, the results are robust to the number of apartments the investor should purchase, as the tangent portfolio of a primary and secondary apartment allocates 100% of the wealth to the primary apartment. Further research should analyze the effect of purchasing secondary housing in a different market, as our results are driven by the lack of diversification benefits from owning two apartments in Oslo. In addition to this, an interesting analysis could be to split the Oslo market into sub-markets and examine whether owning a secondary home in a different area in the same city yields sufficient diversification benefits for there to be an allocation to the secondary apartment. The decision of what constitutes the "market" for an apartment is challenging, as some may argue that Oslo constitutes one housing market, while others may define a housing market on a more granular level such as a city area or postal code.

While we find that our results are not sensitive to wealth or property value, they are indirectly sensitive to income level. Higher leverage yields higher portfolio efficiency in our model, and higher leverage is constrained by higher income for many. Furthermore, we find that the optimal level of leverage is the maximum level permitted. Similarly to our PRE risk and return, this is most likely a result of the period examined where the mortgage rate has been abnormally low. The effect of an increased mortgage rate is not analyzed and can be a topic for further research.

Though the adjustments made in this thesis remove several of the market imperfections that distinguish the investment in an apartment in Oslo from the financial assets, non-financial differences remain. We recognize that the purchase of a primary home is not purely a financial decision, and that the retail investors in this market may be especially prone to irrational behavior. Some participants attribute a value to the security of knowing that they own their housing, while others prefer to rent as they do not have to worry about maintenance or repair. In addition to this, the fact that an individual needs to consume housing services somehow, distinguishes PRE from the financial assets. When housing prices increase, the rent should also increase. An investor who owns a home is hedged against this price increase, while a renter is not. As most Norwegian adults purchase a home at some point in life, renting is the equivalent of being short the housing market. This is risky and can increase the relative attractiveness of the PRE asset compared to the financial assets.

The general findings in this thesis should not be interpreted as specific investment advice on the individual level. The optimal portfolio for an individual investor is a combination of the risk tolerance, wealth level, preferences, liabilities, expectations and so on, of that individual. Though our results do not serve as investment advice, they have implications for the individual investor. The thesis analyzes the characteristics of PRE in increments, defining what drives the risk and return of this asset, and is thereby contributing with information that should be used as input in the investment decision. While we find strong evidence that investing in PRE is superior to investments in the traditional financial assets if the asset is held for five years, this return is achieved at a much higher risk than what is expected by the investors.

On a final note, the results can be used to argue for policy implementation. Due to the large investment needed to gain access to the primary home market, (especially in Oslo), not all individual investors can gain this exposure due to the restrictions on mortgages. We therefore propose the implementation of financial products such as the ones described in Døskeland (2014), which can able investors to gain exposure to the primary home market, without purchasing an entire primary home. These products should also be able to diversify away the individual house risk, one of the market imperfections with the largest detrimental effect on the attractiveness of PRE as an asset.

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Appendix

Table A0.1: Model Data Descriptions

Model	Input	Data Sources
Financial Assets	Global stock and bond funds 10 year government bond	SKAGEN Fondene DNB Asset Management ODIN Bloomberg Norges Bank
Capital Appreciation Individual Risk	Same as above, PRE market WRS freehold HPI	Eiendomsverdi
Liquidity Risk	Same as above, days from listing to sale in Oslo	Eiendomsverdi Real Estate Norway
Imputed Rent	Same as above, market rent for two-room apartments	The Municipality of Oslo
Leverage	Same as above, average loan to value and mortgage rate	Financial Supervisory Authority of Norway Statistics Norway
Owner Cost	Same as above, insurance cost, municipal cost, maintenance cost, document duty,registration fees, mortgage certificate, house-purchase insurance, owner change fee, broker fee, house-sale insurance, appraisal, styling, photography, internet ads, open house	Kartverket Statistics Norway Eurodata Financial Supervisory Authority of Norway Lovdata Huseiernes Landsforbund Takstmannen AS OBOS Söderberg & partners DNB Smarte Penger
Tax Five-Year Freehold	Same as above, general income tax, risk-free interest rate, factor for upward adjustment	The Norwegian Tax Administration The Ministry of Finance
Cooperative Five-Year Cooperative	Same as above, mortgage rate for common debt, registration fees	Kartverket Lovdata BORI
Primary and Secondary Home	Same as above, wealth tax rate, valuation of secondary homes	The Norwegian Tax Administration Lovdata
Sensitivities	Property tax rate, wealth valuation of stock fund, wealth valuation of bond fund	The Norwegian tax Administration

Table A0.2: Correlations

Before-Tax	Stock Fund	Bond Fund
Stock Fund	1	0.27
Bond Fund	0.27	1
Private Real Estate	0.47	0.04
Private Real Estate with Individual Risk	0.27	0.02
Private Real Estate with Liquidity Risk	0.26	0.02
Private Real Estate with Imputed Rent	0.26	0.04
Private Real Estate with Leverage	0.37	0.06
Private Real Estate with Owner Cost	0.37	0.05
After-Tax	Stock Fund	Bond Fund
Stock Fund	1	0.27
Bond Fund	0.27	1
Private Real Estate	0.36	0.05
Private Real Estate (Cooperative)	0.35	0.04
Five Year Holding Period	Stock Fund	Bond Fund
Stock Fund	1	0.28
Bond Fund	0.28	1
Private Real Estate	0.71	0.09
Private Real Estate (Cooperative)	0.63	0.08

Table A0.3: Acronyms

Acronym	**Meaning**
AR	Appraisal Ratio
BF	Bond Fund
CAP	Capital Appreciation
CAPM	Capital Asset Pricing Model
CDMR	Common Debt Mortgage Rate
CDTV	Common Debt To Value
CPI	Consumer Price Index
FUA	Factor-for Upwards Adjustment
GITR	General Income Tax Rate
HPI	House Price Index
IR	Information Ratio
LTV	Loan To Value
MM	Modigliani and Miller
MPT	Modern Portfolio Theory
MR	Mortgage Rate
NY	Net Yield (Y-OC)
OC	Operation Cost
PMR	Private debt Mortgage Rate
PRE	Private Real Estate
PTR	Property Tax Rates
R_{F_1}	10-year Government Bonds
R_{F_2}	Risk-free interest rate for "Skjermingsfradrag"
ROE	Return On Equity
RS	Repeat Sales
SF	Stock Fund
TC	Transaction Cost (purchase and sales cost)
TOM	Time On Market
VFF	Norwegian Fund and Asset Management Association
WRS	Weighted Repeat SalesWTR
WTR	Wealth Tax Rate
WV*B/S/H/SH	Wealth Value Bond/Stock/Primary Home/Secondary Home
Y	Imputed Rent Yield