Norwegian School of Economics

Bergen, Spring 2020

Illiquidity in Asset Pricing and as Investment Strategy

An Empirical Analysis

Irene Haumer and Rubing Mao

Supervisor: Francisco Santos

Master Thesis, MSc in Economics and Business Administration, Finance

NORWEGIAN SCHOOL OF ECONOMICS

Acknowledgements

We thank our supervisor, Francisco Santos, for his guidance and advice during the writing of this thesis. His comments and insights are highly appreciated. We would also like to thank Tore Leite, who sparked the interest in the topic with his course *Trading, Liquidity, and Pricing in Securities Markets*. In addition, one of us would also like to thank her mother – Elisabeth – for her unconditional support and her ability to listen.

Abstract

This Master's thesis examines the illiquidity premium. In the first part of the thesis, we analyse whether a traded illiquid-minus-liquid (*IML*) return factor helps in explaining the cross-section of expected returns. In the second part, we investigate whether the illiquidity premium can be captured in practice. In our asset pricing tests, we find some evidence in favour of adding *IML* to both, the Fama and French three- and five-factor model. For most test portfolios, *IML* improves the description of average excess returns. The improvements are larger when switching from the three-factor model to its *IML*-augmented version than when adding *IML* to the five-factor model. With regards to how implementable an illiquidity strategy is in practice, we find that the illiquidity premium is largely concentrated among small firms. This pattern does not change over time. When considering market-adjusted returns, we show that the illiquidity premium is driven mostly by the long side, though not entirely. The contribution of the long and short side changes over time. Further, we present some evidence that the contribution of the long and the short side varies across firm size. For the smallest firms, shorting is less important than for the biggest firms. Moreover, we find that the illiquidity premium has decreased over time. Given our results, we conclude that it is highly unlikely that the illiquidity premium can be captured in practice.

Contents

List of Tables

List of Figures

1. Introduction

Liquidity is a complex concept that is often simplified described as the ease of trading a security (Amihud, Mendelson & Pedersen, 2005). When referring to an asset as being liquid, one means that it can be traded fast, at a price close to its fundamental value, and with little price impact. This implies that liquidity has several dimensions. The sources of illiquidity are manifold. They include asymmetric information about fundamentals or the order flow, inventory risk, search costs, and exogenous transaction costs. Illiquidity is costly and investors demand to be compensated for these costs. In addition, illiquidity is time-varying (Acharya & Pedersen, 2005), and a risk-averse investor will additionally demand to be compensated for bearing illiquidity risk. Consequently, illiquidity affects the cost of capital by raising the required return of investors.

With our thesis we aim at contributing to the most recent stream of liquidity literature. In the first part of the thesis, we replicate the illiquid-minus-liquid return factor (*IML)*, as presented by Amihud (2019). We then use this factor in formal asset pricing test to examine whether it helps in explaining the cross-section of expected returns. Our motivation for using *IML* in asset pricing tests is the following: As Fama and French (henceforth FF) (1993) note, empirical evidence should motivate the choice of factors. They constructed their factors after it has been shown empirically that the variables used in the construction are priced across stocks. Illiquidity well fulfils this requirement; the following examples of literature shall act as evidence for it.

Amihud and Mendelson (1986) prepared the ground for studying the effects of liquidity on asset pricing. Their paper provides two major contributions: First, they show that expected asset returns increase in illiquidity costs. The authors find that a 1% increase in the bid-ask spread raises the monthly risk-adjusted excess return by 0.21%. Second, they show that this relationship is concave. This is due to the clientele effect: Short term-investors, who more frequently incur the high trading costs inherent in illiquid assets, substantially discount them. Long-term investors, on the other hand, incur the high trading costs less frequently; making it possible for them to harvest an illiquidity premium in excess of their expected transaction costs. Hence, in equilibrium, short-term investors hold liquid assets, while long-term investors hold illiquid ones. Further evidence on the existence of an illiquidity premium is presented by Brennan and Subrahmanyam (1996). They show that the stocks of the lowest liquidity quintile outperform the stocks of the highest liquidity quintile by 6.6% per year. In his seminal 2002 paper, Amihud analyses the effects of illiquidity over time and on the cross-section of stock returns. The author finds a positive and significant effect on stock returns for the sample period 1963-1997.

In a more recent study, using a sample of 45 countries, Amihud et al. (2015) document that the illiquidity premium is positive and significant also in stock markets other than the U.S. This can be seen as evidence for the robustness of the illiquidity premium, as it indicates that the relation between illiquidity and average returns is no chance result and that there is no sample-specific explanation for it. An ideal candidate for being a critical input in describing the cross-section of expected returns stands out due to its pervasiveness and robustness. Hence, the evidence presented in Amihud et al. (2015), further strengthens our reasoning for why we use *IML* in asset pricing tests. Additionally, FF (1993, 2015) point out that the most severe problems for asset pricing are the small stocks. Given that size and illiquidity are related (Stoll & Whaley, 1983; Amihud, 2002), one can argue that the higher average return for small firms is in fact an illiquidity premium. Therefore, an illiquidity factor might improve the performance of asset pricing models.

Using data for the U.S. stock market and a sample period from 1964 to 2019, we find that the average monthly return on *IML* is 0.498% (t = 4.15). The return remains positive and significant after controlling for risk. In our asset pricing tests, we follow FF (1993, 2015) and run time-series regressions. We consider four different models: (i) the FF three-factor model; (ii) the FF three-factor model augmented by *IML*; (iii) the FF five-factor model; and (iv) the FF five-factor model augmented by *IML*. Our test portfolios are a variety of FF portfolios sorted on known anomalies. The most famous of these anomalies are size, value, investment and profitability; but we also consider less often covered anomaly variables, such as daily variance, net share issues, accruals, and market beta. We judge the absolute performance of the models, as well as their relative performance. For judging the absolute performance, we use the F-test of Gibbons, Ross and Shanken (1989). The *GRS* statistic tests the hypothesis that the intercept obtained in a regression of an asset's excess return on factor returns should be indistinguishable from zero if an asset pricing model completely captures expected returns. In order to judge the relative performance and the improvement gained by adding *IML*, we use summary statistics for regression intercepts in addition to the *GRS* statistic. The summary statistics are the average absolute intercept, $A|\alpha_i|$, and $A|\alpha_i|/A|\overline{r_i}|$, which estimates the proportion of the cross-section of expected returns left unexplained by the model tested.

We find some evidence in favour of *IML*. For most test portfolios, *IML* improves the description of average excess returns. We prefer the *IML*-augmented FF five-factor model over the FF five-factor model, the FF five-factor model over the *IML*-augmented FF three-factor model and the *IML*-augmented FF three-factor model over the FF three-factor model. The *GRS* statistic, the average absolute intercept and the proportion of the cross-section of expected returns left unexplained decrease when moving from the least preferred model to the most preferred. In terms of $A|\alpha_i|$, the improvements are larger when switching from the FF three-factor model to its *IML*-augmented version than when adding *IML* to the five-factor model. The improvements gained from adding *IML* to the FF five-factor model are rather small. However, for two of our test portfolios (the 25 *B/M-INV* and the 25 *Size-Market Beta* portfolios), we clearly prefer an *IML*-augmented asset pricing model. For both of them, the *IML*-augmented FF three-factor model fares best, followed by the *IML*-augmented five-factor model. For the 25 *Size-Market Beta* portfolios we even find that the *IML*-augmented three-factor model is a description of the average excess returns.

In the second part of our thesis, we shed light on the question of whether an illiquidity strategy is implementable in practice. We follow Israel and Moskowitz (2013) in their methodology of assessing the implementability of value and momentum strategies. The tests seek to answer three questions. First, how much does short selling contribute to the profitability of an illiquidity strategy? Second, what is the role of firm size with regards to the efficacy of the strategy? Third, how have the returns and the role of shorting and firm size varied over time? Small stocks are more costly and difficult to trade. In addition, shorting them comes with substantial costs. Consequently, if the majority of the returns comes from small stocks, this might hinder investors from capturing any illiquidity premium. Similarly, if shorting is an important driver of profits, this is also bad news for investors. Short positions are costly to maintain. Moreover, some investors are even restricted from taking short positions. Therefore, the net of trading costs returns might be considerably reduced or not accessible at all to investors. Hence, we believe that the answers to these questions help us in better understanding the illiquidity premium and that they can also provide some guidance for potential illiquidity investors.

With regards to how implementable an illiquidity strategy is in practice, we find that the illiquidity premium is largely concentrated among small firms. It is considerably lower for the biggest firms. While we find a monthly *alpha* of 0.76% for the smallest size quintile, the monthly *alpha* of the biggest size quintile is 0.08% and not statistically significant. This pattern does not change over time. When considering market-adjusted returns, we find that the illiquidity premium is driven mostly by the long side, though not entirely. For raw returns, the return premium is dominated by the contribution from long positions. Further, we find some evidence that the contribution of the long and the short side varies across firm size. For the smallest firms, shorting is less important than for the biggest firms. For the full sample period, we report that the contribution of the long side equals 59.21% for the smallest firms. For the biggest firms, the contribution is only 37.5%. However, there is no monotonic pattern in the relation between shorting and firm size. The contribution of the long and short side changes over time. For the most recent subperiod (2002-2019), we find that most profits come from the short side. Shorting remains more important among big firms, though. Moreover, we show that the illiquidity premium decreased over time. For the most recent subperiod, it is insignificant in each of the size groups. We therefore conclude that is highly unlikely that the illiquidity premium can be captured in practice.

In our thesis, we seek to continue this most recent illiquidity literature. A newer stream of literature measures systematic risk with respect to a return factor that is estimated as the differential return of illiquid and liquid stocks. As an extension to his 2002 study, Amihud (2019) presents the illiquid-minus-liquid (*IML*) return factor, which provides a time series of the illiquidity premium. Amihud (2019) finds a positive and significant risk-adjusted expected return on *IML*, confirming prior evidence that an illiquidity premium exists across stocks. We add to Amihud (2019) in that we continue to examine *IML*. Using *IML* in asset pricing tests seems like the next logical step, given that *IML* has a significant and positive risk-adjusted return and it also fulfils the other requirements for being an explanatory return factor in asset pricing tests.

Earlier research (Acharya & Pedersen, 2005; Pástor & Stambaugh, 2003), on the other hand, estimates systematic risk with respect to innovations in market liquidity. One of the most influential papers about asset pricing and liquidity risk is written by Acharya and Pedersen (2005). While prior papers (Amihud & Mendelson, 1986a; Brennan & Subrahmanyam, 1996; Amihud, 2002), present evidence on the positive relation between illiquidity costs and expected returns, Acharya and Pedersen (2005) present evidence on the pricing of systematic illiquidity risk. In order to account for liquidity risk, the authors adjust the standard capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) by augmenting it with three additional betas, each representing its own form of liquidity risk. They show that expected stock returns are increasing in expected stock illiquidity, increasing in the covariance between its illiquidity and the market illiquidity, decreasing in the covariance between the asset's return and the market illiquidity, and decreasing in the covariance between the asset's illiquidity and market return. Acharya and Pedersen (2005) document a significant combined effect in U.S. equities of 4.6% per year for the period 1963-1999. Pástor and Stambaugh (2003) also propose that stocks with higher systematic liquidity risk should earn a higher return. They also show that liquidity risk is priced: For the 1966 to 1999 sample period, the risk-adjusted average return on stocks with a high sensitivity to market-wide liquidity outperforms that on socks with low sensitivity by 7.5% annually.

While we also pursue asset pricing tests, we differ from Acharya and Pedersen (2005) and Pástor and Stambaugh (2003) in that we use the differential return of illiquid and liquid stocks for our analysis. Additionally, in contrast to Acharya and Pedersen (2005), we account for illiquidity in another way. While Acharya and Pedersen (2005) augment the standard CAPM, we use a traded return factor to augment the FF three- and five-factor model. Hence, our thesis also adds to this stream of liquidity literature.

Furthermore, our thesis complements papers that dissect return factors and analyse their implementability when used as an investment strategy. Most of these papers focus on prominent strategies, such as value and momentum. Hence, we also contribute to literature of this kind (see for example FF, 2008; Israel & Moskowitz, 2013), in that we expand the analysis to the illiquidity strategy.

The rest of the thesis is organized as follows. Section 2 describes the data used and the replication of *IML*. Section 3 analyses our asset pricing tests. Section 4 examines the role of shorting, firm size, and time on the illiquidity premium. Section 5 concludes.

2. Data and the replication of *IML*

We download data from the Center for Research in Security Prices (CRSP). The sample period in Amihud (2019) stretches from 1964 to 2017. We extend this period by 2 years to 2019. We use the CRSP daily and monthly file for the time period December 1962 to December 2019. We need those additional 13 months at the beginning in order to calculate the variables on which the first portfolios are formed in January 1964. The daily file contains data on return, price, and trading volume and is used to calculate the sorting variables – the standard deviation of daily returns and $Illiq_{j,y}$. The monthly file contains data on return, delisting return, delisting code, price and shares outstanding. Amihud (2019) uses only NYSE and AMEX securities. We identify them using their exchange code and download data only for stocks with an exchange code of 1 or 2, i.e. stocks traded at the New York Stock Exchange (NYSE) or the American Stock Exchange (AMEX). The reason for why stocks trading on the National Association of Securities Dealers Automated Quotations (NASDAQ) are excluded is that trading on this exchange was done through market makers during part of the sample period. During that time trading volume was counted twice. Since the illiquidity measure employed features the trading volume, it would not be consistent across stocks traded at either NYSE or AMEX and stocks traded at NASDAQ. The sample is restricted to common (ordinary) shares, i.e. shares with share code 10 or 11.

2.1 Measuring illiquidity

As already discussed, illiquidity is a complex concept. Consequently, in terms of measuring illiquidity, a wide variety of measures exist. Often used measures include the bid-ask spread, which captures the trading cost dimension, turnover, which captures the trading quantity dimension, and measures that capture the price impact by estimating the price reaction to trading volume. Clearly, hardly a single measure exists that captures all dimensions. Some of the measures rely on high-frequency data and are therefore not easily implementable. A widely used measure is the Amihud (2002) illiquidity measure, *Illiq*. It is calculated as the ratio of daily return to daily dollar volume, which is then averaged over some period. A stock is deemed illiquid if its value of *Illiq* is high, as this indicates that a stock's price moves a lot in response to a given dollar volume. While finer measures, such as the bid-ask spread or the probability of informed trading, exist, they require microstructure data on transactions and quotes, which is unavailable in many stock markets and for long time periods. The use of *Illiq* is appealing, as it uses daily data, making it easily obtainable and usable for long time series and most stock markets. Several studies (Amihud, 2002; Goyenko, Holden & Trzcinka, 2009; Hasbrouck, 2009) show empirically that *Illiq* is highly and significantly correlated with the Kyle (1985) price impact measure, λ, for the US, but also across countries (Fong, Holden & Trzcinka, 2017). Additionally, *Illiq* is closely correlated to the Amivest illiquidity measure, another popular measure that has been employed in a wide range of empirical microstructure literature. In Amihud (2019), *Illiq* of stock *j* on day *d* is defined as

$$Illiq_{j,d} = \frac{|return_{j,d}|}{dollar\ volume_{j,d}}. \tag{1}$$

$|return_{j,d}|$ is the absolute value of the return on day *d* for stock *j*. $Dollar\ volume_{j,d}$ is the trading volume in US dollars on day *d* for stock *j*. It is calculated by multiplying the shares traded on given day by the share price on the same day. The daily values of $Illiq_{j,d}$ are averaged for each stock over a 12-month period that ends in November each year. Some filters are applied for the calculation of the annual value $Illiq_{j,y}$. Days with a negative price, a trading volume of less than 100 shares, or a return of -100% are removed. After having calculated $Illiq_{j,d}$, the highest value is deleted for each stock in each 12-month period.

2.2 The construction of *IML*

In order to construct *IML*, we next need to form portfolios. The sorting is based on two variables, the standard deviation of daily returns and $Illiq_{j,y}$. In order for a stock to be included in a portfolio, it needs to pass some requirements. Its price has to be between \$5 and \$1,000 and it needs to have more than 200 days of valid return and volume data during the same 12-month period that is used for calculating $Illiq_{j,y}$. We interpret data to be valid if it passes the above requirements for the deletion of days, and does not show *NA*. Lastly, potential outliers – stocks with a value of $Illiq_{j,y}$ in the top 1% - are removed from the sample each year. The second sorting variable - the standard deviation of daily returns of stock *j* - is also calculated over the 12-month period. Amihud (2019) does not clearly state whether all days are used for the calculation or whether days are deleted according to the filters above as well. Hence, we have to make an assumption. We use every day in the 12-month period for calculating the standard deviation. Portfolios are sorted monthly. Stocks that satisfy the requirements and exist at the end of the previous month are sorted into portfolios in *y+1* based on $Illiq_{j,y}$ and the standard deviation of daily returns of year y. We use conditional sorts, first sorting stocks into three portfolios on their standard deviation, then sorting each tercile further into quintiles on $Illiq_{j,y}$. This gives a total of 15 portfolios. The double sorting is done because return volatility impacts a stock's expected return and has been shown to be positively correlated with *Illiq* (Amihud, 2002; Stoll, 1978). Figure 1 illustrates the construction of the 15 portfolios.

Figure 1
The construction of IML

Using conditional sorts, we first sort stocks into three portfolios on their standard deviation (Std). We then sort each tercile further into quintiles on $Illiq_{j,y}$. This results in a total of 15 portfolios.

	Illiq				
Std	Low	1	2	3	High
Low	L/L	L/1	L/2	L/3	L/H
Medium	M/L	M/1	M/2	M/3	M/H
High	H/L	H/1	H/2	H/3	H/H

We then calculate monthly value-weighted returns for each portfolio using the market capitalization of the prior month as weight. The market capitalization is calculated by multiplying shares outstanding by the absolute value of the price for each stock. Returns are adjusted in order to correct for the delisting bias (Shumway, 1997).[1] Average monthly returns, standard deviations and t-statistics for the average monthly returns are reported in Table 1 in Section 2.4 for each of the 15 portfolios. *IML* represents the average of the monthly returns of the highest illiquidity quintile portfolios minus the average of the monthly returns of the lowest illiquidity quintile portfolios across the three standard deviation portfolios,

$$IML = 1/3\ (L/H + M/H + H/H) - 1/3\ (L/L + M/L + H/L). \quad (2)$$

The replication results showing summary statistics for *IML* and the risk-adjusted illiquidity premium are reported in Table 2 in Section 2.4.

[1] The last month's return of a delisted stock is either the last return available from CRSP or the delisting return. If neither one is available and the deletion code is in the 500s, the delisting return is assigned to be -30%.

2.3 Test portfolios and other factors

The factors we use throughout this thesis in addition to *IML* are the FF three factors, the FF five factors and the momentum factor (*MOM*). We download the excess return on the market (*MKT*), *SMB*[2], *HML*, *RMW*, *CMA*, *MOM*, as well as the risk-free rate from Ken French's data library.[3] In order to avoid potential arbitrariness or data mining concerns, we do not construct our own test portfolios for the formal asset pricing tests. Rather, we use readily available portfolios. The test portfolios for our asset pricing tests are also download from Ken French's data library. We discuss them in more detail in Section 3.3.

2.4 Replication results

Table 1 shows the average monthly returns, standard deviations and *t*-statistics for the average monthly returns for each of the 15 portfolios. Looking at the mean returns (Panel A), one sees that illiquid stocks outperform the liquid ones. For each of the standard deviation terciles, the average return is higher for the highest illiquidity group than for the lowest illiquidity group. In particular the long side of *IML*, i.e. the most illiquid stock quintile, is highly statistically significant. The pattern of mean returns across illiquidity quintiles is monotonic for the two lower standard deviation terciles. It is, however, not as monotonic for the highest standard deviation tercile.

Table 2 shows the replication results. Panel A reports the mean value of *IML* for the period January 1964 to December 2019, as well as for two subperiods. The first subperiod (1964-1997) is the same as in Amihud 2019. The second subperiod (1998-2019) is extended to the present. Also shown in Panel A are the *t*-statistics of the average returns. The average return on *IML* is 0.498% per month for the full sample period. It is significant with a *t*-statistic of 4.15. Importantly, the *t*-statistic not only exceeds the usual threshold for establishing significance, but it also survives the higher hurdle proposed by Harvey, Liu and Zhu (2016). The authors argue that it is a considerable mistake to use usual statistical significance cutoffs

---

[2] The construction of *SMB* has changed for the five-factor model. The five-factor model *SMB* also includes *SMB* obtained from constructing the operating profitability and investment factors. Depending on whether we use the three-factor or the five-factor model, we use the different versions of *SMB*.

[3] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

in asset pricing test. The threshold for establishing significance for new factors should be at least 3.0, given the inevitable data mining inherent in the vast increase in factors aimed at explaining the cross-section of expected returns. The mean *IML* return for the first subperiod (1964-1997) is 0.6% per month with a t-statistic of 4.04. For comparison, Amihud (2019) reports an average return on *IML* of 0.635% (t = 4.47) per month for the same period. For the second subperiod (1998-2019), we find an average *IML* return of 0.341% per month that is not statistically significant. Hence, one might be worried that the illiquidity premium has vanished in recent times. For the corresponding shorter period 1998-2017, Amihud (2019) reports a mean *IML* return of 0.43% (t = 2.14) per month. Any differences between Amihud's and our results might be due to the assumptions we had to make in constructing *IML*, and for the second subperiod due to the extension of the period to the present. Amihud (2019) does not report results for the combined period.

Panel B reports the risk-adjusted average return on *IML* (*Alpha*) obtained from regressions of *IML* on different risk factors. In Panel B1 we only adjust for the market, i.e. we use only *MKT* as risk factor. *IML* loads negatively on *MKT*. Consequently, the average risk-adjusted return on *IML* is higher than the average raw return. *Alpha* equals 0.59% (t = 5.08) per month for the period 1964 to 2019. For the first and the second subperiod, *alpha* is equal to 0.67% (t = 4.70) per month and 0.47% (t = 2.38) per month, respectively. While we find that the market-adjusted return is significant for the full sample period, as well as for the first subperiod, it does not survive the findings of Harvey, Liu and Zhu (2016) in the most recent subperiod. The majority of the profits seem to come from the first subperiod. Again, the question as of the profitability of *IML* arises. We will further examine this in Section 4. Amihud (2019) reports a monthly market-adjusted return of 0.714% (t = 5.18) for the earlier period and a monthly *alpha* of 0.552% (t = 2.86) for his recent period.

In Panel B2 we adjust for the FF three factors and *MOM*. We again find a positive and significant risk-adjusted illiquidity premium for the full sample period and for the first subperiod. With regards to the significance in the second subperiod, we observe the same results as previously. The average *IML* adjusted for the FF three factors and *MOM* is equal to 0.35% (t = 4.43) per month for the period 1964-2019. For the first and the second subperiod, *alpha* is equal to 0.4% (t = 3.99) per month and 0.35% (t = 2.67) per month, respectively. Amihud (2019) reports a monthly risk-adjusted return of 0.372% (t = 3.80) for the first period and a monthly *alpha* of 0.403% (t = 3.12) for his second period. Since *IML* loads positively on *SMB* and *HML*, it is naturally lower than the market adjusted return. The slope coefficient of *MOM* is not significant. Our findings regarding the loadings are consistent with the original results in Amihud (2019). We also run a regression of *IML* on the FF five factors and *MOM*. For the full sample period, *alpha* equals 0.3% (t = 3.66) per month. For each of the two subperiods we clearly find that *alpha* is not significant. We relegate the regression results for the full sample period as well as the subperiods to Table A1 of the Appendix. We do not include them here, as we include the five-factor regression for the whole sample period in our asset pricing tests in Section 3.2.

As a final note, we have shown that neither the FF three-factor model, nor the FF five-factor model can explain the returns on *IML*. *IML* remains statistically significant after controlling for risk, indicating that *IML* statistically improves the FF model. This motivates the asset pricing tests in the next section.

Table 1
Summary statistics for portfolios formed on standard deviation and $Illiq_{j,y}$

The sample period is January 1964 to December 2019. We use NYSE and AMEX stocks and apply some filters. Every month, stocks are sorted into three portfolios based on their standard deviation of daily returns. Each standard deviation portfolio is then further sorted into five illiquidity portfolios based on the average of daily values of $Illiq_{j,d} = |return_{j,d}|/dollar\ volume_{j,d}$. Each of the variables is calculated over a 12-month period that ends in November of the previous year. The procedure results in a total of 15 portfolios. Returns and standard deviations are in monthly percentages. Panel A reports the average monthly returns. Panel B reports the *t*-statistics of the monthly returns. Panel C reports the standard deviations of monthly returns.

Panel A: Average monthly returns

	Illiq				
Std	L	2	3	4	H
L	0.87	1.02	1.07	1.15	1.25
2	0.83	1.11	1.14	1.31	1.45
H	0.68	1.00	0.97	1.18	1.18

Panel B: *t*-statistics of average returns

	Illiq				
Std	L	2	3	4	H
L	6.02	6.67	7.04	7.60	8.20
2	3.92	5.25	5.28	6.12	6.90
H	2.37	3.55	3.36	4.25	4.28

Panel C: Average standard deviations

	Illiq				
Std	L	2	3	4	H
L	3.76	3.96	3.96	3.91	3.96
2	5.47	5.48	5.59	5.57	5.43
H	7.46	7.32	7.53	7.21	7.14

Table 2
Raw and risk-adjusted returns on an illiquid-minus-liquid (*IML*) portfolio

IML is the return on an illiquid-minus-liquid portfolio. Every month, stocks are sorted into three portfolios based on their standard deviation of daily returns. Each standard deviation portfolio is then further sorted into five illiquidity portfolios based on the average of daily values of $Illiq_{j,\,d} = |return_{j,d}|/dollar\ volume_{j,d}$. Each of the variables is calculated over a 12-month period that ends in November of the previous year. *IML* is then calculated as the return differential between stocks in the highest illiquidity quintile and stocks in the lowest illiquidity quintile across the three standard deviation portfolios. We use NYSE and AMEX stocks and apply some filters. Returns are in monthly percentages. *t*-statistics of average returns are reported in parenthesis. The full sample period stretches from January 1964 to December 2019. The first subperiod covers the years 1964-1997, the second subperiod the years 1998-2019. The *t*-statistics of the coefficients employ robust standard errors (White, 1980), as in Amihud (2019). Panel A shows the statistics for *IML*. Panel B reports *alphas*, coefficients and the corresponding *t*-statistics from a regression of *IML* on the FF factors. The risk factors used are the FF three factors and *MOM*. The three-factor time-series regression including *MOM* is

$$IML_t = \alpha + bMKT_t + sSMB_t + hHML_t + mMOM_t + \varepsilon_t.$$

Panel A: Statistics for *IML*

Period	1964-2019	1964-1997	1998-2019
Mean	0.498	0.600	0.341
(*t*-statistic)	(4.15)	(4.04)	(1.69)

Panel B1: Regressions of *IML* on the market excess return

	Dependent variable:		
		IML	
	1964-2019	1964-1997	1998-2019
Alpha	0.59	0.67	0.47
	(5.08)	(4.70)	(2.38)
MKT	-0.17	-0.14	-0.22
	(-4.48)	(-2.58)	(-4.45)

Panel B2: Regressions of *IML* on the FF three factors and *MOM*

		IML	
	1964-2019	1964-1997	1998-2019
Alpha	0.35	0.40	0.35
	(4.43)	(3.99)	(2.67)
MKT	-0.27	-0.23	-0.31
	(-11.91)	(-7.64)	(-8.37)
SMB	0.77	0.78	0.74
	(20.24)	(17.12)	(13.36)
HML	0.35	0.36	0.36
	(8.84)	(6.61)	(6.36)
MOM	0.04	-0.03	0.07
	(1.45)	(-0.74)	(2.10)
Observations	672	408	264

3. Formal Asset Pricing Tests

In this section, we use *IML* in formal asset pricing tests to examine whether *IML* helps in explaining cross-sectional returns. The other factors used are the FF factors: *MKT*, *SMB*, *HML*, *RMW*, and *CMA*. Before we turn to the results of the asset pricing tests, we consider it important to first examine the factors in more detail. In particular, we discuss some summary statistics and results from regressions of each factor on the other ones.

3.1 Summary statistics for factor returns

Table 3 shows summary statistics for the explanatory returns of our asset pricing regressions. Panel A displays the monthly mean return, the standard deviation of monthly returns, the *t*-statistic of the returns, and the Sharpe ratio for *MKT, IML*, *SMB*, *HML*, *RMW*, and *CMA* for the period 1964-2019. *MKT* exhibits the highest mean return, 0.54% ($t = 3.16$) per month, very closely followed by *IML*, whose mean return equals 0.5% ($t = 4.15$) per month. The standard deviation of *IML* is, however, lower than that of *MKT,* resulting in a higher Sharpe ratio for *IML*. *IML*'s monthly Sharpe ratio equals 0.16, making it the highest one of the six factors. *IML* also exhibits the strongest *t*-statistic. *SMB* produces the lowest mean and Sharpe ratio of all factors and is not statistically significant ($t = 1.74$) for the sample period. Not shown in the table are the statistics for the five-factor *SMB*. We can report though that the five-factor *SMB* is similar to the three-factor *SMB* on all metrics. Merely the *t*-statistic is higher, but not bigger than 3.0 either. The mean returns for *HML*, *RMW* and *CMA* exhibit a similar economic magnitude, about 0.3% per month. *RMW* and *CMA* are statistically significant; *HML* does not survive the higher hurdle suggested by Harvey, Liu and Zhu (2016).

Table 3, Panel B shows the correlation matrix for the different factors. Not quite surprisingly, *IML* is strongly correlated with *SMB*. Size and illiquidity are related, as a larger stock issue comes with a minor price impact for a given order flow and a smaller bid-ask spread (Amihud, 2002; Stoll & Whaley, 1978). *IML*'s correlation with the three-factor *SMB* is 0.58 and its correlation with the five-factor *SMB* is even higher, it amounts to 0.62. Since *SMB* is constructed without controlling for illiquidity, the average *SMB* return likely is in part an illiquidity premium. We will revisit this fact later when we run factor spanning regressions. The collinearity of *SMB* and *IML* has implications for our asset pricing tests: It tends to weaken the individual impact of the other. Panel B also shows that *IML* is positively correlated with

HML, although this correlation is much lower. This is not surprisingly either, as size enters in the denominator of *B/M*. A high *B/M* firm has a low value of market equity with respect to its book value of equity. As market equity and size are synonymous, a low market equity translates into small size. The reason for why size and illiquidity are related, has already been noted. *IML* and *CMA* are positively correlated, while *IML* and *RMW* are negatively correlated, although the latter correlation is very low. *IML*'s correlation with the market is negative. When the economy performs badly and investors incur losses, they reduce their positions. This drives prices away from fundamentals, thereby reducing market liquidity. As liquidity dries up, investors demand a higher premium in order to be compensated for bearing liquidity risk (Acharya & Pedersen, 2005). This will in particular affect illiquid stocks, since illiquid stocks are highly exposed to liquidity risk as shown by Acharya and Pedersen (2005). They argue that these stocks exhibit a communality in liquidity with market liquidity, a lot of return sensitivity to market liquidity, and a lot of liquidity sensitivity to market returns. This induces a "flight to liquidity": In times of liquidity crises, illiquid assets suffer the most. Finally, Table 3 also shows that no factor is perfectly correlated with any of the other factors. Therefore, no factor can be substituted for any of the others.

Table 3
Summary statistics for factor returns

The period stretches from January 1964 to December 2019, resulting in a total of 672 months. *MKT* is the excess return on the market over the one-month Treasury bill rate. *IML* is the illiquidity factor; *SMB*, *HML*, *RMW* and *CMA* are the FF factors. Panel A shows the average monthly percent returns (Mean), the standard deviation of monthly returns (Std), the *t*-statistics for the average returns and the Sharpe ratios (Sharpe). In Panel A, *SMB* shown is the traditional three-factor *SMB*. Panel B shows the Pearson correlations between the different factors. In Panel B, *SMB 3F* is the three-factor model *SMB*, whereas *SMB 5F* denotes the five-factor model *SMB*.

Panel A: Averages, standard deviations, *t*-statistics and Sharpe ratios for monthly returns

	MKT	*IML*	*SMB*	*HML*	*RMW*	*CMA*
Mean	0.54	0.50	0.20	0.30	0.26	0.28
Std	4.40	3.12	3.05	2.82	2.16	2.00
(*t*-statistic)	(3.16)	(4.15)	(1.74)	(2.79)	(3.08)	(3.63)
Sharpe	0.12	0.16	0.07	0.11	0.12	0.14

Panel B: Correlations between different factors

	MKT	*IML*	*SMB 3F*	*SMB 5F*	*HML*	*RMW*	*CMA*
MKT	1.00						
IML	-0.24	1.00					
SMB 3F	0.30	0.58	1.00				
SMB 5F	0.28	0.62	0.99	1.00			
HML	-0.25	0.26	-0.19	-0.06	1.00		
RMW	-0.23	-0.11	-0.40	-0.35	0.06	1.00	
CMA	-0.39	0.27	-0.17	-0.11	0.70	-0.03	1.00

3.2 Regressions of each factor on the other ones

So far, we have shown that *IML* is an intuitive and powerful factor that has a significant *alpha*, relative to the FF factor models. It is now interesting to see how *IML* affects the *alphas* and interpretations of standard factors. Table 4 shows regressions of each of the factors on the other ones for the period 1964 to 2019. For each of the FF factors, we run two regressions, one including *IML* and one without. Panel A reports regression results for regressions without *IML*, Panel B for regressions with *IML*. The reason for why we run two regressions for each factor is that this way one can see the effects of adding *IML*. Before we begin our analysis of the impact of adding *IML*, we have to dedicate a few words to the effect of the transition from the FF three-factor model to the FF five-factor model. In untabulated results, we find that *SMB*'s *alpha* is strengthened due to the *profitability* factor, *RMW*. *SMB* has a strong and negative loading on *RMW* (-0.43, $t = 8.39$). As a result, the *alpha* of *SMB* is almost doubled, and it also shows a stronger *t*-statistic. The result is the same, no matter if one uses the three-factor *SMB* or the five-factor *SMB* - if anything, the effect is even stronger for the three-factor *SMB*. We further find that *HML*'s *alpha* is not significant due to the inclusion of *RMW* and *CMA*, just as FF (2015) find for the sample period 1963-2013.

We now turn to discussing the effects of *IML*. *SMB* has a large and positive exposure to *IML* (0.71, $t = 29.53$). When showing that *IML* and *SMB* are highly positively correlated, we argued that the average *SMB* return likely in part is an illiquidity premium. Indeed, this seems to be the case. Before including *IML*, *SMB*'s *alpha* is 0.3% (t = 2.66) per month. After including *IML*, we find a monthly *alpha* of -0.1% (t = -1.37). Arguably, - considering the new hurdle proposed by Harvey, Liu and Zhu (2016) - one might already worry about the significance of *alpha* before the inclusion. Still, after adding *IML*, we find that the *t*-statistic has been extremely lowered. Hence, while we might not have full support to argue that the average return to *SMB* is captured by the exposure of *SMB* to *IML*, we can at least argue that adding *IML* pushes the size effect even more away from existence. Illiquidity as explanation for the size effect has been discussed in other literature before. Small firms offer a compensation for the limited availability of information on them (Barry & Brown, 1984). Based on these findings, Amihud (2002) offers an explanation for why illiquidity can explain the size effect: As illiquidity increases in asymmetric information (Glosten & Milgrom, 1985; Kyle, 1985), illiquidity is higher for small firms. We further examine the relation of firm size and illiquidity in Section 4.

Table 4
Regressions of each factor on the other ones

The sample period stretches from January 1964 to December 2019, totalling 672 months. *MKT* is the return on the market minus the one-month Treasury bill rate, *IML* is the illiquidity factor, *SMB*, *HML*, *RMW* and *CMA* are the FF factors. Returns are in monthly percentages. The *t*-statistics of the intercepts and the coefficients are reported in parentheses. Panel A reports results for regressions without *IML*. Panel B reports results for regressions that include *IML*.

Panel A: Regressions without *IML*

	Dependent variable:				
	SMB	*HML*	*RMW*	*CMA*	*MKT*
Alpha	0.30	-0.04	0.40	0.24	0.80
	(2.66)	(-0.48)	(5.13)	(4.53)	(5.23)
MKT	0.13	0.03	-0.10	-0.11	
	(4.78)	(1.26)	(-5.10)	(-8.67)	
HML	0.07		0.14	0.45	0.09
	(1.27)		(3.63)	(23.86)	(1.26)
SMB		0.04	-0.22	-0.03	0.25
		(1.27)	(-8.39)	(-1.63)	(4.78)
RMW	-0.43	0.14		-0.13	-0.38
	(-8.39)	(3.63)		(-5.16)	(-5.10)
CMA	-0.13	1.01	-0.29		-0.91
	(-1.63)	(23.86)	(-5.16)		(-8.67)
R^2	0.17	0.49	0.17	0.55	0.24

Panel B: Regressions including IML

	SMB	*HML*	*RMW*	*CMA*	*MKT*	*IML*
Alpha	-0.10	-0.09	0.36	0.21	0.90	0.32
	(-1.37)	(-1.10)	(4.58)	(3.88)	(6.56)	(4.14)
MKT	0.24	0.06	-0.07	-0.09		-0.25
	(12.92)	(2.92)	(-3.25)	(-5.90)		(-13.11)
IML	0.71	0.16	0.11	0.09	-0.81	
	(29.53)	(4.02)	(2.68)	(3.36)	(-13.11)	
HML	-0.08		0.12	0.43	0.19	0.15
	(-2.17)		(3.15)	(21.78)	(2.92)	(4.02)
SMB		-0.09	-0.30	-0.10	0.85	0.79
		(-2.17)	(-7.53)	(-3.61)	(12.92)	(29.53)
RMW	-0.26	0.12		-0.14	-0.22	0.10
	(-7.53)	(3.15)		(-5.45)	(-3.25)	(2.68)
CMA	-0.19	0.96	-0.30		-0.58	0.19
	(-3.61)	(21.78)	(-5.45)		(-5.90)	(3.36)
R^2	0.64	0.51	0.18	0.56	0.39	0.62

MKT loads negatively on *IML*, which implies that controlling for illiquidity increases the *alpha* of the market. The rest of the factors load positively on *IML*. Consequently, their respective *alphas* are reduced due to including *IML*, although the effect is not large for neither of them. Clearly, the largest loadings on *IML* are found for *SMB* and *MKT*. After having examined the factors in greater detail, we now present our findings from the formal asset pricing tests.

3.3 Results for formal asset pricing tests

The FF three-factor model and even the FF five-factor model fail to fully explain the cross-section of expected returns. Motivated by this fact and the significant *alpha* we find when regressing *IML* on the FF three and five factors, we seek to test whether the FF factor models fare better if we augment them by *IML*. To be precise, we consider four different models: (i) The FF three-factor model; (ii) the FF three-factor model augmented by *IML*; (iii) the FF five-factor model; and (iv) the FF five-factor model augmented by *IML*. Hence, the explanatory returns include - depending on which factor model we consider - *MKT*, *SMB*, *HML*, *RMW*, *CMA*, and *IML*. We follow FF (1993, 2015) and run time-series regressions. Tests of the FF three-factor model augmented by *IML* focus on the following time-series regression,

$$r_{i,t} - r_{F,t} = \alpha_i + b_i MKT_t + s_i SMB_t + h_i HML_t + i_i IML_t + \varepsilon_{i,t}. \quad (3)$$

When considering the FF three-factor model, we run this regression without including *IML*. The same holds true for the FF five-factor model, whose *IML*-augmented version focuses on the following time-series regression,

$$r_{i,t} - r_{F,t} = \alpha_i + b_i MKT_t + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + i_i IML_t + \varepsilon_{i,t}. \quad (4)$$

The dependent variables are returns on portfolios formed to produce large spreads of the average return to be explained. In order to avoid potential arbitrariness or data mining concerns, we refrain from constructing our own test portfolios. Neither do we consider illiquidity sorted portfolios as test portfolios for evaluating the models. We are not interested in testing whether *IML* can explain the returns on illiquidity sorted portfolios – returns on portfolios it was constructed to explain. Even when these were constructed on a finer grid than *IML*, *IML* would still play a home game. Hence, finding that *IML* can explain the returns on other portfolios formed on illiquidity, might not be a very surprising result. We seek a bigger challenge and are more interested in whether *IML* helps in explaining the cross-section of expected returns on portfolios sorted on other known anomalies. We only consider portfolios from multivariate sorts, as value-weighted portfolios from univariate sorts are typically dominated by big stocks. We are – in other words - further increasing the challenge, as the main problems of asset pricing are in small stocks (FF, 1993, 2015). The portfolios used are well-known FF test portfolios as well as a variety of other portfolios sorted on anomaly variables, all downloaded from Ken French's data library. The portfolios include: the 25 and 100 portfolios sorted on *Size* and *Book to Market* (*B/M*), the 25 and 100 portfolios sorted on *Size* and *Operating Profitability* (*OP*), the 25 and 100 portfolios sorted on *Size* and *Investment* (*INV*), the 25 portfolios sorted on *B/M* and *OP*, the 25 portfolios sorted on *B/M* and *INV*, the 25 portfolios sorted on *OP* and *INV*, the 32 portfolios sorted on *Size*, *B/M* and *OP*, the 32 portfolios sorted on *Size*, *B/M* and *INV*, the 32 portfolios sorted on *Size*, *OP* and *INV*, the 25 portfolios sorted on *Size* and *Daily Variance*, the 25 portfolios sorted on *Size* and *Residual Daily Variance*, the 25 portfolios sorted on *Size* and *Accruals*, the 25 portfolios sorted on *Size* and *Market Beta* and the 25 portfolios sorted on *Size* and *Net Share Issues*.

We show the absolute performance of the models considered, i.e. whether a model is rejected. If all models are rejected, we are interested in identifying the model that yields the best – albeit imperfect – results. Moreover, we are not only interested in whether the IML-augmented models are a complete description of average excess returns. We are even more interested in examining whether IML – despite being still rejected - improves the FF models. Hence, we additionally discuss the relative performance. In order to judge the absolute performance, we use the F-test of Gibbons, Ross and Shanken (1989). If an asset pricing model completely captures expected returns, the intercept obtained in a regression of an asset's excess return on factor returns should be indistinguishable from zero. The *GRS* statistic tests this hypothesis for combinations of test portfolios and factors. The *GRS* statistic in combination with summary statistics for regression intercepts is used for judging the relative performance and the improvement gained by adding IML. The first such summary statistic is the average absolute intercept, $A|\alpha_i|$. The smaller this value, the better the model does. The other summary statistic is $A|\alpha_i|/A|\overline{r_i}|$, the average absolute intercept over the average absolute value of $\overline{r_i}$. This ratio estimates the proportion of the cross-section of expected returns left unexplained by the model tested. The numerator measures the dispersion of the intercepts produced by a combination of test portfolios and factors; the denominator measures the dispersion of expected returns for given test portfolios. $\overline{r_i}$ is portfolio i's deviation from the cross-section average and is calculated as $\overline{R_i} - \overline{R}$, where $\overline{R_i}$ denotes the time-series average excess return on portfolio *i* and

$\bar{R}$ is the cross-section average of $\bar{R}_i$. Again, the smaller $A|\alpha_i|/A|\overline{r_i}|$, the better the model does in explaining the cross-section of expected returns.

Table 5 presents summary statistics for the formal asset pricing tests. Reported are the *GRS* statistic, *f(GRS)*, its *p-value*, *p(GRS)*, $A|\alpha_i|$, $A|\alpha_i|/A|\overline{r_i}|$, and the average R^2 ($A(R^2)$). In order to judge the contribution of *IML*, we report them for each of the original FF models and their *IML*-augmented versions. For completeness, we also report the results for a model made up of only *MKT* and *IML*, although we do not expect exactly this model to fare best. The results are shown for all test portfolios considered.

The *GRS* test rejects all, but five, of the model-test portfolios combinations. For the 25 *OP-INV* portfolios (Panel F) and the 25 *B/M-OP* portfolios (Panel D), both the five-factor model and the *IML*-augmented five-factor model are not rejected. However, we must note that in both cases the five-factor model does better, as it yields a lower *GRS statistic*, $A|\alpha_i|$ *and* $A|\alpha_i|/A|\overline{r_i}|$. For the 25 *Size-Market Beta* portfolios (Panel Q), the three-factor model that includes *IML* is not rejected. The second-best model for this test portfolio is the *IML*-augmented five-factor model, so the models including *IML* are clearly preferred over the ones without. For this test portfolio we - not surprisingly - see big improvements when moving from the FF models to *IML*-augmented versions of them. $A|\alpha_i|$ decreases by 6.8 basis points when including *IML* in the three-factor model and by 2.9 basis points when adding *IML* to the five-factor model. The remaining model-test portfolios combinations are all rejected by the *GRS* statistic. Hence, what follows now is a discussion about the relative performance of the models for each of the test portfolios.

For the 25 *B/M-INV* portfolios (Panel E) we also clearly prefer an *IML*-augmented model over the original FF factor models. The *IML*-augmented three-factor model fares best, since it shows the lowest *GRS* statistic, $A|\alpha_i|$ *and* $A|\alpha_i|/A|\overline{r_i}|$. Second comes the five-factor model including *IML*. For some of the remaining test portfolios (25 *Size-B/M* (Panel A), 25 *Size-INV* (Panel C), 100 *Size-B/M* (Panel J), 100 *Size-INV* (Panel L), 32 *Size-B/M-INV* (Panel H), 32 *Size-OP-INV* (Panel I), 25 *Size-Daily Variance* (Panel O), and 25 *Size-Residual Daily Variance* (Panel P)) an interesting pattern emerges: The *IML*-augmented three-factor model fares better than the FF three-factor model, the FF five-factor model does a better job than the *IML*-augmented three-factor model and the *IML*-augmented five-factor model outperforms the original five-factor model. The improvements are larger, though, when switching from the FF three-factor model to its *IML*-augmented version than when adding *IML* to the five-factor model. In terms of $A|\alpha_i|$, the biggest reduction when adding *IML* to the FF three-factor model is found for the 25 *Size-Daily Variance* portfolios. It equals 8.5 basis points. In contrast, the biggest gains in moving from the five-factor model to its *IML*-augmented version is a reduction by only 2.7 basis points. It is yielded by the 25 *Size-Residual Daily Variance* portfolios. For the other portfolios, those numbers are a bit smaller. The improvement for the *IML*-augmented three-factor model ranges from 2.2 basis points for the 32 *Size-OP-INV* portfolios to 1.2 basis points for the 25 *Size-INV* portfolios. The gains from adding *IML* to the FF five-factor model range from a reduction by 2.4 basis points for the 25 *Size-Daily Variance* portfolios to a reduction by 0.2 basis points for the 32 *Size-OP-INV* and the 25 *Size-B/M* portfolios.

We observe a change in the pattern, however, when the sorting variables for the test portfolios combine *Size* and *OP*. Then the pattern looks as follows: For the 25 *Size-OP* portfolios (Panel B) and the 32 *S-B/M-OP* portfolios (Panel G), the five-factor model outperforms the *IML*-augmented five-factor model and yields the best relative performance. The *IML*-augmented three-factor model, on the other hand, still fares better than the FF three-factor model. We inspected whether the improvement here is lower than that found for the other test portfolios, but we find no evidence of this. For the 100 *Size-OP* portfolios (Panel K), the FF five-factor model and its *IML*-augmented version are almost equal in performance. Both models yield the same *GRS* statistic, the difference in $A|\alpha_i|/A|\overline{r_i}|$ is one basis point and the difference in $A|\alpha_i|$ is less than one basis point.

When adding *INV* to the sorting variables -resulting in the 32 *Size-OP-INV* portfolios (Panel I) - the *IML*-augmented five-factor model again does a better job in explaining average excess return, albeit the improvements are very small. Obviously, the combination of *OP* and *INV* are favourable for *IML*, as we also see the *IML*-augmented five-factor model not being rejected for the 25 *OP-INV* portfolios.

The results for the 25 *Size-Net Share Issues* portfolios (Panel N) look the same as for the 25 *Size-OP* and the 32 *Size-B/M-OP* portfolios. For the 25 *Size-Accruals* portfolios (Panel M) both *IML*-augmented versions fare worse than their original counterparts.

Table 5
Summary statistics for asset pricing tests

The sample period is from January 1964 to December 2019, 672 months in total. Tested is the ability of different factor models to explain monthly average excess returns on a variety of test portfolios. Considered are the FF three-factor (FF 3F) and the FF five-factor (FF 5F) model, as well as *IML*-augmented versions of them. Shown are results for the 25 *Size-B/M* portfolios (Panel A), the 25 *Size-OP* portfolios (Panel B), the 25 *Size-INV* portfolios (Panel C), the 25 *B/M-OP* portfolios (Panel D), the 25 *B/M-INV* portfolios (Panel E), the 25 *OP-INV* portfolios (Panel F), the 32 *Size-B/M-OP* portfolios (Panel G), the 32 *Size-B/M-INV* portfolios (Panel H), the 32 *Size-OP-INV* portfolios (Panel I), the 100 *Size-B/M* portfolios (Panel J), the 100 *Size-OP* portfolios (Panel K), the 100 *Size-INV* portfolios (Panel L), the 25 *Size-Accruals* portfolios (Panel M), the 25 *Size-Net Share Issues* portfolios (Panel N), the 25 *Size-Daily Variance* portfolios (Panel O), the 25 *Size-Daily Residual Variance* portfolios (Panel P), and the 25 *Size-Market Beta* portfolios (Panel Q). For each set of test portfolios, the table shows the different models used. The first row shows results for a model that uses only *MKT* and *IML*. Beginning from the second row for each panel, if *IML* is written next to a model, it means that *IML* is added to the factors used in the model above. Also shown are the *GRS* statistic, *f(GRS)*, testing whether the expected values of all intercept estimates are jointly zero, its *p-value*, *p(GRS)*, the average absolute value of the intercept, $A|\alpha_i|$, $A|\alpha_i|/A|\overline{r_i}|$, the average absolute value of the intercept over the average absolute value of $\overline{r_i}$, which is calculated as the average return on portfolio *i* minus the average of the portfolio returns, and the average R^2 ($A(R^2)$).

Panel A: 25 *Size-B/M* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|$	$A(R^2)$
MKT + *IML*	3.88	0.00	0.172	1.10	0.82
FF 3F	3.87	0.00	0.098	0.63	0.91
FF 3F + *IML*	3.24	0.00	0.082	0.53	0.91
FF 5F	3.14	0.00	0.087	0.56	0.92
FF 5F + *IML*	2.87	0.00	0.085	0.54	0.92

Panel B: 25 *Size-OP* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|$	$A(R^2)$
MKT + *IML*	3.26	0.00	0.166	1.22	0.86
FF 3F	3.15	0.00	0.108	0.79	0.90
FF 3F + *IML*	3.30	0.00	0.089	0.65	0.90
FF 5F	2.47	0.00	0.066	0.48	0.93
FF 5F + *IML*	2.78	0.00	0.079	0.58	0.93

Panel C: 25 *Size-INV* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|$	$A(R^2)$
MKT + *IML*	4.80	0.00	0.169	1.13	0.85
FF 3F	4.54	0.00	0.111	0.74	0.92
FF 3F + *IML*	4.18	0.00	0.099	0.66	0.92
FF 5F	3.33	0.00	0.083	0.55	0.93
FF 5F + *IML*	3.31	0.00	0.080	0.53	0.93

Panel D: 25 *BM-OP* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|\|$	$A(R^2)$
MKT + *IML*	1.80	0.01	0.177	0.99	0.72
FF 3F	1.89	0.01	0.125	0.69	0.78
FF 3F + *IML*	1.57	0.04	0.112	0.62	0.78
FF 5F	1.25	0.19	0.097	0.54	0.80
FF 5F + *IML*	1.32	0.14	0.100	0.56	0.80

Panel E: 25 *BM-INV* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|\|$	$A(R^2)$
MKT + *IML*	1.83	0.01	0.119	1.18	0.75
FF 3F	2.16	0.00	0.101	1.00	0.81
FF 3F + *IML*	1.84	0.01	0.089	0.88	0.81
FF 5F	2.05	0.00	0.098	0.97	0.82
FF 5F + *IML*	1.94	0.00	0.095	0.95	0.82

Panel F: 25 *OP-INV* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|\|$	$A(R^2)$
MKT + *IML*	3.01	0.00	0.186	1.57	0.78
FF 3F	2.54	0.00	0.152	1.27	0.80
FF 3F + *IML*	2.50	0.00	0.138	1.16	0.80
FF 5F	1.27	0.17	0.070	0.59	0.83
FF 5F + *IML*	1.52	0.05	0.078	0.65	0.83

Panel G: 32 *Size-B/M-OP* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|$	$A(R^2)$
MKT + *IML*	2.48	0.00	0.203	0.99	0.75
FF 3F	2.33	0.00	0.133	0.65	0.83
FF 3F + *IML*	2.19	0.00	0.116	0.57	0.83
FF 5F	1.75	0.01	0.102	0.50	0.85
FF 5F + *IML*	1.80	0.00	0.106	0.52	0.85

Panel H: 32 *Size-B/M-INV* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|$	$A(R^2)$
MKT + *IML*	3.08	0.00	0.146	0.89	0.79
FF 3F	3.27	0.00	0.124	0.76	0.87
FF 3F + *IML*	2.87	0.00	0.109	0.67	0.87
FF 5F	2.44	0.00	0.094	0.57	0.88
FF 5F + *IML*	2.29	0.00	0.090	0.55	0.88

Table 5 - continued

Panel I: 32 *Size-OP-INV* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|$	$A(R^2)$
MKT + *IML*	3.89	0.00	0.207	1.07	0.81
FF 3F	3.94	0.00	0.169	0.88	0.86
FF 3F + *IML*	3.51	0.00	0.147	0.76	0.86
FF 5F	2.62	0.00	0.090	0.47	0.89
FF 5F + *IML*	2.48	0.00	0.088	0.46	0.89

Panel J: 100 *Size-B/M* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|$	$A(R^2)$
MKT + *IML*	2.61	0.00	0.332	0.95	0.70
FF 3F	2.65	0.00	0.271	0.78	0.78
FF 3F + *IML*	2.46	0.00	0.261	0.75	0.78
FF 5F	2.33	0.00	0.257	0.74	0.79
FF 5F + *IML*	2.28	0.00	0.254	0.73	0.79

Panel K: 100 *Size-OP* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|$	$A(R^2)$
MKT + *IML*	2.03	0.00	0.181	1.11	0.75
FF 3F	1.96	0.00	0.145	0.89	0.79
FF 3F + *IML*	1.90	0.00	0.132	0.81	0.79
FF 5F	1.75	0.00	0.119	0.73	0.81
FF 5F + *IML*	1.75	0.00	0.121	0.74	0.81

Panel L: 100 *Size-INV* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|$	$A(R^2)$
MKT + *IML*	3.12	0.00	0.259	1.03	0.74
FF 3F	3.27	0.00	0.208	0.83	0.80
FF 3F + *IML*	3.05	0.00	0.201	0.80	0.80
FF 5F	2.83	0.00	0.193	0.77	0.81
FF 5F + *IML*	2.73	0.00	0.196	0.78	0.81

Panel M: 25 *Size-Accruals* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|$	$A(R^2)$
MKT + *IML*	3.97	0.00	0.188	1.63	0.85
FF 3F	3.50	0.00	0.108	0.94	0.91
FF 3F + *IML*	3.65	0.00	0.120	1.04	0.91
FF 5F	3.72	0.00	0.117	1.01	0.91
FF 5F + *IML*	3.83	0.00	0.130	1.13	0.92

Panel N: 25 *Size-Net Share Issues* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|$	$A(R^2)$
MKT + *IML*	4.73	0.00	0.199	1.14	0.81
FF 3F	4.43	0.00	0.136	0.78	0.87
FF 3F + *IML*	4.21	0.00	0.126	0.72	0.87
FF 5F	3.36	0.00	0.103	0.59	0.88
FF 5F + *IML*	3.45	0.00	0.110	0.63	0.88

Panel O: 25 *Size-DailyVariance* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|$	$A(R^2)$
MKT + *IML*	5.08	0.00	0.270	1.21	0.82
FF 3F	5.28	0.00	0.237	1.06	0.87
FF 3F + *IML*	4.44	0.00	0.152	0.68	0.88
FF 5F	4.35	0.00	0.140	0.63	0.89
FF 5F + *IML*	3.88	0.00	0.116	0.52	0.90

Panel P: 25 *Size-Residual Daily Variance* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|$	$A(R^2)$
MKT + *IML*	5.70	0.00	0.270	1.13	0.83
FF 3F	6.05	0.00	0.237	0.99	0.88
FF 3F + *IML*	5.21	0.00	0.163	0.68	0.89
FF 5F	4.79	0.00	0.129	0.54	0.90
FF 5F + *IML*	4.35	0.00	0.102	0.43	0.91

Panel Q: 25 *Size-Market Beta* portfolios

	f(GRS)	*p(GRS)*	$A\|\alpha_i\|$	$A\|\alpha_i\|/A\|\overline{r_i}\|$	$A(R^2)$
MKT + *IML*	2.08	0.00	0.167	1.45	0.83
FF 3F	2.09	0.00	0.134	1.16	0.88
FF 3F + *IML*	1.51	0.05	0.066	0.58	0.89
FF 5F	1.97	0.00	0.087	0.75	0.89
FF 5F + *IML*	1.66	0.02	0.058	0.50	0.90

3.3.1 Regression details

In order to gain further insides into the performance of our models, we now shortly examine regression results in a more detailed manner. Table 6 to Table 11 present average values of the intercepts and the corresponding *t*-statistics for a selection of model-test portfolio combinations.[4] We present results for the 25 *Size- Market Beta* portfolios, for which the *IML*-augmented three-factor model is not rejected. We further present results for the 25 *Size-B/M* portfolios, the 25 *Size-INV* portfolios, and the 25 *Size-Daily Variance* portfolios, which represent the group of test portfolios, for which we find an improvement when adding *IML* to the FF three-factor model and the FF five-factor model, respectively, but for which we still prefer the FF five-factor model over the *IML*-augmented three-factor model. For these test portfolios, we always show the results for the winning *IML*-version, as well as the corresponding without-*IML* FF model. In doing so, we are not only able to report that we find improvements, but to actually exactly locate them. Hence, we will see which of the problematic *alphas* are most affected by adding *IML*.

However, as we perceive it important to be unbiased in presenting our findings, we also discuss the results for test portfolios, for which we not necessarily find improvements from adding *IML*. We show results for the 25 *Size-OP* portfolios that represent the group of portfolios, for which the *IML*-augmented FF five-factor model fares worse than the FF five-factor model. Hence, for this group of test portfolios, we report results for the outperformed *IML*-augmented five-factor model and the FF five-factor model. Additionally, we report the intercepts and corresponding *t*-statistics for the 25 *Size-Accruals* portfolios, for which adding *IML* brings about worse performance for both, the FF three-factor and the FF five-factor model.

[4] A more detailed table presenting also the coefficients and their *t*-statistics is available upon request. Also available upon request are the results for the remaining model-test portfolio combinations.

Table 6
Regressions for 25 *Size-B/M* portfolios

January 1964 to December 2019, 672 months. The dependent variables of the time-series regressions are the monthly excess returns on the 25 *Size-B/M* portfolios. The independent variables are *MKT*, *SMB*, *HML*, *RMW*, *CMA*, and *IML*. Results in bold indicate significance at the conventional 5% level. Panel A shows the five-factor intercepts and *t*-statistics produced by *MKT*, *SMB*, *HML*, *RMW*, and *CMA*. Panel B shows the intercepts and *t*-statistics for the *IML*-augmented regression equation,

$$r_{i,t} - r_{F,t} = \alpha_i + b_i MKT_t + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + i_i IML_t + \varepsilon_{i,t}.$$

Panel A: FF five-factor intercepts

α

	L	2	3	4	H
S	**-0.28**	**0.14**	-0.02	**0.17**	**0.12**
2	-0.07	-0.02	0.01	0.01	-0.05
3	0.03	0.02	-0.08	0.00	-0.01
4	**0.19**	**-0.17**	-0.13	0.03	-0.11
B	**0.11**	-0.09	-0.07	**-0.24**	0.00

$t(\alpha)$

	L	2	3	4	H
S	-3.23	2.16	-0.45	3.23	2.02
2	-1.15	-0.39	0.24	0.28	-0.92
3	0.47	0.34	-1.28	0.05	-0.18
4	3.10	-2.59	-1.89	0.50	-1.40
B	2.69	-1.67	-1.06	-3.86	-0.01

Panel B: FF five-factor + *IML* intercepts

α

	L	2	3	4	H
S	**-0.24**	**0.14**	-0.05	**0.13**	0.08
2	-0.01	-0.01	0.00	-0.02	-0.04
3	0.07	0.04	-0.06	-0.01	0.03
4	**0.23**	**-0.15**	-0.12	0.04	-0.06
B	**0.10**	-0.08	-0.09	**-0.23**	0.08

$t(\alpha)$

	L	2	3	4	H
S	-2.82	2.11	-1.00	2.53	1.41
2	-0.22	-0.13	0.09	-0.40	-0.76
3	1.19	0.58	-1.03	-0.11	0.36
4	3.89	-2.31	-1.71	0.57	-0.73
B	2.37	-1.53	-1.33	-3.59	0.89

The first results we discuss are the ones for the 25 *Size-B/M* portfolios. Table 6 shows the intercepts and *t*-statistics for each of them. Panel A reports the FF five-factor model results. The biggest problem for the model are the usual suspects – small, low *B/M* firms. Even for the five-factor model, this portfolio remains problematic, as it still exhibits a large negative monthly *alpha* of – 0.28% (*t* = - 3.23). Adding *IML* is most successful in reducing the *alphas* of small firms and high *B/M* firms, though the changes are not that big in economic terms. If we apply the higher threshold for statistical significance as proposed by Harvey, Liu and Zhu (2016), *IML* actually removes the *alpha* produced by small, low *B/M* firms.

Table 7
Regressions for 25 *Size-INV* portfolios

January 1964 to December 2019, 672 months. The dependent variables in the time-series regressions are the monthly excess returns on the 25 *Size-INV* portfolios. The independent variables are *MKT*, *SMB*, *HML*, *RMW*, *CMA*, and *IML*. Results in bold indicate significance at the conventional 5% level. Panel A shows the five-factor intercepts and *t*-statistics produced by *MKT*, *SMB*, *HML*, *RMW*, and *CMA*. Panel B shows the intercepts and *t*-statistics for the *IML*-augmented regression equation,

$$r_{i,t} - r_{F,t} = \alpha_i + b_i MKT_t + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + i_i IML_t + \varepsilon_{i,t}.$$

Panel A: FF five-factor intercepts

	α					$t(\alpha)$				
	L	2	3	4	H	L	2	3	4	H
S	**0.17**	**0.12**	**0.13**	0.04	**-0.32**	2.26	2.33	2.32	0.62	-5.04
2	-0.03	0.02	0.09	0.03	**-0.13**	-0.56	0.31	1.77	0.57	-2.68
3	0.02	**0.11**	-0.01	0.06	-0.03	0.34	1.96	-0.11	1.03	-0.49
4	-0.10	-0.07	0.03	0.08	0.12	-1.46	-1.08	0.56	1.35	1.84
B	-0.05	-0.05	-0.06	0.02	**0.19**	-0.73	-1.08	-1.38	0.42	3.48

Panel B: FF five-factor + *IML* intercepts

	α					$t(\alpha)$				
	L	2	3	4	H	L	2	3	4	H
S	**0.17**	0.08	0.08	0.02	**-0.31**	2.24	1.60	1.36	0.30	-4.81
2	0.00	0.00	0.08	0.03	**-0.11**	0.00	0.01	1.55	0.54	-2.16
3	0.10	**0.12**	-0.02	0.07	0.01	1.48	2.09	-0.33	1.23	0.17
4	-0.07	-0.06	0.02	0.10	**0.20**	-1.04	-1.05	0.36	1.62	3.13
B	-0.01	-0.05	-0.06	0.02	**0.20**	-0.14	-1.13	-1.51	0.45	3.67

Next, we discuss results for the 25 *Size-INV* portfolios. The intercepts and *t*-statistics for the FF five-factor model and the *IML*-augmented version of it are shown in Table 7. Most of the problematic *alphas* for the FF five-factor model are located in the smallest size group. In addition, high investment firms pose a challenge. The largest *alpha* in absolute terms is the negative *alpha* found for the portfolio that is formed as the intersection of the two. While we see some improvements for the microcap portfolios, adding *IML* does not help in explaining average excess returns for the high investment portfolios. The most problematic *alpha* is hardly reduced in economic terms. FF (2015) find that low *B/M* firms are high investment firms. Earlier, we have argued that low *B/M* stocks are considered liquid. Therefore, it is reasonable to think of high investment stocks as the liquid stocks. This could explain why *IML* does not bring about an improvement for the high investment portfolios.

Table 8
Regressions for 25 *Size-Daily Variance* portfolios

January 1964 to December 2019, 672 months. The dependent variables in the time-series regressions are the monthly excess returns on the 25 *Size-Daily Variance* portfolios. The independent variables are *MKT*, *SMB*, *HML*, *RMW*, *CMA*, and *IML*. Results in bold indicate significance at the conventional 5% level. Panel A shows the five-factor intercepts and *t*-statistics produced by *MKT*, *SMB*, *HML*, *RMW*, and *CMA*. Panel B shows the intercepts and *t*-statistics for the *IML*-augmented regression equation,

$$r_{i,t} - r_{F,t} = \alpha_i + b_i MKT_t + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + i_i IML_t + \varepsilon_{i,t}.$$

Panel A: FF five-factor intercepts

α

	L	2	3	4	H
S	**0.29**	**0.16**	0.05	-0.17	**-0.83**
2	**0.16**	0.06	0.04	**-0.15**	**-0.44**
3	0.07	-0.02	0.01	**-0.15**	**-0.20**
4	0.06	-0.02	-0.07	-0.11	-0.08
B	-0.03	-0.03	-0.08	-0.11	0.10

$t(\alpha)$

	L	2	3	4	H
S	4.70	2.46	0.67	-1.78	-5.47
2	2.74	1.03	0.56	-2.20	-4.30
3	1.10	-0.42	0.21	-2.15	-2.19
4	0.77	-0.34	-1.14	-1.52	-0.86
B	-0.39	-0.54	-1.61	-1.84	1.10

Panel B: FF five-factor + *IML* intercepts

α

	L	2	3	4	H
S	**0.21**	0.10	0.01	-0.15	**-0.74**
2	0.07	0.01	0.02	-0.12	**-0.30**
3	-0.02	-0.06	0.01	-0.10	-0.05
4	-0.05	-0.07	-0.07	-0.06	0.11
B	-0.10	-0.05	-0.10	-0.05	**0.26**

$t(\alpha)$

	L	2	3	4	H
S	3.58	1.48	0.13	-1.62	-4.84
2	1.32	0.20	0.27	-1.72	-3.06
3	-0.40	-1.04	0.12	-1.41	-0.58
4	-0.70	-1.10	-1.09	-0.75	1.28
B	-1.63	-1.06	-1.86	-0.93	3.13

Results for the 32 and 100 portfolios are not discussed here, as they confirm the results for the 25 portfolios. We rather discuss in more detail the results for portfolios formed on a different second variable, namely the 25 *Size-Daily Variance* portfolios. Intercepts and *t*-statistics are shown in Table 8. The FF five-factor model leaves statistically significant *alphas* for small to medium sized firms. The largest *alphas* in absolute terms are the negative *alphas* found in the high variance portfolios. The *alphas* showing up in the highest variance group and the *alpha* found for the portfolio formed as the intersection of small size and low variance remain problematic also for *IML*. However, *IML* reduces them considerably. The reduction ranges from 8.0 basis points for the microcap, low variance portfolio to 14.0 basis points for the portfolio of second smallest, high variance firms. This is one of the largest reductions in *alphas* we find in our asset pricing tests.

Table 9
Regressions for 25 *Size-OP* portfolios

January 1964 to December 2019, 672 months. The dependent variables in the time-series regressions are the monthly excess returns on the 25 *Size-OP* portfolios. The independent variables are *MKT*, *SMB*, *HML*, *RMW*, *CMA*, and *IML*. Results in bold indicate significance at the conventional 5% level. Panel A shows the five-factor intercepts and *t*-statistics produced by *MKT*, *SMB*, *HML*, *RMW*, and *CMA*. Panel B shows the intercepts and *t*-statistics for the *IML*-augmented regression equation,

$$r_{i,t} - r_{F,t} = \alpha_i + b_i MKT_t + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + i_i IML_t + \varepsilon_{i,t}.$$

Panel A: FF five-factor intercepts

	α					$t(\alpha)$				
	L	2	3	4	H	L	2	3	4	H
S	-0.10	0.07	-0.05	0.00	**-0.22**	-1.33	1.24	-0.90	-0.06	-3.22
2	-0.01	-0.08	-0.05	**-0.11**	-0.04	-0.15	-1.41	-1.04	-2.00	-0.72
3	0.12	-0.01	-0.02	-0.09	0.00	1.68	-0.15	-0.47	-1.59	0.01
4	0.13	0.06	-0.08	-0.06	0.03	1.68	0.93	-1.31	-1.01	0.52
B	0.05	**-0.12**	0.06	-0.02	**0.08**	0.69	-2.12	1.06	-0.42	1.98

Panel B: FF five-factor + *IML* intercepts

	α					$t(\alpha)$				
	L	2	3	4	H	L	2	3	4	H
S	-0.10	0.02	-0.08	-0.05	**-0.20**	-1.43	0.38	-1.43	-0.81	-2.91
2	0.04	-0.09	-0.07	**-0.12**	-0.02	0.62	-1.63	-1.39	-2.30	-0.25
3	**0.14**	0.01	-0.02	-0.07	0.05	2.06	0.15	-0.35	-1.35	0.84
4	**0.18**	0.08	-0.07	-0.05	0.09	2.31	1.17	-1.19	-0.94	1.43
B	0.09	**-0.14**	0.06	-0.02	**0.09**	1.29	-2.55	1.10	-0.50	2.09

Next, we examine the intercepts of test portfolios for which we do not solely find improvements from adding *IML*. Table 9 shows results for the 25 *Size-OP* portfolios. This is one of the test portfolios for which the FF five-factor model performs better than the *IML*-augmented five-factor model. The FF five-factor model leaves four significant *alphas*. The largest in absolute terms is -0.22% per month and is found for the smallest size and highest *OP* portfolio. Given its location in the smallest size group, *IML* helps to slightly reduce it to -0.20% per month. For the other alphas – which are all located in bigger size groups - we find that adding *IML* increases them. Including *IML* even produces two new *alphas*. Both are found in the lowest *OP* group among the bigger firms.

Table 10
Regressions for 25 *Size-Accruals* portfolios

January 1964 to December 2019, 672 months. The dependent variables in the time-series regressions are the monthly excess returns on the 25 *Size-Accruals* portfolios. The independent variables are *MKT*, *SMB*, *HML*, and *IML*. Results in bold indicate significance at the conventional 5% level. Panel A shows the three-factor intercepts and *t*-statistics produced by *MKT*, *SMB* and *HML*. Panel B shows the intercepts and *t*-statistics for the *IML*-augmented regression equation,

$$r_{i,t} - r_{F,t} = \alpha_i + b_i MKT_t + s_i SMB_t + h_i HML_t + i_i IML_t + \varepsilon_{i,t}$$

Panel A: FF three-factor intercepts

	α					$t(\alpha)$				
	L	2	3	4	H	L	2	3	4	H
S	-0.04	0.11	0.00	**0.12**	**-0.30**	-0.53	1.59	0.06	2.03	-4.45
2	-0.03	0.08	0.08	-0.02	**-0.17**	-0.53	1.28	1.30	-0.32	-2.64
3	0.10	**0.13**	**0.19**	0.05	**-0.19**	1.39	1.97	2.94	0.82	-2.38
4	0.09	0.08	0.02	**0.17**	0.00	1.15	1.22	0.35	2.64	0.01
B	**0.24**	**0.16**	0.09	0.05	**-0.20**	3.22	3.03	1.62	0.88	-2.42

Panel B: FF three-factor + *IML* intercepts

	α					$t(\alpha)$				
	L	2	3	4	H	L	2	3	4	H
S	-0.01	0.10	-0.01	0.10	**-0.30**	-0.12	1.50	-0.20	1.58	-4.27
2	0.00	0.09	0.08	-0.03	**-0.16**	-0.03	1.47	1.26	-0.42	-2.40
3	**0.19**	**0.16**	**0.21**	0.07	**-0.17**	2.59	2.45	3.31	1.10	-2.03
4	0.13	0.13	0.03	**0.21**	0.08	1.74	1.94	0.44	3.32	1.07
B	**0.27**	**0.17**	0.06	0.06	**-0.20**	3.56	3.15	1.07	1.01	-2.31

Table 10 shows the results for the 25 *Size-Accruals* portfolios. For this test portfolio, *IML* fares worst. Both *IML*-augmented FF models underperform the original FF models. Since the FF three-factor model performs better on this test portfolio than the FF five-factor model, we report the results for the three-factor model and its *IML*-augmented version. Any rare reduction in the values of *alpha*, is completely offset by the fact that *IML* increases most of the remaining ones. At best, it leaves them unchanged.

Table 11
Regressions for 25 *Size-Market Beta* portfolios

January 1964 to December 2019, 672 months. The dependent variables in the time-series regressions are the monthly excess returns on the 25 *Size-Market Beta* portfolios. The independent variables are *MKT*, *SMB*, *HML*, and *IML*. Results in bold indicate significance at the conventional 5% level. Panel A shows the three-factor intercepts and *t*-statistics produced by *MKT*, *SMB*, and *HML*. Panel B shows the intercepts and *t*-statistics for the *IML*-augmented regression equation,

$$r_{i,t} - r_{F,t} = \alpha_i + b_i MKT_t + s_i SMB_t + h_i HML_t + i_i IML_t + \varepsilon_{i,t}.$$

Panel A: FF three-factor intercepts

	α						$t(\alpha)$				
	L	2	3	4	H		L	2	3	4	H
S	0.11	**0.14**	0.05	0.08	**-0.26**	S	1.73	2.23	0.86	1.39	-3.02
2	0.07	**0.15**	**0.17**	0.01	**-0.28**	2	1.08	2.76	2.81	0.21	-3.78
3	**0.14**	**0.23**	0.10	-0.04	**-0.17**	3	1.97	3.90	1.57	-0.53	-2.03
4	**0.17**	**0.17**	0.07	**-0.16**	-0.11	4	2.26	2.63	1.03	-2.12	-1.09
B	**0.14**	0.08	-0.02	-0.14	**-0.29**	B	2.11	1.56	-0.36	-1.87	-2.24

Panel B: FF three-factor + *IML* intercepts

	α						$t(\alpha)$				
	L	2	3	4	H		L	2	3	4	H
S	0.02	0.05	-0.01	0.04	**-0.21**	S	0.33	0.78	-0.21	0.75	-2.47
2	0.00	0.07	0.11	0.01	**-0.17**	2	-0.03	1.34	1.91	0.12	-2.36
3	0.04	**0.16**	0.08	-0.01	-0.02	3	0.64	2.79	1.34	-0.14	-0.24
4	0.08	0.12	0.05	-0.11	0.08	4	1.07	1.84	0.69	-1.43	0.82
B	0.06	0.04	-0.03	-0.03	-0.05	B	0.88	0.73	-0.49	-0.41	-0.39

Lastly, we turn to the more detailed discussion of results for test portfolios for which we clearly prefer an *IML*-augmented model. Table 11 shows intercepts and *t*-statistics for the 25 *Size-Market Beta* portfolios, for which we find that the *IML*-augmented three-factor model describes average excess returns. We present the results for this model and the FF three-factor model to discuss the contribution of *IML*. The FF three-factor model leaves statistically significant *alphas* across all size and beta portfolios. The largest in absolute terms are the negative intercepts for high beta stocks. *IML* removes almost all of them, even the ones that show up for big firms. It leaves only an *alpha* for medium sized firms of the second lowest beta group and for the highest beta groups in each of the two portfolios of the smallest firms. However, *IML* yields a noticeable reduction in those *alphas* between 11.0 and 5.0 basis points.

To sum the discussion of this section up, we find some evidence in favour of including *IML*. For the majority of test portfolios, the addition of *IML* results in improved model performance for the FF three-factor model as well as the FF five-factor model. We have to note, though, that in particular for the transition from the FF five-factor model to its *IML*-augmented version, the improvements are rather small for most of the test portfolios. For two test portfolios, we clearly find that an *IML*-augmented model is preferred. For one of them, the *IML*-augmented three-factor model explains the average excess returns. Finally, the combination of *Size* and *OP* as sorting variables lets the FF five-factor model perform better than when adding *IML* to it. A pattern that is only reversed again, once portfolios are also sorted on *INV*. This concludes our formal asset pricing tests. We next turn to exploring the question of how implementable an illiquidity strategy is in practice.

4. The role of shorting, firm size, and time on the illiquidity premium

In Section 2 we have shown that an illiquidity premium exists. In this section, we now analyse whether the illiquidity premium can be captured in practice. In doing so, the paper of Israel and Moskowitz (2013) serves as foundation. We use some of their tests and apply them to the illiquidity anomaly. The tests aim at examining the role of shorting, firm size, and time on the profitability of an illiquidity strategy. First, we will examine how much short selling contributes to the profitability of this investment style. Second, we examine the efficacy of the strategy with regard to firm size. Third, we analyse how its returns and the role of shorting and firm size have varied over time. The answers to these questions are of importance, as the results will provide us with information about the implementation costs of an illiquidity strategy.

4.1 The importance of shorting and time

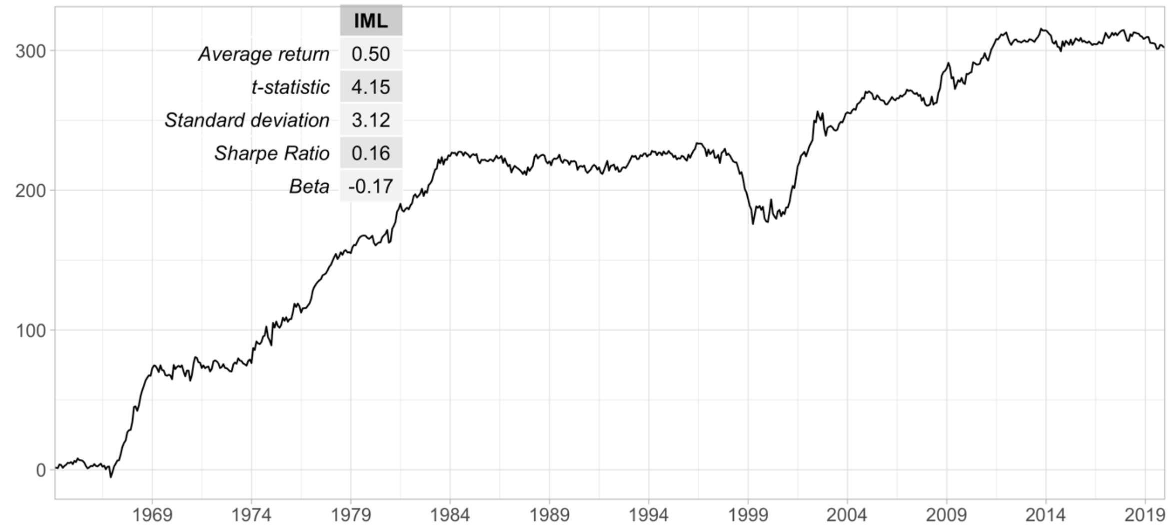
We start by analysing the importance of time and short selling for the profitability of an illiquidity strategy. As a first exercise, we look at the profits to an illiquidity long-short strategy using *IML*.

Figure 2, Panel A plots the cumulative (sum of log) returns of *IML*. Over the 1964 to 2019 sample period, the average monthly return to *IML* equals 0.498% (t = 4.15) with a standard deviation of 3.12%. *IML* yields a monthly Sharpe ratio of 0.16. Figure 2, Panel B plots the cumulative raw excess returns of the long-only component of *IML*, *I*. The long-only illiquidity portfolio generates on average 0.91% per month in excess of T-bills (t = 4.48). The standard deviation of monthly returns is 5.29% and the Sharpe ratio is 0.17. Also shown is the unconditional market beta of *I*. We report it because long-only portfolios are dominated by general stock market exposure. The long-only illiquidity portfolio has a market beta of 0.99.

Figure 2
Cumulative returns to an illiquidity portfolio

Plotted are the monthly cumulative sum of log returns on an illiquidity portfolio from January 1964 to December 2019. Panel A plots the cumulative raw returns of a long-short portfolio. Panel B plots the cumulative raw excess returns to a long-only portfolio.

Panel A: Cumulative raw returns to an illiquidity long-short portfolio



Panel B: Cumulative raw excess returns to an illiquidity long-only portfolio

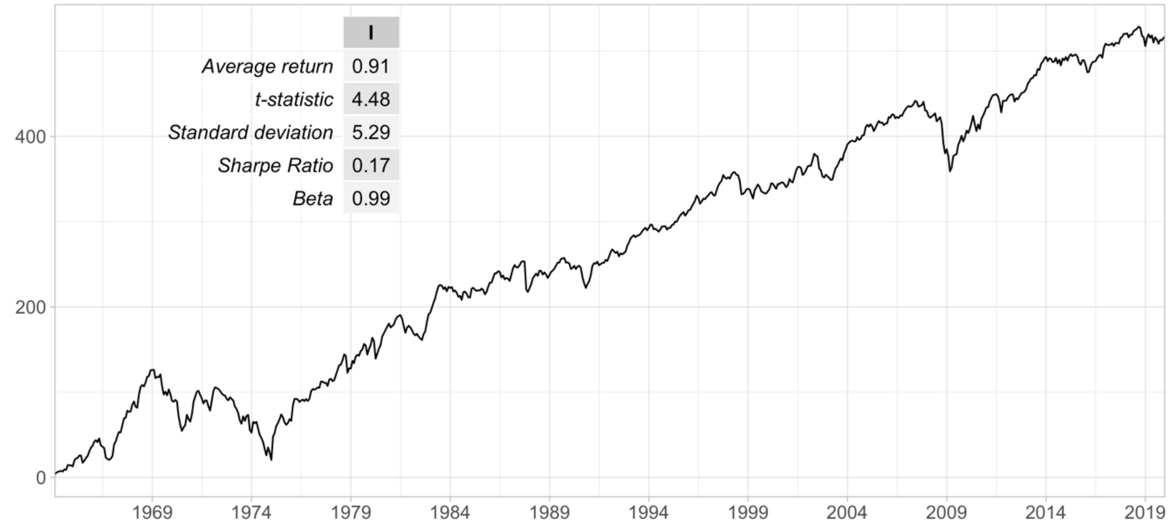


Table 12
Capital Asset Pricing Model (CAPM) *alphas* of illiquidity portfolios over time

Reported are the CAPM *alphas* of *IML*, representing illiquidity long-short portfolio returns, as well as the long and short sides of each portfolio for the full sample period from January 1964 to December 2019 and five subperiods: January 1964 to December 1997, January 1998 to December 2019, January 1964 to December 1982, January 1983 to December 2001, and January 2002 to December 2019. *Alphas* are expressed in monthly percentages. *t*-statistics are reported in parentheses.

CAPM a*lphas* (*t*-statistics)

	1964-2019	1964-1997	1998-2019	1964-1982	1983-2001	2002-2019
Illiquid	0.38	0.48	0.24	0.79	0.30	0.14
	(3.28)	(3.23)	(1.27)	(3.59)	(1.66)	(0.77)
Liquid	-0.21	-0.19	-0.23	-0.18	-0.05	-0.35
	(-3.73)	(-2.96)	(-2.33)	(-1.84)	(-0.59)	(-3.70)
IML	0.59	0.67	0.47	0.97	0.34	0.49
	(5.00)	(4.57)	(2.38)	(4.49)	(1.76)	(2.51)

Table 12 shows the CAPM *alphas* of the long, short, and the long-minus-short returns over a variety of subperiods, using the *IML* factor portfolio. The first column reports *alphas* for the full sample period of 1964 to 2019. The second and third columns report results for the subperiods that Amihud (2019) uses when constructing *IML*.[5] *IML* generates a significant *alpha* of 0.59% per month (*t*-statistic = 5.0) for the full sample period. Comparing this value with the average monthly return generated by *IML*, we find that adjusting for market beta increases the illiquidity premium. For the first subperiod from 1964 to 1997, we find a significant *alpha* of 0.67% per month (*t*-statistic = 4.57). In the second subperiod from 1998 to 2019, *alpha* is lower with a value of 0.47% per month. It is, however, not statistically significant anymore when judged using the higher threshold of 3.0 as proposed by Harvey, Liu and Zhu (2016).

The long side delivers a positive abnormal monthly return of 0.38% (*t*-statistic = 3.28) relative to the market over the full sample period. Hence, for an investor restricted to holding long-only investments, an illiquidity strategy still offers an additional return premium above the general market return. However, all of it comes from the first subperiod, for which we find a 0.48% monthly *alpha* (*t*-statistic = 3.23). After 1997, the long side exhibits an insignificant

[5] The original second subperiod ends in 2017. We extend it until the present.

0.24% monthly *alpha*. Looking at both, the long and short sides, we find that *IML* is driven mostly by the long side, though not entirely. For the most recent subperiod from 2002 to 2019, we find that the contribution of the long and short side has changed. The big majority of profitability comes from the short side.

The last three columns report results obtained for three subperiods split into roughly equal 19-year intervals. The results paint an even gloomier picture for *IML*: The average monthly *alpha* of 0.97% found for the period 1964-1982 is the only significant *alpha*. The *alpha* produced in the period 2002 to 2019 does not survive the higher hurdle for establishing statistical significance (Harvey, Liu & Zhu, 2016). The same applies to the long-only illiquidity portfolio. When looking at the finer time grids, it emerges that all of the profit comes from the period 1964-1982.

We next turn to gaining further insights into the role of shorting on the efficacy of an illiquidity strategy. For this purpose, we form portfolios on finer sorts of illiquidity. In constructing the portfolios, we use the same stocks and apply the same filters as in the construction of *IML*. We sort stocks into decile portfolios by their illiquidity (as defined in Section 2). The sorting takes place monthly, just as for *IML*. We then calculate value-weighted monthly excess returns and construct differences between Deciles 10 and 1, as well as differences between Deciles 9 through 10 and 1 through 2, 8 through 10 and 1 through 3, and 7 through 10 and 1 through 4. We further calculate the Sharpe ratios and run time-series regressions of the excess returns for each decile portfolio, as well as the difference portfolios, on *MKT* in order to get *alpha*. Table 13 reports the results. Shown are average monthly raw returns in excess of the T-bill rate, Sharpe ratios and CAPM *alphas* of value-weighted decile portfolios sorted on illiquidity over the period January 1964 to December 2019. Also reported are the differences between Deciles 10 and 1 (10-1), the differences between the average of Deciles 10 and 9 and the average of Deciles 1 and 2 (9-2), 8 through 10 and 1 through 3 (8-3), and 7 through 10 and 1 through 4 (7-4). These differences help us in judging the importance of the long and short side for an illiquidity strategy. Further, they provide insights into whether returns exhibit any asymmetries. Precisely, we are interested in knowing whether extreme portfolios behave any differently and how monotonic the relation between illiquidity and returns is.

Table 13
Decile portfolios based on illiquidity from January 1964 to December 2019

Shown are the average raw returns in excess of the one-month Treasury bill rate, Sharpe ratios, and CAPM *alphas* of value-weighted decile portfolios formed on illiquidity using the Amihud illiquidity measure. Also reported are the differences between Deciles 10 and 1 (10-1), as well as the differences between the average of Deciles 9 and 10 and the average of Deciles 1 and 2 (9-2), the average of Deciles 8, 9 and 10 and the average of Deciles 1,2 and 3 (8-3), as well as the average of Deciles 7, 8, 9 and 10 and the average of Deciles 1, 2, 3 and 4 (7-4).

	Decile portfolios based on illiquidity										Differences			
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P10-1	P9-2	P8-3	P7-4
Raw excess	0.45	0.58	0.65	0.71	0.72	0.78	0.84	0.83	0.87	0.93	0.48	0.39	0.32	0.27
Sharpe ratio	0.11	0.13	0.14	0.14	0.14	0.15	0.15	0.15	0.15	0.16	0.11	0.11	0.10	0.10
Alpha	-0.03	0.06	0.10	0.15	0.16	0.21	0.25	0.25	0.29	0.38	0.41	0.32	0.26	0.22
(t-statistic*)*	(-0.73)	(1.17)	(1.60)	(2.16)	(1.90)	(2.31)	(2.56)	(2.09)	(2.31)	(2.47)	(2.36)	(2.28)	(2.18)	(2.16)

Regarding our findings, we document a monotonic relation between illiquidity and average raw excess returns. Moving from the lowest to the highest decile, average returns increase consistently. Also, the spread in returns from Decile 10-1 through Deciles 7-4 drops monotonically. The Sharpe ratios, in contrast, are quite flat across the deciles. This means that the higher returns found in the higher deciles are offset by higher volatility. In other words, an illiquidity strategy does not provide added return per unit of volatility when moving towards the higher deciles. As table 13 shows, market *alphas* are stronger for the long side, though their statistical significance is questionable (Harvey, Liu & Zhu, 2016).

4.2 Interaction of firm size and the illiquidity strategy

The following tests shed light on the interaction of size and the profitability of an illiquidity strategy. We examine whether the illiquidity premium varies with firm size. Additionally, we highlight whether the contribution to profits from the long and the short side is different for the various size groups.

For this task, we form 25 portfolios based on size and illiquidity. We use the same stocks, the same sample period, apply the same filters, and use the same measure for illiquidity as in the construction of *IML* in Section 2. Portfolios are formed monthly. We first sort stocks on size, then on illiquidity. In calculating the size breakpoints, we follow the literature and use only NYSE stocks. As a next step, we calculate value-weighted monthly returns for the 25 portfolios. We also calculate the 5-1 spread by subtracting the returns of illiquidity portfolio Quintile 1 from the returns of Quintile 5. This is done for each of the five size groups. We then calculate the monthly excess returns for illiquidity Quintile 5 - the long side - by subtracting the one-month Treasury bill rate. Next, we run a time-series regression of the 5-1 spread and separately for the long-only excess returns on *MKT* in order to get *alpha*. Finally, we calculate the difference between size Quintile 1 and 5. Table 14 presents the results. Panel A reports monthly raw excess returns for the 5-1 spread and the long side (Quintile 5), as well as the percentage of profits coming from the long side. Also shown are the differences between size Quintiles 1 (smallest) and 5 (largest). Panel B reports market *alphas* for the 5-1 spread and for the long side. Presented are also the percentage of alpha coming from the long side and the differences between size Quintiles 1 and 5.

Table 14
Profitability of long and short side of illiquidity across size quintiles

Panel A shows the average raw returns in excess of the one-month Treasury bill rate. Panel B reports CAPM *alphas* of illiquidity sorted portfolios across size quintiles. Based on NYSE breakpoints, stocks are first sorted into size quintiles and then sorted into quintiles based on illiquidity. The difference between the top and bottom illiquidity quintiles (5 and 1) are reported for each size quintile. The *t*-statistics (in parentheses) of the return differences, the average excess returns generated by the long-only strategy (Quintile 5) and their *t*-statistics (in parentheses), as well as the percentage of 5-1 profits coming from the long side are presented. Also reported are the differences between size Quintiles 1 (smallest) and 5 (largest). Returns and *alphas* are expressed as monthly percentages. Results are generated for value-weighted portfolios over the sample period January 1964 to December 2019.

Panel A: Raw excess returns	Smallest				Largest	
	Size 1	Size 2	Size 3	Size 4	Size 5	Size 1-Size 5
5-1 spread	0.51	0.36	0.18	-0.07	0.11	0.40
	(2.82)	(2.29)	(1.40)	(-0.57)	(1.25)	(1.91)
Long side	1.00	0.89	0.87	0.61	0.53	0.46
	(4.25)	(4.25)	(4.52)	(3.30)	(3.13)	(2.72)
Percent long side	196.70	244.98	470.32	-895.68	481.80	

Panel B: *Alphas*						
	Size 1	Size 2	Size 3	Size 4	Size 5	Size 1-Size 5
5-1 spread	0.76	0.58	0.34	0.03	0.08	0.68
	(4.68)	(4.04)	(2.78)	(0.29)	(0.85)	(3.65)
Long side	0.45	0.34	0.33	0.08	0.03	0.42
	(2.81)	(2.89)	(3.66)	(1.03)	(0.53)	(2.44)
Percent long side	59.21	58.62	97.06	266.67	37.50	

We begin by analysing the raw returns of the 5-1 spread. While it appears that the returns to an illiquidity long-short portfolio are stronger for the smallest quintile of stocks, the difference is insignificant. A statistically significant difference exists, however, for the *alphas* of the long-short strategy. Here we clearly find that they are strongest in the smallest quintile of stocks and weakest among the biggest firms. In fact, the 5-1 spread *alpha* is economically and statistically insignificant for the biggest size quintiles. In addition, we find that the *alphas* decrease monotonically as firm size increases.

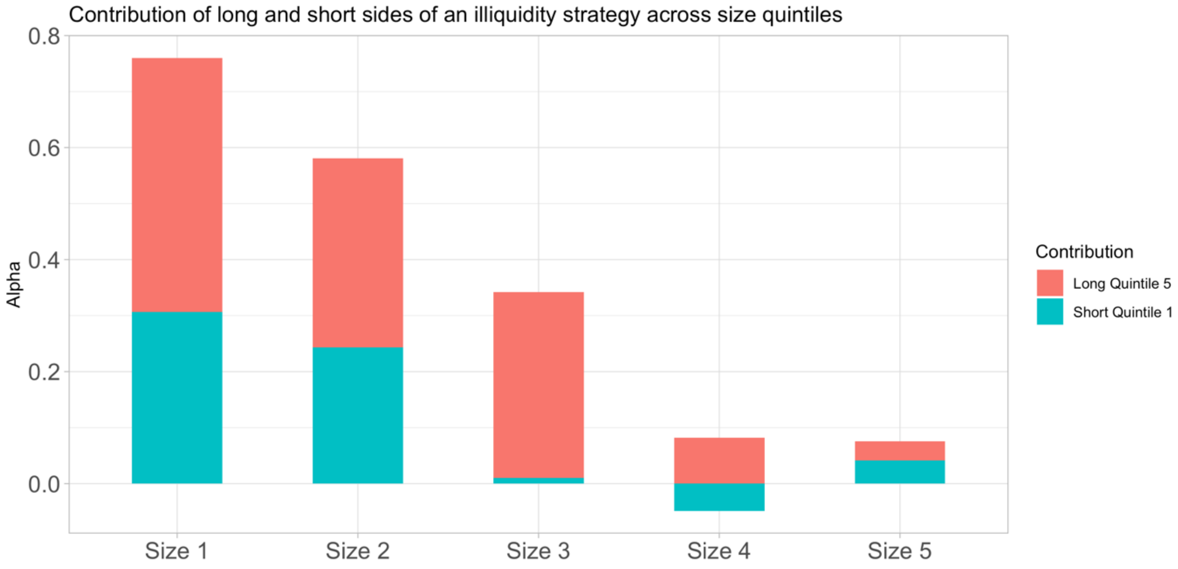
With regards to the long-only portfolios, we find that statistically significant differences across size quintiles exist for both, the raw returns and the *alphas*. Both decrease as firm size increases. The long-only illiquidity strategy is also strongest among the smallest firms and weakest for the biggest firms. Looking at the raw returns, the long-only returns are statistically significant in each size quintile. However, regarding the alphas, the long-side-only does not produce significant risk-adjusted returns for the two largest size quintiles. One might also worry concerning the significance in the smallest size quintile, as it would not pass the higher hurdle suggested by Harvey, Liu and Zhu (2016).

The contribution of the long side to the 5-1 raw return is bigger than 100% for each size quintile, except size Quintile 4. Hence, an investor who does not care about market risk, would only go long, without shorting liquid stocks. After adjusting for the market, the short side also becomes important. The contribution varies with firm size. The long side contributes more to the profitability of the illiquidity strategy among the two small stock quintiles than it does for the largest size quintile, the exception being size Quintile 3 and 4. The long side accounts for 59.21% of the profits for small cap stocks, whereas only 37.5% of the large cap illiquidity profits come from the long side. Since trading costs are high for small firms and shorting is costly and sometimes even constrained, it is a positive feature that shorting seems less important among small firms. However, one should note that even in the smallest size quintile, the profitability of the illiquidity strategy still relies on the short side to a great extent. Around 41 % of the profits come from the short side.

The results presented in Table 14 are visualized in Figure 3, which plots the 5-1 Quintile spread *alphas* across size quintiles. The long side *alpha* is generated by Quintile 5; the short side *alpha* is the negative of the *alpha* generated by Quintile 1. The contributions of each are highlighted. As size increases, the illiquidity premium decreases. For small caps, shorting makes up around 41% of the profitability. Shorting is more important for the biggest size quintile. For the biggest firms, we find that the contribution of shorting equals 62.5%. There is, however, no clear pattern for the contribution of shorting across size quintiles. The contribution is lowest for size Quintiles 3 and 4.

Figure 3
Contributions of long and short sides of an illiquidity strategy across size quintiles

Plotted are the CAPM *alphas* of the difference between Quintile 5 and Quintile 1 value-weighted portfolios formed on illiquidity within size quintiles over the period January 1964 to December 2019. The contributions to total profits from the long (Quintile 5) and the short side (Quintile 1) are highlighted on the graph.



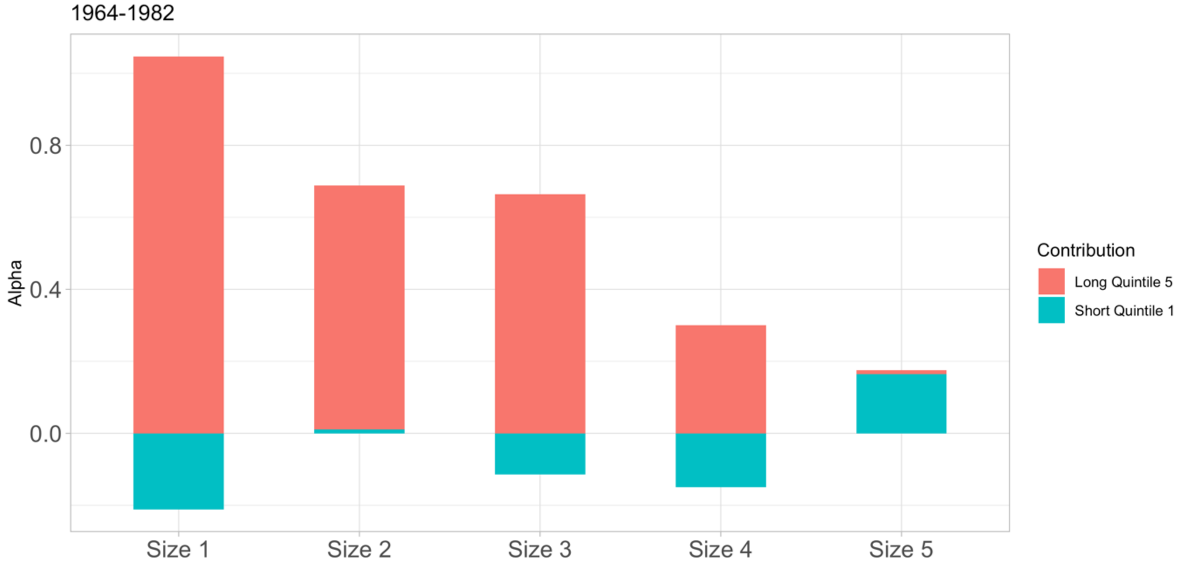
4.3 Variation over time

As a last exercise, we turn to analysing the returns to an illiquidity strategy over time. In addition, we will examine how the contribution to profitability from the long and short side varies over time.

Figure 4 splits the results plotted in Figure 3 into three subperiods: January 1964 to December 1982, January 1983 to December 2001, and January 2002 to December 2019. Plotted are again the CAPM *alphas* generated by an illiquidity strategy within size quintiles using value-weighted portfolios. The illiquidity premium is strongest among small stocks and then decreases gradually as firm size increases. This pattern is found for each of the three subperiods. For the earlier two subperiods, we find a statistically significant illiquidity premium for the two smallest size quintiles with *t*-statistics larger than 3.0. In the first subperiod, size Quintile 3 also exhibits a significant *alpha*. We find no reliable illiquidity premium among large cap stocks in any of the subperiods. The illiquidity premium for the two largest size quintiles is never significant. In the most recent subperiod we do not find a statistically significant illiquidity premium in any of the size quintiles.

Figure 4
Illiquidity long and short side alphas across size quintiles over time

Plotted are the CAPM *alphas* of the difference between Quintile 5 and 1 portfolios formed on illiquidity within size quintiles over three subperiods: January 1964 to December 1982, January 1983 to December 2001, and January 2002 to December 2019. The contributions to profits from the long side (Quintile 5) and the short side (Quintile 1) are highlighted on each graph.



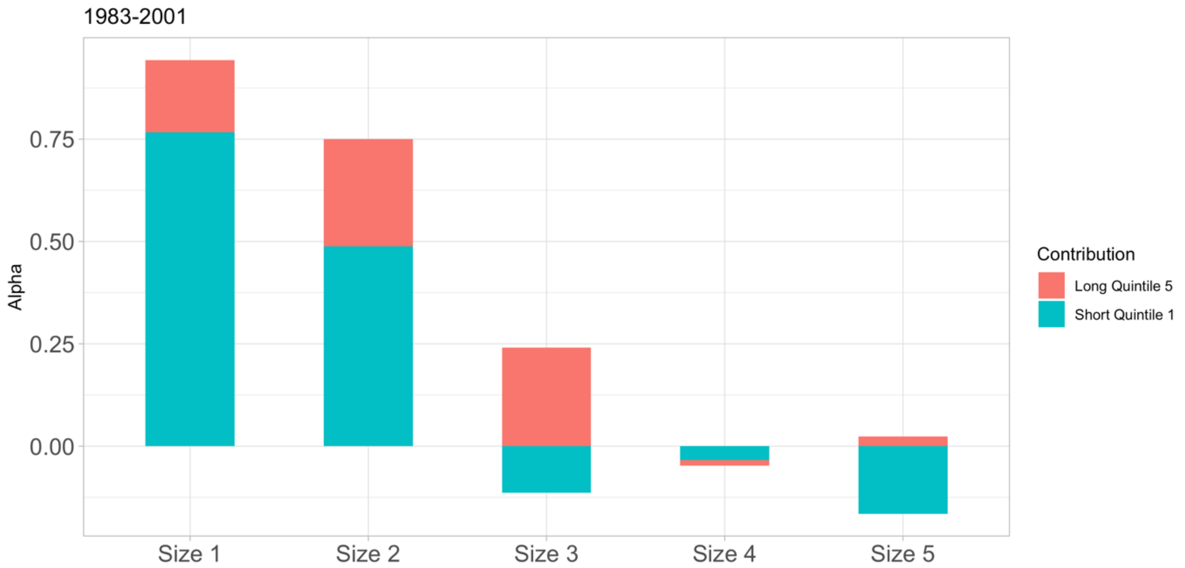
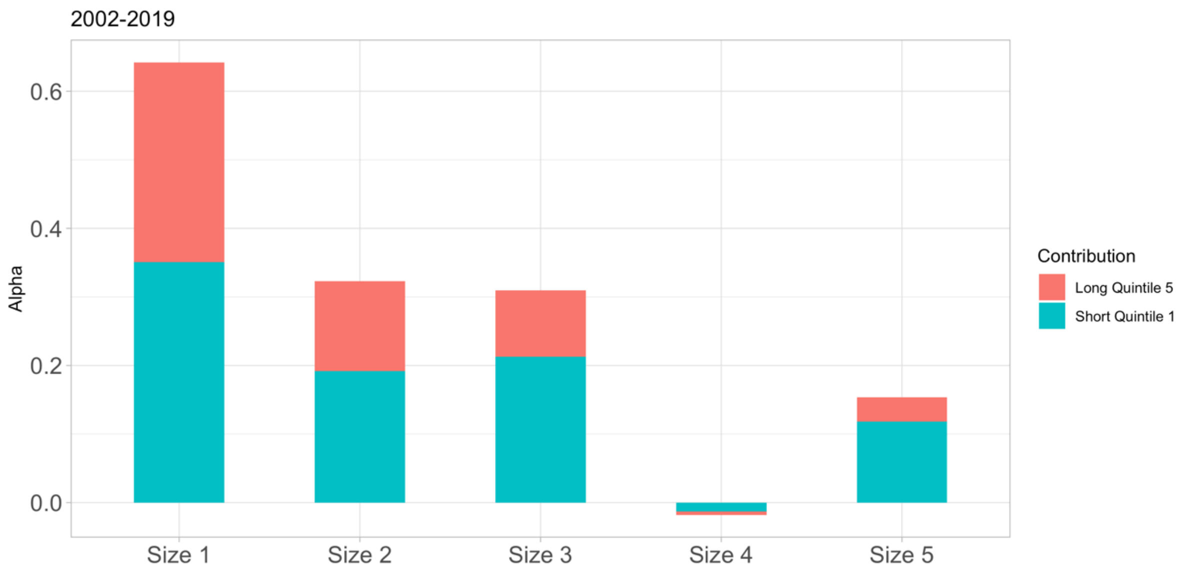


Figure 4 - continued



The contribution to profits from the long and short side changes over time. The vast majority of profitability comes from the long side for the period 1964 to 1982. As firm size increases, shorting becomes more important, and for the biggest size group the contribution of the short side is bigger than the contribution from the long side. This picture changes completely in the period to follow. In the years 1983 to 2001, the bulk of profits stems from the short side, with the exception of size Quintile 3. The contribution of the long side for the smallest size quintile is almost equal to that found for the biggest size quintile, and it is lower than for the three middle size quintiles. For the most recent time period, we also find that the short side leads the ranking of importance. In contrast to the previous subperiod, we clearly find that shorting becomes more important as firm size increases. While it is in general good news that shorting is less important for the smallest size quintiles, shorting still contributes more than half (55%) to the profits for size Quintile 1.

The illiquidity premium declines over time. It is largest in the period 1964 to 1982 for each of the five size groups. The most recent period shows the lowest illiquidity premium; it is statistically insignificant in each of the size groups.

5. Conclusion

In our thesis we examine the illiquidity premium. In the first part of the thesis, we analyse whether a traded illiquid-minus-liquid (*IML*) return factor helps in explaining the cross-section of expected returns. In the second part, we investigate whether the illiquidity premium can be captured in practice. With our thesis we aim at contributing to this most recent stream of the illiquidity literature.

Using data for the U.S. stock market and a sample period from 1964 to 2019, we find that the average monthly return on *IML* is 0.498% (t = 4.15). The return remains positive and significant after controlling for risk. In our asset pricing tests, we follow FF (1993, 2015) and run time-series regressions. We consider four different models: (i) the FF three-factor model; (ii) the FF three-factor model augmented by *IML*; (iii) the FF five-factor model; and (iv) the FF five-factor model augmented by *IML*. Our test portfolios are a variety of portfolios sorted on known anomalies. We judge the absolute performance of the models, as well as their relative performance. We find some evidence in favour of *IML*. For most test portfolios, *IML* improves the description of average excess returns. We prefer the *IML*-augmented FF five-factor model over the FF five-factor model, the FF five-factor model over the *IML*-augmented FF three-factor model and the *IML*-augmented FF three-factor model over the FF three-factor model. In terms of $A|\alpha_i|$, the improvements are larger when switching from the FF three-factor model to its *IML*-augmented version than when adding *IML* to the five-factor model. For two of our test portfolios (the 25 *B/M-INV* and the 25 *Size-Market Beta* portfolios), we clearly prefer an *IML*-augmented asset pricing model. For both of them, the *IML*-augmented FF three-factor model fares best, followed by the *IML*-augmented five-factor model. For the 25 *Size-Market Beta* portfolios we even find that the *IML*-augmented three-factor model is a description of the average excess returns.

In the second part of our thesis, we shed light on the question of whether an illiquidity strategy is implementable in practice. We follow Israel and Moskowitz (2013) in their methodology of assessing the implementability of value and momentum strategies. The tests seek to answer three questions. First, how much does short selling contribute to the profitability of an illiquidity strategy? Second, what is the role of firm size with regards to the efficacy of the strategy? Third, how have the returns and the role of shorting and firm size varied over time? With regards to how implementable an illiquidity strategy is in practice, we find that the illiquidity premium is largely concentrated among small firms. It is considerably lower for the biggest firms. This pattern does not change over time. When considering market-adjusted returns, we find that the illiquidity premium is driven mostly by the long side, though not entirely. For raw returns, the return premium is dominated by the contribution from long positions. Further, we find some evidence that the contribution of the long and the short side varies across firm size. For the smallest firms, shorting is less important than for the biggest firms. However, there is no monotonic pattern in the relation between shorting and firm size. The contribution of the long and short side changes over time. For the most recent subperiod (2002-2019), we find that the big majority of profitability comes from the short side. The contribution of the short side remains more important among big firms. Moreover, we show that the illiquidity premium decreased over time. For the most recent subperiod, it is insignificant in each of the size groups. We therefore conclude that is highly unlikely that the illiquidity premium can be captured in practice. Our results have implications for understanding the illiquidity premium and for its implementation in practice.

References

Acharya, V., & Pedersen, L., H. (2005). Asset Pricing with Liquidity Risk. *Journal of Financial Economics*, 77(2), 375-410.

Amihud, Y., & Mendelson, H. (1986). Asset pricing and the bid-ask spread. *Journal of Financial Economics*, 17(2), 223-249.

Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets*, 5(1), 31-56.

Amihud, Y., Mendelson, H., & Pedersen, L.H. (2005). Liquidity and Asset Prices. *Foundations and Trends in Finance*, 1(4), 269-364.

Amihud, Y., Hameed, A., Kang, W., & Zhang, H. (2015). The illiquidity premium: International evidence. *Journal of Financial Economics*, 117(2), 350-368.

Amihud, Y. (2019). Illiquidity and Stock Returns: A Revisit. *Critical Finance Review*, 8, 203-221.

Barry, C. B., & Brown, S.J. (1984). Differential information and the small firm effect. *Journal of Financial Economics*, 13(2), 283-294.

Brennan, M.J., & Subrahmanyam, A. (1996). Market Microstructure and Asset Pricing: On the Compensation for Illiquidity in Stock Returns. *Journal of Financial Economics*, 41(3), 441-464.

Fama, E. F., & French, K.R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1), 3-56.

Fama, E. F., & French, K.R. (2008). Dissecting Anomalies. *Journal of Finance*, 63(4), 1653-1678.

Fama, E. F., & French, K.R. (2015). A Five-Factor Asset Pricing Model. *Journal of Financial Economics*, 116(1), 1-22.

Fong, K. Y. L., Holden, C. W., & Trzcinka, C. (2017). What Are the Best Liquidity Proxies for Global Research? Review of Finance, 21(4), 1355-1401.

Gibbons, M. R., Ross, S.A., & Shanken, J. (1989). A Test of Efficiency of a Given Portfolio. *Econometrica*, 57(5), 1121-52

Glosten, L. R., & Milgrom, P.R. (1985). Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics*, 14(1), 71-100.

Goyenko, R. Y., Holden, C. W., & Trzcinka, C. A. (2009). Do Liquidity Measures Measure Liquidity? *Journal of Financial Economics*, 92(2), 153-181.

Harvey, C. R., Liu, Y., & Zhu, H. (2016). … and the Cross-Section of Expected Returns. *The Review of Financial Studies*, 29(1), 5-68.

Hasbrouck, J. (2009). Trading Costs and Returns for U.S. Equities: Estimating Effective Costs from Daily Data. *Journal of Finance*, 64(3), 1445-1477.

Israel, R., & Moskowitz, T. J. (2013). The role of shorting, firm size, and time on market anomalies. *Journal of Financial Economics*, 108(2), 275-301.

Kyle, A. S. (1985). Continuous auctions and insider trading. *Econometrica*, 53(6), 1315-1335.

Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Review of Economics and Statistics*, 47(1), 13-37.

Pástor, L., & Stambaugh, R. F. (2003). Liquidity Risk and Expected Stock Returns. *The Journal of Political Economy,* 111(3), 642-685.

Sharpe, W.F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *Journal of Finance*, 19(3), 425-442.

Shumway, T. (1997). The Delisting Bias in CRSP Data. *Journal of Finance*, 52(1), 327-340.

Stoll, H. R. (1978). The pricing of security dealer services: an empirical study of NASDAQ stocks. *Journal of Finance*, 33(4), 1153-1172.

Stoll, H. R., & Whaley, R. E. (1983). Transaction costs and the small firm effect. *Journal of Financial Economics*, 12(1), 57-79.

White, H. (1980). A Heteroscedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity. *Econometrica*, 48(4), 817-838.

Appendix

Table A1
Regressions of *IML* on the FF five factors and *MOM*

IML is the return on an illiquid-minus-liquid portfolio. Every month, stocks are sorted into three portfolios based on their standard deviation of daily returns. Each standard deviation portfolio is then further sorted into five illiquidity portfolios based on the average of daily values of $Illiq_{j,\,d} = |return_{j,d}|/dollar\ volume_{j,d}$. Each of the variables is calculated over a 12-month period that ends in November of the previous year. *IML* is then calculated as the return differential between stocks in the highest illiquidity quintile and stocks in the lowest illiquidity quintile across the three standard deviation portfolios. We use NYSE and AMEX stocks and apply some filters. Returns are in monthly percentages. *t*-statistics of average returns are reported in parenthesis. The full sample period stretches from January 1964 to December 2019. The first subperiod covers the years 1964-1997, the second subperiod the years 1998-2019. The *t*-statistics of the coefficients employ robust standard errors (White, 1980), as in Amihud (2019). Reported are *alphas*, coefficients and the corresponding *t*-statistics from a regression of *IML* on the FF factors. The risk factors used are the FF five factors and *MOM*. The corresponding time-series regression is,

$$IML_t = \alpha + bMKT_t + sSMB_t + hHML_t + rRMW_t + cCMA_t + mMOM_t + \varepsilon_t.$$

	Dependent variable:		
		IML	
	1964-2019	1964-1997	1998-2019
Alpha	0.30	0.42	0.19
	(3.66)	(3.95)	(1.46)
MKT	-0.25	-0.21	-0.24
	(11.07)	(7.02)	(6.41)
SMB	0.79	0.76	0.87
	(20.52)	(16.01)	(17.68)
HML	0.16	0.14	0.10
	(2.89)	(1.56)	(1.29)
RMW	0.09	-0.09	0.30
	(2.01)	(1.10)	(4.47)
CMA	0.18	0.26	0.03
	(2.25)	(2.44)	(0.34)
MOM	0.03	-0.04	0.06
	(1.05)	(1.02)	(1.93)
Observations	672	408	264