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Replicating Nordic Private Equity

Capture Private Equity return and risk in the Nordic stock market

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Abstract

Private equity is widely known as an asset class delivering highly impressive returns. Still, critics like to point out the excessive fee structure in the industry. This paper investigates whether a passive stock portfolio, mimicking the asset selection and leverage level of private equity funds, to a lower cost can emulate the risk and return yielded by Nordic private equity. We find that buyout funds have a tilt towards selecting relatively small firms within specific sectors of the economy. Further, we find that buyout targets tend to be relatively more leveraged, relatively more capital-efficient and to have a relatively lower asset growth turnover than comparable Nordic stocks.

Overall, two of our 24 characteristics-matched and leveraged-matched replicating portfolios offer returns that exceed the attractive returns yielded by Nordic private equity in the period June 2006 to June 2018. A five-year buy-and-hold portfolio, selecting stocks based on size, sector, EBITDA and asset growth turnover, yielded an annualised excess return of 18.6% in the investment period, outperforming the pre-fee private equity return of 17.2%. After accounting for fees and transaction costs, 13 of the 24 replicating portfolios earned a higher return than the benchmark.

However, none of the passive replicating portfolios can reproduce the risk-adjusted return of private equity. Our analysis indicates that the lower risk of private equity may be explained by i) the active management approach, ii) beneficial interest rates and loose covenants of their long-term corporate debt, and iii) the existence of return smoothing. Nevertheless, we conclude that a replicating portfolio offers a cheap and accessible investment strategy for investors that deny paying the excessive fees of private equity and can accept large fluctuations in portfolio values.

Preface

This thesis marks the end of our master’s degree in Economics and Business Administration with a major in Financial Economics at the Norwegian School of Economics (NHH).

There are several people we wish to thank for advice and support during this research. We would particularly like to express our sincere gratitude to our supervisor, Associate Professor Carsten Gero Bienz. We thank him for always being hands-on with critical insights, new perspectives, timely feedback, and valuable discussions. Your guidance has contributed to making this work challenging but highly educational and exciting!

Our gratitude extends to our friends and family who have supported us in numerous ways, as well as Senior Associate Jon Vassengen at Argentum Asset Management for preparing data on Nordic private equity return. From the industry, we would also like to thank Nick Schmitz from the US hedge fund Verdad Capital for giving us key insight on trading costs and fees.

We sincerely hope you enjoy the reading!

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Henrik Fladvad & Jørgen Tvedt

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1. Introduction

Private equity has, for a long time, been an asset class associated with sophisticated investors earning high returns. However, the strong performance comes at the cost of exorbitant fees charged by the private equity funds. The last years, several quant funds attempting to capture the private equity's attractive risk properties in the public market have emerged in the US. Jeffrey Knupp, President of the investment firm DSC Quantitative Group, states in Institutional Investor (2019): "All the magic that the [private equity] managers are doing at the portfolio companies is incorporated into the benchmark index". Furthermore, he says that he does not believe in the value creation of private equity managers because he can replicate it. The returns provided by DSC, among other funds, have outperformed Cambridge Associates' private equity index[1]. This paper investigates whether an outside investor can capture the risk-return profile of buyout funds in the Nordic stock market by seeking an answer to the following research question:

Is it possible to replicate the long-term risk and return of the Nordic buyout industry through passive investments in the Nordic stock market by applying similar investment selection and leverage?

To proxy the returns achieved by investors with a diversified allocation to the Nordic buyout market, the Nordic Private Equity Index offered by Argentum Asset Management is used. Over the period June 2006 to June 2018, the mean return on the private equity index, before fees, is 17.2% per year with an annualised volatility of 17.2% and a market beta of 0.94.

To mimic Nordic private equity, we construct six different characteristics-matched and leveraged-matched portfolios, each invested in four distinct investment strategies, forming a total of 24 different replicating portfolios. Overall, only two of the 24 replicating portfolios can capture the pre-fee returns yielded by Nordic private equity during the investment period. After accounting for fees, 13[2] of the 24 replicating portfolios earn a higher return than private equity. However, the relatively high risk of the replicating portfolios makes them unable to reproduce the risk-adjusted return of private equity. Our analysis indicates that the lower risk of private equity may be driven by their active management and favourable debt financing,

[1] The index is net of fees and tracks self-reported returns of private equity funds.
[2] Assuming a trading cost of 100bps and leverage level mimicking private equity.

combined with the existence of return smoothing in their reporting process. We also stress that some buyout firms convey characteristics not present in the public market or not captured by our selection criteria, potentially resulting in an imperfect replication of the risk properties of buyout targets. Nevertheless, for investors that seek similar exposure as private equity funds and can accept substantial fluctuations in portfolio values, we conclude that a replicating portfolio provides a relatively cheap and accessible investment strategy.

Somewhat contrary to our findings, most existing research (i.a. Robinson & Sensory, 2011; Ljungqvist & Richardson, 2003) shows that the buyout industry has outperformed the stock market, at least before fees. Existing studies (i.a. Kaplan, 1989; Scholes, Siegel, Wilson, & Wright, 2012) have found improvement in productivity and profitability for firms that belong to private equity portfolios. According to Jensen (1989), the main drivers to private equity's value increasing are their applied financial, governance, and operational engineering to their portfolio companies. In financial engineering, buyout funds give equity incentives to their portfolio firms' management teams. Also, leverage pushes managers to optimise the money spending (Gompers, Kaplan, & Mukharlyamov, 2015). In governance engineering, buyout funds control the portfolio firms' boards and have a more active involvement in governance relative to listed firm directors and public shareholders (Gompers et al., 2015). In operational engineering, Leland and Pyle (1977) show that private equity's specialised knowledge develops industries and operating expertise, which furthermore add value to their portfolio firms. Given that these are unique advantages in equity investing, the returns of a passively managed portfolio consisting of comparable listed investments are expected to underperform compared to the pre-fee returns of buyout funds.

However, buyout funds outperforming the public market may not be so clear as literature claims. Studying the portfolio firms of private equity funds, Phalippou (2013) documents that buyout funds primarily invest in small and value firms with an average buyout fund performance similar to small-cap indices. If levering up the small-cap index to a level equivalent to private equity, an average buyout fund underperforms by 3.1% p.a. Furthermore, by selecting small, value listed firms with modest amounts of leverage and hold-to-maturity accounting of portfolio value, Stafford (2016) also manages to mimic buyout funds' risk and pre-fee-returns in the US. Additionally, recent trends in the private equity industry, including record-high levels of undeployed capital and rising valuation multiples, continue to challenge the superior position of private equity (Bain, 2019).

To study the asset selection by Nordic private equity funds, we have assembled a dataset of 594 Nordic buyout transactions in the period 2005 to 2016, provided by Argentum Centre for Private Equity at the Norwegian School of Economics. Targeted firms are matched with accounting data from Amadeus, a comprehensive database of European companies provided by Bureau van Dijk. Unlike Stafford, that only has a sample of public-to-private transactions in the US, we have data for private-to-private deals. The availability of private-to-private transactions is due to EU's law demanding private companies' balance sheets and income statement to be publicly available, which contrasts with the rules in the US (European Commission, 2020). As private-to-private deals are accounting for a vast majority of private equity deals, we argue that our sample is a more accurate representation of buyout targets.

We find that Nordic buyout funds consistently tend to target relatively small firms within industrials and information technology. Interestingly, our data suggest that most buyout targets have relatively higher working capital turnover and a relatively low asset growth turnover than a comparable stock at the time of the buyout. We also find that a typical buyout transaction increases a firm's leverage, measured as the ratio of debt-to-equity, by 33% on average. This result is consistent with research on US buyout transactions (Axelson, Jenkinson, Strömberg, & Weisbach, 2013), as well as Bienz, Thorburn and Walz (2016), both showing that leverage is an essential component of private equity's investment thesis.

Applying the principles of Modigliani and Miller (1958), we can adjust the replicating portfolio leverage to mimic the levered private equity level. As the post-transaction buyout targets tend to be more leveraged than comparable Nordic stocks, we use a brokerage margin account to add leverage to the replicated portfolio by investing a multiple of his equity capital. Due to high collateralisation and the ability to use marking to market, borrowing rates in the replicating portfolios will be close to the risk-free rate (Stafford, 2016). On the other hand, the leverage will not yield the incentive and tax effects that characterise the corporate debt applied in private equity portfolio companies.

The private equity market differs from the public market in several other ways. The private market imposes numerous restrictions in terms of how funds can invest, in contrast to the public market where the same restrictions are almost non-existing. By introducing higher liquidity, lower transaction costs and a passive management approach, the public market involves fewer constraints in terms of holding time and portfolio rebalancing. Furthermore, the replicating portfolios purchase small parts of the targeted stocks, enabling them to avoid paying the control premium associated with full-firm transactions. On the other side, the private market offers an advantage by providing broad access to small firms with annual revenue below $2 million, that, due to their size, are almost absent in the public market.

Moreover, the illiquidity of the private market relative to the public market gives rise to a notable difference in the return reporting process. Due to the lack of transaction-based measures, measured private equity returns are subject to manager discretion in the marking of portfolio net asset values (Getmansky, Lo, & Makro, 2004; Cassar & Gerakos, 2011). In contrast, the replicating portfolios are subject to continuous transaction-based valuations as they consist of publicly traded firms. Literature (Ang, Chen, Goetzmann, & Phalippou, 2018; Stafford, 2016) has found that private equity returns are considerably more volatile than those reported using the appraisal approach preferred by the industry. To examine how differences in reporting processes can affect the measured risks, we apply two different accounting schemes to report portfolio net asset value. In the first accounting scheme, all holdings are reported to their market value each month. The second approach, based on a hold-to-maturity rule, tries to mimic the appraisal method by measuring all holdings at cost until they are realised. This scheme may understate volatility as it will give the replicated portfolio a conservative approximation of value in times where the market is rising.

Overall, our results provide some support to earlier research that characteristics-matched and leveraged-matched replicating portfolios can reproduce the annualised mean return yielded by private equity, in particular after accounting for fees. However, the replicating portfolios face substantially higher measured risk than the benchmark, causing our approach to underperform on a risk-adjusted basis. The private market appears to have advantages over the public market in terms of active management, attractive debt financing and access to very small firms. These advantages seem to more than compensate for the benefits of a public market portfolio, including increased flexibility of rebalancing, lower transaction costs and the absence of bidding wars. Nevertheless, for investors not willing to pay the excessive fees of private equity and can tackle massive movements in portfolio values, the replicating portfolios offer an easily accessible alternative to private equity.

The rest of the paper is organised as follows. The following chapter presents raw data, including our sample of buyout targets, Nordic stocks and historical private equity return. The raw data creates the fundament for Chapter 3, in which we construct an investment universe in the Nordic stock market based on the asset selection choices of Nordic buyout funds.

Chapter 4 presents our methodology for replicating private equity across three pillars: replicating asset selection, market structure and leverage. The following chapter summarises the performance of the replicating portfolios with regards to different choices of asset selection, leverage and market structure. The chapter also examines how portfolio performance is sensitive to transaction costs, the initial year of investment and the choice of an accounting scheme. Chapter 6 discusses the implications of our findings and provides reflections on the advantages of private equity and those of a replicating portfolio. Chapter 7 presents the limitations of our analysis before Chapter 8 finally concludes the paper.

2. Raw data

Our paper uses data that are collected from various sources. This section presents our raw data of private equity return in the Nordics, in addition to our sample of buyout targets and Nordic stocks. The data preparation is done in Excel, while all data handling and performance analysis are performed in R. To ease the data handling process, we have used several packages as credited in the Appendix in Section 9.4.

2.1 Data on Nordic buyout targets

The data on private equity deals in the Nordics are provided from the Argentum Centre for Private Equity at the Norwegian School of Economics. It includes buyouts in the period 1990 to 2016 and entails name and organisational number (ISIN) of the targeted firms, its financial buyer and year of investment. Missing data on ISIN and initial year of investment were covered through Google searches to increase the completeness of the data set. In total, the data set counts 3714 observations with unique ISIN-numbers, whereby only 2530 contain information about the initial year of investment.

Both listed and unlisted firms located in countries that are affected by the EU's laws and regulations, which includes all the Nordic countries, are required to have public balance sheets (European Commission, 2020). That allows us to analyse the accounting data of companies involved in both private-to-private and public-to-private deals. The targeted firms' accounting data were extracted from Bureau van Dijk by matching them on organisational numbers. The database provides data on various metrics from the targeted firms' balance sheets and income statement, as well as the industry sector of the firms. However, the data source only retains financial data for companies for a rolling period of 8 years. When a new year of data is added, the oldest year is dropped, meaning that only the most recent data for each company are available. As we are reliant on accounting data in our analysis, we drop observations were this information is missing. The number of buyouts with corresponding accounting data in the deal year is 773, whereby only 594 occurred during our relevant time frame from 2005 to 2016.

Year	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**
Number of buyouts	5	14	38	92	57	68	56	45	44	69	68	40

Table 1: Number of buyout transactions in our sample each year from 2005 to 2016.

The table reports the number of observations in which the investment year is known, and accounting data in the deal year are available.

2.2 Data on Nordic stocks and market index

Data on listed Nordic firms in the period 2006 to 2018 were extracted from Bloomberg Terminal. All stocks traded on an exchange in Norway, Sweden, Finland or Denmark are included in the data set. In total, the data set consists of 2,049 stocks traded at a Nordic exchange from 2006 to 2018. Accounting data information for these firms are on an annual level and include key metrics from the balance sheet and the income statement. Bloomberg also provides information on industry sectors of the stocks and stock returns, applying the common global classification GICS (Global Industry Classification Standard). We observe that the number of stocks with available accounting data rises considerably over the period, reflecting both better accounting information in Bloomberg and an increase in the number of stocks at Nordic exchanges. Further, we rely on monthly stock prices which are adjusted for dividends and spin-offs to calculate the returns of the stocks.

To proxy the market index in the period, we use an equally weighted portfolio consisting of all tradable stocks on Nordic exchanges from 2006 to 2018. This provides us with an estimate of the return yielded by the general Nordic stock market since 2006. Furthermore, all returns are calculated in excess of the risk-free rate, as proxied by the American Federal Reserve's 3-Month Treasury Bill. The time series from 2005 to 2018 is provided at a monthly level from the Economic Research Division of Federal Reserve Bank of St. Louis. In our investment period from June 2006 to June 2018, the general market has yielded an annualised excess return of 8.1% with a standard deviation of 16.2%.

Year	2006-2007	2007-2008	2008-2009	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	2015-2016	2016-2017	2017-2018
Return	30.3%	-12.5%	-23.0%	23.3%	15.2%	-7.2%	16.2%	26.3%	13.1%	8.4%	15.8%	5.4%

Table 2: Yearly excess return of the general market in our investment period.
The general market is here defined as all tradable equities in the Nordic stock market each year. All stocks have been given equal weights. The yearly periods are defined from June to June the following year, e.g. June 2006 to June 2007.

2.3 Returns to be replicated

To benchmark the performance of our replicating portfolios, we apply an index provided by Argentum Asset Management showing returns yielded by Nordic buyout funds since 2005. The index is based on an end-to-end internal rate of return, which is the most common method for measuring performance in private equity (Phalippou, 2009). As the sample consists of 41 private equity funds in the Nordics, arguably the index can be stated as representative for the Nordic private equity market. The index entails an internal rate of return at an annualised and post-fee level. At its raw format as provided by Argentum, the index is calculated in USD. The returns are currency-adjusted from USD to NOK, using historical exchange rates offered by Norges Bank, to proxy the returns earned by a Norwegian investor with a diversified allocation to Nordic private equity. Since the Norwegian Krone has depreciated relative to the American Dollar in our investment period, the currency adjustment results in a rise in returns. Annualised post-fee excess return increases from 9.6% to 11.1% when adjusting for currency.

Formula:	$Currency\ adjusted\ excess\ return = (1 + Unadjusted\ return) \times (1 + Currency\ change) - Risk\ free\ rate - 1$
Applied example:	Assuming that • Unadjusted return = 10% • Currency change = 5% (USD appreciates with 5% compared to NOK) • Risk-free rate = 2% The formula gives the following return for a Norwegian investor: $Currency\ adjusted\ excess\ return = (1 + 0.10) \times (1 + 0.05) - 0.02 - 1 = 15.5\%$

Table 3: Formula for calculating currency-adjusted returns.
A Norwegian investor with an allocation to a private equity portfolio measured in USD must adjust the return for currency fluctuations. The adjusted return is calculated as the product of the USD-measured private equity return and the currency change in the period. A depreciation (appreciation) of NOK relative to USD results in a relative increase (decrease) in return, since the portfolio assets in USD is worth relatively more (less) when transferred to NOK. The risk-free rate is subtracted to calculate the excess return.

Since the limited partners pay the general partners meaningful fees (Kaplan & Strömberg, 2008), it is vital to be aware of its magnitude. Phalippou (2009) and Phalippou, Rauch, and Umber (2018) estimate the fees for US buyout funds to be 6-7% on average of the invested capital. Further, between 2000 and 2010, Bienz, Thorburn, Walz (2016) found a “2-20-8”-fee structure of Nordic buyout funds, meaning 2% management fee and 20% carry for returns above a hurdle rate of 8%. This finding is consistent with the typical compensation structure found by Metrick & Yasuda (2010), who studied 238 funds raised between 1993 and 2006. They find that private equity funds typically also include a catch-up clause that significantly alters the fees for returns that are above the hurdle rate. The catch-up clause authorizes the general partner, once the hurdle rate is reached, to receive all profits until hitting a predetermined percentage of total profits, typically 20%. For Nordic private equity that generated post-fee returns of 11.1%, a “2-20-8”-fee structure with catch-up provision corresponds to annual fees at 4.8%[3]. However, in accordance with most existing literature and as a conservative measure to not understate pre-fee benchmark returns, we assume fees to be 6% p.a. This is in line with the fees employed by Stafford when replicating private equity return in the US. Hence, 6% is added to the post-fee returns provided by Argentum in order to get an estimate of the pre-fee returns. The table below reports the yearly Nordic private equity returns from 2006 to 2018 when adjusting for the estimated fees paid by the investor.

Period	2006 - 2007	2007 - 2008	2008 – 2009	2009 - 2010	2010 - 2011	2011 - 2012	2012 - 2013	2013 - 2014	2014 - 2015	2015 - 2016	2016 - 2017	2017 - 2018
Post-fee return (USD)	38.2%	12.9%	-35.9%	18.1%	65.6%	-2.2%	12.1%	28.1%	-11.7%	-8.8%	14.0%	8.6%
Currency USD/NOK	-3.5%	-14.4%	24.3%	1.4%	-15.9%	10.6%	-2.5%	3.1%	29.1%	6.4%	1.8%	-4.1%
Post-fee return (NOK)	33.2%	-3.9%	-20.2%	19.7%	39.2%	8.2%	9.3%	32.0%	14.0%	-3.0%	16.1%	4.1%
Pre-fee return (NOK)	39.2%	2.1%	-14.2%	25.7%	45.2%	14.2%	15.3%	38.0%	20.0%	3.0%	22.1%	10.1%

Table 4: Nordic Private equity returns with adjustments for fees and currency
The table reports post-fee and pre-fee excess return of the private equity benchmark, with adjustments for currency fluctuations in the period. All returns are measured in excess of the three-month US Treasury bill return.

This paper seeks to examine if a portfolio of public firms sharing similar characteristics as buyout targets, achieves returns similar to what private equity funds generate before fees. However, it also seeks to investigate whether the returns earned by an investor of the replicating portfolios is similar to the returns yielded by an investor with a diversified allocation to Nordic private equity. As the premise of this paper is twofold, both pre-fee and post-fee returns are applied in the analysis. Overall, in our investment period, private equity has delivered an annualised pre-fee and post-fee excess return of 17.2% and 11.1%,

[3] Distribution of returns given a post-fee return of 11.1% and a “2-20-8”-fee structure. Note that the example provided is simplified and for estimating purposes only.

Fee type	Management fee	Hurdle rate at 8%	Catch-up carry	80/20 split to LP/GP	Total distribution
Return interval	**Always charged**	**2% - 10%**	**10 – 12%**	**12% - 15.9%**	**15.9%**
Return to limited partner LP	0%	8%	0%	3.1%	**11.1%**
Fees to general partner GP	2%	0%	2%	0.8%	**4.8%**

respectively, with an annualised standard deviation of 17.2%. This corresponds to a pre-fee and post-fee annualised Sharpe-ratio of 1.00 and 0.65, respectively.

	Pre-fee returns	**Post-fee returns**
Annualised excess return Private equity index	17.2%	11.1%
Annualised standard deviation	17.2%	17.2%
Sharpe ratio	1.00	0.65
Market beta	0.94	0.94
Alpha	10.4%	4.4%

Table 5: Key performance metrics for the Nordic private equity benchmark 2006–2018.
Pre-fee numbers represent the performance of private equity funds before taking fees into account, while the post-fee numbers are the numbers relevant for investor net of fees. All returns are measured in excess of the three-month US Treasury bill return. Sharpe ratio is measured as the annualised pre-fee excess return divided by the standard deviation of the portfolio. The market beta represents how the benchmark is responding to swings in the general market. The alpha expresses the excess return of the private equity benchmark relative to the return of the general market.

3. Identifying investment universe: From raw data to the selected sample

In this section, we move from raw data to the selected data. Raw data entails all available data, while the selected sample consists of data that we treat in the analysis. In our case, the selected sample can be seen as the public investment universe for private equity; public stocks and buyout targets that share the same characteristics. We hypothesize that only a fraction of public stocks has the same characteristics as buyout targets. Similarly, we expect that only a fraction of buyout targets shares the same attributes as public stocks, as illustrated in the figure below. We start the section by presenting the methodology for identifying the investment universe.

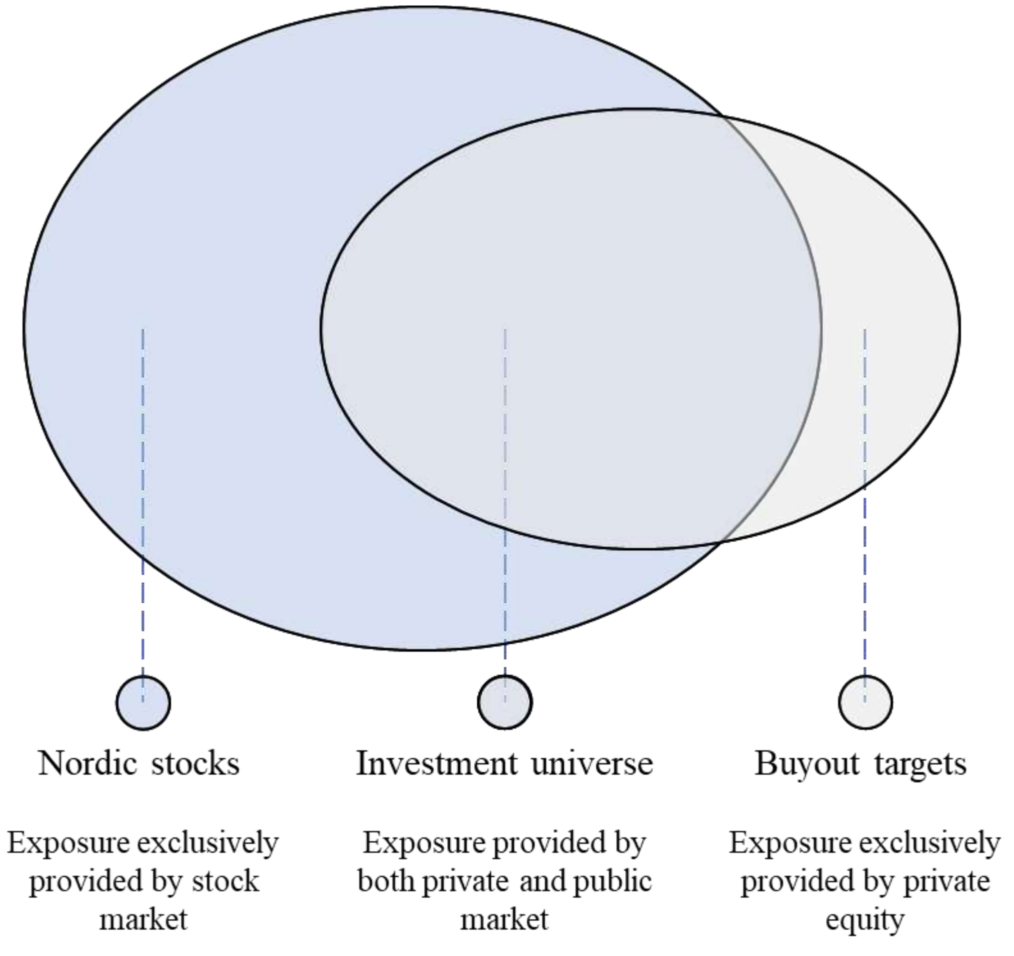

Figure 1: The investment universe.
We expect only a fraction of both the private market and public market to be overlapping with respect to our chosen characteristics. This fraction represents the investment universe in which our replicating portfolios can select stocks. The word "exclusively" in the figure expresses the degree to which companies are offered in the market, taking only the stock market and private equity market into account. Note that the figure is for illustrative purposes only.

3.1 Methodology: From raw data to the selected sample

Two fundamental requirements need to be satisfied in order to identify an investment universe and facilitate a strong replication of a private equity portfolio. Firstly, there need to be some identifiable patterns in the asset selection choice of Nordic buyout funds. Such characteristics can be many and can include factors related to risk premiums, but also characteristics that do not have systematic risk exposure. Secondly, there needs to exist companies in the stock market that share the same characteristics as buyout targets. Given that private equity funds tend to select companies sharing specific characteristics, and these are transferable to the stock market, this may contribute to a robust replication. Consequently, our methodology for going from raw to selected data is split into two phases. The first considers the method for identifying patterns in the asset selection of Nordic buyout funds, while the second phase relates to identifying similar companies in the Nordic stock market.

Considering the first phase, asset selection choices of buyout funds have little empirical evidence to support our analysis. Most studies have examined data related to private equity investments at the fund level. These studies include sector allocation, leverage and cash inflows and outflows at the fund level. Some studies (Chingono & Rasmussen, 2015; Stafford, 2016) have investigated public equities taken private, but these are limited to the US. These studies suffer as investments in public firms are not fully representative of a typical buyout deal. For example, most of the public targets are significantly larger than private targets. Since we are reliant on private targets in the Nordics, our sample is different from most of the existing literature.

Existing research (i.a. Gottschalg, Hadass, Talmor, & Vasvari, 2013) states that the industry sector and firm size may be necessary for explaining private equity returns and risk. As the asset selection choice of private equity funds can change from year to year, we include the year as a dummy variable to control for prevailing market conditions in the deal year. Industry sector as a selection criterion will enable the replicating portfolios to reflect the sector mix of a representative private equity portfolio. It will yearly change in alignment with the buyout industry's composition, ensuring that our portfolio captures the trends in the asset class. Furthermore, Argentum's report (2018) shows that the average deal size in the Nordics is relatively small, despite a clear trend for increasing average deal size the last decade. The deal size and trend are consistent with data covering other geographic regions, e.g. the US (Phalippou, 2014; Linley, 2019). Literature (i.a. Daniel & Titman, 2006; Stafford, 2016) has found that company size is statistically reliable for private equity-selection and its returns. In respect of the latter, size as a risk factor and related to subsequent returns has strong support from the well-established Fama-French 3-Factor Model (Fama & French, 1973). Furthermore, a study by Chingono and Rasmussen (2015) interestingly found that improving asset turnover, measured as the ratio between the percentage sales growth and the percentage asset growth, has statistically power explaining private equity returns. Such asset turnover improvements can be caused by increased revenue that may imply better market conditions for the firm's products, in addition to more efficient operations.

Further, previous studies have found other characteristics that can be reliable predictors of private equity-portfolios. In addition to firm size, Stafford (2016) finds that operating cash flow (EBITDA) multiple is highly reliable for private equity-selection and its subsequent returns. In his paper, the multiple is more potent to source a value premium in companies than the book-to-market ratio, which is a more commonly used metric in the literature for the same purpose (Stattman, 1980; Rosenberg, Reid, & Lanstein, 1985; Fama & French, 1993). Unfortunately, as the private firms included in our sample are not publicly tradable, we are unable to include any market-based variables that potentially are relevant in capturing common characteristics of buyout targets.

Our process of identifying common characteristics across buyout targets will be based on variables presented in the existing literature. However, as the literature is limited, we argue that there is a value related to finding new, potentially relevant variables in our Nordic data set. Common characteristics are identified by comparing summary statistics of buyout targets with corresponding data on public stocks. Our main hypothesis is that the size of a representative buyout target is relatively smaller than the typical stock and that a buyout target typically operates within some specific industry sectors.

The second phase of our methodology for identifying the investment universe is to select stocks sharing the same characteristics as found in the first phase. To account for size variations, we select all stocks with annual revenues ranging between the 20%-percentile and 80%-percentile of the size of buyout targets. Since the size of a typical deal may vary significantly over the years, we adjust the selection for yearly variations by selecting stocks based on buyout activity the previous year. We further restrict the sample by only selecting stocks operating within one of the four most common sectors in buyout transactions. As the funds' targeted sectors vary over time, we adjust our selection, so it accounts for yearly variations in sector allocation. Matching on size and industry sector, we form an investment universe which we can restrict further by adding more criteria based on the asset selection characteristics found in the first phase.

3.2 Asset selection of Nordic private equity funds

The discussion in the last section suggests that there may be some pattern in the asset selection choices of private equity funds. In this section, we will study if these asset characteristics are relevant for identifying buyouts in the Nordics. Additionally, as we handle with a sample that has been an object for a limited amount of research, we will examine if there exist other asset characteristics that separate Nordic buyout targets from a Nordic stock. At first, we will create an investment universe based on size and sector, before continuing the discussion with other characteristics that are relevant for the Nordic market.

3.2.1 Size

The average deal size in the Nordic buyout market has been rising significantly in the last decade. Table 6 illustrates how the market for private equity deals has emerged since 2005. There is substantial volatility in the deal size from year to year, mainly due to the relatively low number of transactions in the Nordic buyout market. However, there is a clear trend towards increased deal size. Measured in yearly revenues, the size of a typical deal has increased from $10 million before 2007 to $30 million from 2015. In the same period, we also observe an increased dispersion in size of buyout targets. These findings are consistent with Argentum's research (2018) on the Nordic market, in addition to a McKinsey report (2018) covering the global market for private equity. According to McKinsey, the main driver for increased deal size is multiple expansion, making each deal more costly.

Further, the table shows two crucial insights regarding variation in the size of Nordic stocks compared to buyout targets. Firstly, it reveals that Nordic buyout funds are seeking companies substantially smaller than the typical stock. The Nordic stock market also has a much higher absolute dispersion in size than the private market. These findings show that to replicate the buyout industry successfully, we are constrained to select stocks from only a fraction of the Nordic stock market.

Secondly, during the last decade, there has been a clear sign of convergence between the typical buyout target and the typical stock. While the typical Nordic buyout target has increased in size, the typical Nordic stock, although higher dispersion, is slightly declining in size. The median Nordic stock's revenue has reduced from $85 million in 2005 to $62 million in 2016, while the 20%-quantile has decreased from $14 million to $4 million in the same period. The main driver for this is the introduction of exchanges like First North that have given smaller companies access to financing through an open market.

This development has exciting implications on how large fraction of the stock market we are entitled to invest in when replicating Nordic buyout funds. Given considerable differences in size between the public and private market, these findings point that it will be necessary to take size into account to replicate Nordic buyout targets successfully. To account for size variations, we select all stocks with annual revenues ranging between the 20%-percentile and 80%-percentile of the size of buyout targets. As the size of a typical deal varies significantly over the years, we adjust the selection for yearly variations by selecting stocks based on buyout activity the previous year. The number of stocks sharing similar size as a representative buyout target has increased significantly in the last decade, growing from 206 in 2006 to 338 in 2016. The drivers for growth are both an increased number of small stocks at Nordic exchanges and increased dispersion in the size of buyout targets.

Sample	Year	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Buyout firms	n	3	9	30	59	33	50	49	38	29	52	44	25
	Median	13.3	1.8	8.7	2.2	2.9	2.2	9.5	19.4	33.2	26.2	36.3	22.5
	P(0.20)	8.2	0.0	0.3	0.1	0.0	0.1	0.1	5.1	6.5	8.9	9.8	3.7
	P(0.80)	19.0	30.2	50.3	23.2	32.7	20.0	39.5	55.7	72.6	84.7	127.9	75.2
	Max	22.7	85.0	12,339	120.2	259.0	199.8	281.0	278.2	10,953	569.7	2,979	388.7
Nordic stocks	n	552	623	683	754	764	817	842	836	867	900	952	1027
	Median	85.6	79.5	88.2	92.6	98.9	82.7	81.7	78.8	79.8	73.5	68.1	62.5
	P(0.20)	13.9	11.4	10.8	11.9	12.0	9.4	8.3	7.3	6.4	5.9	4.6	3.7
	P(0.80)	686.6	571.9	617.1	622.5	683.5	575.3	567.4	586.0	582.6	531.8	554.0	532.8
	Max	45,153	60,627	66,129	89,282	61,513	73,901	87,249	60,272	59,088	47,418	96,542	57,778
# Stocks satisfying size criterion		206	239	257	202	226	206	295	224	246	265	317	338

Table 6: Distribution of annual revenues for our sample of buyout targets and Nordic stocks.

The table reports the median, 20%-percentile P(0.20), 80%-percentile P(0.80) and maximum revenue by our sample of buyout targets and Nordic stocks in the time period. Companies having zero revenues are excluded from the analysis. Further, the last row of the table evaluates the number of stocks included in the investment universe, counting all stocks with a size in between the 20%- and 80%-percentile of buyout targets the given year.

Plotting the cumulative share of companies subject to their revenue, Figure 2 illustrates the large gap in size between buyout targets and stocks. The orange line in the figure shows that the 80%-percentile of buyout targets, representing the upper limit in our size criterion, corresponds to the 45%-percentile of the public market. This implies that 55% of the Nordic stocks have a size larger than eight out of 10 buyout targets. Further, more than 40% of public companies have a larger size than 90% of Nordic buyout targets. Thus, by investing in private equity or our replicating portfolios, the investor lacks the exposure to the big companies exclusively provided by the exchanges.

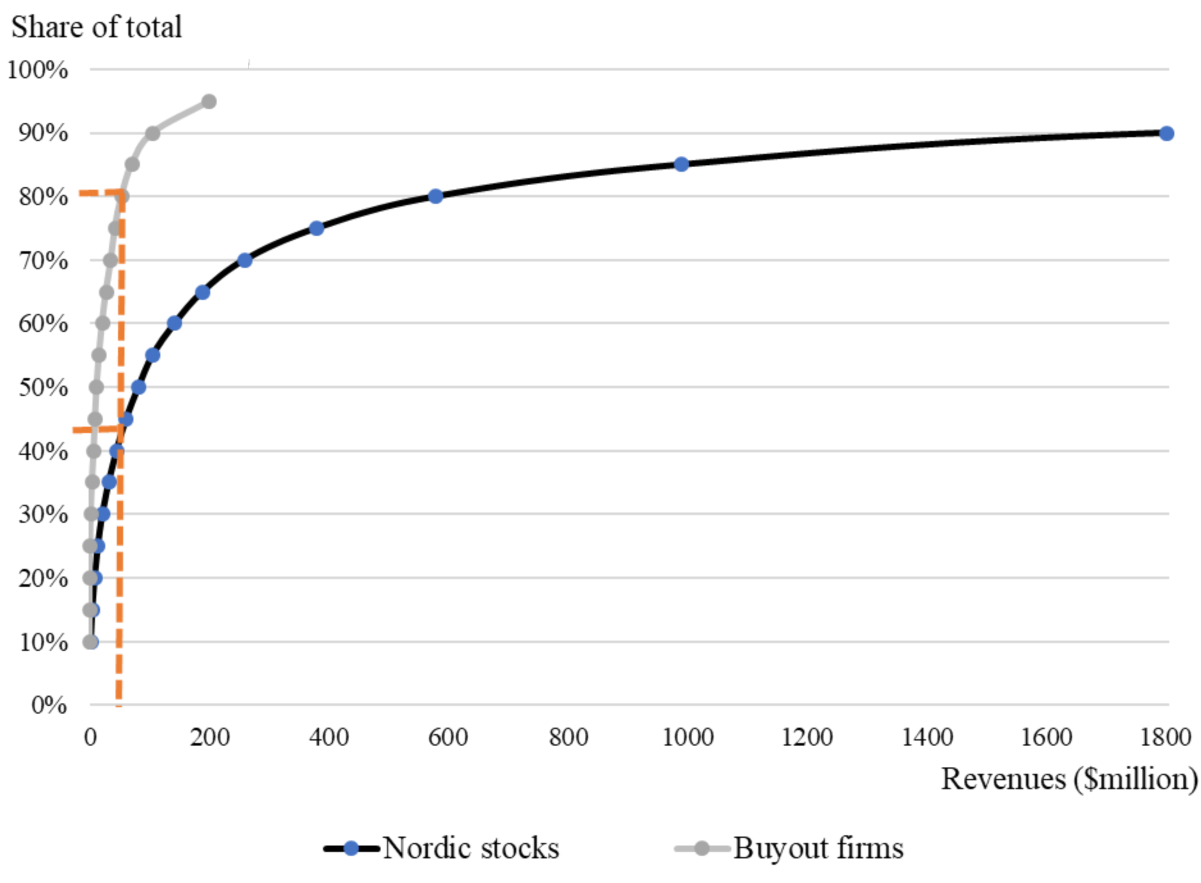

Figure 2: Cumulative share of companies below a given size.
Size is measured as yearly revenues. The black line represents Nordic stocks, while the grey line represents buyout targets. The dotted line expresses that the size of the 80%-percentile of buyout targets is equal to the 40%-percentile of Nordic stocks, at around $50 million in yearly revenue.

However, studying the cumulative share of companies given size below $25 million, Figure 3 shows that the private equity market offers an almost exclusively exposure to companies with annual revenue below $2 million. The figure reveals that 3 of 10 buyout targets have revenue below the 10%-percentile of Nordic stocks, implying that public market portfolios offer limited exposure to this part of the private equity market. By investing in the public market, the investor, thus, to a great extent, lacks exposure to one-third of the private equity market. This share represents the part of private equity that cannot be fully replicated through a replicating portfolio investing in the stock market. However, we observe that the slopes of the curves are approaching each other right after this point, reflecting the stock market's ability to replicate private equity in terms of size above the 30%-percentile.

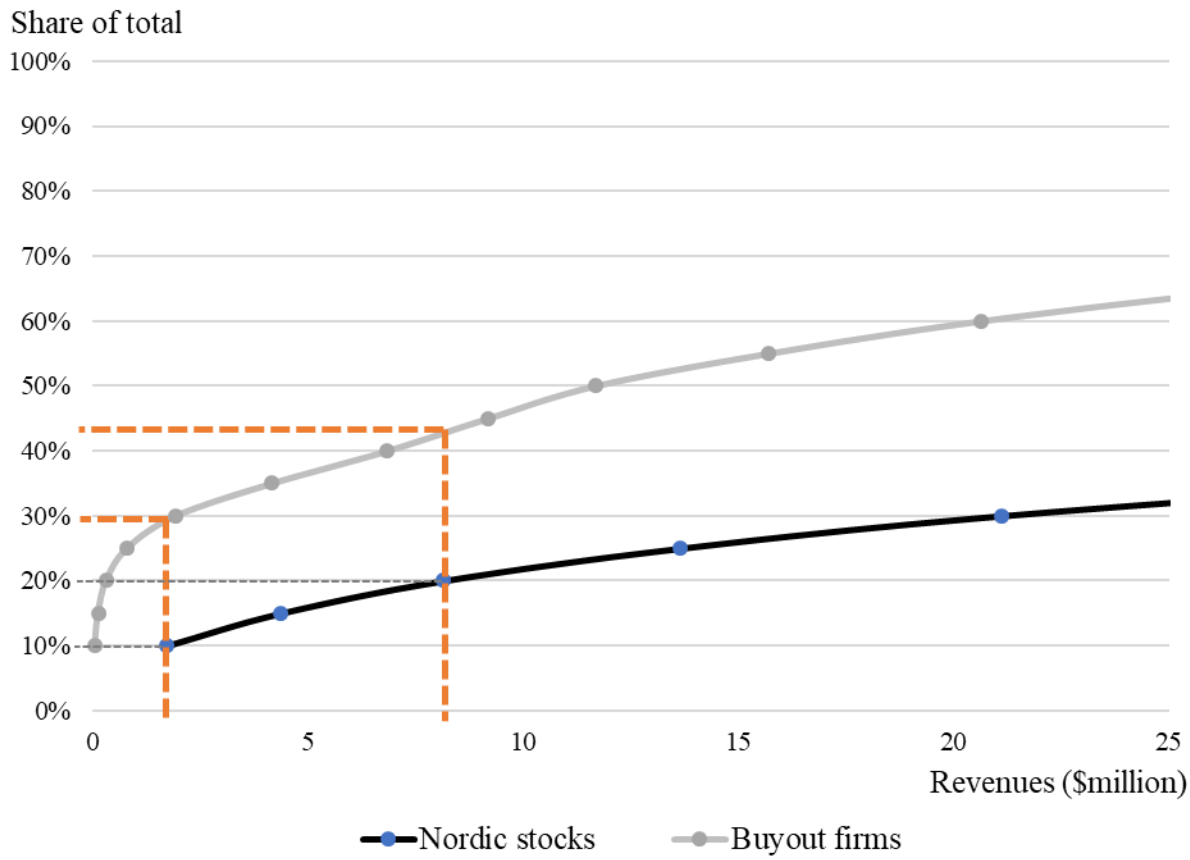

Figure 3: Cumulative share of companies below a given size. Capped at $25 million.
Size is measured as yearly revenues. The black line represents Nordic stocks, while the grey line represents buyout targets. The dotted lines express how the 30%-percentile and 44%-percentile of private equity firms correspond to the 10%-percentile and 20%-percentile of Nordic stocks, respectively.

3.2.2 Industry sector

As seen in the previous section, the buyout industry is seeking companies with a relatively small size compared to the stock market. Another critical characteristic of Nordic private equity funds is how they target funds towards specific sectors of the economy. Since 2005, Nordic buyout transactions have been tilted towards specific sectors like industrials, information technology, energy and consumer discretionary. Industrials stands out as the preferred sector for Nordic buyout funds, accounting for more than 30% of the total number of buyouts in the Nordics since 2005. Buyouts within information technology have experienced a slight drop in popularity recently, but are still accounting for about 20% of all buyouts in the Nordics. In contrast, consumer discretionary has experienced a positive development since 2007, almost doubling their share of total buyouts from 10% to 20%. Nordic buyout funds apparent tilt towards specific sectors is interesting in the perspective of mimicking a typical buyout portfolio.

Industrials and information technology seem to be the two most influential sectors both in the Nordic private equity and the Nordic stock market. However, while accounting for more than 60% of all Nordic buyout transactions in the last decade, these two sectors only account for about 30% of the Nordic stock market. Thus, the private equity market seems more concentrated in terms of sector choice. The table below confirms that a typical Nordic buyout portfolio is deviating from the index in terms of sector allocation.

As Nordic private equity funds tend to pick companies within specific sectors, a mimicking portfolio should have restrictions in terms of which sectors they select stocks. Hence, we choose all stocks operating within one of the four most common sectors in buyout transactions. As the funds' targeted sectors vary over time, we adjust the selection, so it accounts for yearly variations in sector allocation. Table 7 shows how the number of stocks satisfying the sector criterion is increasing from 321 in 2006 to more than 687 in 2017. An increased number of stocks at Nordic exchanges combined with better data availability on type of sector is driving the sharp rise.

Sample	**Year**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**2015**	**2016**	**Mean**
Buyout	Industrials	33.3	22.2	40.0	28.8	30.3	24.0	32.7	34.2	24.1	28.8	31.8	52.0	31.9
	Information Technology	33.3	11.1	20.0	27.1	36.4	28.0	18.4	23.7	20.7	9.6	18.2	16.0	21.9
	Consumer Discretionary	33.3	22.2	10.0	06.8	12.1	10.0	20.4	7.9	13.8	19.2	18.2	16.0	15.8
	Energy	0	22.2	16.7	15.3	12.1	22.0	06.1	21.1	27.6	19.2	9.1	12.0	15.3
	Others	0	22.2	13.3	22.0	09.1	16.0	22.4	13.2	13.8	23.1	22.7	04.0	15.2
Stock market	Industrials	27.3	20.8	15.8	26.1	20.4	30.6	23.9	15.8	28.9	22.8	36.7	32.3	24.0
	Information Technology	13.3	10.9	7.6	13.3	9.9	15.4	13.0	20.3	15.4	12.2	22.1	19.8	13.8

Consumer Discretionary	6.9	6.1	10.2	7.4	5.6	10.0	8.1	12.5	9.8	7.3	14.8	12.4	9.0
Energy	4.3	4.3	6.9	5.6	4.0	7.8	6.0	9.2	7.9	13.0	9.8	7.4	6.8
Others	48.2	57.9	59.5	47.6	60.1	36.2	49.0	42.2	38.0	44.7	16.6	28.1	46.4
# stocks satisfying industry-sector criterion	321	356	388	410	413	451	492	468	498	552	609	687	321

Table 7: Industry sector distribution for our sample of buyout funds and Nordic stocks.
All numbers in percent (except second last row). The number of stocks included in the investment universe counts all traded stocks with industry sector equal to one of the four most prevalent sectors among buyout targets the given year. The percentage in the last row expresses the number of stocks in the investment universe in relation to the total number of stocks in the Nordics (including stocks with no information about sector).

3.2.3 Identifying the investment universe and other characteristics

The reasoning beyond points out that Nordic buyout funds target small firms with a running revenue of less than \$80 million, mainly within industrials and information technology. As seen in Table 8, 32.3% and 38.0% of Nordic stocks in the investment period satisfy our size and industry-sector criteria, respectively. Combining the size criterion with the sector criterion, we identify an investment universe consisting of 365 unique stocks, as seen in equation iii) in the table below. These 365 stocks thus represent the part of the Nordic stock market having the same characteristics as Nordic buyouts in terms of size and sector. However, as previous research demonstrates, more variables potentially can explain the investment behaviour of buyout funds. Thus, we want to use this investment universe of 365 stocks to identify if there exist other characteristics that differentiate a typical Nordic buyout target from Nordic listed companies. Table 9 reports key statistics for buyout targets and Nordic stocks in aspects of four different performance metrics.

If N = Number of stocks traded at Nordic exchanges from 2006 to 2018,

P(Size) = Probability that a stock satisfies the size-criterion

P(Industry) = Probability that a stock satisfies the industry sector-criterion

P(Industry | Size) = Probability that a small sized stock satisfies the industry sector-criterion,

Then the size of the investment universes are:

i) Number of stocks satisfying size-criterion, N(Size)

$$\mathrm{N(Size)} = \mathrm{N} \times \mathrm{P(Size)} = 2{,}049 \times 32.3\% = 661 \text{ stocks}$$

ii) Number of stocks satisfying industry sector-criterion, N(Industry)

$$\mathrm{N(Industry)} = \mathrm{N} \times \mathrm{P(Industry)} = 2{,}049 \times 38.0\% = 778 \text{ stocks}$$

iii) Number of stocks satisfying both size-and sector-criterion, N(Size ∩ Industry)

$$\mathrm{N(Size \cap Industry)} = \mathrm{N(Size)} \times \mathrm{P(Industry \mid Size)} = 661 \text{ stocks} \times 55.1\% = 365 \text{ stocks}$$

Table 8: Calculating the size of the investment universe.

The size of the investment universe is calculated using conditional probability. It shows the total number of unique stocks that satisfy the criteria during the investment period from 2006 to 2018.

Comparing this public investment universe, N(Size ∩ Industry), with our sample of buyout targets, we find that buyout targets tend to have a higher leverage ratio and a higher working capital turnover than a listed company that has similar size and sector. The higher leverage ratio, which will be closely examined in the next section, is consistent with prior research (Axelson et al., 2013) that shows how private equity funds are implementing debt to their portfolio companies to boost returns. The relatively high working capital turnover ratio may be rooted in private equity funds search for capital-efficient companies that have a limited amount of capital tied to the operations. However, since our numbers are based on the year-end post-transaction company, it may also be due to private equity funds making changes in the working capital level in the months following the transaction.

Additionally, we find that a typical buyout target has a lower asset growth turnover than an average stock at the time of the buyout. In light of the study by Rasmussen and Chingono (2015), this is an interesting result. They found that improving asset turnover has statistically power explaining private equity returns. In combination, this may indicate that buyout funds are creating value by buying low asset-turnover-firms and make improvements in terms of revenues and efficiency in operations to improve the turnover (Akguc, Choi, & Kim, 2015). Furthermore, we also find weak evidence for buyout targets having a slightly lower EBITDA than a typical stock with similar revenue. This finding is consistent with Vinten (2007), suggesting the expectation that private equity firms are more willing to acquire low-profitability targets due to their expertise on operational improvements and consequent capability to increase target profitability.

		Nordic buyout targets				**Comparable Nordic stocks**			
	Period	**n**	**Median**	**P(25%)**	**P(75%)**	**n**	**Median**	**P(25%)**	**P(75%)**
Asset growth turnover	2005-2012	57	0.26	-0.39	1.14	493	0.60	-0.31	1.77
	2013-2016	95	0.25	-0.41	0.81	296	0.52	-0.11	1.35
Working capital turnover	2005-2012	261	3.30	-0.10	7.66	486	2.44	0.56	4.83
	2013-2016	143	6.87	3.01	13.59	339	3.96	1.24	9.21
EBITDA	2005-2012	243	0.32	-0.22	2.40	533	0.54	-0.39	2.50
	2013-2016	187	2.92	1	7.02	169	4.49	1.80	8.43
Debt-to-equity-ratio	2005-2012	235	1.07	0.35	3.50	434	0.22	0.07	0.71
	2013-2016	183	0.59	0.10	1.71	309	0.58	0.15	1.21

Table 9: Performance metrics of buyout targets and Nordic stocks.

Asset growth turnover is calculated as Revenue growth / Asset growth. Working capital turnover is calculated as Revenue / Working capital. The metrics are calculated separately for buyout targets and comparable Nordic stocks over two different time periods (2005-2012 and 2013-2016). Comparable Nordic stocks include all stocks that satisfy both the size selection criterion (size in between the 20%- and 80%-percentile of buyout targets) and the industry sector criterion (sector equal to one of the four most prevalent sectors among buyout targets) in the given year.

4. Methodology for replicating private equity asset selection, strategies and leverage

The last section, showing the asset selection choices of private equity, including the tilt towards small stocks within specific sectors in the economy, are crucial insights to take into account when constructing a replicating portfolio. The same applies to leverage choices of private equity and the differences in market structure between a public and a private market. This section presents our methodology in these aspects, starting with our method for replicating asset selection.

4.1 Replicating asset selection

The discussion in the last section points out that many variables potentially are relevant for replicating the asset selection of the buyout industry. Therefore, it is interesting to construct several portfolios, based on different combinations of characteristics, to identify the added value of each. However, the inclusion of more variables that increases the identification power of a representative buyout target comes at the cost of a reduction in the investment universe. Mainly, this is a concern in the Nordic stock market compared to larger samples, such as the US public market. As discussed, our sample of listed firms consists of relatively few companies comparable to typical private equity-backed firms, particularly in terms of firm size. Since our investment universe is relatively small, the trade-off cost is higher, making our sample more vulnerable for adding more reliable characteristics.

Based on the discussion in the previous section, we construct six portfolios, as seen in the table below. Portfolio 1 is restricted to only invest in companies with a similar size as Nordic buyout targets. This investment universe accounts for more than 661 unique stocks in the investment period from 2005 to 2017. Portfolio 2 adds the sector as an additional selection criterion, reducing the number of stocks in the investment universe to 365 in total, as calculated in Table 8 in the last section. The third portfolio is further restricted by only investing in stocks with an EBITDA similar to a typical buyout target, further declining the number of stocks in the investment universe to 245. Portfolio 4 makes asset selection based on size, sector, EBITDA and asset turnover, representing a universe of 141 stocks. The fifth portfolio, replacing asset turnover with working capital turnover, consists of 187 unique stocks in the investment period. Finally, portfolio 6 selects stocks based on size, sector, EBITDA and leverage, accounting for a total of 87 stocks. Thus, there is a clear trade-off between the number of selection criteria and the number of stocks included in the portfolio, whereby an increased number of criteria is reducing the number of stocks in the portfolio significantly.

We construct the portfolios so that they in each year from 2006 to 2017 invest in stocks that satisfy all the given criteria applicable for the different portfolios. All of the selection criteria are formed based on the identified asset selection choice of Nordic private equity in the year prior to the investment. Moreover, due to lags in the firms' annual reports, the portfolios are formed in June each year. This delay prevents look-ahead bias.

	Portfolio 1	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**
Size	✓	✓	✓	✓	✓	✓
Industry sector		✓	✓	✓	✓	✓
EBITDA			✓	✓	✓	✓
Asset Turnover				✓		
Working cap. turnover					✓	
Debt-to-equity ratio						✓
Number of stocks	661	365	245	141	187	87
Share of total stocks	32.3%	17.8%	12.0%	6.9%	9.1%	4.2%

Table 10: The asset selection choices underlying each replicating portfolio.
The size criterion selects all stocks with a size between the 20%- and 80%-percentile of buyout targets' size the previous year. The sector criterion selects all stocks operating in the same industry sector as the four most prevalent sectors among buyout targets. Further, the asset turnover criterion, working capital turnover criterion and debt-to-equity-criterion are all capped at 20%- and 80%-percentile, selecting all stocks falling within that range. The last row expresses the number of unique stocks included in each portfolio during the investment horizon.

4.2 Replicating private equity market restrictions

The last two sections shed light on how we select stocks based on the asset selection choices of Nordic private equity players. In this section, we consider how we want to invest these portfolios and which market structure restrictions that govern them. The main idea is that the private equity market imposes many restrictions in terms of how funds can invest. This contrasts to the public market, where the same restrictions are almost non-existing.

Firstly, the active management of private equity and the relatively high transaction costs compared to the public market set restrictions concerning holding time. Bain & Company (2019) finds that the median holding period of a global private equity fund varied between four and five years in the last decade. By introducing higher liquidity, lower transaction costs and a passive management approach, the stock market involves fewer constraints in terms of holding time.

Secondly, the private market differs from the public market concerning the feasibility for rebalancing the portfolio throughout time. The buyout industry doubtfully characterizes as a particularly stable asset class since changes in sector mix, average deal size, etc. can significantly alter the weights of new investments compared to previous ones. Additionally, "winning-assets" in the portfolio will make up a larger share of the overall portfolio if not being divested or rebalanced. Consequently, the asset allocation of a typical private equity portfolio in the entry year can change significantly from the allocation in the exit year. In the public market, it is easier to divest the winners and rebalance the portfolio throughout time.

Given how the private market differs from the public market in two critical dimensions, we will present four different investment strategies that account for these differences. The first investment strategy seeks to mimic the restrictions imposed in the private market concerning a long holding time and limited possibility of rebalancing. Precisely, in 2006, we will construct a five-year buy-and-hold strategy whereby the selected stocks will be held until 2011 without any rebalancing. In 2011, the portfolio will select new stocks that will be held for the next five years, before the process repeats again in 2016. In this way, we invest our portfolios within a strategy that underlies the same restrictions that govern private equity managers.

The second investment strategy maintains the limitation of a long holding time but incorporates rebalancing of the portfolio each year. In this way, we examine if the increased flexibility of rebalancing offered by the public market relative to the private market adds value to the portfolios. We create dynamic portfolios in the period 2006 to 2018 that held stocks for five years and rebalance yearly. The third investment strategy mirrors the second but is incorporating monthly rebalancing instead of yearly rebalancing. Finally, the fourth investment framework seeks to eliminate the restrictions related to both holding time and rebalancing. Consequently, we create dynamic portfolios in which we each year invest in stocks that satisfy the criteria for being selected to the different portfolios. This implies that stocks that no longer fulfil the selection criteria are being divested, independent of how long they have been in the portfolio. All stocks in the portfolios under the fourth investment strategy are rebalanced yearly. Table 15 below summarizes the four different approaches.

Market restriction	**Investment strategy 1**	**Investment strategy 2**	**Investment strategy 3**	**Investment strategy 4**
Supply of new stocks	Each 5th year	Each year	Each year	Each year
Holding time	5 year	5 year	5 year	Minimum 1 year
Frequency of rebalancing	Each 5th year	Yearly	Monthly	Yearly

Table 11: Key characteristics of the different investment strategies.
Supply of new stocks expresses the frequency of which new stocks are added to the portfolio according to the asset selection criteria underlying it. The first strategy adds stocks with a frequency of five years (2006, 2011, 2016), while the other strategies implement a yearly frequency. Holding time is the length of the period in which the stocks are held in the portfolios. Lastly, the frequency of new stocks expresses how often the portfolios are rebalanced to achieve an equal-weighted portfolio.

4.3 Replicating leverage

Our study of Nordic buyout targets in Section 3 identified a pattern to which buyout targets tend to be relatively highly leveraged at the end of the entry year. Such a pattern may be due to private equity targeting highly leveraged firms. However, it may also be due to private equity managers adding leverage to their portfolio companies after they have been bought. The table below provides more clarity by studying the development in the debt-to-equity ratio in the years before and following a buyout transaction.

	Number of observations	**Median D/E-ratio**	**25%-percentile D/E-ratio**	**75%-percentile D/E-ratio**
Targets 2 years before buyout	104	0.58	0.22	1.43
Targets 1 year before buyout	160	0.59	0.18	1.45
Targets in buyout year	**418**	**0.82**	**0.21**	**2.44**
Targets 1 years after buyout	418	0.81	0.24	2.28
Targets 2 years after buyout	417	0.81	0.24	1.99
Targets 3 years after buyout	368	0.75	0.24	1.85
Comparable public stocks	**335**	**0.62**	**0.18**	**1.32**

Table 12: Distribution of the debt-to-equity (D/E-ratio) ratio of buyout targets in the years before and after the transaction.
The row "Comparable public stocks" here represents stocks comparable to Nordic buyout targets in terms of size and industry sector, meaning our initial investment universe N(Size ∩ Industry). Number of observations represents how many companies in which we have available data on the debt-to-equity ratio.

The table reveals three interesting findings. Firstly, in the years before the buyout transaction, buyout targets tend to be slightly less leveraged than similar-sized stocks operating within the same industry sectors. This observation denies the hypothesis that private equity funds are seeking highly leveraged companies. Secondly, during the year of entry, buyout targets tend to increase their leverage significantly, making them more leveraged than comparable stocks. As aforementioned, this corresponds well with previous research. Thirdly, the leverage ratio tends to be stable in the years following the transaction, suggesting that private equity funds are changing the capital structure early in the investment period.

As buyout targets after the time of entry tend to be more leveraged than comparable Nordic stocks, the returns of a private equity portfolio are more leveraged than an unlevered replicating portfolio. Hence, a private equity portfolio will tend to yield relatively high returns in good times and relatively low returns in bad times. To mimic the relatively high debt of private equity portfolio companies, we add leverage to the replicating portfolios. Leverage is added by modifying the median debt-to-equity ratio of the portfolio, using five different levels of imposed leverage, as presented in the table.

		Portfolio 1	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**
Median unlevered D/E-ratio		0.60	0.62	0.62	0.68	0.62	0.74
Leverage target		**Level of imposed portfolio leverage relative to unlevered**					
1. Unlevered D/E	As is	0%	0%	0%	0%	0%	0%
2. Private equity D/E	0.82	36.7%	32.3%	32.3%	20.6%	32.3%	10.8%
3. D/E = 1	1.00	66.7%	61.3%	61.3%	47.1%	61.3%	35.1%
4. ~1.5 x Unlevered D/E	1.02	70.0%	64.5%	64.5%	50.0%	64.5%	37.8%
5. ~2.0 x Unlevered D/E	1.36	126.7%	119.4%	119.4%	100%	119.4%	83.8%

Table 13: Median unlevered debt-to-equity ratio and level of imposed leverage applied in our sensitivity analysis

The first row expresses the median debt-to-equity ratio of the replicating portfolios during the investment period. For our sensitivity analysis of leverage, we will apply five different leverage levels, as explained by the column "Leverage target". Levels of imposed portfolio leverage are calculated as the ratio between the imposed portfolio D/E-ratio and the unlevered D/E-ratio. The first level for the sensitivity is the unlevered D/E-ratio of the portfolios; hence the imposed leverage is 0%. The second level implies imposed leverage so that the D/E-ratio matches the leverage level of buyout targets at D/E equal to 0.82. The third level represents a D/E-ratio equaling 1.0. The last two levels are calculated as 1.5x and 2.0x the D/E-ratio of the unlevered Portfolio 4. As Portfolio 6 are making asset selection based on D/E-ratio, this portfolio has higher leverage than the other portfolios.

Our approach thus uses the gap in debt-to-equity ratio between buyout targets and the stocks of our replicating portfolios to proxy the sufficient level of imposed leverage. Increased portfolio leverage contributes to amplify returns, applying the following formula:

Formula:	$$Leveraged\ return = \frac{(1 + Levered\ D/E)\ x\ Unlevered\ return}{(1 + Unlevered\ D/E)} - Interest\ costs$$
Applied example:	Assuming that • Levered D/E-ratio = 0.82 • Unlevered return = 18% • Unlevered D/E-ratio = 0.60 The formula gives $$Leveraged\ return = \frac{(1 + 0.82)\ x\ 18\%}{(1 + 0.60)} - Interest\ costs = 20.5\% - Interest\ costs\ in\ \%$$

Table 14: Calculation of leveraged return.
The unlevered D/E-ratio is reported in the first row of the same table. The unlevered return is the return of the portfolio without imposing leverage. Interest costs are the percentage cost associated with having a bank loan to finance the imposed leverage, see Table 15.

Interest rates represent the costs associated with having a bank loan to finance the imposed leverage. To assess for differences in interest rates, we will apply rates in the range between 0% and 9%. The importance of interest rates increases in line with rising leverage since the investor must pay interests on a larger bank loan.

Formula:	$Interest\ costs = Interest\ rate \times (Levered\ D/E - Unlevered\ D/E)$
Applied example:	Assuming that • Interest rate = 4% • Levered D/E-ratio = 0.82 • Unlevered D/E-ratio = 0.60 The formula gives $Interest\ costs = 4\% \times (0.82 - 0.60) = 0.88\%$

Table 15: Formula for calculating the interest rate.
The interest rate represents the yearly cost of having a loan in percentage terms. The interest rate is multiplied by the difference between the levered and unlevered D/E-ratio as the investor only needs to pay interests on the imposed leverage. This is due to that interest on the debt associated with the D/E-ratio of the unlevered portfolios are paid by the companies themselves.

4.4 Returns to the investor

The earnings earned by an investor with an allocation to the replicating portfolios are calculated as pre-fee returns subtracted transaction costs. In order to determine the constructed portfolios' trading costs, we will apply a sensitivity analysis based on multiple research reports. As estimated fees vary highly among research, we will consider a selection of studies viewed to be reliable in the context of our buyout sample, as well as representing both the lower and upper ground of costs found in research. For this purpose, we will use both purely academic research on small- and micro-cap stocks (Novy-Marx & Zekelman, 2019) and papers from practitioners analysing proprietary data (i.a. Hurst, Ooi, & Pedersen, 2017).

The level of transaction costs for a portfolio heavily depends on the liquidity of the selected stocks, size of assets under management and the turnover assumptions underlying the portfolios. Table 16 creates a fundament for calculating the portfolio trading cost.

Formula:	$$Portfolio\ trading\ cost\ = \sum_{i=1}^{n} \frac{Stock\ trading\ cost_i}{n} \times Portfolio\ turnover$$
Applied example:	Assuming that • Average trading cost per stock is 100 basis points (bps) • The portfolio liquidates all positions and buys new stocks to the portfolio so that the yearly turnover is 2.0. The formula gives $Portfolio\ trading\ cost\ = 1.0\% \times 2.0 = 2.0\%\ per\ year$

Table 16: Formula for calculating trading costs.
Stock trading costs represent the percentage costs for selling or buying a single stock, including bid-ask-spread, broker fees and potential market impact. Portfolio turnover expresses the frequency in which stocks are rebalanced in the portfolio, see Table 17.

Concerning the first part of the formula, trading costs for a single stock are highly dependent on the explicit costs of broker fees and the size of the bid-ask-spread. Novy-Marx and Zekelman (2019) analyse the effective bid-ask spreads from 1975–2016 for large institutional investors. Given his definition of small-cap firms, most of our investment universe can be considered as small-caps. For a trailing 12-month average, they find the small-cap firms' transaction costs to be approximately 30bps since 2000. This corresponds with Franzini, Israel and Moskowitz's research (2012) covering 19 countries (including the Nordics except for Finland) from 1998 - 2011, estimating small developed markets-caps trading costs to be around 20bps yearly.

Both studies, supported by most other studies (i.a. Menkveld, 2016) show that the trading costs of the small- and micro-cap firms declined massively after introducing the decimalization and widespread adoption of electronic trading at the turn of the century. Further, a small reduction has continued the last decade, illustrated by Novy-Marx and Zekelman (2019) finding from

2010 to 2015 an average of 27bps and 48bps for the small and microcap stocks, respectively, slightly lower than the estimated trading costs of 30bps and 50bps from 2000 to 2010.

In addition to brokerage fees and bid-ask-spread, trading costs for a single stock depend on the degree to which the investor move the market. The investor's influence on market prices is determined by the size of the assets under management and the liquidity of the stock. According to research from the hedge fund Verdad (Schmitz, 2019), a reasonably close to no-impact equals around $100-200 million for an equal-weighted levered small value strategy in the US. Verdad estimates the corresponding number for Europe to be half the ceiling in the US. Considering the Nordic stock market to be less liquid than the big European stock exchanges, we assume a ceiling here to equal $30 million. Simplified, this means funds or individuals having assets under management below $30 million will have close to no market impact.

For our purposes, we will apply a sensitivity analysis using average stock trading cost in the range of 0bps to 150bps, as presented in the figure below. The upper limit of our sensitivity analysis at 150bps is considerably higher than our findings from research, making it only representative in cases where the investor has a reasonable market impact.

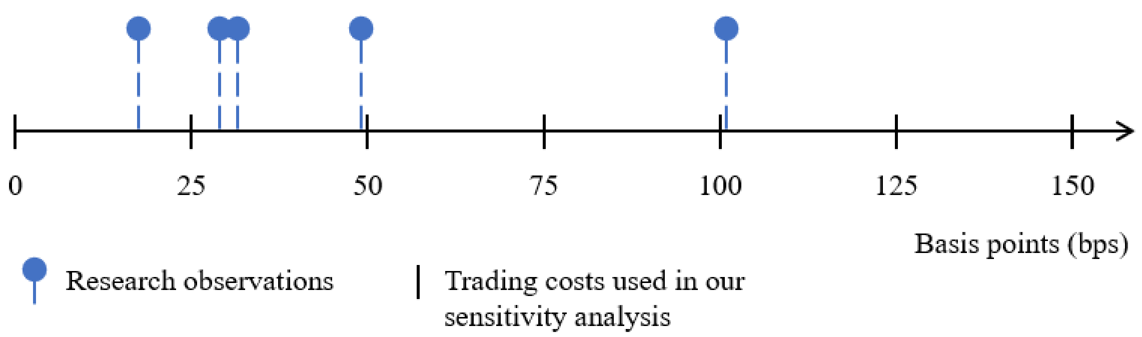

Figure 4: Applied estimates of annually trading costs used in the sensitivity analysis.
30bps and 20bps are from respectively Novy-Marx and Zekelman, and Frazzini, Israel and Moskowitz. Transaction costs of 49bps and 101bps are from research findings based on pure academia (Comerton-Forde, Gallagher Nahhas, & Walter, 2010; Chen, Goldstein, & Jiang, 2007). For our sensitivity analysis, we will apply trading costs at seven data points in the interval between 0bps and 150bps, as illustrated with the vertical black lines.

Concerning the second part of the formula for calculating portfolio trading costs in Table 17, turnover rates are sensitive to the frequency of rebalancing and replacement of stocks, as well as the dispersion in returns among stocks in the portfolio. Portfolio turnover is calculated by dividing the total amount of new stocks purchased each year by the total net asset value of the portfolio, as represented in the table below. For example, a portfolio selling 100% of the portfolio value and then buys new stocks for the same amount, generates a portfolio turnover of 2.0 (100% + 100%). Applying the formula in Table 17, we hypothesize that a monthly rebalanced portfolio will be loaded with trading costs higher than a corresponding portfolio rebalanced yearly. However, we expect the increase in turnover ratio to be less than proportional to the number of rebalancing events. Higher frequency of rebalancing may be correlated with less dispersion in stock returns, implying that a lower fraction of the portfolio needs to be rebalanced to achieve equal weights.

Formula:	The turnover for a single stock in period t is (Abs means absolute value): $Stock\ turnover\ in\ period\ t = Abs(Equal\ weight\ target_t - Market\ weight_t)$ The turnover of each stock must be summarized in order to get the portfolio turnover: $Portfolio\ turnover\ in\ period\ t = \sum_{i=1}^{n} Stock\ turnover\ in\ period\ t_i$
Applied example:	Assuming that • Equal weight target = 2% (given that the portfolio consists of 50 stocks) • Current market weight of stock is 3% due to high return in the previous period The formula gives $Stock\ turnover\ in\ period\ t = Abs(2.0\% - 3.0\%) = 0.01$ This turnover must be summarized across all portfolio stocks to get the portfolio turnover.

Table 17: Formula for calculating stock and portfolio turnover.
The portfolio turnover ratio measures the rate with which the assets under management have changed over the past period, either by rebalancing portfolio weights or liquidating/buying portfolio stocks.

Table 18, studying portfolio turnover on a non-levered portfolio, confirms our hypothesis. Increased frequency of rebalancing tends to increase portfolio turnover, implying a rise in transaction costs. A change in frequency from yearly to monthly rises turnover ratio from 0.5 to 1.6, as shown in the table below. Thus, a monthly rebalanced portfolio is charged with fees at a considerably higher level than a portfolio rebalanced less frequently, illustrating the cost associated with increased frequency of rebalancing. However, the finding also confirms our hypothesis that the increase in portfolio turnover is under-proportional to the frequency of rebalancing, as monthly rebalancing allows for periodically smaller adjustments than yearly rebalancing. Further, we observe that Strategy 1 requires the lowest amount of turnover, as expected from the strategy that implements the less frequent rebalancing and replacements of stocks.

Strategy	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Strategy 1	1.00	0	0	0	0	1.40	0	0	0	0	1.62	0
Strategy 2	1.00	0.39	0.36	0.43	0.41	0.41	0.49	0.49	0.66	0.50	0.43	0.43
Strategy 3	2.08	1.23	1.58	1.51	1.37	1.70	1.83	1.86	1.73	1.60	1.45	1.39
Strategy 4	1.00	0.39	0.37	0.43	0.44	0.42	0.42	0.41	0.40	0.37	0.34	0.34

Table 18: Portfolio turnover for different investment strategies.
The numbers for 2006 include the turnover implied by the initiation of the portfolio, whereby we buy 100% new shares. Turnover for our last year 2018, including a potential liquidation of portfolio assets, is excluded from the table.

To sum up this discussion of portfolio trading costs, we see that trading costs for a single stock lie in the range of 20bps and 101bps. Moreover, the yearly portfolio turnover varies from 0.34 to 2.06, dependent on the degree to which the portfolio is rebalanced. For our analysis, we will apply the actual numbers of portfolio turnover and a set of trading costs varying in the interval from 0bps to 150bps. Using our formula, we see that annual portfolio trading costs will vary in the range of 0.068% (20bps x 0.34) in the most optimistic case and 3.09% (150bps x 2.06) in the most pessimistic case. As a comparison, the hedge fund Verdad charges their clients 1.5% annually for a fund seeking to replicate private equity through investments in the stock market (Schmitz, 2019), giving support to our findings on portfolio trading costs. However, since there is not a robust market for private equity replication, this fee can not likely be considered as a proxy for market equilibrium net of fees. However, this can indicate what a constructed fund could charge their investors and provide additional support to our analysis of trading costs.

4.5 Illustration of portfolio design

Figure 5 illustrates the methodology as has been discussed in this section. Our methodology seeks to replicate the investment pattern of buyout funds by applying six portfolios that mimic the asset selection choices and taking into account the effect of additional leverage. Each portfolio is then invested using four distinct investment strategies, imposing different restrictions in terms of market structure. Six different portfolios, each invested in four different investment strategies, make a total of 24 different replicating portfolios. In this way, we seek to replicate a typical private equity portfolio concerning asset selection, leverage and investment restrictions, as summarised in Figure 5 below. We hypothesize that both increased accuracy in asset selection and a loosening of investment restrictions results in improved portfolio performance.

6 replicating portfolios: Increased accuracy in asset selection

Selection criteria	1	2	3	4	5	6
Size	✓	✓	✓	✓	✓	✓
Industry sector		✓	✓	✓	✓	✓
EBITDA			✓	✓	✓	✓
Asset turnover				✓		
Working cap. turnover					✓	
Leverage						✓

Adding leverage

Invested in 4 investment regimes: Less restrictions imposed

Market restriction	1	2	3	4
Supply of new stocks	Each 5th year	Each year	Each year	Each year
Holding time	5 year	5 year	5 year	Min 1 year
Rebalancing frequency	Each 5th year	Yearly	Monthly	Yearly

Figure 5: The three-pillar-based methodology for replicating Nordic private equity asset selection, market structure and leverage.

We will apply six different replicating portfolios which implement different selection criteria gathered from our and previous research on private equity asset selection. All portfolios are then exposed to different levels of leverage to mimic the private equity capital structure. The levered portfolios are then exposed to Nordic exchanges using four different investment strategies, setting different restrictions in terms of rebalancing, holding time of stocks and the frequency of buying new stocks.

5. Results and discussions

This section evaluates the performance of the replicating portfolios compared to the Nordic private equity benchmark. The section is divided into six parts. We begin by presenting how portfolio performance relates to changes in underlying market restrictions. After that, we discuss how portfolio performance is sensitive to the asset selection choices underlying the portfolio. Then we discuss how changes in imposed leverage and loan costs affect portfolio performance. Next, we study returns earned by a well-diversified investor by introducing transaction fees in both private equity and replicating portfolios. We follow with a discussion on how portfolio performance is sensitive to changes in the starting year of the portfolio. Finally, we present the strategy yielding the best performance during our investment period and discuss how the choice of return reporting process affects portfolio risk.

5.1 How market structure affects portfolio performance

A portfolio manager must adhere to an extended set of trading restrictions when investing in private equity compared to public stocks. Due to their active management and high transaction costs, private equity typically has relatively less frequent rebalancing and relatively longer holding times than what is standard in the public market. This section evaluates how different sets of trading restrictions, as presented in Section 4.2, affect portfolio performance.

Table 19 reveals insights regarding the effect of varying trading restrictions on portfolio returns. Firstly, we find that increased frequency of rebalancing tends to enhance portfolio returns. A monthly rebalanced strategy yields higher returns than a yearly rebalanced strategy across all portfolios, as represented by the difference in return between Strategy 3 and Strategy 2 in the table. Surprisingly, we also find that increased frequency of rebalancing is associated with reduced risk, measured by both standard deviation, maximum portfolio drawdown and the market beta of the portfolios. By periodically rebalancing, we avoid that "winning-stocks" in the portfolio account for a larger share of the overall portfolio. These findings indicate that the limited possibility of rebalancing that is present in the private equity market structure can be a disadvantage for private equity investors.

However, studying Strategy 1, whereby all stocks are held for five years without rebalancing, the picture is more nuanced. Portfolio 1, 2 and 6 deliver a higher return in Strategy 1 than in

Strategy 3, where the stocks are rebalanced monthly. Additionally, the portfolios tend to yield its highest Sharpe ratio in Strategy 1 compared to the other investment strategies. These findings may imply that “winning stocks” in one year, accounting for a larger share of the portfolio in the next period, in average keep continue to outperform the average stock return the following year. However, the findings may also be due to deviation in the selected stocks in the different strategies. Strategy 1 is supplied with new stocks each fifth year in contrast to Strategy 3, which selects new stocks on an annual basis. Stocks eligible for the replicating portfolios in 2006 or 2011, the years when Strategy 1 selects new stocks, may have outperformed stocks that are selected in other years of the investment period. This potential bias in the selection of stocks for Strategy 1 makes it difficult to isolate the effect of rebalancing between these strategies. For Portfolio 3, 4 and 5, however, Strategy 3 yields higher returns than Strategy 1. Hence, in these portfolios, we find a consistent tendency towards the increased frequency of rebalancing being correlated with increased portfolio returns.

Further, we find that a strategy under the restriction of five-year holding time outperforms a strategy without the same restriction. This is interesting given that an uninterrupted five-year holding time imposes limitations in the investment behaviour of portfolio managers. Our results point out that the increased flexibility of holding time offered by the stock market fails to add value to the portfolios. There are mainly two reasons for the elimination of holding time restriction being associated with weak portfolio returns. The first considers the effect of a financial crisis without the limitation of holding time. Following a massive drop in asset values during the financial crisis, the strategy ended up selling underperforming shares as they no longer satisfied the criteria for being selected in the portfolio. The second reason considers that portfolio managers of private equity funds typically have a five-year time horizon when choosing targets to invest in. This implies that a choice of sector and size in 2006 is a choice for the following five years. To illustrate, the Nordic private equity fund Hitecvision’s purchase of 12 oil service companies after the financial crisis was a bet for improved market conditions over a longer time horizon. By implementing a strategy where we refrain from the holding time of private equity, we miss out the long-time perspective underlying the investments. Our findings indicate that such an investment strategy is associated with weaker portfolio performance, both in terms of lower returns, lower Sharpe-ratio, and higher maximum drawdowns.

	Strategy	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6	Benchmark PE
Annualised pre-fee excess return	1	12.2%	12.5%	14.8%	14.2%	13.8%	10.3%	**17.2%**
	2	6.1%	10.5%	13.9%	17.9%	15.5%	4.9%	
	3	6.6%	11.2%	14.9%	18.6%	15.8%	7.7%	
	4	7.1%	7.4%	11.6%	14.9%	10.9%	-2.9%	
Annualised st. deviation	1	23.3%	27.0%	24.5%	24.8%	26.5%	27.6%	**17.2%**
	2	17.2%	27.1%	32.1%	38.8%	35.5%	20.5%	
	3	14.2%	22.9%	26.8%	34.6%	30.5%	23.5%	
	4	19.0%	21.8%	24.2%	29.3%	28.3%	25.6%	
Sharpe ratio	1	0.51	0.46	0.60	0.57	0.51	0.37	**1.00**
	2	0.34	0.38	0.43	0.46	0.43	0.23	
	3	0.44	0.48	0.55	0.53	0.51	0.32	
	4	0.36	0.33	0.47	0.50	0.38	-0.12	
Maximum drawdown	1	-32.7%	-38.0%	-31.2%	-33.2%	-49.3%	-44.3%	**-14.2%**
	2	-34.6%	-36.4%	-35.6%	-38.4%	-42.2%	-45.3%	
	3	-29.1%	-32.8%	-33.1%	-33.3%	-38.2%	-40.6%	
	4	-36.5%	-42.0%	-37.4%	-39.4%	-50.0%	-74.2%	
Market Beta	1	1.17	1.33	1.45	1.36	1.63	1.37	**0.94**
	2	1.07	1.42	1.69	2.00	1.86	1.12	
	3	0.91	1.29	1.53	1.85	1.65	1.32	
	4	1.07	1.25	1.35	1.46	1.55	0.92	

Explanations

x.x%	Outperforming the PE-benchmark
x.x%	Underperforming the PE-benchmark

Table 19: How portfolio performance varies with underlying market restrictions and underlying asset selection choices.
Maximum drawdown expresses the percentage decrease from the minimum portfolio value relative to its previous maximum value. Sharpe ratio is measured as the annualised pre-fee excess return divided by the standard deviation of the portfolio. Variations in market restrictions are given row-wise by applying four different investment strategies, while variations in asset selection choices are given column-wise by applying six different replicating portfolios. Please see Figure 5 above for an explanation of the investment strategies and the replicating portfolios. Furthermore, all returns are pre-fee and measured in excess of the three-month US Treasury bill return. The numbers in the table are based on an imposed portfolio of 0.82 to mimic private equity and 3% loan interest rate. See Appendix for more tables using different assumptions on the level of imposed leverage.

5.2 How asset selection affects portfolio performance

Our six different replicating portfolios seek to mimic the asset selection choices of Nordic private equity in six distinct ways. This section evaluates how asset selection choices affect portfolio performance. The results, as illustrated in Table 19 above, show that increased accuracy in mimicking the asset selection of private equity tends to increase returns. This pattern is consistent across all set of investment restrictions. In the five-year yearly rebalanced strategy, a portfolio screening based on the size as a single criterion yields an annualised excess return of 6.1%, increasing to 10.5% when adding a sector criterion. In comparison, the Nordic stock index yielded 8.1% in annualised excess return during the same period. The inclusion of EBITDA as the third criterion results in an additional increase in return, reaching 13.9% for the yearly rebalanced strategy and 14.9% for the monthly rebalanced portfolio, not far off the private equity benchmark yielding an annualised excess return of 17.2%. This result is particularly striking due to the simplicity of the strategy, indicating that three simple key characteristics of the asset selection of private equity can, combined with modest leverage, explain most of the variation in private equity returns.

Portfolio 4-6 add a fourth selection criterion. By matching on asset growth turnover as the fourth selection criterion in Portfolio 4, the returns further increases to 17.9% and 18.6% for the yearly and monthly rebalanced strategy, respectively. As seen in Table 19, these are the only two of our 24 replicating portfolios that yield a higher return than the Nordic private equity benchmark. This indicates that the relatively low asset growth turnover of buyout companies is contributing positively in terms of returns the following years. This is interesting given earlier research that shows that increased asset growth turnover is as a key contributing factor to private equity returns. Further, the market beta of Portfolio 4 is ~2.0, rising from ~1.7 for the unlevered portfolio, as seen in Table A.1 in the Appendix.

Portfolio 5, matching on working capital turnover as the fourth selection criterion, yields 15.5% and 15.8% in annualised excess returns in the yearly and monthly rebalanced portfolio, respectively. Our results thus suggest that relatively high working capital turnover is contributing slightly positive in terms of returns. Thus, we predict high capital efficiency to be associated with high returns when combined with the size- and sector selection done by private equity. This may be due to that relatively high working capital turnover gives the company increased flexibility in spending and contributes to reducing financial trouble in case of falling sales.

The increased portfolio returns when increasing the accuracy of the replication, come at the cost of rising risk, measured both as the standard deviation and market beta of the portfolios. Interestingly, Portfolio 3 and Portfolio 4 consistently tend to yield the best Sharpe ratio among the replicating portfolios, implying an argument in favour of high accuracy in the asset selection replication. However, a Sharpe ratio of 0.6, which is the highest one achieved by the replicating portfolios, is far off the ratio of Nordic private equity of 1.0. This implies that, although the return of Portfolio 4 is relatively higher, the lower risk of the private equity benchmark seems to more than compensate for the inferior return.

Furthermore, Portfolio 6, selecting small stocks with relatively high leverage ratios, tends to underperform massively compared to the other portfolios. The reason why including leverage as a matching criterion results in lower returns may be related to a bias in the selection process. Our data indicate that Nordic buyout funds tend to buy firms with a relatively high debt-to-equity ratio and add additional leverage within the end of the entry year. As the private equity funds make changes in the capital structure of the portfolio companies within the entry year, we do not capture the leverage level of the pre-transaction company. Consequently, by selecting stocks based on their leverage ratio in the end of the entry year, our approach selects firms that have a higher debt-to-equity ratio than the pre-transaction buyout target.

Additionally, given that buyout funds choose companies with stable cash flows which more securely can cope with debt, a portfolio matching on leverage may choose firms with a higher probability of financial distress. Leverage amplifies the returns, which most research (i.a. Chingono & Rasmussen, 2015) suggests result in performance improvement in the buyout industry. When using leverage as a selection criterion, potentially heavily indebted firms that jeopardise its ability to repay its debt are selected. Consequently, the unattractive highly indebted companies that are a part of the asset selection is expected to reduce the portfolio’s return. This bias in the selection process provides an argument for imposing leverage at portfolio level instead of selecting stocks based on leverage. The portfolio-level approach enables us to mimic the leverage of a private equity portfolio without selecting stocks that do not share the same characteristics of a pre-transaction buyout target.

5.3 How leverage affects portfolio performance

The last two sections showed how a monthly and yearly rebalanced portfolio matching on size, sector, EBITDA and asset growth turnover has achieved a higher mean return than the private equity benchmark since 2006. The results were based on applied portfolio leverage corresponding to a debt-equity ratio of 0.82, which mimics the post-transaction leverage ratio of buyout targets. In this section, we evaluate how portfolio performance changes in response to change in the level of imposed leverage. For this purpose, we apply leverage level in the range from no imposed leverage to two times unlevered debt-to-equity ratio, as well as loan interest rate ranging between 0% and 9%.

As presented in Table 19 and bold-marked in Table 20, the pre-fee excess annualised return of a slightly levered Portfolio 4, yearly rebalanced, is 17.9% (applying a 3% loan interest rate). The corresponding return for a non-levered version of the portfolio is 17.2%. Thus, the portfolio equals the private equity benchmark of 17.2% without applying leverage to the portfolio. This is a remarkable result, showing that the portfolio can replicate private equity return despite consisting of portfolio companies with lower debt levels. Furthermore, the table shows that increased applied portfolio leverage contributes to amplify portfolio returns but also to increase the cost of the interest rate. Additionally, higher leverage results in increased risk, both in terms of a rise in maximum portfolio drawdowns and market equity beta.

	Annualised excess return					Maximum drawdown				
Interest rate	No leverage	D/E= 0.82	D/E= 100%	~1.5x leverage	~2x leverage	No leverage	D/E= 0.82	D/E= 100%	~1.5x leverage	~2x leverage
0%	17.2%	18.4%	19.8%	19.7%	22.0%	-34.7%	-37.6%	-41.0%	-40.9%	-46.9%
1%	17.2%	18.3%	19.4%	19.4%	21.3%	-34.7%	-37.9%	-41.5%	-41.4%	-47.9%
2%	17.2%	18.1%	19.1%	19.0%	20.6%	-34.7%	-38.1%	-42.0%	-41.9%	-48.8%
3%	17.2%	**17.9%**	18.7%	18.7%	19.9%	-34.7%	**-38.4%**	-42.5%	-42.4%	-49.8%
4%	17.2%	17.8%	18.4%	18.3%	19.2%	-34.7%	-38.6%	-43.1%	-42.9%	-50.9%
5%	17.2%	17.6%	18.0%	18.0%	18.5%	-34.7%	-38.9%	-43.6%	-43.4%	-52.8%
6%	17.2%	17.4%	17.7%	17.6%	17.8%	-34.7%	-39.1%	-44.1%	-43.9%	-54.7%
7%	17.2%	17.3%	17.3%	17.3%	17.1%	-34.7%	-39.4%	-44.6%	-44.4%	-56.5%
8%	17.2%	17.1%	16.9%	16.9%	16.4%	-34.7%	-39.6%	-45.6%	-45.4%	-58.2%
9%	17.2%	16.9%	16.6%	16.6%	15.6%	-34.7%	-39.9%	-46.7%	-46.4%	-59.9%

Private equity benchmark

Annualised pre-fee excess return	17.2%
Maximum drawdown	-14.2%

Explanations

[dashed box]	Base case from Table 19.
x.x% (green)	Outperforming PE-benchmark
x.x% (red)	Underperforming PE-benchmark

Table 20: How portfolio performance varies with changes in the leverage level and the loan interest rate. *The table applies five different levels of imposed leverage, as described in the methodology and specified column-wise. Ten different levels of bank loan interest rate, ranging from 0% to 9%, are applied, as specified row-wise. All returns are pre-fee and measured in excess of the three-month US Treasury bill return. Maximum drawdown expresses the percentage decrease from the minimum portfolio value relative to its previous maximum value. The numbers in the table are based on a yearly rebalanced portfolio (Strategy 2) and a portfolio based on size, sector, EBITDA and asset growth turnover (Portfolio 4). See Appendix for more tables applying different strategies and portfolios.*

The increased portfolio drawdowns mark a considerable cost of portfolio leverage. Increased leverage implies rising vulnerability for margin call in case of an economic turmoil. This margin call risk would be a big concern during the 2008 financial crisis, where the replicating portfolios experienced a massive drawdown in portfolio values. For portfolios breaking the margin covenants, such drawdowns may trigger a sizeable margin call whereby the supplier of the loan would require the investor to deposit new cash or securities, or exit the portfolio. However, the degree to which margin calls would appear to be a concern for the investors depends on 1) how levered the private equity portfolio is, and 2) how diversified the investors are.

As discussed, increased portfolio leverage implies increased dispersion in portfolio returns, making the rises higher and drawdowns greater. Accurately, a portfolio with applied leverage corresponding to 2.0x debt-equity-ratio experiences a drop in portfolio value in the range of 47% and 60% during the financial crisis, compared to 35% for a non-levered portfolio. However, to mimic the leverage level of private equity, our replicating portfolios only need a modest level of imposed portfolio leverage. The reason is that the replicating portfolios target small stocks that, on average, are more leveraged than a typical stock index. Stocks included in Portfolio 1 to 6 have a mean debt-to-eq|uity ratio in the range of 0.60 and 0.70, not far off the post-transaction debt-to-equity ratio of 0.82 among buyout targets. Leverage level equivalent to a debt-to-equity ratio of 0.82 increases mean annualised return by up to 1.2% and maximum drawdown with 5% from the unlevered level. The maximum drawdown of the replicating portfolios applying this level of leverage is 40%, which takes place from 2007 to 2009 on the back of the financial crisis. Thus, the relatively high debt on the portfolio stocks' balance sheets reduces the margin call risk of the replicating portfolios by limiting the level of imposed leverage.

Secondly, the risk of margin calls also depends on the degree to which the investor is diversified. The high betas of the replicating portfolios indicate that the portfolios are highly correlated with the market return. This means that a drawdown in the replicating portfolio often coincides with a decrease in the value of a broad public equity portfolio. The mix between a replicating portfolio and a public equity portfolio thus offer limited diversification effect. Instead, the allocation mix between bonds and public equity is the main determinant for the degree of diversification, and thus margin call risk, in a portfolio. Table 21 below examines the margin call risk in four different scenarios with deviating investment mixes.

	Classic portfolio mix	***-Of which public equity***	***-Of which bonds***	**Share in replicating portfolios**	**Exposure to public market**	**Exposure to bond market**	**Return in period**
Highly diversified	**90%**	*60%*	*40%*	**10%**	69%	31%	**-26.7%**

Medium diversified	**80%**	*60%*	*40%*	**20%**	78%	22%	**-30.5%**
Lowly diversifed	**80%**	*100%*	*0%*	**20%**	110%	-10%	**-44.3%**
High PE-exposure	**60%**	*100%*	*0%*	**40%**	120%	-20%	**-48.6%**

Return assumptions		**Explanation**	
Return in general market	-40%	[dashed box]	Stafford example
Return bonds	3%		

Table 21: How the portfolio mix affects the risk of margin call.
The table shows how sensitive the return of the overall portfolio is to changes in the investment mix. The calculation is based on Stafford (2016) which showed that a 20% buyout allocation can be considered as a levered value stock investment with market risk exposure equal to 30% listed firms and -10% bonds. We also assume a return in the broad market portfolio of negative 40% and 3% return of bonds.

The table clearly shows how the portfolio drawdown and the risk of margin calls are highly dependent on how diversified the investor is. The risk is considerably higher for a pure equity investor than for an investor with a sizeable allocation to bonds. To illustrate, Stafford (2016) considered a portfolio consisting of investments of 20% in private equity and 80% in a classic mix of 60% in public equity and 40% in bonds. Stafford shows that a 20% buyout allocation can be considered as a levered value stock investment with market risk exposure equal to 30% listed firms and -10% bonds. This means the overall portfolio's exposure consists of 78% to public equity and 22% to bonds, and consequently, Stafford argues that the risk for suffering from margin call is low.

However, many investors may not be that diversified as the example provided by Stafford assumes. For a pure equity investor, the risk of margin call may be considerably higher. For a pure equity investor with an 80% allocation to public equity and the remaining 20% in the replicating portfolios, the estimated drop in portfolio value is 44.3% given a drop in the broad market by 40%. The drawdown rises to 48.6% if the allocation mix is 60% in public equity and 40% in replicating portfolios. Such drawdowns will likely trigger a margin call from the bank issuing the loan to the replicating portfolio. Since the value of the public market portfolio also has decreased significantly, by 40%, the investor will likely need to realize stocks at a loss to cover the margin call. Thus, the expected cost of margin call for a pure equity investor differs considerably from a highly diversified investor, illustrating the importance of diversification when dealing with imposed leverage.

The downfall of the hedge fund Long Term Capital Management (LTCM) in 1998 provides an adequate illustration of the potential consequences of portfolio leverage. By undertaking highly risky positions in numerous leveraged derivative products, the fund yielded a return on equity of 43% and 41% the two first years of operation (Shirreff, 2000). However, the speculative bets turned out as a catastrophe, as unfortunate price movements in the underlying assets were multiplied by considerable leverage. In August 1998 alone, the fund reported a loss of $2.1 billion, corresponding to 44% of its value that year. LTCM's counterparties started to call for more collateral to cover their positions, forcing them to liquidate positions at a sizeable loss. The losses and margin calls continued the following months until LTCM finally became bailed out by a consortium of 13 banks. According to press reports, investors of the fund accrued a total loss of $2.2 billion on their engagement in the fund. Despite being considerably more levered and exposed to riskier asset classes than our replicating portfolios, the story of LTCM gives a precautionary warning to investors applying portfolio leverage.

To sum up this part, we find that a non-levered replicating portfolio matching on size, sector, EBITDA and asset growth turnover, earns higher pre-fee return than the private equity benchmark. Adding leverage to the portfolios tends to increase the mean return in the investment period, depending on the interest rate level charged by the creditor. However, imposing leverage makes the investor vulnerable to margin calls, which, particularly for low-diversified investors, can mitigate returns. Nevertheless, as the imposed leverage to the replicating portfolios is low, we argue that the risk of margin call mainly is determined by the allocation mix. We argue that a well-diversified investor has a low risk for margin calls. In contrast, the risk is considerably higher for a pure equity investor with a moderate allocation to the replicating portfolios.

5.4 How fees affect returns to the investor

Until this point, we have compared pre-fee portfolio returns to the estimated pre-fee private equity benchmark. By introducing fees and transaction costs, this section examines the earnings generated by an investor. The annualised post-fee excess return earned by an investor with a well-diversified allocation to Nordic private equity since 2006 is 11.1%, well below the estimated post-fee annualised excess return of 17.2%. As the fees in private equity are sizeable, a public market portfolio is likely to generate transaction costs in short of the private equity fees. As presented in Section 4, transaction costs for a public portfolio are sensitive to assets under management and stock liquidity as well as turnover assumptions.

Selected findings from earlier research on trading costs estimate costs in the range of 20bps and 101bps for small- and micro-cap stocks, dependent on the degree to which the investor move the market. In our case, these costs accrue in the event of rebalancing, when the investor buy or sell shares to reach an equal-weighted portfolio. Thus, we expect a monthly rebalanced portfolio to be loaded with trading costs higher than a corresponding portfolio rebalanced yearly. Table 22, studying how fees affect the portfolio returns, confirms our hypothesis. Increased frequency of rebalancing, as associated with a rise in turnover rates, are likely to increase transaction costs. An increase in trading costs with 100bps decreases the annualised return of up to 0.5% and 2.0% in the yearly and monthly rebalanced portfolios, respectively. This implies that the increased pre-fee return associated with increased frequency of rebalancing, as presented in Section 5.1, comes at the costs of higher transaction costs.

		Unlevered portfolios					**PE-replicated leverage (D/E = 0.82)**				
Trading cost →		0bps	25bps	75bps	100bps	150bps	0bps	25bps	75bps	100bps	150bps
Strategy 1	Portfolio 1	11.5%	11.4%	11.3%	11.1%	11.0%	12.2%	12.1%	11.9%	11.7%	11.6%
	Portfolio 2	12.0%	11.9%	11.7%	11.5%	11.4%	12.5%	12.4%	12.2%	12.1%	12.0%
	Portfolio 3	14.0%	13.9%	13.7%	13.5%	13.4%	14.8%	14.7%	14.5%	14.3%	14.2%
	Portfolio 4	13.6%	13.5%	13.3%	13.1%	13.0%	14.2%	14.1%	13.9%	13.7%	13.7%
	Portfolio 5	13.1%	13.1%	12.9%	12.7%	12.6%	13.8%	13.7%	13.5%	13.3%	13.2%
	Portfolio 6	10.2%	10.1%	9.9%	9.8%	9.7%	10.3%	10.2%	10.0%	9.9%	9.8%
Strategy 2	Portfolio 1	6.1%	6.0%	5.8%	5.6%	5.5%	6.1%	6.0%	5.7%	5.5%	5.3%
	Portfolio 2	10.1%	10.0%	9.8%	9.6%	9.5%	10.5%	10.4%	10.1%	9.9%	9.7%
	Portfolio 3	13.3%	13.2%	13.0%	12.8%	12.8%	13.9%	13.8%	13.6%	13.4%	13.2%
	Portfolio 4	17.2%	17.1%	16.9%	16.7%	16.6%	17.9%	17.8%	17.6%	17.4%	17.3%
	Portfolio 5	14.8%	14.7%	14.5%	14.3%	14.2%	15.5%	15.4%	15.2%	15.0%	14.8%
	Portfolio 6	5.0%	5.0%	4.8%	4.6%	4.6%	4.9%	4.8%	4.6%	4.4%	4.4%
Strategy 3	Portfolio 1	6.5%	6.2%	5.5%	4.9%	4.6%	6.6%	6.2%	5.3%	4.5%	4.1%
	Portfolio 2	10.7%	10.4%	9.7%	9.1%	8.8%	11.2%	10.8%	10.0%	9.2%	8.9%
	Portfolio 3	13.9%	13.6%	13.0%	12.4%	12.1%	14.9%	14.5%	13.8%	13.0%	12.6%
	Portfolio 4	17.2%	16.9%	16.3%	15.8%	15.5%	18.6%	18.3%	17.6%	16.9%	16.5%
	Portfolio 5	14.7%	14.4%	13.9%	13.3%	13.0%	15.8%	15.4%	14.7%	13.9%	13.6%
	Portfolio 6	7.3%	7.0%	6.5%	5.9%	5.6%	7.7%	7.3%	6.6%	5.9%	5.6%
Strategy 4	Portfolio 1	7.0%	6.9%	6.7%	6.6%	6.5%	7.1%	7.0%	6.8%	6.5%	6.4%
	Portfolio 2	7.3%	7.2%	7.0%	6.8%	6.7%	7.4%	7.3%	7.1%	6.8%	6.7%
	Portfolio 3	11.1%	11.0%	10.9%	10.7%	10.6%	11.6%	11.5%	11.3%	11.1%	11.0%
	Portfolio 4	14.2%	14.1%	14.0%	13.8%	13.8%	14.9%	14.9%	14.7%	14.5%	14.4%
	Portfolio 5	10.4%	10.3%	10.2%	10.0%	9.9%	10.9%	10.8%	10.6%	10.4%	10.3%
	Portfolio 6	-2.2%	-2.2%	-2.4%	-2.5%	-2.6%	-2.9%	-3.0%	-3.2%	-3.3%	-3.4%

Private equity benchmark		**Explanations**	
Annualised post-fee excess return	11.1%	x.x%	Outperforming PE-benchmark
Annualised standard deviation	17.1%	x.x%	Underperforming PE-benchmark

Table 22: How portfolio return varies with changes in trading costs and imposed leverage.
The table applies trading costs in the range of 0bps and 150bps, as argued in the methodology and specified column-wise. The table reports the portfolio returns for both non-levered portfolios and portfolios with D/E-ratio of 0.82 to mimic private equity leverage. The table evaluates the results using all four investment strategies and all six replicating portfolios. All returns are pre-fee and measured in excess of the three-month US Treasury bill return. For the levered portfolios, a loan interest rate of 3% is applied.

Another valuable insight considers the size of fees in our replicating portfolios compared to private equity. In light of empirical findings, the assumption of an annual fee of 6% charged by private equity funds appears to be reasonable. This fee is significantly higher than the fees generated by a replicating portfolio. As a comparison, a monthly rebalanced portfolio with a turnover cost of 150bps is generating total trading costs of up to 2.5% annually. Hence, replicating portfolios can be managed to a significantly lower cost than what charged by private equity managers. The massive deviation in fees represents an advantage of public equity compared to the private market. Notably, Portfolio 3, 4 and 5 are, in all the four different investment strategies, yielding a higher post-fee return than the private equity benchmark of 11.1%. Assuming a trading cost of 100bps and leverage level mimicking private equity, 13 of our 24 replicating portfolios yield a higher return than the benchmark.

To sum up this part, even applying high-frequent rebalancing and the highest level of trading costs gathered from relevant research papers, the transaction costs of the replicating portfolios are considerably lower than private equity fees. Thus, replicating portfolios that underperform compared to the pre-fee private equity benchmark are more competitive when accounting for fees. In fact, Portfolio 3 to 5 yield superior returns relative to the post-fee private equity benchmark.

5.5 How the initial year of portfolio affects portfolio performance

The last sections discussed the performance of the replicating portfolios compared to the return yielded by Nordic private equity when taking the entire investment period from 2006 to 2018 into consideration. This section demonstrates how the replicating portfolio performance is subject to massive dispersion within the investment period. We will base the discussion on a non-rebalanced five-year buy-and-hold-strategy initiated in each year from 2006 to 2013. By this approach, we seek to reveal how portfolio performance is sensitive to the starting year of investment.

Our results propose that private equity funds outperformed the replicating portfolios from 2007, the start of the financial crisis, to 2012. However, since 2012 the replicating portfolios have been superior to the private equity benchmark in terms of returns. The return smoothing process of private equity may contribute to explain this pattern, as such a process tends to report relatively high returns in bad times and relatively low returns in good times. The attractive returns yielded by the general market since 2012 have benefited the returns of both high-beta replicating portfolios and private equity. However, the return smoothing process may, in such an upturn, understate private equity return, while the public market replicating portfolios always report their real market-based values. A second potential explanation for private equity underperformance since 2011 is the high inflow and record-high deployment of capital to the asset class in this decade (Bain, 2019). The rush of capital has led to an increased number of private equity funds competing on a limited set of targets, resulting in increasing valuations. Currently, private equity is experiencing a record high commitment yet to be deployed, which may imply a risk for returns to continue relatively low in the years ahead.

The wide dispersion in returns makes the portfolio performance highly sensitive to the initial year of investment. Table 23 demonstrates this, showing that the success of a replicating portfolio is highly sensitive to where we are in the economic cycle. The table shows how the returns of a five-year buy-and-hold portfolio vary over time. A five-year buy-and-hold Portfolio 4 invested in 2007 achieves an annualised excess return of negative 2.7%, while the same strategy invested from 2013 yields a return of 31%, exceeding the private equity benchmark. In fact, all the five-year-portfolios initiated before 2011 underperform relative to the private equity benchmark. Portfolios starting between 2006 and 2008 particularly underperform due to the massive drawdown during the financial crisis. On the other side, most of the portfolios initiated in or after 2011 yield higher return than the benchmark. Overall, there is a clear pattern to which the replicating portfolios outperform private equity returns in good times but significantly underperform in times when the market drops.

Metric	**Portfolio**	**2006 - 2011**	**2007 - 2012**	**2008 - 2013**	**2009 - 2014**	**2010 - 2015**	**2011 - 2016**	**2012 - 2017**	**2013 - 2018**
Annualised pre-fee return	Portfolio 1	2.6%	-3.9%	2.5%	11.9%	11.1%	19.5%	25.1%	22.2%
	Portfolio 2	0.7%	-2.8%	1.5%	16.2%	14.6%	24.6%	32.5%	24.1%
	Portfolio 3	9.0%	-7.2%	3.0%	22.7%	23.9%	21.4%	38.9%	30.1%

	Portfolio 4	9.4%	-2.7%	5.7%	21.5%	21.4%	18.0%	38.3%	31.2%
	Portfolio 5	3.4%	-14.3%	1.7%	20.5%	22.3%	25.1%	44.8%	27.3%
	Portfolio 6	2.2%	-7.9%	13.1%	-3.3%	8.2%	16.8%	14.6%	14.7%
	PE Benchmark	**17.4%**	**12.8%**	**15.6%**	**27.1%**	**25.9%**	**17.6%**	**19.2%**	**18.1%**
Annualised st. deviation	Portfolio 1	21.2%	13.4%	19.2%	19.7%	19.6%	26.2%	14.2%	9.8%
	Portfolio 2	22.3%	15.7%	22.8%	31.4%	32.6%	31.4%	22.3%	13.4%
	Portfolio 3	27.2%	7.6%	24.1%	35.8%	41.3%	26.2%	38.0%	18.3%
	Portfolio 4	31.0%	16.3%	25.9%	42.6%	84.9%	23.5%	38.1%	27.0%
	Portfolio 5	34.6%	3.9%	33.6%	25.5%	38.3%	21.5%	48.0%	22.0%
	Portfolio 6	31.6%	16.3%	27.8%	14.3%	35.5%	27.3%	21.3%	15.8%
	PE Benchmark	**25.1%**	**22.6%**	**21.5%**	**13.7%**	**14.2%**	**12.8%**	**12.6%**	**13.3%**
Sharpe ratio	Portfolio 1	0.12	-0.29	0.13	0.61	0.57	0.75	1.77	2.26
	Portfolio 2	0.03	-0.18	0.06	0.52	0.45	0.78	1.46	1.80
	Portfolio 3	0.33	-0.95	0.12	0.63	0.58	0.82	1.02	1.65
	Portfolio 4	0.30	-0.17	0.22	0.51	0.25	0.77	1.01	1.15
	Portfolio 5	0.10	-3.67	0.05	0.80	0.58	1.17	0.93	1.24
	Portfolio 6	0.07	-0.49	0.47	-0.23	0.23	0.62	0.68	0.93
	PE Benchmark	**0.65**	**0.52**	**0.68**	**1.90**	**1.76**	**1.30**	**1.44**	**1.28**
Maximum drawdown	Portfolio 1	-22.2%	-29.0%	-14.3%	-31.8%	-17.5%	-15.4%	-18.1%	0.0%
	Portfolio 2	-28.4%	-29.9%	-23.5%	-37.1%	-29.3%	-18.1%	-18.7%	0.0%
	Portfolio 3	-26.0%	-27.7%	-10.6%	-30.4%	-13.0%	-30.4%	-31.3%	0.0%
	Portfolio 4	-30.0%	-31.8%	-19.4%	-32.5%	-20.2%	-18.4%	-12.9%	-27.5%
	Portfolio 5	-38.7%	-32.9%	-23.4%	-48.6%	-33.5%	-48.2%	-53.7%	0.0%
	Portfolio 6	-31.2%	-41.8%	-43.0%	-44.0%	-36.4%	-34.3%	-33.9%	-15.4%
	PE Benchmark	**-14.2%**	**-14.2%**	**-14.2%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**	**0.0%**

Explanations

x.x%	Outperforming PE-benchmark
x.x%	Underperforming PE-benchmark

Table 23: How portfolio performance varies with the initial year of investment.
All portfolios are held for five years without applying to rebalance. All returns are pre-fee and measured in excess of the three-month US Treasury bill return. Sharpe ratio is measured as the annualised pre-fee excess return divided by the standard deviation of the portfolio. Maximum drawdown expresses the percentage decrease from the minimum portfolio value relative to its previous maximum value. The numbers in the table are based on imposed leverage of 0.82 to mimic private equity and 3% loan interest rate.

5.6 How the return reporting process affects portfolio performance

The results in the discussions beyond reveal interesting insights in several dimensions. A key takeaway relates to the risk involved in the replicating portfolios. The measured risk of the replicating portfolio is strikingly higher than in the case of Nordic private equity. As pointed out earlier, private equity funds can hide some volatility since their return reporting process offers the opportunity for subjective measures of asset prices. Due to the lack of transaction-based measures, the performance of private equity funds can only be measured with reasonable accuracy towards the end of its life when they realise the proceeds from investment, typically late in the fund's life cycle. In contrast, the replicating portfolios are subject to continuous transaction-based valuations as they consist of publicly traded firms. To examine how the differences in valuing portfolios may affect portfolio risk, Stafford introduced an accounting scheme in which all portfolio assets are valued at cost until they are sold in the public market. The approach is based on the idea that the distribution of returns over the entire holding time is less volatile than the distribution of returns over a shorter time period, e.g. monthly. By applying this approach, Stafford abled to eliminate most of the replicating portfolios' risk.

However, by introducing a hold-to-maturity accounting scheme in our sample, we are not able to reproduce the risk reduction achieved by Stafford. Contrastingly, as seen in the table below, the measured risk of the replicating portfolios generally increases when applying a hold-to-maturity approach. For example, applying a book-value approach on Portfolio 3, the measured risk raises from 23.8% and 28.9% to 44.5% and 48.8% for the monthly and yearly rebalanced portfolios, respectively. This pattern applies to Portfolio 3 to 6, the riskiest portfolios in terms of standard deviation. Interestingly, when applied to the less risky portfolios, Portfolio 1 and 2, the hold-to-maturity approach gives a slight reduction in measured risk.

Further, the annualised mean return with a hold-to-maturity approach is smaller than with a market-based rule. The reason is that the hold-to-maturity accounting rule values stocks at book value before they are sold, creating a gap between reported value and market value of the portfolio. For our replicating portfolios, the discount varies between 10% and 30%, signalling the magnitude of the gap. As the private equity benchmark is measured at book value both in the initial year of investment, in 2006, and the final year, in 2018, we assume that the net discount of the private equity benchmark is close to zero.

		Portfolio 1	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**
Annualised pre-fee return (HTM-approach)	Strategy 2	5.2%	7.9%	11.8%	14.9%	12.9%	2.9%
	Strategy 3	5.5%	8.1%	12.0%	14.5%	12.6%	5.2%
Portfolio value discount (HTM-approach)	Strategy 2	-10.2%	-20.2%	-16.3%	-24.1%	-18.7%	-22.5%
	Strategy 3	-9.3%	-23.3%	-21.4%	-29.7%	-23.5%	-22.8%
Annualised st. deviation (HTM-approach)	Strategy 2	10.7%	23.1%	48.8%	53.4%	61.1%	24.2%
	Strategy 3	12.6%	22.6%	44.5%	38.8%	58.0%	27.6%
Annualised st. deviation (MB-approach)	Strategy 2	15.1%	24.1%	28.9%	35.4%	31.6%	19.6%
	Strategy 3	12.6%	20.3%	23.8%	30.3%	26.8%	21.1%

Table 24: The risk and return of the replicating portfolios under a hold-to-maturity approach.
HTM denotes a hold-to-maturity accounting scheme, while MB denotes a market-based accounting scheme. All returns are pre-fee and measured in excess of the three-month US Treasury bill return. Portfolio value discount expresses the percentage difference between in portfolio value of a book-value HTM approach and the market-based value. The numbers in the table are based on non-leveraged replicating portfolios, using investment strategy 2 and 3.

The failure of the hold-to-maturity approach in our sample is interesting as it shows that a long holding period of portfolio stocks is not sufficient to mask portfolio volatility. A long holding time of portfolio stocks needs to be combined with a long investment period to increase the number of return observations. Our choice of a relatively long holding period of five years and a relatively short investment horizon of 13 years, results in only eight observations of five-year returns. The low number of observations makes the data set vulnerable to outliers that can massively affect the measured risk. We, therefore, argue that our investment period is too short for providing a representative measure of the effect of a hold-to-maturity approach.

Our results thus fail to provide an answer to what is the real risk of private equity. However, given Stafford's findings and the nature of private equity funds, the existence of return smoothing may be legitimate. The findings of Stafford are consistent with Ang et al. (2018), who introduced a methodology to estimate returns to investment in private equity funds based on a Bayesian Markov Chain Monte Carlo approach. They found that their estimated private equity returns are considerably more volatile than those measured using the appraisal approach preferred by the industry. Moreover, they conclude that appraisal-based private equity indices exhibit "smoothing induced by a conservative appraisal process or by a delayed and partial adjustment to market prices". In fact, the volatility of their estimated returns is at least as volatile as broad equity market indices. These arguments are quickly summarized by the following quote from David Swenson, the author of the book "Pioneering Portfolio Management" (2000):

> "… the low risk evident in data describing past returns from private investing constitutes a statistical artifact.… If two otherwise identical companies differ only in the form of organization — one private, the other public — the infrequently valued private company appears much more stable than the frequently valued publicly traded company. Although both companies react in identical fashion to fundamental drivers of corporate value, the less volatile private entity boasts superior risk characteristics, based solely on mismeasurement of the company's true underlying volatility*"* (p. 226).

The abovementioned research papers show that several private equity funds included in an index enable opportunities for severe risk reduction by applying return smoothing. This reasoning argues that the real risk of private equity is higher than reported, providing support to the view that private equity risk may be closer to the risk of a portfolio of comparable public equities. However, due to our limited investment period, we are unable to conclude on the real risk of private equity accurately. That being said, one could also argue that the active management of portfolio companies in private equity contributes to reducing the risk, implying it may be reasonable to believe in private equity risk below the risk of a public portfolio of comparable companies. Additionally, the corporate debt approach of private equity provides some benefits over the portfolio-leverage approach, which possibly can explain some of the lower risks the buyout industry achieves. The next section will, among other reflections, further discuss the implications of active management and beneficial debt financing.

6. Further reflections on results

The results show how some characteristics-matched and leverage-matched replicating portfolios can, particularly after accounting for fees, yield higher return than Nordic private equity. This is especially interesting as the replication is simple, applying identifiable patterns in buyout funds' choice of asset selection, leverage, and holding time. Our findings indicate that main value proposition offered by private equity managers is not their active management, but the asset selection process, including their ability to choose small-sized stocks within specific industry sectors and with specific characteristics. However, despite introducing a hold-to-maturity accounting scheme, the replicating portfolios are not able to reproduce the attractive risk profile of private equity. Thus, the interpretation of the results is sensitive to the degree to which the benchmark returns have been smoothed. This section discusses the identifiable advantages of private equity and public equity, but we will start the section by discussing further the main implications of our results.

6.1 Implications of our findings

Our results show that, since 2006, 13 of 24 portfolios mimicking the asset selection choices of private equity have achieved a higher return than the average Nordic private equity fund after accounting for fees. Surprisingly, these replicating portfolios outperform the post-fee private equity return without applying imposed leverage. These results have interesting implications particularly to the degree we can assume that our asset and leverage replication also is a convincing representation of the real risk in private equity. Given this assumption, our results indicate that diversified private equity investors in the Nordics significantly overpay for the capital allocation services of private equity managers. Instead of allocating capital to costly private equity managers, investors can achieve similar returns to a lower cost by investing in comparable companies in the stock market.

For two of the 24 replicating portfolios that earned higher returns than the benchmark before accounting for fees, the implications go even further. Under the assumption of an accurate representation of risk, the results indicate that the value creation of active management conducted by private equity managers is unable to compensate for the purchase price premium associated with gathering majority ownership. Instead, under this assumption, our results suggest that private equity managers are considerably overpaying for the opportunity to manage the companies.

However, the measured risk of the replicating portfolios is significantly higher than the measured risk of the private equity benchmark, making them unable to reproduce the risk-adjusted return of private equity. This may imply that (i) our characteristics- and leverage-matched approach fails to capture the appropriate risk level in private equity, or (ii) differences in the return reporting process causes a bias in the measured risk. The first point indicates that the assumption employed in the two last paragraphs, that our replication approach is a strong representation of the risk in private equity, may be weak. The second point, however, does not reject this assumption since the measured risk of private equity may be understated.

Considering the first point, our replication approach may fail to reproduce the risk properties of private equity due to the fundamental differences between a private and public market. Notably, our passive investments in the stock market lack both the active management approach and favourable corporate debt applied by private equity. Additionally, as seen in Section 3.2.1, by investing in the public market, the replicating portfolios lack exposure to the smallest firms targeted by private equity funds. This implies that a passive stock portfolio can not fully replicate the asset choices of a private equity portfolio. These advantages of private equity, which will be further discussed under the next headline, may contribute to a reduction of risk that a passive stock portfolio is not able to copy.

However, as discussed, the degree of volatility masking in private equity adds uncertainty in the assessment of differences in risk. Our data set is not sufficiently large to debunk the hypothesis that the measured risk of private equity is representative of the real underlying risk of the asset class. Nevertheless, earlier research has found that private equity returns are considerably more volatile than those measured using the appraisal approach applied by the buyout industry. Hence, we cannot claim whether the considerable gap in measured risk implies that private equity truly is less risky. We also stress that differences in risk level may be provoked by an imperfect selection of stocks to the replicating portfolios. Some private firms may convey characteristics that are present in the public market, but not captured by our selection criteria. There may exist risk reducing-characteristics of buyout targets that our selection leaves unnoticed due to limited data availability on private-to-private deals, for instance, recent capital issuance.

To further nuance the discussion, the investment period, from 2006 to 2018, makes portfolio performance vulnerable for short-time fluctuations. Our results could have drastically changed if a different or more extended investment period was applied. During our investment period, the Nordic stock market has, despite the occurrence of the financial crisis, performed well. As the high betas of our replicating portfolios indicate that they perform relatively well during good times and relatively bad during bad times, the attractive returns of the general market in our investment period may have resulted in an upward bias in the replicating portfolio returns. If exposed to an investment period in which the market falls, however, our findings suggest that the strategy may considerably underperform the private equity benchmark and the general market. Nevertheless, the fact that some of our replicating portfolios perform well, despite being exposed to the most severe financial crisis since the Great Depression, adds some robustness to the strategy. We can also rely on the Stafford paper suggesting that the real underlying risk of private equity is similar to the risk of a replicating portfolio, implying that the drawdowns of both asset classes are comparable.

Another critical subject relates to the feasibility of the strategy. As evaluated in the data section, the asset selection choices of private equity firms, with special emphasize to size and industry sectors, vary considerably over time. This underlines an essential skill of private equity fund managers, namely the ability to select e.g. industry sectors that will perform well in the coming years. However, the investments in the replicating portfolios are based entirely on publicly available deal information from the year prior to investment. We argue that this backwards-looking approach makes our strategy feasible for an outside investor. Still, the relatively high portfolio turnover, particularly for the monthly rebalanced portfolios, require the investor to exert serious effort to keep the portfolio equally weighted over time.

Several research papers have placed the recent performance of private equity into perspective. Ilmanen, Chandra and McQuinn (2019) concluded that "private equity does not seem to offer as attractive a net-of-fee return edge over public market counterparts as it did 15-20 years ago from either a historical or forward-looking perspective.". Further, Bain Capital (2019) concludes, in their annual private equity report, that US public equity essentially has matched the returns from US buyouts since 2009. Our findings somewhat support and provide an additional factual basis for the view that private equity has struggled to deliver superior returns in the last decade, particularly after fees. The current dynamics in the private equity market, including record-high dry powder of $2.3 trillion in the first quarter of 2019, pressure to do deals and a decade-long trend of rising valuation multiples (McKinsey, 2020), make an argument in favour of the replicating portfolios. However, our results state that the attractiveness of replicating portfolios in the years ahead is highly dependent on the performance of the general market in the period. Our results also show that the asset selection and investment strategy choice of the replicating portfolios are crucial, as some of our replicating portfolios clearly underperform the benchmark. Additionally, even our top-performing replicating portfolios fail to mimic private equity in some aspects, which will be discussed in the following.

6.2 The advantages of the buyout industry

In alignment with most research findings and prevailing view, it is believed that the buyout industry has some unique benefits which create value. This includes the investment selection and active management, which Jensen (1989) sums up to be the private equity's applied financial, governance, and operational engineering. These benefits contribute to research findings which indicate that buyout-backed firms, on average, increase their operating margins (Kaplan, 1989). Our passive constructed portfolios attempt to replicate the investment selection but are not able to carry out the active management. Given the financial, governance and operational engineering adds value also in the Nordic private equity market, one should expect private equity to deliver a higher pre-fee return than a passive portfolio that only replicate the asset selection. Our results indicate that the active management adds value but not to the extent that it fully covers the management fees of the funds.

Further, the leverage in the buyout industry works differently in numerous aspects that the constructed portfolios cannot fulfil. In financial engineering, general partners provide strong equity incentives to the management teams of their portfolio firms. Leverage put pressure on managers to optimise the money spending (Gompers et al., 2015). By imposing leverage at the portfolio level, the replicating portfolios fail to provide incentives to the management of the portfolio companies. As aforementioned, our second approach, in which we selected highly levered stocks, also fails to replicate the incentive effect since the firms may suffer from costs of financial distress. The leverage in these cases may not have an equivalent reliable incentive power for the management to create value because of the obstacle of the bankruptcy risk they are facing. Since the buyout industry lever up companies with a free cash flow that healthier can cope with debt costs, the incentivising, which is believed to add value, works more efficiently here.

As our second approach of selecting highly leveraged stocks suffer from costs associated with financial distress, our first approach of adding additional leverage at portfolio level seems most fortunate to mimic private equity leverage. However, as this added leverage is applied through short-term debt in a broker account, this approach fails to fully replicate the long-term corporate debt of private equity. The corporate debt applied by private equity funds offers several benefits over debt imposed at the portfolio level. A conceivable advantage of leverage at a firm-level compared to portfolio level is linked to the virtue of the fact that the debt is raised by the firms. This approach means the entire portfolio will not be sorted of risk if any individual public firm collapse. This differs from our approach in which leverage is imposed at the portfolio level through a broker account. Shiller (2013) showed that market prices are 20 times more volatile than fundamentals. Consequently, the debt of the replicating portfolios is exposed to highly volatile price movements of stocks, which can trigger the loan covenants, making the portfolios vulnerable for margin calls in case of drawdowns in the broad market. Notably, as our investment universe consists of relatively risky stocks with relatively high price movements, the replicating portfolios would less efficiently support the use of high applied portfolio leverage, as shown in Section 5.3 (Baker & Wurgler, 2015). Thus, the private equity approach with corporate debt on the firms' balance sheets, where it depends on the companies' earnings, would, in this case, be expected to give an advantage.

Furthermore, Ivashina and Kovner (2010) argue that private equity funds enhance additional value, beyond incentivising management to improve operating performance, by lowering the cost of financing on the corporate debt. They find that the long-term nature of the applied corporate debt combined with private equity funds' repeated interactions with banks enable the portfolio companies to borrow on more beneficial terms. A strong relationship with the banks reduces the costs associated with asymmetric information, enabling the banks to offer more favourable rates and to loosen the financial covenants. The opportunity for cross-selling other fee-based services to the funds' investment portfolios contributes to an additional improvement in the loan terms. The reduced loan rates directly enhance portfolio returns by lowering financial costs, while the loosening of covenants reduces renegotiation costs and makes the portfolio companies less vulnerable to bankruptcy risk. During the financial recession 2008-2009, Thomas (2010) finds that private equity portfolios had a default rate of 2.8%, while comparable firms' default rate equalled 6.2%. These results are consistent with other studies conducted on the default rates of private equity (i.a. Bank for International Settlements, 2008; Kaplan & Strömberg, 2008). The empirical support for low default rates may contribute to the banks' willingness to propose favourable loan terms to private-equity backed firms. Considering the replicating portfolios, on the other side, most of the companies may lack both the bank relationship and potential for cross-selling, making them subject to relatively higher loan rates and relatively stricter covenants. Thus, as illustrated in Figure 6, the replicating portfolios suffer from both relatively worse loan terms on the corporate debt and the margin call risk associated with short-term brokerage debt.

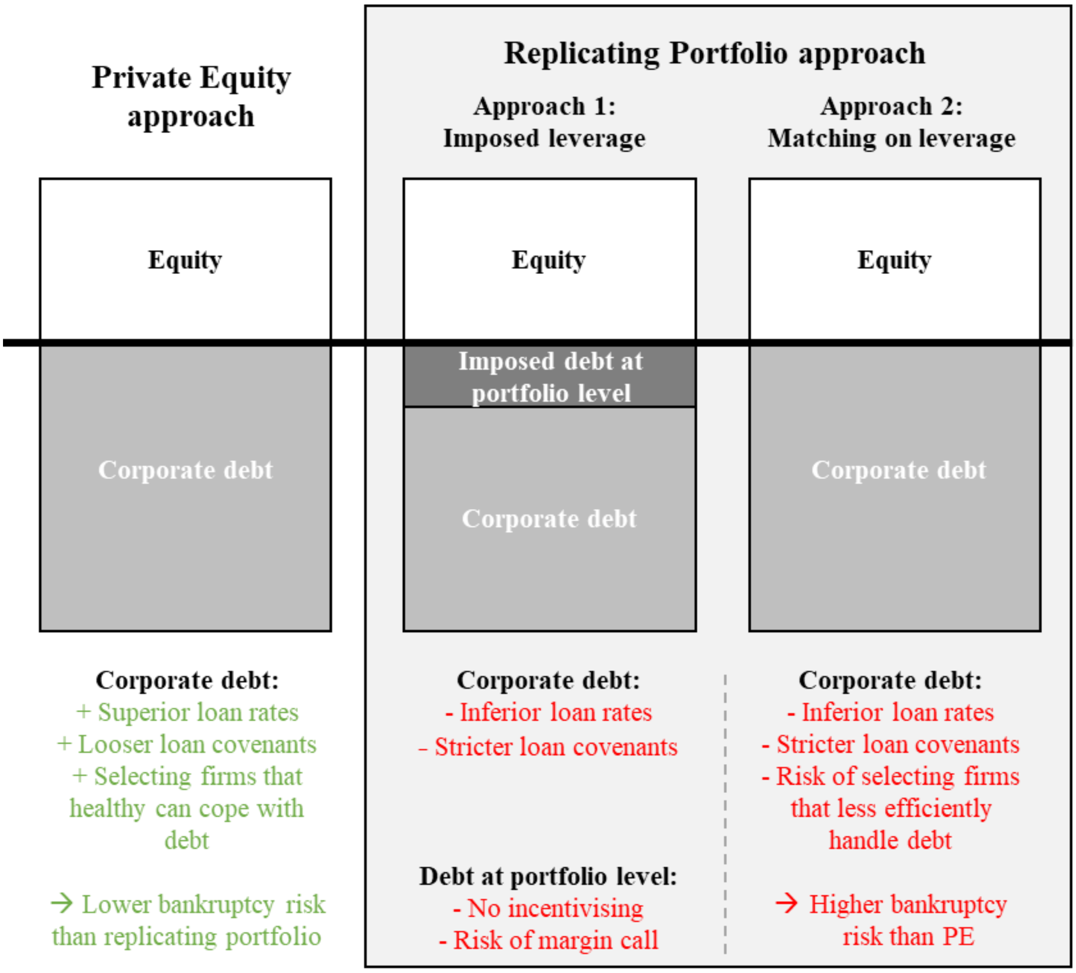

Figure 6: Capital structure differences between private equity and replicating portfolios.
The figure illustrates the capital structure of a typical private equity-backed company in comparison to our two-folded approach. The leverage in both our approaches is similar to the private equity level, but deviates in the inclusion of debt at portfolio level. The figure is for illustrative purposes only.

Furthermore, as presented in Section 3.2.1, the private market provides, compared to the stock market, almost exclusive access to small firms with annual revenue below $2 million. Replication in the stock market is forced to be imperfect in a situation where buyout targets carry characteristics that are absent among most public companies. Such a deviation in asset characteristics underlying investments in our replicating portfolio and private equity funds may give rise to differences in the underlying systematic risk of the portfolios.

Summing up, our results show that our best performing replicating portfolios in terms of results have a measured standard deviation of up to 38.8% compared to the private equity benchmark of 17.2%. The private equity advantages of active management, beneficial debt financing and access to very small firms, may contribute to explain some of the gap in measured risk. Moreover, the possibility of hiding volatility may be seen as an advantage of the private market compared to public equity. This may relate to investor relations, as they might be tempted by the return profile, but also to creditor relations, whereby private equity funds, on the back of low measured risk, possibly enable their debt-givers to proceed with financing at attractive terms.

6.3 The advantages of the replication portfolios

As stated in the last section, a replicating portfolio fails to mimic the active management and attractive long-term debt financing of portfolio companies. The avoided focus on active management, however, enables the replicating portfolio to acquire small fractions of companies instead of a majority interest. Firstly, a replicating portfolio thus avoids paying the price premium associated with acquiring a majority stake in a company. Secondly, it enables a replicating portfolio to include a higher number of assets than a portfolio of a single private equity fund. To the extent Nordic buyout portfolios entail some idiosyncratic risk, this may contribute to a positive diversification effect for the replicating portfolios. Jeffrey Knupp, president of the investment firm DSC, states: “Most PE investors diversify across vintages, and they make sure they have a lot of managers in each vintage…. But that’s a lot of work and you have to be a very large player to do that well.” (Segal, 2019). Thirdly, the passive approach of a replicating portfolio contributes to reducing fees paid by the investor. Our results clearly claim that private equity fees are considerably higher than the costs of a replicating portfolio. Lastly, the absence of active management implies that a replicating portfolio is less restrictive in terms of holding time of stocks. One could expect that the removal of such a restriction increases portfolio return, but, contrastingly, our results indicate that a shorter holding time destroys value when applied to a replicating portfolio, as discussed earlier.

There are additional characteristics with the public market that can provide an advantage over the more stringent private market. The first advantage relates to the higher possibility of rebalancing offered by the public market relative to private equity, which suffers from higher transaction costs. Our results indicate that increased frequency of rebalancing creates value in the replicating portfolios, even when accounting for rising transaction costs. The second advantage considers the long investment cycle of typical private equity funds. A representative private equity portfolio has an investment period of up to three to five years, typically. During this period, the fund managers are searching for potential targets, resulting in a period whereby the investors need to be prepared for capital calls to finance investment opportunities. This contrasts to a public market portfolio which easily can be invested 100% without requiring the capital locked for several years. An imminent aspect of our passive investment approach thus relates to the relatively low fees charged to the investor as well as the avoided cost of inefficient practices like committing capital over time.

Furthermore, a commonly claimed advantage private equity appears to have is its illiquidity premium because of its less liquid market (i.a. Kelleher, 2013). This view is supported by a report from Argentum (2018), finding distinctly lower entry multiples (EV/EBITDA) in private equity deals compared to all transactions for the last ten years in the Nordics. The entry multiple gaps over the years can indicate that an illiquidity premium should exist. Nevertheless, the BlackRock Investment Institute (2019) challenges the commonly viewed illiquidity premium in private equity. They argue that it may be a misconception where the potential premium earned could be more strongly related to complexity and higher governance costs, which can support some of this paper’s findings. Our result that the private equity benchmark seemingly earns a higher risk-adjusted return than our replicating portfolios, advocates the view for the existence of an illiquidity premium in private equity. However, since we cannot conclude how much of private equity's risk-adjusted outperformance that is due to return smoothing, our findings fail to provide strong evidence for neither the existence or absence of an illiquidity premium.

Furthermore, the portfolios purchase small parts of the targeted firms’ equity, which enable them to avoid participation in the processes related to premia occurring in full-firm transactions. They are, therefore, not exposed to bidding-war-process in the private market and avoid paying control premia. Notably, this is an essential concern with the latest trends in the private equity market in which the asset class attracts capital faster than they can deploy it (Bain, 2019). On the back of increasing dry powder in the last years, a more competitive auction process has taken place. Likely, this leads to higher price premiums in the private equity market. This factor contributes to mitigate the value private equity funds would otherwise capture with their investments and serves the replicating portfolios with a relatively more favourable purchase price.

Lastly, a fundamental difference between a replicating public portfolio and private equity is the accessibility for investors. The public market is accessible for everyone, while the private market requires typically a very high initial investment with which are locked to the fund for several years. According to McKinsey (2020), no ETF equivalent gives exposure to passive investment in the asset class. A replicating portfolio thus attempts to give non-professional investors exposure to private equity returns without being subject to the significant barriers to entry that exist in the private market. On the other hand, if many investors and funds start to adopt the strategies for our replicating portfolios, this will ultimately cause a problem for the strategy. The strategy involves investing in a limited number of stocks that appear to be small and illiquid, making the stock prices vulnerable for a massive inflow of capital. Eventually, this will hamper the returns of the replicating strategies and eliminate the possibility to replicate private equity returns.

6.4 How our approach relates to the Stafford paper

As Stafford, we find that a characteristics-matched portfolio applied with the leverage level of private equity funds can reproduce the returns yielded by a pre-fee buyout benchmark. The crucial differences between our findings lie in the initial sample of buyout targets, the asset selection choices and the success in reproducing the attractive risk profile of the benchmark.

Both listed and unlisted firms located in countries that are affected by the EU's laws and regulations, which includes all the Nordic countries, are required to have public balance sheets. That allows us to analyse both private-to-private and public-to-private deals, in which 400 deals were found. Our sample is different from the US sample of 711 deals identified by Stafford (2016) over the period 1983 to 2014, as his only covers public targets taken private by private equity companies. By utilizing a sample of private-to-private deals, our approach differs considerably from the sample of public-to-private deals underlying the Stafford paper. We argue that our sample provides a picture that more accurately represents the aggregate buyout market. Additionally, by incorporating trading costs, our paper introduces a comparison to the post-fee private equity benchmark in contrast to the Stafford paper, which neglected trading costs.

Interestingly, the use of private-to-private deals forces us to restrict which characteristics to be used in asset selection. Stafford found that private equity funds tend to invest in relatively small firms with low net equity issuance and relatively low EBITDA multiples. As recent equity issuance and market values are not available for our treated private companies, we are limited to use other characteristics. Like Stafford, we find that there is a tendency for buyout targets to be relatively small with a relatively low EBITDA. However, we further find that buyout funds target specific sectors and that matching on the relatively high working capital turnover or relatively low asset growth turnover of buyout targets, boosts portfolio performance. We believe our study of characteristic-tendencies in private-to-private deals and the following asset selection choices provide an added understanding of the asset selection of private equity managers.

The massive portfolio risk measured when applying a marking-to-market accounting scheme in our replicating portfolio is consistent with the results of Stafford. However, by applying a hold-to-maturity approach, Stafford managed to eliminate most of the measured risk in the portfolios. A hold-to-maturity approach on our replicating portfolios is not able to reproduce this finding, probably partly due to the limited investment horizon. However, our paper may increase the understanding of the importance of a long holding period and long investment period to smooth volatility through this method. Additionally, our findings add robustness to the conclusion that a portfolio replicating private equity is exposed to high risk.

Furthermore, we find that our approach deviates from the Stafford paper concerning the imposed level of leverage to the portfolios. Stafford imposes fixed leverage of 2x, resulting in a doubleng of the portfolio market beta from 1 to 2. In contrast, although incorporating several levels of imposed leverage, our base case is leverage of ~1.3x, which is sufficient to copy the leverage of Nordic buyout targets. A plausible reason for the difference in imposed leverage may be a deviation in the riskiness of the selected stocks of our approach and Stafford's. Stafford's unlevered replicating portfolio had a market beta close to 1, in contrast to beta in the range 1 and 1.7 for our unlevered portfolios (see Table 1 in the Appendix). Given our private-to-private-approach, the relatively high beta of our replicating portfolios may be due to that we select smaller and riskier stocks than Stafford, who is based on larger public-to-private deals. Interestingly, our best performing portfolio, Portfolio 4, has a market beta of 2.0[4] when imposed our base case level of leverage, which is aligned with the market beta of Stafford's portfolio. This shows that although the distribution of leverage between imposed

[4] Portfolio 4 has a market beta of 2.0 and 1.85 under investment strategy 2 and 3, respectively. See Table 2 in the Appendix.

and company-bearing debt is different between our approaches, the portfolio risk in terms of market beta remains similar.

To sum up, we argue that our approach, utilizing the benefits of a private-to-private sample, add value by supplement and add robustness to the findings of Stafford. It also provides a strong starting point for researching how the replicating portfolio performs over an extended time horizon. Lastly, our results are by far the first in the Nordics, providing a unique insight into how Nordic private equity funds have performed relative to a comparable portfolio of listed firms.

7. Limitations

Due to several conditions, this paper may be subject to numerous limitations. The most profound limitation is related to the restrictions in data availability. The nature of private companies restricts us from applying some of the asset characteristics that previous research has found govern the asset selection choice of private equity. This relates to characteristics involving market prices and capital issuances as such information is not available for private companies. We are further restricted to apply book values of debt and equity which can differ considerably from market values. The lack of several market-based metrics, may trigger an imperfect replication in which the underlying systematic risk of the replicating portfolios differ from private equity funds.

Furthermore, the sample entails a relatively small period because of limited data on Nordic buyout targets. Bureau van Dijk only retains financial data for companies for a rolling period of 8 years. When a new year of data is added, the oldest year is dropped, meaning that only the most recent data for each company is available. This implies that firms involved in a private equity transaction before 2010, and still operating under the same organisational number, most likely are excluded from our data set.

Further, there may be some limitations concerning the private equity benchmark. Argentum reports the number of buyout funds included in the index to be 41 over the period 2005 to 2019. However, there are some uncertainties regarding the number of funds included in the early stages of the period. As the Nordic benchmark returns seem reasonable compared to Argentum's index of private equity return in Western Europe, we argue that the sample of 41 buyout funds is sufficient. However, by comparing the return profile of the replicating portfolio with a benchmark consisting of 41 different funds, our paper overlooks the fact that the benchmark belies substantial variation in performance among individual funds.

Limitations in the availability of both accounting data and benchmark returns result in a reduced length of the investment period. In contrast to Stafford, that invests the replicating portfolios from 1983 to 2014, we are restricted to cover 2006 to 2018. Our limited investment period makes Stafford's sample of buyout targets larger, despite only examining public-to-private deals. Additionally, the longer time horizon makes the portfolio performance less dependent on outliers and randomness, giving his findings more ballast.

An additional limitation relates to the sector classification GICS being applied in Bloomberg. The sector classification involves some missing values going back in time, potentially introducing a bias for which stocks that are missing. As our replicating portfolios invest in stocks operating within specific industry-sectors, stocks with missing information of sector are excluded from the investment universe. Hence, we need to assume that Bloomberg has no selection biases in which stocks they have industry sector information available.

8. Conclusion

This paper investigates whether a passive stock portfolio, mimicking the asset selection and leverage level of private equity funds, to a lower cost can emulate the risk and return yielded by Nordic private equity. We find that buyout funds tend to select relatively small firms within specific sectors of the economy. Further, we provide evidence that buyout targets tend to be relatively more leveraged, relatively more capital-efficient and to have a relatively lower asset growth turnover than comparable Nordic stocks. Additionally, our findings indicate that the public market appears to have advantages over the private market in terms of increased flexibility in rebalancing portfolios, lower transaction costs and the absence of bidding wars.

Overall, two of our 24 characteristics- and leveraged-matched replicating portfolios were able to offer returns that exceed the attractive returns yielded by Nordic private equity in the period June 2006 to June 2018. A five-year buy-and-hold portfolio, selecting stocks based on size, sector, EBITDA and asset growth turnover, yielded an annualised excess return of 18.6% in the investment period, exceeding the pre-fee private equity benchmark of 17.2%. After accounting for fees and transaction costs, 13 of the 24 replicating portfolios earned higher return than private equity.

However, the measured risk of the replicating portfolios is considerably higher than the measured risk of private equity. None of the replicating portfolios is able to reproduce the risk-adjusted private equity return. Our analysis indicates that the lower risk of private equity may be explained by i) the active management approach, ii) the beneficial interest rates and loose covenants of their long-term corporate debt making them less vulnerable to bankruptcy risk, and iii) the existence of return smoothing understating risk. We also stress that some private firms convey risk-reducing characteristics not present in the public market or not captured by our selection criteria. Hence, we cannot exclude that such an imperfect asset replication may explain some of the deviation in risk.

Summing up, our findings indicate that a compelling value proposition of private equity funds is their ability to select small stocks within specific sectors of the economy. Nevertheless, our findings show that an outside investor can capture this return, absent the inefficiencies related to the exorbitant fee structure and multiyear lockup, by merely replicating their asset selection and leverage in the stock market. However, investors paying the high fees of private equity may be exposed to less risk on the back of the risk reduction characteristics provided by private equity's active management and attractive debt financing. Nevertheless, the critical message provided by this paper to investors considering an allocation to either private equity or replicating portfolio relates to risk. Investors in the replicating portfolio must be aware of the tendency towards substantial underperformance in bad times. Investors in private equity should recognise the existence of risk beyond what being reported in the benchmark. Thus, our paper supports previous research that an illiquid private equity portfolio, with funds being locked for several years, may carry a considerably higher risk than what suggested by the return data. Our paper is however not to say that investors should preclude an allocation to private equity as a part of their portfolio. Instead, the findings indicate that a replicating portfolio offers a cheap and accessible alternative investment strategy for investors that deny paying the exorbitant fees of private equity funds and can accept large fluctuations in portfolio values.

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9. Appendix

9.1 How market structure and asset selection affect portfolio performance

The following tables evaluate how portfolio performance is sensitive to changes in market restrictions and asset selection choices. This section represents an extension to Table 19 using different assumptions on the level of imposed leverage. The following tables apply different level of leverage as specified in the headline of each table, starting with a table for unlevered portfolios.

Table A.1. Unlevered portfolios.
Variations in market restrictions are given row-wise by applying four different investment strategies, while variations in asset selection choices are given column-wise by applying six different replicating portfolios. Please see Figure 5 above for an explanation of the investment strategies and the replicating portfolios. Furthermore, all returns are pre-fee and measured in excess of the three-month US Treasury bill return. Sharpe ratio is measured as the annualised pre-fee excess return divided by the standard deviation of the portfolio. Maximum drawdown expresses the percentage decrease from the minimum portfolio value relative to its previous maximum value.

	Strategy	**Portfolio 1**	**Portfolio 2**	**Portfolio 3**	**Portfolio 4**	**Portfolio 5**	**Portfolio 6**
Annualised pre-fee excess return	1	11.5%	12.0%	14.0%	13.6%	13.1%	10.2%
	2	6.1%	10.1%	13.3%	17.2%	14.8%	5.0%
	3	6.5%	10.7%	13.9%	17.2%	14.7%	7.3%
	4	7.0%	7.3%	11.1%	14.2%	10.4%	-2.2%
Annualised st. deviation	1	20.6%	24.0%	22.0%	22.7%	23.6%	26.3%
	2	15.1%	24.1%	28.9%	35.4%	31.6%	19.6%
	3	12.6%	20.3%	23.8%	30.3%	26.8%	21.1%
	4	16.8%	19.2%	21.6%	26.4%	24.7%	23.2%
Sharpe ratio	1	0.56	0.50	0.64	0.60	0.56	0.39
	2	0.40	0.42	0.46	0.49	0.47	0.26
	3	0.51	0.53	0.59	0.57	0.55	0.35
	4	0.42	0.38	0.51	0.54	0.42	-0.09
Maximum drawdown	1	-28.2%	-33.3%	-27.4%	-29.8%	-43.9%	-42.0%
	2	-45.2%	-31.8%	-31.5%	-34.7%	-37.2%	-47.8%
	3	-42.6%	-27.6%	-29.6%	-30.2%	-35.1%	-34.5%
	4	-39.0%	-40.1%	-33.0%	-36.1%	-43.6%	-79.1%
Market Beta	1	0.99	1.12	1.24	1.20	1.38	1.27
	2	0.88	1.19	1.43	1.74	1.55	0.98
	3	0.81	1.15	1.36	1.64	1.45	1.18
	4	0.89	1.04	1.16	1.25	1.30	0.78

Table A.2. Leverage mimicking private equity (D/E-ratio = 0.82)

	Strategy	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6
Annualised pre-fee excess return	1	12.2%	12.5%	14.8%	14.2%	13.8%	10.3%
	2	6.1%	10.5%	13.9%	17.9%	15.5%	4.9%
	3	6.6%	11.2%	14.9%	18.6%	15.8%	7.7%
	4	7.1%	7.4%	11.6%	14.9%	10.9%	-2.9%
Annualised st. deviation	1	23.3%	27.0%	24.5%	24.8%	26.5%	27.6%
	2	17.2%	27.1%	32.1%	38.8%	35.5%	20.5%
	3	14.2%	22.9%	26.8%	34.6%	30.5%	23.5%
	4	19.0%	21.8%	24.2%	29.3%	28.3%	25.6%
Sharpe ratio	1	0.51	0.46	0.60	0.57	0.51	0.37
	2	0.34	0.38	0.43	0.46	0.43	0.23
	3	0.44	0.48	0.55	0.53	0.51	0.32
	4	0.36	0.33	0.47	0.50	0.38	-0.12
Maximum drawdown	1	-32.7%	-38.0%	-31.2%	-33.2%	-49.3%	-44.3%
	2	-34.6%	-36.4%	-35.6%	-38.4%	-42.2%	-45.3%
	3	-29.1%	-32.8%	-33.1%	-33.3%	-38.2%	-40.6%
	4	-36.5%	-42.0%	-37.4%	-39.4%	-50.0%	-74.2%
Market Beta	1	1.17	1.33	1.45	1.36	1.63	1.37
	2	1.07	1.42	1.69	2.00	1.86	1.12
	3	0.91	1.29	1.53	1.85	1.65	1.32
	4	1.07	1.25	1.35	1.46	1.55	0.92

Table A.3. D/E-ratio = 100%

	Strategy	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6
Annualised pre-fee excess return	1	12.6%	13.0%	15.5%	14.9%	14.3%	10.4%
	2	6.1%	10.8%	14.5%	18.7%	16.2%	4.7%
	3	6.6%	11.6%	15.6%	19.6%	16.5%	7.7%
	4	7.1%	7.4%	12.0%	15.6%	11.0%	-4.0%
Annualised st. deviation	1	25.6%	29.6%	26.9%	27.3%	29.2%	30.3%
	2	18.8%	29.8%	35.3%	42.6%	39.0%	22.6%
	3	15.6%	25.2%	29.5%	38.0%	33.5%	25.9%
	4	20.9%	24.0%	26.6%	32.2%	31.1%	28.1%
Sharpe ratio	1	0.49	0.43	0.57	0.54	0.48	0.34
	2	0.31	0.35	0.40	0.43	0.41	0.20
	3	0.39	0.44	0.51	0.50	0.48	0.28
	4	0.33	0.30	0.44	0.48	0.35	-0.15
Maximum drawdown	1	-36.3%	-42.0%	-34.8%	-37.0%	-54.0%	-49.0%
	2	-38.4%	-40.3%	-39.5%	-42.5%	-46.5%	-49.7%
	3	-32.5%	-36.5%	-36.9%	-37.0%	-42.3%	-44.8%
	4	-41.4%	-47.2%	-41.4%	-44.4%	-54.9%	-78.9%
Market Beta	1	1.29	1.46	1.59	1.49	1.79	1.50
	2	1.18	1.56	1.85	2.20	2.04	1.23

	3	1.00	1.42	1.68	2.03	1.81	1.45
	4	1.16	1.37	1.48	1.60	1.70	1.01

Table A.4. Portfolio leverage = ~1.5 x Leverage

	Strategy	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6
Annualised pre-fee excess return	1	12.6%	13.0%	15.5%	14.9%	14.3%	10.4%
	2	6.1%	10.7%	14.5%	18.7%	16.1%	4.7%
	3	6.6%	11.6%	15.6%	19.5%	16.5%	7.7%
	4	7.1%	7.4%	12.0%	15.5%	11.0%	-4.0%
Annualised st. deviation	1	25.6%	29.5%	26.9%	27.2%	29.1%	30.2%
	2	18.8%	29.7%	35.2%	42.5%	38.9%	22.5%
	3	15.6%	25.1%	29.4%	37.9%	33.4%	25.8%
	4	20.8%	23.9%	26.5%	32.1%	31.0%	28.1%
Sharpe ratio	1	0.49	0.43	0.57	0.54	0.48	0.34
	2	0.31	0.35	0.40	0.43	0.41	0.20
	3	0.39	0.44	0.51	0.50	0.48	0.28
	4	0.33	0.30	0.44	0.48	0.35	-0.15
Maximum drawdown	1	-36.2%	-41.9%	-34.7%	-36.9%	-53.9%	-48.8%
	2	-38.3%	-40.2%	-39.4%	-42.4%	-46.4%	-49.6%
	3	-32.4%	-36.4%	-36.8%	-36.9%	-42.2%	-44.6%
	4	-41.2%	-47.0%	-41.3%	-44.3%	-54.8%	-78.8%
Market Beta	1	1.28	1.45	1.59	1.49	1.78	1.50
	2	1.18	1.55	1.85	2.19	2.03	1.23
	3	1.00	1.41	1.67	2.03	1.80	1.44
	4	1.16	1.37	1.48	1.60	1.70	1.01

Table A.5. Portfolio leverage = ~2x Leverage

	Strategy	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6
Annualised pre-fee excess return	1	13.4%	13.6%	16.7%	15.9%	14.9%	10.4%
	2	6.0%	11.1%	15.3%	19.9%	17.1%	4.1%
	3	6.7%	12.2%	16.7%	21.1%	17.7%	7.5%
	4	7.1%	7.3%	12.6%	16.5%	11.0%	-6.2%
Annualised st. deviation	1	29.8%	34.5%	31.3%	31.7%	33.9%	35.2%
	2	21.9%	34.7%	41.1%	49.5%	45.3%	26.2%
	3	18.2%	29.3%	34.3%	44.2%	39.0%	30.1%
	4	24.3%	27.9%	30.9%	37.5%	36.2%	32.7%
Sharpe ratio	1	0.44	0.39	0.53	0.50	0.43	0.29
	2	0.26	0.31	0.37	0.40	0.37	0.15
	3	0.31	0.38	0.46	0.45	0.42	0.22
	4	0.28	0.25	0.40	0.43	0.30	-0.20
Maximum	1	-42.7%	-49.0%	-41.2%	-43.6%	-62.0%	-57.1%

drawdown	2	-45.0%	-47.2%	-46.3%	-49.8%	-54.0%	-57.3%
	3	-38.5%	-43.1%	-43.4%	-43.7%	-49.3%	-52.0%
	4	-49.6%	-55.9%	-48.4%	-53.0%	-63.6%	-85.7%
Market Beta	1	1.50	1.69	1.85	1.74	2.08	1.75
	2	1.37	1.81	2.15	2.56	2.37	1.43
	3	1.17	1.65	1.95	2.37	2.10	1.68
	4	1.35	1.59	1.73	1.86	1.98	1.18

9.2 How leverage affects portfolio performance

This section shows how the choice of imposed leverage affects portfolio return and risk. The tables provide a supplement to Table 20, applying different investment strategies and different replicating portfolios. The applied investment strategy and replicating portfolio are specified in the headline of each table, starting with Strategy 1 and Portfolio 2.

Table A.6. Portfolio 2 and Strategy 1

The table applies 5 different levels of imposed leverage, as described in the methodology and specified column-wise. 10 different levels of bank loan interest rate, ranging from 0% to 9%, are applied, as specified row-wise. All returns are pre-fee and measured in excess of the three-month US Treasury bill return. Maximum drawdown expresses the percentage decrease from the minimum portfolio value relative to its previous maximum value.

	Annualised excess return					**Maximum drawdown**				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	12.0%	13.1%	14.1%	14.1%	15.8%	-33.3%	-37.0%	-40.2%	-40.1%	-45.9%
1%	12.0%	12.9%	13.8%	13.7%	15.1%	-33.3%	-37.3%	-40.8%	-40.7%	-46.9%
2%	12.0%	12.7%	13.4%	13.3%	14.4%	-33.3%	-37.6%	-41.4%	-41.3%	-47.9%
3%	12.0%	12.5%	13.0%	13.0%	13.6%	-33.3%	-38.0%	-42.0%	-41.9%	-49.0%
4%	12.0%	12.3%	12.6%	12.6%	12.9%	-33.3%	-38.3%	-42.6%	-42.4%	-50.0%
5%	12.0%	12.1%	12.2%	12.2%	12.1%	-33.3%	-38.6%	-43.1%	-43.0%	-51.9%
6%	12.0%	11.9%	11.8%	11.8%	11.4%	-33.3%	-38.9%	-43.7%	-43.6%	-53.9%
7%	12.0%	11.7%	11.4%	11.4%	10.7%	-33.3%	-39.2%	-44.3%	-44.1%	-55.8%
8%	12.0%	11.5%	11.0%	11.0%	9.9%	-33.3%	-39.5%	-45.4%	-45.2%	-57.7%
9%	12.0%	11.3%	10.6%	10.6%	9.2%	-33.3%	-39.8%	-46.6%	-46.4%	-59.5%

Table A.7. Portfolio 3 and Strategy 1

	Annualised excess return					**Maximum drawdown**				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	14.0%	15.4%	16.7%	16.6%	18.9%	-27.4%	-30.3%	-33.0%	-33.0%	-37.9%
1%	14.0%	15.2%	16.3%	16.3%	18.2%	-27.4%	-30.6%	-33.6%	-33.5%	-39.0%
2%	14.0%	15.0%	15.9%	15.9%	17.4%	-27.4%	-30.9%	-34.2%	-34.1%	-40.1%
3%	14.0%	14.8%	15.5%	15.5%	16.7%	-27.4%	-31.2%	-34.8%	-34.7%	-41.2%

4%	14.0%	14.6%	15.2%	15.1%	16.0%	-27.4%	-31.5%	-35.4%	-35.3%	-42.2%
5%	14.0%	14.4%	14.8%	14.8%	15.3%	-27.4%	-31.8%	-36.0%	-35.9%	-43.3%
6%	14.0%	14.2%	14.4%	14.4%	14.6%	-27.4%	-32.1%	-36.6%	-36.4%	-44.3%
7%	14.0%	14.1%	14.0%	14.0%	13.9%	-27.4%	-32.4%	-37.2%	-37.0%	-45.4%
8%	14.0%	13.9%	13.7%	13.7%	13.1%	-27.4%	-32.7%	-37.7%	-37.6%	-46.4%
9%	14.0%	13.7%	13.3%	13.3%	12.4%	-27.4%	-33.0%	-38.3%	-38.2%	-47.4%

Table A.8. Portfolio 4 and Strategy 1

	Annualised excess return					**Maximum drawdown**				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	13.6%	14.7%	15.9%	15.9%	18.0%	-29.8%	-32.4%	-35.4%	-35.3%	-40.5%
1%	13.6%	14.6%	15.6%	15.5%	17.3%	-29.8%	-32.7%	-35.9%	-35.8%	-41.6%
2%	13.6%	14.4%	15.2%	15.2%	16.6%	-29.8%	-33.0%	-36.4%	-36.3%	-42.6%
3%	13.6%	14.2%	14.9%	14.9%	15.9%	-29.8%	-33.2%	-37.0%	-36.9%	-43.6%
4%	13.6%	14.1%	14.5%	14.5%	15.2%	-29.8%	-33.5%	-37.5%	-37.4%	-44.6%
5%	13.6%	13.9%	14.2%	14.2%	14.6%	-29.8%	-33.7%	-38.1%	-37.9%	-45.6%
6%	13.6%	13.8%	13.8%	13.8%	13.9%	-29.8%	-34.0%	-38.6%	-38.4%	-46.6%
7%	13.6%	13.6%	13.5%	13.5%	13.2%	-29.8%	-34.2%	-39.1%	-39.0%	-47.5%
8%	13.6%	13.4%	13.2%	13.2%	12.5%	-29.8%	-34.5%	-39.6%	-39.5%	-48.5%
9%	13.6%	13.3%	12.8%	12.8%	11.8%	-29.8%	-34.8%	-40.2%	-40.0%	-49.5%

Table A.9. Portfolio 5 and Strategy 1

	Annualised excess return					**Maximum drawdown**				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	13.1%	14.4%	15.4%	15.4%	17.2%	-43.9%	-48.5%	-52.5%	-52.3%	-59.3%
1%	13.1%	14.2%	15.1%	15.0%	16.4%	-43.9%	-48.8%	-53.0%	-52.9%	-60.2%
2%	13.1%	14.0%	14.7%	14.6%	15.7%	-43.9%	-49.0%	-53.5%	-53.4%	-61.1%
3%	13.1%	13.8%	14.3%	14.3%	14.9%	-43.9%	-49.3%	-54.0%	-53.9%	-62.0%
4%	13.1%	13.6%	13.9%	13.9%	14.2%	-43.9%	-49.6%	-54.5%	-54.4%	-62.9%
5%	13.1%	13.4%	13.5%	13.5%	13.4%	-43.9%	-49.9%	-55.0%	-54.9%	-63.8%
6%	13.1%	13.2%	13.1%	13.1%	12.7%	-43.9%	-50.2%	-55.6%	-55.4%	-64.6%
7%	13.1%	13.0%	12.7%	12.7%	11.9%	-43.9%	-50.5%	-56.1%	-55.9%	-65.5%
8%	13.1%	12.7%	12.3%	12.3%	11.2%	-43.9%	-50.7%	-56.6%	-56.4%	-66.3%
9%	13.1%	12.5%	11.9%	11.9%	10.4%	-43.9%	-51.0%	-57.1%	-56.9%	-67.1%

Table A.10. Portfolio 2 and Strategy 2

	Annualised excess return					**Maximum drawdown**				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	10.1%	11.1%	11.9%	11.9%	13.3%	-31.8%	-35.4%	-38.6%	-38.5%	-44.1%
1%	10.1%	10.9%	11.5%	11.5%	12.6%	-31.8%	-35.7%	-39.2%	-39.1%	-45.2%
2%	10.1%	10.7%	11.1%	11.1%	11.9%	-31.8%	-36.1%	-39.8%	-39.6%	-46.2%

3%	10.1%	10.5%	10.8%	10.7%	11.1%	-31.8%	-36.4%	-40.3%	-40.2%	-47.2%
4%	10.1%	10.3%	10.4%	10.4%	10.4%	-31.8%	-36.7%	-40.9%	-40.8%	-48.3%
5%	10.1%	10.1%	10.0%	10.0%	9.7%	-31.8%	-37.0%	-41.5%	-41.4%	-49.3%
6%	10.1%	9.9%	9.6%	9.6%	8.9%	-31.8%	-37.4%	-42.1%	-42.0%	-50.3%
7%	10.1%	9.6%	9.2%	9.2%	8.2%	-31.8%	-37.7%	-42.7%	-42.5%	-51.3%
8%	10.1%	9.4%	8.8%	8.8%	7.4%	-31.8%	-38.0%	-43.3%	-43.1%	-52.3%
9%	10.1%	9.2%	8.4%	8.4%	6.7%	-31.8%	-38.3%	-43.8%	-43.7%	-53.3%

Table A.11. Portfolio 3 and Strategy 2

	Annualised excess return					Maximum drawdown				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	13.3%	14.5%	15.6%	15.6%	17.5%	-31.5%	-34.7%	-37.8%	-37.7%	-43.2%
1%	13.3%	14.3%	15.2%	15.2%	16.8%	-31.5%	-35.0%	-38.4%	-38.3%	-44.3%
2%	13.3%	14.1%	14.8%	14.8%	16.0%	-31.5%	-35.3%	-38.9%	-38.8%	-45.3%
3%	13.3%	13.9%	14.5%	14.5%	15.3%	-31.5%	-35.6%	-39.5%	-39.4%	-46.3%
4%	13.3%	13.7%	14.1%	14.1%	14.6%	-31.5%	-35.9%	-40.1%	-40.0%	-47.3%
5%	13.3%	13.5%	13.7%	13.7%	13.9%	-31.5%	-36.2%	-40.6%	-40.5%	-48.3%
6%	13.3%	13.4%	13.3%	13.3%	13.1%	-31.5%	-36.5%	-41.2%	-41.1%	-49.3%
7%	13.3%	13.2%	13.0%	13.0%	12.4%	-31.5%	-36.8%	-41.8%	-41.6%	-50.3%
8%	13.3%	13.0%	12.6%	12.6%	11.7%	-31.5%	-37.1%	-42.3%	-42.2%	-51.3%
9%	13.3%	12.8%	12.2%	12.2%	11.0%	-31.5%	-37.4%	-42.9%	-42.7%	-52.3%

Table A.12. Portfolio 4 and Strategy 2

	Annualised excess return					Maximum drawdown				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	17.2%	18.4%	19.8%	19.7%	22.0%	-34.7%	-37.6%	-41.0%	-40.9%	-46.9%
1%	17.2%	18.3%	19.4%	19.4%	21.3%	-34.7%	-37.9%	-41.5%	-41.4%	-47.9%
2%	17.2%	18.1%	19.1%	19.0%	20.6%	-34.7%	-38.1%	-42.0%	-41.9%	-48.8%
3%	17.2%	17.9%	18.7%	18.7%	19.9%	-34.7%	-38.4%	-42.5%	-42.4%	-49.8%
4%	17.2%	17.8%	18.4%	18.3%	19.2%	-34.7%	-38.6%	-43.1%	-42.9%	-50.9%
5%	17.2%	17.6%	18.0%	18.0%	18.5%	-34.7%	-38. 9%	-43.6%	-43.4%	-52.8%
6%	17.2%	17.4%	17.7%	17.6%	17.8%	-34.7%	-39.1%	-44.1%	-43.9%	-54.7%
7%	17.2%	17.3%	17.3%	17.3%	17.1%	-34.7%	-39.4%	-44.6%	-44.4%	-56.5%
8%	17.2%	17.1%	16.9%	16.9%	16.4%	-34.7%	-39.6%	-45.6%	-45.4%	-58.2%
9%	17.2%	16.9%	16.6%	16.6%	15.6%	-34.7%	-39.9%	-46.7%	-46.4%	-59.9%

Table A.13. Portfolio 5 and Strategy 2

	Annualised excess return					Maximum drawdown				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	14.8%	16.2%	17.3%	17.3%	19.3%	-37.2%	-41.3%	-44.9%	-44.7%	-51.1%
1%	14.8%	16.0%	17.0%	16.9%	18.6%	-37.2%	-41.6%	-45.4%	-45.3%	-52.1%

2%	14.8%	15.7%	16.6%	16.5%	17.8%	-37.2%	-41.9%	-46.0%	-45.9%	-53.0%
3%	14.8%	15.5%	16.2%	16.1%	17.1%	-37.2%	-42.2%	-46.5%	-46.4%	-54.0%
4%	14.8%	15.3%	15.8%	15.8%	16.3%	-37.2%	-42.5%	-47.1%	-47.0%	-55.0%
5%	14.8%	15.1%	15.4%	15.4%	15.6%	-37.2%	-42.8%	-47.6%	-47.5%	-55.9%
6%	14.8%	14.9%	15.0%	15.0%	14.8%	-37.2%	-43.1%	-48.2%	-48.0%	-56.9%
7%	14.8%	14.7%	14.6%	14.6%	14.1%	-37.2%	-43.4%	-48.7%	-48.6%	-57.8%
8%	14.8%	14.5%	14.2%	14.2%	13.3%	-37.2%	-43.7%	-49.3%	-49.1%	-58.7%
9%	14.8%	14.3%	13.8%	13.8%	12.6%	-37.2%	-44.0%	-49.8%	-49.7%	-59.6%

Table A.14. Portfolio 2 and Strategy 3

	Annualised excess return					**Maximum drawdown**				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	10.7%	11.8%	12.8%	12.8%	14.4%	-27.6%	-31.8%	-34.7%	-34.6%	-39.9%
1%	10.7%	11.6%	12.4%	12.4%	13.7%	-27.6%	-32.1%	-35.3%	-35.2%	-41.0%
2%	10.7%	11.4%	12.0%	12.0%	13.0%	-27.6%	-32.5%	-35.9%	-35.8%	-42.1%
3%	10.7%	11.2%	11.6%	11.6%	12.2%	-27.6%	-32.8%	-36.5%	-36.4%	-43.1%
4%	10.7%	11.0%	11.2%	11.2%	11.5%	-27.6%	-33.1%	-37.2%	-37.0%	-44.2%
5%	10.7%	10.8%	10.8%	10.8%	10.8%	-27.6%	-33.5%	-37.8%	-37.6%	-45.3%
6%	10.7%	10.6%	10.5%	10.5%	10.0%	-27.6%	-33.8%	-38.4%	-38.2%	-46.3%
7%	10.7%	10.4%	10.1%	10.1%	9.3%	-27.6%	-34.1%	-39.0%	-38.8%	-47.4%
8%	10.7%	10.2%	9.7%	9.7%	8.6%	-27.6%	-34.4%	-39.6%	-39.4%	-48.4%
9%	10.7%	10.0%	9.3%	9.3%	7.8%	-27.6%	-34.8%	-40.2%	-40.0%	-49.4%

Table A.15. Portfolio 3 and Strategy 3

	Annualised excess return					**Maximum drawdown**				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	13.9%	15.5%	16.7%	16.7%	18.9%	-29.6%	-32.2%	-35.1%	-35.0%	-40.2%
1%	13.9%	15.3%	16.3%	16.3%	18.2%	-29.6%	-32.5%	-35.7%	-35.6%	-41.3%
2%	13.9%	15.1%	16.0%	15.9%	17.5%	-29.6%	-32.8%	-36.3%	-36.2%	-42.4%
3%	13.9%	14.9%	15.6%	15.6%	16.7%	-29.6%	-33.1%	-36.9%	-36.8%	-43.4%
4%	13.9%	14.7%	15.2%	15.2%	16.0%	-29.6%	-33.4%	-37.5%	-37.3%	-44.5%
5%	13.9%	14.5%	14.8%	14.8%	15.3%	-29.6%	-33.7%	-38.0%	-37.9%	-45.5%
6%	13.9%	14.3%	14.5%	14.5%	14.6%	-29.6%	-34.1%	-38.6%	-38.5%	-46.5%
7%	13.9%	14.1%	14.1%	14.1%	13.9%	-29.6%	-34.4%	-39.2%	-39.0%	-47.5%
8%	13.9%	13.9%	13.7%	13.7%	13.1%	-29.6%	-34.7%	-39.7%	-39.6%	-48.5%
9%	13.9%	13.8%	13.3%	13.4%	12.4%	-29.6%	-35.0%	-40.3%	-40.2%	-49.5%

Table A.16. Portfolio 4 and Strategy 3

	Annualised excess return					**Maximum drawdown**				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	17.2%	19.1%	20.6%	20.6%	23.2%	-30.2%	-32.5%	-35.4%	-35.3%	-40.6%

1%	17.2%	19.0%	20.3%	20.2%	22.5%	-30.2%	-32.7%	-36.0%	-35.9%	-41.6%
2%	17.2%	18.8%	19.9%	19.9%	21.8%	-30.2%	-33.0%	-36.5%	-36.4%	-42.7%
3%	17.2%	18.6%	19.6%	19.5%	21.1%	-30.2%	-33.3%	-37.0%	-36.9%	-43.7%
4%	17.2%	18.5%	19.2%	19.2%	20.4%	-30.2%	-33.5%	-37.6%	-37.5%	-44.7%
5%	17.2%	18.3%	18.9%	18.8%	19.7%	-30.2%	-33.8%	-38.1%	-38.0%	-45.7%
6%	17.2%	18.1%	18.5%	18.5%	19.0%	-30.2%	-34.0%	-38.6%	-38.5%	-46.6%
7%	17.2%	18.0%	18.1%	18.1%	18.3%	-30.2%	-34.3%	-39.2%	-39.0%	-47.6%
8%	17.2%	17.8%	17.8%	17.8%	17.5%	-30.2%	-34.5%	-39.7%	-39.5%	-48.6%
9%	17.2%	17.6%	17.4%	17.4%	16.8%	-30.2%	-34.8%	-40.2%	-40.1%	-49.5%

Table A.17. Portfolio 5 and Strategy 3

	Annualised excess return					Maximum drawdown				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	14.7%	16.4%	17.7%	17.7%	19.9%	-35.1%	-37.3%	-40.5%	-40.4%	-46.3%
1%	14.7%	16.2%	17.3%	17.3%	19.2%	-35.1%	-37.6%	-41.1%	-41.0%	-47.3%
2%	14.7%	16.0%	16.9%	16.9%	18.4%	-35.1%	-37.9%	-41.7%	-41.6%	-48.3%
3%	14.7%	15.8%	16.5%	16.5%	17.7%	-35.1%	-38.2%	-42.3%	-42.2%	-49.3%
4%	14.7%	15.6%	16.1%	16.1%	16.9%	-35.1%	-38.5%	-42.9%	-42.7%	-50.3%
5%	14.7%	15.4%	15.7%	15.7%	16.2%	-35.1%	-38.8%	-43.4%	-43.3%	-51.3%
6%	14.7%	15.2%	15.3%	15.3%	15.5%	-35.1%	-39.2%	-44.0%	-43.9%	-52.3%
7%	14.7%	15.0%	14.9%	14.9%	14.7%	-35.1%	-39.5%	-44.6%	-44.4%	-53.3%
8%	14.7%	14.8%	14.6%	14.6%	14.0%	-35.1%	-39.8%	-45.1%	-45.0%	-54.3%
9%	14.7%	14.6%	14.2%	14.2%	13.2%	-35.1%	-40.1%	-45.7%	-45.5%	-55.2%

Table A.18. Portfolio 2 and Strategy 4

	Annualised excess return					Maximum drawdown				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	7.3%	8.0%	8.6%	8.6%	9.5%	-40.1%	-40.0%	-43.6%	-43.5%	-49.9%
1%	7.3%	7.8%	8.2%	8.2%	8.8%	-40.1%	-40.7%	-44.8%	-44.7%	-52.0%
2%	7.3%	7.6%	7.8%	7.8%	8.0%	-40.1%	-41.4%	-46.0%	-45.9%	-53.9%
3%	7.3%	7.4%	7.4%	7.4%	7.3%	-40.1%	-42.0%	-47.2%	-47.0%	-55.9%
4%	7.3%	7.2%	7.0%	7.0%	6.6%	-40.1%	-42.7%	-48.3%	-48.2%	-57.7%
5%	7.3%	7.0%	6.6%	6.6%	5.8%	-40.1%	-43.3%	-49.5%	-49.3%	-59.5%
6%	7.3%	6.8%	6.2%	6.3%	5.1%	-40.1%	-44.0%	-50.6%	-50.4%	-61.2%
7%	7.3%	6.6%	5.8%	5.9%	4.4%	-40.1%	-44.6%	-51.7%	-51.5%	-62.9%
8%	7.3%	6.4%	5.5%	5.5%	3.6%	-40.1%	-45.3%	-52.8%	-52.5%	-64.5%
9%	7.3%	6.2%	5.1%	5.1%	2.9%	-40.1%	-45.9%	-53.8%	-53.6%	-66.1%

Table A.19. Portfolio 3 and Strategy 4

	Annualised excess return					Maximum drawdown				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage

0%	11.1%	12.2%	13.1%	13.1%	14.7%	-33.0%	-36.5%	-39.8%	-39.7%	-45.4%
1%	11.1%	12.0%	12.8%	12.7%	14.0%	-33.0%	-36.8%	-40.3%	-40.2%	-46.4%
2%	11.1%	11.8%	12.4%	12.4%	13.3%	-33.0%	-37.1%	-40.9%	-40.8%	-47.4%
3%	11.1%	11.6%	12.0%	12.0%	12.6%	-33.0%	-37.4%	-41.4%	-41.3%	-48.4%
4%	11.1%	11.4%	11.6%	11.6%	11.9%	-33.0%	-37.7%	-42.0%	-41.9%	-49.4%
5%	11.1%	11.2%	11.3%	11.3%	11.2%	-33.0%	-38.0%	-42.6%	-42.4%	-50.4%
6%	11.1%	11.0%	10.9%	10.9%	10.4%	-33.0%	-38.3%	-43.1%	-43.0%	-51.4%
7%	11.1%	10.9%	10.5%	10.5%	9.7%	-33.0%	-38.6%	-43.7%	-43.5%	-52.3%
8%	11.1%	10.7%	10.1%	10.2%	9.0%	-33.0%	-38.9%	-44.2%	-44.1%	-54.0%
9%	11.1%	10.5%	9.8%	9.8%	8.3%	-33.0%	-39.2%	-44.8%	-44.6%	-55.9%

Table A.20. Portfolio 4 and Strategy 4

	Annualised excess return					Maximum drawdown				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	14.2%	15.4%	16.6%	16.6%	18.6%	-36.1%	-37.8%	-41.2%	-41.1%	-47.1%
1%	14.2%	15.3%	16.3%	16.2%	17.9%	-36.1%	-38.3%	-42.3%	-42.2%	-49.1%
2%	14.2%	15.1%	15.9%	15.9%	17.2%	-36.1%	-38.9%	-43.4%	-43.2%	-51.1%
3%	14.2%	14.9%	15.6%	15.5%	16.5%	-36.1%	-39.4%	-44.4%	-44.3%	-53.0%
4%	14.2%	14.8%	15.2%	15.2%	15.8%	-36.1%	-39.9%	-45.5%	-45.3%	-54.8%
5%	14.2%	14.6%	14.9%	14.8%	15.1%	-36.1%	-40.5%	-46.5%	-46.4%	-56.5%
6%	14.2%	14.5%	14.5%	14.5%	14.4%	-36.1%	-41.0%	-47.6%	-47.4%	-58.3%
7%	14.2%	14.3%	14.2%	14.2%	13.7%	-36.1%	-41.5%	-48.6%	-48.4%	-59.9%
8%	14.2%	14.1%	13.8%	13.8%	13.0%	-36.1%	-42.0%	-49.6%	-49.4%	-61.5%
9%	14.2%	14.0%	13.4%	13.5%	12.3%	-36.1%	-42.5%	-50.6%	-50.3%	-63.1%

Table A.21. Portfolio 5 and Strategy 4

	Annualised excess return					Maximum drawdown				
Interest rate	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage	No leverage	PE leverage	1x leverage	1.5x leverage	2x leverage
0%	10.4%	11.5%	12.2%	12.2%	13.3%	-43.6%	-49.2%	-53.3%	-53.2%	-60.6%
1%	10.4%	11.3%	11.8%	11.8%	12.6%	-43.6%	-49.5%	-53.9%	-53.7%	-61.5%
2%	10.4%	11.1%	11.4%	11.4%	11.8%	-43.6%	-49.7%	-54.4%	-54.2%	-62.4%
3%	10.4%	10.9%	11.0%	11.0%	11.0%	-43.6%	-50.0%	-54.9%	-54.8%	-63.6%
4%	10.4%	10.7%	10.6%	10.6%	10.3%	-43.6%	-50.3%	-55.6%	-55.5%	-64.7%
5%	10.4%	10.5%	10.2%	10.2%	9.5%	-43.6%	-50.6%	-56.3%	-56.1%	-65.8%
6%	10.4%	10.2%	9.8%	9.8%	8.8%	-43.6%	-51.0%	-57.0%	-56.8%	-66.9%
7%	10.4%	10.0%	9.4%	9.5%	8.0%	-43.6%	-51.4%	-57.6%	-57.4%	-68.3%
8%	10.4%	9.8%	9.0%	9.1%	7.3%	-43.6%	-51.7%	-58.3%	-58.1%	-69.7%
9%	10.4%	9.6%	8.6%	8.7%	6.5%	-43.6%	-52.1%	-58.9%	-58.7%	-71.1%

9.3 How fees affect portfolio performance

This section considers how different levels of trading cost affect portfolio performance. The tables provide an extension to Table 21, which examined the trading costs effect on both non-levered portfolios and portfolios with a fixed D/E-ratio of 0.82. In this section, we examine the effect of trading costs by applying different levels of leverage.

Table A.22: Unlevered portfolios
The table applies trading costs in the range of 0bps and 150bps, as argued in the methodology and specified column-wise. The table evaluates the results using all 4 investment strategies and all 6 replicating portfolios. Strategy 1 applies a five-year holding time without rebalancing, Strategy 2 a five-year holding time with yearly rebalancing, Strategy 3 a five-year holding time with monthly rebalancing, and Strategy 4 a yearly rebalancing with no holding time. The performances of all 6 different portfolios are evaluated for each of the 4 strategies. All returns are pre-fee and measured in excess of the three-month US Treasury bill return. For the levered portfolios, a loan interest rate of 3% is applied.

Trading cost →		0bps	25bps	75bps	100bps	150bps
Strategy 1	Portfolio 1	11.5%	11.4%	11.3%	11.1%	11.0%
	Portfolio 2	12.0%	11.9%	11.7%	11.5%	11.4%
	Portfolio 3	14.0%	13.9%	13.7%	13.5%	13.4%
	Portfolio 4	13.6%	13.5%	13.3%	13.1%	13.0%
	Portfolio 5	13.1%	13.1%	12.9%	12.7%	12.6%
	Portfolio 6	10.2%	10.1%	9.9%	9.8%	9.7%
Strategy 2	Portfolio 1	6.1%	6.0%	5.8%	5.6%	5.5%
	Portfolio 2	10.1%	10.0%	9.8%	9.6%	9.5%
	Portfolio 3	13.3%	13.2%	13.0%	12.8%	12.8%
	Portfolio 4	17.2%	17.1%	16.9%	16.7%	16.6%
	Portfolio 5	14.8%	14.7%	14.5%	14.3%	14.2%
	Portfolio 6	5.0%	5.0%	4.8%	4.6%	4.6%
Strategy 3	Portfolio 1	6.5%	6.2%	5.5%	4.9%	4.6%
	Portfolio 2	10.7%	10.4%	9.7%	9.1%	8.8%
	Portfolio 3	13.9%	13.6%	13.0%	12.4%	12.1%
	Portfolio 4	17.2%	16.9%	16.3%	15.8%	15.5%
	Portfolio 5	14.7%	14.4%	13.9%	13.3%	13.0%
	Portfolio 6	7.3%	7.0%	6.5%	5.9%	5.6%
Strategy 4	Portfolio 1	7.0%	6.9%	6.7%	6.6%	6.5%
	Portfolio 2	7.3%	7.2%	7.0%	6.8%	6.7%
	Portfolio 3	11.1%	11.0%	10.9%	10.7%	10.6%
	Portfolio 4	14.2%	14.1%	14.0%	13.8%	13.8%
	Portfolio 5	10.4%	10.3%	10.2%	10.0%	9.9%
	Portfolio 6	-2.2%	-2.2%	-2.4%	-2.5%	-2.6%

Table A.23: Leverage mimicking private equity (D/E-ratio = 0.82)

Trading cost →		0bps	25bps	75bps	100bps	150bps

Strategy 1	Portfolio 1	12.2%	12.1%	11.9%	11.7%	11.6%
	Portfolio 2	12.5%	12.4%	12.2%	12.1%	12.0%
	Portfolio 3	14.8%	14.7%	14.5%	14.3%	14.2%
	Portfolio 4	14.2%	14.1%	13.9%	13.7%	13.7%
	Portfolio 5	13.8%	13.7%	13.5%	13.3%	13.2%
	Portfolio 6	10.3%	10.2%	10.0%	9.9%	9.8%
Strategy 2	Portfolio 1	6.1%	6.0%	5.7%	5.5%	5.3%
	Portfolio 2	10.5%	10.4%	10.1%	9.9%	9.7%
	Portfolio 3	13.9%	13.8%	13.6%	13.4%	13.2%
	Portfolio 4	17.9%	17.8%	17.6%	17.4%	17.3%
	Portfolio 5	15.5%	15.4%	15.2%	15.0%	14.8%
	Portfolio 6	4.9%	4.8%	4.6%	4.4%	4.4%
Strategy 3	Portfolio 1	6.6%	6.2%	5.3%	4.5%	4.1%
	Portfolio 2	11.2%	10.8%	10.0%	9.2%	8.9%
	Portfolio 3	14.9%	14.5%	13.8%	13.0%	12.6%
	Portfolio 4	18.6%	18.3%	17.6%	16.9%	16.5%
	Portfolio 5	15.8%	15.4%	14.7%	13.9%	13.6%
	Portfolio 6	7.7%	7.3%	6.6%	5.9%	5.6%
Strategy 4	Portfolio 1	7.1%	7.0%	6.8%	6.5%	6.4%
	Portfolio 2	7.4%	7.3%	7.1%	6.8%	6.7%
	Portfolio 3	11.6%	11.5%	11.3%	11.1%	11.0%
	Portfolio 4	14.9%	14.9%	14.7%	14.5%	14.4%
	Portfolio 5	10.9%	10.8%	10.6%	10.4%	10.3%
	Portfolio 6	-2.9%	-3.0%	-3.2%	-3.3%	-3.4%

Table A.24: D/E-ratio = 1.0

Trading cost →		0bps	25bps	75bps	100bps	150bps
Strategy 1	Portfolio 1	12.6%	12.5%	12.4%	12.2%	12.1%
	Portfolio 2	13.0%	12.9%	12.7%	12.5%	12.4%
	Portfolio 3	15.5%	15.4%	15.2%	15.0%	15.0%
	Portfolio 4	14.9%	14.8%	14.6%	14.4%	14.3%
	Portfolio 5	14.3%	14.2%	14.0%	13.8%	13.7%
	Portfolio 6	10.4%	10.3%	10.2%	10.0%	9.9%
Strategy 2	Portfolio 1	6.1%	5.9%	5.7%	5.4%	5.3%
	Portfolio 2	10.8%	10.6%	10.4%	10.1%	10.0%
	Portfolio 3	14.5%	14.4%	14.1%	13.9%	13.8%
	Portfolio 4	18.7%	18.6%	18.4%	18.2%	18.0%
	Portfolio 5	16.2%	16.0%	15.8%	15.6%	15.5%
	Portfolio 6	4.7%	4.6%	4.4%	4.2%	4.1%
Strategy 3	Portfolio 1	6.6%	6.2%	5.4%	4.6%	4.2%
	Portfolio 2	11.6%	11.2%	10.4%	9.6%	9.2%
	Portfolio 3	15.6%	15.2%	14.5%	13.7%	13.3%
	Portfolio 4	19.6%	19.2%	18.5%	17.8%	17.4%
	Portfolio 5	16.5%	16.1%	15.4%	14.7%	14.3%
	Portfolio 6	7.7%	7.3%	6.6%	5.9%	5.6%

Strategy 4	Portfolio 1	7.1%	7.0%	6.8%	6.6%	6.5%
	Portfolio 2	7.4%	7.3%	7.1%	6.8%	6.7%
	Portfolio 3	12.0%	11.9%	11.7%	11.5%	11.4%
	Portfolio 4	15.6%	15.5%	15.3%	15.1%	15.0%
	Portfolio 5	11.0%	10.9%	10.7%	10.5%	10.4%
	Portfolio 6	-4.0%	-4.1%	-4.3%	-4.4%	-4.5%

Table A.25: Portfolio leverage = ~1.5x leverage

Trading cost →		0bps	25bps	75bps	100bps	150bps
Strategy 1	Portfolio 1	12.6%	12.5%	12.4%	12.2%	12.1%
	Portfolio 2	13.0%	12.9%	12.7%	12.5%	12.4%
	Portfolio 3	15.5%	15.4%	15.2%	15.0%	14.9%
	Portfolio 4	14.9%	14.8%	14.6%	14.4%	14.3%
	Portfolio 5	14.3%	14.2%	14.0%	13.8%	13.7%
	Portfolio 6	10.4%	10.3%	10.2%	10.0%	9.9%
Strategy 2	Portfolio 1	6.1%	5.9%	5.7%	5.4%	5.3%
	Portfolio 2	10.7%	10.6%	10.4%	10.1%	10.0%
	Portfolio 3	14.5%	14.3%	14.1%	13.9%	13.8%
	Portfolio 4	18.7%	18.6%	18.4%	18.1%	18.0%
	Portfolio 5	16.1%	16.0%	15.8%	15.6%	15.4%
	Portfolio 6	4.7%	4.6%	4.4%	4.2%	4.1%
Strategy 3	Portfolio 1	6.6%	6.2%	5.4%	4.6%	4.2%
	Portfolio 2	11.6%	11.2%	10.4%	9.6%	9.2%
	Portfolio 3	15.6%	15.2%	14.4%	13.7%	13.3%
	Portfolio 4	19.5%	19.2%	18.5%	17.8%	17.4%
	Portfolio 5	16.5%	16.1%	15.4%	14.6%	14.3%
	Portfolio 6	7.7%	7.3%	6.6%	5.9%	5.6%
Strategy 4	Portfolio 1	7.1%	7.0%	6.8%	6.6%	6.5%
	Portfolio 2	7.4%	7.3%	7.1%	6.8%	6.7%
	Portfolio 3	12.0%	11.9%	11.7%	11.5%	11.4%
	Portfolio 4	15.5%	15.4%	15.3%	15.1%	15.0%
	Portfolio 5	11.0%	10.9%	10.7%	10.5%	10.4%
	Portfolio 6	-4.0%	-4.1%	-4.3%	-4.4%	-4.5%

Table A.26: Portfolio leverage = ~2x leverage

Trading cost →		0bps	25bps	75bps	100bps	150bps
Strategy 1	Portfolio 1	13.4%	13.3%	13.1%	13.0%	12.9%
	Portfolio 2	13.6%	13.5%	13.3%	13.1%	13.0%
	Portfolio 3	16.7%	16.6%	16.4%	16.2%	16.1%
	Portfolio 4	15.9%	15.8%	15.6%	15.4%	15.3%
	Portfolio 5	14.9%	14.8%	14.6%	14.4%	14.3%
	Portfolio 6	10.4%	10.4%	10.2%	10.0%	9.9%
Strategy 2	Portfolio 1	6.0%	5.8%	5.6%	5.3%	5.2%
	Portfolio 2	11.1%	11.0%	10.8%	10.5%	10.4%

	Portfolio 3	15.3%	15.2%	15.0%	14.7%	14.6%
	Portfolio 4	19.9%	19.8%	19.6%	19.4%	19.2%
	Portfolio 5	17.1%	17.0%	16.7%	16.5%	16.4%
	Portfolio 6	4.1%	4.0%	3.8%	3.7%	3.6%
Strategy 3	Portfolio 1	6.7%	6.3%	5.5%	4.7%	4.3%
	Portfolio 2	12.2%	11.8%	11.0%	10.2%	9.8%
	Portfolio 3	16.7%	16.4%	15.6%	14.8%	14.5%
	Portfolio 4	21.1%	20.7%	20.0%	19.3%	18.9%
	Portfolio 5	17.7%	17.3%	16.5%	15.8%	15.4%
	Portfolio 6	7.5%	7.2%	6.5%	5.8%	5.4%
Strategy 4	Portfolio 1	7.1%	7.0%	6.8%	6.6%	6.5%
	Portfolio 2	7.3%	7.2%	6.9%	6.7%	6.6%
	Portfolio 3	12.6%	12.5%	12.3%	12.1%	12.0%
	Portfolio 4	16.5%	16.4%	16.2%	16.0%	15.9%
	Portfolio 5	11.0%	10.9%	10.7%	10.5%	10.4%
	Portfolio 6	-6.2%	-6.3%	-6.5%	-6.7%	-6.7%

9.4 R-code

The code below excludes the coding applied for the creation of tables and charts. Some of the packages mentioned in the code, which are used to make graphics, are thus not necessary to run the code. Wide margins to simplify reading.

```
rm(list=ls()) # clears environment

# Install/load packages ---------------------------------------------------

# install packages
install.packages("stringr")
install.packages("ggplot2")
install.packages("psych")
install.packages("matrixStats")
install.packages("PerformanceAnalytics")
install.packages("reshape2")
install.packages("data.table")
install.packages("dplyr")
install.packages("formattable")
install.packages("tidyr")
install.packages("rlist")
install.packages("ggpubr")
install.packages("stargazer")


# load packages
library(data.table)
library(dplyr)
library(formattable)
library(tidyr)
library(stringr)
library(reshape2)
library(ggplot2)
library(psych)
library(reshape2)
library(matrixStats)
```

```
library(PerformanceAnalytics)
library(rlist)
library(ggpubr)
library(stargazer)


# Load data sets ----------------------------------------------------------

   ## data on all Nordic stocks in each year in investment period
   pm_revenue <- read.csv("pm_revenue.csv",na.strings = "null",sep=",")[,-2] # annual revenues
   pm_ebitda <- read.csv("pm_ebitda.csv",na.strings = "null",sep=",")[,-2] # annual EBITDA
   pm_sector <- read.csv("pm_sector.csv",na.strings="null",sep=";") # industry sector
   adj_prices_monthly <- read.csv("adj_prices_monthly.csv",na.strings="null",sep=",") # yearly adjusted
stock price
   pm_working_capital <- read.csv("pm_working_capital.csv",na.strings = "null",sep=",")[,-2] # working
capital in balance sheet (BS)
   pm_total_assets <- read.csv("pm_total_assets.csv",na.strings = "null",sep=",")[,-2] # total assets in BS
   pm_lt_borrowing <- read.csv("pm_lt_borrowing.csv",na.strings = "null",sep=",")[,-2] # long-term debt in
BS
   pm_olt_borrowing <- read.csv("pm_olt_borrowing.csv",na.strings = "null",sep=",")[,-2] # other long-term
debt in BS
   pm_st_debt <- read.csv("pm_st_debt.csv",na.strings = "null",sep=",")[,-2] # short term debt in BS
   pm_equity <- read.csv("pm_total_equity.csv",na.strings = "null",sep=",")[,-2] # book value of equity
   pm_employees <- read.csv("pm_employees.csv",na.strings = "null",sep=",")[,-2] # yearly number of
employees
   pm_cf <- read.csv("pm_fcf.csv",na.strings = "null",sep=",")[,-2] # annual free cash flow
   pm_index <- read.csv2("index_return",na.strings = "null") # monthly return on Nordic equities (using
equal weights of all stocks in our data set)
   rf_rate <- read.csv2("rf_rate.csv",na.strings = "null")[-(145:162),] # monthly risk-free rate

   ## data on Nordic buyout targets in buyout year
   pe_entry <- read.csv("Entry Year.csv",na.strings = "null",sep=",")[,1:18]
   pe_names <- c("org_nr","company_name","work_cap","olt_debt","lt_debt", "st_loans",
"st_creditors","cash","ebit", "employees",    "cf","revenue","ebitda","equity","industry",
"accounting_year", "entry_year","total assets")
   names(pe_entry) <- pe_names
   pe_returns <- read.csv("pe_returns.csv",na.strings = "null",sep=",")
   usd_nok <- read.csv2("usd_nok_nb.csv",na.strings = "null", sep=";") # exchange rate USD to NOK
   pe_entry$employees <- read.csv("employees.csv",na.strings = "null",sep=",") # number of employees in
buyout target

   ## data on Nordic buyout targets in years before and after buyout year
   pe_entry_minus1 <- read.csv("Entry Year (-1).csv",na.strings = "null",sep=",")[1:15]
   names(pe_entry_minus1) <- pe_names[c(1:14,18)]
   pe_entry_minus2 <- read.csv("Entry year (-2).csv",na.strings = "null",sep=",")[1:15]
   names(pe_entry_minus2) <- pe_names[c(1:14,18)]
   pe_entry_plus1 <- read.csv("Entry Year (+1).csv",na.strings = "null",sep=",")[1:15]
   names(pe_entry_plus1) <- pe_names[c(1:14,18)]
   pe_entry_plus2 <- read.csv("Entry year (+2).csv",na.strings = "null",sep=",")[1:15]
   names(pe_entry_plus2) <- pe_names[c(1:14,18)]
   pe_entry_plus3 <- read.csv("Entry year (+3).csv",na.strings = "null",sep=",")[1:15]
   names(pe_entry_plus3) <- pe_names[c(1:14,18)]
   pe_entry[pe_entry$employees=="#VALUE!",] <- NA


# Presenting data ---------------------------------------------------------

   ## NORDIC PUBLIC EQUITY INDEX ####

rf_yearly <- sapply(0:11,function(x) mean(rf_rate$rf_rate[(x*12):(x*12+12)])) # finding yearly risk-free
rate

index_monthly <- rbind(0,matrix(pm_index$V1 ,ncol=1)) # monthly return on Nordic equities
index_yearly <- matrix(cumprod(1+index_monthly),ncol=1)[seq(13,145,12),] /
   matrix(cumprod(1+index_monthly),ncol=1)[seq(1,133,12),] - 1 # yearly return on Nordic equities

index_y_cum_return <- data.frame(rbind(1,data.frame(cumprod(1+index_yearly)))) # cumulative return
index_y_an_stdev <- sd(index_yearly) # annual standard deviation
index_y_an_ret <- (index_y_cum_return[nrow(index_y_cum_return),1])^(1/(nrow(index_y_cum_return)-1)) - 1 #
annual return
index_y_sharpe <- (index_y_an_ret) / index_y_an_stdev # Sharpe ratio
index_y_max_drawdown <- min(matrix(unlist(lapply(2:13,function(x) index_y_cum_return[x,1] /
                                       max(index_y_cum_return[1:(x-1),1]) - 1)),ncol=1)) # maximum drawdown

   ## NORDIC PRIVATE EQUITY RETURN
############################################################################
```

```
fee <- 0.06  # setting fee to 6% each year

nok_usd <- cbind.data.frame(usd_nok,data.frame(  # currency exchange rates from USD to NOK
   str_split_fixed(usd_nok$TIME_PERIOD, "-", 2)))
nok_usd_yearly <- nok_usd[nok_usd$X2=="06",][-1,4] / nok_usd[nok_usd$X2=="06",][-13,4] - 1

pe_returns_usd <- cbind.data.frame(pe_returns[complete.cases(pe_returns[,3]),c(1,3)],
                                   data.frame(str_split_fixed(pe_returns[complete.cases(pe_returns[,3]),1],
" ", 2)))[,-1]
pe_returns_usd_yearly <- pe_returns_usd[pe_returns_usd$X1=="Q2",][3:14,]
pe_returns_usd_yearly_excess <- pe_returns_usd_yearly$Nordics - rf_yearly
pe_ret_yearly_postfee <- (1 + pe_returns_usd_yearly$Nordics )  * (1 + nok_usd_yearly) - rf_yearly - 1 #
adjusted from USD to NOK
pe_ret_yearly_prefee <- ( (1 + pe_returns_usd_yearly$Nordics )  * (1 + nok_usd_yearly) ) + fee - rf_yearly
- 1 # adjusted for fees

pe_cum_return_postfee <- rbind(1,data.frame(cumprod(1+pe_ret_yearly_postfee))) # PE postfee return metrics
pe_an_stdev_postfee <- sd(pe_ret_yearly_postfee)
pe_an_ret_postfee <-
(pe_cum_return_postfee[nrow(pe_cum_return_postfee),1])^(1/(nrow(pe_cum_return_postfee)-1)) - 1
pe_sharpe_postfee <- (pe_an_ret_postfee) / pe_an_stdev_postfee
pe_coef_postfee <- lm(pe_ret_yearly_postfee ~ index_yearly)$coef
pe_max_drawdown_postfee <- min(matrix(unlist(lapply(2:13,function(x) pe_cum_return_postfee[x,1] /
                                                   max(pe_cum_return_postfee[1:(x-1),1]) - 1)),ncol=1))

pe_cum_return_prefee <- rbind(1,data.frame(cumprod(1+pe_ret_yearly_prefee))) # PE prefee return metrics
pe_an_stdev_prefee <- sd(pe_ret_yearly_prefee)
pe_an_ret_prefee <- (pe_cum_return_prefee[nrow(pe_cum_return_prefee),1])^(1/12) - 1
pe_sharpe_prefee <- (pe_an_ret_prefee) / pe_an_stdev_prefee
pe_coef_prefee <- lm(pe_ret_yearly_prefee ~ index_yearly)$coef
pe_max_drawdown_prefee <- min(matrix(unlist(lapply(2:13,function(x) pe_cum_return_prefee[x,1] /
                                              max(pe_cum_return_prefee[1:(x-1),1]) - 1)),ncol=1))

pe_rolling5year <- unlist(lapply(1:8, function(x) (pe_cum_return_prefee[x+5,] / pe_cum_return_prefee[x,]) ^
(1/5) - 1)) # PE rolling 5 year return metrics
pe_rolling_sd <- unlist(lapply(1:8, function(x) sd(pe_ret_yearly_prefee[x:(x+4)])))
pe_rolling_reg <- lapply(1:8, function(x) lm(pe_ret_yearly_prefee[x:(x+4)] ~ index_yearly[(x:(x+4))]))
pe_rolling_beta <- sapply(pe_rolling_reg, coef)[2,]
pe_rolling_alpha <- sapply(pe_rolling_reg, coef)[1,]
pe_rolling_sharpe <- (matrix(unlist(pe_rolling5year),nrow=1) - 0.01) / matrix(unlist(pe_rolling_sd),nrow=1)
pe_rolling_maxdrawdown <- apply(matrix(unlist(
   lapply(6:13, function (i)
      matrix(unlist(lapply(1:6,function(x)
         matrix(unlist(pe_cum_return_prefee[(i-5):i,1]),ncol=1)[x,] /
            max(matrix(unlist(pe_cum_return_prefee[(i-5):i,1]),ncol=1)[(1:x),]) -
1)),ncol=1))),ncol=8),2,min)


   ## NORDIC STOCK RETURN
#####################################################################################

adj_prices_monthly <- adj_prices_monthly[,order(ncol(adj_prices_monthly):1)][,(101):(101+144)] # monthly
prices
adj_prices_monthly[c(1982,2008),] <- NA
adj_prices_yearly <- adj_prices_monthly[,seq(1,145,12)] # yearly prices
adj_prices_yearly_help <- matrix(unlist(lapply(1:145, function(x) # ensuring that the last available price
   ifelse(adj_prices_monthly[,x] == 0,adj_prices_monthly[,max(1,x-1)], # is registrered in case
          adj_prices_monthly[,x]))),ncol=145)[,seq(1,145,12)] # stocks are not tradeable in next period
length(which(matrix(apply(adj_prices_yearly,1,sum,na.rm=T),ncol=1)>0))
adj_prices_monthly[adj_prices_monthly==0] <- NA
adj_prices_yearly[adj_prices_yearly==0] <- NA
adj_prices_yearly_help[adj_prices_yearly_help==0] <- NA

stock_ret_m <- ((adj_prices_monthly[1:nrow(adj_prices_monthly),2:ncol(adj_prices_monthly)] / ## monthly
returns
                     adj_prices_monthly[1:nrow(adj_prices_monthly),1:(ncol(adj_prices_monthly)-1)]) - 1)  -
   t(unlist(matrix(rf_rate$rf_rate/12,144,3413)))
stock_ret_y <- ((adj_prices_yearly_help[1:nrow(adj_prices_yearly_help),2:ncol(adj_prices_yearly_help)] / #
yearly returns
                    adj_prices_yearly_help[1:nrow(adj_prices_yearly_help),1:(ncol(adj_prices_yearly_help)-
1)]) - 1) -
                  t(unlist(matrix(rf_yearly,12,3413)))

stock_ret_m[stock_ret_m>10] <- NA
stock_ret_y[stock_ret_y>120] <- NA
```

```
return_available_yearly <- data.frame(ifelse(na.pass(adj_prices_yearly[,1:12])!=0 | # check if return data are available
                                          na.pass(adj_prices_yearly[,1:12])==0,TRUE,FALSE))
return_available_yearly[is.na(return_available_yearly)] <- FALSE
return_available_monthly <- matrix(unlist(lapply(1:12,function(x)
replicate(12,return_available_yearly[,x]))),ncol=144)
return_available_monthly[is.na(return_available_monthly)] <- FALSE

returns_monthly2 <- stock_ret_m
returns_monthly2[is.na(returns_monthly2)] <- 0
returns_yearly2 <- stock_ret_y
returns_yearly2[is.na(returns_yearly2)] <- 0


# Criteria for stock selection -------------------------------------------

   ## Size criterion ###############
pe_revenue_df <- data.frame(name=pe_entry$company_name,
                            revenue=pe_entry$revenue,
                            time=pe_entry$entry_year,
                            industry=pe_entry$industry)
pe_revenue_df$market <- "PE"

pm_revenue_df <- pm_revenue[,c(1,c(5:16))]
pm_revenue_df = reshape(pm_revenue_df, direction="long", varying=2:13, sep="")
names(pm_revenue_df) <- c("isin","time","revenue","id")

size_crit_fill <- list(c()) # checking if stocks have revenue between
for (i in (2005:2016)) { # the 20%- and 80% percentile of Nordic buyout targets
   size_crit_fill[[i]] <- ifelse(
         pm_revenue_df[pm_revenue_df$time==i,]$revenue>=
         quantile(pe_revenue_df[pe_revenue_df$time==max(i,2007) &
pe_revenue_df$revenue>0,]$revenue,probs=c(0.20),na.rm=TRUE) &
         pm_revenue_df[pm_revenue_df$time==i,]$revenue<=
         quantile(pe_revenue_df[pe_revenue_df$time==max(i,2007) &
pe_revenue_df$revenue>0,]$revenue,probs=c(0.80),na.rm=TRUE),
         "pm_pe_universe","other")
}

size_crit_full <- data.frame(matrix(unlist(size_crit_fill[2005:2016]),nrow=3413))

pm_revenue_df$market <-
rbind(matrix(reshape(size_crit_full,varying=12:1,direction="long",sep="")$X,ncol=1))

crit_size <- data.frame(ifelse(size_crit_full=="pm_pe_universe",TRUE,FALSE)) # controls which stocks that
crit_size[is.na(crit_size)] <- FALSE # satisfy the size criterion

   ## Sector criterion ##############
crit_sector_fill <- list(c()) ## checking if stocks operate in one of the 4 sectors that are most populated among buyout targets
for (i in (2005:2016)) {
crit_sector_fill[[i]] <- pm_sector$Sector %in% data.frame(sort(table(
   pe_revenue_df[pe_revenue_df$time==max(i,2007) &
pe_revenue_df$revenue>0,]$industry),decreasing=TRUE)[1:4])$Var1
}

crit_sector <- matrix(unlist(crit_sector_fill[2005:2016]),nrow=3413) # controls which stocks that satisfy the sector criterion


   ## EBITDA criterion ########
pm_ebitda_long <- melt(pm_ebitda[,5:16])
pm_ebitda_long$variable <- as.numeric(gsub("X","",pm_ebitda_long$variable))

ebitda_crit_fill <- list(c()) # checking if stocks have EBITDA between the 20%- and 80%- percentile
for (i in (2005:2016)) { # of Nordic buyout targets
   ebitda_crit_fill[[i]] <- ifelse(
         pm_ebitda_long[pm_ebitda_long$variable==i,]$value>=
         quantile(pe_entry[(pe_entry$entry_year==max(i,2007) &
pe_entry$ebitda!=0),]$ebitda,probs=c(0.20),na.rm=TRUE) &
         pm_ebitda_long[pm_ebitda_long$variable==i,]$value<=
         quantile(pe_entry[(pe_entry$entry_year==max(i,2007) &
pe_entry$ebitda!=0),]$ebitda,probs=c(0.80),na.rm=TRUE),
      "pm_pe_universe","other")
}

ebitda_crit_full <- data.frame(matrix(unlist(ebitda_crit_fill[2005:2016]),nrow=3413))
```

```
crit_ebitda <- data.frame(ifelse(ebitda_crit_full=="pm_pe_universe",TRUE,FALSE)) # controls which stocks that
crit_ebitda[is.na(crit_ebitda)] <- FALSE # satisfy the EBITDA criterion

   ## ASSET TURNOVER CRITERION ########
pm_revenue_growth <- ((pm_revenue[1:nrow(pm_revenue),2:(ncol(pm_revenue)-1)] - # calculating revenue growth
                          pm_revenue[1:nrow(pm_revenue),3:(ncol(pm_revenue))]) /
                         pm_revenue[1:nrow(pm_revenue),3:(ncol(pm_revenue))])
pm_revenue_growth[pm_revenue_growth==-1 | pm_revenue_growth==0 | pm_revenue_growth==Inf] <- NA

pm_asset_growth <- ((pm_total_assets[1:nrow(pm_total_assets),2:(ncol(pm_total_assets)-1)] - # calculating asset growth
                        pm_total_assets[1:nrow(pm_total_assets),3:(ncol(pm_total_assets))]) /
                       pm_total_assets[1:nrow(pm_total_assets),3:(ncol(pm_total_assets))])
pm_asset_growth[pm_asset_growth==-1 | pm_asset_growth==0 | pm_asset_growth==Inf] <- NA

pm_asset_turnover <- round(pm_revenue_growth / pm_asset_growth,4) # calculating asset turnover for stocks
pm_asset_turnover <- melt(pm_asset_turnover[15:4])

pe_entry$revenue_growth <- (pe_entry$revenue / pe_entry_minus1$revenue) - 1
pe_entry$revenue_growth <- ifelse(pe_entry$revenue_growth==-1 |
                                  pe_entry$revenue_growth==0 |
                                  pe_entry$revenue_growth==Inf,NA,pe_entry$revenue_growth)
pe_entry$asset_growth <- (pe_entry$`total assets` / pe_entry_minus1$`total assets`) - 1
pe_entry$asset_turnover <- pe_entry$revenue_growth / pe_entry$asset_growth # asset turnover for buyout targets

pm_asset_turnover$variable <- as.numeric(gsub("X","",pm_asset_turnover$variable))

asset_turnover_crit_fill <- list(c()) # checking if stocks have asset turnover between the 20%- and 80%-percentile
for (i in (2005:2016)) { # of Nordic buyout targets
   asset_turnover_crit_fill[[i]] <- ifelse(
      pm_asset_turnover[pm_asset_turnover$variable==i,]$value>=
         quantile(pe_entry[(pe_entry$entry_year<=max(i,2007)),]$asset_turnover,probs=c(0.20),na.rm=TRUE) &
         pm_asset_turnover[pm_asset_turnover$variable==i,]$value<=
         quantile(pe_entry[(pe_entry$entry_year<=max(i,2007)),]$asset_turnover,probs=c(0.80),na.rm=TRUE),
      "pm_pe_universe","other")
}

asset_turnover_crit_full <- data.frame(matrix(unlist(asset_turnover_crit_fill[2005:2016]),nrow=3413))

crit_ast <- data.frame(ifelse(asset_turnover_crit_full=="pm_pe_universe",TRUE,FALSE)) # controls which stocks that
crit_ast[is.na(crit_ast)] <- FALSE # satisfy the asset turnover criterion

   ## WC TURNOVER CRITERION ###########
pe_entry$wcturnover <- (pe_entry$revenue / pe_entry$work_cap)
pe_entry$wcturnover <- ifelse(pe_entry$wcturnover==-1 |
                              pe_entry$wcturnover==0 |
                              pe_entry$wcturnover==Inf,NA,pe_entry$wcturnover)

pm_wcturnover <-
list(replace(pm_revenue[,2:26],is.na(pm_revenue[,2:26]),0),replace(pm_working_capital[,2:26],is.na(pm_working_capital[,2:26]),0))
pm_wcturnover <- Reduce('/',pm_wcturnover)
pm_wcturnover[pm_wcturnover==0 | pm_wcturnover==-1 | pm_wcturnover==Inf | pm_wcturnover==-Inf] <- NA
pm_wcturnover_long <- data.frame(melt(pm_wcturnover[,15:4]))

pm_wcturnover_long$variable <- as.numeric(gsub("X","",pm_wcturnover_long$variable))

wc_turnover_crit_fill <- list(c()) #  checking if stocks have working capital turnover between the 20%- and
for (i in (2005:2016)) { # 80%- percentile of Nordic buyout targets
   wc_turnover_crit_fill[[i]] <- ifelse(
      pm_wcturnover_long[pm_wcturnover_long$variable==i,]$value>=
         quantile(pe_entry[(pe_entry$entry_year<=max(i,2007)),]$wcturnover,probs=c(0.20),na.rm=TRUE) &
         pm_wcturnover_long[pm_wcturnover_long$variable==i,]$value<=
         quantile(pe_entry[(pe_entry$entry_year<=max(i,2007)),]$wcturnover,probs=c(0.80),na.rm=TRUE),
      "pm_pe_universe","other")
}
quantile(pe_entry[(pe_entry$entry_year<=2016),]$wcturnover,probs=c(0.75),na.rm=TRUE)
wc_turnover_crit_full <- data.frame(matrix(unlist(wc_turnover_crit_fill[2005:2016]),nrow=3413))

crit_wcturnover <- data.frame(ifelse(wc_turnover_crit_full=="pm_pe_universe",TRUE,FALSE)) # checking which
crit_wcturnover[is.na(crit_wcturnover)] <- FALSE # stocks that satisfy the working capital turnover criterion
```

```
   ## DEBT / EQUITY criterion ############
pe_entry$total_debt <- pe_entry$st_loans+pe_entry$lt_debt+pe_entry$olt_debt
pe_entry$debtequity <- (pe_entry$total_debt / pe_entry$equity)
pe_entry$debtequity <- ifelse(pe_entry$debtequity==0,NA,pe_entry$debtequity)

pm_total_debt <-
list(replace(pm_lt_borrowing,is.na(pm_lt_borrowing),0),replace(pm_olt_borrowing,is.na(pm_olt_borrowing),0),
replace(pm_st_debt,is.na(pm_st_debt),0))
pm_total_debt <- Reduce('+',pm_total_debt)
pm_debtequity <- list(pm_total_debt[,16:5],pm_equity[,16:5])
pm_debtequity <- Reduce('/',pm_debtequity)
pm_debtequity[pm_debtequity==0 | pm_debtequity==Inf | pm_debtequity==-Inf] <- NA
pm_debtequity_long <- melt(pm_debtequity)

pm_debtequity_long$variable <- as.numeric(gsub("X","",pm_debtequity_long$variable))

debtequity_crit_fill <- list(c()) #  checking if stocks have leverage ratio between the 40%- and
for (i in (2005:2016)) { # 80%- percentile of buyout targets
   debtequity_crit_fill[[i]] <- ifelse(
      pm_debtequity_long[pm_debtequity_long$variable==i,]$value>=
         quantile(pe_entry[pe_entry$debtequity>0,]$debtequity,probs=c(0.40),na.rm=TRUE) &
         pm_debtequity_long[pm_debtequity_long$variable==i,]$value<=
         quantile(pe_entry[pe_entry$debtequity>0,]$debtequity,probs=c(0.80),na.rm=TRUE),
      "pm_pe_universe","other")
}

debtequity_crit_full <- data.frame(matrix(unlist(debtequity_crit_fill[2005:2016]),nrow=3413))

crit_debtequity <- data.frame(ifelse(debtequity_crit_full=="pm_pe_universe",TRUE,FALSE)) # controls which
crit_debtequity[is.na(crit_debtequity)] <- FALSE # stocks that satisfy the leverage ratio criterion

   ## Leverage Nordic buyout ##########

# finding leverage ratio among buyout targets in the end of the buyout year
lev_pe <- matrix(unlist(lapply(2005:2016,function(x) describe(pe_entry[pe_entry$entry_year>=2005 &
pe_entry$debtequity>0 &
                                                        pe_entry$entry_year<=2016,]$debtequity,na.rm=T)[5])),ncol=1)
lev_pe[2] <- (lev_pe[1]+lev_pe[3]) / 2
lev_pe_med <- describe(pe_entry[pe_entry$entry_year>=2005 & pe_entry$debtequity>0 &
                                    pe_entry$entry_year!=2006,]$debtequity,na.rm=T)[5]


# CONSTRUCTING PORTFOLIOS ######################################################################

   # Identifying stocks that satisfy different criteria in each period
   portfolio <- list(ifelse(crit_size==1,1,0), # portfolio 1: only size criterion
                  crit_size * crit_sector, # portfolio 2: size and sector
                  crit_size*crit_sector*crit_ebitda, # portfolio 3: size, sector and ebitda
                  crit_size*crit_sector*crit_ebitda*crit_ast, # portfolio 4: size, sector, ebitda and asset
turnover
                  crit_size*crit_sector*crit_ebitda*crit_wcturnover, # portfolio 5: size, sector, ebitda
and working capital turnover
                  crit_size*crit_sector*crit_ebitda*crit_debtequity, # portfolio 6: size, sector, ebitda
and leverage
                  data.frame(t(replicate(3413,replicate(12,1)))) # all stocks. Used in our market index
proxy of Nordic market returns
                  )

portfolio_help <- list() # identifies all stocks that satisfy portfolio criteria and are tradeable for each
period
portfolio_cumulative <- list() # counting number of years since investment in stock
portfolio_in <- list() # initial year of investment
portfolio_max <- list() # finding number of years from initial investment to exit for each stock and each
portfolio
stocks_count <- list()
stocks_count_year <- list()
stocks_count_unique <- list()
for (i in (1:7)) {
   portfolio_help[[i]] <-
data.frame(data.frame(ifelse(t(apply(matrix(unlist(portfolio[i]),ncol=12),1,cumsum))>=1,1,0)) *
                                    return_available_yearly)
   portfolio_cumulative[[i]] <- data.frame(t(apply(matrix(unlist(portfolio_help[i]),ncol=12),1,cumsum)) *
                                          return_available_yearly)
   portfolio_in[[i]] <- data.frame(ifelse(matrix(unlist(portfolio_cumulative[i]),ncol=12)==1,TRUE,FALSE))
   portfolio_max[[i]] <- data.frame(apply(matrix(unlist(portfolio_cumulative[i]),ncol=12),1,max))
```

```
   stocks_count[[i]] <- data.frame(matrix(unlist(portfolio[i]),ncol=12)*ifelse(returns_yearly2!=0,1,0))
   stocks_count_year[[i]] <- apply(matrix(unlist(stocks_count[i]),ncol=12),2,sum)
   stocks_count_unique[[i]] <- length(which(apply(matrix(unlist(stocks_count[i]),ncol=12),1,sum)>0))
}

portfolio_max_frame <- matrix(unlist(portfolio_max[1:7]),ncol=7)
portfolio_max_frame[,-7][portfolio_max_frame[,-7]>5] <- 5 # implementing maximum holding time of stocks at
5 years
apply(portfolio_max_frame,2,max,na.rm=T) # control


   # Strategy 1. 5 YEAR BUY AND HOLD #################################

cumret <- list() # cumulative return of each stock on a rolling five year basis
cumret_complete <- list() # adjusting for maximum 5 year holding time; no additional return after exit of
stocks
for (i in (1:12)) {
   cumret[[i]] <- ( adj_prices_yearly[1:nrow(adj_prices_yearly),i:(min(13,i+5))] /
                    adj_prices_yearly[1:nrow(adj_prices_yearly),i] ) - 1 -
                  t(unlist(matrix( rbind(1,matrix(cumprod(1+rf_yearly),ncol=1))[(i:min(13,i+5)),1] /
                    rbind(1,matrix(cumprod(1+rf_yearly),ncol=1))[i,1] - 1,min(13,i+5)-i+1,3413)))
   cumret_complete[[i]] <- cbind(matrix(unlist(cumret[i]),ncol=min(13,i+5)-i+1),
                              replicate(6-(min(13,i+5)-i+1),
                                        matrix(unlist(cumret[i]),ncol=min(13,i+5)-i+1)[,min(13,i+5)-
i+1]))
}

cumret <- rapply(cumret_complete, f=function(x) ifelse(x>120,0,x), how="replace" ) # removing extreme
values
cumret <- rapply(cumret_complete, f=function(x) ifelse(is.na(x),0,x), how="replace" )

strat1_invested <- list() # identifies all stocks that we invest in in each period for each portfolio
weights_strat1 <- list() # calculates equal weights on each stock
for (i in (1:7)) {
   strat1_invested[[i]] <- data.frame(matrix(unlist(portfolio[i]),ncol=12) *
return_available_yearly)[,c(1:12)]
   weights_strat1[[i]] <- matrix(unlist(lapply(1:12,function(x)
1/apply(matrix(unlist(strat1_invested[i]),ncol=12),2,sum)[x]
                                                        *
matrix(matrix(unlist(strat1_invested[i]),ncol=12)[,x],ncol=1))),ncol=12)
}

   # rolling five year
cumret_strat1 <- list() # cumulative return for rolling five years periods, e.g. 2006-2011 and 2007-2012.
ret_strat1 <- list() # yearly return in the rolling five year model
   # strategy 1
ret_strategy <- list() # yearly return for strategy 1 for each portfolio
cumret_strategy <- list() # cumulative return for strategy 1 for each portfolio
ret5_strategy <- list() # cumulative return for each stocks at time of exit (in 2011, 2016 and 2018)
new_weights_strategy <- list() # weights in the end of each period before rebalancing
new_weights_strategy_normalised <- list() # updated weights before rebalancing, forcing sum of weights to
be 1
turnover_strategy_help <- list()
turnover_strategy <- list() # turnover of each stock in each period based on difference between equal- and
end of period-weights
for (i in (1:7)) {
   cumret_strat1[[i]] <- lapply(1:12,function(x) data.frame(colSums(
      matrix(t(data.frame(matrix(unlist(weights_strat1[i]),ncol=12)[,x])),nrow=1) %*%
         matrix(unlist(cumret[x]),ncol=6))+1))
   ret_strat1[[i]] <- data.frame(matrix(unlist(cumret_strat1[i]),ncol=12)[-1,] /
                                    matrix(unlist(cumret_strat1[i]),ncol=12)[-6,] - 1)

   ret_strategy[[i]] <- matrix(matrix(unlist(ret_strat1[i]),ncol=12)[,seq(1,12,5)],ncol=1)[1:12,]
   cumret_strategy[[i]] <- cumprod(1+rbind(0,matrix(unlist(ret_strategy[i]),ncol=1)))
   ret5_strategy[[i]] <- matrix((1 + matrix(unlist(cumret[seq(1,12,5)]),ncol=6)[,6]),ncol=3) - 1
   new_weights_strategy[[i]] <- matrix(unlist(weights_strat1[i]),ncol=12)[,seq(1,12,5)] *
      (1 + matrix(unlist(ret5_strategy[[i]]),ncol=3))
   new_weights_strategy_normalised[[i]] <-
apply(matrix(unlist(new_weights_strategy[i]),ncol=3),2,function(x) {x/sum(x)})
   turnover_strategy_help[[i]] <- rbind(1,
matrix(apply(abs(matrix(unlist(new_weights_strategy_normalised[i]),ncol=3)[,-3] -
                                   matrix(unlist(weights_strat1[i]),ncol=12)[,seq(6,12,5)]),2,sum),ncol=1))
   turnover_strategy[[i]] <-
c(unlist(turnover_strategy_help[i])[1],replicate(4,0),unlist(turnover_strategy_help[i])[2],
                               replicate(4,0),unlist(turnover_strategy_help[i])[3],0)
}
```

```
   # Strategy 2. 5 YEAR DYNAMIC WITH YEARLY REBALANCING ####################

strat_2_exit <- list() # identifies exit year of stocks
strat_2_invested_help <- list()  # identifies stocks being invested in
strat_2_invested <- list() # identifies all stock investments in each period
weights_strat2 <- list() # calculates equal weights of stocks in each period
ret_strat2 <- list() # returns for stocks in the portfolios in each period
cumret_strat2 <- list() # cumulative returns for stocks in the portfolios in each period after initial
investment
new_weights_strat2 <- list() # weights in the end of each period before rebalancing
new_weights_strat2_normalised <- list() # updated weights before rebalancing, forcing sum of weights to be
1
turnover_strat2 <- list() # turnover of each stock in each period based on difference between equal- and
end of period-weights
for (i in (1:7)) {
   strat_2_exit[[i]] <- data.frame(ifelse(
      matrix(unlist(portfolio_cumulative[i]),ncol=12) == matrix(unlist(portfolio_max_frame),ncol=7)[,i] &
      matrix(unlist(portfolio_cumulative[i]),ncol=12) >  0,
      TRUE,FALSE))
   strat_2_invested_help[[i]] <-
rev(data.frame(ifelse(t(apply(matrix(unlist(strat_2_exit[i]),ncol=12)[,12:1],
                                                              1,cumsum))>=1,TRUE,FALSE)))
   strat_2_invested[[i]] <- matrix(unlist(strat_2_invested_help[i]),ncol=12) *
matrix(unlist(portfolio_help[i]),ncol=12)

   weights_strat2[[i]] <- matrix(unlist(lapply(1:12,function(x)
1/apply(matrix(unlist(strat_2_invested[i]),ncol=12),2,sum)[x]
                                                        *
matrix(matrix(unlist(strat_2_invested[i]),ncol=12)[,x],ncol=1))),ncol=12)
   ret_strat2[[i]] <- data.frame(colSums(matrix(unlist(weights_strat2[i]),ncol=12) *
         returns_yearly2))
   cumret_strat2[[i]] <- rbind.data.frame(1,
                                        data.frame(cumprod(1+matrix(unlist(ret_strat2[i]),ncol=1))))
   new_weights_strat2[[i]] <- matrix(unlist(weights_strat2[i]),ncol=12) * (1 + returns_yearly2)
   new_weights_strat2_normalised[[i]] <- apply(matrix(unlist(new_weights_strat2[i]),ncol=12),2,function(x)
{x/sum(x)})
   turnover_strat2[[i]] <- rbind(1,
matrix(apply(abs(matrix(unlist(new_weights_strat2_normalised[i]),ncol=12)[,-12] -
      matrix(unlist(weights_strat2[i]),ncol=12)[,-12]),2,sum),ncol=1))
}

   # Strategy 3. 5 YEAR DYNAMIC WITH MONTHLY REBALANCING ###################

portfolio_monthly <- list() # changing from yearly to monthly data
portfolio_m <- list()
portfolio_help_m <- list() # identifies all stocks that satisfy all portfolio criteria and are tradeable
for each period
portfolio_cumulative_m <- list() # calculates number of months after initial investment
portfolio_in_m <- list() # identifies month of initial investment in stock
portfolio_max_m <- list() # calculates the number of months from initial investment to exit
for (i in (1:7)) {
   portfolio_monthly[[i]] <- lapply(1:12,function(x)
replicate(12,matrix(unlist(portfolio[i]),ncol=12)[,x]))
   portfolio_m[[i]] <- list.cbind(portfolio_monthly[[i]])
   portfolio_help_m[[i]] <-
data.frame(data.frame(ifelse(t(apply(matrix(unlist(portfolio_m[i]),ncol=144),1,cumsum))>=1,1,0)) *
                                        return_available_monthly)
   portfolio_cumulative_m[[i]] <-
data.frame(t(apply(matrix(unlist(portfolio_help_m[i]),ncol=144),1,cumsum)) *
                                              return_available_monthly)
   portfolio_in_m[[i]] <-
data.frame(ifelse(matrix(unlist(portfolio_cumulative_m[i]),ncol=144)==1,TRUE,FALSE))
   portfolio_max_m[[i]] <- data.frame(apply(matrix(unlist(portfolio_cumulative_m[i]),ncol=144),1,max))
}

portfolio_max_frame_m <- matrix(unlist(portfolio_max_m[1:7]),ncol=7)
portfolio_max_frame_m[,-7][portfolio_max_frame_m[,-7]>5*12] <- 5*12 # ensures maximum holding time of 5
years (60 months)
apply(portfolio_max_frame_m,2,max,na.rm=T)

strat_3_exit <- list() # identifies exit month of each stock in each portfolio
strat_3_invested_help <- list()
strat_3_invested <- list() # identifies all stock investments in each period
weights_strat3 <- list() # calculates equal weigths for each stock in each period
ret_strat3_m <- list() # monthly return for each stocks being invested in
ret_strat3 <- list() # yearly return for each stocks being invested in
```

```
cumret_strat3 <- list() # yearly cumulative return for each stocks being invested in
cumret_strat3_m <- list() # monthly cumulative return for each stocks being invested in
new_weights_strat3 <- list() # weights in the end of each period before rebalancing
new_weights_strat3_normalised <- list() # updated weights before rebalancing, forcing sum of weights to be
1
turnover_strat3m <- list() # monthly turnover of each stock in based on difference between equal- and end
of period-weights
turnover_strat3 <- list() # yearly turnover of each stock
for (i in (1:7)) {
   strat_3_exit[[i]] <- data.frame(ifelse(
      matrix(unlist(portfolio_cumulative_m[i]),ncol=144) ==
matrix(unlist(portfolio_max_frame_m),ncol=7)[,i] &
         matrix(unlist(portfolio_cumulative_m[i]),ncol=144) >  0,
      TRUE,FALSE))
   strat_3_invested_help[[i]] <-
rev(data.frame(ifelse(t(apply(matrix(unlist(strat_3_exit[i]),ncol=144)[,144:1],
                                                          1,cumsum))>=1,TRUE,FALSE)))
   strat_3_invested[[i]] <- matrix(unlist(strat_3_invested_help[i]),ncol=144) *
matrix(unlist(portfolio_help_m[i]),ncol=144)
   weights_strat3[[i]] <- matrix(unlist(lapply(1:144,function(x)
1/apply(matrix(unlist(strat_3_invested[i]),ncol=144),2,sum)[x]
                                                *
matrix(matrix(unlist(strat_3_invested[i]),ncol=144)[,x],ncol=1))),ncol=144)
   ret_strat3_m[[i]] <- data.frame(colSums(matrix(unlist(weights_strat3[i]),ncol=144) *
                                          returns_monthly2))
   cumret_strat3[[i]] <- rbind.data.frame(1,

matrix(data.frame(cumprod(1+matrix(unlist(ret_strat3_m[i]),ncol=1)))[seq(12,144,12),],ncol=1))
   cumret_strat3_m[[i]] <- rbind(1,matrix(cumprod(1+matrix(unlist(ret_strat3_m[i]),ncol=1)),ncol=1))
   new_weights_strat3[[i]] <- matrix(unlist(weights_strat3[i]),ncol=144) * (1 + returns_monthly2)
   new_weights_strat3_normalised[[i]] <- apply(matrix(unlist(new_weights_strat3[i]),ncol=144),2,function(x)
{x/sum(x)})
   turnover_strat3m[[i]] <- rbind(1,
matrix(apply(abs(matrix(unlist(new_weights_strat3_normalised[i]),ncol=144)[,-144] -
                                        matrix(unlist(weights_strat3[i]),ncol=144)[,-
144]),2,sum),nrow=143))
   turnover_strat3[[i]] <- lapply(0:11,function(x)
sum(matrix(unlist(turnover_strat3m[[i]]),nrow=144)[max(1,12*x):(12*x+12)]))
   }

for (i in (1:7)) {
   ret_strat3[[i]] <- lapply(1:12,function(x) matrix(unlist(cumret_strat3[i]),ncol=1)[x+1,] /
                              matrix(unlist(cumret_strat3[i]),ncol=1)[x,] - 1 )
}

   # Strategy 4. DYNAMIC WITH YEARLY REBALANCING ####################################

strat_4_invested <- list() # identify stocks that satisfy criterias and are tradeable for each period
weights_strat4 <- list() # calculates equal weight for all stocks in the portfolios in each period
ret_strat4 <- list() # returns for stocks in the portfolios in each period
cumret_strat4 <- list() # cumulative returns for stocks in the portfolios in each period after initial
investment
new_weights_strat4 <- list() # weights in the end of each period before rebalancing
new_weights_strat4_normalised <- list() # updated weights before rebalancing, forcing sum of weights to be
1
turnover_strat4 <- list() # turnover of each stock in each period based on difference between equal- and
end of period-weights
for (i in (1:7)) {
   strat_4_invested[[i]] <- data.frame(matrix(unlist(portfolio[i]),ncol=12) * return_available_yearly)
   weights_strat4[[i]] <- matrix(unlist(lapply(1:12,function(x)
1/apply(matrix(unlist(strat_4_invested[i]),ncol=12),2,sum)[x]
                                                *
matrix(matrix(unlist(strat_4_invested[i]),ncol=12)[,x],ncol=1))),ncol=12)
   ret_strat4[[i]] <- data.frame(colSums(matrix(unlist(weights_strat4[i]),ncol=12) *
                                          returns_yearly2))
   cumret_strat4[[i]] <- rbind.data.frame(1,
                                        data.frame(cumprod(1+matrix(unlist(ret_strat4[i]),ncol=1))))
   new_weights_strat4[[i]] <- matrix(unlist(weights_strat4[i]),ncol=12) * (1 + returns_yearly2)
   new_weights_strat4_normalised[[i]] <- apply(matrix(unlist(new_weights_strat4[i]),ncol=12),2,function(x)
{x/sum(x)})
   turnover_strat4[[i]] <- rbind(1,
matrix(apply(abs(matrix(unlist(new_weights_strat4_normalised[i]),ncol=12)[,-12] -
                                        matrix(unlist(weights_strat4[i]),ncol=12)[,-12]),2,sum),nrow=11))
}


# Portfolio performance ---------------------------------------------------
```

```
   ## Post-fee returns ###############
postfee_ret_strategy_nolev <- list() # annual portfolio returns for different level of trading costs and no
leverage
postfee_ret_strat2_nolev <- list()
postfee_ret_strat3_nolev <- list()
postfee_ret_strat4_nolev <- list()
for (i in (1:7)) {
   postfee_ret_strategy_nolev[[i]] <- sapply(seq(0,0.015,0.0025), function(z)
      matrix(unlist(ret_strategy[[i]]),ncol=1) - z * matrix(unlist(turnover_strategy[i]),ncol=1))
   postfee_ret_strat2_nolev[[i]] <- sapply(seq(0,0.012,0.002), function(z)
      matrix(unlist(ret_strat2[[i]]),ncol=1) - z * matrix(unlist(turnover_strat2[i]),ncol=1))
   postfee_ret_strat3_nolev[[i]] <- sapply(seq(0,0.012,0.002), function(z)
      matrix(unlist(ret_strat3[[i]]),ncol=1) - z * matrix(unlist(turnover_strat3[i]),ncol=1))
   postfee_ret_strat4_nolev[[i]] <- sapply(seq(0,0.012,0.002), function(z)
      matrix(unlist(ret_strat4[[i]]),ncol=1) - z * matrix(unlist(turnover_strat4[i]),ncol=1))
}

postfee_cumret_strategy_nolev <- list() # cumulative returns for different level of trading costs and no
leverage
postfee_an_return_strategy_nolev <- list() # annualised mean return for different level of fees
postfee_cumret_strat2_nolev <- list()
postfee_an_return_strat2_nolev <- list()
postfee_cumret_strat3_nolev <- list()
postfee_an_return_strat3_nolev <- list()
postfee_cumret_strat4_nolev <- list()
postfee_an_return_strat4_nolev <- list()
for (i in (1:7)) { # Cumulative portfolio returns adjusted for leverage
   postfee_cumret_strategy_nolev[[i]] <- sapply(1:7, function(y) rbind.data.frame(1,
      data.frame(cumprod(1+matrix(unlist(postfee_ret_strategy_nolev[[i]]),ncol=7)[,y]))))
   postfee_an_return_strategy_nolev[[i]] <- sapply(1:7, function(y)
      matrix(unlist(postfee_cumret_strategy_nolev[[i]]),ncol=7)[13,y] ^ (1/12) - 1)

   postfee_cumret_strat2_nolev[[i]] <- sapply(1:7, function(y) rbind.data.frame(1,
      data.frame(cumprod(1+matrix(unlist(postfee_ret_strat2_nolev[[i]]),ncol=7)[,y]))))
   postfee_an_return_strat2_nolev[[i]] <- sapply(1:7, function(y)
      matrix(unlist(postfee_cumret_strat2_nolev[[i]]),ncol=7)[13,y] ^ (1/12) - 1)

   postfee_cumret_strat3_nolev[[i]] <- sapply(1:7, function(y) rbind.data.frame(1,
      data.frame(cumprod(1+matrix(unlist(postfee_ret_strat3_nolev[[i]]),ncol=7)[,y]))))
   postfee_an_return_strat3_nolev[[i]] <- sapply(1:7, function(y)
      matrix(unlist(postfee_cumret_strat3_nolev[[i]]),ncol=7)[13,y] ^ (1/12) - 1)

   postfee_cumret_strat4_nolev[[i]] <- sapply(1:7, function(y) rbind.data.frame(1,
      data.frame(cumprod(1+matrix(unlist(postfee_ret_strat4_nolev[[i]]),ncol=7)[,y]))))
   postfee_an_return_strat4_nolev[[i]] <- sapply(1:7, function(y)
      matrix(unlist(postfee_cumret_strat4_nolev[[i]]),ncol=7)[13,y] ^ (1/12) - 1)

   }

   ## HOLD-TO-MATURITY ACCOUNTING SCHEME ######
      ## Strategy 2
htm_weights_strat2 <- list() # adjusted weights in each period for a hold-to-maturity approach
      ## Calculating total returns under Hold-To-Maturity-accounting
htm_stock_return_strat2 <- list()
htm_stock_cumreturn_strat2 <- list()
htm_stock_realised_strat2 <- list()
htm_ret_strat2 <- list()
      ## Adjust for book value in the last period
htm_strat2_exit2 <- list()
htm_strat2_exit <- list()
htm_stock_realised_strat2_disc <- list()
htm_ret_strat2_disc <- list()
for (i in (1:7)) {
   htm_weights_strat2[[i]] <- matrix(unlist(lapply(1:12,function(x)
matrix(unlist(cumret_strat2[i]),ncol=1)[x] *

matrix(matrix(unlist(weights_strat2[i]),ncol=12)[,x],nrow=1))),ncol=12)
   htm_stock_return_strat2[[i]] <- data.frame(matrix(unlist(htm_weights_strat2[i]),ncol=12) *
returns_yearly2)
   htm_stock_cumreturn_strat2[[i]] <- t(apply(matrix(unlist(htm_stock_return_strat2[i]),ncol=12),1,cumsum))
   htm_stock_realised_strat2[[i]] <- matrix(unlist(strat_2_exit[i]),ncol=12) *
matrix(unlist(htm_stock_cumreturn_strat2[i]),ncol=12)
   htm_ret_strat2[[i]] <- apply(matrix(unlist(htm_stock_realised_strat2[i]),ncol=12),2,sum)

   htm_strat2_exit2[[i]] <- matrix(unlist(strat_2_exit[i]),ncol=12)[,12] *
      matrix(unlist(strat_2_invested[i]),ncol=12)[,8]
```

```
   htm_strat2_exit[[i]] <- cbind(matrix(unlist(strat_2_exit[i]),ncol=12)[,-12],
                              matrix(unlist(htm_strat2_exit2[i]),ncol=1))
   htm_stock_realised_strat2_disc[[i]] <- matrix(unlist(htm_strat2_exit[i]),ncol=12) *
      matrix(unlist(htm_stock_cumreturn_strat2[i]),ncol=12)
   htm_ret_strat2_disc[[i]] <- apply(matrix(unlist(htm_stock_realised_strat2_disc[i]),ncol=12),2,sum)
}

htm_cumret_strat2 <- apply(matrix(unlist(htm_ret_strat2[1:7]),ncol=7),2,cumsum) + 1 # Real value last
period
htm_cumret_strat2_disc <- apply(matrix(unlist(htm_ret_strat2_disc[1:7]),ncol=7),2,cumsum) + 1 # Book value
all periods



## Strategy 3
htm_weights_strat3 <- list()
htm_stock_return_strat3 <- list()
htm_stock_cumreturn_strat3 <- list()
htm_stock_realised_strat3 <- list()
htm_ret_strat3 <- list()

htm_strat3_exit2 <- list()
htm_strat3_exit <- list()
htm_stock_realised_strat3_disc <- list()
htm_ret_strat3_disc <- list()
htm_ret_strat3_disc_yearly <- list()

for (i in (1:7)) {

   ## Calculating total returns under Hold-To-Maturity-accounting
   htm_weights_strat3[[i]] <- matrix(unlist(lapply(1:144,function(x)
matrix(unlist(cumret_strat3_m[i]),ncol=1)[x] *

matrix(matrix(unlist(weights_strat3[i]),ncol=144)[,x],nrow=1))),ncol=144)
   htm_stock_return_strat3[[i]] <- data.frame(matrix(unlist(htm_weights_strat3[i]),ncol=144) *
returns_monthly2)
   htm_stock_cumreturn_strat3[[i]] <-
t(apply(matrix(unlist(htm_stock_return_strat3[i]),ncol=144),1,cumsum))
   htm_stock_realised_strat3[[i]] <- matrix(unlist(strat_3_exit[i]),ncol=144) *
matrix(unlist(htm_stock_cumreturn_strat3[i]),ncol=144)
   htm_ret_strat3[[i]] <- apply(matrix(unlist(htm_stock_realised_strat3[i]),ncol=144),2,sum)

   ## Adjust for book value in the last period
   htm_strat3_exit2[[i]] <- matrix(unlist(strat_3_exit[i]),ncol=144)[,(133:144)] *
      matrix(unlist(strat_3_invested[i]),ncol=144)[,85:96]
   htm_strat3_exit[[i]] <- cbind(matrix(unlist(strat_3_exit[i]),ncol=144)[,-(133:144)],
                              matrix(unlist(htm_strat3_exit2[i]),ncol=12))
   htm_stock_realised_strat3_disc[[i]] <- matrix(unlist(htm_strat3_exit[i]),ncol=144) *
      matrix(unlist(htm_stock_cumreturn_strat3[i]),ncol=144)
   htm_ret_strat3_disc[[i]] <- apply(matrix(unlist(htm_stock_realised_strat3_disc[i]),ncol=144),2,sum)
   htm_ret_strat3_disc_yearly[[i]] <- lapply(0:11, function(x) sum(matrix(unlist(htm_ret_strat3_disc[i]),
                                                   ncol=144)[,(1+x*12):(x*12+12)]))
}

htm_cumret_strat3 <- apply(matrix(unlist(htm_ret_strat3[1:7]),ncol=7),2,cumsum)[seq(12,144,12),]  + 1 #
Real value last period
htm_cumret_strat3_disc <- apply(matrix(unlist(htm_ret_strat3_disc[1:7]),ncol=7),2,cumsum)[seq(12,144,12),]
+ 1 # Book value all periods

an_stdev_strat2_htm <- list()
an_return_strat2_htm <- list()
an_stdev_strat3_htm <- list()
an_return_strat3_htm <- list()
an_return_strat2_htm_real <- list()
test <- list()
for (i in (1:7)){
an_stdev_strat2_htm[[i]] <- sd(matrix(unlist(htm_ret_strat2_disc[i]),ncol=1)[5:12,1])
an_return_strat2_htm[[i]] <- matrix(unlist(htm_cumret_strat2_disc),ncol=7)[12,i] ^ (1/12) - 1

an_stdev_strat3_htm[[i]] <- sd(matrix(unlist(htm_ret_strat3_disc_yearly[i]),ncol=1)[5:12,1])
an_return_strat3_htm[[i]] <- matrix(unlist(htm_cumret_strat3_disc),ncol=7)[12,i] ^ (1/12) - 1

}

discount_strat2_htm <- htm_cumret_strat2_disc[12,] / matrix(unlist(cumret_strat2[1:7]),ncol=7)[12,] - 1
discount_strat3_htm <- htm_cumret_strat3_disc[12,] / matrix(unlist(cumret_strat3[1:7]),ncol=7)[12,] - 1
```

```
   ## CONTROLLING WEIGHTS #############
weight_4control_m <- list()
weight_4control <- list()
weight_3control <- list()
weight_2control <- list()
weight_1control <- list()
for (i in (1:7)) { # controlling weights equal to 1
   weight_4control[[i]] <- apply(matrix(unlist(weights_strat4[i]),ncol=12),2,sum)
   weight_3control[[i]] <- apply(matrix(unlist(weights_strat3[i]),ncol=144),2,sum)
   weight_2control[[i]] <- apply(matrix(unlist(weights_strat2[i]),ncol=12),2,sum)
   weight_1control[[i]] <- apply(matrix(unlist(weights_strat1[i]),ncol=8),2,sum)
}

   ## Checking number of stocks in portfolio ##########
numb_stocks_1 <- list() # Number of stocks in portfolio
numb_stocks_2 <- list()
numb_stocks_3 <- list()
numb_stocks_4 <- list()
unique_stocks_2 <- list() # Number of unique stocks in portfolio
unique_stocks_3 <- list()
unique_stocks_4 <- list()
for (i in (1:7)) {
   numb_stocks_1[[i]] <- data.frame(colCounts(matrix(unlist(weights_strat1[i]),ncol=8)>0))
   numb_stocks_2[[i]] <- data.frame(colCounts(matrix(unlist(weights_strat2[i]),ncol=12)>0))
   numb_stocks_3[[i]] <-
data.frame(colCounts(matrix(unlist(weights_strat3[i]),ncol=144)>0))[seq(1,144,12),]
   numb_stocks_4[[i]] <- data.frame(colCounts(matrix(unlist(weights_strat4[i]),ncol=12)>0))
   unique_stocks_2[[i]] <- cumsum(data.frame(colCounts(matrix(unlist(portfolio_in[i]),ncol=12)>0)))
   unique_stocks_3[[i]] <-
cumsum(data.frame(colCounts(matrix(unlist(portfolio_in_m[i]),ncol=144)>0)))[seq(1,144,12),]
   unique_stocks_4[[i]] <- cumsum(data.frame(colCounts(matrix(unlist(portfolio_in[i]),ncol=12)>0)))
}


   ## Portfolio performance before leverage ########
an_return_strategy <- list() # annualised mean return
an_return_strat2 <- list()
an_return_strat3 <- list()
an_return_strat4 <- list()
an_stdev_strat1 <- list() # annualised standard deviation
an_stdev_strategy <- list()
an_stdev_strat2 <- list()
an_stdev_strat3 <- list()
an_stdev_strat4 <- list()
an_stdev_strat3_m <- list()
max_lose_strategy <- list() # maximum drawdown
max_lose_strat2 <- list()
max_lose_strat3 <- list()
max_lose_strat4 <- list()
for (i in (1:7)){
   an_return_strategy[[i]] <- matrix(unlist(cumret_strategy[i]),ncol=1)[13,] ^ (1/12) - 1
   an_return_strat2[[i]] <- matrix(unlist(cumret_strat2[i]),ncol=1)[13,] ^ (1/12) - 1
   an_return_strat3[[i]] <- matrix(unlist(cumret_strat3[i]),ncol=1)[13,] ^ (1/12) - 1
   an_return_strat4[[i]] <- matrix(unlist(cumret_strat4[i]),ncol=1)[13,] ^ (1/12) - 1
   an_stdev_strategy[[i]] <- sd(matrix(unlist(ret_strategy[i]),ncol=1)[,1])
   an_stdev_strat2[[i]] <- sd(matrix(unlist(ret_strat2[i]),ncol=1)[,1])
   an_stdev_strat3[[i]] <- sd(matrix(unlist(ret_strat3[i]),ncol=1)[,1])
   an_stdev_strat4[[i]] <- sd(matrix(unlist(ret_strat4[i]),ncol=1)[,1])
   an_stdev_strat3_m[[i]] <- sd(matrix(unlist(ret_strat3_m[i]),ncol=1)[,1]) * sqrt(12)
   max_lose_strategy[[i]] <- min(
      matrix(unlist(lapply(2:13,function(z) matrix(unlist(cumret_strategy[[i]]),ncol=13)[,z] /
                        max(matrix(unlist(cumret_strategy[[i]]),ncol=13)[,1:z]))),ncol=12)) - 1

   max_lose_strat2[[i]] <- min(
      matrix(unlist(lapply(2:13,function(z) matrix(unlist(cumret_strat2[[i]]),ncol=13)[,z] /
                        max(matrix(unlist(cumret_strategy[[i]]),ncol=13)[,1:z]))),ncol=12)) - 1

   max_lose_strat3[[i]] <- min(
      matrix(unlist(lapply(2:13,function(z) matrix(unlist(cumret_strat3[[i]]),ncol=13)[,z] /
                        max(matrix(unlist(cumret_strategy[[i]]),ncol=13)[,1:z]))),ncol=12)) - 1

   max_lose_strat4[[i]] <- min(
      matrix(unlist(lapply(2:13,function(z) matrix(unlist(cumret_strat4[[i]]),ncol=13)[,z] /
                        max(matrix(unlist(cumret_strategy[[i]]),ncol=13)[,1:z]))),ncol=12)) - 1

}
```

```
sharpe_strat1 <- (matrix(unlist(an_return_strat1),ncol=7)) / matrix(unlist(an_stdev_strat1),ncol=7) #
Sharpe ratio
sharpe_strategy <- (matrix(unlist(an_return_strategy),ncol=7)) / matrix(unlist(an_stdev_strategy),ncol=7)
sharpe_strat2 <- (matrix(unlist(an_return_strat2),ncol=7)) / matrix(unlist(an_stdev_strat2),ncol=7)
sharpe_strat3 <- (matrix(unlist(an_return_strat3),ncol=7)) / matrix(unlist(an_stdev_strat3),ncol=7)
sharpe_strat4 <- (matrix(unlist(an_return_strat4),ncol=7)) / matrix(unlist(an_stdev_strat4),ncol=7)

reg_strategy <- lapply(1:7, function(x) lm(matrix(unlist(ret_strategy),ncol=7)[,x] ~
matrix(unlist(ret_strat2),ncol=7)[,7]))
reg_strat2 <- lapply(1:7, function(x) lm(matrix(unlist(ret_strat2),ncol=7)[,x] ~
matrix(unlist(ret_strat2),ncol=7)[,7]))
reg_strat3 <- lapply(1:7, function(x) lm(matrix(unlist(ret_strat3),ncol=7)[,x] ~
matrix(unlist(ret_strat3),ncol=7)[,7]))
reg_strat4 <- lapply(1:7, function(x) lm(matrix(unlist(ret_strat4),ncol=7)[,x] ~
matrix(unlist(ret_strat4),ncol=7)[,7]))

coef_strategy_unlev <- sapply(reg_strategy, coef) # beta and alpha
coef_strat2_unlev <- sapply(reg_strat2, coef)
coef_strat3_unlev <- sapply(reg_strat3, coef)
coef_strat4_unlev <- sapply(reg_strat4, coef)


   ## Portfolio performance post-leverage ################
lev_strat1 <- list() # average leverage level for the different portfolios for each year
lev_strat2 <- list()
lev_strat3 <- list()
lev_strat3m <- list()
lev_strat4 <- list()
for (i in (1:7)) {
   lev_strat1[[i]] <- describe(ifelse(matrix(unlist(strat1_invested[i]),ncol=8)==1,1,NA) *
pm_debtequity[,1:8],na.rm=T)[,5]
   lev_strat2[[i]] <- describe(ifelse(matrix(unlist(strat_2_invested[i]),ncol=12)==1,1,NA) *
pm_debtequity,na.rm=T)[,5]
   lev_strat3[[i]] <- describe(ifelse(matrix(unlist(strat_3_invested[i]),ncol=144)[,seq(1,144,12)]==1,1,NA)
* pm_debtequity,na.rm=T)[,5]
   lev_strat3m[[i]] <- lapply(1:12,function(x) replicate(12,matrix(unlist(lev_strat3[i]),ncol=1)[x,]))
   lev_strat4[[i]] <- describe(ifelse(matrix(unlist(strat_4_invested[i]),ncol=12)==1,1,NA) *
pm_debtequity,na.rm=T)[,5]
   }

lev_strat1_med <- list() # median leverage level for the different portfolio for all years in total
lev_strat2_med <- list()
lev_strat3_med <- list()
lev_strat4_med <- list()
for (i in (1:7)) { # mean leverage ratio
   lev_strat1_med[[i]] <- describe(melt(ifelse(matrix(unlist(strat1_invested[i]),ncol=8)==1,1,NA))$value *
melt(pm_debtequity[,1:8],na.rm=T)$value)[5]
   lev_strat2_med[[i]] <- describe(melt(ifelse(matrix(unlist(strat_2_invested[i]),ncol=12)==1,1,NA))$value
* melt(pm_debtequity,na.rm=T)$value)[5]
   lev_strat3_med[[i]] <- describe(melt(ifelse(matrix(unlist(strat_3_invested[i]),ncol=12)==1,1,NA))$value
* melt(pm_debtequity,na.rm=T)$value)[5]
   lev_strat4_med[[i]] <- describe(melt(ifelse(matrix(unlist(strat_4_invested[i]),ncol=12)==1,1,NA))$value
* melt(pm_debtequity,na.rm=T)$value)[5]
}

lev_pe_m <- matrix(unlist(lapply(1:12,function(x) replicate(12,lev_pe[x,]))),ncol=1) # leverage = median
debt/equity for buyout targets
lev_100 <- matrix(replicate(12,1),ncol=1) # leverage equal to debt/equity = 1
lev_x2 <- matrix(replicate(12,unlist(lev_strat2_med[4])*2),ncol=1) # leverage = 2 x unlevered level
lev_x1.5 <- matrix(replicate(12,unlist(lev_strat2_med[4])*1.5),ncol=1) # leverage = 1.5 x unlevered level
leverage_level <- list(lev_pe,lev_100,lev_x1.5,lev_x2) # all five levels of applied leverage

ret_strat1_lev <- list() # pre-fee returns for portfolios with different level of leverage and fees
ret_strategy_lev <- list()
ret_strat2_lev <- list()
ret_strat3_lev <- list()
ret_strat4_lev <- list()
for (i in (1:4)) { # Portfolio returns adjusted for leverage each year
   ret_strategy_lev[[i]] <- lapply(1:7, function(j) sapply(seq(0.00,0.09,0.01),function(y)
   ( (1 + matrix(unlist(leverage_level[i]),ncol=1)) * matrix(unlist(ret_strategy[j]),ncol=1) /
   ( 1 + replicate(12,matrix(unlist(lev_strat2_med[j]),ncol=1))) -
      y * ( matrix(unlist(leverage_level[i]),ncol=1) -
replicate(12,matrix(unlist(lev_strat2_med[j]),ncol=1)) )
   )))

   ret_strat2_lev[[i]] <- lapply(1:7, function(j) sapply(seq(0.00,0.09,0.01),function(y)
```

```
      ( (1 + matrix(unlist(leverage_level[i]),ncol=1)) * matrix(unlist(ret_strat2[j]),ncol=1) /
        ( 1 + replicate(12,matrix(unlist(lev_strat2_med[j]),ncol=1))) -
        y * ( matrix(unlist(leverage_level[i]),ncol=1) -
replicate(12,matrix(unlist(lev_strat2_med[j]),ncol=1)) )
   )))

   ret_strat3_lev[[i]] <- lapply(1:7, function(j) sapply(seq(0.00,0.09,0.01),function(y)
      ( (1 + matrix(unlist(leverage_level[i]),ncol=1)) * matrix(unlist(ret_strat3[j]),ncol=1) /
          ( 1 + replicate(12,matrix(unlist(lev_strat3_med[j]),ncol=1))) -
        y * ( matrix(unlist(leverage_level[i]),ncol=1) -
replicate(12,matrix(unlist(lev_strat2_med[j]),ncol=1)) )
   )))

   ret_strat4_lev[[i]] <- lapply(1:7, function(j) sapply(seq(0.00,0.09,0.01),function(y)
      ( (1 + matrix(unlist(leverage_level[i]),ncol=1)) * matrix(unlist(ret_strat4[j]),ncol=1) /
          ( 1 + replicate(12,matrix(unlist(lev_strat4_med[j]),ncol=1))) -
        y * ( matrix(unlist(leverage_level[i]),ncol=1) -
replicate(12,matrix(unlist(lev_strat2_med[j]),ncol=1)) )
   )))
}

# Pre-fee metrics for leveraged portfolios
cumret_strategy_lev <- list() # cumulative return for different level for leverage
an_return_strategy_lev <- list() # annualised mean return
an_stdev_strategy_lev <- list() # annualised standard deviation
max_drawdown_strategy_lev <- list() # maximum portfolio drawdown
cumret_strat2_lev <- list()
an_return_strat2_lev <- list()
an_stdev_strat2_lev <- list()
max_drawdown_strat2_lev <- list()
cumret_strat3_lev <- list()
an_return_strat3_lev <- list()
an_stdev_strat3_lev <- list()
max_drawdown_strat3_lev <- list()
cumret_strat4_lev <- list()
an_return_strat4_lev <- list()
an_stdev_strat4_lev <- list()
max_drawdown_strat4_lev <- list()
coef_strategy <- list() # beta and alpha of portfolios
coef_strat2 <- list()
coef_strat3 <- list()
coef_strat4 <- list()
sharpe_strategy_lev <- list() # Sharpe ratio of portfolios
sharpe_strat2_lev <- list()
sharpe_strat3_lev <- list()
sharpe_strat4_lev <- list()
for (i in (1:4)) { # Cumulative portfolio returns adjusted for leverage
   cumret_strategy_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y) rbind.data.frame(1,

data.frame(cumprod(1+matrix(unlist(ret_strategy_lev[[i]][[x]]),ncol=10)[,y])))))
   an_return_strategy_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y)
      matrix(unlist(cumret_strategy_lev[[i]][[x]]),ncol=10)[13,y] ^ (1/12) - 1))
   an_stdev_strategy_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y)
      sd(matrix(unlist(ret_strategy_lev[[i]][[x]]),ncol=10)[,y])))
   max_drawdown_strategy_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y) min(
      matrix(unlist(lapply(2:13,function(z) matrix(unlist(cumret_strategy_lev[[i]][[x]]),ncol=10)[z,y] /
                              max(matrix(unlist(cumret_strategy_lev[[i]][[x]]),ncol=10)[1:(z),y]) -
1)),ncol=10))))

   cumret_strat2_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y) rbind.data.frame(1,
         data.frame(cumprod(1+matrix(unlist(ret_strat2_lev[[i]][[x]]),ncol=10)[,y])))))
   an_return_strat2_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y)
      matrix(unlist(cumret_strat2_lev[[i]][[x]]),ncol=10)[13,y] ^ (1/12) - 1))
   an_stdev_strat2_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y)
      sd(matrix(unlist(ret_strat2_lev[[i]][[x]]),ncol=10)[,y])))
   max_drawdown_strat2_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y) min(
      matrix(unlist(lapply(2:13,function(z) matrix(unlist(cumret_strat2_lev[[i]][[x]]),ncol=10)[z,y] /
                              max(matrix(unlist(cumret_strat2_lev[[i]][[x]]),ncol=10)[1:(z),y]) -
1)),ncol=10))))

   cumret_strat3_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y) rbind.data.frame(1,
               data.frame(cumprod(1+matrix(unlist(ret_strat3_lev[[i]][[x]]),ncol=10)[,y])))))
   an_return_strat3_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y)
      matrix(unlist(cumret_strat3_lev[[i]][[x]]),ncol=10)[13,y] ^ (1/12) - 1))
   an_stdev_strat3_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y)
      sd(matrix(unlist(ret_strat3_lev[[i]][[x]]),ncol=10)[,y])))
   max_drawdown_strat3_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y) min(
```

```
      matrix(unlist(lapply(2:13,function(z) matrix(unlist(cumret_strat3_lev[[i]][[x]]),ncol=10)[z,y] /
                                max(matrix(unlist(cumret_strat3_lev[[i]][[x]]),ncol=10)[1:(z),y]) -
1)),ncol=10))))

   cumret_strat4_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y) rbind.data.frame(1,
      data.frame(cumprod(1+matrix(unlist(ret_strat4_lev[[i]][[x]]),ncol=10)[,y])))))
   an_return_strat4_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y)
      matrix(unlist(cumret_strat4_lev[[i]][[x]]),ncol=10)[13,y] ^ (1/12) - 1))
   an_stdev_strat4_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y)
      sd(matrix(unlist(ret_strat4_lev[[i]][[x]]),ncol=10)[,y])))
   max_drawdown_strat4_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y) min(
      matrix(unlist(lapply(2:13,function(z) matrix(unlist(cumret_strat4_lev[[i]][[x]]),ncol=10)[z,y] /
      max(matrix(unlist(cumret_strat4_lev[[i]][[x]]),ncol=10)[1:(z),y]) - 1)),ncol=10))))

   coef_strategy[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y) lm(matrix(
      matrix(unlist(ret_strategy_lev[[i]][[x]]),ncol=10)[,y],ncol=1) ~ matrix(index_yearly,ncol=1))$coef))
   coef_strat2[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y) lm(matrix(
      matrix(unlist(ret_strat2_lev[[i]][[x]]),ncol=10)[,y],ncol=1) ~ matrix(index_yearly,ncol=1))$coef))
   coef_strat3[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y) lm(matrix(
      matrix(unlist(ret_strat3_lev[[i]][[x]]),ncol=10)[,y],ncol=1) ~ matrix(index_yearly,ncol=1))$coef))
   coef_strat4[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y) lm(matrix(
      matrix(unlist(ret_strat4_lev[[i]][[x]]),ncol=10)[,y],ncol=1) ~ matrix(index_yearly,ncol=1))$coef))

   sharpe_strategy_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y)
      matrix(unlist(postfee_an_return_strategy_lev[[i]][[x]]),ncol=10)[,y] /
         matrix(unlist(postfee_an_stdev_strategy_lev[[i]][[x]]),ncol=10)[,y] ))
   sharpe_strat2_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y)
      matrix(unlist(postfee_an_return_strat2_lev[[i]][[x]]),ncol=10)[,y] /
         matrix(unlist(postfee_an_stdev_strat2_lev[[i]][[x]]),ncol=10)[,y] ))
   sharpe_strat3_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y)
      matrix(unlist(postfee_an_return_strat3_lev[[i]][[x]]),ncol=10)[,y] /
         matrix(unlist(postfee_an_stdev_strat3_lev[[i]][[x]]),ncol=10)[,y] ))
   sharpe_strat4_lev[[i]] <- lapply(1:7, function(x) sapply(1:10, function(y)
      matrix(unlist(postfee_an_return_strat4_lev[[i]][[x]]),ncol=10)[,y] /
         matrix(unlist(postfee_an_stdev_strat4_lev[[i]][[x]]),ncol=10)[,y] ))
}

postfee_ret_strategy <- list() # postfee annual return of each portfolio for different leverage level and
trading costs
postfee_ret_strat2 <- list()
postfee_ret_strat3 <- list()
postfee_ret_strat4 <- list()
for (i in (1:4)) { # Portfolio returns adjusted for leverage each year
   postfee_ret_strategy[[i]] <- lapply(1:7, function(x) sapply(seq(0,0.015,0.0025), function(z)
      matrix(unlist(ret_strategy_lev[[i]][[x]])[,4],ncol=1) - z *
matrix(unlist(turnover_strategy[x]),ncol=1)))
   postfee_ret_strat2[[i]] <- lapply(1:7, function(x) sapply(seq(0,0.015,0.0025), function(z)
      matrix(unlist(ret_strat2_lev[[i]][[x]])[,4],ncol=1) - z * matrix(unlist(turnover_strat2[x]),ncol=1)))
   postfee_ret_strat3[[i]] <- lapply(1:7, function(x) sapply(seq(0,0.015,0.0025), function(z)
      matrix(unlist(ret_strat3_lev[[i]][[x]])[,4],ncol=1) - z * matrix(unlist(turnover_strat3[x]),ncol=1)))
   postfee_ret_strat4[[i]] <- lapply(1:7, function(x) sapply(seq(0,0.015,0.0025), function(z)
      matrix(unlist(ret_strat4_lev[[i]][[x]])[,4],ncol=1) - z * matrix(unlist(turnover_strat4[x]),ncol=1)))
}

# post-fee metrics for leveraged portfolios
postfee_cumret_strategy_lev <- list() # cumulative post-fee return for different level for leverage and
trading fees
postfee_an_return_strategy_lev <- list() # annualised post-fee return
postfee_an_stdev_strategy_lev <- list() # annualised standard deviation
postfee_max_drawdown_strategy_lev <- list() # maximum drawdown
postfee_cumret_strat2_lev <- list()
postfee_an_return_strat2_lev <- list()
postfee_an_stdev_strat2_lev <- list()
postfee_max_drawdown_strat2_lev <- list()
postfee_cumret_strat3_lev <- list()
postfee_an_return_strat3_lev <- list()
postfee_an_stdev_strat3_lev <- list()
postfee_max_drawdown_strat3_lev <- list()
postfee_cumret_strat4_lev <- list()
postfee_an_return_strat4_lev <- list()
postfee_an_stdev_strat4_lev <- list()
postfee_max_drawdown_strat4_lev <- list()
postfee_coef_strategy <- list() # alpha and beta
postfee_coef_strat2 <- list()
postfee_coef_strat3 <- list()
postfee_coef_strat4 <- list()
postfee_sharpe_strategy_lev <- list() # Sharpe ratio
```

```
postfee_sharpe_strat2_lev <- list()
postfee_sharpe_strat3_lev <- list()
postfee_sharpe_strat4_lev <- list()
for (i in (1:4)) { # Cumulative portfolio returns adjusted for leverage
   postfee_cumret_strategy_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y) rbind.data.frame(1,

data.frame(cumprod(1+matrix(unlist(postfee_ret_strategy[[i]][[x]]),ncol=7)[,y])))))
   postfee_an_return_strategy_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y)
      matrix(unlist(postfee_cumret_strategy_lev[[i]][[x]]),ncol=7)[13,y] ^ (1/12) - 1))
   postfee_an_stdev_strategy_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y)
      sd(matrix(unlist(postfee_ret_strategy[[i]][[x]]),ncol=7)[,y])))
   postfee_max_drawdown_strategy_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y) min(
      matrix(unlist(lapply(2:13,function(z)
matrix(unlist(postfee_cumret_strategy_lev[[i]][[x]]),ncol=7)[z,y] /
                              max(matrix(unlist(postfee_cumret_strategy_lev[[i]][[x]]),ncol=7)[1:(z),y]) -
1)),ncol=7))))

   postfee_cumret_strat2_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y) rbind.data.frame(1,

data.frame(cumprod(1+matrix(unlist(postfee_ret_strat2[[i]][[x]]),ncol=7)[,y])))))
   postfee_an_return_strat2_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y)
      matrix(unlist(postfee_cumret_strat2_lev[[i]][[x]]),ncol=7)[13,y] ^ (1/12) - 1))
   postfee_an_stdev_strat2_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y)
      sd(matrix(unlist(postfee_ret_strat2[[i]][[x]]),ncol=7)[,y])))
   postfee_max_drawdown_strat2_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y) min(
      matrix(unlist(lapply(2:13,function(z) matrix(unlist(postfee_cumret_strat2_lev[[i]][[x]]),ncol=7)[z,y]
/
                              max(matrix(unlist(postfee_cumret_strat2_lev[[i]][[x]]),ncol=7)[1:(z),y]) -
1)),ncol=7))))

   postfee_cumret_strat3_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y) rbind.data.frame(1,

data.frame(cumprod(1+matrix(unlist(postfee_ret_strat3[[i]][[x]]),ncol=7)[,y])))))
   postfee_an_return_strat3_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y)
      matrix(unlist(postfee_cumret_strat3_lev[[i]][[x]]),ncol=7)[13,y] ^ (1/12) - 1))
   postfee_an_stdev_strat3_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y)
      sd(matrix(unlist(postfee_ret_strat3[[i]][[x]]),ncol=7)[,y])))
   postfee_max_drawdown_strat3_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y) min(
      matrix(unlist(lapply(2:13,function(z) matrix(unlist(postfee_cumret_strat3_lev[[i]][[x]]),ncol=7)[z,y]
/
                              max(matrix(unlist(postfee_cumret_strat3_lev[[i]][[x]]),ncol=7)[1:(z),y]) -
1)),ncol=7))))

   postfee_cumret_strat4_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y) rbind.data.frame(1,

data.frame(cumprod(1+matrix(unlist(postfee_ret_strat4[[i]][[x]]),ncol=7)[,y])))))
   postfee_an_return_strat4_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y)
      matrix(unlist(postfee_cumret_strat4_lev[[i]][[x]]),ncol=7)[13,y] ^ (1/12) - 1))
   postfee_an_stdev_strat4_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y)
      sd(matrix(unlist(postfee_ret_strat4[[i]][[x]]),ncol=7)[,y])))
   postfee_max_drawdown_strat4_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y) min(
      matrix(unlist(lapply(2:13,function(z) matrix(unlist(postfee_cumret_strat4_lev[[i]][[x]]),ncol=7)[z,y]
/
                              max(matrix(unlist(postfee_cumret_strat4_lev[[i]][[x]]),ncol=7)[1:(z),y]) -
1)),ncol=7))))

   postfee_coef_strategy[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y) lm(matrix(
      matrix(unlist(postfee_ret_strategy[[i]][[x]]),ncol=7)[,y],ncol=1) ~
matrix(index_yearly,ncol=1))$coef))
   postfee_coef_strat2[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y) lm(matrix(
      matrix(unlist(postfee_ret_strat2[[i]][[x]]),ncol=7)[,y],ncol=1) ~ matrix(index_yearly,ncol=1))$coef))
   postfee_coef_strat3[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y) lm(matrix(
      matrix(unlist(postfee_ret_strat3[[i]][[x]]),ncol=7)[,y],ncol=1) ~ matrix(index_yearly,ncol=1))$coef))
   postfee_coef_strat4[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y) lm(matrix(
      matrix(unlist(postfee_ret_strat4[[i]][[x]]),ncol=7)[,y],ncol=1) ~ matrix(index_yearly,ncol=1))$coef))

   postfee_sharpe_strategy_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y)
      matrix(unlist(postfee_an_return_strategy_lev[[i]][[x]]),ncol=7)[,y] /
         matrix(unlist(postfee_an_stdev_strategy_lev[[i]][[x]]),ncol=7)[,y] ))
   postfee_sharpe_strat2_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y)
      matrix(unlist(postfee_an_return_strat2_lev[[i]][[x]]),ncol=7)[,y] /
         matrix(unlist(postfee_an_stdev_strat2_lev[[i]][[x]]),ncol=7)[,y] ))
   postfee_sharpe_strat3_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y)
      matrix(unlist(postfee_an_return_strat3_lev[[i]][[x]]),ncol=7)[,y] /
         matrix(unlist(postfee_an_stdev_strat3_lev[[i]][[x]]),ncol=7)[,y] ))
   postfee_sharpe_strat4_lev[[i]] <- lapply(1:7, function(x) sapply(1:7, function(y)
      matrix(unlist(postfee_an_return_strat4_lev[[i]][[x]]),ncol=7)[,y] /
```

```
    matrix(unlist(postfee_an_stdev_strat4_lev[[i]][[x]]),ncol=7)[,y] ))
}
```