

# Mixed Member Proportional with faithful accounting

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## Mixed Member Proportional with faithful accounting

### ABSTRACT

With present (2021) tally rules, the number of seats in the Bundestag is highly volatile. In 2017 it got 709 seats, 111 of them extra-ordinary. The rules may double the influence of a voter who splits the ballot, contrary to a goal of equal influence; increased assembly size is a concomitant. The paper explains when and how this happens.

A ballot's combination of Erststimme and Zweitstimme is information now ignored; the tally is as if they were collected in separate ballot boxes. *Faithful accounting* requires this information, but there is strong reason to expect it will reduce the 2017 assembly size to the ordinary 598 seats stated in the Federal Elections Act.

Tallying 2017 votes with the present rules, but with CDU&CSU as a recognized coalition, reduces the size by 41 seats.

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**Word count:** 7811.

## Introduction

In a *Leitsatz* to a ruling of July 3<sup>rd</sup> 2008, the German Federal Constitutional Court emphasized the *Grundsätze der Gleichheit und der Unmittelbarkeit*. New rules came in 2017 for good reasons: Complicated allocation of party seats to states and parties was changed. This improved transparency – an important achievement.

The 2013 election gave the Bundestag 631 seats. With 2017 tally rules, the 2013 assembly size would have been the 598 ordinary seats, even if a lower threshold had let FDP and AfD in (both narrowly missed it in 2013). However, with new tally rules, the Bundestag in 2017 got 709 seats, 111 of them extra-ordinary. What caused the increase? Transparency facilitates an analysis.

The new rules do not cope with some well-known violations of the *Grundsatz der Gleichheit*: The tally still ignores a ballot's *combination* of Erststimme and Zweitstimme. One might as well collect Erststimme and Zweitstimme in separate boxes; the tally would be the same. Ignoring this information keeps an unequal influence, which in turn increases the assembly size.

Changes in the combination pattern together with the proportionality requirement explains the 111 extra-ordinary seats. "*Faithful accounting*" is suggested to avoid unequal influence; there is reason to expect it gives ordinary assembly size, 598 seats.

CSU runs only in Bavaria and CDU only in the other fifteen states; with present tally rules, this explains part of the 111 seat excess. Tallying them as a single party C\*U with 185+46 constituency seats (Table 1) reduces the 111 extra-ordinary seats by 42, to 69.

All information on the 2017 voting and tally needed in this paper, appears in Table 1:

j	P <sub>j</sub>	ERSTSTIMME		SEATS			ZWEITSTIMME		Zwst/tot
		%	#	LOCAL	TOTAL	LISTE	#	%	
1	CDU	30.2	14030751	185	200	15	12447656	26.8	62238
2	SPD	24.6	11429231	59	153	94	9539381	20.5	62349
3	Afd	11.5	5317499	3	94	91	5878115	12.6	62533
4	FDP	7.0	3249238	0	80	80	4999449	10.7	62493
5	Linke	8.6	3966637	5	69	64	4297270	9.2	62279
6	Grüne	8.0	3717922	1	67	66	4158400	8.9	62066
7	CSU	7.0	3255487	46	46	0	2869688	6.2	62385
97-		44966765	299	709	410	44190959	95-		

**TABLE 1** 38 parties were on the ballot in at least one state; the table includes those 7 that passed the 5% threshold. Sometimes a party does not nominate a candidate in a constituency. The Erststimme columns show Erststimme received by *all* constituency *candidates* of a party; the LOCAL column shows the distribution of *winning candidates*.

## 1 The 2017 Bundestag election

**First observations** Erststimme tallies in 299 constituencies find the winners of the local seats with the Plurality method (“first-past-the-post”). The LOCAL column of Table 1 has accounts for the local seats won by *candidates* from each party, e.g. 59 from SPD. Another entry in the same line shows there were 9539381 voters in  $\Lambda(\text{SPD})$ ;  $\Lambda(P_j)$  is the group of voters who give Zweitstimme to party  $P_j$ 's list.

Which voters won those 59 constituencies? More than 1.8 million Erststimme for SPD-candidates must have come from other Zweitstimme groups than  $\Lambda(\text{SPD})$ , and those among them that came in the 59 constituencies supported an SPD-winner. Similarly of course, some voters in  $\Lambda(\text{SPD})$  may have given Erststimme to a candidate *not* from SPD.

Table 1 has “non-human legal entities”, parties  $P_j$  instead of  $\Lambda(P_j)$ ;  $\Lambda$  is for *Leute* (people). The first 0 in Table 1 illustrates this: Party FDP had no success in Erststimme tallies, but data indicate that voters in  $\Lambda(\text{FDP})$  had considerable success with Erststimme in “split” ballots:

$\Lambda(\text{FDP})$  has 4999449 members, but FDP-candidates got only 3249238 Erststimme. Thus, over 1.75 million voters in  $\Lambda(\text{FDP})$  gave Erststimme to candidates *not* from FDP, helping many winners from CDU/CSU in  $185+46 = 231$  constituencies. Since about 2 million Erststimme for CDU/CSU-candidates *cannot* have come from  $\Lambda(\text{CDU})$  or  $\Lambda(\text{CSU})$ ; they *must* have come in split ballots from other Zweitstimme groups, including  $\Lambda(\text{FDP})$ .

Obviously,  $\Lambda(\text{FDP})$  has more than 0 success with its Erststimme, while  $\Lambda(\text{CDU})$  and  $\Lambda(\text{CSU})$  provide less than 100% of the success indicated by  $185+46$  in the LOCAL column of Table 1.

In *faithful accounting*, defined in section 2, the basic legal entity is  $\Lambda(P_j)$ , with voting *Leute* who influence the outcome through their ballots, not an *organization*  $P_j$  expected just to accept the result. Technically, the Zweitstimme tally will be as now, but the new basic entities change the LOCAL column, and thereby also the outcome. A new LOCAL column will show the success the Zweitstimme groups  $\Lambda(P_j)$  have with their Erststimme. Data are already in the ballots.

**The tally of 2017** The TOTAL column in Table 1 shows *proportional distribution*, with good approximation, of all seats according to the sizes  $|\Lambda(P_j)|$  of the Zweitstimme groups  $\Lambda(P_j)$ . The right hand column of Table 1 illustrates the accuracy obtained with integers instead of “ideal” fractions, e.g.:

$$\begin{aligned}
|\Lambda(\text{CSU})| / 46 &= 2869688 / 46 = 62385 \text{ Zweitstimme per seat} \\
|\Lambda(\text{CDU})| / 200 &= 12447656 / 200 = 62238 \text{ Zweitstimme per seat} \\
|\Lambda(\text{FDP})| / 80 &= 4999449 / 80 = 62493 \text{ Zweitstimme per seat}
\end{aligned}$$

With the simplified rules of 2017, only the LOCAL column and the comprehensive Zweitstimme column of Table 1 influence the TOTAL column: 200, 153, ... .

**The pivotal party** One party,  $P_m$ , is seen to have a *pivotal* status. This is the party with *most local seats per Zweitstimme*. Thus,  $P_m = P_7 = \text{CSU}$  in the 2017 election (Table 1).

**Critical assembly size** Zweitstimme and the LOCAL column entry for  $P_m$  determine the *critical assembly size*, i.e. *the smallest size where proportional distribution includes all local winners*. CSU got 46 seats and 2869688 Zweitstimme; all seven parties together have 44190959 Zweitstimme. The proportionality rule gives the *critical size* as 709 seats but, *ideally*,

$$(*) \quad 46 \times 44190959 / 2869688 = 708.364154... \text{ seats.}$$

Proportionality is behind many appearances below of this classical “*Regula de tribus*”:

**Tribus value** Four quantities are connected by proportionality,  $a/b = c/d$ ; if any 3 of them are known, then the 4<sup>th</sup> is found by a calculation like (\*). Thus,

when P already has b seats, and out of c Zweitstimme in the tally, P receives d,  
then  $a = b \times c/d$  will here be called the *Tribus value* of the triple (b, c, d).

Thus, the critical size in 2017 is the *Tribus value* =a for the pivotal party (CSU) with b constituency winners, b=46, and inverse Zweitstimme share  $c/d$ , *rounded upwards* (to 709).

**The seat price** By analogy,  $d/b = 2869688/46 \approx 62385$  Zweitstimme/seat is interpreted as a “price” set by  $P_m$  (= CSU), which each party is *committed* to “pay”. CSU cannot afford more than 46 seats; the others pay for LISTE seats too. The seven parties together afford

$$b \times c/d = c/[d/b] \approx 44190959/62385 \approx 708.36 \text{ seats; } 709 \text{ at reduced price } 62329.$$

The *purchasing power* is  $709/44190959 = 0.0000160440... \text{ seats per Zweitstimme}$ .

**Completing the tally** By law, the work is done by the “*engine*” of *Sainte-Laguë* (1910). This is an algorithm that is given the Zweitstimme number  $|\Lambda(P_j)|$  for each  $P_j$  that passed the 5% threshold (or alternatively, won 3 LOCAL seats). It deals out seats to those parties, one by one, and is used to produce the TOTAL column of Table 1. It follows a simple rule:

**Contest number** With party  $P_j$  is associated a *contest number* which is reduced every time  $P_j$  receives a seat from the engine. While  $P_j$  is in contest for its  $s^{\text{th}}$  seat, its contest number is:

$$|\Lambda(P_j)| / (2 \times s - 1)$$

At each step a seat is dealt out to the party with the *currently highest contest number*.

The engine is kept working at least until critical size is reached. Let  $P_j$ -s be the  $s^{\text{th}}$  seat dealt to party  $P_j$ . Thus, CSU receives its 46<sup>th</sup> seat, CSU-46, with contest number

$$|\Lambda(\text{CSU})| / (2 \times 46 - 1) = 2869688 / 91 \approx 31535.$$

CSU-46 is number 700 from the engine, which works until critical size at 709. The last nine seats come after CSU-46. Enumerated by Sainte-Laguë, and with contest numbers, they are:

701	702	703	704	705	706	707	708	709
CDU-198	SPD-152	FDP-80	AfD-94	Linke-69	CDU-199	SPD-153	Grüne-67	CDU-200
31513	31483	31443	31434	31367	31354	31277	31266	31197

**TABLE 2** Sainte-Laguë deals out CSU-46 prematurely; it is in “*overhang*” until Tribus value 708.364154... and price 62385 are passed. Then CSU affords to buy 46 seats; 6 parties buy compensation, reaching the numbers in the TOTAL column of Table 1. From early on, the engine approximates proportionality well, but overhang of local seats is illegal, and 9 more seats follow. Full payment is enforced for local seats only; CDU-200, Grüne-67, and Linke-69 are not fully paid.

Sainte-Laguë is widely used to approximate proportionality in elections to legislatures. One might expect the tally to end when the last constituency seat CSU-46, already won by Plurality, was re-distributed as seat number 700. However, Tribus values are, directly and intuitively, linked to proportionality. Sainte-Laguë continues until critical size, 709. Arguably, a sequence according to Tribus value would have been more natural: If  $P_j$  is in contest for its  $b^{\text{th}}$  seat with Tribus value  $b \times c/d$ ,  $d = |\Lambda(P_j)|$ , then four seats come after CSU-46:

705) CSU-46 at Tribus value 708.364154... ; and then follow 706) SPD-153 708.768915... ;  
707) Linke-69 709.559740... ; 708) CDU-200 710.028692... ; 709) Grüne-67 712.003235... .

**The size of the pivotal party** CSU is pivotal, but also smallest. With its small size, its pivotal status lets proportionality blow up one extra seat for CSU to much larger increases in the TOTAL column: Before an imagined CSU-47 (contest number 30856) is *paid* at seat 724, come fourteen more compensation seats.

**REMARK 1** CSU runs only in Bavaria, CDU only in the 15 other states. For CDU/CSU as one party C\*U, with  $12447656 + 2869688 = 15317344$  Zweitstimme and  $185 + 46 = 231$  LOCAL seats, the ratio (price) is  $15317344 / 231 \approx 66309$  Zweitstimme per seat. C\*U is pivotal; thus the critical size is 667 seats (by price,  $44190959 / 66309 \approx 666.44$ ) with respectively

231, 144, 89, 75, 65, 63 seats for C\*U, SPD, AfD, FDP, Linke, Grüne.

The final nine-seat Sainte-Laguë stretch with contest numbers is as follows:

659	660	661	662	663	664	665	666	667
FDP-75	C*U-229	SPD-143	C*U-230	Linke-65	Grüne-63	SPD-144	C*U-231	AfD-89
33553	33517	33472	33371	33312	33267	33238	33226	33209

**TABLE 3** Sainte-Laguë deals out seats frequently to a large pivotal unit like C\*U. The overhang ends shortly after C\*U-231 is recognized by Sainte-Laguë with seat 666. The assembly size is reduced by 42 seats: With its Zweitstimme, CDU helps to “pay” for CSU’s 46 local seats.

**Overhang coalition** Instead of fusion to C\*U, an “overhang coalition” CDU&CSU is almost as efficient. Extend Table 2 backwards. It visualizes that CDU gets seats more often than CSU:

- at seat 668, CDU pays for CDU-188;
- at seat 663, CSU pays for CSU-43.

The *coalition’s* commitment is 188+43=231; payment of 231 seats ends the tally at seat 668 instead of 709. A tally with CDU/CSU as one party C\*U or as an overhang coalition CDU&CSU will have significant effect, but excessive size clearly remains a problem to be handled.

**Comparison of two elections** The previous election, of 2013, gave a Bundestag with 631 seats. Was the increase to 709 seats in 2017 due to the tally rules of 2017 or to a change in voter behavior? The critical size in 2013 was 511 seats.

However, FDP and AfD narrowly missed the 5% threshold in 2013. Counterfactually including them for fair comparison, there are 41007935 Zweitstimme. CSU wins 45 local seats, receives 3243569 Zweitstimme, and is clearly pivotal.

Thus, with the same seven parties, the “ideal” critical size is the Tribus value,

$$a = b \times c/d = 45 \times 41007935 / 3243569 = 568.927954... \text{ seats.}$$

The critical size is 569 seats, with proportionality factor

$$41007935 / 569 \approx c/a = d/b = 3243569 / 45 \approx 72070 \text{ Zweitstimme per seat.}$$

Sainte-Laguë deals out 29 more seats proportionally to fill all 598 ordinary seats.

**REMARK 2** Although FDP and AfD are counterfactually included in 2013, there are in 2017 *more* Zweitstimme in sum c, while CSU got *less* Zweitstimme d but won *more* local seats b. All changes pull in the same direction, increasing the Tribus value  $b \times c/d$

$$\text{from } 45 \times 41007935 / 3243569 \text{ to } 46 \times 44190959 / 2869688, \\ \text{and critical size from 569 to 709 seats.}$$

Also counterfactual in 2013, simplified allocation rules create transparency. The tasks get clearer: What motivates ballot changes that cause a 140 seat increase? What can be done?

## 2 Equal influence and faithful accounting

**Matters of accounting** Roughly estimated, 7 of 15 voters support the local winner:

CSU runs only in Bavaria and *won all constituencies*; thus, Table 1 shows on average

$3255487/46 \approx 70771$  Erststimme behind each constituency winner,

and  $70771 \times 299 = 21160529$ . Thus, if Bavaria is typical, about 21 million out of 45 million Erststimme (Table 1) support the local winner, i.e. 7 of 15. With a Majority method in every constituency, there would be at least  $70771 \times 7.5/7 \approx 75826$  Erststimme per winner.

One intended use of the Zweitstimme is to distribute 598 seats proportionally,

at a price  $44190959 / 598 \approx 73898$  Zweitstimme *per seat*.

However, support for local winners comes also from  $\Lambda(P_j)$  that do not pass the threshold: Below, another estimate (by *faithful accounting*), sets their real influence at 5 *seat equivalents*. Thus,

*adjusted price* is  $R = 44190959 / (598-5) \approx 74521$  Zweitstimme *per seat equivalent*.

Arguably,  $\Lambda(\text{CSU})$ 's commitment should then be  $2869688/74521 \approx 38.51$  instead of 46.

With the present accounting system in the LOCAL column of Table 1, Sainte-Laguë's sequence is observed until CSU-46 is finally paid at seat 709; by Table 1 there are then only

$44190959 / 709 \approx 62329$  Zweitstimme per seat.

The 111 extra-ordinary LISTE seats are due to the accounting system in the LOCAL column.

### **Erststimme, the core of MMP**

In Bundestag elections and other MMP-elections

(Mixed Member Proportional), Erststimme tallies concentrate many voters around each winner.

The Duvergerian mechanism is natural in single-seat elections with the Plurality method: Voters get an incentive to include *electability* as a criterion and support *the best liked front-runner* with an *instrumental* Erststimme. For many voters, combining it with an *expressive* Zweitstimme for *the best liked party* should be their natural adaptation to MMP.

Support from a large plurality, preferably a majority, legitimizes the representative for a constituency; vote-splitting may help to obtain it. However, vote-splitting may also exploit unequal influence and blow up the assembly size. Faithful *accounting* counteracts these consequences by redefining the LOCAL column in Table 1, so that it shows how well *each Zweitstimme group*  $\Lambda(P_j)$  does with its Erststimme, as opposed to how well *the party*  $P_j$  does.

Erststimme from members of  $\Lambda(P_j)$  are spread on many parties, and often support the local winner. A member who supports a local winner gets *a share in the seat*: These shares are



aggregated in terms of *seat equivalents* on an account for  $\Lambda(P_j)$  in a new LOCAL column. This faithful accounting requires a thorough scrutiny of the ballots in each constituency.

**Unequal influence** The tally rules of 2017 link excessive assembly size to unequal influence. Ideally, equality means that all voters have the same influence on the TOTAL column. It turns out that some vote splitting leads to unequal influence. Four parties get particularly strong support in split ballots, shown by differences in Table 1:

FDP:	4999449	-	(3249238	-	f)	=	1750211	+	f
AfD:	5878115	-	(5317499	-	a)	=	560616	+	a
Grüne:	4158400	-	(3717922	-	g)	=	440478	+	g
Linke:	4297270	-	(3966637	-	l)	=	330633	+	l

**TABLE 4** The sum is the number of split ballots with Zweitstimme to the indicated party;  $f, a, g, l$  are the unknown numbers of split ballots with Erststimme to the party.

**Ballot splitting and the pivotal party** In  $\Lambda(\text{FDP})$ , more than 1.75 million support a local candidate from another party, typically voters who are confident of Erststimme success, mainly for CDU/CSU that won 77% of the constituencies. For illustration, assume 1.4 million actionists with successful Erststimme, 1.1 million to CDU and 0.3 million to CSU, did split their ballot and moved to  $\Lambda(\text{FDP})$ . Before the action, Zweitstimme numbers and Tribus values were

$$|\Lambda(\text{CDU})| = 12447656 + 1100000 = 13547656, \quad |\Lambda(\text{CSU})| = 2869688 + 300000 = 3169688;$$

$$185 \times 44190959 / |\Lambda(\text{CDU})| = 603.449586\dots, \quad 46 \times 44190959 / |\Lambda(\text{CSU})| = 641.319938\dots$$

The action made Tribus values grow to, respectively, 656.776457... and 708.364154... .

With 46 of 642 seats, CSU was pivotal also before the action reduced CSU's Zweitstimme supply. Reduced supply required  $709 - 642 = 67$  new extra-ordinary seats with price reduction, so that CSU could pay for its 46 constituency seats. The initial seat distribution plus changes are:

$$\text{CDU } 197+3; \text{ SPD } 139+14; \text{ AfD } 85+9; \text{ FDP } 52+28; \text{ Linke } 63+6; \text{ Grüne } 60+7; \text{ CSU } 46+0.$$

The action makes CDU/CSU relatively weaker. The combined relative strength of CDU, CSU and FDP does not change. Table 4 indicates that similar moves e.g. from  $\Lambda(\text{SPD})$  to  $\Lambda(\text{Grüne})$  are common, but they just reduce the number of LISTE seats for SPD and increase it for Grüne as long as CSU remains pivotal, since they do not change data for CSU: Numbers  $b=46, c=44190959, d=|\Lambda(\text{CSU})|$  determine CSU's Tribus value and the assembly size.

**REMARK 3** If actionists help the recipient across the 5% threshold, a "jackpot effect" increases the number  $c$  of Zweitstimme, CSU's Tribus value  $b \times c / d$ , and the assembly size.

**A micro-example** Consider two voters, A and B, both supporting a local winner from CSU. A also gives Zweitstimme to CSU, helping to pay for its 46 seats. Giving Zweitstimme to FDP, B runs away from the bill and helps to buy party seats for FDP.

Supporting CSU's local candidate, B has the same influence as A, *but in addition* B gets the same influence on the representation of FDP, reduces  $d = |\Lambda(\text{CSU})|$  and increases  $a = b \times c/d$ .

**Constitutionality** This option of doubled influence is manifestly a pathological side of the present rules. Voters like B also cause many more seats to all five parties *not* supported in their ballot and reduce the relative strength of CSU.

Under old rules and special conditions, extra Zweitstimme to party P (without other changes) could even *reduce* P's number of seats, an effect *opposite* to the voter's obvious intention. The German Federal Constitutional Court found this unconstitutional. An introductory *Leitsatz* states that this *Negatives Stimmgewicht* "verletzt die Grundsätze der Gleichheit und der Unmittelbarkeit der Wahl" (Bundesverfassungsgericht 2008).

**Faithful accounting** The Zweitstimme groups  $\Lambda(P_j)$  are the basic entities in *faithful accounting*, which requires numbers  $N(j,k)$  from constituency  $k$ ,  $1 \leq k \leq 299$ :

$N(j,k)$  voters from  $\Lambda(P_j)$  give Erststimme to the local winner,  $1 \leq j \leq p$  ( $p = 38$  in 2017).

The local winner receives  $E(k) = N(1,k) + N(2,k) + \dots + N(p,k)$  Erststimme.

Each of the  $N(j,k)$  members in  $\Lambda(P_j)$  wins one  $1/E(k)$ -share in the constituency seat.

$N(j,k)$  can only be found by closer scrutiny of the ballots than is done today.  $\Lambda(P_j)$ 's success in the Erststimme tallies is measured by another sum,

$$S(j) = N(j,1) / E(1) + N(j,2) / E(2) + \dots + N(j,299) / E(299) \text{ seat equivalents};$$

clearly,  $S(1) + S(2) + \dots + S(p) = 299$  seat equivalents.

$\Lambda(P_8), \dots, \Lambda(P_p)$  did not pass the threshold in 2017, but got Erststimme success  $F$ :

$$F = S(8) + S(9) + \dots + S(p) \text{ seat equivalents.}$$

Below,  $F$  is estimated to  $F \approx 5$ ; thus, there are  $598 - 5$  ordinary seats to be distributed. The adjusted seat price  $R = 74521$  means that *purchasing power* is  $1/R$ :

$$1/R = 1/74521 \approx 593 / 44190959 \text{ seat equivalents per Zweitstimme.}$$

A natural idea is to let each ballot from  $\Lambda(P_j)$  which supports the winner in constituency  $k$ , carry an "Erststimme success increment"  $1/E(k)$  to the new LOCAL commitment account for  $\Lambda(P_j)$ . This counteracts unequal influence, and seems fair. But there is a snag:

**Variations in  $E(k)$**  A small  $E(k)$  means a large increment  $1/E(k)$  to  $\Lambda(P_j)$ 's commitment account. Unfortunately, it will not always be compensated by the voter's membership in  $\Lambda(P_j)$ : The seat distribution may end at the ordinary assembly size (598 seats), *with still insufficient purchasing power*, i.e.  $1/R < 1/E(k)$ . With Zweitstimme to  $P_j$  the ballot *then gives negative net support to the party that the voter intends to support*.

This is *Negatives Stimmgewicht*, although less complicated than the version that the Federal Constitutional Court found *unconstitutional* in 2008.

**Modified increment** In faithful accounting, a ballot from  $\Lambda(P_j)$ , with Erststimme supporting the winner in constituency  $k$ , carries a *modified increment*  $M(k)$  to  $\Lambda(P_j)$ 's account,

$$M(k) = \min[1/E(k), 1/R] \leq 1/R.$$

At ordinary assembly size, 598, the ballot's *purchasing power* is  $1/R$  and covers the *commitment increment*  $M(k)$ . No party is harmed by receiving a Zweitstimme. Thus,

*Negatives Stimmgewicht cannot occur.*

By aggregation, the Zweitstimme supply covers the commitments of all voters in  $\Lambda(P_j)$  who give Erststimme to the winner of their constituency.  $\Lambda(P_j)$  pays for its due proportion of seats; there is no lack of Zweitstimme supply to force a price reduction. Thus,

*there are no extra-ordinary LISTE seats.*

**A distribution algorithm** With faithful accounting, commitments are expressed in seat *fractions*. In a practical distribution sequence of LISTE seats, Zweitstimme group  $\Lambda(P_j)$  with commitment  $\text{com}(j)$  is in contest for its  $r^{\text{th}}$  LISTE seat with a Tribus value as contest number:

$$[r + \text{com}(j)] \times 44190959 / |\Lambda(P_j)|.$$

Critical size, where all commitments are paid, is obtained with ordinary seats only.

**Constituency seats and faithful accounting.** Modified commitment increments avoid uncontrolled growth of the number of LISTE seats. But it creates a new problem, fortunately limited to a relatively small number of constituency seat equivalents: In constituencies where  $E(k) < R$ , an Erststimme success  $1/E(k) - 1/R$  per ballot *is not entered in the commitment account*. Thus, with  $E(k)$  ballots, an aggregated seat fraction *not entered*, is:

$$E(k) \times [1/E(k) - 1/R] = 1 - E(k)/R \text{ seat equivalents}$$

Thus, the Zweitstimme groups pay for commitments that add up to less than 299 constituency seat equivalents, and real constituency winners need 299 real seats.

**EXAMPLE** The number of Erststimme  $E(k)$  in the 46 constituencies of Bavaria varies from 45340 to 94252 (in constituencies  $k=244$  and  $k=253$ ), while purchasing power is  $1/R = 1/74521 = 0.0000134190\dots$  seat equivalents per Zweitstimme.

Assume faithful accounting: No extra-ordinary seats are distributed.

253 Augsburg-Land:

$$E(253) = 94252 \text{ (47.8\%), } 1/E(253) = 0.0000106098\dots \text{ (Runner-up SPD, 27813)}$$

$$M(253) = \min [1/R, 1/E(253)] = 0.0000106098\dots$$

Each of the 94252 ballots contributes  $1/R - M(253)$  to the net purchasing power of its Zweitstimme group, i.e. 0.0000028092 above the modified commitment  $M(253)$ .

Thus if, say, 80000 of these 94252 voters are in  $\Lambda(\text{CSU})$ , they also contribute net purchasing power to buy a fraction  $80000 \times (0.0000134190 - 0.0000106098) \approx 0.22478$  of a party seat for CSU.

244 Nürnberg-Nord:

$$E(244) = 45340 \text{ (31.3\%), } 1/E(244) = 0.0000220555\dots \text{ (Runner-up SPD, 37068)}$$

$$M(244) = \min [1/R, 1/E(244)] = 0.0000134190\dots$$

Each of the 45340 ballots with Erststimme to constituency winner  $W$  neutralizes exactly one modified commitment  $M(244)=1/R$ ; thus, no purchasing power is left to buy party seats. To avoid Negatives Stimmgewicht,  $1/E(244) - 1/R$  of the ballot's Erststimme success is unaccounted. Aggregated overhang is  $45340 \times (0.000022055 - 0.000013419) \approx 0.39156$  of the constituency seat..

The 299 constituency seats appear as seat equivalents *in three categories*:

The 1<sup>st</sup> category consists of Erststimme success of those  $\Lambda(P_j)$  that did not pass the threshold, below assessed to  $F=5$  seat equivalents in 2017 ( $8 \leq j \leq p$ ,  $p=38$ ).

The 2<sup>nd</sup> category consists of Erststimme success entered by faithful accounting as commitments  $1/E(k)$  in the new LOCAL column. If  $E(k) > R$ , each ballot then contributes  $1/R - 1/E(k)$  of net purchasing power to their Zweitstimme group, as in  $k=253$ .

The 3<sup>rd</sup> category consists of Erststimme success in form of positive seat fractions  $1/E(k) - 1/R$  *not entered* into commitment accounts; here  $E(k) < R$  Erststimme support the winner, as in  $k=244$ .

When 593 seats have been distributed, 598 seats are filled; seat prices have dropped, and every  $\Lambda(P_j)$  has sufficient Zweitstimme supply to pay for its 2<sup>nd</sup> category commitments. Increased support  $E(k)$  for winner  $W$  in constituency  $k$  will move Erststimme success from 3<sup>rd</sup> to 2<sup>nd</sup> category. If it switches from  $E(k) < R$  to  $R < E(k)$ , all of it goes to the 2<sup>nd</sup> category.

A Majority method in the Erststimme tallies will lead to many such switches. “W-U” is a Majority method that requires only the same ballot information as Plurality elections require:

***W-U, a Majority rule for Plurality ballots*** An Erststimme neither for winner W nor for runner-up U, counts as if the voter gave half an Erststimme for each; this is a common principle for treating indifference in preferential elections. Ballots with Erststimme for U do not increase the commitment account for their Zweitstimme group.

In Bavaria, 26 constituencies have  $E(k) < R$ : Part of their Erststimme success is in the 3<sup>rd</sup> category. With W-U, most of them move all of it to the 2<sup>nd</sup> category, but a few keep a small part in the 3<sup>rd</sup> category. Two neighbors, with similar political landscapes, land close to R, on different sides:

- In  $k=244$ , of the Example above,  $E(244) = 45340$ , and 62515 did not support W or U;  
W-U makes  $E(244) = 45340 + 62515/2 = 76597.5 > R$ ;  
the local election board keeps nothing in the 3<sup>rd</sup> category.
- In  $k=245$  (Nürnberg-Süd),  $E(245) = 46511$ , U has 34621 Erststimme, others have 49493.  
W-U makes  $E(245) = 46511 + 49493/2 = 71257.5 < R$ ;  
a tiny seat fraction  $1 - E(245)/R \approx 0.040$  is left in the 3<sup>rd</sup> category.

***The assembly size*** At critical size, all commitment is covered by Zweitstimme resources. The aggregated commitment coincides with the 2<sup>nd</sup> category. Critical size may be reached well before ordinary size (598 seats). Thus, when 2017 rules are applied to 2013 data, the critical size is 569; a fusion C\*U or coalition CDU&CSU would have made it even smaller.

Voter behavior, that caused the 2017 spike, was very different in 2013, but in 2017 faithful accounting would have avoided the spike. It seems likely that, with faithful accounting, critical size with the desired proportionality will be reached well before the ordinary size. The 3<sup>rd</sup> category is relatively small, and can also be made even smaller (with the W-U method). The gap from critical to ordinary size should be enough for bringing the number of constituency seats up to 298, and then reach ordinary size with continued proportional distribution.

***Voters' adaptation*** Some split “QP”-ballots, with Erststimme to party Q and Zweitstimme to another party P, are certainly “straight”, P being the best liked party and Q having the best liked candidate. The voters' perception of each candidate's personal suitability is a recognized reason for having MMP-elections (Personalisiertes Wahlrecht).

When a “QP”-voter has changed an original plan to vote “PP” or “QQ”, a likely background is that P and Q are political allies. Common reasons for splitting the ballot are hopes that:

- changed Erststimme helps Q instrumentally: P is seen as too weak in the constituency.
- changed Zweitstimme helps P to pass the 5% threshold: Winning a jackpot helps the allies.

Both changes highlight a *weakness in the present accounting system* when Q is pivotal. Move backwards to the original “PP” or “QQ” with Q=CSU, P=FDP in the 2017 election:

- If returns from QP to PP in a constituency reduce Q’s number b of constituency seats in 2017 from 46 to 45, it will reduce the Tribus value  $a = b \times c / d$ ,  
and thereby reduce the assembly size from 709 to 693.

- With b=46, Tribus value a=693 would have been obtained with returns from QP to QQ and

$$d = b \times c / a = 46 \times 44190959 / 693 \approx 2933311; \text{ thus, the assembly size becomes } 693 \text{ when } 2933311 - 2869688 = 63623 \text{ voters return to } \Lambda(\text{CSU}).$$

### *The purpose of faithful accounting*

Moving from QQ to QP is a natural and

acceptable voter action to protect a coalition, consistent with the aim of MMP, not just tolerated as being inside the law. With instrumental Erststimme, voters moving from PP to QP concentrate around candidates accepted across party borders. This is laudable: Concentration is one main purpose of the MMP-method, on level with proportionality.

With 2017 rules, ballot splitting causes “doubled influence” and increased assembly size because of an accounting defect. Faithful accounting avoids an unfortunate side effect and is likely to give ordinary assembly size (598). It should keep natural voter adaptation as a way to serve the dual goal of the election method.

**REMARK 4** A Majority method will strengthen the incentive for instrumental Erststimme in order to concentrate voters around candidates widely acceptable in the constituency.

### *Erststimme success in $\Lambda(P_8), \dots, \Lambda(P_p)$*

The number of ballots with Erststimme but

not Zweitstimme to one of the parties that passed the threshold is, by Table 1,

$$44966765 - (44190959 - g) = 775806 + g;$$

g ballots were split the other way (Zweitstimme but not Erststimme to the larger party). Most likely, very small parties get ideological Zweitstimme, while the ballot’s Erststimme may be pragmatic; in many constituencies, a small party does not even have a candidate.

In 2017, g is most likely to be insignificant; we set g=0. About 7/15 of these 775806 support a local winner:  $775806 \times 7/15 \approx 362043$ . With 70771 Erststimme behind each constituency winner, one should, on the new LOCAL commitment accounts for  $\Lambda(P_8), \dots, \Lambda(P_p)$ , expect about

$$362043 / 70771 \approx 5 \text{ constituency seat equivalents.}$$

### **3 Background and discussion**

***The single seat part*** Surveying parliament elections, Reynolds & al (2005) listed 72 systems of Proportionality based on party lists, and 91 of Plurality/Majority in single-seat constituencies; Plurality dominates. Besides 2-day elections (ordinary runoff), IRV (Instant Runoff Voting) may be the best known Majority system, practiced for 100 years in Australia.

Majority methods cost more money, time, and voter effort, so the dominance of Plurality is not surprising. With Plurality, instrumental voting is of central importance, as it usually results in a race between two major candidates and a winner with wide support, although not always from a majority. This is the Duvergerian mechanism.

Presidential elections are important enough to justify extra costs: The fact that 78 countries use a 2-day method (e.g. France) and only 22 use Plurality, indicates that winning with majority support is important for legitimacy. In MMP, the W-U method is a less expensive alternative.

***MMP and other mixed systems*** Only 30 systems are “mixed”. They may be of the parallel kind, where party seats and local single seats are won independently of each other. Also MMP allows voters to split the ballot, cast an instrumental vote in the local election and get a winner with wider support.

A third kind of mixed system is used in Hungary. It is a “one-vote contest”, not allowing split ballots: The concentration part is by Plurality in the constituencies, while “wasted” votes from the local elections are transferred to the Proportionality part of the tally, giving their voters higher influence there as compensation. Csato (2016) studies this transfer mechanism.

The first MMP-election was for the Bundestag in 1949, but the rules have changed several times. Regarding the German experience as positive, some other countries have introduced their own MMP-variations. Their experiences however, have not always been good (Linhart & al, 2019).

***MMP in OSCE/ODIHR*** The Organization for Security and Cooperation in Europe, through OSCE Office for Democratic Institutions and Human Rights, observes and reports on elections in the 57 member countries, after invitation from the government.

***OSCE and Albania 2005*** The election July 3<sup>rd</sup> 2005 was an MMP-variation for an assembly with 100 constituency seats and only 40 “supplemental” party seats. OSCE (2005) writes:

*... the objective of proportionality in the composition of parliament is hampered by a combination of four factors:*

- 1. The number of supplemental mandates is fixed rather than variable.*
- 2. The number of supplemental mandates is relatively small (40) and thus may not be sufficient to achieve proportionality.*
- 3. The impossibility of 'taking away' any of the single seats won by a party candidate; and*
- 4. The provision that the election is a two-ballot contest (Electoral Code, art 90).*

Other MMP-variations, proposed, tried, or presently used in some countries, have rules alluded to in Points 3 and 4. Points 1 and 2 should be seen in light of the Bundestag experience from 2017: With 299 local seats and 410 party seats to obtain proportionality, it is evident that 40 supplemental party seats are far from sufficient to accompany 100 local seats.

Moreover, suppose that ballot-splitting to support FDP had been popular among voters with Erststimme to CSU: If more of those 3255487 voters had joined the action, given Zweitstimme to e.g. FDP and cut down  $|\Lambda(\text{CSU})|$  by 75%, from 2869688 to 717422, then the assembly size had quadrupled, from 709 to  $2836 \approx 46 \times 44190959/717422$ . (Faithful accounting prevents it.)

OSCE's final report on Albania 2005 also explains explicitly how vote-splitting works:

*Using various methods, political parties can circumvent the aim of the election system and distort the allocation of supplementary mandates in their favour. For example, if supporters of a party (Party A) systematically split their votes between the candidate representing Party A and the election list of another, formally or informally allied party (Party B), then the combined number of mandates won by the two parties is likely to be increased. While this strategy might be within the law, it is problematic for a number of reasons: ... ..*

Mudambi and Navarra (2004) describe the same mechanism with Italian examples. No doubt the "strategy" of ballot splitting is done in order to increase a political camp's number of seats; it clearly works against the aim of MMP, but only because the law also does so. One must assume that the actionists considered the ballot splitting to be within the law, in Albania 2005 as in Germany 2017. In fact, the action is based on unequal influence created by law.

In an "Executive summary" of the final report, OSCE writes:

*While there is room for further improvement of the legal framework, the law is overall conducive for the conduct of democratic elections. However, the major political parties are yet to demonstrate political will and responsibility commensurate with the broad authority granted to them for the electoral process. This was particularly evident in the parties'*



*approach to the election administration, the full respect for citizens' fundamental freedoms, and the implementation of electoral strategies to maximise electoral gains."*

This is an unfortunate shift of responsibility from those who make the rules, to political parties who advise their supporters. The rules give voters, who support a local winner from a party without party seats, the option to exercise double influence by splitting the ballot. There were two such parties, a situation similar to the one before Sainte-Laguë dealt out CDU-185 in 2017. The splitting has no conspicuous consequences – when done to small extent. Done a bit more, it explains the 111 extra-ordinary Bundestag seats in 2017.

In the politically polarized society of Albania, two dominating parties with their supporters were confined in a “duel arena”, and by law armed with the option of ballot splitting and possibly doubled influence (if their local candidate won), to grab as many as possible of the 40 party seats by means of a small auxiliary party.

***OSCE and the Bundestag election 2017*** In the final report (OSCE 2017), four sentences from section IV, *Legal framework*, describe changes in German rules that were made after the Albania report. Footnotes from the report are here re-inserted in curly {}:

*Jurisprudence of the Federal Constitutional Court (FCC) has significantly shaped the electoral legal framework over the years. {In 2008 and 2012 the FCC ruled that elements of the electoral system such as the ‘negative voting weight’ and the number of overhang mandates respectively were unconstitutional. In 2012, it ruled on the requirements for the right to vote of Germans living abroad.}*

*The latest major legal reform took place ahead of the 2013 federal elections when significant changes were made to the electoral system. {The changes pertained to the method of allocation of seats and came into effect to address the disproportionality caused by components of the electoral system.}*

*The legal framework is robust and constitutes a solid basis for the conduct of genuine and democratic elections. However, some OSCE/ODIHR EET interlocutors expressed concerns that the electoral system, and specifically the compensation mechanism, could lead to an ever-expanding Bundestag, potentially affecting the efficiency of the legislative body. {The FEA provides that the Bundestag shall consist of 598 members, subject to variations, which, in practice, due to the electoral system, means that the actual number may be larger. The newly-elected Bundestag has 709 members.}*

With reference to the jurisprudence of the Federal Constitutional Court, and specifically to the ruling of 2008 (Bundesverfassungsgericht 2008), it should have been interesting with a more specific comment to para 92: *Aus dem Grundsatz der Wahlgleichheit folgt für das Wahlgesetz, dass die Stimme eines jeden Wahlberechtigten grundsätzlich den gleichen Zählwert und die gleiche rechtliche Erfolgschance haben muss. Alle Wähler sollen mit der Stimme, die sie abgeben, den gleichen Einfluss auf das Wahlergebnis haben.*

While certainly concerning legality, the formulation highlights legitimacy. In particular, how does this apply to voters A and B in *the micro-example* above? By Table 1, there are at least  $3255487 - 2869688 = 385799$  disjoint pairs like {A, B}, where one split ballot gives Zweitstimme to another party. B's action is legal, but so is now unequal influence, which violates the principle One-Person-One-Vote. Ballot splitting should, and can, be legal without harming legitimacy.

**Legitimacy of purpose** Properties of election methods tend to raise questions about legality and legitimacy, perhaps at the same time. Hettlage (2018) is concerned about the following aspect: *A seat goes to a politician who was not a candidate for it.* The candidacy is clear in a single-seat election, but more or less depersonalized when party lists contend for a known set of seats. However, the 111 extra-ordinary seats did not even exist before the Bundestag election 2017. The purpose of elections is to choose among candidates for positions that are known to exist; the notion of candidacy hinges on it.

In Hettlage's words: *Ohne Kandidat kein Mandat.* The more citizens think that extra-ordinary seats harm legitimacy, the more legitimacy is harmed. Whether extra-ordinary seats also raise legal questions, is perhaps too much of a "technical" issue for voting laypersons.

**Constituency size** The  $E(k)$  depend on constituency size, voter turnout, and election method. Modification of large  $1/E(k)$  avoids creation of extra-ordinary seats, but allows over-representation of constituencies with small  $E(k)$ . The  $E(k)$  are published, but the  $N(j,k)$  must come from tallies with faithful accounting. Regulation of constituency size will have an impact, and so will use of a Majority method and of mandatory participation (as in Australia).

**Concealment of symptoms** Excessive assembly size may be avoided by changing the ratio of local seats to party list seats. 2017-rules give the Bundestag a " $299+(299+x)$ " structure with  $x$  extraordinary seats. Suppose it is safe to assume  $x \leq 598$ , i.e. at most 897 party list seats.

With an official “ $299+(897+x)$ ” structure, Sainte-Laguë continues a *proportional* distribution of seats 710, 711, ..., 1196. The transition point at seat  $y$ , where the last constituency seat is “paid”, becomes invisible ( $y=709$ ; in general  $299 \leq y \leq 1196$ ).  $x > 0$  is just a theoretical possibility. Fusions of constituencies may scale the size down, e.g. to “ $150+(450+x)$ ”.  $x=0$  is a safe prediction.

***The concentration part of MMP*** The many split ballots in 2017 illustrate that many voters accept and deal with the basic theme of MMP, the balance between concentration and proportionality, between Erststimme for pragmatic compromise and Zweitstimme for political ideology. If the voters’ incentive to split the ballot instrumentally gets weaker, the electoral basis for local winners will be reduced.

Weinmann and Grotz (2020) consider reducing the ratio of local seats to ordinary party list seats while keeping voting and tally as they are now. How invasive into the functioning of MMP such a change will be, depends of course on how the term “invasive” is defined. If the concentration component in MMP is seen as important, a reduced ratio should indeed be seen as very invasive:

When it was difficult to establish a government with parliamentary basis on the left or on the right, then there was a solid Bundestag basis for a Grand Coalition. Representatives elected directly from their own constituency, with a broad electoral basis, have a special legitimacy when they support that solution. Even voters who support the local runner-up will usually consider the winner as a legitimate representative for the constituency. Arguably, democracy should work that way, and arguably, it does in Bundestag elections.

By an estimate above, 7 of 15 voters support the local winner, indicating that the concentration part of MMP is important to voters. The history of MMP starts in the late 1940s. Those who worked out the constitution and election system remembered the political fragmentation of the Reichstag in the Weimar time, under a proportionality rule alone.

***Simplicity and common understanding*** Voters’ understanding of their election method is a theme of Jankowski & al (2020) and Behnke (2015). It is central for MMP’s legitimacy. Without some understanding, an assembly size obviously out of control will, unfortunately and easily, be perceived as a sign that something must be wrong with the very aim of MMP.

***Explanation problems*** The price/accounting analogy should activate common knowledge and improve understanding of MMP. But explaining is easier if there is not an excessive assembly size that must also be explained. Starting with Table 1, how do we explain entries 0 and 46 in the commitment accounts? Obviously, the entries are in lines that inform about voter groups  $\Lambda(\text{FDP})$  and  $\Lambda(\text{CSU})$ , as they are identified by size. One expects, e.g., that the 0-entry must somehow be related to ballots from  $\Lambda(\text{FDP})$ . Unfortunately,

- 0 for FDP is as it would be if  $\Lambda(\text{FDP})$  had no success at all with their *Erststimme*;
- 46 for CSU is as it would be if  $\Lambda(\text{CSU})$  brought all the *Erststimme* CSU-winners got.

Obviously, 1.75 million voters in  $\Lambda(\text{FDP})$  gave *Erststimme* to other parties than FDP. Expected *Erststimme* success was their background for joining  $\Lambda(\text{FDP})$  with *Zweitstimme* success too.

Under faithful accounting, what  $\Lambda(P_j)$  obtains with its *Erststimme*, is *measured* in the local tallies and entered *into its commitment account* in the new LOCAL column. Data on *Erststimme* from each *Zweitstimme* group  $\Lambda(P_j)$  exist in the ballots, but are now ignored.

How to explain an election method to voters, politicians, courts, media, and among research colleagues, is often a matter of difficult problem solving. Didactic guidelines do exist, e.g. in “*How to Solve it*”, p.xvi: *Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?* (Polya 1945, 1957). Removing complicated allocation rules was essential, as seen in the application of 2017 rules to 2013 data. Consequences of moves from the pivotal  $\Lambda(\text{CSU})$  to  $\Lambda(\text{FDP})$  are motivation for this paper.

The distribution of constituency seats in Table 1 is typical for Plurality elections, with CDU/CSU and SPD dominating. It is influenced by differences in constituency size, and voter preferences usually have a regional component. Over-representation of some regions is sometimes intended; in MMP it may also be regulated through the number of seats elected from regional party lists.

## 4 Summary and conclusion

The proportionality approximation in the Zweitstimme tally is a blend of two methods. A sequence according to *Tribus values*  $a = c/[d/b]$  is d'Hondt's method (calculating the  $d/b$ -values);  $b$  is the number of seats currently allocated to party  $P$ ,  $d$  and  $c$  the number of Zweitstimme, respectively to  $P$  and to all qualified parties. The assembly size comes *directly* ( $a = 708.364154\dots$ ) and tells when to stop the Sainte-Laguë sequence (Table 2).

The sequence of Tribus-d'Hondt and the less intuitive one of Sainte-Laguë may give *very different* results in elections of a few candidates in a multi-seat constituency. With hundreds of seats, both sequences approximate proportionality well enough for practical purposes, and occasionally give the same result for each party (as e.g. in 2017).

In 2017, a high number of split ballots, with Erststimme to the pivotal CSU and Zweitstimme to FDP, occurred in a natural voter action to help political neighbor FDP across the threshold after failing it in 2013. The actionists happened to exploit the unfortunate *doubled influence*, which is described in the OSCE report from 2005.

The spike of 111 extra-ordinary seats in 2017 is a concomitant effect. Sister parties CDU and CSU work together in the Bundestag, but are entirely separate in elections. A fusion C\*U would have reduced the spike by 42; an *overhang coalition* CDU&CSU by 41.

Faithful accounting avoids doubled influence, and the assembly size would, quite likely, stay at 598 (ballot data are not available). Federal Elections Act states that the "*Bundestag shall, subject to variations resulting from this Act, consist of 598 members*". But the 111 member variation results from doubled influence, and (in part) also from a pivotal one-state party.

Collection of data from the 299 Erststimme tallies will give more information. So may simulation of different scenarios: They may have unusual features, but still be realistic and provide better background for understanding the rôle of Majority methods in the Erststimme elections. (In further analysis of collected or simulated data, a W-U idea may be adapted to the constituency landscape: When Erststimme is to  $P$ , not  $W$  or  $U$ , look *inside*  $\Delta(P)$ ; let  $r_Q$  be the fraction there of Erststimme to  $Q$ , and count  $r_W/[r_W+r_U]$  for  $W$ ,  $r_U/[r_W+r_U]$  for  $U$ .)

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