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The Profitability of the Reverse Mortgage

A Profitability Analysis of the Norwegian Reverse Mortgage From a Lender's Perspective

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NORWEGIAN SCHOOL OF ECONOMICS

Abstract

The Norwegian population is getting older, and future generations are expected to receive less pension than generations before them. However, many seniors have large savings tied up in home equity, which can be released through the reverse mortgage product. The reverse mortgage is a non-recourse loan, guaranteeing borrowers a lifelong residency without having to make interest or instalment payments. Interests are added to the loan balance, exposing lenders to risk of the loan exceeding the value of the mortgaged property. This risk is associated with potential costs for the lenders.

This study aims to analyse the profitability of the reverse mortgage product on the Norwegian market, and how the profitability is influenced by key parameters affecting the lenders' exposure to risk. An essential part of studying the product's profitability is to find the costs related to the embedded guarantee. For this purpose, we adopt a modified version of the Black-Scholes model and termination probabilities, using plausible input data for the Norwegian market. By calculating the potential income and costs related to a reverse mortgage loan, we obtain the lenders' expected day one profit. We perform various sensitivity analyses in order to study the loan's profitability in different scenarios. Our findings suggest that reverse mortgages are highly profitable, and that younger borrowers are the most profitable customer segment. The results further exhibit that large deviations from the baseline scenario must occur in order for the lenders to experience negative day one profits, which suggests that the lenders could increase the loan amount offered and decrease the current interest rate in order to expand the Norwegian reverse mortgage market.

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1. Introduction

The reverse mortgage product was introduced in the Norwegian mortgage market in 2005 and is commonly referred to as “seniorlån” or “LittExtra”. It is offered to people aged 60 and above and allows the borrower to release home equity. The loan is collateralized by the value of the borrower’s property, and the borrower is not obliged to make any interest or instalment payments during the duration of the loan; the interest is simply added to the loan balance. This product appeals to those who are “house rich, cash poor”; having large savings in their property but limited cash to live by. Many seniors experience that their income decreases when going into retirement, and the reverse mortgage loans can be a solution for seniors who would otherwise not be eligible for a loan to obtain more liquidity. All reverse mortgages in Norway are currently organised by the company LittExtra and issued through nine different commercial banks[1].

For the time being, the reverse mortgage market is a relatively small part of the Norwegian mortgage market. However, the product might become increasingly relevant as demographic developments induce implications for pension systems. Nobel laureate Robert C. Merton refers to the reverse mortgage as one of the helping hands in the transition to a population consisting of an ever-increasing number of elderly (Guerin, 2016).

The reverse mortgage loan is non-recourse, giving the borrower a lifelong right to live in the home without being liable for covering the costs if the loan exceeds the property value. This guarantee is commonly referred to as the “no-negative equity guarantee”, hereinafter referred to as the NNEG. Offering this guarantee exposes the lenders to a so-called “crossover risk”, which is the risk of the loan exceeding the property value. This can trigger future income loss for the lender, which affects the product’s profitability.

The reverse mortgages are known to have high interest rates on the Norwegian market, and several of the lenders state that the rate is high in order to provide the guarantee of lifelong right to live in the home without having to make interest or instalment payments (BN Bank, n.d.-a; KLP, n.d.-c; Sparebanken Vest, n.d.-b). However, one of the Norwegian lenders, BN

[1] In the Norwegian market, the reverse mortgage product is offered by the following banks: Bien Sparebank, BN Bank, Fornebu Sparebank, Jbf bank og forsikring, KLP, Lillestrømbanken, OBOS-banken, Sparebanken Vest and Strømmen Sparebank.

Bank, reported in 2013 that they had yet to experience any loss related to this product since they started offering it in 2007 (Öberg, 2013). The initial loan amounts offered are substantially lower than the value of the mortgaged property, which raises the question of whether the high rates are justified by the crossover risk associated with providing the NNEG, or whether the product is actually quite profitable.

The reverse mortgage is, to our knowledge, untouched by Norwegian researchers prior to this study. For that reason, we seek to extend on previous international literature on the subject by assessing the Norwegian product. The objective of this master's thesis is to examine the reverse mortgage product's profitability through a series of sensitivity analyses. We will examine how changes in key parameters, which determine the value of the NNEG, affect the lenders' expected profits. This study seeks to answer the following research question:

To what extent is the Norwegian reverse mortgage product profitable for the lenders, and how is the profitability influenced by changes in key parameters affecting the no-negative equity guarantee?

To answer the research question, we will price the NNEG as a series of European put options, applying a modified version of the Black-Scholes model (Black & Scholes, 1973; Merton, 1973) for dividend-paying assets with plausible input data for the Norwegian market. There are many ways to price the NNEG and analyse the reverse mortgage product's profitability. Our study will provide a proposal of how this can be done and aims to create a foundation for further research in the Norwegian market.

1.1 Existing Research

There has been conducted theoretical and empirical research on reverse mortgage loans internationally. Many have had the objective to value the NNEG, applying various approaches for this purpose. In addition, several researchers have conducted risk and profitability analyses of the reverse mortgage product. In this section, we will provide an overview of the research that has proved the most important for our choice of valuation methods and understanding of the dynamics behind the product's profitability.

A common approach for valuing the NNEG is to apply a version of the Black-Scholes model. Ji et al. (2012) mainly focused on modelling reverse mortgage terminations by using a semi-

Markov multiple state model[2], and applied the model to simulate NNEG prices in the UK and insurance premiums in the US. The NNEG prices in the UK were simulated using a modified version of the Black-Scholes formula, adjusted for dividends. Furthermore, Dowd et al. (2019) deemed another version of the Black-Scholes model, the Black '76 model[3], as the natural choice for NNEG valuation. In addition, they claimed to be the first to value equity release mortgages (ERMs) in the UK, which is the British reverse mortgage product. By conducting sensitivity analyses, Dowd et al. (2019) found that NNEG valuations are more subject to mortality model risk than ERM valuations. Additionally, they found that the value of the ERM relative to NNEG is more robust to changes in other key input parameters. Moreover, the Black-Scholes formula has also been used to value reverse mortgages in South Korea. Choi et al. (2020) explored the option value of reverse mortgages in South Korea from the borrower's perspective, where the borrower's payoff was described as a long straddle[4]. By conducting sensitivity analyses of key variables, they got results that were in accordance with economic rationales of the option pricing model.

In February 2019, the Actuarial Research Centre in the UK published a report from Tunaru and Quaye concerning the NNEG valuation from the lender's view. The report argued that using the Black '76 model is not theoretically appropriate for the use of NNEG valuations, and also outlined the issues related to valuing the NNEG using other methods. Tunaru and Quaye (2019) offered an approach using risk neutral techniques to value the NNEG that is free of the reliance on option pricing. In 2010, Li et al. applied a risk-neutral approach to value the NNEG for the UK market, similar to the proposed method later set out by Tunaru and Quaye (2019). They applied a statistical model in order to model conditional variance of the house price returns, while stochastic mortality models were used to model the uncertainty to the time of termination. In addition to deriving a pricing formula, they performed sensitivity analyses to assess the cost of the NNEG under different scenarios and found that the NNEG can be a

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[2] A multiple state Markov-model is a stochastic model used to model the probabilities of different states and the rates of transitions among them. A semi-Markov multiple state model means that the transition probability depends on the time since the previous transition, and not solely on the current state and time. For more information of its use in reverse mortgage research, see Ji et al. (2012).

[3] The Black '76 model is a variant of the Black-Scholes formula, referring to pricing European options on futures. See Appendix 2 for similarities to the standard Black-Scholes formula and Black-Scholes formula for dividend-paying assets in the case of NNEG valuation.

[4] A long straddle strategy is a long position in both the European call and put option.

significant financial burden for the lender on the basis of historical price returns. Furthermore, Chen et al. (2010) made risk assessments for the leading reverse mortgage programme in the US, where the house price index was modelled through a risk neutral approach and found the product to be sustainable.

Cho et al. (2013) valued the NNEG for the Australian reverse mortgage market, extending the research by Alai et al. (2013). They analysed and compared the lender's risk and profitability of reverse mortgages with two different payout designs; lump sum and income stream payments, while Alai et al. (2013) compared reverse mortgages to home reversion contracts with the same perspective. Instead of pricing the NNEG using the Black-Scholes formula, both studies used risk-adjusted stochastic discount factors and conducted a set of sensitivity analyses to show how key drivers impact a lender's financial position. The sensitivity analysis set out by Alai et al. (2013) confirmed that the borrower's age had a significant impact on payoffs and risks for providers of equity release products. Cho et al. (2013) found that lump sum reverse mortgages were the most profitable and the least risky from the lender's perspective, and that the loan-to-value ratio, borrower's age and mortality improvements were important drivers of lender's risk and profitability.

In addition to academic literature, a series of regulatory documents has been set out by the UK Prudential Regulation Authority (PRA)[5]. The PRA has provided details of expectations in respect to how firms investing in ERMs should assess the risk associated with the NNEG sufficiently and has recommended using the Black-Scholes option pricing model for this purpose (PRA, 2020, p. 12).

[5] The PRA is a part of The Bank of England and is responsible for the regulation and supervision of financial services at the level of the individual firm (Bank of England, n.d.).

1.2 Approach of the Study

This study will conduct a profitability analysis of the reverse mortgage product on the Norwegian market and investigate how the profitability is affected by changes in key parameters. The analysis will be conducted through a series of calculations and sensitivity analyses, mainly inspired by the works of Ji et al. (2012) and Buckner and Dowd (2019). We construct our own tables and figures for the analysis using Microsoft Excel.

Our approach used for answering the research question is firstly to present the characteristics of the reverse mortgage and its importance in light of demographic changes in Norway. In order to understand the dynamics of the NNEG, we will also examine the risks associated with this guarantee. Furthermore, for finding the product's profitability, three components are necessary: an option pricing model, termination probabilities and a profitability measure. The option pricing model is used to price the NNEG, as it is reminiscent of a series of European put options. Since the termination date is arbitrary, each put option is weighed based on the probability of termination. In order to price the NNEG, we apply a modified version of the Black-Scholes model for dividend-paying assets, along with termination probabilities. The profitability measure is further found by subtracting the value of the NNEG and the initial loan amount from the potential income of issuing a reverse mortgage, and is referred to as the "day one profit". The methodology is presented in more detail in Chapter 4. Our input data used for the Black-Scholes model and the termination probabilities, along with its plausibility, will be accounted for before conducting the profitability analysis.

1.3 Disposition

This paper is further structured as follows. Chapter 2 will present the characteristics of the reverse mortgage product, both in Norway and internationally, and consider the product's importance in light of demographic changes. Next, Chapter 3 will examine the crossover risk associated with issuing this product. Furthermore, our methodology is presented in Chapter 4. Chapter 5 will thereafter present the input data necessary for analysing the Norwegian reverse mortgage product. Then, Chapter 6 will analyse the product's profitability through various sensitivity analyses and discuss the results. Finally, Chapter 7 summarizes the most important findings and provides a conclusion, along with suggestions for further research.

2. Presentation of the Reverse Mortgage

The Norwegian population is getting older, and the proportion of elderly is expected to increase rapidly over the coming decades. According to population projections by Statistics Norway (2020e), the proportion of retirees in the population will increase from its current 15% to around 25% by 2050, given the normal retirement age of 67, as presented in Figure 1. These changes are expected to put a greater burden on the work force to finance the elderly, and reverse mortgages can therefore be a good contribution to overcome these demographic challenges.

Figure 1: *The Proportion of Citizens Aged 67 and Above*

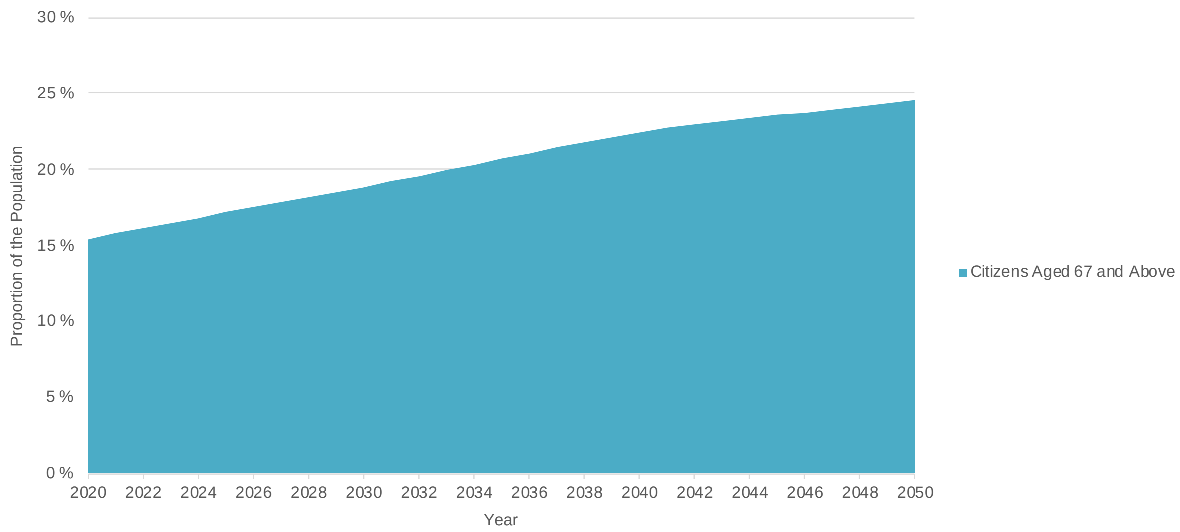

Figure 1. The projected proportion of citizens aged 67 and above in Norway from 2020 to 2050. Source: Statistics Norway (2020e). Authors' calculations.

In this chapter, we will introduce the concept of reverse mortgages by presenting the Norwegian product and accounting for its importance in relation to the demographical changes. In addition, we will present the most relevant particularities for similar products internationally.

2.1 The Reverse Mortgage in Norway

2.1.1 Contract Design

The interest rates of the Norwegian reverse mortgages, hereinafter referred to as the contract rate, are variable (Nyhus, 2019). These loans are instalment-free, and the interests are added to the loan balance as long as the borrower or longest-living spouse remains in the home. Even without making interest payments, the accumulated interest gives the borrower tax deduction (LittExtra, n.d.-b). Furthermore, the property is used as collateral while the borrower retains ownership of the home throughout the duration of the loan. At termination, the borrower or heirs can keep the property in exchange for repaying the loan.

The reverse mortgages differ from traditional mortgages in many aspects. As its name implies, in reverse to making required monthly payments to the lender, the lender makes one or several payments to the borrower. Furthermore, traditional mortgages require the borrower to have a certain credit quality, while the reverse mortgage has no such requirements (LittExtra, n.d.-b). In addition, the reverse mortgages are limited to a specific customer group, namely those aged 60 and above with substantial home equity. Moreover, all Norwegian reverse mortgage loans provide the borrower with the NNEG (LittExtra, n.d.-b). This guarantee makes the reverse mortgages riskier than other mortgage products, as the lender may not seize the collateral before the termination of the loan.

The reverse mortgage has different payment structures, varying between countries and lenders. In Norway, the loan amount is disbursed in one of three ways: as a lump sum payment at the beginning of the contract, as monthly income stream payments, or as a combination of the two (LittExtra, n.d.-b). How the interest accumulates will depend on which payment structure the borrower chooses. An illustrative example of this is presented in Figure 2.

Figure 2: *Lump Sum and Income Stream Reverse Mortgages*

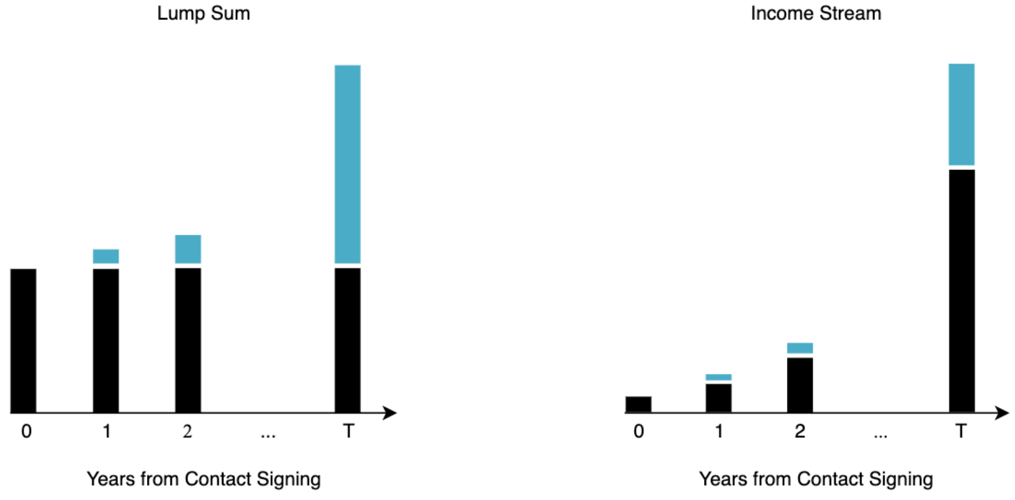

Figure 2. An illustrative example of the loan balance's development from contract signing to termination for a lump sum and an income stream reverse mortgage. The loan principal is given in black, while the accumulated interest is given in blue. Authors' illustration.

Our profitability analysis will be based on the lump sum reverse mortgage, as most research we have examined on the subject focuses on this payment structure. Also, in order to limit our study, we leave analysing other payment structures to further research. Moreover, we assume that no instalment or interest payments are made during the loan's time frame, as such payments would be arbitrary and challenging to incorporate.

2.1.2 Determinants of the Lump Sum

There are four factors determining the lump sum issued by the lender: the borrower's age, the housing market, whether the loan is given to one or two borrowers and whether the borrower has joint debt in a housing cooperative (LittExtra, n.d.-a). Age is an important indicator for life expectancy and will affect the duration of the loan. For that reason, younger borrowers will receive smaller lump sums. However, the lump sums do not increase for borrowers aged 80 and above. Furthermore, the development in the housing market will determine whether the loan might exceed the property value. Therefore, the lenders adjust the lump sums based on what extent the borrower's property is considered to be in a location where properties are easily marketable in a normally functioning housing market. Some lenders do not offer loans to certain areas at all. The lump sum is also dependent on whether the loan is issued to a single borrower or a couple, as the loan is not terminated until the longest-living spouse permanently exits the property. In the case of a couple, the lump sum is therefore slightly reduced if the borrowers are of similar ages. Lastly, if the property already serves as collateral, the existing mortgage on the property must be repaid, which can be done with the proceeds from the reverse mortgage lump sum. This will thus not affect the lump sum distributed by the lender. Joint debt on housing cooperatives, however, will decrease the lump sum issued.

How the above factors contribute to the lump sum is somewhat dependent on the issuer. All the Norwegian lenders have mortgage calculators for the reverse mortgage on their websites. By applying these calculators on the following scenario: a single borrower living in an urban area in a property worth NOK 2 000 000 without joint debt related to a housing cooperative, we find that the lump sums offered are similar, but not identical. Approximately half of the lenders offer loan-to-value ratios[6] varying from 22% to a 60-year-old to 44% to an 80-year-old (BN Bank, n.d.-c; Lillestrømbanken, n.d.; OBOS-banken, n.d.-b and Strømmen Sparebank, n.d.-a). The other half offer ratios varying from 23.5% to 47% for the same age gap (Bien Sparebank, n.d.-b; KLP, n.d.-a; Fornebu Sparebank, n.d.-a; Jbf bank og forsikring, n.d-a; Sparebanken Vest, n.d.-b).

2.1.3 Pension Scheme Developments

Simplified, the Norwegian government is facing three choices on how to meet the demographic challenges related to a population consisting of more elderly: increase tax revenue, reduce pension payments or change priorities in the national budget (Dahl, E. 2010, p. 46). Previous pension reforms have involved a combination of the above, which means that retirees are expected to receive less than the generations before them. Due to the demographic challenges, the same effects should also be expected going forward, which could contribute to an increased demand for alternative ways to fund retirement.

The Norwegian pension system is divided into three parts: retirement pension from the National Insurance Scheme, retirement pension from employers, and private pension savings (NAV, 2015). The retirement pension from the National Insurance Scheme is the basic pension scheme and ensures benefits for everyone, which in simplified terms are financed by current taxes. Furthermore, employees receive occupational pensions, which are pension schemes established for participants in working life. The occupational pension in the private sector is

[6] When using the term loan-to-value ratio, we refer to the ratio between the lump sum and the appraised value of the property at contract signing.

mainly in the nature of defined benefit (DB) or defined contribution (DC)[7]. Today, most employees in the private sector are covered by DC schemes. As retirement pension from DC schemes depend on pension fund returns, most employees are faced with uncertainties in relation to their future pension payments.

According to the European Commission's (2012, p. 127) Aging Report from 2012, it is possible to assess what effect pension reforms will have in terms of future pension adequacy by considering two indicators: the benefit ratio and the replacement rate[8]. The most recent projections of these indicators can be found in the Aging Report from 2018. In Norway, the public pension replacement rate was projected to decline by 10% between 2016 and 2070 (European Commission, 2018, p. 84), meaning that the ratio between pensions and average wages is declining. In addition, pensions were projected to increase less than annual wages, illustrated by the benefit ratio being projected to decrease by 14.9% between 2016 and 2070 (European Commission, 2018, p. 82). This will intensify the imbalance between pensions and wages, and reveals that the public pension systems will, to a lesser extent, be adequate for future retirees.

In light of the projected demographic trends, pension schemes are under pressure and pension benefits will inevitably be reduced in the coming years. The responsibility to fund retirement has shifted more towards the individual and it is necessary to find other alternatives to fund retirement in order to tackle the negative consequences retirees will face due to less generous pensions. One alternative is the reverse mortgage. A major benefit with this product is that it does not change people’s saving behaviour over the life cycle, as people are already using their properties as a way of saving.

Nobel laureate Professor Robert C. Merton visited BI Norwegian Business School in 2015, holding a lecture about the global challenge of funding retirement. He proposed a well-designed reverse mortgage as a partial solution to the global retirement issue and said that “the purpose of a reverse mortgage is to extract the value of the house when we no longer need it,

[7] DB schemes are guaranteed by the employer as a share of the final salary and are therefore predictable for the employee. In DC schemes, however, the employer pays a fixed percentage of the salary each year to a pension account. According to a report published by Statistics Norway (2019a, p. 10), DB schemes are lifelong, while DC schemes end at age 77.

[8] The public pension benefit ratio is the average pensions in relation to average wages, while the replacement rates are measured as the very first pension benefit relative to the last wage before retirement (European Commission, 2018, p. 77 & 83).

when it becomes a financial asset, and move it back in time to be used to help provide the benefits in retirement when we do need it" (BI, 2015, 20:45). The reverse mortgage can be a partial solution to fund retirement also in Norway, because home ownership represents a main source of personal savings for a majority of the population. In 2019, 90.3% of people aged 67-79 and 80.1% of those aged 80 or above lived in owner-occupied households (Statistics Norway, 2020c). In 2018, the primary residence as a share of gross wealth was 54% for households in which the main income earner was aged between 67 and 79, and 61.1% for households in which the main income earner was aged 80 or above (Statistics Norway, 2019c). As housing wealth is such a large wealth component for the Norwegian population, reverse mortgages have the potential to become an essential part of funding retirement in the years to come. It is therefore fairly reasonable to assume that more financial institutions will supply this product in the future.

2.1.4 Alternatives to the Reverse Mortgage

Although reverse mortgages can be a good contribution to an impaired financial situation at retirement, there are also other options available. Some of the main options are to sell, extend the repayment period of an existing mortgage or get a home equity line of credit (HELOC).

The reverse mortgage is reminiscent of a HELOC, which goes under the names "rammelån", "fleksilån" and "boligkreditt" in the Norwegian market. It allows the homeowner to borrow against the equity on their property, up until a certain credit limit. The similarities between the reverse mortgages and the HELOCs are mainly that the borrower may turn home equity into cash, is not obliged to pay instalments and may use the proceeds freely. However, there are also some major differences. HELOCs have lower interest rates than reverse mortgages. In addition, HELOCs require the borrower to make interest payments and are dependent on the borrower's credit quality. Reverse mortgages do not make such requirements. Furthermore, the reverse mortgage borrowers are protected by the NNEG, while borrowers of a HELOC carry the risks associated with changes in interest rates, property value and their ability to make necessary payments. Lastly, the Norwegian HELOC usually offer a credit limit of 60%, the maximum limit permitted by the mortgage regulations[9] (Boliglånsforskriften, 2019, §5),

[9] The term "mortgage regulations" refers to "boliglånsforskriften".

which is significantly higher than the loan-to-value ratios offered through the reverse mortgage product.

Due to the seniors' impaired financial situation, these three alternatives are somewhat risky and incomparable to the reverse mortgage product. As the reverse mortgage does not have any credit requirements, it is accessible for those who would not be able to make monthly rent or interest payments. In addition, selling might not be an option due to home attachment. It is therefore reasonable to assume that reverse mortgages can play an important role for seniors in the years to come.

2.1.5 Regulations and Exemptions for Capital Release Credits

In the continuation of the mortgage regulations by the Ministry of Finance in 2018, exemptions were added for so-called "capital release credits"[10], which includes the reverse mortgage product. The exemptions are stipulated by the Ministry of Finance, emerging from a consultation memorandum, which emphasises that banks bear the risk of the loan exceeding the value of the home upon relocation or death (Finanstilsynet, 2018). Capital release credits are exempt from the regulations of requirements for financial capability, debt ratio, instalments and flexibility (Boliglånsforskriften, 2019, §1). These exemptions imply that borrower's income does not affect the loan application, and that capital release credits must not be included in the value of granted loans when lenders calculate the flexibility quota, which concerns how many loans financial institutions can grant that contravene with certain requirements in the regulation.

The consultation memorandum also states that capital release credits should be covered by the mortgage regulations' requirements for a maximum loan-to-value ratio. However, the assessment of the maximum loan-to-value ratio differs from other loans secured by housing. It is to be understood as the expected loan-to-value ratio at the time of realization of the mortgage instead of when the loan is granted, given a reasonable assessment of life expectancy, future house price development and expected interest rate level (Finanstilsynet, 2018, p. 31).

[10] In Norwegian: Kapitalfrigjøringskreditter

2.2 The Reverse Mortgage Internationally

Similar products to the Norwegian reverse mortgage are offered around the world including the United States, the United Kingdom, Australia, Canada, South Korea and Japan, to name a few. The following section presents the most relevant particularities of these products from a few selected countries, and how they differ from the Norwegian reverse mortgage product.

2.2.1 The United States

The primary reverse mortgage product in the US is the Home Equity Conversion Mortgage (HECM), which was introduced by the Department of Housing and Urban Development in 1988. According to Shan (2011), the product covers over 90% of reverse mortgage products in the country. HECM loans are insured by the sub-agency Federal Housing Administration's insurance programme within the Department of Housing and Urban Development. The programme insures the borrower against the risk of the lender defaulting and being unable to pay upcoming contracted payments, in addition to insuring the lender against the risk of the loan amount exceeding the property value.

HECMs require the borrower to have a minimum age of 62 years and are offered with both variable and fixed interest rates. When choosing a fixed interest rate, the borrower is restricted to receiving a lump sum, whilst payment plans for HECMs with variable interest rate are more flexible[11] (Department of Housing and Urban Development, n.d.-b). According to Baily et al. (2019), 91% of borrowers took out variable-rate draws in 2018. The principal limit factor is what we refer to as the loan-to-value ratio and is decided by the age of the borrower and the expected interest rate on the loan. For an expected interest rate of 4%, the principal limit factor ranges from 0.47 to 0.75 for the ages of 62 to 99 (Department of Housing and Urban Development, n.d.-a).

2.2.2 Australia

Reverse mortgages were introduced on the Australian market in the early 1990's, under this name. During the financial crisis of 2008, the number of lenders drastically decreased, mainly

[11] The following payment plans are available for variable interest rate HECMs: Tenure, Term, Line of Credit, Modified Tenure and Modified Term.

because non-bank lenders relied upon funding from collapsing capital markets (Seniors First, 2019). To our understanding, there are only three small lenders left on the Australian market. By examining their websites, we find that the minimum age of the borrower ranges from 60 to 70 years, whilst the loan-to-value ratio ranges from 15% for 60-year-olds to 50% for 90-year-olds and above (Heartland Seniors Finance, 2020; IMB Bank, n.d.; P&N Bank, n.d.). Furthermore, the lenders offer reverse mortgages with variable interest rates and an NNEG, as is done in Norway.

2.2.3 The United Kingdom

In the UK, reverse mortgages are referred to as equity release mortgages (ERMs) or Lifetime Mortgages and were first introduced in the mid to late 1980s. Products by members of the Equity Release Council, a self-regulatory industry association representing the interest of 90% of the equity release sector, must include the NNEG (Equity Release Council, n.d.; Equity Release Council, 2020). According to the Equity Release Council's (2020) market report on ERMs, the average maximum loan-to-value on ERMs vary from 18.8% for 55-year-olds to 49% for 90-year-olds, where interest rates are either fixed or variable. The variable interest rate ERMs have an upper limit which is fixed for the life of the loan.

The UK has the largest reverse mortgage market in Europe and accounted for three-quarters of the European business in 2009 (European Mortgage Federation, 2009). There is a high variation in the supply of different types of reverse mortgage products across European countries. Overall, the reverse mortgage business is relatively small and accounted for less than 1% of the total mortgage business across EU members in 2009 (European Mortgage Federation, 2009).

Comparing reverse mortgages in Norway with the countries above, displays clear similarities as well as significant differences. As an example, all countries include the NNEG, but in the US the lender is also insured by the Federal Housing Administration's insurance program. Furthermore, all reverse mortgages in Norway have variable interest rates, while other countries offer both variable and fixed interest rates. Moreover, it seems that Australia and the UK offer similar loan-to-value ratios as Norway, whereas the loan-to-value ratios are significantly higher in the US.

3. The Crossover Risk

Since the interest is accumulating on the loan balance, the loan grows swiftly. Due to the NNEG, the reverse mortgage lenders will not receive more than the value of the collateral. This poses the lenders to risk of receiving a lower cash flow than anticipated, namely the crossover risk.

To provide an illustrative example of the crossover risk, we assume the initial property value to be NOK 2 000 000 and the loan-to-value ratio to be 40%, which is illustrated in Figure 3. Given the hypothetical path of the property value and the roll-up reverse mortgage, the crossover will occur in approximately 38 years. If the loan terminates after that point, lenders do not receive a full repayment. If the loan-to-value ratio is higher, for example 50%, the crossover will occur earlier, namely after 29 years, as also illustrated in Figure 3.

Figure 3: *Illustration of Crossover Risk*

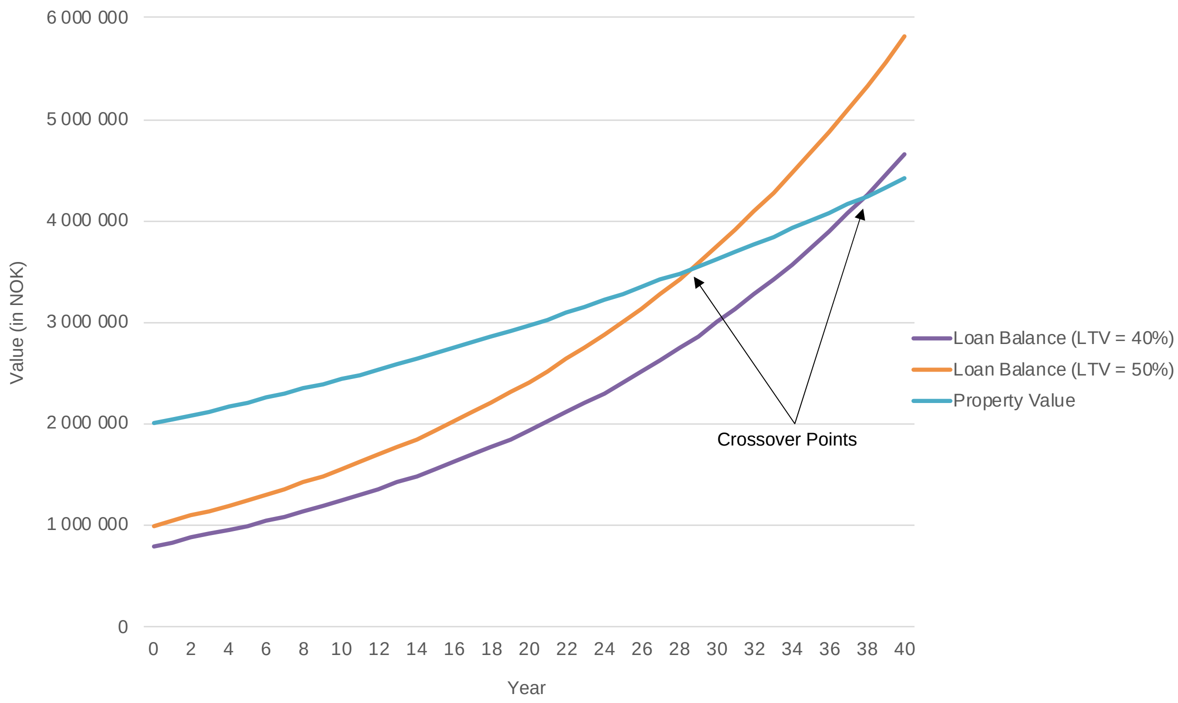

Figure 3. An illustration of crossover risk for two reverse mortgages with loan-to-value ratios of 40% and 50%, given a hypothetical growth in the property value. Authors' illustration.

There are mainly three factors that enforce the crossover risk: the interest rate, the house price and the termination date. The compounding contract rate makes the loan grow forcefully and possibly exceed the property value. If the property value diminishes or grows at a lower rate than the contract rate, there is more chance of this happening. In addition, the termination date determines when the loan is settled. The longer the duration of the loan, the more chance there is that the loan outgrows the property value. In other words, there is a lot of uncertainty associated with these three factors, and a miscalculation can have dire consequences. However, some margin of error is inevitable as they are dependent on future events. These three factors will be examined in more detail in this chapter.

3.1 Interest Rate Risk

The accumulated interest grows at a considerable rate. When market rates rise, variable contract rates might become higher than anticipated, which increases the possibility of a crossover. Contrarily, for reverse mortgages with a fixed interest rate, there will not be any surprises in the accumulation of the rate over time. Furthermore, the accumulated interest presents an interesting paradox. Reverse mortgages have higher interest rates than other mortgage loans in order to compensate for the crossover risk, simultaneously as the contract rate increases the crossover risk. In other words, the lenders need to find a rate that balances the two.

In order to issue reverse mortgage loans, the lenders need funding. According to the European Banking Federation (Cook, 2019), the lenders' most important sources of funding are deposits and covered bonds. These come with a cost. If the reverse mortgage has a fixed interest rate, and the lenders' financing cost does not, the lenders are exposed to the risk of the variable rate varying from the fixed rate and even surpassing it. This may not induce a crossover, but it will affect the lenders financially. However, such a position can easily be hedged through swapping the variable interest rate for a fixed one. In Norway the loans' interest rates are variable, so this will not be of great consequence.

3.2 House Price Risk

Lenders of reverse mortgages are subject to house price risk; the risk that the sales proceeds at termination is less than anticipated. If house prices grow at a lower rate than anticipated or even declines, the net liquidation value of the property becomes less than expected. This will lead to the lenders experiencing lost income, which is exhibited in Figure 4. If the reverse mortgage contract terminates after 30 years, the lost income is given by the gap within the brackets, which is the difference between the outstanding loan balance and the property value in the year of loan termination. If the house price declines at a higher rate, the lender's lost income will be even greater.

Figure 4: *Illustration of the Impact of House Price Depreciation*

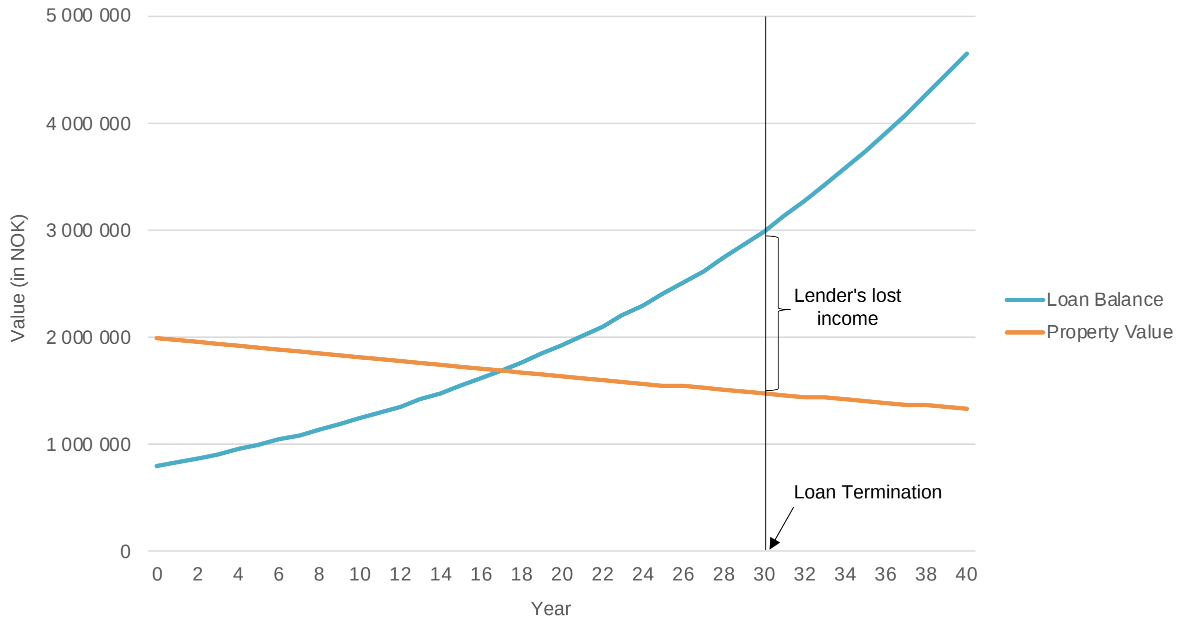

Figure 4. An illustration of how house price depreciation may impact the lender's income due to the NNEG.

3.2.1 External Risk Factors

House price returns tend to exhibit positive autocorrelation, meaning that house price movements have a strong tendency to follow previous movements in the same direction. Case and Shiller (1989) showed that the market for owner-occupied housing is far from efficient and pointed out that prices do not follow a random walk, but rather predictable patterns. It is not surprising that autocorrelation is found in this market, because a key part of the valuation is to refer to comparable houses. Therefore, the presence of positive autocorrelation will likely increase the lenders' income loss under scenarios where the house price growth is lower than the lenders' expectations.

There has been a clear upward trend in the housing market, which is revealed when comparing property price time series for four regions: Oslo including Bærum, Trondheim, Stavanger and Bergen and the national average. The times series are retrieved from Statistics Norway (2020g) and are presented in Figure 5. They are all non-seasonally adjusted quarterly time series. The period examined is from the first quarter of 2005 to the second quarter of 2020, which was the maximum period available for all regional indices. Keep in mind that these observations are house price indices and might not fully explain house price developments for individual houses.

Figure 5: *National and Regional House Price Index*

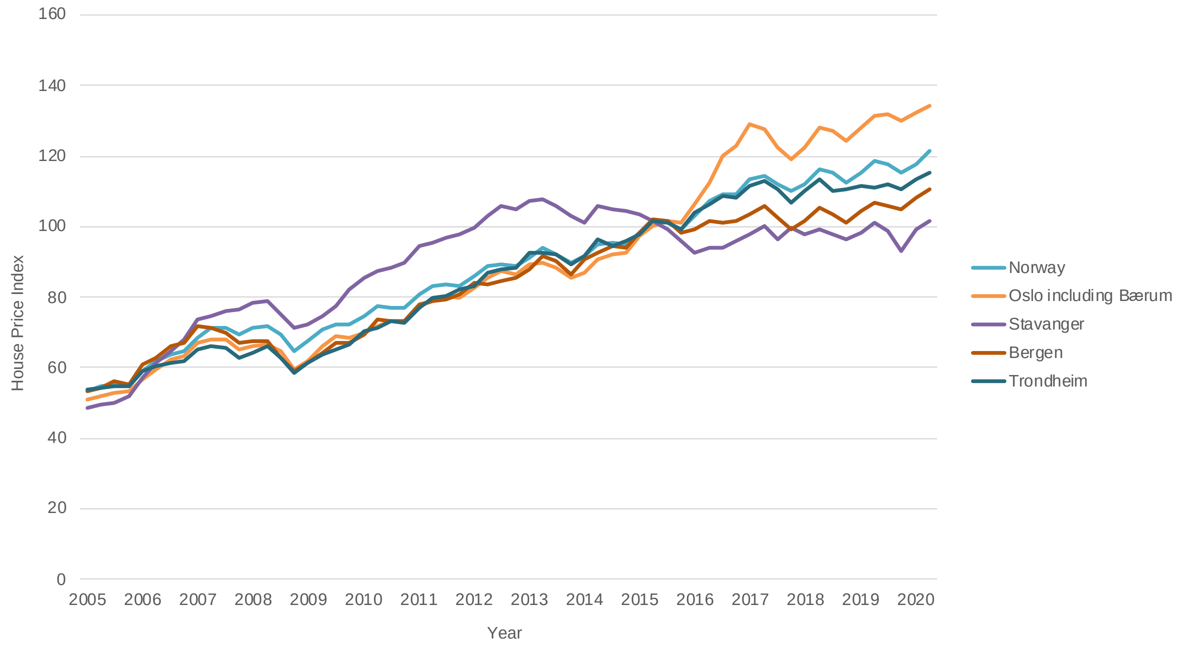

Figure 5. The house price index on a national level and for selected regions. (2015=100). Source: Statistics Norway (2020g).

The housing market is unpredictable. Before 2007, few anticipated that the financial crisis of 2007-2008 was right around the corner, which gave a temporary fall in house prices at the turn of the year 2008-2009. This affected house prices on a national level. There are also regional differences. As an example, when the oil price collapsed in 2014, house prices fell in the city of Stavanger, where the petroleum industry is essential. As we can observe from Figure 5, the housing market in Stavanger has recovered somewhat since but has still not reached old heights. Similar price collapses and financial crises may occur in the future. In other words, there is no guarantee that house prices will continue to follow historical developments. Also, we do not know the long-term effects of the ongoing Covid-19 pandemic on the housing market, where prices are driven by factors such as unemployment, mortgage rates and economic growth. Such external factors can affect house prices nationally, regionally and locally, and make the lenders highly exposed to house price depreciation risk.

3.2.2 Internal Risk Factors

In addition to external factors, lenders are also faced with internal factors. Such an internal factor could be maintenance risk, which is subject to borrowers' influence. To our knowledge, reverse mortgage lenders in Norway do not impose any maintenance requirements. The lenders would naturally prefer that the borrowers invest in maintenance to upkeep the property value, but it would be difficult to standardize such requirements and monitor them properly.

Young homeowners usually have a long-term interest in maintaining and renovating their property, either for their own use or in order to obtain a higher sales value. It is likely that this incentive is not as significant for elderly homeowners who are planning to live in their properties for the remaining part of their lives. Especially when it comes to renovation, elderly are not known to follow the newest trends and technology. This will have a negative effect on the property value over time. Even if an elderly homeowner wishes to conduct maintenance and renovation, it might become tiresome and difficult in the later years of life. Also, reverse mortgage borrowers tend to be cash poor and thus have less funds for maintenance and renovation.

3.2.3 Risk of Incorrect Initial Valuation

The true value of a property will only reveal itself when sold, as it depends on what people are actually willing to pay for it. When the reverse mortgage contract is signed, however, it is valued by an appraiser. No matter how skilled the appraiser might be, there will always be some margin of error in their valuation. In other words, the true market value upon contract signing will be unknown. Lenders might then give a higher lump sum than intended, and thereby be exposed to a greater risk than anticipated.

3.3 Termination Risk

Due to the reverse mortgage being repaid at termination, the termination date becomes essential in order to assess the crossover risk. The challenge is that the termination date is arbitrary and might occur for different reasons. According to Ji et al. (2012, p. 239), the main modes for termination are mortality, entrance into a long-term care facility, moving out for non-health related issues and refinancing. In order to assess the value of the reverse mortgages, the lenders need to find good estimates of the probabilities for termination at different points in time. However, even the best calculations will never fully reflect reality, and lenders are thus faced with uncertainty related to the termination date. This may lead to lenders experiencing income loss.

Out of the four modes of termination proposed, there are only three modes that pose risk for the lenders. Refinancing is not correlated with any income loss, as the borrower will need to repay the loan in full in order to keep the mortgaged property.

The termination date is crucial because it affects both the interest rate risk and the house price risk. In order for the lenders to experience income loss related to the embedded NNEG, the loan has to exceed the property value. The loan grows swiftly due to the interest rate being compounded, and the longer the loan grows, the more likely it is to exceed the property value. It is important to note, however, that a prolonged duration of the loan might also be beneficial to the lenders through the accumulated interest. Furthermore, predicting the evolution of the property value over time is demanding, and with an uncertain termination date, it is even more so. In addition, unless the borrower performs constant maintenance and renovation during the duration of the loan, the property is expected to decay over time.

3.3.1 Longevity Risk

Longevity risk, also referred to as mortality risk, is related to the borrower living longer than expected. This risk is evidently associated with the termination mode mortality. In addition, it might also be associated with the borrower’s entrance into a long-term care facility. In Norway, most people who move into a nursing home remain there until their passing, for whom the average time of living in the nursing home is two years (The Norwegian Directorate of Health, 2017). In other words, the time of moving into a long-term care facility is on average relatively close to the time of death, and it is reasonable to assume that borrowers with low life expectancy will need such care sooner than borrowers with a high life expectancy.

According to a report by Zhai (2000, p. 7 & 8), there are several factors that affect the longevity risk, such as age, gender, location, demographics, health-care conditions, medical history, race, personal habits and social and economic factors. Additionally, the following mortality tendencies are expected among the reverse mortgage borrowers; (1) the borrowers are self-selected and tend to live longer than the general population, (2) female borrowers are expected to live longer than male borrowers, (3) married borrowers tend to live longer than their single counterparts, (4) the joint mortality rate for a couple can be significantly lower than for each individual, and (5) the improvements in standards of living, healthcare and education have increased life expectancies in recent years (Zhai, 2000, p. 6). For these reasons, Zhai (2000, p. 13) insists that using average life expectancy is oversimplified and will mislead the cash flow assessments. Although some of these factors and tendencies may not be applicable in Norway, incorporating these factors and tendencies when calculating the expected time of termination would decrease the calculation error in relations to the expected future cash flow. In addition, lenders could lower the crossover risk by tailoring each loan for the individual borrower, thus giving different contract rates and lump sums based on the borrowers' life expectancy. However, Dowd (2018, p. 17) claims that "even if we could predict the average time of death of a large cohort of people of the same age, gender etc. the timing of death of any individual, i.e., our customer, is still highly uncertain". In other words, individual assessments can diminish the calculation errors, but longevity risk will always remain.

Although life expectancies can vary immensely between borrowers due to many factors, it is considered controversial to discriminate based on them. For example, Zhai (2000, p. 8) highlights that discriminating based on race and medical history is not considered politically correct. Furthermore, even though females tend to live longer than men, a Council Directive from 2004 by the European Union states that "the use of sex as a factor in the calculation of premiums and benefits for the purposes of insurance and related financial services shall not result in differences in individuals' premiums and benefits" (EUR-Lex, 2004). This law was later legislated in Norway in 2014 (Stortinget, 2014).

In Norway, the only personal characteristics that determine the size of the lump sum is the borrower's age and marital status. Younger borrowers get smaller lump sums than older borrowers, and joint borrowers of similar ages get smaller lump sums than those who are single. According to Zhai (2000, p. 7), age is the primary factor for mortality, and Institute of Actuaries (2005) claims that "couples living together will tend to care for one another, thereby significantly delaying any care-entry compared to the case were the individuals living alone" (p. 20). In other words, taking these factors into consideration when calculating life expectancies and determining lump sums will decrease the lenders' crossover risk, but variations within ages and across marital statuses will still exist.

3.3.2 Mobility Risk

Furthermore, mobility risk is related to a postponement of the termination date due to the borrower moving and has the same effect on reverse mortgage products as the mortality rate. A non-health related move-out is certainly included in mobility risk and moving to a long-term care facility may also be considered in this category, in addition to being a longevity risk.

Seniors with a reverse mortgage loan do not have much incentive to move due to non-health related conditions. Firstly, senior homeowners have strong preference for staying in their home due to emotional ties and the various difficulties associated with moving (Alai et al., 2013). Secondly, after receiving the reverse mortgage, the loan diminishes the proceeds from the property. In a scenario where the loan has exceeded the property value, it is unlikely that a borrower would move, as they would rely on additional funds to be able to acquire a new home. If the interest rate was fixed, one can imagine that the borrower would want to refinance the loan when the rates are low. For variable interest rates, however, which are offered in the Norwegian market, this should not be an incentive as long as the interest rates follow the market rates. In other words, although the lenders are faced with mobility risk, we do not consider it to be significant.

4. Methodology

This chapter will present the methodology applied in the profitability analysis in Chapter 6. In order to calculate the reverse mortgage loan's profitability, we need the following three components: an option pricing model, termination probabilities and a profitability measure. The option pricing model is used to value the NNEG, and limitations of its use will be accounted for. Furthermore, the termination of a reverse mortgage is arbitrary, which is why termination probabilities are needed both for valuing the NNEG and finding the loan's profitability. Lastly, our profitability measure will be defined.

4.1 The NNEG as a European Put Option

The payoff from the NNEG is reminiscent of the payoff from a European put option. Denote L_t as the outstanding loan balance at the time of loan termination, H_t as the property value and γ as the assumed proportional transaction costs of selling the property. At termination t, the lender will receive the outstanding loan balance, L_t, if $H_t(1-\gamma) > L_t$. If $H_t(1-\gamma) < L_t$, the lender will only receive $H_t(1-\gamma)$. The repayment to the lender upon death of the homeowner is therefore given by

$$Repayment_t = min[H_t(1-\gamma),\ L_t] = \ L_t - max[L_t - H_t(1-\gamma), 0]. \tag{1}$$

Furthermore, the payoff from the NNEG at the time of loan termination, t, can be defined as

$$NNEG_t = max[L_t - H_t(1-\gamma), 0]. \tag{2}$$

The payoff in Eq. (2) is similar to the payoff function for a European put option, which is given by

$$P = max[K - S_t, 0], \tag{3}$$

where K is the exercise price of the option and S_t is the price of the underlying asset at maturity. In the case of the NNEG, L_t is the exercise price, and $H_t(1-\gamma)$ is the price of the underlying.

From Eq. (1), we observe that the repayment to the lender is the outstanding loan balance less the payoff from the NNEG. This implies that the lender holds an equity position and a short position in the European put option, while the borrower holds a debt position and a long position in the European put option. If the loan amount exceeds the net property value at termination, the option expires “in-the-money”, and the option will be exercised. The risk of the option expiring “in-the-money” is the crossover risk.

4.2 The Black-Scholes Model

In order to value the NNEG, we will apply a modified version of the Black-Scholes model for dividend-paying assets. The Black-Scholes model is internationally used for this purpose, as presented in Section 1.1, and it is the recommended approach in the UK by the PRA (2020, p. 12). As the reverse mortgages in Norway are quite similar to the ones issued in the UK, we see it as natural to apply the same method to the Norwegian market.

The Black-Scholes formula was first introduced in 1973, when Fischer Black and Myron Scholes published a model for valuing dividend-protected European options. Robert C. Merton published a paper expanding their mathematical understanding, accounting for dividend pay-outs and introducing the term “Black-Scholes options pricing model”.

As specified by Black and Scholes (1973, p. 640) in *The Journal of Political Economy*, the Black-Scholes formula assumes “ideal conditions” for the underlying asset and the option, where:

- the short-term interest rate is known and constant;
- the asset price follows a random walk in continuous time;
- the variance rate of the asset is constant;
- the option is European, which means it can only be exercised at maturity;
- there are no transaction costs associated with trading the option or the underlying asset;
- it is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate;
- there are no penalties to short selling.

In addition, the dividend-adjusted Black-Scholes formula assumes the underlying asset to pay a continuous flow of income (Bodie et al., 2018, p. 721). These assumptions indicate market completeness, where assets can be traded continuously. It refers to the situation where the payoff of an option can be obtained as the terminal value of a dynamic and self-financing portfolio. In other words, due to the no-arbitrage principle, the price of the option at any time before the expiration date must be the value of the replicating portfolio at that time. Accordingly, it is possible to create a hedged position for a call option, consisting of a long position in the underlying and a short position in the option.

When the underlying asset pays a constant and continuous dividend yield, the asset value will decrease by the amount of the dividend on the ex-dividend date. For that reason, the asset value is discounted by the dividend yield in order to find the deferment price of the asset, which is defined as the present value of obtaining possession of the asset at some point in the future.

When presenting the Black-Scholes model for dividend-paying assets, the following notations will be used:

- C_0: current call option value;
- P_0: current put option value;
- S_0: current asset price;
- K: exercise price;
- r: risk-free interest rate;
- t: time to expiration, in years;
- σ: standard deviation of the continuously compounded rate of return of the asset;
- g: constant and continuous dividend yield;
- $N(d_1)$, $N(d_2)$: cumulative distribution functions of the standard normal distribution.

The value of a call option is found as follows:

$$C_0 = S_0 e^{-gt} \times N(d_1) - Ke^{-rt} \times N(d_2). \tag{4}$$

Next, we can derive the value of a put option by using the put-call parity theorem[12]. The value can be expressed as the following function: $P(t, S_0, K, r, g, \sigma)$, and is given by

$$P_0 = C_0 + Ke^{-rt} - S_0 e^{-gt}. \tag{5}$$

[12] The put-call parity theorem represents the proper relationship between put and call prices. If violated, there is an arbitrage opportunity. The more general formulation of the put-call parity is: *P = C – S + PV(K) + PV(Div)* (Bodie et al., 2018, p. 676-677).

By inserting the expression for C_0 from Eq. (4) into Eq. (5), and simplifying the expression, we get the price of a European put option:

$$P_0 = Ke^{-rt} \times N(-d_2) - S_0 e^{-gt} \times N(-d_1), \tag{6}$$

where $d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - g + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$ and $d_2 = d_1 - \sigma\sqrt{t}$.

4.2.1 Black ‘76

Dowd (2018, p. 18) claims that many firms use projections of future house price growth in their NNEG valuation, for which he says there is no justification. This is in line with statements from the PRA (2020, p. 12) in the UK, saying the future growth of the property will not be relevant, as the investor (the lender) “will receive the benefit of future property growth (or suffer any property depreciation) because they will own the property at the end of the deferment period”. Dowd (2018, p. 16) further argues that the natural choice for valuing the NNEG is the Black ‘76 model, which applies the forward price of the house instead of its current price. This is also the model recommended by the PRA (2020, p. 12). However, the Black 76’ model is identical to that of the Black-Scholes options pricing model for dividend-paying assets, and therefore applying it in order to value the NNEG will give the same results. A proof of this is provided in Appendix 2.

4.3 Limitations of Using the Black-Scholes Model

As listed in Section 4.2, the Black-Scholes model relies on several underlying assumptions. Although the Black-Scholes formula is commonly used for valuing the NNEG, the use of it is also questioned and criticised. According to a report from the PRA (2018b), many reverse mortgage lenders in the UK raised concerns over the Black-Scholes model’s appropriateness and argued that the underlying assumptions do not hold in practice. In this section, we will highlight some of these controversies and explain why we have decided to apply the Black-Scholes model despite these controversies.

4.3.1 Hedged Position

The Black-Scholes formula is built on the prerequisite that it is possible to replicate the portfolio and thus create a hedge portfolio. According to the PRA (2018b), the most common argument among firms for not using the Black-Scholes formula is that it rests on conceptual assumptions such as the availability of liquid hedging instruments. This is in line with Tunaru and Quaye (2019, p. 14), which claim that NNEG put options are not tradable instruments, because they are embedded in the reverse mortgage contract. Moreover, they argue that the forward contract on a house price cannot be calculated using Black 76', because it is not possible to short sell the value of a house. The inability to replicate the NNEG means that any value derived from the Black-Scholes model, "can at best be regarded as a lower bound for the "market consistent" cost of the NNEG" (Institute of Actuaries, 2005, p. 26). However, Li et al. (2010, p. 519) states that although the NNEG is infrequently traded, it can be dynamically hedged by forming a hedge portfolio that has a price sensitivity profile similar to the NNEG.

4.3.2 European Option

Furthermore, the Black-Scholes model is used for European options, and it is disputed whether the NNEG might be considered as such. In the survey conducted by the PRA (2018b, p. 9), respondents questioned whether the NNEG can be considered as a put option, since the exercise date is not determined by the borrower and the borrower gets no benefit from exercising the NNEG. Nevertheless, the PRA (2018b, p. 9) claims that the NNEG has the economic substance of a put option in the hands of the borrower, seeing that it effectively allows the borrower to sell the property to the lender for full settlement of the debt regardless of the market price of their property. In addition, valuing the guarantee as an option is widespread practice within the UK life insurance industry (PRA, 2018b, p. 9).

4.3.3 Geometric Brownian Motion

The Black-Scholes model further assumes that the logarithm of the randomly varying underlying asset follows a continuous-time stochastic process with a drift. This is known as the geometric Brownian motion, and it is somewhat controversial whether this process holds true for properties. The PRA (2018b, p. 8) states that this assumption is one of the reasons why firms find the Black-Scholes model inappropriate. Under this assumption, asset prices can get arbitrarily close to zero, which they claim to be unrealistic for residential UK properties. In fact, respondents indicated that property markets tend to be backward-looking, subject to inertia and government intervention, which lead to autoregressive property returns. This view is also supported by Li et al. (2010, p. 511), who affirm that the house price returns have strong autocorrelation and time varying volatility.

Although problematic, the violation of this assumption does not necessarily suggest that the Black-Scholes model cannot be used for the purpose of valuing the NNEG. Dowd et al. (2019, p. 3) agrees that the house prices are autocorrelated but confirms that this does not make Black ’76 inapplicable, “rather that care needs to be taken with the volatility calibration”. Moreover, the PRA (2018b, p. 9) states that there is academic research showing that the Black-Scholes formula, if calibrated appropriately, gives reasonable answers regardless of whether this assumption is satisfied. In addition, they add that “individual properties have sometimes traded for very small amounts, which is consistent with the behaviour of geometric Brownian motion” (p. 9).

4.3.4 Dividend Yield

The Black-Scholes model for dividend-paying assets further assumes that the underlying asset pays a continuous flow of income. As a measure for the dividend yield in the Black-Scholes model, several studies use the net rental yield (Dowd et al., 2019; Institute of Actuaries, 2005; Ji et al., 2012; Li et al., 2010), which is the use benefit of living in the property or the net rental income associated with renting the property. More specifically, it is gross rental paid by tenants “less the costs incurred by the lessor such as management, maintenance and the expected costs of void or empty periods while the property is being re-let” (Buckner & Dowd, 2019, p. 33).

The assumption that the net rental yield affects a property value the same way a dividend yield affects a stock price, is somewhat problematic. As stated by a report from Tunaru and Quaye (2019, p. 30), there is no evidence that rental yields are driving future house prices, and the expected property values at future long horizons cannot be determined with growth models in the same way expected share prices is determined with growth models linked to dividends. In other words, simply discounting the property value by the net rental yield, might not give an accurate present value of the underlying asset. In addition, house prices are known to have a positive growth trend, which will affect net rental yields. As the dividend yield is assumed to be constant, this is not taken into consideration in the Black-Scholes model.

However, as mentioned in Section 4.2.1, the PRA considers future property growth to be irrelevant. Furthermore, the PRA (2020) uses four principles for assessing the allowance made for the NNEG risk against its view of the underlying risks retained by the firm, and the third principle states that “the present value of deferred possession of property should be less than the value of immediate possession” (p. 9). In other words, taking ownership over the property today is more valuable than taking ownership later. For this to hold true, the property must be discounted by a positive number. The PRA calls this discount rate the “deferment rate”, and as proven by Buckner and Dowd (2019, p. 33), the deferment rate and the net rental yield are mathematically identical.

4.4 Pricing the NNEG

Despite the controversies surrounding the use of the Black-Scholes model for valuing the NNEG, it is recommended by the PRA (2020, p. 12) and applied by several international studies. We therefore consider it as a natural choice of model but note that our results may have weaknesses. In order to price the NNEG, we have added a few parameters to the Black-Scholes model for dividend-paying assets, as well as calculated termination probabilities. These adjustments deviate from the model presented in Section 4.2, and will be addressed before presenting the pricing model for the NNEG.

The exercise price of an option is usually fixed throughout the option’s lifetime. However, when applying the Black-Scholes model for valuing the NNEG, the exercise price is the loan balance at time t. We assume that no interest payments are made by the borrower, which implies that the loan balance will be compounded by the contract rate up until termination. The exercise price is thus dependent on the time to maturity; the longer the duration of the loan, the higher the exercise price. We therefore need to adjust the Black-Scholes model for dividend-paying assets by compounding the exercise price with the contract rate u. Adding the contract rate in this manner has also been done in the works of Dowd et al. (2019), Ji et al. (2012) and Li et al. (2010).

Furthermore, we need to consider the time delay associated with selling a property. Unlike stocks, selling a property takes time due to processes like estimating the property value, preparing the property for sale, advertising the property, and conducting viewings and bidding rounds. In addition, we assume that all terminations happen mid-year. Due to these two factors, we add δ and 0.5 to the maturity date t, as is done in other research that prices the NNEG as a put option (Chen et al., 2010; Gonçalves, 2017; Ji et al., 2012; Li et al., 2010).

In Section 4.2, assumptions of the Black-Scholes model were presented. Among these, there was the assumption of no transaction costs when trading the underlying asset or the option.

Since it is unrealistic to presume that selling a property comes without transaction costs, such as hiring a real estate agent and buying advertisement, we have decided to add transaction cost to our model. The transaction cost only occurs when the property is sold at termination. This approach has also been applied in similar research (Chen et al., 2010; Gonçalves, 2017; Ji et al., 2012).

In the case of a normal European put option, a maturity date is agreed upon at the time of entering the contract. This is not the case for a reverse mortgage as the lender provides the borrower with the guarantee of residency until death or permanent move out. The termination time t is thus arbitrary, so the structure of the NNEG can be described as a series of European put options (Chinloy & Megbolugbe, 1994). Therefore, we need to compute the present value of the NNEG for different maturities t; from the year the loan is signed to the maximum attainable age w. These values are weighted based on the probability that the contract loan is terminated between time t and $t + 1$ for a borrower aged x at the time of the contract signing, denoted $q_{x,t}$. The approach for finding the termination probability is presented in Section 4.5.

4.4.1 Notations

For our model, we operate with the following notations, similar to the notations in Ji et al. (2012):

- r: the continuously compounded risk-free interest rate;
- u: the continuously compounded interest rate on the reverse mortgage;
- g: the continuously compounded net rental yield;
- L_t: the value of the reverse mortgage at time t; $L_t = L_0 e^{ut}$;
- H_t: the value of the mortgaged property at time t;
- δ: the average time-delay from home exit until the actual sale of the property;
- γ: the transaction cost associated with selling the property;
- $q_{x,t}$: the probability of termination in time t for a borrower initially aged x;
- w: the maximum attainable age of the borrower.

4.4.2 The NNEG Pricing Model

The present value of the NNEG for a loan with a fixed termination date, $NNEG_t$, is found by using Eq. (6) from Section 4.2 and applying the following adjustments: the contract rate u, the average time delay δ along with 0.5 years, and the transaction cost γ. In addition, since the termination date is arbitrary, we need to calculate a series of NNEGs with termination t and multiply each NNEG value with the termination probability $q_{x,t}$. The sum of these constitutes the expected present value of the NNEG, NN, given by

$$NN = \sum_{t=1}^{w-x} q_{x,t} \times NNEG_t \,, \tag{7}$$

where $NNEG_t = P(t + 0.5 + \delta, H_0(1-\gamma), L_0 e^{ut}, r, g, \sigma)$.

$NNEG_t$ is found by applying our adjustments to Eq. (6), and is given by

$$NNEG_t = L_0 e^{(u-r)(t+0.5+\delta)} \times N(-d_2) - H_0(1-\gamma)e^{-g(t+0.5+\delta)} \times N(-d_1), \tag{8}$$

where $d_1 = \frac{\ln\left(\frac{H_0(1-\gamma)}{L_0}\right) + \left(r-u-g+\frac{\sigma^2}{2}\right)(t+0.5+\delta)}{\sigma\sqrt{t+0.5+\delta}}$ and $d_2 = d_1 - \sigma\sqrt{t + 0.5 + \delta}$.

Eq. (7) will be applied in order to find the profitability measure in Section 4.6. As seen from Eq. (8), it is the interest rate margin $u - r$ which proves important for valuing the NNEG, not the contract rate u and the risk-free rate r independently.

4.5 Termination Probability

The NNEG causes the reverse mortgages to have more termination risk than other mortgage loans, as the loan is repaid at maturity. $\mathrm{NNEG_t}$ increases with the time to maturity, whilst property growth is not taken into account when applying the Black-Scholes model. In other words, the chances of the loan exceeding the property value increases with the life expectancy of the borrower, which makes the uncertain termination date a driving force behind the crossover risk. Without knowing the maturity date, one must rely on termination probabilities.

In this study, we assume that termination finds place at the time of death of the borrower and use Statistics Norway’s (2020f) projected probability of death by sex and age to find the probability of loan termination. This assumption and the input data used for calculating the termination probabilities are further presented in Section 5.9. The probability retrieved from Statistics Norway's data is the probability of a borrower aged $x + t$ to die between time t and $\mathrm{t} + 1$, which we define as $q_{x+t,t}$. However, we need the probability that a loan given to a borrower initially aged x terminates between time t and $\mathrm{t} + 1$, which is defined as

$$q_{x,t} = P(t < T_x \leq t + 1) \text{ for } t = 1, 2, \dots, w - x, \tag{9}$$

where $\mathrm{T_x}$ is the contract termination time for a borrower initially aged x and w is the maximum attainable age. This probability may be derived by using the projected probabilities from Statistics Norway (2020f), $q_{x+t,t}$, and the probability of a borrower initially aged x to survive up until time t, denoted as $p_{x,t}$. Then $q_{x,t}$ is given by:

$$q_{x,t} = p_{x,t} \times q_{x+t,t}. \tag{10}$$

If $x = 70$ and $t = 2$, then $q_{x,t}$ represents the probability of a 70-year-old dying in two years, which is equal to the probability of a 70-year-old surviving until the age of 72 multiplied by the probability of a 72-year-old dying at the age of 72. This is the approach we have used when finding the distribution of $q_{x,t}$, which is also the approach used in Dowd et al. (2019) for calculating exit probabilities.

In order to calculate $q_{x,t}$ from Eq. (10), we need to define $p_{x,t}$. For this purpose, we apply the following equation:

$$P[T_x > s + t] = P[T_x > t] \times P[T_{x+t} > s], \tag{11}$$

where $t = 0, 1, \dots, w - x - 1$, and $s = 1$. The equation is retrieved from Aase (1996, p. 32) and adjusted to be applicable for our methodology. In order to explain the Eq. (11), let us consider that $x = 60$. For $\mathrm{t} = 1$, the equation states that the probability of a borrower initially aged 60 to live beyond 2 more years $(\mathrm{s} + \mathrm{t})$, thus beyond the age of 62, is equal to the probability of a 60-year-old to live beyond one year multiplied by the probability of a 61-year-old to live beyond one year.

Since $P[T_x > s + t]$ is the probability of surviving beyond time t, while $p_{x,t}$ is the probability of surviving until time t; $p_{x,t}$ is equal to $P[T_x > t]$. Note that as long as $s = 1$, then $P[T_x > t]$ is equal to $P[T_x > s + t]$ in the previous period, and $P[T_{x+t} > s]$ is equal to $1 - q_{x+t,t}$. In other words, $p_{x,t}$ can be defined as

$$p_{x,t} = p_{x,t-1} \times (1 - q_{x+t-1,t-1}). \quad (12)$$

As we assume that all borrowers live through $t = 0$, implying that $p_{x,0} = 1$, we can use the data provided by Statistics Norway (2020f) to find $p_{x,t}$, and thereby also find $q_{x,t}$.

4.6 Profitability Measure

In order to assess the reverse mortgage product's profitability, we will value the product in the same manner as done by Dowd et al. (2019). According to Dowd et al. (2019), the present value of the reverse mortgage RM is the present value of a risk-free loan L, subtracted by the costs of the NNEG, NN. This follows the second principle from the PRA (2020, p. 9), which states that the economic value of the reverse mortgage cash flow cannot be greater than the value of an equivalent loan without an NNEG. In other words, RM cannot be greater than L. One might consider L to be the expected present value of the lender's potential income from the reverse mortgage, and NN to be the expected present value of the lender's potential costs from the embedded NNEG. The present value of the reverse mortgage is the present value of the lender's expected cash flow, defined as

$$RM = L - NN, \quad (13)$$

where NN is calculated using Eq. (7) presented in Section 4.4.2, and L is defined as the expected present value of the reverse mortgage in a scenario where the lender is guaranteed to be repaid in full. L can be calculated as follows:

$$L = \sum_{t=1}^{w-x} q_{x,t} \times L_0 e^{(u-r)(t+0.5+\delta)}. \quad (14)$$

The lender receives the rate u from the borrower and faces the opportunity cost r from providing the loan. Even though the lender is guaranteed to be repaid in full, there is still uncertainty associated with the time at which the repayment takes place. For this reason, the loan is accumulated for different expiration dates and weighted with the probability of loan termination through the parameter $q_{x,t}$.

In order to assess the reverse mortgage loan's profitability, we can calculate how much the loan is worth in excess of the initial value of the loan at contract signing by merely subtracting the lump sum from the present value of the reverse mortgage. We call this measure the "day one profit", which is also briefly used by Buckner and Dowd (2019, p. 128). The day one profit can be calculated as follows:

$$Day\ one\ profit = RM - L_0. \tag{15}$$

The day one profit will be an essential part of our profitability analysis. When the day one profit is positive, the present value of the loan exceeds the lump sum, and the loan is thereby considered profitable. If the day one profit is negative, the lender is at loss. It is important to note that the lender will face additional costs related to the reverse mortgage, other than the NNEG costs. As an example, since the reverse mortgage product is considered riskier than other mortgage products, it is reasonable to assume that the lenders are facing higher financing costs. However, we do not consider these costs to be of great impact on the product's profitability, and reverse mortgages generating day one profits above zero will thereby be considered profitable in this study.

We have found no previous research using this profitability measure to conduct a profitability analysis of the reverse mortgage product, and this study therefore seeks to contribute with a new perspective to the existing literature on the subject.

4.7 Criticial Aspects of the Study

Our study has several critical aspects which include but are not limited to the following. Firstly, our analysis will only account for the lump sum reverse mortgages, although there exist other payment structures that will have a different level of risk and profitability. In fact, Cho et al. (2013) found lump sum reverse mortgages to be more profitable and less risky from the lender's perspective than income stream reverse mortgages in the Australian market. Secondly, we will not address other costs than the potential costs of the NNEG. Including additional costs will make the loan less profitable than what is indicated through our analysis. Thirdly, we only account for mortality as a cause for termination despite the fact that termination occurs at any permanent exit from the property or when the loan is fully repaid. Lastly, when analysing how the profitability is affected by individual parameters in our analysis, we presuppose the remaining parameters to be at their baseline values. This requires the baseline values to be reasonable and the parameters to be uncorrelated, which is not necessarily true.

5. Input Data

In order to value L, NN, RM and $q_{x,t}$ by applying the methodology presented in the previous chapter, we need to find reasonable parameter values for the Norwegian market. The valuations calculated from using these parameter values will constitute our baseline scenario for the sensitivity analyses. In this chapter, we will account for our chosen parameter values, and consider their relevance for our study.

5.1 Initial Age

Age is a key determinator for crossover risk, and it is the main personal characteristic that determines the lump sum. Therefore, we find it relevant to study the profitability of a reverse mortgage given to borrowers of different ages, instead of limiting the analysis to a single age group. Since the minimum age of receiving a reverse mortgage is 60, and the lump sum offered does not increase for borrowers over the age of 80, we find it most relevant to analyse the dynamics of key parameters within this age gap. We therefore choose the ages of 60, 70 and 80, which would cover the main spectrum of potential borrowers.

5.2 Initial Property Value

The lump sum is correlated with the initial house price through the loan-to-value ratio. According to a living condition survey conducted by Statistics Norway (2018), for a person above the age of 65 that lives alone, the average useful area is 60-79 square meters. The nation-wide average price per square meter for used detached houses was NOK 25 691 in 2019 (Statistics Norway, 2020b). Multiplying this with 79 square meters, gives an estimate of approximately NOK 2 000 000. It is important to note that this is a very simplified method of finding the property value, since the square meter price will be affected by the size of the property; smaller properties are likely to have higher square meter prices.

We will use this baseline value in order for the profitability analysis to be applicable to a real scenario. However, the baseline property value is only meant to be illustrative and the value in itself will not be of a great consequence. L, NN and RM are in fact directly proportional to the property value. As an example, if our profitability analysis is applied to another scenario, where the property value is twice as high as our baseline value, the valuations of L, NN and RM may simply be multiplied by the factor of 2.

5.3 Loan-to-Value Ratio

The loan-to-value ratios offered to borrowers, hereinafter referred to as LTV, is affected by different factors, as presented in Section 2.1.2. These are incorporated in the loan calculators that the lenders provide online. As these calculators indicate slightly different LTVs, we will use the loan calculator provided by LittExtra (n.d.-a) for determining the baseline values of LTV, which is meant to illustrate what the borrower may expect to receive from the Norwegian lenders.

In our baseline scenario, the initial property value is NOK 2 000 000. The loan calculator indicates that a single borrower of 60 years with this property value will receive an LTV of 22%. Furthermore, a single 80-year-old with the same property value might expect an LTV of 44%[13]. Whether the borrower is a single individual or a couple does not have any significant impact on the LTV. By studying the loan calculator from LittExtra (n.d.-a), we find that the age of the youngest borrower determines the lump sum offered to couples, unless the borrowers are of similar ages. With a small age difference, the lump sum is only slightly less than what the youngest borrower would have received alone. Since the effect on LTV is minimal, adjusting the LTV baseline values to account for marital status will not have a great impact on the baseline valuations. We will therefore look past marital status and assume that couples receive the same LTV as the youngest partner would receive alone.

From testing different property values and ages in LittExtra’s (n.d.-a) loan calculator, we find that the LTV does not change at a fixed rate, as shown in Appendix 1, which can be due to the lump sum being rounded to the nearest NOK 10 000. These deviations imply that the loan calculator must be used to get the exact LTV offered on the market. In order to facilitate the process of finding LTVs applicable for different ages, we have developed an equation for this purpose. The rates retrieved from the equation are approximations, and thereby not identical

[13] Retrieved October 5, 2020 from LittExtra (n.d.-a) loan calculator, assuming that the borrower lives in the city centre of Oslo with postal code 0150 and not in a housing associating with joint debt. Using a postal code in Oslo is not of great consequence for the results, as the loan calculator gives the same results for all properties that are considered easily marketable in a normally functioning housing market.

to the rates offered on the market. It is based on the outputs for different property values and ages, presented in Appendix 1, assuming the borrower to be single and live in an urban area without having joint debt related to a housing cooperative.

Let LTV_0 be the minimum LTV offered and ΔLTV the percentage one-year change in LTV based on the initial age of the borrower x. The LTV will only grow up until the age of 80, and then stagnate. The equation is given by:

$$LTV = LTV_0 + min(x - 60, 20) \times \Delta LTV \tag{16}$$

where the average ΔLTV is 1.1%, which is based on our own calculations. By using the Eq. (16), we find that LTVs are 22%, 33% and 44% for the ages 60, 70 and 80 respectively.

5.4 Average Time Delay

An essential part of the Black-Scholes model is the time of maturity, which in our case is the time of death. t refers to the start of each year, but it is unrealistic to assume that this is when all deaths occur. A more realistic approach is to assume that deaths are evenly distributed during the year. We incorporate this assumption into the model by adding 0.5 to each t. Thus, $t + 0.5$ refers to the time of death, which on average is considered to be mid-year. In addition, it takes time for a property to be sold. For that reason, we need to add this time delay to the time of maturity, denoted by δ. The time of maturity for the put option will therefore be $t + 0.5 + \delta$.

It is difficult to find a good estimate for this parameter in Norway. According to Eiendom Norge[14] (2020), the average turnover time for a property in Norway was 47 days in September 2020. The problem with this estimate is that it only estimates the time between starting the selling process and the sale, which will not be sufficient for our case. Since the property is sold due to the borrower's death, it will likely take some time before the selling process begins. This could be due to renovations and other maintenance work, and one would assume that the heirs need time to make the property ready for sale. How long these processes last will vary, and there are no good estimates on this. Due to these uncertainties, we will lean on previous

[14] In English: Real Estate Norway

research for a reasonable estimate. The studies that use the same approach, presented in Section 4.4, apply an average time delay of 0.5 years, which we apply as our baseline parameter value.

5.5 Transaction Costs

Sale transaction costs are included in the Black-Scholes formula to get a more realistic view of the proceeds from the property. By examining Finn.no (n.d.), Norway's largest online marketplace, we find that only 2.5%[15] of the total housing market is sold privately without a real estate agent. In addition, sellers are usually faced with additional costs, such as appraiser costs and change of ownership insurances. According to Andre Øren (Edvardsen, 2020), Chain manager in DNB Real Estate, the provision-based brokers fee is normally between 1% and 3.6%. Furthermore, an article from Hus & Bolig (Sodeland, 2011), states that the total transaction costs of selling a property for NOK 2 000 000 will probably account for more than NOK 70 000, which is 3.5% of the property value. Transaction costs will vary depending on location, value of the property, the brokerage firm etc. Because of these variations and possible additional costs, we have decided to use a slightly higher estimate, namely a transaction cost of 4%.

5.6 House Price Volatility

In order to price the NNEG, we need an annual estimate for the volatility of an individual house price. This parameter will vary immensely across properties and time, and volatility estimates on owner-occupied housing on an individual level is to our knowledge non-existing on the Norwegian market. These factors make it difficult to find a reasonable parameter value, which is why we will examine this parameter thoroughly.

The Black-Scholes model depends on the possibility to dynamically hedge or replicate the underlying. For NNEGs this is not possible, as housing derivatives are to our knowledge non-existing. Dynamic hedging involves the existence of liquid markets giving the ability to adjust the hedge portfolio continuously in the underlying and the option. Illiquid markets have less

[15] 953 housing ads marked as privately sold divided by the total 37 510 ads, both listed by agencies and private per 15.10.2020.

market participants to absorb fluctuations, which lead to prices more easily being affected by buyers and sellers. Therefore, illiquid markets tend to exhibit greater volatility than liquid markets. Property as an asset class is less liquid then listed financial assets, such as shares and bonds. The usual turnover of residential housing has been found to be between 4 to 5 years, but might range from months to decades (Ang, 2014, p. 414). In contrast, the time between transactions of public equities can be within seconds. Illiquidity arises due to market imperfections, such as transaction costs, search frictions, asymmetric information and funding constraints.

The usual approach to find the volatility factor is to use the implied volatility, which is the volatility level for the underlying asset implied by the option price (Bodie et al., 2018, p. 718). It can be backed out of an option-pricing-model by finding the volatility that makes the option's value equal to its observed price (Bodie et al., 2018, p. 735). Unfortunately, the market for housing derivatives in Norway is to our knowledge non-existing, making it impossible to compute an implied volatility. Even in the case of existing housing derivatives, the derivatives would most likely be based on a broad-based index and not on individual housing. As we cannot derive market-implied volatility, we use the following equation to find the historical standard deviation of the Norwegian housing market, retrieved from Bodie et al. (2018, p. 718):

$$\sigma = \sqrt{\frac{n}{n-1}\sum_{t=1}^{n}\frac{(r_t - \bar{r})^2}{n}}, \tag{17}$$

where $r_t = ln\left(\frac{House\ Price\ Index_t}{House\ Price\ Index_{t-1}}\right)$.

The historical volatility will show how the growth in house prices in a given area varies from period to period compared to the average growth throughout the period. Historical volatility therefore measures the spread in housing inflation. If the historical volatility is zero, the housing inflation each period is exactly the same as the average housing inflation. We calculate historical volatility of the natural logarithms of the house price index. For this purpose, we used Statistics Norway's (2020g) quarterly price indices of existing dwellings over the 15-year period from the first quarter of 2005 until the last quarter of 2019, which was the longest period available for all regions. The data consists of both national and regional indices that are not seasonally adjusted in order to capture a higher degree of variation. The indices consist of 33 sub-indices for 11 regions, and measure the value development of the housing stock, based on current price information on second-hand homes sold on the open market. To obtain the annualized historical volatility, we simply multiply the quarterly standard deviation with the square root of the number of quarters in a year. The results are presented in Figure 6. On average, the regional indices are more volatile than the national index, varying from 4.92% in Vestfold, Telemark and Viken excluding Akershus, to 7.24% in Trøndelag excluding Trondheim. As mentioned in Section 3.2, house price returns are likely to be autocorrelated. This imposes challenges for annualizing the volatility as it requires adjusting for the autocorrelation, therefore the result in Figure 6 needs to be regarded with caution.

Figure 6: *Annualized Volatility*

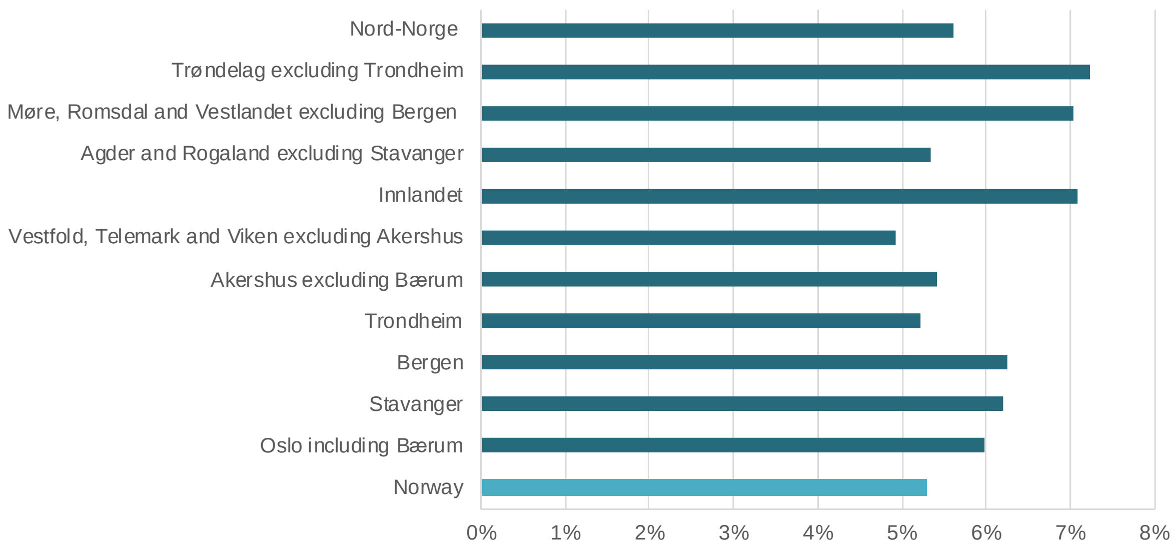

Figure 6. The annualized volatility of the house price index on a national and regional level based on quarterly data. Source: Statistics Norway (2020g). Authors' calculations.

If we move from a regional level to individual houses, we expect to find significantly greater volatilities. As stated by Ang (2014, p. 428): "For illiquid asset classes, an investor can simply not receive the return on some index". All properties are sold and bought individually in local markets. National and regional indices are, on the contrary, averages or medians calculated from thousands of local transactions. Such indices can mask local trends, because events that occur locally will have little or no impact on the national or even regional price averages. If lenders rely on estimates on a national or regional level, the volatility will be underestimated.

In addition to local trends, characteristics of the house and the owner might be connected to the housing volatility. By using the American Housing Survey, Zhou and Haurin (2010) found that the variance of house value is higher for houses at both ends of the quality distribution, and for long-residence and elderly householders. As explained in Section 3.3.2, reverse mortgage borrowers do not have much incentive to move due to non-health related condition, and we therefore assume them to be long-residence householders. Furthermore, houses with reverse mortgages might also be in the lower end of the quality distribution, as elderly have less incentive to maintain their property than other age groups. Our volatility estimate should therefore be adjusted to take these factors into account.

Even though we are interested in the long-term annual volatility for individual housing as our volatility estimate, the expected long-term volatility for the international commercial property may serve as a guideline. Real estate investments are usually expected to have a long-term volatility that lies between global bonds and equities. The Ministry of Finance (Meld St. 10. 2009-2010, p. 119) expects a long-term annual volatility of 12% for international commercial property, while specifying that the estimate is subject to particularly high uncertainty. According to Kan et al. (2004), the volatility for commercial property exceeds that of private housing, based on regional data from the US markets. Therefore, it is realistic to assume that our volatility estimate should be somewhat lower than that of the Ministry of Finance. However, since our study takes volatility of individual housing into account, the volatility should exceed that of a regional perspective and be higher in relation to regional commercial properties found by Kan et al. (2004).

It is difficult to determine the "correct" volatility assumption. In our calculations of the annualized volatilities, presented in Figure 6, we find that the average regional volatility in Norway is 6%. As done in Hosty et al. (2008), we might raise the regional volatility by 3 percentage points to cover the shift from index to individual property, which gives us a volatility estimate of 9%. However, we consider this to be a bit conservative, as the findings from Zhou and Haurin (2010) suggest that we need to adjust the volatilities for reverse mortgage contracts based on the property and the owner's characteristics. In the UK, the PRA (2018b, p. 11) proposes a volatility estimate of 13% when valuing the NNEG, and firms have generally provided the PRA volatility estimates ranging from 10% to 15% (Buckner & Dowd, p. 49). This gives us an indication of where our volatility estimate should lie. Furthermore, the research by Ji et al. (2012) applies an estimate of 12% to value the NNEG, which corresponds with the long-term volatility for international commercial property provided by the Ministry of Finance (2010). Taking all these factors into account, we consider 12% to be a reasonable estimate.

It is important to mention that there is much uncertainty related to this estimate. Firstly, volatilities will vary greatly across reverse mortgage contracts. For example, we might expect the property market in rural areas to be less liquid than that in urban areas, as there are fewer market participants. This will lead to higher volatilities. Secondly, options' volatility will in general vary based on the time to maturity, even if the underlying asset is the same. However, since the purpose of this research is to analyse the dynamics of different parameters, we consider a 12% volatility to be a reasonable baseline value and will apply it for all put options regardless of termination date.

5.7 Net Rental Yield

The dividend yield of the underlying asset is represented by the property's net rental yield. In the Black-Scholes model, the dividend yield is assumed to be constant. It is important to note, however, that this assumption will not fully reflect the market, as the net rental yield will vary over the course of time. In order to include this parameter into the model, we need a realistic estimate applicable for the Norwegian market. In this section, we will use the terms "net rental yield" and "deferment rate" interchangeably, as they are mathematically identical.

Finding a good estimate for the net rental yield in Norway is not straight-forward, as it varies immensely based on the property's location and characteristics, and over time. In the UK, the PRA (2018a, p. 21) has assessed the deferment rate to be 2%, and a minimum of 1%. Many studies on reverse mortgage in the UK apply the PRA's suggestion of 2% (Dowd, 2018; Institute of Actuaries, 2005; Ji et al., 2012; Li et al., 2010), while Dowd et al. (2019) applies a net rental yield of 4.2%. Furthermore, one might find estimates of the net rental yield in Norway, although we have found no research that applies them to a similar case to ours. The lenders of reverse mortgages in Norway are largely connected to Norway's largest cities: Oslo and Bergen[16]. According to estimations by DNB (Iversen, 2019), a two-room apartment in

[16] The following reverse mortgage lenders are connected to Oslo and Bergen: Bien Sparebank, Fornebu Sparebank, Strømmen Sparebank, Lillestrømbanken and Sparebanken Vest.

Oslo has a net rental yield of 2.5%, excluding maintenance cost. Krogsveen (Laugen, 2016) calculated the same scenario including maintenance cost, and got a net rental yield of 2.3%.

It is reasonable to assume that the rent does not proportionally increase with the size of the property, and thus the net rental yield will be smaller for larger properties. This is supported by estimates from the Global Property Guide (2018), which provides yearly gross rental yield of apartments in Oslo of 45, 75 and 120 square meters, which are 4.50%, 3.48% and 3.09% respectively. As the initial property value in our baseline scenario is based on residence of 79 square meters, the gross rental yield for a 75 square meter home will be most appropriate. According to the Global Property Guide (2018), the net rental yields tend to be 1.5 to 2 percentage points lower than the gross rental yield, which leaves us with a net rental yield in the range of 1.48% to 1.98%.

Furthermore, we also conducted our own estimations for the net rental yield. The net rental yield might be retrieved as exhibited in Eq. (18), using estimates of net rental income and property value:

$$Net\ Rental\ Yield = \frac{Net\ Rental\ Income}{Value\ of\ Property} = \frac{Rental\ Income\ -\ Costs\ of\ Ownership}{Value\ of\ Property}. \qquad (18)$$

A report from Samfunnsøkonomisk Analyse AS[17] (Benedictow & Gran, 2018) calculated the costs of living in a property in Norway, which were property tax, municipal taxes, energy cost, interest expense, insurance and maintenance. We do not, however, consider energy costs to be relevant for the net rental yield, as these costs will usually fall on the tenant. Benedictow & Gran’s (2018) calculations are conducted for the year 2017, based on a self-owned detached house of 120 square meters, as this was the average property size in Norway in 2015. Statistics Norway (2019b) supplies us with a predicted monthly rent based on a hedonic price regression model. In 2017, the average monthly rental income for all properties of 120 square meter was 14 616 NOK. The annual rent, along with the estimated costs, are presented in Table 1.

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[17] In English: Economics Norway

Table 1: Net Rental Income

		All values in NOK
Rental income		175 392
Property tax	–	3 199
Municipal taxes	–	10 972
Interest expense	–	39 086
Insurance	–	5 825
Maintenance	–	22 644
Net rental income		*93 666*

Table 1. The calculation for the annual net rental income, using secondary data for rental income and costs of ownership. Source: Benedictow & Gran (2018) and Statistics Norway (2019b).

Having an estimate of the net rental income in place, an estimate of the property value is needed to calculate the net rental yield. For the sake of continuity, we will apply the same estimates as in Section 5.2; a square meter price of NOK 25 691 for a detached house in 2019 (Statistics Norway, 2020b). Assuming that house prices grow proportionally with the size, the price of a 120 square meter house will be NOK 3 082 920.

Entering the above estimates into Eq. (18), we find a net rental yield of approximately 3%. These numbers are based on averages in Norway and will thus not necessarily represent the customer group of reverse mortgages.

In comparison to research practice in the UK and the net rental yield calculations for Norway provided by the Global Property Guide, DNB and Krogsveen, we consider our estimated net rental yield to be somewhat high. There may be additional costs associated with owning a property that are not reflected in Table 1. For our baseline scenario, we will therefore use a net rental yield of 2%, which corresponds well with the presented sources from Norway and the UK.

5.8 The Interest Rate Margin

In the Black-Scholes model, the lump sum is compounded by the difference between the contract rate and the risk-free rate. The option price is thus not affected by the rates themselves, but the interest rate margin represented by the expression $u - r$. According to Store Norske Leksikon (Nyhus, 2019), Norway's largest website for research communication, the reverse mortgage rates in Norway change in line with the mortgage rates, indicating that there is a fixed margin between the two. The mortgage rates in Norway are mostly variable, just like the zero-coupon government bonds. This would further suggest that the margin between the reverse mortgage contract rate and the risk-free rate is fixed, and that $u - r$ is constant.

However, it is important to highlight that assuming the interest rate margin to be constant over the duration of the loan will not hold in real-life. One would expect there to be some delay in the rates' movements, and they may be affected by factors that make them move differently. Nevertheless, due to the Black-Scholes model's assumption of market completeness, this study will not take this into consideration.

5.8.1 The Contract Rate

As of October 2020, there are nine lenders of the reverse mortgage in Norway, which are presented in Table 2. The table also displays their respective nominal rates for the reverse mortgage, which we will apply to find a reasonable estimate for the Norwegian contract rate. It is important to note that there are additional costs that come with the loan, such as establishment fees and registration fees, which will affect the loan's effective rate. For the analysis, we have decided to look past these additional costs, as they make up a small percentage of the lump sum. It is also important to note that some lenders have separate contract rates for members and non-members. Since memberships usually come with membership fees, it would give a false impression to use the membership rates. For that reason, we use the contract rates offered to non-members. The average of the contract rates listed in Table 2 rounds down to 3.80%, which will be the contract rate applied for the baseline scenario.

Table 2: *Nominal Contract Rates from Norwegian Reverse Mortgage Lenders*

Lenders	Nominal Contract Rates
Bien Sparebank	3.95%
BN Bank	3.95%
Fornebu Sparebank	3.60%
Jbf bank og forsikring	3.85%
KLP	3.90%
LillestrømBanken	—
OBOS-banken	3.85%
Sparebanken Vest	3.65%
Strømmen Sparebank	3.85%
Average contract rate	*3.83%*

Table 2. Nominal contract rates for reverse mortgages offered in Norway retrieved October 16, 2020. The contract rate from Lillestrømbanken was not available at the time of retrieval. Source: Bien Sparebank (n.d.-a), BN Bank (n.d.-b), Fornebu Sparebank (n.d.-b), Jbf bank og forsikring (n.d.-b), KLP (n.d.-b), OBOS-banken (n.d.-a), Sparebanken Vest (n.d.-a) and Strømmen Sparebank (n.d.-b).

5.8.2 The Risk-Free Rate

The risk-free rate is the rate an investor would expect to receive when making a risk-free investment and is assumed to be known and constant. However, it is important to note that a fully risk-free rate does not exist in practice, as all investments do carry some degree of risk.

There are different perceptions on what a good estimate of risk-free rate is. Each year, PwC conducts research on the risk premium in the Norwegian market, in cooperation with the Norwegian Society of Financial Analysts. In the research from 2019, analysts and economists with experience from the Norwegian financial market were asked what rate should be applied as a risk-free rate in the required return on equity for Norwegian companies (PwC, 2019). 34% of the respondents answered the 10-year government bond, which made it the most commonly used measure (p. 7).

The Norwegian Accounting Standards Board (NASB) (2020) also applied estimated government bond yields as the basis for the risk-free interest rate. The NASB has predicted a zero-coupon interest rate curve based on the data for government bond yields up to the longest available Norwegian government bond yield in NOK, which is 10 years. Beyond this point, the yield curve is extrapolated using market data on the Norwegian swap yield curve obtained from Bloomberg. The predicted zero-coupon interest rate curve retrieved from the NASB is presented in Figure 7.

Figure 7: *Zero-Coupon Interest Rate Curve*

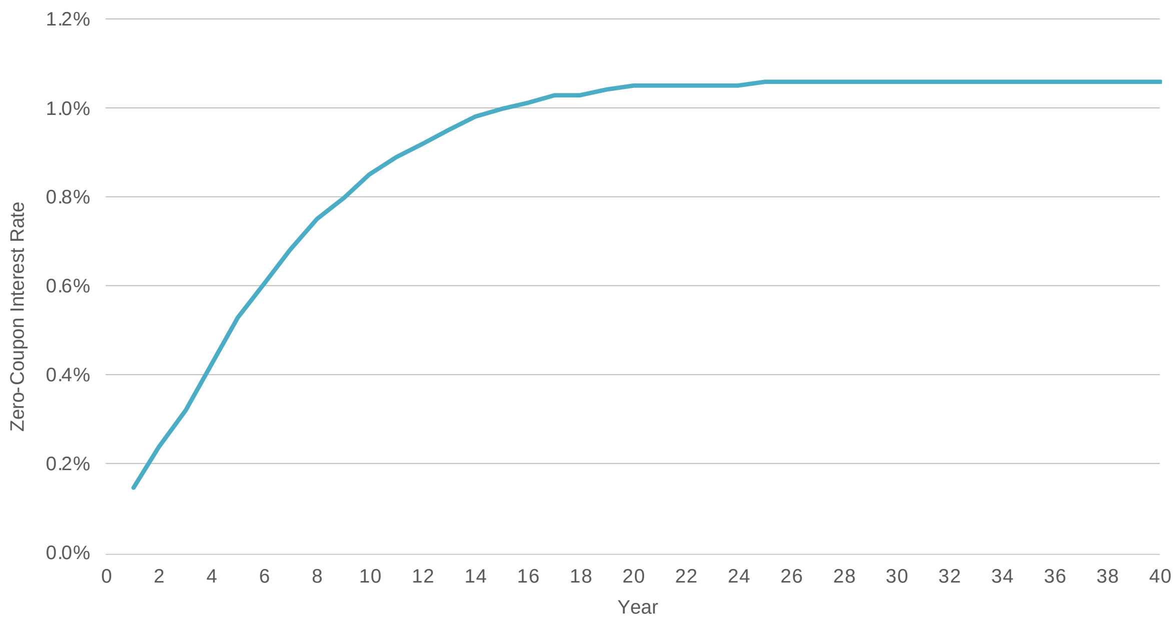

Figure 7. The figure is constructed by the authors. Source: Norwegian Accounting Standards Board (2020).

Ideally, we would apply the discount rate corresponding with time to maturity of the put option when valuing the NNEG. However, as we do not have an equivalent interest rate curve for the contract rates, and the interest rate margin is considered to be fixed, we need a single estimate of the risk-free rate which may determine the interest rate margin. NASB (2020, p. 19) has estimated weighted average interest rates that can be used as approximations to the method of using different discount rates. It is done by calculating the interest rate which gives the same present value of a pension obligation as when discounting annual payments with the zero-coupon interest rate. These calculations give discount rates of 1.0% and 1.1%. Furthermore, the NASB also finds the risk-free interest rate with a 15-year horizon to be approximately 1% per August 31, 2020, on the basis of the swap interest rate market. Due to these estimates, we will consider the risk-free rate to be 1% in our baseline scenario.

5.8.3 The Interest Rate Margin

With a contract rate of 3.8% and a risk-free rate of 1%, the interest rate margin would be 2.8%. This is the estimate that will be adopted as the baseline value for this parameter. However, this interest rate margin is not necessarily fully representative of the reverse mortgage market in Norway. As this study was conducted, the world was in the middle of the Covid-19 pandemic, which had a huge effect on market rates. In March of 2020, a report by Nordea Markets (Mouland et al., 2020) stated that higher credit premiums for banks and problems in the US money market suggested that interest rates on mortgages and corporate loans would not necessarily fall in line with Norges Bank's interest rate cuts. It is therefore reasonable to assume that our interest rate margin is somewhat overestimated.

5.9 Termination

One of the main uncertainties that the lenders face when issuing a reverse mortgage is the termination date, due to the NNEG. In this section, the assumptions for loan termination will be accounted for, along with the data for calculating the loan termination probabilities.

5.9.1 Assumptions

The main triggers for termination are mortality, entrance into a long-term facility, moveout for non-health related issues or refinancing. As explained in Section 3.3, we do not deem moveout for non-health related issues or refinancing to be associated with much crossover risk to the lender. Furthermore, we consider entrance into a long-term facility to be closely related to a person's life expectancy. For that reason, we will only take mortality into consideration when calculating termination dates. This assumption has been done in Dowd et al. (2019), and according to a study in the US by Jiang et al. (2018), death is the main reason for loan termination for older borrowers.

Although females are expected to live longer than men, as affirmed in Section 3.3.1, the lenders' online loan calculators do not take this into account when estimating the lump sum. This is in line with the Norwegian law legislation (Stortinget, 2014), where calculations of premiums and benefits for the purposes of financial services cannot be done on the basis of sex. We have therefore decided to apply average mortality rates for men and women combined, as we do not know the actual distribution of men and women in the lenders' reverse mortgage portfolios. It is meant to represent the average of the reverse mortgage portfolio and will therefore lead to over- and underestimations of the mortality rates of each individual loan based on the gender.

Furthermore, as also affirmed in Section 3.3.1, single seniors tend to live shorter than those with a partner. This is incorporated in the online loan calculators, where a couple of similar ages get slightly lower lump sums than the youngest spouse would get on their own. However, this difference is so small, that we do not consider it necessary to account for life expectancy differences for couples in our termination probabilities. It is nevertheless important to note that not taking couples into consideration will lead to slight underestimations of the value of the NNEGs in question.

Moreover, we only include termination probabilities up to a certain age. We use Statistics Norway's (2020f) table for projected probability of death when calculating the probability of loan termination. This table projects the probability of death from 2020 to 2100, for all ages up until the age of 100. Since the estimates stop at the age of 100, we have decided to set this as the maximum attainable age for all borrowers. This implies that the probability of a borrower to reach the age of 101 is zero. We consider this a reasonable assumption, considering that by January 1, 2020 there were 1 119 people in Norway over the age of 100, which only represents 0,09% of the Norwegian population aged 60 and above (Statistics Norway, 2020a). In other words, this assumption will not have a considerable impact on the results.

5.9.2 Termination Data

As the assumed reason for loan termination is the borrower's passing, we rely on mortality rates to compute our baseline valuations. According to Zhai (2000, p. 6), reverse mortgage borrowers tend to live longer than the general population. However, we do not have mortality rates available for reverse mortgage borrowers especially. We therefore employ Statistics Norway's (2020f) projected probability of death by sex and age for the entire Norwegian population. The data retrieved from Statistics Norway was divided for the two sexes, and we therefore had to calculate the average mortality rates in order to find the mortality rates for men and women combined.

As stated by Statistics Norway's (2020d, p. 65) report on national population projections, the probabilities are estimated using the Lee-Carter model. More precisely parameters are obtained by a product-ratio variant of the Lee-Carter model based on the historical evolution of mortality and forecasted by an ARIMA-model. According to Chen et al. (2010, p. 375) "the Lee–Carter model has been widely used in mortality fitting and projection because of its simplicity and robustness in the context of linear trends in age-specific death rates."

The distributions of $q_{x,t}$ for loans to borrowers aged 60, 70 and 80 are presented in Figure 8. The probabilities will increase after contract signing since the probability of a borrower passing increases with age, and will peak after 33, 22 and 10 years for loans to 60-, 70- and 80-year-olds respectively. Thereafter, the termination probabilities decrease, as it becomes more and more unlikely that a borrower would reach such an old age. Since the maximum attainable age is assumed to be 100, the probability of death at all ages beyond the age of 100 will be captured by the last termination probability for each age, as seen in Figure 8.

Figure 8: *Termination Probabilities*

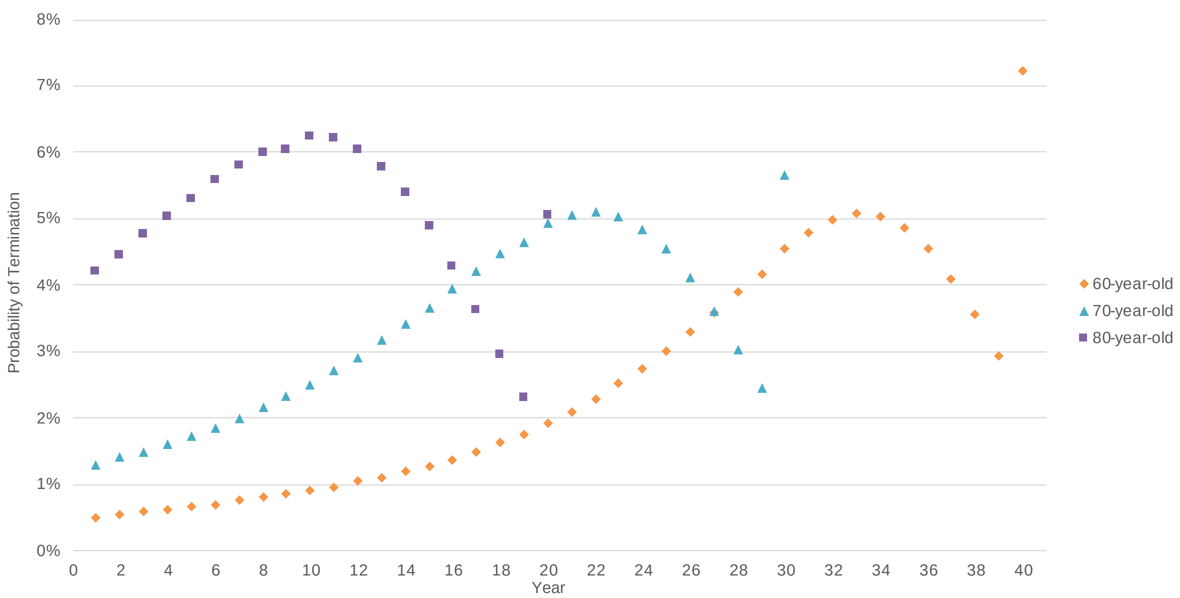

Figure 8. The probability distribution of $q_{x,t}$ for reverse mortgage loans to borrowers initially aged 60, 70 and 80. Source: Statistics Norway (2020f). Authors' calculations.

In addition to providing projections based on age and sex, Statistics Norway's projections differentiate on life expectancy. Since future mortality is uncertain, they calculate an 80% prediction interval around the main alternative: the medium life expectancy. In our baseline scenario, we will assume a medium life expectancy, as it is the main alternative.

6. Profitability Analysis and Discussion of Reverse Mortgages

In order to make an assessment on how profitable reverse mortgages are for the Norwegian lenders, we will conduct various sensitivity analyses. Our objective is to analyse how different values of key parameters in the pricing framework affect the profitability of a loan to three different age groups, both separately and combined. Firstly, the baseline valuations using the input data accounted for in Chapter 5 will be presented and analysed. Secondly, we will analyse the effects of volatility and net rental yield, the most uncertain values in the pricing framework. These values will differ immensely across properties and time and may therefore pose a great risk to the lender when miscalculated. Thirdly, the profitability's sensitivity to LTV and interest rate margin will be examined, the two parameters determined based on data from the lenders themselves. Finally, the results from the profitability analysis will be discussed and seen in light of previous research.

When analysing these dynamics, the remaining parameters will be kept at their baseline values, which are summarized in Table 3. It is important to emphasise that several of these parameters will differ for each individual loan, depending on the loan's characteristics. However, our results are meant to represent an average profitability over a large reverse mortgage portfolio.

Table 3: *Baseline Parameter Values*

Parameters	**Notations**	**Baseline Values**
Initial age	x	60, 70, 80
Initial property value	H_0	NOK 2 000 000
Loan-to-value ratio	LTV	22%, 33%, 44%
Average time delay	δ	0.5 years
Transaction cost	γ	4%
House price volatility	σ	12%
Net rental yield	g	2%
Interest rate margin	$u - r$	2.8%
Life expectancy		Medium

Table 3. The baseline values of the parameters used as input data.

6.1 Baseline Valuations

Using Eq. (8) from Section 4.4.2, we have computed the present value of the NNEG for a series of loans, $NNEG_t$, with different times to maturity t for the ages 60, 70 and 80. The results are illustrated in Figure 9. As long as the interest rate margin is positive, the value of the put option will be a strictly increasing function of t. Due to the lump sums offered being different for the three age groups, the value of $NNEG_t$ will differ between the ages. $NNEG_t$ grows faster for the older borrowers, illustrated by the steep slope, as the higher lump sum leads to a higher annual interest accumulation.

Figure 9: *Value of the No-Negative Equity Guarantee*

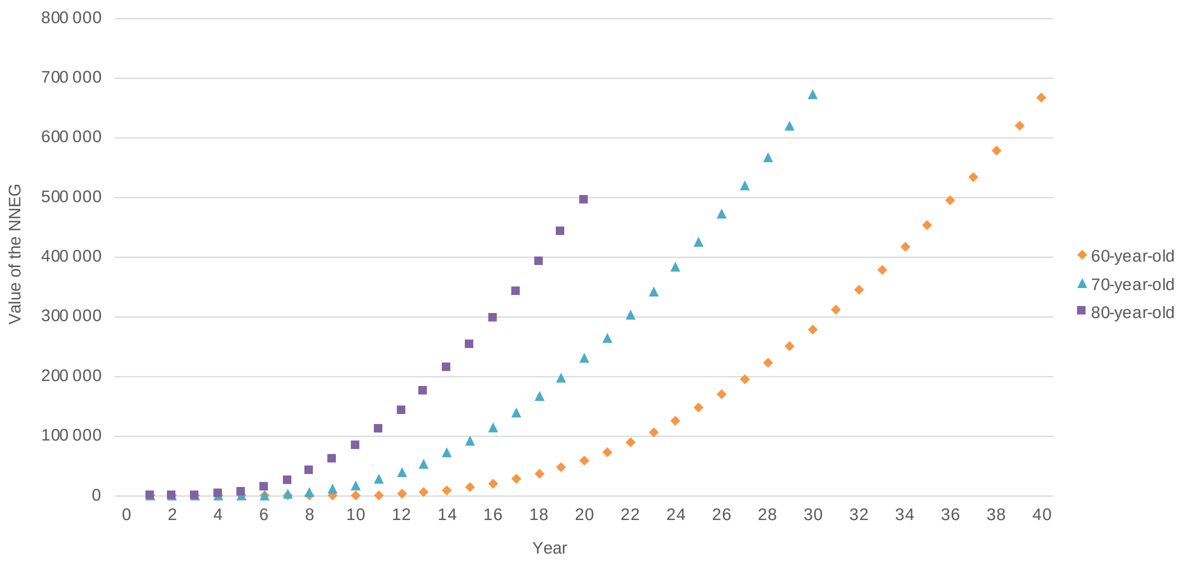

Figure 9. The value of the NNEG for a series of loans to borrowers at the age of 60, 70 and 80 with different maturities. Author's calculations.

The expected present value of the lenders' potential income, L, and the expected present value of the lenders' potential costs related to the NNEG, NN, constitute the lender's expected cash flow associated with issuing a reverse mortgage loan, and thus the loan's present value: RM. By subtracting the lump sum from RM, we find the day one profit, which is our measure for the loan's profitability.

Our baseline valuations of L, NN and RM for the three initial ages are shown in Table 4. These results are reported in NOK, along with the values relative to the lump sum. The latter is added in order to more easily compare the value of the reverse mortgage across ages, as the lump sum differs.

The borrower's age affects the NNEG in two competing ways. Loans to younger borrowers have a longer expected duration, which increases the crossover risk, simultaneously as the lump sum is lower and thus decreases the crossover risk. The baseline valuations suggest that the effect of the duration is dominant, as the NNEG is higher for younger borrowers. In fact, a loan to a 60-year-old borrower gives an NN valuation of 66% of the lump sum offered, which is significantly larger than the NN valuation of 16% for a loan to an 80-year-old. These effects also influence the expected profit, where a longer duration gives the interest more time to accumulate, while a lower lump sum makes the interest accumulate on a smaller amount. Again, the duration of the loan is the dominant effect, exhibited by profits being 44 percentage points higher for a loan to a 60-year-old compared to that of an 80-year-old.

Table 4 exhibit considerable differences in the baseline valuations for the different ages. We observe that NN relative to the lump sum is four times higher for a loan to a 60-year-old than an 80-year-old. At the same time, L relative to lump sum for a loan to a 60-year-old is not even twice the size of that to an 80-year-old. This indicates that loans to younger borrower have more potential costs relative to income than loans to older borrowers.

Furthermore, we take notice of the high values of RM, especially for younger borrowers. The results imply that a lender may invest NOK 440 000 in a loan to a 60-year-old that is in fact worth NOK 729 035. That suggests a 66% yield, which make the reverse mortgage loans seem extremely profitable. The expected yield for a loan to an 80-year-old is a modest 22% compared to that of the 60-year-old but is also noticeably high.

Table 4: *Baseline Valuations*

	60-year-old			**70-year-old**			**80-year-old**		
All values in NOK	Lump Sum = 440 000			Lump Sum = 660 000			Lump Sum = 880 000		
	L	*NN*	*RM*	*L*	*NN*	*RM*	*L*	*NN*	*RM*
Baseline	1 017 311	288 276	729 035	1 154 532	241 517	913 014	1 213 097	136 572	1 076 525
Baseline / Lump Sum	231%	66%	166%	175%	37%	138%	138%	16%	122%

Table 4. The baseline valuations of L, NN and RM in absolute terms and as a percentage of the lump sum. L is the present value of the loan component, NN is the present value of the NNEG and RM is the present value of the reverse mortgage. Authors' calculations.

6.2 Volatility and Net Rental Yield

As specified in Sections 5.6 and 5.7, volatility and net rental yield vary immensely across properties, based on their characteristics, and over time. Volatility is especially difficult to determine for an individual property. To our knowledge, the Norwegian lenders do not make individual assessments of each property's net rental yield and volatility in order to adjust the LTV and the interest rate margin for each loan accordingly. However, they do differentiate based on location. Some lenders only supply reverse mortgages to elders in urban areas, while others offer reduced lump sums in rural areas. In this section, we will explore the volatility and net rental yield's individual and joint effect on the profitability of a reverse mortgage loan.

6.2.1 Volatility

Volatility represents risks associated with house price movements. In order to examine how sensitive a reverse mortgage loan's profitability is to changes in volatility, we can examine the elasticity of the loan's potential income, costs and present value, presented in Table 5. Expressed in elasticity forms, it is evident that the volatility affects the lender's potential costs of the NNEG, but the effect differs between the ages. For a loan to a 60- and 70-year-old, NN is inelastic to changes in volatility from its baseline value, while for a loan to an 80-year-old, NN is elastic. However, although elastic, the lender is exposed to a significantly higher amount of potential costs related to the lump sum for loans to younger borrowers, as seen from the baseline valuations.

As a consequence of a higher volatility inducing an increase in NN, the present value RM will decrease, which can be a motivation for why lenders discriminate based on location. RM's sensitivity to volatility, as shown in Table 5, implies that the present value of the reverse mortgage is more stable for higher ages. This could be an argument for risk-averse lenders to solely supply the loan to older borrowers in rural areas.

Table 5: *Sensitivities of Valuations to Volatility in Elasticity Form*

	60-year-old			70-year-old			80-year-old		
Elasticity	*L*	*NN*	*RM*	*L*	*NN*	*RM*	*L*	*NN*	*RM*
σ	0.00	0.73	-0.29	0.00	0.82	-0.22	0.00	1.12	-0.14

Table 5. The elasticity of L, NN and RM by using a 1% increase in volatility from the baseline value. L is the present value of the loan component, NN is the present value of the NNEG and RM is the present value of the reverse mortgage. Authors' calculations.

Although *RM* is most stable for older borrowers, the sensitivity differences are not major. This is evident from examining the slopes in Figure 10, which illustrate how the day one profit changes for different volatilities. The lender's day one profit is highest when the volatility is low, and the borrower is young. Given that the other baseline values are reasonable, it appears from Figure 10 that the lender faces a negative day one profit if the volatility of the individual property is 29%, 27% and 26% for loans to borrowers aged 60, 70 and 80 respectively. It is unrealistic that volatilities of this magnitude will occur, as it is significantly higher than any of the volatility estimates presented in Section 5.6. This suggests that reverse mortgage products are highly profitable even in housing markets that exhibit great price fluctuations.

Figure 10: *Volatility's Effect on Day One Profit*

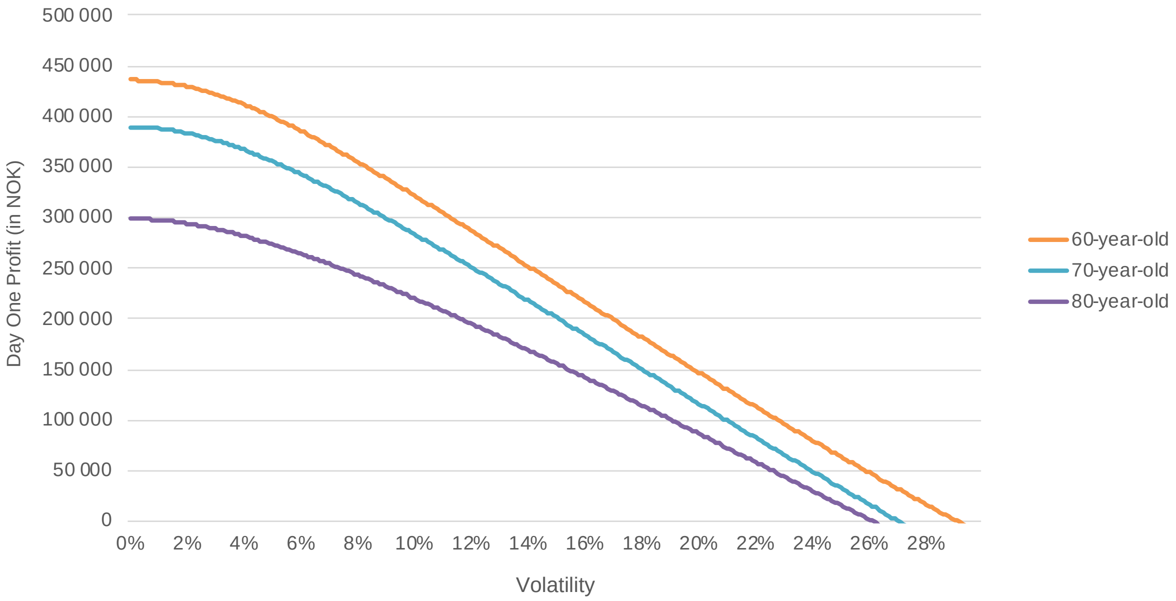

Figure 10. The lender's day one profit for given levels of volatility for loans to borrowers aged 60, 70 and 80. Authors' calculations.

6.2.2 Net Rental Yield

Valuing the NNEG involves finding the deferment price of the property, which is done by discounting it by the net rental yield. The higher the net rental yield, the lower the deferment price. Table 6 presents the sensitivities of the valuations to changes in the net rental yield. A 1% increase in the net rental yield is associated with a reduction in RM, and the percentage change is largest for loans to younger borrowers. This is solely driven by the increase in potential costs NN, as the net rental yield does not have any effect on the potential income L. Nevertheless, RM is inelastic to changes in net rental yield for all ages. Furthermore, NN is also considered inelastic to changes in the net rental yield and is more affected by changes in the net rental yield than RM. However, the difference in elasticity for NN is not grand between the age groups.

Table 6: *Sensitivities of Valuations to Net Rental Yield in Elasticity Form*

	60-year-old			70-year-old			80-year-old		
Elasticity	*L*	*NN*	*RM*	*L*	*NN*	*RM*	*L*	*NN*	*RM*
g	0.00	0.77	-0.30	0.00	0.71	-0.19	0.00	0.69	-0.09

Table 6. The elasticity of L, NN and RM by using a 1% increase in the net rental yield from the baseline value. L is the present value of the loan component, NN is the present value of the NNEG and RM is the present value of the reverse mortgage. Authors' calculations.

Figure 11 exhibits what day one profits the lender might expect for different net rental yields. The curves intersect the x-axis at net rental yields of 4.69%, 4.80% and 5.52% for 60-, 70- and 80-year-olds respectively. This implies that the net rental yield must be significantly larger than the baseline parameter value of 2% in order for the day one profit to be negative. Although our baseline parameter value does not necessarily hold true in all cases, we view these yields as high, especially considering the low current market rates.

Figure 11: *Net Rental Yield's Effect on Day One Profit*

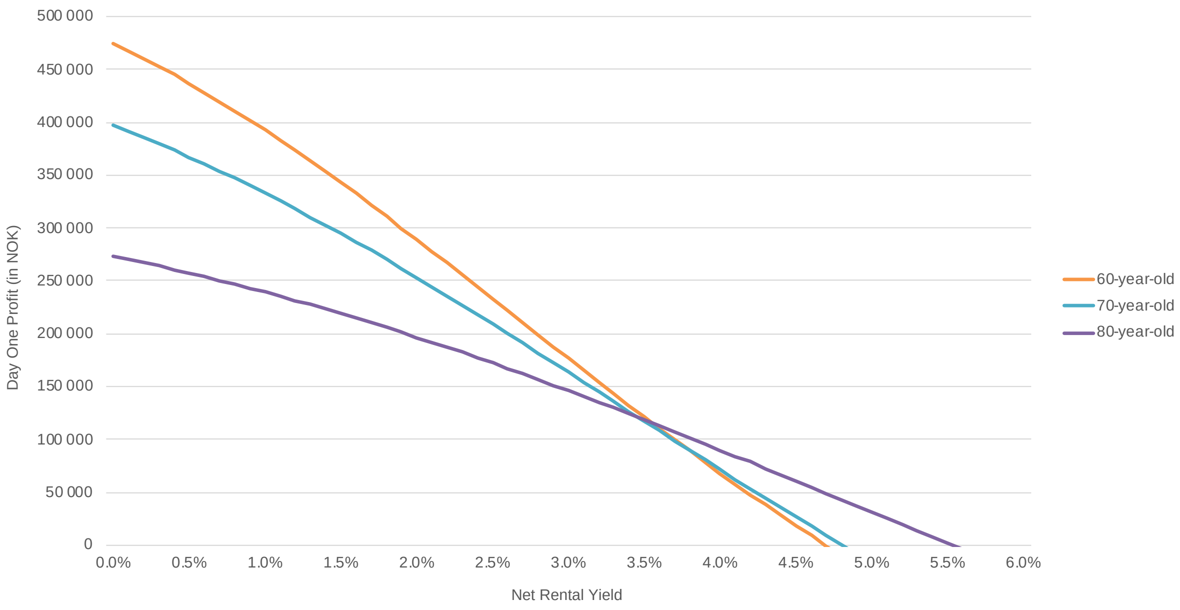

Figure 11. The lender's day one profit for given levels of net rental yield for loans to borrowers aged 60, 70 and 80. Authors' calculations.

Younger borrowers are the most profitable until the net rental yield reaches approximately 3.5%, at which point the curves cross and the older customer segment becomes the most profitable. This illustrates that changes in net rental yield may have great impacts on the lender's investment, and the preferred customer segment. The intersection of the curves is caused by the slope being steeper for younger borrowers, where the sharp decrease in day one profits for this age group is solely due to the duration of the loan giving the net rental yield more time to affect the deferment price, which increases the NNEG costs.

6.2.3 Joint Effect

As discovered in the above sensitivity analyses, drastic increases in the net rental yield and volatility are needed in order for the day one profit to be negative. Figure 12 exhibits how the day one profit changes for a loan to a borrower aged 70 due to changes in both of these parameters. We only consider values above the baseline scenario, as we have already judged the values under the baseline to be profitable. It is evident that the net rental yield and the volatility jointly need to be high in order for there to be no profit for the lender. In a scenario where the net rental yield is 4%, thus twice the size of our baseline value, the volatility would have to be 18% in order for the day one profit to be zero. In a more likely scenario, where the net rental yield is 3%, the volatility would have to be 23%, which is considered unrealistically high.

Figure 12: Volatility and Net Rental Yield's Joint Effect on Day One Profit

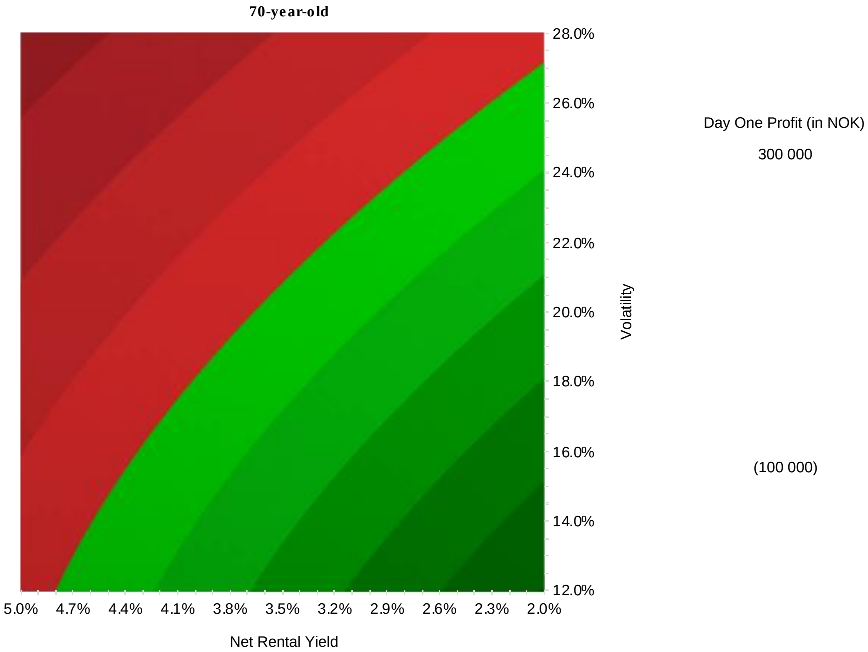

Figure 12. The day one profit for a loan to a 70-year-old borrower for different values of volatility and net rental yield. The day one profit is shown in intervals of NOK 50 000, where the green area represents positive day one profits, and the red area represents negative day one profits. Authors' calculations.

6.3 Loan-to-Value Ratio and Interest Rate Margin

There are only two parameters in the Black-Scholes model whose baseline values were based on data from the lenders themselves: the LTV and the contract rate. These are the mechanisms the lenders can use in order to adjust their exposure to risk and increase their profits. The current contract rates on the Norwegian market are considered high, whereas the lump sums offered are substantially lower than the initial property value. In this section, we will analyse the profitability's sensitivity to the interest rate margin and LTV and assess what interest rate margin and LTV values might be justified. In addition, we will explore how changes in volatility and net rental yield might influence the results.

6.3.1 Loan-to-Value Ratio

Table 7 shows the valuations' sensitivities to LTV in elasticity form. The results suggest that NN is highly elastic to changes in LTV for all ages, and most sensitive to loans given to older borrowers. RM also increases due to changes in LTV but is substantially less sensitive than NN. Since RM is inelastic to a 1% increase in LTV, the LTV increases more than the present value of the loan. This implies that the lender’s day one profit will be reduced if the LTV is increased from its baseline values.

Table 7: *Sensitivities of Valuations to LTV in Elasticity Form*

	60-year-old			70-year-old			80-year-old		
Elasticity	*L*	*NN*	*RM*	*L*	*NN*	*RM*	*L*	*NN*	*RM*
LTV	1.00	2.16	0.54	1.00	2.53	0.60	1.00	3.33	0.70

Table 7. The elasticity of L, NN and RM by using a 1% increase in the loan-to-value ratios from the baseline values. L is the present value of the loan component, NN is the present value of the NNEG and RM is the present value of the reverse mortgage. Authors’ calculations.

Despite NN being more sensitive to older borrowers, NN relative to L is much higher for younger borrowers, as seen in Table 8. The NN-to-L ratio exhibits whether the lender's risk of potential costs for higher LTVs is compensated through additional interest income. As the ratio increases with higher LTVs for all ages, the additional amount of potential costs is not fully compensated by additional accumulated interest. Lenders should therefore bear in mind the amount of risk they are willing to take when setting LTVs for different age groups.

Furthermore, NN might be more sensitive to increases in LTV for older borrowers, but the amount of potential costs associated with loans to younger borrowers is much higher. It would therefore be harder to justify issuing higher LTVs to this customer segment, than it would be for older borrowers.

Table 8: *Potential Costs Relative to Potential Income for Different LTVs*

	60-year-old	**70-year-old**	**80-year-old**
LTV	NN-to-L Ratio		
20%	25%	8%	1%
30%	39%	18%	4%
40%	49%	27%	9%
50%	57%	36%	15%
60%	63%	43%	21%
70%	67%	49%	28%
80%	71%	54%	34%

Table 8. NN relative to L for loans to borrowers aged 60, 70 and 80. L is the present value of the loan component and NN is the present value of the NNEG. Authors’ calculations.

By viewing the lender's day one profit in Figure 13, we can observe that the day one profit becomes negative for LTVs of 50%, 60% and 70% for loans to a 60-, 70- and 80-year-old respectively. This is substantially above the LTVs currently offered on the Norwegian market, indicating that the lender could raise the LTVs without facing negative profits, assuming that the other baseline values are reasonable.

Figure 13: *LTV's Effect on Day One Profit*

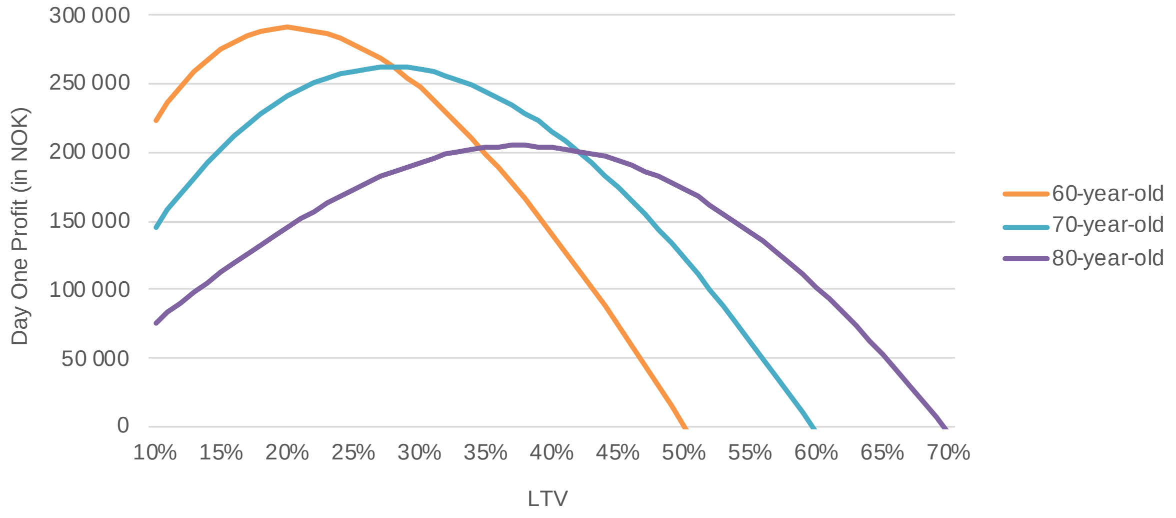

Figure 13. The lender's day one profit for given loan-to-value ratios for loans to borrowers aged 60, 70 and 80. Authors' calculations.

As also displayed in Figure 13, the day one profits peak at LTVs of 20%, 28% and 38% for loans to borrowers aged 60, 70 and 80 respectively, slightly lower than the LTVs currently offered on the Norwegian market. Furthermore, an LTV of 41% to a 70-year-old borrower exceeds the maximum profit from an 80-year-old borrower, and LTVs of 27% and 34% to a 60-year-old exceeds the maximum profit attainable for loan to a 70- and 80-year-old respectively. In other words, the lender's profitability will be higher for loans to younger borrowers than older borrowers, even with a higher LTV than current practice, which confirms that the lender will maximise profits when most borrowers are in the lower part of the age span.

Lenders supply the reverse mortgage on different markets, meaning that their exposure to house price volatility differs. From Figure 14, we can observe how much LTV can be increased for a reverse mortgage loan in order to maintain the same potential costs as in our baseline scenario given different observed market values of volatility. The results imply that in stable housing markets, where volatility is low, the lender can lend a higher proportion of the property value than in housing markets with higher price fluctuations. This is also partly done by Norwegian lenders, as they discriminate lending based on location. In an ideal situation, the LTV should be based on the presumed volatility of each borrower's property, but as explained in Section 5.6, this will be highly difficult as such estimates are nearly impossible to predict.

Figure 14: *Increase in LTV for Different Volatilities Holding NNEG Costs Constant*

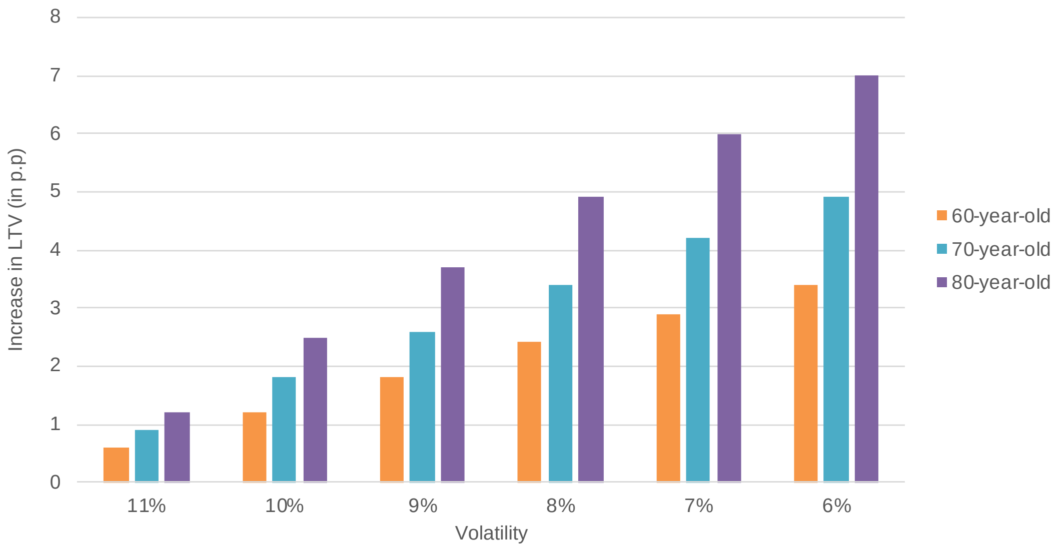

Figure 14. Increase in the loan-to-value ratio in percentage points from the baseline scenario given different annual volatilities, by holding the present value of the NNEG equal to the baseline valuation. Authors' calculations.

Furthermore, the net rental yield also affects the optimisation of LTVs offered. Our 2% baseline value for the net rental yield is largely based on calculations made for apartments in Oslo, and as mentioned in Section 5.7, the lender might expect different net rental yields based on the property's characteristics and location. The highest LTVs are offered in urban areas and cities, and LTVs of our baseline scenario would not necessarily be available to borrowers in other areas. As already established, results show that the loan is profitable for net rental yields below the parameter's baseline value. We therefore want to examine how the profitability changes for net rental yields above the baseline value and LTVs below the baseline value for a loan to a 70-year-old. The results are exhibited in Figure 15. Negative day one profits only occur when both the LTV and net rental yield are considerably high. In other words, as long as the LTVs are low, there is not much chance of loss related to properties generating high net rental yields. Furthermore, for a certain net rental yield, there is a range of LTVs that would give day one profits within the same gap of NOK 50 000. This indicates that it could be beneficial for the lender to lower the LTVs from the baseline values for certain net rental yields in order to maximise the ratio between the day one profit and the lump sum. These results can explain why lower LTVs are offered in certain areas.

Figure 15: *Net Rental Yield and LTV's Joint Effect on Day One Profit*

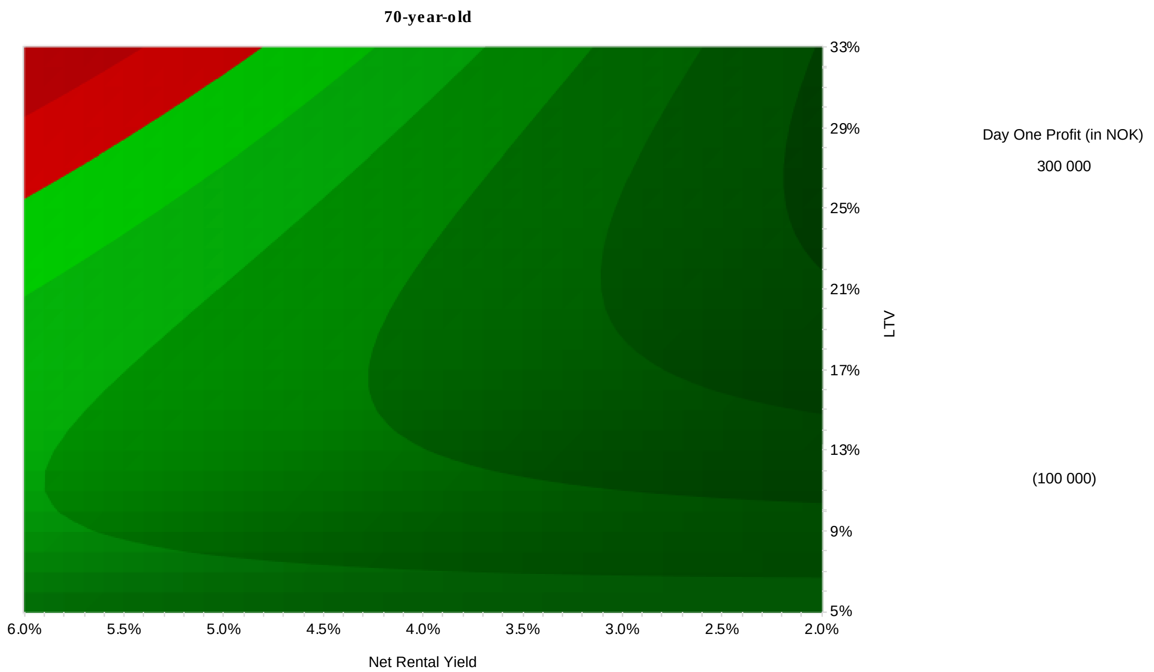

Figure 15. The day one profit for a loan to a 70-year-old borrower for different values of net rental yield and loan-to-value ratio. The day one profit is shown in intervals of NOK 50 000, where the green area represents positive day one profits, and the red area represents negative day one profits. Authors' calculations.

Even though Norwegian lenders do not discriminate based on gender, it is interesting to assess the impact of the borrower's gender on profitability. Figure 16 exhibits the day one profit for different LTVs for a 70-year-old male and female borrower where both individuals are expected to follow Statistics Norway's (2020f) medium life expectancy path. Due to higher life expectancies for female borrowers, the expected duration of the reverse mortgage will be longer, which increases both the lender's potential accumulated interest income and the potential costs from the guarantee. For low LTV values, the benefit from more accumulated interest exceeds that of potential costs, making loans to female borrowers more profitable. This changes when LTV is higher than 37%, where the lender would prefer male borrowers.

The lender's day one profit is maximised when the LTV is 27% for a female borrower, and 29% for a male borrower. In order to solely maximise profits, the lender should discriminate based on gender, giving higher LTVs to male borrowers than female borrowers. However, as mentioned in Section 3.3.1, they are prevented to do so by EU regulations. In addition, the difference between expected day one profits for men and women is not grand, which gives the lender less reason to do so.

Figure 16: *Day One Profit for Male and Female Borrowers*

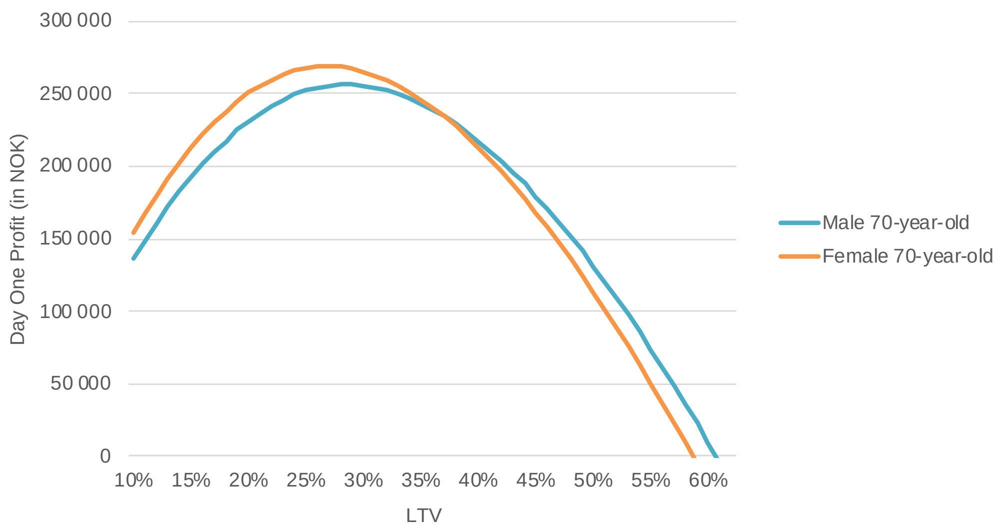

Figure 16. The day one profits for loans to a 70-year-old male and female borrower with medium life expectancy for different loan-to-value ratios. Authors' calculations.

6.3.2 Interest Rate Margin

Since the contract rates in Norway are variable, the lender might be subject to interest rate risk if the market rates rise unexpectedly. From Table 9, we observe that the potential income L is inelastic to changes in the interest rate margin, while the potential costs NN is highly elastic. However, it is important to emphasize that although a high interest rate margin increases the crossover risk, it is not necessarily associated with more costs to the lender. The interest rate margin covers both lender expenses and generates profit. If the lender is experiencing costs due to a higher interest rate margin, it might simply be the additional profit from the interest rate margin increase that is lost.

Moreover, all valuations are the most sensitive to younger borrowers, which indicates that the lender could prioritize younger borrowers in order to maximise profit but would have to accept a fairly large amount of additional risk to do so.

Table 9: *Sensitivities of Valuations to Interest Rate Margin in Elasticity Form*

	60-year-old			70-year-old			80-year-old		
Elasticity	*L*	*NN*	*RM*	*L*	*NN*	*RM*	*L*	*NN*	*RM*
$u - r$	0.87	2.07	0.40	0.58	1.71	0.28	0.33	1.43	0.19

Table 9. The elasticity of L, NN and RM by using a 1% increase in the interest rate margin from the baseline value. L is the present value of the loan component, NN is the present value of the NNEG and RM is the present value of the reverse mortgage. Authors' calculations.

Figure 17 exhibits how the day one profit is affected by different values of this parameter. The interest rate margin needs to be exceptionally small for the day one profit to be negative, namely below 0.21%, 0.29% and 0.31% for a loan to a 60-, 70- and 80-year-old respectively. These rates are even below the interest rates of current traditional mortgage loans offered on the Norwegian market[18]. Beyond these interest rate margins, the day one profit rises dramatically: the higher the interest rate margin, the higher the expected future cash flow. However, although profitable, the lender might not simply increase the interest rate margin in

[18] The average variable interest rate for outstanding loans secured on dwellings offered by Norwegian banks and mortgage companies in October 2020 was 1.86% (Statistics Norway, 2020h). With a risk-free rate of 1%, the interest rate margin would be 0.86%, thus significantly higher than 0.21%, 0.29% and 0.31%.

order to raise their expected profits. Today's interest rate margin is already considered high, and a higher interest rate margin would likely diminish the demand.

Figure 17: *Interest Rate Margin's Effect on Day One Profit*

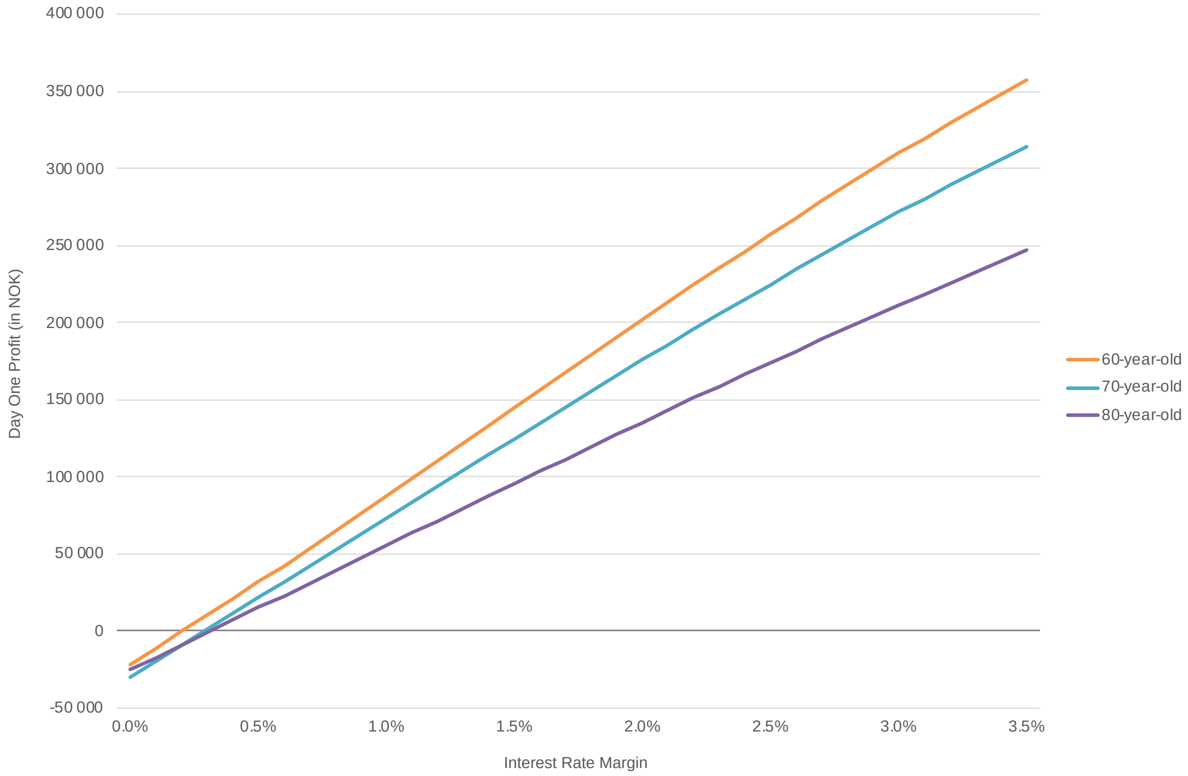

Figure 17. The lender's day one profit for different interest rate margins for loans to borrowers aged 60, 70 and 80. Authors' calculations.

Even if the lender could increase the interest rate margin, such a conclusion cannot be made without seeing the day one profit in context of the potential NNEG costs. Figure 18 exhibits the day one profit-to-NN ratio for different interest rate margins. For low values of the interest rate margin, loans to younger borrowers give the highest day one profit per unit of expected NNEG costs, but this fact changes when the interest rate margin is greater than 1.6%. In that case, a loan to an 80-year-old has the highest day one profit-to-NN ratio. When the interest rate margin is over 2.5%, this measure is also higher for loans to 70-year-olds. In other words, when the interest rate margin is at its baseline value, the lender can expect to receive higher profits relative to costs for older borrowers.

Figure 18: *Day One Profit-to-NN Ratio*

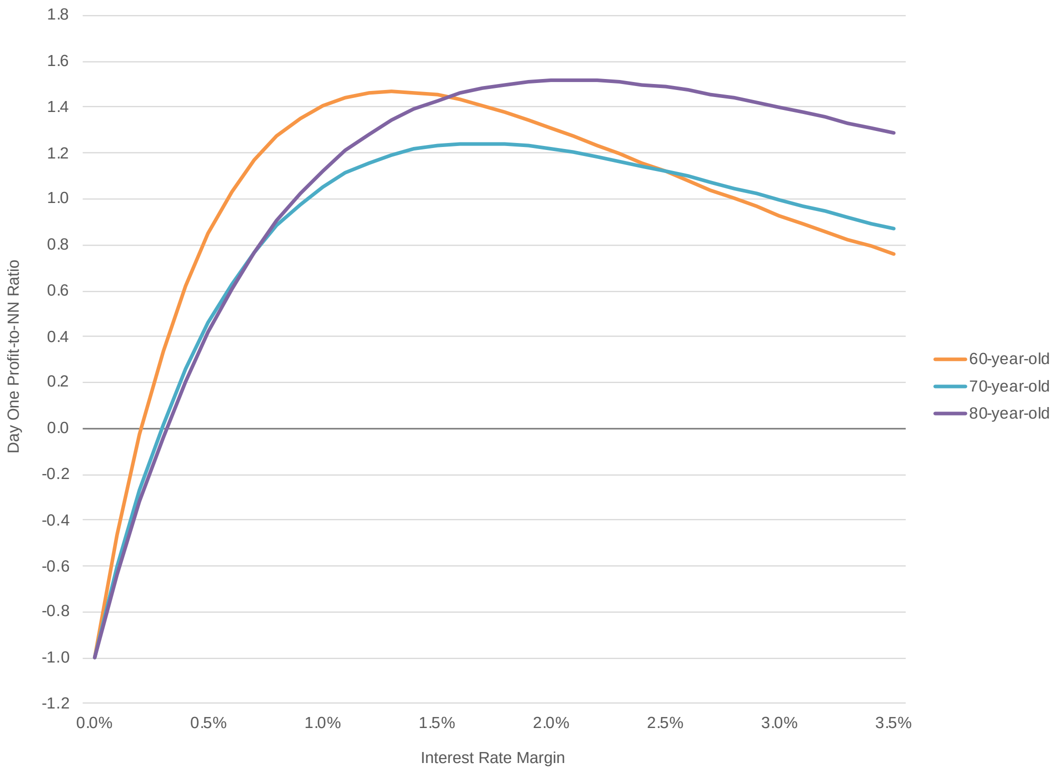

Figure 18. The day one profit-to-NN ratio for different values of the interest rate margins for loans to borrowers aged 60, 70 and 80, where NN is the present value of the NNEG. Authors' calculations.

The results further show that if the lender wants to maximise the day one profit-to-NN ratio, they have to set an interest rate margin of approximately 1.35%, 1.70% and 2.05% for loans to 60-, 70- and 80-year-olds respectively. However, this does not mean that the lender cannot benefit from increasing the interest rate margin. Up until the interest rate margins 2.81%, 2.99% and 4.72% for borrowers at the age of 60, 70 and 80 respectively, for each unit of potential cost the lender is exposed to, the lender receives more than one unit of return. This is illustrated in Figure 18 through the day one profit-to-NN ratio being greater than 1.

6.3.3 Joint Effect

Figure 19 exhibits the joint effect of interest rate margin and LTV on the day one profit for a loan to a 70-year-old. As expected from the above analyses, the interest rate margin drives the day one profit upwards, while the LTV drives them downwards. For any LTV, the lender will increase profits from increasing the interest rate margin. The lender can therefore use the interest rate margin to compensate for giving higher LTVs in order to capture more of the customers' willingness to pay. However, the lender would have to make sure to be within the mortgage regulations if deciding to do so.

Figure 19: *LTV and Interest Rate Margin's Joint Effect on Day One Profit*

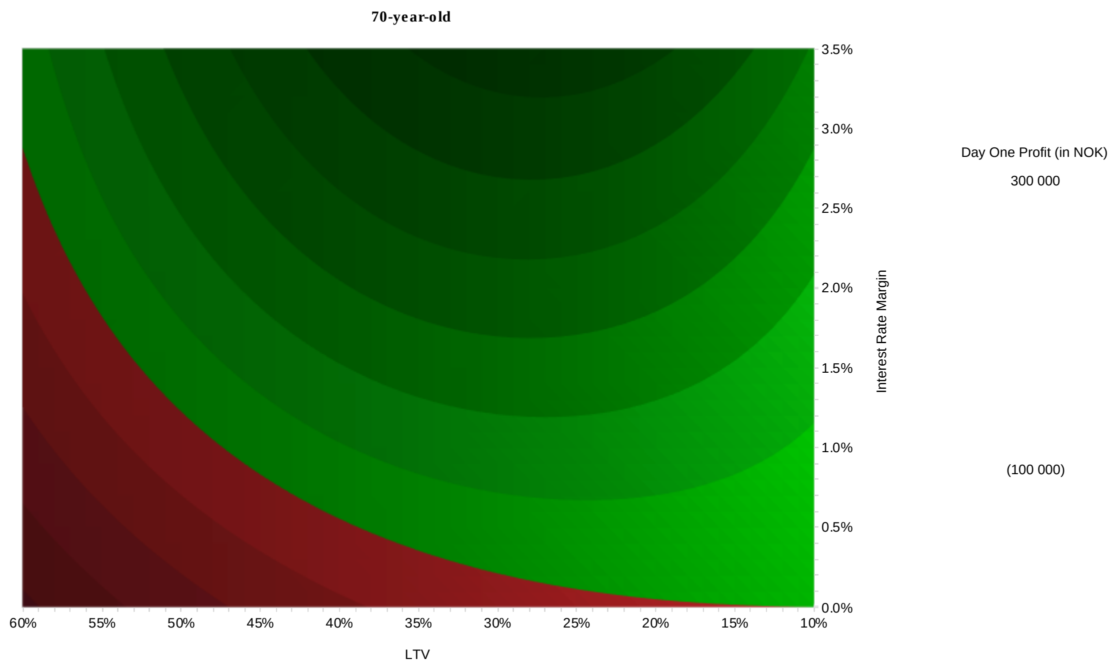

Figure 19. The day one profit for a loan to a 70-year-old borrower for different values of LTV and interest rate margin. The day one profit is shown in intervals of NOK 50 000, where the green area represents positive day one profits, and the red area represents negative day one profits. Authors' calculations.

6.4 Discussion

6.4.1 Initial Age

Our results from the baseline scenario indicate that younger borrowers are the most profitable customer segment. This is also found by Alai et al. (2013), Cho et al. (2013) and Gonçalves (2017), although these studies apply a different profit measure. Younger borrowers are the most profitable due to the interest accumulating over a longer expected time frame, which compensates for the increased potential costs caused by issuing loans to this customer group. In fact, for LTVs below 27%, the day one profit of loans to a 60-year-old will exceed the maximum profit attainable for the other age groups. Only for high values of the net rental yield and LTV, older borrowers become the most profitable. This happens because the loans to younger borrowers will contain so much potential costs that it surpasses the potential gain from the long duration of the loan.

Although most profitable, loans to younger borrowers exhibit a remarkably high risk relative to the lump sum offered to this age group, as shown in the baseline valuation of NN. Alai et al. (2013) and Cho et al. (2013) also found the present value of the NNEG to decrease with age in the Australian reverse mortgage market. This is supported by Gonçalves (2017) using Portuguese data. Indeed, the NN-to-lump sum ratio is over four times larger for a 60-year-old than an 80-year-old in our baseline scenario. This large decrease resonates well with the simulation results from Chen et al. (2010) that found the embedded guarantee in US HECM programmes to drop dramatically with the increase of the borrower's initial age.

Despite the large differences in NN-to-lump sum ratio between the three age groups, our results indicate that the lender might not expect the same deviations in day one profits. In addition, we find the day one profit to be more sensitive to changes in key parameters for this customer group. In other words, the lenders would have to accept a substantial amount of risk issuing loans to borrowers in the lower age span. For interest rate margins over 1.6%, our results indicate that the lenders can expect more profit per unit of potential cost for a loan to an 80-year-old. It could therefore seem that older borrowers are the better investment if the goal is to maximize profits relative to potential costs. However, if the goal is to solely maximize profits, our findings imply that lenders should invest in younger borrowers.

6.4.2 Volatility and Net Rental Yield

Similar to the sensitivity analyses conducted by Dowd et al. (2019), we find that increases in the volatility and net rental yield lead to higher potential costs and lower present value of the loan. Although the lenders are offering different LTVs to borrowers of equal age based on their location, they do not adjust the LTV and interest rate margin to match the assumed volatility and net rental yield of each individual loan. Making such adjustments would give the lenders the opportunity to compensate for the reduction in day one profits caused by higher volatility and net rental yields. However, it is difficult to find true estimates of these parameters. Additionally, these parameter values will change over the loan's lifetime, and even if projections of these parameters were incorporated in the calculations, there will always be uncertainty related to projected values. Due to these uncertainties, all reverse mortgages issued will deviate to some extent from the expectations of these parameter values.

In order to maximise profit, the lenders would naturally prefer the volatility and net rental yield values to be as low as possible, which can explain why the LTVs offered differ based on location. Nevertheless, the results from the profitability analysis indicate that the lenders do not have to be too worried about a loss due to changes in these two parameters, as the volatility and the net rental yield must deviate substantially from the baseline scenario in order for the day one profit to be negative. This is in line with the results from Cho et al. (2013), who found the volatility to be the major contributor to negative equity events, simultaneously as the lenders of lump sum reverse mortgages on average do not face the risk of a negative equity. This implies that lenders could offer the same LTVs to locations with significantly higher volatility and net rental yield, and still expect profits. Although risky and not as profitable, this outlines an opportunity to expand their customer base and in turn increase their income.

6.4.3 Loan-to-Value Ratio

Our results show that higher LTVs cause NN to rise, as is also found by Cho et al. (2013), Institute of Actuaries (2005) and Gonçalves (2017). In the sensitivity analysis conducted by Cho et al. (2013), the NN increases dramatically, which is in line with our results showing that NN is highly elastic to changes in LTV for all ages. However, our analysis further display that LTVs must be substantially higher than current practice for the day one profits to be negative, so it seems that the lenders are certainly protecting themselves against a potential crossover. There might be risk of a crossover associated with one individual loan, but our results indicate that this is not the case for loans in general as long as the LTVs are as current practice.

Furthermore, we find that RM rises for higher LTVs, as is also found by Dowd et al. (2019). Moreover, RM is inelastic in our results, which means that the day one profit will decline for LTVs above our baseline values. The day one profits peak at lower LTVs than currently offered on the Norwegian market, and our sensitivity analysis indicates that solely increasing the LTV is not beneficial if the objective is to maximise profits for each individual loan. Nevertheless, since our results imply that LTVs must be substantially higher than current practice for the lenders to experience loss, the lenders could raise the LTV in order to increase the product's attractiveness within the mortgage regulations' requirements for a maximum LTV.

Moreover, we examined how the borrower's gender affected the profits for different LTVs. Since the lenders are restricted from discriminating based on gender, profits can be affected negatively if most borrowers are female. However, our results indicate that the differences in expected profits are not of great consequence.

6.4.4 Interest Rate Margin

Similar to the other parameters, an increase in the interest rate margin increases the risk of a crossover. This has also been shown through the sensitivity analyses conducted by Dowd et al. (2019) and Li et al. (2010). In fact, Li et al. (2010, p. 518) found that an increase in the contract rate by one percentage point would raise the value of the NNEG by more than 200% for a loan to a 60-year-old male borrower. Although not directly comparable, our elasticity results likewise exhibit drastic changes in the NNEG cost for changes in the interest rate margin.

The interest rate margin is the only parameter we have studied from which values beyond the baseline scenario led to higher day one profits. Our sensitivity analyses suggest that lenders can use this parameter to compensate for high values of volatility and net rental yield in order to ensure that the reverse mortgage loans are profitable, but that the parameter values must be jointly quite high for such a compensation to be necessary. Additionally, the interest rate margin appears to do more than to compensate for the risk of loss. In fact, the reverse mortgage loans would be profitable even for exceptional small values of this parameter. The lenders undoubtedly do not want to break-even, but the lenders' expected day one profit in the baseline scenario indicates extremely high yields for loans to borrowers of all ages. We expect borrowers to accept that the lenders take high yield for high risk, but our analysis indicates that the chance of loss is not grand, and therefore the yields from our baseline scenario seem a little excessive. These results imply that the lenders could lower the interest rate margin along with increasing the LTVs offered, in order to expand the Norwegian market for this product. The results of Alai et al. (2013) gave them premise to draw a similar conclusion for the Australian market. Furthermore, these findings indicate that the reverse mortgage product is an attractive financial product, which can motive other financial institutions to enter the reverse mortgage market.

7. Conclusion

The objective of this master's thesis was to analyse the profitability of reverse mortgage products in the Norwegian market. Inspired by international research, a modified version of the Black-Scholes model was adopted to value the cost of the NNEG. Using an existing approach for valuing the reverse mortgage product, along with termination probabilities, we were able to define our applied measure for profitability: the expected day one profit. As our intent was to study the reverse mortgage product in Norway, we made baseline valuations using input data that was considered realistic for the Norwegian market. This served as the basis for our sensitivity analyses, through which we studied how changes in initial age, gender, volatility, net rental yield, LTV and interest rate margin influence the day one profit, both separately and combined.

Due to the demographic changes in Norway, we believe that products such as the reverse mortgage will prove important in the coming years. Large parts of personal savings go into homes, and without a product for releasing home equity, these savings become unavailable for those who wish to remain in their property towards the end of their lives. Liquidity usually declines when going into retirement, and as future generations are expected to receive smaller pension payments than the generations before them, the demand for alternative ways to fund retirement will rise.

Having in mind that applying the Black-Scholes model on the housing market in order to find the NNEG costs is disputed, we found the reverse mortgage product to be highly profitable for the lenders, and that loss only occurs in extreme scenarios. In order for the product to be unprofitable, the key parameters must deviate largely from their baseline values.

When studying how the product's profitability for the lenders change depending on the borrower's age, our results show that investing in younger borrowers is significantly more profitable than investing in older borrowers. However, lenders can expect a higher profit-to-cost ratio for older borrowers when the reverse mortgage contract rates are at the current level offered on the Norwegian market.

Our profitability analysis further indicates that with the LTVs offered on the market today, there is on average not much chance of loss for the lender; the LTVs need to be remarkably higher to experience negative day one profits. However, in order to maximise the day one profits, the LTVs should actually be lowered from current practice. So, unless the reduction of the day one profit due to an increase in LTV can be compensated by an increase in product attractiveness, we found that increasing the LTV would not be wise from a profit maximising perspective.

Furthermore, the contract rates for reverse mortgage loans are higher than other mortgage loans due to the embedded guarantee. Although this may sound fair to the borrowers, our results imply that the lenders can expect high profits on these products. According to our findings, the interest rate margin could in fact be lower than that of current traditional mortgage loans, and the reverse mortgages would still be profitable. This suggests that the current contract rate does more than compensate for the risk of a crossover, and that Norwegian lenders could decrease the contract rate in order to expand the reverse mortgage market.

To our knowledge, this study is the first to analyse this product on the Norwegian market and to use the day one profit as a measure for assessing the reverse mortgage profitability. Our research may therefore set the grounds for further research on this subject. Several different approaches have been used internationally to price the NNEG, and these approaches could also be applicable in Norway. Further research that wants to apply the Black-Scholes framework, can extend our research by including time-dependent estimates of house price volatility, net rental yield and interest rates. For this purpose, a vector autoregressive model can be used to simulate relevant economic variables, taking their interdependencies into account. This model can also be applied to derive stochastic discount factors, and the results can be compared with the results from an option pricing framework. However, projecting economic scenarios with statistical tools over a long time horizon should be interpreted with caution. Moreover, further research could account for more modes of termination, for example by assessing how the profitability is affected by using a multiple state model. We also encourage further studies to include measurements such as value at risk and conditional value at risk in a sensitivity analysis in order to quantify lenders' level of financial risk associated with investing in reverse mortgages in Norway. This can bring meaningful insight in light of the product's profitability and thereby sustainability on the financial market.

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Appendix 1

Table A.1: Lump Sum Results from LittExtra's Loan Calculator

Age	Property value: 5 000 000	Property value: 4 000 000	Property value: 3 000 000	Property value: 2 000 000	Property value: 1 000 000
60	22.20%	22.25%	22.33%	22.00%	22.00%
60-61	+ 0.80%	+ 0.75%	+ 0.67%	+ 1.00%	+ 1.00%
61-62	+ 0.80%	+ 0.75%	+ 1.00%	+ 1.00%	+ 1.00%
62-63	+ 0.80%	+ 1.00%	+ 0.67%	+ 0.5%	+ 1.00%
63-64	+ 1.00%	+ 0.75%	+ 1.00%	+ 1.00%	+ 1.00%
64-65	+ 0.80%	+ 1.00%	+ 0.67%	+ 1.00%	+ 0.00%
65-66	+ 1.00%	+ 1.00%	+ 1.00%	+ 1.00%	+ 1.00%
66-67	+ 1.00%	+ 0.75%	+ 1.00%	+ 1.00%	+ 1.00%
67-68	+ 1.00%	+ 1.25%	+ 1.00%	+ 1.00%	+ 1.00%
68-69	+ 1.00%	+ 1.00%	+ 1.00%	+ 1.00%	+ 1.00%
69-70	+ 1.00%	+ 1.00%	+ 1.00%	+ 1.00%	+ 1.00%
70-71	+ 1.20%	+ 1.00%	+ 1.33%	+ 1.00%	+ 2.00%

71-72	+ 1.20%	+ 1.25%	+ 1.00%	+ 1.00%	+ 1.00%
72-73	+ 1.20%	+ 1.25%	+ 1.33%	+ 1.50%	+ 1.00%
73-74	+ 1.20%	+ 1.00%	+ 1.00%	+ 1.00%	+ 1.00%
74-75	+ 1.20%	+ 1.25%	+ 1.33%	+ 1.50%	+ 1.00%
75-76	+ 1.20%	+ 1.50%	+ 1.33%	+ 1.00%	+ 2.00%
76-77	+ 1.40%	+ 1.25%	+ 1.33%	+ 1.50%	+ 1.00%
77-78	+ 1.40%	+ 1.25%	+ 1.33%	+ 1.50%	+ 1.00%
78-79	+ 1.40%	+ 1.50%	+ 1.33%	+ 1.00%	+ 2.00%
79-80	+ 1.40%	+ 1.50%	+ 1.33%	+ 1.50%	+ 1.00%
80 →	+ 0.00%	+ 0.00%	+ 0.00%	+ 0.00%	+ 0.00%
80	44.20%	44.25%	44.00%	44.00%	44.00%

Table A.1: LTVs offered to a borrower aged 60 and 80, and the increase in LTV due to a one-year increase in age withing this age gap, for different property values. The outputs are retrieved from using LittExtra's (n.d.-a) loan calculator. The results were retrieved September 18, 2020 and are applicable for a single borrower, not living in a housing association with joint debt, with postal code 1050.

Appendix 2

Unlike the standard Black-Scholes model for assets paying dividends, the Black '76 model uses forward prices for finding the NNEG.

The price of a put option using the Black-Scholes model dividend-paying assets is given by

$$P_0 = Ke^{-rt} \times N(-d_2) - S_0e^{-gt} \times N(-d_1), \tag{A.1}$$

where $d_1 = \frac{\ln\left(\frac{S_0}{K}\right)+\left(r-g+\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$ and $d_2 = d_1 - \sigma\sqrt{t}$.

Furthermore, the Black 76' formula for pricing a put option is expressed as

$$P_t = e^{-rt}[K_tN(-d_2) - F_tN(-d_1)], \tag{A.2}$$

where $d_1 = \frac{\ln\left(\frac{F_t}{K}\right)+\left(\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$ and $d_2 = d_1 - \sigma\sqrt{t}$.

K_t is the exercise price given by the accumulated loan amount by period t, which grows with the contract rate: u. It is given by

$$K_t = K_0e^{ut}. \tag{A.3}$$

F_t is the forward price, thus the price agreed now to be paid on possession in period t. It is defined as

$$F_t = S_0e^{(r-g)t}, \tag{A.4}$$

Where g is the continuous deferment rate, which is equal to the net rental yield of the property.

If we add Eq. (A.3) and (A.4) into (A.2), it gives us the following equation:

$$P_t = e^{-rt}[K_0 e^{ut} N(-d_2) - S_0 e^{(r-g)t} N(-d_1)],$$

$$\text{where } d_1 = \frac{\ln\left(\frac{S_0 e^{(r-g)t}}{K_0 e^{ut}}\right) + \left(\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \text{ and } d_2 = d_1 - \sigma\sqrt{t}. \tag{A.5}$$

Simplifying Eq. (A.5) gives

$$P_t = K_0 e^{(u-r)t} N(-d_2) - S_0 e^{-gt} N(-d_1)], \tag{A.6}$$

$$\text{where } d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r-u-g+\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \text{ and } d_2 = d_1 - \sigma\sqrt{t}.$$

The only difference between Eq. (A.1) and (A.6) is the incorporation of the contract rate u, and thus for $u = 0$ the equations are identical. However, u is also included in studies valuing the NNEG using the Black-Scholes model both with and without dividend yields, because the exercise price grows with the contract rate u (Ji et al., 2012; Li et al., 2010; Tsay et al., 2014). Black '76 and the Black-Scholes model for dividend-paying assets give the same NNEG valuations for $g \neq 0$, and both these models will give the same NNEG valuations as the standard Black-Scholes model for $g = 0$.