# Faithful accounting in MMP-elections

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## ABSTRACT

In MMP-elections for legislatures, political parties compete for a voter's support in two ways: for a *first vote* to the party's candidate in a single-seat constituency and for a *second vote* to the party's list of candidates. To obtain party representation proportional to the second votes, a nationwide second vote tally compensates with list seats to parties with a sub-proportional number of constituency seats.

The German Bundestag has 299 constituency seats, and 299 list seats is the legal norm, but it got 410 list seats in the 2017 election. This deviation also violates a principle, stated by the federal constitutional court, that voters have equal influence.

First and second vote may support different parties, but in the same ballot. The ballots' *combinations* of first and second vote are essential for approximation of both proportionality and the legal norm. Unfortunately, the combinations are ignored: The result would have been the same if first and second votes had been collected in separate ballot boxes.

Compensation based on *"faithful accounting"* uses the combinations to represent the set $\Lambda(P)$ of voters with second vote to party P according to the set's size, rather than representing P itself, while keeping the number of list seats close to the norm.

Another seat reduction, of 42 seats with 2017 data, is obtained if CDU and CSU, working as one party in the Bundestag, also run as one party in elections.

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Faithful accounting in MMP-elections:

## INTRODUCTION

MMP (Mixed Member Proportional) is a class of methods to elect legislatures, with c representatives from single seat constituencies, and q from candidate lists from r parties $P_j$ ($1 \le j \le r$). List seats are for those p parties $P_j$ that pass a threshold criterion ($1 \le j \le p \le r$). The first MMP election was in W-Germany 1949.

A ballot supports, with an *Erststimme* (first vote), the *constituency* candidate from a party $P_j$, and, with a *Zweitstimme* (second vote), the candidate *list* from *any party* $P_h$. *Split ballots*, with $j \neq h$, but *both Stimme in the same ballot paper*, are legal from 1953.

The German electoral law requires $299 = c \le q$, but establishes c=q as a norm. Each of the p parties that pass the threshold, wins a number of seats *proportional* to its number of Zweitstimme; p=7 in the 2017 election (Table 1). Ranked by Zweitstimme, these seven parties (with the number of seats won in the constituencies) are:

(0.1) CDU (185), SPD (59), AfD (3), FDP (0), Linke (5), Grüne (1), CSU (46).

Added list seats give proportionality with the Zweitstimme:

$$(0.2)\quad \frac{185+14.531}{12447656} = \frac{59+93.913}{9539381} = \frac{3+91.224}{5878115} = \frac{0+80.139}{4999449} = \frac{5+63.884}{4297270} = \frac{1+65.658}{4158400} = \frac{46+0.000}{2869688}$$

The Zweitstimme sum is 44189959. Only CSU got no list seats. See (Table 1).

The numerators' sum in (0.2) is $c+q=299+409.348\ldots=708.348\ldots$ seats, which is the *critical assembly size*, i.e. the smallest assembly size that allows proportionality.

With 2013 data (Table 1), critical size is 511.484... seats; see (1.10). The 2017 rules would have approximated proportionally at c=q=299, giving CSU 7 list seats.

In the real 2013 result however, (c, q)=(299, 332): Complicated rules for geographic allocation of list seats overruled the norm c=q; thus CSU got 56 seats (11 list seats).

The critical size changed, from 511.484... (2013) to 708.348... (2017), due to changed voting pattern; however, the size of the difference, 196.8... seats, stems from a tradition of regarding *parties* as "account owners" in the Const columns of Table 1:

*Traditional accounting* regards the $j^{th}$ entry as *party* $P_j$'s success in constituency elections, although it is the success of all *voters with Erststimme* to a winner from $P_j$.

*Faithful accounting* regards the set $\Lambda(P_j)$ of *voters* with Zweitstimme to $P_j$ as account owner. In reality, the $7^{th}$ Const column entry of Table 1 (2017), (=46, i.e. *all* seats where $P_7$ ran), is the success of 3255487 voters with Erststimme to their constituency winner. Members of $\Lambda$(CSU) contribute, *at most*, 2869688 of these Erststimme.

Under faithful accounting, all voters carry (a part of) their Erststimme success to the account of their $\Lambda(P_j)$. Faithful accounting records, *at most*, a proportional share, i.e.

(0.3) $46 \times 2869688 \cdot 3255487^{-1} \approx 40.549$ seats on $\Lambda$(CSU)'s account. [1]

Also this is too high: Many of the 2869688 voters in $\Lambda$(CSU) split their ballot, supporting a constituency candidate *not* from CSU, often for *non-political* reasons. [2]

Much less than 40.549... are then left on the account with $\Lambda$(CSU) as new owner. Replacing CSU's 46 in (0.1) by 39 (say) for $\Lambda$(CSU), the critical size drops,

(0.4) from $46 \times 44189959 \cdot 2869688^{-1} \approx 708.348$ to $39 \times 44189959 \cdot 2869688^{-1} \approx 600.556$.

Faithful accounting is proposed as a remedy, both against an assembly size out of control and against a concomitant flaw, which lets some ballot splitting *double* a voter's *influence* on the outcome. Faithful accounting rules are in section 2.

**[1] CSU won all 46 constituencies where it ran (i.e. in Bavaria), so all the 3255487 ballots were successful.**

**[2] The amendment of 1953 is called "*personalisierte* Verhältniswahl" (personalized proportional election).**

# 1 TRADITIONAL ACCOUNTING

***Notation*** In an MMP-election, there are

(1.1) c *single-seat constituencies* $C_k$ $(1\le k\le c)$ and r *parties* $P_j$ $(1\le j\le r)$.

Each ballot contains two *Stimme* (votes); one *ErSt*, i.e. Erststimme (first vote), for a candidate in the voter's $C_k$, and one *ZwSt*, i.e. Zweitstimme (second vote), for a list of candidates from a party $P_j$ in a nation-wide tally.

Usually there is a Plurality election (first-past-the-post) in $C_k$; other methods may be considered. If parties must pass a *threshold* in order to qualify for list seats, they may be enumerated so that, for some p $(p\le r)$, $P_j$ passes when $j\le p$:

(1.2) $\Lambda(P_j)$ is the set of voters with valid ZwSt for $P_j$ $(1\le j\le r)$;

$\Omega = \Lambda(P_1) \cup \Lambda(P_2) \cup \cdots \cup \Lambda(P_r)$ is the set of voters with a valid ZwSt;

$\Omega^* = \Lambda(P_1) \cup \Lambda(P_2) \cup \cdots \cup \Lambda(P_p)$ is the subset in contest for party list seats.

A set X has $|X|$ members. A *common threshold requirement* is that $|\Lambda(P_j)|$ is above a certain percent of the $|\Omega|$ valid ZwSt.

***Constituency success for $P_j$*** Set an indicator variable $\delta(j,k)=1$ if a candidate from $P_j$ won the constituency seat in $C_k$. Party $P_j$'s *constituency success* $\boldsymbol{\omega}(j)$ is defined as the number of constituency winners from $P_j$ $(1\le j\le r)$:

**(1.3)** $$\boldsymbol{\omega}(j) = \delta(j,1) + \delta(j,2) + \cdots + \delta(j,c)$$

***Proportionality*** For given $\boldsymbol{\omega}(j)$, the ZwSt-tally "ideally" distributes $\alpha(j)$ party list seats to $P_j$ to obtain proportionality, i.e. so that the ratio $\rho$ of ZwSt per seat,

(1.4) $\rho = |\Lambda(P_j)|\cdot[\alpha(j) + \boldsymbol{\omega}(j)]^{-1}$, *is the same for all* j $(1\le j\le p)$.

See (0.2). Solutions $(\rho, \alpha(1), \ldots, \alpha(p))$ of (1.4) have non-negative real numbers: $\rho$ may be decreased arbitrarily; properly increased $\alpha(j)$ give other solutions. Similarly, $\rho$ may be increased until $\alpha(j)=0$ for some j, and $\rho$ gets its *maximal value* $\rho^*$.

More explicitly, *define* m so that $P_m$ has *most constituency winners* $\boldsymbol{\omega}(j)$ *per* ZwSt:

(1.5) $$\boldsymbol{\omega}(m)\cdot|\Lambda(P_m)|^{-1} = \max_j [\boldsymbol{\omega}(j)\cdot|\Lambda(P_j)|^{-1}] \qquad (1\le j\le p).$$

Proportionality factor $\rho$ in (1.4) is maximal at $\alpha(m)=0$. For proof, rewrite (1.4) as

$$\rho^{-1} = |\Lambda(P_j)|^{-1}\cdot\alpha(j) + |\Lambda(P_j)|^{-1}\cdot\boldsymbol{\omega}(j) = |\Lambda(P_m)|^{-1}\cdot\alpha(m) + |\Lambda(P_m)|^{-1}\cdot\boldsymbol{\omega}(m)$$

By definition (1.5), $|\Lambda(P_j)|^{-1}\cdot\boldsymbol{\omega}(j) \le |\Lambda(P_m)|^{-1}\cdot\boldsymbol{\omega}(m)$; therefore (1.4) implies

(1.6) $$|\Lambda(P_m)|^{-1}\cdot\alpha(m) \le |\Lambda(P_j)|^{-1}\cdot\alpha(j)$$

If $\alpha(m)>0$, then, by (1.6), $\alpha(j)>0$ for $j\neq m$. Thus, $\rho$ grows to $\rho^*$ as $\alpha(m)$ drops to 0. (See e.g. (0.2), where $P_m=P_7=$CSU.) By (1.4),

(1.7) $$\rho^* = |\Lambda(P_m)|\cdot\boldsymbol{\omega}(m)^{-1}$$

By (1.7) and (1.4), at $\rho=\rho^*$,

(1.8) $$[\alpha(j) + \boldsymbol{\omega}(j)] = |\Lambda(P_j)|\cdot[\rho^*]^{-1} = |\Lambda(P_j)|\cdot|\Lambda(P_m)|^{-1}\cdot\boldsymbol{\omega}(m).$$ [3]

This is the smallest solution of (1.4). Aggregation in (1.8) ($1\le j\le p$), gives

(1.9) the "critical" assembly size $= \boldsymbol{\omega}(m)\cdot|\Omega^*|\cdot|\Lambda(P_m)|^{-1}$ seats, with $\alpha(m)=0$

***Pivotality*** Data *specific* for $P_m$ determine $\rho^*$ by (1.7). Together with $|\Omega^*|$, they also, by (1.9), determine the minimal assembly size which is compatible with the proportionality requirement (1.4). For this reason, $P_m$ will be called *pivotal*. See (0.3).

***Critical and ordinary size*** Germany's federal legislature, Bundestag, has q list seats; q is variable with a legal bound: $c\le q$, but "ordinary size" is $c=q=299$. A seat distribution algorithm is run until the assembly has reached or passed critical size, and all constituency seats have been "re-distributed". If critical size is below ordinary size, also the pivotal party may get some list seats. (Table 1 and (1.10) show one critical size far above the ordinary 598 (2017) and one far below (2013), but the geographic allocation caused distribution of 631 seats in 2013. [4]

**[3] Obviously, α(j) is rational but, with high probability, not an integer in a solution of (1.4).**

**[4] A feedback of geographic allocation on the assembly size gave the Bundestag 631 seats in the 2013 election, although the critical size, by (1.10), was only 511.484… . With 2017 rules, 85 more party seats would have brought the size from 513 (Table 2) to 598.**

***Geographic allocation*** of all $\alpha(j)$ seats from $P_j$'s party lists, $1 \le j \le p$, gives each state the sum it is entitled to. From 2017, allocation is separate from the ZwSt-tally.[5]

| | | 2013 | | | | 2017 | | | |
|---|---|---|---|---|---|---|---|---|---|
| j | $P_j$ | ErSt | Const | Lis | ZwSt | ErSt | Const | Lis | ZwSt |
| 1 | CDU | 16233642 | 191 | 17 | 14921877 | 14030751 | 185 | 15 | 12447656 |
| 2 | SPD | 12843458 | 58 | 99 | 11252215 | 11429231 | 59 | 94 | 9539381 |
| 3 | AfD | – | – | – | – | 5317499 | 3 | 91 | 5878115 |
| 4 | FDP | – | – | – | – | 3249238 | 0 | 80 | 4999449 |
| 5 | Linke | 3585178 | 4 | 48 | 3755699 | 3966637 | 5 | 64 | 4297270 |
| 6 | Grüne | 3180299 | 1 | 50 | 3694057 | 3717922 | 1 | 66 | 4158400 |
| 7 | CSU | 3544079 | 45 | 0 | 3243569 | 3255487 | 46 | 0 | 2869688 |
| Qual'd | | 39386656 | 299 | 214 | 36867417 | 44966765 | 299 | 410 | 44189959 |
| 3 | AfD | 810915 | 0 | – | 2056985 | | – | – | – |
| 4 | FDP | 1028645 | 0 | – | 2083533 | | – | – | – |
| | SmPoGr | 2398826 | 0 | – | 2718921 | 1422850 | 0 | – | 2325533 |
| | INV/BL | 684883 | – | – | 583069 | 586726 | – | – | 460849 |
| | TOTAL | 44309925 | 299 | 214 | 44309925 | 46976341 | 299 | 410 | 46976341 |

**TABLE 1** Integer approximations of the seat distribution *at critical size* are shown for Bundestag elections of 2017 and 2013. With traditional accounting, the $j^{th}$ entry of Const is the ErSt-success $\omega(j)$ of *party* $P_j$. However, SmPoGr are many small political groups influencing the constituency results with "split ballots".

***Integer approximation*** to a solution of (1.4) is usually delivered by an algorithm. Elections to Bundestag use the algorithm of Daniel Webster (1832) and André Sainte-Laguë (1910).

Applied also to 2013 data, (1.9) illustrates the volatility of the critical size: [6]

**(1.10)** $\omega(m)\cdot|\Omega^*|\cdot|\Lambda(P_m)|^{-1}$

$= 45\cdot 36867417\cdot 3243569^{-1} = 511.484\ldots$ (2013); [7]

$= 46\cdot 44189959\cdot 2869688^{-1} = 708.348\ldots$ (2017)

[5] **The separation improves transparency. Studies of the election method no longer depend on allocation rules.**

[6] **Without the legal lower bound (598), the 2017 tally rules give assembly size 513 in 2013.**

[7] **Also counterfactually, for fair comparison with 2017, one may lower the threshold requirement from 5% to 4% of $|\Omega|$=43726856, including AfD and FDP in the 2013 tally; then $|\Omega^*|$=41007935: With seven parties, the critical size goes, from 45×41007935/3243569=568.927… in 2013, to 46×44189959/2869688 in 2019.**

If critical size is not above 598, distribution by 2017 rules gives at least 299 list seats to parties; also $P_m$ may receive some. However, in 2013 a complicated feedback, of geographic allocation on the assembly size, gave the Bundestag 631 seats.

***Approximation and accuracy*** Ratios $|\Lambda(P_j)|\cdot[\alpha(j)+\boldsymbol{\omega}(j)]^{-1}$, in 2013 spread around $36867417\cdot[299+214]^{-1}\approx 71866$ (Table 1), illustrate the approximation task:

$$|\Lambda(\text{CDU})|\cdot[191+17]^{-1} = 14\,921877\cdot 208^{-1} \approx 71740 \text{ ZwSt/seat}$$
$$|\Lambda(\text{SPD})|\cdot[58+99]^{-1} = 11\,252215\cdot 157^{-1} \approx 71670 \text{ ZwSt/seat}$$
$$|\Lambda(\text{Linke})|\cdot[4+48]^{-1} = 3\,755699\cdot 52^{-1} \approx 72225 \text{ ZwSt/seat}$$
$$|\Lambda(\text{Grüne})|\cdot[1+50]^{-1} = 3\,694057\cdot 51^{-1} \approx 72432 \text{ ZwSt/seat}$$
$$|\Lambda(\text{CSU})|\cdot[45+0]^{-1} = 3\,243569\cdot 45^{-1} \approx 72079 \text{ ZwSt/seat}$$

An analogue: Each $P_j$ is committed to "pay" for its seats in ZwSt-currency. With 2013 data, a *price reduction* stops at *ordinary size* 598: $36867417\cdot 598^{-1}\approx 61651$ ZwSt/seat. The pivotal $P_m$=CSU then affords 52 seats: $3243569\cdot 61651^{-1}\approx 52.688$.

***Webster/Sainte-Laguë*** Based on the data vector $(\boldsymbol{\omega}(1), \boldsymbol{\omega}(2),\dots, \boldsymbol{\omega}(p))$, this method deals out seats one-by-one to the parties.

Party $P_j$ contests for its $s^{th}$ seat $P_j$-s with *contest number* $|\Lambda(P_j)|\cdot[2\cdot s+1]^{-1}$. Thus, n 2013, Webster/Sainte-Laguë deals seat 502 to CSU-44 with contest number

$$|\Lambda(\text{CSU})|\cdot[2\cdot 44+1]^{-1} = 3243569\cdot 89^{-1} \approx 36445.$$

| 505 | 506 | 507 | 508 | 509 | 510 | 511 | 512 | 513 |
|---|---|---|---|---|---|---|---|---|
| SPD-155 | CDU-206 | CDU-207 | SPD-156 | Grü-51 | CDU-208 | Lin-52 | SPD-157 | CSU-45 |
| 36181 | 36130 | 35956 | 35950 | 35865 | 35784 | 35769 | 35721 | 35644 |

**TABLE 2** This stretch of seats dealt out by Webster/Sainte-Laguë is from the Bundestag election 2013. By (1.10), critical size is 511.484... . With 511 seats, CSU is, by proportionality, entitled to 44 seats only; CSU-45 is still in *overhang*. [8]

At 512, CSU-45 is out of overhang, but loses to SPD-157. Winning a constituency, CSU-45 is assembly member by law anyway, but also happens to get seat 513.

Then all constituency seats are both paid and re-distributed.

[8] **At any stage of distribution, party P has some *overhang seats* if strict proportionality only entitles it to less.**

However, 85 more list seats are required to obtain q=c=299: With 2017 rules, seats 514, ..., 598 would follow to five parties in proportion to their ZwSt, i.e. $|\Lambda(\text{CDU})|$, $|\Lambda(\text{SPD})|$, $|\Lambda\,\text{Linke})|$, $|\Lambda(\text{Grüne})|$, and $|\Lambda(\text{CSU})|$

***Ballot-splitting*** A split "QP"-ballot has ErSt to party Q and ZwSt to another party P. Some QP-voters simply regard P as the best party and Q as the party with the best candidate in the voter's constituency. Which party that benefits from their decisions may vary randomly across c constituencies. In Table 1, however (c=299). differences between each party's ErSt- and ZwSt-entries show that millions of split ballots have CDU, SPD, and CSU as Q and AfD, FDP, Linke, and Grüne as P.

Many QP-ballots come from voters who split an originally intended PP or QQ ballot, with effect according to (1.9). When Q is pivotal, the effect is particularly important:

Assume first that some QP-ballots came from PP, giving Q (=CSU=$P_m$) instrumental help to win u of the 46 constituency seats in the Bundestag 2017. Originally, the factor in (1.9) had been $\boldsymbol{\omega}(m)=46-u$, $u>0$. The action raised critical size,

(1.11) from $[46-u]\cdot|\Omega^*|\cdot|\Lambda(P_m)|^{-1}$ to $\mathbf{46}\cdot|\Omega^*|\cdot|\Lambda(P_m)|^{-1} = 708.348\ldots$ .

Assume next that x QP-ballots came from QQ and helped some party P to win list seats by increasing $|\Lambda(P)|$. But this action also *reduced* $|\Lambda(Q)|$ in (1.9) by x (here Q =$P_m$=CSU); it also raised critical size,

(1.12) from $\boldsymbol{\omega}(m)\cdot|\Omega^*|\cdot|2869688+x|^{-1}$ to $\boldsymbol{\omega}(m)\cdot|\Omega^*|\cdot|2869688|^{-1} = 708.348\ldots$ .

## 2 FAITHFUL ACCOUNTING

***Constituency success for* $\Lambda(P_j)$** Definitions: In constituency $C_k$,

(2.1) $N(j,k)$ voters in $\Lambda(P_j)$ give ErSt to the winner; thus, the winner in $C_k$ receives $E(k)$ ErSt, where

$$E(k) = N(1,k) + N(2,k) + \cdots + N(r,k).$$

The $N(j,k)$ require a scrutiny in $C_k$ of ballots with ErSt to the constituency winner. [9]

A voter with ErSt to the winner in $C_k$ wins a *seat share* $E(k)^{-1}$. Thus, the voter set

(2.2) $\Lambda(P_j)$ wins $N(j,k)\cdot E(k)^{-1}$ seat shares in $C_k$ $(1\le j\le r)$; each ballot carries its success as a commitment $E(k)^{-1}$ to the voter's ZwSt set $\Lambda(P_j)$.

$N(j,k)\cdot E(k)^{-1}$ is a measure of $\Lambda(P_j)$'s *ErSt success* in $C_k$. In (1.3), the analogue under traditional accounting is $\delta(j,k)\in\{0,1\}$, it measures $P_j$'s ErSt success in $C_k$.

The total ErSt-success $\xi(j)$ of $\Lambda(P_j)$ is a rational number, aggregated over the $C_k$:

(2.3) $$\xi(j) = N(j,1)\cdot E(1)^{-1} + N(j,2)\cdot E(2)^{-1} + \cdots + N(j,c)\cdot E(c)^{-1} \text{ seats.}$$

This is an analogue to $\omega(j)$ under traditional accounting in Table 1; see (1.3).

Aggregation of (2.3) over all $\Lambda(P_j)$, $1\le j\le r$, includes all c constituency seats:

(2.4) $$c = \xi(1) + \xi(2) + \cdots + \xi(r)$$

***Small* $|\Lambda(P_j)|$** Parties that fail to pass the threshold, win $f$ constituency seats, but in 2013 and 2017, $f=0$ (Table 1). However, faithful accounting aggregates tiny seat shares, $E(k)^{-1}$, so one must expect $\xi(j)>0$ for all j in (2.3), $1\le j\le r$. Define $f$ as

(2.5) $$f = \xi(p+1) + \xi(p+2) + \cdots + \xi(r)$$

(Roughly estimated from 2017 data, $\Lambda(P_j)$ for $p<j\le r$, have ErSt-success $f\approx 5$ seats.)[10]

With faithful accounting, a distribution algorithm starts from $c-f$ constituency seats:

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**[9] N(j,k) is ballot information ignored with traditional accounting; one might as well collect ErSt and ZwSt in separate ballot boxes. The tally in 2017 would still have been the same as in Table 1.**

**[10] Of 7441006 ErSt in Bavaria 2017, 3255487 (43.3475%) supported winners, with 3255487/46≈70771 ErSt/seat (CSU won *all* 46 constituencies). By Table 1, *at least* 776806=44966765–44189959 ballots with ErSt to parties that passed the threshold gave ZwSt to a party that failed. If Bavaria is typical, *at least* 0.433475×776806≈336726 of them supported a constituency winner, winning 336726/70771≈4.76 constituency seat shares.**

***Seat distribution under faithful accounting*** The $f$ seats in (2.5) are associated with voters outside $\Omega^*$. An algorithm starts from its *data vector* ,

(2.6) $$(\xi(1),\ \xi(2),\ \cdots,\ \xi(p)).$$

The component sum is c–$f$ constituency seats; $\xi(j)$ of (2.3) is $\Lambda(P_j)$'s commitment in the contest for list seats. Unlike **ω(j) of (1.3),** $\xi(j)$ is not an integer. The algorithm is a divisor method and distributes seats one by one, like Webster/Sainte-Laguë, but *list seats only*. For simplicity, divisors use the sequence of natural numbers like in d'Hondt's method, not just the odd ones as in Webster/Sainte-Laguë's):

(2.7) $\Lambda(P_j)$ contests for its $s^{th}$ list seat with contest number $|\Lambda(P_j)|\cdot[\xi(j)+s]^{-1}$, $(1\le j\le p)$. [11]

With present rules, distribution stops when critical size has been passed, and also a minimal number $q=q_{min}$ of list seats has been distributed ($q_{min}$ =299 in the Bundestag). We now consider the consequences of imposing a fixed number $q=q_{max}$ of list seats.

***Negatives Stimmgewicht*** After distribution of q list seats,

(2.8) the price has become $R = R(q) = |\Omega^*|\cdot[c - f + q]^{-1}$ ZwSt/seat, and the purchasing power of *each ZwSt* is $R(q)^{-1} = [c - f + q]\cdot|\Omega^*|^{-1}$ seats.

Let $R=R(q_{max})$. By (2.3), E(k) influences many commitments $\xi(j)$, but this is not visible from the algorithm's data vector (2.6). With ErSt to the winner in $C_k$, a ballot carries

- one increment, $E(k)^{-1}$, to the *commitment account* of the voter set $\Lambda(P_j)$, and
- another increment $R(q_{max})^{-1} = [c - f + q_{max}]\cdot|\Omega^*|^{-1}$ to $\Lambda(P_j)$'s *purchasing power*.

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**[11] Used on data from Table 1, with integers ω(j) instead of $\xi$ (j), (2.7) gives the same result as d'Hondt's method.**

**With a small number of seats to be distributed, d'Hondt and Webster/Sainte-Laguë are significantly different:**

**A small party P contests for its 1st seat with λ votes against a bigger party Q with μ votes, contesting for its 2nd seat with μ/2 (d'Hondt) or μ/3 (Webster/Sainte-Laguë).**

**For a legislature's internal elections of committee members, this difference is relevant. On suggestion from Hans Schepers (1980), the Bundestag switched from d'Hondt to Webster/Sainte-Laguë. With hundreds of seats, there is no noticeable difference.**

If now, in some constituency $C_k$,

(2.9) $$E(k)^{-1} > R(q_{max})^{-1},$$

then $C_k$'s report from the ErSt tally increases $\Lambda(P_j)$'s commitment $\xi(j)$ more than its purchasing power at $q=q_{max}$, in fact giving all ZwSt from $C_k$ *negative voting weight*. If the ballot is split and carries a large $E(k)^{-1}$ to $\Lambda(P_j)$'s account, it may harm $\Lambda(P_j)$ and $P_j$ with its ZwSt for $P_j$'s list. An expected ErSt-victory with small E(k) could then, conceivably, be used *against* $\Lambda(P_j)$ and $P_j$ by casting a split ballot with ZwSt *for* $P_j$, actually in a *"Trojan Horse strategy"*.

In Germany, there is reason to ask if this is *unconstitutional*.[12]

***Reduced increments*** In case of (2.9), the ballot's ErSt success is a sum:

(2.10) $$E(k)^{-1} = \left[E(k)^{-1} - R(q_{max})^{-1}\right] + R(q_{max})^{-1}$$

In order to avoid Negatives Stimmgewicht, $C_k$ reports increments $R(q_{max})^{-1}$ of the commitments $\xi(j)$ in (2.6), leaving $E(k)^{-1}$–$R(q_{max})^{-1}$ *unaccounted*. Aggregated over all E(k) ballots with ErSt to the winner, the *unaccounted success* from $C_k$ amounts to

(2.11) $$E(k)\cdot\left[E(k)^{-1} - R(q_{max})^{-1}\right] = 1 - E(k)\cdot R(q_{max})^{-1}$$

A fraction $E(k)\cdot R(q_{max})^{-1}$ of $C_k$'s seat is then spread on the commitment accounts.

If the full success $E(k)^{-1}$ was reported as commitment from all $C_k$,, one would avoid all cases of (2.9) by continuing the distribution (2.7) of list seats, increasing $q_{max}$ until

(2.12) $$|\Omega^*|\cdot E(k)^{-1} \le |\Omega^*|\cdot R(q_{max})^{-1} = [c - f + q_{max}], \quad (1\le k\le c).$$

However, one small E(k) is enough to make $q_{max}$ intolerably high. Examples of one small and one large E(k) from the Bundestag election 2017 (not extreme) are

**[12] A ZwSt that harms the party it is supposed to support, has "Negatives Stimmgewicht" (negative vote weight). The German Federal Constitutional Court has declared that the original and much more complicated version of Negatives Stimmgewicht is unconstitutional (Bundesverfassungsgericht 2008).**

(2.13) $E(245) \approx 46511$, and (2.12) implies $q_{max} \geq |\Omega^*| \cdot 46511^{-1} - c + f = 656.096\ldots$ ;

$E(216) \approx 90383$, and (2.10) implies $q_{max} \geq |\Omega^*| \cdot 90383^{-1} - c + f = 194.918\ldots$ .

Increased E(k) will also help to satisfy (2.12): With 2017 data and $f=5$,

(2.14) the law's norm, $299=c=q_{max}$ is, by (2.12), obtained when

$E(k) \geq R(q_{max}) = |\Omega^*| \cdot [c - f + q_{max}]^{-1} = 44189959 \cdot 593^{-1} \approx 74519$ ZwSt $\quad (1 \leq k \leq 299)$.

A majority method in the ErSt-talies helps to increase the E(k). *"W-U"* is a majority method *based on first-past-the-post ballots*, suggested here for consideration:

***The W-U method*** requires three numbers of ErSt:

w, u, and a, for respectively winner W, runner-up U, and all others together.

(2.15) Ballots with ErSt to a candidate *not* in {W, U}

count as half an ErSt for W and half an ErSt for U.

This is a common principle for treating indifference in ranked choice elections.[13]

(2.16) In the examples of (2.13) from Bavaria, W-U works as follows:

In $C_{245}$ (Nürnberg-Süd): (w, u, a) = (46511, 34621, 49493); w is low.

W-U increases E(245) from 46511 to $46511 + a \cdot 0.5 = 71257.5$.

With $q_{max} = 299$ and $R(q_{max}) \approx 74519$ ZwSt/seat, see (2.14), the *unaccounted* part

drops from $1 - 46511 \cdot 74519^{-1} \approx 0.376$ to $1 - 71257.5/74519 \approx 0.044$;

see (2.11), (2.14). The majority defined by W-U reduces the unaccounted success, although not to 0 in $C_{245}$.

In $C_{216}$ (Ingolstadt): (w, u, a) = (90383, 24799, 67290); w is high.

W-U increases E(216) from 90383 to $90383 + a \cdot 0.5 = 124028$.

Since 90383>74519, the full ErSt success is accounted, both with and without W-U.

The E(k) voters in $C_{216}$ carry a surplus of purchasing power to their ZwSt sets

**[13] Another and perhaps more important argument for a majority method is that it improves the legitimacy of the winner as representative for *all* voters in a constituency. Preferably, it should be a ranked choice method. However, when citizens are interested in the representation of their own constituency, and many cast an instrumental ErSt, then the critical size will perhaps not be problematic.**

$$E(k)\cdot\left[R(q_{max})^{-1} - E(k)^{-1}\right] = E(k)\cdot R(q_{max})^{-1} - 1 = 90383\cdot 74519^{-1} - 1 \approx 0.213;$$

it sems clear that Λ (CSU) will receive enough surplus from constituencies with high E(k) to afford at least one list seat. This means that the critical size will have q below the $q_{max}$ considered here (299).

Small E(k) come for different reasons: A small constituency; a small voter turnout; a close contest with three or more candidates. In the latter case, a conventional majority method will make a clear difference. A real majority behind the winner, e.g. after a final runoff, will help to legitimize the constituency winner. However, even W-U should reduce unaccounted ErSt success (2.11) to a tolerable level, and it requires no change in the voting procedure.

***Conventional majority methods*** in the ErSt-tallies will increase most E(k): 2-day elections (Two Round System) are common. Australia has more than 100 years' experience with a ranked choice method. Ireland uses it for presidential elections.

Another, smaller but still substantial reduction of critical size will be obtained, both under traditional and under faithful accounting, if the sister parties CDU and CSU cooperate in elections. (They form one party group in the Bundestag.) Tables 3 and 4 below visualize two ways to cooperate.

# 3 BACKGROUND AND DISCUSSION

***Structure matters*** For every possible set of non-negative entries in the Const column (Table 1), the proportionality requirement (1.4) *implies* that there is a *pivotal entity*, see (1.5), and a *critical size* (1.9). These facts about the solutions of (1.4) are one structural feature of MMP.

The Const column has *accounts*, showing how many constituency seats account owners, i.e. parties $P_j$ or their ZwSt supporters in $\Lambda(P_j)$, are committed to "pay" for from their ZwSt-supply $|\Lambda(P_j)|$. The ownership is a basic structural feature of MMP: In *traditional accounting*, $P_j$ is an entity. In *faithful accounting*, $\Lambda(P_j)$ is an entity, and arguably the natural choice: The $j^{th}$ line of Table 1 includes $|\Lambda(P_j)|$;

$\Lambda(P_j)$ consists of *voters*, while $P_j$ just *nominates* candidates.

A circumstance *special for Bundestag elections* is that the pivotal $P_m$=CSU is small (being a single-state party). (1.9) shows how $|\Lambda(P_m)|$ in 2017 might, by being smaller, have made the critical size even higher. A pivotal CSU will not get any list seat unless critical size is below ordinary size (which happened in 2013). In 2017 however, an increased $|\Lambda(\text{CSU})|$ would give a list seat only if it made another party (CDU) pivotal. The "competition" between CSU and CDU for pivotal status, is tied when

$$46\cdot|\Omega^*|\cdot|\Lambda(\text{CSU})|^{-1} = 185\cdot|\Omega^*|\cdot|\Lambda(\text{CDU})|^{-1}, \text{ i.e.}$$
$$|\Lambda(\text{CSU})| = |\Lambda(\text{CDU})|\cdot 46\cdot 185^{-1} = 12447656\cdot 46\cdot 185^{-1} = 3095092.8\ldots;$$

see (1.9). For CDU to become pivotal, x voters must return to $\Lambda(\text{CSU})$ in (1.12), with x>3095092−2869688=225404. A structural feature of this special political landscape is CSU's *stable pivotal status* – with *high volatility of the critical size*.

An extra ZwSt for CSU seems "wasted". A voter with ErSt for CSU's constituency

winner will not give it to a competitor, but make use of the *doubled influence* offered by present rules, and give the ZwSt to a political ally. If doubled influence pushes the receiving allied party across the 5%-threshold, a jackpot effect may save a government coalition. A side effect is increased assembly size and smaller relative size of CSU in the assembly.[14]

A pivotal entity exists because of (1.4), and the choice of accounting principle defines the election method. Thus, it also decides what incentives voters get, given their preferences and the political landscape, nationwide and in their constituency.

Public discourse on MMP concerns assembly size, constitutionality, legitimacy, and legality. Below, we look at these themes in the context of structural features.

***Proportionality and concentration*** The history of MMP starts in W-Germany in the late 1940s. Those who made the new electoral system remembered the political fragmentation of the Reichstag in Weimar time, under a proportionality rule alone.

They wanted proportionality together with concentration around parties with strong local support. With split ballots (allowed from 1953), instrumental ErSt strengthen the concentration part of MMP. Unfortunately, traditional accounting ignores the ballot's combination of ErSt and ZwSt. Waste of information has two serious side effects, connected to a change in the ZwSt-support of the pivotal party; see (1.12).

Most conspicuous in 2017 was the effect when x voters with ErSt to the pivotal CSU gave ZwSt to former coalition partner FDP, (perhaps thinking, correctly, that they were too few to help CSU to get list seats anyway). If x=250000, they reduced CSU's ZwSt-

**[14] In 2017, there were many more moves from Λ(CSU) to Λ(FDP) than FDP needed to pass the threshold. Luckily, the reduction of |Λ(CSU)| stopped at 2869688 and the increase of critical size at 708.348… ; see (1.9).**

supply $|\Lambda(\text{CSU})|$ from 3119688 to 2869688; see (1.12). Their action also raised the assembly's critical size from 651.598... to 708.348... seats. [15]

The other side effect is *doubled influence*: Two voters, A and B give ErSt to a constituency winner from CSU. A stays in $\Lambda(\text{CSU})$, contributing one ZwSt to pay CSU's commitment (46 constituency seats). As one of many voters, moving from $\Lambda(\text{CSU})$ to $\Lambda(\text{FDP})$, B pays one ZwSt for one more list seat to FDP, see (1.12):

A is in the electoral basis for *one* seat winner; B for *two* seat winners.

***Constitutionality*** With traditional accounting, nothing is exceptional about B's doubled influence: There is always a pivotal party $P_m$. If critical size is above ordinary Bundestag size (598 seats), $P_m$ gets no list seat, and there is doubled influence.[16]

Germany's Federal Constitutional Court, in a ruling of July 3rd 2008, was clear about unequal influence. Obiter dictum, it states a principle of equal influence in para 92. [17]

***No reason for surprise*** It seems that the Bundestag's 111 extra-ordinary list seats in 2017 came as a surprise to most people. However, *two numbers specific for the pivotal party* $P_m$ (CSU) determine the assembly's critical size;

- its *account's number* of constituency winners, i.e. $\boldsymbol{\omega}(m) = 46$, m=7, and
- its percent of the ZwSt, i.e. $|\Lambda(\text{PSU})|\cdot|\Omega^*|^{-1} = 0.064938...$ ; thus,

by (1.9), the critical size is $46\cdot 0.064938...^{-1} = 708.348...$ .

It might be hard to predict these two numbers, but with the obvious incentives for

**[15] CSU was pivotal all the way: Also $|\Lambda(\text{CDU})|$ was reduced when $\Lambda$(CDU)-members left and joined $\Lambda$(FDP), thus, CDU did not take over the role as pivotal party.**

**[16] Moreover, if $P_m$ has a few list seats only, an action to help a political ally is likely to raise critical size above ordinary size; then voters like B will exploit their doubled influence.**

**[17] Aus dem Grundsatz der Wahlgleichheit folgt für das Wahlgesetz, dass die Stimme eines jeden Wahlberechtigten grundsätzlich den gleichen Zählwert und die gleiche rechtliche Erfolgschance haben muss. Alle Wähler sollen mit der Stimme, die sie abgeben, den gleichen Einfluss auf das Wahlergebnis haben.**

supporters of FDP and CDU/CSU to cooperate through split ballots, the result should not have come as a surprise.

CSU's clear pivotal position is a mixed blessing for a small party: CDU, its closest "competitor" for pivotal status, has a reliable ZwSt-source in constituencies dominated by other parties, mainly by SPD: In the 2017 election,

- CDU has 12447656/185 ≈ 67285 ZwSt / constituency seat;
- CSU has only 2869688/46 ≈ 62385 ZwSt / constituency seat; thus,

the pivotal CSU "set a low price"; 709 seats were "bought".

***The basic flaw of traditional accounting*** is to open an account for *party* $P_j$. Ballot information comes from *each member of* $\Lambda(P_j)$. The *electoral basis* for the 46 constituency winners in Bavaria consists of 3255487 voters who win 46 seats *out of 46 possible*: With no ErSt wasted, there are, on average,

$3255487 \cdot 46^{-1} \approx 70771$ *voters in the electoral basis* for a CSU-seat.

Under *faithful accounting*, $\Lambda(\text{CSU})$ will be recorded with a much smaller commitment than the fictitious 46 seats, even below the 40.549... of (0.3). A significant number of voters In $\Lambda(\text{CSU})$ give ErSt (without success) to candidates *not* from CSU, but even for a rough assessment of it, traditional accounting is insufficient; see (0.4).

Faithful accounting uses ballot information from each of the 3255487 voters with ErSt to CSU, finds the voter's ZwSt-set $\Lambda(P_j)$, and records commitment on its account:

Traditional accounting wastes a lot of essential ballot information.

***Problems with and for a small pivotal party*** The sister parties CSU (Bavaria) and CDU (the 15 other states) receive seats with very different frequencies.

In the real seat distributions of Tables 2 and 4, CSU is pivotal ($P_m$=CSU), and smallest in terms of ZwSt. After *fusion* to a new (pivotal) party $P_m$=C*U in 2017, CDU/CSU would afford to pay for their 231 constituency winners at seat 667: [18]

| 659 | 660 | 661 | 662 | 663 | 664 | 665 | 666 | 667 |
|---|---|---|---|---|---|---|---|---|
| FDP-75 | C*U-229 | SPD-143 | C*U-230 | Linke-65 | Grüne-63 | SPD-144 | C*U-231 | AfD-89 |
| 33553 | 33517 | 33472 | 33371 | 33312 | 33267 | 33238 | 33226 | 33209 |

**TABLE 3** This final stretch of Webster/Sainte-Laguë has a counterfactual C*U. The last constituency seats out of overhang are C*U-229, C*U-230, and C*U-231, re-distributed as seat 660, 662, and 666; critical size is 666.426... .

An *"overhang coalition"* CDU&CSU is an alternative to C*U:

Extend Table 4 backwards; as separate parties, CSU pays for CSU-43 at seat 663, and CDU pays for CDU-188 at seat 668; in an overhang coalition they pay their joint commitment of 231 constituency winners at seat 668. [19]

A unification, C*U or CDU&CSU will be useful even under faithful accounting; both make use of the different frequencies of seats to CDU and to CSU: All 299 real constituency winners should, preferably, be seated within ordinary size.

In 2017, natural ballot splitting, with ErSt to CSU and ZwSt mainly moved to FDP, blew up the assembly size and reduced CSU's relative strength. Imagine x=4057 voters returning their ZwSt to CSU; see (1.12): The critical size is reduced by 1;

$$46\times 44189959\cdot[2869688+4057]^{-1} = 707.348...$$ ; new assembly size will be 708.

The real final 9-seat stretch of Webster/Sainte-Laguë after CSU-46 at seat 700, in 2017, shows some details; critical size is 708.348... :

**[18] By (1.9) and Table 1, ω(m)×|Ω*|/|Λ(C*U)| = [185+46]×44189959/[12447656+2869688] = 666.426... .**

**[19] By (1.9) and Table 1, ω(7)×|Ω*|/|Λ(CSU)| = 43×44189959/2869688 = 662.152... ($P_7$=CSU pays at size 663); ω(1)×|Ω*|/|Λ(CDU)| = 188×44189959/12447656 = 667.411... ($P_1$=CSU pays at size 668).**

| 701 | 702 | 703 | 704 | 705 | 706 | 707 | 708 | 709 |
|---|---|---|---|---|---|---|---|---|
| CDU-198 | SPD-152 | FDP-80 | AfD-94 | Linke-69 | CDU-199 | SPD-153 | Grüne-67 | CDU-200 |
| 31513 | 31483 | 31443 | 31434 | 31367 | 31354 | 31277 | 31266 | 31197 |

**TABLE 4** Webster/Sainte-Laguë, 2017 election. If 4057 voters return from Λ(FDP) to Λ(CSU), critical size drops by 1.000; CDU-200, at seat 709 vanishes. With more returns, Grüne-67 vanishes, etc. With smaller |Λ(FDP)|, all FDP-s will be delayed.

Unification as CDU&CSU or C*U is a significant way to reduce excessive assembly size (by 41 or 42 seats) under traditional accounting. It will be a useful contribution even under faithful accounting, which is a much stronger remedy.

Other remedies have also been suggested. Behnke (2020) considers the electoral law of October 2020, with three "mechanisms to contain the Bundestag's size", but expresses doubt (based on simulations) about their efficiency, and about their constitutionality. One of the three is a reduction from 299 to 280 constituencies. Weinmann and Grotz (2020) focus on such reductions.

***Robustness of MMP*** Through changes of rules and despite flaws, the German MMP versions show an essential robustness: When it was difficult to establish a government with parliamentary basis on the left or on the right, then there was a basis for a Grand Coalition, CDU/CSU and SPD. Representatives elected directly from their own constituency, preferably with a broad electoral basis, have a special legitimacy when they support that solution.

Balance between proportionality and concentration gives the theme *"ideology vs. pragmatism"* a place in the political discourse, on level with the ubiquitous *"left vs. right"*. Despite the lopsided 410 list seats vs. 299 constituency seats in 2017, there was still a majority (CDU/CSU and SPD with 399 of 709 seats) sufficient for the Grand

Coalition, which then continued, although after protracted negotiations.

***The balance norm*** The law's norm, q=c, is followed only if the critical size is ≤598¸ then the Bundestag gets 598 seats. However, q–c=111 was accepted in 2017, while a critical size with q<c (as in 2013) is followed up with more list seats for "compensation", even though proportionality is already achieved. It is a fact that c<q is accepted (to avoid overhangs) but q<c is never accepted even if proportionality is achieved. This fact strengthens a common perception that list seats are more important than constituency seats, and ZwSt more important than ErSt.

However, in a party group of size $\alpha(j)+\boldsymbol{\omega}(j)$, see (1.4), $\boldsymbol{\omega}(j)$ *persons* are elected *by ErSt only*, while $\alpha(j)$ "compensation seats" are due *to ErSt and ZwSt together*. All $\alpha(j)+\boldsymbol{\omega}(j)$ seats are for party representatives from $P_j$, but media link assembly size and composition primarily to the $\alpha(j)$, to their sum q, and to ZwSt. [20]

***Other MMP-versions*** Regarding the German experience as positive, some other countries consider, use, or have used their own variations over the MMP-theme. Their experiences however, have not always been good (Linhart & al, 2019).

Mixed member methods may be of the parallel kind, i.e. with no connection between constituency and party list tallies: Parties with a sub-proportional share of constituency seats cannot reach an overall proportional assembly share.

Hungary's single-vote variation transfers, to the second tally from the first tally, "surplus votes" for the winner and votes for losing candidates (Csato, 2016).

[20] **Mit der Zweitstimme werden demgegenüber Parteien gewählt. Die Zweitstimme entscheidet über die Zusammensetzung des Bundestages. Sie ist daher eindeutig wichtiger als die Erststimme.**

MMP-variations reach overall proportionality if there is no upper bound on the number of seats dealt to party-lists. An Albanian variation had 100 constituencies and only 40 list seats. In an election report, OSCE-Odihr (2005) writes: [21]

> *The number of supplemental mandates is relatively small (40) and may not be sufficient to achieve proportionality.*

OSCE describes well the mechanism behind the "doubled influence strategy". [22]

The same description fits MMP in Italy during 1994-2005, with 475 seats to constituencies and 155 to party lists (Mudambi and Navarra, 2004).

However, in an "executive summary" of its final report, OSCE brings up the responsibility for organized use of a *"doubled influence strategy"*:

> *While there is room for further improvement of the legal framework, the law is overall conducive for the conduct of democratic elections. However, the major political parties are yet to demonstrate political will and responsibility commensurate with the broad authority granted to them for the electoral process.*

Conduciveness of the law is a dubious claim: 40 list seats cannot compensate for 100 constituency seats. Albanias ratio was 0.40, but at critical size in the Bundestag, this ratio was 0.711 in 2013 and 1.369 in 2017, see (1.10).

No more than it did in Germany 2017, did ballot splitting in Albania 2005 show any lack of "political will and responsibility" in the major parties. When normal and desired voter behavior has unacceptable consequences, attention should be on the lawmakers' work,

[21] **Organization for Security and Co-operation in Europe**

[22] ***"Using various methods, political parties can circumvent the aim of the election system and distort the allocation of supplementary mandates in their favour. For example, if supporters of a party (Party A) systematically split their votes between the candidate representing Party A and the election list of another, formally or informally allied party (Party B), then the combined number of mandates won by the two parties is likely to be increased. While this strategy might be within the law, it is problematic for a number of reasons:…***

not on parties and their supporters adapting to the rules. [23]

Ballot splitting in Bundestag elections came in 1953, and made instrumental voting easier. Instrumental votes are essential for the concentration part of MMP. In all Plurality elections, instrumental votes help to elect a winner with legitimacy as representative for the whole constituency (Dowding and Van Hees, 2008); majority methods would be an improvement.

***Voters' perception of MMP*** What voters know about the properties of their election methods, what they should understand, and what they would like to understand, are themes that raise special questions when the method is MMP.

One theme is that the assembly size is obviously out of control. The 111 extra-ordinary list seats in 2017 did not even exist before the election. The very notion of candidacy hinges on the existence of an office. Lists of "candidates" for offices that are not known to exist, raise problems of conceptuality and legitimacy. In the words of Hettlage (2018): *Ohne Kandidat, kein Mandat*.

How to explain that about 4000 voters by moving from $\Lambda(P_j)$ make the assembly size grow by 1 (and *thus* reduce $P_j$'s relative strength) – if and only if $P_j$=CSU? To offer an explanation (pivotality and critical size) is not likely to improve legitimacy.

Should lawmakers understand the effect if the sister parties join in an "overhang

**[23] Reporting from Germany's 2017 election, OSCE-Odihr drops charges that parties lack *"political will and responsibility"*, seen in its Albania report of 2005, but also drops the (correct) description of *doubled influence*.**

**In Albania, *two major parties could not win party list seats*. They found themselves in a position like CSU in Germany 2017: Both got a partner, like CDU/CSU got FDP.**

**Partnership is a natural adaptation to present rules, but a twoedged sword. However, the fixed number of *"supplemental mandates"* in Albania (40 of 140) prevented excessive assembly size.**

**In the German version, ballot splitting also pushed up the critical size, which had to be passed with an integer approximation. In the Albanian version, a duel between the two rivals in CSU's role ensued; they fought it over a short supply of 40 list seats, with doubled influence as weapons provided by the lawmaker.**

coalition" CDU&CSU (extend Table 4 backwards) or fuse to C*U (see Table 3)?

Voters in $\Lambda(P_j)$ may accept that the influence of their ZwSt is reduced if their ErSt supports a winner, and reduced more if the ErSt won a large share (e.g. $50000^{-1}$) than if it won a small share (e.g. $100000^{-1}$).

Behnke (2015) and Linhart & al (2020) consider other sides of public understanding.

**MMP in the World** Reynolds & al (2005) survey the use of various families of methods in elections for legislatures (as of 2004):

91 cases of "Plurality/Majority"; 72 of "Proportionality"; 30 of "Mixed systems".

Of the latter, 21 were classified as "Parallel". Only 9 were "MMP", including Italy and Albania, but both had abandoned it by 2006.

For future introductions of MMP, the perception of how it works in Germany will be essential. Support varies over regions and stabilizes variations over time for nationwide parties: A ZwSt resource in constituencies which party P cannot win helps it to pay commitments. With traditional accounting and a single-state pivotal $P_m$=CSU, factors $|\Lambda(P_m)|^{-1}$ and $\boldsymbol{\omega}(m)$ in (1.9) makes the critical size volatile. Moreover, with a pivotal party without list seats comes doubled influence.

A fusion, or an overhang coalition that works like a nationwide pivotal party, will be a limited but noticeable contribution to bring the assembly size under control.

Proportionality to $|\boldsymbol{\Lambda}(P_j)|$, which carries no ErSt information, cannot function properly. Faithful accounting is another, and more potent remedy. It represents the *voter set* $\boldsymbol{\Lambda}(P_j)$ according to ballot information that is now wasted.

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