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On the Choice of Uniform or Personalized Prices in the Digital Economy

Spatial Price Policy in Two-Sided Markets

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Abstract

In this thesis, we examine firms' choice of price policy, uniform or personalized prices, in the digital economy. We focus on firms operating in the digital economy as the prevalence of big data allows firms in this industry the opportunity to apply personalized pricing. We survey relevant literature on product differentiation, two-sided markets and first-degree price discrimination in spatial models. Further, we propose a Hotelling model of a two-sided market consisting of symmetric firms, consumers and advertisers. The firms choose their pricing policies, invest in product quality and set the prices in the advertiser and consumer market. To our knowledge, the model is the first of its kind to combine the choice of price policy, the presence of a two-sided market and quality investments in the same framework. We present two versions of the model, a one-sided duopoly model and a two-sided duopoly model. In the one-sided model we find that the firms will always choose to price discriminate as it constitutes a dominant strategy, complementing the early literature on the field. In contrast, we show that in the two-sided model with an adequately large advertiser market, the optimal price policy strategy is characterized by a mixed strategy. Thus, the firms will not always choose to price discriminate with the introduction of an advertiser market. For the firms, a large advertisement market drives costly quality investments. This has implications on the choice of price policy as pricing uniformly softens the competition in quality relative to personalized pricing.

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1 Introduction

Before the 1860s, prices of goods and services in the retail sector in the Unites States and Western Europe were typically set through interaction between an individual seller and an individual buyer (Phillips, 2012). For instance, when a potential customer entered a clothing store looking for a new coat, the salesperson would attempt to assess the customer's willingness to pay for the coat, before offering the customer a price for the coat. If the salesperson noticed that the customer wore expensive clothing items, the salesperson might presume that the customer was willing to pay a higher price for the coat. The salesperson would also try to figure out whether the customer was in a hurry or willing to stay and negotiate the price.

This type of price setting can be described as a form of price discrimination, namely personalized pricing: offering an individual price to each customer. The downfall of this type of price setting was that in the case of large numbers of customers, it was inefficient for the stores to spend time negotiating and gathering information about each individual's reservation price (Phillips, 2012). However, around this time we saw the introduction of departments stores and price tags. These innovations normalized uniform prices, alleviating the retailers from the inefficient price setting process of personalized prices.

Today, personalized pricing is becoming relevant again. This happens as our everyday lives are becoming increasingly digitalized. Through the internet and other technological solutions, we leave digital traces that allow firms to collect information about us. People generate data all day long. After the alarm goes off in the morning, we typically check our phones, scrolling through various social networks and websites. Instead of reading the newspaper about yesterday's news in hard copy, we often prefer to read today's news online. On our way to work, firms can track the music we listen to and the ticket for public transportation is commonly registered through a mobile app. Furthermore, firms are able to track our location through mapping software. At home, some people habitually order food from Foodora, and eat the meal while watching the latest show on Netflix. Additionally, in recent years, online shopping has become increasingly frequent, enabling firms to track our retail purchase history. By means of all of these generated data, firms are able to track and collect unfathomable amounts of data on an individual level (White House, 2015).

All this data creates large, varied and complex data sets, typically referred to as "big data" (Sagiroglu and Sinanc, 2013). Big data and machine learning algorithms have lowered the firms' cost of collecting and analysing data about their customers' behavior. Through the use of big data, the digital economy provides information on consumer preferences, thereby enabling targeting of pricing, marketing and product characteristics to individual consumers unlike seen before (Choe et al., 2018). Particularly relevant to our paper, the firms may use this data to estimate individuals' willingness to pay for a product (Choe et al., 2018). What once was a slow dance between a salesperson and a customer in clothing shops in the early nineteenth century, has in the digital economy become a highly efficient and scalable process. Furthermore, before the advent of big data, firms could possibly need to wait months before they could analyze the data, which made predictions on sales in near future impossible. Now, the volume, speed and the variety of the data allow the firms to analyze data in real time, granting high precision and adaptability to predict consumer and market characteristics.

There are both empirical and theoretical examples of possible applications of big data and its advantages. A real-life example is the Netflix show "House of Cards". The successful show was created based on analyses of big data on consumer preferences. The show is a result of combining the preferences for the actor Kevin Spacey, the producer David Fincher and the previous BBC show "House of Cards" (Erevelles et al., 2016). Shiller (2013) offers a more theoretical approach. He uses consumer data from 2006, including almost 5000 potential website browsing explanatory variables, to examine if consumer data can be used to raise firms' profits. Shiller (2013) concludes that Netflix's profits can be increased by 12.2 percent by using website browsing behaviors for personalized pricing. In contrast, the paper estimates that Netflix can raise their profits only by 0.8 percent if they use demographics to price discriminate the consumers.

To recapitulate, we have a newfound relevance of personalized pricing due to big data. Intriguingly, even though firms have obtained these advanced ways to offer personalized prices, consumers still experience personalized pricing to a relatively limited degree in the digital economy. We will survey existing economic literature, which offers insights to why this is the case. Generally, the economic literature looks at the firms' incentives to offer personalized prices under different competitive settings, as well how these incentives change with different extensions.

We contribute to the literature by presenting a duopoly location model using the traditional spatial model with linear transportation costs as proposed by Hotelling (1929). The model consists of consumers, firms and advertisers; thereby, it also includes the element of two-sided markets.[1] Furthermore, we model that the firms have the ability to invest in product quality, and to choose whether to price discriminate or to have uniform prices in the consumer market.

We find it suitable to include two-sidedness in the model when studying price discrimination in the digital economy, as firms operating in the digital economy often are characterized by their capability to connect users and advertisers. Moreover, with the introduction of two-sided markets, we may get contrasting pricing implications compared to when simply considering a one-sided market. Examples of two-sidedness in the digital economy include online newspapers, who typically earn revenue from both readers and advertisers, social media such as Facebook and search engines such as Google. In addition, the e-commerce part of the company Amazon is increasingly dependent on advertisement revenue (Perrin, 2018). In fact, Amazon might be the most relevant real-life case to our model as it has a business model that easily allows for either uniform prices or personalized pricing. However, it is debatable whether or not the e-commerce part of Amazon is operating in a duopoly or if it is a monopoly.

We analyse two iterations of our model. In the first iteration, we disregard the advertiser market, thereby examining a one-sided model. In the second iteration, we include the advertiser market, enabling us to compare the results with and without two-sidedness. Thus, the question arises: how will the introduction of a two-sided market affect the firms' decision of pricing policy when the firms have the ability to invest in quality?

Our thesis is structured as follows. In chapter 2, we offer an overview of the existing economic literature on product differentiation, price discrimination and two-sided markets. In chapter 3, we present a duopoly location model. In chapter 4, the equilibria are derived

[1] A possible definition of a two-sided market is that the firms connect at least two distinct groups of users, the consumer market and the advertiser market. Additionally, at least one of the groups exhibits network effects on the other. More on this is in section 2.3 Two-sided markets.

and analysed. Chapter 5 discusses the results and chapter 6 concludes.

2 Literature review

We begin the literature review by giving a brief introduction of firms' price setting. Secondly, we investigate oligopoly price competition and product differentiation in section 2.2. In that section we also present a basic Hotelling model which we will later expand on. Thereafter, we complement this exploration by reviewing the relatively new theoretical field of two-sided markets. Finally, we look specifically at first-degree price discrimination in a monopoly and a duopoly situations. We expand on the review of first-degree price discrimination in a duopoly situation by examining different aspects to consider when studying this type of competition. The literature review also relates to the later chapters, as our model is based upon several of the elements presented in this chapter. Also, it provides a theoretical foundation to discuss our own model and its contribution to the literature.

2.1 Price setting

Any given firm will strive towards a pricing policy that maximizes its profits.[2] A pricing policy is a firm's method of setting a price on its products based on market characteristics. A firm operating in a market with perfect competition does not have any market power and in the long run it faces a market price equal to its marginal cost of producing the good. On the contrary, a firm having market power is able to influence its own price or its quantity produced, not taking them as given (see e.g. Franck and Peitz, 2019).

Economic theory usually divides firms with market power into three main forms of market structures: monopoly, oligopoly and monopolistic competition. A monopolist operates in a market absent of any actual or potential competitors. The two other market structures are seen as less extreme, with characteristics from both perfect competition and monopoly.

If we depart from the market characteristics and look at price setting specifically, a natural division of pricing policies is uniform pricing and discriminatory pricing. Uniform pricing is as the phrase suggests: a policy of pricing the provided good at the same price for every buyer. The theoretically optimal way to price uniformly is to produce the quantity

[2] It is possible to argue that the managers of firms might have other objectives. Nonetheless, this is a premise is made by most economic theory. For a further discussion see *The Theory of Industrial Organization* (Tirole, 1988).

where the firm's marginal cost equals its marginal revenue and charging the corresponding price.[3] In contrast to uniform pricing, price discrimination is when the same product is sold by a firm at different prices. Formally defined by Stigler (1987) as *"... when two or more similar goods are sold at prices that are in different ratios to marginal costs"* (Varian, 1989, p. 598). A firm must have market power to be able to price discriminate. A firm without market power is a price-taker, rather than a price-setter, and therefore price discrimination will not be an option anyway.

If a firm chooses to price discriminate, the firm may earn a greater profit than a firm with uniform prices. This is possible by charging a higher price for consumers that have the highest willingness to pay (WTP). The outcome of price discrimination differs depending on what kind of market structure the firm operates in. In some cases, price discrimination leads to a higher producer surplus. While in other cases, price discrimination leads to a higher consumer surplus. The coming sections of the literature review will look further into this.

In standard economic theory, if a firm has market power and prices uniformly, the firm must take several considerations into account when setting a price with the aim of maximizing profits. In the most simplistic approach, a higher price yields a higher revenue from each consumer, while it will also scare away some consumers. Conversely, a lower price will attract more consumers while yielding lower revenue per consumer. In the case of personalized pricing, these mechanics cease to exist. One is no longer interested to see how a given price affects the total demand of the good, as the price is tailored to the specific individual's willingness to pay. The price offered to one individual does not affect the price given to another individual, which will further affect the price setting and competition between firms as we will see later.

A requirement for a successful price discrimination, is that the firm must prevent resale (Varian, 1989). If a consumer can buy the product at one price and then resale the same product to another consumer at a higher price, the price discriminatory policy will not work. The low-price consumer will earn profit from resale, and the high-price consumer can buy the product at a lower price than from the firm directly. Further requirements for

[3] If the firms operate in a two-sided market they would need to correct the marginal revenue for the existence of cross-group network externalities. The concept of cross-group network externalities will be looked into in section 2.3 Two-sided markets.

price discrimination are that the firm needs different customers with different willingness to pay, and a way to identify these differences or to make them self-select into different groups. The firm will not be able to charge different prices if all the customers have the same price sensitivity of demand.

Price discrimination is seen in different forms. Economic theory commonly classify price discrimination into three categories, following the traditional classification of Pigou (1920): first-degree, second-degree, and third-degree price discrimination (Varian, 1989).

First-degree price discrimination occurs when each consumer receives an individual price, also called personalized prizing.[4] In most instances, it is naturally difficult for a firm to identify every single consumer's willingness to pay. Due to the impracticality of first-degree price discrimination in most markets, at least before the arrival of big data, the two other types have been by far most prevalent. Nonetheless, personalized pricing has been seen to some extent in industries such as hotel and airline agencies (Mohammed, 2017). Further, the e-commerce part of Amazon is suggested to previously having experimented with personalized pricing (Wallheimer, 2016).

The firms also have the option to offer slightly different products at different prices, which often is easier than identifying a single consumer's reservation price. This differentiation between the products can be found in both quality and quantity. This is described as second-degree price discrimination. Digital companies, such as the music streaming service Spotify and the online video platform YouTube, use pricing policies such as the freemium model, which is a form of second-degree price discrimination (Sato, 2019). The freemium model is based on offering different versions of the product, one ad-based, free version and an ad-free, higher quality, subscription version of the product.

The last category of price discrimination is characterized by offering a different price for the same product to different consumer groups, which is called third-degree price discrimination. An example of third-degree price discrimination is to divide the consumers into different age groups and offer different prices to the different groups (Armstrong,

[4]There is a distinction between the terms first-degree price discrimination and personalized pricing. First-degree price discrimination refers to charging the consumers their full reservation value, while personalized pricing involves charging the consumers different prices, however not necessarily the full reservation price (Shiller, 2013). We have used the term personalized pricing upon till now, as it is more precise. Practically, it is far-fetched to consider situations in which the consumers' full reservation prices are identifiable. However, for theoretical purposes, first-degree price discrimination is often a more suitable term. We will use the terms interchangeably throughout the rest of this paper.

2008). For instance, Spotify make use of third-degree price discrimination by offering students a lower price. In some ways first-degree and third-degree price discrimination are becoming more indistinguishable (Varian, 2018). New technology enables digital companies to personalize the user-products. Following the example of Spotify, they would no longer have to settle for a discriminatory practice between students and other listeners. They can, in theory, tailor the product to each listener.

The firm's decision of which type of price discrimination to implement depends on the type of information the firm has access to. If the firm has sufficient information about each customer's demand curves before the transaction, the firm can charge each customer a personalized price. If the firm can identify the demand of different customers groups, it can practice third-degree discrimination. When the firm cannot differ between the types of consumers before the purchase, the firm can offer different pricing packages and the customers will self-select into their own preferred option. The last example mentioned will be second-degree price discrimination through offering different packages containing quantity discount and different versions of the product.

2.2 Product differentiation

In between the two extremes of perfect competition and a monopoly market lies oligopoly competition. Oligopoly competition is composed of competition among a few firms with some degree of market power. Whereas the economic theory regarding the two extremes can be described to be more general and absolute, the theory of how industries are organized is more complex, and the theoretical modelling needs to be fitted to the characteristics of a specific industry. A classic disjunction in industrial economics is made between price competition and quantum competition, drawing on the influential works of economists Joseph Bertrand and Antoine Cournot respectively. The difference between the two types of competition regards the firms' choice variable: whether they set and compete in prices or quanta.

Assuming homogeneous products, the consumers will always buy the cheapest product. Therefore, Bertrand competition leads to the famous result that firms will undercut each other's prices until they reach their marginal costs and there are no profits left, given that their marginal costs are equal. This result has been named the Bertrand paradox as the high market concentration and seemingly weak competition can become extremely intense and the firms compete away super-normal profits with the addition of only one rival from the monopoly situation. Given the unfortunate nature of this type of competition the firms are eager to seek away from this situation.

Chamberlin (1933) argues that firms in monopolistic competition can have market power as long as the firms do not have homogeneous products. Thus, firms can avoid the Bertrand paradox by offering differing product characteristics, namely product differentiation. Lancaster (1979) defines two types of differentiation: vertical- and horizontal differentiation. Vertical differentiation is when the goods have different quality and every single consumer would prefer one good over the other good if they were sold at the same price. That is to say the product is objectively better. On the other hand, horizontal differentiation refers to a differentiation between goods where consumers cannot agree which good they would prefer at equal prices. In this case the firms differentiate their products to appeal to the consumers' subjective preferences.

2.2.1 Hotelling's linear city

Harold Hotelling (1929) conceptualizes the idea of horizontal differentiation using a spatial model. His model, often referred to as Hotelling's linear city, consists of two firms (firm 1 and 2) competing for consumers uniformly distributed along a line. We will consider a line normalized to have length 1. The distribution of consumers along this line represents their heterogeneity in preferences. The firms' locations on this line characterize their degree of product differentiation. Given that firm 1 is located (weakly) to the left of firm 2, we denote their respective distance to the end points, 0 and 1, as a and b. Implying that $a, b \geq 0$ and $1 - a - b \geq 0$. If they are located at the exact same spot along the line, we have $a = b = \frac{1}{2}$ and the products will consequently be homogeneous. Thus, we are back at the Bertrand paradox.

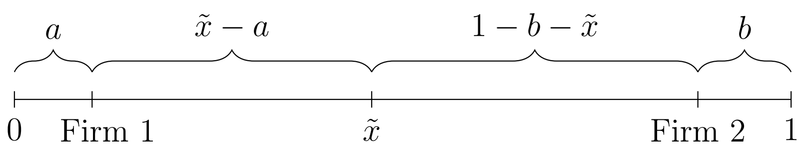

Figure 2.1: Hotelling's linear city

When deciding which product to buy, the consumers' preferences will deviate from the specifications of a firm's product. The consumers experience a transportation cost which can be deemed as the mismatch between the consumers' preferences and the characteristics of a firm's product. The transportation cost is increasing in the difference between the consumers' preferences and the actual product characteristics. This is where spatial models prove valuable. If the spatial model represents actual geographical space, naturally you experience a higher transportation cost the further you travel. Obviously, you would prefer buying the same bag of groceries for the same price at the supermarket closest to you as you would find this less of a hassle. In the same way, deviating further away from your preferred variety of a product would be costly.

The brief presentation above provides the foundation for the most basic version of the model. The utility of buying from firm 1 is depicted by:

$$U_1 = v - p_1 - t|a - x|$$

Correspondingly, the utility a consumer would get from buying from firm 2 would be:

$$U_2 = v - p_2 - t|(1 - b) - x|$$

Where v represents the consumer's gross willingness to buy the product. p_i is the price paid to firm i $(i = 1, 2)$. The transportation costs of travelling to, meaning purchasing, from either firm 1 or 2, are $t|a-x|$ or $t|(1-b)-x|$ respectively. The consumer will purchase from the firm providing him/her the highest utility. In this case, this will be equivalent to the lowest generalized cost. Figure 2.1 above also depicts the indifferent consumer, $\tilde{x}$, who obtains the same level of utility from buying from each firm. Mathematically, the indifferent consumer can be found by setting $U_1 = U_2$ and solving for x. Consequently, the area spanning from the indifferent consumer to each side of the line represents each firm's demand. The demand for firm 1 will then be $\tilde{x}$ and $(1 - \tilde{x})$ is the demand for firm 2.[5]

Formally, the demand functions can be written as:

[5] We assume that consumers are single-homing, meaning that they buy one, and only one, unit of the good.

$$x_1(p_1, p_2) = \tilde{x} = \frac{p_2 - p_1}{2t} + \frac{1 + a - b}{2}$$

$$x_2(p_1, p_2) = 1 - \tilde{x} = \frac{p_1 - p_2}{2t} + \frac{1 + b - a}{2}$$

The firms maximize the following profit functions:[6]

$$\pi_i = x_i(p_i, p_j) \cdot (p_i - c)$$

By differentiating the profit function with respect to prices and utilizing the expressions for the demand functions, it can be shown that the price setting will be given by the following reaction functions:

$$p_1^R(p_2) = \frac{p_2 + c}{2} + \frac{t(1 + a - b)}{2}$$

$$p_2^R(p_1) = \frac{p_1 + c}{2} + \frac{t(1 + b - a)}{2}$$

Lastly, we can maximize firm 1's profit with respect to its location.

$$\frac{d\pi_1}{da} = \frac{p_1 - c}{4} > 0$$

We obtain the result that a firm has incentives to localize inwards in the line. This result arises because of Hotelling's assumption of a linear transportation cost, each firm will earn a marginally larger market share if they move towards the middle of the line. As a result of the model, both firms would locate in the middle of the line next to each other. This is the reasoning for *the principle of minimum differentiation*.

The observant reader would notice that the principle of minimum differentiation pushes the firms back to the Bertrand paradox, creating a stability issue for this equilibrium, which is pointed out by the influential work of d'Aspremont et al. (1979). They show that

[6] c stands for the firms' marginal cost, whereby we assume $c_1 = c_2 = c$.

two conflicting effects are influential in the firms' price setting. On the one hand, the firms will try to differentiate as little as possible as that gives the largest possible market size, however they will also want to push toward differentiation as that enables them to set higher prices. The effects are illustrated in figure 2.2:

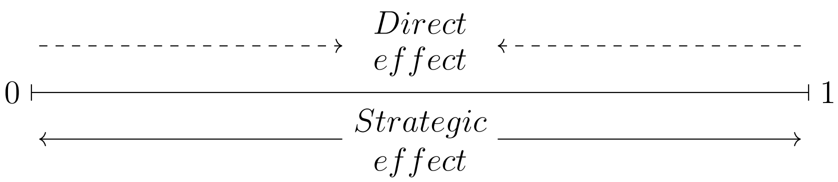

Figure 2.2: The direct effect versus the strategic effect

D'Aspremont et al. (1979) demonstrate that if the linear transportation costs are substituted with quadratic transportation costs, the firms will now locate at each end of the line.[7] This originates the *principle of maximum product differentiation*. In this scenario the strategic effect dominates the direct effect.

The findings of d'Aspremont et al. were later confirmed by Economides (1986). However, he points out that the specific degree of differentiation found by d'Aspremont et al. only holds for quadratic forms of the utility function, and therefore, the principle of maximal differentiation is not a general result. Rather, he shows that if one uses a less convex, but not linear utility function ($t(d_i) = t(1 - a - b)^{\alpha}$ with $1 < \alpha < 2$), the equilibrium outcome is an interior location on the line.

This illustrates the importance of the parameter t, which in the digital economy can be said to model the consumer's choosiness or loyalty for a specific brand. In models of spatial differentiation, the consumer's preference, or more precisely disutility of deviating from its preference, is what allows the firms to escape the Bertrand paradox. The larger the $t(d_i)$, the more it costs the consumer in terms of utility loss to deviate from its preference, and the larger market power the firm has over its consumers.

2.3 Two-sided markets

A defining feature of many digital companies is that they can be characterized as two-sided markets. A two-sided market can be described as a market (i.e. a platform) that connects

[7]In fact, they show that the firms will prefer to locate outside the line at $a = b = -1/4$ if allowed for.

at least two distinct groups of users, where at least one of the groups exhibit cross-group network externalities (also referred to as cross-group network effects). This means the group of users' value of the platform is dependent on the number of users of the other group. Following the phrasing of Evans (2003), the platform internalizes the externalities between the different groups of users. Furthermore, for the market to be considered as a two-sided market, the two different groups of users cannot be able to connect to each other without the platform.

Although two-sided markets have existed in various forms throughout history, it was only just after the last change of millennium that it emerged as a new theoretical field in economics. One of the first articles on the field is the paper by Rochet and Tirole (2003) regarding the economics of payment card associations. A credit card provider is the intermediary between cardholders and the merchants who accept the card as payment. Implying that we have two distinct user-groups, as well as they display network externalities from one side to the other side, consumers will want to carry cards that are accepted by as many merchants as possible and merchants will seek to offer the predominant payment solution among the consumers. Lastly, following the definition, the cards offered by companies such as Visa and MasterCard, operate as matchmakers and internalize these externalities. Sun and Tse (2007) provide a further specification of the role of the different agents in two-sided markets. In their terminology, it would be more precise to denote Visa and MasterCard as distributors of a network. The distributors produce and sell network-specific products to network participants. Sun and Tse (2007) call the group operating the network for the platform sponsor, which in the case of payment card networks, would be the banks issuing the cards. In many two-sided markets, the role of the distributor is absent, as the network platform sponsor takes on this role by itself.

Subsequent to the birth of the theory, Evans and Schmalensee (2005) provides a categorization of different types of two-sided markets: These are exchanges (platforms facilitating between what we typically think of as buyers and sellers), advertiser-supported media (traditionally media such as newspapers and free television, and now increasingly digital platforms), transaction devices (such as described with payment card associations), and software platforms (for instance video games platforms such as PlayStation and Xbox). Judging based on the development of the literature, this categorization seems to still hold up well. However, the e-commerce part of Amazon, a traditional case of an exchange type two-sided market, is now blurring the lines between the categories as advertising becomes an increasingly bigger part of its business strategy. Even though Amazon's marketplace does not fall under the media category directly, the business model of Amazon today shares many similarities with this field on the literature on two-sided markets.

A key element in the literature on media markets in relation to two-sidedness comes from the interplay between the media consumers and the advertisers. Here, negative cross-group externalities might be present. The literature on the field is ambiguous regarding whether advertisements are increasing or decreasing the consumer utility of using a platform. Naturally, advertisers experience positive network externalities from consumers; however, consumers may experience negative network externalities from advertisers since consumers may dislike advertisements. Papers such as Kind et al. (2009) model this as a nuisance cost of advertisement for the consumers. This leads to asymmetric network externalities and have pricing implications in two-sided markets.

From a theoretical perspective, there is a common distinction in economic theory between informative and persuasive advertising. To some degree, all advertising can be said to have both an informative and persuasive character. Although, the motive and the degree of persuasiveness in advertising is different (Kaldor, 1950). Given that consumers have perfect information about the product, Kaldor argues that advertising may be socially bad. However, informative advertisement may provide information about a product's quality and may therefore have a positive contribution to a consumer's utility. On the other hand, persuasive advertisements are often seen as less appreciated by readers.

The sign of the externality has also been empirically studied. Kaiser and Song (2009) test if media consumers dislike advertising by using data from German consumer magazines. Their study found little evidence that readers of the magazines dislike advertisement. In addition, Kaiser and Song show that consumers appear to like advertising to a higher degree the more informative it is. Wilbur (2008) studies a two-sided, empirical model of television advertising. Wilbur finds empirical evidence for viewers to have an inclination to be averse to advertising. He estimates that a decrease of 10 percent in advertising level will result in a 25 percent increase of audience size.

As we can see, the literature on advertising in markets with network externalities is ambiguous with respect to advertisement's effect on consumer utility, and is likely dependent on its type, the setting and individual preferences. Related to the case of the e-commerce platform Amazon, the advertising on this platform have both informative and persuasive elements to it. Amazon can provide targeted and rather informative advertisement based on your purchasing history. However, if you search for headphones of a specific brand and must toggle through numerous unknown "knock-off" brands, that might be seen as an inconvenience. Becker and Murphy (1993) argue that utility-decreasing advertisements are less likely to be placed in newspapers and magazines because readers can easily ignore advertisements in this type of media.

Naturally, advertisers want to reach out to as many potential customers as possible, however we have seen that the media consumers do not necessarily reciprocate this sentiment. This imposes consequences as to how the platform should construct its pricing structure. They need to balance the advertisers' need for consumers with the consumer's potential distaste of advertising. Armstrong (2006a) summarizes the pricing structure in two-sided markets and states that it will depend on three factors:

1) The relative size of the cross-group externality

Armstrong (2006a) expands on this by stating that "*If a member of group 1 exerts a large positive externality on each member of group 2, then group 1 will be targeted aggressively by platforms*" (p. 668). Aggressive targeting can be thought of as a downward pressure on prices compared to what would have been the case with no network externalities. In fact, one might end up in a situation where the one side is so heavily subsidized that prices fall below marginal cost to zero or even negative prices. This is seen in many of today's business models for digital platforms. In a United States congressional hearing, Facebook's CEO Mark Zuckerberg, famously answered "Senator, we run ads", to the question on how they sustain a business model in which users do not pay for Facebook's services (The Washington Post, 2018). This is in line with the point Armstrong is making: subsidize the users as they exert relatively large externalities on the advertisers.

2) Fixed fees or per-transaction charges

This factor is regarding the pricing scheme itself, and is related to Rochet and Tirole's (2003) distinction between subscription fees (synonymous with fixed fees) and transaction fees. A platform will choose the pricing schemes which suits them best. It might even apply different schemes to different user-groups, as well as allowing users in a group to opt for the preferred scheme.

The essential difference between the pricing schemes is how they affect the cross-group externalities. Armstrong (2006a) states that cross-group externalities are weaker with per-transaction based pricing, the reason being that a part of the marginal benefit of interacting with an extra agent on the other side is eroded by the extra payment incurred. In other words, when an agent only pays the platform per successful interaction, it is not equally important for the agent that the platform first has gotten the other side "on board". The consequence is that the platform profits might be higher under per-transaction charges due to weaker externalities. As we have seen intuitively, the platform profits are highly dependent upon the externalities between the groups.

3) Single- or multi-homing

In the presentation of the Hotelling model, we assume that the consumers are single-homing, meaning that they only buy one good. However, opening for users to buy from several platforms, will impact the outcome. Generally speaking, there are three cases to consider: either both user-groups single-home, one single-homes and the other multi-homes, or lastly, both multi-home.

Armstrong (2006a) shows that single-homing consumers in two-sided markets lead to the concept of "competitive bottlenecks". That is, in the case of digital platforms, if an advertiser wants to reach out to a user of a digital platform, the advertiser must deal with that specific user's choice of platform. The advertiser has no other way of reaching that user. In turn, this gives the platform monopoly power over access to this user. Naturally, this will lead to higher prices being charged to the multi-homing side, and aggressive competition for the single-homing users in the first place. Furthermore, the high profits from the advertiser side may be passed on to the consumer in form of low prices. Empirical studies have found evidence of this in form of subsidization of the readership side in the market for magazines (e.g. Kaiser and Wright, 2006).

2.4 First-degree price discrimination

The literature survey of *Recent Developments in the Economics of Price Discrimination*, by Armstrong (2006b), states that the nature and effects of price discrimination are crucially dependent upon which kind of information that is available for the firm. In fact, some information will cause the firm to make higher profits, and some may lower the prices, which favors the consumers. "Information" can be thought of as which variable of the consumers' utility function the firm can discriminate over. The literature mainly considers types of information such as information of the consumers' valuation v, their choosiness/loyalty t or their location/brand preference x.

It is possible to show that, if the firm has perfect information over consumers' valuation v, price discrimination on this variable will not affect the equilibrium price at all (Armstrong, 2006b). Price discrimination on the choosiness variable t differs from what we get with v. Equilibrium prices are in duopoly frameworks typically dependent on t, and it can be seen that this type of price discriminatory practice can both increase and decrease prices dependent on other factors. However, we will focus on situations in which the firms can price discriminate on the consumers' location/brand preference x, which has been the predominant focus in the literature on the field.

2.4.1 First-degree price discrimination in a monopoly

A monopoly who is able to first-degree price discriminate (FDPD) will charge each consumer exactly its willingness to pay. Thereby, the monopolist will extract all the surplus from the trade and the consumers are left with nothing (Armstrong, 2006b). This holds as long as the consumer's WTP is above the firm's marginal cost of producing the product. In contrast to uniform prices, the market is efficient when a monopoly practices FDPD. The quantity sold is the same as in a perfectly competitive market, therefore no potential economic surplus is lost from a reduction in quantity. Although the deadweight loss is avoided, the consumer surplus is entirely gone. The situation is illustrated in figure 2.3.

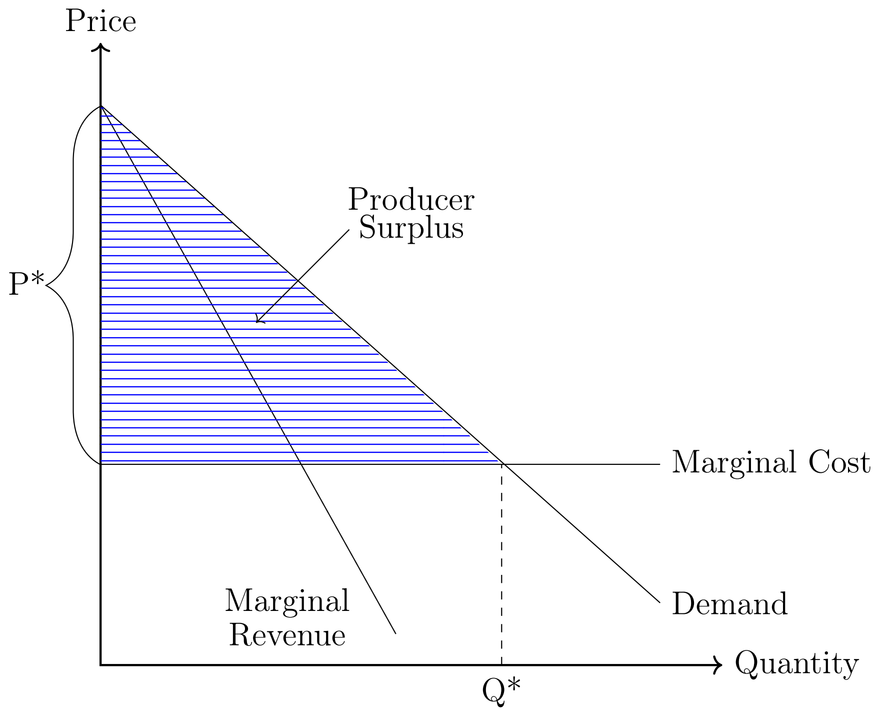

Figure 2.3: First-degree price discrimination in a monopoly

The result heavily relies on the premise of a monopoly situation. As we will see, the outcome changes considerably if we introduce rivaling firms.

2.4.2 First-degree price discrimination in a duopoly

A seminal paper on first-degree price discrimination in a competitive setting is the paper "On the Strategic Choice of Spatial Price Policy" by Thisse and Vives (1988). The paper, which looks at a spatial competition situation, offers contrasting implications to the scenario presented with FDPD in the monopoly situation. Thisse and Vives study the firms' incentives to price discriminate among their consumers, rather than offering a uniform price. In addition, they look at the implication of the price policy on consumer surplus and firm profits.

Thisse and Vives consider two separate cases, either the firms choose their price policy simultaneously or the firms have the option to commit to a price strategy in advance of the price competition. Regardless of what case they consider, they find a general conclusion that "*there is a robust tendency for firms to choose the discriminatory policy...*" (p. 134) over uniform pricing. This is analogous with FDPD in the monopoly situation; however, the firms may be worse off from the discriminatory policy than what they would have been without the possibility to price discriminate. This seemingly counterintuitive result stems from the fact that price discrimination creates a prisoner's dilemma situation whereby each firm would be better off unilaterally implementing personalized pricing, despite the fact they would have been better off pricing uniformly collectively. We will take a deeper look into the model to achieve an understanding of the firm's incentives and the implications on price setting.

We will initially present a simplified version of Thisse and Vives' model, inspired by Foros et al. (2017), thereafter complement the model with the more intuition-based analysis from the original article. The Hotelling line is akin to what we already have presented when looking at spatial competition. The firms are symmetric and thought to have perfect information of the location of all consumers, which enable the firms to provide personalized prices based on the location, x, instead of a uniform price. In the Thisse and Vives set-up, the firms are exogenously located at each end of a Hotelling line. Using our original Hotelling model, presented in section 2.2.1 Hotelling's linear city, this implies that $a, b = 0$. Hereby, we get the following demand functions:

$$x_1(p_1, p_2) = \tilde{x} = \frac{1}{2} - \frac{p_1 - p_2}{2t}$$

$$x_2(p_1, p_2) = 1 - \tilde{x} = \frac{1}{2} - \frac{p_2 - p_1}{2t}$$

The firms profit functions are equivalent as before:

$$\pi_i = x_i(p_i, p_j) \cdot (p_i - c)$$

First, we look at the equilibrium price during uniform pricing. We get the equilibrium price:

$$p_i^* = c + t$$

Further, we have that:

$$\frac{dp_i}{dt} = 1$$

We see that prices are monotonously increasing in the transportation cost. This means as the consumers have higher preferences or loyalty for one of the firms, the consumers are less price sensitive and the firms have more market power over their consumers.

By plotting the price into the profit function, we obtain the profit for each of the firms with uniform pricing:

$$\pi_i^* = \frac{t}{2}$$

Now, we will consider how first-degree price discrimination affect prices and profits in our simplified version. When the firms are price discriminating, the consumers choose to buy from the firm offering the lowest generalized price. The consumer located at point x will buy from firm 1 if:

$$p_1(x) + tx \leq p_2(x) + t(1-x)$$

Contrary to the uniform price setting, the firms now offer a personalized price to each and every consumer. The consumers are characterized by their location x. The firms will offer lower prices to those consumers further away, as they have less market power over them, and the rivalling firm can increasingly match their price. In the middle of the line, none of the firms exhibit any locational advantages.

The firms will not in any scenario offer any less than their marginal cost, however they will be willing to price as low as their marginal cost if that means they will attract that consumer. In other words, the firms will always charge a price that reflects the firms' incremental value of the consumer.

Below, we see a depiction of the equilibrium prices in figure 2.4.[8]

[8]Inspired by Foros et al. (2017).

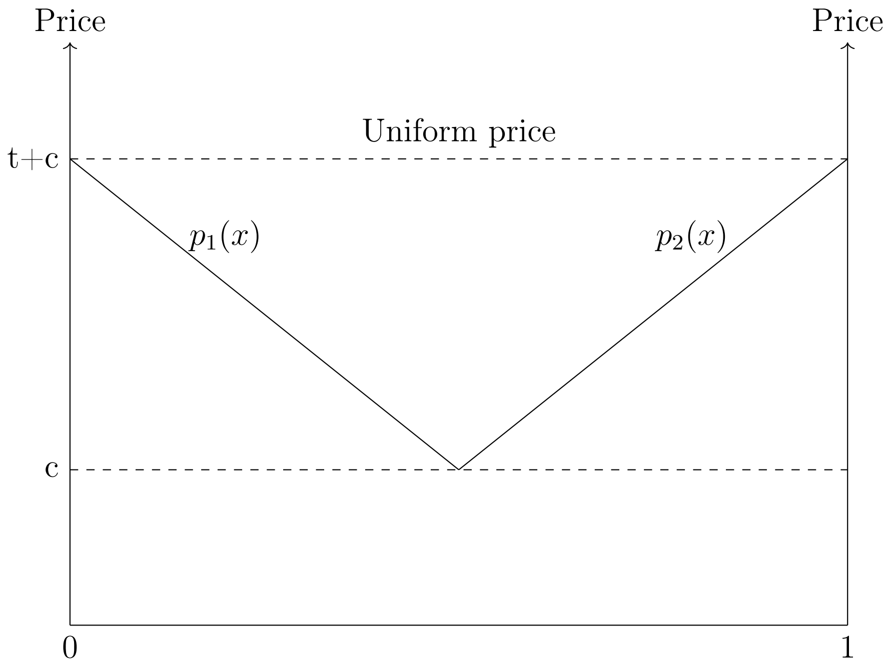

Figure 2.4: Equilibrium market price schedule

As we can see in figure 2.5 below, the firms have, in this example, halved their profits from $\frac{t}{2}$ to $\frac{t}{4}$ by price discriminating rather than pricing uniformly. For firm 1, the areas A and B in the figure make up its profit when pricing uniformly, but when price discriminating, B is transferred to the consumers (Foros et al., 2017).

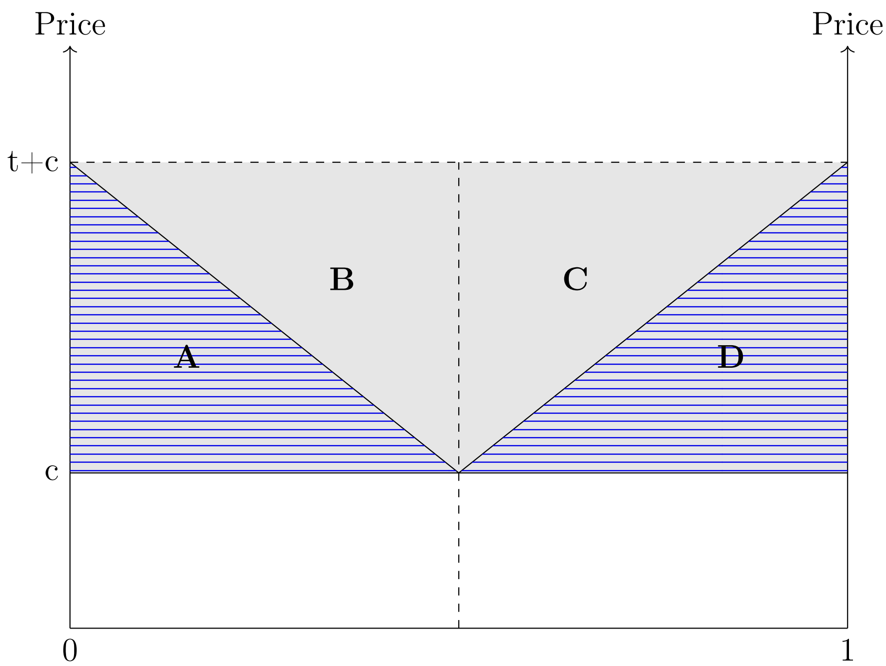

Figure 2.5: Profits under uniform pricing and price discrimination

To see how the firms choose their pricing policies, we will turn over to the two cases presented in Thisse and Vives (1988), the simultaneous and the sequential cases. They provide a more intuition-based understanding of the price setting.

In the case of simultaneous price setting, the paper provides a logical explanation as to why it will always be in firm i's interest to price discriminate given what the competing firm does. They segment the Hotelling line by what type of price setting which will be optimal for the firms at given locations.

In a segment of locations, firm i will charge the monopoly price, $p_i^M(x)$, to the consumers if they are able to. This segment of locations exist if $p_i^M(x)$ does not exceed the total marginal costs of the rival firm j. In this case, firm j cannot price under marginal cost and firm i will price similarly as to what they would have without a competitor.

Another segment is characterized by one firm having a cost advantage over the other firm, meaning the advantageous firm can charge the profit maximizing price given the rival's total marginal cost. The last segment of the line will be when the firm is in an inferior position and will simply prize at its total marginal cost, denoted as $mc_i(x)$.

For each segment of the line, the abovementioned pricing policy is the dominant policy. The monopoly pricing will be the optimal pricing policy for the firm in the interval they have the opportunity to do so. Thereafter, we define $\bar{p}_i(x)$, as the optimal price given the firms cost-advantage position. As for the last segment, the rival firm will in any case be able to provide the best price for consumers and the firm will earn zero profits regardless.

As the different segments comprise the potential market area, Thisse and Vives show intuitively that price discriminating will be the dominant strategy. No matter what the rival does, the firm will always benefit from price discriminating rather than pricing uniformly for any given location of a consumer.

$$p_i^*(x) = \begin{cases} p_i^M(x), & \text{if firm } i \text{ has a monopoly position at } x \\ \bar{p}_i(x), & \text{if firm } i \text{ does not have monopoly position,} \\ & \text{but it has a locational advantage} \\ mc_i(x), & \text{otherwise} \end{cases}$$

The second case presented by Thisse and Vives (1988) is a two-period model where the firms in the first period have the possibility to commit themselves to a uniform price, thereafter, compete in prices in the next period. A way to commit yourself to uniform prices in the digital economy, would be to not collect data in the first period (Matsumura and Matsushima, 2015). If you choose to not collect data about your users, you will not be able to personalize the prices. Conversely, if a firm decides to collect data, one expects it would be in the firm's interest to capitalize on this one way or another. Data collection in the digital economy often occurs through placing cookies onto the user's computer. The cookie stores information about the user's behavior on the internet, and over time it has the potential to build a long-term picture of the individuals Internet browsing history (White House, 2015).

In this game, Thisse and Vives find that it is a dominant strategy to choose the unrestricted price schedule at the first stage. However, as we have briefly touched upon, the game is a prisoner's dilemma in which both parties would be better off committing to uniform prices. However, at the same time they will have incentives to deviate from this strategy. We describe the unrestricted price schedule as "dominant" for this very reason, as the firms by not committing to uniform prices would not regret their decision after learning what the rivalling firm chose. It constitutes a Nash equilibrium.

To understand how the option to price discriminate creates this unfortunate situation for the firms, we need to consider the nature of the competition during price discriminatory practices. The essential difference we observe in how the firms compete under the two price policies is that under uniform pricing the firms compete for the marginal consumer, while that under personalized pricing the competitive field is broadened, and the firms compete for each consumer individually. In the monopoly situation, price discrimination served as a mean to extract the consumer surplus, whereas in the spatial context it is more of a curse with intensified price competition. In fact, in the framework of Thisse and Vives (1988), the two firms compete away their surplus and the consumers gain what the firms loose. Accordingly, the total economic surplus is equal in both cases, and the pricing policy simply affects the distribution of the surplus between the firms and the consumers.

2.4.3 Uniform pricing puzzle

Given that price discrimination constitutes a dominant strategy, we would expect to see more examples of this in the real world. It is suggested that price discrimination is observed unproportionally less than expected in markets where firms can personalize their prices. The rarity of price discrimination has originated the uniform pricing puzzle in the academic literature.

For instance, DellaVigna and Gentzkow (2019) show that most U.S. retail chains choose to price close to uniformly across stores, even though there is a wide variation in customer demographics and competition. Another example of this is Richardson and Stähler (2016), who consider the uniform pricing puzzle in the recorded music industry. The recorded music industry, and specifically the sale of CDs, is characterized by uncertainty among customers as to the type and quality of the music and hence the value they should place upon it. The paper finds uniform pricing to be the optimal action for the firms, as it constitutes a pooling equilibrium across different quality types. Meaning, the firms who are producing records of different qualities choose the same action. The explanation is similar to the findings of Orbach (2004), who studies the motion picture industry. He argues that there is uncertainty regarding the success of newly released movies and there is a notion that prices would be interpreted as quality signals, thus discriminating the sales of low-quality goods.

Furthermore, in the wake of Thisse and Vives (1988), several papers contribute to expand the field of literature on first-degree price discrimination. Thereby, the papers provide several explanations to this puzzle. Besides the academic literature, as FDPD has become more relevant with the increasing prevalence of big data, it has become more of a discussed topic in other fora as well. We will now review different aspects to consider when studying FDPD and expansions to the original Thisse-Vives model.

2.4.4 Aspects to consider when studying price discrimination

Dynamic or static framework

Several studies present a dynamic framework to study price discrimination, in opposition to the static model of Thisse and Vives (1988). The models are dynamic in the sense that they explicitly incorporate the stage in which firms gather customer information through interacting with them, before being able to price discriminate in later stages. In relation to the literature on the field of price discrimination, these models may also be referred to as behavior-based pricing (BBP). The Thisse-Vives model is described as static since the firms simply have the choice to price discriminate or not, without having to interact with users in earlier stages.

In the literature survey of "Recent Developments in the Economics of Price Discrimination" by Armstrong (2006b), the author suggests that BBP models in competitive settings can be categorized as two different cases. The first case is when firms react to previous consumer decisions, whereby the firms in the second period try to "poach" the consumers who preferred the rivalling firm's product, also called ex-post pricing as the price discrimination happens after the first period. The second case is when the firms in the first period offer explicit loyalty schemes to the consumers, named ex-ante pricing as the firms offer pricing schemes before the consumers do anything. In other words, in the first case the firms do not commit to prices beforehand, but they do in the latter.

When dealing with dynamic models such as these, it is necessary to make assumptions as to whether consumer preferences change between the periods. On the one end, one could assume correlation in brand preference between the periods. Alternatively, consumer preferences can be considered independently distributed over the two periods. Further, models such as Chen (1997) considers switching costs as well.

Armstrong (2006b) shows how both practices may involve welfare losses. The first case (ex-post pricing) may lead to excessive switching between firms, while the second case (ex-ante pricing) may lead to excessive loyalty. Armstrong (2006b) states that the key difference is that in ex-post pricing, the aggressive pricing is aimed at the rival's consumers. While the ex-ante loyalty scheme involves pricing low at your own customer base.

In case of ex-post pricing, excessive switching implies that too many consumers switch which firm they purchase from in between the periods, thereby purchasing from the less preferred firm. On the other hand, excessive loyalty implies that the rivalling firm will have inferior conditions to compete for the other firm's customers in the second period, thereby inhibiting what can be described as "healthy" competition. Specific outcomes are dependent upon assumptions made in each case, however, generally said, the aggressive pricing indicates that these types of competitions might be beneficial for the consumer, and detrimental for profits of the firms (Armstrong, 2006b). Also, in general, we expect that when firms price discriminate in the ex-post case, profit is decreasing in the extent of correlation of brand preference, while consumers benefit from a higher correlation. The reason being that there will not be any motives to try to poach the consumers if the consumer brand preference is fully uncorrelated between the periods. However, in case of ex-ante pricing, the more uncorrelated they are, the firms must provide the consumers more attractive loyalty offers. This implies the firms' profits are increasing in the extent of correlation in brand preference in the ex-ante case, which is the opposite to what we saw under ex-post pricing.

We will look a bit more into the case of ex-post pricing. The main part of the early literature on BBP considers third-degree price discrimination after the firms have collected data on the consumers. Fudenberg and Tirole (2000) offer such a framework with stable consumer brand preferences, in which firms learn where all consumers purchased in the first period. However, the firms do not get any more specific details about the consumer preferences. They find that more information about the consumers triggers aggressive competition and a prisoner's dilemma situation occurs (Foros et al., 2018).

This framework is evolved by Choe et al. (2018) to regard personalized pricing in the second period. Choe et al. study the implication of big data on price discrimination. Their focus is "pricing with cookies", a way of tracking consumer data and collecting information in the digital economy. Information collection can provide highly personalized pricing. They point out that the collection of information by firms typically will be asymmetric, as the information a firm retrieve from tracking the behavior of a consumer naturally is only available to that specific firm. On the other hand, the firms are endowed with symmetric information about the consumers in the Thisse-Vives model. Furthermore, Choe et al. find that the asymmetric information leads to equilibria in which the firms try to exploit their acquired information to increase their market share in the second period. This allows them to charge higher prices to consumers in second period. However, both firms are worse off with personalized pricing compared to third-degree price discrimination (Foros et al., 2018). This relates to how the flexibility of personalized pricing can be more of a curse than a blessing.

The significance of information

Armstrong (2006b) points out that the industry profit in the static Thisse-Vives model, which we have seen to be $\frac{t}{2}$ with price discrimination, is slightly lower than the industry profit in a related model by Bester and Petrakis (1996) (which can be calculated to be $\frac{5}{9}t$). In the Bester-Petrakis' model the firms simply observes whether a given consumer has a brand preference for their own firm or the rival. This means whether x is greater or less than $\frac{1}{2}$. The set-up drives the firms to set two price schedules, a higher price to those on their own "turf" and a lower price for those on the rival's turf. Hence, the consumers closest to the middle, will choose to buy from the firm they are the least loyal to, since it can offer the lowest generalized prize. This is an inefficient solution, similar to what we saw under ex-post pricing, as consumers split in half and a proportion of them are travelling across the middle of the Hotelling line. The higher profits for the firms in Bester-Petrakis model are a result of the softened competition because of the less precise information compared to the perfect information in the Thisse-Vives model.

Moreover, Armstrong refers to Liu and Serfes (2004) who propose that these two models can be thought of as extremes, having either perfect information of the consumers' locations or being able to simply identify consumer's preferred firm. Furthermore, they find that if the firms have somewhat less than perfect information, then the firms will obtain less profit than if they had perfect information. Under even less information the profit will start increasing again, meaning that the profit as a function of information is a non-monotonic function with a U-shape.

We can relate these findings to the increased information of consumers with the use of big data. Liu and Serfes (2004) show that the consumer welfare has an inverted U-shape over the information precision. This implies that a middling information amount over consumers will be beneficial for consumers, but increasing or decreasing the degree of information from this level, the consumers will become worse off. By this logic, the effect of big data and increased information, will depend on the ex-ante level of information. Minimalist architect Ludwig Miles van der Rohe popularized the phrase "Less is more". Evidently, that can be applied to industrial economics as well in terms of firm's access to information in duopoly competition.

Consumer preferences regarding fairness

A possible explanation to the uniform pricing puzzle is consumers' moral distaste for price discrimination. An example of this is when the e-commerce vendor Amazon sold the same DVDs at different prices to different customers based on their purchase history. The pricing policy was discovered by their customers and Amazon received a backlash. Hence, Amazon stopped the policy and gave thousands of customers refunds (Wallheimer, 2016). Notably, they avoided denoting it as dynamic pricing, rather they referred to it as a price test to estimate consumers' response to different discount levels.

From a theoretical point of view, Steinberg (2020) argues that price discrimination "*unfairly undermines consumers' ability to benefit from the market, which is the very point of having a market*" (p. 97). A counterargument to this statement may be that a market is facilitating a transaction between buyer and seller, and the point of a market would be the transaction itself, which offers value to at least one of the parties. However, as we have seen, FDPD may distort the distribution of the economic surplus. Here one may present normative assessments of what a fair solution would be, which we will not go further into. Steinberg's statement is problematized by considering the following scenario presented in the Chicago Booth Review-article "Are you ready for personalized pricing" (Wallheimer, 2016), referring to Dubé and Misra (2017): A monopolistic firm, which sets a uniform price, knows customers are willing to pay between two and five dollars for a juice cartoon. The profit maximizing price for the firm may then be four dollars, and thereby excluding customers willing to only pay less than four dollars. In this way, price discrimination expands the market, allowing more consumers to benefit from a marketplace, and somewhat contradicts the original statement by Steinberg.

Even so, theorizing ethical concerns regarding FDPD only get us so far. In many ways, what counts for the firm, is simply the consumers' perception of these practices. Li and Jain (2016) extend the literature by implementing fairness concerns into the dynamic, two-period duopoly model of Fudenberg and Tirole (2000) where firms can price discriminate based on past behavior. By doing so, they find that BBP might be more profitable than not practicing BBP. This is conditional on consumers having sufficiently strong fairness concerns. This contrasts previous studies showing that BBP often yields lower profits than without applying BBP. The result arises because the fairness concerns ease the aggressive competition we otherwise would have seen in the second period in models such as Fudenberg and Tirole (2000).

Market size

An aspect yet to be studied, is the significance of market size. The market size is relevant both on the supply and demand side of the market. On the supply side, the increased market size will presumably have positive effects on the costs of production. The intuition is that the larger size will reduce the unit cost of every product, commonly known as economies of scale. According to Cachon and Harker (2002), this may lead to more aggressive pricing by the firms in both the uniform and price discriminatory situation (Foros et al., 2017).

Nonetheless, this thesis focuses on how firms can apply the capabilities of big data on the demand side. In industries reliant on big data, an additional user is not only valuable as for the extra income the user brings itself. The user also provides value in form of more data that will be valuable for the firm over every other user. By the same intuition as for the cost side, the firm has extra incentives to compete harder for the marginal user and leads to more aggressive behavior.

As the name suggest, big data is applicable for the firms with access to large amounts of data. However, according to Foros et al. (2018), smaller firms may instead buy access to data from third-party providers. Such competition may be intense (Varian, 2018). High production costs and cheap reproduction costs are characteristically for information-based industries. Furthermore, a key feature in digital platforms is network effects. As we described the cost advantages due to the scale of their operation as economies of scale, the network effects have fittingly been termed economies of scale on the demand side (Shapiro and Varian, 1998).

The network effects may apply across groups, as we have seen gives rise to two-sided markets. A general framework of price discrimination and two-sided markets is presented by Liu and Serfes (2013). In their model two platforms are competing and facilitating for two groups of agents exhibiting positive externalities to each other. We may expect from the competitive nature of FDPD in a duopoly and what we know from the literature on two-sided markets that combining these two results will give an even more competitive environment with low prices and profits. However, under certain conditions, FDPD is increasing the platform profits and decreasing the consumer surplus. This arises when the marginal cost is low relative to the cross-group network effects, which is characteristic for our situation with digital platforms.

Liu and Serfes (2013) present the intuition behind the result. In two-sided markets, FDPD has two effects on competition. First, and equivalent to one-sided markets, the prices charged by the firms are differing by the difference in transportation costs for the consumer. This will be the optimal price setting by the firms, since if a firm sets a price differential lower than the differential in transportation costs, the firm will be relinquishing additional income. On the other hand, if the differential is too large, the consumer will naturally opt for the rivalling firm. This constraint on price setting is a negative effect on firms' profits as it reduces their flexibility. However, FDPD may, under the stated conditions, have a positive effect as well. This is because under uniform pricing the prices will be dependent on the cross-group externality, and FDPD may make the cross-group externality irrelevant in equilibrium. Through the high externalities, the firms will price aggressively for the consumers located towards the center of the line, since an extra consumer is valuable to every agent in the other group. However, there is a threshold as to how aggressive they can price. In their framework there is a "limit price" of zero as they assume that prices must be non-negative.

When pricing uniformly, one must price lower to all consumers to attract the extra consumers, while with a price discriminatory practice one can extract all the surplus from those located the closest to oneself. By this, firms will not compete away their profits. They simply cannot do that because they are restricted by the limit price. In so, the cross-group externality will be irrelevant in this scenario. Hence, liberating the firms from this effect, as we do when going from uniform to discriminatory pricing, makes a positive impact by softening competition.

Kodera (2015) also presents a spatial model of price discrimination in two-sided markets. The main purpose of the study is to consider whether price discrimination in media markets is favorable for media platforms, consumers and advertisers. In contrast to Liu and Serfes (2013), Kodera's study allows negative prices and assumes asymmetric network effects. In Liu and Serfes' model, the platforms simultaneously offer discriminatory prices for two groups and allow platforms to price discriminate on both sides. In Kodera's model, the platforms can only offer uniform prices to consumers, while the platforms can choose to price discriminate over the advertisers.[9] Under certain conditions, the results of Kodera extend the findings of Liu and Serfes (2013). As long as the negative network effects are large enough, a platform's profit is higher under price discrimination than with uniform prices. On the other hand, if the negative network effects are moderate while platforms price discriminate the advertisers, both the consumers and the platforms are worse off.

Quality

Investments in quality can be included in duopoly models. We will investigate two different ways of modelling quality investments. Generally, following Choudhary et al. (2005), "quality" is any features that may affect a consumer's WTP.

First, Choudhary et al. (2005) consider a vertically differentiated duopoly, i.e. the two firms can differentiate themselves in a quality space. This diverges from the Thisse-Vives model, which regards horizontal differentiation. Naturally, to avoid the Bertrand paradox, the firms are assumed to differentiate themselves into a high- and a low-quality firm. This is consistent with the findings of Grossman and Shapiro (1984).

Choudhary et al. (2005) show that a higher quality firm can be worse off with personalized pricing, contrasting the seminal Thisse-Vives model and the model of Shaffer and Zhang (2002).[10] The rationale behind this result is that FDPD induces the rival to improve its product quality relative to when pricing uniformly. For the higher quality firm, it would be damaging to induce the rival to increase its quality as the firms then will move closer in the quality space and thereby cause fiercer competition. Therefore, it might be in the higher quality firm's interest to opt for uniform pricing to mitigate the strategic incentives the rival has to increase the quality of its product.

Secondly, Matsumura and Matsushima (2015) present a framework more akin to the familiar set-up from Thisse and Vives (1988). They extend the Thisse-Vives model by allowing firms to invest in reduced marginal cost after determining their pricing policies.

[9] The paper also makes an extension which considers price discrimination on both sides.

[10] This is a model of spatial competition with firms that offer products of different quality. They show that the firm with the high-quality products can benefit from personalized pricing through an increased market share.

This will be mathematically equivalent to an increase in the consumer's utility functions by a sufficiently large, positive constant, v. Additionally, their model separates itself from Thisse and Vives by having asymmetric firms, such that one of the firms have an ex-ante quality (cost) advantage over the other firm. As it is more relevant to our model, we will consider investments to be in quality, not to achieve a cost advantage. Opposing Choudhary et al. (2005), Matsumura and Matsushima (2015) find that the firm with the cost advantage will always prefer to price discriminate, while the firm providing the inferior product will choose to price uniformly if the difference in quality is sufficiently large.

Investments in increased quality are shown to be strategic substitutes. Meaning, if one of the firms increase its investments, it would be optimal for the rivalling firm do the opposite and reduce its investment level. However, prices are strategic complements as usual in Bertrand competition. Furthermore, the strategic aspects of FDPD (or more accurately the absence of them) come to play here as well. When both firms employ FDPD, one firm's quality improving activities do not affect the prices of the opposing firm, rather it alters the market shares. The higher the quality they can provide, the larger number of consumers will prefer their product. Following, the larger number of consumers they serve, the greater the returns from quality improving activities. This holds conversely as well and implies that the greater the ex-ante quality difference, the less incentives there are for the inferior firm to improve its quality.

On the other hand, if the inferior firm prices uniformly, a quality improvement from the superior firm will not only affect the distribution of consumers they serve, but also the price of the uniform firm. The quality improvement accelerates competition, as the firm's original market power from consumer loyalty becomes relatively less important. Therefore, the effect on the rivalling firm's price setting weakens the incentive of the superior firm to invest in quality compared to before. Analogous as to before, this effect will be proportionally larger with the ex-ante cost difference.

Summing up, quality investments from both firms accelerate the competition, which hurts the inferior firm the most, gradually more with larger ex-ante quality differences. The lower quality firm therefore wishes to reduce the investments in quality and soften the competition. Thus it will opt for uniform pricing for this strategic reason.

The qualitatively different results in Choudhary et al. (2005) and Matsumura and Matsushima (2015) stem from the different model set-ups. Choudhary et al. (2005) do not allow for horizontal differentiation and they differ in how quality improvements affect costs. However, Matsumura and Matsushima (2015) point out that the most important difference lies in how an increase in product quality increases consumers' WTP. In Matsumura and Matsushima (2015), an increase in quality increases the WTP equally for all consumers as the quality improvements are modelled similarly for all consumers as the constant v. However, in Choudhary et al. (2005), the consumers differ in their valuation of a quality improvement.

Non-price variables

We have seen throughout the literature review that extending the original framework of Thisse and Vives (1988) and loosening up on their assumptions, the result of a prisoner's dilemma situation and inclination towards price discrimination do not hold for every situation.

Furthermore, Foros et al. (2018) show that the more general result of the prisoner's dilemma situation is entirely dependent on the assumption that prices are the firms only choice variable. Departing from this and opening for other non-price variables, they find that it might be a dominant strategy to not employ FDPD.

In the model of Foros et al. (2018), the firms choose whether to commit to uniform prices in stage 0, before they do anything else. Moreover, the prices offered under price discrimination to each consumer are irrelevant of the strategic effect which is present under uniform pricing. It draws on the same intuition as we observe in Matsumura and Matsushima (2015), where choosing the pricing policy affects the quality decision of the rivaling firm. Therefore, the decision in stage 0 can be said to be a choice of *"... whether to give the rival strategic incentives to soften competition through non-price variables"* (Foros et al., 2018, p. 3). In turn, this may invalidate the prisoner's dilemma outcome.

3 Model

In this section we present a duopoly location model using the spatial model with linear transportation costs as proposed by Hotelling (1929). We expand on the simplified version of the Thisse-Vives model presented in the literature review, by including an advertisement market and the opportunity to invest in quality into our model. Other models include either vertical differentiation or two-sidedness in a setting similar to the Thisse-Vives model. Nonetheless, our model is to our knowledge unique in the way that it includes both of these elements in an unified model.

We consider a market with symmetric duopolists, $i = 1, 2$, selling horizontally differentiated goods. The two firms are exogenously located at each end of a Hotelling line with length normalized to 1. We consider the firms to be two platforms, which connect two distinct groups of users, advertisers and consumers. In addition to selling a product to consumers, the platforms sell ad-inserts to advertisers.

The model offers a framework to study firms that have the ability to price discriminate (employ personalized prices) among consumers through utilization of big data (see e.g. Choe et al., 2018). We consider a static model, similarly as Thisse and Vives (1988), implying that the firms are equipped with perfect information over the consumers' location x. Furthermore, we include firms' ad-revenues into our model, as many firms operating in the digital economy are increasingly dependent on this feature (see e.g. Perrin, 2018). Our modelling of ad-revenues is basic, nonetheless it illustrates how the increased relevance of ad-revenues affect the decisions of the firms. Lastly, firms are able to improve their product or services, which we model as quality investments, in order to become more attractive to the consumers. Our model is closely related to Matsumura and Matsushima (2015) in the way it models and combines horizontal and vertical product differentiation, opening up for first-degree price discrimination.

3.1 Consumer preferences

We model the consumers to care about several different aspects when deciding which product to buy. Each consumer buys one unit of the product, either from firm 1 or firm 2.

The consumers' utility of buying a product from either firm is expressed by the following utility functions:

$$U_1 = v + q_1 - p_1 - tx \tag{3.1}$$

$$U_2 = v + q_2 - p_2 - t(1 - x) \tag{3.2}$$

Firstly, consumers consider how close the product is to their preferences. To represent the diversity of consumer preferences, we assume that the consumer preferences are uniformly distributed along a Hotelling line in the interval [0,1], and the density of the consumer distribution is 1. Consumers do not usually find a product that is a perfect fit, and therefore the mismatch can be represented as a transportation cost measuring the cost of the deviation from their preferences. A consumer located at x in the range [0,1] experiences a linear transportation cost of tx or $t(1 - x)$ when buying the product respectively from firm 1 or 2. Related to the digital economy, the transportation cost can be considered a measure of the consumer's choosiness or loyalty for a specific brand.

Secondly, each consumer derives a positive intrinsic value from consuming the product, denoted by the letter v. We assume that v is sufficiently large to ensure every consumer purchases one unit of the goods. Further, we assume single-homing consumers. Anecdotally, we expect that we should see at least some multi-homing consumers in the online retail economy, however we disregard this possibility to simplify our analysis. Even so, Haucap and Heimeshoff (2014) argue that multi-homing is to some degree difficult for buyers.[11]

We enable firms to invest in the quality of the product. We model the quality investments as a quality component, q_i, similarly as Gabszewicz and Thisse (1979). The consumers consider the relative quality difference between the firms, $q_i - q_j$, when deciding which firm they are going to purchase from. A higher quality of the product increases the consumers' utility of the product and makes the product more attractive relative to the other product. We include the quality parameter because in many economic decisions the quality component bears as much on the outcome of the choice as the quantity component. In some cases where the consumer buys one unit of the product, only the quality component plays a role in the decision making by the consumer (Gabszewicz and

[11]Haucap and Heimeshoff (2014) explain that this is because it might be difficult to build up a good reputation on several platforms. A user's reputation is determined by the number of transactions conducted over the platform and can potentially be a factor determining prices and whether a transaction takes place.

Thisse, 1979). This can relate to our model where we only allow the consumer to buy one unit of the product.

As we have seen in the literature review, the literature is ambiguous regarding the effect of advertisement on consumer utility. While Anderson and Coate (2005) assume that viewers are affected negatively from advertising intensity, Gabszewicz et al. (2001) assume consumers to be indifferent to ads. Our model does not include any nuisance cost of advertisement, meaning the consumers' utility function does not get affected by consumers' annoyance of advertisement.[12] Anecdotally, advertisement on platforms such as the example of Amazon is often seen as a nuisance. It would be a hassle to scroll through several cheap rip-off versions of a product that is advertised if you search for a specific high-quality pair of headphones, for instance. On the other hand, advertisements may have an informative and positive value on the consumer utility. Consequently, it is not obvious what the sign of the effect should be on consumer utility.

If the firms opt to use personalized pricing, each consumer is offered the product at an individual price of $p_i(x)$, where x is the location of the consumer on the Hotelling line. In contrast, if the firms use uniform pricing, all of the consumers are offered the same price independently of the location, $p_i(x) = p_i$. Given that firm 1 and 2 set the prices $p_1(x)$ and $p_2(x)$, consumer x buys the product from firm 1 if $p_1(x) + tx - q_1 \leq p_2(x) + t(1-x) - q_2$. In the case of $p_1(x) + tx - q_1 = p_2(x) + t(1-x) - q_2$, we assume that the consumer randomly chooses one of the products.

3.2 Advertisers

The models of Liu and Serfes (2013) and Kodera (2015) are examples of models that incorporate two-sidedness into a duopoly model that allows for first-degree price discrimination. In Liu and Serfes (2013), the two sides exhibit positive network effects on each other, while Kodera (2015) operates with asymmetric network effects between an advertiser and a consumer side. Furthermore, in Liu and Serfes (2013) the firms can discriminate both consumer groups, while in Kodera (2015) the firms can discriminate on the advertiser side.

[12] A more elaborate review of the consumer's sentiment to advertisement is found in the section Two-sided markets 2.3, in the literature review.

However, we specifically look at situation with consumers on one side and an advertisement market on the other, and we allow for price discrimination over the consumers. The consumers exhibit positive network effects on the advertiser side.

There are several ways of modelling advertisers in two-sided markets. Gal-Or et al. (2012) distribute the advertisers along the same Hotelling line as the consumers and assume heterogeneity among the advertisers. Thereby, the advertisers prefer to advertise to the consumers closest aligned to their own location as these consumers are most likely to consider their products. In Ellman and Germano (2009), readers get influenced by advertisement differently, and consequently the advertisers choose to target the consumers that are the most receptive to advertisement.

Our model closely follows the assumptions by Gabszewicz et al. (2001, 2002) regarding the advertisers.[13] Similarly to Gabszewicz et al. (2001, 2002), we model that the advertisers differ in the intensity of their preferences of buying advertisement. The advertisers' intensity for buying an ad-insert is represented by θ in the range of [0,1]. Moreover, a larger number of consumers buying from a firm results in a larger number of people that will receive the advertisement message. Naturally, an advertiser's utility increases by the number of consumers. Thus, an advertiser's valuation of advertising at firm i is given by the product of the number of consumers, x_i, and the intensity of preferences for buying the ad-insert, θ.

Also similar to Gabszewicz et al. (2001, 2002), we include the aspect of perfectly elastic demand for advertisement. Perfectly elastic demand for advertisement implies that the firms do not need to set the level of advertisement. Contrasting, Peitz and Valletti (2008) assume imperfectly elastic advertisement demand.

Advertisers can buy ad-inserts at firm i at the cost of r_i. Therefore, an advertiser's net utility of buying an advertisement at firm i is represented by the following expression:

$$u_i^A = \theta x_i - r_i \tag{3.3}$$

To simplify impending calculations, we follow Gabszewicz et al. (2001, 2002), assuming that the density of advertisers of type θ is constant and equal to $4k$. We further assume

[13]The way we model the advertisement market is inspired by Hansen and Tho (2017).

that advertisers cannot engage in multi-homing. In contrast to us, Gabszewicz et al. (2001, 2002) allow multi-homing. Nonetheless, the market outcome does not change because of the assumption of perfectly elastic demand for advertisement. Assuming θ to be equal to $4k$ implies that k is a representation of the population size of advertisers. Consequently, a larger k will increase a firm's advertising revenue, all else equal.

3.3 Firms

The firms earn their revenues from two different sources, by selling their product to consumers and by selling ad-inserts to advertisers. When pricing uniformly, the total revenue from consumers will be the price multiplied by the demand for the product, $p_i x_i$. When price discriminating, they will take the highest price possible from each consumer they serve. In the next chapter we will look at how this price is set. To find the consumer revenue when price discriminating, we will need to integrate a price function over their market share of consumers. By integrating we find the area under the price function over the relevant range of x, which equals their consumer revenue.

The firms' revenue from the advertiser side A_i is the price r_i per ad-inserts sold times the demand for advertisement a_i. We make the simplifying assumption of zero marginal costs, both from selling the product and advertisements. The firms can invest in quality to make their product more attractive to consumers and thereby increase their market share. Quality improvements in the digital economy might be considered to be improved user interface, better customer services, shortened delivery time, etc. When investing in quality, the firms incur an investment cost of $C(q_i) = \frac{\phi}{2} q_i^2$. We assume that the constant $\phi > 0$ is sufficiently large to fulfill all second order conditions for profit maximization.

We assume, as Anderson et al. (2017), the cost function to be sufficiently convex to ensure the existence of a stable, symmetric equilibrium. The investment cost function satisfies the criteria of $C(q_i) \geq 0$, $C'(q_i) > 0$ and $C''(q_i) > 0$. Generally, the profit of firm i is hence given by:

$$\pi_i = (p_i(x))x_i + r_i a_i - \frac{\phi}{2} q_i^2$$

3.4 Timeline

We consider a four-stage game with two firms in a two-sided market. The game developes in the following order:

Stage 1: The firms choose between the pricing policies uniform prices (UP) and personalized pricing (PP).

Stage 2: The firms invest in quality. Firm 1 chooses the level of quality q_1, and firm 2 chooses the level of quality q_2.

Stage 3: The two firms set the prices (r_1, r_2) in the advertisement market.

Stage 4: In the last stage, the firms set the prices $(p_1(x), p_2(x))$ in the consumer market.

4 Analysis

To find the subgame perfect Nash equilibrium, we solve the sequential game by backward induction. Therefore, we start the analysis with the last stage, i.e. the price game.

Throughout the analysis we need to consider the different combinations of pricing policies by the firms. We can consider three cases since we have symmetric firms. Either both firms price uniformly, both firms price discriminate or they differ, where one chooses to price discriminate and the other prices uniformly. In the latter case, we consider firm 1 to be the firm pricing uniformly and firm 2 to be the firm price discriminating (UP,PP).

Throughout, we will use the notations in table 4.1 to indicate the pricing policy of each firm.

Combinations of pricing policies	Type of notation
Both firms set uniform prices	UP,UP
Firm 1 sets a uniform price, while firm 2 employs personalized prices	UP,PP
Both firms employ personalized prices	PP,PP

Table 4.1: Notations of pricing policies

We will only consider cases where the second order conditions for quality investments are satisfied. Therefore, we assume: $t\phi < \frac{1}{2}$.[14]

4.1 One-sided market

The one-sided case is characterized by not having any advertisement market and therefore we disregard the third stage in the presented model set-up. The firms maximize the following profit function:

$$\pi_i = p_i(x)x_i - \frac{\phi}{2}q_i^2$$

[14]See appendix for calculations of second order conditions for quality investments.

4.1.1 Price game

UP,UP

First, we consider the case where both firms price uniformly. Before we can find the optimal prices, we need to derive the demand functions faced by each platform. We solve for the indifferent consumer by setting the utility functions equal to each other (equations 3.1 and 3.2). This gives us:

$$\tilde{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t} + \frac{q_1 - q_2}{2t} \tag{4.1}$$

The demand for firm 1 is $x_1 = \tilde{x}$, while the demand for firm 2 is $x_2 = 1 - \tilde{x}$. Firm i (i=1,2) will attract more consumers by offering either a lower price ($\frac{\partial x_i}{\partial p_i} < 0$) or a higher quality relative to the rival ($\frac{\partial x_i}{\partial q_i} > 0$), everything else equal.

The firms maximize the following profit function when choosing their optimal price:

$$\pi_i = p_i x_i - \frac{\phi}{2} q_i^2 \tag{4.2}$$

Differentiating their profit with regards to their price, we obtain the expression:

$$\frac{\partial \pi_i}{\partial p_i} = x_i + \frac{\partial x_i}{\partial p_i} p_i = 0$$

An increased price results in increased revenue per unit a firm sells, but lowers the number of units it will sell. Solving for p_i, gives us:

$$p_i = \frac{t}{2} + \frac{p_j}{2} + \frac{q_i - q_j}{2} \tag{4.3}$$

We get the usual result that prices are strategic complements, $\frac{\partial p_i}{\partial p_j} > 0$.

The second order condition for a maximum is easily verified:

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -\frac{1}{t} < 0$$

Substituting the symmetric price p_j into p_i (equation 4.3) and solving for the two firms' prices simultaneously, we find the optimal price:

$$p_i = t + \frac{q_i}{3} - \frac{q_j}{3} \tag{4.4}$$

We find that the optimal uniform prices of the firms are increasing in their product quality and decreasing in the rival's product quality ($\frac{\partial p_i}{\partial q_i} > 0$, $\frac{\partial p_i}{\partial q_j} < 0$), all else equal. Furthermore, we get the typical result that by a higher value of t, the firms can take a higher price as the competitive pressure falls.

Substituting the optimal prices (equation 4.4) into the expression for the indifferent consumer (equation 4.1), we can express the indifferent consumer by the variable q:

$$\tilde{x} = \frac{1}{2} + \frac{q_i - q_j}{6t} \tag{4.5}$$

UP,PP

The next case we will consider, is when the firms differ in their pricing schemes. We consider the case where firm 1 is employing UP and firm 2 is employing PP. In this situation, firm 1 sets its price first, while firm 2 sets its prices after observing the rival's choice.

The consumers decide which firm they buy from by comparing the utility they receive from purchasing from either firm. Consumer x will buy from firm 2 if:

$$p_1 + tx - q_1 > p_2(x) + t(1 - x) - q_2$$

$$p_2(x) < p_1 + t(2x - 1) + q_2 - q_1$$

If the right-hand side of the latter inequality is less than zero, firm 2 will not be able to attract a consumer regardless the price it set, given that we have non-negative prices. However, if it is greater than 0, firm 2 will set a price incrementally lower than the mentioned right-hand side of the latter inequality, $p_2(x) = p_1 + t(2x - 1) + q_2 - q_1 - \varepsilon$. We will regard ε as sufficiently small and omit this value. Hence, the optimal price of firm 2 at point x is given by:

$$p_2(x) = \begin{cases} p_1 + t(2x-1) + q_2 - q_1, & \text{if } p_1 + t(2x-1) + q_2 - q_1 > 0 \\ \\ 0, & \text{if otherwise} \end{cases}$$

The indifferent consumer, $\tilde{x}^{UP,PP}$, is implicitly given by:

$$p_1 + t(2\tilde{x} - 1) + q_2 - q_1 = 0$$

Solving for $\tilde{x}$, we find:

$$\tilde{x} = \frac{1}{2} - \frac{p_1}{2t} + \frac{q_1 - q_2}{2t}$$

We find the uniform price of firm 1 by differentiating the profit function (equation 4.2) and solving for p_1.

$$p_1 = \frac{t}{2} + \frac{q_1 - q_2}{2} \tag{4.6}$$

The second order condition for a maximum is easily verified:

$$\frac{\partial^2 \pi_1}{\partial p_1^2} = -\frac{1}{t} < 0$$

We observe that we move from dividing the market in half to an expression that is based around $x = \frac{1}{4}$, further diverting from this level by the relative quality difference:

$$\tilde{x} = \frac{1}{4} + \frac{q_1 - q_2}{4t} \tag{4.7}$$

PP,PP

The last case we will consider, is when both of the firms are price discriminating. We use a similar set-up as in the previous case. Consumers will buy from firm 1 if, $p_1(x) + tx - q_1 < t(1-x) - q_2$. The optimal price of firm 1 and 2 at point x is given by:

$$p_1(x) = \begin{cases} t(1-2x) + q_1 - q_2, & \text{if } t(1-2x) + q_1 - q_2 > 0 \\ \\ 0, & \text{if otherwise} \end{cases}$$

$$p_2(x) = \begin{cases} t(2x-1) + q_2 - q_1, & \text{if } t(2x-1) + q_2 - q_1 > 0 \\ \\ 0, & \text{if otherwise} \end{cases}$$

The indifferent consumer, $\tilde{x}^{PP,PP}$, is given by: $t(1-2x) + q_1 - q_2 = t(2x-1) + q_2 - q_1$. Generalized, we have:

$$\tilde{x} = \frac{1}{2} + \frac{q_i - q_j}{2t} \tag{4.8}$$

4.1.2 Quality game

Next, we solve for the optimal quality investments by the firms.

UP,UP

Combining the optimal price (equation 4.4) and the expression for the indifferent consumer (equation 4.1) into the profit function (equation 4.2), we obtain the profit function as an expression of the quality levels:

$$\pi_i = \frac{(3t + q_i - q_j)^2}{18t} - \frac{\phi}{2} q_i^2$$

Differentiating the profit function with respect to q_i and solving for q_i gives us the firms' optimal quality investment levels:

$$q_i = \frac{1}{3\phi} \tag{4.9}$$

Inserting the optimal quality investment (equation 4.9) back into the price function (equation 4.4), we get the optimal price:

$$p_i = t$$

The profit functions around the optimal value of q will be:

$$\pi_i = \frac{t}{2} - \frac{1}{18\phi}$$

As the firms are symmetric and both are pricing uniformly, they divide the market equally between themselves and we get the price, t. This is equivalent to what we present in the literature review as an example of the original Thisse-Vives model. However, their profits are lowered by $\frac{1}{18\phi}$. This is because of the quality investments. The consumers are the beneficiaries of the introduction of ability to invest in quality. They pay the same prices as in the simplified Thisse-Vives model, while they at the same time get a better product.

We get the intuitive result that the higher the investment cost, the lower the optimal quality level ($\frac{\partial q_1}{\partial \phi} < 0$). Consequently, an increased investment cost will be advantageous for the firms in terms of the weakened pressure to invest in quality, all else equal. The firms weigh the marginal benefit of an extra consumer up against the marginal cost of quality investments when considering optimal quality levels. Given an increased cost of investments they would need to capture a relatively larger market share to uphold the relative size of quality level to the cost of investments.

UP,PP

Inserting firm 1's optimal uniform price (equation 4.6) and the quantity demanded of firm 1 (equation 4.7) into their profit function (equation 4.2), we find:

$$\pi_1 = \frac{(t + q_1 - q_2)^2}{8t} - \frac{\phi}{2} q_1^2 \tag{4.10}$$

The profit of the price discriminating firm (firm 2) is given by the following expression:[15]

$$\pi_2 = \int_{\tilde{x}^{UP,PP}}^{1} (p_1 + t(2x - 1) + q_2 - q_1)\, dx - \frac{\phi}{2} q_2^2 \tag{4.11}$$

$$\pi_2 = \frac{(q_1 - q_2 - 3t)^2}{16t} - \frac{\phi}{2} q_2^2 \tag{4.12}$$

[15]Remember, firm 2 observes the uniform price of the rival, p_1, before it set its prices.

The optimal quality levels are found by maximizing the firms' profit functions (equations 4.10 and 4.12) with respect to the quality investments. We get:

$$q_1 = \frac{t - q_2}{4\phi t - 1} \tag{4.13}$$

$$q_2 = \frac{3t - q_1}{8\phi t - 1} \tag{4.14}$$

Combining the reaction functions (equations 4.13 and 4.14), we get the optimal quality levels as functions of the exogenous parameters t and ϕ:

$$q_1 = \frac{2\phi t - 1}{8\phi^2 t - 3\phi} \tag{4.15}$$

$$q_2 = \frac{3\phi t - 1}{8\phi^2 t - 3\phi} \tag{4.16}$$

Given that the second order conditions for every combination of pricing policies are fulfilled we have that $q_1 < q_2$. This follows the intuition presented by Matsumura and Matsushima (2015), that a firm with a larger market size will be more inclined to invest in quality as it has a greater output over which it can apply the added quality level. Firm 2 has the greater market share ($x_1 = \frac{2\phi t-1}{8\phi t-3}, x_2 = 1 - \frac{2\phi t-1}{8\phi t-3}$), given that our second order conditions are fulfilled.

The optimal quality levels (equations 4.15 and 4.16) give us the following profit expressions:

$$\pi_1 = \frac{(2\phi t - 1)^2(4\phi t - 1)}{2\phi(8\phi t - 3)^2}$$

$$\pi_2 = \frac{(3\phi t - 1)^2(8\phi t - 1)}{2\phi(8\phi t - 3)^2}$$

We can compare the optimal price for firm 1 under (UP,UP) and (UP,PP), given the optimal quality levels, Under (UP,UP) we have that $p_1 = t$ while under (UP,PP) we have $p_1 = \frac{t}{2} - \frac{1}{2(8\phi t-3)}$. Given that the second order conditions are fulfilled, we have that when the rival is price discriminating instead of pricing uniformly, a firm will set a lower uniform price. This is needed to meet the aggressive competition of the rival.

PP,PP

We solve the following expressions to find the profits when both firms are price discriminating:

$$\pi_1 = \int_0^{\tilde{x}} (t(1-2x) + q_1 - q_2)\,dx - \frac{\phi}{2}q_1^2$$

$$\pi_2 = \int_{\tilde{x}}^1 (t(2x-1) + q_2 - q_1)\,dx - \frac{\phi}{2}q_2^2$$

This results in:

$$\pi_i = \frac{(t + q_i - q_j)^2}{4t} - \frac{\phi}{2}q_i^2 \tag{4.17}$$

Noteworthy, the functions which the firms integrate, are not dependent on the prices set by the rival, while under (UP,PP), the price discriminating firm's profit is dependent on the rival's uniform price (see equation 4.11 which consists of p_1).

The optimal quality levels are found by maximizing their profits (equation 4.17) with respect to the quality investments and solving the expressions for q_i:

$$q_i = \frac{t - q_j}{2\phi t - 1}$$

Thereafter, we get the optimal level of quality investments as a function of ϕ by substituting the symmetric reaction functions into each other:

$$q_i = \frac{1}{2\phi}$$

Comparing the quality levels under different pricing schemes, we see that there are higher quality levels when both firms are price discriminating relative to when both are pricing uniformly ($q_i^{UP,UP} < q_i^{PP,PP}$). This follows the same reasoning we had in the original Thisse-Vives model; under price discrimination there is harder competition for the consumers, as it enables the firms to compete for each consumer individually. However, in this framework, the fiercened competition takes shape in higher quality levels.

Furthermore, we observe another feature consistent with the Thisse-Vives findings; when both firms price uniformly, they obtain higher profit than when they both price discriminate:

$$\pi_i^{UP,UP} > \pi_i^{PP,PP}$$
$$(\frac{t}{2} - \frac{1}{18\phi}) > (\frac{t}{4} - \frac{1}{8\phi})$$

4.1.3 Choice of pricing policy

We set up the following payoff matrix given each firm's choice of pricing policy:

Firm 1 \ Firm 2	UP	PP
UP	$\frac{t}{2} - \frac{1}{18\phi}$, $\frac{t}{2} - \frac{1}{18\phi}$	$\frac{(2\phi t-1)^2(4\phi t-1)}{2\phi(8\phi t-3)^2}$, $\frac{(3\phi t-1)^2(8\phi t-1)}{2\phi(8\phi t-3)^2}$
PP	$\frac{(3\phi t-1)^2(8\phi t-1)}{2\phi(8\phi t-3)^2}$, $\frac{(2\phi t-1)^2(4\phi t-1)}{2\phi(8\phi t-3)^2}$	$\frac{t}{4} - \frac{1}{8\phi}$, $\frac{t}{4} - \frac{1}{8\phi}$

Figure 4.1: Payoff matrix in the one-sided market

The differences $\pi_2^{UP,PP} - \pi_2^{UP,UP}$ and $\pi_2^{PP,PP} - \pi_2^{PP,UP}$ are positive for any values of ϕ and t that satisfies our second order conditions. Meaning, employing PP dominates employing UP regardless of what the rival chooses. This, combined with the fact the profit is collectively higher when both are pricing uniformly ($\pi_i^{UP,UP} > \pi_i^{PP,PP}$), implies that we have the same prisoner's dilemma situation as in the original Thisse-Vives.

Proposition 1: *Under the one-sided market, the optimal pricing policy of the firms will be to employ personalized prices.*

Logically, the same intuition is present here as well. Going from UP to PP expands the market for the individual firm. We have one further effect compared to the Thisse-Vives model, which is also seen in the model of Matsumura and Matsushima (2015). The choice of pricing policy is also a choice of quality level. When a firm opts for personalized prices, it will adopt a higher quality level. We conclude that the introduction of quality investments does not alter the choice of pricing policy compared to our example from the Thisse-Vives model under a one-sided market.

4.2 Two-sided market

In this section we model duopoly competition between firms selling goods in a two-sided market. In a two-sided market we have to take into consideration the earnings the firms have from selling ad-inserts to advertisers, A_i. Therefore, the firms in the two-sided market will maximize the following profit function:

$$\pi_i = p_i(x)x_i + A_i - \frac{\phi}{2}q_i^2 \tag{4.18}$$

4.2.1 Price game

In the last stage, the firms set the prices in the consumer market. The firms maximize profits by setting $\frac{\partial \pi_i}{\partial p_i} = 0$. The first order condition is given by:

$$\frac{\partial \pi_i}{\partial p_i} = x_i + \frac{\partial x_i}{\partial p_i} p_i + \underbrace{\frac{\partial A_i}{\partial p_i}}_{=0} = 0$$

Advertisements have already been sold when the prices in the consumer market are to be set in the last stage. As a consequence prices will not effect the advertisement level and we can set $\frac{\partial A_i}{\partial p_i} = 0$. For this reason, we can skip the calculations for the price game and refer to the equilibrium solutions we present in section 4.1.1 Price game.

4.2.2 Advertisement game

In the third stage of the model, the firms are going to set the optimal prices for advertisement. By using the advertisers' net utility function of buying an ad-insert, $u_i^A = \theta x_i - r_i$ (equation 3.3), we can find the location of the indifferent advertiser. The advertiser which is indifferent between buying an ad-insert or not is located such that:

$$u_i^A = 0 \Leftrightarrow \theta = \frac{r_i}{x_i}$$

As stated earlier, the density of advertisers is equal to $4k$, which implies that the demand for advertisement at platform i is given by:

$$a_i = 4k(1 - \frac{r_i}{x_i}) \tag{4.19}$$

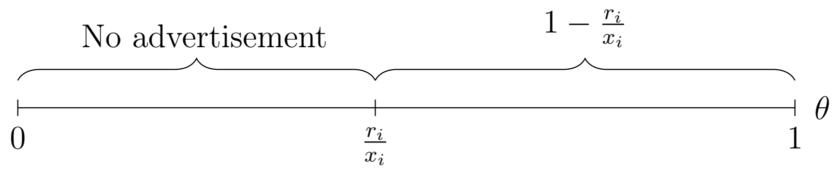

Figure 4.2: Demand for advertisement

The demand for advertisement is independent of the rival's price because of the assumption of perfectly elastic demand for advertisement. Thus, each firm maximizes its advertisement revenues, $A_i = a_i r_i$, independent of the competing firm's price for ad-insert, r_j. We get the following expression for advertisement revenue by substituting the demand for advertisement (equation 4.19) into the expression for the firms' revenue in the advertisement market, A_i:

$$A_i = r_i a_i = r_i 4k(1 - \frac{r_i}{x_i})$$

The optimal price of advertisement is found by solving the first order condition:

$$\frac{\partial A_i}{\partial r_i} = 4k(1 - \frac{r_i}{x_i}) - 4k\frac{r_i}{x_i}$$

$$r_i^* = \frac{x_i}{2}$$

This gives us the equilibrium revenue of:

$$A_i^* = x_i k \tag{4.20}$$

The second order condition is verified by:

$$\frac{\partial^2 A_i}{\partial r_i^2} = -\frac{8k}{x_i} < 0$$

Both the price and the advertisement revenue increase in the demand for a given product, $(\frac{\partial r_i^*}{\partial x_i} > 0$ and $(\frac{\partial A_i^*}{\partial x_i} > 0)$. We have that when the demand for the product increases, the platform get more users and in turn increasing the advertisers' valuation of advertising.

4.2.3 Quality game

In the second stage, we solve for the firms' optimal investments in quality. Similarly, as to the one-sided case, the prices are given from section 4.1.1 Price game, thus we must also in this stage consider the three separate cases. However, with the introduction of an advertisement market, we need to add the firms' advertisement revenue. The advertisers are indifferent to the level of quality per se; however, a higher level of quality increases the firm's market share and consequently the advertisers' potential outreach.

UP,UP

By substituting the equilibrium advertisement revenues (equation 4.20) into the firms' profit functions (equation 4.18), we can express the firms' profit function in a two-sided market as:

$$\pi_i = p_i x_i + k x_i - \frac{\phi}{2} q_i^2$$

$$\pi_i = x_i(p_i + k) - \frac{\phi}{2} q_i^2 \tag{4.21}$$

Substituting the corresponding price and demand expressions (equations 4.4 and 4.5) into the profit function (equation 4.21), we get:

$$\pi_i = (\frac{1}{2} + \frac{q_i - q_j}{6t})(t + \frac{q_i - q_j}{3} + k) - \frac{\phi}{2} q_i^2$$

By the same approach as before, we find the optimal levels of quality:

$$q_i = \frac{6t - 2q_j + 3k}{18\phi t - 2}$$

$$q_i = \frac{k + 2t}{6\phi t}$$

We find that the investment levels increase with the size of the two-sided market ($\frac{\partial q_i}{\partial k} = \frac{1}{6\phi t}$). A larger advertisement market increases the competition for the consumers as they become increasingly valuable.

We obtain the profit function given by the optimal level of quality investments:

$$\pi_i = \frac{t}{2} + \frac{k}{2} - \frac{(k + 2t)^2}{72\phi t^2}$$

By this intuition, we would also expect lower prices offered to the consumers compared to the one-sided case, while in our model set-up there are not any changes in the prices. Lower prices because of the introduction of an advertisement market is a typical feature in two-sided markets (Armstrong, 2006b). As more thoroughly discussed in the literature review, this characteristic stems from a subsidization of the market that exerts the greatest externalities on the other side. Nonetheless, we do not observe this in our model, the reason being that the timing of the game does not allow for the firms to consider advertisement revenue when they are setting their prices.

In our model, the increased competition through larger advertisement market size manifests itself in higher product quality rather than lowered prices. As we remember, the prices are set before we consider the advertisement level and they are therefore not affected by the advertisement size, k. Contrastingly, the quality investments are higher in a two-sided market than in a one-sided market. We can observe this by comparing the quality levels when both firms are pricing uniformly. While we have $q_i = \frac{1}{3\phi}$ in a one-sided market, it increases to $q_i = \frac{1}{3\phi} + \frac{k}{6\phi t}$ in a two-sided market. The result of higher quality investments in a two-sided market than in a one-sided market holds for all three combinations of pricing policies.

UP,PP

Inserting firm 1's optimal uniform price (equation 4.6), the demand of firm 1 (equation 4.7) and the equilibrium advertisement revenue (equation 4.20) into the profit function (equation 4.18), we find:

$$\pi_1 = \frac{(t+q_1-q_2)^2}{8t} + k(\frac{1}{4} + \frac{q_1-q_2}{4t}) - \frac{\phi}{2}q_1^2 \tag{4.22}$$

Following the same approach as under the one-sided market, and including the advertisement revenue, kx_2, we can express the profit function for the price discriminating firm as the following:

$$\pi_2 = \int_{\tilde{x}^{UP,PP}}^{1} (p_1 + t(2x-1) + q_2 - q_1)\,dx + kx_2 - \frac{\phi}{2}q_2^2 \tag{4.23}$$

$$\pi_2 = \frac{(q_1-q_2-3t)^2}{16t} + k(1-(\frac{1}{4} + \frac{q_1-q_2}{4t})) - \frac{\phi}{2}q_2^2 \tag{4.24}$$

Differentiating with regards to q_i and solving for q_i gives the following quality levels:

$$\frac{\partial \pi_1}{\partial q_1} = \frac{t+q_1-q_2}{4t} + \frac{k}{4t} - \phi q_1$$

$$q_1 = \frac{t+k-q_2}{4\phi t - 1}$$

$$\frac{\partial \pi_2}{\partial q_2} = \frac{3t-q_1+q_2}{8t} + \frac{k}{4t} - \phi q_2$$

$$q_2 = \frac{2k+3t-q_1}{8\phi t - 1}$$

Combining the reaction functions of the quality levels gives us the optimal levels of quality:

$$q_1 = \frac{8k\phi t + 8\phi t^2 - 3k - 4t}{32\phi^2 t^2 - 12\phi t} \tag{4.25}$$

$$q_2 = \frac{8k\phi t + 12\phi t^2 - 3k - 4t}{32\phi^2 t^2 - 12\phi t} \tag{4.26}$$

Substituting the optimal quality levels into the price function for firm 1, we get the same expression as in the one-sided case, $p_1 = \frac{t}{2} - \frac{1}{2(8\phi t - 3)}$. This implies that the uniform price is independent of k.

By plotting the optimal quality levels (equations 4.25 and 4.26) into the profit functions (equations 4.22 and 4.24), we get the following expressions:

$$\pi_1 = \frac{256\phi^3 t^5 + (512k\phi^3 - 320\phi^2)t^4 + (-576k\phi^2 + 128\phi)t^3 + (-64k^2\phi^2 + 208k\phi - 16)t^2 + (48k^2\phi - 24k)t - 9k^2}{32\phi t^2(8\phi t - 3)^2}$$

$$\pi_2 = \frac{1152\phi^3 t^5 + (1536k\phi^3 - 912\phi^2)t^4 + (-1280k\phi^2 + 224\phi)t^3 + (-64k^2\phi^2 + 328k\phi - 16)t^2 + (48k^2\phi - 24k)t - 9k^2}{32\phi t^2(8\phi t - 3)^2}$$

PP,PP

We follow the same approach as under the one-sided market. Additionally, we include the advertisement revenue, $A_i = kx_i$, where x_i can be expressed as $\frac{1}{2} + \frac{1}{2t}(q_i - q_j)$ (see equation 4.8). Thereby, when both firms are price discriminating, the profit expressions can be expressed as:

$$\pi_1 = \int_0^{\tilde{x}} (t(1-2x) + q_1 - q_2)\, dx + kx_1 - \frac{\phi}{2}q_1^2$$

$$\pi_2 = \int_{\tilde{x}}^1 (t(2x-1) + q_2 - q_1)\, dx + kx_2 - \frac{\phi}{2}q_2^2$$

$$\pi_i = \frac{(t + q_i - q_j)^2}{4t} + k(\frac{1}{2} + \frac{1}{2t}(q_i - q_j)) - \frac{\phi}{2}q_i^2$$

Differentiating and solving for q_i, gives:

$$\frac{\partial \pi_i}{\partial q_i} = \frac{t + q_i - q_j}{2t} + \frac{k}{2t} - \phi q_i$$

$$q_i = \frac{k + t - q_j}{2\phi t - 1}$$

Substituting the symmetric quality levels into each other gives us the optimal level of quality investments:

$$q_i = \frac{k+t}{2\phi t}$$

We get the same result as in the one-sided case, that there are higher quality levels when both firms are price discriminating compared to when pricing uniformly ($q_i^{PP,PP} > q_i^{UP,UP}$).

Substituting the optimal level of quality into the profit function gives us:

$$\pi_i = \frac{t}{4} + \frac{k}{2} - \frac{(k+t)^2}{8\phi t^2}$$

Comparing the profits when the firms choose matching pricing policies, the profits under uniform pricing are larger than under price discrimination ($\pi_i^{UP,UP} > \pi_i^{PP,PP}$). Further, the difference is increasing in k.

$$\pi_i^{UP,UP} - \pi_i^{PP,PP} = \frac{18\phi t^3 + 8k^2 + 14kt + 5t^2}{72\phi t^2}$$

$$\frac{\partial(\pi_i^{UP,UP} - \pi_i^{PP,PP})}{\partial k} > 0$$

4.2.4 Choice of pricing policy

In the first stage, the platforms choose between employing a uniform price or to price discriminate. We set up the following payoff matrix, depicting the profits under different combinations of pricing policies:

		Firm 2	
		UP	PP
Firm 1	UP	$\frac{t}{2} + \frac{k}{2} - \frac{(k+2t)^2}{72\phi t^2}$ $\frac{t}{2} + \frac{k}{2} - \frac{(k+2t)^2}{72\phi t^2}$	$\frac{1152\phi^3t^5+(1536k\phi^3-912\phi^2)t^4+(-1280k\phi^2+224\phi)t^3}{32\phi t^2(8\phi t-3)^2} + \frac{(-64k^2\phi^2+328k\phi-16)t^2+(48k^2\phi-24k)t-9k^2}{32\phi t^2(8\phi t-3)^2}$ $\frac{256\phi^3t^5+(512k\phi^3-320\phi^2)t^4+(-576k\phi^2+128\phi)t^3}{32\phi t^2(8\phi t-3)^2} + \frac{(-64k^2\phi^2+208k\phi-16)t^2+(48k^2\phi-24k)t-9k^2}{32\phi t^2(8\phi t-3)^2}$
	PP	$\frac{256\phi^3t^5+(512k\phi^3-320\phi^2)t^4+(-576k\phi^2+128\phi)t^3}{32\phi t^2(8\phi t-3)^2} + \frac{(-64k^2\phi^2+208k\phi-16)t^2+(48k^2\phi-24k)t-9k^2}{32\phi t^2(8\phi t-3)^2}$ $\frac{1152\phi^3t^5+(1536k\phi^3-912\phi^2)t^4+(-1280k\phi^2+224\phi)t^3}{32\phi t^2(8\phi t-3)^2} + \frac{(-64k^2\phi^2+328k\phi-16)t^2+(48k^2\phi-24k)t-9k^2}{32\phi t^2(8\phi t-3)^2}$	$\frac{t}{4} + \frac{k}{2} - \frac{(k+t)^2}{8\phi t^2}$ $\frac{t}{4} + \frac{k}{2} - \frac{(k+t)^2}{8\phi t^2}$

Figure 4.3: Payoff matrix in the two-sided market

In the one-sided market we find that the profits are always higher when price discriminating compared to when pricing uniformly, regardless of the rival's pricing policy ($\pi_2^{UP,PP} - \pi_2^{UP,UP} > 0$, $\pi_2^{PP,PP} - \pi_2^{PP,UP} > 0$). This holds for any positive values of ϕ and t that satisfies our second order conditions. Therefore, price discrimination constitutes a dominant strategy for the firms. However, this is not necessarily the case anymore with the introduction of two-sided markets.

When plotting the expressions for the differences in profits for firm 2 when $\phi, t = 1$, we get the inverse U-shaped graphs in figure 4.4. The red, solid graph shows the situation in which firm 1 is pricing uniformly. The blue, dashed graph shows when firm 1 employs personalized prices.

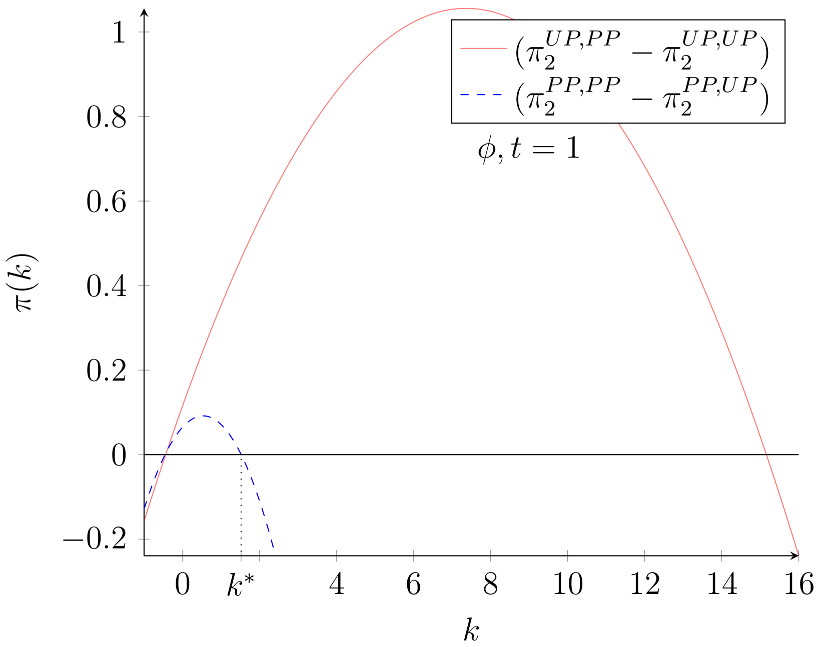

Figure 4.4: Differences in profits for firm 2 when pricing uniformly or price discriminating, given the pricing scheme of firm 1

We observe that the difference $\pi_2^{PP,PP} - \pi_2^{PP,UP}$ (the blue, dashed line) is positive for $k = 1$, while it turns negative before $k = 2$. A negative $\pi_2^{PP,PP} - \pi_2^{PP,UP}$ implies that firm 2 would do better pricing uniformly than price discriminating, given that firm 1 price discriminates. Technically, we denote the positive value of k where $\pi_2^{PP,PP} - \pi_2^{PP,UP}$ becomes negative as k^*. This indicates that if the advertisement market is sufficiently large, relative to the consumer market, we have duopoly competition in which price discrimination is not necessarily the dominant strategy.

If k is larger than k^*, approximately 1.5, given the parameter values of $\phi, t = 1$, and the rival price discriminates, the firm will earn a higher profit by choosing a uniform price than choosing to price discriminate. The firms' payoff structures do not any longer resemble the previous prisoner's dilemma situation we have under the one-sided market. Rather, the firms will now prefer to choose opposite actions of each other. Moreover, each firm would ideally see the rivalling firm price uniformly and be the firm which price discriminates itself.

We have two asymmetric pure strategy equilibria and one symmetric mixed strategy equilibrium. This situation forces the firms' optimal strategies to be a mixed strategy, rather than a pure strategy as previously. A mixed strategy implies that a firm chooses either to price discriminate or to price uniformly randomly, given by a probability distribution over each choice. Therefore, we can state that price discrimination will not always be the firms' chosen pricing policy.

Proposition 2: *Under the two-sided market, the optimal pricing policy of the firms will be to employ personalized prices for values of k up to k^*. When $k > k^*$, the optimal pricing policy of the firms will be determined by a mixed strategy.*

Before we go any further into the economic rationale for these findings of our model, we need to address a significant feature of our model. We have throughout this analysis seen that in the two-sided market, the optimal investment levels are monotonically increasing in k ($\frac{\partial q_i}{\partial k} > 0$). Moreover, when firms are maximizing their profits with regards to quality investments, they incur investment costs that grow convexly over the investment level. As a consequence of the convex investment costs, at an adequately high k, the investment costs will be so high that the profits will be negative. We will neglect ranges of k in which the profits of the firms are negative. Implicitly, this can be considered as a limit as to how high their quality investment levels can be.

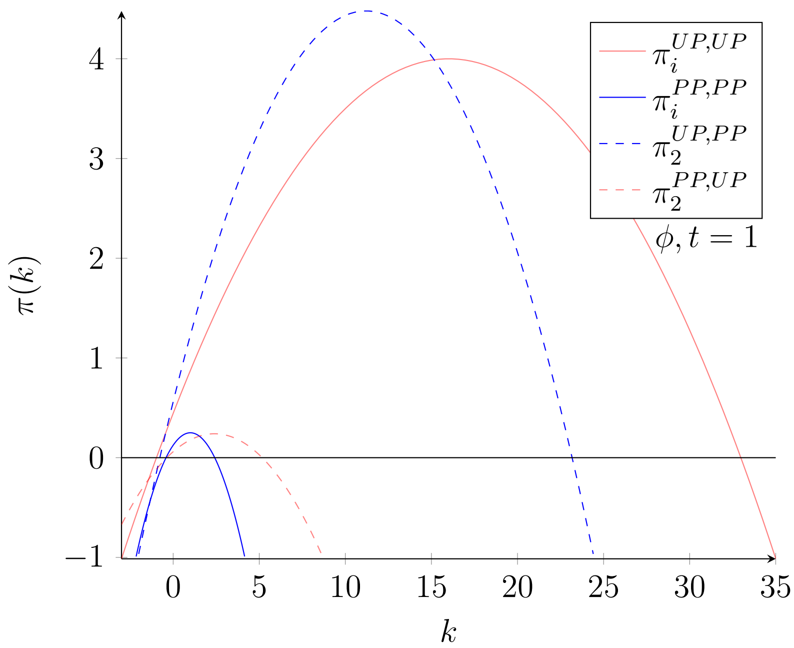

Figure 4.5: Profit expressions under the two-sided market

We have that the expression $\pi_2^{PP,PP} - \pi_2^{PP,UP}$ (figure 4.4) turns negative, before that of $\pi_i^{PP,PP}$ (figure 4.5), when k increases. This entails that we are within a permissible range of q. As for $\pi_2^{UP,PP} - \pi_2^{UP,UP}$(figure 4.4), this expression is positive for all permissible values of q, implying that when the rival prices uniformly the firm will always prefer to price discriminate.

To understand why firms are no longer consistently opting to price discriminate when we have a two-sided market, we need to consider what the effects are when a firm goes from offering a uniform price to price discriminating when the rival price discriminates (i.e. from (UP,PP) to (PP,PP)). Furthermore, it is essential to see how these effects depend on the relative importance of the advertisement market (k). First, we will examine how a firm's incentives to invest in quality differ under the two pricing schemes. Secondly, we will show how the rival, firm 2, is affected by the pricing policy by firm 1.

Price discrimination increases the firm's market size, as it enables them to compete harder for the consumers of the rivalling firm without having to set lower prices for its own target group. Given the second order conditions are fulfilled, we have that the firm will have a larger market share when price discriminating, given that the rival price discriminates ($x_1^{UP,PP} < x_1^{PP,PP}$). Notably, the increase in market share is independent of k (($x_1^{UP,PP} = \frac{2\phi t-1}{8\phi t-3}$),($x_1^{PP,PP} = \frac{1}{2}$)).

A larger market share increases the firm's incentive to invest in quality, as the investments can be applied to all consumers. Further, a higher k increases the incentives to invest in quality as the consumers are increasingly more valuable, as they exhibit positive network effects on the advertiser market. These features entail that the difference in quality investments under the two pricing schemes of firm 1 will be greater the larger advertisement market gets ($\frac{\partial(q_1^{PP,PP}-q_1^{UP,PP})}{\partial k} = \frac{1}{4\phi t} > 0$), as firm 1 profits on corresponding market expansion over more advertisers.

As we see, a higher k leads to fiercer competition in quality. This is costly for the firms as an increased quality level comes with convexly growing quality costs ($C(q) = \frac{\phi}{2}q^2$). On the other hand, if firm 1 opts for uniform prices, it relinquish itself from the added quality level induced by price discrimination relative to employing uniform prices, in a situation where we have a larger market share. For low values of k, the benefit of lower quality investments will be overshadowed by the loss in profit from a reduced market share. However, with a higher k, the effect of lower investment costs will have relatively more importance compared to the loss in market share. Thus, a firm might be prompted to choose uniform prices for the reason of weakening its own incentives to make costly quality investments.

Now, we can discuss the effects of firm 1's choice of pricing policy on firm 2 and how this in turn affect firm 1. If firm 1 chooses to employ uniform prices, it weakens firm 2's incentives to invest in quality, as firm 2 must account for the effects of quality investments on the uniform price of firm 1 (see equation 4.23). This effect gets stronger over k ($\frac{\partial(q_2^{PP,PP} - q_2^{UP,PP})}{\partial k} = \frac{1}{4\phi t} > 0$), following the same reason as before. There are higher incentives to invest in quality to attract consumers when the relative importance of the advertisement market increases.

As before, a larger market share increases the firms' incentive to invest in quality as the investments can be applied to all consumers. Thus, a higher k increases the incentives to choose a pricing policy that lowers the incentives of the rival to invest in quality, thereby softening the competition for market shares and lowering the pressure on the prices charged to consumers.

Summing up, we have a direct effect of lowered quality costs stemming from weakened incentives to invest in quality when pricing uniformly. In addition, we have an indirect effect of softened competition from the rivalling firm through turning on the strategic effect on the rival's price setting by employing uniform prices.

Naturally, if the firms are choosing different pricing schemes, none of the firms would wish to be the one choosing uniform prices, whilst the other price discriminates. If we consider firm 2 to be pricing uniformly, the positive effects for firm 1 of going from (UP,UP) to (PP,UP) will outweigh the induced additional investment costs from price discriminating rather than employing uniform prices for every k that we consider. As we remember, we cannot have values of k that induce quality investments which give the firms negative profits. We can see this in the figure of the different profit expressions (figure 4.5).

5 Discussion

The purpose of our model is to study the choice of pricing policy for firms in the digital economy. We focus on the digital economy as the entry of big data has enabled personalized pricing in this industry. To emulate the digital economy, we have included the characteristic elements quality investments and an advertisement market into our model. Still, our model is clearly a simplified depiction of reality. In this chapter, we discuss how our model adds to the literature on the field and the limitations of our model.

Matsumura and Matsushima (2015) offer a model related to ours, by considering the same choice of pricing policy, as well as allowing for horizontal differentiation and quality investments by the firms. Moreover, we follow their model closely in terms of how quality investments are modelled. We therefore see some of the same mechanisms in our model as in theirs. Particularly, there is the strategic effect on the rival when choosing to price uniformly, which ceases to exist when price discriminating. However, there are two main differences between our and their model set-ups. First of all, Matsumura and Matsushima (2015) consider a situation in which one of the firms has an ex-ante quality advantage over the other firm. In their model, it is the size of this ex-ante quality difference that potentially may induce the inferior firm to opt for uniform prices for the purpose of weakening the incentives of the rivalling firm to invest in quality. By inducing the rival to invest relatively less in quality, the firm achieves a softened competition. Comparably, we consider symmetric firms. Moreover, we contrast Matsumura and Matsushima (2015) by adding an advertiser market into our model. We analyse two iterations of the model. We consider both a one-sided and a two-sided market. We see that our result drastically hinges on whether we include the advertiser market.

In the one-sided case, we achieve a result analogous with that of Thisse and Vives (1988). Namely, the firms end up in a prisoner's dilemma situation where both firms choose to price discriminate, even though pricing uniformly is the preferred option collectively. We compare the one-sided case of our model with a simplified version of the Thisse-Vives model presented in the literature review. There is only the firms' ability to invest in quality that separates these two models. We find that the consumers benefit from the introduction of quality investments, while the firms' profits decrease. Further, the firms' profits increase in the investment cost ($\frac{\partial \pi_i}{\partial \phi} > 0$).

There are seemingly only a few other papers studying the effect of two-sided markets in spatial models of first-degree price discrimination. In the literature review we examine two such papers, Liu and Serfes (2013) and Kodera (2015). However, the firms are not incentivized to deviate from the choice of personalized prices in either of the models, as seen in Thisse and Vives (1988). Rather, both articles find that the profits of the firms may be higher with price discriminatory practices than when offering uniform prices.

In our model, there is a crucial interplay between quality investments and the advertiser market. An increased market size of advertisers, k, intensifies the competition for consumers as they are increasingly valuable due to the positive network externality they exert on the advertiser market. This is because the advertisers' willingness to pay for ad-inserts increases in consumer demand for the product. Essentially, a firm would have two ways to attract consumers with the choice variables included in our model; either it offers lower prices or a higher quality good. The timing of our game does not allow the firms to offer lower prices when the advertiser market increases, which one typically would see in models of two-sided markets. This implies that the firms would invest more in quality to attract consumers. The neglection of the effect of advertiser market size on consumer prices is a flaw of our model. Anyway, our model still illustrates how the addition of an advertising market intensifies the competition for the consumers and how this may transpire through quality investments. In fact, we obtain the result that the escalated quality competition prompts the firms to choose a mixed strategy of pricing policies. The profit-damaging competition in quality resembles the see-saw effect in prices described by Armstrong (2006b). In this situation, the firms compete away all of the additional revenue from the addition of an advertisement market. In our model, we do not see the same profit-ruining competition in prices, however we see some of the same mechanisms with competition in quality.

Price competition in a duopoly setting such as considered has the natural downward limit of non-negative prices. This inhibits the firms to compete away all their profits when they operate in a two-sided market, as we see in the discussion by Liu and Serfes (2013). On the contrary, quality investments do not exhibit any such limit. This induces intense competition to offer the highest quality product. Remarkably, the competition becomes so intense that it alters what pricing policy will be optimal. Related to the original Thisse-Vives model, we saw how price discrimination fiercened the competition and lowered profits. Nonetheless, it would still be the dominant strategy for the firms to pursue. However, in our model, a large advertisement market size makes it more profitable to avoid price discrimination if the rival price discriminates, through the softening of the competition in quality.

Now, we go over to our modelling of the advertiser market. In addition to enabling personalized prices in the consumer market, the entry of big data also provides personalized marketing opportunities. For instance, Amazon offers advertisers various different ways to advertise through their e-commerce service. One of these ways is offering advertisers the option to buy top spots at the product search results. Amazon labels this type of advertisement as sponsored products, which promotes individual products listings on Amazon. Buying advertisement such as sponsored products, will make the product more visible for customers, thereby influencing the customers to discover and purchase products that are sponsored (Amazon, n.d.). In contrast, our model constructs a highly simplified way to model the effect of an advertiser market. We make the assumption of perfectly elastic demand for advertisement, which is contrasted by several other models (see e.g. Gal-Or et al., 2012; Ellman and Germano, 2009). In fact, because of our general modelling of the advertisement market it is possible to view the modelling of the advertisement as a proxy for more broad sources of revenue stemming from the number of consumers the firms reach out to.

Further, we only allow for price discrimination in the consumer market, in opposition to Kodera (2015), who focuses on discrimination on the advertiser side. Moreover, Kodera (2015) considers that the advertiser side yield negative network externalities on the consumer side. Due to the ambiguous literature on the field, we disregard the consumers' sentiment to advertisement in our model. Nonetheless, the advertisement market has become an essential component of the business models of companies in the digital economy, and alternative ways of modelling advertisement demand in this type of framework are a possibility for future research.

Various literature put emphasis on how the modelling of price discrimination as a dynamic process, rather than as a static model, offers additional insights. Specifically, there is customarily a stage in which the firms collect information, and an ensuing stage in which they can apply this information. This contrasts our strict assumption of firms having perfect information over all consumers, which is debatable in the real world. Arguably, it would be more precise to require that the firms need to interact with the consumer to be able to collect consumer specific information.

Moreover, many firms in the digital economy operate in industries with strong network effects, modelled by us in the case of a two-sided market. Strong network effects might potentially create a "winner takes it all"-situation (Navon et al., 1995). Further, a theme throughout this paper has been how the outcome of price discrimination crucially depends upon the market structure the firms operate in (e.g. a monopoly or duopoly situation). Additionally, as discussed by Matsumura and Matsushima (2015) and ourselves, the relative market shares of the firms make up a pivotal part of the firms' decision of pricing structure.

A related aspect is the location incentives of the firms. Our model is an extension of a basic Hotelling model, where firms are located exogenously at each end of the line. By doing so, we neglect the location incentives of the firms, which we would be able to study if we modelled x as an endogenous variable. Similar to relevant literature, we find that when firm j is employing PP, firm i alleviates itself from the strategic effect its price setting has on the rival. Foros et al. (2018) find that the firm will locate closer to the rival that uses PP compared to one that uses uniform pricing for this very reason. Furthermore, the literature on two-sided markets (see e.g. Kind et al., 2013), suggests that by extending our Hotelling model from a one-sided to a two-sided market we should see firms locating even closer to the center of the line. This is because when adding a positive cross-group externality to our model, the relative importance of the business stealing effect compared to the strategic effect is increased. Thus, including location incentives may add insights we have not considered in our model.

In the model, we assume that the firms' marginal costs of selling their product is constant and equal to zero. This is mainly due to simplicity of calculations. Moreover, it is a conventional way to construct models in the literature, as it typically does not provide any additional insights. Noteworthy, in other models of vertical differentiation, such as Choudhary et al. (2005), the cost of quality investments is modelled as increased marginal costs, rather than as a fixed cost, which we do. Related to our case, it is nonetheless a characteristic feature of several industries within the digital economy to have close to negligible marginal costs (see e.g. Liu and Serfes, 2013). This is naturally dependent upon which specific industry one considers.

The literature on the choice of pricing policy in spatial models is often concerned with which parties gain and loose from different pricing policies. We have seen that in specific monopoly situations, the firms might be able to extract all of the consumer surpluses from purchasing the product. This might change drastically through the introduction of only one rival in a spatial competition setting, as we have seen in the presentation of the simplified Thisse-Vives model. The consumers in our model achieve a higher consumer surplus compared to the presented simplified Thisse-Vives model as the firms invest in quality and accelerate the competition for the consumers. This holds for both the one-sided and the two-sided market, and by increasing the size of the advertisement market, the consumer surplus increases further. With an adequately large advertisement market, we have the result that the firms' pricing policies will be decided by a mixed strategy. Choosing to employ uniform prices lowers the accompanying quality investments by a firm. Consequently, the consumers will be better off in a situation in which both firms price discriminate.

6 Conclusion

In this thesis, we aim to study the choice of pricing policy, employing either personalized or uniform prices, in the digital economy. We review existing literature on the topic of first-degree price discrimination and propose a theoretical model of spatial competition in a two-sided market with quality investments. We focus on the digital economy as the prevalence of big data allows firms in this industry to come much closer to apply personalized pricing than before.

Seminal literature on the field of price discrimination, such as Thisse and Vives (1988), suggests that a price discriminatory practice by firms is the optimal choice of pricing policy in a duopoly competition with horizontal differentiation. However, in real-life, we have seen proportionately less personalized pricing than what we should expect from the theory, thereby constituting a uniform pricing puzzle. Various papers complement the early literature on the field by making extensions and loosening up on the assumptions made by Thisse and Vives (1988) and others. These papers provide rationals as to why firms might opt for uniform prices rather than personalized pricing. Our model contributes to this literature by studying a Hotelling model in which firms are enabled to invest in quality to attract consumers, as well as including an advertisement market into the framework.

We study two iterations of our model. In the first iteration, we disregard the advertiser market, thereby examining a one-sided model. In the second iteration, we include the advertiser market. In the one-sided model, we find that the firms have a dominant strategy of choosing to employ personalized prices. This complements the findings of Thisse and Vives (1988). However, in the two-sided market our result contrasts these findings when we extend the framework to include an adequately large advertisement market. We find that the firms' choice of pricing policy will be given by a mixed strategy, implying that they will not always choose to price discriminate. An adequately large advertisement market forces the firms to make high quality investments, for the reason of attracting the increasingly valuable consumers, as they are crucial for the firms' ability to earn advertisement revenue. This competition for the consumers through investing in quality is costly for the firms. In the model, we observe how the choice of pricing policy also can be considered a choice of optimal quality investments by the firms. The different pricing policies affect a firm's and its rival's incentives to invest in quality differently. This way, the firms will choose their pricing policy based on a mixed strategy, with the purpose of weakening the competition in quality.

Few papers combine the choice of pricing policy and the presence of a two-sided market in duopoly competition. We construct a model which serves as a simplified and general framework to study this type of competition. Our findings contribute to help explain the uniform pricing puzzle. More specifically, it highlights the role of two-sided markets in the choice of pricing policy.

Naturally, many different aspects come into play when firms are choosing pricing policies in the digital economy. Furthermore, the digital economy is still a fairly young industry and is continuously developing. Consequently, there are several potential ways to expand on this model and make various specifications.

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Appendix

A1 Second order conditions for quality investments

One-sided market

UP,UP

$$\frac{\partial \pi_i}{\partial q_i} = \frac{3t + q_i - q_j}{9t} - \phi q_i$$

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = \frac{1}{9t} - \phi < 0$$

$$\phi t > \frac{1}{9}$$

UP,PP

$$\frac{\partial \pi_1}{\partial q_1} = \frac{t + q_1 - q_2}{4t} - \phi q_1$$

$$\frac{\partial^2 \pi_1}{\partial q_1^2} = \frac{1}{4t} - \phi < 0$$

$$\phi t > \frac{1}{4}$$

$$\frac{\partial \pi_2}{\partial q_2} = \frac{3t - q_1 + q_2}{8t} - \phi q_2$$

$$\frac{\partial^2 \pi_2}{\partial q_2^2} = \frac{1}{8t} - \phi < 0$$

$$\phi t > \frac{1}{8}$$

PP,PP

$$\frac{\partial \pi_i}{\partial q_i} = \frac{t + q_i - q_j}{2t} - \phi q_i$$

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = \frac{1}{2t} - \phi < 0$$

$$\phi t > \frac{1}{2}$$

Two-sided market

UP,UP

$$\frac{\partial \pi_i}{\partial q_i} = \frac{3t + q_i - q_j}{9t} + \frac{k}{6t} - \phi q_i$$

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = \frac{1}{9t} - \phi < 0$$

$$\phi t > \frac{1}{9}$$

UP,PP

$$\frac{\partial \pi_1}{\partial q_1} = \frac{t + q_1 - q_2}{4t} + \frac{k}{4t} - \phi q_1$$

$$\frac{\partial^2 \pi_1}{\partial q_1^2} = \frac{1}{4t} - \phi < 0$$

$$\phi t > \frac{1}{4}$$

$$\frac{\partial \pi_2}{\partial q_2} = \frac{3t - q_1 + q_2}{8t} + \frac{k}{4t} - \phi q_2$$

$$\frac{\partial^2 \pi_2}{\partial q_2^2} = \frac{1}{8t} - \phi < 0$$

$$\phi t > \frac{1}{8}$$

PP,PP

$$\frac{\partial \pi_i}{\partial q_i} = \frac{t + q_i - q_j}{2t} + \frac{k}{2t} - \phi q_i$$

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = \frac{1}{2t} - \phi < 0$$

$$\phi t > \frac{1}{2}$$