Norwegian School of Economics

Bergen, Spring, 2021

Opportunities for Retail Investors in Alternative Investments

An investigation of whether liquid alternatives can increase diversification and enhance returns in retail investors' portfolios

Mikael Lassooy & Lars Kåre Helgesen

Supervisor: Aksel Mjøs

Master thesis, MSc in Economics and Business Administration,

Financial Economics

NORWEGIAN SCHOOL OF ECONOMICS

Abstract

Today's low-interest environment implies that expected stock and bond returns will be lower than average over the next decade. When traditional assets are unable to meet investors' needs, alternative assets have received increasing attention. Previous research has concentrated on alternative investments from the perspective of institutional investors. We take a retail investor's perspective and examine their opportunities in the universe of alternative investments. Contrary to the belief that alternative assets are always illiquid, a growing number of so called ‘liquid alternatives’ have been listed on public stock exchanges. Business Development Companies, Listed Private Equity, Real Estate Investment Trust, High Yield bonds, and Commodities have demonstrated in this paper that they are capable of meeting new demand that stocks and bonds cannot. This study demonstrates that while listed alternatives have a higher risk, especially during market turbulence, they can still help increase portfolio diversification and risk adjusted return. Our results suggest that regardless of the investor's objective, including alternatives enhances retail investors' portfolios and beats those that merely include traditional assets.

Keywords: Retail Investors, Modern Portfolio Theory, Alternative Investments, Liquid Alternatives, Portfolio Optimization.

We want to thank our supervisor Aksel Mjøs for the helpful guidance and support in completing this master’s thesis. In particular, we appreciate his insights and practical advice throughout the writing process and empirical analysis.

Table of Contents

List of Figures

List of Tables

1. Introduction

1.1 Motivation and Purpose

For more than 20 years, well-established institutions and extremely wealthy individuals have added alternative assets to their portfolios, with the main idea of diversifying their exposures and enhancing long-term returns. Alternative assets are often defined as investments that are not categorized as traditional assets accessible to the majority of investors, such as stocks, bonds, or cash. The term "alternative investment" refers to private equity, private debt, real estate, commodities, and high-yield bonds in this paper. Alternatives have generally been considered an exclusive luxury of institutions and ultrarich individuals alone. As entry tickets vary between millions of dollars and regulators demand professional accreditation, these investments are hardly accessible for traditional retail investors directly.

Over the past few decades, the demand for these products has been stronger than ever, particularly among institutional investors. Since 2000, alternative assets have risen from just one trillion dollars combined to over $10 trillion. Furthermore, Preqin has forecasted this value to reach $17 trillion by 2025. While institutional investors initially fueled the growth, retail investors have also been clamoring for a piece of the action during the past decade. (Holmes, 2019; Lowery, 2020.)

And not for nothing, according to a study presented by the U.S. securities and exchange commission (SEC), the average 20-year return to 2016 in private equity (PE) was around 12% after fees, while a corresponding number for S&P500 was merely 8% (CCMR 2018; Finley, 2019). Furthermore, PE showed a lower level of volatility during the same period compared to S&P500. If this is true, retail investors are unarguably cut out of major return potential in the public markets. (Bricker & Johnson, 2015; Finley, 2019.) Nonetheless, returns and volatility in PE are debatable, and many researchers get varying conclusions about the true returns when risk and cost are included.

While assets under management (AUM) have increased amongst the alternatives, so has the supply of investment targets. According to World Bank's study in 2018, the number of listed companies has decreased 39% in the past 25 years, while the market cap has increased 492%. More and more companies are tapping private financing to skirt the bureaucratic hassle of going public. As the chairman of the SEC, Jay Clayton, put it, "retail investors have less access to the market nowadays than ever. And more importantly, less access to companies that are well-established, but still growing." (Finley, 2019). Considering the decline in the number of public companies, achieving true diversification without allocating to private markets is problematic (Døskeland & Strömberg. 2018).

However, as Keynes' law states, demand creates its supply. Financial institutions' ability to package financial products to meet investor demand is, after all, one of the cornerstones of the industry. A solution for the increased retail demand for the private markets has been so-called "liquid alternatives." These products began to rise after the financial crisis in 2008 and are built to mimic their institutional brethren, such as private equity, real estate, and private debt. However, unlike private partnerships, these products are sold via registered vehicles like mutual funds, closed-end funds, and exchange-traded funds, offering daily liquidity (Morningstar, 2019.). According to J.P. Morgan's capital market expectations report, asset managers are creative in their approach to developing financial vehicles that should serve as alternative assets for ordinary investors. They observe that a growing number of semiliquid structures, such as interval funds and closed-end real estate investment trusts (REITs), are finding their way into the portfolios of the average investor (J.P Morgan, 2021).

Interest in such products was quickly proved by the market demand, especially in the European cross-border markets. Assets under management in these vehicles grew from 2008's $50 billion to over $510 billion by 2019. Similarly, the number of such funds has risen by 76%, which equates to 2,663 live open-ended funds in Morningstar's database as of 2019. This growth is faster than for any other asset class. (Morningstar, 2019.)

Despite their high level of demand, retail investors' portfolios remain highly unallocated in alternative assets compared to other investors. On average retail investors allocate around 5% of their portfolios to alternative investments, while the corresponding number for pension funds and endowments is around 30% (MMI & Dover, 2015; Willis Towers Watson 2016; Finley, 2019). The returns are also notably different, with institutions averaging a 6,9% annualized return against individual investors' 3,4%[2] between 1997 and 2017 (Wilshire Compass, 2018; FS Investment Solutions, 2019). Surely, an institution's wealth

[2] Individual investor is calculated as 60% average equity fund investor and 40% average fixed income investor. Data for average equity fund investor and average fixed income fund investor are from the DALBAR Report. Data for average institutional investor is from Wilshire TUCS®, defined as the median total return, gross of fees, of master trusts — all plans.

management capabilities and returns are not directly comparable to individuals, but that does not alone explain the massive allocation differences. As the barriers to enter such markets are being torn down, individuals may now have a chance to reach their investment goals by allocating more of their portfolios to alternative assets, mimicking the investment strategies of larger institutions (FS Investment Solutions, 2019).

The possible benefits of alternative assets are clear. They enable better diversification and ideally give a chance for enhanced returns. Thus, the benefit could come from two sources: better downside protection during market downturns or higher returns during market upswings, or both. These seem to be the benefits of traditional alternative assets, but it is important to note that liquid alternatives are structurally different. Thus, the interesting question is whether the current liquid alternatives offer similar uncorrelated or enhanced returns against the public markets that the underlying private funds offer. (Finley, 2019.) Furthermore, do they carry the same disadvantages and risks as the private alternatives, such as illiquidity and high costs?

1.2 Research Question and Methods

This thesis intends to investigate the opportunities retail investors have in alternative assets and what allocation – if any – of such products would be optimal in their portfolios. First, we introduce potential alternative investment products retail investors can access. Following that, we analyze the performance and characteristics of these assets both individually and as components of a portfolio. The goal of this thesis is to find answers to the following questions:

- What alternative asset classes are available to retail investors?
- How the liquid alternatives perform against their private counterparts and other asset classes in high and low economic conditions?
- Is it rational for retail investors to allocate capital in alternative assets?

This thesis is based on a literature review and an empirical study consisting of a quantitative approach. The quantitative study investigates the optimal allocation of alternatives in retail investors' portfolios using historical data and market expectations from various asset classes. Additionally, this thesis compares the individual performances of liquid alternatives to their private counterparts and traditional assets (stocks and bonds). The intention is only to consider instruments that are accessible for most retail investors, not solely the accredited high net worth individuals or institutions. Further, the study is done on a global scope as all of these instruments are accessible worldwide.

The quantitative section begins with Markowitz's (1952) modern portfolio theory (MPT). The theory suggests variance and historical average returns to be used as risk and return estimators. However, as Rosadi et al. (2020) mention, these estimators rely on assumptions that are rarely fulfilled in real applications and that in the portfolio analysis, the results are never optimal with uncertain parameters. Thus, alternative risk and return estimators are introduced to present a more comprehensive analysis of the topic. More precisely, in addition to historical returns, we use marked assumptions for expected returns from various experts.

Further we use expected shortfall (Conditional Value at Risk) as an alternative risk estimator and Bayes-Stein estimator as an alternative for expected returns. Additionally, a simulation approach using various lengths of rolling windows is used to present a more dynamic model to estimate optimal allocations. Lastly, we stress-test a subset of portfolios.

There is substantial literature on this subject from an institutional standpoint, but relatively little from a retail point of view. Research papers from Fischer and Lind-Braucher (2010) and Bekkers et al. (2009) fall somewhat in between the two and provide great theoretical grounds for the quantitative methodology of this thesis. They present studies of optimal portfolio allocations with multiple asset classes, including alternative assets. The choices between portfolio optimization models and statistical measures are influenced heavily by Jorion's research (1985, 1986, 1991, 1996) regarding these topics. Furthermore, alternative optimization objectives are implemented based on studies by Estrada (2010) and Choueifaty and Coignard (2008). They propose diversification and geometric return maximization as alternatives to the classic goals of risk minimization and Sharpe Ratio maximization.

This paper begins with a literature review chapter in which terms such as 'listed private equity and 'REITs' are explained. This is followed by empirical literature chapter in which the historical performances of these assets are introduced. Further, the fourth chapter presents the theoretical frameworks used in the quantitative part of this study.

Chapters 5-7 represent the quantitative part of this thesis. The fifth chapter introduces the data sources and samples in detail. After that, the sixth chapter explains the methodology and how the data is analyzed to answer our research questions. The results of the quantitative study are presented in the seventh chapter, followed by a discussion of the results in the eight chapter. Here we compare the past research and literature of the topic to the findings of this study. The last chapter then consists of conclusions that are made based on the discussion.

2. Literature Review

2.1 Alternative Investments

The following sub-chapter introduces and explains the alternative assets that are available for retail investors. In this paper, the alternative assets included in the analysis are Private Equity, Private Debt, Real Estate, and Commodities. Typically, these assets are considered to be traded only in private markets, accessible to a small percentage of market participants. However, there exist comparable assets available for retail investors listed on public stock exchanges. For private equity (PE), Listed private equity (LPE) is a publicly traded counterpart to traditional non-listed private equity. For private debt, a public alternative is investments in Business Development Companies (BDCs). For risky debt, one can invest in High Yield Bonds. For traditional real estate, real estate investment trusts (REITs) are a widely considered public alternative. Lastly, for commodities, one can invest in commodities futures indexes in the public markets. While the underlying funds are similar in many ways, the structure and liquidity of these instruments are quite different. Thus, it is intriguing to investigate whether the public and private counterparts of such assets behave similarly and the most noticeable differences between them. Based on prior literature, this chapter will clarify how these assets work and what they consist of.

2.1.1 Private Equity

Private equity, in general, means investments in privately held companies with some sort of potential. PE companies are established as limited partnerships for the sole purpose of raising capital to invest in private equity and exit their investments 5-10 years later (Berk & DeMarzo, 2020). Traditionally, a private equity firm raises capital through the creation of a private equity fund. The majority of funds are "closed-end" funds, which typically have institutional investors such as hedge funds, insurance firms, or very wealthy individuals as investors.

Private equity companies use a variety of different strategies for generating returns. These strategies are presented in the Figure 1 below.

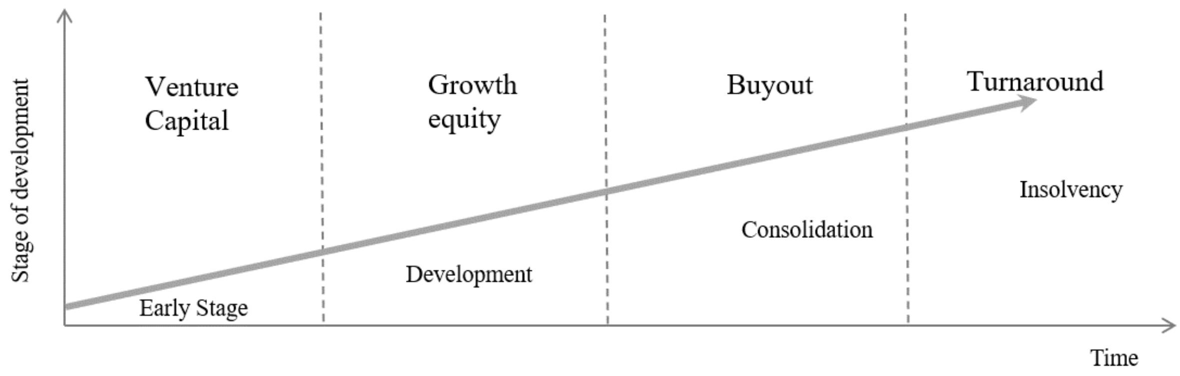


Figure 1 Private equity strategies

A buyout strategy entails identifying an established business with strong cash flows and acquiring a large equity interest. Another approach is to invest in emerging businesses and startups; this is most widely referred to as venture capital (VC). Growth equity is an investment strategy that focuses on firms that have progressed beyond the venture stage but are still experiencing rapid growth. The last big division of private equity is a strategy devoted to turnarounds, often referred to as distressed private equity. (Strömberg & Døskeland, 2018).

The sector has drawn tremendous attention due to its high fees and lack of transparency. Warren Buffet has long slammed the industry for charging excessive fees and inflating returns. Metric and Yasuda (2007) demonstrate that management fees are 2% of invested capital and that 92 % of funds have a 20% carry. Døskeland & Strömberg (2018) estimate total annual fees at about 6-7%. However, they argue that general partners (GPs) in private equity generate real value through active management and corporate governance, which are difficult to recreate in a public context. Empirical research suggests that private equity activity, on average, generates economic value (Kaplan & Strömberg, 2009). Further, the question is who gets this value. Ludovic Phalippou is a notable critic of private equity; he argues that PE generates returns comparable to public markets while charging excessive fees (Phalippou, 2020). More on returns in chapter 3.1.

Listed Private Equity

Whereas non-listed private equity is inaccessible to most investors, retail investors can participate in the market through listed private equity. Figure 2 illustrates the two dominant structures of listed PE.

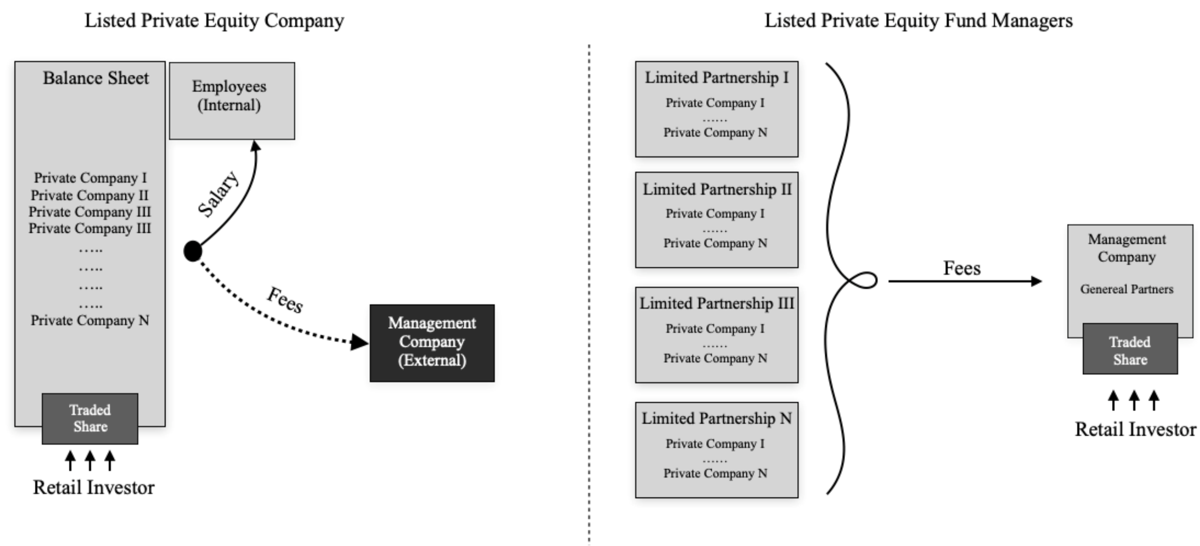


Figure 2 - Key structures of listed PE

According to Bergmann, Christopher, Huss, and Zimmermann (2009), listed private equity exposes investors to a diversified private equity portfolio. The structure of such investment is displayed left in figure 2 above. Moreover, investors can also benefit from the fees received by general partners in a situation where the management company is traded. This structure is presented on the right in figure 2 above and further explain in this chapter.

In recent years, institutional investors, have started to invest in listed private equity to control their private equity allocation more effectively (Cumming, Fleming, & Johan, 2011). In 1995, 25 private equity funds were publicly traded. By 2008, it had increased to 121 (Bergmann, Christophers, Huss, & Zimmermann, 2009), and there are now about 300 funds available to retail investors worldwide (Oakley Capital, 2020). The market capitalization of listed private equity in a global context is illustrated in Figure 3.

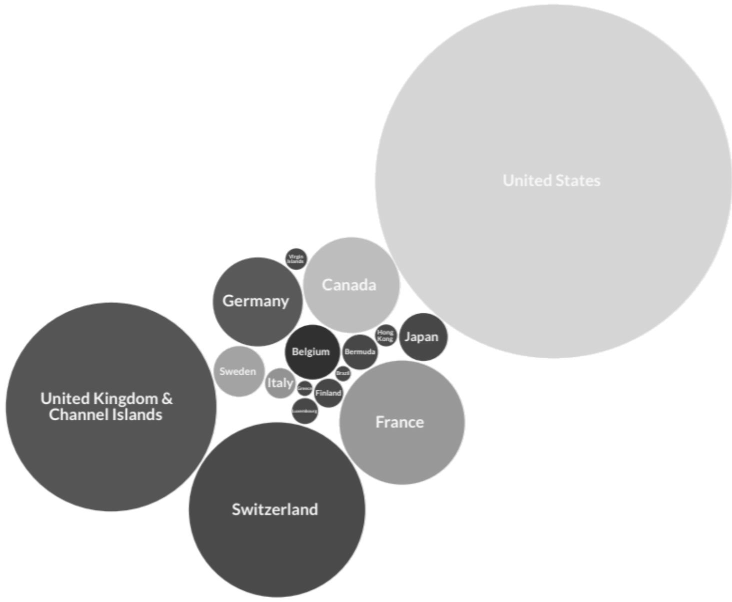


Figure 3 - Market Cap of Listed Private equity (Source: LPX & LPeC, 2020)

As illustrated in the figure above, listed PE is concentrated in the United States, but it is also growing in Europe. The growth in Europe is illustrated in figure 4 below.

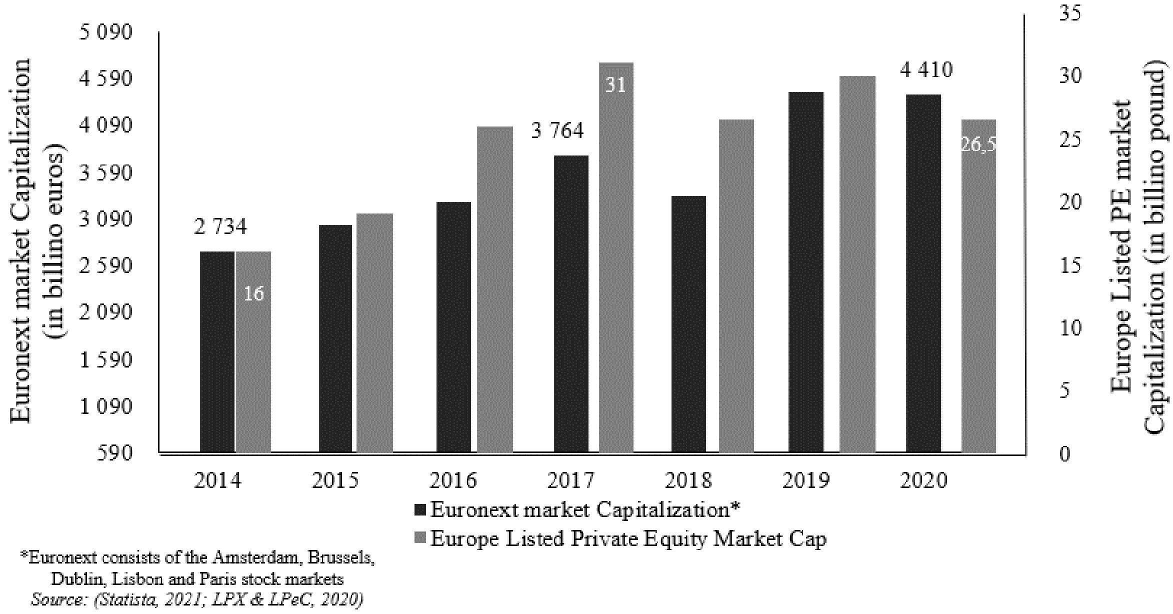


Figure 4 - Listed Private Equity versus Euronext Market Capitalization in Europe

The listed universes can be classified into three groups: Listed private equity investment companies, listed indirect private equity investment companies (fund of funds), and listed private equity fund managers (Bergmann, Christophers, Huss, & Zimmermann, 2009). The key characteristics of these structures are presented in table 1.

Table 1 - Key characteristics of listed PE structures

	Listed direct PE	Listed indirect PE (fund of fund)	Listed PE fund managers
Revenue Streams	Returns from one private equity fund	Returns from numerous private equity funds	PE managers fees
Key Assets	Investment in unlisted companies through one PE fund	Multiple limited partnerships in different PE funds	Human capital
Value propositions	Investor gains access to a diversified pool of unlisted companies	Investor gains access to a diversified pool of limited partnership	Investor gains access to management firm's future performance and dividends generated by PE managers' high fees.
Examples	3i Group plc	Pantheon International Participations PLC	The Blackstone Group LP
Key Costs	50 % of the listed vehicle have external management; they charge a 2% fee and 20 % carry from the listed fund. 50% have internal management; they mimic the high fees through options & bonuses.	The investor is subject to a double fee structure: one through LP in each PE fund, and another for the fund of funds manageres. Fund of funds managers usually charge 1-2 percent in addition to the 2% fee and 20% carry charged by the underlying PE funds.	Salaries

Listed Direct Private Equity Investment Companies

Direct private equity is a listed fund that invests directly in private companies. The intrinsic value of the stock would be the sum of the future performance of the private investments. Bergmann et al. (2009) discovered that half of the listed direct PE had internal management (management is like employees), while the other half had external (controlling the firm from an extern entity).

The reasoning behind internal or external is little discussed in the literature. However, the most logical understanding is that it comes down to preferences from the management team for how they want to be compensated. If the firm is managed externally, the fees are arranged similarly to traditional non-listed private equity; usually, a 2% annual fee on committed capital and a 20% carry that is paid out to the external management firm. This will reduce the profits for the listed PE firm, which again reduces the return for shareholders. If the managers are in-house, the fee structure is different. According to Metric and Yasuda (2007), in-house managers often attempt to replicate the carry-fee structure by combining stock options and large bonuses. Again, this reduces the return for shareholders.

Listed Indirect Private Equity Investment Companies (Fund of Funds)

Listed indirect private equity investment companies are PE funds that invest in other PE funds, therefore called funds of funds. Globally, according to a 2009 survey, there were 27 publicly traded listed PE funds of funds, 25 of which had external management. The external management firm will charge management fees, creating a "double fee structure" for the listed fund. First, one must pay fees to direct private equity funds for being an LP, and secondly, fees to investment managers' that manage the listed fund of fund (Bergmann, et.al, 2009). According to Ljungqvist and Richardson (2003), a listed fund of funds' fee structure is usually 1-2% in addition to the fees paid as a limited partner that is mostly 2 % and a 20 % carry.

Listed Private Equity Fund Managers

The last structure is where the management firm is publicly traded. Bergmann et al. (2009) estimate that this is a minority of the described listed PE universe. Since management firms do not invest in private companies directly, owning a share in them exposes investors to the management firm's future performance and dividends generated by PE managers' high fees.

2.1.2 Real Estate Investment Trusts

REITs are mutual funds investing in real estate and mortgages. They were introduced in the 1960s after implementing tax laws that would exclude REITs from federal corporate income taxes (Schulkin, 1971). The aim was to allow retail investors to invest in real estate and mortgages in the same way they can invest in public stocks through mutual funds. Mull and Soenen (1997) conclude that REITs can be an important asset class for a diversified portfolio of professionally managed real estate due to their high liquidity. Schulkin (1971) classified REITs into two categories: long-term investments and construction and development loans.

Hoesli and Lizieri (2007) concluded in a report for the Norwegian government that listed real estate securities are associated with the long-term success of the underlying real estate fund. However, in the short run, the overall market conditions influenced the volatility and price movements.

To qualify as a REIT, the REIT's total assets must consist of at least 75% real estate, cash, and government securities. Additionally, at least 75% of gross revenue must be derived from rents, mortgage interest, and capital gains on real estate. Finally, 90% of REITs' profits must be distributed to shareholders (Schulkin, 1971). The criteria are intended to limit management's influence on investment decisions to maximize investors' exposure to underlying real estate returns (Hoesli & Lizieri, 2007).

REITs managers are mostly structured through an external company that charges fees, normally 2 % on invested capital and 20 % carry. The management firm acts as the REIT's advisor, manages daily operations, and presents investment opportunities (Schulkin,1971). Commercial banks, financial conglomerates, mortgage bankers, and insurance firms are often named advisors to REITs.

Compared to private real estate indices, using REITs in asset allocation is more favorable (1995, Frott). Private data is often based on appraisal-based return, in which the appraiser determines the value of the real estate. The same accounts for the private equity sector, where the valuations for the underlying investments are based on subjective valuations by an appraiser. Utilizing such indices for mean-variance asset allocation could introduce biases (Giliberto, 1988). Geltner (1991) argues that appraisal-based returns would understate real estate volatility. Froot (1995) suggested using REITs in asset allocation because the returns are dependent on real transaction prices, resulting in higher-quality REIT return data. Thus, REITs are considered to be superior to private real estate in terms of portfolio construction (1995, Frott).

2.1.3 Business Development Companies

A Business Development Companies (BDCs) operates similarly to venture capital or private equity company in several ways; they serve on the board of directors, recruit senior executives, review growth strategies, and engage in strategic decisions (Anson, 2004). The 1980 act authorized BDCs to apply leverage up to a ratio of 1:1 to the fund's net assets, in other words, allowed 100% leverage. Compared to traditional mutual funds, which can only lend up to one-third of their assets, BDCs carry greater risks but still have the potential for greater returns (Anson, 2004).

BDCs generate income primarily from interest on private debt or hybrid instruments, but they may also earn revenue through profits from private equity investments. One reason this is an attractive investment opportunity for retail investors is that BDCs are required by securities law to pay 90% of their profits to shareholders in the form of dividends. However, the investor's return is limited by high management fees. Since administration is mostly outsourced to an external firm, that typically charges 2% of invested capital plus a 20% carry (Anson, 2004).

BDCs was founded with the objective of investing in both private equity and private debt. However, the majority of BDCs now invest exclusively in debt instruments. BDCs target the underserved middle market, whom banks find too expensive to lend to, allowing BDCs to lend at relatively high rates. Figure 5 below details a selection of BDCs' (IPOs) and their key investment strategy.

Name of BDC	Manager of Fund	IPO Amount (millions)	Target investments
Apollo Investment Corporation	Apollo Investment Group	$930	Long term Sub.Debt and Senipr Secured Loans
Blackridge Investment Corporation	Blackstone Group	$850	Mezzaine, Senior Secured Bank Loans
KKR BDC Inc	Kohlberg Kravis Roberts	$750	Secured and Unsercured Senior & Junior Debt
BlackRock Kelso Capital	BlackRock/Kelso & Co	$750	Long term Sub.Debt and Senipr Secured Loans
Gleacher Investment Corp	Erik Gleacher	$500	Mezzaine, Senior debt
THL Investment Capital Corp	Thomas H Lee Partners	$500	Long term Sub.Debt and Senipr Secured Loans
Evercore Investment Corporation	Evercore Group	$460	High Yield, Distressed Debt, Bridge Loans
Ares Capital Corporation	Ares Capital Management	$450	1st & 2nd Lien Senir Loans & Mezzanine
Gores Investment Corporation	Gores Technology Group	$250	Senior Debt, Mezzaine and Debtor-in-Possession

Figure 5 - Selection of BDCs IPO

The fundamental strategies described in figure 5 involve lending to small and medium-sized businesses. Apollo Investment Corporation explains that its strategy is to provide private middle-market companies with tailored funding strategies (Apollo, 2021). BDCs also use senior unsecured debt, subordinated securities and unsecured debt, high yield, convertible bonds (Anson, 2004).

Retail investors should find BDCs appealing due to their high dividend yield and exposure to private markets, especially private debt. Along with dividend yields, share price volatility affects investment returns. Since the BDCs pay out 90% of their earnings as dividends, the opportunity for capital gains is minimal, and as a result, their listed stock remains relatively stable. The share price fluctuates in response to the performance of the underlying investments, especially when loans default. Additionally, it depends on how the firm raises new capital and how the investors perceive management's latest acquisitions and loans.

As of now, BDCs only have jurisdiction to operate in the US. European managers are increasingly adopting this model; however, their biggest hurdle is the region's lack of regulatory harmonization (Dechert LLP, 2019). Retail investors worldwide can still access BDCs listed on exchanges in the US through various brokerage platforms.

However, BDCs should be treated cautiously because their underlying investments are often financially distressed or growing businesses with uncertain potential cashflows. Nonetheless, private investments result in a lack of publicly available information, thus reducing market transparency (Anson, 2004).

2.1.4 Commodities

Institutional investors investing in commodity futures date back to the 1800s (Ankrim & Hensel, 1993). Commodities enable investors to invest in real assets in favor of, for example, real estate. Investors often invest in commodity indices that track the development of the commodity's underlying prices. Commodity investing is studied in this paper through the context of long positions, infrequent selling, and broad exposure to commodities.

Retail investors can invest in commodities through commodity indexes; this instrument provides access to a basket of the sectors in figure 6. The portfolio's weights are mostly determined by the global market capitalization of each asset. By the use of futures contracts, the index generates returns comparable to long positions in commodities (Tang & Xiong, 2012). Retail investors may gain exposure to commodities through commodity index swaps, exchange-traded funds, and exchange-traded notes.

Figure 6 below presents the allocation weights of the S&P GSCI Index[3]; the weights are based on world production weights, based on each asset as of 2021 (S&P GSCI, 2021).

[3] The S&P GSCI Index is a widely recognized as the leading measure of generegal commodity price movements in the world economy. Index is calculated on a world production weights basis, comprised of the principal psyical commodities future contracts.

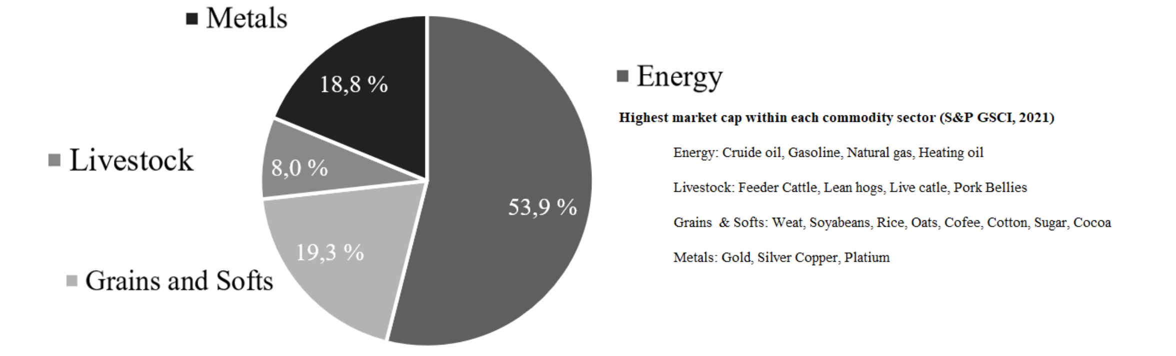


Figure 6 - Composition of S&P GSCI commodity index
Source: S&P GSCI (2021)

Institutional investors have increased their exposure to commodity futures since the early 2000s. Investments in commodity-related instruments rose from $15 billion in 2003 to $200 billion in 2008 (CFTC, 2008). Institutions invest in commodities for diversification purposes (Buyuksahin and Robe, 2014; Singleton, 2014).

However, Basak & Pavlova (2016) discovered that the presence of institutional investors has the power to influence commodity prices and dynamics. Additionally, Irwin & Sanders (2011) discovered that increased institutional investor involvement, also known as the financialization of commodities, increased commodity correlation. The asset's demand and supply do not solely determine the underlying commodity price. Nonetheless, non-energy commodities were found to be correlated with the price of oil and gas (Tang & Xiong, 2012), weakening the diversification advantages of owning all commodities.

2.1.5 High Yield Bonds

High yield bonds, also known as "junk bonds", are bonds with a credit rating below the S&P BBB (Berk & DeMarzo, 2020). Retail investors can still maintain control over the risk of high yield bonds if the investor diversifies by holding an ETF of these products. Several well-known high yield ETFs include Barclays High Yield Bonds and iShares iBoxx High Yield corporate bonds issued by BlackRock, the world's largest private equity company. However, there are costs associated with investors trading ETFs. For instance, Barclay charges an annual fee of 0.15% on invested capital. However, low compared to PE (Meziani, 2006).

Daniel Jark (2020) argues that the term "junk bonds" is misleading. While some bonds will always be risky and default, the quality of the majority of bonds is fairly high. Additionally, he emphasizes how much more stable high yield bonds are than the stock market, placing their returns and volatility between bonds and stocks. Additionally, in the event of a corporation's bankruptcy, bondholders collect their capital before equity holders. Thus, most institutional and accredited investors diversify their portfolios by investing in high-yield bonds (Jark, 2020). Regardless, there will still be an element of uncertainty around high yield since they bear a higher chance of default than traditional bonds. According to Houwelingn (2012), junk bonds appear to fall rapidly during recessions, which this paper will analyze in greater depth later.

3. Empirical literature

The following chapter reviews the empirical literature on alternative assets' historical performance and their treatment in portfolio optimization. Additionally, the risks and performance of alternatives during past economic crises will be covered in this chapter.

3.1 Past Performance of Alternative Assets

3.1.1 Private Equity

Finding and calculating accurate returns for non-listed private equity can be difficult. Numerous commercial platforms gather performance data; however, the data source is often complex, resulting in return sample bias (Harris, Jenkinson & Kaplan, 2014). Harris, Jenkinson, and Kaplan (2014) analyzed a Burgiss dataset containing 1400 buyout and venture capital firms in the United States. They discovered that buyout strategies outperformed the S&P 500 by more than 3% a year after fees. In the 1990s, venture capital outperformed the S&P 500 but stalled in the 2000s. Additionally, they compared their findings to performance data from other datasets from Cambridge Associates and Preqin and discovered that their findings were consistent.

Phalippou and Gottschalg (2009), on the other hand, discovered the opposite. They discovered that the private equity market underperformed the S&P500 by 3% per year and 6% after risk factors were considered. In a newer study by Phalippou in 2013, he found that buyout funds underperformed the DFA micro-cap [4]index by 3.1 % annually, but comparing his findings against the S&P500, buyout outperformed by 5 % after fees. Another research done by Hwang, Quigley, and Woodward (2005) indicates that PE returns after fees are very similar to those of the S&P500. The literature is divided, and actual returns are difficult to obtain in private equity, where fees can destroy much of the actual investor value. Despite this, the average empirical evidence indicates that private equity has outperformed public markets on returns after fees, although this cannot be verified with certainty (Moskowitz & Vissing-Jørgensen 2002; Ljungqvist & Richardson 2003; Guo, Hotchkiss, & Song 2008; Jagadeesh,

[4] *DFA Micro Cap index is offers broad exposure to the smallest stocks listed in the U.S. market*

Kraussl & Pollet 2009; Harris, Jenkinson, & Kaplan, 2014; Harris, Jenkinson, & Kaplan, 2016).

3.1.2 Listed Private Equity

A crucial investment element in listed private equity is determining if it will generate the same returns as the unlisted version. According to empirical studies, the primary risk factor affecting returns in traditional PE is liquidity risk, which is discussed in further detail in Chapter 3.4. However, listed PE does not face the same liquidity constraints. If the risk factors are not identical, how can the return capabilities be equal, further can listed PE outperform common stocks in terms of returns?

Martin and Petty (1983) published the first research on the topic in 1983. They discovered that listed private equity delivered better returns than the stock market between 1973 and 1979. Zimmermann, Bilo, Christophers, and Degosciu (2005) discovered that their sample of 229 publicly traded private equity funds correlated 0.4 with the MSCI world index and just 0.02 the global bond market. Michel Degosciu (2012) questioned whether listed private equity's net asset value (NAV) return was equal to unlisted private equity. The NAV reflects the valuation of the underlying investments of private companies. This is often determined by ongoing valuations or by the price paid by the PE fund for the private company. The correlation between listed PE NAV and unlisted PE NAV was 0.94 between 2003 and 2011 and statistically cointegrated. He said that long-term investors should remain neutral on whether to invest in listed PE or unlisted PE.

Since listed private equity is subject to daily valuations and pricing, stock values do not necessarily correspond to NAV. Michel Degosciu (2012) identified that NAV indexes are significantly less volatile than listed price indexes. Figure 7 illustrates the market capitalization and net asset value (NAV) of European-listed private equity firms. As can be seen, the NAV is continuously below the market capitalization.

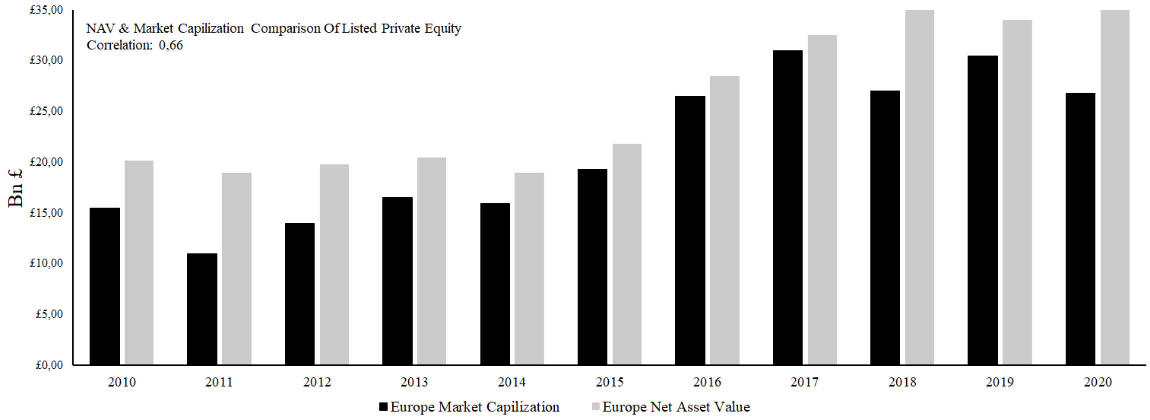


Figure 7 - European listed private equity market cap and NAV(Source: LPX & LPeC, 2020)

Huss (2005) discovered that unlisted private equity funds outperformed listed funds in terms of average returns. However, the median return of unlisted funds was somewhat lower than that of listed PE funds. He concludes that there is no noticeable difference in performance between listed and unlisted private equity. As a result, Huss casts doubt on his hypothesis that the illiquid premium is the primary driver of private equity's superior returns.

In chapter 7.2, using the most recent data available from Preqin private database, the relationship between listed and unlisted PE will be investigated. The historical return will be further investigated in chapter 7.1. However, figure 8 below presents listed VC, PE, and buyout development from 2010 to 2021 against the MSCI world stock index.

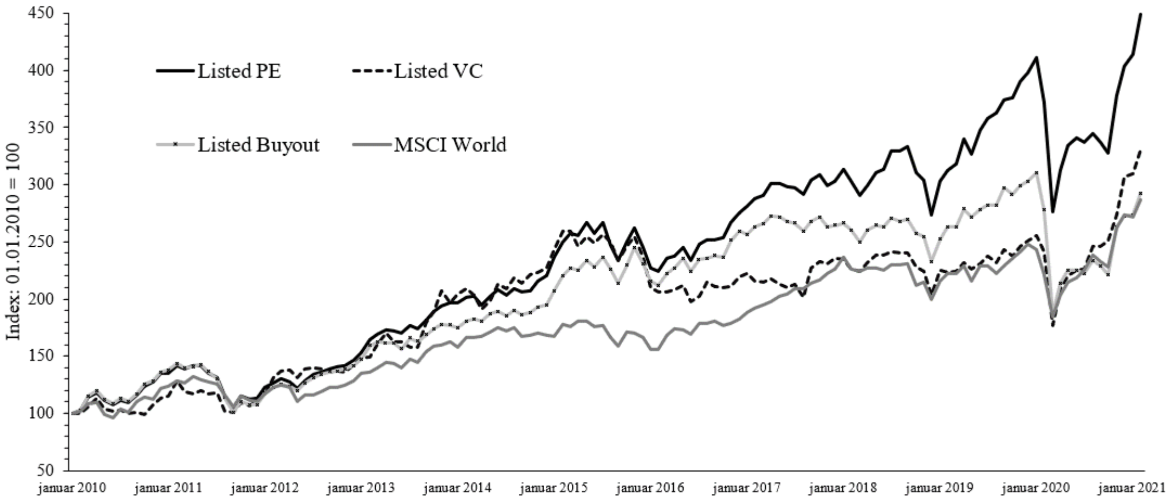


Figure 8 - Comparison of Listed PE and MSCI world index (Source: Bloomberg)

3.1.3 Business Development Companies

Between 1980 and 1990, Kleiman and Shulman (1992) analyzed 26 BDCs. They discovered a beta of 1.07 and discovered that they underperformed the NASDAQ index between 1980 and 1986 but outperformed it between 1986 and 1990. According to Anson (2004), back then, BDCs paid dividends between 13 and 18 percent. Today, according to a dataset comprising 40 BDCs, the median and average annual dividend yields are about 9%, with the top quartile yielding 11%. (Ciura, 2021). Kallenos and Nishiotis (2019) examined the BDC market's characteristics and discovered a 0.55 correlation between the BDC's net asset value and the global private equity market. They claim that BDCs and the S&P 500 have a low correlation. Additionally, they observe that BDC returns are cyclical in the same way that other empirical literature on PE returns has documented.

The dividend yield offered by Cliffwater[5] is illustrated in Figure 9. The average dividend yield of this index has been 10%, compared to the S&P 500's 2% (Shiller, 2021). As of March 2021, the dividend yield for BDCs was 8.4% (Cliffwater, 2021), whereas the S&P 500 yielded 1.58%. Although stocks do not have similar obligations to pay out dividends as BDCs, the difference is still noteworthy. It also illustrates well the different cash-generating features BDCs have compared to stocks.

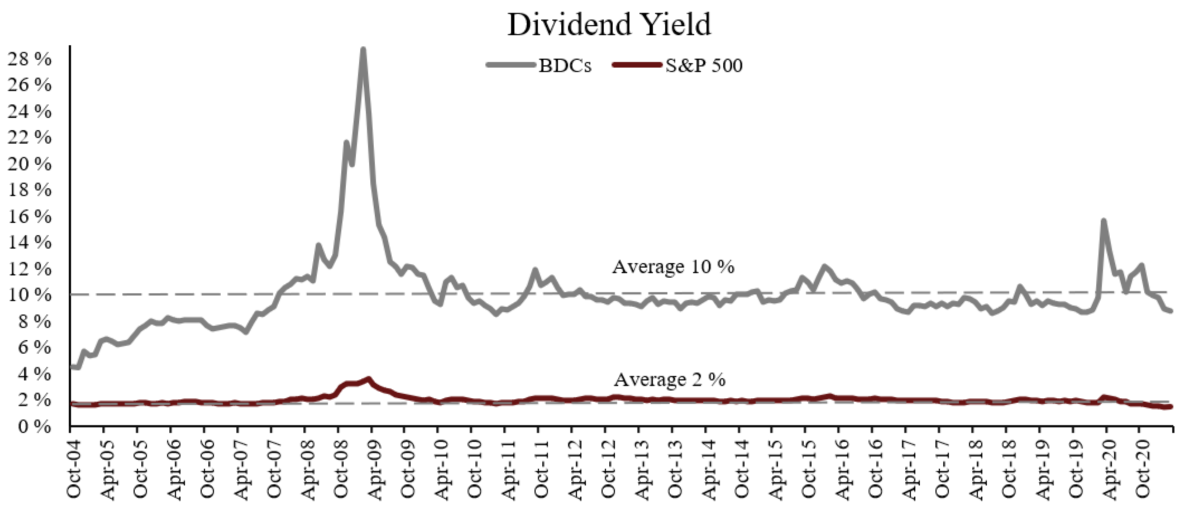


Figure 9 - Comparison of Dividend Yields S&P500 vs. BDCs (Shiller, 2021; Cliffwater, 2021)

[5] Cliffwater is the largest lending-oriented exchange-traded BDC index. Today, their index includes 38 publicly traded BDCs

3.1.4 Real Estate Investment Trusts

Between the late 1980s and 2000, REITs attracted significant attention. Grissom, Kuhle, and Walther (1987); Gilberto (1993); Froot (1995); and Eichholtz (1996) are only a few of the empirical studies that indicated real estate should be a key component of an investor's portfolio. They argue that REITs have superior returns to bonds and stocks and show low correlation. However, the studies are old.

Mull & Soenen (1997) suggest that US REITs offer diversification and inflation hedge to a traditional portfolio and significantly increase their risk-adjusted returns. From 1985 to 1994, REITs yielded a total return of 10.86 %, with a standard deviation of 12,06 %, simultaneous the S&P 500 return of 14%, but with a higher standard deviation of 15,25 %. They discovered that REITs underperformed the S&P 500 by 8% over this estimation period. Corgel and Djoganopulos (2000) argued that in the majority of cases, the equity beta should be less than 0.4. They attribute the low beta to the long-term real estate contracts, generating consistent and stable cash flows.

Figure 10 compares the dividend yield of S&P 500 and US REITs. Between 1972 and 2021, US REITs averaged a dividend yield of 7.34% (NAREITs, 2021), compared to the S&P average of 2.84% (Shiller, 2021). As of December 2021, the average dividend yield of REITs was 3.84% (Nareits,2020).

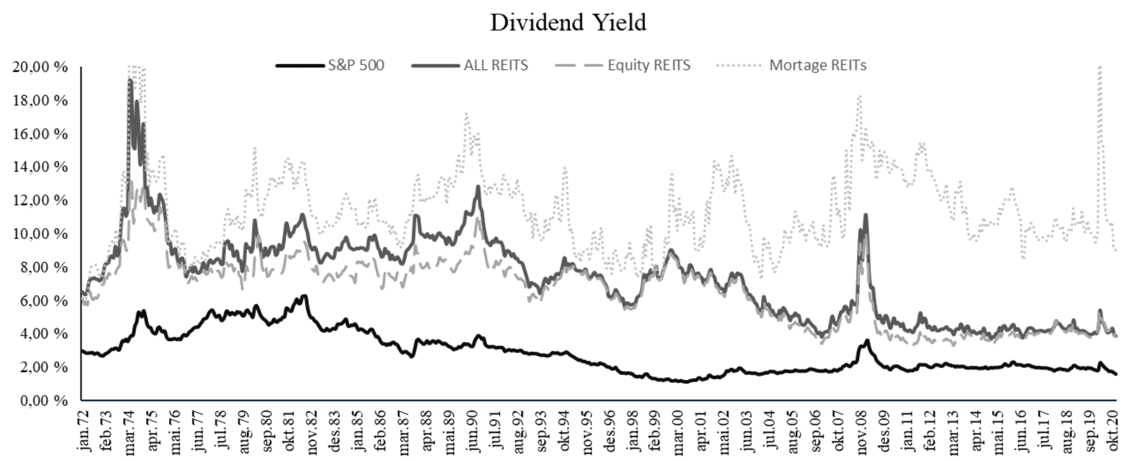


Figure 10 - Comparison of Dividend Yields S&P500 vs. US REITs (Shiller, 2021; NAREITS, 2021)

Since 2000, the literature has focused on REITs' risk factors. Bond & Xue (2017) discovered that profitability is a strong predictor of REITs returns. Chuei, Titman, and Wei (2003) observe that the returns of REITs are influenced by momentum. Ling and Naranjo (2015) established that leverage has a significant effect on returns. According to a follow-up report by Shen (2020), the REITs industry has a higher bankruptcy rate than the general market. Between 2007 and 2008, REITs suffered a massive share price decline due to their high leverage level (Ling & Naranjo 2015).

3.1.5 Commodities

Bodie and Rosansky (1980) and Gorton and Rouwenhorst (2006) propose that long-only portfolios of commodity futures result in equity-like returns. Erb & Harvey (2006) investigated commodities performance from 1969 until 2004. They found that S&P 500 had an annualized compounded return of 11.20 % with a standard deviation of 15.64 %. In comparison, the Goldman Sachs Commodity Index (GSCI) returned 12,24 %, with a standard deviation of 18.35 %. The two assets had a negative correlation of -0.03.

The literature mentioned above is not the first to discover and discuss a negative correlation between commodity and equity returns. Bodie & Rosansky (1980); Fortenbery and Hauser (1990); Ankrim & Hensel (1993), and Forton and Rouhenhorst (2006) are notable works of literature that all have the same result. Diversification benefits have been reported to vanish in recent literature due to commodity financialization through institutional investors' entrance (Cheung & Miu, 2010). Figure 11 compares the S&P Commodity Index with the MSCI world index.

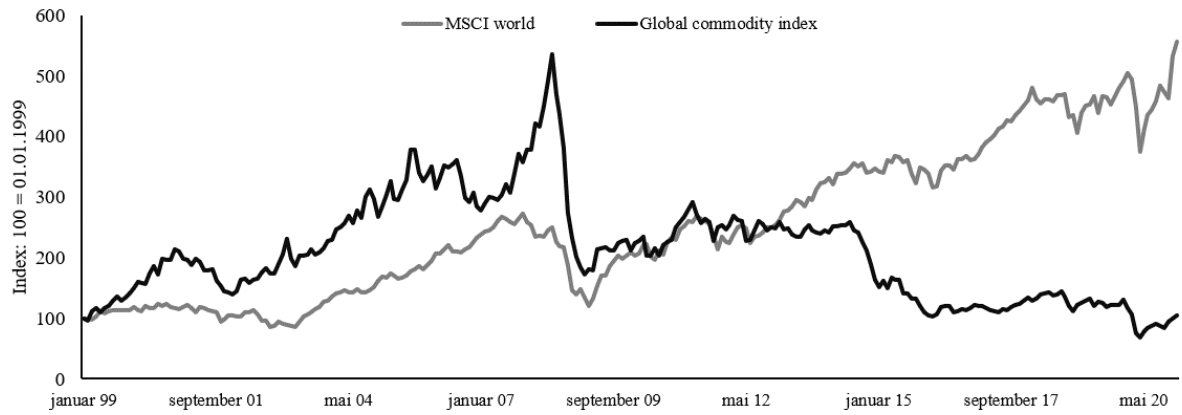


Figure 11 - Comparison of commodity index and MSCI world (Source: Bloomberg)

From January 2005 to June 2008, Daskalaki & Skiadopoulos (2011) conclude that supplementing commodities to a US investor's portfolio is beneficial. From 2008 until 2020, the S&P 500 has outperformed the broad commodity index (Figure 11). The reasoning behind this is because WTI crude oil has dropped more than 50 % since 2008, and oil is a big part of the index. Therefore, investing in the broad commodity index includes high risk since the index is driven by oil price development (Gagnon, Manseau, & Power, 2020)

Finally, Gagnon, Manseau, and Power (2020) argue that commodities offer significant diversification benefits but are more beneficial to investors with a high-risk tolerance. Additionally, they confirm that indices with a lower allocation to the energy sector provide better diversification and return.

3.1.6 High Yield Bonds

Hernandez (2020) claims that High Yield, compared to the stock market, provides less volatility in the long run and produces higher returns than traditional bonds. In today's low-interest-rate environment, traditional bonds offer a meager return. Turning to High-yield can result in a relatively high and steady income to a portfolio. Moriarty (2019) argues that High-yields are riskier than Treasuries or Investment-grade bonds.

In terms of past performance, according to Reif (2021), the average over the last 12 months is 7.7%, while the S&P 500 has achieved 19.8%. To put it in a more historical context, Bekkers, Doeswijk & Lam (2009) use a consensus estimate based on previously reported studies to determine High Yield bonds' risk premium. They reported an annual risk premium for high yield of 2.5 - 3.2% on average and a volatility of 11%. Traditional bonds were assigned a risk premium of 0.75% with a volatility of 7% in the same report. As a result, high yield generates a higher return per unit of risk.

Altman (1998) conducted a study on High Yield in the United States. He used data from 1978 to 1997 and arrived at a risk premium of 2.5%, with a standard deviation of 5.2%. Comparing traditional bonds with this estimate, Altman (1998) settled that Investment grade bonds had a risk premium of 0.8%, but a slightly higher standard deviation than High-yield. Jong and Driessen (2005) explain the difference between High Yield and Bonds with a liquidity premium and a default risk premium. Elton et al. (2001) further describe the yield-spread between Treasuries and High Yield from tax premium and systematic risk premium.

Figure 12 below visualizes the development of global High Yield Corporate Bonds[6] compared to the index of global investment-grade bonds[7] from 2007 until 2021.

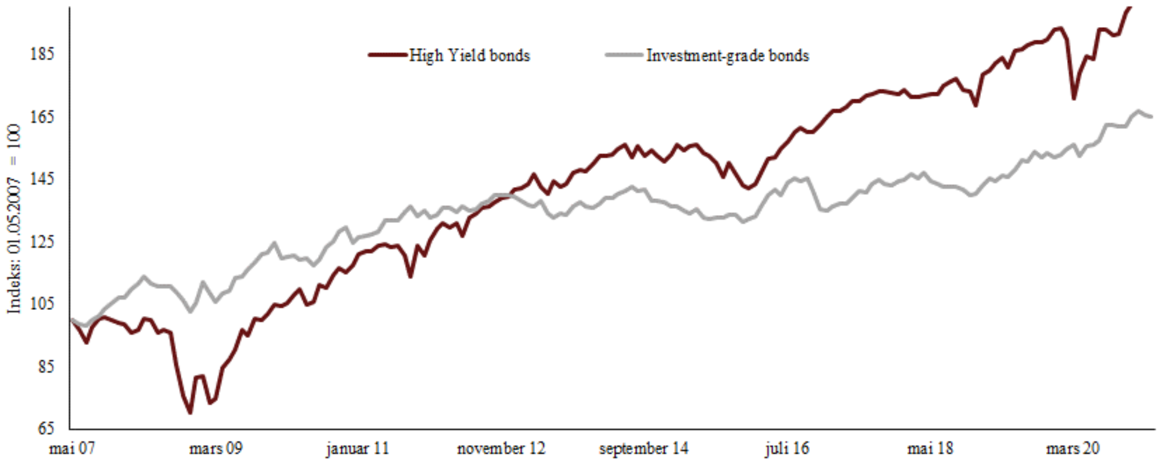


Figure 12 - Comparison between High Yield bonds and Investment grade bonds (Source: Bloomberg)

[6] BlackRock provides the index, which is a High Yield Corporate Bond ETF. The ETF seeks to replicate the performance of an index of High Yield Corporate Bonds.

[7] The index is the Bloomberg Barclays Global Aggregate Index, which serves as a benchmark for investment-grade debt globally.

3.2 Optimal Allocation to Alternative Assets

The past literature is limited when it comes to the optimal allocation of retail investors in alternative investments. There is research about this from the perspectives of institutions and high net worth (HNW) individuals. However, since retail investors do not have the same access as they do, this research is not completely applicable to our setting. However, such studies do an excellent job shedding light on the evolution of the role of alternative assets in retail investors' portfolios.

Fischer and Lind-Braucher (2010) investigated an optimal portfolio (from 1999-2009) of retail and institutional investors and were interested in the potential benefits of including alternative assets. The results show clearly that investors would be better off adding some alternative investments to their traditional portfolios of stocks and bonds. More specifically, their maximum performance portfolios were allocated roughly over half in bonds, and the rest divided amongst hedge funds and managed futures - specific weights depending on the model.

The investigation from Fischer and Lind-Braucher (2010) was conducted using various risk estimators and return estimators[8] (see Tables 2 & 3). Thus, they also investigated how the choices between them affect the weights of an MRP or MRPP. They found that the choice of risk measure did not make much of a difference regarding the weights of MRPP, while the choice of the return estimator did.

[8] Fischer and Lind-Braucher (2010) Find optimal portfolios using data from 1999 to 2009. They use volatility, VaR, CvaR, mVaR and mCVaR as various risk estimators. Furthermore, their models are based on historical returns, Bayes-Stein and Black-Litterman estimators and CAPM.

Table 2 - Comparison of Minimum Risk Portfolios (MRPs) (Fischer & Lind-Braucher, 2010)

					Traditional Investments		Alternative Investments				
Estimator	Goal Function		Expected Return in % p.a.	Risk Measure in % p.a.	Stocks in %	Bonds in %	Hedge Funds in %	Managed Futures in %	Real Estate in %	Private Equities in %	Commodities in %
historical	Volatility	normal distributed	2.52	2.78	0.00	84.35	14.77	0.88	0.00	0.00	0.00
	VaR		4.01	-1.25	0.00	57.98	25.00	17.02	0.00	0.00	0.00
	CVaR		1.61	-7.80	1.09	97.84	0.00	0.00	0.00	1.07	0.00
	mVaR	non-normal distributed	2.12	1.63	0.00	91.75	0.00	5.55	2.70	0.00	0.00
	mCVaR		1.63	-4.32	2.71	97.29	0.00	0.00	0.00	0.00	0.00
Bayes-Stein	Volatility	normal distributed	2.77	2.78	0.00	84.35	14.77	0.88	0.00	0.00	0.00
	VaR		3.27	-1.57	0.00	70.19	25.00	4.81	0.00	0.00	0.00
	CVaR		2.35	-8.32	1.19	94.51	4.31	0.00	0.00	0.00	0.00
CAPM	Volatility	normal distributed	0.81	2.78	0.00	84.35	14.77	0.88	0.00	0.00	0.00
	VaR		0.85	-3.75	0.00	81.53	18.43	0.04	0.00	0.00	0.00
	CVaR		0.77	-6.53	0.00	86.60	11.85	1.55	0.00	0.00	0.00
Black-Litterman	Volatility	normal distributed	2.00	2.78	0.00	84.35	14.77	0.88	0.00	0.00	0.00
	VaR		2.01	-2.57	0.00	83.37	15.20	1.43	0.00	0.00	0.00
	CVaR		2.00	-7.74	0.00	85.13	14.42	0.45	0.00	0.00	0.00

Table 3 - Comparison of Maximum Relative Performance Portfolios (MRPPs) (Fischer & Lind-Braucher, 2010)

					Traditional Investments		Alternative Investments				
Estimator	Goal Function		Expected Return in % p.a.	Risk Measure in % p.a.	Stocks in %	Bonds in %	Hedge Funds in %	Managed Futures in %	Real Estate in %	Private Equities in %	Commodities in %
historical	Volatility	normal distributed	4.51	3.56	0.00	49.65	25.00	25.00	0.00	0.00	0.35
	VaR		4.51	-1.34	0.00	49.65	25.00	25.00	0.00	0.00	0.35
	CVaR		4.54	-11.95	0.00	49.25	25.00	25.00	0.00	0.00	0.75
	mVaR	non-normal distributed	3.46	0.49	0.00	67.33	18.70	12.74	1.22	0.00	0.00
	mCVaR		3.08	-6.90	0.00	74.96	13.05	11.32	0.67	0.00	0.00
Bayes-Stein	Volatility	normal distributed	3.43	3.07	0.00	66.05	25.00	8.95	0.00	0.00	0.00
	VaR		3.43	-1.63	0.00	66.05	25.00	8.95	0.00	0.00	0.00
	CVaR		3.46	-9.77	0.00	65.21	25.00	9.79	0.00	0.00	0.00
CAPM	Volatility	normal distributed	3.17	9.75	40.00	44.01	0.75	0.00	2.12	3.44	9.68
	VaR		2.98	-21.76	40.00	43.48	5.32	0.00	0.93	1.32	8.96
	CVaR		3.28	-24.39	40.00	40.05	2.84	0.00	2.68	4.04	10.39
Black-Litterman	Volatility	normal distributed	2.07	2.88	2.29	82.08	11.81	3.31	0.00	0.00	0.50
	VaR		2.07	-8.02	2.34	82.15	11.71	3.37	0.00	0.00	0.43
	CVaR		2.09	-8.10	2.86	82.06	10.79	3.70	0.00	0.00	0.58

Other published empirical studies, such as Kaiser et al. (2008) and Schweizer et al. (2008), show positive portfolio diversification effects if alternative assets are added to a traditional portfolio. Schweizer et al. (2008) found allocations in alternatives as high as 60% in aggressive portfolios and 77% in conservative portfolios using data from 1999 and 2009. Furthermore, they found that private equity was allocated with the maximal portfolio weight of 40% depending on the risk-aversion parameter. Private Equity was found to play a more significant role in defensive portfolios compared to offensive ones. Either way, it had the highest allocation among alternative assets overall.

Lee and Stevenson (2005) investigated the value of adding REITs to a traditional mixed-asset portfolio. Their findings show that REITs consistently provide diversification benefits with substantial allocations in efficient portfolios. This was especially the case with the long investment horizon. Further, they found that the rationale behind REITs inclusion alters as investors move across the efficient frontier. REITs are seen as a return enhancer for low-risk-return portfolios and as diversifiers for high-risk-return portfolios. This trend is considered to arise due to the low correlation it presented relative to both stocks and bonds and its risk-return profile being in between stocks and bonds.

In the light of past literature, Kuhle (1987) and Mueller et al. (1994) find contradicting results regarding whether REITs add value to a common stock portfolio. Later, Mull & Soenen (1997) show that the value of REITs as an asset class is time-dependent, meaning that whether it offers improvements in a mixed-asset context depends on the period and the holding period. Furthermore, between 1972 and early 2000, REITs earned a premium over the S&P 500 of 1.6%. Meanwhile, the correlation between the two was only 0.56. (Marston, 2007.) Based on such performance measures, many people, such as David Swensen, the director of the Yale Endowment, believe that real estate could help improve the performance of an investor's portfolio.

In addition, Marston (2007) introduces portfolios including all alternatives assets that are available for high net worth (HNW) [9]and ultra [10]HNW individuals. He analyses performance differences of such portfolios, including real estate, hedge funds, private equity, and venture capital funds, compared to traditional portfolios available. These portfolios tend to receive around 10 to 20 percent allocations to such alternatives each and without questions outperform traditional portfolios. However, the difference is only an extra 0.7 to 1 percent return adjusted for risk, which is not much considering the added illiquidity of such portfolios.

This raises the question of how it is then possible that some wealthy institutions, such as Yale and Rockefeller, have made huge returns by investing in alternative assets. Yale, for one, earned an impressive 5.7% risk-adjusted excess return compared to Russell 3000 [11] throughout 1986 to 2009. (Marston, 2007.) Whether this succession is due to Yale's devotion to alternative investments or superior manager selection is an interesting question. Marston

[9] High net worth: People or households who own liquid assets valued between $1 million and $5 million (O'Connell, 2021).

[10] Ultra HNW: People or households who own more than $30 million in liquid assets (O'Connell, 2021).

[11] The Russell 3000E™ Index includes approximately 4,000 of the largest US equity securities (Russell, 2021).

(2007) examines this by creating a portfolio with the same asset allocation that Yale did from 1986 to 2009, but with each asset invested in an index rather than in the managers chosen by Yale. Such a portfolio would have earned on a risk-adjusted basis 2.3% more than an investment in the traditional portfolio. Thus, Yale's choice of managers added an extra 2% to its performance. According to Marston, the extra 1.3% such portfolio would have earned compared to the portfolios of ultra HNW individuals can then be explained by superior access of an institution and Yale's reliance on alternative investments.

A study by Bekkers et al. (2009) adds to the literature of portfolio optimization using ten asset classes simultaneously in a mean-variance framework. Their study suggests that adding real estate, commodities, and high yield to the traditional asset mix delivers the most efficiency improving value for investors. Private equity is discovered to be somewhat similar to stocks but shows up in riskier portfolios when moving up the efficient frontier. The optimal portfolio, including all assets, has a 26% weight in stocks, 26% in real estate, 13% in commodities, 7% in high yield, and 28% in bonds. Additionally, in the lower spectrum of the efficient frontier, alternative assets play a major role, with allocations in real estate, commodities, and high yield. When moving up to riskier portfolios, private equity replaces bonds, real estate, commodities, and stocks – in that order.

3.3 Alternative Assets in Crises

By now, it is well studied how liquid assets such as equities, bonds, and credit perform in an economic downturn or crisis. In general, stocks perform poorly, whereas bonds provide downside protection. The question is, then, where do alternative assets fit in this environment.

While alternative assets have increased their allocations in investor portfolios, bonds have done the opposite. Nowadays, not only do they have reduced weight in most portfolios, their ability to deliver sufficient return protection for the next downturn is being questioned. Thus, alternative investments can carry an increased load in many portfolios during the next downturn.

Christoph Junge and Frank Hvid Petersen (2020) investigated the performance of alternative investments during the current Covid-19 crisis and past crises during the previous decades. Real estate has, in general, been a good diversifier during economic downturns except when valuation is extremely high, like in the early 1990's recession or the Great Recession in 2008. However, the performance has been mixed across historical crises and sub-asset classes like retail, office, and industrial. Additionally, as non-listed real estate is a heterogenous and seldom traded asset class, its low volatility during crises and good performance can often be due to a lack of observations (Cho, Hwang, & Lee, 2013).

Private Equity, on the other hand, has not given any shelter in times of crisis. In general, PE is not a good diversifier as it posts some heavy losses like equities in an economic downturn as the same economic factors influence them. However, like real estate, it depends on the business model and style of private equity how hard the crisis hits. For example, Venture Capital can be expected to increase diversification slightly. Some tech-driven businesses are profiting while restaurant chains, event businesses, travel, or oil-related businesses have been severely hit. Such findings were also reported by Nielsen (2010) and Brammer and Rants (2015).

The study by Junge and Petersen (2020) was done using the listed liquid replication benchmark as the index for private equity index, which might overstate the impact over short time frames. Marston (2011) additionally noted that as public assets such as stocks and REITs are priced daily on exchanges, the same cannot be said for private assets such as private equity. He also argued that the listed liquid replicas of real estate and private equity react quicker to financial crises than their private counterparts. This is because such private assets are based on appraisals and not market prices(Cho, Hwang, & Lee, 2013). For example, during the Great Recession in 2008, the FTSE/NAREIT return on listed REITs was down 63.4 %. Meanwhile, the NCREIF index reflecting institutional real estate holdings was down only 10.5 %. Surely the difference between commercial real estate held in REITs and those held in institutional portfolios cannot be that large. While such stale values may appear to protect investors against market downturns, these seem certain far from the truth. REITs and non-listed real estate have the same average leverage, around 40 % (Kempen, 2017); leverage can neither explain the difference.

While it is probable that REITs are more influential due to higher leverage, Kempen (2017), discovered that both listed and unlisted real estate had roughly the same leverage, around 40% leverage to the total value.

Looking at the past financial crises and comparing the traditional stock-bond portfolios to those that include alternative assets, the latter has outperformed the prior. Alternative assets, such as hedge funds and private equity, have been shown to soften the blow. Although, as discussed above, some of that cushioning was more apparent than real, considering the appraisal issues (Marston, 2011.) These comparisons, however, are done in regards to the true private alternative assets and not their modern liquid counterparts. Thus, it remains to be seen how the modern liquid alternatives will perform and protect retail investor's portfolios during future economic turmoil.

3.4 Liquidity risk

3.4.1 Alternative Assets

Market liquidity is important to the success of any financial market. Moreover, market liquidity has many forms - the size or volume of the market, the bid-ask spread, the spread's volatility, and the trading speed. Chaudhry, Maheshwari, and Webb (2004) define liquidity as converting assets to cash quickly and without loss of value. Anson (2010) market described liquidity risk as the threat associated with investing in an investment that cannot be rapidly sold or traded at a significant discount. On the contrary, whereas large listed companies are extremely liquid, meaning investors can convert their stocks to cash in a matter of seconds, the case is not the same for traditional non-listed alternative assets.

Liquidity constraints in many alternative assets are related to an inability to locate a counterparty to trade with. Appropriate counterparties are not always easy to find (Ang, Papanikolau & Westerfied, 2014). Furthermore, the time needed to wait for another trade opportunity is unknown. The timing of exit and reinvestment in private equity and venture capital is stochastic and depends on the IPO or M&A markets (Ang, Papanikolau & Westerfied, 2014). Due to the illiquid asset's inability to be sold for an indefinite amount of time, the investor faces unhedged risk.

Real estate is among the most challenging asset groups to liquidate. Levitt and Syverson (2008) suggest that the usual time to sell a property in real estate markets is between 110 and 135 days following the first listing. Further, the normal duration of venture capital or private equity portfolio is three to ten years. While the investment horizon is technically defined, partnerships frequently return investor capital before the partnership's official 10-year expiration. For example, the median investment length in private equity in four years, with 16% returning within two years and 26% returning after six years (Lopez, Phalippou & Gottschalg 2010).

Investors want a liquidy premium to compensate for the liquidity risk. The premium is the portion of an investor's return earned to provide capital to an asset class with a long holding period (Anson, 2010). As Ashish, Pedersen & Hoffmann (2012) have shown, liquidity changes over time and has similar characteristics across securities and asset types.

Historically, private equity has been viewed as a source of excess returns and diversification (Franzoni, Nowak, and Phalippou, 2012). These advantages, however, may be less than expected, as private equity is significantly exposed to liquidity risk. Franzoni et al.(2012) found that the liquidity risk premium for private equity is around 3% each year. Additionally, the addition of this liquidity risk premium decreases alpha to zero in a four-factor model.

Due to low market liquidity, private equity managers may have trouble refinancing their investments. They may be obliged to sell assets or accept higher borrowing fees during these periods. Due to their high leverage, private equity investments are vulnerable to the capital constraints of the debt lenders to private equity, which are predominantly banks and hedge funds. As a result, periods of low market liquidity are likely to correlate with periods when private equity managers face financing challenges, resulting in reduced returns for this asset class Franzoni et al. (2012).

3.4.2 Listed Alternative Assets

REIT's principal objective is to increase the liquidity of illiquid real estate assets (Blau, Nguyen, & Whitby, 2015). Market illiquidity arises if investors cannot sell stocks of REITs. This might be due to a lack of market depth, as these securities are often followed by fewer analysts (Chaudhry, Maheshwari, & Webb, 2004). According to the 2015 research by Blau, Nguyen, and Whitby, REITs are more liquid than direct real estate investments since they are publicly traded. They further question if REITs have as good liquidity as ordinary stocks. Blau, Nguyen, and Whitby (2015) discovered that using bid-ask spreads as a proxy for liquidity, REITs had a higher average and variance of bid-ask spreads than common stocks. Thus, if market conditions weaken, REITs are more likely to face liquidity constraints than common stocks. Bertin, Kofman, Michayluk, and Prather (2005) found similar results in an earlier study, stating that REITs have less liquidity than ordinary stocks and that there is a 15-25 % higher chance of affecting the price by investing in REITs than common stocks.

Cherkes, Sagi & Stanton (2008) claim that Listed PE provides a service to investors by making illiquid assets liquid through its listed entity. They do, however, discover that many Listed PE entities are rather small and hence more illiquid than ordinary stocks. Further, Lahr & Herschke (2009) finds that Listed PE stocks trade thinly and exhibit autocorrelation in their returns, indicating illiquidity. They conclude that listed PE appears to be impacted in the short run by illiquidity. Additionally, they found wide bid-ask spreads and strong autocorrelation, attributed to the absence of weekly trading in Listed PE.

The same may be said about BDCs, which are structured similar to listed PE. To our knowledge, no study on the market liquidity of BDCs has been done. However, Anson (2004) notes that trading in the outstanding shares may be limited following a BDC's IPO. Nonetheless, comparing to their unlisted counterparts - private debt, private equity, or real estate - finding a buyer on the exchange is far quicker than in private markets, lowering the risk of market liquidity. However, retail investors will remain exposed to the underlying liquidity risks involved with the private investments made by the BDCs, REITs, or Listed PE.

4. Theoretical Frameworks

The following chapters introduce the chosen theoretical frameworks used later in the empirical part of this paper. Measuring and comparing the performance of different asset classes has been done for decades, and thus, the models and methods vary a lot. The choice between one another depends strongly on the purpose of the study and the qualities of the sample data.

First, this chapter introduces basic performance measures such as volatility and returns. After that, Markowitz's portfolio theory is explained. In the end, modifications to the classic performance measures and the modern portfolio theory (MPT) are presented. This results in a more comprehensive comparison of different asset classes and potentially enhances our estimations' accuracy.

4.1 Performance Measures For Different Asset Classes

The study of investment performance is a major part of investment analysis. Such performance is often called efficiency, which includes simultaneous analysis of the rate of return on investment and the risk that accompanies this rate of return. Traditionally the higher the risk associated with a given investment, the higher the expected return should be. (Potrykus, 2018.)

According to Potrykus (2018), the choice of the measure of effectiveness does not affect the assessment of individual investments. When comparing different ratios such as Sharpe, Calmar, and Information Ratio (IR), the rankings of the assets remain mostly the same. Thus, in the context of this paper, it is not necessary to include multiple measures in this sense. This paper chooses to use Sharpe as the main indicator of investment efficiency, which will be introduced in more depth in the following chapter.

Sharpe (1964) complemented Markowitz's (1952, 1959) insight about advocating focus on mean and variance and the selection of portfolios with the lowest risk (volatility) for a target level of return, or the highest return for a target level of risk. This was also the birth of the modern portfolio theory (MPT), which will be better introduced in the following chapters. Sharpe (1964) argued that, given a risk-free rate, the optimal combination of risky assets is given by the market (or tangency) portfolio, which is the one that maximizes returns in excess of the risk-free rate per unit of volatility risk. Selecting the portfolio of risky assets so that the Sharpe ratio is maximized has been the standard criterion for academics and practitioners ever since. Sharpe is calculated by dividing the expected risk premium of the portfolio by the standard deviation of the portfolio's returns. Below is the standard formula for the Sharpe ratio:

$$SR = \frac{E(r_p) - r_f}{\sigma_p} \tag{1}$$

Regarding the main components of the Sharpe ratio, return, and volatility risk, modern research has presented some alternative methods of calculations. Traditionally Sharpe ratio uses the arithmetic mean as the return component and standard deviation as the risk component (Sharpe, 1964).

The most common alternatives for this, which this study includes, would be to change the return component or risk component and calculate the return-risk ratio. Instead of the traditional measures, this paper additionally uses geometric returns and Bayes-Stein estimators for returns and CVaR for risk when analyzing the return-risk ratios of different asset classes and portfolios. These will be introduced more formally in the following chapters.

In general, means can be calculated in two ways, arithmetically or geometrically. Whether one is better than the other relies strongly on the context of the study. When it comes to returns, the arithmetic mean return is the sum of a series of returns divided by the count of that series of returns. This works especially well when the series consists of independent events and when the data follows a more-or-less normal distribution with no outliers. (Missiakoulis et al., 2010.) However, in the context of finance and especially investment analysis, this is hardly the case.

Poterba and Summers (1988) argue that most returns in finance are correlated, mean-reverting, and exhibit serial correlation, in which geometric mean return works better, especially over multiple periods. The geometric mean considers compounding and is hence considered a more accurate measure of average returns of an investment portfolio over multiple periods (Francis & Ibbotson, 2002; Missiakoulis et al., 2010).

The biggest differences between the arithmetic and geometric mean arise in the presence of high volatility, skewness, and outliers (Missiakoulis et al., 2010). Investment strategies with significant volatility have lower geometric means than arithmetic means. In the

Sharpe ratio, the standard deviation used in the denominator already captures the effect of higher volatility and lowers the ratio. Thus, a major argument for the arithmetic means is that the higher risk of high volatility should not be penalized twice, as would be the case with geometric Sharpe. Consequently, although both can be used, the arithmetic Sharpe is viewed as a more intuitive measure than the geometric Sharpe. It is to be noted that geometric mean overall tends to perform poorly while estimating risk-adjusted appraisal measures, mainly for double penalizing for volatility. (Gilligan, 2019.) Geometric mean works better for measuring historical returns compounded and reinvested over multiple periods, while arithmetic works for future-oriented analysis where expected short-term values are appropriate. (Missiakoulis et al., 2010.)

Hence, both of these measures have their biases and advantages, which should be considered when using them. As standardized as Sharpe is in investment analysis, Estrada (2010) found that in addition to being interested in the risk-adjusted ratios, investors seem to lay significantly more emphasis on whether or not their investment capital grows and at what rate. Furthermore, fund management companies tend to summarize performance with the mean compound return of their funds. For both these reasons, then, a potential plausible goal for portfolio managers to adopt would be to grow the capital entrusted to them at the fastest possible rate; that is, to maximize the geometric mean return of their portfolios. (Estrada, 2010.)

In a study presented by Estrada (2010), Sharpe ratio maximization (SRM) works better as a performance measure for the relatively risk-averse investors who are uncertain about their holding period. On the contrary geometric mean maximization (GMM) suits better for less risk-averse investors, those with a long investment horizon, and those likely to stick to their expected holding period. As the superiority of one measure over another is based on investor preferences, both will be included when evaluating the performances of asset classes independently and as components of portfolios. Thus, we can obtain a more comprehensive understanding of the qualities of each asset.

4.2 Portfolio Theory

4.2.1 Modern Portfolio Theory

The modern portfolio theory (MPT), also known as the mean-variance theory, was pioneered by Harry Markowitz (1952, 1956). He developed a quantitative method that takes into account the diversification benefit in portfolio allocation. This model allows us to determine the optimal allocation of each asset in a portfolio. MPT is a theory on how investors can construct portfolios to maximize expected return based on a given level of market risk.

The theory is built upon an argument that an investment's risk and return characteristics should not be viewed alone but rather evaluated by how the investment affects the overall portfolio's risk and return. It shows that an investor can construct a portfolio of multiple assets that will maximize returns for a given level of risk, also known as maximizing the risk-adjusted returns. Likewise, an investor can build a portfolio with the lowest possible risk using the same method. These two portfolios are more commonly called the tangency portfolio and the minimum variance portfolio (MVP).

In MPT, certain assumptions are required for the model to work and determine risk, return, and covariances parameters. Some of these assumptions are questioned to hold in real market conditions, leading to inapplicable results regarding each asset's optimal weights. This will be discussed more in the later chapters and suggestions on approaching this issue. Although not explicitly stated, the assumptions implied by the model are as follows (Brennan, 1971; Beyhaghi & Hawley, 2013):

1. Returns from the assets are stochastic, following a normal bell curve distribution.
2. Market information is symmetric, as all market participants have access to the same information, which is immediately reflected in the prices on the market.
3. Investors are rational and risk-averse, and risk-aversion (the risk-return trade-off) is linear.
4. Investors always prefer a portfolio with a higher expected return over another portfolio with a lower expected return.
5. Taxes and transaction costs do not exist.
6. All the investors have the same views on the expected rate of return.

7. Individual investors are not sizeable and capable enough to influence the prices prevailing in the market.

It is easy to concur that not all of these assumptions are admissible in the real world. Still, the point of these molds is not to explain the true nature of the markets but to minimize the portfolio allocation model's complexity while still gaining interpretable results regarding the portfolio's optimal allocation. (Elton et al. 2014.)

Markowitz's model implies that the optimal portfolio is decided by the securities return, risk, and correlation with each other. In Markowitz's model, variance is used as a risk measure and classic historical mean as a return measure for each asset and the portfolio. Variance is the square of standard deviation, a commonly used measure for an asset's level of risk or volatility. It is used to determine how widely spread out the asset movements are over time. Calculating the expected return is the sum of the weighted expected returns of each of the securities:

$$\overline{R}_P = \sum_{i=1}^{N} \overline{R}_i \tag{2}$$

The portfolio's risk is measured based on the covariance, variance, and weight of each asset. This is based on the idea that as long as the assets in the portfolio are not perfectly correlated, one can minimize the assets' idiosyncratic risk. Idiosyncratic risk means the risk specific to each asset, while in contrast, systematic risk refers to the risk that is common for the entire market. In this context, the risk that remains after diversification is systematic risk, while firm-specific risk has been eliminated. This is the true benefit of diversification. (Elton et al. 2014.)

To calculate the portfolio variance, one has to know the variance, weight, and covariance of each asset at hand. The following formula is used for the calculation, where w_i and w_j represent weights of each asset (Markowitz, 1952):

$$\sigma_P^2 = VaR(R_P) = VaR\left(\sum_{i=1}^{N} w_i R_i\right) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{3}$$

where

$$\sigma_{ij} = \textit{the covariance between assets i and j}$$

$$R_i = \textit{the return of asset i}$$

$$w_i = \textit{the weight of asset i}$$

The variance of portfolio return is greater when the two assets' covariance is positive and similarly smaller when negative. (Markowitz, 1952.)

Markowitz (1952) develops a method to identify the efficient frontier – the subset of investment portfolios that maximize return and minimize risk. Portfolios that lie on the efficient frontier are thus called efficient portfolios, and portfolios that lie below are sub-optimal because they do not provide enough return for the level of risk. An illustration of the efficient frontier is presented below:

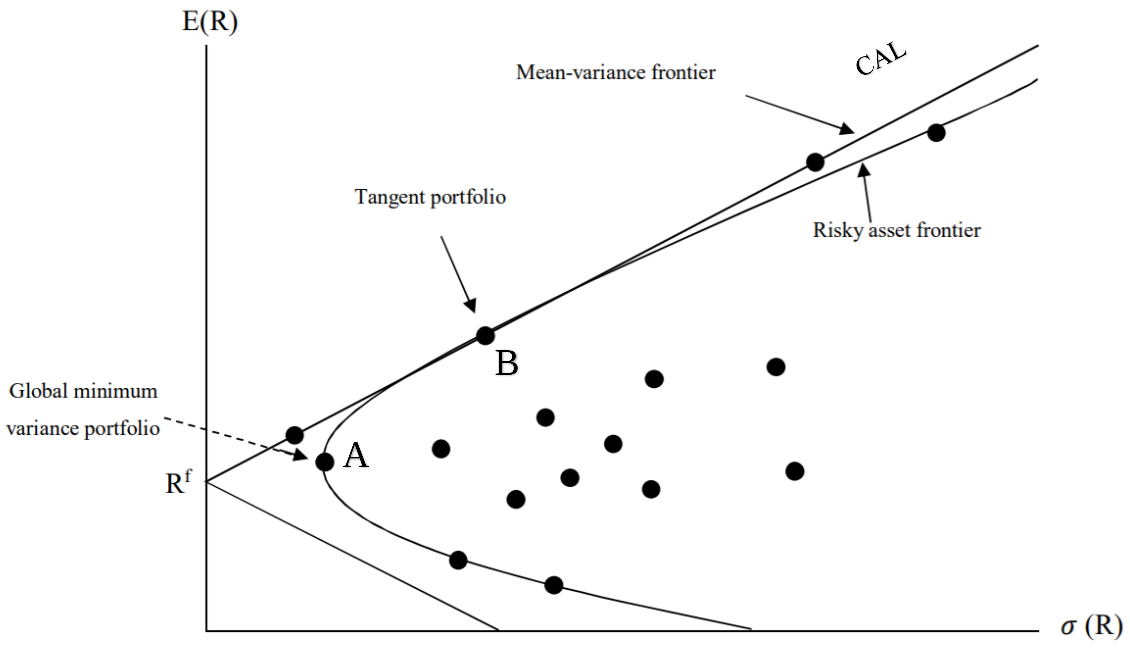


Figure 13 - Illustration of the efficient frontier (Cochrane, 2005)

The upper portion of the curve (point A onwards) is the efficient frontier. Point A on the graph represents the minimum variance portfolio, meaning the combination of risky assets that minimize standard deviation. On the other hand, Point B is the optimal market portfolio, which yields the best combination of risky assets, which optimizes the return and risk on a given risk-free rate. It is depicted by the line called the capital allocation line (CAL) that is tangent to the efficient frontier. The slope of this line is called the Sharpe (1964) ratio, which represents the risk-reward ratio

The allocation between the risk-free asset and the tangent portfolio is then based on the investor's risk preference. A more risk-loving investor might borrow on the risk-free rate and leverage their position in the tangent portfolio. In contrast, a more risk-averse investor might lend on the risk-free rate and invest a smaller amount in the tangent portfolio. Thus, by using the risk-free rate and the tangent portfolio, investors can construct the most efficient portfolio for the given level of risk they prefer. One assumption behind this theory is that one can both lend and borrow at the same risk-free rate. In real market conditions, this is hardly true, and there are ways to adapt the model to solve this issue (Brennan, 1971). However, this paper does not consider such modifications in the quantitative analysis. A theoretical point of departure here is the MPT model introduced above, to which additional modifications are made. These modifications concern certain assumptions and issues in the model, which will be presented in the following chapters.

4.2.2 Modifications to MPT

As brilliant as the theory is in its simplicity and clarity, years of examination have led to a discussion about whether the assumptions upon which the model depends reflect real market conditions. More importantly, whether its conclusion can be transposed to actual portfolio management. (Fama 1970; Cheoueifaty & Coignard 2008; Francis & Dongcheol 2013.)

A notorious problem with Markowitz's theory is that it relies on the unobservable parameters of covariances and expected excess returns. Hence, it unarguably involves estimation risk, in the sense that the estimated parameters differ from the true values. Including approximated values into the above-mentioned formulas, therefore, leads to non-optimal weights. In particular, the expected returns are especially difficult to estimate. According to numerous research papers, the use of historical averages for estimating expected returns leads to poor out-of-sample performance (Merton, 1980; Jorion, 1985; Jorion, 1991; Kondor et al., 2007).

The considerable difficulty to estimate the returns accurately has drawn interest for alternative weighting schemes that do not use this parameter. These schemes include maximum decorrelation, risk parity (Maillard et al., 2010), and maximum diversification (Choueifaty and Coignard, 2008). The corresponding portfolios can be computed with the sole knowledge of covariances, making them insensitive to the estimation errors in the parameter of expected return. In this paper, the maximum diversification (MD) portfolio is included in the quantitative analysis for this very reason.

The estimation error in the expected return is not the sole issue, however. Estimation errors in the covariance matrix can be large too. The significance of such estimation error comes largely down to the ratio between the number of assets (N) and the number of observations (T) (Kan and Zhou, 2007; Fan et al., 2008). When the ratio N/T is large, Kan and Zhou (2007) show that estimation errors in the sample covariance matrix contribute more than estimation errors in expected returns to the loss of efficiency of the proxied mean-variance efficient portfolio. However, when the sample size is large relative to the universe size (T at least equal to 3N[12]), the sample covariance matrix is suitable for portfolio optimization, as was shown in a study by Pantaleo et al. (2011). This applies to the data in this paper and is why the MPT is considered a solid base case for the analysis in the empirical study.

Considering the MPT from a behavioral finance perspective, we notice that the estimation error in the covariance matrix comes largely down to the other flaw of the model that lies in the risk measure itself. It simply assumes that a variance of returns is the correct risk indicator. Meaning, no matter the direction of the fluctuation, the higher the volatility – the greater the risk. It has been studied that investors usually do not perceive risk in that sense. The return fluctuations above the minimum they must earn to achieve their investment goals do not carry the same weight as the return fluctuations below the minimum. (Estrada, 2006.) In other words, investors do not view fluctuations above a particular benchmark as bad or risky. Sharpe (1964) additionally comments on this assumption: "under certain conditions, the mean-variance (MV) model can be shown to lead to unsatisfactory predictions of investor's behavior."

In addition to variance and mean as risk and return estimators, this paper introduces CVaR, and Bayes-Stein estimators as additional ways to estimate risk, the covariance matrices, and return vectors. CVaR are measures of downside risk, which is the financial risk associated with losses. Downside risk measures are used to understand an investment's potential to suffer

[12] Number of obervations equals three times the number of assets.

a decline in value or the amount of that decline if market conditions change. The theory and fundamentals of the estimators are shortly explained in the following chapters.

4.3 Alternative Risk and Return Estimators

4.3.1 Conditional Value at Risk

Alternative risk measures have been studied for decades after risk practitioners and researchers noticed that the gap between market practice and theoretical progress had widened enormously (Acerbi & Tasche, 2002). Artzner et al. (1997, 1999) were among the first to ignite a discussion thriving to define in a clear-cut way what properties a statistic of a portfolio should have for it to be considered a sensible risk measure. The answer to this question was given through a complete characterization of such properties via the concept of coherent risk measure. This is where the value at risk (VaR) was introduced. As a better alternative, the expected shortfall (ES), better known as conditional value at risk (CVaR), was added, passing the bar to be regarded as a coherent risk measure. VaR is truly coherent only based on the standard deviation of normal distributions (Rockafellar & Uruyasev, 2000). To define CVaR, we start by first explaining VaR.

VaR is a downside measure of the risk, which is the expected worst loss over a given horizon at a given confidence level (Jorion, 1996). Confidence level relates to the probability that a parameter will fall between values around the mean, and for VaR and CVaR, 95% and 99% are the most common confidence levels to use. VaR ultimately provides a single number summarizing the global exposure to market risks and the probability of adverse moves in financial variables. The great thing about it is that it measures risk using the same units as the bottom line – dollars. So, in other words, VaR tells us what the expected maximum dollar loss on an investment with a predetermined confidence interval and time horizon is. (Jorion, 1996). Figure 14 visualises VaR and CvAr.

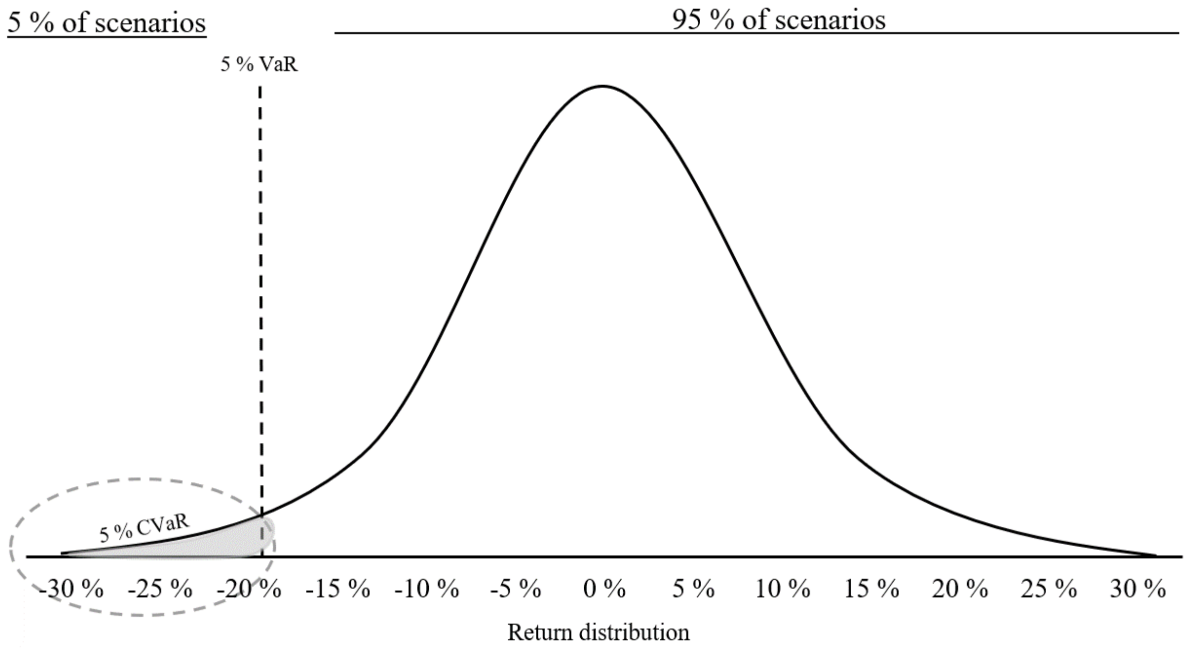


Figure 14 - Vizual presentation of CVaR and VaR

To formally define VaR, two quantitative factors must be chosen. The first is the length of the time horizon, and the second is the confidence level, further one assumes normal distribution of returns. Different choices of horizon and confidence level will naturally lead to trivially different VaR numbers. VaR can be used for various purposes, such as measuring the risk of an index, entire portfolio, single stock, or financial risk within a firm. It can be calculated using various methods such as the historical and variance-covariance methods and Monte Carlo simulation (Jorion, 1996; Stambaugh, 1996; Bucay and Rosen, 1999). The formal way to define VaR is presented below (Artzner et al., 1999):

$$VaR_{\alpha}(X) = -\inf\{x|\ \mathrm{P}[X \leq x \cdot r] < \alpha\} \tag{4}$$

Where $\alpha \in]0, 1[$ and represents the quantile, r is the reference instrument, X is the final net worth and P is the probability distribution. This provides a solid understanding of the methodology behind the measure, no matter the calculation technique.

Now, a more comprehensive alternative measure is the conditional value at risk (CVaR), which has some advantages compared with the VaR (Acerbi & Tasche, 2002; Artzner et al., 1999). The formal way to define CVaR, also known as ES, is as follows (Acerbi & Tasche, 2002):

$$ES_{\alpha}(X) = \frac{1}{\alpha}\int_{0}^{\alpha} VaR_{u}(X)du \tag{5}$$

As we can see from (5), CVaR is similar to VaR because it is derived from it. After it was better introduced by Rockafellar and Uryasev (2000), it quickly became considered a more consistent risk measure than VaR (Sarykalin et al., 2008). By definition, concerning a specified probability level β, the β-VaR of a portfolio is the lowest amount α such that, with probability β, the loss will not exceed α, whereas the β-CVaR is the conditional expectation of losses above that amount α (Rockafellar & Uryasev, 2000). In other words, CVaR is a weighted average of the "extreme losses" in the tail of the distribution of possible returns. Thus, it also succeeds better in including tails in measuring risk (Rockafellar & Uryasev 2000; Sarykalin et al., 2008).

Rockafellar and Uryasev (2000) and later Sarykalin et al., 2008 showed that CVaR is superior to VaR, especially in optimization applications, as VaR can be relatively difficult to optimize. Another major issue with VaR compared to CVaR is that it does not control scenarios exceeding itself. In other words, one can significantly increase the largest loss exceeding VaR, but the VaR risk measure will not change. This is not necessarily good or bad, but it may lead to incomprehensive results regarding portfolio optimization. (Sarykalin et al., 2008.)

One of the main problems with such risk measures, according to Jorion (1996), is that they are generally based on historical data and thus will inevitably be affected by 'estimation risk. However, such risk will always be entailed, no matter the measure. Furthermore, the measures are still good at evaluating the performance of alternative assets compared to each other.

4.3.2 Bayes-Stein Estimator

In 1986 Jorion presented an alternative estimator for estimating the expected returns. The model is based on the inaccuracy of the historical mean as the estimator, ignoring information in other series. According to multiple papers, and as mentioned above, in portfolio analysis, the results are never optimal with uncertain parameters (He & Litterman 1999, Jorion 1986, 1991). While all estimators entail some level of uncertainty, the measure of Bayes-Stein is one approach that strives to enhance the accuracy of the estimation.

Instead of the sample mean, the Bayes-Stein estimator is obtained by “shrinking” the means toward a common value. A shrinkage estimator applies the effects of shrinkage, that is, the reduction in distance of some sort. Such estimators indicate that the estimate is transformed and closer to some predetermined value than the original estimate. The most well-known application in finance of such method is the utilization of James-Stein estimators (Jobson & Korkie, 1980). This method means shrinking future returns towards the average expected return based on the volatility of an asset and the distance of its expected return from the average. This should decrease the estimation error in the case of two or more assets in the portfolio.

Later, Jorion developed the Bayes-Stein (1986) estimator, a similar technique to the James-Stein estimators, but instead brings future return estimates closer towards the minimum variance portfolio. The idea behind this method can be traced to the inadmissibility of the sample mean, which was proved by Stein (1955) and Brown (1966). Traditionally, portfolio selection and statistical estimation have been kept separate, mostly because portfolio choice has been analyzed in the “certainty equivalence” framework, which assumes underlying motions to be known. However, as Jorion (1986) shows, this two-step procedure is not optimal from an estimation viewpoint and the Bayes-Stein estimator challenges. Instead, in Bayes-Stein, the estimation error for all assets is summarized into one loss function, which should be minimized as a whole rather than each component separately. According to Jorion’s (1986) study, simple mean works great when the data consists of one variable alone, but the shrinkage estimators seem superior when the number of unknown means is more than two. The formal way to define the Bayes-Stein estimator, and the way it is used in the empirical part of this paper, is as follows (Jorion, 1991):

$$E(r_j)^{BS} = (1-w)\cdot E(r_j)^{hist} + w\cdot E(r_{MVP})\,, \tag{6}$$

where

$E(r_j)^{hist} = the\ historical\ return$

$E(r_{MVP}) = the\ global\ minimum\ variance\ return$

$w = the\ Shrinkage\ Factor$

$$w = \frac{N+2}{(N+2) + T\left(E(r_j) - E(r_{MVP})\underline{1}\right)' \Sigma^{-1}\left(E(r_j) - E(r_{MVP})\underline{1}\right)},$$

where

$\Sigma = \langle \sigma_{ij} \rangle$

$N = the\ number\ of\ assets$

$T = the\ number\ of\ periods\ of\ estimation$

$1 = the\ vector\ of\ ones$

$\Sigma^{-1} = the\ inverse\ of\ the\ variance\ covariance\ matrix$

Since the variance-covariance matrix is not known in practice, it is replaced by (Avramov & Zhou, 2010):

$$\hat{\Sigma} = \frac{T}{T-N-2}\sigma_{ij} \tag{7}$$

The main objective of a Bayes-Stein estimator is to minimize the impact of estimation risk on optimal portfolio choice. Further, the Bayes-Stein estimator implies that estimation risk implies a loss of investor utility, which should be viewed as a function of the estimator and the true parameter values (Jorion, 1986).

In the context of portfolio optimization, Jorion (1986) shows that the Bayes-Stein estimator is always shown to outperform the classical sample mean, and the gains are often substantial. He also evaluated the out-of-sample performance of various estimators based on actual stock return data and found that shrinkage significantly outperformed the classical sample mean. However, when a large set of samples was included (T=200), the outperformance of Bayes-Stein reduced significantly. Additionally, it is stated that for large sets of samples (T), the gains and differences between shrinkage estimates and definitive rule tend to zero, in which cases sample means are accurate estimates of the expected returns.

4.4 Cointegration

This paper will look for a significant relationship between unlisted and listed alternative investments through cointegration. Cointegration was introduced in 1987 by Engle and Granger and was designed to test whether two-time series had a long-term relationship. Cointegration determines if two-time series are integrated so that their long-term individual average does not diverge. If the T value is beyond its critical value of -3,41, it can be concluded that the two variables have a long-term and stable relationship (Engle & Granger 1987).

It is essential to distinguish between cointegration and correlation. Correlation is a good measure if two variables move in the same direction. However, cointegration aims to see if the difference between the average values of the two time-series remains constant. Two variables trending in the same direction may have a strong correlation; this does not always imply that they are cointegrated.

The Dickey-Fuller test would be used to determine unit roots in this paper. A time series that has unit-roots are not cointegrated. If the time series $\mathrm{X_t}$ and $\mathrm{Y_t}$ are cointegrated, the difference $\mathrm{Y_t - \theta X_t}$ must be stationary. If the time series $\mathrm{Y_t - \theta X_t}$ is nonstationary, the variables are not cointegrated. Since the θ, usually referred to as beta, is unknown, it must be calculated using an OLS regression. The OLS regression residuals are equal to $\mathrm{Y_t - \theta X_t}$.Which needs to be stationary for significant cointegration. The OLS regression is expressed as follows (Engle & Granger 1987).

$$Y_t = \alpha + \theta X_t + z \qquad (8)$$

The Dickey-Fuller test is then used to test whether the residuals, "zt" are stationary (Engle & Granger 1987). If the T-value is greater than the critical value, the null hypothesis that the variables are nonstationary is rejected, implying proof of cointegration. Thus, the time series will depend on each other, and the time series share the same underlying stochastic trend. If the variables are not cointegrated, the time series will follow a "Random Walk" and will be independent of one another.

5. Data

In this section, we will describe the data used in the empirical part of this paper. The data set includes eight indexes, one ETF, and six indexes provided by Preqin to compare listed and unlisted funds.The data will serve as the foundation for historical individual asset performance and portfolio construction. This data source contains only securities that are traded on official stock exchanges, and so meets the retail investor access requirement.

5.1 Data Source

All the financial data in this paper is collected from Bloomberg and Preqin except one index is from Yahoo Finance. This is because Yahoo Finance presents it in the form of a Total Return index which Bloomberg does not. Yahoo Finance is the world's biggest business news platform (LinkedIn, 2020). They provide all information "as is", and states that their data is provided for information purposes only. It is widely considered an unbiased and reliable data source in the financial industry and is often used as an alternative to products such as Bloomberg Terminal by professionals.

According to Bloomberg (2020), their instrument reference data is an industry-leading source of the detailed terms, and conditions data firms require for security identification, creation, trading and settlement. Thus, it is considered reasonable to employ Bloomberg Terminal as the main data source in our analysis.

5.2 Data Sample

The data in this paper consists of 11 series of index data from June 2007 to January 2021. Data is included from the following seven asset classes: public equity, public debt, high yield, commodities, real estate, private equity, private debt. All the series are total return series and the study is undertaken from the view of global investors investing in U.S. Dollars (USD). Monthly historical data is used for the calculations.

For public equity, MSCI World Index (M2WOEW) is chosen. The MSCI World Index captures large and mid-cap representation across 23 Developed Markets (DM) countries. With

1,585 constituents, the index covers approximately 85% of the free float-adjusted market capitalization in each country. (MSCI, 2020.) Thus, it can be considered as an appropriate proxy for the global developed public equity market.

For public debt, “the Bloomberg Barclays Global Aggregate Index (LEGATRUU) is chosen. It is a flagship measure of global investment grade debt from 24 local currency markets. This multi-currency benchmark includes treasury, government-related, corporate, and securitized fixed-rate bonds. (Bloomberg, 2020.) Hence, it works great as a proxy for the public debt market.

For commodities, the S&P GSCI index is chosen (SPGSCITR.IND). It is widely recognized as the leading measure of general commodity price movements and comprised of the principal physical commodities futures contracts (Bloomberg, 2020).

BDCs are considered to be their own asset class because of their unique nature, but still to work as a decent proxy for the private debt market, which is their primary focus. The Cliffwater BDC Index (CWBDC) is chosen to represent BDCs. It measures the general performance of the lending-oriented exchange-traded BDCs (Bloomberg, 2020).

For real estate, FTSE EPRA/NAREIT Global Index (TENHGU) is chosen. It is designed to represent general trends in eligible real estate equities worldwide.

For private equity, the LPX50 Private Equity index (LPX50TR) is chosen. This index is provided by the LPX Group and contains the largest private equity companies listed on global stock exchanges. The index is well diversified across listed private equity categories, styles, regions, and vintage years and can hence be considered an excellent proxy for the listed PE market. (Bloomberg, 2020.) Additionally, LPX Venture Listed Private Equity index (LPXVENTR), LPX Buyout Listed Private Equity index (LPXABOTR) are included to analyze further the differences amongst listed PE styles and their private equivalent.

Unlike for all the other series, an ETF was chosen for high yield. This is because for retail investors to invest in high yield through exchanges, one has no other choice but to invest through a fund structure and is thus inevitably forced to pay the fees associated with that fund. All the other asset classes can be invested independently without a fund structure. This study uses iShares iBoxx High Yield Corporate Bond ETF (HYG) to proxy for the high yield market.

It is one of the most widely used high yield bond ETFs based on 20-day average volume, and it has the largest AUM amongst high yield ETFs (Bloomberg, 2020).

As a risk-free rate of return, the monthly average of the 1-month London Interbank Offered Rate (LIBOR) rate is used. The data series was exported from Federal Reserve Economic Data (FRED). Table 3 below summarizes the included asset classes as well as their corresponding databases.

Table 4 - Investment Universe

Investment Universe			
Traditional Investments		**Alternative Investments**	
Asset Class	**Database**	**Asset Class**	**Database**
Stocks	MSCI World	Real Estate	FTSE/EPRA Global Real Estate
Bonds	Bloomberg Barclay's Global Aggregate Index	Private Equities	LPX50
		Commodities	S&P GSCI
		Private Debt (BDC)	Cliffwater BDC
		High Yield	iShare iBoxx (HYG)

Most series provide data from many decades back, but the shortest time series is from High Yield, which states back to 2007. Thus, the other series were cut to match this length. This study is most interested in the value of modern instruments in alternative assets, and many of these instruments were invented after the 2008 financial crisis. Thus, although a longer time horizon generally provides more significant data, it does not necessarily provide relevant data for this study. The data after 2007 still includes two economic downturns in the series, which allows a thorough analysis of each asset’s performance throughout a full economic cycle.

Finally, chapter 7.2 will compare publicly traded and privately held alternative investments using data from Preqin. Preqin is the industry leader in alternative investment data, and its indexes can be used to compare public and private alternatives. According to Preqin, the index is a time-weighted index that enables investors to compare the returns on various private assets and their publicly traded counterparts. We will use their indices to measure the PE market as a whole and specific sectors such as buyouts and venture capital. Similarly, we will use their private debt and real estate indexes to analyze the differences there. These indexes are calculated using cash flow transactions and net asset values at the fund level, which can be expressed mathematically using the following formula (Preqin, 2013):

$$Percentage\ change\ in\ quarter = \frac{NAV\ at\ end\ of\ quarter\ +\ distribution\ during\ quarter}{NAV\ at\ start\ of\ quarter\ +\ contributions\ during\ quarter} - 1 \quad (9)$$

“Contribution during the quarter” refers to capital raised by the fund managers from limited partners during one quarter. While “distribution during the quarter” is about the capital distributed to limited partners throughout the quarter (Preqin, 2013).

6. Methodology

To analyze the performance of different asset classes, several models are used to construct optimal portfolios. More specifically, minimum risk portfolios (MRP), tangency portfolios, geometric mean maximization (GMM) portfolios, and maximum diversification portfolios (MDP) are constructed. These optimal portfolios will then be compared to traditional stock-bond portfolios. Additionally, the portfolios built are evaluated through a sensitivity analysis using a simulation approach, and lastly, a subset of portfolios are stress-tested.

Additionally, the analysis is done around multiple time horizons, considering time windows including and excluding the 2008 Great Recession and 2020 Covid-19 crises. The objective for this is to determine how the assets look without the impact of abnormal market situations. However, it is also critical to examine the assets' performance during times of stress, so two historical estimation periods are employed in this paper.

Both Excel and R programming are used in the analysis. The data was first exported to Excel, where the initial processing of the samples was done. After that, the more advanced models were built in R, which performs better in complex data analysis than Excel.

As the first part of our analysis, the different asset classes are analyzed individually. The main goal is to identify specific qualities that each of them has regarding returns and risks. In the second part, we look into the allocations of each asset from a portfolio perspective. This consists of establishing capital market expectations, followed by deriving the efficient frontier and finding the optimal asset mix. The goal is to identify optimal asset allocations regarding various investor preferences. More specifically, several models are built to minimize risk, maximize return, maximize diversification, or mix all of these. Throughout, we compare the optimal portfolios to corresponding benchmark portfolios consisting only of stocks and bonds. This helps us to quantify the potential benefits of adding alternative assets to a retail investor’s portfolio.

We start by estimating all statistical measures in monthly rates, based on which the analysis is done further on. We also provide annualized measures to better visualize the performances on a yearly scale. Monthly returns are annualized by assuming compounding, that is, by calculating the geometric return based on the monthly average. Sharpe and risk measures such as Standard Deviation and CVaR, on the other hand, are scalable by multiplying by the square root of twelve. Formulas below:

$$Annualized\ return = (Monthly\ return + 1)^{12} - 1)$$
$$Annualized\ Standard\ Deviation(SD) = Monthly\ SD * \sqrt{12}$$
$$Annualized\ Sharpe\ Ratio(SR) = Monthly\ SR * \sqrt{12}$$
$$Annualized\ CVaR = Monthly\ CVaR * \sqrt{12}$$

Although such annualized measures differ from the actual measures calculated with yearly data, they still provide accurate representations of the measures per annum for this study. One of the biggest flaws of such annualizations is that they assume a normal distribution, which seldom holds in real-life settings.

It is to be noted that all of the expected portfolio returns are presented in excess returns, also known as risk premium, if not stated otherwise. In other words, the monthly risk-free rates are subtracted from the monthly portfolio returns, resulting in monthly risk premiums.

We take the perspective of an asset-only investor searching for the optimal portfolio, meaning liabilities are not taken into account. The investment horizon is long-term, and the opportunity set consists of seven asset classes presented above. The investor pursues either wealth maximization, risk minimization, or a mix of these, and no other investment goals are considered. Additionally, this paper considered global retail investors, as all of these securities can be invested regardless of the nation in which one resides. All products are available for online trading through platforms such as Nordnet in Norway or larger worldwide platforms such as Etoro. The impact of varying tax policies in different countries on returns is not considered in this paper.

6.1 Portfolio Optimization Models

The different models used in portfolio optimization are presented below. First portfolios are optimized based on capital marked predictions (CMP). Multiple CMP portfolios with different goals are optimized using Markowitz's standard mean-variance framework to satisfy various investors. Additionally, historical portfolios for two time periods are examined: July 2007 –

February 2021 and August 2009 – December 2019. As a result, we find comparable and complementary results for optimal portfolios for both crisis and non-crisis periods. The difference between CMP portfolios and portfolios constructed using historical data is mostly the returns. It is worth noting that CMP portfolios are based on the last ten years' historical risk and correlations; however, they are distinct from historical portfolios in that the estimated period varies.

In comparison to the CMP portfolios, historical portfolios are calculated using a variety of different methods. The first three historical models (MV, Mean-CVaR, Bayes-Stein) all use the same risk-minimizing and relative return maximizing principles when constructing the optimized portfolios, thus providing comparable results between them. The last two, on the other hand (GMM & MD), have different optimization objectives, which makes their results rather complementary than comparable. Based on the analysis of these results, we hope to understand whether and which liquid alternatives are rational additions for retail investor's portfolios and based on which objectives.

6.1.1 Mean-Variance

The mean-variance (MV) model is the most classical approach to portfolio optimization. It assumes that investors prefer portfolios with high expected returns in relation to risk. The MV model is one of the building blocks of Markowitz's modern portfolio theory. The main principles of MPT is explained in the theory chapter above. This model uses the arithmetic mean and standard deviation as return and risk estimates. The variance covariance matrix is then calculated based on these measures. Formulas for the arithmetic mean and standard deviation are presented below:

$$Arthmetic\ mean\ = \frac{X1 + X2 + X3 + \ldots\ldots.Xn}{n} \tag{10}$$

$$Standard\ Deviation\ = \sqrt{\frac{\sum(X - \bar{x})^2}{n - 1}} \tag{11}$$

where

$$X\ =\ The\ value\ in\ the\ data\ distribution$$

$$\bar{x} = The\ sample\ mean$$
$$n = Total\ number\ of\ observations$$

These statistical metrics are derived from holding period returns (HPR), which are defined by the following formula:

$$\text{HPR} = \frac{\text{P1} - \text{P0} + \text{D1}}{P0} \tag{12}$$

where

$$P0 = Beginning\ Price$$
$$P1 = Ending\ price$$
$$D1 = Dividend$$

Different portfolios are constructed here, first and foremost the minimum variance portfolio (MVP) and the tangency portfolio. The tangency portfolio is found by maximizing the Sharpe Ratio (1). The MVP is found by minimizing the portfolio variance (3). The weights that maximize and minimize these equations give the optimal allocations of each asset in a portfolio. Further using MV the efficient frontier and portfolios with risk and return constraints are constructed and analyzed. The optimization using the MV framework is done in R and Excel.

6.1.2 Mean-CVaR

A similar approach to the MV model is the Mean-CVaR model. The theoretical background of CVaR is presented in detail in the theoretical frameworks section above. In short, the biggest difference to MV is that CVaR includes tail risk and is more sensitive to the tail behavior of the distribution function. Regarding optimization, it is not limited to elliptical distributions as MV is.

Again, two portfolios are constructed using the Mean-CVaR model, the Minimum Risk Portfolio (MRP) and the Maximum Relative Performance Portfolio (MRPP). The optimization approach is identical to MV in minimizing risk and maximizing risk-adjusted returns, but instead of volatility as the measure of risk, this model uses CVaR. Because this model uses CVaR instead of volatility, the maximization portfolio is called MRPP instead of tangency, although the ideology is the same. These portfolios are constructed using the "fPortfolio" package in R (Wuertz, Chalabi, Chen & Ellis, 2010)

6.1.3 Bayes-Stein

While Mean-CVaR uses alternative estimates for risk, the Bayes-Stein model focuses on enhancing the return estimation. Equation (6) on page 53 shows the formal definition of the math on which this model is based.

The optimal portfolios using the BS model were built in R using the framework explained originally by Jorion (1986, 1991) and later modified by Avramov and Zhou (2010). Hence, we find a new return vector for the assets using the BS shrinkage estimator and a new variance-covariance matrix based on these returns. The variance-covariance matrix is calculated using the formula below:

$$\Sigma^{BS} = \left(1 + \frac{1}{T + \hat{\lambda}}\right)\widehat{\Sigma} + \frac{\hat{\lambda}}{T\left(T + 1 + \hat{\lambda}\right)} \cdot \frac{\underline{1} \cdot \underline{1}'}{\underline{1}\widehat{\Sigma}^{-1}\underline{1}} \tag{13}$$

where

$$\hat{\lambda} = (N+2)/\left[\left(E\left(r_j\right) - E(r_{MVP})\underline{1}\right)' \widehat{\Sigma}^{-1}\left(E\left(r_j\right) - E(r_{MVP})\underline{1}\right)\right],$$

$$\widehat{\Sigma} = \frac{T}{T - N - 2}\sigma_{ij}$$

Using the new estimated returns and covariances, we construct two portfolios, the MRP and the tangency portfolio. The portfolio construction is done using the same principles as before, but in this case, using the BS estimators for inputs.

6.1.4 Geometric Mean Maximization

GMM was proposed as an alternative to mean-variance optimization. The main difference being that GMM considers wealth maximization as the main objective of investors rather than the optimization of risk-adjusted returns.

As shown by numerous researchers, the maximization of a portfolio's geometric mean (GM) return can be done in numerous ways (Estrada, 2010). The method proposed by Estrada (2010), which is the one used in our analysis, is constructed using historical observations. The formal way to define this geometric mean maximization is presented below:

$$\begin{aligned} Max\ GM_p &= exp\left\{ln\left(1+\mu_p\right) - \frac{\sigma_p^2}{2\left(1+\mu_p\right)^2}\right\} - 1 \\ &= \exp\left\{ln\left(1+\sum\nolimits_{i=1}^{n} x_i\mu_i\right) - \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij}}{2(1+\sum_{i=1}^{n} x_i\mu_i)^2}\right\} - 1 \end{aligned} \qquad (14)$$

where

$x_i =$ $proportion\ of\ the\ portfolio\ in\ asset\ i$

$\sigma_{ij} = the\ covariance\ between\ assets\ i\ and\ j$

$\mu_i = the\ expected\ return\ of\ asset\ i$

$\sigma_p^2 = the\ variance\ of\ a\ portoflio$

$\mu_p =$ $the\ expected\ return\ of\ a\ portfolio$

Subject to $\sum_{i=1}^{n} x = 1$ and $x_i \geq 0$ for all i. This is the formal expression of the model referred to in this paper as GMM. Equation (14) shows the role volatility in the GMM model compared to the MV model. In the MV, volatility is undesirable because it is synonymous with risk; in the GMM model, volatility is also undesirable, not because it means risk but because it lowers the geometric mean return. In other words, it lowers the rate of growth of the capital invested, ultimately lowering the expected terminal wealth which the model is designed to maximize.

6.1.5 Maximum Diversification

Maximum diversification portfolio (MD) was first introduced by Choueifaty and Coignard (2008) and gave the portfolio that maximizes the diversification ratio. Its true value is derived from informing investors about the degree of diversification available within that investment universe. This portfolio establishes a theoretical maximum level of diversification, giving insight into which assets provide the most diversification benefit when establishing a portfolio.

However, according to Choueifaty & Coignard (2008), the most diversified portfolio should not be considered an equilibrium model. It does not generally meet the objectives of most investors and thus should rather be seen as an idealized target. Additionally, Theoren and Vuuren (2017) investigated previous claims that the MD portfolio's performance would be superior to MVP or tangency portfolios. They concluded that the MD portfolio outperforms the MVP portfolio in out-of-sample performance but falls short behind the tangency portfolio in total performance. This was especially the case in terms of cumulative returns.

This model is implemented using the same inputs as previous models: Holding period returns and standard deviations. A formal definition of the diversification ratio of a portfolio P is as follows:

$$D(P) = \frac{P' \cdot \Sigma}{\sqrt{P'VP}} \,, \tag{15}$$

where

$$\Sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_N \end{bmatrix} = the\ vector\ of\ asset\ volatilites$$

$$P = \left(w_{p1}, w_{p2}, \ldots, w_{pN}\right)$$
$$V = Covariance\ matrix\ of\ assets$$

Thus, as can be seen from (15), the diversification ratio is the ratio of the weighted average of volatilities divided by the portfolio volatility. By maximizing (15), we find the weights that provide the highest diversification benefit for an investor.

6.2 Portfolio Simulation

Philippe Jorion (1992) proposes a simulation approach for determining the optimal portfolio distribution. The rationale is to gain a more comprehensive understanding and remove some of the misconceptions associated with mean-variance analysis. Traditionally, portfolio optimization is built on a single estimation period. However, the optimal portfolio is strongly dependent on the time frame used for estimation. A way to correct this is by using Jorion's simulation model. Because this simulation is run over several time windows, it can provide insight into the optimal portfolio that performs best out of sample.

Jorion (1992) breaks down the simulation into four stages. First, the simulation begins by choosing one random sample of returns from the entire estimation period. Subsequently, it utilizes the sample to calculate the mean returns and covariance matrix. Following that, it uses the input parameters to determine the tangent portfolio. The simulation outcome represents a single observation in the optimal portfolio distribution. Repeat all steps until the distribution is approximated to a sufficient degree of accuracy. Jorion (1992) simulated 1000 different portfolios, from one sample period, in his research paper.

The process will determine if the optimal portfolio's weights are consistent with the estimation period and will provide insight into the distribution's average weights for each asset. According to Jorion (1992), this could provide investors with a deeper understanding of whether the optimized portfolios make sense. In other words, consider this analysis to be a sensitivity analysis to the tangent portfolios discussed in this paper.

6.3 Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) is a mathematical model that depicts the link between systematic risk and expected return on assets, most notably equities. CAPM is commonly used in finance to price risky assets and calculate projected returns on assets given their risk (Fama & French, 2004). The following formula represents CAPM:

$$Ri = Rf + \beta i \cdot (Rm - Rf) \tag{16}$$

where

$$Ri = Is\ the\ expected\ return\ on\ the\ investment$$
$$Rf = Is\ the\ risk - free\ rate$$
$$\beta i = Is\ the\ beta\ of\ the\ investment$$
$$Rm = Is\ the\ expected\ return\ of\ the\ market$$

In this research, the CAPM will determine the expected return for listed PE and BDCs since no credible analysis provides forecasts for these assets. An important estimate in the CAPM is the beta. It measures how much a stock fluctuates in price compared to the market. CAPM only measures systematic risk; it assumes that the investor is diversified and therefore is not exposed to idiosyncratic risk. The beta can be measured the following way:

$$\beta i = \frac{Cov(ri, rm)}{Var(rm)}$$

where

$$COV(ri, rm) = Covariance\ between\ the\ asset\ and\ market$$
$$ri = Is\ the\ expected\ return\ of\ the\ asset$$
$$\beta i = Is\ the\ beta\ of\ the\ investment$$
$$rm = Is\ the\ expected\ return\ of\ the\ market$$

However, as noted in Chapter 3.4 regarding liquidity, even when the alternative assets are traded on exchanges, they are still subject to illiquidity due to infrequent trading patterns. When the standard beta coefficient is applied to an illiquid stock, an unreasonably low beta may result. To overcome this, Lahr and Herschke used the Dimson beta. Dimson (1979) suggests that when a financial asset is subject to infrequent trade, an unbiased beta calculation should be performed. Dimson recommends adjusting for illiquidity by performing amultiple regression on historical returns against previous and present market returns. The following formula can express the regression:

$$ri = \alpha + \beta^{-1} rm_{t-1} + \beta^{0} rm_{t} + \varepsilon t \tag{17}$$

$$D\beta = \beta^{-1} + \beta^{0}$$

where

$\beta^{-1} = Is\ the\ first\ lag\ beta\ of\ the\ investment$

$\beta^{0} = The\ current\ beta\ of\ the\ investment$

$\varepsilon t = \quad Is\ the\ error\ term$

$D\beta = Dimson\ (1979) beta$

We apply his methodology and calculate Dimson's beta for alternative assets, specifically listed PE, BDCs, and REITs. According to the literature, High Yield and commodities are not affected by infrequent trading.

7. Analysis

7.1 Individual Historical Performances

The following chapters present an analysis of individual asset class performances regarding our estimation period of 2007-2021. Additionally, the performances have been analyzed independently for the years 2009-2019 and examined their performance only during times of crisis. The individual analysis allows us to investigate how each asset class has performed against one another and, more specifically, how the alternative assets of interest have performed compared to traditional investments. Analysis from multiple periods provides a more thorough understanding of how much individual economic phenomena can affect statistical measures during certain periods. Especially, as these measures are further used as inputs in many of the portfolio optimization models. Additionally, professional capital market expectations are used as inputs to eliminate some of the biases present in the historical data sample. Although the monthly rates are used in further calculations and models, in this part, we mostly talk about the annualized measures. This is because such measures are more interpretable.

7.1.1 Historical Returns

First, we present each asset's historical performance and analyze two different periods. Risks and returns are presented using multiple measures. Although they all represent risks and returns, they enable a deeper analysis of specific qualities such as volatility risk and downside risk regarding each asset. Figure 15 presents the development for both traditional and alternative assets from 2007 until 2021.

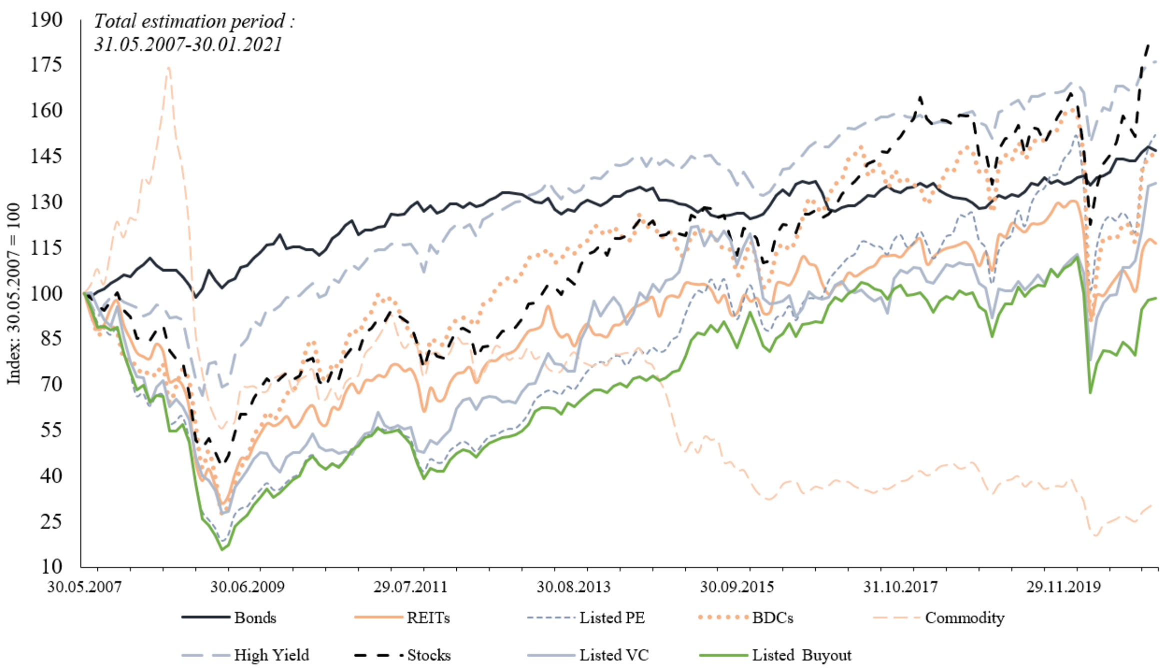


Figure 15 - Development of alternative and traditional assets total estimation period – 2007-2021 (Data Source: Bloomberg)

2007-2021

The best performers were BDCs, which averaged 6.4% per year, and stocks and listed PE, which averaged 6.2 % per year when it comes to average arithmetic returns. The worst returns were commodities with negative 5.3%. While BDCs had the highest returns, they were also among the ones with the highest risks, with 25.2% standard deviation (SD) and 70.7% CVaR per annum. Comparing this to bonds, which had the lowest risks during the entire period, corresponding numbers were 5.4% in SD and 11.6% in CVaR, which are significantly lower. Table 5, shows how the volatility of the assets raises the arithmetic returns. This can especially be seen in BDCs, having the highest arithmetic return but only the fourth highest geometric return.

Table 5 – Monthly and annualized performance measures, 2007-2021 (Data source: Bloomberg)

2007-2021	*Return Measures*			*Risk Measures*				*Sharpe*	
Assets	Geometric Excess Return	Arithmetic Excess Return	*Annualized return	Standard Deviation	*Annualized Sd	CVaR	*Annualized CVaR	Arithmetic Sharpe	*Annualized Sharpe
Stocks	0,4 %	0,5 %	6,2 %	5,2 %	18,1 %	12,4 %	42,8 %	0,096	0,331
Bonds	0,2 %	0,2 %	3,0 %	1,6 %	5,4 %	3,3 %	11,6 %	0,158	0,546
REITs	0,1 %	0,3 %	3,4 %	6,0 %	20,7 %	15,3 %	52,9 %	0,047	0,162
BDC	0,2 %	0,5 %	6,4 %	7,3 %	25,2 %	20,4 %	70,7 %	0,072	0,248
Listed PE	0,3 %	0,5 %	6,2 %	6,9 %	23,7 %	17,4 %	60,1 %	0,073	0,252
Listed VC	0,2 %	0,4 %	5,1 %	6,7 %	23,3 %	15,7 %	54,2 %	0,062	0,215
Listed Buyout	0,0 %	0,3 %	3,5 %	7,4 %	25,8 %	20,0 %	69,2 %	0,038	0,132
Commodities	-0,7 %	-0,5 %	-5,3 %	7,0 %	24,1 %	17,5 %	60,6 %	- 0,065	- 0,226
High Yield	0,3 %	0,4 %	4,9 %	3,2 %	11,3 %	7,7 %	26,6 %	0,123	0,425

*Returns are annualized by calculating the geometric return based on the monthly average. Risk measures are annualized by multiplying by the square root of twelve

Hereby, it can be concluded that bonds significantly outperformed all other assets regarding risks. BDCs had the highest arithmetic returns, but stocks have the highest geometric mean return. From the values of CVaRs, one can see that the alternative assets had slightly higher downside risks than stocks. Commodities had the worst performance by far. This is largely due to the heavy allocation towards oil, which saw a steep drop in 2014. Although not being the best in any category, the high yield had excellent performance overall.

2009-2019

This period has been generally free of major crises; in other words, it may be considered stable for these assets, shown by the statistics in Table 6 below. Since two crises have been cut out, the risks associated with alternative asset classes have decreased, most notably the downside risk associated with equity-like assets. Moreover, while before BDCs had the highest risk statistics, now commodities had significantly higher. All alternatives had very similar volatilities, all falling between 14-14.5%. However, in terms of CVaRs, commodities had a CVaR of 43%, whereas other alternatives had around 30%. Stock’s risk statistics were very close but slightly below the values of listed PE. High yield, on the other hand, was similar to bonds in terms of risk.

Table 6 – Monthly and Annualized Performance measures, 2009-2019 (Data source: Bloomberg)

2009-2019	*Return Measures*			*Risk Measures*				*Sharpe*	
Assets	Geometric Excess Return	Arithmetic Excess Return	*Annualized return	Standard Deviation	*Annualized Sd	CVaR	*Annualized CVaR	Arithmetic Sharpe	*Annualized Sharpe
Stocks	0,7 %	0,8 %	9,8 %	3,9 %	13,4 %	8,4 %	29,1 %	0,204	0,705
Bonds	0,1 %	0,16 %	1,9 %	1,4 %	4,8 %	3,2 %	11,1 %	0,114	0,396
REITs	0,7 %	0,8 %	9,9 %	4,0 %	14,0 %	7,8 %	27,2 %	0,195	0,677
BDC	0,8 %	0,9 %	11,8 %	4,2 %	14,7 %	8,0 %	27,6 %	0,221	0,764
Listed PE	1,2 %	1,2 %	15,9 %	4,2 %	14,5 %	8,6 %	29,6 %	0,296	1,025
Listed VC	0,7 %	0,8 %	10,1 %	4,9 %	16,9 %	9,3 %	32,1 %	0,164	0,569
Listed Buyout	1,0 %	1,1 %	13,5 %	4,2 %	14,4 %	8,3 %	28,8 %	0,255	0,884
Commodities	-0,6 %	-0,3 %	-3,5 %	4,2 %	14,4 %	12,7 %	43,8 %	- 0,072	- 0,249
High Yield	0,5 %	0,5 %	6,6 %	2,0 %	6,9 %	3,5 %	12,0 %	0,266	0,922

*Returns are annualized by calculating the geometric return based on the monthly average. Risk measures are annualized by multiplying by the square root of twelve.

Listed PE has the highest returns and Sharpe with 15.9% in annualized returns and Sharpe of 1.025. After listed PE, high yield and BDCs came next in terms of Sharpe Ratio. The worst Sharpe, again, was in commodities with a negative value.

To conclude, it can be said that listed PE had the best overall performance during the non-crisis period, especially in regards to returns. BDCs outperformed stocks as well in terms of returns and Sharpe Ratio during this period. High yield had a great overall performance, being second best in terms of Sharpe and risks. Figure 16 below compares the two periods in a risk-return diagram. It is easy to observe that the non-crisis period increases the return for assets and lowers the risk. However, the ones with the highest returns are still BDCs and Listed PE in both periods. Bonds carry the lowest risk in both periods.

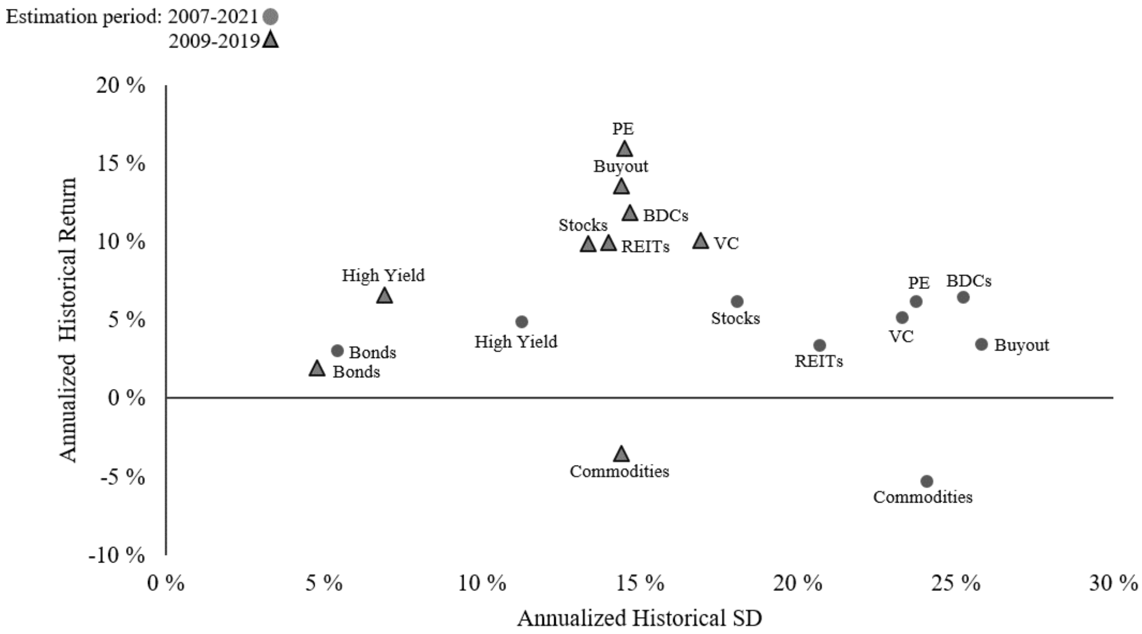


Figure 16 - Traditional and alternative asset classes' risk and return, for both estimation periods

7.1.2 Historical Correlations

From a diversification point of view, it is relevant to analyze the correlations between the asset classes. The smaller the correlation between the assets, the larger the potential diversification benefit.

Based on our data, it seems that the strongest correlation is between stocks and REITs, with a value of 0.89. REITs correlate strongly with every other asset class except commodities and bonds. Despite this, they seem to have the highest correlation to bonds and commodities of every asset class. This would imply that REITs are a bad diversifier in a traditional portfolio of stocks and bonds. Moreover, they neither seem to provide good diversification when mixed with non-traditional assets such as listed PE or high yield. The entire correlation matrix is presented below in table 7 with significance.

Table 7 - Correlation matrix 2007 – 2021

Asset	Bonds	REITs	Listed PE	Listed VC	Listed Buyout	BDCs	Commodity	High Yield	Stocks
Bonds	1.00								
REITs	0.54***	1.00							
Listed PE	0.13	0.80***	1.00						
Listed VC	0.08	0.69***	0.85***	1.00					
Listed Buyout	0.15	0.81***	0.98***	0.83***	1.00				
BDCs	0.32***	0.82***	0.84***	0.73***	0.87***	1.00			
Commodity	0.26***	0.49***	0.47***	0.39***	0.46***	0.49***	1.00		
High Yield	0.50***	0.82***	0.69***	0.62***	0.69***	0.77***	0.43***	1.00	
Stocks	0.44***	0.89***	0.84***	0.73***	0.82***	0.81***	0.61***	0.78***	1.00

*p<0.05 **p<0.01 ***p<0.001

The lowest correlation was found between bonds and listed PE, with a value of 0.11. This being nearly zero implies that listed PE would be a great diversifier when mixed with bonds. Bonds are also found to be the least correlated asset class overall with the others. The second least correlated asset class overall, according to our results, is commodities.

When alternative asset correlations are compared to traditional asset, commodities tend to be the best diversifier, followed by listed PE, BDCs, and high yield, and lastly REITs. Although listed PE has a relatively high correlation with the stock market, 0.83, it is so uncorrelated with bonds that it makes a great overall diversifier. This is especially the case considering that all alternatives are quite similarly correlated with stocks, commodities, and high yield having the lowest correlations followed by BDCs, listed PE, and REITs.

As shown in Table 7, listed PE seems to be composed mostly of listed buyout funds, having a correlation of 0.978 with them. This is likely because these funds tend to be the largest and thus obtain most of the weight in the index. Listed VC, however, seems to be less correlated with other asset classes than listed buyout, which makes it an excellent diversifying component in an investor's portfolio and in the listed PE index.

The relative order of the correlations remains somewhat the same no matter the time horizon. The most notable difference is that REITs become relatively less correlated with other asset classes and most correlations fall when excluding crises. These differences can be seen by comparing Tables 7 and 8.

Table 8 - Correlation matrix 2009/9 - 2019/12

Assets	Bonds	REITs	Listed PE	Listed VC	isted Buyou	BDCs	Commodity	High Yield	Stocks
Bonds	1.00								
REITs	0.54***	1.00							
Listed PE	-0.03	0.62***	1.00						
Listed VC	-0.15	0.43***	0.73***	1.00					
Listed Buyout	-0.03	0.60***	0.96***	0.67***	1.00				
BDCs	0.19*	0.66***	0.76***	0.56***	0.77***	1.00			
Commodity	0.22*	0.39***	0.34***	0.23**	0.34***	0.51***	1.00		
High Yield	0.45***	0.78***	0.66***	0.48***	0.66***	0.73***	0.58***	1.00	
Stocks	0.33***	0.81***	0.79***	0.56***	0.76***	0.77***	0.59***	0.79***	1.00

*p<0.05 **p<0.01 ***p<0.001

Since 2007, figure 17 compares the alternative assets to stocks on a rolling 24-month correlation basis. As the time series begins on 29.05.2007, the first observation occurs on 29.05.2009. Furthermore, the correlation is calculated monthly until 31.01.2021 using data from the previous 24 months.

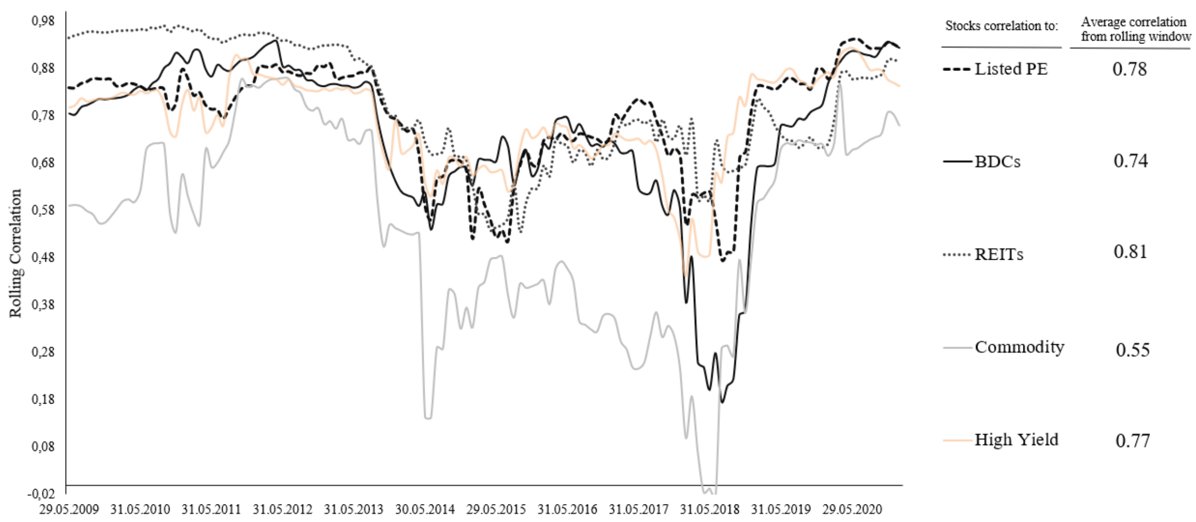


Figure 17 - Stocks 24-month rolling window correlation against alternative assets.

The first thing to observe is that commodities have a highly variable correlation with stock; from mid-2016 to mid-2018, they had a negative correlation with stocks, but by the start of 2021, they had a correlation close to 0.8. However, the rolling window's average correlation is 0.55, the lowest of all alternative assets.

Between 2013 and 2017, all alternative assets exhibited a low correlation to stocks. However, from 2017 to the start of 2021, all assets are advancing in the same direction, increasing the correlation. In other words, diversification appears to fade away. It should be noted, that by excluding commodities, BDCs have the lowest average rolling window correlation with stocks. What is clear is that correlations fluctuate from year to year, illustrating the criticism directed to the MV analysis.

All in all, many of the alternatives, seem to show great diversifying benefits when combined with traditional assets and other alternative assets. Most alternatives show lower correlations with bonds than stocks do. Thus, solely from a diversifying point of view, they seem to have clear potential in a retail investor’s portfolio.

7.1.3 Performance During Economic Crisis

It is essential to analyze how assets are affected by uncertainty and market shocks, such as financial instability, political tension, or unexpected events. In this chapter, the traditional assets will be compared against the alternatives in the most significant crises in recent times, namely the Great Recession and the Covid-19.

The Great Recession

The chairman of the federal reserve from 2006 until 2014, Ben Bernanke, said that September and October of 2008 were the worst financial crisis in global history. Figure 18 visuals the development of the assets from the start of the great recession in mid-2007 to the end in mid-2009. The performance of the asset is presented in table 9, where the assets are ranked from highest to lowest, based on geometric monthly return.

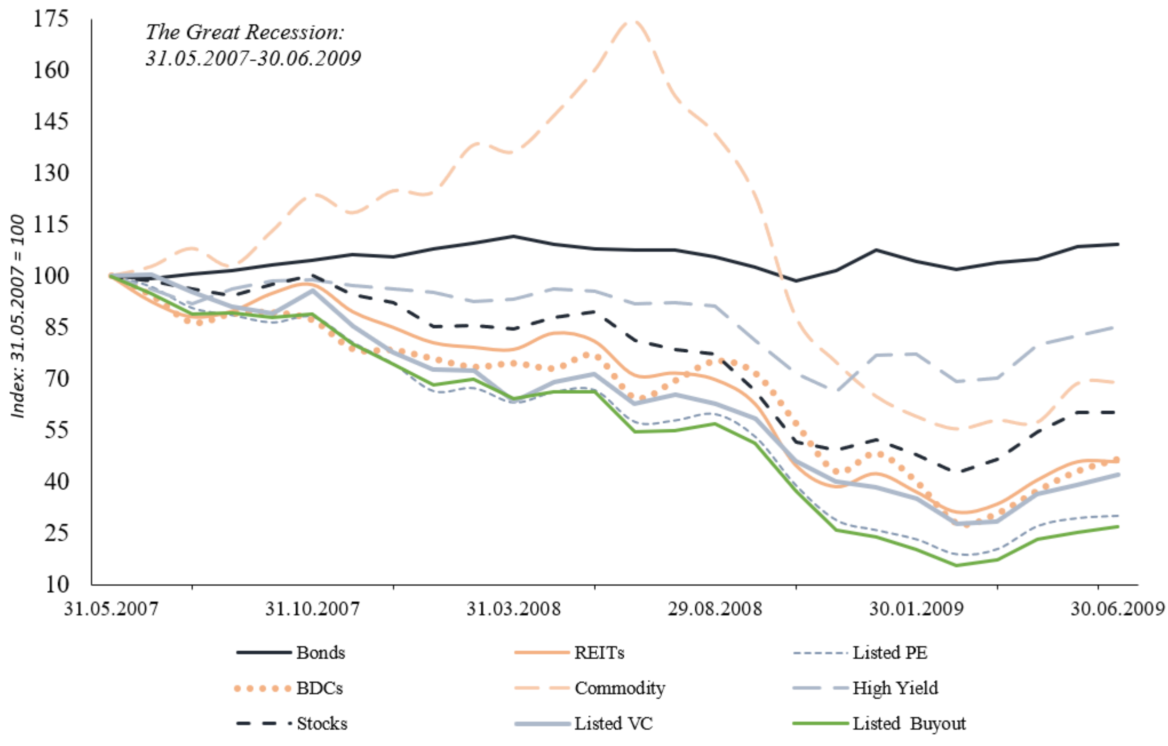


Figure 18 - The Great Recession (source: Bloomberg)

Bonds, being the only asset with a positive return, outperformed all the other assets. High Yield became second in terms of returns. The different Listed PE strategies are ranked last, in which listed buyout performed the worst, with negative 40.3% in annualized return.

CVaR sheds light on which assets were worst hit by the crisis. The listed buyout and BDCs were particularly hit by the great recession, with a monthly CVaR of approximately 30%. BDCs had the same CVaR as listed buyout, but recovered more quickly, resulting in BDCs outperforming listed PE and finishing just behind stocks in the return rankings.

Table 9 – Monthly and annualized asset class performances in the Great Recession (Source: Bloomberg)

2007-2009 (The great recession)

	Return Measures			***Risk Measures***			***Sharpe***	
Assets	Geometric monthly Excess Return	Arithmetic monthly Excess Return	*Annualized return	Monthly Standard Deviation	*Annualized Sd	CVaR	Arithmetic Sharpe	*Annualized Sharpe
Bonds	0,3 %	0,4 %	4,6 %	2,3 %	8,1 %	-3,9 %	0,160	0,553
High Yield	-0,6 %	-0,4 %	-5,2 %	6,4 %	22,2 %	-11,8 %	-0,069	-0,240
Commodities	-1,5 %	-0,9 %	-10,2 %	10,6 %	36,8 %	-28,4 %	-0,084	-0,290
Stocks	-2,0 %	-1,7 %	-18,3 %	8,2 %	28,3 %	-22,4 %	-0,204	-0,708
BDC	-3,0 %	-2,2 %	-23,4 %	12,6 %	43,7 %	-30,1 %	-0,174	-0,602
REITs	-3,1 %	-2,5 %	-26,5 %	10,4 %	36,0 %	-28,4 %	-0,244	-0,845
Listed VC	-3,4 %	-2,9 %	-29,6 %	10,6 %	36,6 %	-20,8 %	-0,273	-0,946
Listed PE	-4,7 %	-4,0 %	-38,7 %	12,1 %	41,8 %	-26,6 %	-0,331	-1,147
Listed Buyout	-5,1 %	-4,2 %	-40,3 %	13,4 %	46,4 %	-30,3 %	-0,314	-1,089

‘Returns are annualized by calculating the geometric return based on the monthly average. Risk measures are annualized by multiplying by the square root of twelve.

Commodities did very well at the beginning of the crisis due to the high demand for investments in something other than financial assets and due to their low correlation with the rest of the market. However, their success did not last throughout the crisis and ended up being only the third best overall performer.

The Covid-19

The financial markets understood the seriousness of the Covid-19 in late February 2020 after various governments placed restrictions worldwide. However, markets recovered extremely fast, which is different compared to the Great Recession. Many investors realized that big tech companies like Spotify, Amazon, and Netflix can still operate in a world where a pandemic is ravaging. Figure 19 visuals the development of the assets from the end of 2019 until first month in 2021. The performances of the assets are presented in table 10.

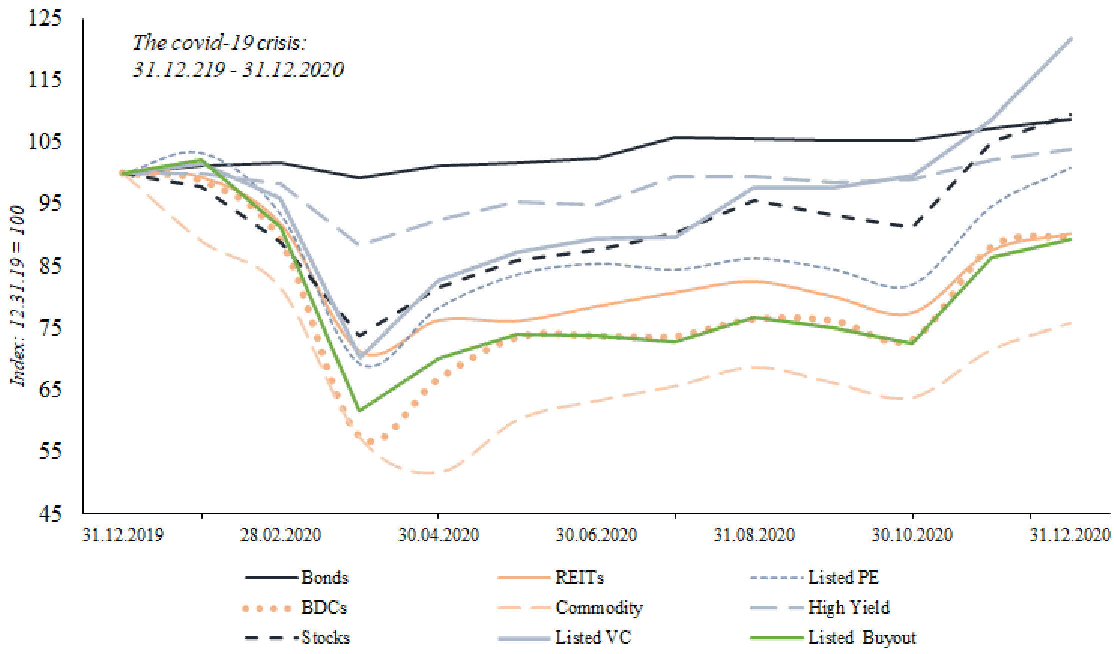


Figure 19 - Covid-19 Crisis (Source: Bloomberg)

During this period, Listed PE performed much better compared to the Great Recession. A component of listed PE, listed VC, had an annualized return of 29.2 % which was the highest of all during this period. However, high return is often accompanied by high risk. Listed VC indeed had a relatively high monthly standard deviation of 10.6 % and a CVaR of 26.7 %. Bonds again top the rankings for the highest Sharpe as they did in the prior crises.

Table 10 – Monthly and annualized asset class performances in the Covid-19 Crisis (Source: Bloomberg)

End 2019 until start 2021 (Covid-19)

	Return Measures			***Risk Measures***			***Sharpe***	
Assets	Geometric monthly Excess Return	Arithmetic monthly Excess Return	*Annualized return	Monthly Standard Deviation	*Annualized Sd	CVaR	Arithmetic Sharpe	*Annualized Sharpe
Listed VC	1,6 %	2,2 %	29,2 %	10,6 %	36,7 %	-26,7 %	0,204	0,706
Stocks	0,7 %	1,0 %	12,2 %	8,1 %	28,2 %	-16,8 %	0,119	0,412
Bonds	0,6 %	0,6 %	7,2 %	1,4 %	4,8 %	-2,3 %	0,417	1,444
Listed PE	0,3 %	0,8 %	10,3 %	10,4 %	35,9 %	-25,8 %	0,079	0,273
High Yield	0,3 %	0,4 %	4,9 %	3,8 %	13,3 %	-10,1 %	0,104	0,359
BDC	-0,7 %	0,3 %	3,7 %	13,5 %	46,7 %	-35,6 %	0,023	0,078
Listed Buyout	-0,8 %	0,0 %	-0,4 %	12,3 %	42,6 %	-32,5 %	- 0,003	- 0,009
REITs	-0,9 %	-0,5 %	-6,0 %	8,3 %	28,7 %	-22,4 %	- 0,062	- 0,215
Commodities	-1,7 %	-1,0 %	-11,5 %	11,9 %	41,3 %	-29,5 %	- 0,085	- 0,296

'Returns are annualized by calculating the geometric return based on the monthly average. Risk measures are annualized by multiplying by the square root of twelve.

Commodities and REITs have still not recovered from the recession as of March 2020 and therefore have the worst performance of all assets during the Covid-19 so far. When it comes to CVaR, BDCs are on the top of that list, meaning that the BDCs had the greatest collapse in March 2020 of all assets. They also had a significantly high CVaR in the Great Recession, leaving this asset, together with listed Buyout, most exposed to market instability. Post-crisis, BDCs have, however, had rapid recoveries, both in Covid-19 and in the Great Recession.

Bonds performed very well during both crises, outperforming all the other assets in terms of Sharpe ratio. Although High Yield Bonds has higher volatility than traditional bonds, it performs very well during crises relative to the equity assets.

In summary, the impression is that bonds and stocks performed better than the alternative assets through the Great Recession. This can be seen through the returns, the standard deviation, and the Sharpe ratio. The Covid-19 was a very different crisis than the financial crisis. However, bonds and stocks were solid there as well. Stocks' CVaR was lower than that of the alternatives, except for high yield, which leaves them less sensitive to market uncertainty and shocks. However, it is to be noted that alternative investments tend to recover fast from the crises, thus making them a competitive alternative to the traditional assets in the long run.

7.2 Relationship With Non-listed Alternatives

As introduced earlier in this paper, non-listed alternatives play an influential role in many institutions' portfolios as either diversifiers or return enhancers. The real question is whether liquid listed alternatives behave the same way as their private counterparts. We investigate this by comparing the private and public vehicles against each other regarding returns, risks, correlations, and cointegration. All the statistics are presented as quarterly measures, and Preqin provides the private data for Private equity, Real Estate and Private Debt.

7.2.1 Private Equity

According to Michael Degosciu (2012) and Huss (2005), there is no noticeable difference between investing in listed and unlisted private equity. However, since both of their research are older, this will be critically evaluated in our analysis using newer data.

Figure 24 illustrates the risk and return characteristics of three different private equity asset classes. Listed PE indexes contain all PE strategies, while its sub sectors VC and buyout strategies are included separately for a more in-depth review. The data sample is comprised of quarterly data from 2001 to the end of 2019. Preqin's[13] non-listed PE data can in theory serve as a historical risk and return benchmark for institutional investors, whereas listed PE can serve as a historical risk and return benchmark for individual investors.

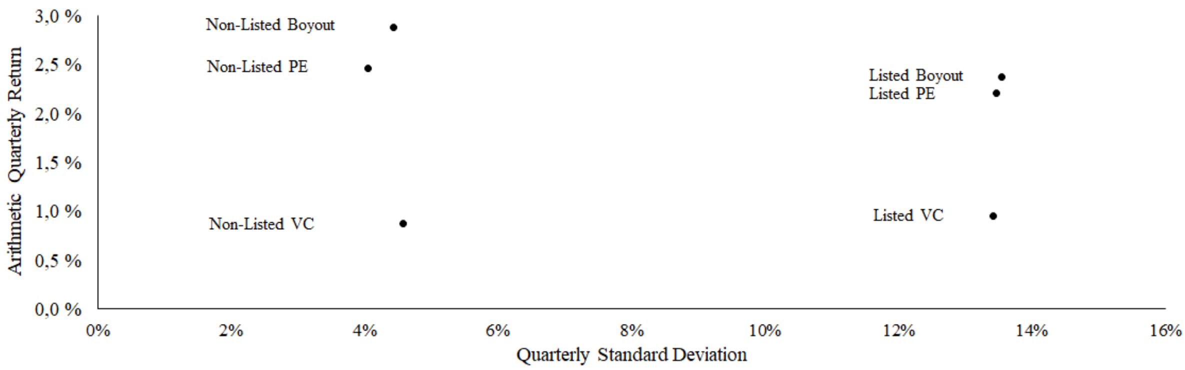


Figure 20 – Period: 2001-2019, Risk and return profile of listed and unlisted PE. (Source: Preqin Pro & Bloomberg)

The Buyout sector has historically produced the highest returns for both listed and unlisted PE, followed by PE and venture capital. In other words, listed and unlisted follow the same pattern in terms of the highest and lowest returns on PE strategies. The arithmetic means returns on listed and unlisted venture capital are comparable. However, unlisted Buyout does better than the listed counterpart, at about 0,4% per quarter.

As volatility is considered, all listed options attain a quarterly standard deviation of about 13.5 %, while unlisted PE achieves under 5 %. However, those observations should be interpreted cautiously since the private indexes are calculated on various assumptions.

[13] It should be noted that Preqin is a data supplier and private equity promoter, and hence is not an objective third party.

Preqin's private equity indices are measured using quarterly net asset value changes from a pool of 4000 global private equity funds. In contrast, the listed PE indices are calculated using real-time global capital market transactions. Quarterly growth in unlisted private equity is determined by various PE firms' reported performance and their portfolio companies' subjective valuations. Numerous articles and researchers argue that unlisted private equity companies intentionally understate their net asset value (NAV) and smooth it out (Jenkinson, Sousa, Stucke 2013). As a consequence, the Preqin index's standard deviation can be skewed and should be viewed carefully.

The graph in figure 25 illustrates a situation in which an investor buys the asset in 2001 and keeps it until the end of 2019 while reinvesting all dividends. Further, this is after cost and fees.

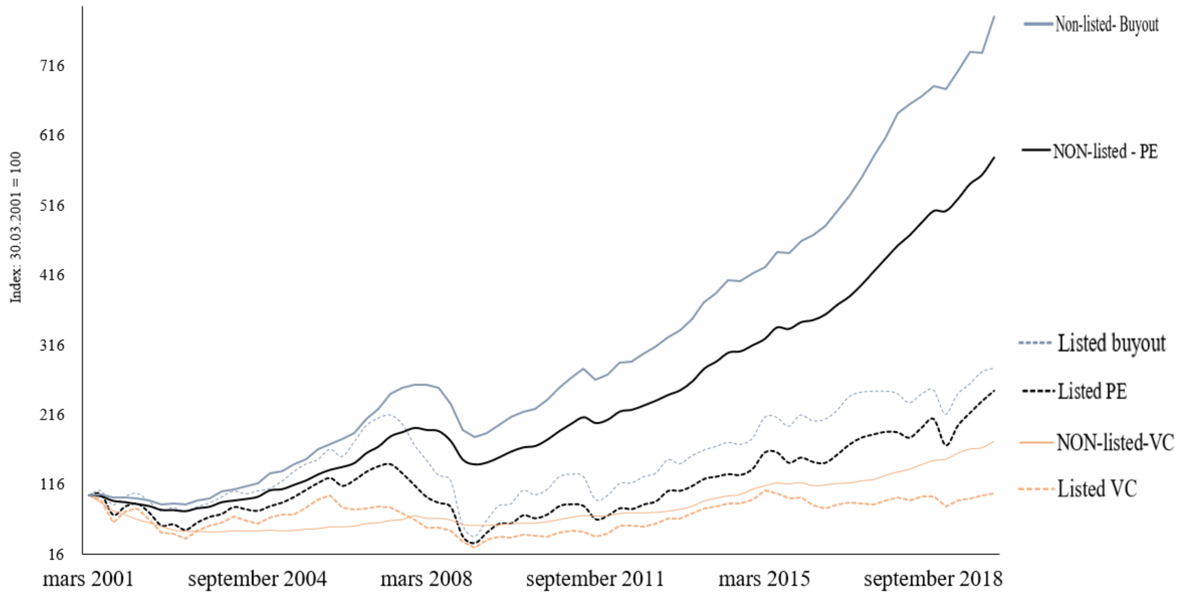


Figure 21 – Time-period: 2001-2019, Listed and non-listed PE comparison,
(Source: Preqin & Bloomberg)

Non-listed buyouts, private equity, and venture capital outperform their publicly traded counterparts in total cumulative returns. The arithmetic average discussed above indicates no substantial differences in return between listed and unlisted alternatives. However, the performance in figure 25 paints a different picture; investment in unlisted companies generates significantly more value. This is since arithmetic means do not take compounding into account.

Figure 26 illustrates the correlations and returns on the various assets. The correlation between listed and unlisted PE is the strongest at 0.71. The key difference is that when the economy is affected by a financial downturn, the listed alternatives appear much more impacted. However, there is again the problem addressed by Jenkinson, Sousa, Stucke (2013) of smoothening valuations of unlisted PE. The relationship between listed and unlisted VC tends to be the lowest, with a correlation of 0.476. Their unstable relationship was most visible between 2001 and 2004. The correlation of listed and unlisted buyouts is 0.6667.

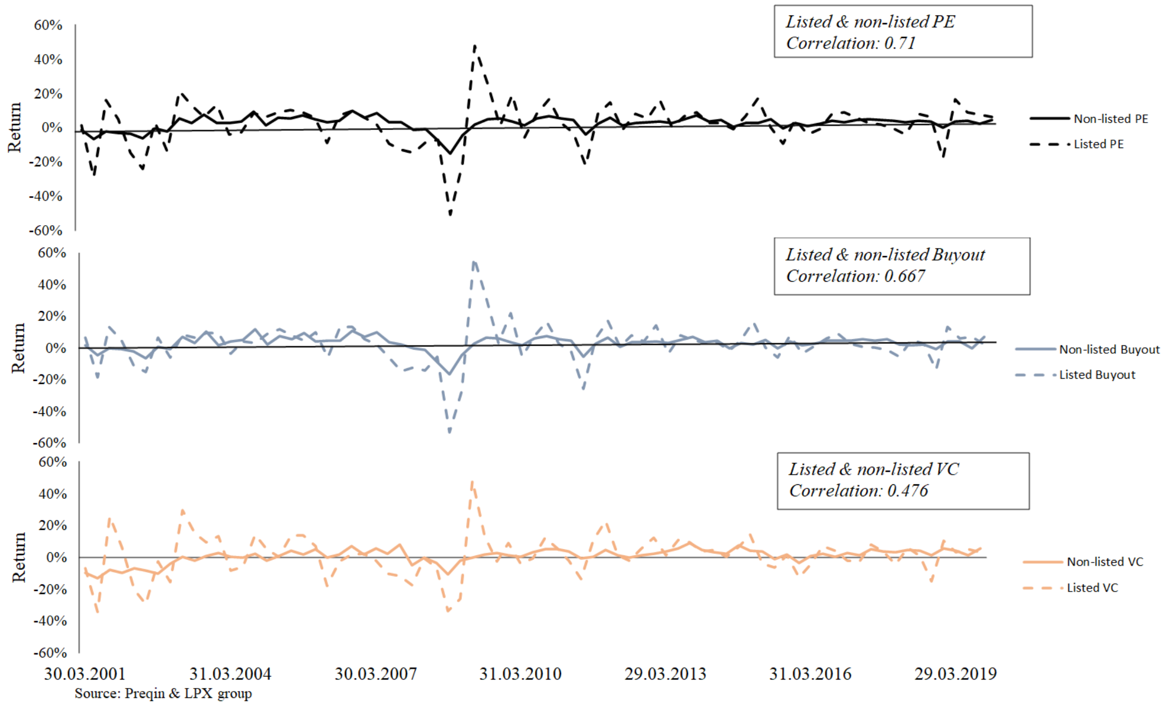


Figure 22 - Return and covariance listed versus unlisted PE (Source: Preqin & Bloomberg)

The cointegration test can be used to test if they have a long-term relationship. It is explained in chapter 4.4, and the test results can be seen in table 12. Since the T-values of each test are non-significant at $p = 0.05$, the null hypothesis is accepted; in other words, neither of the time series is cointegrated. One cannot prove a long-term relationship between the index of listed PE and Preqin non-listed PE.

Table 11 - Augmented Dickey-Fuller Test for Cointegration

	Listed and Non-listed **VC**	Listed and Non-listed **PE**	Listed and Non-listed **Buyout**
Type: ADF Test*			
T-Value	-2,15	-2,00	-2,302
Critical values: 10 %	-3,12	-3,12	-3,12
Critical values: 5 %	-3,41	-3,41	-3,41
Significant	NO	NO	NO

To summarize, listed PE and Preqin unlisted PE shows a reasonable correlation and roughly equivalent historical arithmetic returns. Non-listed options outperform listed options in terms of historical value creation. Unlisted PE carries a smaller risk than listed PE. However, caution is advised when interpreting the volatility of non-listed PE. Overall, this study demonstrates that there is a difference in investing in listed and unlisted private equity. However, conclusions are challenging to draw since the analysis is limited to the Preqin private index, constructed based on subjective valuations rather than actual transactions.

7.2.2 Real Estate

First, comparing the returns of REITs and the private real estate sector between 2005-2019, we see many similarities. The arithmetic return of REITs is higher at 2.30% compared to the private index's 1.91%. However, looking at the geometric mean, the values are extremely close to one another, REITs with 1.775% and private index with 1.770%. The reason why geometric means are so close but arithmetic show a difference is because the public index has a significantly higher standard deviation. Private data is often based on appraisal-based return, in which the appraiser determines the value of the real estate. Giliberto (1988) and Geltner (1991) argue that appraisal-based returns would understate real estate volatility. REITs have a 10.24% standard deviation, whereas the private index has 5.24%. The difference in volatility can also be easily visualized by looking at Figure 27, which shows the performances of the two indices.

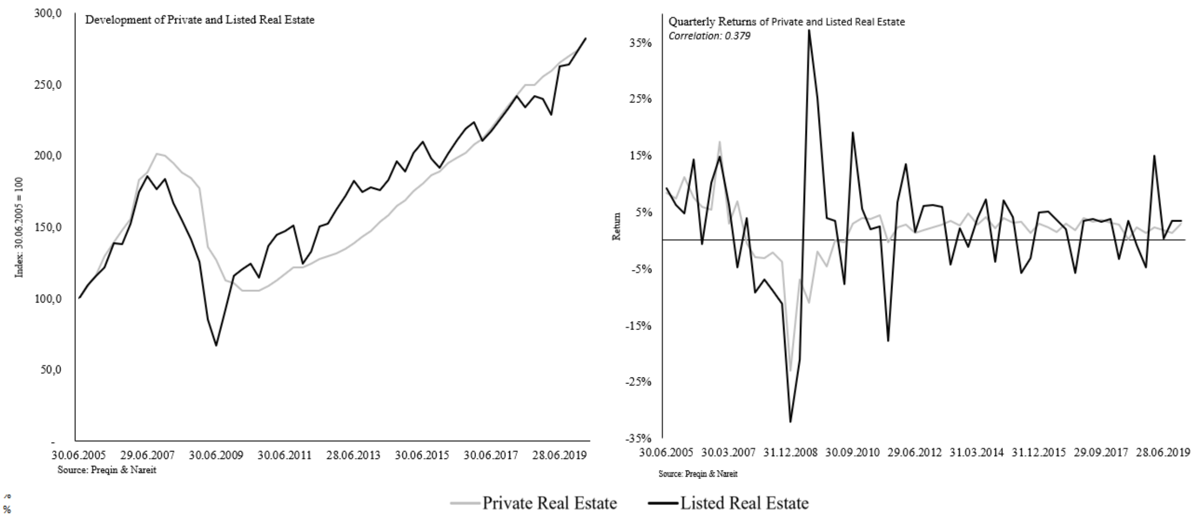


Figure 23 - Performance of Listed and Unlisted Real Estate (Source: Preqin & Bloomberg)

As can be seen from these figures, there is a low correlation between the assets. REITs and the private RE have a correlation of only 0.379. Although it is clear to see that the standard deviations are not alike and they lack correlation, the long-term relationship is still unclear. Moreover, it seems that the upward trend in the two seems similar. To test this, we ran an Augmented Dickey-Fuller (ADF) test on the series, which tests for cointegration. The results indicate that REITs and Preqin private real estate would not be cointegrated. In other words, the series does not seem to correlate either in the short or long term.

Thus, although the returns of the series are very similar, other characteristics differ a lot. To conclude, according to our results, REITs do not seem to be a suitable proxy for the private real estate sector. Moreover, our results show that between the years 2005 and 2019, REITs have provided slightly higher cumulative returns than private real estate.

7.2.3 Private Debt

Although BDCs do not solely invest in the private debt market, the Cliffwater BDC index chosen for this study comprises lending-oriented BDCs firms. Therefore, it could be argued to be the closest equivalent to a diversified portfolio of private debt to which retail investors can access.

From 2008 until 2019, the arithmetic average return for BDCs was 2.77%, whereas it was 1.81% for private debt (figure 28). The geometric returns are again somewhat closer, BDCs with 1.85% and private debt with 1.73%. Like REITs, this is largely due to the significantly higher standard deviation in BDCs, which may be explained by the appraisal-based returns common in non-listed alternatives. BDCs had a standard deviation of 13.72 %, whereas private debt had a standard deviation of 4.06 %,

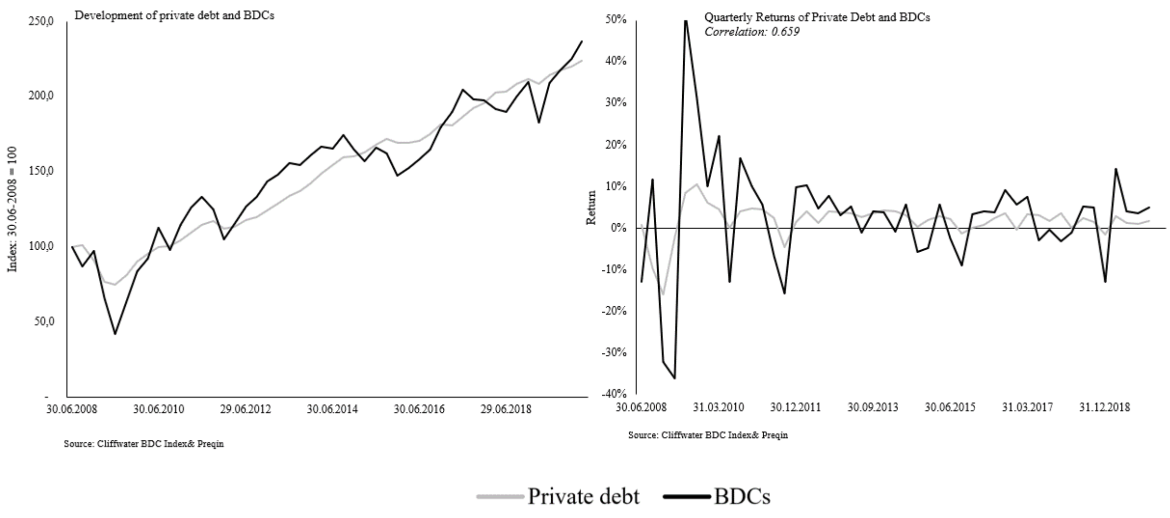
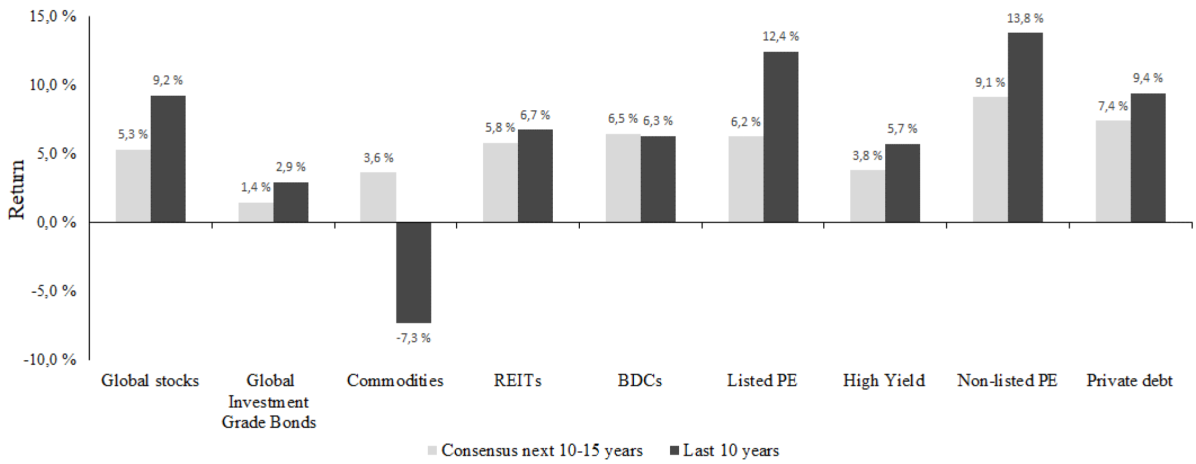


Figure 24 - Performance of Private Debt and BDCs (Source: Preqin & Bloomberg)

The correlation between BDCs and private debt is semi-strong, with a value of 0.659. So while the correlation is not perfect, BDCs still seem to move somewhat similar to the private debt market. Additionally, while testing for long-term correlations through the ADF test, these two series do not show significant cointegration. Thus, it seems that the series is somewhat correlated in the short term but does not have a strong long-run relationship.

7.3 Capital Market Expectations And Assumptions

It is insufficient only to use historical input measures when constructing optimal portfolios for the future. The performance during the past ten years is no proof of the performance of the next decade. Hence, numerous analysts, including JPMorgan, Blackrock, and Vanguard, set targets for future returns considering the current market outlook. They estimate returns for different asset classes over the next ten to fifteen years using data provided by banks, policy markets, and other economic indicators. The average forecast from 13 different analysts will yield a consensus estimate of expected returns in this study. Figure 20 compares the historical 10-year returns with the consensus expectation for the next decade. The outlook by experts is clear, bond returns will stay at historic lows, and equity returns are anticipated to be below average.



Source: (BlackRock, 2021; J.P.Morgan, 2021; Benz, 2021; Reasearch Affliliates, 2021; Vanguard, 2021; Northern Trust, 2021; Robeco, 2020; Invesco, 2021; Horizon, 2020; Pimco, 2020; Twosigma, 2020; BMO, 2020; MFS, 2021)

Figure 25 - Expected versus 10-year historical return

The global pandemic of 2020 brought about the sharpest economic downturn and fastest recovery in history. Experts agree that government spending and ficial stimulate will affect markets in the coming decades (BlackRock, 2021; J.P.Morgan, 2021; Vanguard, 2021). Moreover, according to them, our economies will continue to be affected by climate change, an aging population, and emerging technology. Analysts predict modest GDP growth over the coming years, with the expectation of avoiding permanent economic scars. In reality, this was a crisis that should not have occurred. It was triggered by an exogenous surprise rather than an underlying problem or imbalance that forced the economy over the edge. JP Morgan's article for capital market assumptions highlights that all investors, retail, and institutions must build a new portfolio for a new decade. They urge investors to look for other opportunities besides the traditional asset classes, stocks, and bonds (J.P.Morgan, 2021).

The analyst estimated returns for different asset groups are shown in Table 11 below. None of the analysts forecast future returns for BDCs or publicly traded PE. At the end of this chapter, we forecast their expected return

Table 12 - Capital market predictions next 10-15 years by experts.

Capital Market expectations									
	Traditional assets		Listed alternative assets					Non-listed alternative assets	
Expert	Global stocks	Global Investment Grade Bonds	Commodities	REITs	BDCs	Listed PE	High Yield	Non-listed PE	Private debt
BlackRock	7,1 %	-0,3 %	-	-	-	-	2,4 %	13,3 %	-
J.P.Morgan	5,6 %	2,5 %	2,3 %	6,4 %	-	-	4,8 %	8,0 %	6,8 %
Morningsstar Investment Management	3,1 %	1,0 %	-	4,7 %	-	-	-	-	-
Reasearch Affliliates	4,1 %	2,2 %	2,5 %	3,5 %	-	-	3,1 %	-	-
Vanguard	6,0 %	1,6 %	-	-	-	-		-	-
Northern Trust	4,9 %	1,6 %	3,6 %	6,3 %	-	-	5,6 %	7,9 %	-
ROBECO	6,3 %	1,0 %	6,5 %	4,5 %	-	-	3,0 %	-	-
Invesco	6,4 %	0,6 %	4,9 %	9,2 %	-	-	2,5 %	-	-
Horizon	6,5 %	2,5 %	3,3 %	5,6 %	-	-	4,8 %	9,0 %	8,0 %
PIMCO`s	4,4 %	2,0 %	-	-	-	-	4,0 %	-	-
Twosigma	5,2 %	1,0 %	4,0 %	-	-	-	2,5 %	7,5 %	-
BMO	7,1 %	2,0 %	2,3 %	5,4 %	-	-	6,0 %	9,0 %	-
MFS	2,3 %	1,0 %	3,3 %	6,6 %	-	-	3,1 %	-	-
Average	**5,3 %**	**1,4 %**	**3,6 %**	**5,8 %**	-	-	**3,8 %**	**9,1 %**	**7,4 %**
Average last 10 years	9,2 %	2,9 %	-7,3 %	6,7 %	6,3 %	12,4 %	5,7 %	13,8 %	9,4 %

Source: (BlackRock, 2021; J.P.Morgan, 2021; Benz, 2021; Reasearch Affliliates, 2021; Vanguard, 2021; Northern Trust, 2021; Robeco, 2020; Invesco, 2021; Horizon, 2020; Pimco, 2020; Twosigma, 2020; BMO, 2020; MFS, 2021)

Investment-grade bonds are trading at an all-time low due to low starting yields (J.P.Morgan, 2021). The forecasts are down over 50% compared to the historical return on investment-grade bonds (figure 20). For the next decade, no analyst predicts such high prices. The economy is still in a recovery phase after the covid-19 crisis; therefore, analysts are not expecting an increase in government bonds in the years to come. Analysts' average expected return of global bonds yields to 1.4% (table 11). This is considerably lower than the previous two years, where investment-grade bonds have yielded an 8 % in annual returns[14]. 10-year US Government bonds are trading at 1.6%; hence, the investment-grade bond risk premium is expected to be negative.

Considering global public equity markets, the consensus reflects outlooks of stabilizing worldwide growth and valuation (Invesco, 2021; Twosigma, 2020; BMO, 2020). High

[14] Returns are from the Bloomberg Barclays Global Aggregate Index, a flagship measure of global investment grade debt.

valuations in equity markets, lower profit margins and a broader focus on stakeholders rather than just shareholders can result in lower returns over the next ten years compared to the previous decade (J.P.Morgan, 2021). Due to high GDP growth, all analysts expect equities in emerging and developed markets to exceed US equity markets' returns. The consensus for expected return for global equities is 5.3 %, which is low compared to the past ten years of 9.2% (figure 20).

During the last decade, commodities have struggled, with and an annual return of average negative 7 %. However, experts predict that the cycle is about to reverse, with a consensus of 3.1% yearly return outlook for the next decade (figure 20). This is primarily driven by supply constraints, especially from capital expenditure restrains in the oil sector and production constraints across many different commodities (J.P.Morgan, 2021). J.P.Morgan (2021) sees further potential in the gold price, driven by growing demand from China and India, in addition to negative real interest rates and high interest from institutional investors seeking protection. Northern Trust (2021) highlights that natural resources are reasonably priced relative to wider equity markets and should still protect against unexpected inflation, which they see as a real risk.

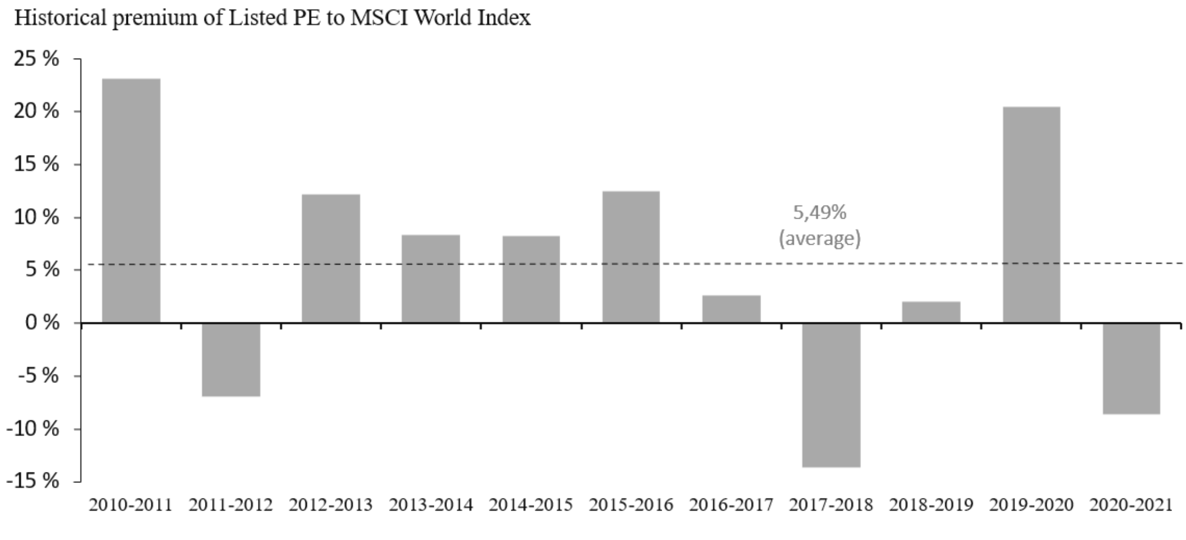
REITs' expectations of 5.8% in annual return are somewhat lower than their historical average of 6.7%. REITs receive higher expectations than stocks from experts (figure 20). The biggest reason for this is the benefits from leverage, and that most of them are trading at discounts on their underlying real estate (J.P.Morgan, 2021; Vanguard, 2021). Overall, especially both J.P.Morgan and Blackrock are bullish on alternative assets in the upcoming decade.

J.P.Morgan is one of the analysts who are optimistic about High Yield's outlook and predicts an annual expected return of 4.8 %. Northern Trust also highlights their strong expectations for High Yield and places it as an alternative to global equities. They expect a return of 5.8 %, which is higher than their global equity forecast of 4.9%. However, the average of the experts forecast for High Yield bonds is 3.8 % (table 11).

Regarding Listed PE, this niche instrument does not get covered in the expert's forecast. Determining the expectations for this instrument, the Capital Asset Price Model (CAPM) is applied. Instead of looking at historical returns, this estimate will consider the low risk-free rate we have today and the risk premium from today's equity expectations.

An important parameter in CAPM is the beta. The traditional beta between listed PE and the MSCI world index is 1.10, calculated from monthly returns from 2008 until 2021. However, Lahr and Herschke (2009) find that Listed PE is influenced by illiquidity in the short run, as discussed in chapter 3.4. Calculating the traditional beta coefficient for an illiquid stock may get an unrealistically low beta; therefore, the Dimson beta will be applied. The sum of beta coefficients from regressions of excess stock returns on the current and lagged market premium results in a beta of listed PE of 1.29. The first lagged market returns are significant predictors for future returns for listed PE due to illiquidity in the stock. Further, we implement the ten-year US government bond as a risk-free rate (1.6%) and expected risk premium [15]for stocks (3.7%), resulting in an expected return of 6.23% for listed PE.

The past ten years, Listed PE have outperformed Stocks with 5.49 % each year in terms of annual return (figure 21). CAPM predicts a 0.96 % premium over the next decade, which may look low compared to the last ten years. However, the average premium above stocks over the previous five years has been only 0.57%, which is more in line with the predicted premium. Given the high expectations for non-listed private equity over the next decade, which is illustrated in table 11, it is reasonable to anticipate that listed private equity will likewise outperform stocks over the next decade.

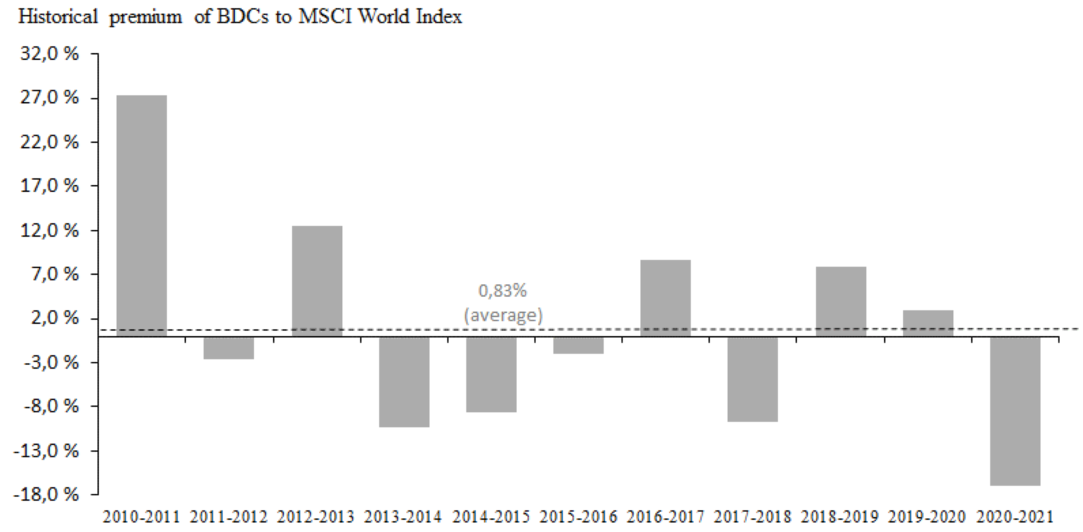


Source: (BlackRock, 2021 J.P.Morgan, 2021. Benz, 2021. Research Affiliates, 2021. Vanguard, 2021. Northern Trust, 2021. Robeco, 2020. Invesco, 2021. Horizon, 2020. Pimco, 2020. Twosigma, 2020. BMO, 2020. MFS, 2021)

Figure 26 - Historical premium of listed PE to MSCI world index

[15] The risk premium for stocks is the average stock return forecast (table 11) minus the risk-free.

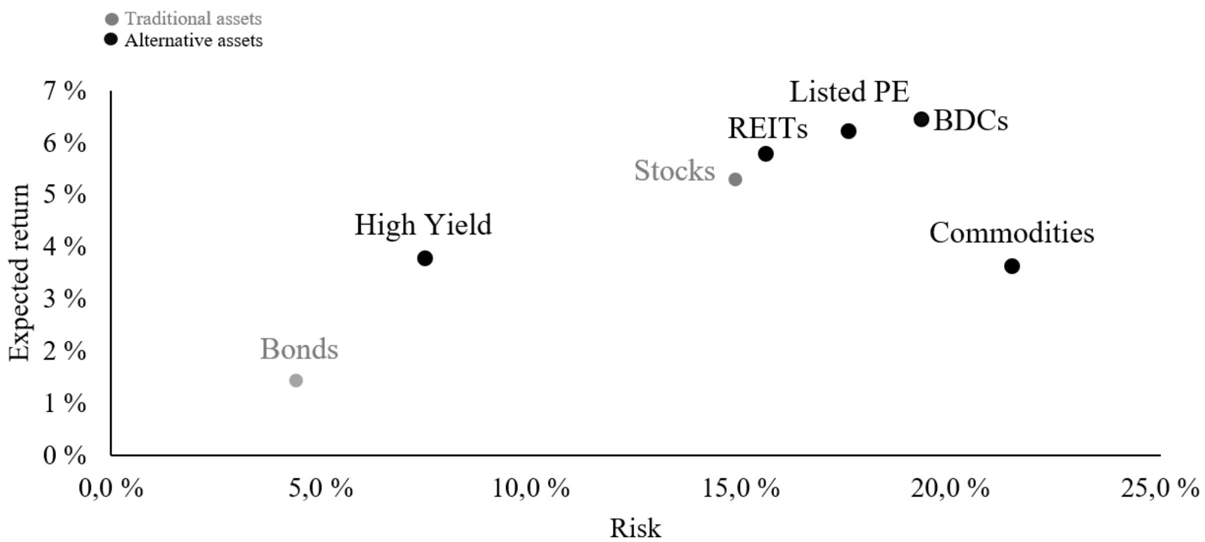
The same methodology is applied to BDCs; they achieve a Dimson beta of 1.31, resulting in an expected return of 6.45%, a premium of 1.16 % over the expected stock return. This is somewhat consistent with their historical 10-year average BDCs premium over stocks of 0.83 % (figure 22).



Source: (BlackRock, 2021 J.P.Morgan, 2021. Benz, 2021. Research Affiliates, 2021. Vanguard, 2021. Northern Trust, 2021. Robeco, 2020. Invesco, 2021. Horizon, 2020. Pimco, 2020. Twosigma, 2020. BMO, 2020. MFS, 2021)

Figure 27 - Historical premium of BDCs to MSCI world index

Figure 23 displays the expected risk-return profile of the alternative and traditional assets. The risk is the last 10-year standard deviation.



Source: (BlackRock, 2021 J.P.Morgan, 2021. Benz, 2021. Reasearch Affliliates, 2021. Vanguard, 2021. Northern Trust, 2021. Robeco, 2020. Invesco, 2021. Horizon, 2020. Pimco, 2020. Twosigma, 2020. BMO, 2020. MFS, 2021)

Figure 28 - Risk-return of expected return for traditional and expected assets

7.4 Portfolio Construction

When deciding to invest in alternative assets, it is not sufficient to look at their individual historical or expected performances. The following chapters would examine how alternative investments could be structured in a portfolio. Both, portfolios using capital market predictions (CMP) from chapter 7.3 and historical data presented in chapter 7.1 are built and analysed to thoroughly investigate optimal allocations in each asset.

7.4.1 Capital Market Prediction Portfolios (CMP)

According to financial theory, the optimal portfolio is the one that maximizes the Sharpe ratio, also known as the tangent portfolio. Figure 29 visualizes the tangent portfolio based on the historical[16] standard deviation and analyst expected returns for the different assets.

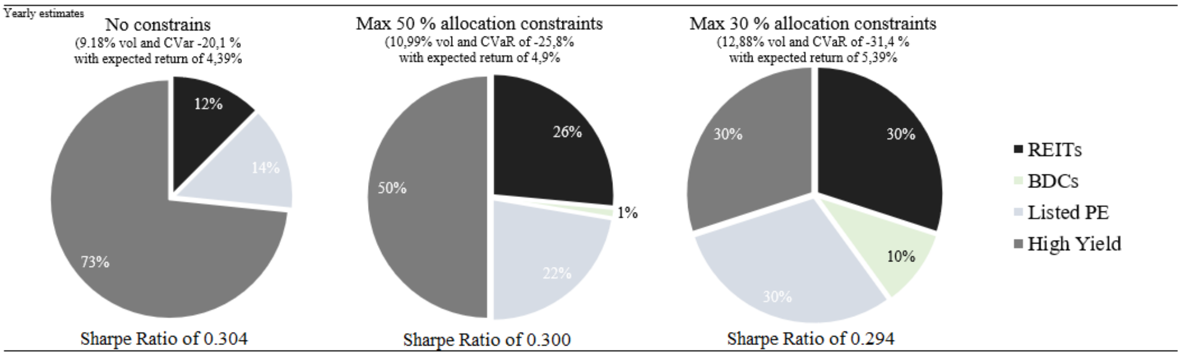


Figure 29 - Tangent portfolio asset allocation using a mean-variance for all assets – including alternatives (Data source: Bloomberg)

The tangent portfolio allocates 73 % towards High Yield Bonds, 12 % towards REITs, and 14 % to Listed PE. The portfolio would have an expected return of 4.39 %, with a yearly standard deviation of 9.18 % and CVaR of 20.1%. However, since the allocation is strongly weighted towards high-yield, other portfolios are presented in which single asset weights cannot exceed 50% and 30%. Those would result in more diversified portfolios but at the cost of higher SD and CVaR. The three portfolios, all excluding traditional assets. JP Morgan

[16] The correlation, CVaR and standard deviation(vol) is calculated based on the last 10 years (2011 until 2021)(Bloomberg, 2021).

emphasized in their capital market expectations that bonds have lost their ability to provide portfolio protection in market downturns. In addition, when there are low market expectations for stocks, all investors should start looking towards alternatives to seek excess returns, fixed income, and diversification (J.P.Morgan 2021). Figure 30 presents the expected efficient frontier for the all-asset portfolio and compares it to the traditional frontier, with respective tangent lines applied.

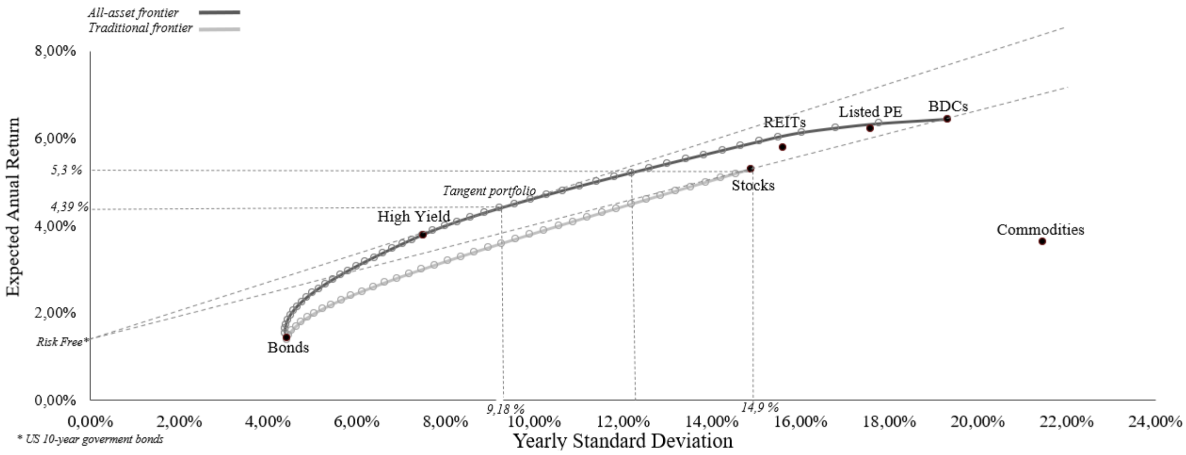


Figure 30 - Expected efficient frontier for the all-asset and traditional portfolio (Data source: Bloomberg)

Since the tangent line for the all-asset frontier has a higher slope than the traditional frontier, it has a higher Sharpe ratio. Additionally, one can see that the all-asset frontier is higher than the traditional frontier, implying that diversifying a portfolio with alternatives can either raise the return for the same level of risk or reduce the risk for the same level of return. Figure 31 visualizes the different weights through the all-asset efficient frontier.

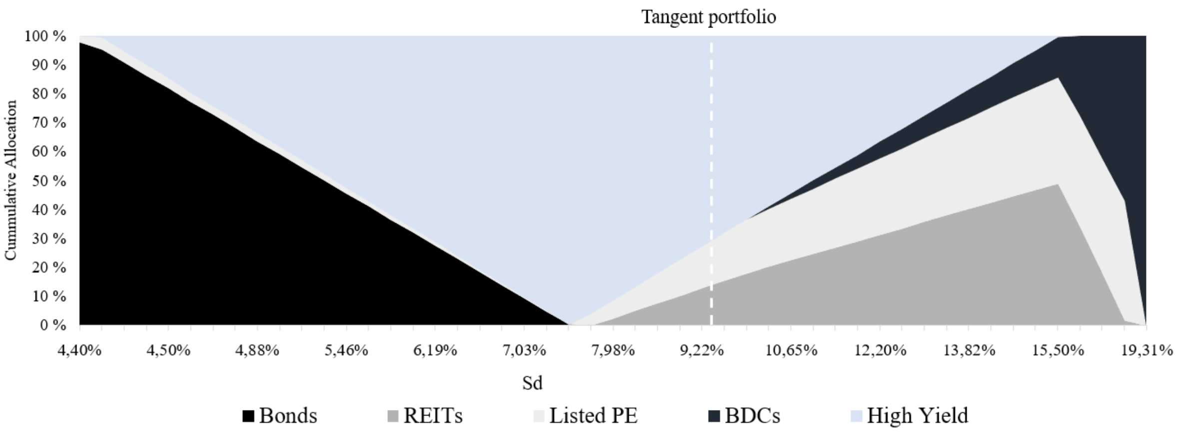


Figure 31 - different weights through the all-asset efficient frontier

(Data Source: Bloomberg)

One can see that until 7 % standard deviation, the tangent line consists of bonds, listed PE, and High Yield. After 7 %, bonds get swapped with BDCs and REITs while Listed PE and High yield remain as key assets.

The tangent portfolio presented above only includes alternative assets. The majority of retail investors will most likely avoid this portfolio because alternatives are new and unfamiliar. Present under in figure 32 is a strategy for retail investors to incorporate alternatives into their allocation. Three different portfolios were constructed with the intention to fit both conservative and aggressive investors. The conservative portfolio consists of at least 70 % towards fixed income, while the rest can be allocated towards equity. The balanced portfolio should constitute 50 % in fixed income and 50 % towards equity, and last, the aggressive portfolio has 10 % in fixed income and 90 % towards equities. Figure 32 presents an allocation the retail investor can use compared to the traditional portfolio by reallocating 50-60 % of the total portfolio towards alternatives. The portfolio characteristics of a conservative, balanced or aggressive are still considered when finding an alternative portfolio to the traditional. In either case, adopting an alternative investment strategy increases the portfolio's expected return and decreases the risk. The optimal allocation has been found through the mean-variance framework to maximize the return at given constraints.

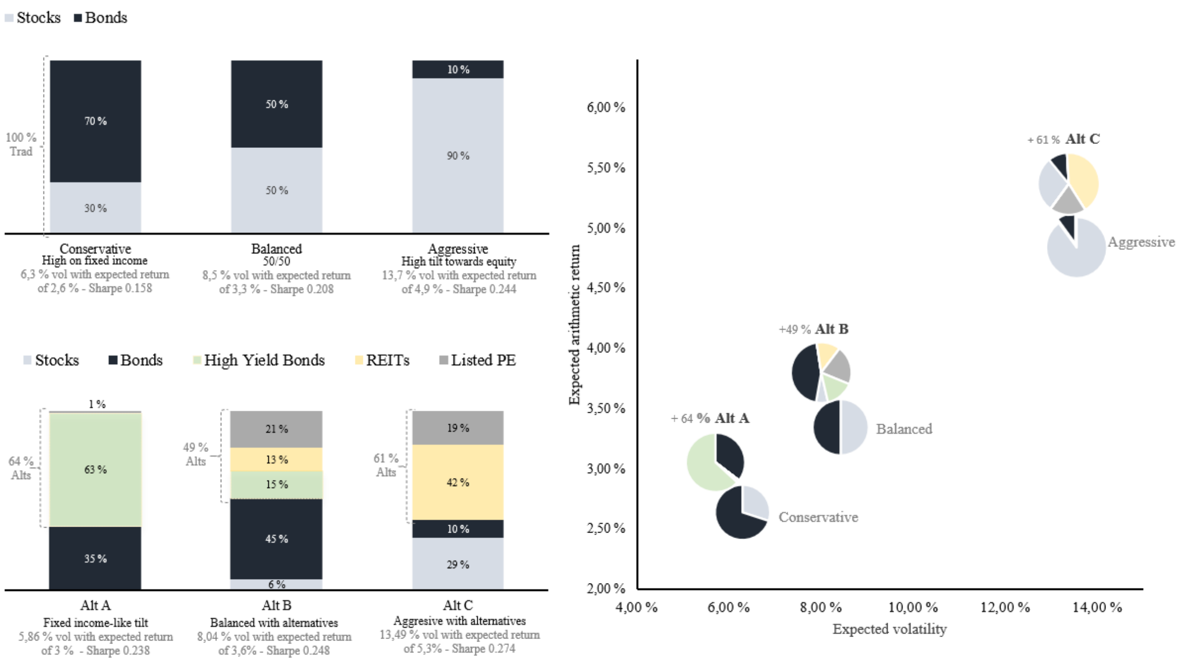


Figure 32 - Strategy for retail investors to incorporate alternatives into their allocation (Data source: Bloomberg)

The traditional conservative portfolio consisting of 70% bonds and 30% stocks has an expected return of 2.3 % with a volatility of 6.3 %. Reallocating stocks with High Yield and 1 % towards listed PE increase the expected portfolio return to 3% with a volatility of 5.8%. The same applies to the balanced and aggressive portfolios. Including High Yield, REITs, and Listed PE increases the return while lowering the standard deviation.

Further, figure 33 shows the tail-risk of the portfolios through CVaR, which can be viewed as the worst-case loss at 95 % confidence level for the portfolios.

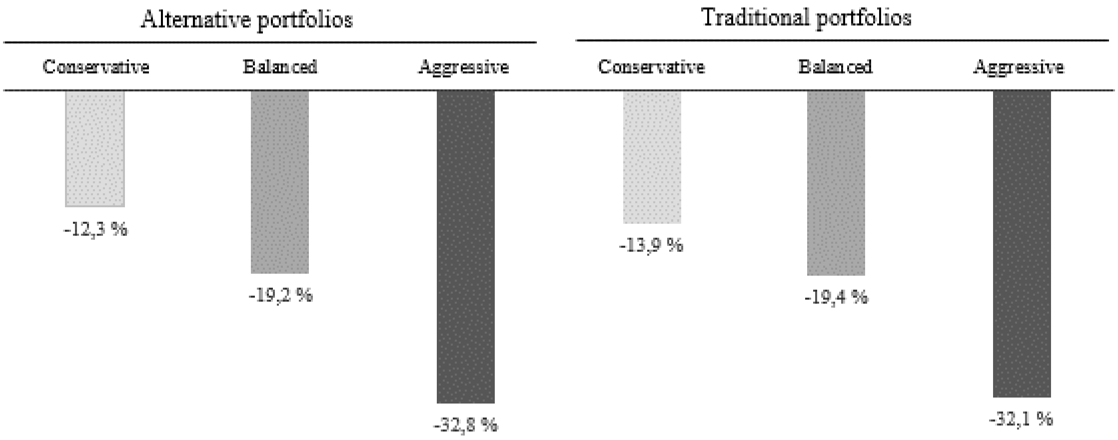


Figure 33 - yearly CVaR for alternative and traditional portfolio (Data Source: Bloomberg)

The tail risk for the conservative and balanced portfolio is lower for the one that's diversified with alternatives. However, the aggressive alternative portfolio has a potential loss of negative 32.8 % at a 5% significance level, compared to the standard portfolio's loss of negative 31.1 %.

The portfolio presented above still includes traditional assets; therefore, the full potential of enhancing the expected returns is not exploited. Investors willing to accept a higher allocation to alternatives can achieve higher expected returns than what is presented in figure 32. However, when investing solely in alternatives, retail investors must tolerate more risk.

Figure 34 below shows the same traditional portfolio of conservative, balanced and aggressive strategies, although the stock-bond portfolio is now compared with a portfolio consisting solely of alternatives. Investment-grade bonds are switched out with high yield, and stocks have been replaced with BDCs, Listed PE, and REITs. The conservative to aggressive portfolios have been allocated using the mean-variance framework to maximize returns under the limits of the fixed-income allocation constraints.

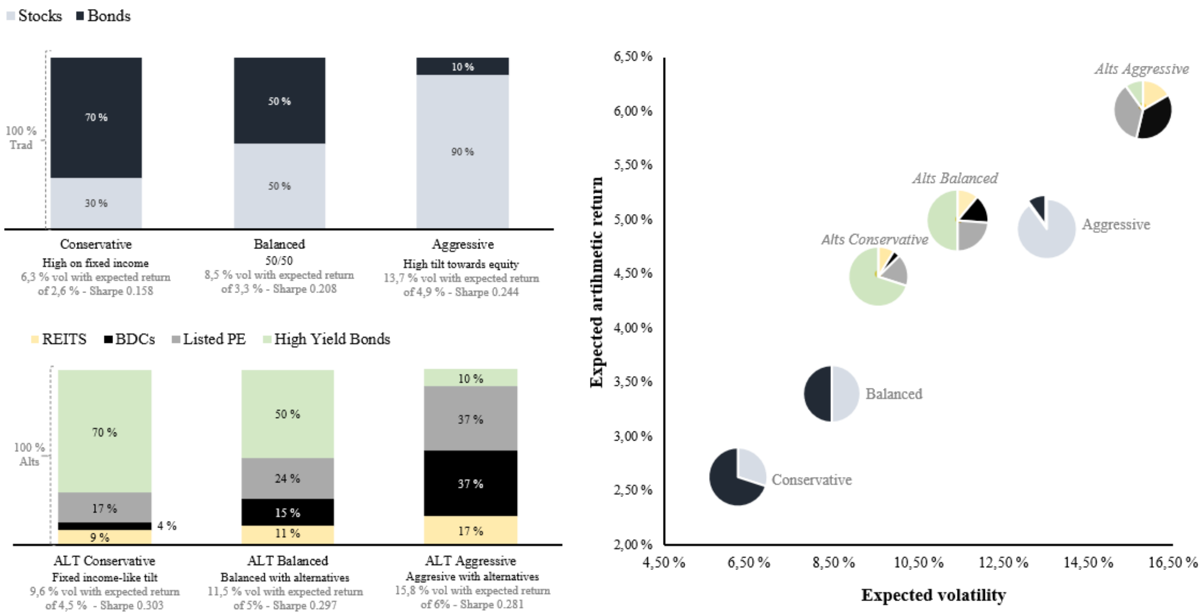


Figure 34 - Alternative portfolios for retail investors (Data Source: Bloomberg)

The ALT conservative portfolio comprises 70% high yield bonds, 17% listed private equity, 4% BDCs, and 9% REITs. It generates a 4.5 % return, has a SD of 9.6 %, and a Sharpe of 0.303. Compared to the traditional portfolio, which was 70% stocks and 30% bonds, the ALT conservative portfolio has a far higher Sharpe ratio.

Commodities do not get allocation into any of the portfolios presented here. Even when analysts expect positive returns for the next decade, the risk associated with commodities has historically been very high, and therefore the mean-variance framework does not include it.

Using the capital market predictions, alternatives seem to increase diversification, boost return, and can reduce portfolio risk compared to the traditional portfolio.By going all-in on alternatives, one obtains a high expected return, however at the cost of risk. Figure 35 below presents the CVaR for each of the presented portfolios in figure 35.

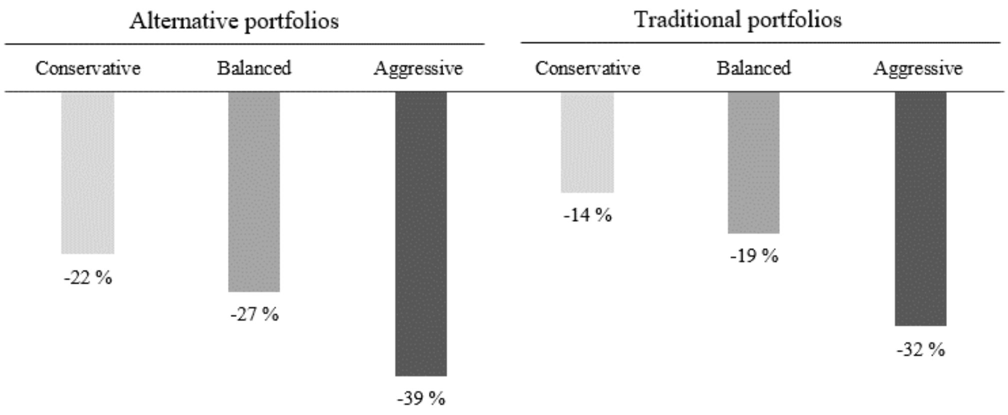


Figure 35 - yearly CVaR for alternative and traditional portfolio.

For the conservative traditional portfolio, a worst-case yearly loss is minus 14 % at 5 % confidence level. However, it is 60 % higher for the conservative alternative portfolio and even higher than the CVaR of the balanced portfolio of traditional assets. When it comes to the aggressive portfolio both the traditional and alternative have over minus 30 % in worst-case scenario losses.

7.4.2 Historical Portfolios

The following chapter examines historical portfolios. A famous saying is that “past performance is not an indicator of future success.” Although the future is unknown, quantitative models, historical models, and psychic models have tried to find the perfect formula to predict the future of returns, but with low success (Stibel, 2009). However, while analyst forecasts are based on subjective opinions, historical returns are not. We will analyze the historical efficient-frontiers that include both the financial and covid-19 crises to understand how these assets work together in turbulent times. We will also look at the efficient historical frontier for a period without any mayor crises.

A drawback of this historical point of view is that it results in an unnaturally large allocation to bonds as a result of their strong performance over the previous 10-15 years. As the bond market has witnessed enormous changes during the most recent years, similar returns are not expected for the next decade. Therefore, the historical tangents portfolio from the mean-variance analysis will not be discussed in-depth, as it gives a somewhat unrealistic picture of the expected optimal portfolio for the upcoming years. However, the historical tangent and minimum risk portfolio calculated on historical returns can be viewed in the appendix 1-4. To reduce some of the biases with the mean-variance framework, mean CVaR and Bayes-Stein estimators have been added to the analysis for portfolio construction.

Historical Portfolios Including the Financial and Covid 19 Crises

Mean-Variance

Figure 36 presents how the individual assets perform compared to the efficient historical frontier and compare the frontiers between the traditional and the all-asset portfolios. This portfolios have constructed on data from 2007 until 2021. More detailed statistics can be seen in appendix 1.

Stocks, Listed PE, and BDCs have had similar returns since 2008. However, stocks have much lower volatility. As a result, and implied by the efficient frontier, including alternatives from 2008 until 2021, does not significantly increase the returns or decrease the volatility. Although the tangent and minimum risk portfolio is highly skewed towards bonds, it still allocated towards listed PE and High yield over stocks.

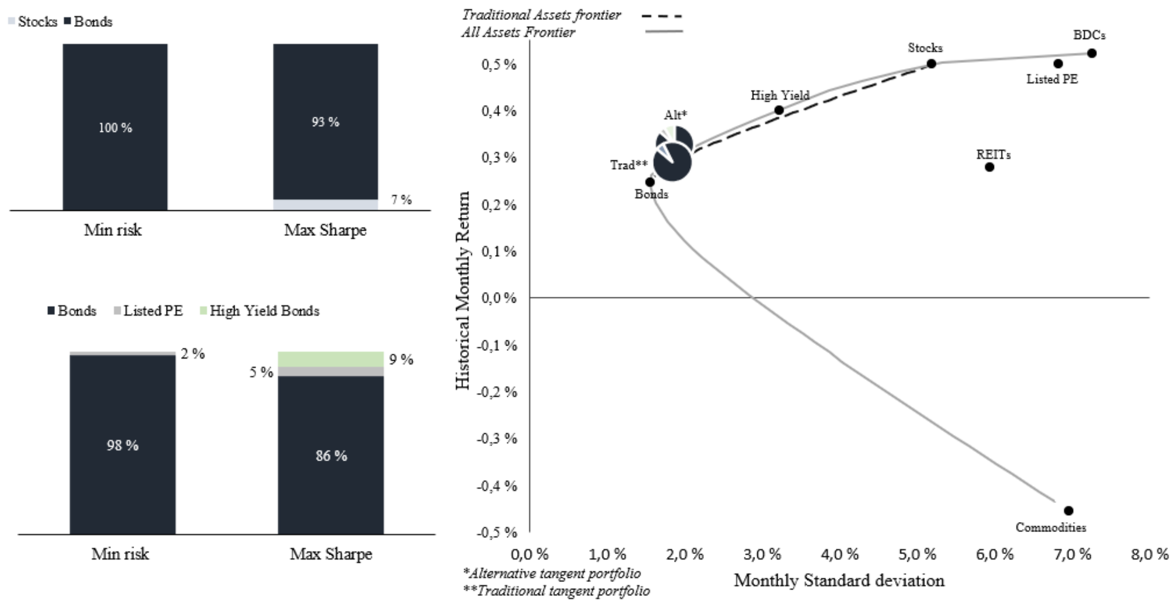


Figure 36 - Efficient frontier for the traditional and all asset portfolio, from 2007 until 2021. (Data Source: Bloomberg)

CVaR and Bayes-Stein

The following section estimates portfolios and the efficient historical frontier with CVaR and Bayes-Stein estimators. These approaches are discussed in detail in the theory and method chapters and comparison of all the portfolios can be seen in appendix 1.

Comparing the CVAR minimum risk (MVP) and tangent portfolio with mean-variance portfolios, listed PE is replaced with High Yield to minimize risk and maximize Sharpe. The explanation behind this is due to the significant price decline Listed PE suffered during the Great Recession, resulting in a high CVaR. Both the historical MVP and tangency portfolio include allocations of 90 % or more towards bonds. Looking forward, based on the expected optimal portfolio presented in chapter 7.4.1, bonds should not receive over 90 % in allocation. On the contrary, the CVaR estimation method emphasizes High Yield as an attractive substitute to stocks and listed PE for risk-averse investors as it lowers the overall tail-risk.

Figure 37 visualizes the CVaR efficient frontier and its weights. The SD has been swapped with CVaR on the X-axis in comparison to figure 36. A notable difference is that listed PE is excluded from every allocation on the efficient frontier using CVaR compared to SD.

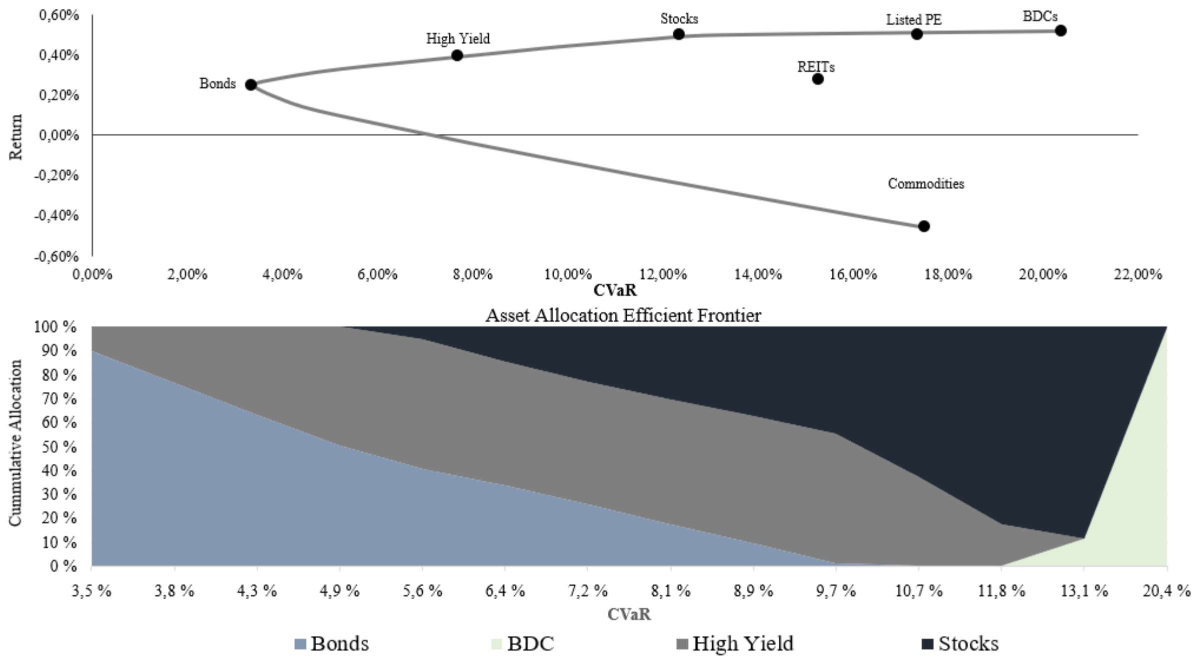


Figure 37 - CVaR efficient frontiers and efficient frontier weights, estimation period: 2007-2021 (Data Source: Bloomberg)

The Bayes-Stein estimator results in similar results as the traditional mean-variance framework. Only difference is that Bayes-Stein appears more conservative, allocating more towards bonds. However, the trend is the same as with SD; listed PE seems to have the best diversification benefits together with Bonds. Portfolios can be seen in appendix 1.

Geometric Mean Maximization and Maximum Diversification portfolios

This section will cover the historical GMM and MD portfolio. The GMM will look beyond risk and focus on maximizing the cumulative return for an investor. The MD portfolio, on the other hand, focuses on finding an optimal allocation that maximizes diversification and do not consider returns.

According to the GMM portfolio, an allocation of 62.5 % in stocks and 37.5 % in high yield would maximize the return from 2008 until 2020. To achieve a more diversified portfolio, weight constraints are placed on the GMM. Contraining weights to 30%, Listed PE, BDCs, and bonds get included with small allocations. In the MD portfolio, bonds, listed PE and commodities are the assets that have the best diversification benefits together. The GMMs and MD portfolio is visualized in figure 38 below. For more detailed statistics, see appendix 2

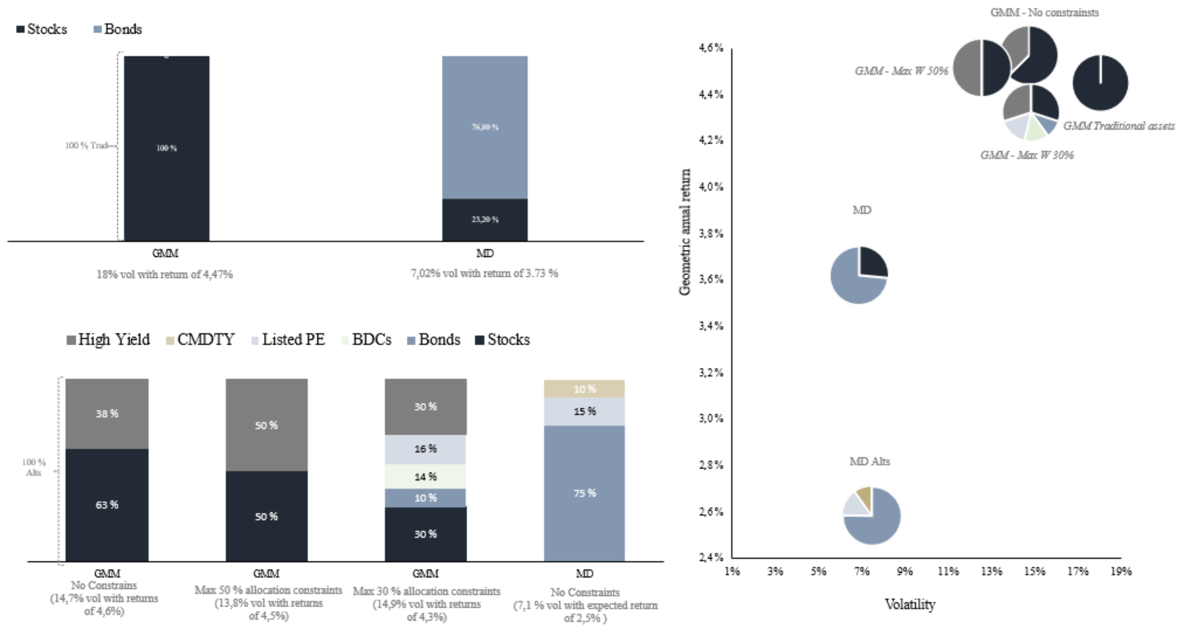


Figure 38 – Historical MD and GMM portfolios from 2007-2021 (Data Source: Bloomberg)

The optimal portfolios found in GMM focus on volatile assets such as BDCs listed PE and stocks rather than less volatile assets such as bonds. Between 2007 and 2021, the monthly SD of the tangent portfolio from MV was 1.63%, while for GMM, it was 4.25%; however, with higher risk, the GMM also has a much higher return.

Comparing the all-asset GMM portfolios and the benchmark GMM portfolios is very straightforward. As the goal is wealth maximization, the benchmark portfolios allocate 100% to stocks in each scenario on each estimation period. According to our results, historically, adding alternative assets seems to enhance the cumulative returns of a portfolio.

Between 2007 and 2021, the portfolio that would be most diversified includes 75.2% in bonds, 14.9% in listed PE, and 9.9% in commodities. The diversification ratio of such a portfolio is 1.41. The benchmark portfolio's equivalent measure is 1.19, meaning that all-asset portfolios outperform stock-bond portfolios in terms of diversification ratio (Appendix 14). The benchmark and all-asset portfolios each have an equal bond allocation of about 75%; however, the remaining 25 % in the alternative portfolio is allocated between listed PE and commodities, while the benchmark allocates it to stocks. This shows that commodities and listed PE have been better diversifiers in the past.

Although adding alternatives delivers great diversification benefits, they do not necessarily enhance the general performance of the portfolios. One of the biggest reasons for this might be the inclusion of commodities in the all-asset MD portfolios. From Table 16, we see that commodities have an extremely low correlation with every other asset class, making it a great diversifier, which must be why it is included in the MD portfolio. However, it also has the lowest average return of all the asset classes, lowering the portfolios' overall returns. It is yet to be noted that with analysts predicting annual returns of 3 % for commodities over the next decade, they could compete with other investments as a great diversifier in optimal portfolios.

Optimal Portfolios for the Non-Crisis Period

The following chapters investigate optimal historical portfolios for a period without any significant economic crises between 2009 and 2019.

Mean-Variance

Appendix 3 presents and compares the traditional and all asset portfolios using the mean-variance, Mean-CVaR, and Bayes-Stein estimators.

The portfolio that minimizes SD has 89.5% in bonds and 10.5 % in listed PE. So listed PE again earns better diversification benefits in a portfolio with bonds than stocks. The tangent portfolio still allocates towards High Yield, Listed PE, and bonds, respectively, 10.2%, 41.4%, and 48.5%.

Compared to the capital market predictions portfolios, the tangent portfolio is very similar, and the only difference is that bonds get swapped with REITs. The tangency line has been added to the two efficient frontiers in figure 39. The slope is greater for the alternative assets, signaling that the Sharpe ratio is higher.

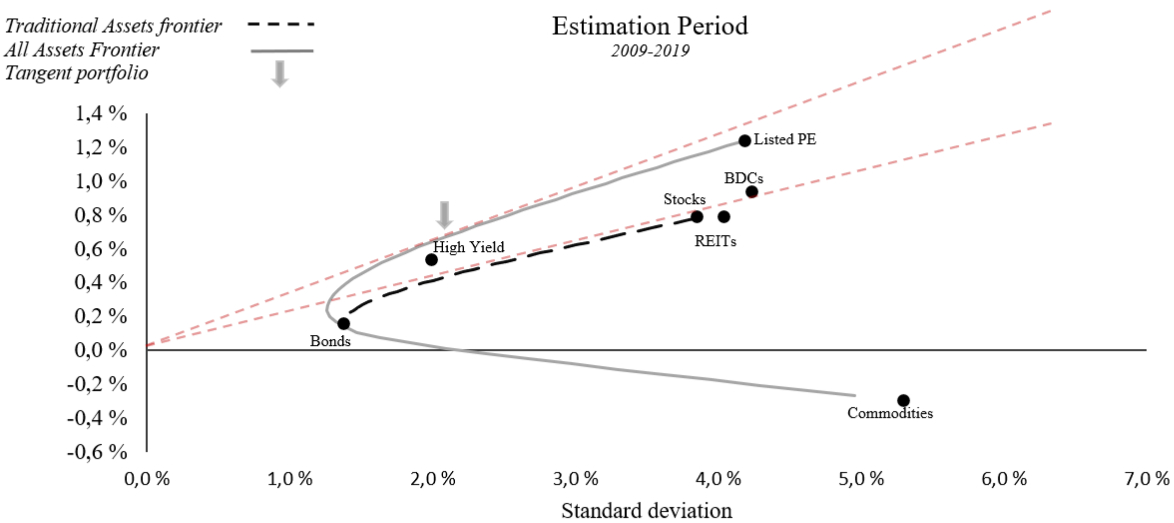


Figure 39 - Comparison of CVaR Efficient Frontier between the traditional and the all asset portfolio. Estimation Period: 2009-2019
(Data Source: Bloomberg)

Figure 40 compares the tangent portfolios from the traditional mean-variance framework discussed in this paper.

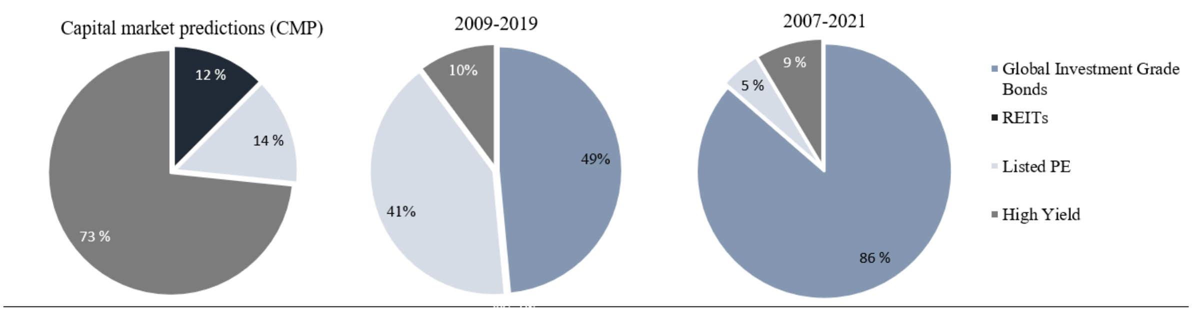


Figure 40 - Summary of tangency portfolios (Data Source: Bloomberg)

All of the three portfolios are highly skewed towards fixed-income securities. The historical tangent portfolio places the most weight on traditional bonds. However, the CME tangent portfolio sets 73 % towards high yield. All three portfolios include a portion of listed PE and high yield, which creates additional trust in those assets.

CVaR and Bayes-Stein

The traditional minimum risk portfolio, consisting of 7 % in stocks and 93 % in bonds, ends in a monthly CVaR of 3.3 %. Shifting towards alternative assets, with a combination of 82 % in bonds and 18% in listed PE, the monthly CVaR falls to 2.64 %.

The diversification benefits and enhanced returns stand out by including alternative assets in the portfolio. At a monthly CVaR of 8 %, a combination of stocks and bonds would yield a monthly return of 0.74 %. Including alternative assets to the portfolio would increase the return to 1.15 % at the same CVaR level of 8 %. This combination would be 83 % in listed PE, 6 % in BDCs, and 11 % in High Yield bonds. In other words, both traditional assets would be excluded from that portfolio.

Figure 41 compare CVaR Efficient Frontier between the traditional and the all asset portfolio.

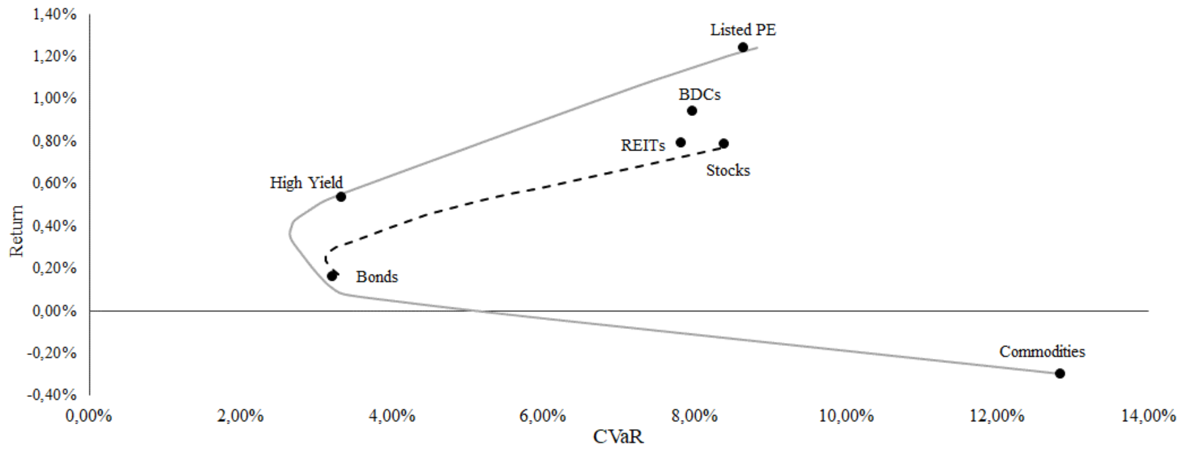


Figure 41 - Comparison of CVaR Efficient Frontier between the traditional and all asset portfolios. Estimation Period: 2009-2019 (Data Source: Bloomberg)

The Bayes-Stein estimators' tangency portfolio results in a portfolio of 65 % in bonds, 29 % in listed PE, and 6 % in High Yield. Thus, the trend is clear. According to the Bayes-Stein model, REITs, Commodities, and BDCs are not preferred assets which are in line with the results found in the traditional MV analysis. The mean-CvaR model, on the other hand, includes BDC as an option for the more risk-seeking investor.

Figure 42 visualizes how the all-asset portfolio outperforms the traditional portfolio to maximize historical Sharpe from 2009-2019 and 2007-2021.

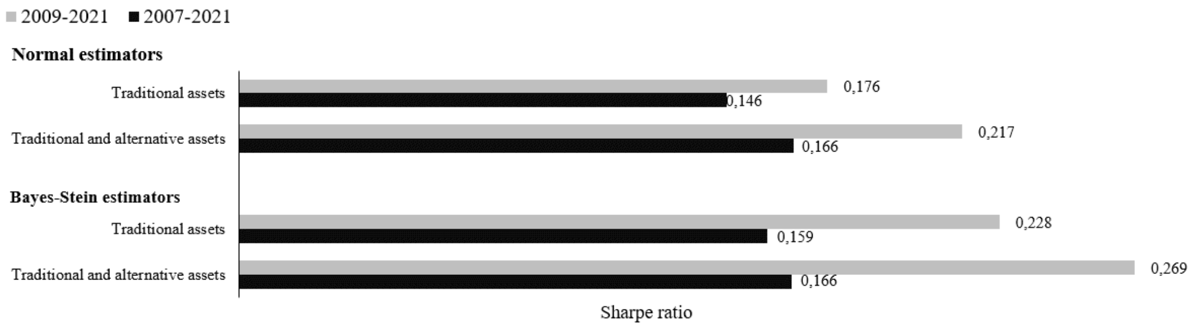


Figure 42 - Sharpe-Ratio comparison between the tangent portfolio of stocks and bonds and the all-asset tangent portfolio (Data Source: Bloomberg)

Geometric Mean Maximization and Maximum Diversification Portfolios

The portfolio that would maximize the total return from 2009 until 2019 is 100% in listed PE. Additionally, if we constrain these weights, BDCs are next to receive allocations. Finally, with weight constraints low enough, stocks and REITs receive the remaining 10 % (Figure 43). For the MD portfolios, the asset choices remain the same regardless of the estimation period. The all-asset MD portfolio beats the benchmark in terms of diversification ratio during every period.

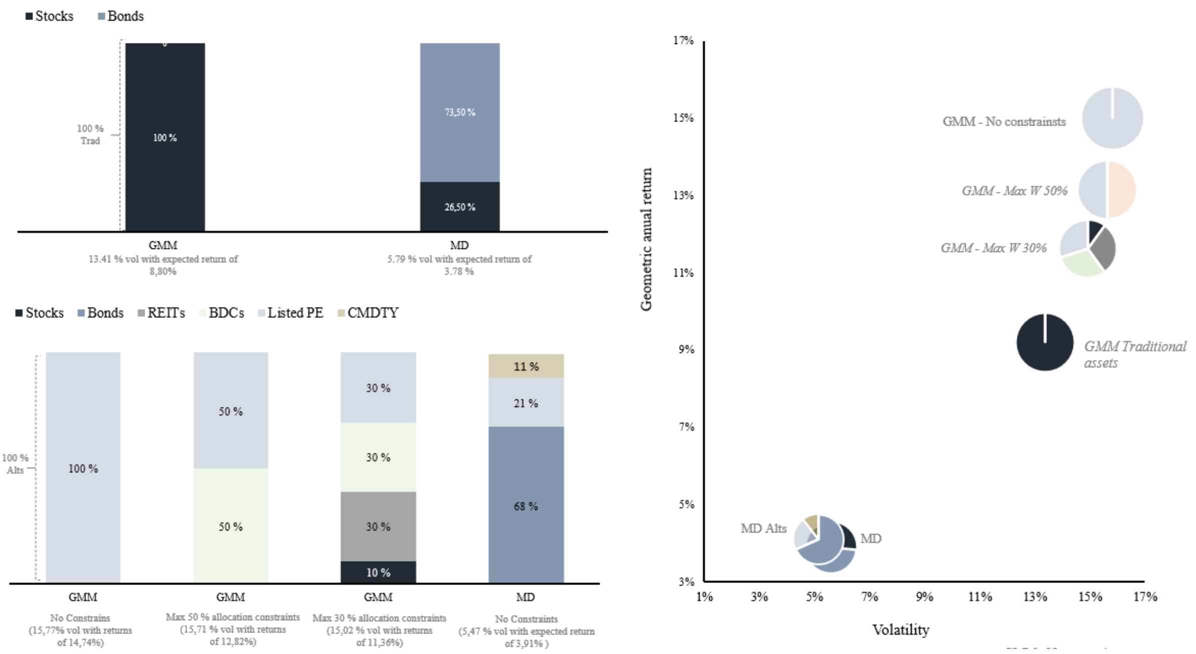


Figure 43 - Historical MD and GMM portfolios from 2009-2019 (Data Source: Bloomberg)

For the GMM portfolio, the geometric return of the portfolio falls if weight constraints are set, which can be seen in figure 43. However, constrained portfolios still beat the benchmark portfolios in terms of Sharpe and returns.

Listed PE dominating the GMM portfolio after the Great Recession, further BDCs seem to be a solid addition for a wealth maximizing investor. Although they rarely have performed best, they tend to be the second-best choice in multiple scenarios (Appendix 4). On the contrary, REITs seem to be a poor return enhancer in a GMM portfolio, receiving allocations only in the most constrained scenarios, and even then, very little.

The monthly Sharpe Ratio of this portfolio is 0.295. Comparing to the tangency portfolio from the mean-variance framework and CVaR models during the same estimation period, it is 0.025 lower. Thus, it is not the most efficient portfolio in the investment universe.

This can also be seen from Figure 44, which demonstrates the position of the GMM and MD portfolios on the historical mean-variance efficient frontier.

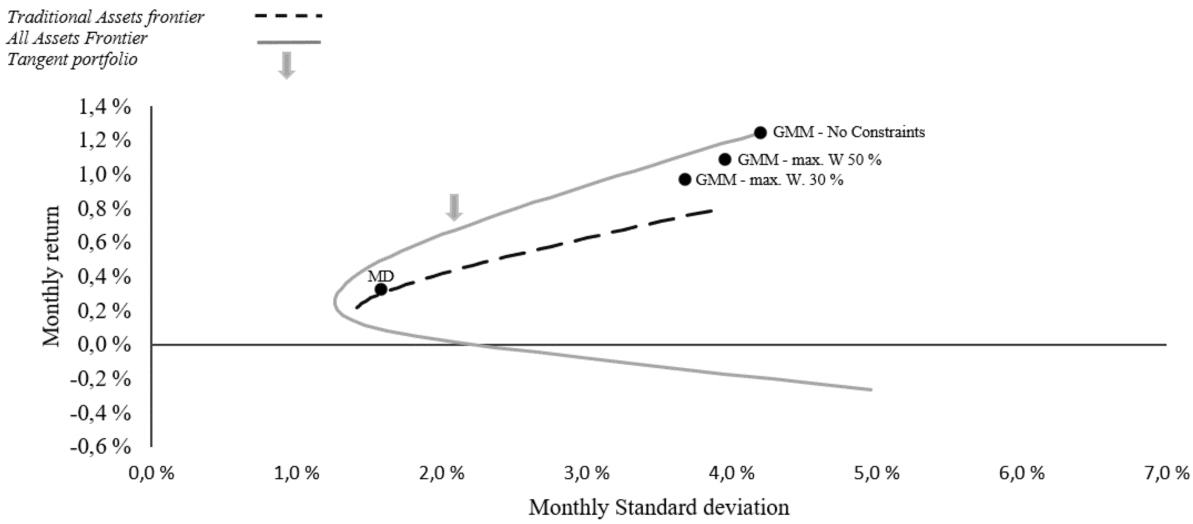


Figure 44 - position of a GMM portfolio on the MV efficient frontier. Estimation perido, 2009-2019 (Data Source: Bloomberg)

For the portfolio that maximizes diversification, adding alternatives provides better diversification and enhances risk-adjusted returns than the MD for traditional assets. Interestingly, these results show that the same three assets receive all of the allocations regardless of the time period. Thus, it can be said that in the given investment universe, listed PE, bonds, and commodities provide the highest diversification benefit in a portfolio. Additionally, it seems that the covariances between these asset classes remain constant regardless of the estimation period and whether crises are included or not. Hence, these portfolios hardly qualify as optimal for most investors despite their great diversification benefits, as can visually be interpreted in Figures 43 and 44.

7.5 Portfolio Simulation

Philippe Jorion (1992) proposes a simulation method for determining the optimal portfolio distribution. A deeper explanation is presented in chapter 6.2. The rationale is to gain a more comprehensive understanding and remove some of the misconceptions associated with mean-variance analysis. Consider this analysis to be a sensitivity analysis of the historical tangent portfolios discussed in this chapter.

The optimal allocation distribution for each asset is seen in figure 45 and the average weights are displayed in figure 46. In this analysis, 2000 tangent portfolios are simulated, implying that each distribution comprises a total of 2000 observations. The histogram portrays the weights assigned to each asset in the simulated portfolios. Subsequently, it divides the weights into intervals ranging from 0 to 20%, 20 to 40%, and 80 to 100% on the X-axis.

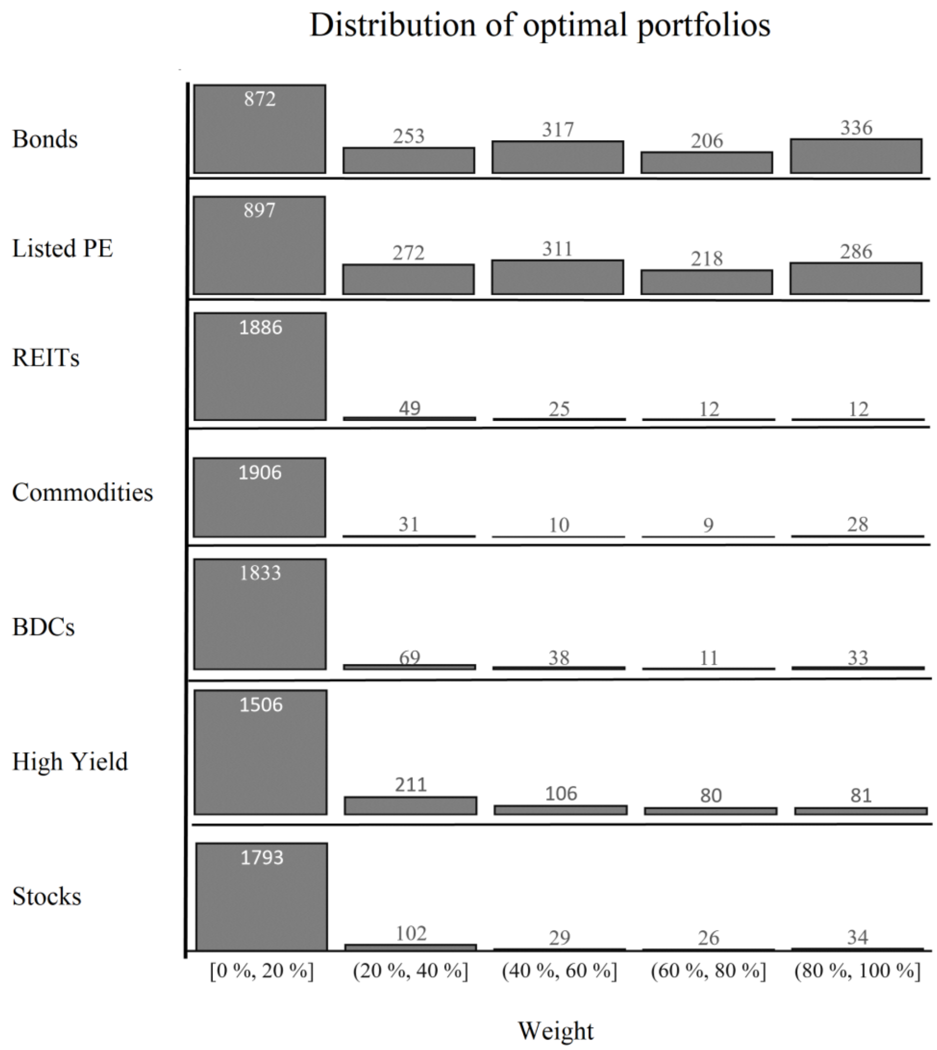


Figure 45 - Distribution of Optimal weights of Traditional and Alternative Assets (Data Source: Bloomberg)

For all assets, the distribution is heavily skewed to the left. This means that all assets had an allocation in the tangent portfolio between 0 and 20 % in most cases. In 83% of the 2000 simulated portfolios bonds had an asset allocation below 80%. The tangent portfolio estimated on the entire sample period (2007-2021) had an allocation of 86% to bonds, but, in light of the simulation findings, that allocation seems inaccurate. Bonds have an average weight of 36% in the 2000 simulated portfolio, which is comparable to the allocation bonds get in the 2009–2019 tangent portfolios. However, still high compared to the portfolios calculated on future predictions, where bonds receive zero percent.

Surprisingly, only 34 optimal portfolios out of 2000 held more than 80% in stocks. BDCs achieved similar results, with just 33 portfolios containing more than 80% in the optimal portfolio. The average weights from the distribution can be seen in figure 46.

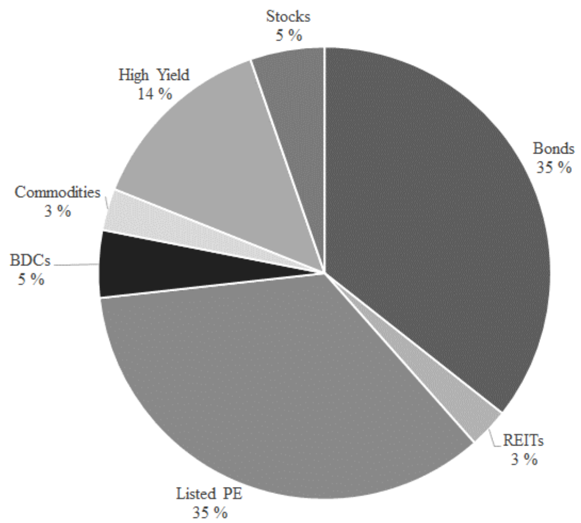


Figure 46 - Average Weights from Simulation (Data Source: Bloomberg)

Stocks and BDCs received an average allocation of 5% in optimal portfolios, while REITs and commodities received the smallest allocations of 3%. Listed PE has an average weight of 35 %. This allocation is consistent with the tangent portfolios estimated for 2009-2019 and the capital prediction portfolios, which allocate between 20% and 40% of the tangent portfolios to listed PE. Furthermore, the simulation shows that in 54% of the portfolios listed, PE had an allocation greater than 20%. Thus, listed PE, along with bonds, is the most frequently used asset class in the simulation. This reinforces a conclusion that listed PE should be included in the portfolios of retail investors. In comparison, only 10% of all simulated portfolio stocks receive an allocation greater than 20%. High Yield has a weight of more than 20% in 23.4% of all portfolios, making it the third most frequently used asset class in the simulation after bonds and listed PE.

The simulation demonstrates that the tangent weights are highly sensitive, making it difficult to find a stable allocation over time. This is also why, outside of the sample, the equally weighted portfolio often outperforms the tangent portfolio (DeMiguel, Garlappi & Uppal, 2009; Plyakha, Uppal, Volkov, 2015; , Malladi & Fabozzi, 2016). To conclude, the simulation utilizes the majority of the same assets as the optimal portfolio discussed previously in this paper, namely listed PE, bonds, and High Yield. When considering the historical tangent portfolio weights discussed in this chapter, the portfolios with more than 80% in bonds can have a poor out-of-sample performance. Portfolios built during non-crisis times seem to be more suited to out-of-sample fitting since they are more compatible with simulation results.

7.6 Stress-testing

A stress test is a risk management tool used to detect the effect on a portfolio of unexpected but often possible events or changes in a series of financial variables (Lopez, 2015). The following section will stress-test a subset of the portfolios discussed earlier in this chapter. First, capital market prediction-based portfolios are tested. Following that, a stress-test on conservative, balanced, and aggressive portfolios comprised entirely of alternative assets will be examined. Their allocation is seen in figure 34. The capital market prediction tangent portfolio is exposed to stress testing both with and without weight constraints; allocation can be seen in figure 29. To supplement the mean-variance framework, the GMM (figure 43) and MD portfolios (figure 43) built using 2009-2019 data are stress-tested for further analysis.

7.6.1 Sensitivity Analysis

A sensitivity analysis is presented to determine the effect of a hypothetical event on the portfolios. The framework is built on a single-factor beta coefficient used to anticipate the expected reaction to a scenario. Table 13 assumed a market correction of 30% and evaluated the effect on seven portfolios over a month.

The beta is estimated using the Dimson beta since alternative assets have demonstrated illiquid stocks, reacting to the stock market more slowly than other assets Lahr & Herschke (2009). Furthermore, all of the betas are significant at the 99% confidence level.

Table 13 - Expected one-month portfolio returns based on global equity scenario. (Data Source: Bloomberg)

Estimation method	Capital market expectations					Historical data: 2009-2019	
Scenario: MSCI world index down by 30 % in one month	Conservative	Balanced	Aggressive	Tangent	Tangent - MAX W: 30 %	MD	GMM
Beta	0,73	0,88	1,18	0,68	0,99	0,46	1,19
T-statistic	26,89*	29,81*	28,73*	25,77*	28,73*	30,55*	32,86*
Expected portfolio return	**-21,79 %**	**-26,27 %**	**-35,46 %**	**-20,50 %**	**-29,75 %**	**-13,79 %**	**-35,63 %**

*Significant at 1 percent level
**Significant at 5 percent level

By examining Table 13, the investor will identify the systematic risk associated with the various portfolios. The MD portfolio, which is expected to be the most diversified, is constructed of commodities, bonds, and listed PE. This portfolio does its job due to its low beta and hence the lowest sensitivity to market movements.

Unsurprisingly, the portfolio that maximizes return (GMM) has the highest sensitivity. This is also true for the aggressive portfolio constructed based on capital market expectations, both of which are predicted to decline by more than 35%.

According to Table 13, beta rises as the amount of equity-like instruments in the portfolio increases. The conservative portfolio, which includes 70% fixed income (High Yield) and 30% in equity-like assets (REITs, listed PE, and BDCs), has a beta of 0.73. While the balanced strategy investing 50% in fixed-income has a beta of 0.88 and the aggressive strategy allocating just 10% in fixed-income has a beta of 1.18. It can be claimed that equity-like alternatives raise the portfolio's systematic risk.

7.6.2 Event Stress-testing

According to a Global Financial System Committee (2005) review of portfolio stress testing techniques, the majority of stress tests are designed around diverse scenarios based on historical events. This thesis will explore this further, looking at four events since 2008 that shock financial markets. Starting with the financial crises in 2008, then the euro crisis in 2011, followed by the Chinese stock market turbulence of 2015, and ending with the covid 19 crises.

However, one should be aware of this technique's drawbacks, which involve analyzing the effect of historical events on a portfolio. Berkowitz (2000), and later Alexander and Sheedy

(2008), critiqued these strategies for failing to engage critical thinking. They are often conducted without the use of a risk model, which complicates evaluating the probability of each scenario. Additionally, there is a high possibility that other severe yet potential events will be completely overlooked.

The Great Recession 2008 and Eurozone Crisis 2011

The great recession was discussed earlier. New to the analysis is the eurozone crisis, also known as the eurozone debt crisis. It began in 2009 when Greece defaulted on its national debt, and the crisis expanded throughout Europe over the next three years (Amadeo, 2020). Finally, by April 2011, it shocks global financial markets with a 20% decline in the MSCI world index.

Figure 47 presents a backtest of the different portfolios during these crises. It is assumed that the weights are continuously rebalanced, and transaction costs are not considered.

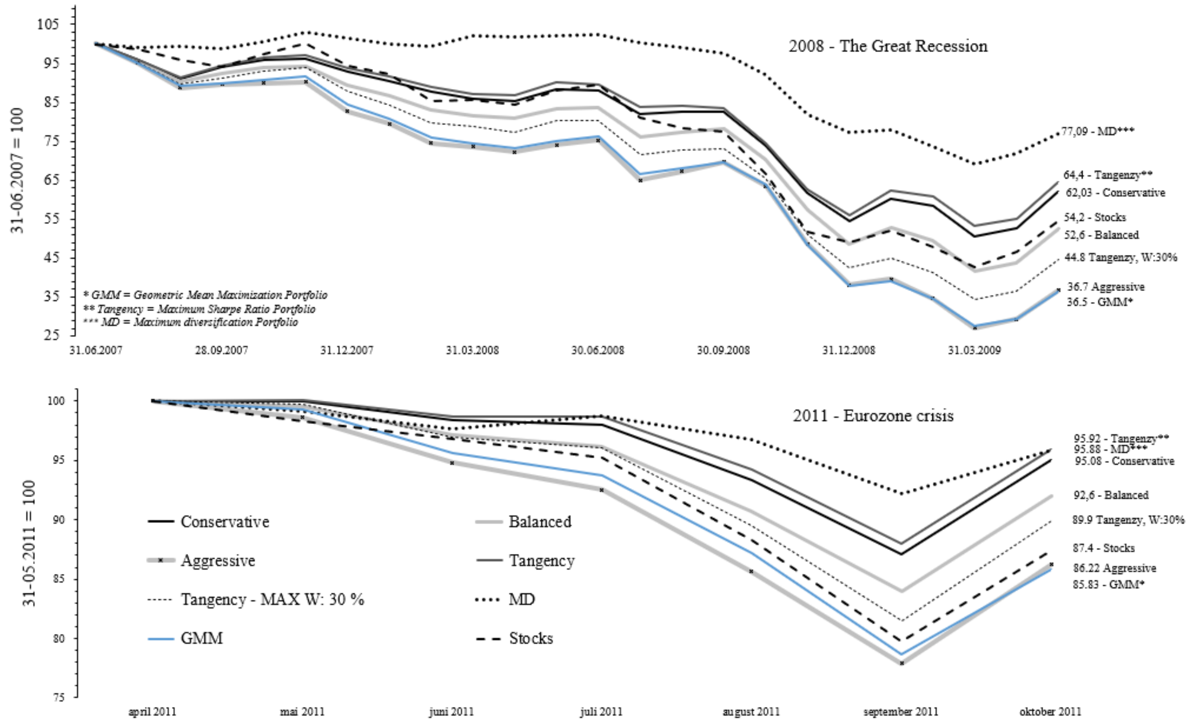


Figure 47 - The Great Recession & Eurozone crisis
(Data Source: Bloomberg)

In every example, the portfolio with the most diversification (MD) outperforms the others during a crisis. Similarly, the tangency and conservative portfolios perform well, particularly during eurozone crises. In comparison to stocks, the more aggressive portfolio does underperform, particularly during the great recession. On the other hand, the tangency (MAX 30%) performs well throughout the eurozone crisis, even though this is an aggressive portfolio comprised entirely of alternative assets.

China Stock Market Turbulence & Covid 19

Furthermore, China's stock market turbulence in 2015 will be investigated (figure 48). Typically, this crisis is understood as a period of rapid capital growth that ended in a bubble burst (Hsu, 2016). Stocks underperform in comparison to previous crises, demonstrating that portfolios with a higher degree of diversification across other asset classes are less affected by this crisis. Although the conservative and MD portfolios are least affected, the more aggressive portfolios track them closely.

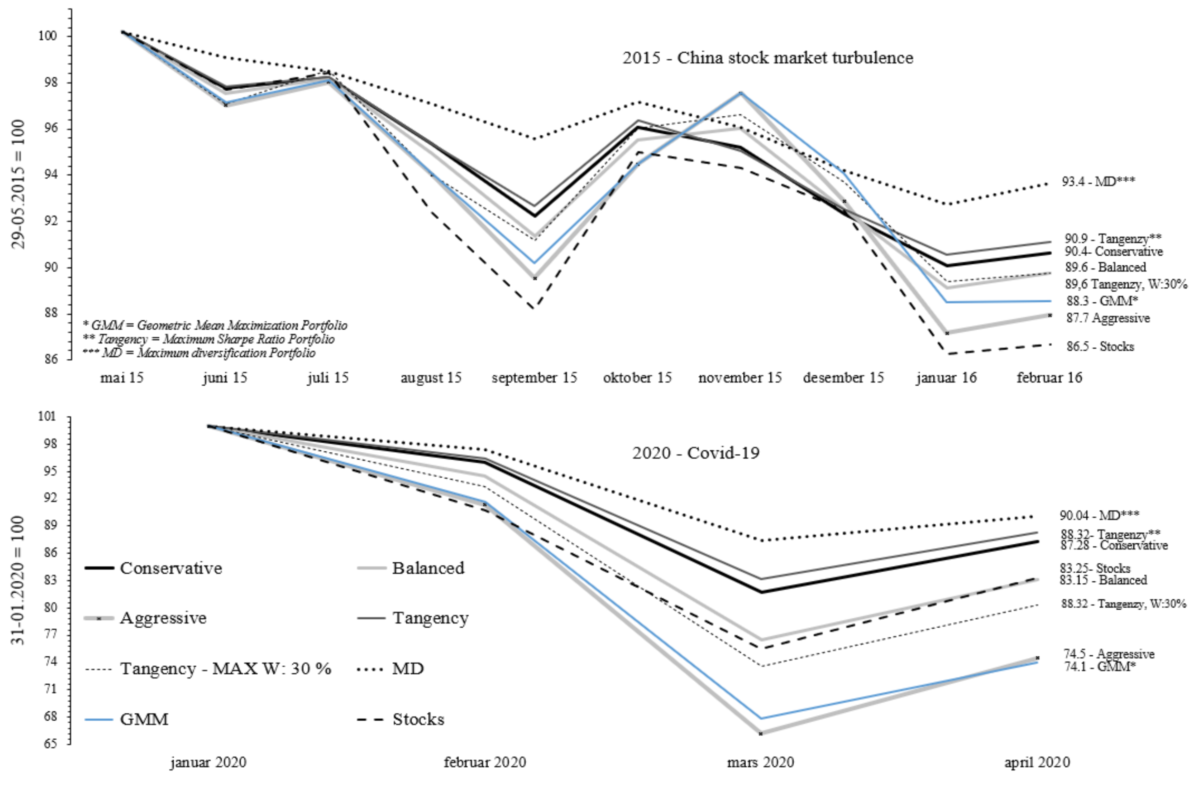


Figure 48 - China stock market turbulence & Covid 19.
(Data Source: Bloomberg)

COVID 19 had a big influence on aggressive portfolios, even more than on stocks. However, the balanced portfolio, which is 50% high yield and remaining towards REITs (11%), BDCs (15%), and listed private equity (24%), appears to be a portfolio capable of handling this crisis. Additionally, the balanced portfolio has had an impressive performance through the other three crises. Furthermore, the four assets in the balanced portfolio are those that many experts believe will outperform stocks in terms of returns over the next decade. Thus, this is a portfolio with a great expected return and the capability to survive crises. Table 14 shows cumulative returns from the peak of the crisis to the bottom.

Table 14 - Stress event test results (Data Source: Bloomberg)

Estimation method	Capital market expectations					Historical data: 2009-2019		
Stress event	Conservative	Balanced	Aggressive	Tangent	Tangent - MAX W: 30 %	MD	GMM	MSCI world stock index
2008 Great recession 31.06.2007-27.02.2009	-49,3 %	-58,5 %	-72,9 %	-46,6 %	-65,6 %	-30,8 %	-72,3 %	-57,3 %
2011 Eurosone crisis 31.05.2011-30.09.2011	-12,9 %	-15,6 %	-21,0 %	-12,1 %	-18,3 %	-7,0 %	-20,8 %	-18,8 %
2015 Chine Equity market correction 29.05.2015 - 29.01.2016	-10,1 %	-11,1 %	-13,0 %	-9,6 %	-10,8 %	-7,5 %	-11,7 %	-13,9 %
2020 Covid-19 31.01.2020 - 29.05.2020	-9,5 %	-13,0 %	-20,6 %	-8,9 %	-16,5 %	-6,8 %	-21,5 %	-12,1 %

The aggressive portfolio would have been hardest hit by the Great Recession and the eurozone crisis. However, this portfolio contains a portion of REITs that were particularly stroked in 2008 due to their high real estate exposure. Furthermore, the turbulence in China's stock markets had the most impact on global stocks, with a 14% decline. Lastly, the GMM portfolio was hardest hit by covid-19. All in all, when an investor chooses an aggressive portfolio with alternatives, the investor must expect the risk of a greater fall in value during recessions than investing solely in stocks. If an investor selects a more balanced or conservative portfolio, the investor can still earn a high expected return in normal times, and the portfolio can do better in times of crisis than stocks.

7.6.3 Continuous Value-at-Risk

Lastly, the expected shortfall through CVaR will be evaluated since it is essential for determining the tail risk of a portfolio. This was discussed in chapter 4.3.1, and Table 15 presents the CVaR at 1 %, 5%, and 10 % confidence level.

Table 15 - Monthly CVAR (Data Source: Bloomberg)

Estimation method	Capital market expectations					Historical data: 2009-2019		
Significant level	Conservative	Balanced	Aggressive	Tangent	Tangent - MAX W: 30 %	MD	GMM	MSCI world stock index
1,0 %	-15,5 %	-18,8 %	-25,5 %	-14,7 %	-21,6 %	-10,8 %	-25,0 %	-22,4 %
5,0 %	-10,1 %	-12,0 %	-16,3 %	-9,6 %	-13,5 %	-6,2 %	-15,8 %	-12,7 %
10,0 %	-7,2 %	-8,7 %	-12,0 %	-6,8 %	-9,9 %	-4,4 %	-11,5 %	-10,2 %

Again, the MD portfolios have the lowest risk of all portfolios. Assuming a normal distribution, there is a 1 % chance that the MD portfolios will suffer a monthly loss of 10.8%.

In comparison, for the same probability, the aggressive portfolio has a 25.5 % potential loss. Overall, it appears that the aggressive and GMM are the portfolios that have the largest tail risk. Additionally, this analysis shows that even though aggressive alternative portfolios are more diversified, they still carry the same or more tail risk than stocks.

8. Discussion of the Results

8.1 Asset Class Performances

Past literature talks about the diversification benefits one can achieve by adding alternative investments to a portfolio. Zimmermann, Bilo, Christophers, and Degosciu (2005), found that their selection of 229 listed PE funds correlated with MSCI world by only 0.4 and with the global bond market by 0.02. Our findings support these findings since, over the last decade, listed PE has had the lowest correlation to the global bond market of all asset classes, with values varying between -0.03 and 0.11 depending on the period. However, according to our findings, listed PE seems to correlated quite heavily with the stock market having a correlation of 0.84, which is quite average amongst all our assets. Thus, it seems that although listed PE provides excellent diversification when combined with bonds, the same does not apply to a combination with stocks. This may explain why they are rarely combined in estimated portfolios.

Continuing on the correlations, Kallenos and Nishiotis (2019) found in their study that BDCs had a low correlation with S&P500 and the public equity market overall. Our results on the contrary, show a rather high correlation between the two, a value of 0.81. Additionally, Grissom, Kuhle, and Walther (1987); Gilberto (1993); Froot (1995), and Eichholtz (1996) argue REITs to also have low correlations with stocks and bonds. Like before, our results point to the contrary. We found REITs to be an asset class most correlated with both bonds and stocks of all our assets, hence being a weak diversifier.

According to our results and past literature, the diversification capabilities of commodities and high yield seem to be somewhat similar. After many others, Forton and Rouhenhorst (2006) show commodities to be a strong diversifier and have even a negative correlation to equities. Although our findings show the low correlations commodities have with other assets, they rarely are the lowest and never negative. Thus, our results are more in line with the most recent study by Cheung & Miu (2010), in which they found diversification benefits to faint after the financialization of commodities through institutional investors' entry.

Our results might differ from the past finding for numerous reasons, one of which might be that we use the“liquid alternatives” rather than the private alternative assets in our analysis. Hence, we need to ask how closely the public proxies follow the underlying private asset classes.

Huss (2005) found that unlisted PE exceeded listed PE on average returns but had a slight underperformance when comparing the median returns. Like Degosciu (2012), he concludes that there is no significant difference between listed and unlisted PE performance during their timeframes. Our results agree with the first finding but show a significant difference between the two indexes. Firstly, the standard deviation of the listed PE is double that of unlisted PE. Secondly, the correlation between the indexes is only around 0.7. Finally, we do not find significant cointegration between the two indexes. Thus, according to our results, listed PE does not perform identically with unlisted PE, and investors should not be indifferent between the two.

The above applies to BDCs and REITs too. None of the private indices show cointegration with the public counterparts. Additionally, BDCs and REITs show even lower correlations with their private indexes than PE. Thus, as opposed to what the past literature says, our results indicate that the inclusion of listed alternative assets has different results than if one would include the real private assets in their portfolio. This, however, is not necessarily a negative thing. It merely means that listed alternatives can not be used the same way as their private counterparts in an asset mix. To evaluate whether it is rational for retail investors to include these assets in their portfolios, this needs to be analyzed from a portfolio optimization perspective.

8.2 Portfolio Optimization

Our analysis used four main optimization criteria: risk minimization, relative performance maximization, wealth maximization, and diversification maximization. Regarding the first two, Fischer and Lind-Braucher (2010) built similar portfolios and found similar results. Like in our MRP and tangency portfolios, they similarly found weights of bonds to vary between 80-97% and 40-60%. Additionally, their portfolios tended to include small amounts of PE in the mix, just like ours. Our capital market prediction(CMP) portfolios differ from previous research, which excluded bonds entirely from optimal portfolios. However, the similarities remain as all the optimal portfolio is divided between fixed income securities and equity-like assets. Where the differences in the CMP portfolios are their preference for high yield bonds over investment-grade bonds.

Unlike in Fischer and Lind-Braucher’s portfolios, commodities and REITs were not included in any historical MRP and tangency portfolios in our study. However, REITs are included in the tangency portfolios estimated on expected returns, not historical. Moreover, earlier studies by Bekkers et al. (2009) found optimal portfolios to have real estate, commodities, and high yield. Commodities were not included in either historical and CPM portfolios because of their high historical SD and CVaR. As Manseau and Power (2020) note, crude oil’s price development has taken the overall commodity index down and added risk.

Our results regarding commodities are that they are included in all of the maximum diversification portfolios, indicating that it indeed provides a hedge against market downturns. With analysts predicting higher returns for commodities, it may be a valuable addition to the portfolio of a retail investor.

The poor performance of REITs in our study compared to past research could be explained by its cyclicality. Mull & Soenen (1997) show that the value of REITs as an asset class is time-dependent, meaning that whether it offers improvements in a mixed-asset context depends on the timing and the holding period. One could argue that after the Great Recession led by the subprime mortgage crisis, the real estate sector has been struggling to get back on its past peaks, which is reflected in its poor risk-return performance during the past decade. That might also be why many analyst analysts believe REITs would outperform stocks over the next 10-15 years and why they are included in the optimal portfolio estimated using expected returns rather than historical returns.

In the study of Bekkers et al. (2009), listed PE was included in the riskier portfolios while moving to the far right on the efficient frontier, finally ousting the other alternatives completely. While this aligns with what we found, our results indicate a better overall performance of listed PE in the MRP and tangency portfolios. According to our finding, while listed PE has the majority of weights in the wealth maximizing and riskiest efficient portfolios, it also shows up with allocations in the minimum risk and tangency portfolios. Thus, our results approve both the diversifying and return enhancing factors of listed PE mentioned in the past literature.

High yield bonds were an asset class that performed extremely well overall in our analysis. Although it is not officially regarded as an alternative asset, we considered it since it behaves quite differently from stocks and bonds and is more uncommon and difficult to access than the assets mentioned above. High yield was included with allocations in many of the portfolios, no matter the optimization objective, and it was the key investment in the tangent portfolio for capital market predictions. It seemed to especially add value when aiming to maximize Sharpe Ratio and risk-adjusted returns. It was also more present in portfolios calculated using CVaR as the risk estimator. This was in line with the finding from Hernandez (2020), showing that high yield, compared to the stock market, provides less volatility in the long run and produces higher returns than traditional bonds. Especially in today's low-interest-rate environment, as traditional bonds offer a low return, high yield could be an excellent core component in a well-diversified portfolio. This was particularly evident in conservative and balanced portfolios, where the Sharpe ratio improved significantly compared to portfolios comprised entirely of traditional assets.

Lastly, although BDCs show the highest historically and expected arithmetic returns in our analysis, it barely appears in any optimized portfolios. This is largely due to its high risk, which is highest regardless of the estimation period or the measure. This is why BDCs are only included in portfolios with weight restrictions in the GMM and capital market predictions portfolio. There is little consensus regarding the position of BDCs in the optimal portfolio due to the insufficiency of research on the topic. However, Kleiman and Shulman (1992) and Kallenos and Nishiotis (2019) find that BDCs sometimes underperform and overperform, consistent with our findings.

8.3 Performance in Crises

When investigating the performance of liquid alternatives during crises, our study presents similar results to the most recent literature. Junge and Petersen (2020) and Brammer and Rants (2015) showed that real estate is very period-dependent during times of crisis, and PE does not give any downside protection. VC, however, was found to give decent protection as the younger and more tech-driven companies it focuses on can navigate better through economic downturns. Our results indicate that listed VC has the lowest correlations with bonds and stocks of all alternatives, highlighting its diversifying abilities. Additionally, it was shown to give the highest returns during the Covid-19 of all assets and better than PE and buyout in the Great Recession.

Overall, looking at the performance of liquid alternatives during crises, the unlisted alternatives witnessed far less decline in value. This agrees with what the past literature states about the biases regarding the timely valuations in private assets. Junge and Petersen (2020) added that liquid alternatives might overstate the impacts of crises on the asset class, which is similar to what Marston (2011) found in his study and we found in our research. This is also what leads to liquid alternatives trading at a high discount compared to their NAVs during crises, making them seem like a valuable investment. Some of this might indeed be an overreaction by the market, but some of it is unarguably due to the lag in the re-valuation of the underlying private assets.

8.4 Limitations

As shortly mentioned in the previous chapters, there are important limitations in this research. Such limitations inevitably challenge the conclusions one can make based on such quantitative analysis and thus need to be taken into consideration.

Firstly, the optimal historical portfolios do not take the out-of-sample performance into account. Instead, our portfolios are based on the in-sample observations. Although some of our models, such as the Bayes-Stein, are chosen based on the great out-of-sample performance found in previous academic research. By using various optimization models in different estimation periods, we can achieve a more comprehensive outlook of how the weights of assets alter depending on the input parameters and the model of choice.

Related to the one above, another issue is the estimation error in the input parameters of the models. Portfolio optimization models generally use expected returns and risks as input parameters when calculating the optimal allocations. For certain parts of the thesis, these expected rates are taken from historical performance, which is not guaranteed future performance. Thus, these parameters inevitably possess estimation risk, which leads to suboptimal portfolios out-of-sample, as discussed above. The effect of such estimation errors is well observed by changing the estimation periods in the analysis.

To get a forward-looking perspective, portfolios are estimated using capital market predictions published by different brokerages or other industry experts. These are based on subjective analyses, which therefore also possess estimation risk.

Another limitation of our study is that hedge funds are not included in our analysis. This is because hedge funds have extremely varying and non-normal distributions of returns depending on the strategy. Using an aggregate index of these strategies would be unreasonable and inaccurate in our context. To be exact, hedge funds are not an asset class itself, rather a legal structure representing a highly divergent group of strategies, many of which have little to no equity exposure. As the main goal of hedge fund investments is to diversify portfolios, often long equity portfolios, the evaluation of optimal allocations to such strategies is extremely complex and case-specific. For these reasons, hedge funds are excluded from our analysis.

The last and certainly most common limitation present in this study is the assumption of efficient markets. The main assumption of most portfolio optimization models is based upon the efficient market hypotheses, which means that investors are, among other things, assumed to be rational, and all new information is assumed to reflect on the prices. However, as proved by many researchers, the financial markets are semi-efficient and exposed to various anomalies. Although this leads to unsatisfactory results, it does not alter the main findings of this paper. It simply means that the results should be interpreted with a certain degree of caution.

After all, the results found in our study are based on the estimation periods of our choice and the data available today. Moreover, as many of the assets are found to behave cyclically, the optimal allocations are inevitably dependant on the exact holding period and timing of investments.

8.5 Future Research

Interesting topics for future research would be to include crowdfunding and crowdlending in the mix. These assets are very similar to the private assets in the way they are constructed but are still available to retail investors. Additionally, as such platforms are still rather young and private, there is very little data on their performance. It would be interesting to investigate how high returns one can achieve from such investments and what level of risk. Furthermore, it would be relevant to study how closely such investments follow other asset classes such as venture capital and private debt.

We would have enjoyed focusing more on the PE sector alone. We scratched the surface on different components of listed PE, such as listed VC, listed buyout and listed distressed capital. However, it would have been intriguing to dig deeper into how these different components perform alone compared to the other asset classes and as part of a portfolio. Additionally, it would have been interesting to more deeply analyse how certain risks such as liquidity risk affects the returns of listed alternatives. More specifically how well the liquidity premium is priced in such assets.

As the markets keep on changing, and every crisis hits differently, it will be interesting to see how these assets perform during the next years. The Covid-19 is hardly over yet and certain sectors are expected to still face turmoil due to the continuing restrictions. It remains to be seen how the listed alternatives that rose after the previous financial crisis, will perform and protect retail investors during future economic turmoil.

9. Conclusions

The main objective of this paper was to investigate whether the liquid alternatives offered to retail investors are rational investments and, if so, under which assumptions. Additionally, this analysis compared the listed alternatives to their unlisted counterparts. These assets were studied both individually and in the framework of portfolio optimization. Seven asset classes were included in the portfolio analysis: Public equity, public debt, high yield bonds, commodities, real estate, private equity, and private debt. Individual performances and optimal portfolio allocations were examined based on global index data from July 2007 to February 2021 and capital market predictions from 14 different analysts and experts.

For the portfolio optimization, three estimators for returns were used, capital market predictions, historical and Bayes-Stein, combined with the two most established risk measures, standard deviation and CVaR. We estimated optimal all asset portfolios with varying objectives as well as comparable benchmark stock-bond portfolios. Input measures for the models were derived from future capital market pxpectations and historical data. The minimum risk portfolio (MRP) and the tangency portfolio use historical returns and Bayes-Stein with CVaR and SD as risk measures. In addition, two separate portfolios - the maximum geometric mean (GMM) and maximum diversification (MD) - were created for a more thorough investigation of allocations regarding different investor preferences. In the end, a simulation approach was executed to analyse the sensitivity of the weights in the historical tangent portfolios. Finally, we stress-tested a subset of the presented portfolios.

Although alternatives come with higher fees than traditional assets, our findings unambiguously show that retail investors would be well advised to add liquid alternatives to their portfolios. This is because our results indicate that portfolios including certain alternative assets outperform traditional portfolios in terms of risk, Sharpe Ratio maximization, and diversification.

According to our analysis, Listed PE performs best of all alternatives. Its main benefit comes from return enhancement rather than downside protection, as was also concluded in previous studies. Overall, listed alternatives performed quite poorly during economic distress, presenting higher levels of downside risk than traditional assets. The only exception is high yield bonds, which are categorized somewhat in between traditional and alternative investments. Moreover, according to past literature, REITs, Listed PE, and BDCs are subject to infrequent trading, resulting in them having lower market liquidity than common stocks, which sets investors up for the additional risk.

Furthermore, analysts anticipate REITs to perform better than before over the next decade, and as a result, they are included in optimal portfolios based on capital market predictions. Given that many experts expect stock returns to be lower over the next decade, alternatives such as listed PE, REITs, and BDCs were found as potential replacements in more aggressive portfolios.

High yield bonds, like listed PE, were present in almost every portfolio regardless of the optimization objective. It showed excellent downside protection with good steady returns, and its risk profile was close to bonds with significantly higher returns. Additionally, it has had a very low correlation with bonds over the last five years, highlighting its diversification benefits. Several industry experts also project high yield bonds to have the best risk-adjusted returns in the coming years. Thus, it also received the highest allocation in the tangent portfolio estimated on capital market predictions. According to our research, high yield bonds are a noteworthy alternative for traditional bonds in retail investors' portfolios.

Regarding the differences between listed and unlisted alternatives, our analysis indicates that they are very different. Compared to unlisted alternatives, listed alternatives are significantly more volatile due to their heavy exposure to the public market as a traded and liquid vehicle. Additionally, there is not significant cointegration nor correlation between the two. While BDCs and REITs provide similar average returns as their private counterparts, listed PE falls significantly behind the private benchmarks in cumulative return. However, in terms of average arithmetic returns, they are very similar.

In the end, the exact allocation to each asset should be determined by the investor's preferences and investment horizon. Our results still show that no matter the investor's objective, the inclusion of alternatives enhances retail investors’ portfolios, hence making them rational investments.

References

Acerbi, C., & Tasche, D. (2002). Expected Shortfall: A Natural Coherent Alternative to Value at Risk. *Economic Notes, 31(2),* 379-388.

Alexander, C., Sheedy, E. (2008). Developing a stress testing framework based on market risk models. *Journal of Banking & Finance, 32(10),* 2220-2236. doi:10.1016/j.jbankfin.2007.12.041

Altman, E. I. (1987). The anatomy of the High-yield bond market. *Financial Analysts Journal, 43*(4), 12-25. doi:10.2469/faj.v43.n4.12

Amadeo, K. (2020). Eurozone debt crisis. Retrieved from https://www.thebalance.com/eurozone-debt-crisis-causes-cures-and-consequences-3305524

Ankrim, E. M., & Hensel, C. R. (1993). Commodities in asset allocation: A real-asset alternative to real estate? *Financial Analysts Journal, 49*(3), 20-29. doi:10.2469/faj.v49.n3.20

Anson, M. J. (2004). Business development companies. *The Journal of Private Equity, 7*(4), 10-16. doi:10.3905/jpe.2004.434762

Artzner, P., Delbaen, F., Eber, J. M., & Heath, D. (1997). Thinking Coherently. *Risk, 10*, 68–71.

Artzner, P., Delbaen, F., Eber, J. M., & Heath, D. (1999). Coherent Measures of risk. *Mathematical Finance*, *9(3),* 203–28.

Avramov, D., & Zhou, G. (2010). Bayesian Portfolio Analysis. *Annual Review of Financial Economics, 2(1),* 25-47.

Basak, S., & Pavlova, A. (2016). A model of financialization of commodities. *The Journal of Finance, 71*(4), 1511-1556. doi:10.1111/jofi.12408

Bekkers, N., Doeswijk, R. Q., & Lam, T. W. (2009). Strategic Asset Allocation: Determining the Optimal Portfolio with Ten Asset Classes. *The Journal of Wealth Management, 12(3),* 61-77.

Benz, C. (2021). Experts forecast stock and bond returns: 2021 edition. Retrieved May 15, 2021, from https://www.morningstar.com/articles/1018261/experts-forecast-stock-and-bond-returns-2021-edition

Bergmann, B., Christophers, H., Huss, M., & Zimmermann, H. (2009). Listed private equity. Retrieved from https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1362390

Berk, J. B., & DeMarzo, P. M. (2020). *Corporate finance*. Harlow, England ; Munich: Pearson.

Berkowitz, J. (2000). A coherent framework for stress-testing. *Journal of Risk, 2,* 1–11.

Bessler W., Wolff D. (2015). Do commodities add value in multi-asset-portfolios? An out-of-sample analysis for different investment strategies. *Journal of Banking and Finance, 60,* 1–20.

Beyhaghi, M., Hawley, J. P. (2012). Modern Portfolio Theory and Risk Management: Assumptions and Unintended Consequences. *Journal of Sustainable Finance & Investment, 3(1),* 17-37.

Bilo, S., Christophers, H., Degosciu, M., & Zimmermann, H. (2005). Risk, returns, and biases of listed private equity portfolios. Retrieved from https://www.researchgate.net/publication/228379204_Risk_returns_and_biases_of_listed_private_equity_portfolios

BlackRock. (2021). Capital market assumptions. Retrieved May 15, 2021, from https://www.blackrock.com/institutions/en-us/insights/charts/capital-market-assumptions

Bloomberg. (2020). *HYG*. Retrieved from Bloomberg: https://www.bloomberg.com/quote/HYG:US

Bloomberg. (2020). *Reference Data: Content and Data.* Retrieved from Bloomberg: https://www.bloomberg.com/professional/product/reference-data/

BMO. (2020). BMO Long-Term Capital Market Assumptions. Retrieved from https://www.bmoetfs.ca/uploads/2020-BMO-Capital-Market-Assumptions-Canada_Final.pdf

Brammer, N. D., & Rants, K. O. K. (2015). Kapitalfonde klarer sig relativt bedst under kriser. *Finans/Invest, 4,* 19-28.

Brennan, M. J. (1971). Capital Market Equilibrium with Divergent Borrowing and Lending Rates. *The Journal of Financial and Quantitative Analysis, 6(5),* 1197-1205.

Bricker, C. J., & Johnson C. M. (2015). *Liquid Alternatives: The Next Dimension in Asset Allocation.* AllianceBernstein L.P. (AB).

Brown, C., & Kräussl, R. (2010). Risk and return characteristics of listed private equity. *SSRN Electronic Journal*. doi:10.2139/ssrn.1676768

Brown, L. D. (1966). On the Admissibility of Invariant Estimators of One or More Location Parameters. *Annals of Mathematical Statistics*, *37*, 1087-1136.

Bucay, N., & Rosen, D. (1999). Credit risk of an international bond portfolio: A case study. *ALGO Research Quarterly, 2(1),* 9-29.

Buyuksahin, B., & Robe, M. A. (2014). Speculators, commodities and cross-market linkages, *Journal of International Money and Finance, 42*, 38-70.

Caselli, S. (2018). *Private equity and venture capital in Europe: markets, techniques, and deals Second edition.* Academic Press, an imprint of Elsevier, London.

CCMR. (2018). *Expanding Opportunities for Investors and Retirees: Private Equity.* Committee on Capital Market Regulations.

CFTC. (2008). Staff Report on Commodity Swap Dealers & Index Traders with Commission Recommendations. *Commodity Futures Trading Commission (September).*

Cho, Y., Hwang, S., & Lee, Y. (2013). The dynamics of Appraisal smoothing. *Real Estate Economics, 42*(2), 497-529. doi:10.1111/1540-6229.12027

Choueifaty, Y., & Coignard, Y. (2008). Toward Maximum Diversification. *Journal of Portfolio Management, 35(1),* 40-51.

Ciura, B. (2020). 2021 BDC List. Retrieved from https://www.suredividend.com/ness devleopment-list/

Cliffwater. (2021). CWBDC Index. Retrieved May 20, 2021, from http://bdcs.com/

Cochrane, J. (2005). *Asset pricing, Revised edition.* Princeton, New Jersey: Princeton University Press.

Committee on the Global Financial System. (2005). A survey of stress tests and current practice at major financial institutions. https://www.bis.org/publ/cgfs24.htm

Conceição, P., & Marone, H. (2008). Characterizing the 21st Century First Commodity Boom: Drivers and Impact, Working Paper, Office of Development Studies/United Nations Development Programme.

Corgel, J. B., & Djoganopoulos, C. (2000). Equity reit beta estimation. *Financial Analysts Journal, 56*(1), 70-79. doi:10.2469/faj.v56.n1.2332

Cumming, D., Fleming, G., & Johan, S. A. (2011). Institutional investment in listed private equity. *European Financial Management, 17*(3), 594-618. doi:10.1111/j.1468-036x.2011.00595.x

Daskalaki C., Skiadopoulos G. (2011). Should investors include commodities in their portfolios after all? New evidence. *Journal of Banking and Finance, 35(10),* 2606–2626.

Dechert LLP. (2019). *FINANCING THE ECONOMY*. https://www.arenaco.com/wp-content/uploads/2019/11/Financing-the-Economy-The_future_of_private_credit__FTE_2019_report_FINAL.pdf.

Degosciu, M. (2012). Listed vs. unlisted private equity (first version). *SSRN Electronic Journal*. doi:10.2139/ssrn.2081997

DeMiguel, V., L. Garlappi, & Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? *Review of Financial Studies, 22*, 1915–1953.

Døskeland, T. M., & Strömberg, P. (2018). Evaluating Investments in Unlisted Equity for the Norwegian Government Pension Fund Global. *Regjeringen*. doi:https://www.regjeringen.no/contentassets/7fb88d969ba34ea6a0cd9225b28711a9/evaluating_doskelandstromberg_10012018.pdf

Eichholtz, Piet. (1996). Does International Diversification Work Better for Real Estate Than for Stocks and Bonds? *Financial Analysts Journal, 52(1),* 56-62.

Elton, E. J., Gruber, M., Brown, S., & Goetzmann, W. (2014). *Modern portfolio theory and investment analysis, 9th edition.* Hoboken, New Jersey: John Wiley & Sons Inc.

Engle, Robert, & Clive Granger. (1987). Co-integration and Error Correction: Representation, Estimation and Testing. *Econometrica 55(2),* 251–76.

Ennis, R. M., & Sebastian, M. D. (2005). Asset allocation with private equity. *The Journal of Private Equity, 8(3),* 81-87.

Erb, C. B., & Harvey, C. R. (2006). The tactical and strategic value of Commodity Futures. *Financial Analysts Journal. 62(2),* 69-97.

Estrada, J. (2006). Downside Risk in Practice. *Journal of Applied Corporate Finance, 18(1),* 117-125.

Estrada, J. (2010). Geometric Mean Maximization: An Overlooked Portfolio Approach? *The Journal of Investing, 19(4).*

Fama, E. F. (1970). Efficient Capital Markets: A Review of Theory and Empirical Work. *The Journal of Finance, 25(2),* 383-417.

Fama, E. F., & French, K. R. (2004). The capital asset pricing MODEL: Theory and evidence. *Journal of Economic Perspectives, 18*(3), 25-46. doi:10.1257/0895330042162430

Fan, J., Fan, Y., & Lv, J. (2008). High dimensional covariance matrix estimation using a factor model. *Journal of Econometrics*, *147(1)*, 186–197.

Finley, J. (2019). *Expanding Retail Access to Private Markets.* Small Business Capital Formation Advisory Committee.

Francis, J. C., & Dongcheol, K. (2013). *Modern Portfolio Theory: Foundations, Analysis, and New Developments, First edition*. Hoboken, New Jersey: Wiley, c2013.

Francis, J.C., & Ibbotson, R. (2002). Investments, A Global Perspective. Upper Saddle River, New Jersey: Prentice-Hall.

Froot, K. A. (1995). Hedging portfolios with real assets. *The Journal of Portfolio Management, 21*(4), 60-77. doi:10.3905/jpm.1995.409527

FS Investment Solutions. (2019). Investing Like an Institution: A Hybrid Approach to Accessing Alternatives. FS Investment Solutions, LLC.

FTSE Russell. (2020). *FTSE EPRA Nareit Global Real Estate Index Series.* Retrieved from FTSE Russell: https://www.ftserussell.com/products/indices/epra-nareit

Gagnon, M., Manseau, G., & Power, G. J. (2020). They're back! post-financialization diversification benefits of commodities. *International Review of Financial Analysis, 71,* 101515. doi:10.1016/j.irfa.2020.101515

Geltner, D. M. (1991). Smoothing in appraisal-based returns. *The Journal of Real Estate Finance and Economics, 4*(3), 327-345. doi:10.1007/bf00161933

Giliberto, M. (1988). A Note on the Use of Appraisal Data in Indexes of Performance Measurement. *Journal of the American Real Estate anid Urban Econiomics Association, 16(1),* 77-83.

Gilligan, S. P. (2019). Arithmetic vs Geometric Mean: Which to use in Performance Appraisal. Retrieved from Longs Peak Advisory Services: https://longspeakadvisory.com/arithmetic-vs-geometric-mean-which-to-use-in-performance-appraisal/

Grissom, T., Kuhle, J., & Walther, C. (1987). Diversification Works in Real Estate, Too. *Journal of Portfolio Management, 13(2*), 66-71.

Hamilton, J. D., & Wu, J. C. (2014). Risk premia in crude oil futures prices, *Journal of International Money and Finance 42*, 9-37.

Harris, R. S., Jenkinson, T., & Kaplan, S. N. (2014). Private equity performance: What do we know? *Journal of Finance, 69(5),* 1851-1882.

Harris, R. S., Jenkinson, T., & Kaplan, S. N. (2016). How Do Private Equity Investments Perform Compared to Public Equity? *Journal of Investment Management 14(3).*

He, G., & Litterman, R. (1999). The Intuition Behind Black-Litterman Model Portfolios. *Working Paper,* Goldman Sachs Quantitative Resources Group.

Hoesli, M., & Lizieri, C. (2007). Real Estate in the Investment Portfolio. Retrieved from Regjeringen: https://www.regjeringen.no/globalassets/upload/fin/statens20pensjonsfond/norway-real-estate-final-report-revised-may-31.pdf

Holmes, F. (2019). *The Barriers To Investing In Private Equity Are Too High.* Retrieved from Forbes: https://www.forbes.com/sites/greatspeculations/2019/10/15/the-barriers-to-investing-in-private-equity-are-too-high/?sh=5e64cfa769a3

Horizon. (2020). Survey of Capital Market Assumptions. Retrieved from https://www.horizonactuarial.com/uploads/3/0/4/9/30499196/rpt_cma_survey_2020_v0716.pdf

Houweling, P. (2012). On the performance of fixed-income exchange-traded funds. The Journal of Index Investing, 3(1), 39-44. doi:10.3905/jii.2012.3.1.039

Hsu, D. H. & Kenney, M. (2005). Organizing Venture Capital: the Rise and Demise of American Research and Development Corporation, 1946-1973. *Industrial and Corporate Change, 14(4)*, 579-616.

Hsu, S. (2016). China's stock market crash: One year later. Retrieved from https://www.forbes.com/sites/sarahsu/2016/07/13/chinas-stock-market-crash-one-year-later/?sh=438ba9705503

Huss, M. (2005). Performance characteristics of private equity. Retrieved from https://www.institutionalassetmanager.co.uk/sites/default/files/import_attachments/Performance%20Characteristics%20of%20Private%20Equity%20-%20An%20Empirical%20Comparison%20of%20Listed%20and%20Unlisted%20Private%20Equity%20Vehicles%20-%20Matthias%20Huss%20University%20of%20Basel.pdf

Hwang, Quigley, & Woodward. (2005). An Index for Venture Capital, 1987-2003. *Contributions to Economic Analysis and Policy, 4.*

Invesco. (2021). 2021 Long-Term Capital Market Assumptions. Retrieved from https://www.invesco.com/content/dam/invesco/apac-master/en/pdf/apac/2020/solutions/2021-LTCMA-USD.pdf

Investopedia. (2020). Confidence Interval. Retrieved from Investopedia: https://www.investopedia.com/terms/c/confidenceinterval.asp

Irwin, S. H., & Sanders, D. R. (2011). Index funds, financialization, and commodity futures markets, *Applied Economic Perspectives and Policy 33*, 1-31.

J.P.Morgan. (2021). Capital Market Assumptions. Retrieved from https://am.jpmorgan.com/content/dam/jpm-am-aem/global/en/insights/portfolio-insights/ltcma/ltcma-full-report.pdf

Jark, D. (2020). The pros and cons of high-yield bonds. Retrieved March 24, 2021, from https://www.investopedia.com/articles/investing/112515/highyield-bonds-pros-and-cons.asp

Jenkinson, T., Sousa, M., & Stucke, R. (2013). How fair are the valuations of private equity funds? *SSRN Electronic Journal*. doi:10.2139/ssrn.2229547

Jensen, M. (1989). Eclipse of the Public Corporation. *Harvard Business Review, 67(5),* 61-74.

Jobson, J. D., & Korkie, B. (1980). Estimation for Markowitz Efficient Portfolios. *Journal of the American Statistical Association*, *75*, 544-554.

Jorion, P. (1985). International portfolio diversification with estimation risk. *Journal of Business*, *58(3),* 259–278.

Jorion, P. (1986). Bayes-Stein Estimation for Portfolio Analysis. *Journal of Financial and Quantitative Analysis, 21(3),* 279-292.

Jorion, P. (1991). Bayesian and CAPM Estimators of the Means: Implications for Portfolio Selection. *Journal of Banking & Finance, 15(3)*, 717–727.

Jorion, P. (1992). Portfolio optimization in practice. *Financial Analysts Journal, 48(1),* 68-74. doi:10.2469/faj.v48.n1.68

Jorion, P. (1996). Risk²: Measuring the Risk in Value at Risk. *Financial Analysts Journal, 52(6),* 47-56.

Jorion, P. (1996). *Value at Risk: A New Benchmark for Measuring Derivatives Risk.* Chicago: Irwin Professional Publishing.

Junge, C., & Petersen, F. H. (2020.) Alternative Investeringer Gennem Kriser. *Finans/Invest, 6*, 1-12.

Kaiser, D. G., Schweizer, D., & Wu, L. (2008). Strategic Hedge Fund Portfolio Construction That Incorporates Higher Moments. *Working Paper*, Available at SSRN: https://ssrn.com/abstract=1080509

Kallenos, T. L., & Nishiotis, G. P. (2019). Market-Based Private Equity Returns. *SSRN Electronic Journal*. doi:10.2139/ssrn.3383461

Kan, R., & Zhou, G. (2007). Optimal portfolio choice with parameter uncertainty. *Journal of Financial and Quantitative Analysis, 42(3),* 621.

Kaplan, S., & Strömberg, P. (2009). Leveraged buyouts and private equity. *The Journal of Economic Perspectives, 23(1),* winter, 121-146. doi:10.3386/w14207

Kempen. (2017). Listed and non-listed real estate investment - why combine. Retrieved May 24, 2021, from http://www.ipfos.eu/wp-content/uploads/2018/11/Kempen-Listed_Nonlisted_RealEstate.pdf

Kondor, I., Pafka, S., & Nagy, G. (2007). Noise sensitivity of portfolio selection under various risk measures. *Journal of Banking & Finance, 31(5),* 1545–1573.

Kuhle, J. (1987). Portfolio Diversification and Return Benefit – Common Stocks vs. Real Estate Investment Trusts (REITs*). Journal of Real Estate Research, 2,* 1-9.

Kuhle, J., Walther, C., & Wurtzebach, C. (1986). The financial performance of real estate investment trusts. *Journal of Real Estate Research, 1*(1), 67-75. doi:10.1080/10835547.1986.12090517

Lahr, H., & Herschke, F. T. (2009). Organizational forms and risk of listed private equity. *The Journal of Private Equity, 13*(1), 89-99. doi:10.3905/jpe.2009.13.1.089

Lee S. T., Stevenson, S. (2005). The Case for Reits in the Mixed-Asset-Portfolio in the Short and Long Run. *Journal of Real Estate Portfolio Management, 11(1),* 55-80.

Listed Private Capital. (2020). Annual Market Review. Retrieved 2020, from https://www.listedprivatecapital.com/media/1945/lpec_annualmarketreview2020.pdf

Lopez, J. (2005). Stress tests: Useful complements to financial risk models. FRBSF Economic Letter, 119–124.

Lou, X., & Sadka, R. (2011). Liquidity level or liquidity risk? Evidence from the financial crisis. *Financial Analysts Journal, 67*(3), 51-62. doi:10.2469/faj.v67.n3.5

Lowery, D. (2020). Future of Alternatives 2025: Preqin Forecasts Alternative AUM Growth of 9.8% through to 2025. Retrieved from Preqin:

https://www.preqin.com/insights/research/blogs/preqin-forecasts-alternative-aum-growth-of-9-8-percent-through-to-2025

Maillard, S., Roncalli, T., & Teïletche, J. (2010). The properties of equally weighted risk contribution portfolios. *The Journal of Portfolio Management, 36(4),* 60–70.

Malladi, R., & Fabozzi, F. J. (2016). Equal-weighted strategy: Why it outperforms value-weighted strategies? Theory and evidence. *Journal of Asset Management, 18*(3), 188-208. doi:10.1057/s41260-016-0033-4

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance, 7(1),* 77-91.

Markowitz, H. (1956). The Optimization of a Quadratic Function Subject to Linear Constraints. *Naval Research Logistics Quarterly, 3(1-2),* 111-133.

Marston, R. C. (2011). *Portfolio Design a Modern Approach to Asset Allocation, First Edition.* Hoboken, New Jersey: Wiley.

Martin, J. D., & Petty, J. W. (1983). An analysis of the performance of publicly traded venture capital companies. *The Journal of Financial and Quantitative Analysis*, *18*(3), 401. doi:10.2307/2330729

Merton, R. C. (1980). On estimating the expected return on the market: An exploratory investigation. *Journalof Financial Economics, 8(4),* 323–361.

Meucci, A. (2010). Quant Nugget 2: Linear vs. Compounded Returns – Common Pitfalls in Portfolio Management. *GARP Risk Professional*, 49–51.

Meziani, A. S. (2006). Exchange-traded funds versus mutual funds — weighting the options. *Exchange-Traded Funds as an Investment Option,* 40-57. doi:10.1057/9780230513372_3

MFS. (2021). MFS Long Term Capital Market Expectations. Retrieved from https://www.mfs.com/content/dam/mfs-enterprise/mfscom/insights/2021/january/mfse_fly_568555/mfse_fly_679695.pdf

Missiakoulis, S., Vasiliou, D., & Eriotis, N. (2010). Arithmetic Mean: A Bellwether for Unbiased Forecasting of Portfolio Performance. *Managerial Finance, 36(11),* 958-968.

MMI, & Dover. (2015). *Distribution of Alternative Investments through Wirehouses*. Money Management Institute and Dover Financial Research.

Moriarty, B. (2019). High-flying high yield, for now. Retrieved May 24, 2021, from https://www.morningstar.com/articles/931882/high-flying-high-yield-for-now

Morningstar. (2019). *Cross-Border Liquid Alternative Fund Landscape 2019: A Fast-Growing Asset Class Still Needing to Prove Its Worth.* Morningstar Manager Research Services.

MSCI. (2020). *MSCI World Index (USD).* Retrieved from MSCI: www.msci.com

Mueller, G. R., Pauley, K. R., & Morrill, W. K. Jr. (1994). Should REITs Be Included in a Mixed-Asset Portfolio? *Real Estate Finance, 11*, 8-23.

Mull, S. R., & Soenen, L. A. (1997). US REITs as an asset class in international investment portfolios. *Financial Analysts Journal, 53*(2), 55-61. doi:10.2469/faj.v53.n2.2072

NAREITs (2021). REIT Indexes. Retrieved from: https://www.reit.com/data-research/reit-indexes/ftse-nareit-us-real-estate-index-historical-values-returns

Northern Trust. (2021). Northern trust capital market assumptions 2021 edition. Retrieved May 15, 2021, from https://www.capitalmarketassumptions.com/

O'Connell, B. (2021). What are high-net-worth individuals? Retrieved May 27, 2021, from https://www.forbes.com/advisor/investing/high-net-worth-individual-hwni/

Pantaleo, E., Tumminello, M., Lillo, F., & Mantegna, R. N. (2011). When Do Improved Covariance Matrix Estimators Enhance Portfolio Optimization? An Empirical Comparative Study of Nine Estimators. *Quantitative Finance, 11(7),* 1067–1080.

Phalippou, L. (2020). An inconvenient fact: Private equity returns and the billionaire factory. *The Journal of Investing, 30(1),* 11-39. doi:10.3905/joi.2020.1.153

Phalippou, L., & Gottschalg, O. (2009). The performance of private equity funds. *Review of Financial Studies, 22*(4), 1747-1776. doi:10.1093/rfs/hhn014

Pimco. (2020). PIMCO's capital MARKET Assumptions, February 2021. Retrieved May 15, 2021, from https://global.pimco.com/en-gbl/insights/viewpoints/in-depth/pimcos-capital-market-assumptions-february-2021

Plyakha, Y., R. Uppal, & G. Vilkov. (2014). Equal or Value Weighting? Implications for AssetPricing Tests. SSRN, http://ssrn.com/abstract=1787045.

Poterba, J., & Summers, L. (1988). Mean Reversion in Stock Prices: Evidence and Implications. *Journal of Financial Economics, 22*, 27-59.

Potrykus, M. (2018). Comparison of Investment Performance Measures Using the Example of Selected Stock Exchanges. *Financial Sciences, 23(2),* 30-46.

Poulsen, T., & Lund-Nielsen, B. (2010). Kapitalfondsejede selskaber klarer tilsyneladende lavkonjunkturen bedre. *Finans/Invest, 7,* 11-17.

Reasearch Affliliates. (2021). Asset allocation Interactive. Retrieved May 15, 2021, from https://interactive.researchaffiliates.com/asset-allocation#!/?currency=USD&model=ER&scale=LINEAR&terms=REAL

Robeco. (2020). Outlook 2021-2025 - 5-year expected returns, brave real world. Retrieved May 15, 2021, from https://www.robeco.com/en/themes/expected-returns/2021-2025/

Rockafellar, R. T., & Uryasev, S. P. (2000). Optimization of conditional value-at-risk. *Journal of Risk*, *2*, 21–42.

Rosadi, D., Setiawan, E. P., Templ, M., & Filzmoser, P. (2020). Robust Covariance Estimators for Mean-Variance Portfolio Optimization with Transaction Lots. *Operations Research Perspective*, *7*, 100-154.

Russell. (2021). Russell US indexes. Retrieved May 27, 2021, from https://www.ftserussell.com/products/indices/russell-us

S&P 500, (2020). Retrieved from: https://www.spglobal.com/spdji/en/documents/indexnews/announcements/20201112-1255559/1255559_spgsci2021cpwindexannouncement.pdf

S&P GSCI. (2021). S&P GSCI Methodology. Retrieved from https://www.spglobal.com/spdji/en/documents/methodologies/methodology-sp-gsci.pdf

Sarykalin, S., Serraino, G., & Uryasev, S. (2008). Value-at-Risk vs Conditional Value-at-Risk in Risk Management and Optimization. *Risk Management and Optimization, Informs 2008*, 270–294.

Schenone, K. (2021). Six styles of high yield bond ETFs. Retrieved March 24, 2021, from https://www.etfstrategy.com/six-styles-of-high-yield-bond-etfs-98547/

Schulkin, P. A. (1971). Real estate investment trusts. *Financial Analysts Journal, 27*(3), 33-40. doi:10.2469/faj.v27.n3.33

Schweizer, D., Hass, L. H., & Cumming, D. J. (2008). Portfolio Optimization with Alternative Investments. Available at SSRN: https://ssrn.com/abstract=1091093

Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance, 19(3)*, 425–442.

Shiller, R. (2021). Online Data Robert Shiller. Retrieved from http://www.econ.yale.edu/~shiller/data.htm

Singleton, K. J. (2014). Investor flows and the 2008 boom/bust in oil prices, *Management Science 60*, 300-3.

Stambaugh, F. (1996). Risk and value-at-risk. *European Management Journal, 14(6)*, 612-621.

Stein, C. (1955). Inadmissibility of the Usual Estimator for the Mean of a Multivariate Normal Distribution. *Proceedings of the 3rd Berkeley Symposium on Probability and Statistics 1, 197-206,* Berkeley: Univ. of Calif. Press, 197-206.

Tang, K., & Xiong, W. (2012). Index investment and the financialization of commodities, *Financial Analysts Journal 68*.

Twosigma. (2020). Street View. Retrieved from https://www.twosigma.com/wp-content/uploads/SV_Estimating-Global-Investor-Views-with-Reverse-Optimization.final_.pdf

Vanguard. (2021). Vanguard economic and market outlook for 2021: Approaching the dawn. Retrieved from https://pressroom.vanguard.com/nonindexed/Vanguard-economic-and-market-outlook-report-2021-120920.pdf

Willis Towers Watson. (2016). Global Pension Assets Study 2016. Willis Towers Watson

Wilshire Compass. (2018). Annual Quantitative Analysis of Investor Behavior Report (“DALBAR Report”). Wilshire Compass.

Wuertz, D., Chalabi, Y., Chen, W., & Ellis, A. (2010). Portfolio Optimization with R/Rmetrics

Yahoo Finance. (2020). *Exchanges and Data Providers on Yahoo Finance.* Retrieved from Yahoo Finance: https://help.yahoo.com/kb/finance-for-web/SLN2310.html?locale=en_US

Appendix

Appendix 1 - The historical minimum risk and tangency from the mean-variance, mean-CVaR and Bayes-Stein models (2007-2021)

Estimation period - 2007 until 2021														
MODEL		Portfolio Performance Measures						Weights in the portfolio						
								Traditional investments		*Alternative investments*				
Minimum risk portfolio	Risk Measure	**Expected return in %**	**Anualized arithmetic return in %**	**Risk Measure in %**	**Anualized σ in %**	**Sharpe Ratio**		**Stocks**	**Bonds**	**REITs**	**BDC**	**Listed PE**	**Commodities**	**High Yield**
	Volatility	0,25 %	3,04 %	1,56 %	5,40 %	0,160		-	97,7 %	-	-	2,3 %	-	-
Historical	Cvar	0,26 %	3,17 %	3,38 %	11,71 %	0,077		-	94,4 %	-	-	-	-	5,6 %
Bayes-Stein	Volatility	0,27 %	3,29 %	1,62 %	5,60 %	0,167		-	97,7 %	-	-	2,3 %	-	-
Minimum risk portfolio Stocks and Bonds		*Benchmark portfolio*												
	Volatility	0,26 %	3,17 %	1,64 %	5,68 %	0,159		-	100,0 %	-	-	-	-	-
Historical	Cvar	0,25 %	3,04 %	3,49 %	12,09 %	0,072		-	100,0 %	-	-	-	-	-
Bayes-Stein	Volatility	0,24 %	2,92 %	1,64 %	5,68 %	0,146		-	100,0 %	-	-	-	-	-
Maximum Relative Performance Portfolios														
Historical	Volatility	0,27 %	3,29 %	1,63 %	5,65 %	0,166		-	86,4 %	-	-	5,1 %	-	8,6 %
	Cvar	0,26 %	3,17 %	3,42 %	11,85 %	0,076		-	91,2 %	-	-	0,0 %	-	8,8 %
Bayes-Stein	Volatility	0,27 %	3,29 %	1,62 %	5,62 %	0,166		-	95,5 %	-	-	4,5 %	-	0,0 %
Maximum Relative Performance Portfolios Stocks and Bonds		*Benchmark portfolio*												
	Volatility	0,26 %	3,17 %	1,64 %	5,68 %	0,159		6,5 %	93,5 %	-	-	-	-	-
Historical	Cvar	0,26 %	3,17 %	3,41 %	11,81 %	0,076		3,6 %	96,4 %	-	-	-	-	-
Bayes-Stein	Volatility	0,24 %	2,92 %	1,64 %	5,68 %	0,146		-	100,0 %	-	-	-	-	-

Appendix 2 - The historical *GMM and MD Portfolios (2007-2021)*

Estimation period - 2008 until 2021														
Model		**Performance Measure**						**Weights in the portfolio**						
								Traditional investments		*Alternative investments*				
All-asset portfolios	**Div. Ratio**	**Geometric return in %**	**Arithmetic return in %**	**Anualized arithmetic return in %**	**σ**	**Anualized σ in %**	**Return/ Risk**	**Stocks**	**Bonds**	**REITs**	**BDC**	**Listed PE**	**CMDTY**	**High Yield**
GMM - No Constraints		0,37 %	0,46 %	5,68 %	4,25 %	14,71 %	0,109	62,5%	-	-	-	-	-	37,5%
GMM - max. W 50%		0,37 %	0,45 %	5,52 %	3,98 %	13,77 %	0,113	50,0%	-	-	-	-	-	50,0%
GMM - max. W 30%		0,35 %	0,45 %	5,50 %	4,31 %	14,93 %	0,104	30,0%	10,0%	-	13,7%	16,3%	-	30,0%
MD	1,41		0,21 %	2,61 %	2,05 %	7,11 %	0,105	-	75,2%	-	-	14,9%	9,9%	
Benchmark portfolios														
GMM		0,36 %	0,50 %	6,16 %	5,19 %	18,00 %	0,096	100,0%	-	-	-	-	-	-
MD	1,19		0,31 %	3,73 %	2,03 %	7,02 %	0,151	23,2%	76,8%	-	-	-	-	-

Appendix 3 - The historical minimum risk and tangent portfolio from the mean-variance, mean-CVaR and Bayes-Stein (2009-2019)

Estimation period - 2009 - 2019													
		Portfolio Performance Measures					Weights in the portfolio						
							Traditional investments		*Alternative investments*				
Minimum risk portfolio	Risk Measure	**Expected return in %**	**Anualized arithmetic return in %**	**Risk Measure in %**	**Anualized σ in %**	**Sharpe Ratio**	**Stocks**	**Bonds**	**REITs**	**BDC**	**Listed PE**	**Commodities**	**High Yield**
Historical	Volatility	0,27 %	3,29 %	1,30 %	4,50 %	0,208	-	89,5 %	-	-	10,6 %	-	-
	Cvar	0,35 %	4,28 %	2,64 %	9,15 %	0,133	-	81,8 %	-	-	18,2 %	-	-
Bayes-Stein	Volatility	0,27 %	3,29 %	1,37 %	4,73 %	0,198	-	89,5 %	-	-	10,6 %	-	-
Minimum risk portfolio Stocks and Bonds		*Benchmark portfolio*											
Historical	Volatility	0,23 %	2,80 %	1,54 %	5,33 %	0,149	-	100,0 %	-	-	-	-	-
	Cvar	0,03 %	0,35 %	3,30 %	11,43 %	0,009	7,0 %	93,0 %	-	-	-	-	-
Bayes-Stein	Volatility	0,22 %	2,67 %	1,54 %	5,33 %	0,143	-	100,0 %	-	-	-	-	-
Maximum Relative Performance Portfolios													
Historical	Volatility	0,64 %	7,96 %	2,00 %	6,93 %	0,320	-	48,5 %	-	-	41,3 %	-	10,2 %
	Cvar	0,52 %	6,42 %	3,12 %	10,81 %	0,167	-	57,0 %	-	-	28,5 %	-	14,5 %
Bayes-Stein	Volatility	0,40 %	4,91 %	1,67 %	5,79 %	0,240	-	65,1 %	-	-	29,3 %	-	5,6 %
Maximum Relative Performance Portfolios Stocks and Bonds		*Benchmark portfolio*											
Historical	Volatility	0,63 %	7,83 %	2,69 %	9,32 %	0,234	51,6 %	48,4 %	-	-	-	-	-
	Cvar	0,46 %	5,66 %	4,11 %	14,24 %	0,112	30,0 %	70,0 %	-	-	-	-	-
Bayes-Stein	Volatility	0,33 %	4,03 %	2,69 %	9,32 %	0,123	26,4 %	73,6 %	-	-	-	-	-

Appendix 4 - The historical GMM and MD Portfolios (2009-2019)

Estimation period - 2009 until 2019														
Model		**Performance Measure**						**Weights in the portfolio**						
								Traditional investments		*Alternative investments*				
All-asset portfolios	**Div. Ratio**	**Geometric return in %**	**Arithmetic return in %**	**Anualized arithmetic return in %**	**σ**	**Anualized σ in %**	**Return/ Risk**	**Stocks**	**Bonds**	**REITs**	**BDC**	**Listed PE**	**CMDTY**	**High Yield**
GMM - No Constraints		1,15 %	1,24 %	15,93 %	4,20 %	14,55 %	0,295	-	-	-	-	100,0%	-	-
GMM - max. W 50%		1,01 %	1,09 %	13,86 %	3,95 %	13,70 %	0,275	-	-	-	50,0%	50,0%	-	-
GMM - max. W 30%		0,90 %	0,97 %	12,26 %	3,68 %	12,75 %	0,263	10,0%	-	30,0%	30,0%	30,0%	-	-
MD	1,51		0,32 %	3,93 %	1,58 %	5,49 %	0,203	-	68,4%	-	-	21,1%	10,5%	-
Benchmark portfolios														
GMM		0,71 %	0,78 %	9,77 %	3,87 %	13,41 %	0,202	100,0%	-	-	-	-	-	-
MD	1,23		0,31 %	3,81 %	1,67 %	5,77 %	0,187	26,5%	73,5%	-	-	-	-	-