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Discussion paper

Future world market prices of milk and feed looking into the crystal ball

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Abstract

Both the world milk price and the world feed price have become more volatile during the last 7-8 years. The ability of dairy farmers to adapt quickly to these changing circumstances will be a key driver for future success, considering that feed is the major cost component in milk production and that the milk market is highly volatile. This development has increased the need for research on price dynamics and price forecasting. The first aim of this paper is to apply the wavelet multi-resolution analysis (MRA) to investigate the cyclical dynamics embedded in and between the world milk and feed prices. Second, the aim is to explore both the long and short interactions and the impulse response functions (IRF) between the two price series in the system of a vector error correction model (VECM). Third, the aim is to produce reliable forecasts for both the milk and the feed price applying a SARIMA model, a VECM model and wavelet MRA.

We collected the world milk price and the world feed price series from 2002 to 2015 from the International Farm Comparison Network (IFCN). The analysis revealed that the two price series contain business cycles of approximately 32 months. Further, the two series share a long-run relationship, they are co-integrated, with the feed price as the leading variable. The results also revealed that a combination of different forecasting models can provide reasonably good forecasts of both prices for a period of one year ahead.

1. Introduction

According to the International Farm Comparison Network (IFCN), the world market prices of milk and feed were fairly stable in the period 1996-2006. However, since 2007 both the prices and the price fluctuations have increased significantly. A typical agricultural price series exhibits considerable variability that originates from factors such as, for example, weather, yield and demand. Understanding of price behavior is a critical element to make decisions in uncertain conditions that significantly influence the return of agricultural market participants (Kantanantha et al., 2010; Peterson and Tomek, 2005). Thus, analyzing the past dynamic movements of the milk and feed prices can reveal the deeper relationship between them, and prepare the ground foundation for reliable forecasts. Forecasts of agricultural prices are intended to be useful for farmers, governments, and agribusiness industries. Because of the special position of food production in a nation's security, governments have become both principal suppliers and main users of agricultural forecasts. In many poor countries, the increases in prices of staple foods since the mid-2000s have raised the real incomes of those selling food, many of whom are relatively poor, while hurting net food consumers, many of whom are also relatively poor (Ivanic and Martin, 2008). However, there is evidence that the overall impact of higher food prices on poverty is generally adverse (Ivanic and Martin, 2008). Thus, reliable forecasts of the world feed and milk prices may put governments in a better position to deal with possible upcoming price increases. Good price forecasts can also improve farm decision making in terms of crop and production decision planning; what and how much feed to grow, when to produce the milk, etc. Most of the farmers' decisions are made at the beginning of the growing season, and not all of them are reversible. Therefore price forecasting is a crucial step, and research on agricultural price forecasting is of great significance (Martin-Rodriguez et al., 2012).

In the 1970s and 1980s, the autoregressive integrated moving average (ARIMA) modeling quickly became the dominant paradigm for nonstructural analysis of agricultural time. This kind of model provides a "reduced form" representation for variables based on historical correlations from time-series data series (Myers et al., 2010). Early applications in agriculture include Larson (1960), Leuthold et al. (1970), and Brandt and Bessler (1981). For recent applications of SARIMA (seasonal ARIMA) and ARIMA models in forecasting agricultural prices and commodities see, for example, Fenyvesa et al. (2010) and Hansen (2015). A particular finding from using the ARIMA model is that unit root behavior is a common phenomenon in commodity prices and therefore the prices need to be differenced to induce stationarity (Myers et al., 2010). However, differencing non-stationary variables is unsatisfactory, partly because differencing removes information about the long-run equilibrium relationship between the variables. To overcome this problem, co-integration and error correction models (ECM), a framework which encompasses both the long-run equilibrium and short term dynamics, appeared in the 1990s. These models soon became popular among agricultural economists (Myers et al., 2010).

The components of the cycles inherent in commodity prices can include business cycles which have a period of about 3-6 years. However, agricultural commodity prices will also be influenced by the cycles related to growth and harvest of crops, such as the weather conditions during the growing season. These cycles have shorter periods. Most of the past research only concentrates on one kind of cycle without considering how the different cycles affect the prices separately. Thus, a contribution of this paper is to decompose the world milk and feed prices into cycles with different periods and analyze them scale by scale according to the length of the period. A recently popular decomposition method, wavelet multi-resolution analyze (MRA), is adopted to achieve the scale decomposition and provide a detailed view of how the different cycles can affect the prices. To the best of our knowledge, few studies have applied wavelet decomposition in studies of agricultural prices. An exception is Bowden and Zhu (2007), who showed that short term cycles in New Zealand dairy farm profits are almost wholly the result of changes in commodity prices. Longer cycles are produced by the interaction of commodity prices with the exchange rate (Bowden and Zhu, 2007).

Despite its importance, little recent research exists on price forecasting in agricultural markets. Few papers have been published in the last 15 years that focus on the specification and estimation of price forecasting methods for livestock and milk markets (see Wang and Bessler, 2004 for an example). The lack of research is somewhat understandable since developing predictive models is challenging in an environment like agriculture where markets are subject to major changes. Commodity price behavior over time is basically a mixture of systematic intra and inter-year fluctuations plus randomness, and the variability of prices depends on information flows regarding supply and demand. Given the complexity of price series, many models of behavior model commodity prices (Peterson and Tomek, 2005; Tomek and Peterson, 2001). Among other factors, the models are likely to differ depending on the commodity being studied (Martin-Rodriguez et al., 2012). In a review of agricultural commodity price forecasting, Allen (1994) emphasizes that price forecasts are largely made by conventional econometric methods, with time series approaches occupying minor roles. Because of the dominance of agricultural economists, there has been an overemphasis on explanation, and little interest in the predictive power of models (Allen, 1994). A forecasting model is a simple approximation to reality that is changing due to shifts in institutions and technology. In practice, this calls for the estimation of a variety of flexible models that allow for different weighting schemes between old and new data and for averaging or weighting of individual forecasts (Allen, 1994). According to Allen (1994), vector autoregression (VAR) has proven to be the best single method. Tashman (2000) also argues strongly for recalibration, or re-optimization, rather than simply updating parameters as new data become available. Similarly, Stock and Watson (2003) suggest that the lag structure of the model should be updated over time. For short-term forecasting, combining methods leads to more accurate forecasts (Stock and Watson, 2003).

The prices of milk and feed are obviously interrelated, and from economic theory we know that they influence both profit and supply. Let us take a profit maximizing firm with one output milk (y), two inputs x_1 (feed) and x_2 (fertilizer), milk price p and input prices w_1 and w_2. The profit is defined as total revenue minus total costs:

$$\pi = py - (w_1 x_1 + w_2 x_2) \quad [1]$$

To find the values of x_1 and x_2 which maximize profit we substitute the production function $y = f(x_1, x_2)$ into [1] to obtain $\pi = pf(x_1, x_2) - (w_1 x_1 + w_2 x_2)$. We then take the first partial derivative of this with respect to each of the input quantities, set these equal to zero and solve simultaneously for x_1 and x_2 to obtain the input demand equations $x_1^* = x_1^*(p, w_1, w_2)$ and $x_2^* = x_2^*(p, w_1, w_2)$ where $x_i^*, i = 1,2$ represents the profit maximizing level of the i^{th} - input and $x_i^*(\cdot), i = 1,2$ represents the functional relationship between x_i^* xi* and the prices. To obtain the output supply equation, these two input demand equations are substituted into the production function $y = f(x_1, x_2)$ to obtain the output supply function $y^* = y^*(p, w_1, w_2)$ (See, for example, Chambers, 1988). Here y^* represents the profit maximizing level of milk produced and $y^*(\cdot)$ represents the functional relationship between y^* and the prices. Thus, we notice that the milk supply is governed both by the milk price and the input prices. When output is fixed, as has been the case with milk quotas, for example, in the EU countries, revenue is also fixed, and profit is maximized by minimizing costs. However, from 2015 the EU has abolished the milk quota system, and hence the optimal adaptation for the dairy farmers will change. In a more dynamic environment reliable information about the price dynamics and the future prices of milk and feed will be even more important for the dairy farmers. The ability of farmers to adapt quickly to changing circumstances will be a key driver for future success, considering that feed is the major cost component in milk production and that the milk market is highly volatile (IFCN,2014). Therefore a study of the world market for dairy products and feed requires an analysis of price patterns and price forecasts.

The rest of the paper is organized as follows: In section 2 we present the development of the world milk and feed prices. Section 3 presents the time series methodologies applied in this paper. In the result section, section 4, we present the co-integration analysis and the vector error correction model (VECM) to analyze the long run relationship between and the short term dynamics of the two prices. The result section also contains the cyclic analysis based on the wavelet method and the forecasted results from three different time series models. Finally, we discuss our results and provide conclusions in section 5.

2. Material – the world milk and feed prices

The monthly data of the world milk and feed prices from January 2002 to December 2014 show an obvious common trend (Figure 1).

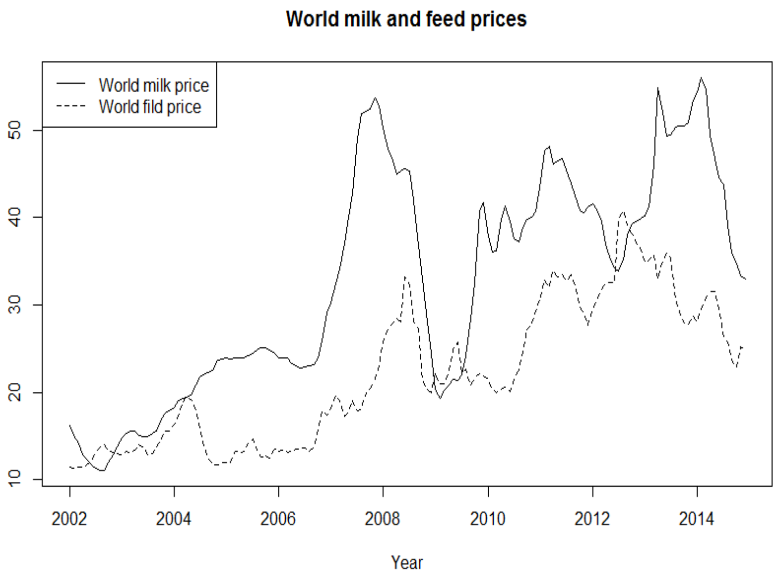

Figure 1. The world milk and feed prices from Jan. 2002 to Dec. 2014

Both prices have a general upward trend with obvious volatility, particularly from 2007. Scholars have identified the following possible causes of price volatility in agricultural commodities: 1) Growing demand for grain-intensive meat production in developing countries (Fuglie, 2008; Hochman et al., 2011); 2) Growing demand for biofuels (Fuglie, 2008; Hochman et al., 2011); 3) Variability of production, consumption and stock demand, and changes in elasticity of supply and demand (Gilbert and Morgan 2010, 2011; Ott, 2014); 4) Climate change (see Peterson et al., 2012 for some examples); 5) New information technology, changed trading practices and new players such as investment funds on the spot and the futures markets (Gilbert, 2010a); 6) Economic growth in emerging economies which increases consumption (Gilbert and Morgan, 2010, 2011); 7) Changes in the oil price and the ethanol price (Myers et al., 2010; Serra et al., 2011; Zhang et al. 2011); and 8) Changes in the interest rate and the US dollar exchange rate (Ott, 2014).

2.1 The world feed price

Animal feeding is the first step in the production of milk, and feed is the main driver of the production costs on dairy farms. Feed is commonly divided into roughage and concentrates. Concentrates may be, for example, grain and oilseeds grown on the farm or they can be purchased off the farm as raw materials, processed feeds or by-products such as, for example, distillers grain, citrus pulp or cottonseed. Comparison of feed-prices worldwide is complicated because it is difficult to compare their contents, for example, energy and protein. The IFCN calculates a comparable world feed price, which is based on a mixture of energy feed, corn or barley, and protein feed, soybean. The price is calculated based on a blend of 70 percent corn or barley, and 30 percent soybean. The advantages of this indicator are that it gives a preliminary idea of regions with high or low feed prices and makes it possible to identify trends. A limitation is that the price is calculated based on USD, which makes it vulnerable to the currency rates in different countries and changes in the exchange rates. Further, in a number of countries dairy compound feed is based on other commodities, which often leads to excessively high price estimates. The data are mainly based on national statistics.

The price of corn spiked in 2008. From 2001 to 2011 world corn prices increased by 150%. According to Hochman et al. (2011), biofuels contributed about 23% of the increase in the price of corn while economic growth and the following growth in demand for meat contributed more than 50%. From 2001 to 2011, the price of soybeans grew by 77%. Growth in income, and thus demand for food, particularly meat, contributed almost 60% to the increase of the price of soybeans (Hochman et al., 2011). The remaining percentage increase in the corn and soybean prices in the period from 2001 to 2011 was, among other factors, due to the consequences of low inventories, weather, large land-use shifts, speculative activity, and export bans (Hochman et al., 2011). In 2013 the world market price of feed was 32.4 USD/100 kg (Figure 1), a decrease from 2012.

2.2 The world milk price

Even though the global dairy trade increases every year, only 14.3 percent of all milk delivered to dairy and 20 percent of all tradable dairy products were traded internationally in 2013 (IFCN 2014). The most commonly traded dairy products are cheese, butter and butter oil, condensed milk and dry products such as milk powder and casein. We collected the world milk price from the IFCN in 2014. The world milk price is based on the weighted average of three indicators: Skimmed milk powder and butter (35%); cheese and whey (45%) and whole milk powder (20%). This product mix reflects the share of each commodity in world market trade, thus the world milk price is a trade weighted average of the major dairy commodities. One advantage with this method is that it reduces the effects of possible detachments of single commodities from the overall world market price. Thus, it makes the world milk price indicator robust to price fluctuations of single dairy commodities and better reflects the real world milk price.

The world market price has exhibited strong fluctuations over the years, ranging from 8.0 USD to 44.5 USD/100 kg from 1981 to 2012 (IFCN, 2014). Similar to the feed price, the milk price peaked in 2007, when the price doubled in one year. The price then fell sharply until 2009. In 2011 the price was 44.5 USD/100 kg, well above the previous peak in 2007. In 2013 the milk price was 50.6 USD /100 kg milk due to a sharp increase of about 38 percent from 2012, and in 2014 the milk price decreased by 21 percent in the first six months. The world market milk price is linked to the farm gate prices in most countries. Thus in 2013, the world market price was reflected by increasing national farm gate prices in 79 of the 100 member countries in the IFCN (IFCN, 2014). The demand for dairy products will continue to grow due to market recoveries and there will be no extra milk on the world market, thus reducing the world dairy stock levels and keeping the milk and feed prices at relatively high levels (IFCN 2014). According to the IFCN, the main drivers for the milk prices are the production process and the world dairy trade (IFCN 2014).

In 2013 the world milk supply was not able to catch up with the rate of change in milk demand. According to the International Farm Comparison Network (IFCN 2014), the demand surplus was the reason for higher milk prices in the world market in 2013. Moreover, the world population is constantly increasing in most of the world's regions, and population increases will probably drive the demand for milk more than per capita consumption (IFCN 2014). To sum up, both milk and feed prices have shown similar price development patterns in recent years and both have been characterized by strong fluctuations.

3. Time series models and forecasting methods

3.1 Co-integration, the Vector Error Correction Model (VECM), and the wavelet method

3.1.1 Co-integration and Vector Error Correction Model (VECM)

From the common development patterns of the milk and feed price in Figure 1 we notice that both series meander randomly, but still show a common trend. To check whether this pattern is caused by a stochastic trend (unit root), or a deterministic trend which changes with *t*, we apply the Augmented Dickey-Fuller test (ADF) (Dickey and Fuller, 1979) to test the stationarity of $\{y_t\}_{t=-\infty}^{\infty}$:

$$\Delta y_t = a_0 + \gamma y_{t-1} + a_1 t + \sum_{i=1}^{p} \phi_i \Delta y_{t-i} + \varepsilon_t \text{ where } \Delta y_t = y_t - y_{t-1} \qquad [2]$$

To choose the number of lags p in equation [2] for the test, we applied Akaike's Information Criterion (AIC) (Akaike, 1969; 1973; 1974) or the Bayesian Information Criterion (BIC) (Schwarz, 1978). The null hypothesis of no significant deterministic trend is $a_0 = a_1 = 0$ and the null hypothesis of non-stationarity or a stochastic trend is $\gamma = 0$. If the null hypothesis of $\gamma = 0$ is not rejected, the series is stochastic non-stationary, and contains unit root. The series will then meander randomly in the long run and the variance of the series will increase with time. Furthermore, for the stochastic non-stationary series, a random shock to the unit root series will have a permanent effect, and the effect of all random shocks will be accumulated. To eliminate the stochastic non-stationarity, one typical procedure is to take the first difference and get a new series $\{\Delta y_t\}_{t=-\infty}^{\infty}$. If $\{\Delta y_t\}_{t=-\infty}^{\infty}$ is stochastic stationary, we say that $\{y_t\}_{t=-\infty}^{\infty}$ is integrated of order 1. If we need to difference $\{y_t\}_{t=-\infty}^{\infty}$ n times to get a stationary series, we see that $\{y_t\}_{t=-\infty}^{\infty}$ is integrated of order n, or contains n unit roots. One restriction in making the non-stationary series stationary by differencing is that the long term property of the original series will be eliminated. However, if we have a group of stochastic non-stationary series, which are all integrated of the same order, we are interested in exploring how the series will meander in the long term. If the series meander randomly but never drift far apart from each other, we still have a long-term equilibrium among the original series. Take the two series $Y = \{y_t\}_{t=-\infty}^{\infty}$ and $Z = \{z_t\}_{t=-\infty}^{\infty}$ as an example. Both are stochastic non-stationary and integrated of the same order. If a linear combination of the two series exists that is stationary, then Y and Z share similar stochastic trends which never diverge too far from each other, and they are co-integrated (Engle and Granger, 1987). The long run equilibrium between Y and Z is modeled in the regression $y_t = \beta_1 + \beta_2 z_t + u_t$, where $\{u_t\}_{t=-\infty}^{\infty}$ is the stationary equilibrium error (Davidson and MacKinnon, 1993) which captures the short-term dynamic deviations from the long-run equilibrium. If we assume that both Y and Z are integrated of order 1, the vector error-correction model (VECM) (Engle and Granger, 1987) explores both the long-term relationships and the short term dynamics between Y and Z. The regression form of the VECM model for Y and Z is:

$$\Delta y_t = \alpha_{10} + \alpha_{11} u_{t-1} + \sum_{i=1}^{p} c_{1i} \Delta z_{t-i} + \sum_{j=1}^{p} d_{1i} \Delta y_{t-i} + \varepsilon_t \qquad [3]$$

$$\Delta z_t = \alpha_{20} + \alpha_{21} u_{t-1} + \sum_{i=1}^{p} c_{2i} \Delta y_{t-i} + \sum_{j=1}^{p} d_{2i} \Delta z_{t-i} + \varepsilon_t \qquad [4]$$

We use equation [3] for a detailed illustration. Equation [3] decomposes Δy_t, or the dynamic adjustments of the variable Y, to changes in Z in two components: first, a long-run equilibrium component given by $u_{t-1} = y_{t-1} - \beta_1 - \beta_2 z_{t-1}$, and second; a short-term component given by $\sum_{j=1}^{p} d_{1i} \Delta y_{t-i}$ and $\sum_{i=1}^{p} c_i \Delta z_{t-i}$. The error correlation term $\alpha_{11} u_{t-1}$ corrects the departure of y_t from its long-run equilibrium. The coefficient α_{11} governs the speed of the adjustment back towards the long-run equilibrium. α_{11} is usually expected to be negative so that a positive (or negative) departure from equilibrium in the previous period will be corrected by a negative (or positive) value in the current period. The Engle-Granger two step method (Engle and Granger, 1987) can be applied to estimate the error correction model. First we use least squares to estimate the co-integrating relationship $y_t = \beta_1 + \beta_2 z_t + u_t$. Then we use the lagged residuals $\Delta\hat{u}_{t-1} = y_{t-1} - \hat{\beta}_1 - \hat{\beta}_2 z_{t-1}$ as the right hand side in equations [3] and [4], estimating equations [3] and [4] with a second least squares regression.

3.1.2 The wavelet method

In the result section we will use the VECM model from section 3.1.1 to analyze the relationship between two stochastic non-stationary series: world milk price (denoted as M) and world feed price (denoted as F) from 2002 to 2013. The inherent non-stationarity of both M and F can be due to several sources exhibiting different dynamics. As mentioned in section 1.1, the agricultural commodities can include cyclic characteristics with different period lengths. If we look closer at Figure 1, both series show cycles with different degrees of fluctuation over the whole period. However, we notice a regime shift around 2007: while small cycles or fluctuations dominate before 2007, larger cycles are more dominant after 2007. This kind of irregular cyclical behavior can be caused by a mix of relatively short agricultural cycles with an annual or seasonal period, and longer business cycles with periods of 3–6 years. The two different cycles have different effects before and after 2007. In the result section, we will adopt wavelet multiresolution analysis (MRA) to decompose the world feed and world milk prices into cyclic components with different periods. We will analyze them scale by scale and also explore how the periodicity of the cycles may change over time. There is increasing interest in adopting the wavelet technique to explore and forecast various dynamic features of economic and financial time series by, for example, Ramsey (1999), Schleicher (2002) and Crowley (2005), Vidakovic (1999), Percival and Walden (2000) and Gençay *et al.* (2001). However, we find few applications in studies of agricultural commodities. This paper will therefore contribute to the literature in the field.

The wavelet method appears recently as a viable and modern tool for investigating the non-stationary dynamics in various scientific fields such as hydrodynamics, geophysics, data processing, image compression, detection of discontinuities, neural networks (Yousefi, 2005). The wavelet methodology represents an arbitrary time series in both time and frequency domains by convolution of the time series with a series of small wavelike functions. Corresponding to the time-infinite sinusoidal waves in the Fourier transform, the time-located wavelet basis functions $\{\psi_{jk} : j, k \in \mathbb{Z}\}$ used in the wavelet transform are generated by translations and dilations of a basic mother wavelet $\psi \in L^2(\mathbb{R})$. The function basis is constructed through $\psi_{jk}(t) = 2^{j/2}\psi(2^j t - k)$, where k is the location index and j is the scale index that corresponds to the information inside the frequency band $(\frac{1}{2^j}, \frac{1}{2^{j-1}})$. For a signal f, its wavelet transform is given by the wavelet coefficients $f^* = \{\gamma(j,k)\}_{k,j\in\mathbb{Z}}$ with $\gamma(j,k) = \langle f, \psi_{jk} \rangle = \int f(t)\psi^*_{jk}(t)dt$, which represent the resolution at time k and scale j.

For a discrete time series vector $Z' = \{Z_t, t = 0,...,T-1\}$, the wavelet coefficients for Z are obtained via, for example, maximum overlap discrete wavelet transform (MODWT). Generally, after the level J MODWT, we can get $J+1$ transformed vectors $\mathbf{W}_1,...,\mathbf{W}_J,\mathbf{V}_J$. The T dimensional vectors $\mathbf{W}_j$ ($j = 0,1,2,...,J$) and $\mathbf{V}_J$ are computed by $\mathbf{W}_j = W_j\mathrm{Z},\ \mathbf{V}_J = V_J\mathrm{Z}$ with $j = 0,1,2,...,J$. The $T \times T$ matrices W_j ($j = 0,1,2,...,J$) can be viewed as the high-pass filter which extract out the higher part of the frequency band in Z. The output from this high-pass filtering are wavelet coefficients $\mathbf{W}_j$, which corresponds to the local fluctuations of scale $\tau_j = 2^{j-1}$. The $T \times T$ matrix V_J is then the low-pass filter which filters out the lowest part of the frequency band in Z. The outputs from this low-pass filtering are wavelet scaling coefficients $\mathbf{V}_J$, which correspond to averages on a scale of $\lambda_J = 2^j$. To reconstruct the original series $\mathrm{Z} = \{Z_t, t = 0,...,t-1\}$ from $\mathbf{W}_1,...,\mathbf{W}_J,\mathbf{V}_J$, we apply the MODWT based synthesis:

$$\mathrm{Z} = \sum_{j=0}^{J} W_j^T \mathbf{W}_j + V_J^T \mathbf{V}_J = \sum_{j=0}^{J} D_j + S_J \qquad [5]$$

From equation [5] the original series Z is decomposed into $J+1$ detail scales $D_0, D_1, \ldots, D_J$ and a smooth scale S_J. D_j ($j = 0,1,2,...,J$) is the j^{th} level MODWT detail which captures the local fluctuations over the whole period in the scale with frequency band $(\frac{1}{2^{j+1}}, \frac{1}{2^j})$ of Z. The scale S_J is the J^{th} level MODWT smooth containing information in the frequency band $(0, \frac{1}{2^J})$ and provides a "smooth" or overall "trend" of the original signal. When adding all the frequency bands of D_j ($j = 0,1,2,...,J$) and S_J together, we get the frequency band (0, $\frac{1}{2}$), which is the frequency band for the original discrete data Z. As all the scales ($D_0, D_1, \ldots,$ D_J, S_J) are still time series data and they each include T values, the time information is preserved in each scale. This time-scale based analysis is the wavelet multi-resolution analysis (MRA). For more information about the MODWT and MRA, we refer to Vidakovic (1999), Percival and Walden (2000), and Gençay et al. (2001).

3.2 Three methods for forecasting the feed and the milk price

To make forecasts for 2014 and to assess the forecasts, we use the dataset from January 2002 to December 2013 as the training set, and 2014 as the test set. However, to make forecasts for 2015 we use the dataset from January 2002 to December 2014 as the training set. We compare three different forecasting methods: The ARIMA model (Shumway and Stoffer, 2010), the VECM and the wavelet method (Grossmann and Morelet, 1984).

3.2.1 Forecast based on the ARIMA model

We first use the ARIMA model to forecast the feed price F and the milk price M. For each series, we tried models with different time lags, and to choose between models we picked the model with the lowest AIC and BIC values. Once we had fitted a suitable model to the historic data, we used the model to forecast future milk delivery. To assess the precision of the forecasts, prediction intervals were calculated. Based on the data from January 2002 to December 2013 the models for the feed price F and the milk price M can be written:

$$F_t - F_{t-1} = \varepsilon_t + 0.200\varepsilon_{t-1}, \quad \varepsilon_t \sim N(0, 2.203) \qquad [6]$$

$$M_t - M_{t-1} - 0.301(M_{t-1} - M_{t-2}) = \varepsilon_t + 0.404\varepsilon_{t-1} - 0.207\varepsilon_{t-12} - 0.084\varepsilon_{t-13} \quad \varepsilon_t \sim N(0, 2.456) \quad [7]$$

Equation [6] and [7] express F_t and M_t as a function of their past value and past random errors. Then at time T, the optimal j-step ahead forecasts of F_{T+j} and M_{T+j} are then defined as the conditional expectations $E(F_{T+j} | \Upsilon_T)$ and $E(M_{T+j} | \Upsilon_T)$, where Υ_T denotes the information till time T and the formation of $E(F_{T+j} | \Upsilon_T)$ and $E(M_{T+j} | \Upsilon_T)$ can be deducted from equation [6] and [7].

3.2.2 Forecast based on the VECM model

The forecasting based on the ARIMA model is a univariate method which produces forecasts for the milk and feed prices separately. However, the two prices are obviously interrelated, and there is feedback or dependence between them. The feedback between the two prices can give valuable information for future forecasts. The VECM introduced in section *3.1.1* treats all variables as jointly exogenous and can capture the feedback among the series. In the VECM system, each variable is allowed to depend on its past realizations and on the past realizations of other variables. Since the feed price and the milk price are tied to each other, this method fits well for forecasting the milk and feed prices. Another advantage of applying the VECM model is that it can generate the impulse response function (IRF) to measure the effects of a random shock to an endogenous variable itself or on another endogenous variable. The IRF shows how those effects develop with time. (Sims, 1980), and we use it to trace out the time path of the series' responses to shocks in the milk price and feed price. Because the underlying shocks in, for example, the feed price are less likely to occur in isolation, we will apply orthogonal impulse responses (Sims, 1980, Enders, 2004) based on a Cholesky decomposition of the covariance matrix of random shocks.

3.2.3 Forecast based on the wavelet MRA

Section *3.1.2* shows that by using wavelet MRA, we can decompose original series Z into $J+1$ detail scales $D_0, D_1, \ldots, D_J$ and a smooth scale S_J. The finest scale D_0 contains most of the random noise or outliers and can be deleted when forecasting, the finer scales $D_1, \ldots, D_J$ capture the fluctuations at higher frequencies, while the coarse scale S_J reveals the long term trend. For forecasting by the wavelet method, we can carry out adaptive extensions separately of D_j ($j=1,2,...,J$) and S_J to forecasts over different forecasting horizons. As the decomposed scales D_j ($j=1,2,...,J$) and S_J show different behavior, we can use the most appropriate methodology for modeling and extension of the corresponding scale separately. As D_1 to D_J contain oscillations behavior, trigonometric models with different frequencies are reasonable choices to fit the data and calculate the extended values. As S_4 represent the long term trend, the regression model on time t is applied to get the extension as the trend prediction. At the last step, all the individual predictions are added together and we get the aggregate forecast, which are the forecasted values for the milk and feed prices in this paper.

3.2.4 Evaluating the forecast results

There are many ways to evaluate the accuracy of forecasting methods. They all involve looking at past data to time $t-1$ and comparing the forecasted value $\hat{y}_{t|t-1}$ at time t using the model and the estimated parameters with the actual observation y_t . Different accuracy measures often give different results (Hyndman et al., 2008). Therefore the choice of accuracy measure must be adapted to the problem at hand. The two most commonly used scale-dependent accuracy measures are based on the absolute errors or squared errors, Hyndman et al. (2008). If we denote the forecast error $e_t = y_t - \hat{y}_{t|t-1}$, the mean absolute error is simply: $\mathrm{MAE} = \frac{\sum_{t=1}^{T} |e_t|}{T}$ while the root mean squared error is: $\mathrm{RMSE} = \sqrt{\frac{\sum_{t=1}^{T} e_t^2}{T}}$. When comparing forecast methods on a single data set, the MAE is popular as it is easy to understand and compute. A difference between the two measures is that because the RMSE squares the errors, it penalizes large errors more severely than the MAE. We will apply both measures.

4. Results

4.1 Co-integration analysis and the VECM model

In this section we use monthly data from January 2002 to December 2013. The world milk price is denoted $M = \{M_t, t = 1,...,144\}$, and the world feed price $F = \{F_t, t = 1,...,144\}$. Based on the ADF test procedure we could not reject either the null hypothesis $\gamma = 0$ or the null hypothesis $a_0 = a_1 = 0$ for both prices. Thus, both the milk and the feed prices contain only stochastic trends. However, the null hypothesis of non-stationarity was rejected for the first differences of both prices, which means that both series are integrated of order 1. In the next step we carry out a co-integration analysis by building a linear regression between the two prices and check if the residuals are stationary. The estimated regression model is:

$$M_t = 7.001 + 1.136 F_t + R_t \qquad [8]$$

The ADF test of the residuals R_t showed that R_t is stationary according to the MacKinnon (1991) adjusted critical values. Thus, we can conclude that the world milk price and the feed price are co-integrated with long-term linkages between them. Equation [8] is super-consistent, which implies that as $t \to \infty$, we do not no need to include R_t in equation [8]. A 1 USD/100 kilogram price increase in the feed price will lead to a 1.14 US/ 100 kilogram price increase in the milk price. We now build VECMs to explore the speed at which the prices return to their long-run equilibrium. The estimated VECMs for the feed price F and the milk price M are:

$$\Delta M_t = \alpha_{10} + \alpha_{11} R_{t-1} + \sum_{i=1}^{p} c_{1i} \Delta F_{t-i} + \sum_{j=1}^{p} d_{1i} M_{t-i} + \varepsilon_{Mt} \qquad [9]$$

$$\Delta F_t = \alpha_{20} + \alpha_{21} R_{t-1} + \sum_{i=1}^{p} c_{2i} \Delta M_{t-i} + \sum_{j=1}^{p} d_{2i} F_{t-i} + \varepsilon_{Ft} \qquad [10]$$

where $R_{t-1} = M_{t-1} - 7.001 - 1.136 F_{t-1}$ is estimated in the first step and estimated coefficients in equation [9] are:

Table 1. Estimated coefficients for equation [9]

α_{11}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}
0.059 (0.018) **	0.330 (0.09) ***	0.212 (0.0937) *	0.013 (0.096)	0.094 (0.096)	0.113 (0.093)	0.786 (0.0854) ***	0.342 (0.111) **	0.202 (0.114)	0.023 (0.110)	0.210 (0.091) *

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

The estimated coefficients in equation [10] are:

Table 2. Estimated coefficients for equation [10]

α_{21}	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}
0.045 (0.018) *	0.097 (0.084)	-0.062 (0.108)	0.086 (0.112)	-0.270 (0.108) *	0.030 (0.089)	0.233 (0.087) **	0.033 (0.092)	0.049 (0.094)	-0.002 (0.094)	0.013 (0.091)

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Table 1 and Table 2 show that in equation [9], both c_{11} and c_{12} are significant while in equation [10], only c_{24} is significant. It shows that in the short term, the feed price will affect the milk price faster and stronger than the effect of the milk price on the feed price. Although feedback exists in milk and feed prices, it is the feed price that leads the milk price.

Due to the autoregressive structure of equations 9 and 10, a random shock ε_{Mt} to the milk price, will affect both the future milk price and the future feed price. A random shock ε_{Ft} to the feed price will also affect future milk and feed prices. Price shocks of milk and feed can be due to different causes. Production shocks are typically influenced by yields, which in turn are typically influenced by weather changes. Further, there are demand shocks due to changes in consumption of milk and feed, exchange rate shocks, shocks due to changes in stock demand, and shocks due to peaks in the crude oil price (Ott, 2014). To explore how the milk and feed prices respond to a shock of +1 USD/ 100 kilogram milk and feed we show the impulse response functions in Figures 2 and 3.

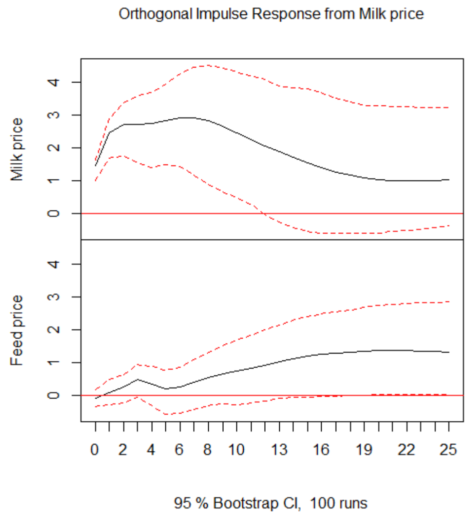

Figure 2. Orthogonal impulse responses of the milk and feed prices to a +1 USD impulse on the milk price

Figure 2 shows the response paths of the milk and feed prices to a + 1 USD/100 kilogram milk shock in the milk price. From the upper panel of Figure 2, we see that the milk price increases from around 1.45 USD after one month to around 2.92 USD after 8 months. Then the milk price starts to gradually decline to a long term level of around 1 USD after a period of approximately 20 months. From the lower panel of Figure 2 we see that the feed price is less affected by a shock in the milk price, and the feed price reaches a long term level of around 1.31 USD after a period of approximately 20 months.

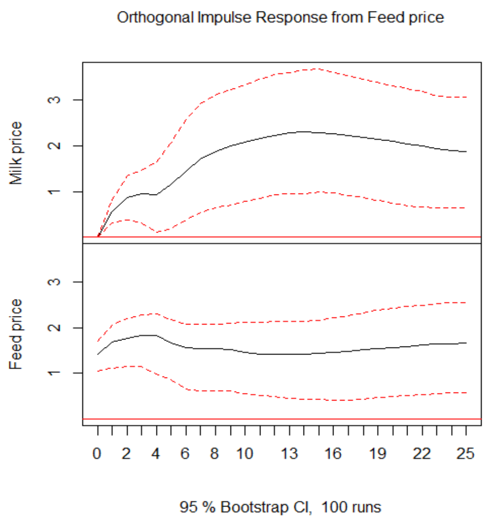

Figure 3. Orthogonal impulse responses of the milk and feed prices to a +1 USD shock in the feed price

Figure 3 shows the response paths of the milk and the feed price to a +1 USD/100 kilogram feed shock in the feed price. From the upper panel of Figure 3 we notice that the milk price responds by gradually increasing to a maximum after 14 months, and reaches a long term level of approximately 1.90 USD after 20 months. Similarly, from the lower panel of Figure 3 we notice that the feed price shows less response as compared to the milk price, and stabilizes at around 1.65 USD after a period of approximately 20 months. Taken together, Figures 2 and 3 show that the milk price responds faster to a shock in the feed price than the feed price responds to a similar shock in the milk price. Thus, the impulse response functions underline the findings from the VECM, that the feed price is the leading variable.

4.2 Analysis of possible cycles and trends

We apply a fourth-level wavelet multi-resolution analysis with $J = 4$ to decompose both the milk and the feed price. The decomposition of the two series to December 2013 is shown in Figure 3.

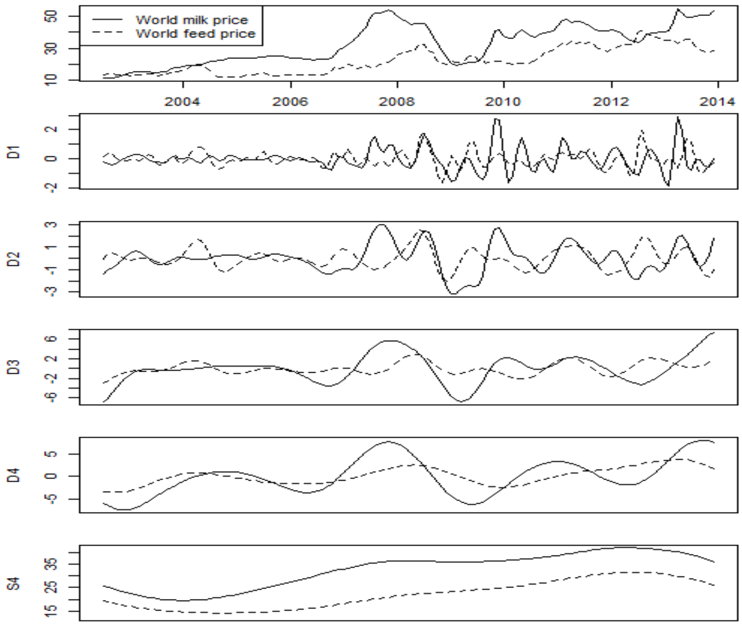

Figure 4. Wavelet decomposition of the world milk and feed prices in USD/ 100 kilogram

In Figure 4, the first line is the original series for world milk and feed prices. The next 5 lines correspond to decomposed series, the detailed scales D_j, ($j = 1, 2, 3, 4$) and the smooth scale S_4. D_j, ($j = 1, 2, 3, 4$) capture the local fluctuations over the whole period in the scale with frequency band $(\frac{1}{2^{j+1}},\frac{1}{2^j})$. As the data are monthly, the frequency band $(\frac{1}{2^{j+1}},\frac{1}{2^j})$ will represent a time period of 2^j to 2^{j+1} months. For $j=1$, we inspect the scale D_1 in the second line of Figure 4. D_1 captures the cyclical variation in the milk price over a period of four months or one quarter. We notice that in these short time cycles the feed price shows larger fluctuations than the milk price before 2007. The feed price fluctuates approximately quarterly before 2007. However, after 2007, both the feed and the milk prices show larger fluctuations. The second detail scale D_2 catches the variation within a time period of 8 months. The overall picture is similar to D_1. The third detail scale D_3 catches the cyclical variation within 16 months. The feed price shows similar fluctuation over the whole period. However, the milk price shows almost no fluctuation from 2003 to 2007 and the curve is quite flat. The fourth detail scale D_4 represents the cyclical variation within a period of 32 months, or almost three years. We can call this a business cycle. Both the milk and the feed prices show cyclical fluctuations with a period of 32 months over the whole period, which means that both prices inherit business cycles. However, all the cycles peak at around January 2008, and are at their lowest at around 2009-2010. The smooth scale S_4 is the long run trend in the frequency band $(0,\frac{1}{2^4})$. Both prices show an upward trend over most of the period. However, both prices show a downward trend from the beginning of 2013.

4.3 Forecasting using the ARIMA, VECM and wavelet models

In this section we first present the MAE and the RMSE for the three models, based on forecasts and actual milk and feed prices for 2014. Then we present a forecast for 2015 where we combine different methods according to the results from 2014. The MAE and the RMSE of the feed price forecasts for 2014 are given in Figures 5 and 6.

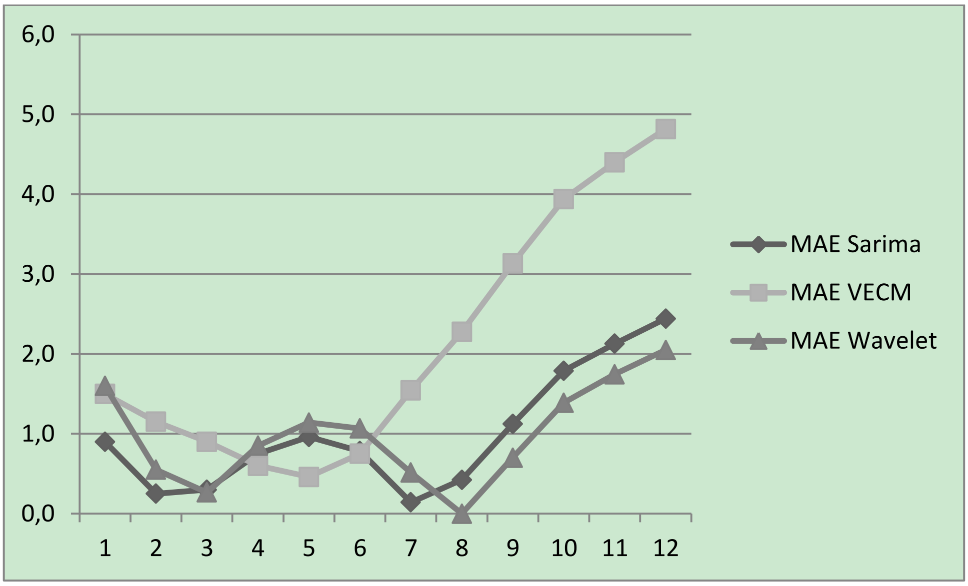

Figure 5. MAE for the feed price forecasts in USD/100 kg milk for the three methods for each month in 2014

From Figure 5 we notice that the SARIMA performs best for the first seven months except for the month of May, but for the rest of the year the Wavelet performs best. The VECM is outperformed by the other methods except for the interval from four to six months. The SARIMA yields a forecast error of less than approximately +/− 1 USD/100 kilograms of milk for the first seven months of 2014. This represents an error of approximately 7%, which is quite good.

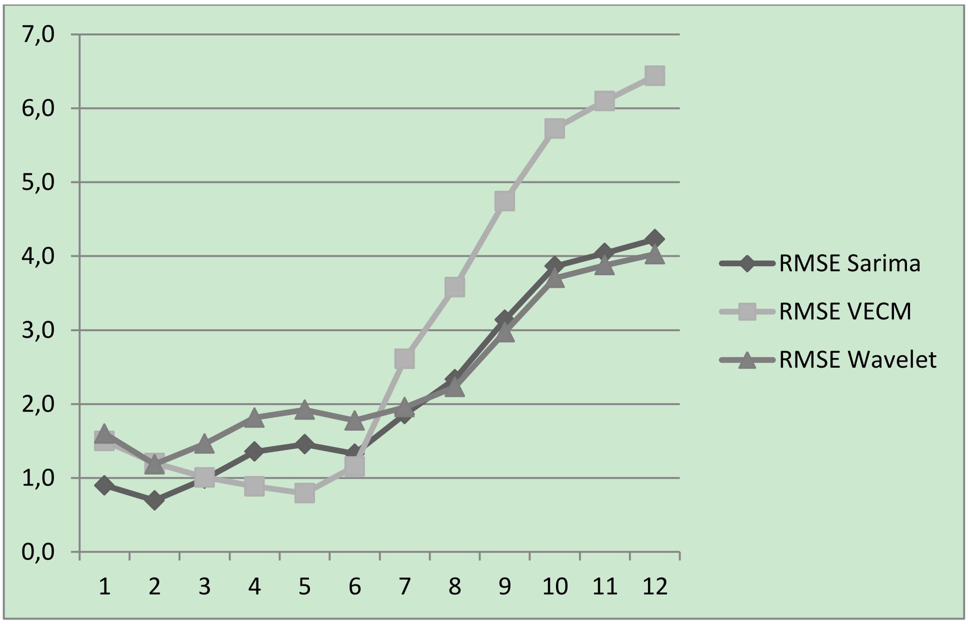

Figure 6. RMSE for the feed price forecasts for the three methods for each month in 2014

From Figure 6 we notice that the VECM and the SARIMA yield the most reliable forecasts for the first six months, while the wavelet performs slightly better than the SARIMA from eight months on.

The forecast errors for the world milk price for the three different methods are given in Figures 7 and 8.

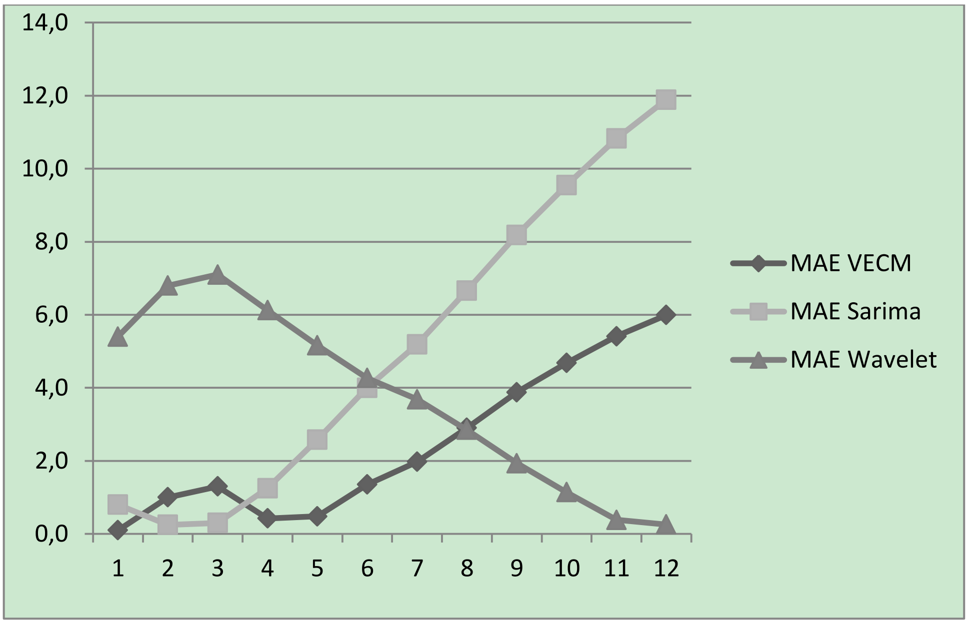

Figure 7. The MAE for the milk price forecasts in USD/100 kg milk for the three methods for each month in 2014

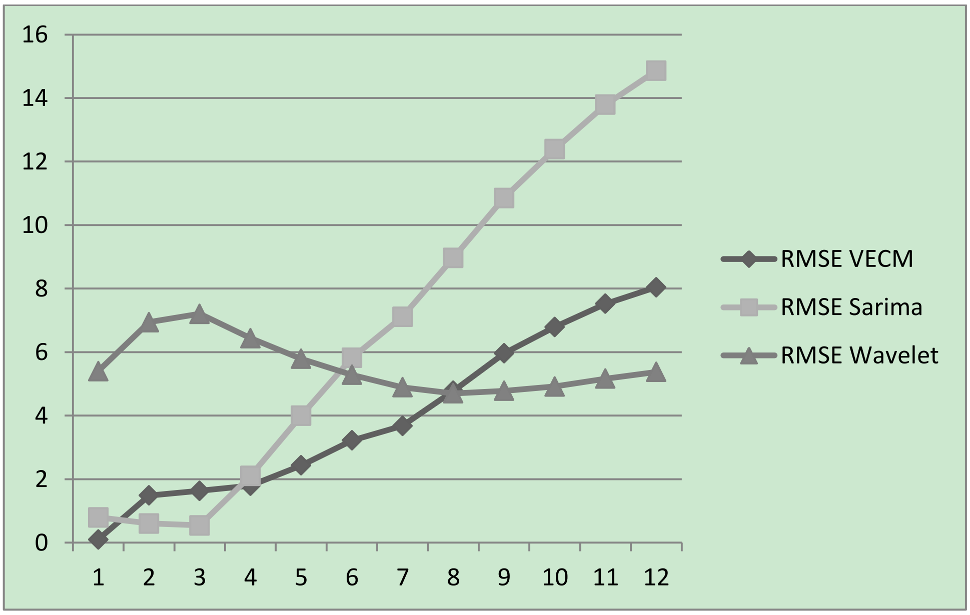

Figure 8. The RMSE for the milk price forecasts for the three methods for each month in 2014

From Figure 7 we notice that, on average, the SARIMA and the VECM perform best for the first four months. The VECM yield forecasts errors of less than +/− 2 USD/100 kilograms of milk for the first seven months of 2014, which represents an error of approximately 8 %. After eight months the wavelet reaches a similar error level. From Figure 8 we notice a similar picture to that in Figure 7. The SARIMA and the VECM perform best for the first four months, and from eight months on the wavelet takes the lead.

Based on the above results, we present a combined forecast for 2015. We apply the SARIMA model to forecast the first seven months for the feed price, and the wavelet model for the remaining five months. Similarly, we apply the VECM to forecast the first eight months for the milk price and the wavelet for the remaining four months. The forecasts are based on the time series from January 2002 to December 2014, and the results are given in Figure 9.

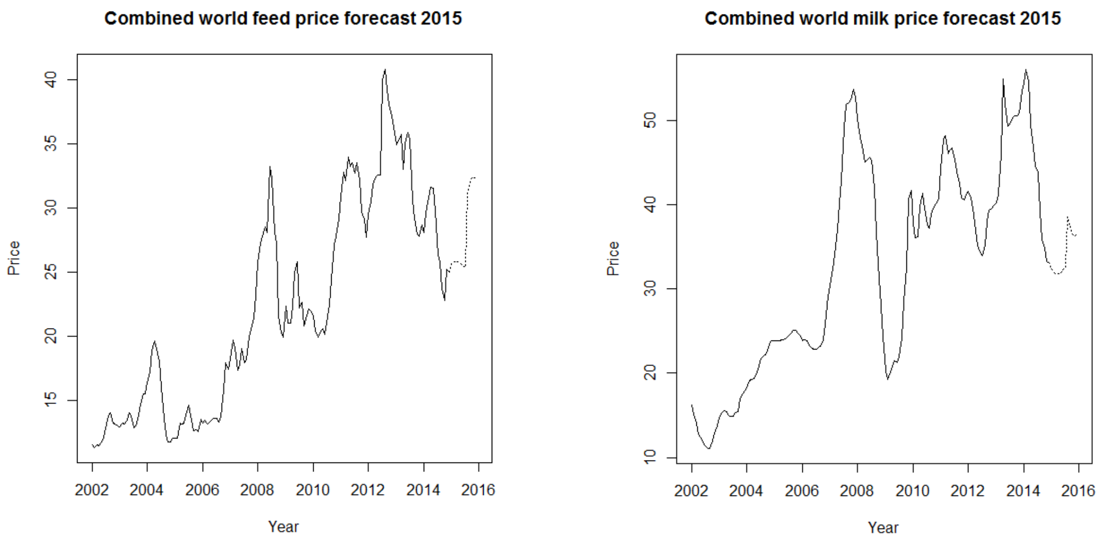

Figure 9. Forecast of the feed price and the milk price from Jan. 2015 to Dec. 2015 based on combinations of different methods.

In Figure 9, we notice that both the feed price and the milk price will increase during 2015. Moreover, Figures 5-8 show that the VECM and the SARIMA perform best in the first seven to eight months, and then the wavelet takes over. Thus the wavelet performs best in the “long term”, while the VECM and the SARIMA have the best “short term” properties. To prove this, we provide a three year ahead forecast using all the three methods (Figure 10)

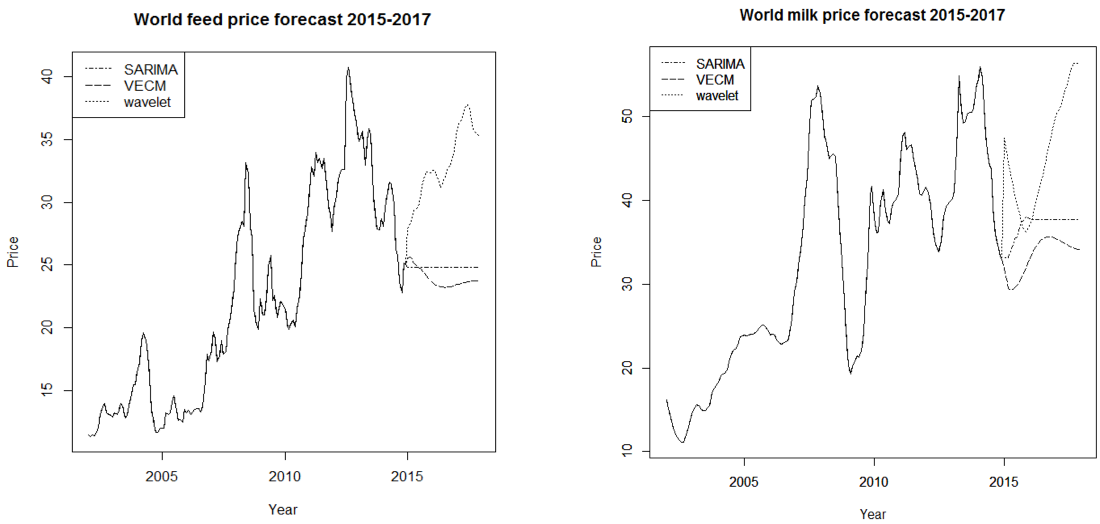

Figure 10. Forecast of the feed price and the milk price from Jan. 2015 to Dec. 2017 based on combinations of different methods.

Figure 10 shows that the wavelet forecast captures the larger cycles and trends while the forecasts from the SARIMA and the VECM converge to a stable level after about 1 to 1.5 years. One can argue that three years is too long for forecasting. However, it can still provide some ideas about how the prices will develop in the period.

5. Discussion and conclusion

The findings reported here show that both the world feed price and the world milk price are non-stationary time series with one unit root. Further, the results show that they are co-integrated, they share similar stochastic trends and never diverge too far from each other. However, according to our findings from the VECM and the IRF, the feed price is the key variable and leads the milk price in the short term dynamics. The forecasts show that a combination of the VECM and the SARIMA produce fairly reliable forecasts of the feed price six months ahead, but after eight months the wavelet performs slightly better. The SARIMA yields a fairly reliable forecast of the feed price for a period of seven months ahead, while the wavelet performs best afterwards. There is a pattern where the VECM and the SARIMA perform best in the first seven to eight months, and then the wavelet takes over. Thus the wavelet performs best in the “long term” and can capture the business cycles while the VECM and the SARIMA have the best “short term” properties. Thus, this study shows that the three different models complement each other. Like Tashman (2000) and Stock and Watson (2003), we therefore recommend to combine different forecasting models, since they all have their strengths and weaknesses. Our finding is somewhat in contrast to Allen (1994) who concludes that there is one best method.

For large dairy exporting regions or countries like the EU-28, the US and New Zealand it is important to have an idea of how the feed and milk prices in the world market will develop. Similarly, large importers of dairy products like China, the Russian Federation, Mexico and Japan also have an interest in monitoring the world feed prices and milk prices closely to make optimal buying decisions. The world feed and milk market prices are characterized by large fluctuations and the degree and timing of changes are different. Due to these changes, both sellers and buyers can suffer great losses. Thus, it is essential for both dairy companies and farmers to have reliable forecasts. For developing countries, it is important to have reliable forecasts of both the milk price and particularly of the feed price, since the feed price leads the milk price. Further, soy bean and corn are used for both human consumption and for feedstuffs. Thus, reliable forecasts can give developing countries early warnings of possible price increases, which may threaten the overall supply of food, especially for the poor. Finally, for feed companies it is also important to be able to forecast future prices of different feed constituents, in order to combine them in an economically and nutritionally optimal manner.

The impulse response functions show that shocks in either the feed price or the milk price have long term effects. Given the long and complicated production chain, it is no surprise that the transfer of an increase in the milk price to the feed price takes some time. The farmers may respond to an increase in the milk price by adapting their feeding schemes and increasing their production, but this takes some time. Further, if the production capacity is already fully utilized, the farmers will not in any way be able to increase their milk production. On the other hand, a price increase in soy bean and corn leaves less scope for farmers to adapt their production, because soy bean and corn are main ingredients in concentrates. Either the farmers have to demand a higher milk price to cover their increased costs and maintain profits, or they have to reduce the milk volume produced.

Our study indicates that there are business cycles of between two to three years in the dairy sector. Future studies could explore these long term business cycles further and explore how they are related to changes in, for example, exchange rates as suggested by Bowden and Zhu (2007). This study also reveals that the two price series contain inter-annual cycles similar to those reported by Ott (2014) in the cereal sector. Future studies could further explore these intra-annual changes in the world milk and feed prices. Future studies could also compare national prices with the world market prices.

As forecasting tasks can vary by many dimensions in terms of the length of forecast horizon, the size of test set, the forecast error measures and the interval of data etc., it is unlikely that, for example, time series models will be better than all other models for all forecasting scenarios. The underlying presumption behind time series models that correlation between adjacent points in time is best explained in terms of a dependence of the current values on past values, means that the models depend heavily on the time period analyzed. Thus, the models assume that the historical patterns will not change during the forecast period, and that future errors remain uncorrelated. Thus, analysis of other periods could produce other models. This dependence makes it necessary to recalibrate and update the models regularly. Thus, our results are very much in line with the recommendations of Stock and Watson (2003) and Tashman (2000).

In conclusion, the world milk and feed prices have shown increased volatility since 2007-2008. Both price series contain business cycles of between two and three years in length, as well as short term cycles of four and eight months in length. In the short term, the feed price fluctuates more than the milk price. The milk and the feed prices are co-integrated, with the feed price as the leading variable. The results show that a combination of ARIMA, VECM and wavelet models yield reasonably good forecasts within a period of one year.

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