NORGES HANDELSHØYSKOLE

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Integration and coordination of supply chain

Case studies in the forestry and petroleum industries

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PhD thesis, Department of Finance and Management Science

NORGES HANDELSHØYSKOLE

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Introduction

1.1 General introduction to planning of the supply chain

A supply chain can be defined in many ways and one by Mentzer et al. (2001) is as follows.

> "A supply chain is a set of three or more entities (organization or individuals) directly involved in the upstream and downstream flows of products, services, finances and/or information from a source to an ultimate customer."

Generally speaking, necessary information should be mutually shared among the firms involved in the supply chain and all the activities within the chain should be performed with a consistent customer focus.

The official definition for supply chain management (SCM) developed by the Council of Supply Chain Management Professionals is

> "Supply chain management encompasses the planning and management of all activities involved in sourcing and procurement, conversion, and all logistics management activities. Importantly, it also includes coordination and collaboration with channel partners, which can be suppliers, intermediaries, third party service providers, and customers. In essence, supply chain management integrates supply and demand management *within and across* companies." (CSCMP 2007)

Mentzer et al. (2008) proposed a framework to more clearly define the scope of SCM decision-making relative to the disciplines of marketing, logistics, production, and operations management. Figure 1.1 portrays the distinct levels of interactivity for each managerial area listed within the functions. While functional management focuses on the most efficient control and execution of existing activities, operations management is concerned with improvement of processes particularly as related to coordination of the *cross-functional* interfaces within a firm. When the scope of decision-making for each individual functional area crosses *organizational* boundaries, it is then considered within the managerial realm of SCM. Stadtler (2005) pointed out that if the organizational units belong to one single enterprise, hierarchical coordination is possible and prevailing in this *intra-organizational* supply chain.

Planning supports decision-making by identifying alternatives of future activities and selecting some good ones or even the best one (Fleischmann et al. 2008). Figure 1.2 illustrates the two-dimensional supply chain planning matrix. The *X*-axis represents the material flow across the activities in a supply chain, i.e., procurement, production, transportation and distribution, and sales. The *Y*-axis expresses the different levels of planning intervals ranging from "aggregated long-term" to "detailed short-term" planning. Advanced planning systems (APS) consist of several software modules, each of them covering a certain range of planning tasks and linked by vertical and horizontal information flows (Meyr et al. 2008). Optimization techniques are applicable in the areas of Strategic Network Planning, Master Planning, Production Planning and Scheduling, and Distribution and Transportation Planning. The remaining areas are typically tackled with statistics (Demand Planning), rules-based algorithms (Demand Fulfillment & ATP), or transactional and/or rules-based processing (MRP) (Kallrath and Maindl 2006).

Figure 1.1 The domain of supply chain management (Mentzer et al. 2008)

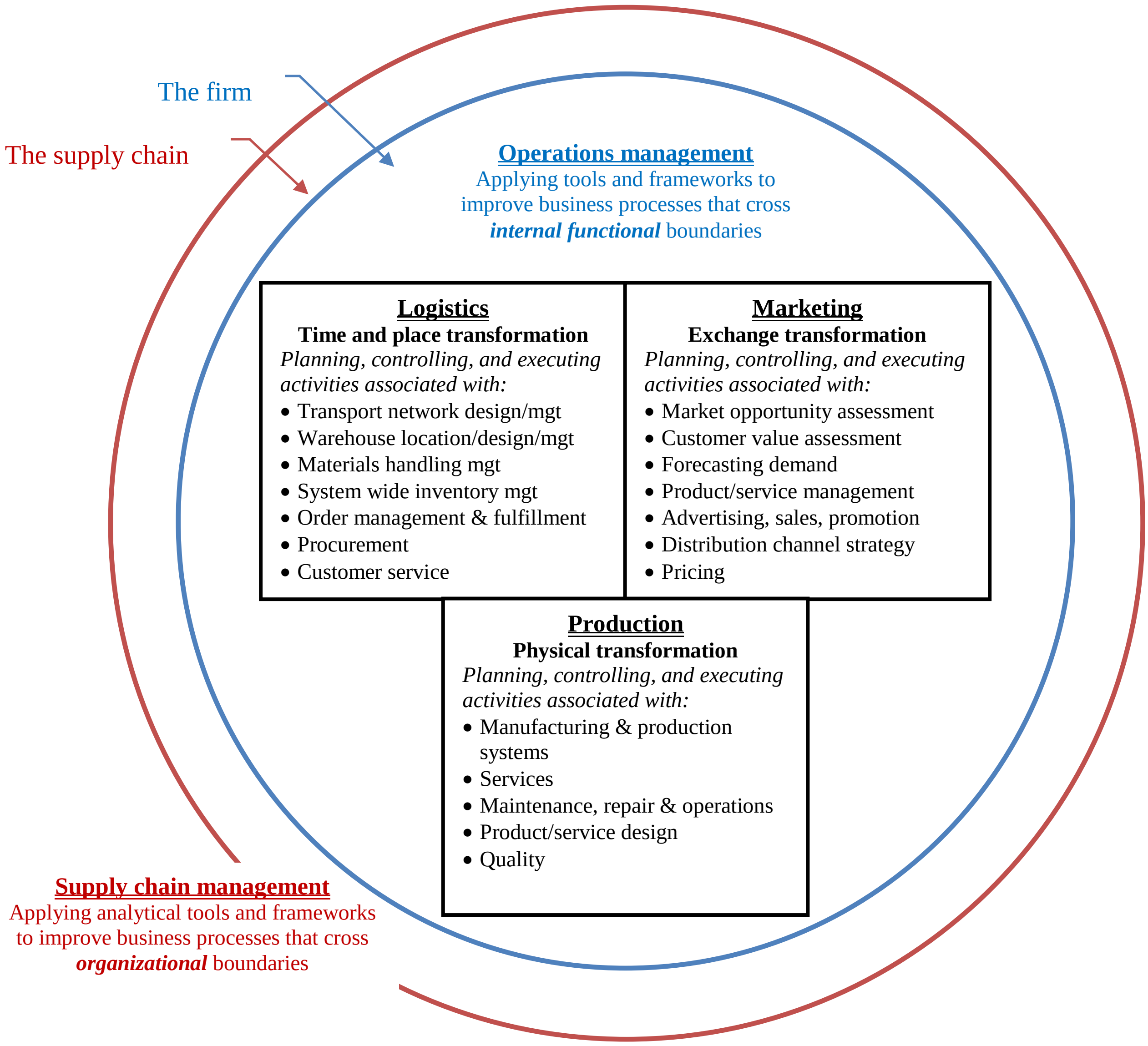
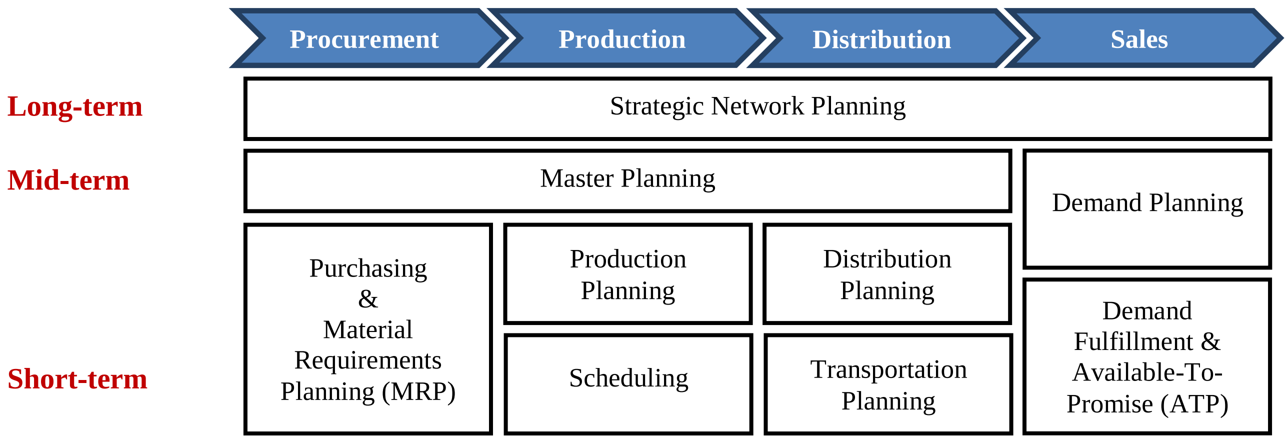


Figure 1.2 Supply chain planning matrix (Meyr et al. 2008)



Giunipero et al. (2008) offered an in-depth analytical review and investigated the trends and gaps in the supply chain literature covering the 10-year-period between 1997 and 2006. Stadtler and Kilger (2008) demonstrated how concepts and ideas of SCM are applied to industrial practice with the support of actual APS. Eight case studies provide valuable insights into the planning processes encountered in specific industries such as chemical, oil, and food and beverages.

The scope of this thesis is modeling large-scale planning problems in the supply chains within forestry and petroleum industries. The methodology is based on operations research and real-world case studies. The first two papers concentrate on the integration in the initial stage of the forestry supply chains while the other two papers focus on coordination issues using the notion of internal/transfer pricing. More specifically， Paper I integrates two supply chains of roundwood and forest biomass and Paper II makes an important extension of the limitation of Paper I. Paper III studies the intra-organizational coordination between the strategic forest management and the tactical industrial production whereas Paper IV investigates the cross-functional coordination within an oil refinery. All the problems are multicommodity flow problems formulated as either mixed integer programming (MIP) or quadratic programming (QP) models, involving typical supply chain activities such as procurement, production, distribution and final sales. In what follows, we will give a brief introduction of both supply chains and highlight the issues about integration and coordination of the corresponding supply chains. This introduction, however, does not pretend to be an exhaustive review of the literature.

1.1.1 The forestry supply chain

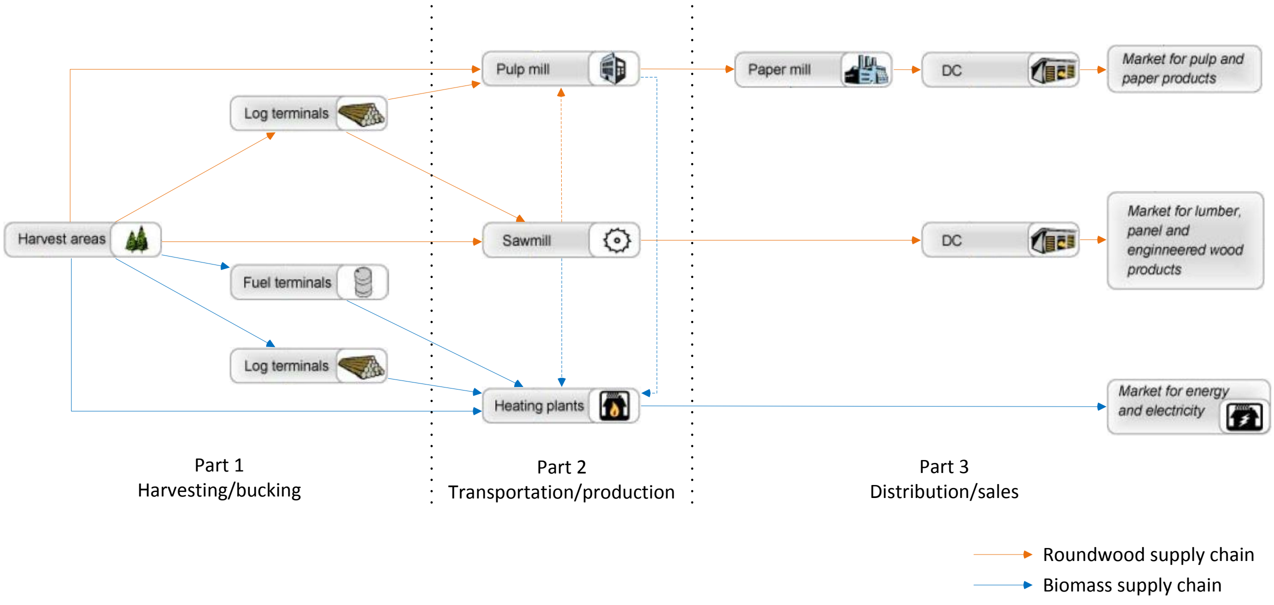
The roundwood and biomass supply chains in the forest products industry provides original forest resource for divergent final uses. The chains start with harvest in the forest where different log types, e.g., sawlogs, pulp logs, and fuel logs, are generated from the bucking process. These logs are then transported to downstream mills, i.e., sawmills, pulp mills, and heating plants, for further production of various final products, i.e., lumber and wood products, pulp and paper, and energy and electricity. Byproducts from wood processing facilities, i.e., chips, sawdust, and bark, will be reutilized and transported to mills for either pulp and paper production or power generation (Figure 1.3).

Forestry supply chain planning encompasses a wide range of decisions, from strategic to operational level. At the strategic level, decisions include forest management, silviculture treatments, road construction, facility location, process investments, and product and market development. Tactical planning integrates the needs of the different parts of supply chains and focuses on how to execute the forest management or production and distribution issues, serving as a bridge between the long-term comprehensive strategic planning and the short-term detailed operational planning that determines real-world operations.

Numerous models based on operations research (OR) have been developed to optimize forestry supply chain planning and to understand the complex functioning of the systems during the last half century. Martell et al. (1998), Rönnqvist (2003), Bettinger and Chung (2004), Weintraub and Romero (2006), D'Amours et al.

(2008), and Carlsson et al. (2009) reviewed the applications and contributions of OR to the forestry industry from different perspectives on the forest products supply chain.

Figure 1.3 Illustration of the main supply chains in the forest products industry



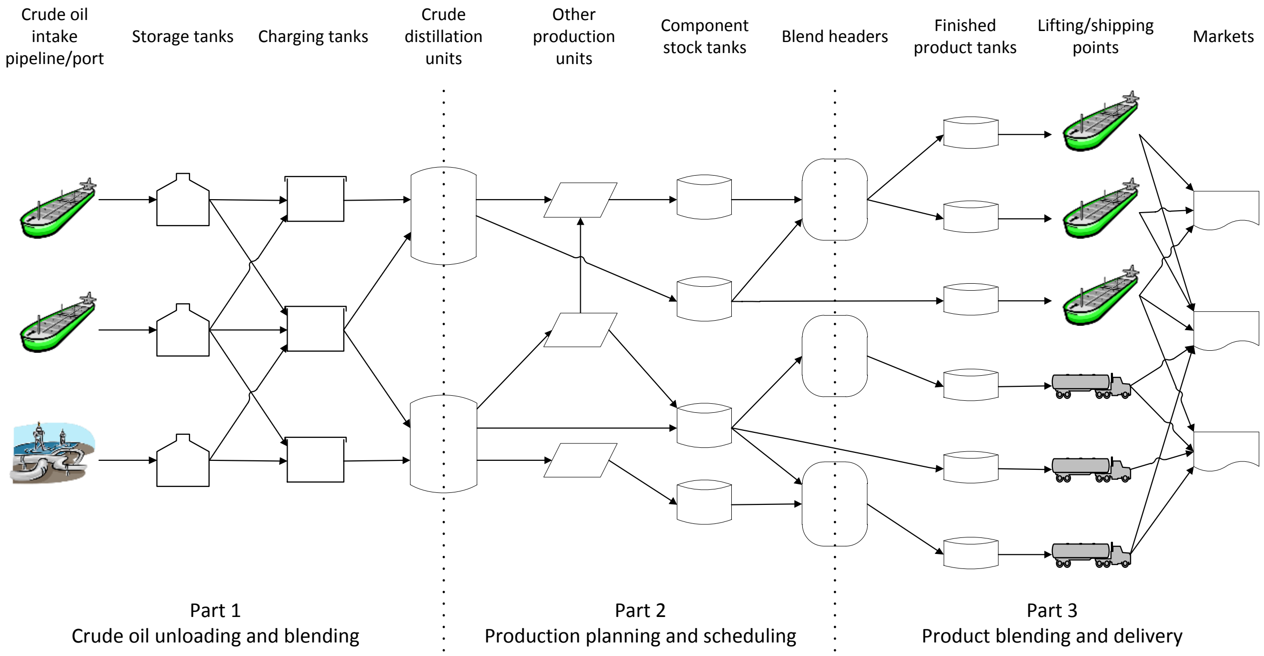
1.1.2 The petroleum supply chain

The petroleum supply chain extends from upstream petroleum exploration where decisions include design and planning of oil field infrastructure, through downstream refinery operations at which crude oil is converted to products, and to distribution centers concerning product transportation. Neiro and Pinto (2004a) presented a general framework for modeling petroleum supply chains. However, the paper in this thesis deals with the refinery system as depicted in Figure 5.1. An oil refinery system, as part of petroleum supply chain, stretches from procurement of crude oil to distribution and sales of petrochemical products, involving many complex processes with various connections. It is usually decomposed into three parts, i.e., crude oil unloading and mixing, production planning and scheduling, and product blending and delivery (Figure 5.1), and modeled and solved independently in a sequence. After these processes, crude oil is transformed into gasoline, diesel, jet fuel, and other hydrocarbon products that can be used as either feedstock or energy source in chemical process industry.

Meanwhile, a hierarchy of decisions also arises in such a supply chain. At the strategic level, the supply chain design includes multisite supply and transportation planning. A tactical planning problem determines how to best allocate the production, distribution, and storage resources in the chain to respond to market demands and forecasts in an economically efficient manner, while a scheduling problem decides the detailed schedule for shorter period, i.e., days or weeks, by taking into account the operational constraints of the system.

See Pinto et al. (2000) for an survey of earlier literature and Bengtsson and Nonås (2010) for an overview of the latest literature on refinery production planning and scheduling models. Shah et al. (2011) presented an extensive literature review of methodologies for addressing scheduling, planning, and supply chain management of oil refinery operations.

Figure 1.4 Illustration of a standard refinery system



1.1.3 Similarities and differences between the two supply chains

Forests cover roughly 31% of the earth's land surface (Martell et al. 1998) while petroleum or oil is found underground storing the sun's energy over millennia past. The two supply chains based on these two crucial natural resources share common problems as well as differ in characteristics.

Although some transformation activities in the supply chains involve many-to-many processes, which consume a set of input products in order to product a set of output products (e.g., recipes in the pulp industry and some product blending at refinery), from the overall viewpoint, both supply chains are divergent as the number of commodities produced increases throughout the chain. Table 1.1 lists the processes and corresponding materials in forestry and petroleum supply chains, which clearly indicate that through a set of production units in the supply chains, one or two raw materials are gradually transformed into a dozen consumer products. How to determine the prices for those intermediate products, as an indicator of cost to the final products or value to the raw materials, becomes an issue. These internal prices count not only for the financial control but also for the coordination between two or more functional divisions.

In addition, coproduction exists in both industries. Because a harvest area is usually composed of multiple tree species, harvesting of one area will result in the coproduction of various resources, e.g., spruce, pine, and birch. Even though in the plantations of one species, e.g., pine or eucalyptus, one bucking pattern will convert trees into various types of log, which are usually defined by the length, diameter, and quality of the timber. In other words, the more one kind of logs are harvested, the more other logs in this area will also be produced and vice versa (Kong et al. 2012). Similarly, in the distillation process at refineries, crude oil is separated into different fractions according to the boiling ranges. Given a run-mode and a type of crude oil, the proportions between the components obtained from the process are fixed and hence, if generation of more of a certain component is desired, then more of the other components obtained in this run-mode will also be generated (Guajardo et al. 2012). Coproduction definitely complicates the planning and trade-offs between production of high-value products and inventory of undesirable outputs always occur.

Byproducts such as bark, wood chips, and sawdust are generated in those wood processing facilities. They will be transported to pulp mills or heating plants for further use. Some sawmills and pulp mills also use the byproducts for internal steam and heating generation for dryers. In the same fashion, the refinery process system produces such byproducts as fuel gas, which cannot in turn be used in the blending of any products. However, the plant can burn fuel gas for its thermal energy.

Table 1.1 Processes and corresponding materials in forestry and petroleum supply chains found in this thesis

	Forestry supply chain	**Petroleum supply chain**
Part 1		
Original resource	Trees (1)	Crude oils (2)
Process	Harvest, bucking, chipping, transportation	Crude oil unloading & blending
Location	Harvest areas, terminals	Refinery
Part 2		
Intermediate product	Logs (7-8)	Components (12)
Process	Saw patterns, pulp recipes, energy conversion	CDU, cracker, reformer, blending recipes
Location	Sawmills, pulp mills, heating plants	Refinery
Part 3		
Final product	Lumber and wood products, pulp and paper, energy and electricity (7)	Petroleum products (8)
Process	Distribution, sales	Lifting, distribution, sales
Location	Markets	Markets
Extra		
Byproduct	Chips, sawdust, bark	Fuel gas (1)
Process	Pulp recipes, energy conversion	Internal thermal energy
Location	Pulp mills, heating plants	Refinery

Besides those commonalities, there are several discrepancies between these two supply chains. First, for forestry as renewable resources, we must consider the sustainability that implies imposing constraints on the model to ensure that the harvest rate of the resource does not surpass its natural regenerative capacity and that we maintain the financial rate of growth (Weintraub and Romero 2006). Nowadays, such environmental concerns as protecting soil, water quality, and scenic beauty, are increasingly took into account. A number of models hence incorporate the maximum open area and adjacency constraints (Constantino et al. 2008; Goycoolea et al. 2005). In native forests in developed countries, sustainability, wildlife, biodiversity, and preservation of nature often play more important roles than timber production (Weintraub et al. 2000). These environmental issues greatly influence the harvesting processes and the consecutive industrial production. By comparison, petroleum as one of the three major forms of fossil fuels is finite in supply. It is also the primary energy that we are drawing on today to fuel the activities of the modern economy. Today, the refinery industry is facing lower-margin profits due to tighter competition, stricter environmental regulations, and uncertain market demands. To survive successfully in a dynamic global marketplace, stress is laid on enterprise-wide optimization in the petroleum industry (Grossmann 2005; Varma et al. 2007). Furthermore, the quality of available crude oil is decreasing, putting more pressures on the efficient production of existing refineries.

Second, these two areas also differ in time horizons considered. For example, in the native forests, standard tree rotation is 60-80 years and the planning horizon commonly covers several rotations. Thus, strategic planning for forest management may span more than 100 years, while operational planning for cutting trees or logs may involve only fractions of a second (D'Amours et al. 2008). In petroleum industry, strategic level decisions are made for decade in a highly uncertain environment. For example, the problem of investment and operations in offshore oil field development is characterized by a long-term planning horizon, typically 10 years (Carvalho and Pinto 2006).

Moreover, although transportation is an important factor in global logistics for both supply chains, the emphasis are placed on different aspects. Harvest in forest involves large geographical areas, which usually cover from a few hectares to several hundred depending on the various restrictions in different countries. How to effectively forward logs from thousands of pickup points to such destinations as sawmills, pulp mills, heating plants, and ports for export is a particular concern in the vehicle routing and scheduling problem for logging trucks (Andersson et al. 2008; Bredström and Rönnqvist 2008). Backhauling is used as an important tool to reduce the cost of transportation (Carlsson and Rönnqvist 2007; Forsberg et al. 2005). However, in the initial stage of refinery supply chain, an oil refinery receives its crude oil through a pipeline that is linked to a docking station, where oil tankers unload. The supply-chain network is composed of shipping via vessel, oil tankers, and pipelines that may run across multiple countries.

Next, there are often several forest companies operating in the same region. Collaborative transportation planning (Frisk et al. 2010) or negotiation-based wood procurement planning (Beaudoin et al. 2010) in a multi-firm context is becoming popular. By contrast, in a typical petroleum supply chain, a set of crude oil suppliers and refineries are interconnected by intermediate and final product streams and a set of distribution centers. Coordination and collaboration in the petroleum supply chain are limited. This is likely because of the intensive competition between a few large competitors. However, there are many examples that oil companies have joint projects to explore the new potential fields. The main reason for this is the huge investment costs and high uncertainties.

Last but not least, as for mathematical programming technologies, in the forestry supply chain, linear programming (LP) applications of resource allocation and timber harvest scheduling have been prevalent in forestry since the early 1970s (Garcia, 1990). MIP is then widely used to represent discrete decisions and determine whether or not to execute an activity, such as which areas should be harvested (Beaudoin et al. 2007), which roads should be built or maintained (Weintraub et al. 1994; Weintraub et al. 1995), whether to accept certain contracts as resource providers (Gunnarsson et al. 2004) or from customers (Gunnarsson and Rönnqvist 2008). Oil refinery production operation is one of the most complex chemical industries. Refinery planning optimization is mainly addressed through a successive LP approach, while nonlinear programming (NLP) models are increasingly becoming a common methodology for the refinery optimization (Pinto and Moro 2000). The numerous nonlinear constraints appear from computing the properties of the products after being processed. Meanwhile, a large number of binary allocation variables are used to consider the discrete scheduling decisions. These factors result in a complex mixed integer nonlinear (MINLP) model. When the planning horizon consists of various time periods, it becomes quite hard to solve real-world instances. See examples in section 1.2.2. In fact, Bengtsson and Nonås (2010) identified the proper treatment of complex MINLP problems as one of the main future challenges.

1.2 Integration of the supply chain

With the onset of globalization, there are needs for efficiency in overall performance. An increasing number of firms across various industries are now stressing a value delivery network that is based on strong alliances alongside significant vertical and horizontal integration (McCutcheon and Stuart 2000; Tan et al. 2002). It is believed that an effective integration can assist a company with an established and sustainable competitive advantage.

1.2.1 Models in the forestry supply chain

Recent years have witnessed growing interest in integrating the different planning problems, i.e., synchronizing the procurement, production, distribution, and sales activities. Philpott and Everett (2001) developed a supply chain optimization model for the Australian paper industry. Gunnarsson et al. (2007) considered an integrated supply chain planning for a major European pulp mill company. Singer and Donoso (2007) presented a model for optimizing tactical planning decisions in the Chilean sawmill industry. Feng et al. (2008) proposed a series of models to coordinate business units along the supply chain in a Canadian Oriented Strand Board manufacturing company.

Furthermore, researchers attempt a greater integration that combines the upstream forest management and harvesting operations with the downstream industrial planning. Barros and Weintraub (1982) incorporated decisions concerning managing forest lands and supply of timber for a pulp plant, a sawmill and export. Gunn and Rai (1987) introduced a model with an integrate viewpoint, which is somewhat similar to that of Barros and Weintraub (1982) but enables calculation of regeneration harvest policies. Jones and Ohlmann (2008) constructed an analytical model by combining perspectives from forest economics and operations management in a vertically integrated paper mill. Troncoso et al. (2011) evaluated the superiority of an integrated strategy where long-term forest management and mid-term downstream production planning are simultaneously determined and the objective is to maximize the net present value (NPV) for the whole forestry supply chains.

In addition to integrating the divergent activities, it is of increasing importance to integrate different levels of decision-making. Although consistency and feasibility arise as problems, there are many successful implementations both in theory and in practice. Cea and Jofre (2000) considered the strategic investment and tactical planning decisions to assist forestry companies. Forsberg et al. (2005) developed a decision support system for strategic and tactical transportation planning in Swedish forestry.

However, most papers that integrate various planning problems or link different decision levels deal with one specific forestry industry, i.e., the wood processing industry, the pulp and paper industry, or heating and power generation. Relatively few exist on how to integrate the whole market. On the other hand, in the past decades, concerns about soaring prices for fossil fuel, trade in carbon emission rights and domestic security of energy have placed emphasis on renewable energy. Forest biomass, as a safe, stable, and renewable energy alternative, receives amounts of attention and becomes one of the most promising and feasible choices for heating and electricity generation in countries with plentiful forest resources. Due to the accelerating promotion of wood-based energy, directly using roundwood, e.g., pulpwood, for energy production is becoming attractive. This trend will likely affect other conventional timber consumers, especially the pulp and paper industry, and generate a competition for forest raw materials. Gunnarsson (2007) saw it as a new and exciting challenge to establish a mathematical model for both forest fuel and pulp products in the forestry supply chain. The first two papers presented in this thesis contribute to the research field by integrating two supply chains of roundwood and biomass and studying the impact of using pulpwood as bioenergy on the supply market for raw materials.

1.2.2 Models in the petroleum supply chain

To drive operational efficiency and improve overall performance, in the past decades, the focus of major refining companies is shifting from individually investigating the various processes to managing supply chain activities as an integrated system (Bengtsson and Nonås 2010). Integrated planning hence becomes one of the main topics studied by recent literature. One of the first researchers to address the supply chain management in the context of an oil company is Sear (1993), who developed a linear programming network model for logistics planning of a downstream oil company. The model involves purchase and transportation of crude oil, processing and distribution of products, and depot operation.

With the advances in powerful computers and efficient modeling techniques, the scope of petroleum supply chain modeling increases. Neiro and Pinto (2004a) established an integrated model for a real-world petroleum supply chain that is composed of oilfield infrastructure, refineries, storage facilities and pipeline network for crude oil supply and product distribution. In Part 1 of the two-part paper, Pitty et al. (2008) first developed a dynamic simulator for the integrated refinery supply chain, which explicitly handles both external supply chain entities and intra-refinery activities. Then in Part 2 Koo et al. (2008) demonstrated the application of the decision support to optimize the refinery supply chain design and operation based on a simulation–optimization framework. Alabi and Castro (2009) presented a mathematical model of the refinery operations characterized by complete vertical integration of subsystems from crude oil procurement through to product distribution with a planning horizon ranging from 2 to 300 days. Guyonnet (2009) explored the potential benefits of an integrated model involving crude oil unloading, production and distribution where the unloading section, the refinery and the distribution center are connected by pipelines and the delivery to final customers is made by truck on a daily basis.

1.3 Coordination of the supply chain

An integrated modeling approach can achieve best performance and avoid suboptimal solutions along the entire supply chain. However, it makes sense in theory, but is not practical in reality for many cases. Instead, the decoupled approach is still widely used, even though plantation lands and downstream mills belong to one company (Troncoso et al. 2011). Most of the oil industry companies also operate their planning, central engineering, upstream operations, refining, and supply and transportation groups as complete separate entities (Shah et al. 2011). Three main reasons can be used to explain the situation:

1) Inherent responsibilities of different divisions

In practice, different constituents in a supply chain are managed by independent companies or divisions with different objectives. For example, sales division typically concentrates on revenue generation whereas production is responsible for cost reduction; Forest management aims to attain social targets in order to meet sustainable socio-economic development whereas the industrial production emphasizes on commercial use of timber and on fulfilling market demands.

2) Limitation of existing decision support systems

Advance planning systems (APS) are developed independently by different software vendors. For example, refineries generally use commercial software packages to support various decisions in the supply chain matrix, which is structured into a number of different software modules. Each module is dedicated to solving a specific problem, such as production planning, distribution planning and sales forecast (Bredström and Rönnqvist 2008) . Moreover, if partners are reluctant to share their data and to feed it into a central data-base while insisting on their own planning domain, modeling supply chain-wide flows by a single APS is still impossible (Stadtler 2005).

3) Complexity of real-world dimension

Due to large size and complexity of real-world dimension, either simultaneous optimization across all the departmental divisions or temporal integration at different decision levels will make the integrated planning models significantly challenging and highly intractable (Shah et al. 2011).

Therefore, it is important and necessary to establish practical coordination mechanisms to lead supply chain partners to operate in ways that are best for the chain as a whole. It is proved that if the coordination scheme is appropriately set and there is no private information, distinct functional divisions with divergent interests will be induced to act in a globally optimal fashion (Karabuk and Wu 2002).

1.3.1 Coordination mechanism

Coordination is the main challenge for distributed decision making in supply chain management (Schneeweiss 2003). There is a significant body of literature on incentive schemes for channel coordination in decoupled supply chains. Interested readers can refer to the current comprehensive reviews by Cachon (2003), Arshinder et al. (2008), Bahinipati et al. (2009) and Chan and Chan (2010). However, the papers included in this thesis do not deal with the coordination mechanisms cross a supply chain consisting of independent self-interested organizations, but focuses on coordination within a firm.

Coordination mechanisms within a firm usually include: 1) accounting-based schemes. Celikbas et al. (1999) developed coordination mechanisms through penalty schemes where marketing is penalized for over-forecasting and manufacturing is penalized for under-supply and compared the performance of centralized and decoupled systems under a stochastic setting. Pekgun et al. (2008) showed that a transfer price contract with bonus payments can motivate the decoupled marketing and production departments in a Make-To-Order firm to match the centralized solution; 2) improved contract design. Chen (2005) designed a menu of linear contracts as incentive schemes to salesman whose private information about market condition is important for the firm's production and inventory planning decisions. By observing which contract the salesman chooses, the firm can attain the knowledge about the market and make proper production decisions to maximize its expected profit; 3) decision making hierarchies. Li and Atkins (2002) studied coordination issues in a firm where replenishment and pricing decisions are made by production and marketing, respectively, in a decoupled fashion and found that having marketing as the leader in a Stackelberg framework can lead to improved performance for both production and marketing, and thus the firm as a whole; 4) internal markets. Kouvelis and Lariviere (2000) placed an internal market between the manufacturing and marketing managers where the transfer prices for the intermediate output from one function can differ when it is sold to another. This incentive scheme allows the system to be successfully decoupled.

1.3.2 Internal pricing

In our papers, we use transfer pricing as a prominent application for internal coordination. Pricing has long been recognized as a significant tool used in the industrial operations to manipulate demand and to regulate the production and distribution of products (Soon 2011). Internal pricing, also known as transfer pricing, is utilized as a communication device between participants in an internal market in order to arrive at the system optimal allocation of resources.

Theoretically, the main stream of research on coordination mechanism with transfer pricing under a decoupled system takes its cue from the economics literature. Game theory is normally implemented to analyze real situations where multiple agents within a firm are involved in a decision process and their actions are inter-related. Hu et al. (2011) introduced an internal price charged by manufacturing department to the sales department to balance the cost pressure of lead-time hedging amount. A Nash game model and a

Stackelberg game model using the internal price can increase the firm's overall profit as compared to the traditional model without coordination. Erickson (2012) included a constant transfer price in a differential game model that allows the coordination of equilibrium marketing and production strategies to achieve a maximum profit for the firm.

Pfeiffer (1999) summarized two common approaches to derive transfer pricing system. One is an economic approach that uses methods of marginal analysis to determine values of intermediate commodities. The other is a mathematical programming approach that is based on the dual Lagrangian principle. In the internal market constructed by Kouvelis and Lariviere (2000), the prices a market maker pays when buying an output and charges when selling it are related to the shadow price of the output's availability constraint. Erickson (2012) pointed out that when there is no market for the transferred product, or when the market is imperfectly competitive, the correct transfer price procedure is to transfer at marginal cost. If there is a competitive market for the transferred product, the appropriate transfer price is the market price. Karabuk and Wu (2002) presented two coordination mechanisms for decoupled semiconductor capacity planning by finding a form of transfer pricing making use of an augmented Lagrangian approach. Stadtler (2005) also noticed that it is challenging to find the accurate setting of (fair) transfer prices in an inter-organizational supply chain.

In the forestry industry where the planning horizon for forest management may cover decades, the planning is typically made in a decoupled structure with three planning problems. First, the long-term forest management model is solved and as a result when and where to harvest is decided. Second, the available volumes of logs for a mid-term period are generated by solving a bucking planning problem. Last, the industrial planner makes the tactical logistic and production planning under restrictions on the supply of logs (Troncoso et al. 2011). A set of internal prices is usually used to guide the production of logs in the bucking process in order to match the supply of timber in the forest to the demand at mills. In most cases, these prices are manually estimated by experience planners and given as parameters, which do not necessarily provide a system optimal solution from the perspective of supply chain optimization.

As for the refinery operations, Bengtsson and Nonås (2010) highlighted the importance of how to determine the proper values of intermediate products, which are commonly presented as a known value in the literature. Guajardo et al. (2012) studied how to coordinate production with sales decisions in a refinery supply chain. The transfer pricing in their model is manually preset by the company to reflect all the costs to produce a product before its shipment to the market. Li et al. (2003) proposed an analytical method called "Marginal Value Analysis (MVA)" to price intermediate products and Li and Hui (2007) extended MVA along with sensitivity analysis and parametric programming to trace the change of marginal values that indicate product values in a multiperiod refinery planning model. Lozano (2009) used a thermo economic analysis based on marginal production costs to obtain unit costs for internal energy flows and final products.

The last two papers in this thesis contribute to the research field by presenting two methods to systematically determine the appropriate setting of internal pricing as effective coordination mechanism. One is obtained from marginal values of flow balance constraints. The other is defined by Lagrangian multipliers originating from Lagrangian decomposition, which will be discussed in next section.

1.4 Lagrangian decomposition

For large-scale optimization problems, arising either because of site types or from multiperiods, a decomposition approach is usually employed to take advantage of the structure of these problems. The idea behind decomposition is to break the overall problem into a number of smaller subproblems that are mathematically solvable and computationally efficient.

Among various decomposition techniques found in the literature, Lagrangian decomposition (LD) is one of the most popular methods applied in both forestry and petroleum planning problems. LD is first proposed by Guignard and Kim (1987) as a generalization of conventional Lagrangian relaxation (LR). It is also known as variable layering (Glover and Klingman 1988) or variable splitting (Jörnsten and Näsberg 1986). Although separability is the goal of both decomposition approaches, the main difference between LR and LD is that instead of relaxing the "complicating" constraints that tie together the problems in LR, common variables involving in two (or more) subproblems are first duplicated and the equality/coupling constraints are then dualized in LD. Thus, LD is usually used to obtain good optimistic bound to the original problem, i.e., upper bound (UBD) for maximization problem and lower bound (LBD) for minimization problem. Guignard and Kim (1987) demonstrated that the bound generated by LD can strictly dominate the one by LR. Guignard (2003) gave a very detailed and comprehensive review concerning these issues.

In forestry, Gunnarsson and Rönnqvist (2008) used LD to relax all storage balance constraints, resulting in one subproblem for each time period. The coordination is obtained by updating the Lagrangian multipliers from the solutions to each subproblem. In another paper (2011), the authors applied the same technique but decomposed the problem into subproblems representing distinct physical stages in the supply chain.

In the refinery planning, Wu and Ierapetritou (2003) accomplished the integration of the three scheduling subproblems by establishing an iterative solution framework that exploits the LBD obtained through the heuristic-based decomposition approaches and the UBD based on LD. Neiro and Pinto (2004b) presented four strategies to solve the long-range multiperiod production planning of petroleum refineries that rely on decomposition techniques such as LD. Neiro and Pinto (2006) exploited the block-diagonal structure of the multiperiod refinery planning model under uncertainty and applied LD on a temporal basis where the planning horizon of T time periods is decomposed in T problems and solved independently.

Most applications of LD, the same as other decomposition approaches, stress in significant improvements in computational efficiency, but as far as we know no attempt has been made to apply LD as a coordination scheme to find internal prices in forestry and refinery planning problems and to allow the decoupled system to perform like a centralized one. Lagrangian multipliers involved in LD give the rate of change in the objective function with respect to the rate of change in the right-hand side of relaxed constraints. They may be interpreted as the (penalty) costs to the objective function paid for the violation. In other words, they represent the shadow prices on the coupling constraints. Therefore, in the proposed methodology in the third and fourth papers, we take related Lagrangian multipliers as internal prices for corresponding commodities.

1.5 Outline of the papers

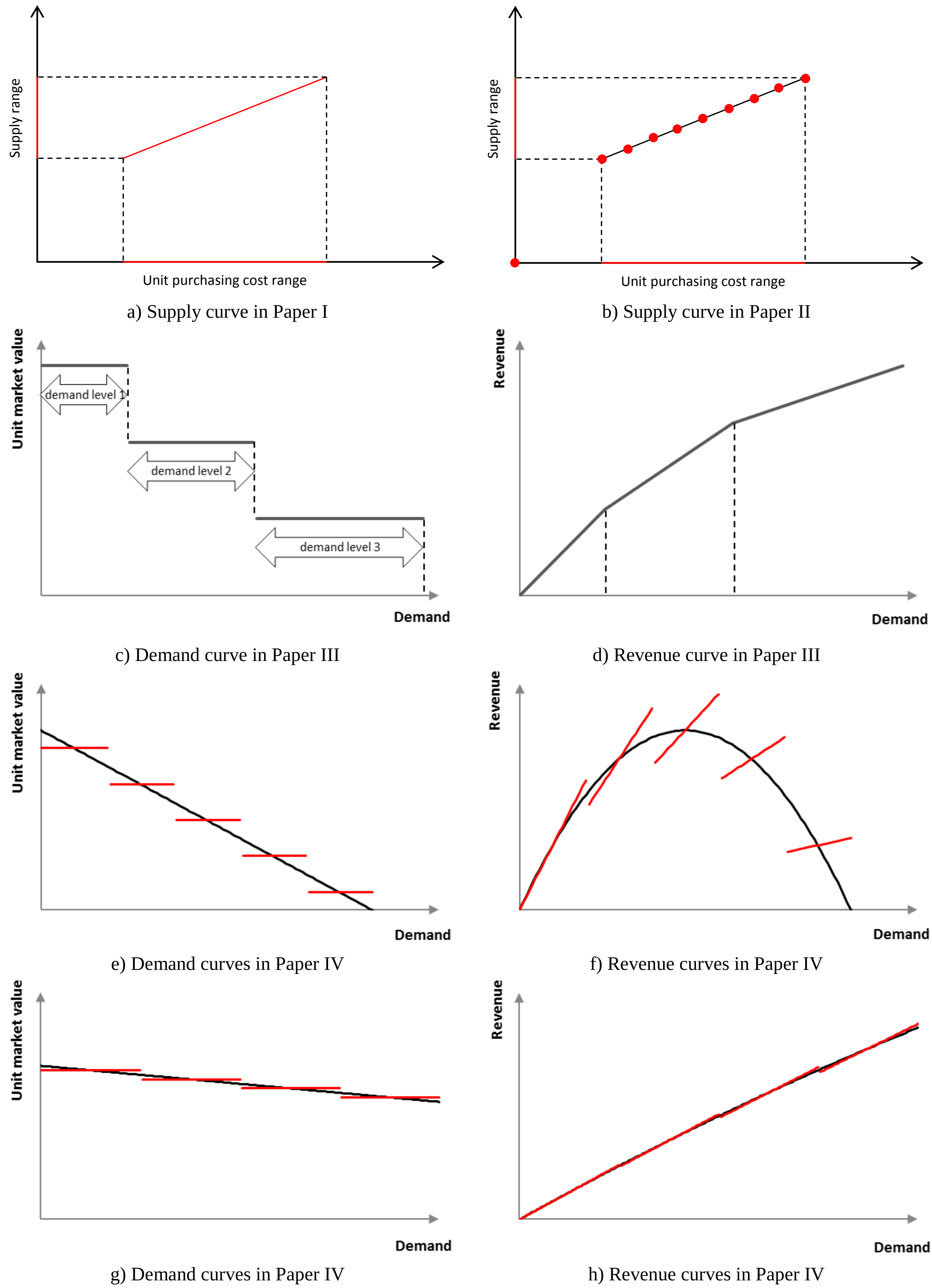
Supply chains are often very complex. Not every detail that has to be dealt with in reality can and should be respected in a plan and during the planning process. According to Fleischmann et al. (2008), the “art of model building” is to represent reality as simple as possible but as detailed as necessary, i.e., without ignoring any serious real world constraints. The purpose of this thesis is to model planning problems in the forestry and petroleum supply chains with different emphases and viewpoints.

In the first two papers, we integrate the two supply chains of roundwood and forest biomass in an optimization model and study the impact of integration on market prices for raw materials. In other two papers, we focus on defining the appropriate setting of internal pricing as coordination mechanism in forestry and petroleum supply chains. All the problems are multicommodity flow problems formulated as either MIP or QP models. The papers concerning forestry are all multiperiod and the one relating to petroleum considers only one time period. Except the third paper that combines strategic and tactical planning levels, others lay stress on tactical planning. Table 1.2 presents the overview of the properties of the problems and size of models in the four papers. It is worth mentioning that all the models developed take into account the dynamic relationship between prices and volumes, either on the supply side or on the demand side (Figure 1.5). These assumptions give us a better understanding of how the price behavior will influence the flows in the entire supply chains.

Table 1.2 Overview of the properties of the problems and size of models in the four papers

	Paper I	Paper II	Paper III	Paper IV
General information				
Industry	Forestry	Forestry	Forestry	Petroleum
Country	Sweden	Sweden	Chile	Norway
Focus	Integration	Integration	Coordination	Coordination
Decision level	Tactical	Tactical	Tactical+Strategic	Tactical
No. of time periods	12(monthly)	12(monthly)	5(yearly)+ 4(aggregated 5-year)	1
No. of harvest areas	234	234	1 226	--
No. of downstream mills	40	40	5	1
No. of original resources	8	8	1	2
No. of intermediate products	--	--	7	12
No. of final products	3	3	7	8
No. of byproducts	2	2	3	1
Supply chain activity				
Procurement	Yes	Yes	Harvest+Bucking	Yes
Distribution	Yes	Yes	Yes	Yes
Production	No	No	Yes	Yes
Sales	Yes	Yes	Yes	Yes
Supply	Linear price	Discretized price	Depend on harvest	Fixed
Demand	Fixed	Fixed	Step price	Linear price; Step price
Model structure				
Type of model	QP	MIP	MIP+LP	LP+QP; LP+MIP
Objective	Minimize cost	Minimize cost	Maximize profit + Maximize profit	Maximize profit +Minimize cost
No. of continues variables	835 598-896 367	873 904	42 819-249 468	318-1 837
No. of binary variables	--	22 464-202 176	8 054	48-480
No. of constraints	54 423-265 152	87 080-168 712	6 117-107 470	100-1 102

Figure 1.5 Various supply curves and demand curves in the four papers



1.5.1 Paper I: Modeling an integrated market for sawlogs, pulpwood and forest bioenergy

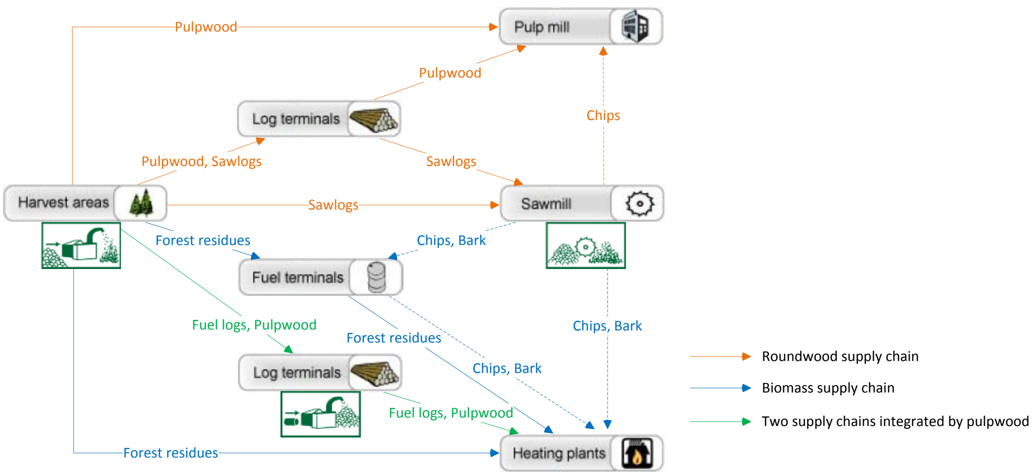
Jiehong Kong, Mikael Rönnqvist, and Mikael Frisk

Canadian Journal of Forest Research, 2012, 42(2), 315-332

Presented in 6th CEMS Research Seminar on Supply Chain Management, Riezlern, Austria, 2009; Geilo seminar, NHH, Norway, 2009 & 2012; NFB conference, Bergen, Norway, 2009; INFORMS Annual Meeting, Charlotte, USA, 2011

Most studies of the forestry supply chain in the past have addressed integration either for various planning problems or for different decision levels, concentrating on one specific forest product industry. This paper, for the first time, deals with an integrated market for all forest raw materials, i.e., sawlogs, pulpwood, and forest residues, and byproducts from sawmills, in the initial stage of the supply chain. These raw materials exist in an integrated market where pulpwood can be sent to heating plants as bioenergy (Figure 1.6).

Figure 1.6 Illustration of an integrated market for raw materials in the initial stage of the forestry supply chains



This problem is described from a supplying company's perspective. The objective is to purchase adequate amounts of raw materials in the harvest areas and byproducts from sawmills in order to satisfy the diverse demands in sawmills, pulp mills, and heating plants at the minimum combined costs of procurement, chipping, inventory, and transportation. The planning horizon is one year and monthly time periods are considered to account for the seasonality, which has a great influence on the whole supply chains. The model represents a multiperiod multicommodity network planning problem with multiple sources of supply, i.e., preselected harvest areas, and multiple types of destinations, i.e., sawmills, pulp mills, and heating plants.

Different from the classic wood procurement problem, we take the unit purchasing costs of raw materials as variables on which the corresponding supplies of different assortments depend linearly (Figure 1.5a). It implies that the higher unit purchasing cost the supplying company offers, the more volume, if possible, the forest owners will provide under constraints of harvest conditions. With this price mechanism, the popularity of harvest areas can be distinguished. The proposed model is a typical QP problem with a quadratic objective function and linear constraints, and it can be efficiently solved by CPLEX.

We use the data provided by the Forestry Research Institute of Sweden to simulate the integrated market. The case study covers the harvest areas in southern Sweden that can annually supply 1.6-2.2 million m^3 of required wood assortments to sawmills, pulp mills, and heating plants. We generate 16 instances to test the proposed model and make seven different scenario comparisons to analyze the results. The main purpose is to study how price restriction, market regulation, harvest flexibility, demand fluctuation, and exogenous change in the price for fossil fuel will influence the entire wood flows. Pairwise comparisons show that in the integrated market, competition for raw materials between forest bioenergy facilities and traditional forest industries pushes up the unit purchasing costs of pulpwood. The results also demonstrate that resources can be effectively allocated by the price mechanism in the supply market. The overall energy value of forest biomass delivered to heating plants is 23% more than the amount in the situation when volumes and unit purchasing costs of raw materials are given as parameters. The results also indicate the strong connection and high dependency among all forest-related industries.

1.5.2 Paper II: Using mixed integer programming models to synchronously determine production levels and market prices in an integrated market for roundwood and forest biomass

Jiehong Kong, Mikael Rönnqvist, and Mikael Frisk

Under second round review at *Annals of Operations Research*

Presented in INFORMS Annual Meeting, Charlotte, USA, 2011; Geilo seminar, NHH, Norway, 2012

In Paper I, we assumed that the set of harvest areas to purchase forest raw materials is predefined. This is a valid assumption on a short-term planning horizon. However, for longer term planning covering more than one year, this assumption is not practical. During such a longer period, there is always a possibility to choose areas from a larger set of available areas. Furthermore, if the fixed cost associated with forwarding operation is considered, one tends to avoid purchasing timber from the areas where volumes of supply are low. Hence, this paper deals with an important extension of the limitation of the previous work.

For a traditional sequential approach, two steps are needed. In the first step, binary variables are introduced to select harvest areas with respect to volume balance and these decisions become constraints for determining prices in the second step. This approach involves two models, a MIP problem and a QP problem. In this paper, we proposed a new synchronous approach that can jointly choose areas and define the discretized price levels for different assortments at the chosen supply points. With this approach, it is possible to solve the entire problem in one MIP model. This new method avoids establishing a non-linear MIP model.

In the earlier model, we took the unit purchasing costs of raw materials as variables on which the corresponding supplies depend linearly. This linear relationship can be approximated by different number of discretized nodes on the cost-supply curve (Figure 1.5b). Once a harvest area is selected to provide certain assortment, the price level of this assortment in the area is also determined. Otherwise, there is no supply in the non-chosen area. To our knowledge, it is the first time to take the variable price factor into consideration in a MIP model of wood procurement problem. With this implementation, different equilibrium prices for roundwood and forest biomass in the forest can be generated and unprofitable supply points, if any, can be excluded.

The data in the case study is the same as those in Paper I, except for the newly introduced fixed cost. The fixed cost occurs by the fact that after harvesting logs have to be forwarded from piles scattered in the harvest area to the pickup point adjacent to forest roads. In reality, it is difficult to obtain the detailed information about the exact rate. Therefore, in this paper, we treat this operating cost as a fixed cost and use sensitivity analysis to study its impact on wood procurement decisions. The computational results show that if no fixed cost concerning log forwarding is counted at each supply point, there is less than 0.01% difference in the optimized objective value between the sequential approach and the synchronous approach. However, the solution time of the synchronous approach is faster than the time used by the sequential approach. With the increase of fixed cost, the synchronous approach has a remarkable advantage over the sequential approach.

1.5.3 Paper III: Coordination between strategic forest management and tactical logistic and production planning in the forestry supply chain

Jiehong Kong and Mikael Rönnqvist

Submitted to *European Journal of Operational Research*

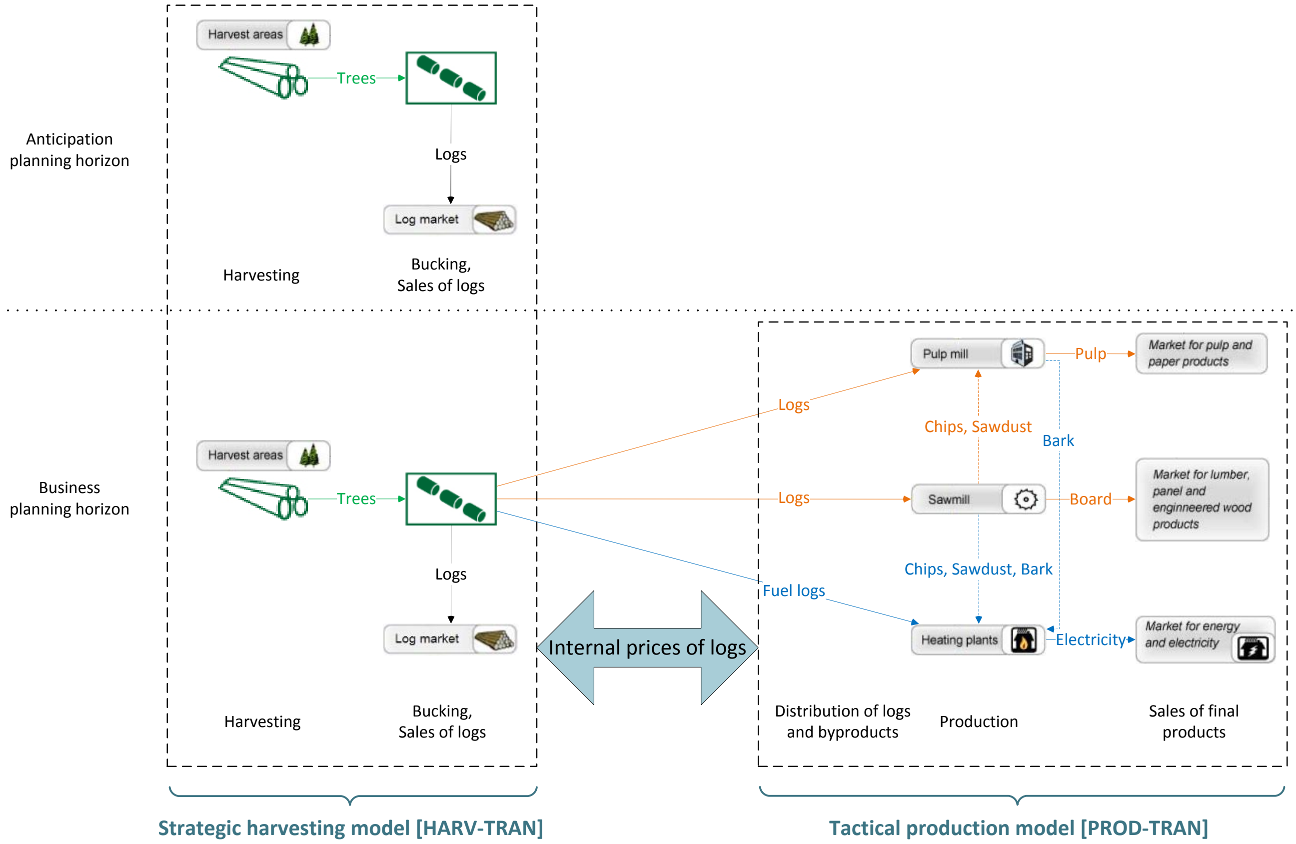
In this paper, we study the coordination mechanism in the forestry supply chain between strategic forest management and tactical logistic and production planning using the notion of internal pricing (Figure 1.7). The structure of the forest products supply chain is similar to the two largest forest holding companies in Chile. These companies own plantation lands and downstream mills. For the long-term forest management, the planner only focus on harvesting and bucking decisions in order to optimize an expected net present value (NPV) of logs. In the mid-term business planning horizon, the supply chain stretches from harvesting operations in the forest to industrial production in mills and to final sales in markets.

We assume that the demand for final products is not fixed. Instead, we use a step price function to better describe the real market behavior toward different volumes of products (Figure 1.5c). The highest unit sale price is only applied to a limited number of volumes and demand beyond this limit pays lower price. There are two break points with three demand limits or levels. Then the total revenue of sold products increases with the volumes in a piecewise-linear fashion, with three pieces as shown in Figure 1.5d.

We first formulate an integrated model to establish a theoretical benchmark for performance of the entire supply chain, including harvest areas, sawmills, pulp mills, heating plants and final markets. This is a MIP model that involves harvesting, bucking, transportation, production and sales decisions for both tactical (5 years) and strategic (25 years) planning levels. We then present two sequential approaches S-A and S-B. Sequential approach S-A is currently used in practice where harvesting in forest is the main driver of the supply chain activities and internal pricing is introduced to control bucking decision in a separate stage.

Sequential approach S-B is proposed in this paper where downstream demand information is taken into consideration and internal pricing directly influences harvesting decision in the first stage.

Figure 1.7 Illustration of coordination between strategic forest management and tactical industrial production



At present, the internal prices, which can be viewed as a coordination mechanism to match the supply of timber in the forest to the demand at mills, are estimated by experienced planners. In order to systematically find the appropriate setting of internal pricing that leads to the system optimum, we suggest two heuristics H-I and H-II. The internal pricing in H-I is based on dual values and in H-II derived from multipliers in a Lagrangian decomposition. Different from the manually estimated internal prices, internal prices obtained in these two heuristics indicate not only the log type but also the location, which in theory can eliminate the symmetry inherent of variable coefficients to achieve a significant computational advantage and in practice can help the forest manager to get a more comprehensive idea of the value of logs in various harvest areas.

A real-life case study in the Chilean forest industry is used to compare the results of different approaches. It is shown that the new sequential approach S-B generates as good feasible solution as that obtained from the integrated approach but in much shorter time. Both heuristics H-I and H-II bring about near-optimal feasible solutions. H-II also provides both optimistic and pessimistic bounds of the optimal objective function value, which can be used as a measure of the solution quality as well as a convergence criterion.

1.5.4 Paper IV: Coordination between production and sales planning at a refinery based on Lagrangian decomposition

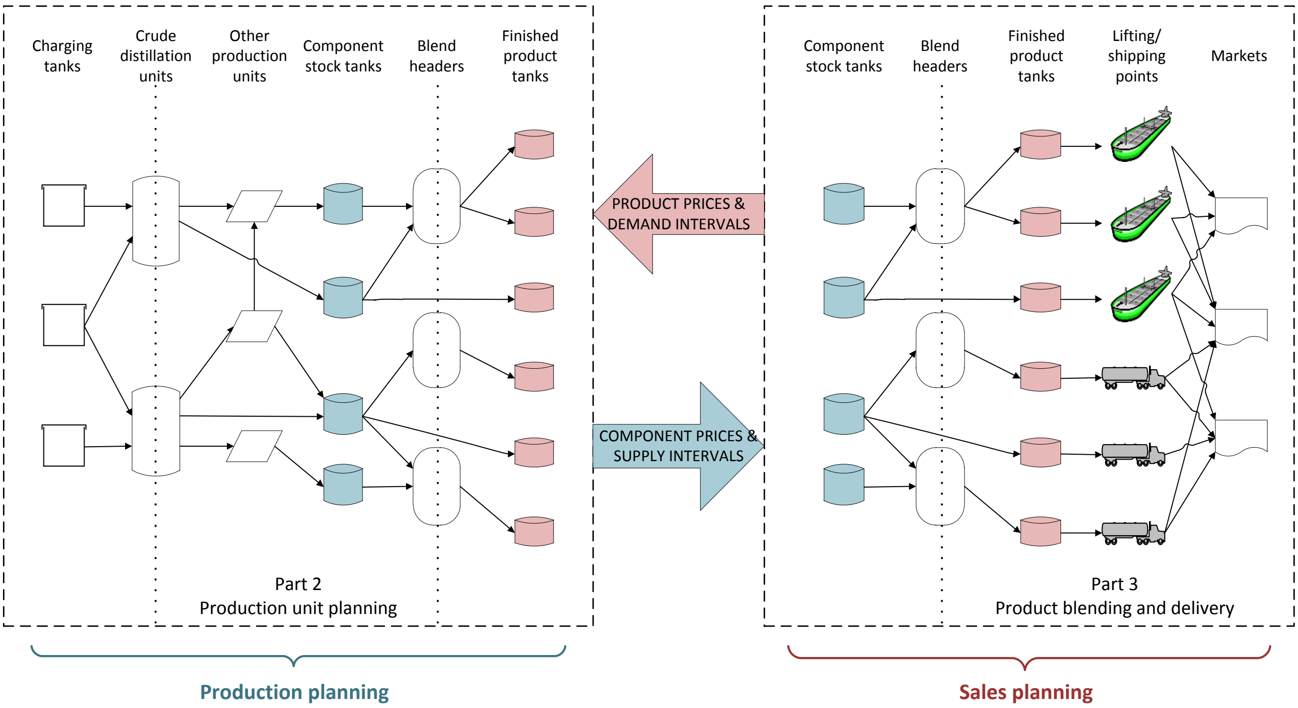
Jiehong Kong and Mikael Rönnqvist

Submitted to *International Journal of Production Economics*

Presented in 25th European Conference on Operational Research, Vilnius, Lithuania, 2012; NFB conference, Kristiansand, Norway, 2012

In practice, production and sales planning at a refinery are performed separately. Each problem has its own objective function seeking for local optimality. To avoid a suboptimal solution, necessary information between production and sales divisions should be transferred. Given the estimated value (internal prices for products) and demand interval for each product, the production manager minimizes the cost by deciding the volumes of crude oil charged, process modes used, components produced and products blended. In the sales division, the planning problem is based on the supply of components and corresponding estimated costs (internal prices for components). The objective is to maximize the expected net profit through sales for products in different markets at different prices, taking into consideration the blending, inventory, transportation and internal component costs (Figure 1.8).

Figure 1.8 Illustration of coordination between the production planning and the sales planning



In this paper, we first formulate two separate planning problems, i.e., production model [P] and sales model [S], both involving blending decisions. These two models represent how the production and sales planning are made in practice. We then present an integrated model [I] that maximizes enterprise's overall profit. This joint model [I] establishes a theoretical benchmark for performance, but is generally not possible to solve due to complexity of real-world dimension.

Then we propose two mechanisms using the notion of internal pricing together with volume constraints as a means to achieve cross-functional coordination between the production and sales divisions that corresponds to the enterprise's optimal performance. These two coordination mechanisms basically differ in how to determine the internal prices and the volumes. In the first mechanism, internal prices are obtained from traditional marginal values, and in the second mechanism, they are defined by the multipliers originating from Lagrangian decomposition.

Four distinct market scenarios representing common demand behavior are tested. In scenario S1, the price for one product in one market is linearly dependent on its demand and the sales revenue is hence a quadratic objective function (Black line in Figure 1.5e and f). In scenario S2, the price is a step function. That is, instead of changing with each amount, the price is fixed at a level within a certain quantity segment and then decreases step by step. Then the sales model becomes a MIP problem (Red line in Figure 1.5e and f). Scenarios S3 and S4 use the corresponding price functions from S1 and S2, but the price curve is much flatter, implying that the price is slightly affected by the demand (Figure 1.5g and h).

A case study based on a refinery in Norway is implemented to illustrate the methodology. It is demonstrated that the first mechanism using marginal values as internal prices provides good pessimistic bounds, (i.e., feasible solution) on the optimal value defined by model [I]. The second one employing Lagrange decomposition generates both optimistic and pessimistic bounds, converging to the global optimal solution to model [I]. We also compare the convergent performance under different market demand scenarios. It is shown that both quadratic programming (QP) model with a linear demand function and mix integer programming (MIP) model with a step price function present good convergence properties by the second coordination mechanism using Lagrangian decomposition.

1.5.5 Contributions

In the first two papers, we focus primarily on integrating the supply chain of roundwood and the supply chain of forest biomass in mathematical programming models. The developed models provide a framework to address scenario-specific issues facing forestry companies and governmental institutions. This is the main contribution from a modeling point of view. In comparison, in the third and fourth papers, we consider a more methodological perspective in pursuit of effective coordination between different divisions. The two systematic methods to determine internal pricing as a coordination mechanism are general enough so that other industries could easily adapt them for cross-functional or intra-organizational coordination. These are the main contributions from the methodology viewpoint. These papers have been presented at international conferences in Austria, USA and Lithuania and internal seminars in NHH, Laval University and NFB.

The main contributions of each paper are summarized as follows.

Paper I:

(*i*) We establish a new model to integrate the two supply chains roundwood and forest bioenergy, including decisions about procurement, transportation, chipping, and inventory planning.

(*ii*) We take the unit purchasing costs as variables and assume that the supplies of raw materials are linear with respect to the unit costs. It allows for the possibility to find equilibrium prices for raw materials in each harvest area and to make the supply chain more efficient.

(*iii*) Geographic maps with price differences can be generated. Supplying companies can use them to make in-depth analysis of the supply market and negotiate with forest owners for better rebates when signing annual contracts.

Paper II:

(*i*) We present a new method where MIP model is employed to select harvest areas and determine the market prices for raw materials at the same time. This synchronous approach can be efficiently solved by standard commercial solvers and has a proved advantage over the traditional sequential approach in terms of both solution time and solution quality.

(*ii*) The impact of possible fixed costs on market prices is taken into account. Sensitivity analysis provides a basis about which area in which period is not profitable to extract certain assortment once the fixed cost at that area increases to certain level. This is particularly useful for forest companies to get a more comprehensive understanding of the value of raw materials in various harvest areas, and hence, to make better wood procurement decisions.

(*iii*) Any geographical map for specific assortment in a particular period can be generated under estimated fixed cost. If several forest companies plan to collaborate, such maps and information can contrastively show the importance of certain supply points and facilitate the negotiation about cost or revenue allocation.

Paper III:

(*i*) We propose a modified sequential approach that is effective in producing near-optimal solution in quick computational time. It keeps the conventional planning method but obtains the same superior effect as in the integrated approach.

(*ii*) For the first time, we apply Lagrangian decomposition as a coordination scheme to find internal prices in forest planning problems. Lagrangian heuristic provides both upper and lower bounds of the optimal value, which can be used as a measure of the solution quality.

(*iii*) Both dual heuristic and Lagrangian heuristic to determine the internal pricing use the existing decoupled modeling setting, independent of the integrated model. It means that only minor change in information exchange rather than reform of the whole planning system is enough to make major progress.

Paper IV:

(*i*) We demonstrate that the coordination allowing the decoupled system to perform like a centralized one is achievable by introducing accurate setting of internal prices and proper restrictions on volumes. Two coordination mechanisms are proposed and have performed very well.

(*ii*) For the first time, we apply Lagrangian decomposition as a coordination scheme to find internal prices for refinery operations. The second mechanism involving Lagrangian decomposition provides both upper and lower bounds of the optimal value, which can be used as a measure of the solution quality. Both QP model with a linear demand function and MIP model with a step price function present good convergence properties using this mechanism.

(*iii*) We check the impact of different parameter assumptions and updating rule on convergence for both mechanisms. We find that an appropriate choice of volume interval magnitude and a decreasing deviation on required volume are essential in convergence in the first mechanism. In the second mechanism using Lagrangian decomposition, the convergence behavior depends on the proper combination of how to update convergence parameter after how many unchanged iterations.

1.5.6 Suggestions for future research

For the integration of the roundwood and forest biomass supply chains, further work includes studying the impact if several companies would collaborate. This is already implemented for roundwood with large savings (Frisk et al., 2010). In such collaboration, the resource of supply is treated as common and the destination of deliver can be changed within companies as long as there is a balance in the wood trade. As the logistic cost is critical for forest biomass, such collaborative plans could improve efficiency. Moreover, backhauling (Carlsson and Rönnqvist, 2007) is traditionally efficient for the transportation planning of roundwood. In the application of forest biomass, any backhauling will be more complex as more types of systems are involved. It would be interesting and challenging to adapt traditional backhauling to the transportation of the forest biomass.

For the coordination in both industries, we would like to check whether the proposed methodology, using the multipliers in Lagrangian decomposition as internal prices, prevails in a greater sophistication of the model that employs non-linear constraints, for example, the complex blending process or pulp production. On the other side, other techniques could be used to update Lagrangian multipliers, for example the various methods reviewed by Bazaraa and Goode (1979) and the recent method based on the Nelder-Mead algorithm (Wu and Ierapetritou 2006). In the computational study of Paper IV, we also found that even though the results are convergent to the same optimal solution, the corresponding internal prices are in dramatic difference. We therefore would like to seek other alternatives to determine a set of stable and identical internal prices to facilitate practical use.

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Part II Papers

2

Modeling an integrated market for sawlogs, pulpwood and forest bioenergy

Jiehong Kong, Mikael Rönnqvist, and Mikael Frisk

Abstract

Traditionally, in the initial stage of the forest supply chain, most applications deal with sawlogs to sawmills, pulpwood to pulp mills, or forest residues to heating plants. In this paper, we develop a model that accounts for all raw materials in the forest, i.e., sawlogs, pulpwood, and forest residues, and byproducts from sawmills. They exist in an integrated market where pulpwood can be sent to heating plants as bioenergy. The model represents a multiperiod multicommodity network planning problem with multiple sources of supply, i.e., preselected harvest areas, and multiple types of destinations, i.e., sawmills, pulp mills, and heating plants. Different from the classic wood procurement problem, we take the unit purchasing costs of raw materials as variables on which the corresponding supplies of different assortments depend linearly. The objective of the problem is to minimize the total cost for the integrated market including the purchasing cost of raw materials. Therefore, it is a quadratic programming problem. A large case study in southern Sweden, under different scenario assumptions, is implemented to simulate the integrated market and to study how price restriction, market regulation, harvest flexibility, demand fluctuation, and exogenous change in the price for fossil fuel will influence the entire wood flows.

Keywords: Forest supply chain, integrated market, bioenergy, wood procurement, wood distribution, quadratic programming[1]

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2.1 Introduction

The forest supply chain provides original forest resource for divergent final uses. It can be viewed as a large network of production units that gradually process raw materials, i.e., sawlogs, pulpwood, and forest bioenergy, into consumer products, i.e., wood products, pulp and paper products, and energy and electricity (D'Amours et al. 2008). The difference between sawlogs and pulpwood is defined by the length, diameter, and quality of the timber. Traditionally, the lower part of the tree, which has a larger diameter with higher value, is sent to sawmills as sawlogs. The upper, thinner part with a lower value is better suited for pulp and paper mills as pulpwood. The remaining tops and branches with least value, treated as residues, are left in the forest as soil nutrient or forwarded to heating plants as forest bioenergy.

The energy crisis of the 1970s, along with later soaring prices of fossil fuels, boosted the development of alternative renewable energy, among which forest bioenergy is one of the most promising and feasible choices for medium- and large-scale heating and electricity generation. Since trees capture and store carbon as part of photosynthesis, the net release of CO_2 into the atmosphere caused by the combustion of forest bioenergy is near zero. It means that using forest bioenergy instead of fossil fuels can reduce CO_2 emissions from existing power production plants. For the countries that own abundant forest resources, appropriate exploitation of forest bioenergy complies with environmental commitments regarding “green” energy as well as relieves their dependence on energy import. Therefore, many countries, such as Sweden (Gunnarsson et al. 2004), Belgium (Van Belle et al. 2003), Austria (Gronalt and Rauch 2007), Ireland (Murphy et al. 2010), and the United Sates (Conrad et al. 2011), have established sustainable energy goals and implemented various subsidies and incentive policies to encourage energy generation from forest-based biomass.

Forest bioenergy normally refers to forest residues that are trivial tree parts left onsite after final felling or thinning, poorly formed logs that cannot be further processed, stubs on the ground, and byproducts that are generated from the wood-processing industries. However, due to the accelerating promotion of wood energy and relatively lower price for forest fuel compared with fossil fuel, directly using pulpwood for power production is becoming attractive. Although pulpwood is more expensive than forest residues, it is more efficient to transport and has higher energy content.

This trend will likely affect other conventional timber consumers, especially the pulp and paper industry, and generate a competition for forest raw materials. Through a mail survey in the southern United States, Conrad et al. (2011) found that although the wood energy facilities and traditional forest industries are not competing for raw materials on a large scale at present, 32% of pulp and paper mills expect that wood-fired power plants will be their largest competitors over the next decade and 55% of wood energy facilities already count pulp and paper mills as their main rivals. As the real price of paper is decreasing over time, the pulp and paper producers have strong desire to reduce the production cost and are thus not willing to pay more for the pulpwood as raw material (Carlgren et al. 2006). However, Lundmark (2006) indicated that in Sweden, if the wood energy consumption exceeds the economic supply of 21 TWh of forest residues, it will be more economical to directly use pulpwood as bioenergy than to further extract forest residues, putting upward pressure on the price for pulpwood. Moreover, Galik et al. (2009) suggested that there will be a dramatic spike in pulpwood price if the demand for bioenergy exceeds the supply of forest residues, which will squeeze out marginal pulpwood consumers. Therefore, concerns increase about competition for forest resources and interaction among traditional forest industries and emerging forest bioenergy facilities. Gunnarsson (2007) saw it as a new and exciting challenge to establish a mathematical model for both forest fuel and pulp products in the forest supply chain.

Numerous models based on operations research have been developed to optimize forest supply chain planning and to understand the complex functioning of the systems during the last half century. Rönnqvist (2003), Bettinger and Chung (2004), Weintraub and Romero (2006), D'Amours et al. (2008), and Carlsson et al. (2009) reviewed the applications and contributions of operations research to the forestry industry from different perspectives on the forest supply chain.

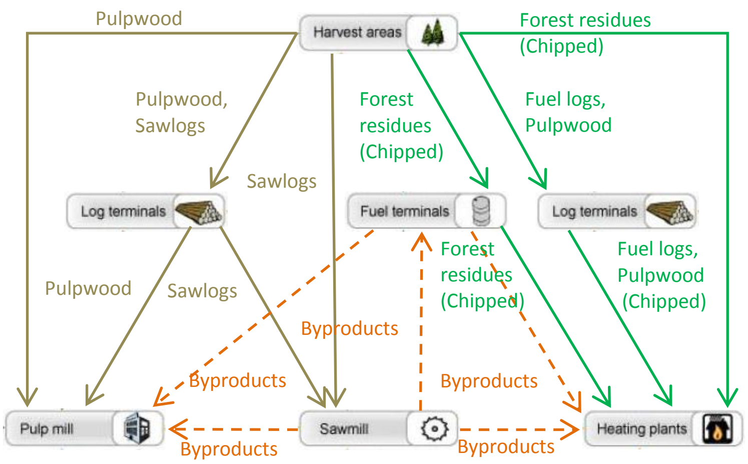
Recent years have witnessed growing interest in integrating the different planning problems, i.e., synchronizing the procurement, production, distribution, and sales activities throughout a set of independent business units or within large international companies that have many interrelated forest product supply chains. Gunnarsson et al. (2004) studied a problem where a supplying company is obliged to deliver a certain amount of forest fuel to several heating plants, involving procurement, conversion, transportation, and terminal location planning. Beaudoin et al. (2007) introduced a centralized annual model to manage the wood flow from the forest to the end market for an integrated forest company that owns many sawmills. Gunnarsson and Rönnqvist (2008) solved an integrated planning of the overall supply chain for one of the world’s largest suppliers of market pulp.

In addition to integrating the divergent activities, it is of increasing importance to integrate different levels of decision-making, ranging from aggregated strategic forest management to detailed operational tasks. Although consistency and feasibility occur as problems under hierarchical planning, there are many successful implementations both in theory and in practice. Cea and Jofre (2000) considered the strategic investment and tactical planning decisions to assist forestry companies. Forsberg et al. (2005) developed a decision support system for strategic and tactical transportation planning in Swedish forestry.

There are many articles that simultaneously deal with various planning problems or link different decision levels, but most focus on one specific forestry industry, i.e., the wood processing industry, the pulp and paper industry, or heating and power generation. Relatively few exist on how to integrate the whole market. In a case study in central Ireland, Murphy et al. (2010) demonstrated that modeling and planning tools can optimize the allocation of wood fiber in a nontraditional market where both forest bioenergy and logs are supplied.

In this paper, we integrate the two value chains of roundwood and forest bioenergy in an optimization model. That is, all raw materials in the forest, i.e., sawlogs, pulpwood, and forest residues, and byproducts from sawmills, i.e., wood chips and bark, exist in an integrated market where pulpwood can be sent to heating plants as bioenergy (Figure 2.1). The model represents a multiperiod multicommodity network planning problem with multiple sources of supply, i.e., preselected harvest areas, and multiple types of destinations, i.e., sawmills, pulp mills, and heating plants. The planning horizon is 1 year and monthly time periods are considered to account for the seasonality, which has a great influence on the whole supply chain. For example, during the summer in the Nordic countries, operations in the forest often focus on silvicultural management and harvest capacity decreases due to holidays, hence affecting the supply of logs. On the demand side, the consumption of heating energy during January-February is much higher than during June-July. All of these concerns imply a need for advance planning. The decisions in the model incorporate purchasing raw materials in harvest areas, reassigning byproducts from sawmills, transporting those assortments to different points for chipping, storing, wood processing or wood fired, and replenishing fossil fuel when necessary.

Figure 2.1 An integrated market for raw materials in the initial stage of the forest supply chain



A combined wood procurement and distribution problem requires a supplying company to determine how to obtain the wood required by mills and how to deliver from sources to destinations, both distributed geographically. If the supplying company owns forest, the decisions involve which blocks to harvest (Beaudoin et al. 2007), what types of harvesting methods to use (Burger and Jamnick 1995), how to allocate the crews (Karlsson et al. 2004), whether to buck trees into logs of specific dimensions in the woods (Carlgren et al. 2006), and how many types of logs to produce onsite (Chauhan et al. 2009). The volumes of assortments at supply nodes will be affected by these factors, but the related harvesting or purchasing costs are normally predefined as parameters. Analysts usually use mixed integer programming (MIP) models to balance the supply of raw materials and the demand for specific products.

To our knowledge, so far no attempt has been made to take the unit purchasing costs as variables on which the corresponding supplies of different raw materials depend linearly. In our paper, we assume that the supplying company purchases raw materials directly from the preselected harvest areas. The higher unit purchasing cost the supplying company offers, the more volume, if possible, the forest owners will provide under constraints of harvest nature. With this price mechanism, the popularity of harvest areas can be distinguished. Given that the demands in mills and delivery prices for assortments are specified, the objective is to minimize the total costs for the integrated market including the purchasing cost of raw materials.

Therefore, the model is a quadratic programming (QP) problem with a quadratic objective function and linear constraints.

We use the data provided by the Forestry Research Institute of Sweden to simulate the integrated market and the separated market, respectively. The difference between these two markets is whether or not pulpwood can be used in heating plants as forest bioenergy. The case study covers the harvest areas in southern Sweden that can annually supply 1.6-2.2 million m^3 of required wood assortments to sawmills, pulp mills, and heating plants. We generate 16 instances to test the proposed model and make seven different scenario comparisons to analyze the results. The main purpose is to study how price restriction, market regulation, harvest flexibility, demand fluctuation, and exogenous change in the price for fossil fuel will influence the entire wood flows. Pairwise comparisons show that in the integrated market, competition for raw materials between forest bioenergy facilities and traditional forest industries pushes up the unit purchasing costs of pulpwood. The results also demonstrate that resources can be effectively allocated by the price mechanism in the supply market. The overall energy value of forest bioenergy delivered to heating plants is 23% more than the amount in the situation when volume and unit purchasing cost of raw materials are fixed. The results also indicate the strong connection and high dependency among all forest-related industries.

The main contribution of this paper is twofold. Firstly, we integrate the two value chains roundwood and forest bioenergy, including decisions about procurement, transportation, chipping, and inventory planning. Secondly, we take the unit purchasing costs as variables and assume that the supplies of raw materials are linear with respect to the unit costs. It allows for the possibility to find equilibrium prices for raw materials in each harvest area and to make the value chain more efficient. The remainder of this paper is as follows. In the next section, a detailed problem description will be given. We then formulate the mathematical model in the third section. In the fourth section, a case study based on real-world data is provided, with numerical results and scenario analysis. The paper ends with some concluding remarks and suggestions for further work in the fifth section.

2.2 Problem description

2.2.1 Supply of raw materials in the harvest areas

Forest in a district is divided into harvest areas that vary in size and in available volumes of assortments. The assortments of raw materials can be classified according to their use. Sawlogs, pulpwood, and forest residues are the major parts of the assortments. Each part can be further divided into several subgroups according to their species, qualities, and dimensions.

The harvesting operation in Sweden, as well as in many other countries, fells trees and delimbs stems. The stems are directly bucked into logs by the harvesters under optimized bucking decisions. Top and limb portions of the tree are left as residues. The high-quality sawlogs and the lower-quality pulpwood are forwarded to storage locations adjacent to forest roads by forwarders. They will be piled temporally and then transported to wood-processing factories. The residues are left for around a year in the woods or adjacent to roads in large piles and then chipped before final delivery. In addition, during the felling operation, defect wood, i.e., decayed or damaged logs, will be found. These logs cannot be further processed either in sawmills or in pulp mills but used as fuel logs for energy generation. They will also be left in the forest for drying, the same as forest residues, and then transported to terminals for storage and chipping.

Seasonality has great influence on harvesting operations. In the Nordic countries, for example, most sawmills are closed for holiday in most of July, meaning that a relatively small proportion of the annual harvesting is done during this period. Operations instead deal with silvicultural management such as regeneration and cleaning activities, which will reduce the supply of logs in that period and consequently affect the availability of byproducts. This implies a need for inventory planning over all of the year.

In this paper, we assume that the supplying company has selected a potential number of harvest areas to purchase forest raw materials. The respective ranges of acceptable unit purchasing costs and corresponding supplies for each assortment are also predefined. The volumes supplied or harvested depend linearly on the unit purchasing costs offered. Additionally, the unit cost ranges of the same assortment are the same in all of the harvest areas, but the supply ranges of that assortment depend on the production level in the area. Table 2.1 gives typical data for four areas in the case study. Therefore, one of the crucial decisions for the supplying company is to determine what the unit costs should be in order to obtain enough raw materials to satisfy demands while minimizing the total procurement cost.

Table 2.1 Unit purchasing cost ranges and supply ranges for forest raw materials in four harvest areas

	Sawlogs		**Pulpwood**			**Fuel logs**	**Forest residues**	
	Pine	**Spruce**	**Pine**	**Spruce**	**Birch**	**Decayed wood**	**Branches**	**Tree parts**
Unit cost range (SEK/m^3)								
All areas	383-518	383-518	213-288	225-305	225-305	128-173	43-58	85-115
Supply range (m^3)								
Area-H1	1822-2464	1240-1678	1034-1400	1138-1540	64-86	189-255	--	--
Area-H2	138-186	95-129	102-138	--	44-60	111-151	--	--
Area-H3	757-1025	672-909	943-1275	46-62	218-294	117-159	1018-1378	1676-2268
Area-H4	23-31	--	201-273	--	17-23	--	141-191	83-113

Since forest residues and fuel logs are already available in the forest after the harvesting of logs from the previous year, their actual supplies are simply determined by the unit purchasing costs the supplying company is willing to pay. In contrast, the actual supplies of sawlogs and pulpwood are not only decided by the unit costs but also limited by harvest nature. Because a harvest area is usually composed of several tree species, harvesting of one area will result in the coproduction of various assortments. That is, if a harvest area consists of 400 m^3 of sawlog "Pine" and 600 m^3 of sawlog "Spruce" and we want to harvest 50% volume of sawlog "Pine", we will end up with 200 m^3 of sawlog "Pine" and 300 m^3 of sawlog "Spruce", respectively. In other words, the more one kind of logs are harvested, the more other logs in this area will also be produced and vice versa.

2.2.2 Supply of byproducts in sawmills

Sawlogs sent to sawmills are transformed into boards to produce lumber and dimension parts or into flakes to produce panels. The process will produce byproducts such as bark, wood chips, and sawdust. Traditionally, except that some byproducts are burned directly to generate steam for wood dryers in sawmills, most of the byproducts, especially wood chips and sawdust, are further transported to pulp mills as raw material for pulp. However, since forest fuel becomes an increasingly attractive alternative for heating plants, wood chips can also be shipped to heating plants.

The supplying company delivers sawlogs to several sawmills. The byproducts with specified prices are then transported, if needed, to pulp mills or heating plants for further use. We assume that the wood products and different types of byproducts are proportionally produced. That is, once the amount of sawlogs processed in sawmills is known, the volumes of different byproducts generated can be exactly measured in each time period. The supplying company is responsible to continuously move away the byproducts, since there is a limited storage capacity for chips and sawdust in sawmills.

2.2.3 Chipping and storage

The residues and fuel logs are piled at the landing until they are chipped for direct delivery or transported to terminals for further process or storage. In this paper, we assume that chipping of residues is carried out in the forest by the mobile machinery. Though chipping onsite is very costly, it is more economical for later transportation since the loading capacity of bulky tree tops and branches is low. Yet chipping of fuel logs and pulpwood, if any, typically occurs at terminals by industrial chippers before they are eventually sent to heating plants. Byproducts from sawmills are already chipped. Furthermore, all the sawlogs sent to sawmills or pulpwood to pulp mills involve no chipping. They are transported as logs all the way from sources to terminals or directly to final destinations.

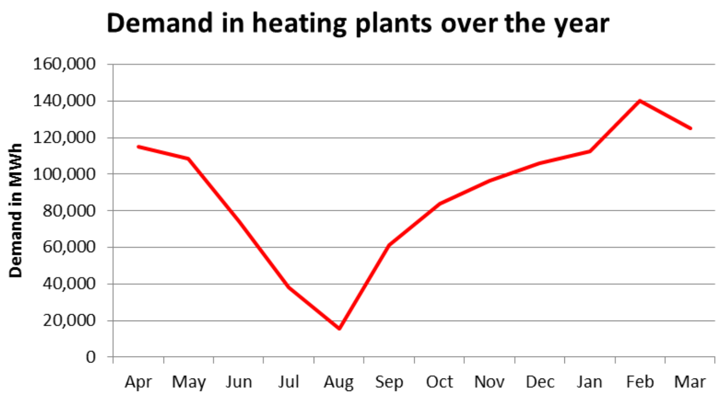
Storage plays an important role in the whole supply chain. It is used to balance the seasonal fluctuation of supply and demand. Our formulation considers two types of storage: roadside in the forest and at terminals. Both locations have certain capacity constraints. However, due to higher quality degradation in the forest, it is typically more expensive to store harvested raw materials in the forest than in the terminals. Moreover, once forest residues are chipped, they have to be shipped to the terminals or heating plants immediately since there are no chip storage bins in the forest. In addition, the variation in the production of sawlogs has a direct impact on the supply of byproducts. We therefore assume that byproducts can also be transported to terminals with chip storage bins for temporary storage.

It is true that not all terminals have chipping ability or storage capacity for chipped forest fuel. In this paper, instead of introducing new sets of variables and constraints to separate the terminals of different types, we model these possibilities by prohibiting the flow of logs sent to heating plants from terminals without chipping equipment and by preventing the flow of chipped bioenergy via terminals without chip storage bins.

2.2.4 Demand at heating plants

Heating plants usually supply residential and industrial sectors with hot water for heating. Therefore, their demands for energy fluctuate with seasons. Figure 2.2 depicts the total demand in 22 heating plants during the whole planning period in the case study. In contrast with the supply of forest fuel given in terms of volume (cubic meters), the demand at the heating plants is specified in energy value (megawatt hours). Therefore, conversion from volume to energy is necessary in the flow conservation constraints. The energy values of assortments depend on their species, moisture content, and the portion of the tree being used, i.e., stem, branches, or bark.

Figure 2.2 Monthly total demand in 22 heating plants



2.2.5 Demand in sawmills and pulp mills

The sawmills and pulp mills for which the supplying company is obliged to provide raw materials are all contract based. Log types and their delivery prices are predefined. Volumes of sawlogs and pulpwood are in specified amounts. Differently, the demand for byproducts in pulp mills is flexible, within a certain interval based on the consumption of pulpwood. The proportion of pulpwood and byproducts used can be adjusted according to the production recipes for specific pulp.

2.2.6 Transportation

The supplying company is responsible for delivering all of the wood assortments required by different facilities. As far as the different assortments are concerned, the density will limit the quantity that a truck can load. A weight limit of 60 tons for trucks corresponds to a maximal loading weight of about 40 tons and a length restriction of 24 meters cannot be violated. Typically, the loading capacity of logs is limited by weight and that of other assortments is by volume. The transportation cost is thereby associated with the types of assortments. With regard to the same assortment in different form, i.e., chipped or nonchipped fuel logs and pulpwood, it is cheaper to transport chips, yet it costs more in loading and unloading. As a result, the transportation costs, including loading and unloading, are similar. As to the distance factor, we use the common assumption that the unit transportation cost is linear with the distance between two points, which is the case in transportation agreements. It is possible to control the flow between any two locations under various assumptions.

2.3 Mathematical formulation

In this section, we present the mathematical model of an integrated market for sawlogs, pulpwood, and forest bioenergy. First, the sets used in the model are introduced. Note that the subscript capital letters indicate the type of raw materials or products and do not carry a value.

A	Set of harvest areas
K	Set of terminals
H	Set of heating plants
S	Set of sawmills
M	Set of pulp mills
R_S	Set of sawlog assortments
R_P	Set of pulpwood assortments
R_F	Set of fuel log assortments
R_G	Set of forest residue assortments
R	Set of raw materials, $R = R_S \cup R_P \cup R_F \cup R_G$
P_S	Set of finished wood products in sawmills
P_B	Set of byproducts in sawmills
P	Set of products processed in sawmills, $P = P_S \cup P_B$
W	Set of fossil fuel alternatives
T	Set of time periods

In the remainder of the paper, we will use index i for nodes of outbound flow (sources), j for nodes of inbound flow (destinations), a for harvest areas, k for terminals, h for heating plants, s for sawmills, m for pulp mills, r for raw materials, p for processed products in sawmills, w for fossil fuel and t for time periods.

The parameters used in the model are as follows. As mentioned in Section 2.2.6, transportation cost includes loading and unloading operational fees.

α_{pt}	Proportion of sawlogs processed into product p in time period t, where $\sum_{p \in P} \alpha_{pt} = 1$
β_h	Minimal percentage of forest bioenergy required to use at heating plant h
γ^L_{mpt}	Minimal percentage of byproduct p demanded in pulp mill m in time period t, $p \in P_B$
γ^U_{mpt}	Maximal percentage of byproduct p demanded in pulp mill m in time period t, $p \in P_B$
c^A_r	Unit chipping cost of raw material r in harvest areas, $r \in R_G$
c^K_r	Unit chipping cost of raw material r at terminals, $r \in R_F \cup R_P$

c_{ijr}^{T} Unit transportation cost of raw material r from source i to destination j, $i \in A \bigcup K$, $j \in K \bigcup H \bigcup S \bigcup M$, $r \in R$

c_{ijp}^{T} Unit transportation cost of byproduct p from source i to destination j, $i \in S \bigcup K$, $j \in K \bigcup H \bigcup M$, $p \in P_B$

d_{ht}^{H} Demand for converted energy at heating plant h in time period t

d_{srt}^{S} Demand for raw material r in sawmill s in time period t, $r \in R_S$

d_{mrt}^{M} Demand for raw material r in pulp mill m in time period t, $r \in R_P$

e_{rt} Energy value of one volume unit of raw material r in time period t, $r \in R_F \bigcup R_G \bigcup R_P$

e_{pt} Energy value of one volume unit of byproduct p in time period t, $p \in P_B$

f_{pt}^{B} Unit purchasing cost of byproduct p in time period t, $p \in P_B$

f_{wt}^{E} Unit energy purchasing cost of fossil fuel w in time period t

f_{art}^{L} Lower bound of unit purchasing cost in harvest area a of raw material r in time period t, $r \in R$

f_{art}^{U} Upper bound of unit purchasing cost in harvest area a of raw material r in time period t, $r \in R$

h_{art}^{A} Unit inventory cost in harvest area a of raw material r in time period t, $r \in R$

h_{krt}^{K} Unit inventory cost at terminal k of raw material r in time period t, $r \in R$

h_{kpt}^{K} Unit inventory cost at terminal k of byproduct p in time period t, $p \in P_B$

n_{srt}^{S} Unit penalty cost in sawmill s of raw material r in time period t, $r \in R_S$

n_{mrt}^{M} Unit penalty cost in pulp mill m of raw material r in time period t, $r \in R_P$

n_{mpt}^{M} Unit penalty cost in pulp mill m of byproduct p in time period t, $p \in P_B$

s_{art}^{L} Lower bound of supply in harvest area a of raw material r in time period t, $r \in R$

s_{art}^{U} Upper bound of supply in harvest area a of raw material r in time period t, $r \in R$

v_{a}^{I} Storage capacity in harvest area a

v_{k}^{I} Storage capacity at terminal k

v_{t}^{C} Total capacity of mobile chippers in harvest area in time period t

v_{kt}^{C} Chipping capacity at terminal k in time period t

v_{k}^{G} Maximal flow capacity at terminal k

The variables will be presented below, in the same order as they are illustrated in Figure 2.3. Note that the initial storage level, given by time index 0, in different districts is known.

F_{art} Unit purchasing cost in harvest area a of raw material r in time period t, $r \in R$

S_{art} Supply in harvest area a of raw material r in time period t, $r \in R$

X_{ijrt} Flow from source i to destination j of raw material r in time period t, $i \in A \bigcup K$, $j \in K \bigcup H \bigcup S \bigcup M$, $r \in R$

I_{art}^{A} Storage in harvest area a of raw material r at the end of time period t, $r \in R$

I_{krt}^{K} Storage at terminal k of raw material r at the end of time period t, $r \in R$

I_{kpt}^{K} Storage at terminal k of byproduct p at the end of time period t, $p \in P_B$

Z_{ijpt} Flow from source i to destination j of byproduct p in time period t, $i \in S \cup K$, $j \in K \cup H \cup M$, $p \in P_B$

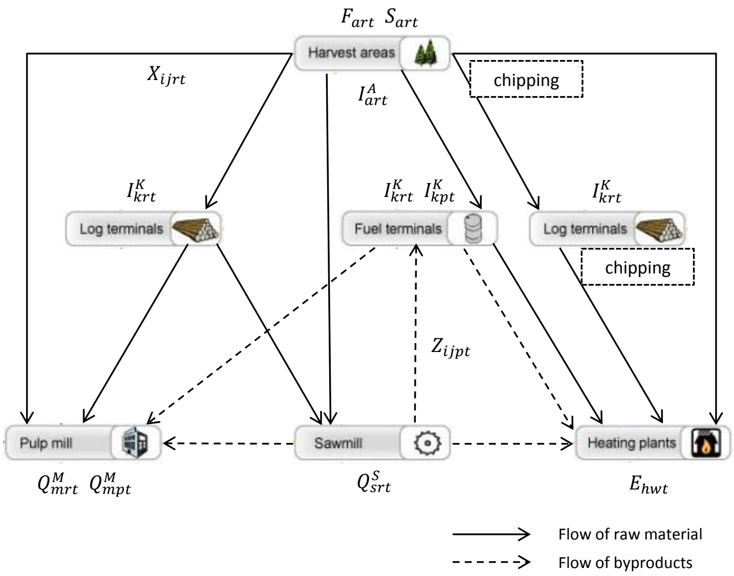
E_{hwt} Fossil fuel w forwarded to heating plant h in time period t

Q^S_{srt} Unsatisfied demand in sawmill s of raw material r in time period t, $r \in R_S$

Q^M_{mrt} Unsatisfied demand in pulp mill m of raw material r in time period t, $r \in R_P$

Q^M_{mpt} Unsatisfied demand in pulp mill m of byproduct p in time period t, $p \in P_B$

Figure 2.3 An illustration of the possible flows in an integrated market



The total cost (TC) of the problem is expressed as

$$\begin{aligned}
\min TC = & \sum_{a\in A}\sum_{r\in R}\sum_{t\in T} F_{art} S_{art} + \sum_{s\in S}\sum_{j\in K\cup H\cup M}\sum_{p\in P_B}\sum_{t\in T} f^B_{pt} Z_{sjpt} + \sum_{h\in H}\sum_{w\in W}\sum_{t\in T} f^E_{wt} E_{hwt} \\
& + \sum_{a\in A}\sum_{j\in K\cup H}\sum_{r\in R_G}\sum_{t\in T} c^A_r X_{ajrt} + \sum_{k\in K}\sum_{h\in H}\sum_{r\in R_P\cup R_F}\sum_{t\in T} c^K_r X_{khrt} \\
& + \sum_{a\in A}\sum_{r\in R}\sum_{t\in T} h^A_{art} I^A_{art} + \sum_{k\in K}\sum_{r\in R}\sum_{t\in T} h^K_{krt} I^K_{krt} + \sum_{k\in K}\sum_{p\in P_B}\sum_{t\in T} h^K_{kpt} I^K_{kpt} \\
& + \sum_{i\in A\cup K}\sum_{j\in K\cup H\cup S\cup M}\sum_{r\in R}\sum_{t\in T} c^T_{ijr} X_{ijrt} + \sum_{i\in S\cup K}\sum_{j\in K\cup H\cup M}\sum_{p\in P_B}\sum_{t\in T} c^T_{ijp} Z_{ijpt} \\
& + \sum_{s\in S}\sum_{r\in R_S}\sum_{t\in T} n^S_{srt} Q^S_{srt} + \sum_{m\in M}\sum_{r\in R_P}\sum_{t\in T} n^M_{mrt} Q^M_{mrt} + \sum_{m\in M}\sum_{p\in P_B}\sum_{t\in T} n^M_{mpt} Q^M_{mpt}
\end{aligned}$$

subject to

$$S_{art} = s^L_{art} + \left(F_{art} - f^L_{art}\right) \cdot \frac{s^U_{art} - s^L_{art}}{f^U_{art} - f^L_{art}}, \quad \forall a \in A, \forall r \in R, \forall t \in T, \tag{1}$$

$$f^L_{art} \le F_{art} \le f^U_{art}, \quad \forall a \in A, \forall r \in R, \forall t \in T, \tag{2}$$

$$\frac{S_{ar_1t} - s^L_{ar_1t}}{s^U_{ar_1t} - s^L_{ar_1t}} = \frac{S_{ar_2t} - s^L_{ar_2t}}{s^U_{ar_2t} - s^L_{ar_2t}}, \quad \forall a \in A, \forall r_1 \in R_S \cup R_P, \forall r_2 \in R_S \cup R_P, \forall t \in T, \tag{3}$$

$$\alpha_{pt}\left(\sum_{i\in A\cup K}\sum_{r\in R_S} X_{isrt}\right) = \sum_{j\in K\cup H\cup M} Z_{sjpt}, \quad \forall s \in S, \forall p \in P_B, \forall t \in T, \tag{4}$$

$$I^A_{ar,t-1} + S_{art} = I^A_{art} + \sum_{j \in K \cup H \cup S \cup M} X_{ajrt}, \quad \forall a \in A, \forall r \in R, \forall t \in T, \tag{5}$$

$$I^K_{kr,t-1} + \sum_{i \in A} X_{ikrt} = I^K_{krt} + \sum_{j \in H \cup S \cup M} X_{kjrt}, \quad \forall k \in K, \forall r \in R, \forall t \in T, \tag{6}$$

$$I^K_{kp,t-1} + \sum_{i \in S} Z_{ikpt} = I^K_{kpt} + \sum_{j \in H \cup M} Z_{kjpt}, \quad \forall k \in K, \forall p \in P_B, \forall t \in T, \tag{7}$$

$$\sum_{i \in A \cup K} \sum_{r \in R_P \cup R_F \cup R_G} e_{rt} X_{ihrt} + \sum_{i \in S \cup K} \sum_{p \in P_B} e_{pt} Z_{ihpt} + \sum_{w \in W} E_{hwt} = d^H_{ht}, \quad \forall h \in H, \forall t \in T, \tag{8}$$

$$\sum_{i \in A \cup K} \sum_{r \in R_P \cup R_F \cup R_G} e_{rt} X_{ihrt} + \sum_{i \in S \cup K} \sum_{p \in P_B} e_{pt} Z_{ihpt} \ge \beta_h d^H_{ht}, \quad \forall h \in H, \forall t \in T, \tag{9}$$

$$\sum_{i \in A \cup K} X_{isrt} + Q^S_{srt} = d^S_{srt}, \quad \forall s \in S, \forall r \in R_S, \forall t \in T, \tag{10}$$

$$\sum_{i \in A \cup K} X_{imrt} + Q^M_{mrt} = d^M_{mrt}, \quad \forall m \in M, \forall r \in R_P, \forall t \in T, \tag{11}$$

$$\gamma^L_{mpt}\left(\sum_{r \in R_P} d^M_{mrt}\right) - Q^M_{mpt} \le \sum_{i \in S \cup K} Z_{impt} \le \gamma^U_{mpt}\left(\sum_{r \in R_P} d^M_{mrt}\right), \quad \forall m \in M, \forall p \in P_B, \forall t \in T, \tag{12}$$

$$\sum_{r \in R} I^A_{art} \le v^I_a, \quad \forall a \in A, \forall t \in T, \tag{13}$$

$$\sum_{r \in R} I^K_{krt} + \sum_{p \in P_B} I^K_{kpt} \le v^I_k, \quad \forall k \in K, \forall t \in T, \tag{14}$$

$$\sum_{a \in A} \sum_{j \in K \cup H} \sum_{r \in R_G} X_{ajrt} \le v^C_t, \quad \forall t \in T, \tag{15}$$

$$\sum_{h \in H} \sum_{r \in R_P \cup R_F} X_{khrt} \le v^C_{kt}, \quad \forall k \in K, \forall t \in T, \tag{16}$$

$$\sum_{i \in A} \sum_{r \in R} X_{ikrt} + \sum_{s \in S} \sum_{p \in P_B} Z_{skpt} \le v^G_k, \quad \forall k \in K, \forall t \in T, \tag{17}$$

$$X_{ijrt} \ge 0 \quad \forall i \in A \cup K, \forall j \in K \cup H \cup S \cup M, \forall r \in R, \forall t \in T, \tag{18}$$

$$Z_{ijpt} \ge 0 \quad \forall i \in S \cup K, \forall j \in K \cup H \cup M, \forall p \in P_B, \forall t \in T, \tag{19}$$

$$F_{art}, S_{art}, I^A_{art}, I^K_{krt}, I^K_{kpt} \ge 0 \quad \forall a \in A, \forall k \in K, \forall r \in R, \forall p \in P_B, \forall t \in T, \tag{20}$$

$$E_{hwt}, Q^M_{mrt}, Q^M_{mpt} \ge 0 \quad \forall h \in H, \forall m \in M, \forall w \in W, \forall r \in R_P, \forall p \in P_B, \forall t \in T, \tag{21}$$

$$Q^S_{srt} \ge 0 \quad \forall s \in S, \forall r \in R_S, \forall t \in T. \tag{22}$$

Because the delivery prices for forest raw materials and byproducts in the mills, as well as the energy prices for bioenergy at heating plants, are covered by preexisting contracts, the revenues associated with delivery are parameters of the problems and thus irrelevant in decisions. Therefore, the objective for the supplying company is to minimize the total cost by procuring wood assortments and byproducts, complementing fossil fuel when necessary, chipping forest fuel, balancing the inventory, and optimizing the wood flows.

The first line in the objective function is the procurement costs that constitute the purchasing cost of raw materials in harvest areas, the purchasing cost of byproducts from sawmills, and the purchasing cost of fossil fuel. Since the supply of each assortment S_{art} depends linearly on the unit purchasing cost F_{art}, the total purchasing cost of raw materials $\sum_{a \in A} \sum_{r \in R} \sum_{t \in T} F_{art} S_{art}$ makes the objective function nonlinear but quadratic. The next line represents chipping costs in the forest and at terminals, respectively. Note that residues in the forest will not be chipped until delivery to terminals or heating plants, the same as logs sent to heating plants as bioenergy. The third line corresponds to the storage costs in different locations. The fourth line is the transportation costs for the whole wood flows in this integrated market and the last line represents the deficit costs.

As mentioned earlier, in our model, the supply of certain raw material in the harvest areas is linear with its purchasing cost, which is expressed as constraint set (*1*). Constraint set (*2*) ensures that the actual unit purchasing cost must be within the cost bounds, together with the supply bounds, which are all predefined under binding contracts between the supplying company and forest owners. Constraint set (*3*) reflects harvest nature that harvested volumes of fresh logs are proportional in any harvest area. As to the supply of byproducts in sawmills, the volume of byproducts available in each time period is based on the volume of sawlogs processed. Constraint set (*4*) stipulates that all of the byproducts will be delivered to different destinations for temporary storage or further use in the same time period when they become available.

Constraint sets (*5*), (*6*) and (*7*) represent classical flow conservation constraints in the harvest areas and at terminals. We assume that the chipping for forest fuel does not influence volumes and thus a change in the form of raw materials will not impact the inventory balance constraints in the harvest areas for residues or those at terminals for logs.

The demands at the heating plants in each time period are specified in terms of energy (megawatt hours), but all raw materials or byproducts transported to the heating plants are expressed in volume unit (cubic meters). We therefore introduce the conversion factors e_{rt} and e_{pt} in constraint set (*8*) to ensure that demand of converted energy is satisfied. Note that different assortments have different energy values that vary from one time period to another. The supplying company will decide how much raw materials and byproducts should be sent to heating plants. In the same period, the company can also provide fossil fuels such as heating oil or coal, specified in energy value, to adapt to the increasing demand during the winter. However, due to environmental regulation, constraint set (*9*) guarantees that a minimal percentage of forest fuel β_h will be used as "green" energy at heating plants. In order to get a robust model, we introduce penalized variables to represent the deviation of the amount delivered from the amount demanded in one period, respectively, in sawmills (constraint set (*10*)) and pulp mills (constraint sets (*11*) and (*12*)). If these variables are not included in the model, it might be impossible to find any feasible solution or to identify the problems. Actually, the unit penalty costs are set large enough to ensure that all of the demands in mills are satisfied.

Constraint sets (*13*) and (*14*) refer to capacity restrictions regarding storing in each district. Constraint set (*15*) gives a restriction on the total volume of forest residues that can be chipped in each time period by the mobile chippers working at harvest areas. Similarly, at every terminal with permanent chipping equipment for fuel logs or pulpwood, the monthly amount that can be chipped is limited by constraint set (*16*). Constraint set (*17*) restricts the throughput or total flow handled at each terminal. All of the variables are continuous and no less than zero, which are specified in constraints (*18*), (*19*), (*20*), (*21*), and (*22*).

2.4 Case study and discussion

In this section, we apply the proposed model to a hypothetical but realistic case study. The Forestry Research Institute of Sweden provides the data from two different real cases of the same company, one for roundwood and one for forest bioenergy. All harvest areas, terminals, forest industries, and wood energy facilities are located in a region in southern Sweden. The geographical distribution of supply and demand nodes is given in Figure 2.4. These harvest areas, corresponding to aggregated standard areas used in the Swedish forest industry, can annually supply 1.6-2.2 million m^3 of required wood assortments to sawmills, pulp mills, and heating plants.

Figure 2.4 Geographical distribution of nodes in the case study

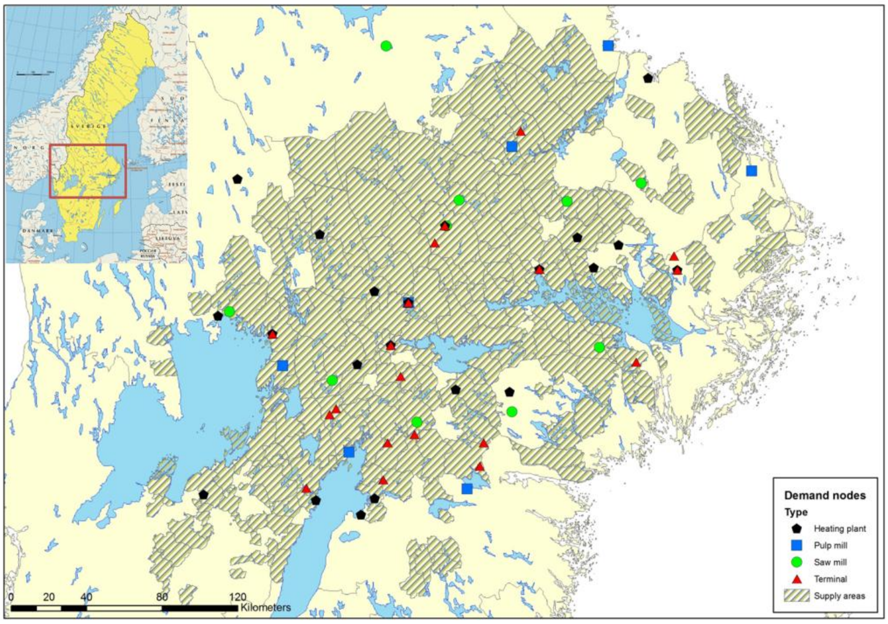


Table 2.2 lists the basic information for this case study. Monthly total supplies of raw materials and byproducts and demands in sawmills, pulp mills, and heating plants are illustrated in Figure 2.5. The volumes of raw materials and byproducts are measured in cubic meters and energy value is in megawatt hours. We notice that the demands (broken line) for sawlogs in sawmills and pulpwood in pulp mills are all within the supply ranges (solid line) whereas the demand (broken line) at heating plants exceeds the maximal available supplies (stacked area) of forest bioenergy during the winter. Moreover, since a relatively small proportion of the annual harvesting for sawlogs is done during the summer (July), the supplies of byproducts in that period are lower than the minimum demands in pulp mills. These observations all imply a need for efficient inventory management during the year.

We generate 16 instances to test the proposed model and make seven different scenario comparisons to analyze the results. The main purpose is to investigate the changes of wood flows in the whole forest raw material market under various assumptions. In the integrated market, it is possible to use pulpwood as forest fuel whereas in the separated market, it is not allowable to send pulpwood to heating plants as bioenergy.

Table 2.3 gives a short description of each instance. Table 2.4 and Table 2.5 list the corresponding computational results. Although the costs of raw materials in the case study are set by the authors, they do reflect the relative value of different assortments based on real market prices. Total costs are in Swedish kronor (SEK) and unit costs are in SEK per m^3; 10 SEK is about 1 euro.

Table 2.2 Information for the case study

Number of	
Harvest areas	234
Terminals	20
Heating plants	22
Sawmills	11
Pulp mills	7
Sawlog assortments	2
Pulpwood assortments	3
Fuel log assortments	1
Forest residue assortments	2
Types of fossil fuel	1
Finished wood products	1
Byproducts	2
Time periods	12

Figure 2.5 Monthly total supplies and demands in the case study

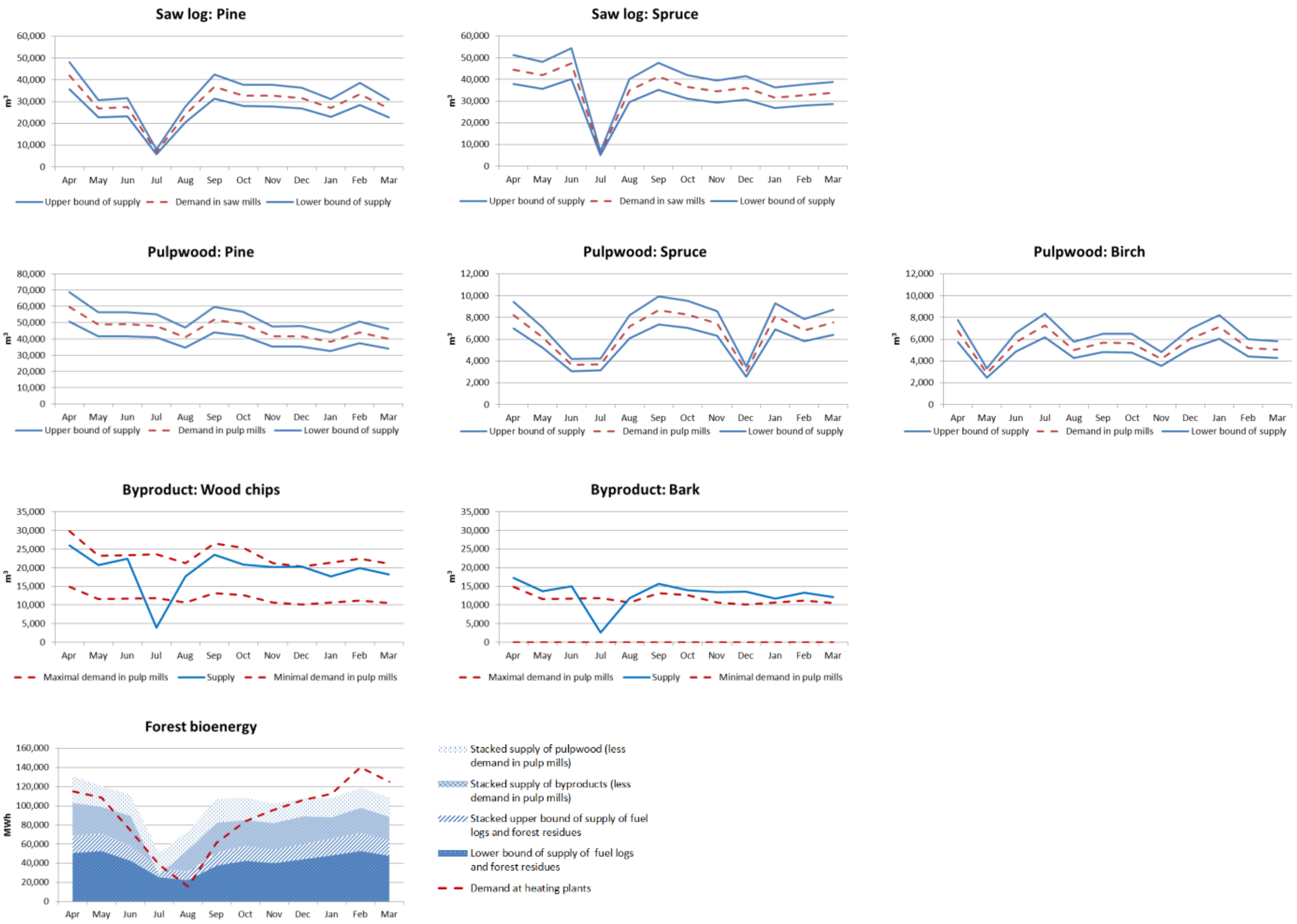


Table 2.3 Basic information for the seven comparisons and 16 instances

Instance	Description	No. of variables	No. of constraints	Solution time (second)
Comparison 1: Integrated market, different price restrictions				
S1	Free prices, integrated market	896 367	64 627	565
S2	Period-same prices, integrated market	896 367	93 268	771
S3	Area-same prices, integrated market	896 367	265 152	785
S4	Fixed prices, integrated market	851 438	31 958	65
Comparison 2: Separated market, different price restrictions				
S5	Free prices, separated market	880 527	64 627	452
S6	Period-same prices, separated market	880 527	93 268	433
S7	Area-same prices, separated market	880 527	265 152	603
S8	Fixed prices, separated market	835 598	31 958	23
Comparison 3: Increased harvest flexibility				
S9	Based on S1, the constraints that assortments are proportionally harvested in any harvest area are relaxed	896 367	54 423	478
Comparison 4: Increased demand at heating plants				
S10	Free prices, separated market, demands at heating plants increase 10%	880 527	64 627	553
S11	Free prices, integrated market, demands at heating plants increase 10%	896 367	64 627	630
Comparison 5: Decreased demand in sawmills				
S12	Period-same prices, separated market, demands in sawmills decrease 10%	880 527	93 268	537
S13	Period-same prices, integrated market, demands in sawmills decrease 10%	896 367	93 268	784
S14	Based on S13, sawlogs can be sent to pulp mills	939 039	93 268	821
Comparison 6: Increased bioenergy proportion at heating plants				
S15	Based on S3, minimal percentage of bioenergy used in heating plants increases from 50% to 80%	896 367	265 152	842
Comparison 7: Change in price for fossil fuel				
S16	Based on S3, the price for fossil fuel changes from 50% less to 50% more	896 367	265 152	796

Table 2.4 Cost comparisons of 15 instances (SEK)

Instance	Purchasing Raw material	Purchasing Byproduct	Purchasing Fossil fuel	Chipping	Storage	Transport	Deficit	Total cost
S1	592 994 210	21 179 263	--	57 671 659	2 204 370	149 859 644	--	823 909 146
S2	589 271 005	21 179 263	5 045 729	59 426 347	2 136 040	151 702 497	--	828 760 880
S3	573 413 187	21 179 263	28 399 276	58 493 879	2 255 646	151 246 790	--	834 988 041
S4	557 821 700	21 179 263	69 518 122	50 614 924	1 236 913	148 733 657	--	849 104 577
S5	569 828 674	21 179 263	35 591 570	58 204 922	2 060 614	149 816 517	--	836 681 559
S6	569 552 898	21 179 263	35 590 499	58 205 252	1 807 814	150 905 306	--	837 241 032
S7	569 070 294	21 179 263	35 586 848	58 206 376	1 978 255	151 589 762	--	837 610 797
S8	557 821 700	21 179 263	69 535 222	50 614 240	1 237 285	148 732 526	--	849 120 236
S9	597 844 938	21 179 263	--	52 755 077	1 841 077	145 418 764	--	819 039 118
S10	569 829 401	21 179 263	76 543 309	58 205 702	1 766 594	149 039 640	--	876 563 908
S11	604 309 076	21 179 263	20 936 894	60 428 911	2 068 770	151 376 049	--	860 298 964
S12	534 971 383	19 061 336	53 587 806	58 205 626	2 898 760	141 902 028	--	810 626 939
S13	534 971 383	19 061 336	53 587 806	58 205 626	2 898 760	141 902 028	--	810 626 939
S14	553 535 469	19 061 336	--	59 259 619	1 895 263	140 147 763	--	773 899 451
S15	573 436 139	21 179 263	28 364 869	58 495 255	2 285 565	151 471 404	--	835 232 495

Table 2.5 Actual total supplies of raw materials of 15 instances (m^3)

Instance	Sawlogs		Pulpwood			Fuel logs	Forest residues	
	Pine	Spruce	Pine	Spruce	Birch	Decayed wood	Branches	Tree parts
S1	348 548	421 607	589 803	83 165	70 833	71 490	351 306	103 486
S2	348 548	421 607	578 313	82 563	69 176	71 490	367 506	106 749
S3	348 548	421 607	558 757	79 229	67 476	71 490	367 523	106 749
S4	348 551	421 616	552 495	78 832	66 588	62 165	319 585	92 825
S5	348 548	421 607	552 489	78 824	66 584	71 490	367 510	106 749
S6	348 548	421 607	552 489	78 824	66 584	71 490	367 513	106 749
S7	348 548	421 607	552 489	78 824	66 584	71 490	367 523	106 749
S8	348 551	421 616	552 495	78 832	66 588	62 165	319 585	92 825
S9	348 548	421 607	606 042	87 685	72 066	71 486	313 756	91 873
S10	348 548	421 607	552 489	78 824	66 584	71 490	367 517	106 749
S11	348 548	421 607	600 489	83 935	72 011	71 490	367 507	106 749
S12	326 826	395 701	552 489	78 824	66 584	71 490	367 516	106 749
S13	326 826	395 701	552 489	78 824	66 584	71 490	367 516	106 749
S14	339 275	408 254	557 784	79 947	67 045	71 490	359 145	104 229
S15	348 548	421 607	558 772	79 249	67 478	71 490	367 523	106 749

We use AMPL as the modeling language and CPLEX 10.0 as the solver. The instances have been solved on a T7300 2.00 GHz processor with 3 GB of RAM. The number of variables and constraints and the solution time of each instance are also included in Table 2.3. After AMPL's presolve phase reduces the size of the instances, the number of variables and constraints of each instance is still very large. However, since the proposed model is a typical QP problem, CPLEX 10.0 solves QP problems well within reasonable time.

2.4.1 Comparison 1: Integrated market, different price restrictions

To begin with, we study the effect of various price restrictions in the harvest areas where the supplying company purchases forest raw materials. It provides insights into the supply-market price behavior, which cannot be obtained by using the conventional wood procurement assumption.

Instance S1

The model presented in Section 2.3 represents the scenario of free prices. That is, there are no temporal or spatial constraints on the unit purchase costs of different raw materials in one period or in any harvest area. In other words, all of the assortments can be purchased at any price within the price range, regardless of the prices of the same assortment in other harvest areas or other periods.

Instance S2

The second instance includes the temporal constraints that in any harvest area, the unit purchasing cost of a certain assortment should be the same all over the year. It is called as scenario of period-same prices. This instance represents the case when the supplying company signs the annual procurement contract with forest owners. Here, we add this instance-specific constraint set into the proposed model:

$$F_{art_1} = F_{art_2}, \quad \forall a \in A, \forall r \in R, \forall t_1, t_2 \in T. \qquad (23)$$

Instance S3

In the third instance, we introduce the spatial constraints that in any period of the planning year, the unit purchasing cost of a certain assortment should be the same among all of the harvest areas. We call it the scenario of area-same prices. It represents that the supplying company itself has this kind of procurement rule. It is actually the current situation in Sweden. Consequently, we insert the following constraint set into the model:

$$F_{a_1rt} = F_{a_2rt}, \quad \forall a_1, a_2 \in A, \forall r \in R, \forall t \in T \tag{24}$$

Instance S4

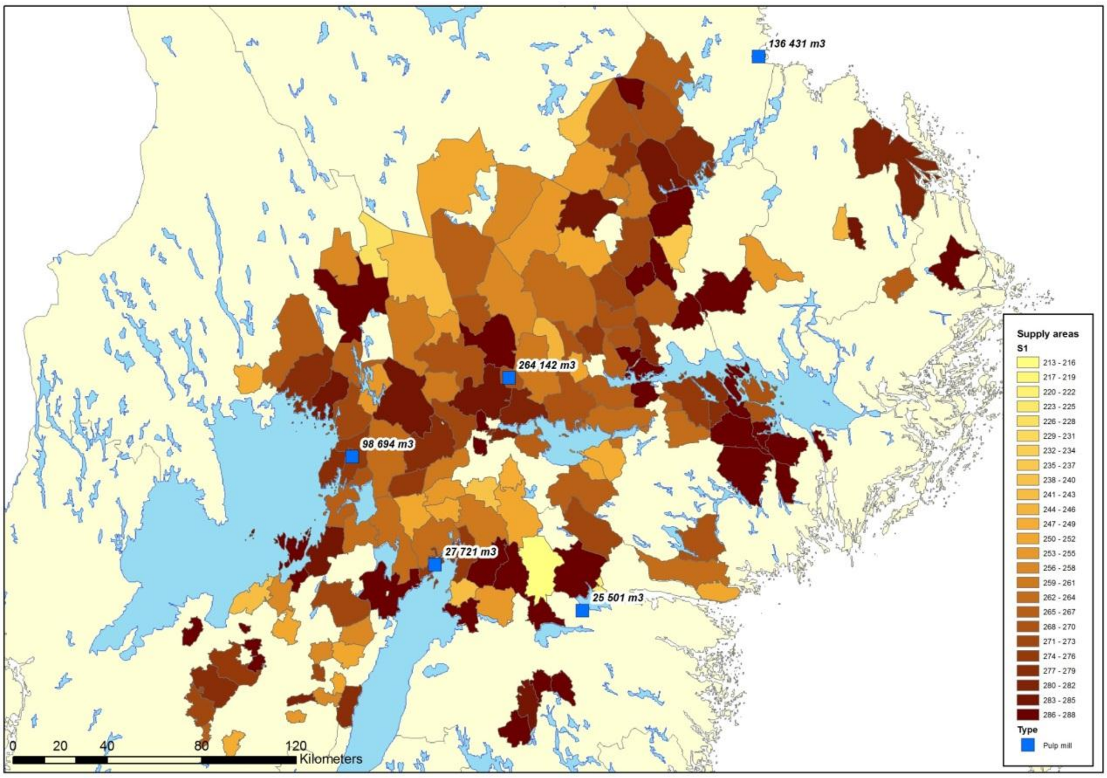
We also study the situation where the purchasing costs and volumes of raw materials are fixed, which is assumed by the majority of models dealing with the wood procurement problem. In this scenario, variables F_{art} and S_{art} become parameters, i.e., the midprice of the cost range and the average amount of the supply range, respectively. The total purchasing cost of raw materials turns into a parameter as well. We also exclude constraint sets (*1*), (*2*), and (*3*) in the proposed model. Therefore, the original optimization model becomes a classic network linear program.

Firstly, we compare the results of Instance S1, Instance S2, and Instance S3. As we expect, the total cost of Instance S1 is the lowest with the least constraints, while that of Instance S3 is the highest with the most constraints. In Instance S2 and Instance S3, less is spent on raw materials in the forest, but much more fossil fuel is sent to heating plants than in Instance S1. These changes stem from the fact that less pulpwood is purchased, and hence, less is forwarded to heating plants in Instance S2 and Instance S3.

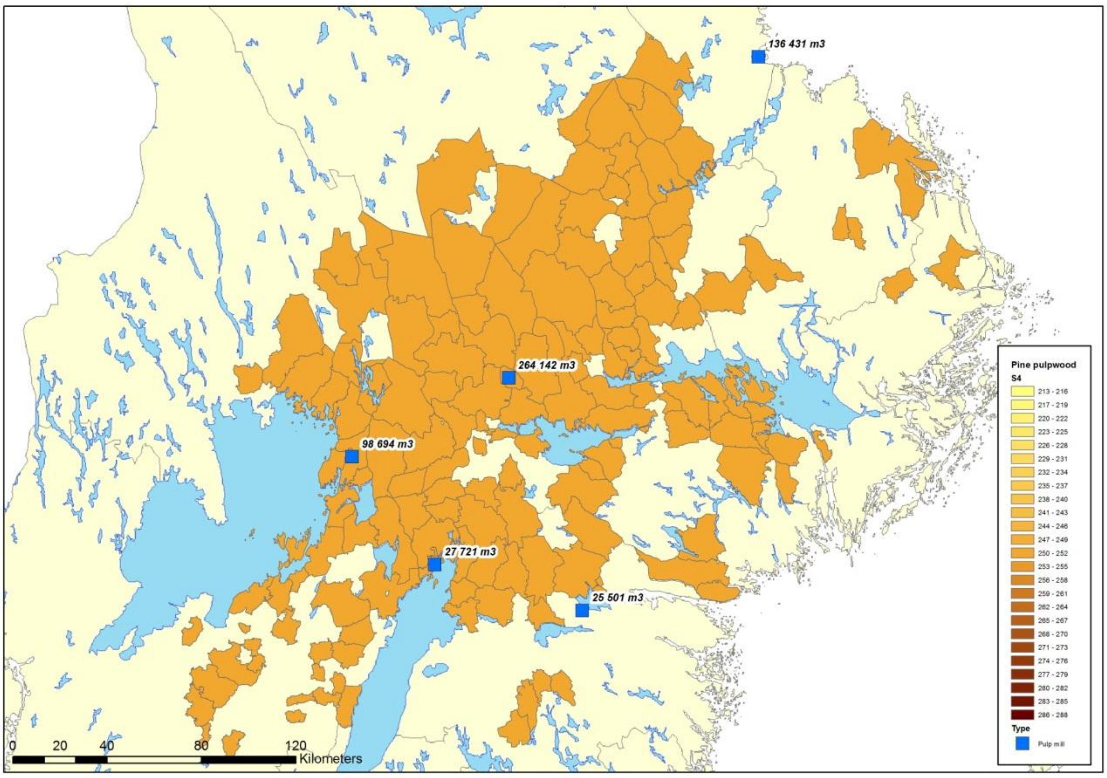
Secondly, we focus on the comparison between Instance S1 and Instance S4. When the supplies of raw materials can change freely within certain ranges and be decided according to the purchasing costs in Instance S1, the price mechanism will effectively allocate the resources and no other uneconomic resources, such as heating oil, are needed. For example, in the first map in Figure 2.6, the gradient color indicates the popularity of the harvest areas for pulpwood “Pine”. The darker the color is, the higher the unit purchasing cost in those areas is, and thus higher volume is offered. The increase in purchasing cost of raw materials is justifiable as long as it can be offset by a reduction in total cost.

By contrast, in Instance S4, because the fixed supplies of raw materials are slightly higher than the demands in the final destinations, the excess or undesirable sawlogs have to be left in the forest and slightly more pulpwood can be sent to heating plants. However, the limited supplies of bioenergy cannot meet the requirements from heating plants. Therefore, a shortage arises that has to be covered by relatively expensive heating oil. We find that the overall energy value of forest bioenergy delivered to heating plants in Instance S1 is 23% more than the amount in Instance S4.

Figure 2.6 Unit purchasing costs of pulpwood "Pine" in the harvest areas



Average unit purchasing cost in S1 (free prices)



Average unit purchasing cost in S4 (fixed prices)

2.4.2 Comparison 2: Separated market, different price restrictions

All of the instances in Comparison 1 are based on the assumption that it is allowable to send pulpwood to heating plants. It can be treated as an integrated market for raw materials, which is the perfect market condition. However, several institutional restrictions in Sweden limit the use of pulpwood in energy generation (Lundmark 2006). Therefore, we modify the assumption and make it forbidden to transport pulpwood to heating plants, which represents a separated market. This change can be achieved by modification of the route design. The above four instances become Instance S5, Instance S6, Instance S7, and Instance S8, correspondingly.

Different from what we observed from Instance S1, Instance S2, and Instance S3, the gaps among the total costs of Instance S5, Instance S6, and Instance S7 are negligible. The supplies of sawlogs and pulpwood in a separated market are exact to the demands in sawmills and pulp mills. No extra pulpwood is sent to heating plants. The supplies of fuel logs and forest residues reach their upper bounds.

However, it is obvious that the total costs in the four instances in Comparison 2 are all higher than their counterparts in Comparison 1. This arises from the fact that expensive fossil fuel has to be used to fill the shortfall at heating plants whereas it can be totally or partially substituted with cheaper pulpwood in an integrated market. Because more pulpwood is desired, no matter what kind of price restrictions are applied, the unit purchasing costs of pulpwood in the integrated market are higher than those in the separated market (Figure 2.7). It is in line with the concern that once it is acceptable to sell pulpwood to heating plants, competition for raw materials between forest bioenergy facilities and traditional forest industries is expected to occur (Conrad et al. 2011).

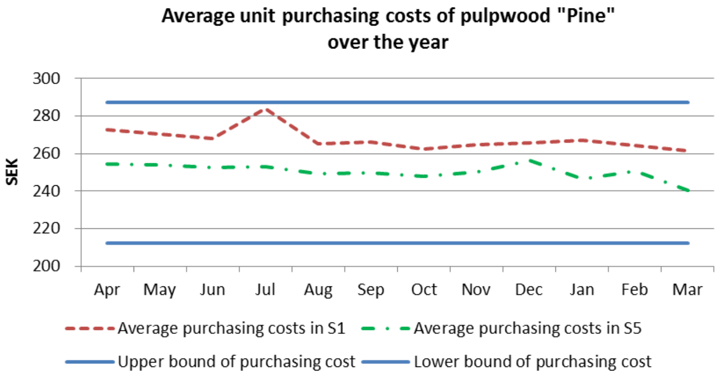
Last but not least, we have to point out that Instance S8 with fixed supply in a separated market actually represents the situation in the real world for some forest companies. The least flexible situation results in the highest total costs, 3% more than the total cost of an integrated market (Instance S1). This reinforces the benefit of integrating a market for all forest raw materials and introducing a price mechanism in the harvest areas.

2.4.3 Additional comparisons

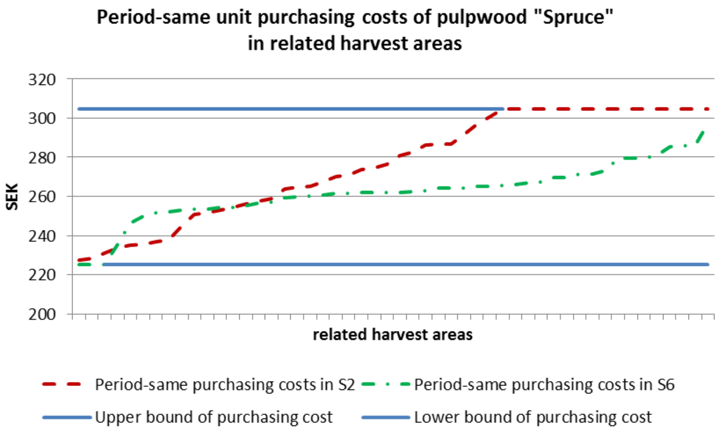
In this section, we will briefly discuss the other five scenarios.

As we mentioned in Section 2.2.1, harvest nature leads to the coproduction of assortments in each harvest area. In order to study the effect of harvest flexibility, in Comparison 3, we assume that any assortment can be freely harvested in one area through, for example, specialized final felling or thinning. To achieve this purpose, we take away the constraint set (*3*) in the proposed model. It is interesting to note that although the total volumes of sawlogs harvested are still the same, the allocations of volumes are different, which is reflected by the color shifting of unit purchasing costs in different areas (Figure 2.8). Once the assortments can be ideally harvested as needed in Instance S9, regardless of the harvesting of other assortments in the same area, the distance from harvest areas to final destination determines the prices of the assortments in the areas. That is, the supplying company is willing to pay more for the supplies of raw materials close to the mills, as the corresponding transportation costs will be lower compared with the supplies farther away. Therefore, the closer the harvest area is to the demand nodes, the higher the unit purchasing cost of the required assortment will be (the second map in Figure 2.8).

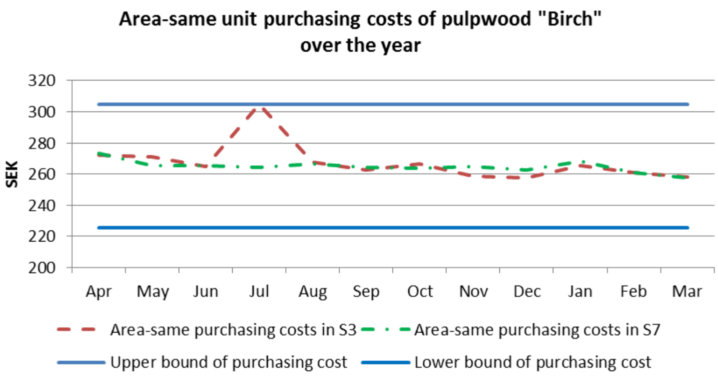
Figure 2.7 Comparisons of unit purchasing costs of pulpwood in an integrated market and those in a separated market



Note: Unit purchasing costs in S1 are all higher than those in S5

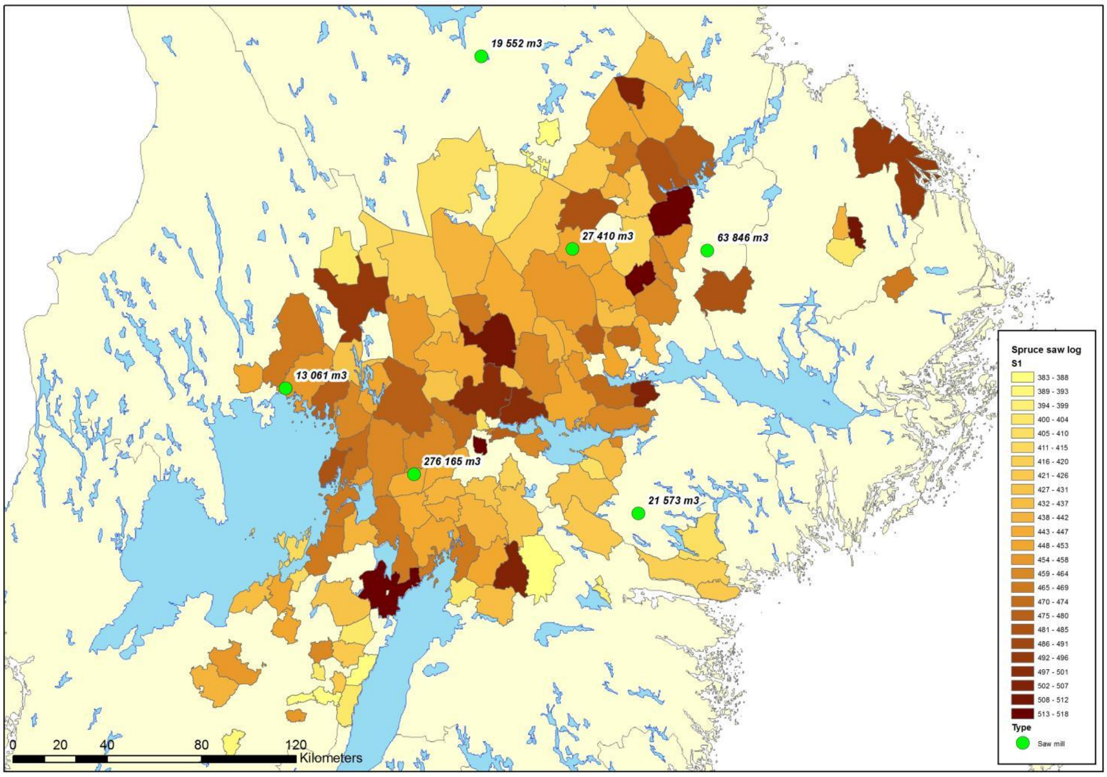


Note: In more harvest areas unit purchasing costs reach the upper bound in S2 than in S6

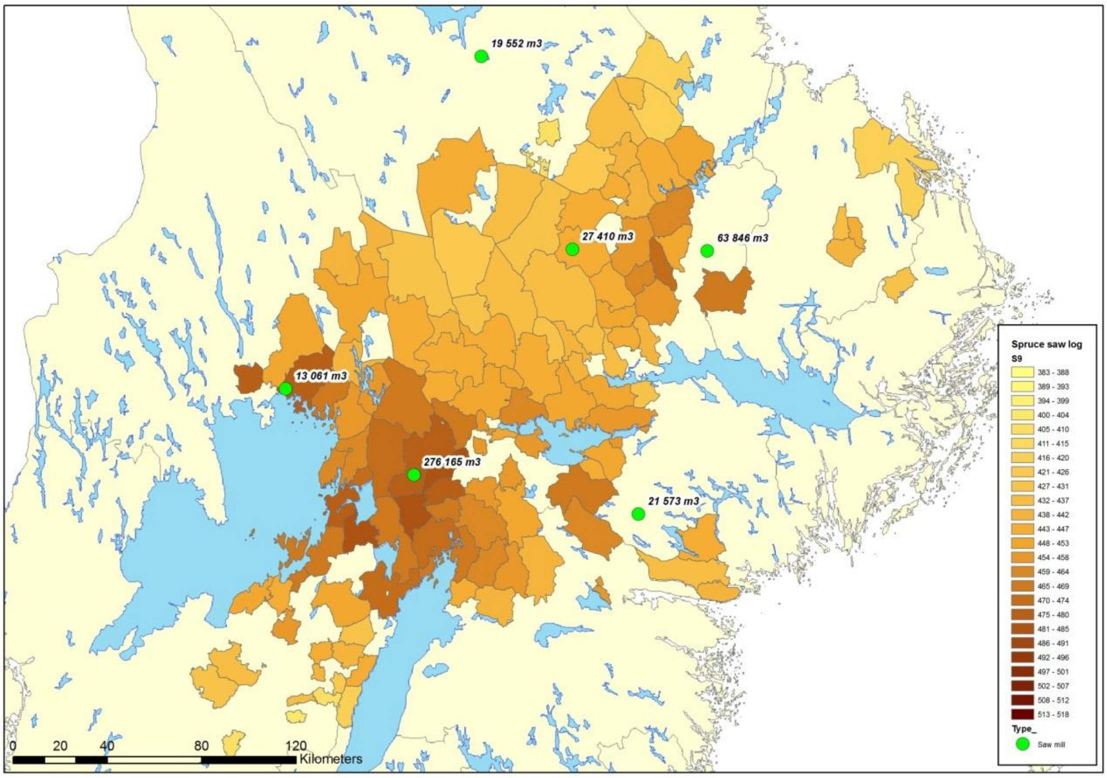


Note: Unit purchasing costs in S3 are slightly higher than those in S7

Figure 2.8 Unit purchasing costs of sawlog "Spruce" in the harvest areas



Average unit purchasing cost in S1
(Assortments within an area have to be proportionally harvested)



Average unit purchasing cost in S9
(Assortments within an area can be freely harvested)

Activities in the forest supply chain are highly interconnected. For example, a decline in exports of finished wood products will influence the demand for sawlogs as raw material and then cut down the availability of byproducts as bioenergy and reduce the wood flows to other facilities. Therefore, the impact of demand changes on the whole wood flows certainly merits special attention..

In Comparison 4, the demands at heating plants increase 10% during the planning period. In a separated market where pulpwood can only be delivered to pulp mills, the supplying company has no other choices but to use up all of the available resources of fuel logs and forest residues. The remaining demand gap has to be filled by fossil fuel, leading to a dramatic increase in the purchasing cost of substitute energy. The severe situation is alleviated in an integrated market. Besides fuel logs and forest residues, more pulpwood is forwarded to heating plants to meet the surging demand. It emphasizes that an integrated market is more flexible to respond to external changes than a separated market.

In Comparison 5, we assume that the demands for sawlogs in all sawmills decrease 10% during the whole planning period. We notice that pulpwood is no longer sent to heating plants even in an integrated market, ending up that the results of Instance S12 and Instance S13 are exactly the same. This arises from harvest nature, which has already been discussed in Comparison 3. When the demands for sawlogs decrease 10%, the supplies of sawlogs correspondingly decrease. Due to the coproduction, the supplies of pulpwood are also influenced and drop to the minimum level that just satisfies the demands in pulp mills. Therefore, the actual supplies of sawlogs are related to the supplies of pulpwood and still exceed the demands in sawmills. The remaining sawlogs have to be left at terminals as storage for future sales. This is the reason that the storage costs of Instance S12 and Instance S13 increase more than 60% when we compare them with their counterparts, namely Instance S6 and Instance S2. In fact, it complies with the real-world phenomena, i.e., strong sawlog markets can stimulate more harvesting and feed more low-grade wood into the pulp and biomass markets whereas weak sawlog markets will make many landowners hold off harvesting, reducing the flows of wood to the other markets.

Furthermore, we assume that if sawlogs and pulpwood are originally the same timber, sawlogs can then be sent to pulp mills as a substitution for pulpwood. Specifically, in our case study, sawlog “Pine” and sawlog “Spruce” can replace pulpwood “Pine” and pulpwood “Spruce” but not pulpwood “Birch”. We hence modify the flow conservation constraint set (*11*) in pulp mills in the proposed model as:

$$\sum_{i \in A \bigcup K} X_{imrt} + Q^M_{mrt} = d^M_{mrt}, \quad \forall m \in M, \forall r \in R_P \setminus R_S, \forall t \in T, \tag{25}$$

$$\sum_{i \in A \bigcup K} X_{imr_1t} + \sum_{i \in A \bigcup K} X_{imr_2t} + Q^M_{mr_1t} = d^M_{mr_1t}, \quad \forall m \in M, \forall r_1 \in R_P \bigcap R_S, \forall r_2 \in R_S, r_1 = r_2, \forall t \in T. \tag{26}$$

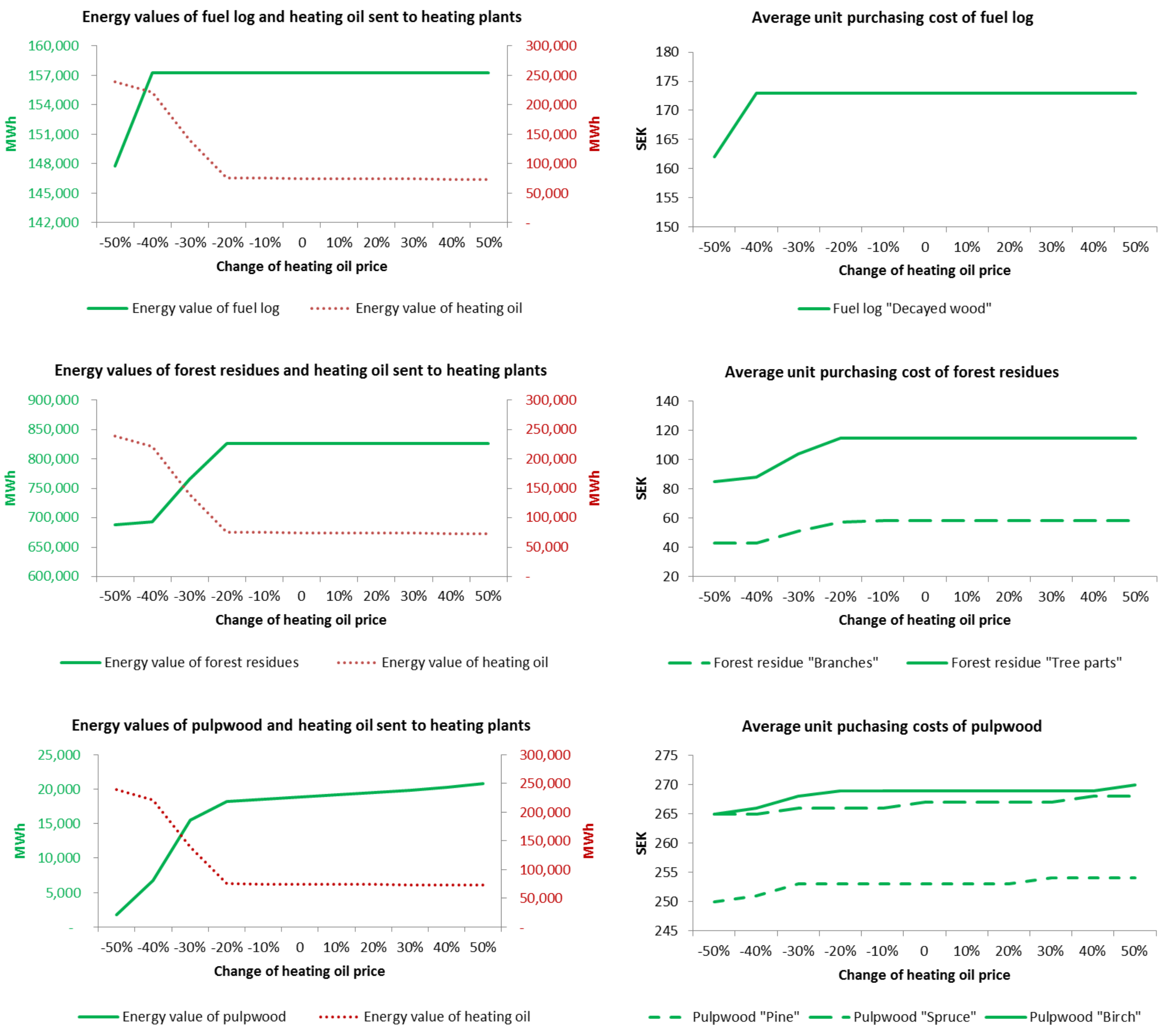
Once it is allowable to forward sawlogs to pulp mills as substitutions for pulpwood, the entire wood flows become more efficient and cost-saving. Figure 2.9 contrastingly shows the different allocations of sawlogs and pulpwood in Instance S13 and Instance S14. In Instance S13, the undesirable sawlogs caused by fixed harvesting proportion have to be left in the forest and the pulpwood purchased is only transported to pulp mills. In contrast, in Instance S14, excess harvested sawlogs are delivered to pulp mills while pulpwood is sent to heating plants as economic bioenergy. No fossil fuel is needed.

Figure 2.9 Different allocations of sawlogs and pulpwood in Instance S13 and Instance S14

In Comparison 6, we investigate whether government policies play a significant role in the raw material market. It is assumed that a new policy is imposed that the share of forest fuel used in heating plants increases from 50% to 80%. We find that there is no significant change in costs. Only 0.12% less fossil fuel is purchased. Actually, in both Instance S3 and Instance S15, among 22 heating plants during 12 planning periods, most slack of the proportion constraints is positive. Only six constraints' slack is zero in Instance S3 and 31 in Instance S15. Indeed, the positive slack implies that these constraints do not bind. Changing the proportion of forest fuel used in heating plants does not substantially affect the optimum. That is, in our case study, the consumption of forest fuel in most heating plants in most periods is under regulation. It should be noted that the results can be sensitive to a change in the value of parameters, such as the price for fossil fuel. In any case, the amount of forest raw materials that can be used for energy generation is encouraged not only by the policies but also by harvest nature and the trade-off between costs of other assortments, as discussed above.

In the last comparison, we concentrate on how a change in the heating oil price impacts the supply market of raw materials and the wood flows in the network. We assume that the price for heating oil changes from 50% less to 50% more by every 10%. Obviously, the consumption of bioenergy in heating plants is driven by prices for possible substitute resources, e.g., heating oil (Figure 2.10). At the beginning, when the price for heating oil is relatively low, heating oil is preferred and not all of the available fuel logs and forest residues are forwarded to heating plants. Some purchased forest residues are even left in the forest. At the same time, little pulpwood is used as bioenergy. Most purchasing costs of forest bioenergy are around or lower than the midprice of the range. However, with the steady increase of heating oil price, the situation changes dramatically. Supplies of fuel logs and forest residues quickly reach the upper bounds with a drastic increase in the corresponding purchasing costs. The higher the price for heating oil becomes, the less heating oil is purchased and more pulpwood is sent to heating plants. The purchasing costs of pulpwood are also pushed up. Because of harvest nature that the supplies of pulpwood are proportional to those of sawlogs, when the price for heating oil rises by 40% or more, even more sawlogs than required will be purchased in order to increase the availability of pulpwood as bioenergy. The trade-off between increase in the costs of forest raw materials (i.e., purchasing cost, chipping cost, transportation cost, and storage cost) and decrease in the consumption of heating oil should be taken into consideration.

Figure 2.10 Total energy values of forest bioenergy sent to heating plants and corresponding unit purchasing costs under the changes in the price for heating oil



2.5 Concluding remarks and future work

Most studies of the forest supply chain in the past have addressed integration either for various planning problems or for different decision levels, concentrating on one specific forest product industry. This paper, for the first time, deals with an integrated market for all forest raw materials in the initial stage of the supply chain. The objective of the proposed problem is to purchase adequate amounts of raw materials in the harvest areas and byproducts from sawmills in order to satisfy the diverse demands in sawmills, pulp mills, and heating plants at the minimum combined costs of procurement, chipping, inventory, and transportation.

We also include the possibility of deciding the unit purchasing costs for different assortments so as to dynamically change the corresponding supplies of raw materials, which depend linearly on the unit costs. This implementation allows the forest companies to make in-depth analysis of the supply market and generate geographical maps with price differences. They can use this information to negotiate with forest owners for better rebates when signing annual supply contracts.

Although the proposed model is developed from a supplying company's perspective, similar modeling approaches could also be applied for (*i*) a forest owner who harvests logs and residues and sells to different customers or (*ii*) a forest association who owns mills and heating plants and wants to meet all of the specific demands and balance the needs for byproducts among its subsidiaries.

The integrated market for all forest raw materials is simulated with data from the Forestry Research Institute of Sweden. The proposed model is a typical QP problem of large size, but it can be efficiently solved by CPLEX as a solver. 16 instances under different assumptions are generated. Pairwise comparisons demonstrate that resources can be effectively utilized with the price mechanism in the supply market. No other uneconomic resources, such as heating oil, are needed to replenish the shortage at heating plants in the winter. The overall energy value of forest bioenergy delivered to heating plants is 23% more than the amount in the situation where volume and unit purchasing cost of raw materials are predefined.

The results also indicate that because, in an integrated market, pulpwood can be used both as raw material for pulp process and as bioenergy for heat generation, the unit purchasing costs of pulpwood in the harvest areas are pushed up. This is in line with the concern that once it is acceptable to sell pulpwood to heating plants, competition for raw materials between forest bioenergy facilities and traditional forest industries is expected to occur. However, an integrated market leads to a considerable cost saving potential in total cost and is more flexible to respond to external changes, i.e., demand fluctuation, than a separated market.

The fact that the harvest of one area will result in the coproduction of various assortments is an important factor that makes the whole market highly interacted. If the assortments can be ideally harvested, regardless of the harvesting of other assortments in the same area, the distance from one harvest area to mills turns out to be the main factor that determines the purchasing costs of the assortments in that area. Additionally, the amount that forest raw materials can be used for energy generation is encouraged not only by the policies but also by harvest nature and trade-off between costs of other assortments. Nevertheless, the exogenously increasing price for possible substitute resources, e.g., heating oil, will boost the consumption of bioenergy in heating plants.

One interesting further step of this research will introduce binary variables in the supply market to exclude the real "unpopular" harvest areas and relax the specified volume and delivery prices in the demand market. We will estimate how these new changes influence the whole market and have a more comprehensive view of the impact of an integrated market on the competition between wood energy facilities and traditional forest industries.

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3

Using mixed integer programming models to synchronously determine production levels and market prices in an integrated market for roundwood and forest biomass

Jiehong Kong, Mikael Rönnqvist, and Mikael Frisk

Abstract

We study a problem of integrating the supply chain of roundwood with the supply chain of forest biomass. The developed optimization model is a multiperiod, multicommodity network planning problem with multiple sources of supply, i.e., harvest areas, and multiple types of destinations, i.e., sawmills, pulp mills, and heating plants. This paper presents an extension of previous work where the set of harvest areas was given and market prices for raw materials had linear relationship with corresponding volumes. In this paper, the assumption of predefined areas is removed, and we must make the selection of harvest areas. Instead of using a traditional sequential approach to first select areas and then determine the prices, we present a new synchronous approach that can jointly choose areas and define price levels for different assortments at those chosen supply points. We test the possible settings of discretized price and use sensitivity analysis to evaluate how the variation of fixed cost concerning log forwarding at each supply point affects the wood procurement decisions. A case study from Sweden is used to analyze the market prices in an integrated market. The computational results also highlight the advantage of the proposed synchronous approach over the sequential one in both solution quality and solution time.

Keywords: Forestry supply chain, integrated market, forest biomass, wood procurement, mixed integer linear programming[2]

[2] We would like to acknowledge the support of NHH Norwegian School of Economics as well as the industrial support of the Forestry Research Institute of Sweden (Skogforsk). We are also indebted to the three referees for many valuable comments that helped to restructure this paper and make it more readable and accessible to both theoreticians and practitioners.

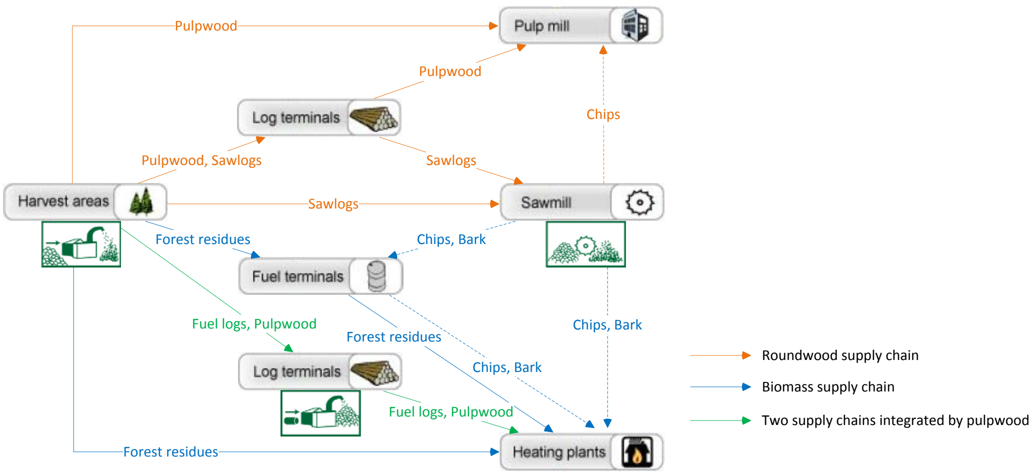
3.1 Introduction

In the past decades, concerns about soaring prices for fossil fuel, trade in carbon emission rights and domestic security of energy have placed emphasis on renewable energy source. Forest biomass, as a safe, stable, and renewable energy alternative, receives a lot of attention and has become one of the most promising and feasible choices for heating and electricity generation in countries with plentiful forest resources, such as Austria (Gronalt and Rauch, 2007), Belgium (Van Belle et al., 2003), Finland (Palander, 2011), Ireland (Murphy et al., 2010), Sweden (Gunnarsson et al., 2004) and the United States (Galik et al., 2009).

Forest biomass normally refers to residues left on-site after thinning or final felling, decayed logs found during harvesting, and bark and wood chips generated from the wood processing industries. However, due to the accelerating promotion of wood-based energy, the direct use of roundwood, e.g., pulpwood, for energy production becomes attractive. Although roundwood in general is more expensive than forest biomass, it has many comparable benefits in terms of harvesting, transportation and energy content. Lundmark (2006) indicated that in Sweden a breakpoint exists beyond which it becomes cheaper to exploit roundwood for energy than to collect and transport residues. Röser et al. (2011) suggested that in Northern Scotland it is cost-effective to chip roundwood for fuel if the advantage of harvesting is taken into account. The growing utilization of roundwood for bioenergy will likely impact other users of the forest resource, especially pulp and paper producers. The potential competition for raw materials between traditional forest industries and emerging forest bioenergy facilities is expected to intensify in the future (Conrad et al., 2011).

Kong et al. (2012) developed an optimization model to integrate the two value chains of roundwood and forest biomass in the initial stage of the forestry supply chains. That is, all raw materials in the forest, i.e., sawlogs, pulpwood, and forest residues, and byproducts from sawmills, i.e., wood chips and bark, exist in an integrated market where pulpwood can be sent to heating plants as bioenergy (Figure 3.1). The unit purchasing costs (stumpage price including harvesting cost) of raw materials are treated as variables, on which the corresponding supplies depend linearly between lower and upper limits. It means that the higher unit purchasing cost the supplying company offers, the more volume, if possible, the forest owners will provide under constraints of the harvest conditions. The objective of the integrated planning problem is to minimize the total cost and decisions incorporate purchasing raw materials in the harvest areas, reassigning byproducts from sawmills, transporting assortments to different points for chipping, storing, wood processing or wood fired, and replenishing fossil fuel when necessary. Because of the variable supply levels depending on the variable prices, the proposed model is a quadratic programming (QP) problem. Pairwise comparisons show that because pulpwood, in an integrated market, can be used both as input for pulp process and as feedstock for heat generation; the competition for raw material drive up the market prices for pulpwood in the harvest areas.

Figure 3.1 Illustration of an integrated market for raw materials in the initial stage of the forestry supply chains



Kong et al. (2012) also assumed that the set of harvest areas to purchase forest raw materials is predefined. This is a valid assumption on a short-term planning horizon. However, for longer term planning covering more than one year, this assumption is not practical. During such a longer period, there is always a possibility to choose areas from a larger set of available areas. Furthermore, if the fixed cost associated with forwarding operation is considered, one tends to avoid purchasing timber from the areas where volumes of supply are low. Hence, this paper deals with an important extension of the limitation of the previous work. For a traditional sequential planning approach (named S-A), two steps are needed. In the first step, binary variables are introduced to select harvest areas with respect to volume balance and these decisions become constraints for determining prices in the second step. This approach involves two models, a mixed integer programming (MIP) problem and a QP problem. In this paper, we propose a new synchronous approach (S-B) that can jointly choose areas and define the discretized price levels for different assortments at the chosen areas. With this approach, it is possible to solve the entire problem in one MIP model. We use sensitivity analysis to evaluate how the variation of fixed cost at each supply point affects the wood procurement decisions. The computational results show that if no fixed cost concerning log forwarding is counted at each supply point, there is less than 0.01% difference in the optimized objective value between S-A and S-B. However, the solution time of S-B is faster than the time used by S-A. With the increase of fixed cost, S-B has a remarkable advantage over S-A.

Linear programming (LP) applications of resource allocation and timber harvest scheduling have been prevalent in forestry since the early 1970s (Garcia, 1990). However, LP is not suitable for site-specific, tactical planning, which normally addresses the allocation rules about when, where and how timber should be harvested, which roads should be built or maintained, as well as where the harvesting machinery or a processing facility should be located. MIP is needed where the binary variables represent discrete decisions and determine whether to execute an activity (Weintraub et al., 1994; 1995). The work by Weintraub and Navon (1976) for the US Forest Service was one of the first attempts at formulating a MIP model that incorporates timber harvesting, road building and transportation activities. Later a wide variety of MIP models from different perspectives on the forest supply chain have been developed (Weintraub, 2007).

Gunnarsson et al. (2004) studied a problem where a supplying company is obliged to deliver a certain amount of forest biomass to several heating plants. Discrete decisions include when and where to convert forest residues into fuel and whether to contract additional harvest areas and sawmills as resource providers. Troncoso and Garrido (2005) presented a MIP model that allows the strategic selection of the optimal location and production capacity of a sawmill. Gunnarsson and Rönnqvist (2008) considered an integrated planning of production and distribution in the pulp and paper industry. Binary variables are associated with production alternative, contract acceptance, terminal location and shipping route choice.

In the roundwood or forest biomass procurement problems, MIP models are used to balance the supply of raw materials in the forest and demand for products from mills. Decisions involve which blocks to harvest (Beaudoin et al., 2007), what types of harvesting methods to use (Burger and Jamnick, 1995), how to allocate the crews (Karlsson et al., 2004), whether to buck trees into logs of specific dimensions in the woods (Carlgren et al., 2006), how many types of logs to produce on-site (Chauhan et al., 2009) and how to remove woody biomass from the forest (Wu et al., 2011). The selection strongly affects the production level of different assortments, which must fulfill the needs of the geographically distributed mills or plants.

The main contribution of this paper is as follows. First, we present a new method where MIP model is employed to not only define the production level for timber but also determine its price level (at the same time). Second, we show that this integrated model can be efficiently solved by standard commercial solvers. Third, the impact of possible fixed costs on market prices can be illustrated in geographical maps. This information can support the planner to get a more comprehensive understanding of the value of raw materials in various harvest areas, and hence, to make better wood procurement decisions. Furthermore, if several forest companies plan to collaborate, such maps and information can contrastively show the importance of certain supply points and facilitate the negotiation about cost or revenue allocation.

The remainder of this paper is organized as follows: Section 3.2 introduces the problem and Section 3.3 presents the mathematical formulation. Sensitivity analyses and computational results are described in Section 2.4. Finally, a conclusion summarizes the proposed approach and suggests some further work.

3.2 Problem description

In this section, we briefly summarize the general planning problem. A detailed description is given in Kong et al. (2012). The problem is described from a supplying company's perspective. This company is responsible to purchase adequate amounts of raw materials in the harvest areas and byproducts from wood processing facilities in order to satisfy the diverse demands in sawmills, pulp mills, and heating plants at the minimum combined costs of procurement, forwarding, chipping, inventory, and transportation. An example of such a company is given in Flisberg et al. (2012a).

3.2.1 Supply of raw materials and byproducts

A forest is divided in harvest areas that vary in size and available volumes of assortments. The harvesting activities in the forest produce roundwood and biomass. The lower part of the tree, which has a larger diameter with higher value, is sent to sawmills as sawlogs. The upper, thinner part with a lower value is better suited for pulp and paper mills as pulpwood. The remaining tops and branches with least value, treated as residues, are forwarded to heating plants as forest bioenergy. In addition, during the felling operation, defect wood, i.e., decayed or damaged logs, will be found. These logs cannot be further processed either in sawmills or pulp mills but used as fuel logs for energy generation. Sawlogs, pulpwood, forest residues, and fuel logs are the main parts of the assortments. Each part can be further divided into several subgroups according to quality, dimensions, and species. In this paper, an assortment is defined as a single product type according to its use and species, e.g., “Sawlog Spruce” and “Pulpwood Birch”.

The planning horizon in this paper is one year and monthly time periods are considered to account for seasonality, which has a large influence on harvesting operations. In the Nordic countries, for example, most sawmills are closed for holiday in July, meaning that a relatively small proportion of the annual harvesting is done during this period. However, pulp mills operate during the whole year except for some planned maintenance. This maintenance is spread out over several months as it is done in sequence for a set of mills. We define a supply point as a combination of a harvest area, an assortment and a period. Information relating to each supply point is the unit purchasing cost (market price in the forest) and corresponding supply of a specific assortment in a geographical area during a particular month.

Sawlogs sent to sawmills are transformed into wood products. This process also produces byproducts such as bark, wood chips, and sawdust, which will be transported to pulp mills or heating plants for further use.

3.2.2 Demand at heating plants, sawmills and pulp mills

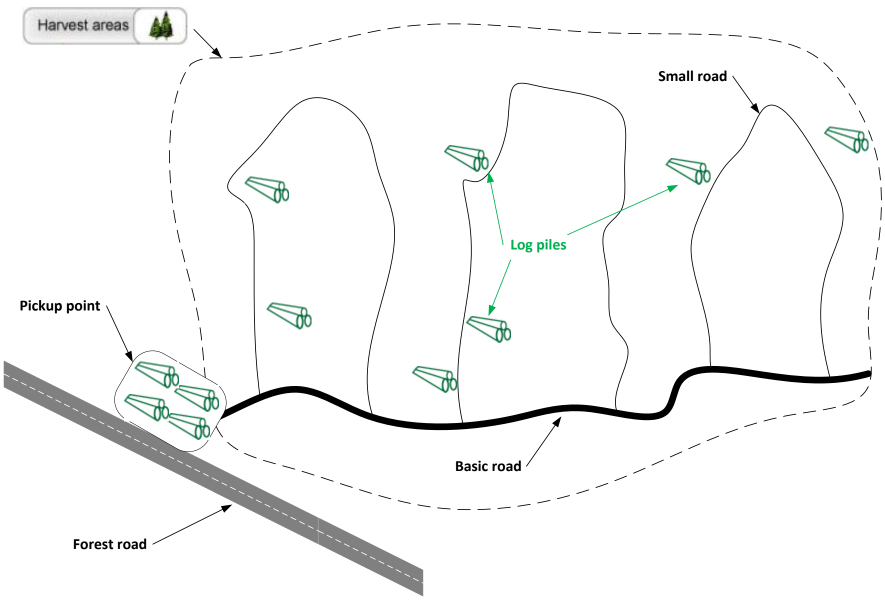
Heating plants usually supply residential and industrial sectors with hot water for heating (Flisberg et al., 2012a, b). Therefore, their demands for energy fluctuate with seasons. Since the demand at the heating plants is specified in energy value (megawatt hours, MWh) while the supply of forest biomass is given in terms of volume (cubic meters), conversion from volume to energy is necessary in the flow conservation constraints. The energy values of assortments depend on their species, moisture content, and the portion of the tree, i.e., stem, branches, or bark.

The volume of sawlogs required in sawmills and pulpwood in pulp mills is given Differently, the demand for byproducts in pulp mills is flexible, often within a certain interval based on the consumption of pulpwood. The proportion of pulpwood and byproducts can be adjusted according to the production recipes for specific pulp.

3.2.3 Log forwarding at harvest areas

Once the harvesting is done, logs are put in different assortment plies depending on their characteristics. These piles are scattered throughout each harvest area. They will be collected by forwarders and transported to the pickup point adjacent to forest roads for further transportation (Figure 3.2). The number of piles can easily exceed 1 000 even for small harvest areas. The overall cost for this operation in Sweden in estimated to be US$200-250 million per annum (Flisberg et al., 2007). This cost exists at any supply point, regardless of assortment, volume, or location. Therefore, when all the harvest areas must be harvested, as assumed in Kong et al. (2012), we can take it as an antecedent fixed cost and do not add it into the objective function. However, if we relax the "preselect" assumption, this fixed cost concerning log forwarding has to be taken into account. It plays a significant role in making decisions about whether or not to purchase timber from certain supply point. If the fixed cost is too high in comparison to the available volume, those areas with low supply of raw materials might be excluded.

Figure 3.2 Illustration of one harvest area and the roads where the forwarder drive to pick log piles



3.2.4 Chipping, storage and transportation

In this paper, we assume that after forest residues are forwarded to the pickup point, chipping is carried out by mobile chippers (Flisberg et al., 2012b). Although chipping on-site is relatively costly as compared to chipping at terminals with larger systems, it is more economic for later transportation since the loading capacity of bulky tree tops and branches is low. Byproducts from sawmills are already chipped. On the other hand, sawlogs sent to sawmills or pulpwood to pulp mills involve no chipping. They are transported as logs all the way from sources to terminals or directly to final destinations.

Two types of storage, roadside in the forest (pickup point) and terminal, are considered. Once forest residues are chipped, they have to be transported to the terminals or heating plants immediately since there are no chip storage possibility in the forest. In addition, byproducts can also be transported to terminals for temporary storage.

The transportation cost is associated with the types of assortments. Sawlogs and pulpwood can be transported by the same type of trucks while chipped biomass has to be shipped by special trucks. Moreover, the unit transportation cost is piece-wise linear with the distance between two points, which is a standard transportation agreement.

3.3 Mathematical formulation

In this section, we begin by describing the sets of variables, and then follow the constraints and the objective function. Afterwards, two approaches S-A and S-B are presented. The mathematical model is developed to integrate a market where roundwood and forest biomass co-exist and interact.

Let A be the set of harvest areas, K the set of terminals, H the set of heating plants, S the set of sawmills, M the set of pulp mills, R the set of raw materials, P the set of products processed in sawmills, W the set of fossil fuel alternatives, T the set of time periods and N the set of approximation nodes. The set of raw materials includes subsets for sawlog assortments R_S, pulpwood assortments R_P, fuel log assortments R_F, and forest residue assortments R_G. The set of products processed in sawmills includes subsets for finished wood products P_S, and byproducts P_B.

In the remainder of the paper, we will use index i for nodes of outbound flow (sources), j for nodes of inbound flow (destinations), a for harvest areas, k for terminals, h for heating plants, s for sawmills, m for pulp mills, r for raw materials, p for processed products in sawmills, w for fossil fuel, t for time periods, and n for nodes on the cost-supply curve. Unless otherwise stated, we assume that definitions, for example, using index a, are valid for all $a \in A$. All the continuous variables are non-negative.

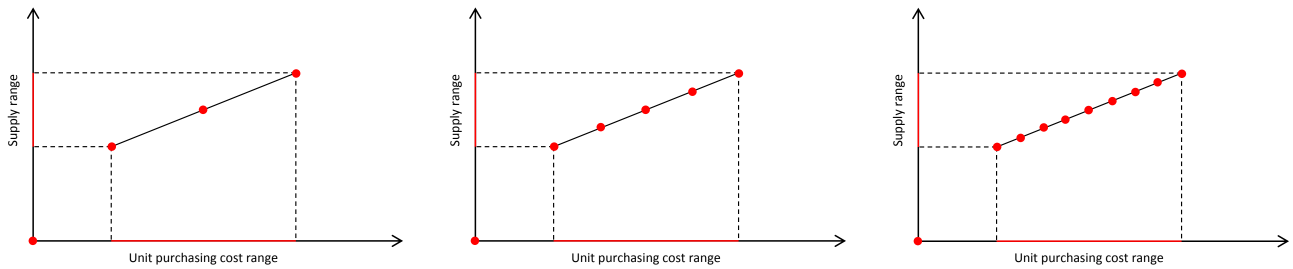
3.3.1 Variables

We use binary variables to identify which harvest area should be chosen to provide a specific assortment in a specific time period (month). Additionally, in earlier model, it is assumed that the supply of one assortment depends linearly on its unit purchasing cost. This linear relationship can be approximated by discretized nodes on the cost-supply curve (Figure 3.3), which are represented by the binary variables as well:

$$U_{art}^{n} = \begin{cases} 1 & \text{if havest area } a \text{ is chosen to provide raw material } r \text{ in time period } t \\ & \text{at node } n \text{ of the cost} - \text{supply curve,} \\ 0 & \text{otherwise.} \end{cases}$$

Once a harvest area is selected to provide certain assortment, the price level of this assortment in the area is also determined. Otherwise, there is no supply in the non-chosen area. It is necessary to discretize price rather than keep it linear. Otherwise, we would need to solve a nonlinear MIP model if we still want to jointly determine volume and price. It is well-known that such models are very hard to solve even for small instances. Moreover, there are no efficient commercial solvers for such models.

Figure 3.3 Discretized unit purchasing cost and corresponding supply with different approximation nodes



a) $N=3$ b) $N=5$ c) $N=9$

The continuous variables are listed below. The flow of raw materials is represented by the variables

X_{ijrt} flow from source i to destination j of raw material r in time period t, $i \in A \bigcup K$, $j \in K \bigcup H \bigcup S \bigcup M$, $r \in R$

whereas the flow of byproducts is expressed in the variables

Z_{ijpt} flow from source i to destination j of byproduct p in time period t, $i \in S \bigcup K$, $j \in K \bigcup H \bigcup M$, $p \in P_B$

Harvested raw materials can be temporarily piled at the roadside in the forest. The storing of raw materials is expressed in the variables

I^A_{art} storage in harvest area a of raw material r at the end of time period t, $r \in R$

Terminals are used for chipping process or storage. The variables representing the storing of raw materials and byproducts are

I^K_{krt} storage at terminal k of raw material r at end of time period t, $r \in R$

I^K_{kpt} storage at terminal k of byproduct p at end of time period t, $p \in P_B$

Note that the initial storage level, given by time index 0, in different districts is assumed to be zero.

If the biomass supply were insufficient to achieve the demands at heating plants during some periods, e.g., extremely cold winter, it is possible to purchase fossil fuels such as heating oil or coal, specified in MWh, to fill the demand gap. This supplementary is described in the variables

E_{hwt} fossil fuel w forwarded to heating plant h in time period t

In order to get a robust model, we introduce penalty variables

Q^S_{srt} unsatisfied demand in sawmill s of raw material r in time period t, $r \in R_S$

Q^M_{mrt} unsatisfied demand in pulp mill m of raw material r in time period t, $r \in R_P$

Q^M_{mpt} unsatisfied demand in pulp mill m of byproduct p in time period t, $p \in P_B$

to represent the deficiency in one period, respectively, in sawmills and pulp mills. If these variables are not included in the model, it might be impossible to find any feasible solution or to identify the problems. The unit penalty costs are set large enough to ensure all the demands in mills satisfied.

3.3.2 Constraints

On the cost-supply curve, the pair parameters f_{art}^{n}, the unit purchasing cost in harvest area a of raw material r in time period t, and s_{art}^{n}, the corresponding supply, are designated as possible choices of node n. In order to describe the selection decisions, we use the constraints

$$\sum_{n \in N} U_{art}^{n} \leq 1, \quad \forall a \in A, \forall r \in R, \forall t \in T \tag{1}$$

to specify that at most one node on the cost-supply curve can be chosen. Then the chosen node interprets the price $\sum_{n \in N} f_{art}^{n} U_{art}^{n}$ and supply level $\sum_{n \in N} s_{art}^{n} U_{art}^{n}$ of assortment r in harvest area a in period t. If a constraint in set (1) is satisfied with strict inequality, no assortment is purchased from that area in that month. Therefore, no crew or equipment needs to be assigned to transport logs and no fixed cost arises at that supply point.

Forest residues are left in the forest after the harvesting of logs from the previous year, but fresh timber can be harvested when needed. Because a harvest area is usually composed of several tree species, e.g., Spruce and Pine, or diameters, e.g., sawlogs and pulpwood, harvesting of one area will result in the coproduction of various assortments, which is modeled by the constraints

$$U_{ar_1t}^{n} = U_{ar_2t}^{n}, \quad \forall a \in A, \forall r_1 \in R_S \bigcup R_P, \forall r_2 \in R_S \bigcup R_P, \forall t \in T, \forall n \in N. \tag{2}$$

where r_1 and r_2 refer to the distinct assortments that are produced simultaneously in one area.

Constraint set (2) implies that the more one type of logs is harvested, the more other kings of logs in this area will also be produced and vice versa. The production and price levels for assortments in one area are hence interrelated.

The supply of byproducts in sawmills is described in constraints

$$\alpha_{pt}\left(\sum_{i \in A \bigcup K} \sum_{r \in R_S} X_{isrt}\right) = \sum_{j \in K \bigcup H \bigcup M} Z_{sjpt}, \quad \forall s \in S, \forall p \in P_B, \forall t \in T. \tag{3}$$

The parameters α_{pt} show that the wood products and different types of byproducts p are proportionally produced in every period t, where $\sum_{p \in P} \alpha_{pt} = 1$. Therefore, once the amount of sawlogs processed in sawmills is known, the volumes of different byproducts can be exactly measured. Constraint sets (3) also stipulate that all the byproducts will be delivered to different destinations for temporary storage or further use in the same period when they become available.

Raw material can be stored at the roadside in the forest or terminals. This is expressed in the classical flow conservation constraints

$$I^{A}_{ar,t-1} + \sum_{n \in N} s^{n}_{art} U^{n}_{art} = I^{A}_{art} + \sum_{j \in K \cup H \cup S \cup M} X_{ajrt}, \quad \forall a \in A, \forall r \in R, \forall t \in T, \tag{4}$$

and

$$I^{K}_{kr,t-1} + \sum_{i \in A} X_{ikrt} = I^{K}_{krt} + \sum_{j \in H \cup S \cup M} X_{kjrt}, \quad \forall k \in K, \forall r \in R, \forall t \in T. \tag{5}$$

We assume that the chipping for forest fuel does not influence volumes, and thus, a change in the form of raw materials will not impact the inventory balance constraints in the harvest areas for residues or those at terminals for logs. If, in some cases, the loss in volume during chipping is not negligible for the overall material balances, we can easily introduce a fractional factor, say δ_{ar}, to indicate the change of volume after chipping. Similarly, the flow balancing constraints for byproducts are

$$I^{K}_{kp,t-1} + \sum_{i \in S} Z_{ikpt} = I^{K}_{kpt} + \sum_{j \in H \cup M} Z_{kjpt}, \quad \forall k \in K, \forall p \in P_B, \forall t \in T. \tag{6}$$

To ensure that demands for converted energy at the heating plants are all satisfied, we need the constraints

$$\sum_{i \in A \cup K} \sum_{r \in R_P \cup R_F \cup R_G} e_{rt} X_{ihrt} + \sum_{i \in S \cup K} \sum_{p \in P_B} e_{pt} Z_{ihpt} + \sum_{w \in W} E_{hwt} = d^{H}_{ht}, \quad \forall h \in H, \forall t \in T. \tag{7}$$

The parameters d^{H}_{ht} define the demand at the heating plant h in each time period t, which are specified in terms of energy (megawatt hours), but all raw materials or byproducts transported to the heating plants are expressed in volume unit (cubic meters). We therefore introduce conversion factors e_{rt} and e_{pt} in constraints (7). They indicate the energy value of one volume unit of raw material r or byproduct p in time period t. Due to environmental regulation, constraints

$$\sum_{i \in A \cup K} \sum_{r \in R_P \cup R_F \cup R_G} e_{rt} X_{ihrt} + \sum_{i \in S \cup K} \sum_{p \in P_B} e_{pt} Z_{ihpt} \geq \beta_h d^{H}_{ht}, \quad \forall h \in H, \forall t \in T \tag{8}$$

guarantee that a minimal percentage of forest fuel β_h must be used as “green” energy at heating plant h.

Constraints

$$\sum_{i \in A \cup K} X_{isrt} + Q^{S}_{srt} = d^{S}_{srt}, \quad \forall s \in S, \forall r \in R_S, \forall t \in T, \tag{9}$$

and

$$\sum_{i \in A \cup K} X_{imrt} + Q^{M}_{mrt} = d^{M}_{mrt}, \quad \forall m \in M, \forall r \in R_P, \forall t \in T, \tag{10}$$

are used to ensure the demands for raw materials in sawmills and pulp mills fulfilled. The demand for raw material r in sawmill s in time period t is denoted by d^{S}_{srt} and that in pulp mill m is by d^{M}_{mrt}.

In pulp mills, different shares of pulpwood and byproducts are needed to produce pulp according to different recipes. The parameters γ^{L}_{mpt} and γ^{U}_{mpt} express the minimum and maximum level of byproduct p needed in pulp mill m at time period t, which depends on the total consumption of pulpwood. This relationship is formulated as

$$\gamma^L_{mpt}\left(\sum_{r\in R_P} d^M_{mrt}\right) - Q^M_{mpt} \le \sum_{i\in S\cup K} Z_{impt} \le \gamma^U_{mpt}\left(\sum_{r\in R_P} d^M_{mrt}\right), \quad \forall m\in M, \forall p\in P_B, \forall t\in T. \tag{11}$$

We also have to consider a number of capacity restrictions regarding storing, chipping, and operating. Let v^I_a denote the storage capacity in harvest area a and let v^I_k denote the storage capacity at terminal k. The constraints with regard to the storage capacity in each district are

$$\sum_{r\in R} I^A_{art} \le v^I_a, \quad \forall a\in A, \forall t\in T, \tag{12}$$

and

$$\sum_{r\in R} I^K_{krt} + \sum_{p\in P_B} I^K_{kpt} \le v^I_k, \quad \forall k\in K, \forall t\in T. \tag{13}$$

The total volume of forest residues that can be chipped in each period is restricted by the chipping capacity of the mobile chippers working in the harvest areas. Let v^C_t denote the total capacity of mobile chippers in harvest area in time period t. On the other hand, the monthly amount of logs that can be chipped at every terminal is limited by the chipping capacity of permanent chipping equipment. Let v^C_{kt} denote the chipping capacity at terminal k in time period t. The capacity constraints on chipping can be expressed as

$$\sum_{a\in A}\sum_{j\in K\cup H}\sum_{r\in R_G} X_{ajrt} \le v^C_t, \quad \forall t\in T, \tag{14}$$

and

$$\sum_{h\in H}\sum_{r\in R_P\cup R_F} X_{khrt} \le v^C_{kt}, \quad \forall k\in K, \forall t\in T, \tag{15}$$

Note that residues in the forest will not be chipped until delivery to terminals or heating plants, the same as logs sent from terminals to heating plants as bioenergy. Additionally, it is true that not all terminals have chipping ability or storage capacity for chipped forest fuel. In this paper, instead of introducing new sets of variables and constraints to separate the terminals of different types, we model these possibilities by prohibiting the flow of logs sent to heating plants from terminals without chipping equipment and by preventing the flow of chipped bioenergy via terminals without chip storage bins.

Finally, let v^G_k denote the maximal flow capacity at terminal k. Then, the constraints

$$\sum_{i\in A}\sum_{r\in R} X_{ikrt} + \sum_{s\in S}\sum_{p\in P_B} Z_{skpt} \le v^G_k, \quad \forall k\in K, \forall t\in T, \tag{16}$$

restrict the throughput or total flow handled at each terminal.

3.3.3 Objective function

The objective is to minimize the total costs of procurement, forwarding, chipping, storage and transportation, which is expressed as

$$\min TC = C^{A-pure} + C^{A-fix} + C^{P-pure} + C^{E-pure} + C^{chip} + C^{stor} + C^{tran} + C^{defi}.$$

The component of C^{A-pure} represents the total purchasing cost of raw materials in harvest areas and consists of

$$\sum_{a\in A}\sum_{r\in R}\sum_{t\in T}\sum_{n\in N} f_{art}^{n} s_{art}^{n} U_{art}^{n}.$$

The component of C^{A-fix} represents the total fixed cost relating to log forwarding operation at supply points and consists of

$$\sum_{a\in A}\sum_{r\in R}\sum_{t\in T} f_{art}^{F}\left(\sum_{n\in N} U_{art}^{n}\right),$$

where the cost coefficients f_{art}^{F} denote the cost to forward raw material r in harvest area a to pickup point in time period t. The component of C^{P-pure} represents the total purchasing cost of byproducts from sawmills and consists of

$$\sum_{s\in S}\sum_{j\in K\cup H\cup M}\sum_{p\in P_B}\sum_{t\in T} f_{pt}^{B} Z_{sjpt},$$

where the cost coefficients f_{pt}^{B} denote the unit purchasing cost of byproduct p in time period t. The component of C^{E-pure} represents the total purchasing cost of fossil fuel and consists of

$$\sum_{h\in H}\sum_{w\in W}\sum_{t\in T} f_{wt}^{E} E_{hwt},$$

where the cost coefficients f_{wt}^{E} denote the unit energy purchasing cost of fossil fuel w in time period t. The component of C^{chip} represents chipping costs and consists of

$$\sum_{a\in A}\sum_{j\in K\cup H}\sum_{r\in R_G}\sum_{t\in T} c_r^{A} X_{ajrt} + \sum_{k\in K}\sum_{h\in H}\sum_{r\in R_P\cup R_F}\sum_{t\in T} c_r^{K} X_{khrt},$$

where the cost coefficients c_r^{A} and c_r^{K} denote the unit chipping cost of raw material r in the harvest areas and at terminals, respectively. The component of C^{stor} represents the storage costs and consists of

$$\sum_{a\in A}\sum_{r\in R}\sum_{t\in T} h_{art}^{A} I_{art}^{A} + \sum_{k\in K}\sum_{r\in R}\sum_{t\in T} h_{krt}^{K} I_{krt}^{K} + \sum_{k\in K}\sum_{p\in P_B}\sum_{t\in T} h_{kpt}^{K} I_{kpt}^{K},$$

where the cost coefficients h_{art}^{A} denote the unit storage cost in harvest area a of raw material r in time period t. The cost coefficients h_{krt}^{K} and h_{kpt}^{K} denote the unit storage cost at terminal k of raw material r and byproduct p in time period t, respectively. The component of C^{tran} represents the transportation costs for the whole wood flows in this integrated market and consists of

$$\sum_{i\in A\cup K}\sum_{j\in K\cup H\cup S\cup M}\sum_{r\in R}\sum_{t\in T} c_{ijr}^{T} X_{ijrt} + \sum_{i\in S\cup K}\sum_{j\in K\cup H\cup M}\sum_{p\in P_B}\sum_{t\in T} c_{ijp}^{T} Z_{ijpt},$$

where the cost coefficients c_{ijr}^{T} and c_{ijp}^{T} denote the unit transportation cost of raw material r and byproduct p from source i to destination j, respectively. The component of C^{defi} represents the deficit costs and consists of

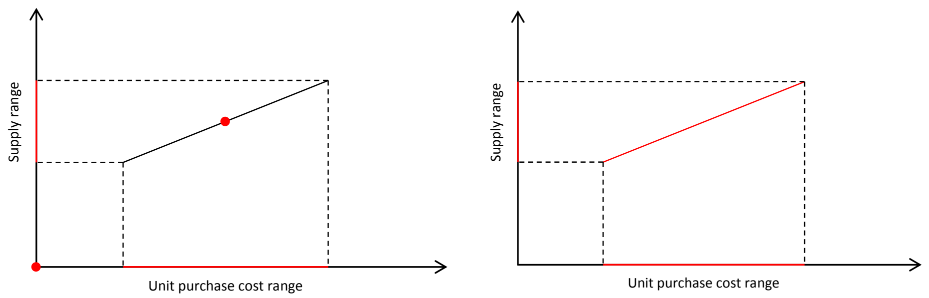
$$\sum_{s\in S}\sum_{r\in R_S}\sum_{t\in T} q^S_{srt} Q^S_{srt} + \sum_{m\in M}\sum_{r\in R_P}\sum_{t\in T} q^M_{mrt} Q^M_{mrt} + \sum_{m\in M}\sum_{p\in P_B}\sum_{t\in T} q^M_{mpt} Q^M_{mpt},$$

where the cost coefficients q^S_{srt} denote the unit penalty cost in sawmill s of raw material r in time period t. The cost coefficients q^M_{mrt} and q^M_{mpt} denote the unit penalty cost in pulp mill m of raw material r and byproduct p in time period t, respectively.

3.3.4 Sequential approach S-A and synchronous approach S-B

In the sequential approach S-A, there are two separate steps. First, we define where and when to purchase raw materials. It is a typical wood procurement problem in which the supply and price of assortment in each area for a period are given as parameters, i.e., the average amount of the supply range and the midprice of the cost range, respectively. Binary variables are used to decide areas from which the total supply of logs or biomass can meet the demand from mills. The problem can be formulated by the proposed MIP model with only one node on the cost-supply curve (Figure 3.4a). After the areas are selected, then the next problem is formulated as a QP model. This second model is the one presented and solved in Kong et al. (2012). In the chosen areas, the supply is linear dependent on the purchasing cost (Figure 3.4b), flexible within certain range.

Figure 3.4 Two models involved in sequential approach S-A



a) MIP model in first step
when harvest areas are not determined

b) QP model in second step
when harvest areas are determined

In contrast, in the synchronous approach S-B, this planning problem is solved once. As a result, we get the full solution about selected supply points and the corresponding volumes and prices. The solution quality replies on the number of approximation nodes we employ on the cost-supply curve.

3.4 Sensitivity analyses and computational results

In this section, we apply the model to a real-life case study. The data are provided by the Forestry Research Institute of Sweden and originate from two cases of the same company, one for roundwood and one for forest biomass. The case study covers harvest areas in southern Sweden that annually can supply 1.6-2.2 million m^3 of required wood assortments for local sawmills, pulp mills and heating plants. Total costs are in Swedish kronor (SEK) and unit costs are in SEK per m^3. 10 SEK is about 1 euro. The geographical distribution of supply and demand points is given in Figure 2.4 and the basic information for the case study is listed in Table 3.1. Figure 3.6 illustrates monthly total supplies of raw materials and byproducts and demands in sawmills, pulp mills, and heating plants.

The mathematical models are programmed by the modeling language AMPL (version 20120217) and solved by commercial solver CPLEX 12.2. The MIP-gap expresses the relative difference between the best feasible solution and the best bound and the default setting in CPLEX is 0.01%, which is used throughout. All computations are performed on a T7300 2.00 GHz processor with 3 GB of RAM.

Figure 3.5 Geographical distribution of terminals, mills and heating plants and district with available harvest areas in the case study

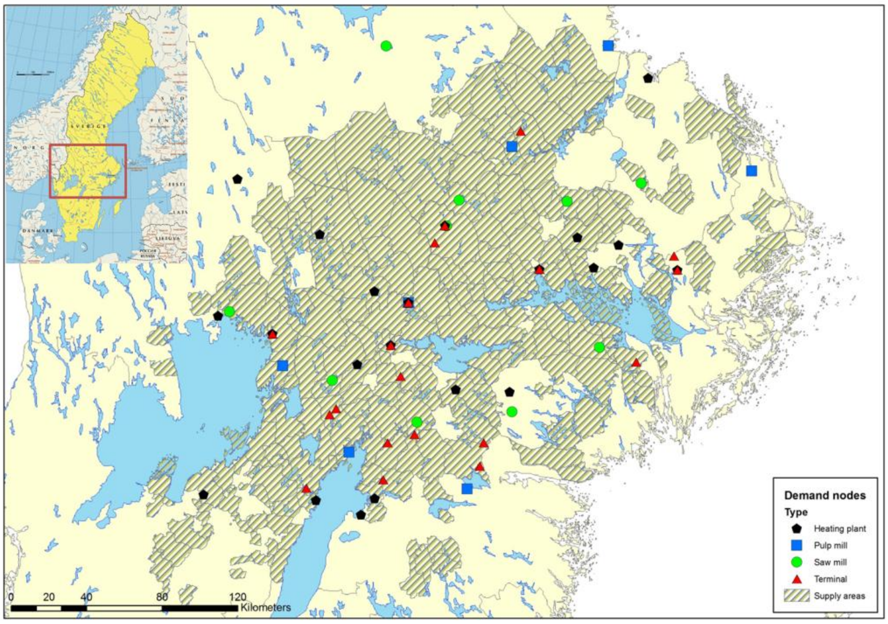
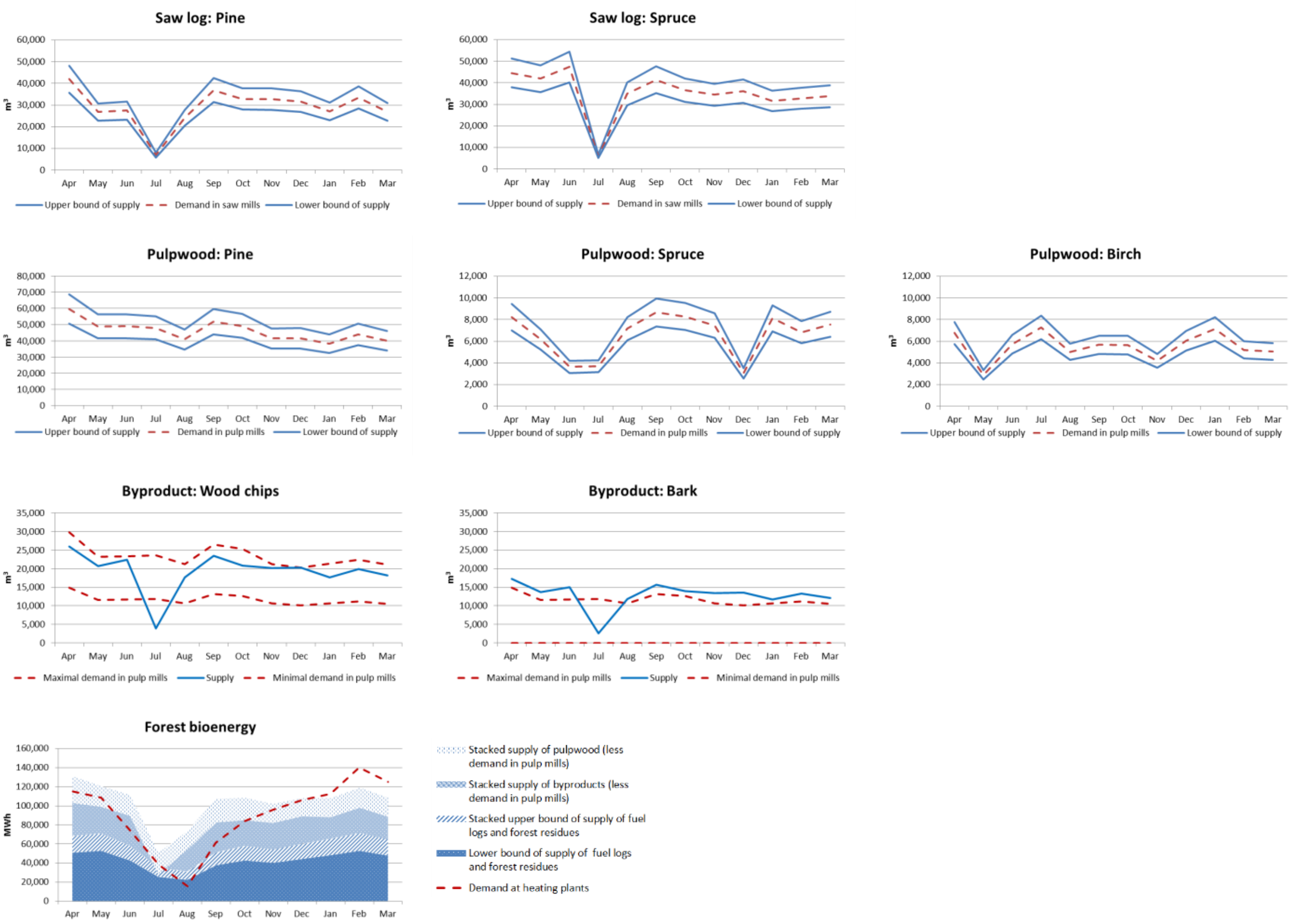


Table 3.1 Information for the case study

Number of	
Harvest areas	234
Terminals	20
Heating plants	22
Sawmills	11
Pulp mills	7
Sawlog assortments	2
Pulpwood assortments	3
Fuel log assortments	1
Forest residue assortments	2
Types of fossil fuel	1
Finished wood products	1
Byproducts	2
Time periods	12
Supply points*	3 899

Note: * A supply point is a combination of a harvest area, an assortment and a period, explained in section 3.2.1.

Figure 3.6 Monthly total supplies and demands in the case study



When harvest in the forest is accomplished in every planning period, crews and forwarders should be assigned to the right places to forward the assortments as discussed in Section 3.2.3. The cost includes three main components: (*i*) cost of forwarding logs or biomass to pickup point and cleaning the harvest area before moving to the next; (*ii*) cost for crews to travel between their home bases and harvest areas; (*iii*) cost of using a trailer to move machines between harvest areas if the distance is over a certain limit (Bredström et al., 2010). All of these operating costs are not trivial, but in reality, it is difficult to obtain the detailed information about the exact rate for each supply point. Therefore, in this paper, we treat these costs as a fixed cost and use sensitivity analysis to study its impact on wood procurement decisions. We assume that the fixed cost at each supply point changes from 0 SEK to 100 000 SEK at 1 000 SEK intervals.

For sequential approach S-A, we first solve MIP model and then fix the chosen points and solve QP model, as displayed in Figure 3.4. For synchronous approach S-B, we test three alternatives to approximate the linear relationship between cost and supply, i.e., using 3 nodes ($N = 3$), 5 nodes ($N = 5$), and 9 nodes ($N = 9$). These nodes are evenly distributed on the cost-supply curve for each supply point when applicable (Figure 3.3). The size of the models in two approaches and computational results when fixed cost is assumed to be zero are presented in Table 3.2.

The quality of the integer solutions is usually measured by the MIP-gap. As can be seen from Table 3.2, the solutions to MIP models are all within 0.01% gap (the default tolerance of optimality) and the solution times are acceptable. The setting of zero fixed cost corresponds to the assumption in Kong et al. (2012), in which fixed cost is not taken into account. The computational results in Table 3.2 show that almost all the supply points are selected to provide raw materials in the MIP models. Especially in the MIP model with $N = 1$ used in S-A, all the supply points are chosen and then the result of the consecutive QP model is exactly the same as the one obtained in the previous work. The reason why all of the supply points are selected is that most monthly demands for raw materials are within the supply ranges (Figure 3.6) and if there is no fixed cost at each supply point, the optimization procedure would choose the supply volumes with favorable unit purchasing costs at every possible area. We also notice that the total cost of the MIP model with 9 approximation nodes in S-B is very close to the result of S-A, less than 0.01% difference, but the computational time is obviously faster.

Table 3.2 Size and results of sequential approach S-A and synchronous approach S-B (with $f_{art}^{F} = 0$)

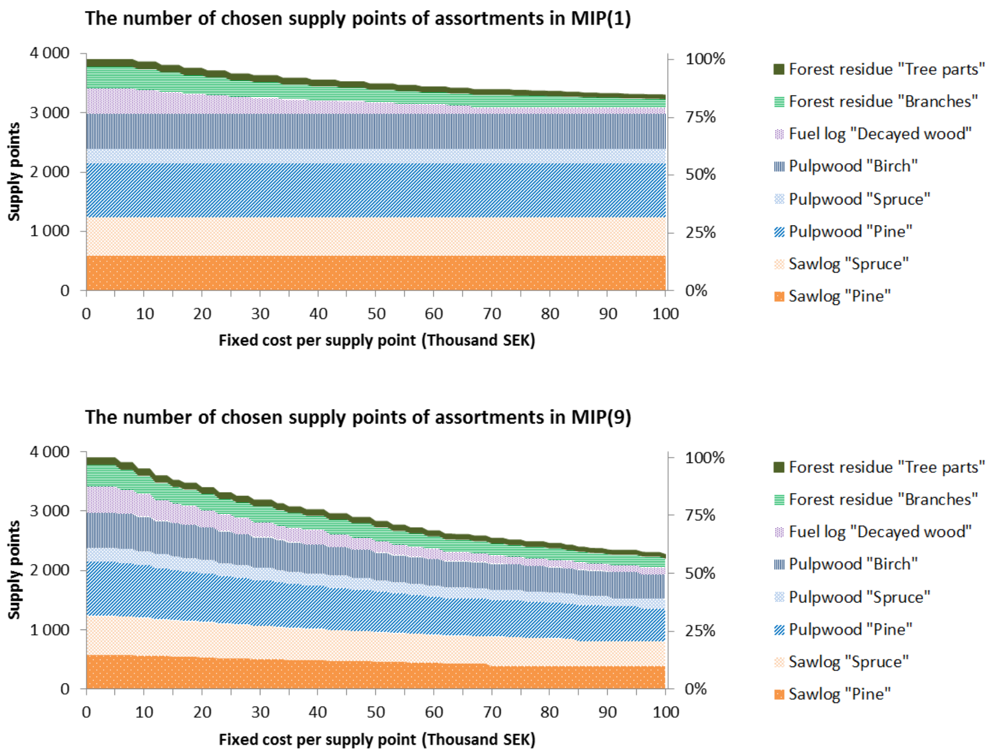
Approach	Continuous variable	Binary variable	Constraint	MIP-gap (%)	CPU time (sec.)	Chosen point	Total cost (MM SEK)
Sequential approach S-A							
MIP(1)	873 904	22 464	87 080	0.0006	54	3 899	--
QP	896 367	--	64 427	--	853	3 899(fixed)	823.91
Synchronous approach S-B							
MIP(3)	873 904	67 392	107 488	0.0023	118	3 897	824.66
MIP(5)	873 904	112 320	127 896	0.0013	219	3 897	824.14
MIP(9)	873 904	202 176	168 712	0.0005	134	3 897	823.96

Table 3.3 lists the results of S-A and S-B under four settings of growing fixed cost. In the table, the field "Change" represents the deviation (in percentage) of the outcome of S-B from the total cost of S-A. As expected, with the increase of the fixed cost, the total number of the chosen supply points in both approaches is decreasing. It reduces up to 42%, from 3 897 to 2 268, in S-B, while the total number only decrease by 15%, from 3899 to 3301, in S-A. Figure 3.7 clearly illustrates the reason. When the supply is fixed as midpoint of the range (MIP model with $N = 1$ in S-A), the total amount for sawlogs and pulpwood is equal to the total demand (Figure 3.6). Therefore, every point for these two types of logs has to be chosen, no matter how high the fixed cost becomes. The only points that can be excluded are those providing biomass. In contrast, there is more freedom for volume choices in MIP model with different approximation nodes ($N = 9$) in S-B. As a result, the total costs of the three MIP models in S-B end up with 5% lower than that in S-A, around 60 million SEK savings. It confirms that S-B has a robust performance and is superior to S-A when fixed cost is considered.

Table 3.3 The number of chosen supply points and total costs (million SEK) of sequential approach S-A and synchronous approach S-B under four settings of possible fixed cost

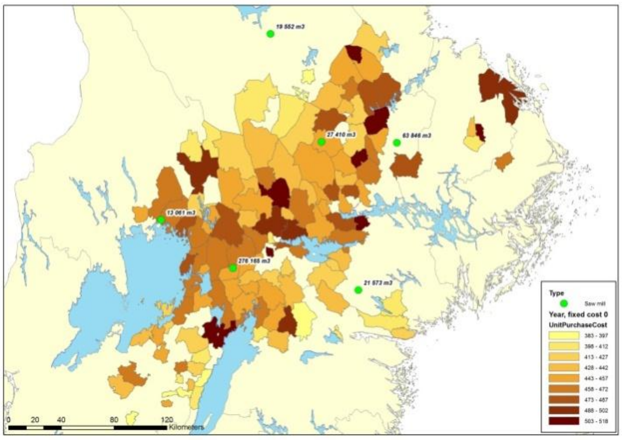
Approach	$f_{art}^{F}=0\,\text{SEK}$		$f_{art}^{F}=33000\,\text{SEK}$		$f_{art}^{F}=66000\,\text{SEK}$		$f_{art}^{F}=100000\,\text{SEK}$	
	Chosen point	**Total cost**	**Chosen point**	**Total cost**	**Chosen point**	**Total cost**	**Chosen point**	**Total cost**
Sequential approach S-A								
MIP(1)+QP	3 899	823.91	3 596	946.24	3 401	1 060.91	3 301	1 175.07
Synchronous approach S-B								
MIP(3)	3 897	824.66	3 119	940.22	2 592	1 033.56	2 268	1 115.83
Change (%)		*0.09*		*-0.64*		*-2.58*		*-5.04*
MIP(5)	3 897	824.14	3 126	939.72	2 592	1 033.08	2 273	1 115.37
Change (%)		*0.03*		*-0.69*		*-2.62*		*-5.08*
MIP(9)	3 897	823.96	3 124	939.66	2 593	1 033.02	2 268	1 115.22
Change (%)		*0.01*		*-0.70*		*-2.63*		*-5.09*

Figure 3.7 Sensitivity analysis of the number of the chosen points in MIP model with $N=1$ of sequential approach S-A and MIP model with $N=9$ of synchronous approach S-B

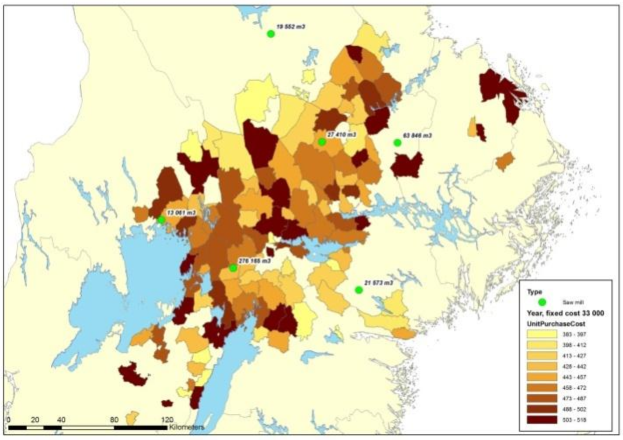


The decreasing number of chosen supply points in Figure 3.7 also implies that the areas, which are no longer profitable to extract assortments in certain period, are gradually excluded. However, the required demands for raw materials must be fulfilled anyway. Less chosen supply points mean more supplies desired in those selected areas, pushing up the market prices for the assortments accordingly. Figure 3.8 illustrates the evolution of chosen supply points and average prices of sawlog "Spruce" in the selected harvest areas over the year. The darker the color is, the higher the unit purchasing cost becomes. Any map for specific assortment in a particular period can be generated under estimated fixed cost. The supplying company can hence obtain an intuitive view when making strategic plans concerning wood and forest biomass procurement.

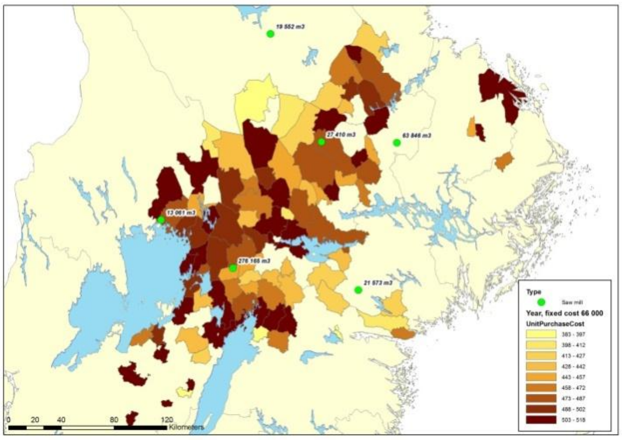
Figure 3.8 Average unit purchasing costs of sawlog “Spruce” in the harvest areas



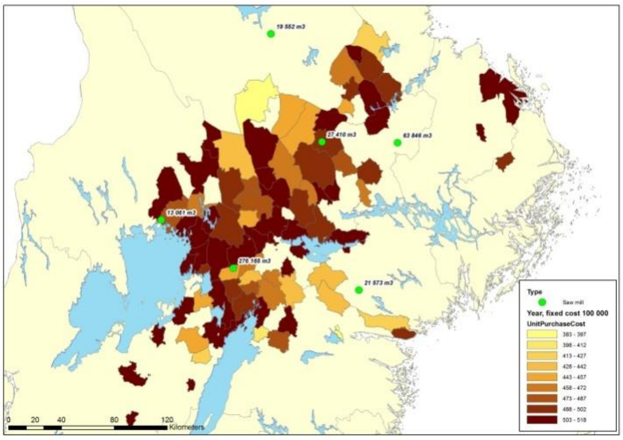
Fixed cost per supply point = 0 SEK
Chosen supply points = 645



Fixed cost per supply point = 33 000 SEK
Chosen supply points = 543



Fixed cost per supply point = 66 000 SEK
Chosen supply points = 462



Fixed cost per supply point = 100 000 SEK
Chosen supply points = 408

Last but not least, we would like to analyze how the integrated market will react if it is not allowable to use fossil fuel, e.g., heating oil, at heating plants as supplementary energy. That is, all the demands at heating plants over the whole planning horizon should be fulfilled by forest biomass and pulpwood. It represents an extreme “green” case. In the proposed model, we simply fix the variables $E_{hwt} = 0$. We use *S1* to represent the original formulation and *S2* for the new assumption. The contrastive sensitivity analyses by fixed cost under these two scenarios are given in Figure 3.9, Figure 3.10, and Figure 3.11. All the calculations are based on the MIP model with $N = 9$ in S-B.

There is no obvious difference in selection decisions between S1 and S2 when fixed cost at each supply point is under 40 000 SEK. After that, more supply points are kept in S2 than in S1 (Figure 3.9). However, the total costs under these two scenarios maintain alike since the increased fixed cost in S2 is replaced with the purchasing cost of heating oil in S1 (Figure 3.10). In Figure 3.11, at the beginning, more and more pulpwood

“Pine” is purchased and sent to heating plants as bioenergy. However, when the fixed cost reaches the breakpoint, i.e., 40 000 SEK, the energy value of pulpwood sent to heating plants starts to decline while the demand for heating oil increases dramatically (S1). By comparison, if only forest biomass can be used in heating plants (S2), there is continuous growth in demand for pulpwood “Pine”. Note that the amount of pulpwood “Birch” sent to heating plants keeps decreasing both in S1 and S2. It arises because the supply of pulpwood “Birch” is much less than pulpwood “Pine”. An increase of fixed cost will exclude the points where the available volume is relatively small. It complies with the real-world phenomena.

Figure 3.9 Sensitivity analysis of the number of the chosen supply points in scenario S1 and scenario S2

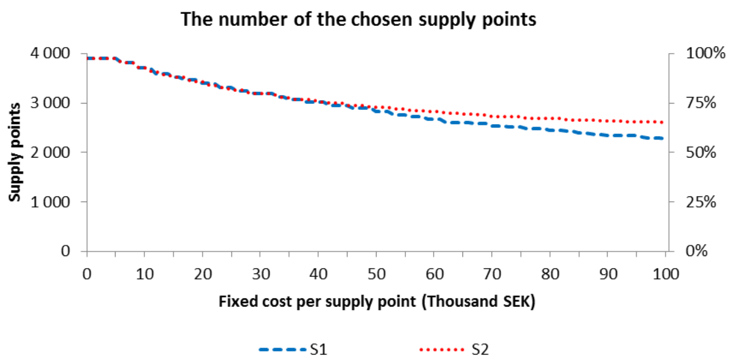


Figure 3.10 Sensitivity analysis of the distinct cost components in scenario S1 and scenario S2

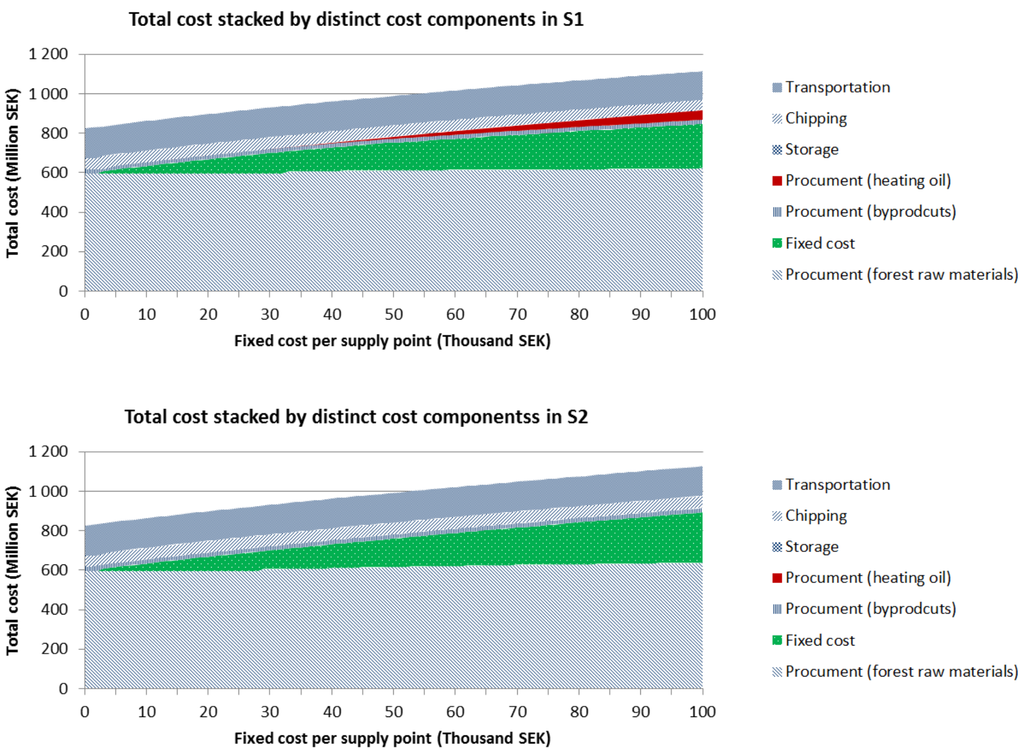
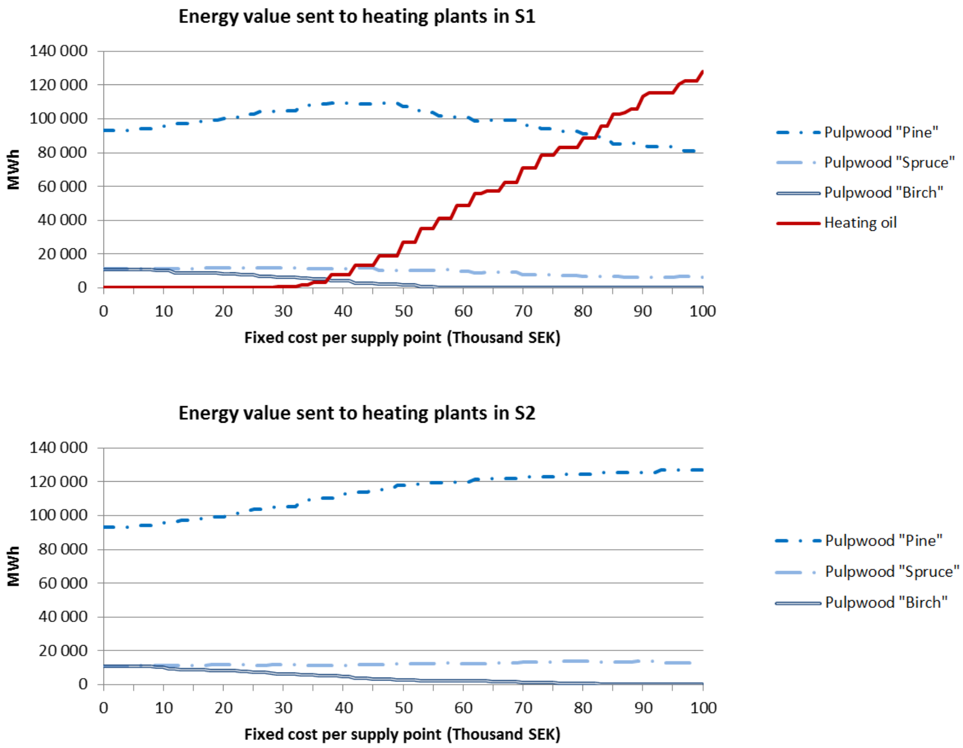


Figure 3.11 Sensitivity analysis of energy value sent to heating plants in scenario S1 and scenario S2



3.5 Concluding remarks and future work

We have introduced a synchronous approach that can select harvest areas and determine the market prices for raw materials at the same time. This new method avoids establishing a non-linear MIP model. It also has a proved advantage over the traditional sequential approach in terms of both solution time and solution quality.

Each binary variable in the synchronous approach represents a discretized price level of a specific assortment in a specific harvest area at a specific period. Solution provides not only whether to purchase raw material from one harvest area but also what the price level of the assortment in this area should be. To our knowledge, it is the first time to take the variable price factor into consideration in a MIP model of wood procurement problem. With this implementation, different equilibrium prices for roundwood and forest biomass in the forest can be generated and unprofitable supply points, if any, can be excluded.

The fixed costs relating to log forwarding after harvest are difficult to estimate, but the proposed model can be used to make trade-offs with different levels of fixed costs. It provides a basis about which area in which period is not profitable to extract certain assortment once the fixed cost at that area increases to certain level. This is particularly useful for forest companies to obtain more detailed resource allocation planning.

Further work includes studying the impact if several companies would collaborate. This is already implemented for roundwood with large savings (Frisk et al., 2010). In such collaboration, the resource of supply is treated as common and the destination of deliver can be changed within companies as long as there is a balance in the wood trade. As the logistic cost is critical for forest biomass, such collaborative plans could improve efficiency. Moreover, backhauling (Carlsson and Rönnqvist, 2007) is traditionally efficient for the transportation planning of roundwood. In the application of forest biomass, any backhauling will be more complex as more types of systems are involved. It would be interesting and challenging to adapt traditional backhauling to the transportation of the forest biomass.

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4

Coordination between strategic forest management and tactical logistic and production planning in the forestry supply chain

Jiehong Kong and Mikael Rönnqvist

Abstract

In this paper, we study the coordination mechanism in the forestry supply chain between strategic forest management and tactical logistic and production planning. We first formulate an integrated model to establish a theoretical benchmark for performance of the entire supply chain, including harvest areas, sawmills, pulp mills, heating plants and final markets. This model is a mixed integer programming (MIP) model that involves harvesting, bucking, transportation, production and sales decisions for both tactical (5 years) and strategic (25 years) planning levels. We then present two sequential approaches S-A and S-B where the coordination is done through internal prices. The approach S-A is the one currently used in practice where harvesting in the forest is the main driver of the supply chain activities and internal pricing is introduced to control bucking decision in a separate stage. The approach S-B is proposed in this paper where downstream demand information is taken into consideration and internal pricing directly influences harvesting decision in the first stage. In order to find the appropriate setting of internal pricing that leads to the system optimum, we suggest two heuristics H-I and H-II. The internal pricing in H-I is based on dual values and in H-II derived from multipliers in a Lagrangian decomposition. A real-life case study in the Chilean forestry industry is used to compare the results of different approaches. It is shown that the new sequential approach S-B generates as good feasible solution as that obtained from the integrated approach but in much shorter time. Both heuristics H-I and H-II bring about near-optimal feasible solutions. H-II also provides optimistic bounds of the optimal objective function value, which can be used as a measure of the solution quality. Internal pricing obtained in these two heuristics indicates not only the log type but also the location, which in theory can eliminate the symmetry inherent of variable coefficients to achieve a significant computational advantage and in practice can help the forest manager to get a more comprehensive idea of the value of logs in various harvest areas.

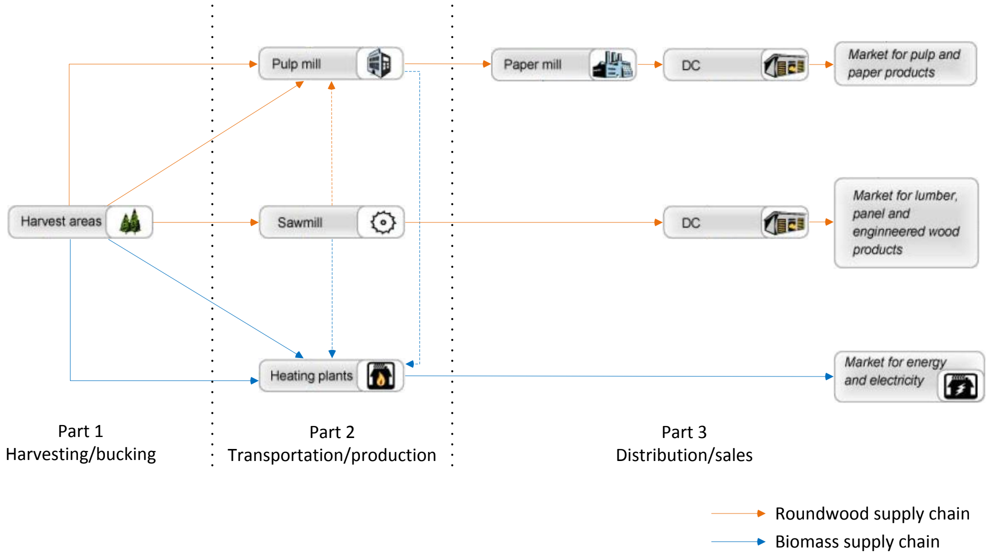
Key word: coordination, internal price, forest management, production planning, Lagrangian decomposition[3]

[3] We would like to acknowledge the support of NHH Norwegian School of Economics and Research Consortium on e-Business in the Forest Products Industry (FORAC), University Laval.

4.1 Introduction

The supply chains of roundwood and biomass in the forest products industry start with harvest in the forest where different log types, e.g., sawlogs, pulp logs, and fuel logs, are generated from the bucking process. These logs are then transported to downstream mills, i.e., sawmills, pulp mills, and heating plants, for further production of various final products, i.e., engineered wood products, pulp and paper, and energy and electricity. Byproducts from wood-processing facilities, i.e., chips, sawdust, and bark, will be reutilized and transported to mills for either pulp and paper production or power generation (Figure 4.1). In addition, forestry supply chain planning encompasses a wide range of decisions, from strategic to operational. At the strategic level, decisions include forest management, silviculture treatments, road construction, facility location, process investments, and product and market development. Tactical planning focuses on how to execute the forest management or production and distribution issues, serving as a bridge between the long-term comprehensive strategic planning and the short-term detailed operational planning that determines the real-world operations (D'Amours et al. 2008).

Figure 4.1 Illustration of the main supply chains, pulp & paper, engineered wood products, and energy, in the forest products industry



Historically, the strategic forest management and the tactical industrial production are managed in a decoupled manner. Forest management models tend to ignore issues of prices and markets for forest products, and similarly, forest productivity does not change with forest management (Gunn 2007). It can be partially explained by the fact that strategic planning may span decades, while tactical planning may deal with a maximum of five years. Another reason is that the strategic decisions of forest management aim to not only maximize the net present value (NPV) but also attain social targets in order to meet sustainable socio-economic development, whereas the emphasis of tactical production is placed on commercial use of timber and on fulfilling market demands. The literature in the forestry domain is hence divided into two categories.

Numerous models based on operations research (OR) have been developed to aid forest managers and public forestry organizations in their decision-making. However, those widely recognized models, for example, timber RAM (Navon 1971), FORPLAN (Johnson et al. 1986) and SPECTRUM (Greer and Meneghin 1997), all used by the United States Department of Agriculture (USDA) Forest Service, FOLPI in New Zealand (Garcia 1984), and JLP in Finland (Lappi 1992), do not take into account the interaction with downstream operations. On the other hand, practitioners as well as researchers proposed abundant OR models to optimize supply chain planning for the forest products. Philpott and Everett (2001) developed a supply chain optimization model for the Australian paper industry. Gunnarsson et al. (2007) considered an integrated supply chain planning for a leading European pulp mill company. Singer and Donoso (2007) presented a model for optimizing tactical planning decisions in the Chilean sawmill industry. Feng et al. (2008) proposed a series of models to coordinate business units along the supply chain in an Canadian Oriented Strand Board manufacturing company. The commonality of these models is that the availability of logs is already decided and treated as constraints and the supply of raw materials is hence taken as exogenous in the supply chain design.

Although relatively few articles combine the upstream forest management and harvesting operations with the downstream industrial planning, researchers have recognized the need for greater integration and the resulting benefits. Two of the first authors to vertically integrate forest industry was Barros and Weintraub (1982), who incorporated decisions concerning managing forest lands and supply of timber for a pulp plant, a sawmill and export. Gunn and Rai (1987) introduced a model with an integrate viewpoint, which is somewhat similar to that of Barros and Weintraub (1982) but enables calculation of regeneration harvest policies. Weintraub et al. (1986) discussed the advantage of a two-level hierarchical approach where the strategic decisions first depend on aggregated plant operation variables and then lead to production guidelines for a tactical model. In order to reconcile the outputs occurring in the aggregation and disaggregation process, various feedback and feed-forward mechanisms are proposed (Weintraub and Bare 1996). In contrast to these operational models, Jones and Ohlmann (2008) constructed an analytical model by combining perspectives from forest economics and operations management in a vertically integrated paper mill. Troncoso et al. (2011) evaluated the superiority of an integrated strategy where long-term forest management and mid-term production planning in sawmills, pulp mills, and heating plants are simultaneously determined and the objective is to maximize the NPV for the whole forestry supply chains. It is shown that the NPV can increase up to 5% when the proposed integrated strategy is implemented as compared to the conventional sequential approach where forest planner and downstream plants optimize separately the operations.

Indeed, in theory, an integrated planning approach can obtain better NPV of the forest products supply chain. However, due to nature of distinct planning levels, inherent responsibilities of different divisions, and limitation of existing decision support systems, as discussed earlier, it is difficult to motivate the managers from various constituents of the supply chain to work together under an integrated approach, even though the forest lands and mills belong to only one company. Therefore, the sequential approach is still widely used in practice (Troncoso et al. 2011). The purpose of this paper is, with minor changes to the currently used planning method, to find effective coordination mechanism using the notion of internal pricing between the strategic forest management and the tactical industrial planning.

Pricing has long been recognized as a significant tool used in the industrial operations to manipulate demand and to regulate the production and distribution of products (Soon 2011). Internal pricing, also known as transfer pricing, is utilized as a communication device between participants in an internal market in order to arrive at the system optimal allocation of resources. Pfeiffer (1999) summarized two common approaches to derive internal pricing system. One is an economic approach that uses methods of marginal analysis to determine values of intermediate commodities. The other is a mathematical programming approach that is based on the dual Lagrangian principle. In those integrated forest planning models, harvest and bucking operations are based on internal prices for desired logs, which can be viewed as a coordination mechanism to match the supply of timber in the forest to the demand at mills. In most cases, these prices are manually estimated or justified, referring to sale prices for the logs on market, and given as parameters. In this paper, we suggest two heuristics H-I and H-II to systematically find the appropriate setting of internal pricing. The internal pricing in the first, H-I, is based on dual values and in the second, H-II, it is derived from Lagrangian decomposition (LD).

For large-scale optimization problems, arising either because of site types or from multiperiods, a decomposition approach is usually employed to take advantage of the structure of these problems. The idea behind decomposition is to break the overall problem into a number of smaller subproblems that are easier to solve and to coordinate these subproblems through a master problem. Hauer and Hoganson (1996) gave a brief comparison of several decomposition techniques used in forest management scheduling models, mainly derived from Lagrangian relaxation (LR). Pittman et al. (2007) applied LR to formulate a hierarchical production planning model and used the Lagrangian multipliers to convey information between forest-level planning and management zone level planning. Gunnarsson and Rönnqvist (2008) used LD to relax all storage balance constraints, resulting in subproblems for each time period. The coordination is obtained by updating the Lagrangian multipliers from the solutions to each subproblem. In another paper (2011), the authors applied the same technique but decomposed the problem into subproblems representing distinct physical stages in the supply chain. Although separability is the goal of both decomposition approaches, the main difference between LR and LD is that instead of relaxing the “complicating” constraints that tie together the problems in LR, common variables involving in two (or more) subproblems are first duplicated and the equality constraints are then dualized in LD. Guignard and Kim (1987) demonstrated that the LD bound can strictly dominate the LR bounds. Most applications of LD, the same as other decomposition approaches, stress in significant improvements in computational efficiency, but as far as we know no attempt has been made to apply LD as a coordination scheme to find internal prices in forest planning problems and to allow the decoupled system to perform like a centralized one.

In this paper, we first formulate an integrated model for the entire supply chain, including harvest areas, sawmills, pulp mills, heating plants and final markets. This is a mixed integer programming (MIP) model that involves harvesting, bucking, transportation, production and sales decisions for both tactical (5 years) and strategic (25 years) planning levels. We then present two sequential approaches S-A and S-B. The approach S-A is the one currently used in practice where harvesting in forest is the main driver of the supply chain activities and internal pricing is introduced to control bucking decision in a separate stage. The approach S-B proposed in this paper takes into account downstream demand information and internal pricing directly influences harvesting decision in the first stage. A real-life case study in the Chilean forest industry (also used in Troncoso et al. (2011)) is implemented to compare the results of different approaches. It is shown that the new sequential approach S-B generates as good feasible solution as that obtained from the integrated approach but in much shorter time. In order to find the appropriate setting of internal pricing that leads to the system optimum, we suggest two heuristics H-I and H-II. The internal pricing in H-I is based on dual values and in H-II derived from multipliers in a Lagrangian decomposition. Both heuristics H-I and H-II bring about near-optimal feasible solutions.

When constructing a MIP representation, analysts are required to formulate not only a *correct* mathematical model of the problem but also a *good* one that yields acceptable result within reasonable time. It is already demonstrated that by disaggregating sets of constraints (Barnhart et al. 1993), adding logical inequalities and lifting the constraints (Andalaft et al. 2003), introducing symmetry-breaking hierarchical constraints (Sherali and Smith 2001), and eliminating unnecessary binary variables, dramatic improvements were obtained in the overall computation performance. In the case study, we also find that if the symmetry inherent of variable coefficients is eliminated, a significant computational advantage can be achieved.

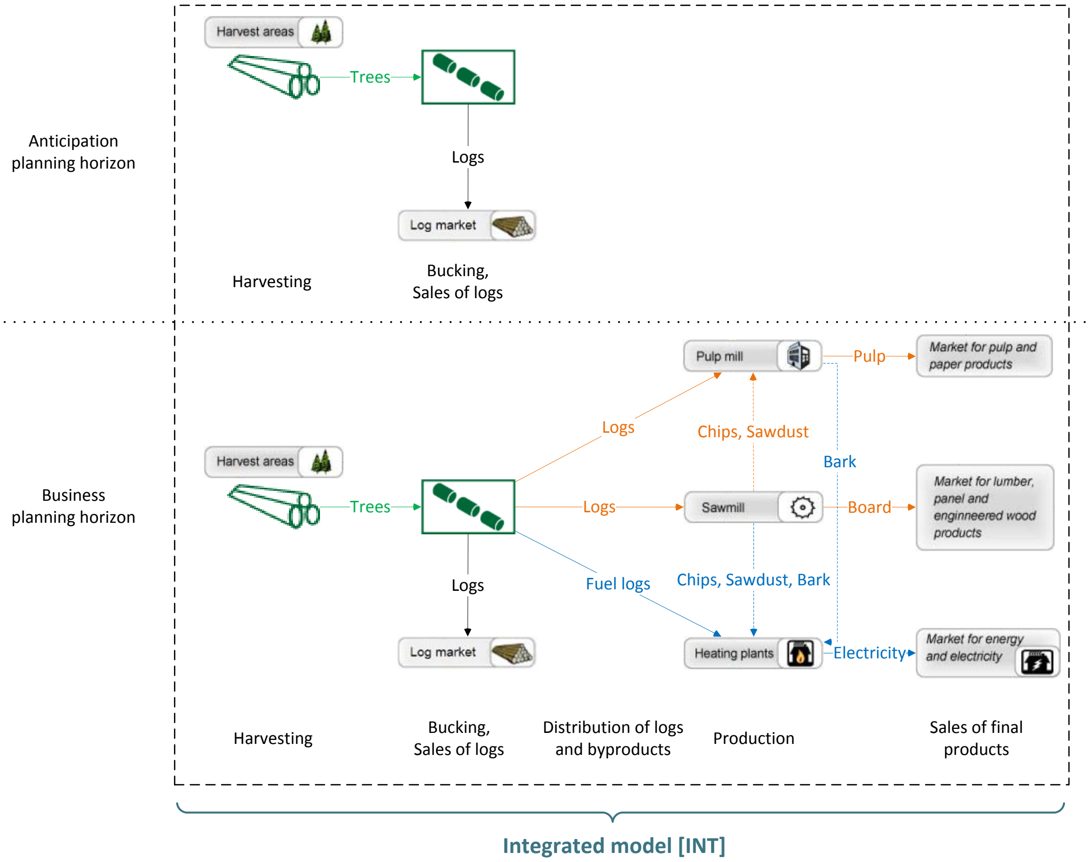
The main contributions of this paper are threefold. First, the modification of the sequential approach is effective in producing near-optimal solution in quick computational time. It keeps the conventional planning method but obtains the same superior effect as in the integrated approach. Second, both heuristics to determine the internal pricing use the existing decoupled modeling setting, independent of the integrated model. It means that only minor change in information exchange rather than reform of the whole planning system is enough to make major progress. Moreover, heuristic H-II also provides optimistic bounds of the optimal value, which can be used as a measure of the solution quality. Last but not least, the methodology to employ LD to find internal prices is general enough so that other industries could easily adapt it for coordination between two functional divisions or organizations.

The outline of the paper is as follows. In next section, the planning decisions concerning the entire supply chain and three planning approaches are described. Then, in Section 4.3, the related mathematical models are formulated. Two heuristics to determine internal pricing are proposed in Section 4.4. Afterwards, the case study and numerical results are presented in Section 4.5. Finally, some concluding remarks are made.

4.2 Problem description

The structure of the forest products supply chain is similar to the two largest forest holding companies in Chile. These companies own plantation lands and downstream mills. For the long-term forest management (upper part of Figure 4.2), the planner only focus on harvesting and bucking decisions in order to optimize an expected NPV of logs. In the mid-term business planning horizon (lower part of Figure 4.2), the supply chain stretches from harvesting operations in the forest to industrial production in mills and to final sales in markets. It is worth mentioning that this supply chain is a typical divergent chain with one-to-many processes. Hence, different planning strategies will result in many diverse outcomes.

Figure 4.2 Illustration of the planning decisions in the entire supply chain



4.2.1 Planning horizon

The entire planning horizon covers a minimum of one full forest rotation, which in the case study originating in Chile corresponds to about 25 years. The selection of at least a full rotation is common for the strategic planning in the forestry industry. In contrast, tactical planning problems are often reviewed annually over a five-year planning period (D'Amours et al. 2008). Therefore, in this paper we separate the planning horizon into two parts. The first part is 5 years, denoted by business planning horizon T^1 of yearly periods. It is dictated by the fact that future demands and sale prices of final products can only be reasonably forecasted for such an interval (Andalaft et al. 2003). The second part spans 20 years, named as anticipation planning horizon T^2 of 5-year periods. The strategic forest management covers all the time periods in both business and anticipation planning horizons, whereas the tactical industrial production only considers the first five years. Figure 4.3 summaries the relationship between planning horizon and planning level.

Figure 4.3 Illustration of the planning horizon separated into business planning and anticipation planning horizons

<table>
<tr><td>Calendar year</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td></tr>
<tr><td>Planning horizon</td><td colspan="5">Business planning horizon (T^1)</td><td colspan="20">Anticipation planning horizon (T^2)</td></tr>
<tr><td>Planning period</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td colspan="5">6</td><td colspan="5">7</td><td colspan="5">8</td><td colspan="5">9</td></tr>
<tr><td rowspan="2">Planning level</td><td colspan="25">Strategic forest management</td></tr>
<tr><td colspan="5">Tactical production</td><td colspan="20"></td></tr>
</table>

4.2.2 Harvesting

The forest is usually divided into a number of so-called stands. In general, a timber stand consists of trees within a contiguous area that share similar characteristics such as age, species composition, or site quality. It also represents the smallest unit to harvest at one time. Since trees are fixed with location, it is possible to gather precise information about age and existing timber volumes and expected future volumes according to growth curves. Normally, a stand becomes more valuable with age as the standing volume and average tree diameter increase. Therefore, the key problem in forestry is to determine which areas and when to harvest. Decision-makers face the choice each year as to whether to keep the trees in the stands for another year as a capital stock or harvest them as output (Grafton et al. 2004).

In this paper, a stand is known as a harvest area which has a given age in the initial state. Each area is supposed to be fully harvested in a single period or not at all. Once a harvest unit is cut, we assume that the stand is immediately replanted. Based on growth curves, we can calculate the volumes for each area during the entire planning horizon. However, the actual harvest level is not only determined by biological growth criteria but also restricted by forest management rules. For example, some areas may not produce commercial timber until the trees reach a certain age; Socio-economic consequences such as non-declining yield also have to be ensured; The status of the forest at the end of the planning period should be similar to the original structure of age class. In our case, we require that the volumes of each age group must be at least the same as the initial state of the forest.

4.2.3 Bucking

Once trees are harvested, they are bucked into logs directly in the forest. In some cases, trees may also be moved to a nearby sawmill or terminal for bucking. The log types are defined through attributes, i.e., diameter, length and quality. Traditionally, the lower part of the tree, which has a larger diameter with higher value, is sent to sawmills as sawlog. The upper, thinner part with a lower value is better suited for pulp and paper mills as pulplog. The remaining tops and branches with least value are left in the forest as soil nutrient or forwarded to heating plants as fuel log. In some articles, the name fuel log may also define low quality logs, and branches and tops are referred to as forest residues. In our case study, we group all forest biomass for energy as fuel logs. The volumes of logs are based on tree characteristics, i.e., species, age and location, as well as bucking patterns. A bucking pattern is a list of possible log types and the proportion of which they are produced when applied to a harvest area with a particular tree age (Table 4.1). One bucking pattern may, for example, produce more "Pruned Log" than other bucking patterns. Decisions following harvesting then turn into choices of bucking pattern for each harvested area at each time period.

Table 4.1 Proportion of different log types with a tree age of 30 years under 7 bucking patterns

	Bucking pattern						
Log type	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Sawlog Type 1	0.18	0.24	0.16	0.24	0.16	0.09	0.05
Sawlog Type 2	0.22	0.24	0.23	0.36	0.17	0.24	0.14
Sawlog Type 3	0.18	0.17	0.16	0.10	0.18	0.09	0.07
Pruned Log	0.12	0.05	0.15	-	0.08	-	0.12
Long Log	-	-	-	-	0.03	0.14	0.10
Pulplog	0.28	0.27	0.28	0.26	0.33	0.36	0.42
Fuel Log	0.02	0.03	0.02	0.04	0.05	0.08	0.10
Sum of proportion	1.00	1.00	1.00	1.00	1.00	1.00	1.00

The long-term forest management solution is evaluated by the expected NPV. The standard approach to calculate the sales value for each log type is to use current market prices minus an average transportation cost throughout the entire planning horizon. These sale prices refer to the value of the logs by roadside in the forest before any transportation is done. If the value for certain log is zero, e.g., "Fuel log", it implies that this type of log will not be sold on the market, but sent to mills or left in the forest. In order to obtain an efficient and profitable result, the forest manager would like to apply bucking patterns that generate a higher volume of logs with better expected prices, e.g., "Pruned Log", and only meet the a small estimated demand for logs with lower expected value, e.g., "Sawlog Type 3". To avoid this situation and to reflect the needs of mills, internal prices as coordination mechanism are hence employed in the business planning horizon. At present, these internal prices are estimated manually, based on prior experience. Table 4.2 lists the sale prices and internal prices used in practice.

Table 4.2 Sale prices and internal prices used in the case study (US$/$m^3$)

Log type	Sale price	Internal price
Sawlog Type 1	38	32
Sawlog Type 2	31	26
Sawlog Type 3	24	22
Pruned Log	76	43
Long Log	42	34
Pulplog	10	6
Fuel Log	0	0

4.2.4 Distribution

In the business planning horizon, after solving the problem of allocating wood for different uses, decisions become a question of distributing the logs to different destinations to satisfy temporal demand for timber. Though some logs might be sold on the market, most are used for further manufacturing processes in downstream facilities, i.e., sawmills, pulp mills and heating plants, operated by the same company (in our case study from Chile). Thus, the harvesting and bucking operations in the forest are merely part of the supply chain for the whole forest products industry. In addition, byproducts such as chips and sawdust from sawmills will be forwarded to pulp mills as raw materials and barks from wood-processing factories will be transported to heating plants as bioenergy.

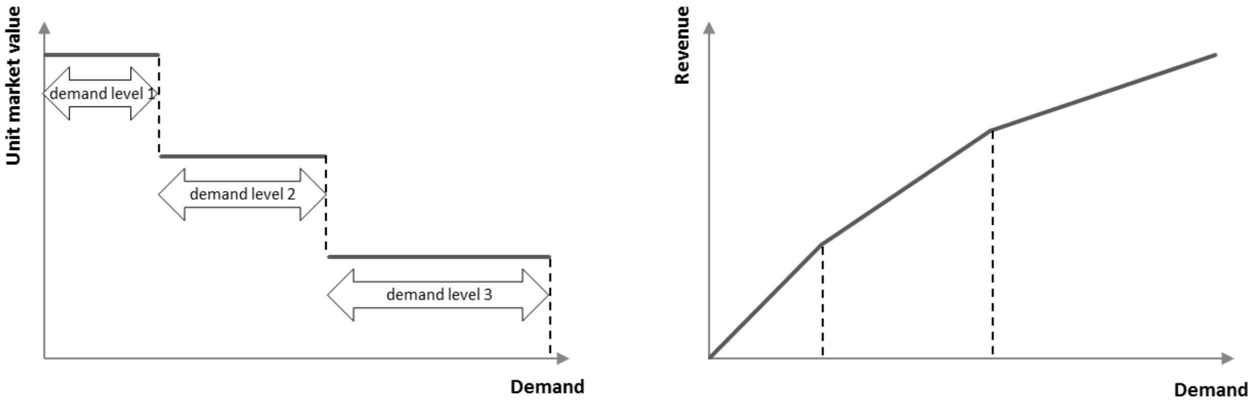
4.2.5 Production

Production is carried out at sawmills, pulp mills and heating plants through various processes during the business planning horizon. At sawmills there exist several sawing patterns which describe how one unit of specific sawlog (m^3) can be broken down into a set of final products (m^3), for example boards of different dimensions, and byproducts (m^3), i.e., chips, sawdust and bark The sawing patterns are based on statistical analysis of past used processes and experiences. We have in the case study access to this information for all pairs of sawing pattern and log type. At pulp mills there are recipes which tie the input of pulp logs (m^3) or chips (m^3) with final production of pulp (tonne). For the heating plant we also have a relationship between fuel logs (m^3) as well as bark or chips (m^3) used and the energy (MWh) produced. All plants have maximum and minimum capacities and processing costs.

4.2.6 Final sales

The customers are represented by the demand curves for the products. We assume that the demand for final products is not fixed. Instead, we use a step price function to better describe the real market behavior toward different volumes of products. The highest unit sale price is only applied to a limited number of volumes and demand beyond this limit pays lower price. There are two break points with three demand limits or levels. Then the total revenue of sold products increases with the volumes in a piecewise-linear fashion, with three pieces as shown in Figure 4.4.

Figure 4.4 Example of a step price function (left) and piecewise-linear revenue function (right) with three demand levels, respectively

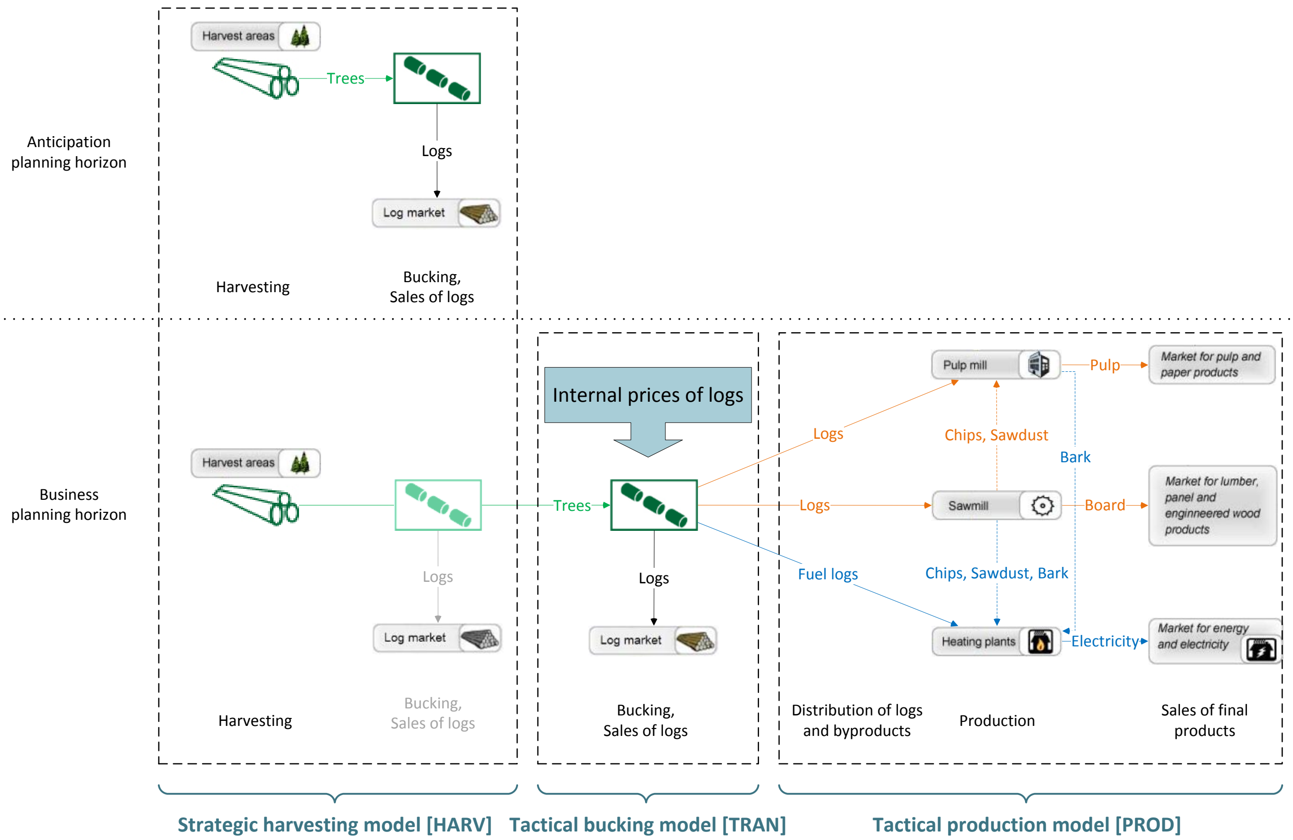


4.2.7 Planning approaches

In this paper, we compare three different planning strategies. The first approach relates to an integrated strategy that was suggested by Troncoso et al. (2011) and illustrated in Figure 4.2. In this integrated approach, the forest planner and the industrial planner centrally coordinate all the operations and maximize the total NPV of the whole supply chain. No internal transfer prices for logs are needed. It refers to the ideal goal that the entire system can achieve and can be served as the theoretical benchmark for performance.

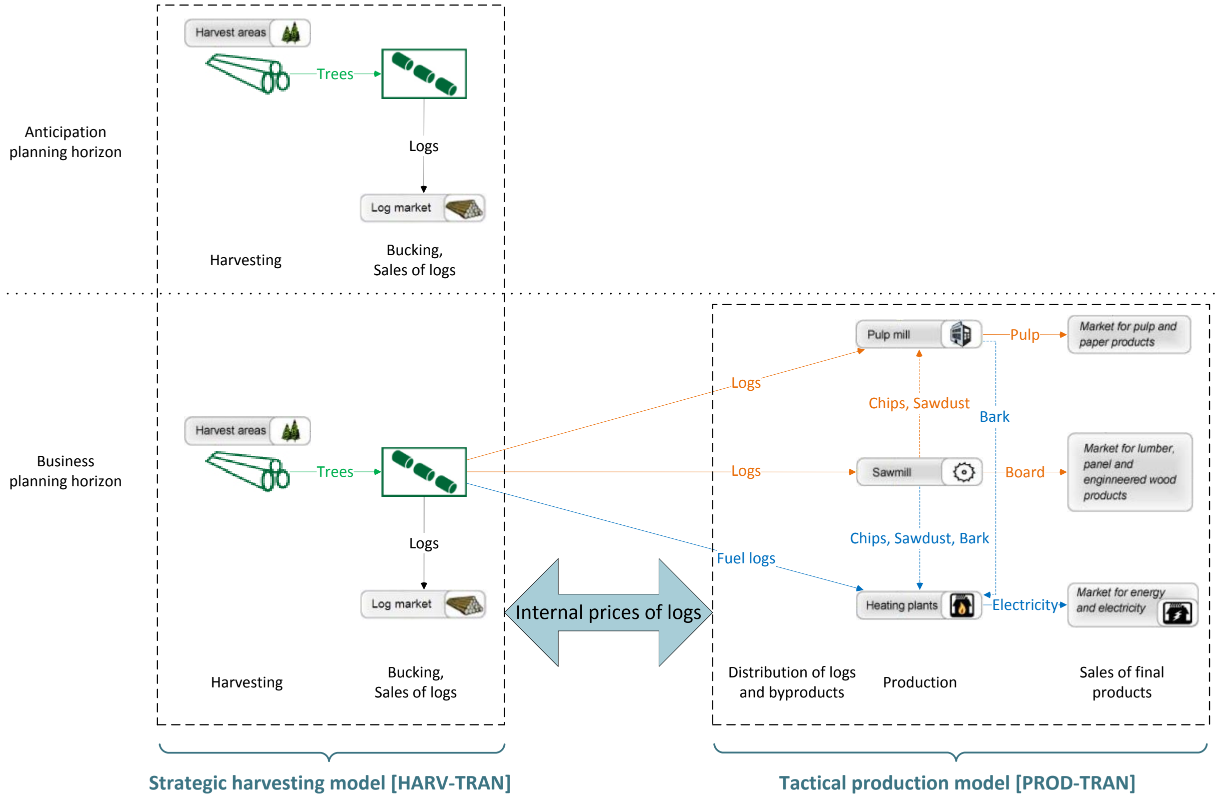
However, in practice, although forest and mills involved are owned by one company, each business unit is managed independently and has his own objective. The only coordination mechanism is the manually estimated internal prices. Sequential approach S-A represents the planning process in practice (Figure 4.5). In this decoupled strategy, the forest planner determines the optimal harvesting and log production decisions by using sale prices for the NPV computations. Then with the fixed availability of trees in specific areas and periods, the planner decides which bucking patterns to use for the first five years (business horizon) by using internal prices. Afterwards, the mill planner optimizes the transportation and production to meet market demands under the volume constraints of several alternative logs that the forest unit produces and sells. Note that this approach S-A corresponds to the decoupled strategy S2 described in Troncoso et al. (2011).

Figure 4.5 Illustration of sequential approach S-A



The third approach, sequential approach S-B (Figure 4.6), is modified from S-A and proposed for the first time in this paper. Instead of using internal pricing in a separate stage, internal pricing aiming at reflecting downstream demand information is directly used in the first stage to control the harvesting and bucking decisions synchronously. By excluding the redundant rescheduling, this approach can better utilize the information and hence facilitate the coordination.

Figure 4.6 Illustration of sequential approach S-B



4.3 Mathematical formulation

In this section mathematical models for the three different approaches mentioned above are developed. We first describe the sets of parameters and variables, and then follow constraints and objective function. For clarity, all the parameters, variables and constraints are grouped into *Forest* and *Mills* according to the structure of the problem. The formulation of the supply chain is based on conditions for the Chilean company in the case study, but can be easily generalized to other forest companies.

4.3.1 Parameters and variables

The indexes and sets used in the models are given below. Along with the parameter and variables, the subscript denotes the indexes and the superscript indicates the activities, locations or materials in an abbreviated form.

$i \in I$	Set of harvest areas
$i \in I_t^{NH}$	Set of harvest areas that cannot be harvested (NH: No Harvesting) during time period t
$j \in J^{SAW}$	Set of sawmills
$j \in J^{PULP}$	Set of pulp mills

$j \in J^{\text{HEAT}}$	Set of heating plants
$j \in J$	Set of mills, $J = J^{\text{SAW}} \cup J^{\text{PULP}} \cup J^{\text{HEAT}}$
$u \in U^{\text{BUCK}}$	Set of bucking patterns in harvest areas
$u \in U_j^{\text{PROC}}$	Set of processes in mill j
$r \in R$	Set of tree types
$l \in L$	Set of log types
$k \in K^{\text{F}}$	Set of final products (F) that are salable in markets
$k \in K^{\text{B}}$	Set of byproducts (B) that will be transported between mills for further use
$k \in K$	Set of all products, $K = K^{\text{F}} \cup K^{\text{B}}$
$m \in M$	Set of demand levels for final products
$t \in T^1$	Set of time periods in the business planning horizon
$t \in T^2$	Set of time periods in the anticipation planning horizon
$t \in T$	Set of all time periods, $T = T^1 \cup T^2$

The parameters are defined as follows:

η_t	Capital discount factor, $\eta_t = 1/(1+\eta)^t$, $t \in T$. If not specially mentioned, the number of t is defined as the mid of time period t. For example, for "Planning period 1" in T^1, $t = 0.5$ and for "Planning period 6" in T^2, $t = 7.5$. If it refers to the end of entire planning horizon, $t = 25$.

Forest:

α_{irult}	Conversion rate for bucking pattern u from tree type r to log type l in harvest area i in time period t, $u \in U^{\text{BUCK}}$, $t \in T$
$\underline{\delta}_t, \bar{\delta}_t$	Min and max percentage of harvesting yield in time period t compared to next period, $t \in T$
$\underline{\delta}^5, \bar{\delta}^5$	Min and max percentage of harvesting yield in one 5-year period compared to average 5-year period throughout the entire planning horizon
h_{it}	Harvest capacity required in harvest area i in time period t, $t \in T$
$\bar{h}_t$	Max harvest capacity in time period t, $t \in T$
s_{irt}	Supply of tree type r in harvest area i in time period t, $t \in T$
s_{it}^{TOT}	Total supply (TOT) in harvest area i in time period t, $t \in T$
v_t^{INI}	Volume of forest under certain age at the beginning of the entire planning horizon (INI: initial). This age corresponds to the age timber will be at the end of entire planning horizon if it is harvested in time period t, $t \in T$
$v^{\text{INI-20}}$	Total volume of forest older than 20 years (the number of years included in the anticipation planning horizon) at the beginning of the entire planning horizon
v_{it}^{END}	Volume of forest in harvest area i at the end of the entire planning horizon (END) if it is harvested in time period t, $t \in T$
v_i^{NH}	Volume of forest in harvest area i at the end of the entire planning horizon if it is not harvested in the entire planning horizon

p_{it}^{END} Value of harvest area i at the end of the entire planning horizon if it is harvested in time period t, $t \in T$

p_i^{NH} Value of harvest area i at the end of the entire planning horizon if it is not harvested in the entire planning horizon

p_{lt}^{L} Unit sale price for log type l in time period t, $t \in T$

p_{lt}^{TRAN} Unit internal/transfer price (manually estimated) for log type l in time period t, $t \in T^1$

p_{ilt}^{TRAN} Unit internal/transfer price (heuristically determined in the proposed approaches) for log type l in harvest area i in time period t, $t \in T^1$

c_{it}^{H} Harvest cost in harvest area i in time period t, $t \in T$

c_{ijlt}^{T} Unit transportation cost of log type l from harvest area i to mill j in time period t, $j \in J, t \in T^1$

Mills:

β_{julk} Conversion rate for process u from input l to product k in mill j, $j \in J, u \in U_j^{\text{PROC}}, l \in L \bigcup K^{\text{B}}, k \in K$

$\underline{e}_{jt}, \overline{e}_{jt}$ Min and max production capacity in mill j in time period t, $j \in J, t \in T^1$

d_{kmt}^{K} Volume limit of product k on demand level m in time period t, $k \in K^{\text{F}}, t \in T^1$

p_{kmt}^{K} Unit market price for product k on demand level m in time period t, $k \in K^{\text{F}}, t \in T^1$

c_{jkt}^{P} Unit production cost of product k in mill j in time period t, $j \in J, k \in K^{\text{F}}, t \in T^1$

c_{ijkt}^{T} Unit transportation cost of byproduct k from mill i to mill j in time period t, $i \in J^{\text{SAW}} \bigcup J^{\text{PULP}}, j \in J^{\text{PULP}} \bigcup J^{\text{HEAT}}, k \in K^{\text{B}}, t \in T^1$

c_{jt}^{N} Unit penalty cost for not fulfilling the min production capacity in mill j in time period t, $j \in J, t \in T^1$

There are two types of variables. We use binary variables to represent the strategic harvesting decisions and non-negative continuous variables for the flows, production and sales decisions.

Forest:

$$\omega_{it} = \begin{cases} 1, \text{if harvest area } i \text{ is harvested in time period } t, t \in T \\ 0, \text{otherwise} \end{cases}$$

$$\omega_i^{\text{NH}} = \begin{cases} 1, \text{if harvest area } i \text{ is not harvested in the entire planning horizon} \\ 0, \text{otherwise} \end{cases}$$

Q_{irut}^{BUCK} Volume of tree type r using bucking pattern u in harvest area i in time period t, $u \in U^{\text{BUCK}}, t \in T$

W_{ilt}^{L} Volume of log type l left in harvest area i in time period t, $t \in T$

A_{ilt}^{L} Volume of log type l sent to mills from harvest area i in time period t, $t \in T^1$

D_{ilt}^{L} Volume of log type l sold to markets from harvest area i in time period t, $t \in T$

Mills:

X_{ijlt}^{L} Flow of log type l from harvest area i to mill j in time period t, $j \in J, t \in T^1$

Q_{jult}^{PROC}	Volume of input l using process u in mill j in time period t, $j \in J, u \in U_j^{\text{PROC}}, l \in L \bigcup K^{\text{B}}, t \in T^1$
Y_{jkt}^{K}	Volume of product k produced in mill j in time period t, $j \in J, k \in K, t \in T^1$
X_{ijkt}^{K}	Flow of byproduct k from mill i to mill j in time period t, $i \in J^{\text{SAW}} \bigcup J^{\text{PULP}}, j \in J^{\text{PULP}} \bigcup J^{\text{HEAT}}, k \in K^{\text{B}}, t \in T^1$
W_{jt}^{N}	Volume not fulfilling the min production capacity in mill j in time period t, $j \in J, t \in T^1$
D_{kmt}^{K}	Volume of product k sold on demand level m in time period t, $k \in K^{\text{F}}, t \in T^1$

4.3.2 Constraints

Constraints concern harvest operations in the forest for the entire planning horizon:

$$\sum_{t \in T} \omega_{it} + \omega_i^{\text{NH}} = 1, \quad \forall i \in I \tag{1}$$

$$\sum_{t \in T} \sum_{i \in I_t^{\text{NH}}} \omega_{it} = 0, \tag{2}$$

$$\underline{\delta}_t \sum_{i \in I} s_{i,t+1}^{\text{TOT}} \omega_{i,t+1} \le \sum_{i \in I} s_{it}^{\text{TOT}} \omega_{it} \le \bar{\delta}_t \sum_{i \in I} s_{i,t+1}^{\text{TOT}} \omega_{i,t+1}, \quad \forall t \in T : t \ne \text{last}(T^1), t \ne \text{last}(T^2) \tag{3}$$

$$\underline{\delta}_{\text{last}(T^1)} \sum_{i \in I} s_{i,\text{first}(T^2)}^{\text{TOT}} \omega_{i,\text{first}(T^2)} \le \sum_{i \in I} \sum_{t \in T^1} s_{it}^{\text{TOT}} \omega_{it} \le \bar{\delta}_{\text{last}(T^1)} \sum_{i \in I} s_{i,\text{first}(T^2)}^{\text{TOT}} \omega_{i,\text{first}(T^2)}, \tag{4}$$

$$\underline{\delta}^5 \frac{\sum_{i \in I} \sum_{t' \in T} s_{it'}^{\text{TOT}} \omega_{it'}}{|T^2| + 1} \le \sum_{i \in I} \sum_{t \in T^1} s_{it}^{\text{TOT}} \omega_{it} \le \bar{\delta}^5 \frac{\sum_{i \in I} \sum_{t' \in T} s_{it'}^{\text{TOT}} \omega_{it'}}{|T^2| + 1}, \tag{5}$$

$$\underline{\delta}^5 \frac{\sum_{i \in I} \sum_{t' \in T} s_{it'}^{\text{TOT}} \omega_{it'}}{|T^2| + 1} \le \sum_{i \in I} s_{it}^{\text{TOT}} \omega_{it} \le \bar{\delta}^5 \frac{\sum_{i \in I} \sum_{t' \in T} s_{it'}^{\text{TOT}} \omega_{it'}}{|T^2| + 1}, \quad \forall t \in T^2 \tag{6}$$

$$\sum_{i \in I} \sum_{t \in T} v_{it}^{\text{END}} \omega_{it} + \sum_{i \in I} v_i^{\text{NH}} \omega_i^{\text{NH}} \ge \sum_{t \in T} v_t^{\text{INI}}, \tag{7}$$

$$\sum_{i \in I} \sum_{t \in T^1} v_{it}^{\text{END}} \omega_{it} + \sum_{i \in I} v_i^{\text{NH}} \omega_i^{\text{NH}} \ge v^{\text{INI-20}}, \tag{8}$$

$$\sum_{i \in I} v_{it}^{\text{END}} \omega_{it} \ge v_t^{\text{INI}}, \quad \forall t \in T^2 \tag{9}$$

$$\sum_{i \in I} h_{it} \omega_{it} \le \bar{h}_t, \quad \forall t \in T \tag{10}$$

$$s_{irt} \omega_{it} = \sum_{u \in U^{\text{BUCK}}} Q_{irut}^{\text{BUCK}}, \quad \forall i \in I, \forall r \in R, \forall t \in T \tag{11}$$

$$\sum_{r \in R} \sum_{u \in U^{\text{BUCK}}} \alpha_{irult} Q_{irut}^{\text{BUCK}} - W_{ilt}^{\text{L}} = A_{ilt}^{\text{L}}, \quad \forall i \in I, \forall l \in L, \forall t \in T^1 \tag{12}$$

$$\sum_{r \in R} \sum_{u \in U^{\text{BUCK}}} \alpha_{irult} Q_{irut}^{\text{BUCK}} - W_{ilt}^{\text{L}} = 0, \quad \forall i \in I, \forall l \in L, \forall t \in T^2 \tag{13}$$

$$W_{ilt}^{\text{L}} \ge D_{ilt}^{\text{L}}, \quad \forall i \in I, \forall l \in L, \forall t \in T \tag{14}$$

$$\omega_{it}, \omega_i^{\text{NH}} \in \{0, 1\}, \quad \forall i \in I, t \in T \tag{15}$$

$$\text{all involved variables} \ge 0. \tag{16}$$

Note that $\text{first}(T^2)$ denotes the first element of set T^2, $\text{last}(T^1)$ indicates the last element of set T^1, and $|T^2|$ represents the cardinality of set T^2.

Constraints relate to production in mills for the business planning horizon:

$$A_{ilt}^{\mathrm{L}} = \sum_{j \in J} X_{ijlt}^{\mathrm{L}}, \quad \forall i \in I, \forall l \in L, \forall t \in T^1 \tag{17}$$

$$\sum_{i \in I} X_{ijlt}^{\mathrm{L}} = \sum_{u \in U_j^{\mathrm{PROC}}} Q_{jult}^{\mathrm{PROC}}, \quad \forall j \in J, \forall l \in L, \forall t \in T^1 \tag{18}$$

$$\sum_{i \in J^{\mathrm{SAW}} \cup J^{\mathrm{PULP}}} X_{ijkt}^{\mathrm{K}} = \sum_{u \in U_j^{\mathrm{PROC}}} Q_{jukt}^{\mathrm{PROC}}, \quad \forall j \in J^{\mathrm{PULP}} \cup J^{\mathrm{HEAT}}, \forall k \in K^{\mathrm{B}}, \forall t \in T^1 \tag{19}$$

$$\sum_{u \in U_j^{\mathrm{PROC}}} \sum_{l \in L \cup K^{\mathrm{B}}} \beta_{julk} Q_{jult}^{\mathrm{PROC}} = Y_{jkt}^{\mathrm{K}}, \quad \forall j \in J, \forall k \in K, \forall t \in T^1 \tag{20}$$

$$\underline{e}_{jt} - W_{jt}^{\mathrm{N}} \leq \sum_{k \in K^{\mathrm{F}}} Y_{jkt}^{\mathrm{K}} \leq \bar{e}_{jt}, \quad \forall j \in J, \forall t \in T^1 \tag{21}$$

$$Y_{jkt}^{\mathrm{K}} \geq \sum_{i \in J^{\mathrm{PULP}} \cup J^{\mathrm{HEAT}}} X_{jikt}^{\mathrm{K}}, \quad \forall j \in J^{\mathrm{SAW}} \cup J^{\mathrm{PULP}}, \forall k \in K^{\mathrm{B}}, \forall t \in T^1 \tag{22}$$

$$\sum_{j \in J} Y_{jkt}^{\mathrm{K}} = \sum_{m \in M} D_{kmt}^{\mathrm{K}}, \quad \forall k \in K^{\mathrm{F}}, \forall t \in T^1 \tag{23}$$

$$D_{kmt}^{\mathrm{K}} \leq d_{kmt}^{\mathrm{K}}, \quad \forall k \in K^{\mathrm{F}}, \forall m \in M, \forall t \in T^1 \tag{24}$$

$$\text{all involved variables} \geq 0. \tag{25}$$

Constraint sets are defined and explained as follows:

(*1*) Each harvest area can be harvested at most once during the entire planning horizon.

(*2*) Trees in the forest cannot be harvested before they reach a certain age.

(*3*) Yield of harvesting is non-declining between current and succeeding periods.

(*4*) Yield of harvesting is non-declining between the aggregated business planning horizon and the first period of anticipation planning horizon.

(*5*) Yield of harvesting in the aggregated business planning horizon is within limit based on average harvest level.

(*6*) Yield of harvesting in each period of anticipation planning horizon is within limit based on average harvest level.

Constraint sets (*3)* to (*6)* are typical forest management strategies regarding non-declining yield. They provide a series of near-equal harvest volumes (even flow) on an annual basis in business planning horizon and on a 5-year basis in anticipation planning horizon. They also guarantee that long-term harvest levels do not drop below a specified level.

(*7*) Total volume of the forest at the end of the entire planning horizon should be no less than the initial volume.

(*8*) Total volume of trees older than 20 years (the number of years included in the anticipation planning horizon) at the end of the entire planning horizon should be no less than the initial volume of trees older than 20 years.

(*9*) Total volume of trees within certain age range at the end of the entire planning horizon should be no less than the initial volume of trees within that age range. This age range corresponds to the age timber will be at the end of entire planning horizon if it is harvested in period *t* of anticipation planning horizon.

Constraint sets (*7)* to (*9)* are forest management strategies regarding sustainable production. They impose the forest to regain its original structure of age class at the end of the entire planning horizon. Specifically, constraint set (*8)* makes sure that no over-cutting for mid-term return occurs during the business planning horizon.

(*10)* Harvesting capacity in the forest for one period is limited.

(*11)* All harvested trees are bucked into logs in the same period.

(*12)* Flow balance of logs is guaranteed in the forest in the business planning horizon.

(*13)* Flow balance of logs is guaranteed in the forest in the anticipation planning horizon. No logs will be sent to mills.

(*14)* Only logs that are not transported to mills can be sold on the market.

(*15)* Variables are binary.

(*16)* Variables concerning harvest operations are non-negativity.

(*17)* Flow balance of logs transported to mills is guaranteed in the forest.

(*18)* Flow balance of logs transported from the forest is guaranteed in mills.

(*19)* Flow balance of byproducts transported from other mills is guaranteed in mills.

(*20)* Flow balance of products is guaranteed in mills.

(*21)* Production of final products is within min and max production capacities. If the min production capacity is not fulfilled, a penalty cost will be incurred.

(*22)* Volume of byproducts transported for further use cannot excess the available volume.

(*23)* Volume of final products is equal to the aggregate demand of different levels.

(*24)* Demand for final products of each level is within corresponding volume limit.

(*25)* Variables concerning productions are non-negativity.

4.3.3 Mathematical model for the integrated approach

For an integrated strategy, all parts of the supply chain, i.e., harvest areas, sawmills, pulp mills, heating plants and final markets, are driven by demands of final products over the business planning horizon and forest management is influenced by an estimation of the forest value over the remaining periods. In terms of modeling, it is a mixed integer programming (MIP) model that involves harvesting, bucking, transportation, production and sales decisions for both tactical (5 years) and strategic (25 years) planning levels. The objective is to maximize the total expected mid-term and long-term profit of the company.

Objective function in the integrated model [INT]:

$$z^{\mathrm{INT}} = \max \eta_{25}\left(\sum_{i\in I}\sum_{t\in T} p_{it}^{\mathrm{END}}\omega_{it} + \sum_{i\in I} p_i^{\mathrm{NH}}\omega_i^{\mathrm{NH}}\right) + \sum_{t\in T}\eta_t\left(\sum_{i\in I}\sum_{l\in L} p_{lt}^{\mathrm{L}} D_{ilt}^{\mathrm{L}} - \sum_{i\in I} c_{it}^{\mathrm{H}}\omega_{it}\right)$$
$$+\sum_{t\in T^1}\eta_t\left(\sum_{k\in K^{\mathrm{F}}}\sum_{m\in M} p_{kmt}^{\mathrm{K}} D_{kmt}^{\mathrm{K}} - \sum_{j\in J}\sum_{k\in K^{\mathrm{F}}} c_{jkt}^{\mathrm{P}} Y_{jkt}^{\mathrm{K}} - \sum_{i\in I}\sum_{j\in J}\sum_{l\in L} c_{ijlt}^{\mathrm{T}} X_{ijlt}^{\mathrm{L}} - \sum_{i\in J^{\mathrm{SAW}}\bigcup J^{\mathrm{PULP}}}\ \sum_{j\in J^{\mathrm{PULP}}\bigcup J^{\mathrm{HEAT}}}\ \sum_{k\in K^{\mathrm{B}}} c_{ijkt}^{\mathrm{T}} X_{ijkt}^{\mathrm{K}} - \sum_{j\in J} c_{jt}^{\mathrm{N}} W_{jt}^{\mathrm{N}}\right)$$

subject to constraint sets (1) to (25).

Terms in the first line are related to forest management and sales of logs for the entire planning horizon $t \in T$:

$\sum_{i\in I}\sum_{t\in T} p_{it}^{\mathrm{END}}\omega_{it}$	Ending value of harvest areas that are harvested in time period t
$\sum_{i\in I} p_i^{\mathrm{NH}}\omega_i^{\mathrm{NH}}$	Ending value of harvest areas that are never harvested
$\sum_{i\in I}\sum_{l\in L} p_{lt}^{\mathrm{L}} D_{ilt}^{\mathrm{L}}$	Sales revenue of logs in harvest areas in time period t
$\sum_{i\in I} c_{it}^{\mathrm{H}}\omega_{it}$	Harvesting cost in harvest areas in time period t

Terms in the second line are about transportation, production in mills and sales of final products for the business planning horizon $t \in T^1$:

$\sum_{k\in K^{\mathrm{F}}}\sum_{m\in M} p_{kmt}^{\mathrm{K}} D_{kmt}^{\mathrm{K}}$	Sales revenue of products in markets in time period t
$\sum_{j\in J}\sum_{k\in K^{\mathrm{F}}} c_{jkt}^{\mathrm{P}} Y_{jkt}^{\mathrm{K}}$	Production cost in mills in time period t
$\sum_{i\in I}\sum_{j\in J}\sum_{l\in L} c_{ijlt}^{\mathrm{T}} X_{ijlt}^{\mathrm{L}}$	Transportation cost of logs from forest to mills in time period t
$\sum_{i\in J^{\mathrm{SAW}}\bigcup J^{\mathrm{PULP}}}\ \sum_{j\in J^{\mathrm{PULP}}\bigcup J^{\mathrm{HEAT}}}\ \sum_{k\in K^{\mathrm{B}}} c_{ijkt}^{\mathrm{T}} X_{ijkt}^{\mathrm{K}}$	Transportation cost of byproducts between mills in time period t
$\sum_{j\in J} c_{jt}^{\mathrm{N}} W_{jt}^{\mathrm{N}}$	Penalty cost if min production capacity in mills is not fulfilled in time period t

Corresponding capital discount factors η_t are used to calculate the NPV of each term. Note that there are no internal prices in the integrated model [INT]. In the business planning horizon, logs are sent directly to mills and the excess of production can be sold on the market. In the anticipation planning horizon, logs are only sold on the markets. All are valued by sale prices.

4.3.4 Mathematical model for sequential approach S-A

In the sequential approach S-A, there are three separate but coherent stages. Stage 1 includes the harvesting and bucking decisions over the full planning period. We view this problem as a forest planning model [HARV]. In Stage 2, internal prices are introduced to generate logs for the first five years. We consider it as a bucking model [TRAN]. In the last stage, we fix the available volumes of logs and solve the planning problem for remaining downstream activities. It is treated as a transportation and production planning model [PROD]. The problem in one stage is constrained by the solutions obtained from the planning model in the previous stage. The total profit of the decoupled strategy is the sum of the NPV for revenues and costs in different models.

Stage 1: strategic level

The main purpose of Stage 1 is to make long-term harvesting decision for the entire planning horizon T. No transportation of logs from forest to mills are included. If the unit sale price for logs, i.e., the market price minus average transportation cost, is positive, all the logs produced by distinct bucking patterns will be sold on the market. In this profit-oriented model, no actual demand from mills is taken into account.

Objective function in the strategic forest planning model [HARV]:

$$z^{\mathrm{HARV}} = \max \eta_{25}\left(\sum_{i\in I}\sum_{t\in T} p_{it}^{\mathrm{END}}\omega_{it} + \sum_{i\in I} p_i^{\mathrm{NH}}\omega_i^{\mathrm{NH}}\right) + \sum_{t\in T}\eta_t\left(\sum_{i\in I}\sum_{l\in L} p_{lt}^{\mathrm{L}} D_{ilt}^{\mathrm{L}} - \sum_{i\in I} c_{it}^{\mathrm{H}}\omega_{it}\right)$$

subject to constraint sets (1) to (16).

As a result, the decision of which stands to harvest in each period for the business planning horizon is known and therefore the supply of trees in harvest areas in each year is specified by fixing variable ω_{it}, $t \in T^1$ in the next stage.

Stage 2: tactical level

Stage 2 only focuses on bucking decisions, based on internal prices, for the first 5-year business planning horizon T^1. The internal prices are used to manipulate the production of specific logs that corresponds to demand from mills. Compared to the bucking decisions made in Stage 1, the new bucking decisions during the business planning horizon can better reflect the needs of the mills if the internal prices are appropriately set. We use p_{lt}^{TRAN} to present the unit internal price for log type l in time period t, $t \in T^1$.

Objective function in the tactical bucking model [TRAN]:

$$z^{\mathrm{TRAN}} = \max \sum_{t\in T^1}\eta_t\left(\sum_{l\in I} p_{lt}^{\mathrm{TRAN}}\left(\sum_{i\in I}\sum_{r\in R}\sum_{u\in U^{\mathrm{BUCK}}}\alpha_{irult} Q_{irut}^{\mathrm{BUCK}}\right)\right)$$

subject to

$$s_{irt}\omega_{it} = \sum_{u\in U^{\mathrm{BUCK}}} Q_{irut}^{\mathrm{BUCK}} \quad \forall i \in I, \forall r \in R, \forall t \in T^1 \tag{11'}$$

Here, variables ω_{it} are fixed with the solution to strategic model [HARV] from Stage 1.

As a result, the volume and type of logs in harvest areas are decided by fixing Q_{irut}^{BUCK}, $t \in T^1$ in next stage.

Stage 3: tactical level

Decisions are shifting from forest to transportation, production and sales planning in mills for the business planning horizon T^1. This optimization problem is then solved from the perspective of mills. Note that the purchase cost of logs is still based on the sale price.

Objective function in the tactical production model [PROD]:

$$z^{\mathrm{PROD}} = \max \sum_{t\in T^1} \eta_t \Bigg(\sum_{k\in K^{\mathrm{F}}} \sum_{m\in M} p^{\mathrm{K}}_{kmt} D^{\mathrm{K}}_{kmt} - \sum_{j\in J} \sum_{k\in K^{\mathrm{F}}} c^{\mathrm{P}}_{jkt} Y^{\mathrm{K}}_{jkt} - \sum_{i\in I} \sum_{j\in J} \sum_{l\in L} c^{\mathrm{T}}_{ijlt} X^{\mathrm{L}}_{ijlt} - \sum_{i\in J^{\mathrm{SAW}} \bigcup J^{\mathrm{PULP}}} \sum_{j\in J^{\mathrm{PULP}} \bigcup J^{\mathrm{HEAT}}} \sum_{k\in K^{\mathrm{B}}} c^{\mathrm{T}}_{ijkt} X^{\mathrm{K}}_{ijkt} - \sum_{j\in J} c^{\mathrm{N}}_{jt} W^{\mathrm{N}}_{jt}$$
$$- \sum_{i\in I} \sum_{j\in J} \sum_{l\in L} p^{\mathrm{L}}_{lt} X^{\mathrm{L}}_{ijlt} \Bigg)$$

subject to

$$\sum_{r\in R} \sum_{u\in U^{\mathrm{BUCK}}} \alpha_{irult} Q^{\mathrm{BUCK}}_{irut} - W^{\mathrm{L}}_{ilt} = A^{\mathrm{L}}_{ilt}, \quad \forall i \in I, \forall l \in L, \forall t \in T^1 \qquad (12')$$

Here, variables Q^{BUCK}_{irut} are fixed with the solution to bucking model [TRAN] from Stage 2;

and constraint sets (*17*) *to* (*25*).

If there are any logs left in harvest areas W^{L}_{ilt}, they will be sold on the market if their sales values are positive. The revenue $\sum_{t\in T^1} \eta_t \left(\sum_{i\in I} \sum_{l\in L} p^{\mathrm{L}}_{lt} W^{\mathrm{L}}_{ilt} \right)$ is not considered in the production planning from the perspective of the mills, but is included in the calculation of the total profit for the entire supply chain.

After sequentially solving these three problems, we fix all the relevant variables and get the right costs and revenues based on the corresponding coefficients. The total profit of the supply chain under sequential approach S-A is given below.

$$z^{\mathrm{INT\text{-}SQ}} = \eta_{25} \left(\sum_{i\in I} \sum_{t\in T} p^{\mathrm{END}}_{it} \omega_{it} + \sum_{i\in I} p^{\mathrm{NH}}_{i} \omega^{\mathrm{NH}}_{i} \right) + \sum_{t\in T^1} \eta_t \left(\sum_{i\in I} \sum_{l\in L} p^{\mathrm{L}}_{lt} W^{\mathrm{L}}_{ilt} \right) + \sum_{t\in T^2} \eta_t \left(\sum_{i\in I} \sum_{l\in L} p^{\mathrm{L}}_{lt} D^{\mathrm{L}}_{ilt} \right) - \sum_{t\in T} \eta_t \left(\sum_{i\in I} c^{\mathrm{H}}_{it} \omega_{it} \right)$$
$$+ \sum_{t\in T^1} \eta_t \Bigg(\sum_{k\in K^{\mathrm{F}}} \sum_{m\in M} p^{\mathrm{K}}_{kmt} D^{\mathrm{K}}_{kmt} - \sum_{j\in J} \sum_{k\in K^{\mathrm{F}}} c^{\mathrm{P}}_{jkt} Y^{\mathrm{K}}_{jkt} - \sum_{i\in I} \sum_{j\in J} \sum_{l\in L} c^{\mathrm{T}}_{ijlt} X^{\mathrm{L}}_{ijlt} - \sum_{i\in J^{\mathrm{SAW}} \bigcup J^{\mathrm{PULP}}} \sum_{j\in J^{\mathrm{PULP}} \bigcup J^{\mathrm{HEAT}}} \sum_{k\in K^{\mathrm{B}}} c^{\mathrm{T}}_{ijkt} X^{\mathrm{K}}_{ijkt} - \sum_{j\in J} c^{\mathrm{N}}_{jt} W^{\mathrm{N}}_{jt} \Bigg)$$

If we compare the composition of objective function z^{INT} with that of $z^{\mathrm{INT\text{-}SQ}}$, the only difference is that the third term $\sum_{t\in T} \eta_t \left(\sum_{i\in I} \sum_{l\in L} p^{\mathrm{L}}_{lt} D^{\mathrm{L}}_{ilt} \right)$ in z^{INT} is separated to two parts $\sum_{t\in T^1} \eta_t \left(\sum_{i\in I} \sum_{l\in L} p^{\mathrm{L}}_{lt} W^{\mathrm{L}}_{ilt} \right)$ and $\sum_{t\in T^2} \eta_t \left(\sum_{i\in I} \sum_{l\in L} p^{\mathrm{L}}_{lt} D^{\mathrm{L}}_{ilt} \right)$ in $z^{\mathrm{INT\text{-}SQ}}$. The latter one together with other terms in the first line are derived from the solution to the strategic model [HARV] in Stage 1 while the former one along with the terms in the second line are based on the result of the tactical production model [PROD] in Stage 3. Internal pricing in Stage 2, as a coordination scheme, is used to manipulate the production of logs through different bucking patterns. It does not act as any cost or revenue terms in the final objective function.

4.3.5 Mathematical model for sequential approach S-B

The main difference between the two sequential approaches S-A and S-B is that the internal price is directly introduced in the first stage in S-B to determine the bucking decision for the business planning period. Therefore, there is no need to put an extra transition stage between the strategic and tactical planning, i.e., Stage 2 in S-A.

Stage 1': strategic level

The prices for logs are divided into two settings. Internal prices p_{lt}^{TRAN} are used in the first five years whereas sale prices p_{lt}^{L} are kept in the remaining 20 years. Therefore the revenue for logs is partitioned into two terms.

Objective function in the strategic harvesting model [HARV-TRAN]:

$$z^{\mathrm{HARV\text{-}TRAN}} = \max \eta_{25}\left(\sum_{i\in I}\sum_{t\in T} p_{it}^{\mathrm{END}}\omega_{it} + \sum_{i\in I} p_i^{\mathrm{NH}}\omega_i^{\mathrm{NH}}\right) + \sum_{t\in T^1}\eta_t\left(\sum_{i\in I}\sum_{l\in L} p_{lt}^{\mathrm{TRAN}} D_{ilt}^{\mathrm{L}}\right) + \sum_{t\in T^2}\eta_t\left(\sum_{i\in I}\sum_{l\in L} p_{lt}^{\mathrm{L}} D_{ilt}^{\mathrm{L}}\right) - \sum_{t\in T}\eta_t\left(\sum_{i\in I} c_{it}^{\mathrm{H}}\omega_{it}\right)$$

subject to constraint sets (1) to (16).

Consequently, the decisions concerning which areas to cut ω_{it} and which bucking pattern to choose Q_{irut}^{BUCK} over the business planning period T^1 are synchronously made in response to the requirements for raw materials in mills.

Stage 2': tactical level

We remove the transition stage and optimize the operations in the industrial plants subject to the availability of logs. Now the purchase cost of logs is based on internal prices.

Objective function in the tactical production model [PROD-TRAN]:

$$\begin{aligned} z^{\mathrm{PROD\text{-}TRAN}} = \max \sum_{t\in T^1}\eta_t\Bigg(& \sum_{k\in K^{\mathrm{F}}}\sum_{m\in M} p_{kmt}^{\mathrm{K}} D_{kmt}^{\mathrm{K}} - \sum_{j\in J}\sum_{k\in K^{\mathrm{F}}} c_{jkt}^{\mathrm{P}} Y_{jkt}^{\mathrm{K}} - \sum_{i\in I}\sum_{j\in J}\sum_{l\in L} c_{ijlt}^{\mathrm{T}} X_{ijlt}^{\mathrm{L}} - \sum_{i\in J^{\mathrm{SAW}}\cup J^{\mathrm{PULP}}}\ \sum_{j\in J^{\mathrm{PULP}}\cup J^{\mathrm{HEAT}}}\ \sum_{k\in K^{\mathrm{B}}} c_{ijkt}^{\mathrm{T}} X_{ijkt}^{\mathrm{K}} - \sum_{j\in J} c_{jt}^{\mathrm{N}} W_{jt}^{\mathrm{N}} \\ & - \sum_{i\in I}\sum_{j\in J}\sum_{l\in L} p_{lt}^{\mathrm{TRAN}} X_{ijlt}^{\mathrm{L}}\Bigg) \end{aligned}$$

subject to

$$\sum_{r\in R}\sum_{u\in U^{\mathrm{BUCK}}} \alpha_{irult} Q_{irut}^{\mathrm{BUCK}} - W_{ilt}^{\mathrm{L}} = A_{ilt}^{\mathrm{L}}, \quad \forall i\in I, \forall l\in L, \forall t\in T^1 \qquad (12')$$

Here, variables Q_{irut}^{BUCK} are fixed with the solution to model [HARV-TRAN] from Stage 1′;

and constraint sets (17) to (25).

When calculating the total profit, we sum up all the related terms that are the same in $z^{\mathrm{INT\text{-}SQ}}$. Similarly as S-A, the components in first line, except for $\sum_{t\in T^1}\eta_t\left(\sum_{i\in I}\sum_{l\in L} p_{lt}^{\mathrm{L}} W_{ilt}^{\mathrm{L}}\right)$, use the solution to model [HARV-TRAN] and the component in second line with $\sum_{t\in T^1}\eta_t\left(\sum_{i\in I}\sum_{l\in L} p_{lt}^{\mathrm{L}} W_{ilt}^{\mathrm{L}}\right)$ come from the result of model [PROD-TRAN].

4.4 Heuristics

The main purpose of this paper is to achieve coordination between the strategic forest management and tactical industrial production that corresponds to the enterprise's optimal performance. Improvement of the planning process made by sequential approach S-B can advance coordination. On the other hand, appropriate setting of coordination mechanism, i.e., internal pricing, is of great importance. In this section, we put forward two heuristics H-I and H-II to determine internal pricing. The internal pricing in H-I is based on dual values and in H-II from Lagrangian decomposition. Deferent from the manually estimated internal prices p_{lt}^{TRAN} adopted in above models, the internal prices obtained in these two heuristics indicate not only the log type but also the location, denoted by p_{ilt}^{TRAN}. That is, even for the same type of logs, the price may vary in different harvest areas. This can support the forest manager with a more detailed and comprehensive description of the logs values. Both new settings are applied to sequential approach S-B as an illustration.

4.4.1 Dual heuristic H-I

In the first heuristic, the internal prices for logs are originated from the dual values of constraint set (*12′*) in model [PROD-TRAN]. A dual value or shadow price is defined as the change in the optimal objective function value when the right-hand-side of the constraint is increased by one unit (Lundgren et al. 2010). If a constraint expresses the quantity balance of a commodity, then its dual value can be interpreted as the marginal value for producing one additional unit of that product. The constant θ expresses how much consideration is taken to the previous information when updating the internal prices. If the constant θ is set as $\theta = 1$, only dual values at current iteration will be considered and if $\theta = 0$, the solution obtained will be the same as the result of first iteration since internal prices remain unchanged. The main steps of the heuristic are summarized as follows.

Algorithmic description of H-I:

Step 0: Set $n = 0$, $\text{LBD}^{(0)} = -\infty$, $n_{\max}$, $\theta \in [0,1]$ and χ;
Choose initial internal prices for logs $p_{ilt}^{\text{TRAN}(1)}$.

Step 1: Solve model [HARV-TRAN];
Then fix Q_{irut}^{BUCK} for $t \in T^1$ in constraint set (*12′*) in model [PROD-TRAN] and solve the model;
Calculate $z^{\text{INT-SQ}}$ with the solutions to model [HARV-TRAN] and [PROD-TRAN];
Update $\text{LBD}^{(n)} = \max\left(z^{\text{INT-SQ}}, \text{LBD}^{(n-1)}\right)$.

Step 2: Check the convergence criterion. If $\text{LBD}^{(n)}$ is not improved during last χ iterations or $n = n_{\max}$, then stop. Return the solutions that generate the current LBD and the corresponding internal prices for logs $p_{ilt}^{\text{TRAN}(n')}$. Otherwise, go to Step 3.

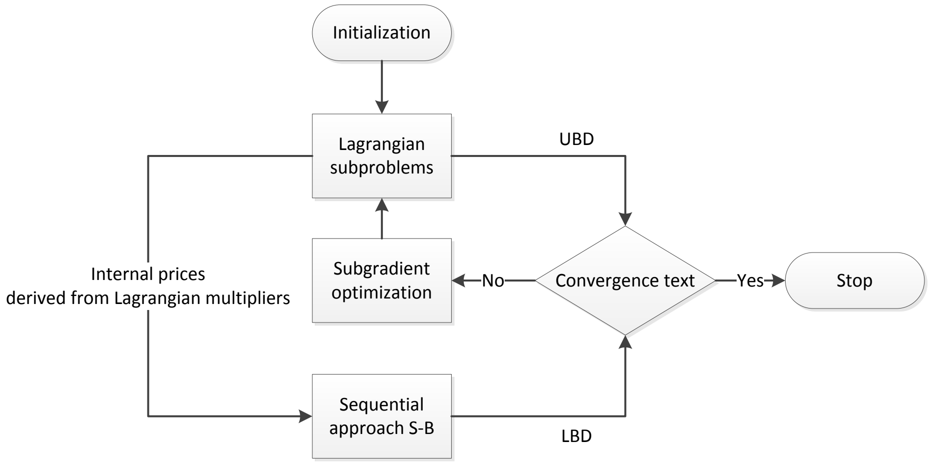
Step 3: Use the dual values of constraint set (*12′*) in model [PROD-TRAN], denoted by $(12').\text{dual}_{ilt}$, and update the internal prices as
$p_{ilt}^{\text{TRAN}(n+1)} = \theta \cdot (12').\text{dual}_{ilt} + (1-\theta) \cdot p_{ilt}^{\text{TRAN}(n)}$;
Set $n = n+1$ and go to Step 1.

This method is straightforward to understand and easy to employ. However, there is no upper bound to measure its quality. That is, if the benchmark z^{INT} is not known a priori or the integrated model does not exist, H-I cannot guarantee whether the LBD obtained is near or far from global optimum. This limitation for H-I will be obviated in H-II based on a more theoretical sound decomposition principle.

4.4.2 Lagrangian heuristic H-II

In the second heuristic, the internal prices take the optimal values of Lagrangian multipliers generated by LD and found by subgradient optimization. Figure 4.7 shows a diagram of the general steps. Lagrangian multipliers give the rate of change in the objective function with respect to the rate of change in the right-hand side of relaxed constraints. They may be interpreted as the (penalty) costs to the objective function paid for the violation. In other words, they represent the shadow prices on the coupling constraints. Therefore, we take related Lagrangian multipliers as internal prices for corresponding logs.

Figure 4.7 Illustration of general steps in Lagrangian heuristic H-II



The basic idea behind LD method is to reformulate the original problem into an equivalent one by duplicating a set of variables and adding a number of coupling constraints. These constraints are then relaxed such that the model decomposes into two subproblems, each having part of the constraints of the original problem. LD provides a systematic way of obtaining optimistic estimate for complex large-scale problems.

Consider the following general optimization problem [A]:

$$z^{\mathrm{A}} = \max cx \quad \text{s.t.}\, Ax = b,\, Bx = d,\, x \in X. \qquad \text{[A]}$$

The first step is to reformulate problem [A] by creating a copy of variable x, denoted by y, and add an equality constraint:

$$z^{\mathrm{EA}} = \max c_1 x + c_2 y \quad \text{s.t.}\, Ax = b,\, By = d,\, x = y,\, x \in X,\, y \in X, \qquad \text{[EA]}$$

where $c = c_1 + c_2$.

Then we relax the coupling constraint $x = y$ with Lagrangian multiplier λ and the problem becomes:

$$z^{\mathrm{DA}}(\lambda) = \max c_1 x + c_2 y + \lambda(x - y) \quad \text{s.t.}\, Ax = b,\, By = d,\, x \in X,\, y \in X, \tag{DA}$$

The resulting problem [DA] can be nicely separated to the following two models that can be solved independently:

$$z^{\mathrm{DA\text{-}1}}(\lambda) = \max\,(c_1 + \lambda)x \quad \text{s.t.}\, Ax = b,\, x \in X. \tag{DA-1}$$

$$z^{\mathrm{DA\text{-}2}}(\lambda) = \max\,(c_2 - \lambda)y \quad \text{s.t.}\, By = d,\, y \in X. \tag{DA-2}$$

It has been proved that $z^{\mathrm{DA}}(\lambda) = z^{\mathrm{DA\text{-}1}}(\lambda) + z^{\mathrm{DA\text{-}2}}(\lambda)$, where $z^{\mathrm{DA\text{-}1}}(\lambda)$ and $z^{\mathrm{DA\text{-}2}}(\lambda)$ are the optimal solutions to problem [DA-1] and [DA-2], respectively, is an optimistic bound of the optimal solution to the original problem [A] for any λ (Fisher 1981). In order to get the tightest bound, the value of λ should be adjusted such that:

$$z^{\mathrm{LD}} = \min_{\lambda} z^{\mathrm{DA}}(\lambda) \tag{LD}$$

This Lagrangian dual problem is a non-differentiable optimization problem. The well-known subgradient method (Poljak 1967) is typically employed to optimize $z^{\mathrm{DA}}(\lambda)$ over λ.

In what follows, the procedure for using the Lagrangian heuristic in this paper is discussed in detail.

4.4.2.1 Reformulation of the integrated model [INT]

One of the major questions to apply the LD approach is to identify which variables to duplicate and how to decompose constraints in the original problem. Because of the inherent planning structure, constraint set (*12)* is flow conservation for logs between forest and mills over the business planning horizon T^1. We hence duplicate all the variables that are involved in constraint set (*12)*. The new variables are:

$A^{\mathrm{L\text{-}b}}_{ilt}$	Volume of log type l sent to mills from harvest area i in time period t, $t \in T^1$
$W^{\mathrm{L\text{-}b}}_{ilt}$	Volume of log type l left in harvest area i in time period t, $t \in T^1$
$Q^{\mathrm{BUCK\text{-}b}}_{irut}$	Volume of tree type r using bucking pattern u in harvest area i in time period t, $u \in U^{\mathrm{BUCK}}, t \in T^1$

The coupling constraint sets will be added into model [INT] due to the duplication:

$$A^{\mathrm{L\text{-}b}}_{ilt} = A^{\mathrm{L}}_{ilt}, \quad \forall i \in I, \forall l \in L, \forall t \in T^1 \tag{26}$$

$$W^{\mathrm{L\text{-}b}}_{ilt} = W^{\mathrm{L}}_{ilt}, \quad \forall i \in I, \forall l \in L, \forall t \in T^1 \tag{27}$$

$$Q^{\mathrm{BUCK\text{-}b}}_{irut} = Q^{\mathrm{BUCK}}_{irut}, \quad \forall i \in I, \forall r \in R, \forall u \in U^{\mathrm{BUCK}}, \forall t \in T^1 \tag{28}$$

Then an identical constraint that is associated with constraint set (*12)* is created:

$$\sum_{r \in R} \sum_{u \in U^{\mathrm{BUCK}}} \alpha_{irult} Q^{\mathrm{BUCK\text{-}b}}_{irut} - W^{\mathrm{L\text{-}b}}_{ilt} = A^{\mathrm{L\text{-}b}}_{ilt}, \quad \forall i \in I, \forall l \in L, \forall t \in T^1 \tag{12b}$$

And the flow balance constraint set (*17)* is modified as below:

$$A^{\mathrm{L\text{-}b}}_{ilt} = \sum_{j \in J} X^{\mathrm{L}}_{ijlt}, \quad \forall i \in I, \forall l \in L, \forall t \in T^1 \tag{17b}$$

At last, several valid constraints are added:

$$D_{ilt}^{\mathrm{L}} = 0, \quad \forall i \in I, \forall l \in L, \forall t \in T^1 \tag{29}$$

$$\sum_{u \in U^{\mathrm{BUCK}}} Q_{irut}^{\mathrm{BUCK\text{-}b}} \le s_{irt}, \quad \forall i \in I, \forall r \in R, \forall t \in T^1 \tag{30}$$

$$A_{ilt}^{\mathrm{L\text{-}b}} \le \sum_{r \in R} \max_{u \in U^{\mathrm{BUCK}}} \left(\alpha_{irult}\right) s_{irt}, \quad \forall i \in I, \forall l \in L, \forall t \in T^1 \tag{31}$$

Since the downstream plants have enough capacities to handle all types of logs produced in the forest, Constraint set *(29)* coincides that no excess logs would be sold on the market in the business planning periods. Constraint sets (*30)* and (*31)* are to limit the sizes of the new variables. No matter whether to harvest, the bucked timber would never exceed the possible supply. Moreover, the availability of each log type is restricted by the maximum proportion it can be produced among all the bucking patterns. Constraint sets *(29)* to (*31)* are redundant in the original problem. They are introduced to strengthen the bounds when model [INT] is decomposed into two subproblems. Actually, these valid constraints in some sense recreate the decomposed structure, that is transfer upstream information to the downstream model and vice versa, but without destroying the separability of the subproblem (Holmberg and Yuan 2000).

4.4.2.2 Lagrangian decomposition

After relaxing the coupling constraint sets (*26)* to (*28)* with a set of Lagrangian multipliers λ_{ilt}, μ_{ilt} and γ_{irut}, respectively, we get the decomposed subproblems model [HAVR-LD] and model [PROD-LD]. Note that all the multipliers are not restricted in sign, because the relaxed constraint sets are equalities. These two models have the similar structures as model [HARV-TRAN] and model [PROD-TRAN] in sequential approach S-B. It implies that the current separate modeling setting with minor modification can be used to establish the coordination mechanism.

Objective function in model [HARV-LD]:

$$\begin{aligned} z^{\mathrm{HARV\text{-}LD}}(\lambda,\mu,\gamma) = \max\ & \eta_{25}\left(\sum_{i \in I}\sum_{t \in T} p_{it}^{\mathrm{END}}\omega_{it} + \sum_{i \in I} p_i^{\mathrm{NH}}\omega_i^{\mathrm{NH}}\right) + \sum_{t \in T}\eta_t\left(\sum_{i \in I}\sum_{l \in L} p_{lt}^{\mathrm{L}} D_{ilt}^{\mathrm{L}} - \sum_{i \in I} c_{it}^{\mathrm{H}}\omega_{it}\right) \\ & + \sum_{i \in I}\sum_{l \in L}\sum_{t \in T^1}\lambda_{ilt}A_{ilt}^{\mathrm{L}} + \sum_{i \in I}\sum_{l \in L}\sum_{t \in T^1}\mu_{ilt}W_{ilt}^{\mathrm{L}} + \sum_{i \in I}\sum_{r \in R}\sum_{u \in U^{\mathrm{BUCK}}}\sum_{t \in T^1}\gamma_{irut}Q_{irut}^{\mathrm{BUCK}} \end{aligned}$$

subject to constraint sets (1) to (16) and (29).

Objective function in model [PROD-LD]:

$$\begin{aligned} z^{\mathrm{PROD\text{-}LD}}(\lambda,\mu,\gamma) = \max \sum_{t \in T^1}\eta_t & \left(\sum_{k \in K^{\mathrm{F}}}\sum_{m \in M} p_{kmt}^{\mathrm{K}} D_{kmt}^{\mathrm{K}} - \sum_{j \in J}\sum_{k \in K^{\mathrm{F}}} c_{jkt}^{\mathrm{P}} Y_{jkt}^{\mathrm{K}} - \sum_{i \in I}\sum_{j \in J}\sum_{l \in L} c_{ijlt}^{\mathrm{T}} X_{ijlt}^{\mathrm{L}} - \sum_{i \in J^{\mathrm{SAW}} \cup J^{\mathrm{PULP}}}\ \sum_{j \in J^{\mathrm{PULP}} \cup J^{\mathrm{HEAT}}}\ \sum_{k \in K^{\mathrm{B}}} c_{ijkt}^{\mathrm{T}} X_{ijkt}^{\mathrm{K}} - \sum_{j \in J} c_{jt}^{\mathrm{N}} W_{jt}^{\mathrm{N}}\right) \\ & - \sum_{i \in I}\sum_{l \in L}\sum_{t \in T^1}\lambda_{ilt}A_{ilt}^{\mathrm{L\text{-}b}} - \sum_{i \in I}\sum_{l \in L}\sum_{t \in T^1}\mu_{ilt}W_{ilt}^{\mathrm{L\text{-}b}} - \sum_{i \in I}\sum_{r \in R}\sum_{u \in U^{\mathrm{BUCK}}}\sum_{t \in T^1}\gamma_{irut}Q_{irut}^{\mathrm{BUCK\text{-}b}} \end{aligned}$$

subject to constraint sets (12b), (17b), (18) to (25), (30) and (31).

The Lagrangian relaxed objective function now can be expressed as

$$z^{\text{INT-LD}}(\lambda,\mu,\gamma) = z^{\text{HARV-LD}}(\lambda,\mu,\gamma) + z^{\text{PROD-LD}}(\lambda,\mu,\gamma).$$

For any given λ, μ, and γ, we will get an optimistic estimate of the value of z^{INT}, i.e., $z^{\text{INT-LD}}(\lambda,\mu,\gamma) \geq z^{\text{INT}}$. In order to find the best optimistic estimate, we solve the following dual problem using subgradient optimization:

$$\min_{\lambda,\mu,\gamma} z^{\text{INT-LD}}(\lambda,\mu,\gamma).$$

4.4.2.3 Subgradient optimization

The procedure of the subgradient optimization is described as follows.

Algorithmic description of H-II:

Step 0: Initialize:

Set $n = 0$, $\text{LBD} = -\infty$, $\text{UBD} = +\infty$, ε, $n_{\max}$, $\sigma \in (0, 2]$, and χ;

Choose initial Lagrangian multipliers $\lambda_{ilt}^{(1)}$, $\mu_{ilt}^{(1)}$ and $\gamma_{inut}^{(1)}$.

Step 1: Solve the Lagrangian subproblems:

Solve model [HARV-LD] and model [PROD-LD] with given $\lambda_{ilt}^{(n)}$, $\mu_{ilt}^{(n)}$ and $\gamma_{inut}^{(n)}$;

Let $z^{\text{INT-LD}} = z^{\text{HARV-LD}} + z^{\text{PROD-LD}}$;

Update $\text{UBD} = \min\left(z^{\text{INT-LD}}, \text{UBD}\right)$.

Step 2: Find a feasible solution:

let $p_{ilt}^{\text{TRAN}} = \lambda_{ilt}^{(n)}$ and solve model [HARV-TRAN];

Then fix Q_{inut}^{BUCK} for $t \in T^1$ in constraint set (*12′*) in model [PROD-TRAN] and solve the model;

Calculate $z^{\text{INT-SQ}}$ with the solutions to model [HARV-TRAN] and [PROD-TRAN];

Update $\text{LBD} = \max\left(z^{\text{INT-SQ}}, \text{LBD}\right)$.

Step 3: Check the convergence criterion:

If $\dfrac{\text{UBD} - \text{LBD}}{\text{LBD}} \leq \varepsilon$ or $n = n_{\max}$, then stop. Return the feasible solutions that generate the current LBD and the corresponding internal prices for logs $\lambda_{ilt}^{(n')}$. Otherwise, go to Step 4.

Step 4: If UBD has not improved for χ iterations, update convergence parameter σ according to the updating rule and go to Step 5. Otherwise, go to Step 5.

Step 5: Use the values of variables from the solutions in Step 1 and compute the subgradient as

$$\Delta_{ilt}^{\lambda(n)} = A_{ilt}^{\text{L-b}} - A_{ilt}^{\text{L}},\ \Delta_{ilt}^{\mu(n)} = W_{ilt}^{\text{L-b}} - W_{ilt}^{\text{L}},\ \Delta_{inut}^{\gamma(n)} = Q_{inut}^{\text{BUCK-b}} - Q_{inut}^{\text{BUCK}}.$$

Step 6: Determine the step length as

$$\kappa^{\lambda(n)} = \frac{\sigma(\text{UBD} - \text{LBD})}{\left\|\Delta_{ilt}^{\lambda(n)}\right\|^2},\ \kappa^{\mu(n)} = \frac{\sigma(\text{UBD} - \text{LBD})}{\left\|\Delta_{ilt}^{\mu(n)}\right\|^2},\ \kappa^{\gamma(n)} = \frac{\sigma(\text{UBD} - \text{LBD})}{\left\|\Delta_{inut}^{\gamma(n)}\right\|^2}.$$

Step 7: Update the Lagrangian multipliers as

$$\lambda_{ilt}^{(n+1)} = \lambda_{ilt}^{(n)} + \kappa^{\lambda(n)}\Delta_{ilt}^{\lambda(n)},\ \mu_{ilt}^{(n+1)} = \mu_{ilt}^{(n)} + \kappa^{\mu(n)}\Delta_{ilt}^{\mu(n)},\ \gamma_{inut}^{(n+1)} = \gamma_{inut}^{(n)} + \kappa^{\gamma(n)}\Delta_{inut}^{\gamma(n)};$$

Set $n = n + 1$ and go to Step 1.

We have tested other modified formula to update the search direction suggested in Crowder (1976) and there is no significant improvement. Therefore, we set the search direction equal to the subgradient. The step length we adopt is widely used in a standard subgradient approach. Given a specific problem, the practical convergence rate relies heavily on the choice of the parameters contained in the subgradient search procedure, such as σ and χ . However, there are no simple rules to determine and justify these parameters (Holmberg and Yuan 2000). In our implementation, we choose $\sigma = 2$ as a start value. If no improvement of UBD in $\chi = 10$ successive iterations, we set $\sigma \leftarrow 0.9\sigma$. Meanwhile, we find that $\sigma = 0.5$ as initial value yields a good starting UBD but much worse convergence behavior. It complies to the observation of Goffin (1977) that for convergence to occur it is necessary to set the initial step size large enough.

Another issue in subgradient optimization is to choose the starting point for Lagrangian multipliers. We try two settings of initial multipliers in the computational study. The first alternative is to start with $\lambda_{ilt}^{(1)} = 0$, $\mu_{ilt}^{(1)} = 0$ and $\gamma_{inut}^{(1)} = 0$. In the second alternative, we replace $\lambda_{ilt}^{(1)} = 0$ with the sales value $\lambda_{ilt}^{(1)} = p_{lt}^{\mathrm{L}}$. As expected, the second one leads to a better convergence behavior.

Ideally, the procedure terminates when the subgradient is zero vector, which means that the dual optimum is found and that the solution is primal feasible. Since all the constraints we relaxed are equalities, the primal feasible solution is also optimal. However, it is unlikely to happen in most cases because the Lagrangian dual problem is solved approximately and a duality gap may exist. Therefore, in practice, we choose to stop the search procedure when the gap between the UBD and LBD is sufficiently small $\varepsilon = 0.01$ or after a quite limited number of iterations $n_{\max} = 200$.

Last but not least, the feasible solution in this algorithm is obtained by sequential approach S-B. In other words, we make use of the recursively updated Lagrangian multiplier $\lambda_{ilt}^{(n)}$ as the internal prices. Once one of the termination criteria is met, we get the coordinated result as well as the quality of solution, which is indicated by the gap. This is the strong advantage of H-II compared to H-I, although H-II is more complicated in terms of algorithm.

4.5 Case study

We divide this section into four parts. First, we outline the background of the case study based on a Chilean forest company. Then we compare three planning approaches through financial performance and solution time. Afterwards, we use the numerical results to illustrate the heuristics H-I and H-II, respectively. All the methods are programmed by the modeling language AMPL (version 20120217) and the MIP models are solved by the commercial solver CPLEX 12.2. We change the default setting of MIP-gap (0.01%) and CPU time limit (1e+75) in CPLEX according to different analytic emphasis. In the specific analysis below we will mention which setting is used. All runs were made on a T7300 2.00 GHz processor with 3 GB of RAM.

4.5.1 Case description

The firm, as one of the largest forest holding companies in Chile, owns several geographically separated forests. In this case study, we only consider the largest one which is composed of spatially located harvest areas, sawmills, pulp mill and heating plants. Figure 4.8 depicts the geographical distribution of the problem. The forest may contain a diverse mix of tree species. However, in our case study of a commercial plantation, one species, radiata pine, is planted and homogeneous harvest areas are distinguished by age. The total planning period is 25 years. As described in Section 4.2, the first 5 years is the business planning horizon. For each year, harvested timber is first bucked into logs on spot and then transported to downstream mills for further production in order to meet market demands. The next 20 years is the anticipation planning horizon during which only harvesting and bucking decisions are considered. Table 4.3 summaries the basic information of the case study.

Figure 4.8 Geographical distribution of mills and part of harvest areas in the case study

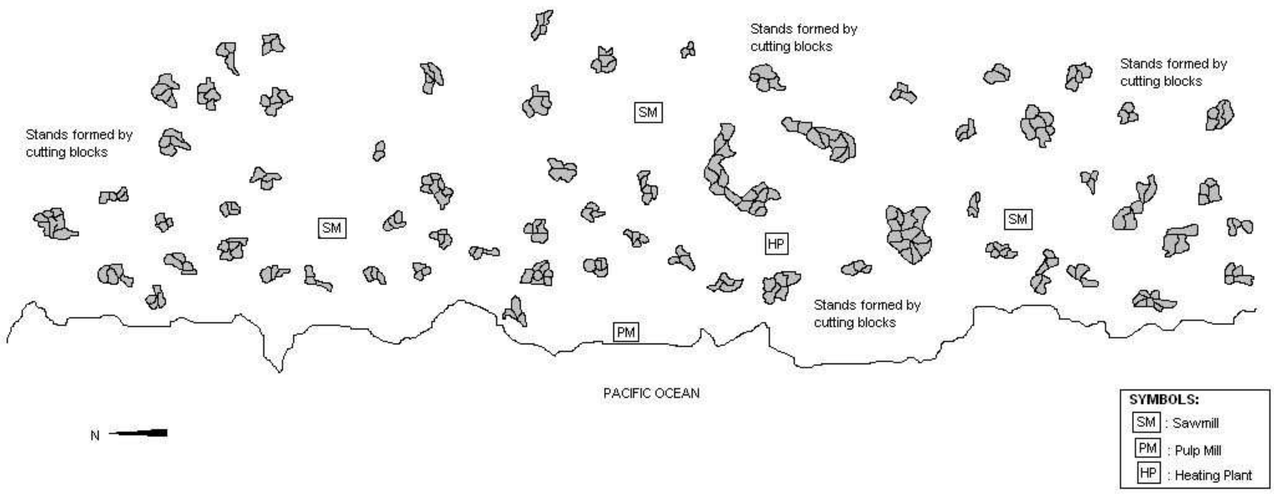


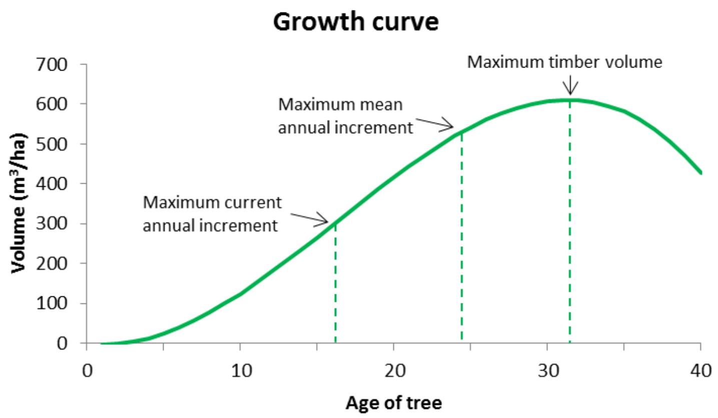
Table 4.3 Information for the case study

Number of	**Content**
Harvest areas	1 226
Bucking patterns	7
Tree types	1 (Radiata Pine)
Log types	7 (Sawlog Type1, Sawlog Type2, Sawlog Type3, Pruned Log, Long Log, Pulplog, Fuel Log)
Sawmills	3
Pulp mills	1
Heating plants	1
Processes	10 (Saw1, Saw2, Saw3, Saw4, Saw5, Saw6, Saw7, Pulp1, Sale1, Heat1)
Final products	7 (Lateral Board, Square Broad, Pruned Board, Long Board, Pruned Log, Pulp, Electricity)
Byproducts	3 (Chips, Sawdust, Bark)
Demand levels	3
Time periods in business planning horizon	5 (yearly)
Time periods in anticipation planning horizon	4 (aggregated 5-year)

In discussions with the firm, we obtained information on the current status of all harvest areas, growth curve for the trees, historical output data on bucking patterns, sawing patterns, pulp recipes and energy conversion rate, production capacities, estimated demand and economic parameters used in their own planning.

Growth-simulator model is used to estimate timber yields over the whole planning periods. The growth function is drawn from Vargas and Sandoval (1998), i.e., $V = -4.970533x + 2.188126x^2 - 0.044911x^3$, where volume V is expressed in cubic meter per hectare (m^3/ha) and x is the tree age. Figure 4.9 illustrates the growth curve. The growth rate increases over time to a maximum, then declines and eventually becomes zero when a unit of land reaches its maximum timber content and afterwards becomes negative due to tree decay resulting from old age and disease. According to the traditional forest management that could achieve the maximum sustained yield, the harvest rotation is around 24.36 years, which corresponds to the assumption in our paper (25 years).

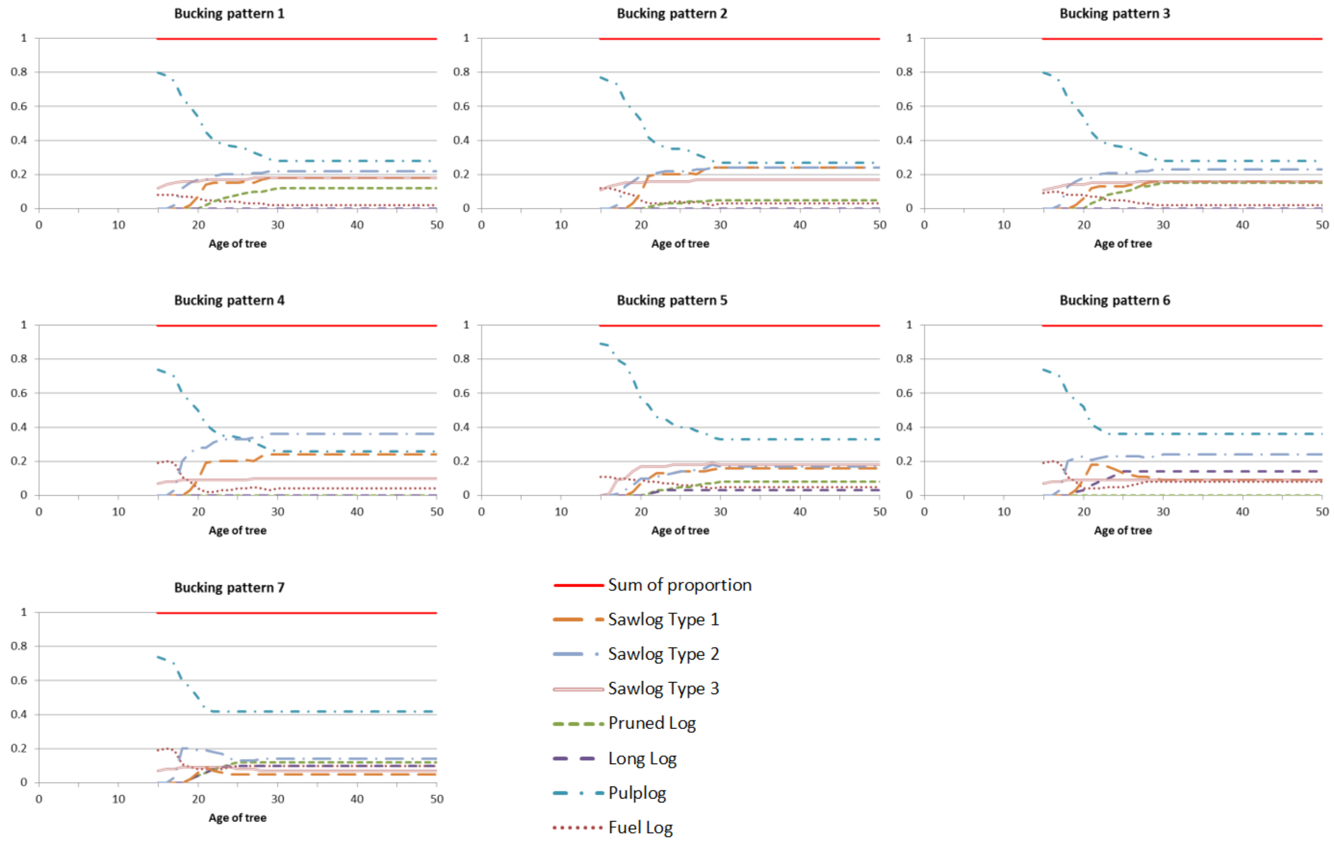
Figure 4.9 Volume of forest per hectare as a growth function of tree age



For the bucking process in the forest, seven bucking patterns define the transformation possibilities when a tree is bucked into different types of logs. The proportion relies on the age of trees, and hence we have different values for each year, from 15 to 50 years old (Figure 4.10). Note that trees cannot be harvested for commercial use until 15 years old. As can be seen from Figure 4.10, on shorter rotations trees can provide fiber suitable for pulp, while longer-term rotations are required if they are to provide fiber for lumber and other solid wood products. It also implies that the per unit volume value of trees increases with age, because the larger volume of logs with higher prices, such as sawlogs, is associated with older trees.

For the various manufacturing processes in downstream mills, there are seven sawing patterns which have different yields according to the used log type. Pulplog, chips and sawdust can be used for pulp production where 1 cubic meter yields 0.33 metric tons of Kraft cellulose. At the heating plant, one cubic meter of biomass yields 0.33 MWh of electricity. Table 4.5 lists the yields of products when applying the different processes at the mills.

Figure 4.10 Change of proportion of different log types with the increase of tree age under 7 bucking patterns



At last, the economic parameters are given. Harvesting and transportation costs are shown in Table 4.4. Sale prices and internal prices for logs are already presented in Table 4.2. A discount rate 8% is used to compute the NPV of revenues can costs.

Table 4.4 Harvesting and transportation costs used in the case study

Cost	**Value**
Harvesting cost (US$/m^3)	6.5
Transportation cost (US$/ m^3·km)	
From forest to mills (logs)	0.11
From mills to mills (byproducts)	0.09

Table 4.5 Yields of various outputs from one unit input under different processes at the mills

Mill	Process	Input	Output	Yield
Sawmill 1	Saw1	Pruned Log	Lateral Board	0.28832
			Square Board	0.26208
			Chips	0.14382
			Sawdust	0.15578
			Bark	0.15
Sawmill 1	Saw1	Sawlog Type 1	Lateral Board	0.26163
			Square Board	0.25502
			Chips	0.150822
			Sawdust	0.187528
			Bark	0.145
Sawmill 1	Saw6	Pruned Log	Pruned Board	0.53
			Chips	0.15
			Sawdust	0.25
			Bark	0.15
Sawmill 2	Saw1	Sawlog Type 2	Lateral Board	0.22704
			Square Board	0.24566
			Chips	0.160992
			Sawdust	0.226308
			Bark	0.14
Sawmill 2	Saw5	Sawlog Type 1	Lateral Board	0.23085
			Square Board	0.25675
			Chips	0.1539
			Sawdust	0.2135
			Bark	0.145
Sawmill 2	Saw6	Pruned Log	Pruned Board	0.53
			Chips	0.15
			Sawdust	0.25
			Bark	0.15
Sawmill 2	Saw7	Long Log	Long Board	0.45
			Chips	0.15
			Sawdust	0.25
			Bark	0.15
Sawmill 3	Saw5	Sawlog Type 2	Lateral Board	0.194016
			Square Board	0.282584
			Chips	0.160992
			Sawdust	0.222408
			Bark	0.14
Sawmill 3	Saw5	Sawlog Type 3	Lateral Board	0.179055
			Square Board	0.261745
			Chips	0.168156
			Sawdust	0.256044
			Bark	0.135
Sawmill 3	Saw7	Long Log	Long Board	0.45
			Chips	0.15
			Sawdust	0.25
			Bark	0.15
Pulp mill 1	Pulp1	Pulp Log	Pulp	0.33
			Bark	0.13
Pulp mill 1	Pulp1	Chips	Pulp	0.33
Pulp mill 1	Pulp1	Sawdust	Pulp	0.33
Pulp mill 1	Sale1	Pruned Log	Pruned Logs	1
Heating plant 1	Heat1	Fuel Log	Electricity	0.33
Heating plant 1	Heat1	Chips	Electricity	0.33
Heating plant 1	Heat1	Sawdust	Electricity	0.33
Heating plant 1	Heat1	Bark	Electricity	0.33

4.5.2 Numerical tests on different approaches

We first concentrate on the comparison between the planning methods. Table 4.6 presents the size of the models in the three approaches. Even after pre-solving, the MIP models are quite large. The degree of solvability of the problems, however, as will be discussed below, depends not just on the size of the models, but on the model structure and on the relative magnitude of prices assigned to the logs.

Table 4.6 Dimensions of the models in three approaches

Model	No. of continuous variables	No. of binary variables	No. of constraints
Integrated approach			
Model [INT]	249 468	8 054	107 470
Sequential approach S-A			
Model [HARV]	248 703	8 054	107 040
Model [TRAN]	42 819	6 117 (all fixed)	6 117
Model [PROD]	146 855 (42 819 fixed)	--	20 821
Sequential approach S-B			
Model [HARV-TRAN]	248 703	8 054	107 040
Model [PROD-TRAN]	146 855 (42 819 fixed)	--	20 821

To gain insight on whether internal pricing plays a significant role in coordination and how it influences the result, we consider four possible settings of internal prices used in two sequential approaches S-A and S-B:

1) Estimated internal price

It is the original internal price that is manually determined and currently used by the firm.

2) Sale price

It is the sales value for logs, reflecting that sale prices are directly used as coordination mechanism in the sequential approaches and no extra internal prices are needed.

3) Aggregated dual value

After solving model [INT] as a linear programming (LP) problem with all binary variables allowed to take any value between 0 and 1, we denote the dual values of constraint set *(12)* as $(12).\text{dual}_{ilt}$. Then the aggregated dual values are calculated by $\sum_{i\in I}\sum_{j\in J}(12).\text{dual}_{ilt}X^{\text{L}}_{ijlt}\Big/\sum_{i\in I}\sum_{j\in J}X^{\text{L}}_{ijlt}$, indicating the log type and period, the same as settings 1 and 2.

4) Dual value

The dual values of constraint set (*12)* obtained from the LP relaxation of model [INT] $(12).\text{dual}_{ilt}$ is directly used as internal prices, representing not only the log type and period but also the location.

Table 4.7 gives the results using the three planning approaches under various settings of internal prices. The stopping criterion is either a MIP-gap 1% or a maximum run time of an hour (3 600 seconds). For the integrated approach, the model can be solved within reasonable time and the MIP-gap of solution is less than 1%.

There is no considerable difference in the results acquired from sequential approach S-A under the four different settings of internal prices. It is caused by the fact that the decisions on where and when to harvest is already made in the first stage and the impact of internal prices on log production in the consecutive stage is hence limited. Note that the currently employed internal prices by the firm, although intentionally applied for better coordination, end up with the worst outcome.

Sequential approach S-B, under the first two settings, is not at an advantage over S-A. Nevertheless, S-B with dual value either aggregated or not prompts the near-optimal solution in less than 20 seconds. It is confirmed that the modification made in S-B from S-A are both effective and efficient. The reason why the solution time is remarkably shortened is that the variation of internal prices based on the dual values in log type, harvest area and periods breaks the symmetry inherent of variable coefficients and helps the branch-and-bound procedure (CPLEX) to reduce the extent of the search region. In contrast, given any more general internal prices such as settings 1 and 2, the algorithmic procedure has to explore and fathom symmetric reflections of various solutions during the search process and gets hopelessly stuck. This is the partial reason that integrated model can be solved much quicker, although the size is larger, than the MIP models in sequential approaches when general internal prices are used since the symmetry inherent in the integrated model is precluded.

In addition, after checking the cost and revenue components, we find that the increase in total profit for both the integrated approach and sequential approach S-B with dual value is generated by the increment in sales value of final products. That is, the production of logs and final products under these two situations is better coordinated with respect to the market demands. It implies that if the internal prices for logs are appropriately set and can reflect the demands in mills, the decoupled planning approach can also lead to centralized optimum.

Table 4.7 Computational results under different approaches with various settings of internal price

		Estimated internal price		**Sale price**		**Aggregated dual value**		**Dual value**	
	Integrated	**S-A**	**S-B**	**S-A**	**S-B**	**S-A**	**S-B**	**S-A**	**S-B**
Profit (million US$)	493.17	459.17	451.03	461.51	463.62	467.07	491.47	468.59	493.08
MIP-gap (%)	0.45	2.29	2.80	2.82	2.29	2.52	0.60	2.43	0.28
CPU time (second)	156	3 603	3 603	3 603	3 603	3 603	20	3 603	17
Revenue (million US$)									
Ending value of forest	22.32	21.68	21.71	21.63	21.68	21.71	22.28	21.66	22.39
Sales value of logs	140.40	144.27	144.90	144.07	144.27	143.92	140.41	143.90	140.07
Sales value of products	651.94	581.72	565.54	586.89	589.46	597.62	650.98	600.92	652.01
Cost (million US$)									
Harvesting cost	60.66	59.69	59.20	59.53	59.69	59.64	60.70	59.73	60.65
Production cost	220.62	190.75	185.32	193.40	193.87	198.55	220.93	199.89	220.58
Transportation cost	40.20	38.06	36.59	38.15	38.22	37.99	40.56	38.27	40.16
Forest volume (million m^3)									
Initial volume	8.40	8.40	8.40	8.40	8.40	8.40	8.40	8.40	8.40
Ending volume	8.62	8.40	8.41	8.40	8.40	8.41	8.62	8.40	8.64
Harvested volume	18.78	18.81	18.75	18.77	18.81	18.77	18.78	18.80	18.75

We further investigate how the result would be improved if the models in the integrated approach and sequential approach S-B are run for longer time. Table 4.8 shows the computational performance. If we run the integrated model for one hour, the feasible solution is less than 0.05% from the optimal objective function value. It is very close to the best result we can obtain before the computer runs out of memory (second column in Table 4.8). The solution to sequential approach S-B with dual value after one hour is trivially improved in comparison with the one solved in a couple of seconds, 494.19 (3 603 seconds) against 493.08 (17 seconds) million US$.

Table 4.8 Computational results under the integrated approach and sequential approach S-B

	Integrated	Integrated	S-B (Dual value)
Profit (million US$)	495.14	495.15	494.19
MIP-gap (%)	0.05	0.05	0.02
CPU time (second)	3 600	11 048	3 603

Finally, the internal prices based on dual values are illustrated in Figure 4.11, one indicating the fluctuation of log prices in different harvest areas for “Period 3” and the other displaying the changes during the business planning horizon. The forest manager can thereby get a more detailed and comprehensive idea of the values of logs. The internal prices for sawlogs and pulp log decrease with time, perhaps because of the fact that the production of most final products goes down as time goes by (Figure 4.12) and less demands for raw materials lead to the decline in prices. Consequently, influenced by fewer byproducts generated from wood-processing factories, more fuel logs are needed in the heating plant and hence the prices for fuel logs increase. In Figure 4.12, we also notice that the production of pulp almost reaches the maximum demand level whereas the generation of electricity is limited to the first demand level. Because the profit of pulp is quite high, all the available pulp logs and related byproducts will be transported to the pulp mill. By comparison, the margin of electricity is very low and only the sale price on the first level, the highest, can cover the production cost. Therefore, even though there is sufficient supply of fuel log in the forest, it will not be transported to heating plant for further use. This result complies with the real-world phenomena. That is, the use of biomass is sensitive to its cost.

Figure 4.11 Internal prices for logs determined by the dual values of constraint set *(12)* obtained from the LP relaxation of model [INT]

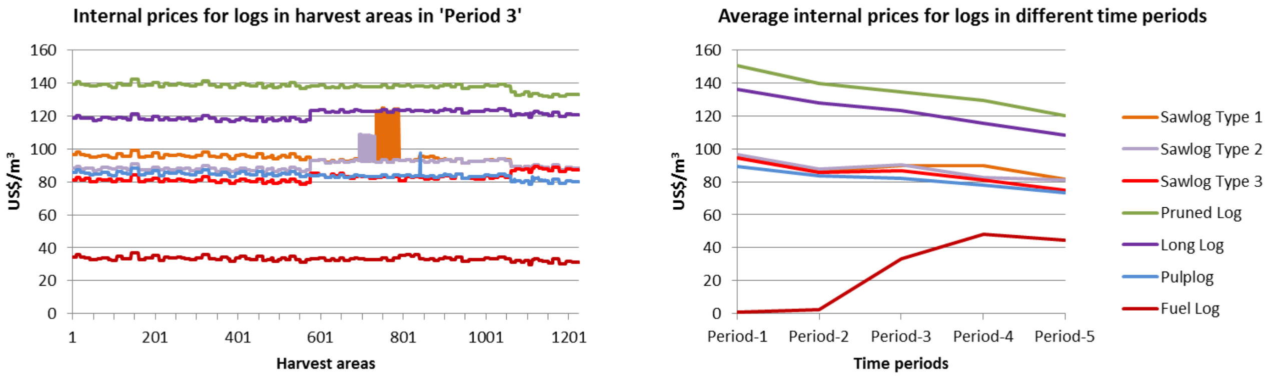
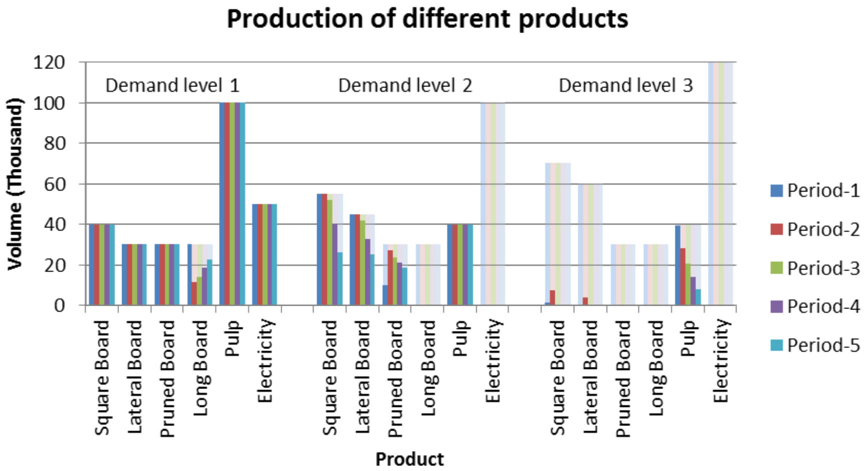


Figure 4.12 Production of final products during the business planning horizon



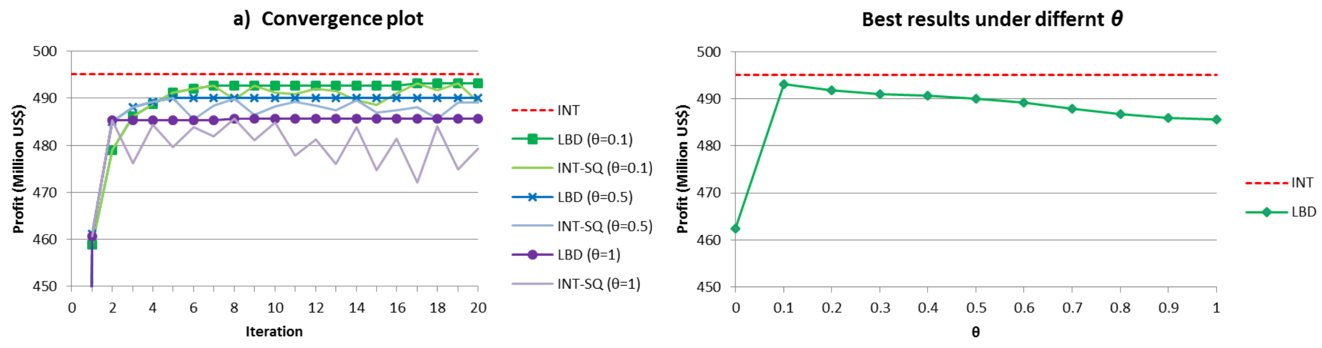
4.5.3 Numerical tests on dual heuristic H-I

In previous section, it is demonstrated that adequate setting of internal prices can result in effective coordination between upstream harvest and downstream production. However, the internal prices that work well in Section 4.5.2 are derived from the dual values of the integrated model. It means that in order to get these prices we have to change the traditional method and establish a new system. Instead, we would like to explore some heuristics that can find proper internal pricing under the current decoupled planning structure. Heuristics H-I and H-II are proposed for this purpose and tested in sequential approach S-B.

In dual heuristic H-I, the internal prices are the dual values of constraint set (*12′*) in model [PROD-TRAN]. We choose the sale prices as initial internal prices and set the constant θ from 0 to 1 with an interval of 0.l. In order to have a general comparison, we run each instance to $n_{\max} = 20$ iterations. From this point on, the tolerance of MIP-gap is 1% and the maximum processing time allowed for each model is 600 seconds.

Figure 4.13a illustrates the convergent plot when $\theta = 0.1$, $\theta = 0.5$, and $\theta = 1$, respectively and Figure 4.13b shows the best achievable results (LBD) under different choices of θ after 20 iterations. As can be seen, except for the extreme case $\theta = 0$ in which internal prices remain unchanged, with the increase of θ, less previous information is considered and worse result is received with greater volatility between each iteration. Nevertheless, all consecutive solutions obtained from iterations are superior to the first one, which refers to the procedure that S-B is just solved once.

Figure 4.13 Convergence plot for H-I and best achievable results under different θ



4.5.4 Numerical tests on Lagrangian heuristic H-II

In Lagrangian heuristic H-II, the internal prices are the optimal values of Lagrangian multipliers. As discussed in Section 4.4.2, H-II generates both UBD and LBD. The notation λ^{null} expresses the first alternative that all the multipliers are set to zero while λ^{sale} represents the second one that sales values are used as the starting point for λ_{ilt}.

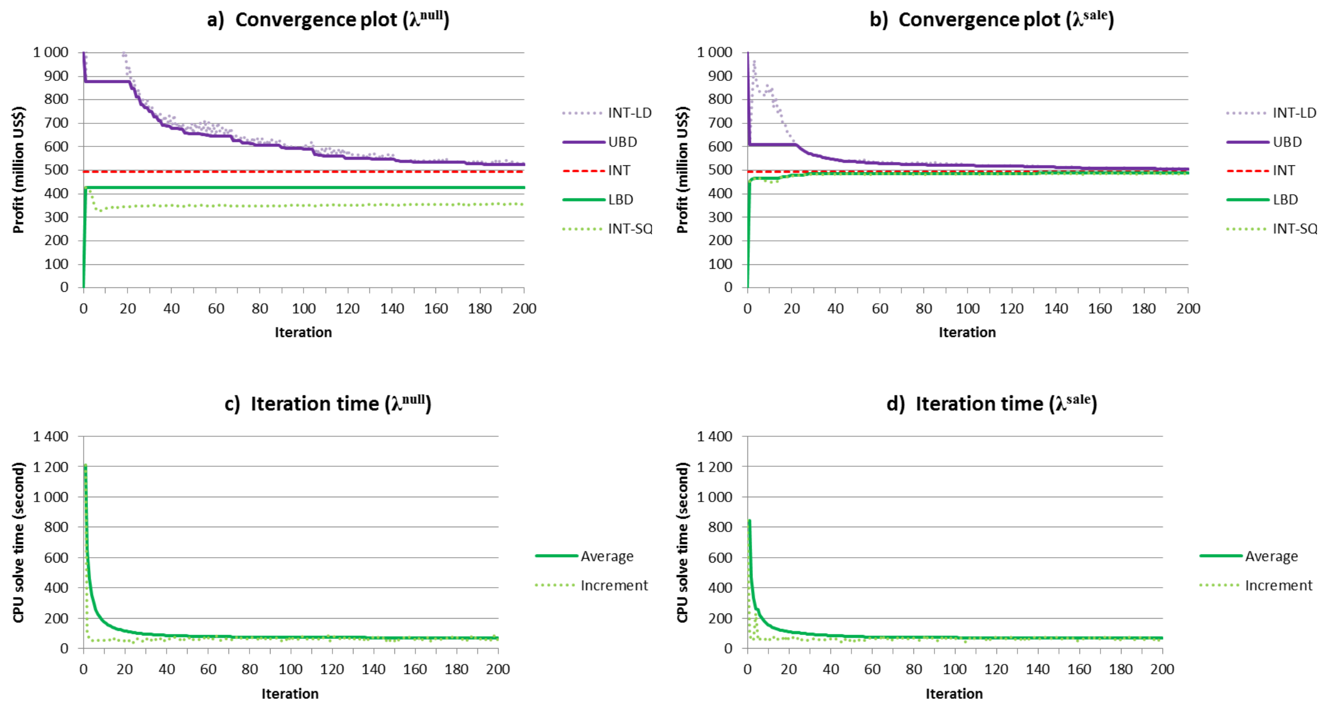
The computational results are presented in Table 4.9, where "Best dual objective" and "Best primal objective" correspond to the UBD and LBD up to the present iteration. Different from "MIP-gap" that is provided by CPLEX, "Gap" here is defined as $\dfrac{\text{UBD}^{(n)} - \text{LBD}^{(n)}}{\text{LBD}^{(n)}}$, which represents the convergence rate and can be used to measure the quality of the coordinated solution. Iteration time for both runs is depicted in Figure 4.14.

Table 4.9 Computational results under H-II with different initial Lagrangian multipliers λ

	λ^{null}	λ^{sale}
After 1 iteration		
Best dual objective	876.66	610.33
Best primal objective	428.27	454.66
Gap (%)	104.70	34.24
Average iteration time (CPU second)	1209	843
After 100 iterations		
Best dual objective	590.53	520.32
Best primal objective	428.27	486.93
Gap (%)	37.89	6.86
Average iteration time (CPU second)	73	72
After 200 iterations		
Best dual objective	524.10	506.14
Best primal objective	428.27	488.18
Gap (%)	22.38	3.68
Average iteration time (CPU second)	70	68

The choice of the starting point for Lagrangian multipliers has an evident effect on the convergence behavior. Starting with zero multipliers (λ^{null}), though UBD is gradually tightened, UBD keeps constant and feasible solution is very poor. In contrast, H-II with sales values as initial multipliers (λ^{sale}) can create near-optimal feasible solution, 488.18 million US$ with a gap of 3.68%. It is verified that by effectively utilizing information generated by the dual procedure, we can obtain good primal solutions (Holmberg and Yuan 2000). Again, the quick drop in solution time for both instances strongly attests to the assertion in Section 4.5.2 that once the symmetry inherent of variable coefficients are broken, the search process will be accelerated, irrespective of whether these internal prices can promote coordination or not.

Figure 4.14 Convergence plot and iteration time for H-II with different initial Lagrangian multipliers λ



4.6 Concluding remarks

In order to find optimal or near-optimal solutions for an entire supply chain, the common approach in theory is to integrate all the parts into one model. However, in practice different parts in a supply chain are often managed by independent companies or divisions within a company or organization. If partners are reluctant to share their data and to feed it into a central data-base while insisting on their own planning domain, modeling the supply chain from the centralistic view is no longer possible (Stadtler 2005). Therefore, it is important and necessary to establish practical coordination mechanisms to support and lead each independent business unit to plan and operate in ways that are best for the chain as a whole.

In the forestry industry where the planning horizon for forest management may cover decades, the planning is typically made in a decoupled structure with three planning problems. First, the long-term forest management model is solved and as a result it is decided when and where to harvest. Second, the available volumes of different log types for a mid-term period are generated by solving a bucking planning problem. Last, the industrial planner makes the tactical logistic and production planning with the availability of logs. A set of manually estimated internal prices (or transfer prices) is used to guide the production of logs in the bucking process. This top-down method to solve a strategic model that provides constraints for tactical problems may suboptimize the overall plan. This is supported by Troncoso et al. (2011) who asserted that the use of internal prices as coordination mechanism is not efficient to align the production of timber with the industry needs.

In this paper, however, we demonstrate that using a modified sequential approach and properly setting the internal prices, coordination between forest and industrial planners corresponding to the global optimum can be achieved in the current decoupled structure. In the improved sequential approach S-B, the forest management planning and bucking planning are integrated into one step. Two heuristics are presented to determine the internal prices. One is based on using straightforward dual values of flow conservation constraints. The other is derived from reformulating the model and employing Lagrangian multipliers in LD where some sets of variables are duplicated. The latter provides both lower and upper bounds of the optimal objective function value that can be used as a measure of the solution quality as well as a convergence criterion.

In the case study, we notice that the internal prices obtained from the proposed approaches contain both spatial and temporal information, which removes the symmetry inherent of variable coefficients and leads to significant improvement in solution time. This information also gives the forest manager a more detailed and comprehensive understanding of the values of logs in different areas and times..

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5

Coordination between production and sales planning at a refinery based on Lagrangian decomposition

Jiehong Kong and Mikael Rönnqvist

Abstract

In this paper, we study a coordination scheme between production and sales planning at a refinery. Decisions include procurement of crude oil, use of process modes, blend of components, and sale of final products in different markets. We first formulate two separate planning problems, i.e., production model [P] and sales model [S], both involving blending decisions. The production model minimizes the cost given the demand of products whereas the sales model maximizes the profit given the supply of components. These two models represent how the production and sales planning are made in practice. We then present an integrated model [I] that maximizes enterprise's overall profit. This joint model [I] establishes a theoretical benchmark for performance, but is generally not possible to solve due to complexity of real-world dimension. We propose two mechanisms using the notion of internal pricing together with volume constraints as a means to achieve cross-functional coordination between the production and sales divisions that corresponds to the system optimum. A computational study is implemented to illustrate the methodology. It is demonstrated that the first mechanism using marginal values as internal prices provides good pessimistic bounds, (i.e., feasible solution) on the optimal value defined by model [I]. The second one employing Lagrange decomposition generates both optimistic and pessimistic bounds, converging to the global optimal solution to model [I]. We also compare the convergent performance under different market demand scenarios. It is shown that both quadratic programming (QP) model with a linear demand function and mix integer programming (MIP) model with a step price function present good convergence properties by the second coordination mechanism using Lagrangian decomposition.

Key word: coordination, production planning, sales planning, internal price, Lagrangian decomposition, refinery[4]

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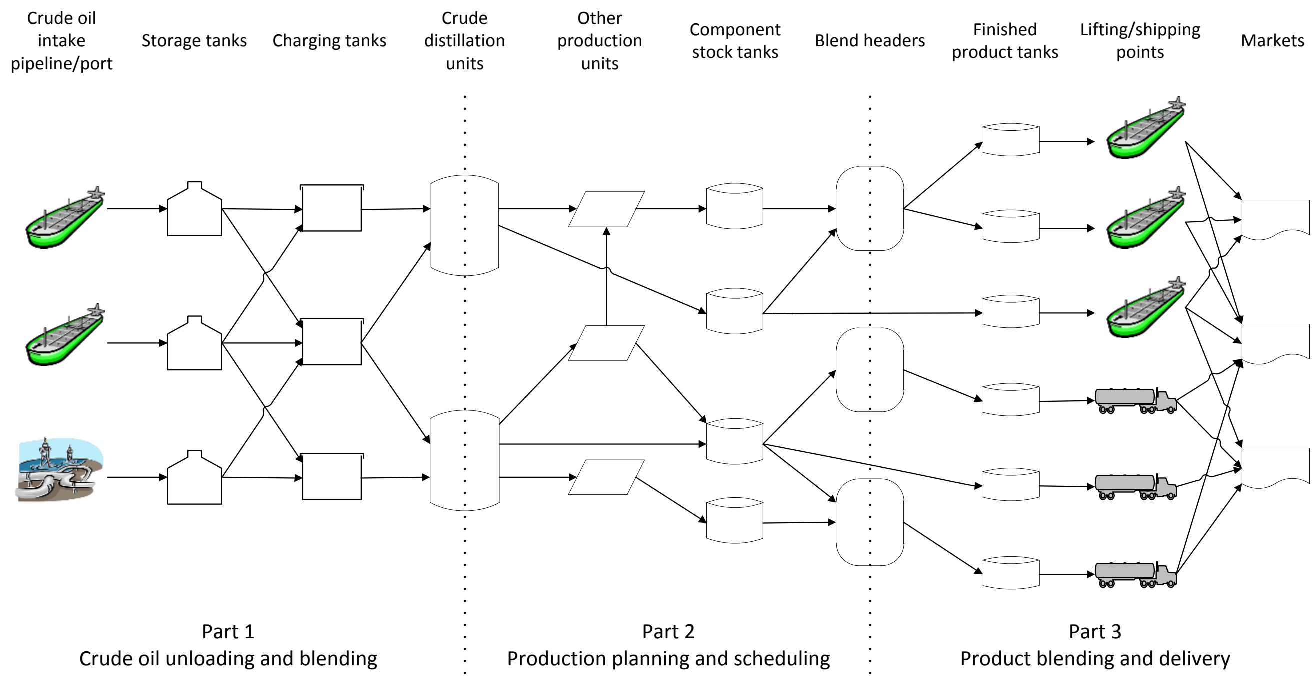
5.1 Introduction

An oil refinery system, as part of petroleum supply chain, stretches from procurement of crude oil to distribution and sales of petrochemical products, involving many complex processes with various connections. It is usually decomposed into three parts, i.e., crude oil unloading and mixing, production planning and scheduling, and product blending and delivery (Figure 5.1), and modeled and solved independently in a sequence.

Figure 5.1 Graphical overview of a standard refinery system



To drive operational efficiency, in the past few years the focus of major refining companies is shifting to managing supply chain activities as an integrated process to improve communication and overall performance (Bengtsson and Nonås 2010). Integrated planning hence becomes one of the main topics studied by recent literature. Neiro and Pinto (2004a) established an integrated model for a real-world petroleum supply chain that is composed of oilfield infrastructure, refineries, storage facilities and pipeline network for crude oil supply and product distribution. In Part 1 of the two-part paper, Pitty et al. (2008) first developed a dynamic simulator for the integrated refinery supply chain, which explicitly handles both external supply chain entities and intra-refinery activities. Then in Part 2 Koo et al. (2008) demonstrated the application of the decision support to optimize the refinery supply chain design and operation based on a simulation–optimization framework. Alabi and Castro (2009) presented a mathematical model of the refinery operations characterized by complete vertical integration of subsystems from crude oil procurement through to product distribution with a planning horizon ranging from 2 to 300 days.

An integrated modeling approach, it is argued, would achieve better functional cooperation between different planning problems and avoid suboptimal solutions and bottleneck along the entire refinery supply chain (Guyonnet et al. 2009). However, due to large size and complexity of real-world dimension, either simultaneous optimization across all the departmental divisions or temporal integration at different decision levels will make the planning and scheduling models significantly challenging and highly intractable (Shah et al. 2011). Therefore, decomposition, as fundamental technique in large-scale optimization, is the main concern of the ongoing research in refinery operations optimization and introduced to build subproblems that are mathematically solvable and computationally efficient. Zhang and Zhu (2000) decomposed the overall refinery model into a site level (master model) and a process level (sub-models). The site level optimization generates operating guidelines for each process while the process optimization returns updated yield performance to the site level for further re-evaluation. The procedure terminates when the product yields for each process from two consecutive iterations are close enough. Jia and Ierapetritou (2004) spatially decomposed the short-term scheduling for overall refinery operations into three domains: the crude oil unloading and blending, the production unit operations and the product blending and delivery. Each subproblem is modeled independently and solved efficiently, based on a continuous time formulation. Alabi and Castro (2009) applied Dantzig-Wolfe and block coordinate-descent decomposition approaches to the large integrated refinery planning problem and obtained either the optimal or an approximate feasible solution.

Among various decomposition techniques found in the literature, Lagrangean decomposition (LD) is one of the most popular methods applied for solving refinery operation models (Chen and Pinto 2008). LD is first proposed by Guignard and Kim (1987) as a generalization of conventional Lagrangian relaxation. By duplicating certain variables and relaxing the coupling constraints, the basic problem is divided into smaller subproblems that are more standardized with simpler structure. Thus, LD is usually used to obtain good optimistic bound to the original problem, i.e., upper bound (UBD) for maximization problem and lower bound (LBD) for minimization problem. Wu and Ierapetritou (2003) accomplished the integration of the three scheduling subproblems, proposed in Jia and Ierapetritou (2004), by establishing an iterative solution framework that exploits the LBD obtained through the heuristic-based decomposition approaches and the UBD based on LD and Lagrangian relaxation (LR). Neiro and Pinto (2004b) presented four strategies to solve the long-range multiperiod production planning of petroleum refineries that rely on decomposition techniques such as LD. Neiro and Pinto (2006) exploited the block-diagonal structure of the multiperiod refinery planning model under uncertainty and applied LD on a temporal basis where the planning horizon of T time periods is decomposed in T problems and solved independently.

As discussed above, most applications of LD to models in refinery supply chain optimization stress in significant improvements in computational efficiency. As far as we know, no attempt has been made to apply LD as a coordination scheme to find internal prices between production and sales planning at a refinery and to allow the decoupled system to perform like a centralized one.

On the other hand, although complete integration can achieve best performance, it makes sense in theory, but is not practical in reality because of inherent roles and responsibilities of different divisions. In practice refineries generally use commercial software packages to support various decisions in the supply chain matrix, which is structured into a number of different software modules. Each module is dedicated to solving a specific problem, such as production planning, distribution planning and sales forecast. Information on prices for components (used as an indicator of cost to blend products) and products (used as an indicator of value for production), quality requirements, and available volumes are therefore needed to facilitate coordination between the distinct software modules (Bredström and Rönnqvist 2008). However, the exchange information is usually estimated manually or depends on internal negotiation, which does not necessarily provide a system optimal solution from the perspective of supply chain optimization. Bengtsson and Nonås (2010) highlighted the importance of how to determine the proper values of intermediate products, which are commonly presented as a known value in the literature. The refinery operations is a divergent process as the number of components increases throughout the chain. Because not all of the component values can be found from an external market and different values can result in different optimal blends, special attention should be made to the selection of values in order to optimize the combined performance of the product blending and shipments.

The primary contribution of this paper is emphasized on the effect of a coordination scheme between production and sales planning at a refinery, i.e., the second part and the third part in Figure 5.1. We propose two mechanisms using the notion of internal prices together with volume constraints as a means to achieve cross-functional coordination that corresponds to the system optimum. Internal prices are based on traditional marginal values in Mechanism I but derived from LD in Mechanism II. We demonstrate that the first mechanism provides good LBD from feasible solutions on the optimal objective function value defined by the integrated model that establishes a theoretical benchmark for overall performance. The second mechanism generates both UBD of the optimal objective function value and LBD, converging to the global optimal solution. Furthermore, we provide an in-depth study of the system performance under four distinct market scenarios that represent some common demand behaviors. It is illustrated that both quadratic programming (QP) model with a linear demand function and mix integer programming (MIP) model with a step price function present good convergence properties by Mechanism II using LD. Because a multiperiod model captures the cumulative impact of coordination and demand evolution and hence makes the comparison of decoupled and integrated systems intricate (Celikbas et al. 1999), we restrict out attention to single period models.

The remainder of the paper is organized as follows. In Section 5.2, we give a literature review to examine the coordination mechanisms in the context of sales-production interface in a generic company. In Section 5.3, the decoupled production and sales planning models are formulated, followed by the conceptual integrated model. Two coordination mechanisms are separately described in Section 5.4 and 5.5. In Section 5.6, we make computational experiments to analyze the impact of parameter choices and compare the convergent performance under various market demand scenarios. Concluding remarks and potential directions for further research are presented in Section 5.7.

5.2 Literature review

Sales and production are two core functional divisions in a firm whose decisions greatly impact the overall financial performance, operational efficiency, and service level. In the literature, the sales-production interface is also terminologically defined as sales-manufacturing (Hu et al. 2011), sales-operations (Feng et al. 2008), marketing-operations (Erickson 2012) or marketing-manufacturing (Porteus and Whang 1991). See Eliashberg and Steinberg (1993) for an insightful review of the earlier literature on analytical models in the area of marketing-manufacturing interface, and Tang (2010) for a recent discussion about quantitative models based on different combinations of marketing and operations factors.

Generally speaking, sales specifies "what" kind of products to offer in "which" location at "what" price, which can be viewed as the "demand" in an internal market, while production examines "how" to deliver this demand by utilizing internal or external resources, treated as "supply". Traditionally, the planning of these two functions is made separately with different objectives. Sales division typically concentrates on revenue generation whereas production is responsible for cost reduction. The decoupled planning often results in suboptimal decisions with little emphasis on the profitability of the entire organization. Therefore, a coordinated plan is developed through an iterative negotiation process among these two functional groups. If the coordination scheme is appropriately set and there is no private information, sales and production with divergent interests will be induced to act in a globally optimal fashion (Karabuk and Wu 2002).

Coordination mechanisms within a firm usually include: 1) accounting-based schemes. Celikbas et al. (1999) developed coordination mechanisms through penalty schemes where marketing is penalized for over-forecasting and manufacturing is penalized for under-supply and compared the performance of centralized and decoupled systems under a stochastic setting. Pekgun et al. (2008) showed that a transfer price contract with bonus payments can motivate the decoupled marketing and production departments in a Make-To-Order firm to match the centralized solution; 2) improved contract design. Chen (2005) designed a menu of linear contracts as incentive schemes to salesman whose private information about market condition is important for the firm's production and inventory planning decisions. By observing which contract the salesman chooses, the firm can attain the knowledge about the market and make proper production decisions to maximize its expected profit; 3) decision making hierarchies. Li and Atkins (2002) studied coordination issues in a firm where replenishment and pricing decisions are made by production and marketing, respectively, in a decoupled fashion and found that having marketing as the leader in a Stackelberg framework can lead to improved performance for both production and marketing, and thus the firm as a whole; 4) internal markets. Kouvelis and Lariviere (2000) placed an internal market between the manufacturing and marketing managers where the transfer prices for the intermediate output from one function can differ when it is sold to another. This incentive scheme allows the system to be successfully decoupled. In what follows, we focus on transfer pricing as a prominent application for internal coordination.

Pricing has long been recognized as a significant tool used in the industrial operations to manipulate demand and to regulate the production and distribution of products (Soon 2011). Transfer pricing, also known as internal prices, is utilized as a communication device between participants in an internal market in order to arrive at the system optimal allocation of resources.

Theoretically, the main stream of research on coordination mechanism with transfer pricing under a decoupled system takes its cue from the economics literature. Game theory is normally implemented to analyze real situations where multiple agents within a firm are involved in a decision process and their actions are inter-related. Hu et al. (2011) introduced an internal price charged by manufacturing department to the sales department to balance the cost pressure of lead-time hedging amount. A Nash game model and a Stackelberg game model using the internal price can increase the firm's overall profit as compared to the traditional model without coordination. Erickson (2012) included a constant transfer price in a differential game model that allows the coordination of equilibrium marketing and production strategies to achieve a maximum profit for the firm.

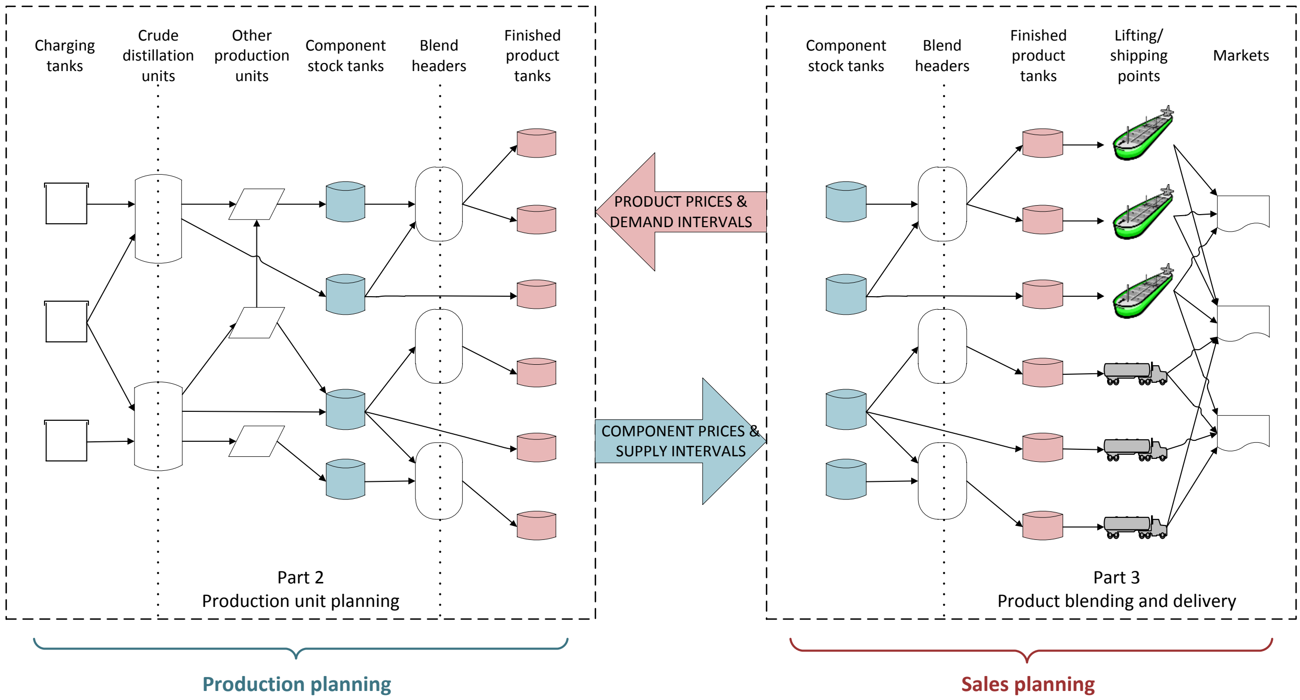
Pfeiffer (1999) summarized two common approaches to derive transfer pricing system. One is economic approach that uses methods of marginal analysis to determine values of intermediate commodities. The other is mathematical programming approach that is based on the dual Lagrangian principle. In the internal market constructed by Kouvelis and Lariviere (2000), the prices a market maker pays when buying an output and charges when selling it are related to the shadow price of the output's availability constraint. Erickson (2012) pointed out that when there is no market for the transferred product, or when the market is imperfectly competitive, the correct transfer price procedure is to transfer at marginal cost. If there is a competitive market for the transferred product, the appropriate transfer price is the market price. Karabuk and Wu (2002) presented two coordination mechanisms for decoupled semiconductor capacity planning by finding a form of transfer pricing making use of an augmented Lagrangian approach.

As for the refinery operations, internal prices are commonly used for coordination. Guajardo et al. (2012) studied how to coordinate production with sales decisions in a refinery supply chain. The transfer pricing in their model is manually preset by the company to reflect all the costs to produce a product before its shipment to the market. Li et al. (2003) proposed an analytical method called "Marginal Value Analysis (MVA)" to price intermediate products and Li and Hui (2007) extended MVA along with sensitivity analysis and parametric programming to trace the change of marginal values that indicate product values in a multiperiod refinery planning model. In this paper, the internal prices in Mechanism I are obtained from marginal values of intermediate materials. Then in Mechanism II the internal prices are defined by Lagrangian dual variables originating from LD.

5.3 Mathematical formulation

Recall that although the three refinery stages in Figure 5.1 are separate, there is a need for information exchange to keep the mass balance consistent in those connecting units between different parts. In this paper, we concentrate on the coordination between the second part and the third part (Figure 5.2). Along with a concise description of the refinery operations, we first define two decoupled planning problems, i.e. the production model [P] and the sales model [S], both involving blending decisions. The overstepping of the blending not only gives the production manager more freedom to plan the activities but also helps the sales trader find out how costly to blend one product. After that, we establish the integrated model [I] that centrally coordinates production and sales decisions, including procurement of crude oil, use of process modes, blend of components, and sale of final products in different markets. As mentioned in Section 5.1, in practice due to computational complexity or organizational structure, such a joint model is generally not possible to solve. However, we can use the optimal solution of this conceptual model as the theoretical goal for coordination in a relatively simple case.

Figure 5.2 Information coordination between production and sales planning



In the paper, *component* refers to the output, intermediate or semi-finished product from process units; *product* means finished commodity that is saleable in the markets; *direct product* represents a component that can either be sold as product or used as ingredient to blend other products; *blending (BLD)* is a pseudo process where components after all the production units are ready for final blending.

To focus on the coordination issues for cross-functional planning, we assume that the plant cannot import or export components from other refineries and no additional chemical additives are introduced into resultant blend. The components after all processes are collected in the stock tanks. Inventory holding cost occurs if the components are left after blending. Moreover, the refinery process system produces liquefied petroleum gas, gasoline, etc., as well as some byproducts, such as fuel gas, which cannot in turn be used in the blending of any products. However, the plant can burn fuel gas for its thermal energy (Zhang and Hua 2007). We separate this type of components from others by giving a specific set. A value is placed on this internally consumed energy.

The required quality attribute of a product is managed by the accurate blending of different components produced in the refining processes. Typical quality specifications relate to the octane number, vapor pressure, density, and sulfur content. In this paper, various blending recipes for one certain product are predefined, according to which the mixed product will meet quality standards established by customers and environmental regulations. The assumption of a constant production recipe is made to keep the mass balance constraint linear (Jia and Ierapetritou 2003).

Sets, parameters and variables used throughout the paper are listed as follows.

Indexes and sets:

$u \in U^{\text{CDU}}$	Set of Crude distillation units (CDU)
$u \in U^{\text{PRC}}$	Set of processes (excluding BLD)
$u \in \underline{G}_u$	Set of processes (including CDUs) directly preceding process u (including BLD)
$u \in \overline{G}_u$	Set of processes (including BLD) directly following process u (including CDUs)
$o \in O$	Set of crude oil
$c \in C^{\text{CDU}}$	Set of components from CDUs
$c \in C_u^{\text{PRC}}$	Set of components in process u
$c \in C^{\text{INT}}$	Set of components that can only be used as internal thermal energy
$c \in C^{\text{BLD}}$	Set of components that can be blended to products
$k \in K$	Set of products
$m \in M$	Set of markets
$r \in R_u^{\text{PRC}}$	Set of possible modes at process u
$r \in R_k^{\text{BLD}}$	Set of possible recipes to blend product k

Parameters:

c_o^{OIL}	Unit purchase cost of crude oil o
c_{ou}^{CDU}	Unit operating cost to use crude oil o at CDU u, $u \in U^{\text{CDU}}$
c_{ur}^{PRC}	Unit operating cost to use mode r at process u, $u \in U^{\text{PRC}}, r \in R_u^{\text{PRC}}$
c_{kr}^{BLD}	Unit operating cost to use recipe r to blend product k, $r \in R_k^{\text{BLD}}$
c_c^{H}	Unit inventory holding cost of component c, $c \in C^{\text{BLD}}$
c_{km}^{T}	Unit transportation cost of product k to market m
p_c	Unit value of component c used as internal thermal energy, $c \in C^{\text{INT}}$
$\underline{f}_{ou}^{\text{CDU}}$	Input of crude oil o at CDU u, $u \in U^{\text{CDU}}$
$\overline{f}_{ouc}^{\text{CDU}}$	Output of component c when using crude oil o at CDU u, $u \in U^{\text{CDU}}, c \in C^{CDU}$
f_{ucr}^{PRC}	Input (–) or output (+) of component c when using mode r at process u, $u \in U^{\text{PRC}}, c \in C_u^{\text{PRC}}, r \in R_u^{\text{PRC}}$
$\underline{f}_{kcr}^{\text{BLD}}$	Input of component c to blend product k when using recipe r, $c \in C^{\text{BLD}}, r \in R_k^{\text{BLD}}$
$\overline{f}_{kr}^{\text{BLD}}$	Output of product k when using recipe r, $r \in R_k^{\text{BLD}}$
$\bar{s}_o$	Max supply of crude oil o

h $\underline{l}_u^{\mathrm{CDU}}, \bar{l}_u^{\mathrm{CDU}}$ Min and max process volume at CDU u, $u \in U^{\mathrm{CDU}}$

$\underline{l}_u^{\mathrm{PRC}}, \bar{l}_u^{\mathrm{PRC}}$ Min and max process volume at process u, $u \in U^{\mathrm{PRC}}$

$\underline{l}_c^{\mathrm{BLD}}, \bar{l}_c^{\mathrm{BLD}}$ Min and max storage volume for each component c, $c \in C^{\mathrm{BLD}}$

$\underline{d}_{km}, \bar{d}_{km}$ Min and max demand of product k in market m

a_{km}, b_{km} Polynomial coefficients to determine the unit value of product k in market m

Variables:

V_o Volume of crude oil o purchased

V_{ou}^{CDU} Volume of crude oil o used at CDU u, $u \in U^{\mathrm{CDU}}$

V_{ur}^{PRC} Volume of mode r used at process u, $u \in U^{\mathrm{PRC}}, r \in R_u^{\mathrm{PRC}}$

$W_{uu'c}$ Volume of component c from process u to process u',

$u \in U^{\mathrm{CDU}} \cup U^{\mathrm{PRC}}, u' \in U^{\mathrm{PRC}} \cup \{\text{"BLD"}\},$

$c \in C^{\mathrm{CDU}} \cup C_u^{\mathrm{PRC}} \cup C^{\mathrm{INT}} \cup C^{\mathrm{BLD}}$

X_c Volume of component c after all the production units, $c \in C^{\mathrm{INT}} \cup C^{\mathrm{BLD}}$

V_{kr}^{BLD} Volume of recipe r used to blend product k, $r \in R_k^{\mathrm{BLD}}$

H_c Volume of component c left as inventory, $c \in C^{\mathrm{BLD}}$

Y_k Volume of product k for sales

Q_{km} Volume of product k transported to market m

P_{km} Unit value of product k in market m

Note that, for clarity, the superscript of all the variables will be indicated by the abbreviation of the models in the following formulations.

5.3.1 The decoupled setting – Production model [P]

Under a decoupled setting, production and sales planning are performed separately. Each problem has its own objective function seeking for local optimality. As illustrated in Figure 5.2, the first decision for production planning is to determine how much and what type of crude oil to use through CDUs where different fractions are generated. Only a few of the outputs can be directly used as components for blending products, while most need to be further processed in other production units, such as the cracker or reformer, to improve or change the quality. Thereafter follows decisions on the choice of blending recipes, in terms of both type and quantity, to get final products up to required demand. To avoid a suboptimal solution, necessary information between production and sales divisions should be transferred. Given the estimated value (internal prices for products) and demand interval for each product, the production model [P] decides the volumes of crude oil charged, process modes used, components produced and products blended.

Extra parameters used in the production model [P]:

p_k^{P}	Unit internal price for product k
$\underline{d}_k^{\mathrm{P}}, \overline{d}_k^{\mathrm{P}}$	Min and max demand of product k

Objective function in the production model [P]:

$$z^{\mathrm{P}} = \max \sum_{k\in K} p_k^{\mathrm{P}} Y_k^{\mathrm{P}} + \sum_{c\in C^{\mathrm{INT}}} p_c X_c^{\mathrm{P}}$$
$$- \sum_{o\in O} c_o^{\mathrm{OIL}} V_o^{\mathrm{P}} - \sum_{o\in O} \sum_{u\in U^{\mathrm{CDU}}} c_{ou}^{\mathrm{CDU}} V_{ou}^{\mathrm{CDU\text{-}P}} - \sum_{u\in U^{\mathrm{PRC}}} \sum_{r\in R_u^{PRC}} c_{ur}^{\mathrm{PRC}} V_{ur}^{\mathrm{PRC\text{-}P}} - \sum_{k\in K} \sum_{r\in R_k^{\mathrm{BLD}}} c_{kr}^{\mathrm{BLD}} V_{kr}^{\mathrm{BLD\text{-}P}} - \sum_{c\in C^{\mathrm{BLD}}} c_c^{\mathrm{H}} H_c^{\mathrm{P}}$$

Constraints concerning the production:

$$V_o^{\mathrm{P}} \le \overline{s}_o, \quad \forall o \in O \qquad (P1)$$

$$V_o^{\mathrm{P}} = \sum_{u\in U^{\mathrm{CDU}}} \underline{f}_{ou}^{\mathrm{CDU}} V_{ou}^{\mathrm{CDU\text{-}P}}, \quad \forall o \in O \qquad (P2)$$

$$\underline{l}_u^{\mathrm{CDU}} \le \sum_{o\in O} V_{ou}^{\mathrm{CDU\text{-}P}} \le \overline{l}_u^{\mathrm{CDU}}, \quad \forall u \in U^{\mathrm{CDU}} \qquad (P3)$$

$$\sum_{o\in O} \sum_{u\in U^{\mathrm{CDU}}} \overline{f}_{ouc}^{\mathrm{CDU}} V_{ou}^{\mathrm{CDU\text{-}P}} = \sum_{u\in U^{\mathrm{CDU}}} \sum_{u'\in \overline{G}_u} W_{uu'c}^{\mathrm{P}}, \quad \forall c \in C^{\mathrm{CDU}} \qquad (P4)$$

$$\sum_{u'\in \underline{G}_u} W_{u'uc}^{\mathrm{P}} + \sum_{r\in R_u^{\mathrm{PRC}}} f_{ucr}^{\mathrm{PRC}} V_{ur}^{\mathrm{PRC\text{-}P}} = \sum_{u'\in \overline{G}_u} W_{uu'c}^{\mathrm{P}}, \quad \forall u \in U^{\mathrm{PRC}}, \forall c \in C_u^{\mathrm{PRC}} \qquad (P5)$$

$$\underline{l}_u^{\mathrm{PRC}} \le \sum_{r\in R_u^{\mathrm{PRC}}} V_{ur}^{\mathrm{PRC\text{-}P}} \le \overline{l}_u^{\mathrm{PRC}}, \quad \forall u \in U^{\mathrm{PRC}} \qquad (P6)$$

$$\sum_{u'\in \underline{G}_u} W_{u'uc}^{\mathrm{P}} = X_c^{\mathrm{P}}, \quad \forall u \in \{\text{"BLD"}\}, \forall c \in C^{\mathrm{INT}} \cup C^{\mathrm{BLD}} \qquad (P7)$$

$$\underline{l}_c^{\mathrm{BLD}} \le X_c^{\mathrm{P}} \le \overline{l}_c^{\mathrm{BLD}}, \quad \forall c \in C^{\mathrm{BLD}} \qquad (P8)$$

$$X_c^{\mathrm{P}} = \sum_{k\in K} \sum_{r\in R_k^{\mathrm{BLD}}} \underline{f}_{kcr}^{\mathrm{BLD}} V_{kr}^{\mathrm{BLD\text{-}P}} + H_c^{\mathrm{P}}, \quad \forall c \in C^{\mathrm{BLD}} \qquad (P9)$$

$$\sum_{r\in R_k^{\mathrm{BLD}}} \overline{f}_{kr}^{\mathrm{BLD}} V_{kr}^{\mathrm{BLD\text{-}P}} = Y_k^{\mathrm{P}}, \quad \forall k \in K \qquad (P10)$$

$$\underline{d}_k^{\mathrm{P}} \le Y_k^{\mathrm{P}} \le \overline{d}_k^{\mathrm{P}}, \quad \forall k \in K \qquad (P11)$$

$$\text{all variables} \ge 0. \qquad (P12)$$

In the objective function, the first line represents the expected total value of products and benefit of thermal energy for internal use. The second line constitutes the costs of purchasing crude oil, operating at CDUs and other processes, blending products, inventory for components.

Constraint set (*P1*) defines the limitation on supply of crude oil and constraint set (*P2*) describes the flow balance for the raw material at the aggregated level. Constraint set (*P3*) is the production capacity restriction at CDUs. The flow conservation for output yield streams from CDUs is presented in constraint set (*P4*). Constraint set (*P5*) states the flow balance for related intermediate streams at other process nodes. Constraint set (*P6*) gives the lower and upper limits on production capacity at those processes. Constraint set (*P7*) indicates the flow balancing for components that are ready for blending of products or converting into internal energy use. The inventory capacity constraint set (*P8*) for stock tanks implies the min requirement and max availability of components. These components can be blended into multiple products according to recipes as well as left as inventory, which is illustrated in constraint set (*P9*). However, the objective function forces the left volumes as small as possible. Constraint set (*P10)* is yield balance for final products, which is regulated by internal restrictions on the demand, i.e. constraint set (*P11)*. Constraint set (*P12)* defines the domain of the decision variables.

5.3.2 The decoupled setting – Sales model [S]

In the sales model [S], the optimization problem is based on the supply of components and corresponding estimated costs (internal prices for components). Once the products are blended, the distribution decision describes the flow of products to a number of geographical regions. The objective is to maximize the expected net profit through sales for products in different markets at different prices, taking into consideration the blending, inventory, transportation and internal component costs.

Extra parameters used in the sales model [S]:

c_c^{S}	Unit internal prices for component c , $c \in C^{\mathrm{BLD}}$
$\underline{s}_c^{\mathrm{S}}, \bar{s}_c^{\mathrm{S}}$	Min and max supply of component c , $c \in C^{\mathrm{BLD}}$

Objective function in the sales planning model [S]:

$$z^{\mathrm{S}} = \max \sum_{k \in K} \sum_{m \in M} P_{km}^{\mathrm{S}} Q_{km}^{\mathrm{S}} - \sum_{k \in K} \sum_{r \in R_k^{\mathrm{BLD}}} c_{kr}^{\mathrm{BLD}} V_{kr}^{\mathrm{BLD\text{-}S}} - \sum_{c \in C^{\mathrm{BLD}}} c_c^{\mathrm{H}} H_c^{S} - \sum_{k \in K} \sum_{m \in M} c_{km}^{\mathrm{T}} Q_{km}^{\mathrm{S}} - \sum_{c \in C^{\mathrm{BLD}}} c_c^{\mathrm{S}} X_c^{S}$$

Constraints concerning the sales:

$$\underline{s}_c^{\mathrm{S}} \le X_c^{\mathrm{S}} \le \bar{s}_c^{\mathrm{S}}, \quad \forall c \in C^{\mathrm{BLD}} \qquad (S1)$$

$$\underline{l}_c^{\mathrm{BLD}} \le X_c^{\mathrm{S}} \le \bar{l}_c^{\mathrm{BLD}}, \quad \forall c \in C^{\mathrm{BLD}} \qquad (S2)$$

$$X_c^{\mathrm{S}} = \sum_{k \in K} \sum_{r \in R_k^{\mathrm{BLD}}} \underline{f}_{kcr}^{\mathrm{BLD}} V_{kr}^{\mathrm{BLD\text{-}S}} + H_c^{\mathrm{S}}, \quad \forall c \in C^{\mathrm{BLD}} \qquad (S3)$$

$$\sum_{r \in R_k^{\mathrm{BLD}}} \overline{f}_{kr}^{\mathrm{BLD}} V_{kr}^{\mathrm{BLD\text{-}S}} = Y_k^{\mathrm{S}}, \quad \forall k \in K \qquad (S4)$$

$$Y_k^{\mathrm{S}} = \sum_{m \in M} Q_{km}^{\mathrm{S}}, \quad \forall k \in K \qquad (S5)$$

$$P_{km}^{\mathrm{S}} = a_{km} + b_{km} Q_{km}^{\mathrm{S}}, \quad \forall k \in K, \forall m \in M \qquad (S6)$$

$$\underline{d}_{km} \le Q_{km}^{S} \le \overline{d}_{km}, \quad \forall k \in K, \forall m \in M \qquad (S7)$$

$$\text{all variables} \ge 0. \qquad (S8)$$

The limitations on the supply of components are described by constraint set (*S1)*. Constraint sets (*S2)* to (*S4)* refer to the blending decision which has the same explanation as (*P8)* to (*P10)* in model [P] and will not be reiterated here. Constraint set (*S5)* defines the sales decisions in distinct markets. The demand description is based on a spatial equilibrium model where the unit value of the product depends linearly on the demand in each market, indicated by constraint set (*S6)*. The sales revenue for products hence makes the objective function quadratic. Since all the constraints are linear, the sales planning model [S] is a quadratic programming (QP) problem. Note that the coefficient b_{km} is negative in accordance with the law of demand.

This profit maximization problem is convex. Constraint set (*S7)* states that the products shipped to markets must satisfy the demand limitations. Non-negativity restrictions on variables are given in constraint set (*S8)*.

It is worth noting that the total cost of the transfer is added as a *revenue* term $\sum_{k\in K} p_k^{\mathrm{P}} Y_k^{\mathrm{P}}$ to the production problem and as a *cost* term $\sum_{c\in C^{\mathrm{BLD}}} c_c^{\mathrm{S}} X_c^{S}$ to the sales problem. If we set the internal prices for products and components to zero, model [P] becomes classical cost-minimization problem for manufacturing and model [S] turns to be revenue-maximization problem for marketing. The production manager would only produce the minimum required demands for products while the sales planner would like to increase the sales quantity up to the maximum availability of blending resource. In other words, the prices or volume constraints for products and components play an important role. If they were incorrect, the production and sales planning would be misled.

5.3.3 The centralized setting – Integrated model [I]

The joint production and sales planning is carried out centrally in the integrated model [I]. The objective is to maximize the global net profit by balancing the sales revenue and supply chain cost subjecting to the flow conservation and capacity constraints. The optimal solution to model [I] is the best achievable result for the whole refinery system, served as the theoretical target of coordination and the benchmark for performance.

Objective function in the integrated model [I]:

$$
\begin{aligned}
z^{\mathrm{I}} = \max \sum_{k\in K}\sum_{m\in M} P_{km}^{\mathrm{I}} Q_{km}^{\mathrm{I}} + \sum_{c\in C^{\mathrm{INT}}} p_c X_c^{\mathrm{I}} \\
- \sum_{o\in O} c_o^{\mathrm{OIL}} V_o^{\mathrm{I}} - \sum_{o\in O}\sum_{u\in U^{\mathrm{CDU}}} c_{ou}^{\mathrm{CDU}} V_{ou}^{\mathrm{CDU\text{-}I}} - \sum_{u\in U^{\mathrm{PRC}}}\sum_{r\in R_u^{PRC}} c_{ur}^{\mathrm{PRC}} V_{ur}^{\mathrm{PRC\text{-}I}} - \sum_{k\in K}\sum_{r\in R_k^{\mathrm{BLD}}} c_{kr}^{\mathrm{BLD}} V_{kr}^{\mathrm{BLD\text{-}I}} - \sum_{c\in C^{\mathrm{BLD}}} c_c^{\mathrm{H}} H_c^{\mathrm{I}} - \sum_{k\in K}\sum_{m\in M} c_{km}^{\mathrm{T}} Q_{km}^{\mathrm{I}}
\end{aligned}
$$

Constraints concerning the production:

$$V_o^{\mathrm{I}} \le \bar{s}_o, \quad \forall o\in O \qquad (I1)$$

$$V_o^{\mathrm{I}} = \sum_{u\in U^{\mathrm{CDU}}} \underline{f}_{ou}^{\mathrm{CDU}} V_{ou}^{\mathrm{CDU\text{-}I}}, \quad \forall o\in O \qquad (I2)$$

$$\underline{l}_u^{\mathrm{CDU}} \le \sum_{o\in O} V_{ou}^{\mathrm{CDU\text{-}I}} \le \bar{l}_u^{\mathrm{CDU}}, \quad \forall u\in U^{\mathrm{CDU}} \qquad (I3)$$

$$\sum_{o\in O}\sum_{u\in U^{\mathrm{CDU}}} \overline{f}_{ouc}^{\mathrm{CDU}} V_{ou}^{\mathrm{CDU\text{-}I}} = \sum_{u\in U^{\mathrm{CDU}}}\sum_{u'\in \overline{G}_u} W_{uu'c}^{\mathrm{I}}, \quad \forall c\in C^{\mathrm{CDU}} \qquad (I4)$$

$$\sum_{u'\in \underline{G}_u} W_{u'uc}^{\mathrm{I}} + \sum_{r\in R_u^{\mathrm{PRC}}} f_{ucr}^{\mathrm{PRC}} V_{ur}^{\mathrm{PRC\text{-}I}} = \sum_{u'\in \overline{G}_u} W_{uu'c}^{\mathrm{I}}, \quad \forall u\in U^{\mathrm{PRC}}, \forall c\in C_u^{\mathrm{PRC}} \qquad (I5)$$

$$\underline{l}_u^{\mathrm{PRC}} \le \sum_{r\in R_u^{\mathrm{PRC}}} V_{ur}^{\mathrm{PRC\text{-}I}} \le \bar{l}_u^{\mathrm{PRC}}, \quad \forall u\in U^{\mathrm{PRC}} \qquad (I6)$$

$$\sum_{u'\in \underline{G}_u} W_{u'uc}^{\mathrm{I}} = X_c^{\mathrm{I}}, \quad \forall u\in \{\text{"BLD"}\}, \forall c\in C^{\mathrm{INT}} \cup C^{\mathrm{BLD}} \qquad (I7)$$

Constraints concerning both the production and the sales:

$$\underline{l}_c^{\mathrm{BLD}} \le X_c^{\mathrm{I}} \le \bar{l}_c^{\mathrm{BLD}}, \quad \forall c\in C^{\mathrm{BLD}} \qquad (I8)$$

$$X_c^{\mathrm{I}} = \sum_{k \in K} \sum_{r \in R_k^{\mathrm{BLD}}} \underline{f}_{kcr}^{\mathrm{BLD}} V_{kr}^{\mathrm{BLD\text{-}I}} + H_c^{\mathrm{I}}, \quad \forall c \in C^{\mathrm{BLD}} \tag{I9}$$

$$\sum_{r \in R_k^{\mathrm{BLD}}} \overline{f}_{kr}^{\mathrm{BLD}} V_{kr}^{\mathrm{BLD\text{-}I}} = Y_k^{\mathrm{I}}, \quad \forall k \in K \tag{I10}$$

Constraints concerning the sales:

$$Y_k^{\mathrm{I}} = \sum_{m \in M} Q_{km}^{\mathrm{I}}, \quad \forall k \in K \tag{I11}$$

$$P_{km}^{\mathrm{I}} = a_{km} + b_{km} Q_{km}^{\mathrm{I}}, \quad \forall k \in K, \forall m \in M \tag{I12}$$

$$\underline{d}_{km} \le Q_{km}^{\mathrm{I}} \le \overline{d}_{km}, \quad \forall k \in K, \forall m \in M \tag{I13}$$

$$\text{all variables} \ge 0. \tag{I14}$$

Compared with model [P] and [S], no estimated internal prices for products or components are used in model [I]. Besides, the internal volume restrictions for products (*P11)* and for components (*S1)* are also excluded. Constraint sets (*I8)* to (*I10)* connect the production and sales planning together concerning blending decision. Similarly as model [S], the integrated model [I] is a QP problem with linear constraints.

5.4 Coordination mechanism I

In this paper, two mechanisms are proposed as a means to achieve coordination between the production and sales divisions that corresponds to the enterprise's optimal performance. They basically differ in how to determine the internal prices and the intervals of the volume produced.

In the first coordination mechanism, the internal prices for products in model [P] p_k^{P} are derived from dual values of the constraint set (*S4)* in model [S], denoted by $S4.\mathrm{dual}_k$, whereas the internal prices for components in model [S] c_c^{S} are based on dual value of the constraint set (*P9*) in model [P]. Dual value or shadow price is defined as the change in the optimal objective function value when the right-hand-side of the constraint is increased by one unit (Lundgren et al. 2010). If a constraint expresses the quantity balance of a commodity, then its dual value can be interpreted as the marginal value for producing one additional unit of that component or product. During the successive iterations, the internal prices are updated according to following method. The reason why to use half of the previous value is to make sure the parameter does not change too much between consecutive iterations. Another motivation is to avoid a zigzag of the prices. This oscillating behavior is common in using steepest descent method and can be prevented by any conjugate gradient approach where the values are based on two consecutive iterations.

$$p_k^{\mathrm{P}(n+1)} = 0.5 p_k^{\mathrm{P}(n)} + 0.5 \cdot S4.\mathrm{dual}_k \tag{E1}$$

$$c_c^{\mathrm{S}(n+1)} = 0.5 c_c^{\mathrm{S}(n)} + 0.5 \cdot P9.\mathrm{dual}_c \tag{E2}$$

As to the demand interval particular for each product $\left[\underline{d}_k^{\mathrm{P}}, \overline{d}_k^{\mathrm{P}}\right]$, it is generated as α % under and over the volume of required products Y_k^{S} from the solution to model [S]. It ensures that the amount produced in the production level is close to that needed in the sales planning. Similarly, the supply interval of component $\left[\underline{s}_c^{\mathrm{S}}, \bar{s}_c^{\mathrm{S}}\right]$ comes from produced components X_c^{P} in model [P]. That is,

$$\underline{d}_k^{\mathrm{P}(n+1)} = \max\left(\left(1-\frac{\alpha}{n}\right)Y_k^{\mathrm{S}}, \sum_{m\in M}\underline{d}_{km}\right) \tag{E3}$$

$$\bar{d}_k^{\mathrm{P}(n+1)} = \min\left(\left(1+\frac{\alpha}{n}\right)Y_k^{\mathrm{S}}, \sum_{m\in M}\bar{d}_{km}\right) \tag{E4}$$

$$\underline{s}_c^{\mathrm{S}(n+1)} = \max\left(\left(1-\frac{\alpha}{n}\right)X_c^{\mathrm{P}}, \underline{l}_c^{\mathrm{BLD}}\right) \tag{E5}$$

$$\bar{s}_c^{\mathrm{S}(n+1)} = \min\left(\left(1+\frac{\alpha}{n}\right)X_c^{\mathrm{P}}, \bar{l}_c^{\mathrm{BLD}}\right) \tag{E6}$$

The scaling factor α represents the magnitude of the flexible production. The larger α is, the larger the volume interval becomes. Then the feasible space is larger. Nonetheless, we use a decreasing deviation, divided by the number of iterations n. Note that when $n \to \infty$ or large enough, the volume interval will shrink to a very small interval. It implies the demand of products or the supply of components becomes exact rather than flexible.

Figure 5.3 outlines the coordination scheme for Mechanism I. It starts by choosing the initial values for prices and volume intervals, known as *starting point*. Then model [P] and [S] are solved optimally and separately. Since the blending decision is duplicated in both model [P] and model [S], when calculating the overall profit for the firm we cannot make it a straightforward combination of production and sales local optimums. Instead, we fix the required volume of products in model [I] with the optimized solution to model [P], i.e., $Y_k^{\mathrm{I}} = Y_k^{\mathrm{P}}$ and then solve model [I] to get the sales decisions. It refers to what the actual enterprise performance can be realized under the proposed production plan. The objective value obtained with fixed production solution is expressed as $z^{\text{I-FP}(n)}$. Another similar objective value, i.e., $z^{\text{I-FS}(n)}$, can be made from the sales perspective, utilizing the sales solution, i.e., $Y_k^{\mathrm{I}} = Y_k^{\mathrm{S}}$, to regular production decision in the same fashion. If solvable, both $z^{\text{I-FP}(n)}$ and $z^{\text{I-FS}(n)}$ are feasible solutions to model [I]. We can get the best achievable result of the coordination mechanism at the n th iteration by updating $\mathrm{LBD} = \max\left(z^{\text{I-FP}(n)}, z^{\text{I-FS}(n)}, \mathrm{LBD}\right)$.

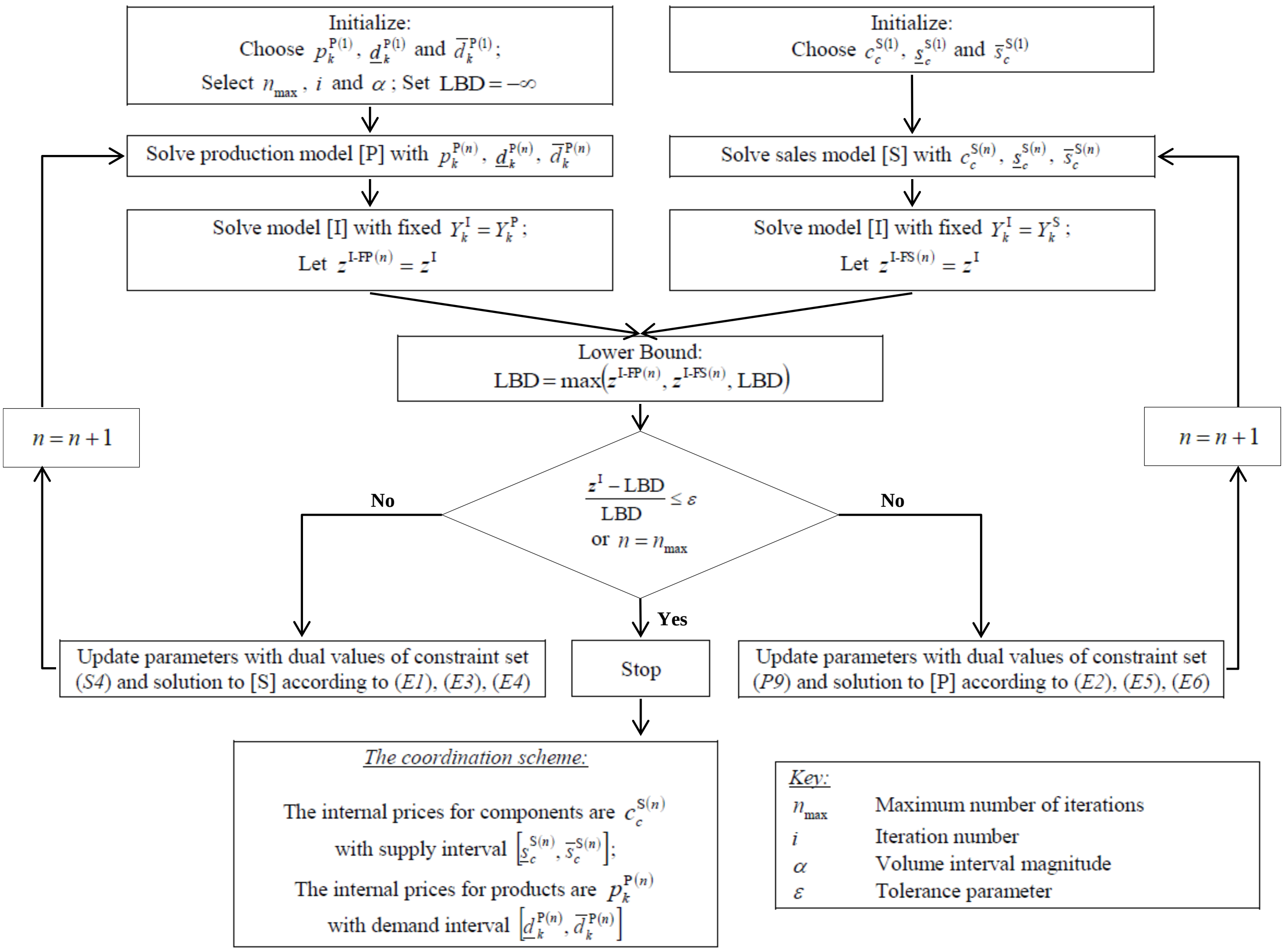
Note that in this paper the integrated model [I] is established as benchmark for comparison and overall result can be obtained by fixing certain variables in model [I] with solution to the decoupled model [P] or [S] as mentioned above. However, if the integrated model cannot be solved with the increase in size and complexity of the real problem, we still can get overall profit $z^{\text{I-FP}(n)}$ by only involving the two decouple models. That is, we first fix the volume of products in model [S] with solution to model [P], then solve model [S], and at last sum up all the objective items of two models except for the duplicated ones. In the similar manner, we can get $z^{\text{I-FS}(n)}$.

Define *profit gap* as the percent deviation between optimum of model [I] z^{I} and current LBD at iteration n , i.e., $\dfrac{z^{\mathrm{I}} - \mathrm{LBD}}{\mathrm{LBD}}$. If profit gap is smaller than a required tolerance ε , we say that the coordination is *convergent*. Define *iteration gap* as the difference between the results of model [I] with fixed production planning or sales planning under consecutive iterations, i.e., $\left|z^{\text{I-FP}(n)} - z^{\text{I-FP}(n-1)}\right|$ and $\left|z^{\text{I-FS}(n)} - z^{\text{I-FS}(n-1)}\right|$. If iteration gap is smaller than an arbitrary number κ, we say that the coordination is *stable*.

Model [P] and [S] are iteratively solved by recursive updating of internal prices and volume intervals in one problem with the solution to the other, i.e., equations *(E1)* to *(E6)*. The procedure stops when LBD is *convergent* to the optimal value of model [I] i.e., $\frac{z^{\text{I}} - \text{LBD}}{\text{LBD}} \le \varepsilon$, or the maximal iterations $n_{\max}$ has been reached. Then the coordination scheme is that internal prices for components are $c_c^{\text{S}(n)}$ with supply intervals $\left[\underline{s}_c^{\text{S}(n)}, \overline{s}_c^{\text{S}(n)}\right]$ or the internal prices for products are $p_k^{\text{P}(n)}$ with demand intervals $\left[\underline{d}_k^{\text{P}(n)}, \overline{d}_k^{\text{P}(n)}\right]$. The choice depends on which scheme leads to a *stable* coordination at the same time.

Last but not least, it is worth noting that if in reality the integrated model does not exist and the benchmark z^{I} is then not known a priori, the termination criterion has to be changed. One choice to stop the procedure is that if LBD is not improved during last i iterations. However, Mechanism I cannot guarantee whether the LBD obtained is near or far from global optimum. This limitation for Mechanism I will be obviated in Mechanism II based on theoretical sound decomposition principle, which will be discussed in next section.

Figure 5.3 Algorithmic procedure for Mechanism I



5.5 Coordination mechanism II

In this section we start by briefly describing Lagrangian decomposition along with subgradient optimization method, then reformulate model [I] and relax constraints leading to decomposition into subproblems, i.e., model [D-P] and model [D-S], and finally provide the algorithm for Mechanism II.

5.5.1 Lagrangian decomposition

The basic idea behind LD method is to reformulate the original problem into an equivalent one by duplicating a set of variables and adding a number of coupling constraints. These constraints are then relaxed such that the model decomposes into two subproblems, each having part of the constraints of the original problem. LD provides a systematic way of obtaining optimistic estimate for complex large-scale problems.

Consider the following optimization problem [A]:

$$z^{\mathrm{A}} = \max cx \quad \text{s.t.}\, Ax = b,\, Bx = d,\, x \in X. \tag{A}$$

The first step is to reformulate problem [A] by creating identical copy of variable x and add equality constraint:

$$z^{\mathrm{EA}} = \max c_1 x + c_2 y \quad \text{s.t.}\, Ax = b,\, By = d,\, x = y,\, x \in X,\, y \in X, \quad \text{where } c = c_1 + c_2. \tag{EA}$$

Then relax the coupling constraint $x = y$ with Lagrangian multiplier λ and the problem becomes:

$$z^{\mathrm{DA}}(\lambda) = \max c_1 x + c_2 y + \lambda(x - y) \quad \text{s.t.}\, Ax = b,\, By = d,\, x \in X,\, y \in X, \tag{DA}$$

Note that λ is not restricted in sign, since the relaxed constraint is an equality. The resulting problem [DA] can be nicely separated to the following two models that can be solved independently:

$$z^{\mathrm{DA\text{-}1}}(\lambda) = \max (c_1 + \lambda)x \quad \text{s.t.}\, Ax = b,\, x \in X. \tag{DA-1}$$

$$z^{\mathrm{DA\text{-}2}}(\lambda) = \max (c_2 - \lambda)y \quad \text{s.t.}\, By = d,\, y \in X. \tag{DA-2}$$

It has been proved that $z^{\mathrm{DA}}(\lambda) = z^{\mathrm{DA\text{-}1}}(\lambda) + z^{\mathrm{DA\text{-}2}}(\lambda)$, where $z^{\mathrm{DA\text{-}1}}(\lambda)$ and $z^{\mathrm{DA\text{-}2}}(\lambda)$ are the optimal solutions to problem [DA-1] and [DA-2], respectively, is an upper bound of the optimal solution to the original problem [A] for any λ (Fisher 1981). In order to get the tightest bound, the value of λ should be adjusted such that:

$$z^{\mathrm{LD}} = \min_{\lambda} z^{\mathrm{DA}}(\lambda) \tag{LD}$$

This Lagrangian dual problem is a non-differentiable optimization problem. An iterative technique such as subgradient method is typically employed to optimize $z^{\mathrm{DA}}(\lambda)$ over λ. This involves the following updating procedure:

Step 0: Choose an initial dual value $\lambda^{(1)}$

Step 1: Solve the Lagrangian subproblems [DA-1] and [DA-2] for the given $\lambda^{(n)}$, which give the solution $x^{(n)}$ and $y^{(n)}$ and the optimal objective function value $z^{\text{DA}}\left(\lambda^{(n)}\right) = z^{\text{DA-1}}\left(\lambda^{(n)}\right) + z^{\text{DA-2}}\left(\lambda^{(n)}\right)$. Update $\text{UBD} = \min\left(\text{z}^{\text{DA}}(\lambda^{(n)}), \text{UBD}\right)$.

Step 2: Determine a feasible solution $\tilde{x}^{(n)}$ based on $x^{(n)}$ or $y^{(n)}$. Update $\text{LBD} = \max\left(\text{z}^{\text{A}}(\tilde{x}^{(n)}), \text{LBD}\right)$.

Step 3: Check the convergence criteria. If $(\text{UBD} - \text{LBD})/\text{LBD} \le \varepsilon$ or $n = n_{\max}$, STOP. Let $\tilde{x}^{(n)}$ be the best found feasible solution

Step 4: Compute a subgradient as $\gamma^{(n)} = x^{(n)} - y^{(n)}$.

Step 5: Determine the step length as $t^{(n)} = \dfrac{\sigma(\text{UBD} - \text{LBD})}{\left\|\gamma^{(n)}\right\|^2}$, where σ is a convergence parameter satisfying $0 < \sigma \le 2$.

Step 6: Update the Lagrangian multipliers as $\lambda^{(n+1)} = \lambda^{(n)} + t^{(n)}\gamma^{(n)}$.

Step 7: Update σ according to certain updating rule if UBD has not improved during the last i iterations. Set $n \leftarrow n+1$ and go to *Step 1*.

Since the subgradient obtained as a solution form the Lagrangian subproblem is not necessarily an ascent direction, the select of the step length should be carefully. To guarantee convergence, the theoretical requirement for the step length is that $t^{(n)} \to 0$ and $\sum_{k=1}^{n} t^{(k)} \to \infty$ when $n \to \infty$. The step length presented in *Step 5* is one of the common choices used in practice. It is also important to decide when to update the step length in order to avoid oscillating behavior or accelerate convergence (Fumero 2001). Furthermore, how to reach a good, feasible solution to the original problem to provide a LBD to the optimal objective value is highly problem-specific. We will discuss these issues in detail in Section 5.5.3 and 5.6.3.

5.5.2 Reformulation of model [I]

One of the major questions to apply the LD approach is to identify which variables to duplicate and how to decompose constraints in the original problem. From the basic description of the mathematical formulation of model [I], we notice that constrain sets *(I8)* to *(I10)* are concerned with both production and sales planning and connected through such variables as component volume X_c^{I}, blending recipe volume $V_{kr}^{\text{BLD-I}}$, inventory H_c^{I}, and product volume Y_k^{I} .Thus we duplicate these variables and add the coupling constraints, i.e., $X_c^{\text{Da}} = X_c^{\text{Db}}$, $V_{kr}^{\text{BLD-Da}} = V_{kr}^{\text{BLD-Db}}$, $H_c^{\text{Da}} = H_c^{\text{Db}}$ and $Y_k^{\text{Da}} = Y_k^{\text{Db}}$, to require the split variables to have the same value. Then the relaxation of the new constraint sets with the Lagrangian multipliers, i.e., λ_c, μ_{kr}, ν_c and δ_k, gives rise to the reformulated model [D]. Note that the related coefficients in the objective function are also evenly split. Constraint sets (*D11)* to (*D13)* expressed by X_c^{Db}, $V_{kr}^{\text{BLD-Db}}$, H_c^{Db} and Y_k^{Db} are copied from constraint sets (*D8)* to (*D10)* by X_c^{Da}, $V_{kr}^{\text{BLD-Da}}$, H_c^{Da} and Y_k^{Da}.

Objective function in the reformulated model [D]:

$$
\begin{aligned}
z^{\mathrm{D}} = \max & \sum_{k\in K}\sum_{m\in M} P_{km}^{\mathrm{D}} Q_{km}^{\mathrm{D}} + \sum_{c\in C^{\mathrm{INT}}} p_c X_c^{\mathrm{Da}} \\
& - \sum_{o\in O} c_o^{\mathrm{OIL}} V_o^{\mathrm{D}} - \sum_{o\in O}\sum_{u\in U^{\mathrm{CDU}}} c_{ou}^{\mathrm{CDU}} V_{ou}^{\mathrm{CDU\text{-}D}} - \sum_{u\in U^{\mathrm{PRC}}}\sum_{r\in R_u^{PRC}} c_{ur}^{\mathrm{PRC}} V_{ur}^{\mathrm{PRC\text{-}D}} - \sum_{k\in K}\sum_{m\in M} c_{km}^{\mathrm{T}} Q_{km}^{\mathrm{D}} \\
& - \sum_{k\in K}\sum_{r\in R_k^{\mathrm{BLD}}} \frac{c_{kr}^{\mathrm{BLD}}}{2} V_{kr}^{\mathrm{BLD\text{-}Da}} - \sum_{k\in K}\sum_{r\in R_k^{\mathrm{BLD}}} \frac{c_{kr}^{\mathrm{BLD}}}{2} V_{kr}^{\mathrm{BLD\text{-}Db}} - \sum_{c\in C^{\mathrm{BLD}}} \frac{c_c^{\mathrm{H}}}{2} H_c^{\mathrm{Da}} - \sum_{c\in C^{\mathrm{BLD}}} \frac{c_c^{\mathrm{H}}}{2} H_c^{\mathrm{Db}} \\
& - \sum_{c\in C^{\mathrm{BLD}}} \lambda_c \left(X_c^{\mathrm{Db}} - X_c^{\mathrm{Da}}\right) - \sum_{k\in K}\sum_{r\in R_k^{\mathrm{BLD}}} \mu_{kr}\left(V_{kr}^{\mathrm{BLD\text{-}Db}} - V_{kr}^{\mathrm{BLD\text{-}Da}}\right) - \sum_{c\in C^{\mathrm{BLD}}} \nu_c \left(H_c^{\mathrm{Db}} - H_c^{\mathrm{Da}}\right) - \sum_{k\in K} \delta_k \left(Y_k^{\mathrm{Db}} - Y_k^{\mathrm{Da}}\right)
\end{aligned}
$$

Constraints concerning the production:

$$V_o^{\mathrm{D}} \le \bar{s}_o, \quad \forall o \in O \tag{D1}$$

$$V_o^{\mathrm{D}} = \sum_{u\in U^{\mathrm{CDU}}} \underline{f}_{ou}^{\mathrm{CDU}} V_{ou}^{\mathrm{CDU\text{-}D}}, \quad \forall o \in O \tag{D2}$$

$$\underline{l}_u^{\mathrm{CDU}} \le \sum_{o\in O} V_{ou}^{\mathrm{CDU\text{-}D}} \le \bar{l}_u^{\mathrm{CDU}}, \quad \forall u \in U^{\mathrm{CDU}} \tag{D3}$$

$$\sum_{o\in O}\sum_{u\in U^{\mathrm{CDU}}} \overline{f}_{ouc}^{\mathrm{CDU}} V_{ou}^{\mathrm{CDU\text{-}D}} = \sum_{u\in U^{\mathrm{CDU}}}\sum_{u'\in \overline{G}_u} W_{uu'c}^{\mathrm{D}}, \quad \forall c \in C^{\mathrm{CDU}} \tag{D4}$$

$$\sum_{u'\in \underline{G}_u} W_{u'uc}^{\mathrm{D}} + \sum_{r\in R_u^{\mathrm{PRC}}} f_{ucr}^{\mathrm{PRC}} V_{ur}^{\mathrm{PRC\text{-}D}} = \sum_{u'\in \overline{G}_u} W_{uu'c}^{\mathrm{D}}, \quad \forall u \in U^{\mathrm{PRC}}, \forall c \in C_u^{\mathrm{PRC}} \tag{D5}$$

$$\underline{l}_u^{\mathrm{PRC}} \le \sum_{r\in R_u^{\mathrm{PRC}}} V_{ur}^{\mathrm{PRC\text{-}D}} \le \bar{l}_u^{\mathrm{PRC}}, \quad \forall u \in U^{\mathrm{PRC}} \tag{D6}$$

$$\sum_{u'\in \underline{G}_u} W_{u'uc}^{\mathrm{D}} = X_c^{\mathrm{Da}}, \quad \forall u \in \{\text{"BLD"}\}, \forall c \in C^{\mathrm{INT}} \cup C^{\mathrm{BLD}} \tag{D7}$$

$$\underline{l}_c^{\mathrm{BLD}} \le X_c^{\mathrm{Da}} \le \bar{l}_c^{\mathrm{BLD}}, \quad \forall c \in C^{\mathrm{BLD}} \tag{D8}$$

$$X_c^{\mathrm{Da}} = \sum_{k\in K}\sum_{r\in R_k^{\mathrm{BLD}}} \underline{f}_{kcr}^{\mathrm{BLD}} V_{kr}^{\mathrm{BLD\text{-}Da}} + H_c^{\mathrm{Da}}, \quad \forall c \in C^{\mathrm{BLD}} \tag{D9}$$

$$\sum_{r\in R_k^{\mathrm{BLD}}} \overline{f}_{kr}^{\mathrm{BLD}} V_{kr}^{\mathrm{BLD\text{-}Da}} = Y_k^{\mathrm{Da}}, \quad \forall k \in K \tag{D10}$$

Constraints concerning the sales:

$$\underline{l}_c^{\mathrm{BLD}} \le X_c^{\mathrm{Db}} \le \bar{l}_c^{\mathrm{BLD}}, \quad \forall c \in C^{\mathrm{BLD}} \tag{D11}$$

$$X_c^{\mathrm{Db}} = \sum_{k\in K}\sum_{r\in R_k^{\mathrm{BLD}}} \underline{f}_{kcr}^{\mathrm{BLD}} V_{kr}^{\mathrm{BLD\text{-}Db}} + H_c^{\mathrm{Db}}, \quad \forall c \in C^{\mathrm{BLD}} \tag{D12}$$

$$\sum_{r\in R_k^{\mathrm{BLD}}} \overline{f}_{kr}^{\mathrm{BLD}} V_{kr}^{\mathrm{BLD\text{-}Db}} = Y_k^{\mathrm{Db}}, \quad \forall k \in K \tag{D13}$$

$$Y_k^{\mathrm{Db}} = \sum_{m\in M} Q_{km}^{\mathrm{D}}, \quad \forall k \in K \tag{D14}$$

$$P_{km}^{\mathrm{D}} = a_{km} + b_{km} Q_{km}^{\mathrm{D}}, \quad \forall k \in K, \forall m \in M \tag{D15}$$

$$\underline{d}_{km} \le Q_{km}^{\mathrm{D}} \le \bar{d}_{km}, \quad \forall k \in K, \forall m \in M \tag{D16}$$

$$\text{all variables} \ge 0. \tag{D17}$$

It is obvious that model [D] can be decomposed into two subproblems, i.e., model [D-P] and model [D-S], which independently solve their portion of the objective function with the separable constraint sets. The structures of constraints in these two models are similar to the ones in model [P] and [S] presented in Section 5.3, but no extra volume restrictions are added either for products in model [D-P] or for components in model [D-S].

Objective function in the model [D-P]:

$$z^{\text{D-P}} = \max \sum_{k \in K} \delta_k Z_k^{\text{Da}} + \sum_{c \in C^{\text{INT}}} p_c X_c^{\text{Da}} + \sum_{c \in C^{\text{BLD}}} \left(\nu_c - \frac{c_c^{\text{H}}}{2} \right) H_c^{\text{Da}} + \sum_{c \in C^{\text{BLD}}} \lambda_c X_c^{\text{Da}}$$
$$- \sum_{o \in O} c_o^{\text{OIL}} V_o^{\text{D}} - \sum_{o \in O} \sum_{u \in U^{\text{CDU}}} c_{ou}^{\text{CDU}} V_{ou}^{\text{CDU-D}} - \sum_{k \in K} \sum_{r \in R_k^{\text{BLD}}} \left(\frac{c_{kr}^{\text{BLD}}}{2} - \mu_{kr} \right) V_{kr}^{\text{BLD-Da}} - \sum_{u \in U^{\text{PRC}}} \sum_{r \in R_u^{PRC}} c_{ur}^{\text{PRC}} V_{ur}^{\text{PRC-D}}$$

subject to constraint sets(D1) to (D10) plus (D17) for related variables.

Objective function in the model [D-S]:

$$z^{\text{D-S}} = \max \sum_{k \in K} \sum_{m \in M} P_{km}^{\text{D}} Q_{km}^{\text{D}}$$
$$- \sum_{c \in C^{\text{BLD}}} \lambda_c X_c^{\text{Db}} - \sum_{k \in K} \sum_{r \in R_k^{\text{BLD}}} \left(\frac{c_{kr}^{\text{BLD}}}{2} + \mu_{kr} \right) V_{kr}^{\text{BLD-Db}} - \sum_{c \in C^{\text{BLD}}} \left(\frac{c_c^{\text{H}}}{2} + \nu_c \right) H_c^{\text{Db}} - \sum_{k \in K} \delta_k Y_k^{\text{Db}} - \sum_{k \in K} \sum_{m \in M} c_{km}^{\text{T}} Q_{km}^{\text{D}}$$

subject to constraint sets (D11) to (D17).

5.5.3 Algorithm

Similar to Mechanism I, the LBD in the second coordination mechanism is also created from the objective value with fixed production solution $z^{\text{I-FP}(n)}$ or fixed sales solution $z^{\text{I-FS}(n)}$. However, The main difference between these two mechanisms is that instead of using the dual values and volumes from results of previous iteration in Mechanism I, the internal prices in Mechanism II are updated by the Lagrangian multipliers and volume intervals are defined by the solution to Lagrangian subproblem [D-P] in current iteration. Therefore, the changes occur both in content and in sequence.

Figure 5.4 shows a diagram of the decomposition steps for Mechanism II. To begin with, the Lagrangian multipliers are initialized. Then the Lagrangian subproblems models [D-P] and [D-S] with given multipliers are solved separately. The objective value $z^{\text{D}} = z^{\text{D-P}} + z^{\text{D-S}}$, where $z^{\text{D-P}}$ and $z^{\text{D-S}}$ are the optimal objective values of model [D-P] and [D-S], respectively, is an upper bound to the original optimal solution to model [I]. The current UBD is determined as $\min(z^{\text{D}}, \text{UBD})$.

Lagrangian multipliers give the rate of change in the objective function with respect to the rate of change in the right-hand side of relaxed constraints. They may be interpreted as the (penalty) costs to the objective function paid for the violation. In other words, they represent the shadow prices on the coupling constraints. Therefore, the multiplier δ_k defined for coupling constraint $Y_k^{\text{Da}} = Y_k^{\text{Db}}$ can be referred to as internal prices for the specific product k. In addition, the volume of products obtained from model [D-S] Y_k^{Db} is used to generate the demand interval in model [P]. The required parameters are updated by

$$p_k^{\text{P}(n)} = \delta_k^{(n)} \qquad (E7)$$

$$\underline{d}_k^{\text{P}(n)} = \max\left(\left(1 - \frac{\alpha}{n}\right) Y_k^{\text{Db}}, \sum_{m \in M} \underline{d}_{km} \right) \qquad (E8)$$

$$\bar{d}_k^{\mathrm{P}(n)} = \min\left(\left(1+\frac{\alpha}{n}\right)Y_k^{\mathrm{Db}}, \sum_{m\in M}\bar{d}_{km}\right) \tag{E9}$$

Upon solving model [P] and getting the optimal value with fixed production planning $z^{\text{I-FP}(n)}$, we update the internal prices for components based on the Lagrangian multipliers λ_c and the supply interval in model [S] derived from X_c^{P} in model [P]:

$$c_c^{\mathrm{S}(n)} = \lambda_c^{(n)} \tag{E10}$$

$$\underline{s}_c^{\mathrm{S}(n)} = \max\left(\left(1-\frac{\alpha}{n}\right)X_c^{\mathrm{P}}, \underline{l}_c^{\mathrm{BLD}}\right) \tag{E11}$$

$$\bar{s}_c^{\mathrm{S}(n)} = \min\left(\left(1+\frac{\alpha}{n}\right)X_c^{\mathrm{P}}, \bar{l}_c^{\mathrm{BLD}}\right) \tag{E12}$$

Then the optimal value with fixed sales planning $z^{\text{I-FS}(n)}$ is similarly determined and update $\mathrm{LBD} = \max\left(z^{\text{I-FP}(n)}, z^{\text{I-FS}(n)}, \mathrm{LBD}\right)$.

Check the convergence criterion. If $\frac{\mathrm{UBD}-\mathrm{LBD}}{\mathrm{LBD}} \le \varepsilon$ or $n = n_{\max}$, then stop and the coordination scheme is that internal prices for components are $\lambda_c^{(n)}$ with supply volume $\left[\underline{s}_c^{\mathrm{S}(n)}, \bar{s}_c^{\mathrm{S}(n)}\right]$ or the internal prices for products are $\delta_k^{(n)}$ with demand interval $\left[\underline{d}_k^{\mathrm{P}(n)}, \bar{d}_k^{\mathrm{P}(n)}\right]$. The choice depends on which scheme leads to a *stable* coordination at the same time.

At last, we use subgradient method to update the values of the Lagrangian multipliers that are in turn used to solve subproblems.

$\lambda_c^{(n+1)} = \lambda_c^{(n)} + t^{\lambda(n)}\gamma_c^{\lambda(n)}$,

where subgradient $\gamma_c^{\lambda(n)} = X_c^{\mathrm{Db}} - X_c^{\mathrm{Da}}$ and step length $t^{\lambda(n)} = \frac{\sigma\left(\mathrm{UBD}^{(n)} - \mathrm{LBD}^{(n)}\right)}{\left\|\gamma_c^{\lambda(n)}\right\|^2}$ (*E13*)

$\mu_{kr}^{(n+1)} = \mu_{kr}^{(n)} + t^{\mu(n)}\gamma_{kr}^{\mu(n)}$,

where subgradient $\gamma_{kr}^{\mu(n)} = V_{kr}^{\text{BLD-Db}} - V_{kr}^{\text{BLD-Da}}$ and step length $t^{\mu(n)} = \frac{\sigma\left(\mathrm{UBD}^{(n)} - \mathrm{LBD}^{(n)}\right)}{\left\|\gamma_{kr}^{\mu(n)}\right\|^2}$ (*E14*)

$\nu_c^{(n+1)} = \nu_c^{(n)} + t^{\nu(n)}\gamma_c^{\nu(n)}$,

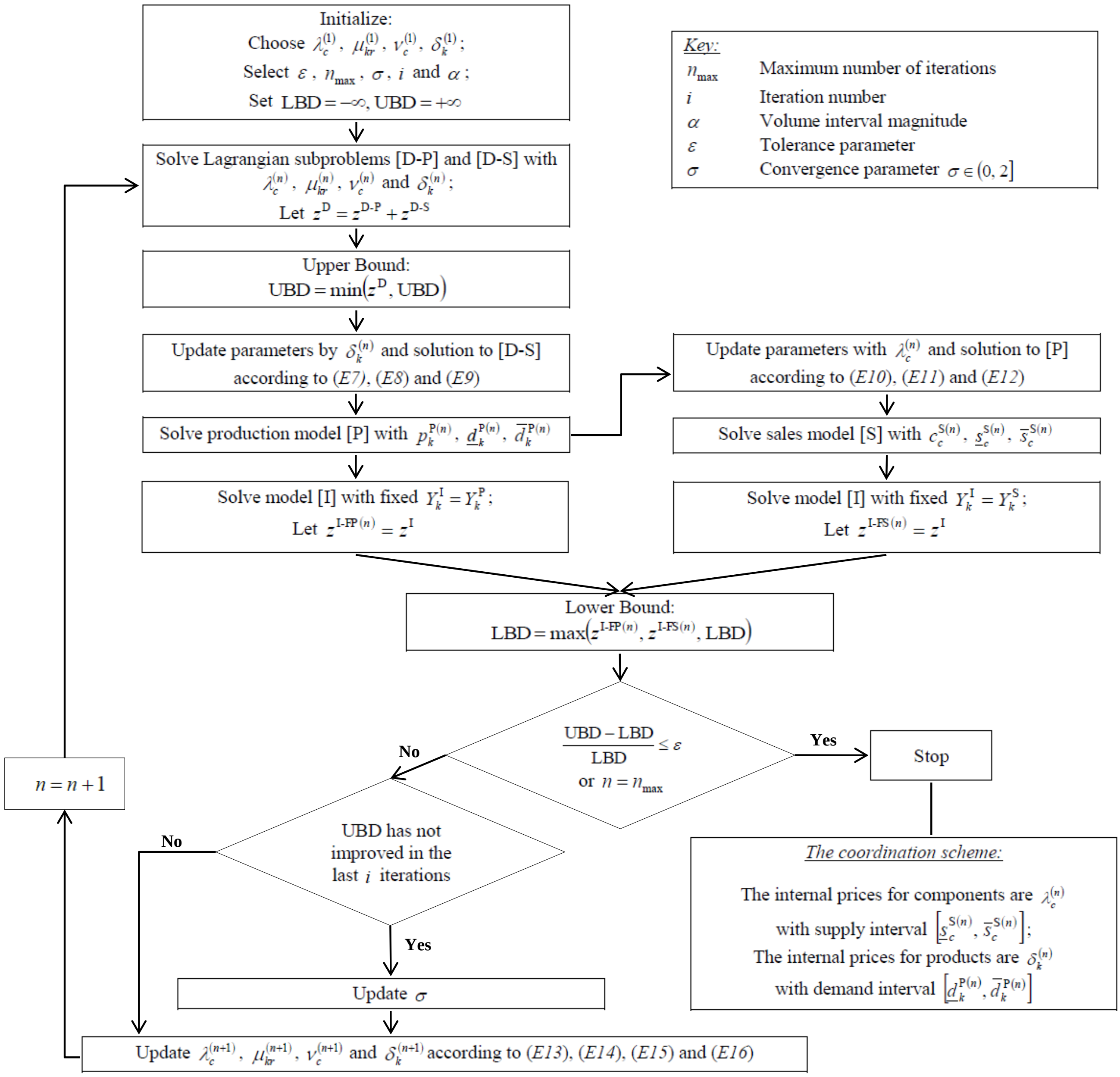
where subgradient $\gamma_c^{\nu(n)} = H_c^{\mathrm{Db}} - H_c^{\mathrm{Da}}$ and step length $t^{\nu(n)} = \frac{\sigma\left(\mathrm{UBD}^{(n)} - \mathrm{LBD}^{(n)}\right)}{\left\|\gamma_c^{\nu(n)}\right\|^2}$ (*E15*)

$\delta_k^{(n+1)} = \delta_k^{(n)} + t^{\delta(n)}\gamma_k^{\delta(n)}$,

where subgradient $\gamma_k^{\delta(n)} = Y_k^{\mathrm{Db}} - Y_k^{\mathrm{Da}}$ and step length $t^{\delta(n)} = \frac{\sigma\left(\mathrm{UBD}^{(n)} - \mathrm{LBD}^{(n)}\right)}{\left\|\gamma_k^{\delta(n)}\right\|^2}$ (*E16*)

Note that if UBD has not improved during the last i iterations, we will update the convergence parameter σ according to the specific updating rule which will be discussed in Section 5.6.3.

Figure 5.4 Algorithmic procedure for Mechanism II



5.6 Case study

We divide the computational study into four parts. First, we give a detailed description of the refinery system and the key data. Next, we separately test the proposed algorithmic parameters in the first coordination mechanism to gain some insights on which parameter accounts more for convergence. Then, we employ the second coordination mechanism to study the impact of various updating rules on convergence behavior. Finally, we test four distinct market demand scenarios. The linear demand function makes the model a QP problem while the step price function changes it into a MIP problem. The convergent performance of Mechanism II is examined.

Though the main termination criterion for Mechanism I is $\frac{z^{\mathrm{I}} - \mathrm{LBD}^{(n)}}{\mathrm{LBD}^{(n)}} \leq \varepsilon$ and the one for Mechanism II is $\frac{\mathrm{UBD}^{(n)} - \mathrm{LBD}^{(n)}}{\mathrm{LBD}^{(n)}} \leq \varepsilon$, in order to have a general comparison between the two mechanisms under all parameter settings and scenario assumptions, we run every situation to maximum 500 iterations. In the specific analysis below we will mention at which iteration the convergence criterion is met.

The mathematical models are programmed by AMPL modeling language (version 20120217). The QP models are solved by MINOS 5.5 and the MIP models by CPLEX 12.2. The default parameters of solvers are used throughout. All computations are performed on a T7300 2.00 GHz processor with 3 GB of RAM.

5.6.1 Case description

We use an adapted version of the case study in Bengtsson et al. (2010). The main differences are that the blending activity of products takes place at a refinery rather than on a ship. The final products will be sent to three markets where demand varies with the product price.

The refinery system involves a CDU that performs atmospheric and vacuum distillation, a reformer, a cracker and four blend headers for gasoline, jet fuel, automotive gas oil and heavy fuel oil (Figure 5.5). The explanation of abbreviations in the flowchart is given in Table 5.1. Note that components that are ready for further blending are in blue boxes and the one can only be burned as internal thermal energy is in green box, both initialized with “COM”.

Figure 5.5 Flowchart of the refinery system in the cast study

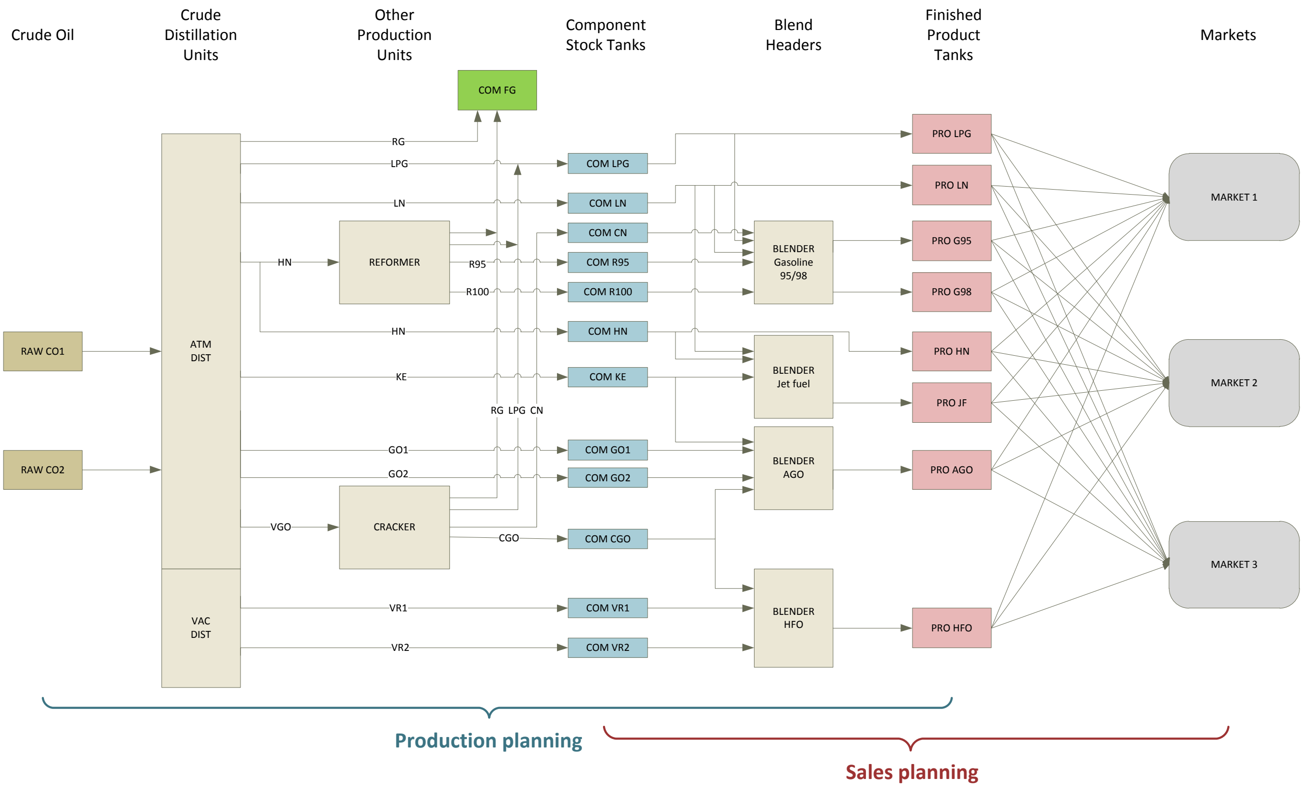


Table 5.1 Explanation of abbreviations for raw materials, components and blended products

Abbreviation	Explanation	Abbreviation	Explanation
Raw material		***Additional components out from reformer***	
CO1	Crude oil 1	R95	Reformate 95
CO2	Crude oil 2	R100	Reformate 100
Components out from CDU			
FG	Fuel gas	***Additional components out from cracker***	
LPG	Liquefied petroleum gas	CN	Cracker naphtha
LN	Light naphtha	CGO	Cracker gas oil
HN	Heavy naphtha		
KE	Kerosene	***Blended products***	
GO1	Gas oil 1*	G95	Gasoline 95
GO2	Gas oil 2**	G98	Gasoline 98
VGO	Vacuum gas oil	JF	Jet fuel
VR1	Vacuum residue 1*	AGO	Automotive gas oil
VR2	Vacuum residue 2**	HFO	Heavy fuel oil

Note: * Originates from crude oil 1
** Originates from crude oil 2

Different crude oil charged into CDU or the same type of feedstock fed to reformer and cracker in separate modes will create distinct output yields in terms of quantity and quality. Table 5.2, Table 5.3 and Table 5.4 show yield coefficients for these three main refinery processes, respectively.

Table 5.2 Yields from CDU for one unit of "Crude oil 1" and "Crude oil 1"

Output yield	Crude oil 1	Crude oil 2
Fuel gas	0.0010	0.0020
Liquefied petroleum gas	0.0401	0.0056
Light naphtha	0.1389	0.0287
Heavy naphtha	0.3182	0.1330
Kerosene	0.1263	0.0920
Gas oil 1	0.2683	-
Gas oil 2	-	0.3540
Vacuum gas oil	0.0926	0.2810
Vacuum residue 1	0.0146	-
Vacuum residue 2	-	0.1037
	1.0000	1.0000

Table 5.3 Yields from reformer for one unit of "Heavy naphtha" in REF95 mode and REF100 mode

Output yield	REF95 mode	REF100 mode
Fuel gas	0.08	0.09
Liquefied petroleum gas	0.09	0.12
Reformate 95	0.83	-
Reformate 100	-	0.79
	1.00	1.00

Table 5.4 Yields from cracker for one unit of "Vacuum gas oil" in Naphtha mode and Gas oil mode

Output yields	Naphtha mode	Gas oil mode
Fuel gas	0.035	0.032
Liquefied petroleum gas	0.063	0.056
Cracker naphtha	0.446	0.391
Cracker gas oil	0.456	0.521
	1.000	1.000

For a particular product, it can be blended in several ways from different components according to the recipes. These predetermined recipes, presented in Table 5.5, Table 5.6, Table 5.7 and Table 5.8, respectively, will make the final product meet required quality specifications. In addition, “Heavy naphtha” as *direct product* exists in the component set C^{BLD} as well as in the product set K . There is a pseudo recipe, “Recipe 0”, with zero blending cost and the input and output coefficient for “Heavy naphtha” is both set to one, i.e., $\underline{f}^{\mathrm{BLD}}_{\mathrm{HN,HN,R0}} = 1$ and $\overline{f}^{\mathrm{BLD}}_{\mathrm{HN,R0}} = 1$.

Table 5.5 Recipes for one unit of “Gasoline 95” and “Gasoline 98”

Output / Input	Gasoline 95			Gasoline 98	
	Recipe 1	**Recipe 2**	**Recipe 3**	**Recipe 1**	**Recipe 2**
Liquefied petroleum gas	0.0509	0.0384	0.04	0.03733	0.04
Light naphtha	-	-	0.01	-	-
Reformate 95	-	0.0662	-	-	-
Reformate 100	0.2997	0.2752	0.32	0.72821	0.73
Cracker naphtha	0.6494	0.6202	0.63	0.23446	0.23
	1.0000	1.0000	1.00	1.00000	1.00

Table 5.6 Recipes for one unit of “Jet fuel”

Output / Input	Jet fuel	
	Recipe 1	**Recipe 2**
Light naphtha	0.05	0.035
Heavy naphtha	0.10	0.075
Kerosene	0.85	0.890
	1.00	1.000

Table 5.7 Recipes for one unit of “Automotive gas oil”

Output / Input	Automotive gas oil		
	Recipe 1	**Recipe 2**	**Recipe 3**
Gas oil 1	0.39268	0.05044	0.10
Gas oil 2	0.30248	0.57093	0.65
Cracker gas oil	0.30484	0.22128	0.25
Kerosene	-	0.15735	-
	1.00000	1.00000	1.00

Table 5.8 Recipes for one unit of “Heavy fuel oil”

Output / Input	Heavy fuel oil		
	Recipe 1	**Recipe 2**	**Recipe 3**
Vacuum residue 1	0.13695	0.04366	0.02
Vacuum residue 2	0.57610	0.65565	0.68
Cracker gas oil	0.28695	0.30069	0.30
	1.00000	1.00000	1.00

The purchase costs and max supply of raw materials and variable operating costs and capacity limitation of all production units are listed in Table 5.9. The value of fuel gas as internal thermal energy is set to \$50 per tonne. The max storage volume for each component is 500,000 tonne while the min is 0 and the corresponding inventory cost is \$50 per tonne. The blending cost for recipes using three inputs is \$6 per tonne and that with four inputs is \$8 per tonne.

Table 5.9 Purchase costs and max supply of raw materials and processing costs and capacities of production units

	Cost ($/tonne)	Min capacity (10^3 tonne)	Max capacity (10^3 tonne)
Raw material			
Crude oil 1	845	-	1 500
Crude oil 2	749	-	1 500
Production unit			
CDU with Crude oil 1	5.8	1 000	2 000
CDU with Crude oil 2	6.2		
Reformer in REF95 mode	5.4	100	400
Reformer in REF100 mode	6.4		
Cracker in Naphtha mode	6.0	200	600
Cracker in Gas oil mode	6.0		

5.6.2 Numerical tests on Mechanism I

We first check the impact of volume interval magnitude α on the convergence behavior under Mechanism I. The initial prices $p_k^{P(1)}$ and $c_c^{S(1)}$ are both set to zero and initial volume intervals are roughly limited by aggregated minimum and maximum demands for the products or supply capacity, i.e., $\left[\underline{d}_k^{P(1)}, \bar{d}_k^{P(1)}\right] = \left[\sum_{m\in M} \underline{d}_{km}, \sum_{m\in M} \bar{d}_{km}\right]$ and $\left[\underline{s}_c^{S(1)}, \bar{s}_c^{S(1)}\right] = \left[\underline{l}_c^{BLD}, \bar{l}_c^{BLD}\right]$. It was believed that if we knew a good *starting point* and allowed for the right magnitude of deviation in the volume control, a reasonable convergence would take place (Bredström and Rönnqvist 2008). However, Figure 5.6 shows that without a good *starting point* the coordination scheme can still result in a perfect convergence. In some situations (Figure 5.6c and d), the appropriate choice of α greatly speeds up the convergence. That is, after 41 ($\alpha = 20\%$) and 8 ($\alpha = 40\%$) iterations, $\frac{z^{I} - \text{LBD}}{\text{LBD}}$ is less than 1%. Meanwhile, once the coordination is *convergent*, it is also *stable*. Therefore, the first mechanism using dual values as internal prices can provide good LBD to the optimal value defined by model [I].

The scaling factor α decides the size of volume intervals at the beginning of iteration, but plays a diminishing role with the increase of iterations due to the division of n. As pointed out in Section 5.4, the increase of n will lessen the volume interval, illustrating in Figure 5.7a. Now we assume that there is no decreasing deviation on volume intervals, which is then updated by following equations. The results are displayed in Figure 5.7b.

$$\underline{d}_k^{P(n+1)} = \max\left((1-\alpha)Y_k^{S}, \sum_{m\in M} \underline{d}_{km}\right) \quad (E3')$$

$$\bar{d}_k^{P(n+1)} = \min\left((1+\alpha)Y_k^{S}, \sum_{m\in M} \bar{d}_{km}\right) \quad (E4')$$

$$\underline{s}_c^{S(n+1)} = \max\left((1-\alpha)X_c^{P}, \underline{l}_c^{BLD}\right) \quad (E5')$$

$$\bar{s}_c^{S(n+1)} = \min\left((1+\alpha)X_c^{P}, \bar{l}_c^{BLD}\right) \quad (E6')$$

Figure 5.6 Convergence plot for Mechanism I with different volume interval magnitude α

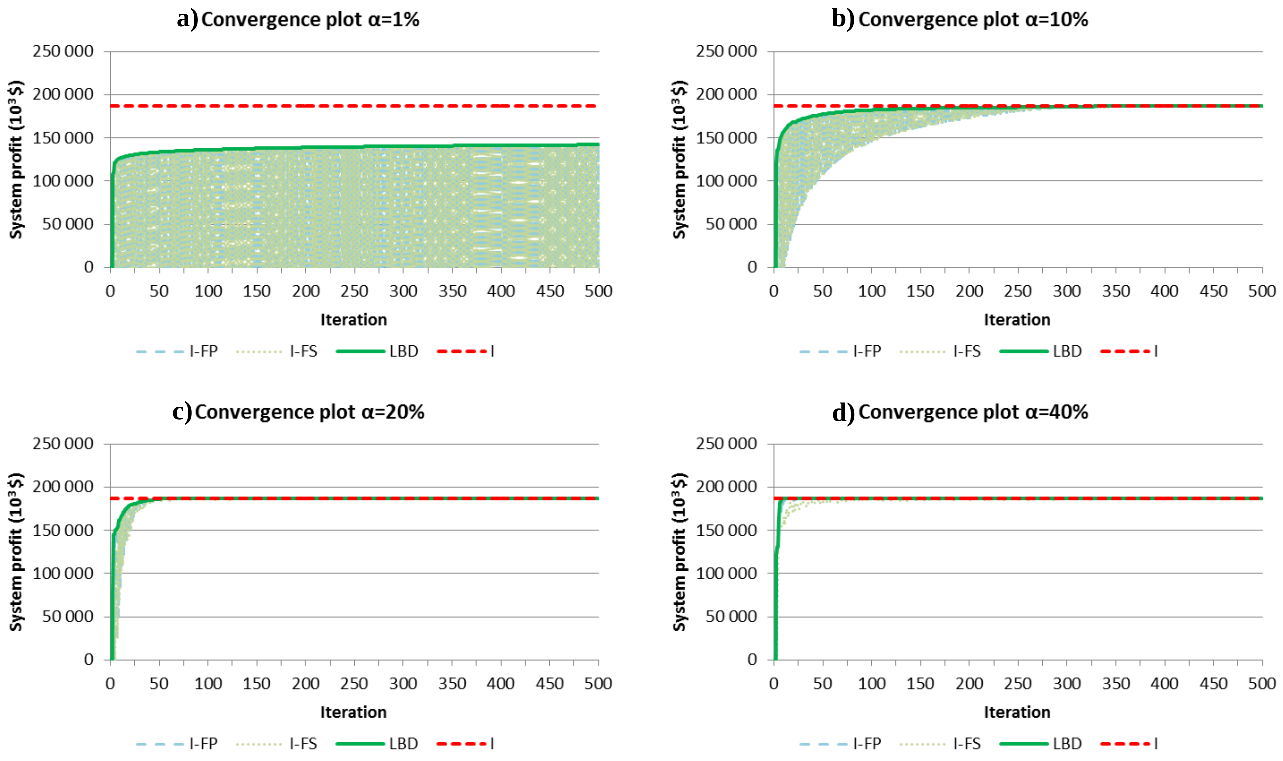
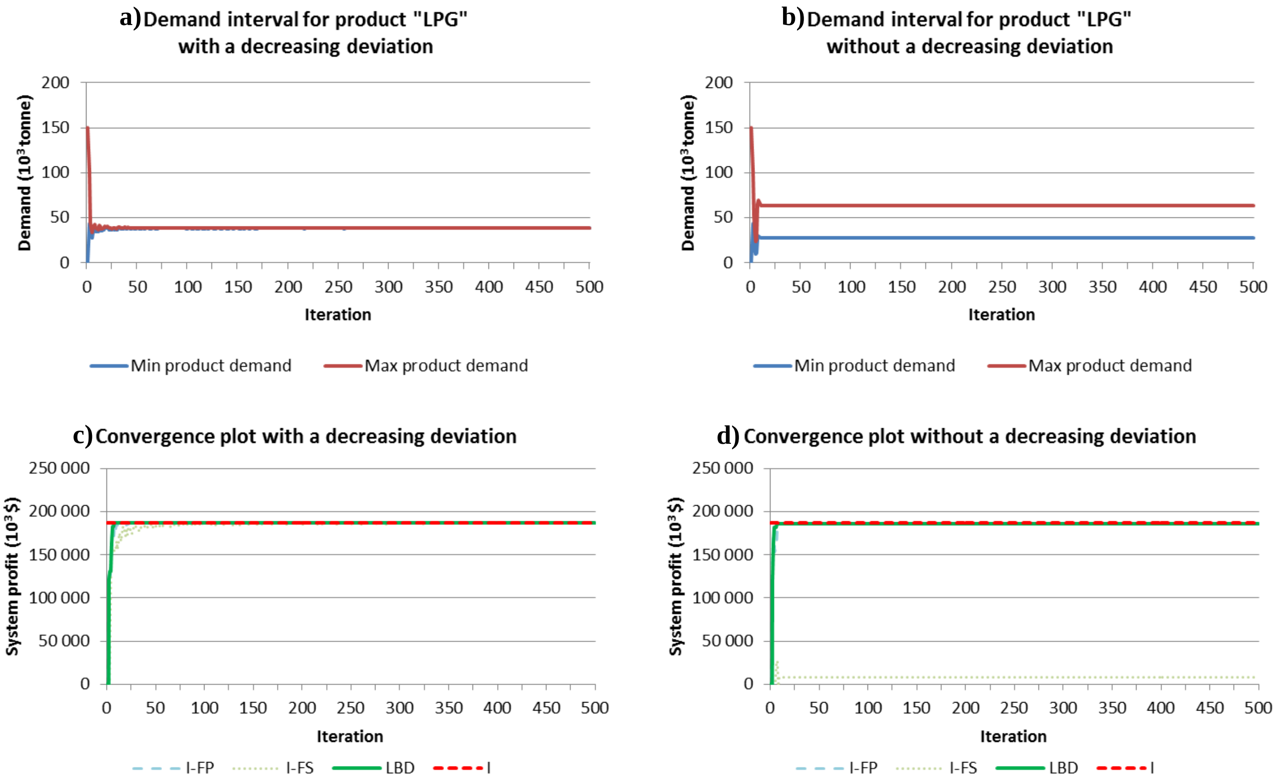


Figure 5.7 Impact of the decreasing deviation on convergence



Note: Volume interval magnitude $\alpha = 40\%$ in both instances.

One interesting observation from the result shown in Figure 5.7b is that although the LBD determined by $z^{\text{I-FP}}$ reaches the optimum quite early, the objective value with fixed sales solution $z^{\text{I-FS}}$ is far from the optimum. It is also proved in Table 5.10 where after 500 iterations the optimized volume for product Y_k^{P} in model [P] and Y_k^{S} in mode [S] are compared with Y_k^{I} obtained directly from model [I]. The solution to model [P] is similar to the optimal result of model [I] (less than 5%) and the objective value with fixed production plan $z^{\text{I-FP}}$ is near optimal. In contrast, because the sales planning maximizes its profit based on a flexible choice of available components, the solution to model [S] is hence much higher than the optimal result (between 10-30%), causing the misalignment for the overall performance $z^{\text{I-FS}}$. A decreasing deviation on required volume is essential to narrow down the range of choice and converge to the global solution.

Table 5.10 Optimized product volumes (10^3 tonne) in different models without a decreasing deviation

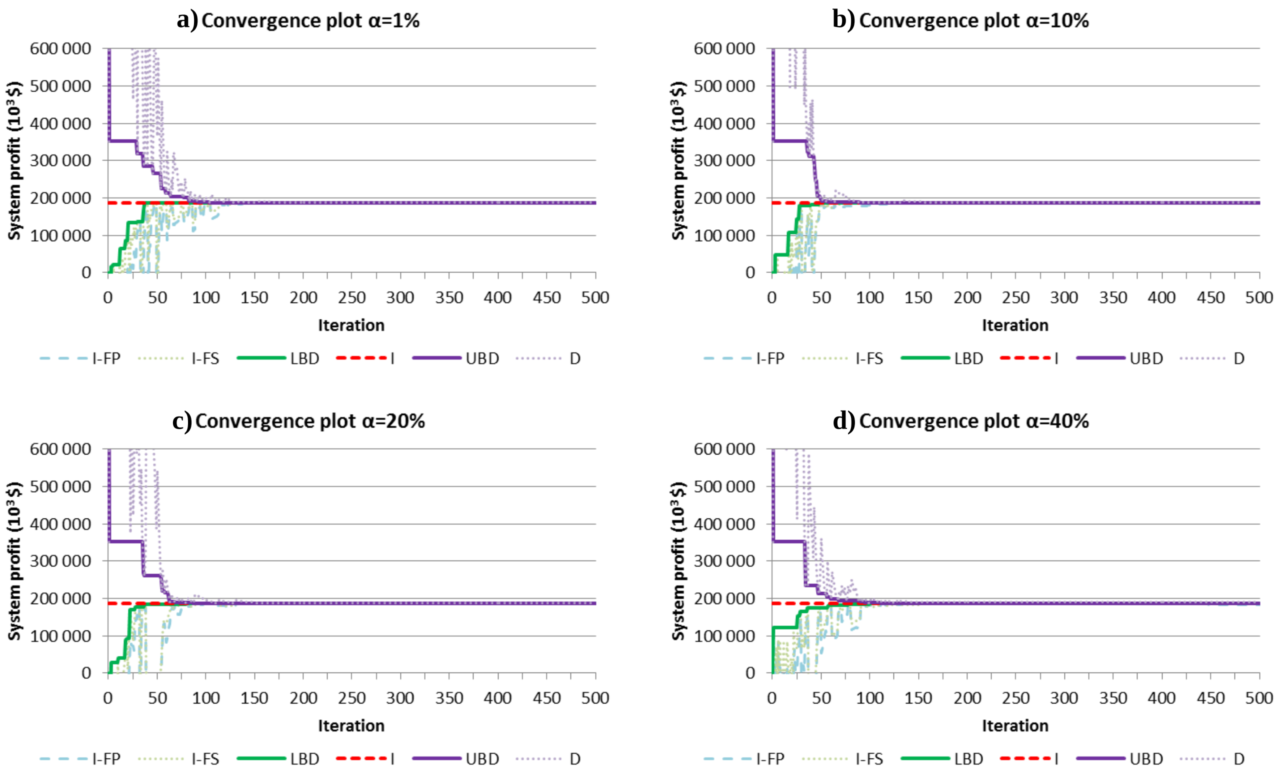
Product	Y_k^{I} **in model [I]**	Y_k^{P} **in model [P]** $(n=500)$		Y_k^{S} **in model [S]** $(n=500)$	
LPG	38.04	37.92	-0.3%	44.90	18.0%
LN	69.80	71.88	3.0%	81.83	17.2%
HN	98.66	103.63	5.0%	109.32	10.8%
G95	124.52	125.20	0.5%	150.61	21.0%
G98	52.05	50.21	-3.5%	68.14	30.9%
AGO	255.52	255.52	0.0%	264.04	3.3%
JF	125.58	119.94	-4.5%	163.79	30.4%
HFO	92.71	92.71	0.0%	120.16	29.6%

5.6.3 Numerical tests on Mechanism II

The second coordination mechanism employing Lagrange decomposition generates both UBD and LBD. In contrast to Mechanism I, the volume interval magnitude α has no effect on the convergence performance in Mechanism II (Figure 5.8). The iteration number is 104 ($\alpha = 1\%$), 91 ($\alpha = 10\%$), 87 ($\alpha = 20\%$), and 111 ($\alpha = 40\%$), respectively when $\frac{\text{UBD} - \text{LBD}}{\text{LBD}}$ is less than 1%. All the coordination is *stable* and volumes of products in model [P] and [S] both accommodate to the optimal solution to model [I].

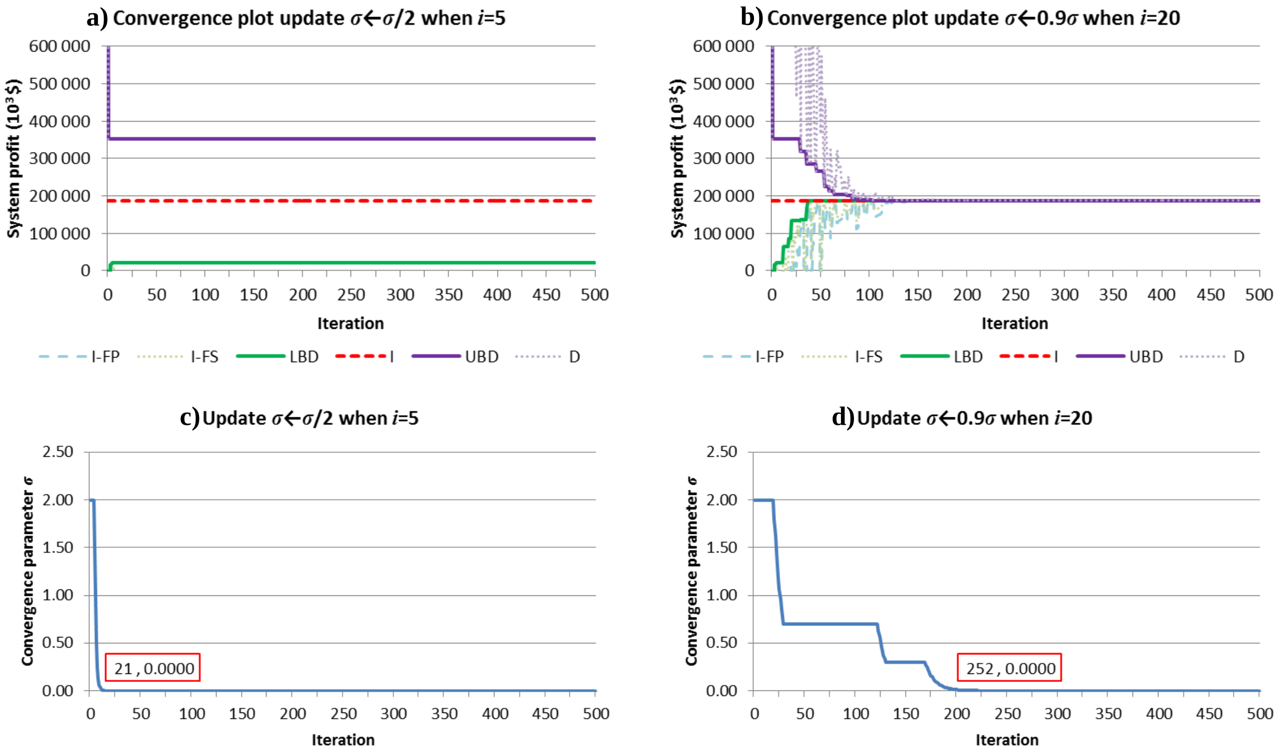
However, the practical convergence rate relies heavily on the heuristic choice of the convergence parameter σ. It determines suitable changes to step length, thus influencing the rate at which the Lagrangian multipliers are updated and enhancing convergence. We test two alternatives. One is to start with $\sigma = 2$ and then update $\sigma \leftarrow \sigma / 2$ if UBD has not improved during the last i iterations. The other is still to begin with $\sigma = 2$ but then update $\sigma \leftarrow 0.9\sigma$ if UBD fails to decrease within i iterations. i is set to 5, 10, 20 or 40, respectively. It turns out that proper combination of how to update σ after how many unchanged i iterations could ensure convergence, given distinct market demand functions. The complete comparison is listed in Appendix A.

Figure 5.8 Convergence plot for Mechanism II with different volume interval magnitude α



Note: Updating rule is to update $\sigma \leftarrow 0.9\sigma$ if UBD has not improved during the last $i = 20$ iterations in all instances.

Figure 5.9 Convergence plot and updating convergence parameter σ with different updating rules

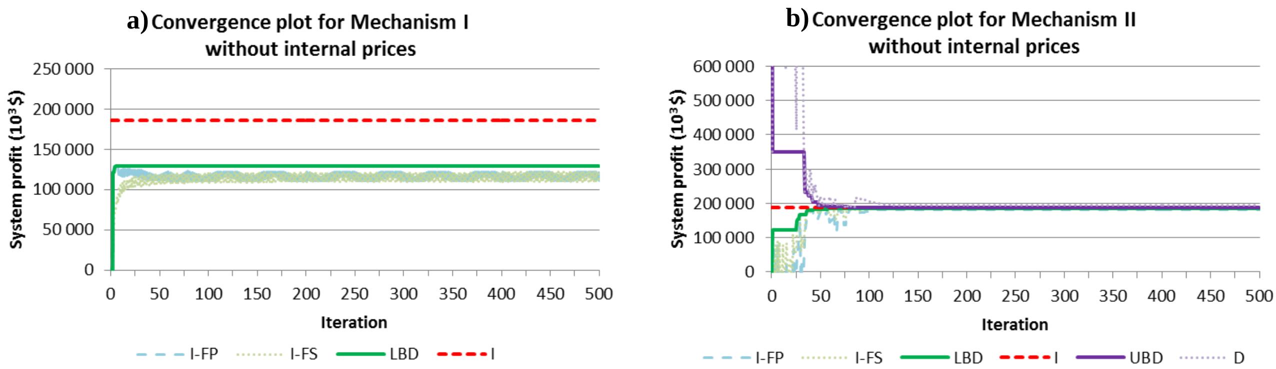


Note: Volume interval magnitude $\alpha = 1\%$ in all instances.

We have found that the better choice of updating rule for the problem studied seems to be $\sigma \leftarrow 0.9\sigma$, which can guarantee convergence in most cases compared to the more common choice $\sigma \leftarrow \sigma / 2$. The reason is that updating $\sigma \leftarrow \sigma / 2$ after small number of iterations, for example $i = 5$, makes the step length quickly drop to zero and then the Lagrangian multipliers become constant. The solutions hence get stuck and keep the poor performance hereafter (Figure 5.9a and c).

Finally, we assume the following situation: the production and sales divisions only exchange information about volumes of products and components but no prices. By comparing Figure 5.10a with Figure 5.6d, we can see that in Mechanism I internal prices have a significant impact on the convergence. Exchanging volume orders alone brings about degradation on overall performance. In contrast, in Mechanism II internal prices do not have the same importance for achieving system optimum (Figure 5.10b). In addition, although the LBD remains unchanged in Mechanism I (Figure 5.10a), we cannot guarantee the quality of LBD that is near or far from global optimum if there is no benchmark. By comparison, Mechanism II generates both LBD and UBD. Once they converge, the solution is optimal. There is no need for extra information to measure the solution quality. This is the main advantage of Mechanism I over II.

Figure 5.10 Convergence plot without internal prices for components and products in information exchange scheme



Note: Volume interval magnitude $\alpha = 40\%$ in both instances.

5.6.4 Variation in market demand scenarios

All the tests above are based on the assumption that the price for one product in one market is linearly dependent on its demand, i.e., $P_{km} = a_{km} + b_{km}Q_{km}, \quad \forall k \in K, \forall m \in M$. Thus, the sales revenue is $\sum_{k \in K} \sum_{m \in M} P_{km}Q_{km}$, resulting in a quadratic objective function. In this section, we assume that the price is a step function. In the real world, instead of changing with each amount, the price is more probably fixed at a level within a certain quantity segment and then decreases step by step. We use J to be the set of steps on the price-demand curve and introduce binary variables ω_{jkm} to identify which quantity segment the actual demand belongs to. That is,

$$\omega_{jkm} = \begin{cases} 1 & \text{if the demand of product } k \text{ in market } m \text{ is within } q_{j-1,km} \text{ and } q_{jkm}, \\ 0 & \text{otherwise.} \end{cases}$$

The unit price for each step p_{jkm} and quantity segment q_{jkm} are predefined as parameters and the demand can be expressed as

$$Q_{km} = \sum_{j \in J} Q_{jkm}, \quad \forall k \in K, \forall m \in M \tag{E17}$$

$$q_{j-1,km}\omega_j \le Q_{jkm} \le q_{jkm}\omega_j, \quad \forall j \in J, \forall k \in K, \forall m \in M \tag{E18}$$

$$\sum_{j \in J} \omega_{jkm} = 1, \quad \forall k \in K, \forall m \in M \tag{E19}$$

Equation (*E18)* and (*E19)* specify that the actual demand is only located in one quantity segment. Then the revenue becomes $\sum_{k \in K} \sum_{m \in M} \sum_{j \in J} p_{jkm} Q_{jkm}$. After the related element in the objective function and corresponding constraints are replaced, the original QP problem becomes a typical MIP problem. The possibility to express the step function is achieved at the expense of an increased number of binary variables.

On the other hand, we simulate two types of price curve. First is steep that refers to a wide range of choices of prices and second is much flatter, implying that the price is slightly affected by the demand. Figure 5.11 illustrates these four market demand scenarios, taking product "LPG" as an example. Note that tests before this section, if not specially mentioned, are all under market demand scenario 1.

Figure 5.12 demonstrates that both QP model and MIP model present good convergence properties by the Mechanism II using Lagrangian decomposition. Yet, the coordination in MIP model is not *stable*. The performance is improved with the increase of steps that approximates the linear demand curve. Overall, the integrated model with fixed sales planning $z^{\text{I-FS}(n)}$ has always produced better feasible solution than the one with fixed production planning $z^{\text{I-FP}(n)}$ because the volume constraints used in sales mode [S] are derived from the optimized production planning [P].

Table 5.11 summaries the performance for each test depicted in Figure 5.12. The first column indicates the iteration number when the gap between UBD and LBD is within the tolerance of 1%, which can be used as termination criterion in practice. The QP models with the linear demand function reach this specific tolerance more quickly than those MIP model with the step price function. For the instances of S2-5 steps and S2-10 steps, the gap is 2.4% and 1.6% after 500 iterations. The second column represents after 500 iterations the percent deviation of the LBD from the optimum by solving model [I] directly. The results show that in all the cases feasible solution is within 0.25% of the optimal solution, and therefore, the coordination under Mechanism II is satisfactory.

Table 5.11 Summary of performance under different market demand scenarios ($n = 500$)

Scenario	**Iteration number** n **when** $(\text{UBD} - \text{LBD}) / \text{LBD} \le 1\%$	**Percentage deviation (%)** $(z^{\text{I}} - \text{LBD}) / z^{\text{I}}$	**LBD (10^3 \$)**
S1 – linear	104	0.06	186 723
S2 – 5 steps	-	0.15	305 198
S2 – 10 steps	-	0.25	258 925
S2 – 20 steps	268	0.00	224 328
S3 – linear	126	0.03	424 595
S4 – 2 steps	138	0.01	484 882
S4 – 4 steps	189	0.21	455 141
S4 – 8 steps	178	0.01	440 424

Figure 5.11 Different market demand scenarios

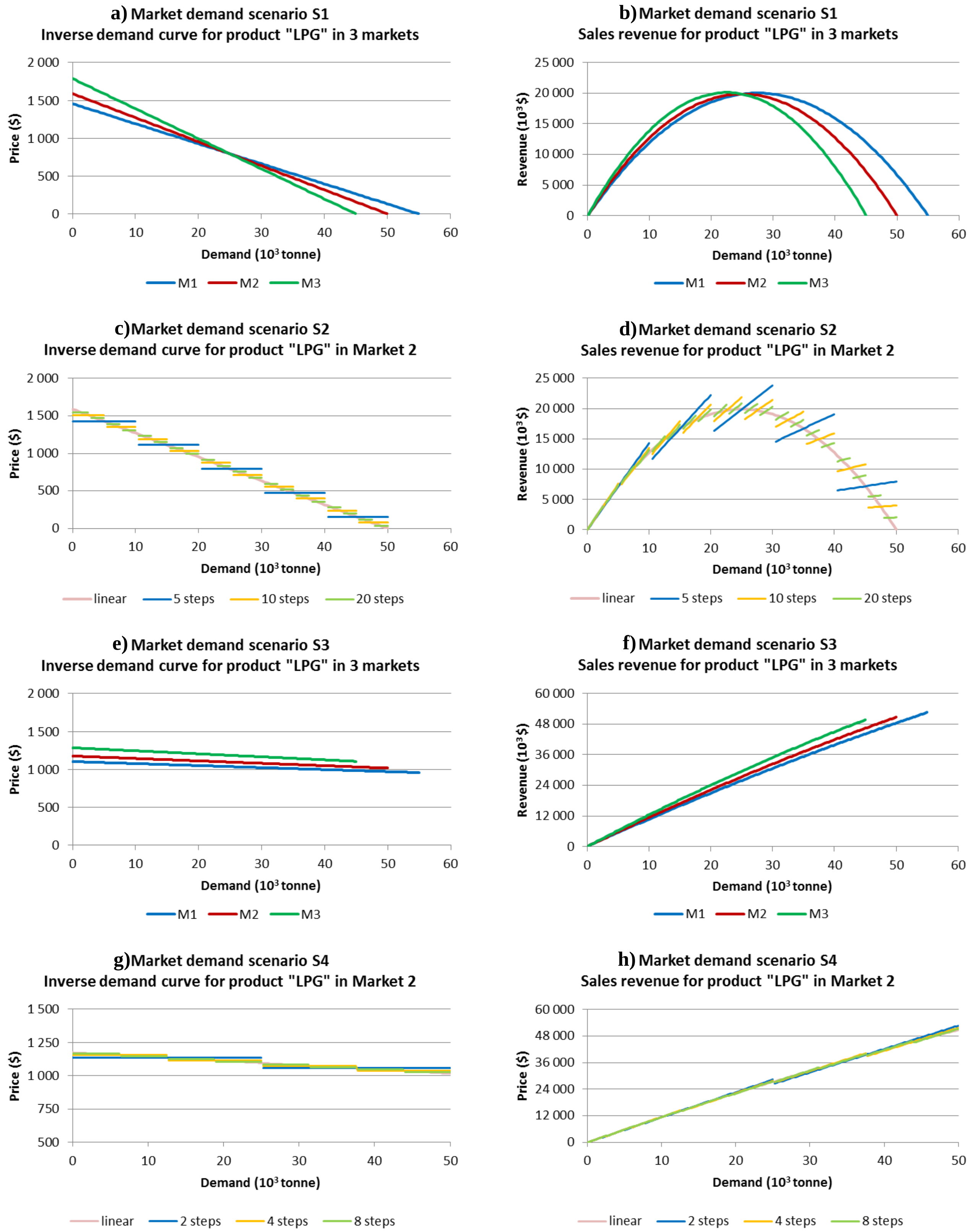
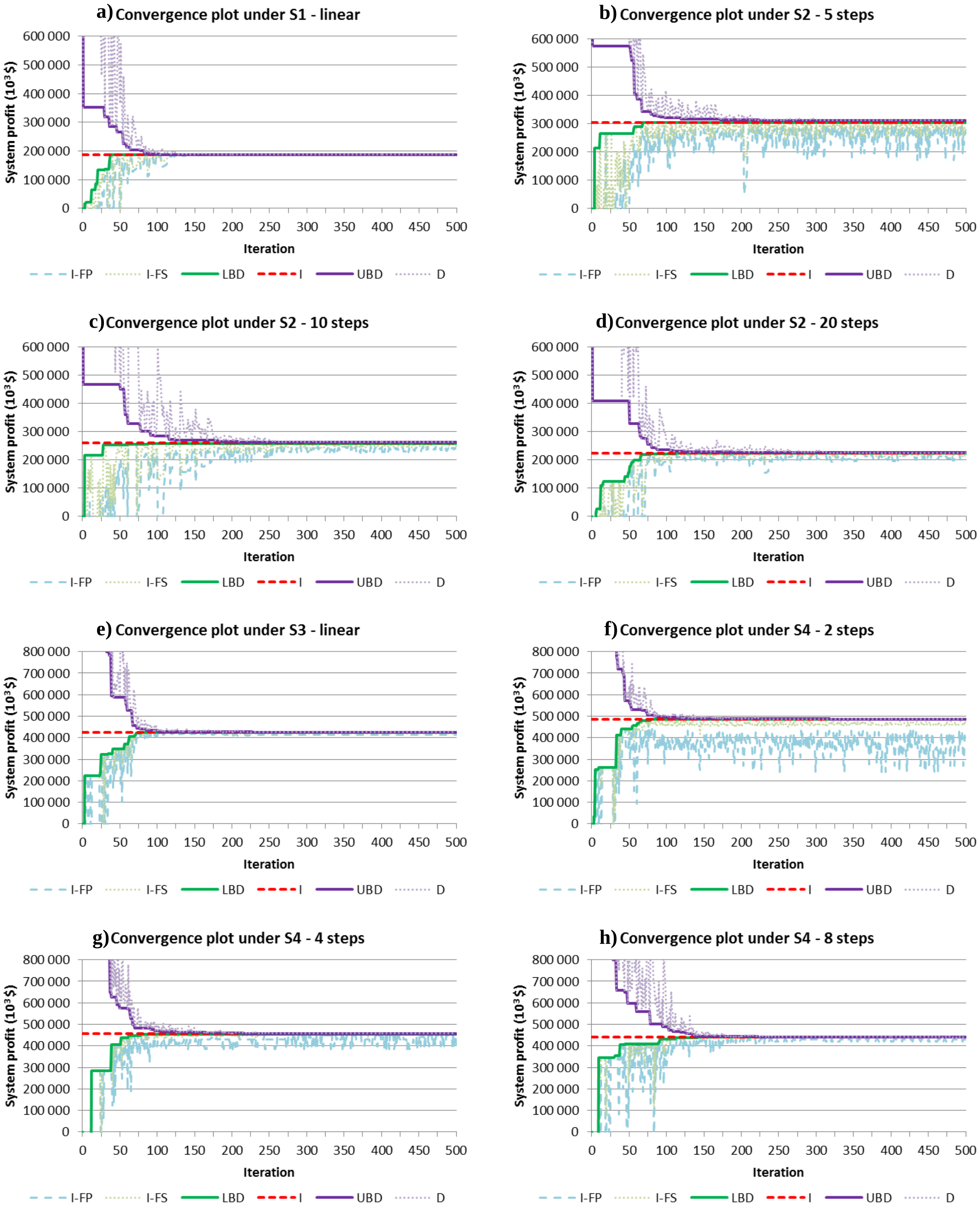


Figure 5.12 Convergence plot under different market demand scenarios



Note: Volume interval magnitude $\alpha = 1\%$ in all instances;
Update $\sigma \leftarrow 0.9\sigma$ if UBD has not improved during the last $i = 20$ iterations in S1, S3 and S4;
Update $\sigma \leftarrow 0.9\sigma$ if UBD has not improved during the last $i = 40$ iterations in S2.

5.7 Conclusions and future work

In this paper, we study the issues about coordination between production and sales planning at a refinery. We model the decision problems from the perspectives of the production manager, sales manager and the firm. It is shown that the coordination allowing the decoupled system to perform like a centralized one is achievable by introducing accurate setting of internal prices and proper restrictions on volumes. Two coordination mechanisms are proposed and have performed very well. They can be used to support both production and sales managers to make more efficient planning.

Mechanism I using dual values provides good pessimistic bound (LBD) on the optimal value defined by the integrated system with appropriate choice of volume interval magnitude. A decreasing deviation on required volume is also essential in convergence.

Due to the fact that Mechanism I cannot guarantee whether the LBD obtained is near or far from global optimum, Mechanism II based on theoretical sound decomposition principle is proposed to obviate this limitation. Lagrangian decomposition (LD) is, for the first time, applied as a coordination scheme to determine internal prices at a refinery. Mechanism II involving LD generates both optimistic (UBD) and LBD. The convergence behavior depends on the proper combination of how to update convergence parameter after how many unchanged iterations. Moreover, internal prices have a significant impact on the convergence in Mechanism I whereas have no importance for achieving system optimum in Mechanism II.

At last, four distinct market demand scenarios, two demand functions and two types of price curve, are tested under Mechanism II. Both quadratic programming (QP) model with a linear demand function and mix integer programming (MIP) model with a step price function present good convergence properties by Mechanism II using LD.

As a follow up for the present work, we would like to check whether the proposed methodology prevails in a greater sophistication of the model that employs non-linear blending restrictions and binary variables determining the use of process modes. Meanwhile, we can explore new methods of calculating marginal values of intermediate materials to better support pricing decisions (Hui 2000). On the other side, other techniques could be used to update Lagrangian multipliers, for example the various methods reviewed by Bazaraa and Goode (1979) and the recent method based on the Nelder-Mead algorithm (Wu and Ierapetritou 2006). In the computational study, we also found that even though the results are convergent to the same optimal solution under both Mechanisms I and II, the corresponding internal prices are in dramatic difference. We therefore would like to seek other alternatives to determine a set of stable and identical internal prices to facilitate practical use.

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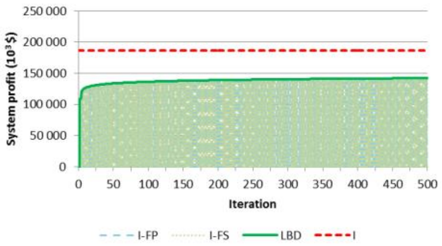
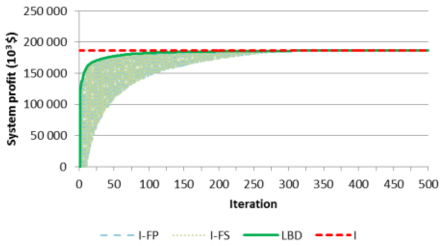
Appendix A Convergence plot under market demand scenario S1

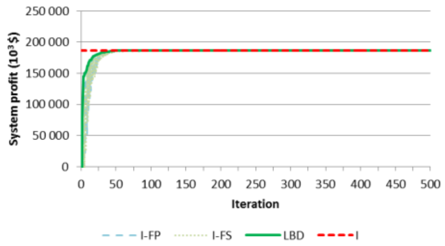
Updating rule	$\alpha = 1\%$	$\alpha = 10\%$	$\alpha = 20\%$	$\alpha = 40\%$

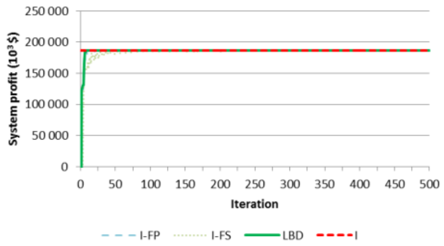
Mechanism I

Dual value



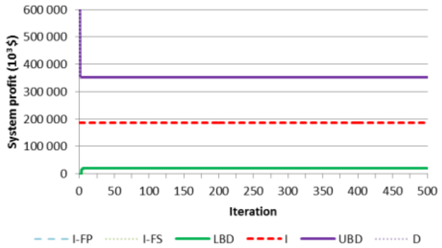


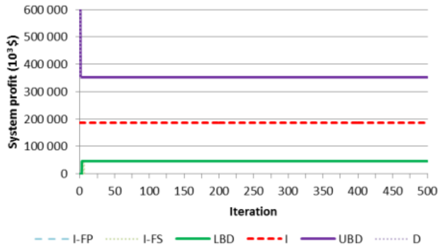


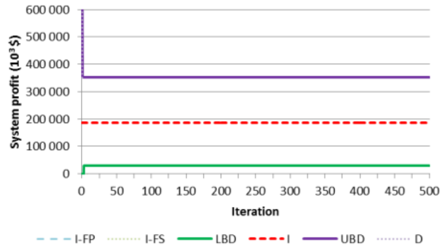


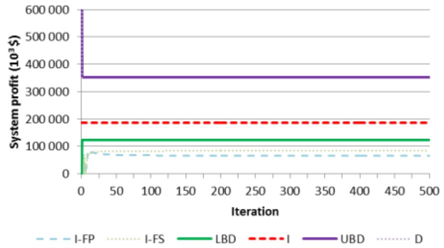
Mechanism II

$\sigma \leftarrow \sigma/2$, $i = 5$









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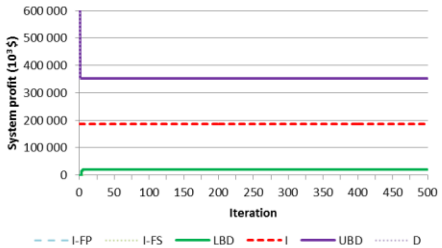

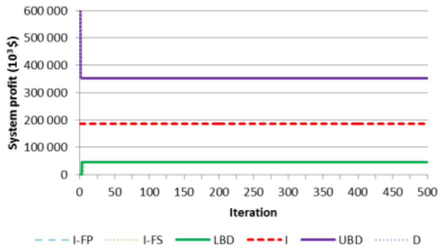

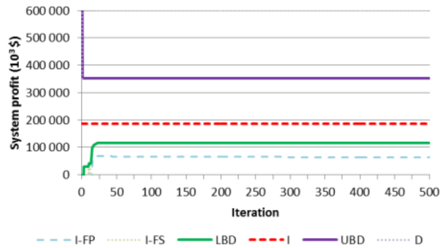

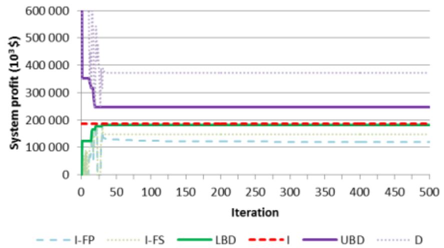

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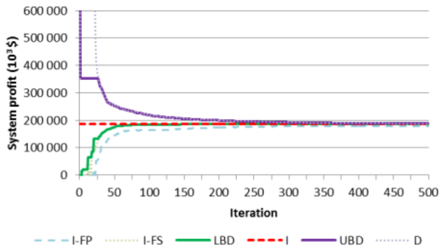

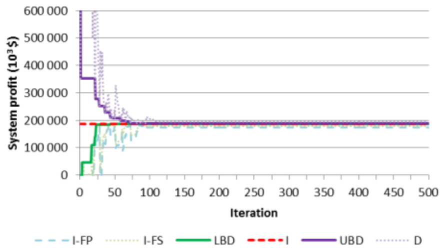

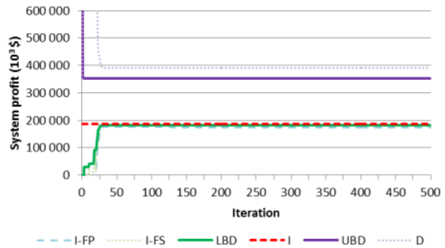

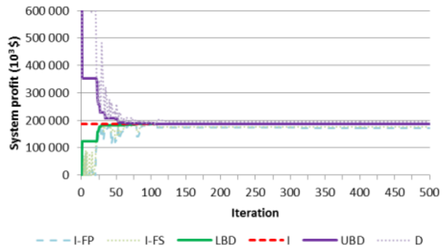

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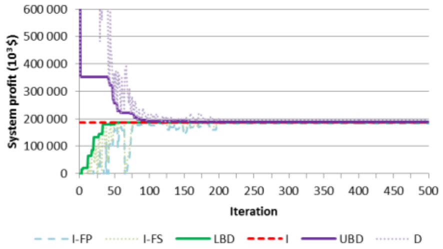
<td>
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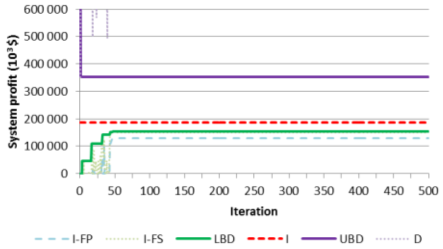
<td>
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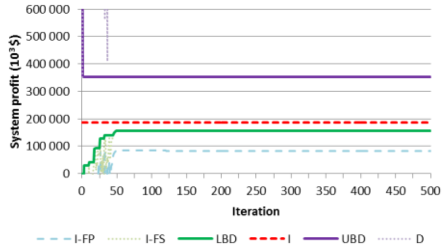
<td>
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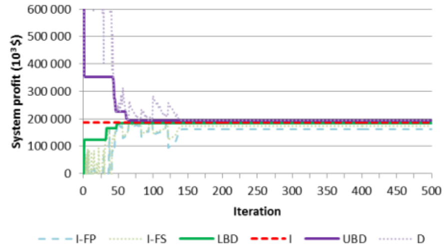
<td>
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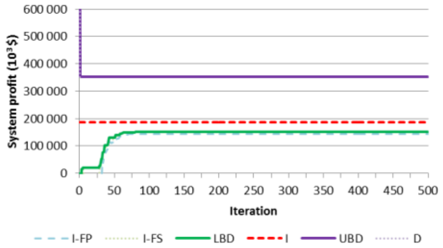
<td>
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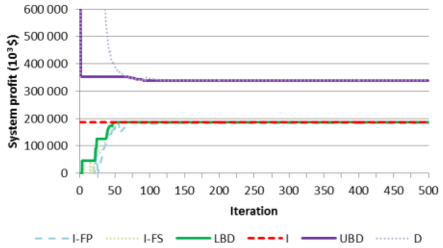
<td>
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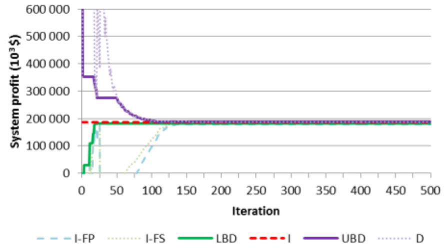
<td>
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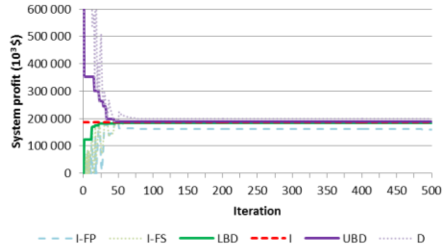
<td>
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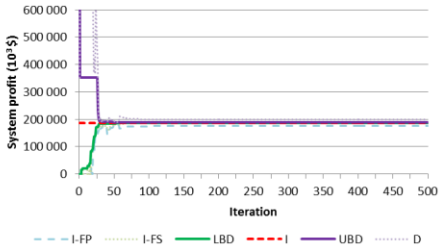
<td>
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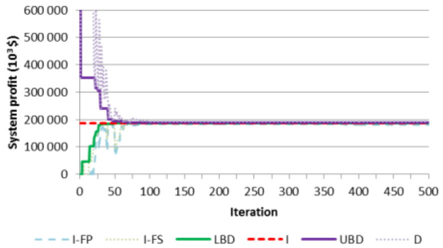
<td>
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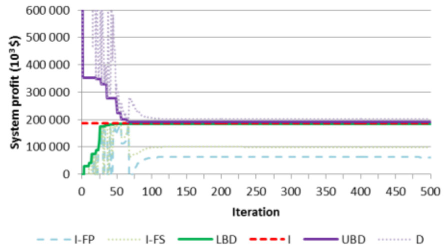
<td>
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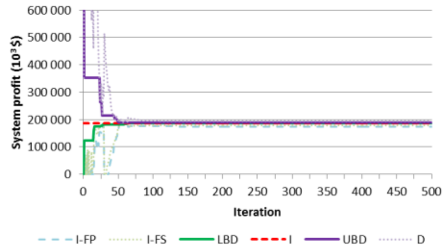
<td>
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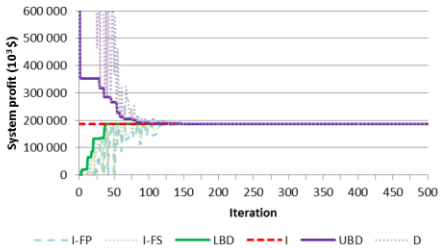
<td>
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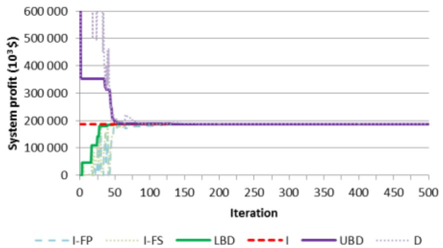
<td>
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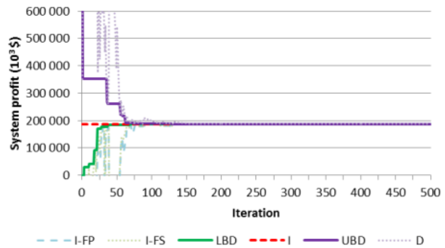
<td>
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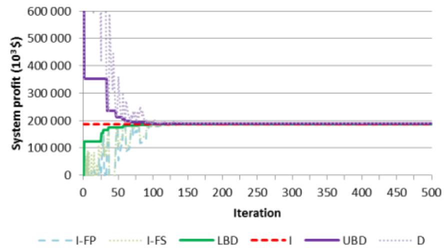
<td>
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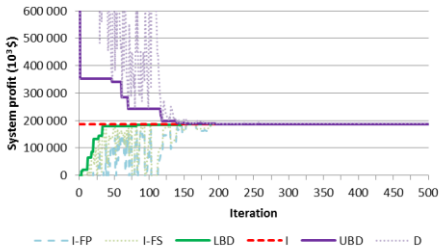
<td>
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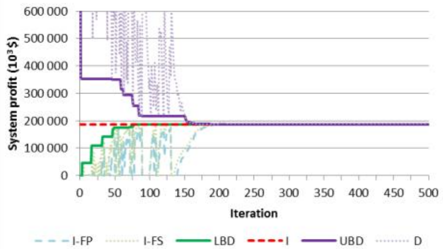
<td>
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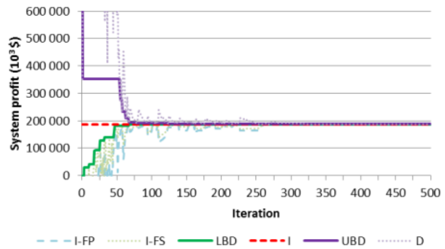
<td>
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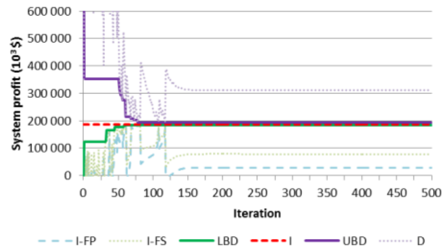
<td>
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