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Analysis of Autocallable Notes

Who reap the benefits? The issuer, facilitator, distributor, or the investor?

Andreas Garborg Sie and Jonas Blom Helmersen

Supervisor: Petter Bjerksund

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During the process of investigating the complex autocallables, we have faced and overcome numerous challenges that have proven to be very educational. We encountered new advanced derivatives concepts and met challenges regards to financial engineering. This have led us to get a better foundation of the derivatives field, in addition to become better at programming. We hope, and strongly believe, that we will benefit from the acquired knowledge after our graduation.

We want to thank our supervisor Petter Bjerksund who has been very helpful offering great discussions and good feedback throughout the process. We will also thank our families and friends for the continuous support.

The thesis assumes basic knowledge of option pricing theory.

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Andreas Garborg Sie

Jonas Blom Helmersen

Abstract

Autocallable structured products are complex instruments incorporating features and conditions that make them difficult to assess for potential investors. Despite this, they have become one of the most popular structured products in Norway. A potential reason is that many investors believe that these notes offer a high fixed coupon combined with limited risk. Finance experts do not share this belief suggesting that investors of these products are either ignorant or idiots.

In this thesis we analyse two autocallable notes offered in the Norwegian market. Our analysis suggest that investors pay price premiums of ca. 50% relative to the present value of the notes. The products have a high probability of negative and strong negative returns. In addition, the sales documents of these products appear biased. Based on this, it seems like the benefits of these notes are not reaped by the investor, but by the issuer, facilitator, and distributor.

Keywords – Autocalls, Structured products, Geometric Brownian motion, NHH

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1 Introduction

Autocallable structured products are a type of structured products that have become increasingly popular the past decade. Common for all structured products is that they are a package consisting of many financial instruments. Structured products have evolved over time resulting in a variety of products with differences in characteristics and complexity. Autocallables however, are a modern type of such structured products, with potentially more complex features, which make them difficult to understand. Despite this, they have become popular among private investors.

Autocallables as an investment alternative have received much criticism in Norway. Most of the criticism is based on the complexity of the products. Firstly, the complexity makes it hard for investors to understand how the products work and what they can expect in terms of returns. Secondly, the complexity makes it difficult for investors to know the true value of the products. Various experts have warned investors of these products. Despite this, there seems to be an attractive market for autocallable structured products in Norway as new notes are frequently offered. Garantum who is a broker of structured products claimed to have over 30,000 customers who in total had invested over 80 billion Swedish kronor in 2018 (Bjørklund, 2018).

In this thesis we will see if the criticism of these products can be justified. Our main goal is to find out who benefits from the autocallable notes. Is it the issuer who constructs the product? The facilitator and distributor who act as brokers? The investor? Or all of them? To give a good answer we will research two specific autocallable notes and answer the following questions:

1. *What are the fair prices of these products?*
2. *What can investors expect in returns from these products?*
3. *How are the products presented to potential investors?*

Outline

Chapter 2 of the thesis will present traditional structured products and autocallable structured products. In the discussion of autocallable notes we will present various features to illustrate how these notes might differ. The goal is to give the reader a clear intuition of how these products work and how they can generate return or losses for investors. We will also present the specific notes that we will value in this thesis.

Chapter 3 will present the valuation model used to value the autocallable notes. This chapter will cover the option pricing theory applied in the model and illustrate how the valuation model works. Chapter 4 will present the data and parameters used to value the notes.

Chapters 5, 6, and 7 will analyse the autocallable notes. In chapter five we will value the notes and compare the fair price derived from our valuation model with the actual price of the notes. Chapter six will identify expected returns and returns distributions of the notes. Whereas chapter seven will look at how the notes are sold in the Norwegian market with great focus on the marketing material used.

2 Structured Products

Structured products are typically composed of three elements. An obligation, a derivatives product, and lastly, one or more underlying assets. Issuers of structured products package these three components to make a single structured product (Osphare-Druilhe, 2021). The only limitations of these products are the fantasy of the issuer and current demands from investors.

Since structured products were first introduced to private investors in 1996 they have grown rapidly in popularity. One explanation for this is that structured products offer retail investors easy access to derivatives where they can invest in products that offer customized exposure to hard-to-reach asset classes. Another reason might be that structured products often are principal protected, meaning that the investor will have a minimum return equal to the initial investment. However, principal protection is not always a feature of structured products.

There are many potential risks that the investors of structured products should be aware of. Firstly, there is a lack of liquidity in the structured products market due to the highly customized nature of the products. Thus, investors should generally invest in structured products with a buy-and-hold attitude as the products are hard getting rid of once invested. Secondly, structured products vary in complexity. Some products are so complex that the risk and return profile becomes unclear for the investor. Lastly, there are risks regarding the issuer's credit quality. If the issuer of the product goes bankrupt, the investor might lose the invested money.

This chapter will present the role structured products have had in Norway and how traditional structure products have developed in to modern structured products such as the autocallable note. The discussion will start with the traditional guaranteed structured products before introducing autocallable structured products. To wrap up the chapter we will present the two specific autocallable notes we will analyse in this thesis.

2.1 Structured products in Norway

Structured products were first introduced in Norway in 1992. Originally, they were only offered to institutional investors, but in 1996 DnB started to offer these products to private investors as well (Bøe, 2007). The structured products offered are categorised as guaranteed structured products, characterised by a certain bond element where the interest paid by the bond is allocated to an uncertain derivatives element. These products are designed such that investors are guaranteed a minimum amount, often equal to the original invested amount, in addition to an uncertain potential return dependent on the derivatives part. The products became popular among private investors, and in 2006 private investors made up 90% of all outstanding structured products (Bøe, 2007). Figure 2.1 displays the total outstanding volume of guaranteed structured products in the time period from 2000 to 2009 using data from Bøe and SSB (2007; 2009). The growth was high and stable between 2000 to 2005. In 2005 the total volume was 47 billion NOK, more than nine times higher than in 2000.

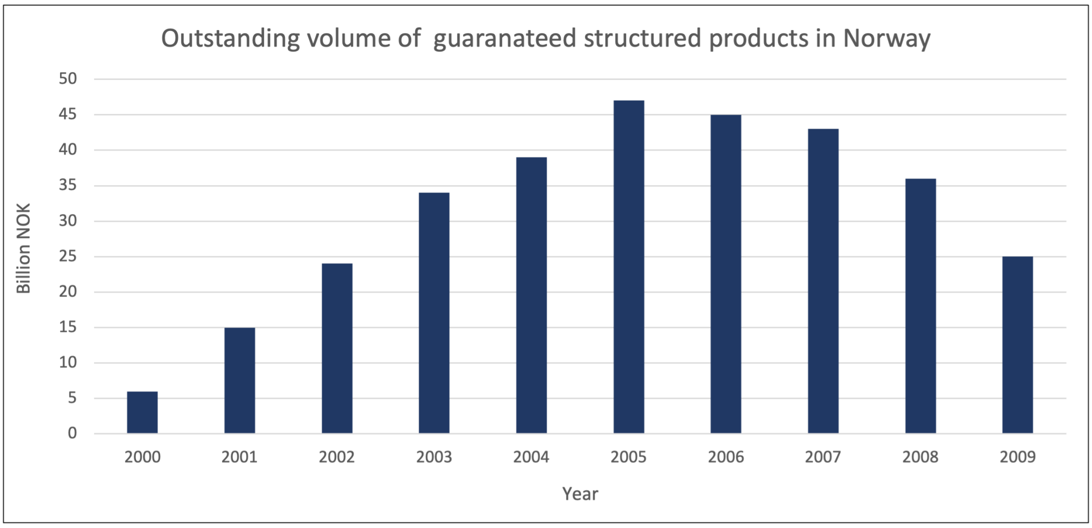


Figure 2.1: Outstanding volume of structured products in Norway 2000-2009

What Figure 2.1 also illustrates is that the growth stagnated between 2005 and 2006. By looking at the numbers we can see that the total volume of outstanding structured products decreased significantly between 2006 and 2009. During this period the products received much critique in media, which was specially directed at the gearing and advertisement of these products. As a result of the critique, regulations of structured products tightened between 2006 to 2008. The tightened regulations required investors to have more knowledge of the structured products, and it was also stated that private investors normally would not have the necessary knowledge to understand the risks of structured products. This resulted in a stop in sales of guaranteed structured products to non-professional investors (Helgerud, 2012).

In the 2010s we have seen a rise of new variants of structured products being offered to private investors in Norway. Modern structured products differ from the guaranteed structured products because of the regulations and interest rate environment. The interest rates in the western world, and Norway, decreased significantly during the 2000s and especially after the financial crisis in 2008. Lower interest rates affected the structured products. Traditionally, the guaranteed structured products packaged a bond which used the interest paid to finance a long position in a derivatives component. Certain modern structured products however, such as the autocallable note, package the bond with both long and short positions of derivatives with the goal of increasing the fixed interest payment. The next section will present autocallable notes in more detail.

2.2 Autocallable structured products

Autocallable structured products are a specific type of structured products. Since first introduced by BNP Paribas in 2003, autocallable structured products have been an influential force in flow derivatives. A flow derivative is an instrument that provides maximum leverage to profit from small movements in the market value of the underlying. Autocallable products typically consist of a fixed income part and two or more barrier options that depend on one or more underlying assets.

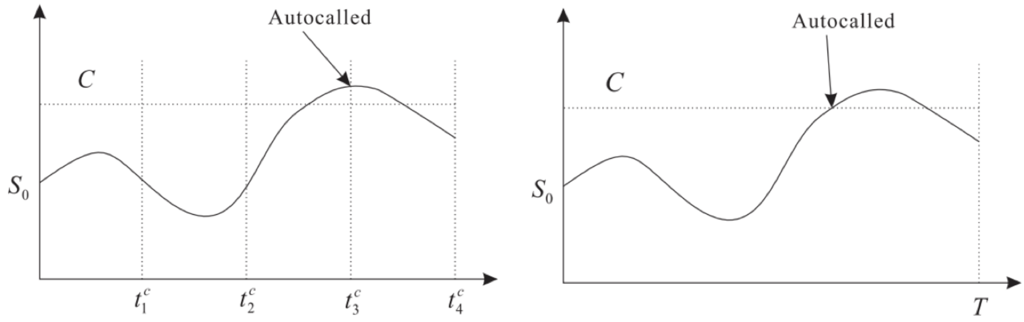
The main idea of autocallable structured products is to pay predetermined coupons at predetermined dates, just like a normal bond. However, the coupon payments for autocallables contain some uncertainty in that they depend on the value of one or more underlying assets. The underlying asset(s) must be equal to or higher than a predetermined reference value named the coupon barrier for the coupons to be paid out. Another aspect of autocallables and the reason behind the name is that they might be terminated or “autocalled” before maturity. When the product is autocalled, the investor is repaid the notional value plus one coupon payment, and all future coupons are cancelled. The autocall occurs when the value of the underlying asset(s) is above or equal to a predetermined reference value called the autocall barrier. Typically, autocallable structured products also have something called a risk barrier. If a product is not autocalled before maturity, and the reference value of the underlying asset(s) is below the risk barrier at maturity, then the investor will not be repaid the notional value. Instead, the investor will receive the reference value of the relevant underlying asset. In the worst case scenario, this reference value is 0 implying that the investor might lose the entire investment. Therefore, we often say that autocallable structured products are not principal protected. The risk barrier can be viewed as a written put option on the bond. Thus, the coupons of the autocallable should increase as the risk/volatility of the underlying asset(s) increases to reflect the higher value of the sold put option.

A question to be asked is why investors invest in autocallable notes. Some investors may view autocallables attractive due to the potential high promised returns offered in low yield environments. If stock markets move sideways the autocallable note may promise higher returns than stocks. The autocall feature may also be perceived as attractive as it offers flexibility if the market rallies. However, a disadvantage of the autocall feature is that high underwriting costs might offset the returns if the note is autocalled early. The obvious disadvantage of the autocallable note is the written put option (risk barrier), which expose the investor to the downside of the underlying stock(s).

2.2.1 Features of autocallable structured products

Features of autocallable structured products vary, resulting in many different variants. The most common variations are tied to discrete/continuous call dates, number of underlying assets, currency of underlying asset(s), and features of the coupon payment structure.

Autocallable notes can have discrete or continuous call dates. Discrete autocallables can only be autocalled if the autocall barrier is crossed at an observation date. An observation date is a date where the prices of the underlying assets are observed to determine if there will be a coupon payment or if the note should be autocalled. Continuous autocallable notes on the other hand are called immediately when the relevant underlying asset hits the autocall barrier. This implies that continuous autocallable notes are more likely to be autocalled compared to the discrete autocallable notes (Deng et al., 2011). The difference between discrete and continuous call dates, for autocallable structured products, is illustrated in Figure 2.2.



(a) Discrete Callable

(b) Continuous Callable

Figure 2.2: Discrete and continuous call dates

Autocallables may be referred to as univariate or multivariate. Univariate autocallables have only one underlying asset, whereas multivariate autocallables have two or more underlying assets. The most popular autocallable today is the worst-of multivariate where the relevant underlying at all times is the worst performing asset. For such products, investors should be aware of the correlation between the underlying assets. If for example, the correlation is low, then there is a higher chance that one of the assets will perform badly.

Some autocallable notes have underlying assets denominated in another currency than the settlement currency. The settlement currency refers to the main currency of the product, i.e., the currency of the notional amount and coupon payments. Autocallable notes where the underlying assets are denominated in another currency than the settlement currency usually have a *quantos* structure. Quantos are known for facilitating foreign returns in domestic currency. For example, if the return of a foreign underlying asset is 20%, then the return of the option would be 20% as well, independent of the exchange rate. This means that the size of the domestic payout is directly dependent of the foreign asset(s)' returns, and thus uncertain (Bjerksund et al., 1999).

Features in the payment structure of coupons vary in that they might be accumulated or not. For coupons that are not accumulative, any missed coupon barriers will result in those coupons being lost. However, if the same note had an accumulative feature, the missed coupons would instead accumulate and been paid out if the coupon barrier once again was crossed.

2.2.2 Autocall Norske Selskaper and Autocall Telekom

Now that we have presented what autocalls are and how they might differ, we want to present two specific types of autocalls that we will focus on in our thesis. The specific autocalls are named *Autocall Norske Selskaper* and *Autocall Telekom* (Garantum Fondkommission, 2021a,b). They are both structured by Goldman Sachs and sold to Nordic customers through the broker firm Garantum. Their main features are summarized in Table 2.1. As the table suggests, they have many similarities, but there are differences in the underlying assets and the coupons.

	Name of Autocall note	
Description	Autocall Norske Selskaper	Autocall Telekom
Starting day	29th October 2021	
Last possible ending day	18th November 2026	
First possible autocall date	31th October 2022 (4th observation date)	
Observation dates	20	
Autocall barrier	90%	
Coupon barrier	80%	
Risk barrier	60%	
Multi/Univariate	Multivariate	
Relevant underlying	Worst-off	
Discrete/Cont. observations	Discrete	
Coupons	Accumulative	
Coupon paid	Quarterly, 14 business days after observation	
Underlying (Ticker)	NHY, YAR, TEL, SALM	NOKIA, ERIC, VOD, TEL
Quantos structure	No	Yes
Quarterly coupons[1]	4.00%	3.65%

Table 2.1: Main features of the autocall notes *Autocall Norske Selskaper* and *Autocall Telekom*.

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[1]Confirmed coupons for the autocallable notes are found in Garantum Fondkommission and Garantum Fondkommission (2021c; 2021d)

Both notes started on 29th of October 2021 and will run until 18th of November 2026, if they are not autocalled at an earlier point in time. They both have notional values of NOK10,000 per certificate and incur significant costs. Firstly, there are underwriting fees of 3%, which means the investor must pay NOK10,300 per certificate. Secondly there are facilitating costs of 6% that are said to be included in the notional value of NOK10,000. Lastly, there are also issuer costs indicated to be 3.05%, also included in the notional value of NOK10,000. In total there are costs of 12.05% based on the notional value. The way we interpret this is that the investor must pay NOK10,300 for something that the issuer values to NOK9,095[1]. However, this value is as we will see in Chapter 7 not communicated directly in the prospect. In later chapters we will value these products to check whether or not the price of NOK10,300 paid by investors is acceptable. We will also analyse the sales process of these products to get an understanding of how these products are presented and sold to investors.

[1] $9,095 = 10,300 - (10,000 \times 12.05\%)$

3 Valuation model for autocallable notes

The autocallable structures are complex in that they are path dependent, have accumulative coupons, and a potential early maturity. Because of this complexity, we cannot value the products with a closed form solution. Instead we must use a numerical method. A popular method to price such complex products is with the Monte Carlo simulation approach. The idea behind the Monte Carlo simulation approach is to simulate a random variable numerous times and using the average value as the estimate for the given variable's expected value (Glasserman, 2003). In our case, to simulate the price of the autocallable notes, we must simulate the underlying stock paths and tie them to the payoff function. As the stock path model is a stochastic model, every unique iteration of the simulation will yield a different price path generation, and thus a different price estimate. If we simulate these prices enough times, then the law of large numbers will ensure that the average price will converge to the correct value of the product.

We will now introduce all relevant theory needed to perform the numerical valuation of the autocallable strucutres using the Monte Carlo approach. The chapter can be summarized as follows; In Subsection 3.1, we present the Black & Scholes model. This model is the underlying reason for why we can produce a fair price estimate of the autocallables. In Subsection 3.2, we present the underlying theory of Monte Carlo simulations. In Subsection 3.3, we present the stock price model we use to model the underlying stock prices. Lastly, in Subsection 3.4, we present the payoff function. In the Chapter 4 we will discuss how the model parameters are defined. In Chapter 5, we will perform the actual valuation of the two autocallable notes.

3.1 The Black & Scholes model

When valuing structured products we operate in a risk-neutral framework called the Black & Scholes model. The risk-neutrality of this model is what allows us to make a fair price estimate of the products, and is the cornerstone of modern option pricing. Brandimarte sums up the key results of the valuation framework in his book *Handbook in Monte Carlo simulation* (2014). Namely, given a risk-free interest rate and a risk neutral probability measure Q, one can price an option by taking the expectation of the discounted future payoffs. The expectation must be taken under the aforementioned risk-neutral measure, which is often called the *equivalent martingale measure*, and the discounting must be done using the risk-free interest rate. These findings are great news for us, as it eliminates the necessity of having to find the subjective expected rate of returns and the associated discounting rates of the underlying assets, in order the price the structured products.

Going from the real probability measure to the equivalent martingale measure is trivial in the Black & Scholes model. This is accomplished by replacing the expected rate of return parameter in the stock price model, with the objective risk-free rate. We will explain this further when we introduce the stock price model in Section 3.3. Utilizing the Monte Carlo simulation method, taking the expectation of discounted future payoffs is also a trivial exercise. Thus, the difficulty in the valuation of the autocallable structures boils down to how we tie the simulated stock prices to the payoff function.

The Black & Scholes model makes a couple of assumptions which must be clarified. These are listed below.

1. The market is complete and there are no arbitrage opportunities.
2. There are no transaction costs, and investors have no restrictions regarding short-sales, nor borrowing.
3. The volatility, expected rate of return, dividend, and the risk-free rate are constant parameters.
4. Trading occurs in continuous time.
5. Stocks follow a geometric Brownian motion.

The first assumption is necessary for the risk-neutral valuation to hold. A risk-neutral investor is indifferent between a certain and an uncertain payout with equal expected return (McDonald, 2014). The law of one price ensures that there will be no arbitrage opportunities which is a crucial element in risk-neutral valuation. Assumption five will be introduced in Section 3.3. As for assumption two, three and four, these will be implemented through the geometric Brownian motion model.

3.2 Monte Carlo method

The Monte Carlo method is a mathematical technique for simulating stochastic systems numerically. As mentioned above, in a risk-neutral valuation framework we can find the option value by taking the expectation of a discounted future payoff. In short, this is precisely what the Monte Carlo technique do. Thus, in cases where there are no easy ways of finding the option value, Monte Carlo simulation, in combination with the risk-neutral valuation framework, is a very handy tool. The pricing of autocallable structures is such a case, as there is no analytical solution to the problem.

Monte Carlo simulation utilizes the randomness of the stock price model to build a probability distribution of possible outcomes (Brandimarte, 2014, p.3). By drawing from the underlying model's probability distribution, the Monte Carlo model generates numerous scenarios. The draws must be individual and identically drawn (IID) to ensure that each scenario is random. This will provide a range of possible outcomes together with their respective probabilities. The key feature with the simulated estimate-range is that the average estimate will, by the law of large numbers (LLN), converge to the model's expected value as the number of simulations increases. Thus, in the aspect of finding the value of a structured product, simulating the underlying stock paths and the associated payoff function thousands of times are essential to get a good estimate of the product's value. Eq. 3.1 shows how the Monte Carlo method utilizes the LLN, where N is the number of iterations, g^n represents the individual discounted payoff, and μ_g is the true value of the payoff.

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} g^n = \mu_g \tag{3.1}$$

According to MIT, the advantage of using Monte Carlo simulation is that the value estimate is unbiased (na). When using the Monte Carlo framework it is important to consider the standard error of the sample mean. In Eq. 3.1, the sample mean converges towards the true expected value when N increases. However, it is worth noting that the standard error of the estimate decreases with the square root of the number of simulations[1].

[1]Equation showing how the standard error of the Monte Carlo estimator $\sigma_{\bar{y}}$ decreases by the square root of the number of simulations N: $\sigma_{\bar{y}} = \frac{\sigma_y}{\sqrt{N}}$

This implies that to decrease the variation of the sample mean by a factor of 10, one must increase the number of simulations by a factor of 100. Thus, it is necessary to consider the computational time of the Monte Carlo simulations. For instance, in complex situations, the number of required simulations could be problematic as it becomes too time consuming. In our case, the computational cost is manageable as we are easily able to simulate enough iterations to reduce the standard error.

3.3 Geometric Brownian motion

3.3.1 Properties

Geometric Brownian motion (GBM) is the most popular model used to model stock prices. As mentioned in Section 3.1, the GBM is the stock price model used in the Black & Scholes model. The GBM models the change in a stock price as a combination of a deterministic drift-term and a stochastic dispersion-term (Hull, 2018, p.331). The drift-term is driven by a drift parameter over the time interval dt, while the dispersion term is driven by a random shock scaled by a volatility parameter σ. The random shock is called a Wiener process, and is nothing more than a normally distributed random variable with mean zero and variance equal to the time interval, $dW_t \sim N(0, dt)$. Eq. 3.2 illustrates the GBM model as a stochastic differential equation where we use the expected rate of return μ as the parameter driving the drift-term. The model needs a starting value, called the initial value, which is given by $S_0 = s > 0$. The equation models how the stock price changes from time t over a very small (infinitesimal) time step dt.

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{3.2}$$

Instead of showing the change in the stock price we can instead utilize two mathematical techniques, called *Itô's Lemma* and the *Feynman-Kac solution*, to compute the final date solution to the process instead (Glasserman, 2003). We can also utilize that the Wiener process is equal in distribution to a random variable z multiplied with the square root of the time interval. Thus, as $W(t) \stackrel{d}{=} \sqrt{t}z$ and $z \sim N(0, 1)$, we can write the solution to the GBM as Eq. 3.3.

$$S_t = S_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}z} \tag{3.3}$$

where S_t is the time t stock price and the S_0 is the initial stock price.

The GBM makes a couple of assumptions. Firstly, the assumptions under the Black & Scholes model are also relevant for the GBM. Secondly, the GBM is nothing more than an exponentiated Brownian motion. Taking the logarithm of the stock price from the GBM will return a Brownian motion (Glasserman, 2003, p.93). As the Brownian motion is normally distributed, it then follows that the geometric Brownian motion is lognormally distributed. This property is important as it stops the stock prices to become negative. Due to the the limited liabilities assumption of the stock market, negative prices are unreasonable (Bodie et al., 2018, p.41). Additionally, the GBM follows a Markov process (Hull, 2018, p.324). This implies that the only information relevant for pricing future stock prices lies in the current price of the stocks. This also imply that price changes are independent of each other. Lastly, the GBM assumes that the expected dollar return is proportional to the stock price level. In other words, if the expected rate of return is 12% when the stock price is \$30, it will still be 12% when the stock price is \$80, *ceteris paribus*. The same assumption applies to the volatility parameter as well. Thus, both the uncertainty and the expected rate of returns grows proportionally to the price level, which is a reasonable assumption according to (Haug, 2021).

3.3.2 GBM under the risk-neutral framework

To value the autocallable notes we must use the risk-neutral valuation framework. As mentioned in Section 3.1, this is a trivial implementation as the only difference from Eq. 3.3 is the replacement of the subjective expected rate of return parameter μ, with the objective risk-free rate r_f. Thus, in a valuation framework, one can use Eq. 3.4 to simulate the time t value of the underlying stock price.

$$S_t = S_0 e^{\left(r_f - \frac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}z} \tag{3.4}$$

3.3.3 Implicit dividends in GBM

It is important to distinguish between total returns and capital gains of an asset when analyzing the autocallable structures. As it is the stock price that determines the payoff and not the total return, we must adjust for this when simulating the stock prices. In the Black & Scholes model it is easy to implement such a implicit dividend rate, as the only implication is that we must subtract the rate in the drift-term. In the Black & Scholes model it is usual to use delta δ as the notation for the continuous dividend yield. However, as the *quantos* structure of the *Autocall Telekom* note will introduce some implications later on in Section 4.4, we must distinguish between the implicit dividend rate and the continuous dividend yield. In this thesis, we will thus use δ as the implicit dividend rate, and λ as the continuous dividend yield. Eq. 3.5 shows the GBM in the risk-neutral valuation framework, where implicit dividend rate is included.

$$S_t = S_0 e^{\left(r_f - \delta - \frac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}z} \tag{3.5}$$

3.3.4 Correlated geometric Brownian motion

The autocallable notes we are investigating have four underlying assets each determining the payoff. Thus, we must account for the correlation between the assets when we model the stock prices. According to Hansson, neglecting this adjustment will bias the value estimate of the autocallable structures (2012). Therefore, to have a robust and credible model, correlation must be taken into account.

Instead of simulating four individual GBMs, we must adjust each individual stock price model such that all four of them have the correct correlation with the three others. To implement this, we must adjust the dispersion-term. More specifically, we will adjust the Wiener process $W(t)$ in each GBM, such that the four Wiener processes have the same correlation matrix p_{ij} as the underlying assets (Glasserman, 2003). According to Glasserman, this can be accomplished by multiplying a vector of IID Wiener processes $(W(t)_1, ..., W(t)_d)$ with any matrix A, where $AA^T = \Sigma$. The subscript d is the number of underlying assets, and the matrix Σ is nothing more than the underlying assets' $d \times d$ covariance matrix. The matrix A^T is transposed of matrix A, and matrix A and A^T are a lower and a upper triangular matrix, respectively.

To find matrix A we must utilize a mathematical technique called the *Cholesky decomposition* (Haugh, 2004). The *Cholesky decomposition* factorizes any positive semidefinite matrix into a lower and a upper triangular matrix, where one is transposed of the other. In most programming languages, including R which we use, the *Cholesky decomposition* is implemented through a built-in function. Thus, finding matrix A in our situation is accomplished by feeding the Cholesky function with the underlying assets' covariance matrix. If the reader is interested in reading more about the theory behind the Cholesky decomposition, then Haugh's lecture slides on the topic is recommended (2004, p. 5).

As in Section 3.3.1, we will utilize that the Wiener process can be written as a standard normal random variable multiplied by the square root of time, $W(t) \stackrel{d}{=} \sqrt{t}z$, and $z \sim N(0,1)$. Eq. 3.6 shows the correlated GBM for asset i.

$$S_t^i = S_0 e^{\left(r_f - \delta_i - \frac{1}{2}\sigma_i^2\right)t + \sqrt{t}a_i z}, \qquad i = 1, .., d, \tag{3.6}$$

where a_i is the ith row of the matrix A, and z a standard normal random variable $z \sim N(0,1)$. For each stock price generation, the z's are IID. The parameter δ_i is the implicit dividend rate for asset i.

3.4 Payout function

In the Monte Carlo simulation method, we compute the value estimate of the structured products by taking the average of N simulated price estimates. In this section we present how the price in each of those different scenarios are derived. As we want to price the autocallables, we operate within the risk-neutral framework, thus using the risk-free rate r_f as the main driver of the drift-term in the stock price model.

The price solely depends on the payout tied to the underlying stock paths. As the autocallable structures to be priced have complex features, it is difficult to make an easy understandable function for the cash flow payouts. Instead we will simplify the illustration by simulating the path of one stock, and show how we tie the payout to the path using numerical vectors. It is worth noting that the only difference between one and four stock paths regarding the payout function is that when having multiple underlying assets, we evaluate the numerical vectors based on the lowest stock price of the four at each valuation date.

Figure 3.1 illustrate a simulated stock path for a stock with initial value of 100, volatility of 20%, and drift equal the risk-free rate. The risk-free rate in this example is set to an arbitrary number of 2%. The stock path in the figure is simulated for each consecutive day for five years. This is done for an illustrative purpose. When doing the valuation of the autocallable notes in Chapter 5, we will simply simulate the stock prices for a quarter at a time, and then evaluate the payouts. The difference in length of simulation steps has no importance on the estimated value of the notes. However, having longer simulation steps (i.e., a quarter at a time) will decrease the computational workload, which is desirable.

The horizontal lines illustrate the different barriers, and the vertical lines illustrate the discrete observations dates. The autocall barrier is set at 90% of initial value, the coupon barrier at 80%, and the risk barrier at 60%. The first possible autocall date is the 4th observation date illustrated by the highlighted vertical line. The autocallable to be valued here has a notional value of NOK10.000 and a coupon of 4.00% (NOK400).

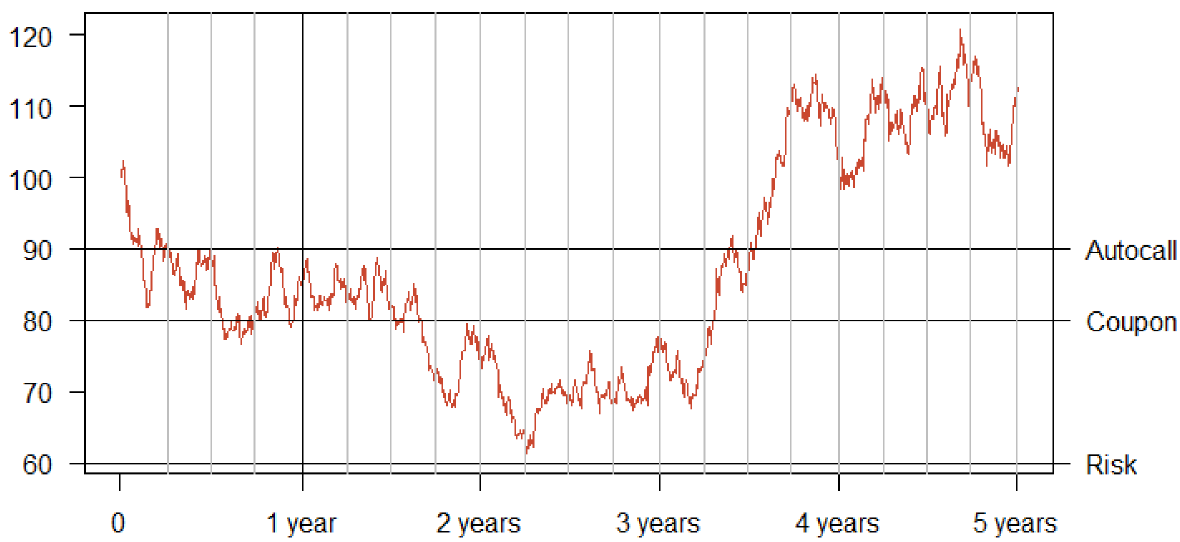

Figure 3.1: One GBM stock path.

The precise stock price at each valuation date is illustrataed in the column *Valuation prices* in Table 3.1 at the end of this subsection. From Figure 3.1 we can see that the autocallable note will be autocalled on the 15th observation date (3rd year, 3 qtr.). This is also verified in Table 3.1 from the aforementioned *Valuation prices* column, where the 15th valuation price is the first price above the autocall barrier of 90 (disregarding the first three valuation dates). In the column *Autocall* the autocall condition is met, illustrated by a vector turning 1 at the 15th valuation date.

To capture the coupon payouts, we use a similar approach as above by constructing a vector that turns 1 when the condition for coupon is met, and 0 otherwise. This is represented in the *Coupons* column in the same table. For coupons we must also adjust for accumulation. The effect of accumulation is captured by counting observation dates where the stock price is below the coupon barrier of 80. If the stock price moves above the coupon barrier at a later valuation date, the counted streak will be added to that date. From the *Coupons* column, we observe that the stock price is below the coupon barrier between date 7 and date 13, resulting in a counting streak of 7. At observation date 14, the stock price is once again above the coupon barrier. The counted streak of 7 is then added to the coupon of date 14, resulting in a coupon payout of $8 \times \text{NOK}400 = \text{NOK}3,200$. The coupon vector must also reflect that there will be no coupons after the note has been autocalled. This effect is captured by connecting the coupon vector to the autocall vector. The coupon vector turns 0 for all remaining values if the autocall column turns 1.

There is also a vector for the risk barrier, represented by the *Risk-barrier* column. This vector can only turn 1 at observation date 20. The conditions for the risk barrier vector to turn 1 is that the sum of the autocall vector must be equal to 0 and that the stock price at the last observation date is below the risk barrier of 60. In the example presented, all the values in the risk barrier are 0, as the note is autocalled at valuation date 15.

With the *Autocall*, *Coupons*, and *Risk-barrier* columns in Table 3.1 identified, it is possible to find the cash flows for the different observation dates, which are represented in the *CF* column. For valuation date 1 to 19, the cash flows are identified by multiplying the notional value of NOK10,000 with the *Autocall* vector and the coupon amount of NOK400 with the value in the *Coupons* vector.

For valuation date 20, there are three different scenarios. Firstly, the cash flow is zero if the note has been autocalled during the first 19 valuation dates. Scenario two and three however, depend on whether the stock price is above or below the risk barrier of 60. This is represented by the value of the *Risk-barrier* column. If the *Risk-barrier* value is 1 the cash flow is computed as the final date stock price divided by the initial stock price, multiplied by the notional amount, $S_t/S_0 \times \text{NOK}10,000$. If the *Risk-barrier* value is 0, the cash flow will be similar to the computations of valuation date 1 to 19. However, the notional amount of NOK10,000 will be paid out regardless of the value of the *Autocall* vector. Thus, the cash flow will be the notional amount of NOK10,000, plus the the coupon amount of NOK400 multiplied with the value in the *Coupons* vector.

The last column of Table 3.1, the *Discounted* column, reflects the discounted cash flows. In this column we discount the cash flows from column *CF* by a continuous discount rate of 2%. All observation dates are discounted according to correct time, represented by the *Observations* column. Thus, the cash flow of observation date 15 is discounted 15 periods back. To find the product's value, we simply sum up all the discounted cash flows, which in this scenario equals a value of NOK14,990.6.

Observations	Valuation prices	Autocall	Coupons	Risk-barrier	CF	Discounted
1	90.60	0	1	0	400.0	398.0
2	88.30	0	1	0	400.0	396.0
3	80.50	0	1	0	400.0	394.0
4	84.80	0	1	0	400.0	392.1
5	82.60	0	1	0	400.0	390.1
6	82.10	0	1	0	400.0	388.2
7	71.30	0	0	0	0.0	0.0
8	74.20	0	0	0	0.0	0.0
9	62.00	0	0	0	0.0	0.0
10	70.20	0	0	0	0.0	0.0
11	68.70	0	0	0	0.0	0.0
12	77.10	0	0	0	0.0	0.0
13	74.90	0	0	0	0.0	0.0
14	86.90	0	8	0	3200.0	2983.7
15	111.50	1	1	0	10400.0	9648.5
16	102.30	0	0	0	0.0	0.0
17	114.90	0	0	0	0.0	0.0
18	112.30	0	0	0	0.0	0.0
19	113.40	0	0	0	0.0	0.0
20	113.50	0	0	0	0.0	0.0
Value						14,990.6

Table 3.1: Table illustrating the vectors used finding the discounted cash flows from one simulated stock path. Column *Observations* represents the different valuation dates. Column *Valuation prices* represents the accompanied stock prices. Columns *Autocall*, *Coupons*, and *Risk-barrier* represent the different conditions for autocall, coupons and risk-barrier, respectively. Column *CF* and column *Discounted* represent the exact cash flows and discounted cash flows at each individual observation date. The value of this particular autocallable note is 14,990.6, which can be seen at the last row. The highlighted row, row 15, illustrate the valuation date which the note is autocalled.

4 In-data parameters

The value of the autocallable notes are determined by a set of different variables. In this chapter we will present each variable, which are the risk-free rate r_f, the discount rate, the expected rate of return μ, the implicit dividend δ, the volatility σ, and the correlation parameter ρ. We will also present the historical data which is used to compute the two latter variables.

4.1 Data

For the volatility parameter σ and the correlation parameter ρ we will use historical data for estimating the values. These parameters can vary substantially depending on the data we estimate them from. Thus, it is necessary to discuss both the frequency and the length of the historical data. We must also asses what type of data we will use in the models. Do we want data that are adjusted for stock splits and dividends, or do we want raw data? We must also consider whether we want to use logarithmic or arithmetic returns in our estimation of the parameter values.

The length of the historical data depends on whether the range is regarded to be relevant for the analysis or not. This is a subjective matter which needs to be evaluated in context of the length of the products simulated. Usually, when simulating stock price behaviour for two years, two years of historical data is considered to be relevant. In our case, this translates to five years of historical data as the structured products have a five-year horizon. There should also be an assessment regarding extraordinary events as well. Extraordinary events create abnormal stock price movements, which can heavily affect the parameter estimates. The last two years have been affected by the COVID-19 pandemic. However, as the COVID-19 *Omicron* variant is on the rise (as of December 2021), we do not believe excluding the COVID-19 data will be representative for the future. Thus, we choose not to exclude any data from the historical data-set.

Regarding the frequency of the data, we believe a monthly frequency is an appropriate frequency to be used. As the autocallable notes are evaluated once a quarter, a daily frequency can create too much noise, as auto-correlation can be an issue. A quarterly frequency will match the data even better. However, this will lead to a data-set consisting of twelve observations per underlying asset. When using historical data to estimate parameters, it is preferable to use log normally distributed returns. However, it is difficult to say anything about the distribution when having so few observations. A monthly frequency will yield 60 observations per asset which we believe are more appropriate.

As we do not want the correlation and volatility estimates to be biased by events like stock splits and dividends, we must use data that are adjusted for this. There are many sources providing such adjusted data. However, we choose to download adjusted closing prices from Yahoo finance. These prices have been adjusted according to the CRSP[1] standards (Yahoo!, na). As mentioned in the last paragraph, we will manipulate the historical data to construct the logarithmic returns for each underlying asset. This will be done in the programming language R.

To summarize, we use monthly adjusted stock prices from November 2016 to November 2021, downloaded from yahoo finance. In the programming language R, we remove any missing values, and compute associated logarithmic returns for each underlying asset. The logarithmic returns are used to estimate volatility and correlation.

4.2 Risk-free rate and discount rate

In a risk-neutral valuation framework, the risk-free rate r_f is used as the parameter responsible for the drift in the stock price model. The risk-free rate is also one of two building blocks in the discounting rate, which is used to discount the simulated value estimates in the Monte Carlo simulation. The second building block in the discounting rate is the credit spread of the issuing bank, which is simply added to the risk-free rate (Hull, 2018).

For the risk-free rate we find the yield of the Norwegian government bond with maturity of five years to be a good proxy. Firstly, governments in stable countries such as Norway

[1] https://www.crsp.org/products/documentation/crsp-calculations

have extremely low, if no, probability of default. Secondly, the settlement currency is NOK. And thirdly, the maturity of the notes is five years.

4.3 Expected rate of return

In Chapter 6, we will investigate what returns the investor can expect when buying the autocallable notes. We will also examine the probabilities of when the notes are autocalled. In both instances we find ourselves outside of the risk-neutral framework. Thus, we must use the subjective expected rate of return as the main driver of the drift when simulating the stock prices.

According to Coval and Shumway, as we assume the underlying stocks follow a GBM we can apply the *Capital Asset Pricing Model* (CAPM) to estimate the expected rate of return (2001). The CAPM describes the relationship between risk and the expected return. The general idea of the model is to compensate the investor in two ways. Through the time value of money, denoted as r_f, and through risk. For taking on risk the investors are compensated by a risk premium r_p, which in the CAPM is represented as the market's expected rate of return minus the risk-free rate, $r_p = r_m - r_f$. Eq. 4.1 below illustrates how the expected rate of return μ is estimated using the CAPM.

$$\mu = r_f + \beta r_p \tag{4.1}$$

where β is the asset's measure of systematic risk, beta. We will use the CAPM to compute the expected rate of return of all the individual underlying assets. As each company reacts differently to the market movement, we must use the individual betas in the computation. For the assets denoted in foreign currencies, we must also use the foreign risk-free rate and the foreign market risk-premium, when calculating the expected rate of returns.

4.4 Implicit dividend rate

As mentioned in Section 2.2.1 there are some implications regarding the *quantos* structure of the *Autocall Telekom* note and the implicit dividend rate δ_i. For notes with *quantos* structure the underlying assets are denominated in another currency than the settlement currency. The return of the autocallable is thus idependent of the development of the exchange rates. As the size of the domestic payout are directly dependent on the foreign assets' return, we must adjust for this when modeling the underlying assets. This adjustment will be implemented through the implicit dividend parameter.

In the article *Pose og Sekk*, the authors Bjerksund et al. illustrate how this adjustment is implemented (1999). In the article, the implicit dividend rate is adjusted to include not only the dividend yield, but also a component for the rate differences between the domestic and foreign risk-free rate. This rate-difference is what distinguish the implicit dividend rate δ_i with the continuous dividend yield λ_i. Eq. 4.2 shows the relationship explicitly.

$$\delta_i = \lambda_i + (r_f - r_i) \tag{4.2}$$

where the term $(r_f - r_i)$ corrects for the foreign assets being denominated in another currency than the settlement currency. r_f is as usually the domestic risk-free rate, and r_i is the $i = 1, .., d$ individual foreign risk-free rates. In the *Autocall Telekom* note, one of the underlying assets is a Norwegian company. In such instances, the correction-term will become zero as the "foreign" risk-free rate will be equal to the domestic risk-free rate. The article of Bjerksund et al. also include a component adjusting for the covariance between the logarithmic return of the foreign assets, and the change in exchange rates. This term is excluded from our equation as we find it to be so small that it has no real affect on the implicit dividend rates.

Thus, for autocallable notes where all the underlying assets are denoted in the domestic currency, the implicit dividend rate δ_i is equal to the continuous dividend yield of the underlying asset λ_i as there is no rate difference. For *Autocall Telekom* which have the *quantos* structure however, the implicit dividends will differ from the individual dividend yields. The correction-term shows that if the domestic risk-free rate is higher than the foreign risk-free rate, then it is expected that the foreign exchange rate will appreciate. Such positive rate-difference will give a higher implicit dividend rate. In Table 5.8 we will see that for the underlying assets in the *Autocall Telekom* note, this adjustment will make a big difference.

4.5 Volatility

Volatility measures the uncertainty of an asset. Plain vanilla call and put options increase in value with increased volatility as increased volatility makes it more probable for such options to be in the money, and deeper in the money. In the geometric Brownian motion model used to model stock paths, the volatility is constant. However, in the real world volatility varies over time. In some instances, like during the financial crisis in 2007-2008 or during the COVID-19 pandemic the two past years, the volatility was higher than normal. In other times it is more subdued. This has led to criticism of the GBM, resulting in solutions such as the Heston model. However, stochastic volatility models, like the Heston model, does not guarantee any better results. An implementation of such models would also incur even higher implementation difficulties, as one would have to calibrate the model as well. Even though constant volatility does not reflect the real world correctly at all times, it works as an okay proxy on average. Thus, we would use a constant volatility in our valuation in the Results from valuation model, Chapter 5. To find the constant volatility used in the GBM we have two approaches. Either we can look at historical data or we could look at the implied volatility of the underlying assets in the options market. As the options markets for the underlying assets are nonexistent or illiquid, we find it unfeasible to use implied volatility. Instead, we will use historical volatility. There are different methods to estimate historical volatility. The simplest method is the historical volatility method. Other more sophisticated methods are the EWMA- and GARCH-model, but these are more difficult to implement. Thus, we consider the simple historical volatility method in this thesis illustrated in Eq. 4.3.

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}u_i^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n}u_i\right)^2} \tag{4.3}$$

where σ is the volatility, u_i the logarithmic daily returns in the interval $i = 1, ..., n$ and $\bar{u}$ is the average logarithmic return.

4.6 Correlation

Correlation measures how the underlying assets move together, and is illustrated by the ρ parameter. The autocallable notes to be valued in this thesis have four underlying assets, where the worst performing asset is the relevant for defining the payoff. In such a structure, the correlation coefficients are very important. The lower the correlation, the more likely it is for one of the assets to perform badly, assuming all other parameters are held constant. In our model, the correlation is implemented through the *Cholesky* factorization of the covariance matrix, as introduced in Section 3.3.4. Eq. 4.4 shows how the covariance between two assets is estimated. x_i and y_i represent the $i = 1, ..., n$ individual logarithmic returns for the two assets, and $\bar{x}$ and $\bar{y}$ represent the mean returns. Eq. 4.5 illustrates the relationship between the covariance and correlation. Here σ_x and σ_y represent the respective standard deviations of the returns.

$$cov_{x,y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n-1} \tag{4.4}$$

$$p_{x,y} = \frac{cov_{x,y}}{\sigma_x \sigma_y} \tag{4.5}$$

5 Results from valuation model

In this chapter we are going to conduct the valuation of the autocallable notes using the methods introduced in Chapter 3. As mentioned in Section 3.1, when valuing a derivatives product we operate in a risk-neutral framework. Thus, we will use the risk-free rate as the parameter responsible for the drift in the stock price model. The framework will also imply the use of the discount rate to discount the value estimates from the Monte Carlo simulation in order to get a present value estimate of the true value.

The chapter will be divided into three parts. In part one, the estimated parameters used in the valuation framework will be declared. In part two, the actual valuation of the autocallable notes *Autocall Norske Selskaper* and *Autocall Telekom* will be executed. Lastly, in part three we will summarise the numerical valuation.

5.1 Parameters

In Chapter 4 we examined how we would define the model parameters. In this section we switch the focus to getting a real number. We distinguish between stock-dependent and stock-independent parameters. For the independent parameters it is only necessary to define the value once, while for the stock-dependent parameters we are forced to declare a value for each underlying asset. We start by defining the stock-independent parameters, which are the initial stock price S_0, the domestic risk-free rate r_f, and the discounting rate. Moving on, we will compute the stock-dependent parameters; volatility σ_i, correlation ρ_{ij}, and the implicit dividend yield δ_i. For the assets denoted in foreign currencies, we must also declare the foreign risk-free rates r_i.

Initial stock price

We simplify the initial price of the underlying assets by setting them all to $S_0 = 100$. In Section 3.3.1 we saw that the GBM assumes expected dollar returns to be proportional to the stock price level. The relative change in the stock prices will therefore be the same independent of the initial value. To clarify, the initial stock price will only be identical at date zero, as from that moment and on wards the stock prices will develop randomly.

Domestic risk-free rate

As mentioned in Section 4.2, we will use the rate of the five-year Norwegian government bond as the proxy for the domestic risk-free rate. This rate is given by Bloomberg to be 1.43% (as of 01.12.2021), equalling a continuous risk-free rate of 1.42%[1] (Bloomberg L.P., 2021).

Discount rate

As discussed in Section 4.2, the discount rate is composed of the domestic risk-free rate and the credit spread of the issuing bank. We find the appropriate credit spread by looking at the credit rating of the issuer, which in our case is Goldman Sachs. In the *sales brochure*, Garantum states Goldman Sachs' ratings from S&P and Moody's, which are A+ and A1 respectively. These ratings equals an appropriate credit spread of 0.980% (Damodaran, 2020). If we adjust this credit default swap rate to be continuous, we get a rate of 0.975%. This result in an appropriate continuous discount rate of 1.420% + 0.975% = 2.395%.

Foreign risk-free rates

To adjust the implicit dividend rate we must also find the risk-free rates for the underlying assets denoted in foreign currencies. These assets are Nokia, Vodafone, and Ericcson, which are listed in Finland, UK, and Sweden, respectively. The risk-free rates of these countries are identified by the five-year treasury yield found using the Bloomberg terminal (Bloomberg L.P., 2021). The foreign risk-free rates in Finland, UK9, and Sweden are -0.030%, 0.707%, -0.407%, respectively. What we see is that all of these rates are lower than the Norwegian rate. This implies a hidden dividend rate which is not favorable for investors. The reason for this is that when we have a *quantos* structure, as in *Autocall Telekom*, the return of the note is not affected by changes in foreign exchange rates. Taking the return of the foreign underlying asset without including the exchange rate implies a swap of returns, where the domestic interest rate is swapped with the foreign interest rate. A positive difference between the domestic and foreign interest rate corresponds to a situation where the foreign currency is expected to appreciate relative to the domestic currency.

[1] Where we use the equation cont.rate = $ln(1 + \text{ annual rate})$ for finding the continuous rates.

Volatility

The volatility parameters are estimated in R using a function that implement Eq. 4.3. As mentioned in section 4.1, the estimates are based on five years of monthly returns from 2016 to 2021. This yields a monthly volatility estimate. To properly implement the volatility in the GBM model we must scale the volatility to be annual. This is easily done by multiplying the monthly volatility with the square root of 12. For the underlying companies in *Autocall Norske Selskaper*, Norsk Hydro, Salmar, Telenor and Yara, we get annual volatility estimates of 0.319, 0.325, 0.167, and 0.195 respectively. Equivalently, for Ericsson, Vodafone and Nokia, the underlying asset of *Autocall Telekom*, we get annual volatility estimates of 0.248, 0.263, and 0.354, respectively. The Norwegian company Telenor is included as an underlying company in both notes.

Covariance

As mentioned in section 4.6, the covariance matrix is fundamental for making correlated stock paths in the GBM model. The covariance matrices for *Autocall Norske selskaper* and *Autocall Telekom* are found in table 5.1. It may be difficult to relate to the values in the covariance matrices which is why we also will present the correlation matrices later. However, as the covariance matrices are crucial inputs in the model it is reasonable to present them. The values in the matrices are found by using a function in the programming language R that estimates the values according to Eq. 4.4.

Autocall Norske Selskaper				
	Norsk Hydro	Salmar	Telenor	Yara
Norsk Hydro	0.12	0.02	0.01	0.04
Salmar	0.02	0.10	0.01	0.01
Telenor	0.01	0.01	0.04	0.01
Yara	0.04	0.01	0.01	0.06

Autocall Telekom				
	Ericsson	Vodafone	Telenor	Nokia
Ericsson	0.11	0.01	0.01	0.05
Vodafone	0.01	0.05	0.02	0.01
Telenor	0.01	0.02	0.04	0.01
Nokia	0.05	0.01	0.01	0.11

Table 5.1: Covariance matrices of the underlying assets

Dividend yield

Finding appropriate estimates for dividend yield is difficult because it is uncertain how companies will allocate their capital in the future. As mentioned in section 4.4, we make a best guess based on the companies' historical dividends. The estimated future dividend yields are illustrated in Table 5.2, with both annual and continuous compounding. According to our estimates, all the companies in *Autocall Norske selskaper* and *Autocall Telekom* have positive dividend yields. Isolated, dividends represents a disadvantage to the investors of the notes because they will decrease the capital gains of the underlying assets. In other words, higher dividends decrease the chance for early autocall and increase the chance of ending below the risk-barrier.

	Annual yield	Continuous yield
Norsk Hydro	3.00%	2.96%
Salmar	4.00%	3.92%
Yara	5.00%	4.88%
Telenor	5.00%	4.88%
Nokia	2.50%	2.47%
Vodafone	7.50%	7.23%
Ericsson	2.00%	1.98%

Table 5.2: Annual and continuous dividend yields

5.2 Model reliability

In this subsection we will test how our stock price model cope when pricing a derivatives product with a well known solution. The intuition of doing this test is to ensure that the model behaves as expected. In such, it can be considered as a robustness test. If the simulated value-estimate converges to the theoretical price given the same parameters, we can be assured that our stock price model is legitimate. We will also simulate 5,000 thousand stock price generations for one of the autocallable notes to assess whether the historical correlation is reflected in the simulated prices or not.

5.2.1 Numerical versus theoretical price

As our model is based on the Black & Scholes framework, we consider the European call option as a good product to use for the robustness test. As this derivative has a closed form solution through the Black & Scholes formula, we can compute it's theoretical price. The Black & Scholes formula for a European call option with dividends, is given by Eq. 5.1.

$$C = S_0 e^{-\delta T} N(d_1) - K e^{-r_f T} N(d_2) \tag{5.1}$$

where

$$d_1 = \frac{log(\frac{S_0}{K}) + (r_f - \delta + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

and $N(d_1)$ and $N(d_2)$ denotes the cumulative distribution function of the standard normal distribution.

For both the Black & Scholes formula and for the numerical model, the arbitrary parameters listed below will be used to calculate the value of the option.

- Initial stock price, $S_0 = 100$
- Strike price, $K = 100$
- Risk-free rate r_f of 1.420%
- Volatility σ of 16.740%
- Implicit dividend rate δ of 4.879%
- Time to maturity T equalling 5 years

Given the arbitrary parameters, the theoretical price of the European Call option using the Black & Scholes formula is 6.625. Table 5.3 shows the numerical results from the stock price model.

N	Price estimate	error margin	Upper c.i.	Lower c.i.
1000	6.190	-0,435	7.177	5.204
10,000	6.723	0.098	7.060	6.386
100,000	6.616	-0.009	6.721	6.511

Table 5.3: Numerical results of a European call option

We observe that the simulated price estimates converges to the theoretical price when the number of simulations increases. This argues that the model is reliable.

5.2.2 Simulated versus historical correlation

To check whether the correlation is reflected in the stock price model or not, we perform a Monte Carlo simulation where we generate N number of stock path generations. For each of these simulations we compute the logarithmic returns for each asset and compute the simulated correlation. Then we can plot the N different correlation estimates in a histogram to assess the distribution of the correlation. As mentioned in 3.2, as the number of simulations N increases the average estimate will converge to the true expected value. Thus, if we plot the historical correlations as a vertical line, we can quickly observe if the simulated correlation distribution seems correct or not. Figure 5.1 shows the simulated correlation distributions between every underlying asset in the *Autocall Norske Selskaper* note, with the associated historical correlations plotted as a vertical red line. We have used 5,000 simulations for the simulated correlations.

From the figure we observe that the average simulated correlations converge to the historical correlation as expected. Thus, we are positive that we have managed to incorporate correlation into our stock price model. Table 5.4 shows both the historical and simulated correlation matrices for the underlying assets in *Autocall Norske Selskaper*. The simulated correlation matrix is constructed using the average of the simulated correlations. We observe that the simulated correlation matrix and the historical correlation is very similar, almost identical. Thus we can argue that the stock price model manages to incorporate the correct correlation when generating stock paths.

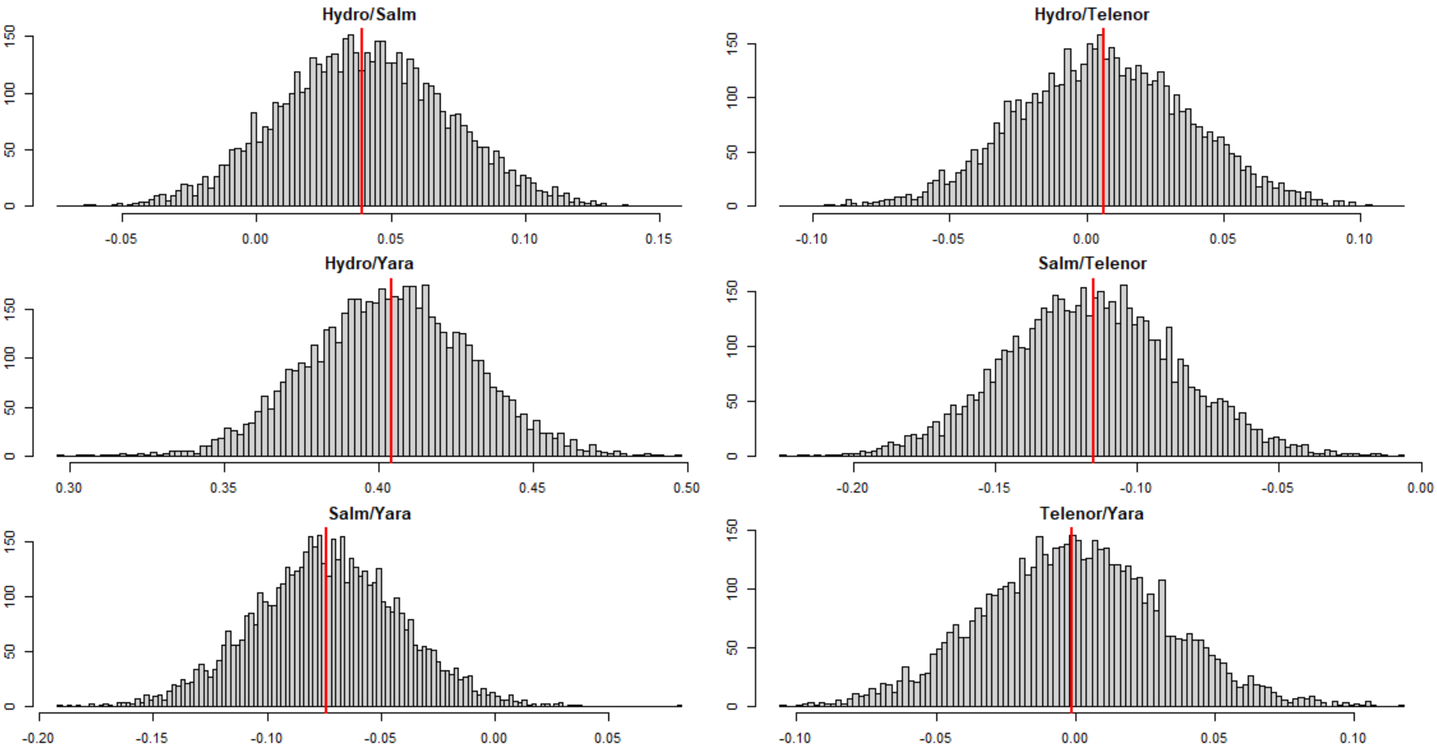

Figure 5.1: Figure illustrating the distribution of 5,000 simulated correlations for the underlying assets in the *Autocall Norske Selskaper* note. The red vertical lines display the historical correlations between the assets.

Historical	Norsk Hydro	Salmar	Telenor	Yara
Norsk Hydro	1.000	0.039	0.006	0.404
Salmar	0.039	1.000	-0.116	-0.074
Telenor	0.006	-0.116	1.000	-0.001
Yara	0.404	-0.074	-0.001	1.000

Simulated	Norsk Hydro	Salmar	Telenor	Yara
Norsk Hydro	1.000	0.040	0.006	0.404
Salmar	0.040	1.000	-0.116	-0.074
Telenor	0.006	-0.116	1.000	-0.001
Yara	0.404	-0.074	-0.001	1.000

Table 5.4: Historical and simulated correlation matrices for *Autocall Norske Selskaper*' underlying assets.

5.3 Autocall Norske Selskaper

In this section we will value *Autocall Norske selskaper* using the quantified parameters outlined above. The parameters are summarised in Table 5.5. The historical correlation matrix identified in Table 5.4 can help the reader to get an intuition of how the underlying assets in this product move together. From Table 5.4, we observe that Yara's correlation with Norsk Hydro is positive with a value of 0.404. While for both Salmar and Telenor, Yara's correlations are almost zero. Thus we can expect that Yara's stock path will move more with Norsk Hydro's stock path, than with other two assets.

	Norsk Hydro	Salmar	Telenor	Yara
Initial stock price	100	100	100	100
Volatility	0.319	0.325	0.167	0.195
Risk-free rate	1.42%	1.42%	1.42%	1.42%
Dividend yield	2.96%	3.92%	4.88%	4.88%

Table 5.5: Parameters used to model paths for underlying assets in *Autocall Norske selskaper.*

The underlying assets' stock paths, together with the payout condition, determines the value of the autocallable note. To get an intuition of the paths of the underlying assets in *Autocall Norske selskaper* we have made figure 5.2. This figure illustrates the price paths of the underlying assets according to the parameters and correlations presented in tables 5.4 and 5.5. The vertical lines in this figure illustrates the observation dates, and the dark vertical line signal the first possible autocall date. In this exact path the note would be autocalled at observation date 5. Note that we for illustrative purposes have used time-steps of one day in this figure. In the actual valuation model we will have time-steps of one quarter for the purpose of increasing the efficiency of the model.

Table 5.6 presents the payoff specific parameters of *Autocall Norske selskaper.* These parameters specify the conditions for payout. At each observation date the prices of the underlying assets will be observed. The observed price of the worst performing asset together with the payout conditions will determine what the cash flow will be. This relationship highlights the fact that the value of the autocallable is determined by the paths of the underlying assets and the payout conditions.

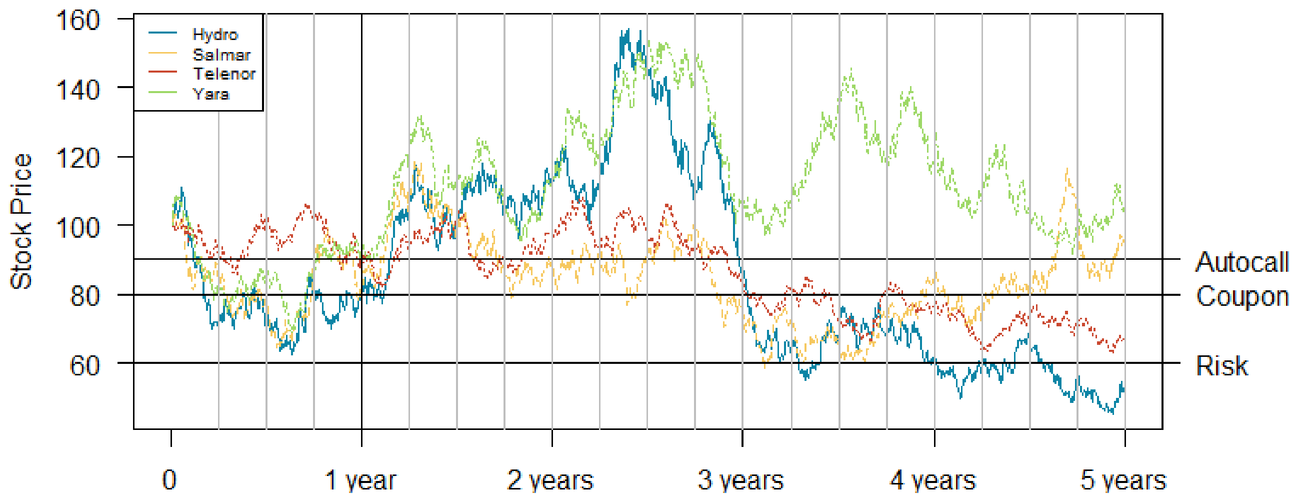

Figure 5.2: One path generation of *Autocall Norske Selskaper*'s four underlying assets

	Payoff specific parameters
Autocall barrier	90%
Coupon barrier	80%
Risk barrier	60%
Notional	NOK10,000
Quarterly accumulative coupon	$Notional \times 4.00\%$
Discount rate	2.40%

Table 5.6: Payoff specific parameters used in the valuation of *Autocall Norske Selskaper*

5.3.1 Results

Table 5.7 displays the results from our valuation algorithm. Each iteration of the valuation model derives different price estimates of the autocallable note. The *price estimate* column in the table illustrates the average price estimate from the number of iterations presented in the *iterations* column. What we see is that the price estimates becomes more precise as the number of iterations increase, just as suggested in Section 3.2. The most precise answer is found from a simulation with 1,000,000 iterations. With 1,000,000 iterations we value *Autocall Norske selskaper* to NOK6,955.7, with a standard error (SE) of NOK4.0 giving us a 95% confidence interval between NOK6947.9 and NOK6963.6.

The results in table 5.7 suggest that *Autocall Norske selskaper* is extremely overpriced. Investors must pay NOK10,300 for something that is only worth NOK6,955.7.

Iterations	Lower c.i.	Price estimate	Upper c.i.	SE	Time
10	$4,166.2$	$6,018.1$	$7,870.0$	944.9	0.020 s.
100	$5,975.1$	$6,773.2$	$7,571.4$	407.2	0.100 s.
$1,000$	$6,752.0$	$6,998.7$	$7,245.4$	125.9	0.960 s.
$10,000$	$6,917.9$	$6,996.6$	$7,075.2$	40.1	9.170 s.
$100,000$	$6,922.9$	$6,947.6$	$6,972.4$	12.6	1.71 min
$1,000,000$	$6,947.9$	$6,955.7$	$6,963.6$	4.0	43.35 min

Table 5.7: Price estimates, confidence intervals, and standard errors of *Autocall Norske Selskaper* from valuation model.

This implies a total price premium of 10,300/6,955.7 - 1 = 48.08%, which is far above the costs indicated in the prospect and *sales brochure*. One reason why the price premium is so much higher than the costs indicated in the prospect might be that the price of constructing the autocallable note exceed the value of the note, due to the highly customized derivatives components comprising the product. However, this does not mean that it is fair to allocate all the costs incurred when constructing the product to the investors. For this reason we believe that the issuer and brokers should clearly communicate what the value of the product is in addition to all the costs related to construct the product. This would ensure more transparency for the investor and give a better foundation for a well informed investment decision. Another reason why our price estimate is so much lower than the price paid by investors might be the uncertainty regarding the estimated parameters. To give a more reliable conclusion regarding the price estimate, we will in the next section perform a sensitivity analysis where we see how sensitive the price estimate is to changes in the parameters.

5.3.2 Sensitivity analysis

As there are uncertainties regarding the estimated parameters used to value *Autocall Norske selskaper* we find it necessary to do a sensitivity analysis. The sensitivity analysis will see how the price estimate change when certain estimated parameters change, such as the correlation and volatility. We will also see how sensitive the price estimate is to parameters where we have had to make a best guess based on information, such as the drift rate that is determined by risk-free rates and dividends. There will also be an assessment of how sensitive the price estimate is to the coupon payments to see how high the coupons must be to justify the sales price of NOK10,000.

Figure 5.3 illustrates how the price would change if either the volatility, or the correlation would change by relative +/− 30%. We observe that even if we have miss-estimated the value of the parameters by +/− 30%, the value of *Autocall Norske Selskaper* would still be much lower than the price paid by investors. This means that even if the estimation error of the volatility or correlation was as high as +/− 30%, our conclusion stating that *Autocall Norske selskaper* is extremely over priced would not change.

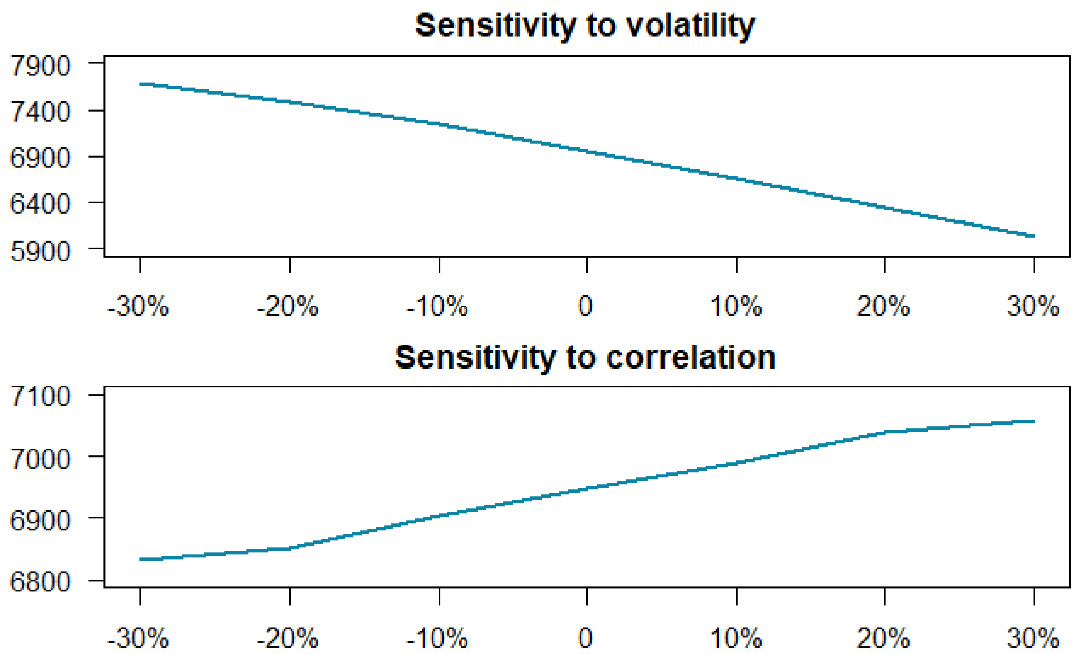

Figure 5.3: Effect of a +/− 30% relative change in volatility and correlation on price estimate.

Figure 5.4 displays how the price estimate of *Autocall Norske selskaper* changes as the drift-term changes. The drift-term in the valuation model is determined by the Norwegian risk-free rate and the dividend yields of the underlying assets. From the figure we see that even if the drift-term was 5% higher in absolute terms, the price estimate would still be significantly below the price of NOK10,300 paid by investors. The price would also still be lower than the price paid by investors minus the costs paid by investors. We find it highly unlikely that the drift-term would deviate more than absolute 5% from our estimate. Thus, we still believe that *Autocall Norske selskaper* is overpriced.

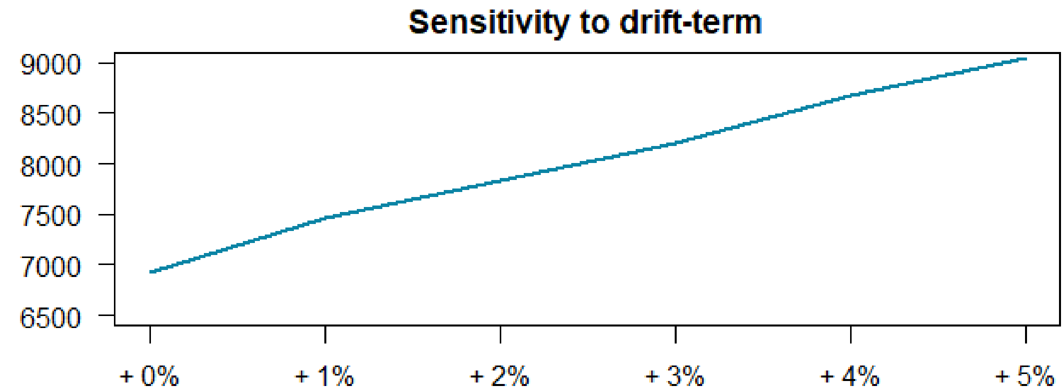

Figure 5.4: Sensitivity analysis of drift, *Autocall Norske selskaper*.

In figure 5.5 we can also see what the coupons would have to be to justify a value of NOK10,300. A coupon of 12% would justify a value of ca. NOK10,300. This represents a coupon three times higher than the actual coupon of 4%.

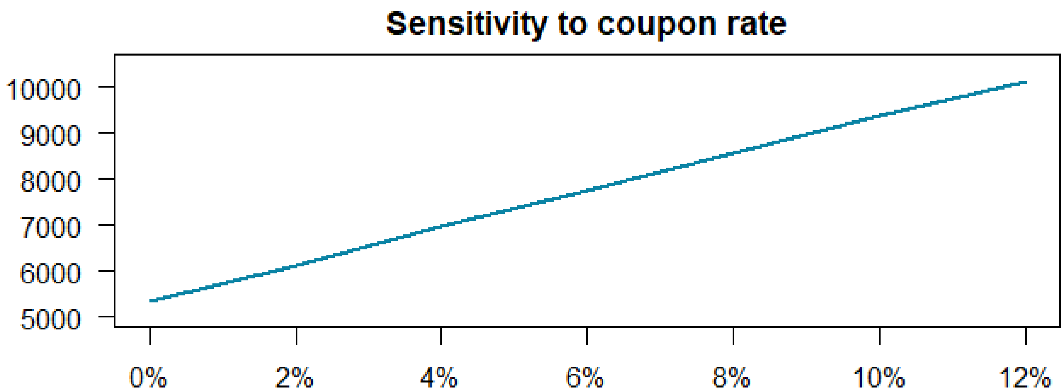

Figure 5.5: Sensitivity analysis of coupon rate, *Autocall Norske selskaper*.

5.4 Autocall Telekom

In this section we will value *Autocall Telekom*. The procedure will be similar as we did above for *Autocall Norske selskaper*. Table 5.8 displays all the parameters used to model the stock paths, and Table 5.9 displays the correlation between the underlying assets. As *Autocall Telekom* have a *quantos* structure there is an additional parameter in the table, the foreign risk-free rate. As mentioned earlier, when the domestic risk-free rate is higher than the foreign risk-free rate we get a higher implicit dividend yield. This is the case for *Autocall Telekom*' foregin underlying assets, as can be seen in Table 5.8 below.

	Ericsson	Vodafone	Telenor	Nokia
Initial stock price	100	100	100	100
Volatility	0.248	0.263	0.167	0.354
Domestic risk-free rate	1.42%	1.42%	1.42%	1.42 %
Foreign risk-free rate	-0.03%	0.71%	1.42%	-0.47%
Dividend yield	1.98%	7.23%	4.88%	2.47%
Implicit dividend yield	3.43%	7.94%	4.88%	4.36%

Table 5.8: Parameters used to model paths for underlying assetsin *Autocall Telekom*

	Ericsson	Vodafone	Telenor	Nokia
Ericsson	1.00	0.10	-0.07	0.18
Vodafone	0.10	1.00	0.22	0.25
Telenor	-0.07	0.22	1.00	0.06
Nokia	0.18	0.25	0.06	1.00

Table 5.9: Correlation matrix of underlying assets in *Autocall Telekom*

To get an intuition of the paths of the underlying assets in *Autocall Telekom* we have made Figure 5.6. This figure illustrates price paths of the underlying assets of *Autocall Telekom* for one simulation by using the parameters and correlation from Table 5.8 and Table 5.9. The vertical lines in this figure illustrate the observation dates, and the dark vertical line signals the first possible autocall date. In the paths displayed in Figure 5.6 the note would be autocalled at observation date 4, which is the first possible autocall date. Note that we also here have used time-steps of one day for illustrative purposes.

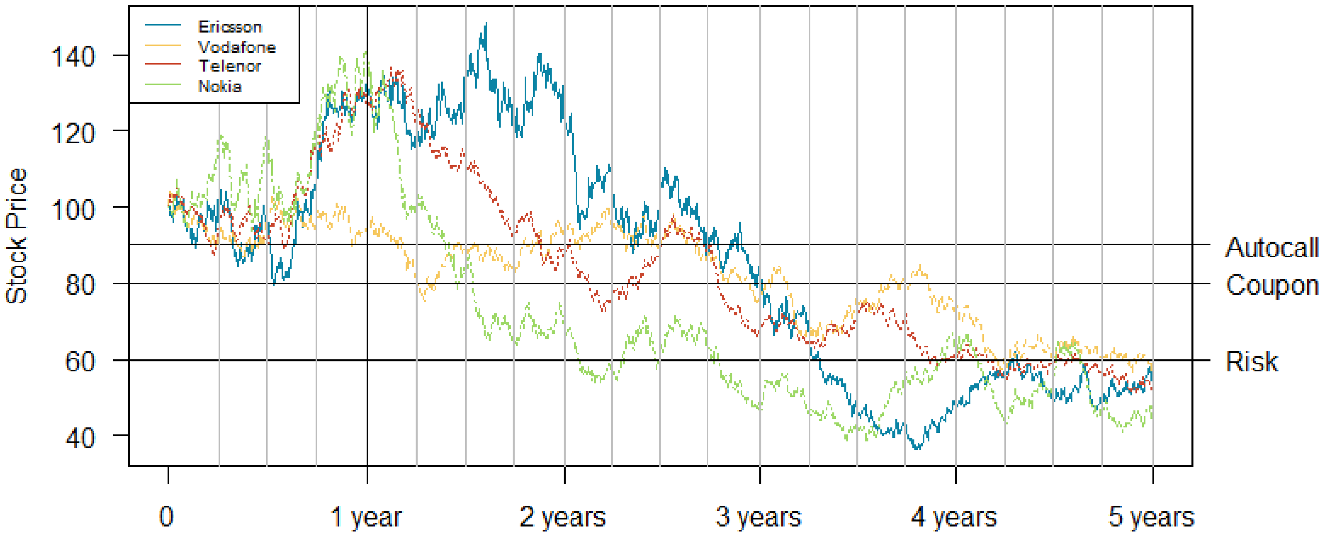

Figure 5.6: One path generation of *Autocall Telekom*'s four underlying assets.

Table 5.10 presents the payoff specific parameters. The payoff specific parameters for *Autocall Telekom* are almost identical as the payoff specific parameters for for *Autocall Norske selskaper*. The only difference is that *Autocall Telekom* pays a lower coupon.

	Payoff specific parameters
Autocall barrier	90%
Coupon barrier	80%
Risk barrier	60%
Notional	NOK10,000
Quarterly accumulative coupon	$Notional \times 3.65\%$
Discount rate	2.40%

Table 5.10: Payoff specific parameters used in the valuation of *Autocall Telekom*.

5.4.1 Results

Table 5.11 below displays the results of the valuation algorithm using the parameters of *Autocall Telekom* presented above. The precision of the price estimates increases with number of iterations. From 1,000,000 iterations we get a price estimate of NOK6633.1, with a standard error (SE) of NOK4.0, which gives us a 95% confidence interval between NOK6625.2 and NOK6640.9.

Iterations	Lower c.i.	Price estimate	Upper c.i.	SE	Time
10	$5,296.5$	$8,056.8$	$10,817.1$	$1,408.3$	0.020 s.
100	$5,593.6$	$6,425.8$	$7,258.0$	424.6	0.110 s.
$1,000$	$6,244.7$	$6,487.9$	$6,731.2$	124.1	0.930 s.
$10,000$	$6,537.4$	$6,615.5$	$6,693.5$	39.8	9.270 s.
$100,000$	$6,611.9$	$6,636.6$	$6,661.3$	12.6	1.71 min
$1,000,000$	$6,625.2$	$6,633.1$	$6,640.9$	4.0	38.77 min

Table 5.11: Price estimates, confidence intervals, and standard errors from our valuation model

These results suggests that also *Autocall Telekom* is extremely overpriced with a price premium of 10,300/6633.1 - 1 = 55.28%. The most likely reasons why the price premium is so much higher than the costs indicated in the prospect, are the same as we discussed for *Autocall Norske selskaper*. It might be that the costs of constructing the product exceeds the value of the product, or it might be that the estimated parameters have some errors. In the next section we will see how sensitive the price estimate of *Autocall Telekom* is to the parameters.

5.4.2 Sensitivity analysis

This section will see how sensitive the price estimate of *Autocall Telekom* is. We will do this in the same manner as we did for *Autocall Norske selskaper* by examining how the price estimate of *Autocall Telekom* changes as the parameters; volatility, correlation, drift-rate, and coupon change.

Figure 5.7 displays how sensitive the price estimate of Autocall Telekom is to relative changes in the volatility and correlation parameters. The figure illustrates that higher volatility gives lower price estimates, and that lower correlation gives lower price estimates. We can also observe the same findings as we did for *Autocall Norske selskaper*. Even with relative changes of $+/-$ 30% in the volatility and correlation parameters, the price estimate would still be far below the price of NOK10,300 paid by investors. Based on this, we still find our conclusion stating that *Autocall Telekom* is extremely overpriced to be reasonable.

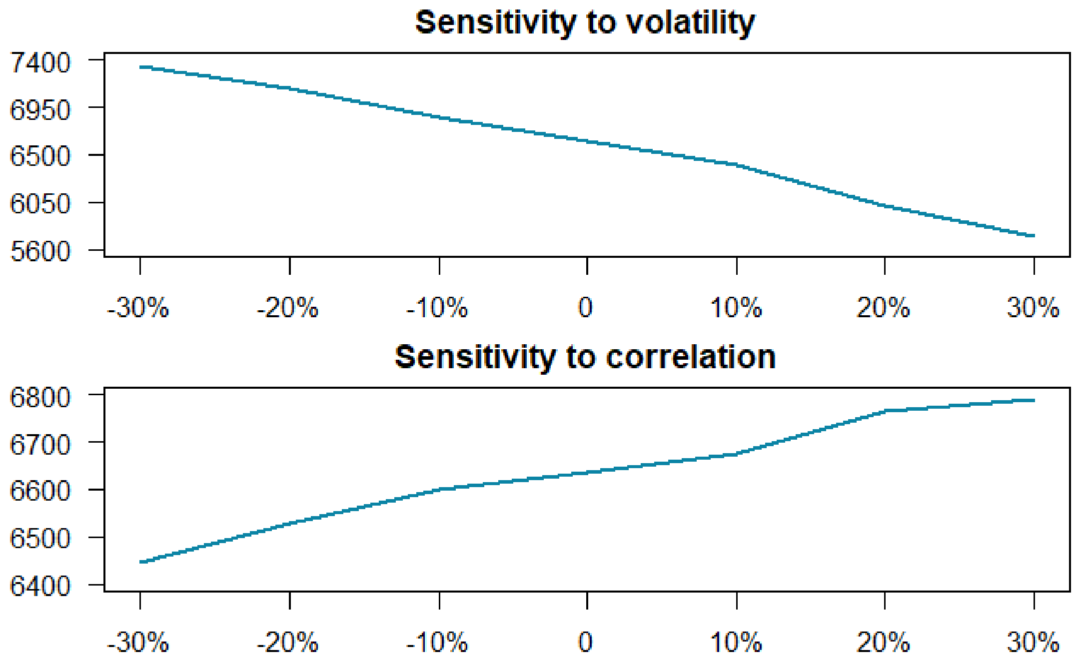

Figure 5.7: Value-effect of a +/- 30% relative change in volatility and correlation.

Figure 5.8 below displays how sensitive the price estimate of *Autocall Telekom* is to absolute changes in the drift-rates of the underlying assets. The drift-rates in the valuation model is determined by the Norwegian risk-free rate, and the implicit dividend yields. From the figure we see that with an absolute increase of 5% in the drift-rates, the price-estimate is NOK8,500. We find it unlikely that the estimated drift-rate parameter will have an error greater than this. Thus, we still find it reasonable to conclude that *Autocall Telekom* is overpriced.

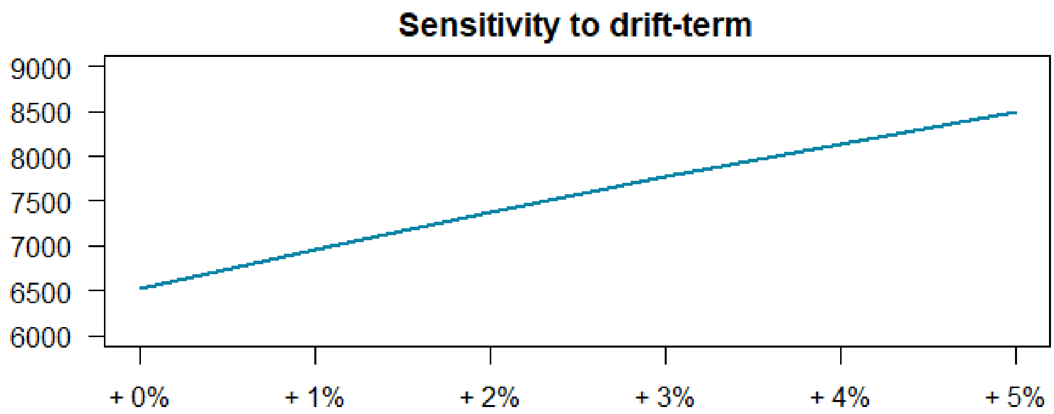

Figure 5.8: Sensitivity analysis of drift, *Autocall Telekom*.

In Figure 5.9 we can see the sensitivity to the coupon rate. As for *Autocall Norske Selskaper*, we can see that quarterly coupons would have to be 12% for the price of *Autocall Telekom* to be ca. NOK10,000. This represents a coupon that is more than three times higher than the actual coupon of 3.65%.

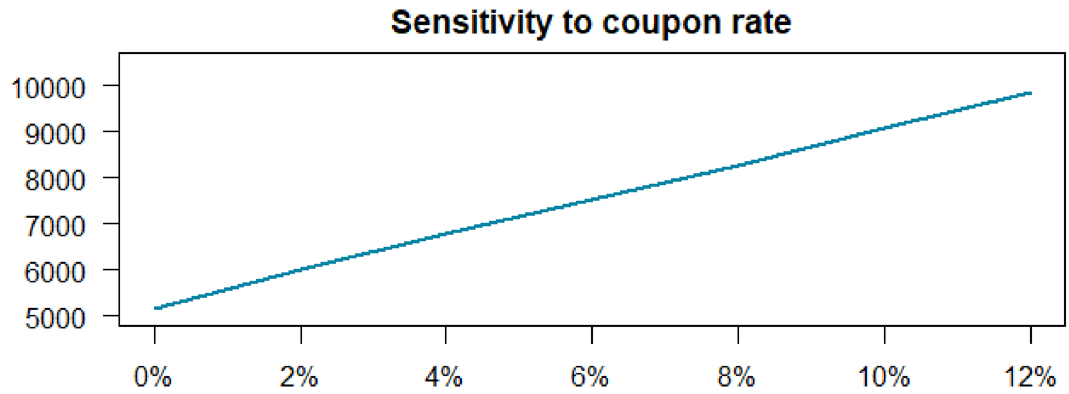

Figure 5.9: Sensitivity analysis of coupon rate, *Autocall Telekom*.

5.5 Summary of results

The numerical results of the valuation model suggests that both *Autocall Norske selskaper* and *Autocall Telekom* are extremely overpriced. Deviations this large could be explained by bad estimates of the input parameters. However, in the sensitivity analysis we saw that even by adjusting the parameters to give a higher value, the price-estimates was still far off the price of NOK10,300 paid by investors. Thus, based on the valuation model, *Autocall Norske selskaper* and *Autocall Telekom* appears extremely overpriced.

6 Expected return and probability distribution

This chapter will examine what investors can expect from buying the autocallable notes. We will investigate the holding period returns (HPR), the internal rate of return (IRR), and the expected lifetime of both the *Autocall Norske Selskaper* and the *Autocall Telekom* notes. Simulating 100,000 future values of the notes will provide a distribution for each measure, which will give us an overview of the development of the notes. The HPR will give insights of what investors can expect of total returns when holding the products. The IRR will provide information of the notes' dollar-weighted returns and the expected annual returns. While the investigation of the maturity will give investors a greater understanding of how long they can expect their funds to be locked in for. Contradictory to Chapter 5 we will not use the risk-neutral valuation framework, thus we must make some adjustments to the pricing model.

Subsection 6.1 below, addresses the model adjustments. In Subsection 6.2 and Subsection 6.3 we investigate each note respectively. Finally, Subsection 6.4 will summarize the findings of the two autocall-specific subsections.

6.1 From risk-neutral to the true probability measure

In the previous chapter we used the risk-neutral valuation framework to simulate the fair price estimates of the autocallable notes. In this chapter we are not interested in pricing the products anymore. Instead, we are interested in how the notes are expected to develop when using the real probability measure. The key difference between the two is that we now must estimate the investors expected rate of return μ_i, and use this variable as the main driver in the stock price model. Thus, we go from using the risk-free rate r_f and the equivalent martingale measure Q in Chapter 5, to the expected rate of return μ_i and the real probability measure P in this chapter. As we are not interested in finding a present value, we do not have to discount the Monte Carlo estimates anymore neither.

6.1.1 CAPM

As mentioned in Section 4.3, we will use the CAPM to estimate the underlying assets' expected rate of returns. The CAPM requires certain parameters as can be seen in Eq. 4.1. These are the risk-free rate, the beta, and the market risk-premium. The risk-free rate is already identified, whereas the betas are found using the Bloomberg terminal. The risk-premium is identified through various sources, and will be discussed in the specific sections below.

6.2 Autocall Norske selskaper

Most of the parameters needed to compute the expected rate of returns are already outlined in Chapter 5, such as the risk-free rate, the implicit dividend, and the volatility. In addition to these parameters we need the beta of each underlying asset and their respective risk premiums. As mentioned, we get the individual betas from the Bloomberg terminal, where the betas are calculated by a regression analysis based on five years of monthly historical returns (Bloomberg L.P., 2021). For the *Autocall Norske Selskaper* note, it is sufficient with the risk-premium of the Norwegian market as all the underlying assets are listed in Norway. From a study by Ødegaard we find the appropriate risk-premium in the Norwegian market to be 5% (2021). This is also confirmed in PWC's annual survey on the matter (PricwaterhouseCoopers, 2020). All parameters used to estimate the expected returns of *Autocall Norske selskaper* are illustrated in Table 6.1. The table also illustrate what the calculated expected rate of returns of each assets are, both annually and continuously. The computations are done using the equation in Subsection 4.3, namely Eq. 4.1.

To compute the holding period returns of the *Autocall Norske Selskaper* note, we run the pricing model from Chapter 3 with the minor adjustment described in Section 6.1. As we use the expected rate of returns, and skip the discounting of the Monte Carlo price estimates, we get N simulated future cash flows. By dividing these future cash flows by the notional value, we get N different HPR estimates.

	Norsk Hydro	Salmar	Telenor	Yara
Domestic risk-free rate	1.43%	1.43%	1.43%	1.43%
Beta	1.39	0.81	0.43	0.77
Risk-premium	5.00%	5.00%	5.00%	5.00%
Expected return	8.38%	5.47%	3.58%	5.27%
Continuous expected return, μ_i	8.05%	5.33%	3.51%	5.13%

Table 6.1: Parameters and expected returns of underlying assets in *Autocall Norske selskaper*.

Figure 6.1 shows a distribution of 100,000 simulated expected holding period returns ($N = 100,000$). Looking at the figure, one can see a big spread in the distribution as the simulated HPRs varies between -90% and 80%. The most frequent HPR is in the interval of 10% - 20% however. The spike at this interval is caused by many notes being autocalled at the first possible autocall-date. These notes will have a future payoff of 11,600, equalling a HPR of 16%. There are also numerous outcomes with negative holding period returns. For our simulation, there are nearly more negative holding period returns than positive! For a note to receive a negative holding period return it must run to maturity and drop below the risk-barrier. Thus, nearly half of the simulated notes dropped more than 40%. It is worth noting that a note that goes below the risk-barrier can still have a positive HPR if the coupon amount received during its lifetime is larger than the final date loss. We also observe a high frequency of negative HPRs, falling between -30% and -70%.

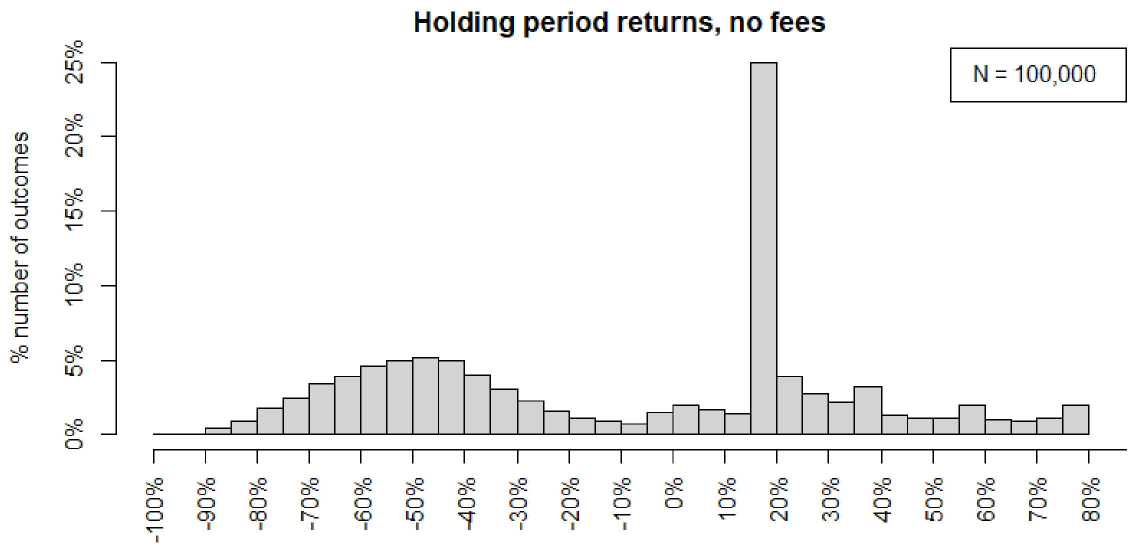

Figure 6.1: Histogram of 100,000 simulated holding period returns of *Autocall Norske Selskaper* note.

The findings from Figure 6.1 can also be illustrated in Figure 6.2. This figure illustrates when, or if, the autocallable notes are autocalled. This statistic can be found by counting the number of times the simulated notes are autocalled within each interval. The figure indicates that 18.3% of the simulated notes were autocalled at the first possible autocall-date, and 12.2% during the second year. The figure gives the intuition that if the notes do not get autocalled during the first two years, they are very likely to run all the way to maturity. 61.7% of the simulated notes did not get autocalled at all, and ran all the way to the final date at year five. Only 13.4% of those ended above the risk-barrier and as much as 48.3% ended below. Thus, 48.3% of the simulated 100,000 notes did not get the notional amount of 10,000 back, which is confirmed by the high frequency of HPRs between -30% and -70%. This illustrate a high probability of loosing a lot of money when investing in the autocallable note.

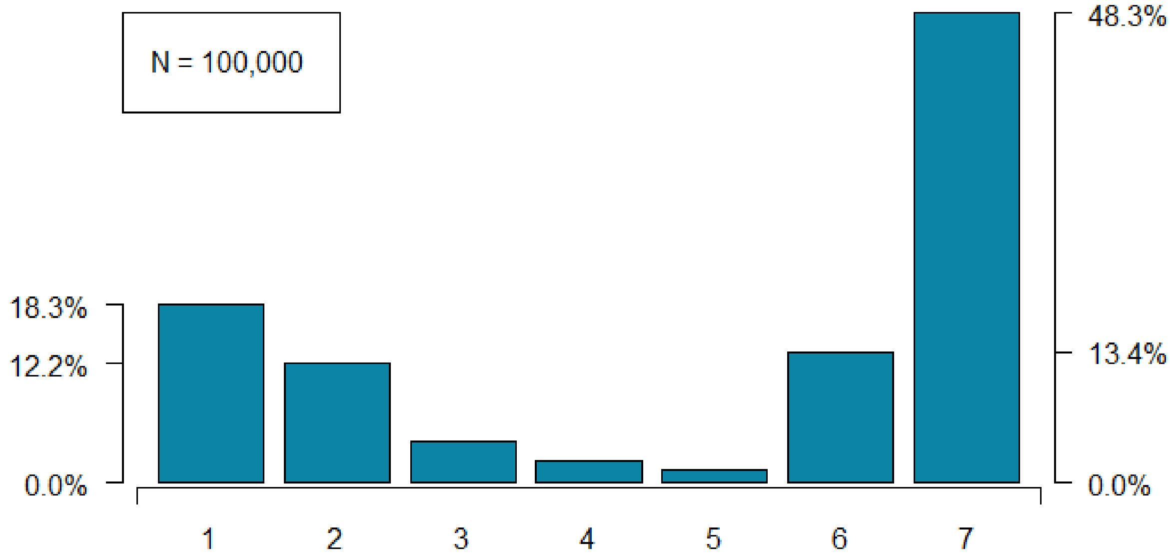

Figure 6.2: Figure illustrating when or if 100,000 simulations of *Autocall Norkse selskaper* are autocalled. Where 1=1st year, 2=2nd year, 3=3th year, 4=4th year, 5=5th year, 6=Not Autocalled, above riskb., 7=Not Autocalled, below riskb.

Knowing what to expect of annual returns when investing in the autocallable notes are clearly in the best interest of the investors. As the lifetime and timing of the cash flows of the autocallable notes are stochastic, we find the internal rate of return[1] to be a good alternative to estimate the annual returns. Thus, we have computed the annual internal rate of returns of 100,000 simulations of *Autocall Norske Selskaper*, illustrated in

Table 6.2. The table show the distribution of those simulated IRR's, where the different IRR outcomes have been grouped into small blocks. From the row *Sum: IRR > 0* we observe that the investors can expect a probability of positive returns of about 52%, and equivalently from the row *Sum: IRR ≤ 0* a probability of 48% for negative returns. The probabilities do not differ much when including the underwriting fees of 3% in the simulation, as can be seen when comparing the two columns in the table. We can also see that the positive returns are in the range of 12% to 18%, whereas the negative returns are mostly in the range of -5% to -30%. The average IRR is however slightly negative. This coincides with the above discussion where we saw that the note is most likely to be autocalled either within the two first years, earning returns similar to the coupon, or run to maturity and end below the risk-barrier earning large negative returns.

Internal rate of return	**Without fees**	**With fees**
≤ -30 %	3.66 %	3.97 %
-30 – -15 %	20.54 %	22.01 %
-15 – -5 %	19.80 %	18.42 %
-5 – 0 %	3.32 %	3.12 %
Sum: IRR ≤ 0	47.32 %	47.52 %
0 – 3 %	4.70 %	5.45 %
3 – 6 %	2.70 %	2.05 %
6 – 9 %	1.60 %	1.5 %
9 – 12 %	2.09 %	2.93 %
12 – 15 %	9.38 %	40.55 %
15 – 18 %	32.20 %	0 %
Sum: IRR > 0	52.68 %	52.48 %
Total:	100 %	100 %
Average IRR:	**- 0.95%**	**- 2.28 %**

Table 6.2: Table showing the distribution of *Autocall Norske Selskaper*'s IRR from 100,000 simulations. With and without underwriting fees of 3%. The last row also displays the average expected IRR.

[1]The internal rate of returns were computed using a built in R-function called *irr*.

6.3 Autocall Telekom

In Chapter 5 we identified the risk-free rate, the implicit dividend, and the volatility of the underlying assets in *Autocall Telekom*. The beta of the underlying assets are derived from the Bloomberg terminal just as for the underlying assets of *Autocall norske selskaper*. Finding the correct risk-premiums requires more work for *Autocall Telekom* as the underlying assets are based in different countries. The Norwegian risk-premium is already identified as 5%. We use a survey by Pablo Fernandez to identify the risk-premium in Sweden, Finland, and UK (2020). The risk-premiums are summarized in Table 6.3. It is worth mentioning that the risk-premium from the survey of Fernandez suggests a risk-premium in the Norwegian market to be 5.8% which differs from the 5% found in Ødegaard's and PWC's surveys.

	Ericsson	Vodafone	Telenor	Nokia
Foreign risk-free rate	-0.03%	0.71%	1.43%	-0.47%
Beta	0.61	1.12	0.43	1.09
Risk-premium	6.10%	5.80%	5.00%	6.50%
Expected return	3.69%	7.22%	3.58%	6.62%
Continuous expected return μ_i	3.62%	6.97%	3.51%	6.41%

Table 6.3: Parameters and expected returns of underlying assets in *Autocall Telekom*

Figure 6.3 shows the distribution of expected holding period returns from 100,000 simulations. The figure illustrates that *Autocall Telekom* have a similar return distribution as the *Autocall Norske selskaper* note. We observe a large spread with HPRs between -90% and 80%. Similarly to *Autocall Norske Selskaper*, we observe a large spike around the interval of 10% - 20%, and frequent HPRs between -35% and -75%. However, by comparing the figure with that of the *Autocall Norske Selskaper*, we observe that the highest HPRs are a bit lower for *Autocall Telekom*. This is due to *Autocall Telekom* having a lower coupon. The same findings can be seen in Figure 6.4 indicating that the note will either autocall within the two first years, or go all the way to maturity. The early autocalls explain the spike in the 10% - 20% HPR interval, as these instances return a HPR of 14.6% or higher. The frequent large negative HPRs are explained by the last column in the figure, indicating that 51.3% of the notes end up going below the risk-barrier at maturity.

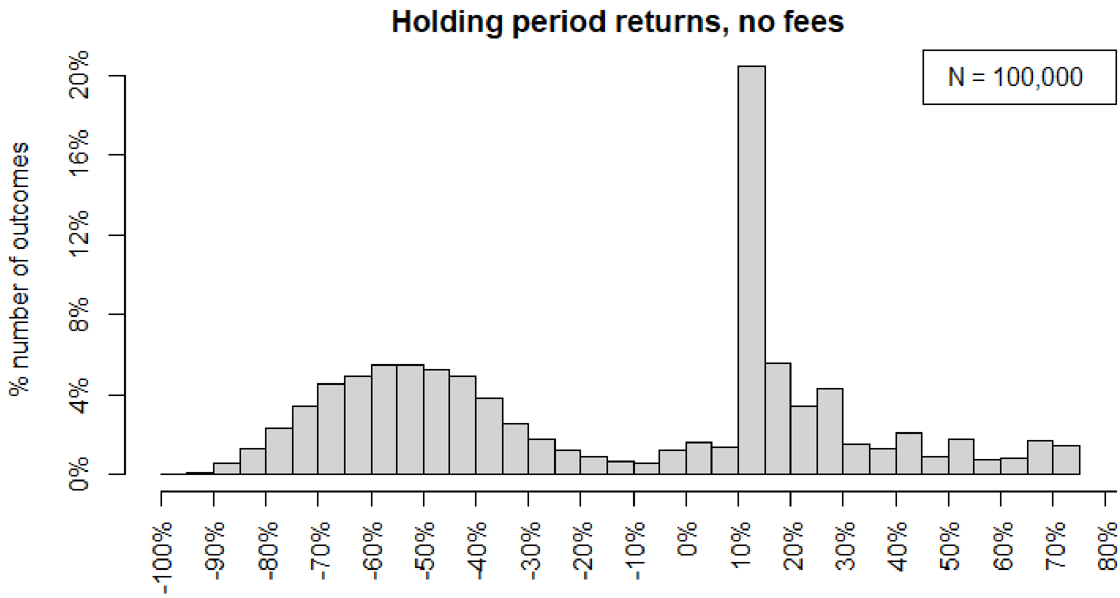

Figure 6.3: Histogram of 100,000 simulated holding period returns for the *Autocall Telekom* note. Disregarding underwriting fees.

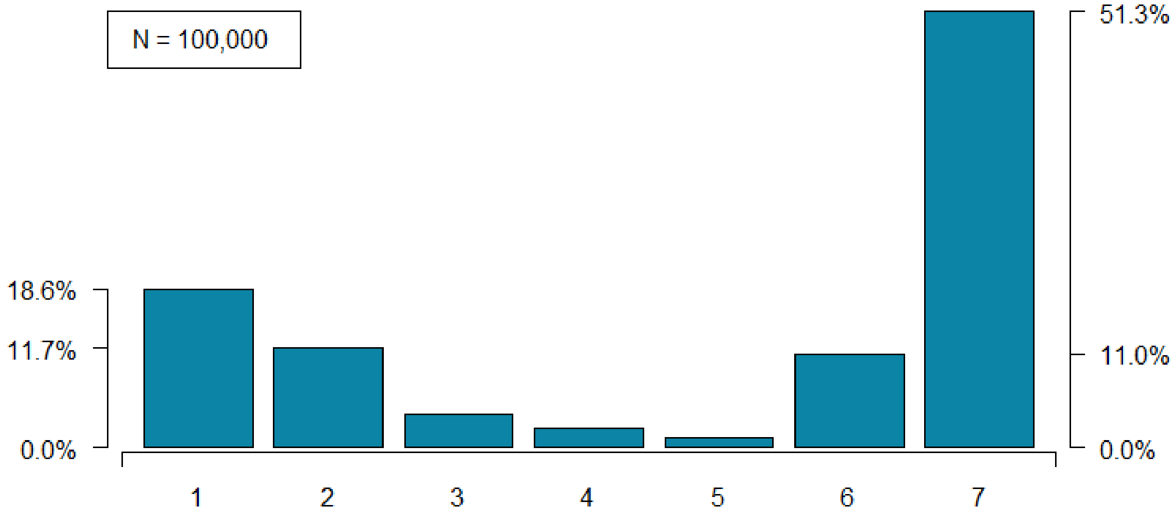

Figure 6.4: Figure illustrating when 100,000 simulated outcomes of *Autocall Telekom* are autocalled, if autocalled at all. Where 1=1st year, 2=2nd year, 3=3th year, 4=4th year, 5=5th year, 6=Not Autocalled, above riskb., 7=Not Autocalled, below riskb.

Table 6.4 illustrates the distribution of IRR from 100,000 simulations of *Autocall Telekom*. The results are again very similar to what we saw for *Autocall Norske Selskaper*. The main difference is that *Autocall Telekom* have no outcomes in the IRR interval of 15% to 18%. This is explained by the lower coupon in *Autocall Telekom*. We can also notice that *Autocall Telekom* have a higher probability of negative IRR than *Autocall Norske selskaper*. The table indicates that there is a higher probability of negative returns than positive returns for *Autocall Telekom*.

Internal rate of return	**Without fees**	**With fees**
≤ -30 %	4.88 %	5.48 %
-30 – -15 %	24.80 %	26.16 %
-15 – -5 %	18.34 %	16.94 %
-5 – 0 %	2.87 %	2.80 %
Sum: IRR ≤ 0	50.89 %	51.38 %
0 – 3 %	3.84 %	4.52 %
3 – 6 %	2.24 %	1.68 %
6 – 9 %	1.13 %	1.20 %
9 – 12 %	2.69 %	30.19 %
12 – 15 %	39.21 %	11.03 %
15 – 18 %	0 %	0 %
Sum: IRR > 0	49.11 %	48.62 %
Total:	100 %	100 %
Average IRR:	**- 3.04%**	**- 4.47%**

Table 6.4: Table showing the distribution of *Autocall Telekom*'s IRR from 100,000 simulations, with and without underwriting fees.

6.4 Summary of results

In this chapter we studied the expected returns and returns distributions of the autocallable notes. Our findings are similar for *Autocall Norske selskaper* and *Autocall Telekom*. The results indicate large variations in potential returns where we have seen holding period returns varying from up to 80% and down to -90%. A reason for the strong negative returns is the exposure to unsystematic risk these notes offer. When there are four underlying assets and the relevant underlying is the worst performing, the exposure to negative returns is high. Another aspect is the relationship between when the notes are autocalled and the returns. The results suggest that the notes are likely to either be autocalled early and earn positive return as all the coupons are paid, if not, run to maturity and end under the risk barrier. Overall, we believe that such relationship between risk and return, where there is limited upside potential and high probability of strong negative returns will not be found attractive by few investors.

7 Analysis of prospect

In the previous chapters, Chapter 5 and Chapter 6, our results indicate that the autocallable notes are not very attractive. The results suggest that the products are sold with price premiums of ca. 50%, and that there are ca. 50% chance for negative returns for both notes. Despite this, there have been sold Autocalls for billions of NOK and there are still new notes frequently being offered in Norway. This suggests that there is an attractive market for these products in Norway, and that people are willing to invest in these products. This chapter will look at the Norwegian market of autocallable notes, identify the sellers and customers, and investigate how the products are presented to potential investors.

7.1 Market participants and their role

Structured products are typically structured by an issuing bank, facilitated by a facilitator, and sold to customers through a distributor. For the autocallables presented in this thesis, the issuer is Goldman Sachs, the facilitator is Garantum Fondskomission and the products are distributed to the Norwegian market through the distributor Garantum Norge. The latter is an agent acting on behalf of Garantum Fondskomission. Therefore we recognise the facilitator and the distributor as one participant, under the name broker. It is important to distinguish between the issuer and broker as they serve different roles and earn profits from different operations.

The role of the issuer is to structure, value, and create prospects of the products. In the process of structuring the products, the issuer faces certain costs related to the structuring itself and hedging. To cover the expenses and earn money, the issuer adds a margin to the constructed product. As we will see, it is hard to determine what this margin really is because the issuer do not give any information regarding the value of the constructed product, nor the costs of constructing it. For investors, this mean they will not be able to know the true value of what they buy. And for the issuer this represent a conflict of interest. The higher the price they are able to sell the product for, the more money they will earn. As an example, imagine that the issuer constructs the product for NOK7,000, and get NOK9,000 of the total sales price of NOK10,300. This would mean earnings of NOK2,000 for the issuer paid by the investor. One could argue that the distributor would not agree to such terms. However, it is the issuer who have the relevant information regarding the value of the product, and the distributor might not be aware of such high margins. Even if the facilitator and distributor is aware of these margins they might find it acceptable because they do not buy the product, they just act as middlemen and operate with profits as long as they are able to sell the products.

The role of the facilitator and the distributor is hard to distinguish. From our understanding, their job is to sell the products to potential investors and act as brokers. Profits are strongly connected to number of notes sold. Based on this it also appears as the distributors have conflict of interest. The more attractive they portray the products the more they will sell and eventually earn. Earlier happenings can back the belief of such conflict. There have for example been instances where Garantum Norge have under communicated the potential risks of loss (Bjørklund, 2018). Another example is Garantum Norge who did not allow media to attend at their investor presentation in 2018 (Jordheim, 2019). Articles in the media also suggest that these products have been aggressively marketed to investor through direct mails and phone calls to potential investors (Bjørklund, 2018).

The distributor's goal is to sell these products to investors. In the prospects of the notes, it is stated that the targeted customers are both professional and non-professional investors. People that invest in these products must understand them and view them as good investments. The main material to get an understanding of the product and an intuition of the products as investments, are the prospects and sales documents. In the following sections we will give a review of the *final terms* documents which is a finalised form of the prospect, and the *sales brochures*.

7.2 Final terms

The *final terms* documents are created by the issuer with purpose of meeting legal requirements in the *EU Prospectus Regulation*. A *final terms* document is a version of the base prospect, but with more specified characteristics. In the *EU Prospectus Regulation* *article 8* they state: “Whereas the base prospectus contains options with regard to the information required by the relevant securities note, the final terms shall determine which of the options is applicable to the individual issue by referring to the relevant sections of the base prospectus or by replicating such information” (Regulation (EU) 2017/1129 of the European Parliament and of the Council, 2017). Most of the information in the *final terms* documents are general. Thus, we will do a common review of the documents for *Autocall Norske selskaper* and *Autocall Telekom*.

As the *final terms* and base prospects are made for legal reasons the main focus seem to be presenting all the characteristics and formalities of the products in a clear manner. It does not present any suggestions of whether or not the product described is a good investment. The reader will however, get a good intuition of how the products work and important formalities. One thing that caught our attention is that the conflict of interest faced by the issuer is declared. On page 30 in both of the *final terms* documents, it is written that the issuer is subject to a number of conflict of interests between its own interests, and those who holds the security (Goldman Sachs International, 2021a,b). One thing that we did miss was a better presentation of the costs incurred, in addition to a discussion regarding the true value of the product.

Overall the *final terms* are very informative, but there is a lot of information and terminology which can be overwhelming for non-professional investors. We see the importance of such a document and believe that potential investors should become familiar with it. However this is a demanding task, as there is a lot of information to process. Thus, the risk of missing vital information is high.

7.3 Sales brochure

The *sales brochure* is created by the distributor Garantum with the purpose of marketing the product. It presents much of the same information as in the *final terms* document, but the information is more condensed with a greater focus on what the notes can offer as an investment. Also for the *sales brochures* we will do a common review of the content in *Autocall Norske selskaper* and *Autocall Telekom* as much of the information is the same.

The content of the *sales brochures* summarise the features, risks, and costs of the autocallable notes. Much of the content is the same as in the *final terms* documents, but the content is more condensed and presented more reader friendly. The brochures present the features, scenarios, risks, and important formalities in a way that gives the reader a better intuition of what to expect when investing in the autocallable notes. However, after we have carefully studied the *sales brochures*, we have made some remarks regarding the presentation of the products.

The main remark is regarding the costs of the product and how they are presented. Information about the costs are first encountered at page 2 in both documents (Garantum Fondkommission, 2021a,b). Here the costs are presented in terms of amount and to whom they are paid. It is specified that there will be an underwriting fee of 3% accrued to the distributor. In addition, there is specified a margin of 6% where maximum 3.5% is accrued to the distributor and the rest to the Garantum Fondkomission AB. We find this presentation confusing as they have not specified who the distributor is and neither have properly specified who Garantum Fondkomission is. In fact, the distributor is Garantum Norge who act as an agent on behalf of Garantum Fondkomission. So, in total Garantum Norge and Garantum Fondkomission charge fees of 9%. The way we interpret this is that Garantum buys a product from Goldman Sachs for 9,400 and add total costs of 9% relative to the notional amount of NOK10,000. At first glance this seem like all the costs. However, if one reads the paragraph carefully, they state that there also is a cost falling to the issuer. This cost is not specified, but they refer to a later section called *Viktige opplysninger* where they better explain this cost. In this section the cost is specified to cover the expenses occurred by the issuer (Goldman Sachs) when constructing, hedging, and distributing the product. The cost is indicated to be 3.05% and 3.03% for *Autocall Norske selskaper* and *Autocall Telekom*, respectively (Garantum Fondkommission, 2021a,b, p. 6). To get a better understanding of the costs of *Autocall Norske Selskaper*:

1. Goldman Sachs makes a product worth NOK9,095. To cover costs incurred when constructing the product, they add a margin of 3.05% * NOK10,000. As a result, they sell the product for NOK9,400 to Garantum.

2. Garantum add facilitating costs of 6% *NOK10,000 and underwriting fees of 3%*NOK10,000. As a result, they sell the product for NOK10,300.

This suggests that the investor pays a total price premium of $10,300/9,095 - 1 = 13.25\%$. Relative to the notioanl value, this imply a total costs of 12.05%. For *Autocall Telekom*, the corresponding price premium and total costs are 13.22% and 12.03%, respectively. There are two additional aspects that make these findings even more interesting. Firstly, they have a section discussing the total costs, but here they exclude the issuer margin. Instead of referring to the total costs as 12.05% and 12.03%, they only refer to the costs accrued to Garantum of 9%. We believe that this can be misleading for the investor. Secondly, there are no discussions regarding the margins charged by the issuer. The margins are indicated as a cost of 3.05% and 3.03%. A margin of 3.05% of the nominal value would imply that Goldman Sachs values the autocallable note to 9,095. However, there is no additional information regarding this value. The higher the value the issuer can justify for the product, the less this indicated cost will be. This illustrates a prime example of the conflicts of interest the issuer face. As we saw in the valuation chapter, the values of both *Autocall Norske Selskaper* and *Autocall Telekom* are far below NOK9,095 and NOK9,097. This suggests that Goldman Sachs charge costs way above the 3,05% and 3.03% indicated in the *sales brochures*. Overall we find the communication of the costs to be very unclear. They are presented more complicated and with more possibilities for misinterpretation than necessary. It would be more transparent if they clearly stated the value of the products and the costs of constructing them. This would give the reader a better intuition of how much they actually pay in costs. A possible explanation to why such information is not included in the prospects, nor in the *sales brochures*, is that it would make the products appear more unattractive.

Another remark we made was regarding a figure presented. On page 4 they present a histogram of when the autocallable note will be autocalled based on simulations using historical data (Garantum Fondkommission, 2021a,b). In the document of *Autocall Norske selskaper* 60% of the simulations are autocalled at the first possible date whereas ca. 10% of the simulations go below the risk barrier. In *Autocall Telekom*'s document 40% of the simulations are autocalled at the first possible date and ca. 20% of the simulations go below the risk-barrier. These results contradict our findings in Chapter 6. The *sales brochures* states that the simulations are based on historical values and are not reliable. If this is true, why are they included in the brochures and why are the data and assumptions used not presented?

What they write about the underlying assets and their associated industries, seems biased. For instance, in the *sales brochure* of the *Autocall Telekom* note, Garantum portray the Telekom industry as a very promising industry. Suggesting that it will revolutionize the way we communicate, leading to a new way of living life (Garantum Fondkommission, 2021b, p. 3). The description of the industry is not necessarily wrong, but it has an immense focus on the positive aspects and do not mention the risks of the Telekom industry at all. Such narrative will potentially lead to a unfortunate intuition of the underlying assets.

The specified coupons in the *sales brochures* and also the *final terms* are only indicative. For *Autocall Norske selskaper* the indicative coupon is 3.5% and for *Autocall Telekom* the indicative coupon is 4%. Both these coupons ended up being adjusted. For *Autocall Norske selskaper* the coupon was adjusted upwards to 4% and for *Autocall Telekom* it was adjusted downwards to 3.65% (Garantum Fondkommission, 2021c,d). In neither the *sales brochures* nor the prospects it is clearly specified why the coupons are indicative. There are also no information regarding which factors determine a potential coupon adjustment. There is most likely a good reason for the indicative coupons but the reason for this should be presented to ensure transparency, especially in the prospects.

7.4 Our thoughts of the supporting documents

Overall the supporting documents of the autocallable notes give an overview of the notes and what to be aware of. The majority of important information is presented clearly which gives the reader a good intuition of the products, especially in the *final terms* documents. For the *sales brochures*, the information are overall clear but there are more room for misinterpretation, and some of the information presented appears more biased. In both the *final terms* and *sales brochures* we find some deficiencies which can result in uninformed investment decisions. First of all, the incurred costs are not communicated clear enough. Secondly, the value of the notes and the associated costs of constructing them, are not specified. We believe that such information would help the investors to make a more informed investment decision. Thirdly, we believe there are important information not emphasised enough, or not emphasised at all. Neither of the documents give a thorough explanation of the effect of dividends, the *quantos* structure, nor the unsystematic risk exposure of the notes. All of these aspects represent a clear disadvantage to investors, and it would perhaps portray the autocallable notes as more unattractive, if the information were included. Both the *final terms* and the *sales brochures* present the autocallable notes in a way that it is possible for potential investors to understand them. However, the biased information suggests that the documents are presented in a way that do not give the investors sufficient knowledge to do a well informed investment decision.

8 Discussion

In this chapter we will discuss and answer the main research question of the thesis. We will also discuss strengths and weaknesses regarding the thesis itself, and regarding the models implemented throughout the thesis.

8.1 Assessment of research questions

This section addresses the main research question of the thesis. Namely: *Who reap the benefits? The issuer, facilitator, distributor, or the investor?* To get a proper foundation to answer this question we must first address the three sub-questions introduced in Chapter 1. The first of which focuses on the price of both the *Autocall Norske Selskaper* and the *Autocall Telekom* notes, and asks what would a fair price of the autocallable notes be. The second question draws the attention to what investors could expect of returns when investing in the notes. The third and final sub-question addresses how the notes are presented to investors. Ultimately, we will conclude on the main research question of the thesis.

8.1.1 What is the fair price of the products?

In Chapter 5 we conducted a numerical valuation of both the *Autocall Norske Selskaper* and the *Autocall Telekom* notes. For both notes our results from the valuation chapter were significantly lower than what are presented in the *sales brochures* by Garantum. For *Autocall Norske Selskaper*, we found a price of NOK6, 955.7, while for *Autocall Telekom* we found a price of NOK6, 633.1. These prices are excluding any margins, thus a fair price of the products are obviously higher than these estimates. As discussed in Section 7.1, the autocallable notes are created by Goldman Sachs, facilitated by Garantum Fondkommission, and finally distributed and sold to Norwegian investors by Garantum Norge. Neither of these market participants do things without getting paid, implying there must incur some fees to the products. Based on our price estimates, the price premium of the *Autocall Norske Selskaper* and *Autocall Telekom* are 48.08% and 55.28%, respectively. As Goldman Sachs get NOK9,400 for the products, this corresponds to

35.14% and 41.71% of the price premiums of *Autocall Norske Selskaper* and *Autocall Telekom* respectively. Garantum Fondkommission and Garantum Norge receive in total NOK900 per autocallable note that are sold to investors. These NOK900 comprise 12.94% and 13.57% of the total price premium of *Autocall Norske Selskaper* and *Autocall Telekom* respectively. The question to be asked is whether the price premius are too high. We will argue that these premiums are enormous. Of course, Goldman Sachs incur extra costs regarding hedging of their exposure, however, premiums of 35.14% and 41.71% seem enormously high. Even Garantum's part of the premium, which is 9% of the notional value is huge in the financial environment today, where funds are considered expensive when exceeding fees of 2-3% (Lorvik, 2021). Especially considering it is Goldman Sachs that bear the market risk of the autocallable structures, and not Garantum.

To summarise the first sub-question. We are in no position to tell what the fair price of the autocallable notes really should be. However, we strongly believe that the autocallable notes distributed and sold by Garantum are hugely overpriced.

8.1.2 What can investors expect in returns from the products?

In Chapter 6 we investigated the development of 100,000 simulated cases of both *Autocall Norske Selskaper* and *Autocall Telekom*. This gave us a wider understanding of what investors can expect when investing in these types of structured products. The analysis included an investigation of the total returns, the annual dollar-weighted returns, and the length of the investment period. The latter analysis also examined whether notes ended above or below the risk-barrier if not getting autocalled at all. This analysis illustrated that almost half of the 100,000 simulated notes ended up below the risk-barrier at year five, which in most cases also led to a negative return for the investor. These results were observable for both of the notes. The analysis for both the holding period return and the analysis of the dollar-weighted annual returns revealed that the notes that fell below the risk-barrier often incurred large losses as well. For holding period returns, the simulation implied negative returns between -30% and -70% to be very common. Similarly, for the dollar-weighted returns, annual returns between -5% and -30% were found to be very likely. There are however opportunities for decent gains for the investors as well. The most frequent annual return for *Autocall Norske Selskaper* was 16%, which is associated with the note being autocalled at the first possible autocall-date about 18% of the times. For *Autocall Norske Selskaper*, we saw that investors would retrieve dollar-weighted annual returns between 12% and 18% around 41% of the time. Similar tendencies were found with the *Autocall Telekom* note.

Thus to summarise the sub-question, we believe that the investors could expect a high probability of huge losses. Although, there is also a high probability of receiving returns of about 12-18%. However, the probability of getting negative returns are larger than for positive. Also the expected losses are much larger than eventual positive returns, leading to a negative expected annual return for both notes.

8.1.3 How are the products presented to potential investors?

To answer the question regarding how investors are presented the autocallable notes, we investigated two of the supporting documents enclosed with the notes, the *final terms* and the *sales brochures*. The first documents we found to be very informative, providing detailed information of most of the key aspects of the notes. Although, the documents are quite large. In addition, there are used a lot of terminology which can be overwhelming for one of the intended customer groups, the non-professional investors. We would also like the documents to be both more clear regarding the incurred costs, and to provide information regarding the true value of the notes. We also found the *sales brochures* to be informative. However, these documents left room for more misinterpretations, and were a little biased. The documents had a larger focus on the upside than for the downside, which can give investors the wrong expectations. The major problem with the supporting documents however, were the lack of information regarding the effects of regular dividends, hidden dividends imposed by the *quantos* structures, and of unsystematic risk exposure. These are elements that represents disadvantages for investors, and that could make the notes less attractive if had been included.

To summarise the discussion regarding this sub-question, we believe that the investors are provided with enough information so that it is possible to understand the autocallable notes. However, the *sales brochures* provide biased information, and key information is either lacking or not included in all of the supporting documents. Thus, we believe that the products are presented in a way that does not give the investors sufficient knowledge to do a well informed investment decision.

8.1.4 Who reap the benefits?

Regarding the main research question: *Who reap the benefits? The issuer, facilitator, distributor, or the investor?* The two autocallable notes analysed in this thesis appear overpriced with unfavorable conditions and biased marketing for the investor. In Chapter 5 we found a value of these products that suggested a price premium of ca. 50% paid by the investor. Further in Chapter 6, we identified unfavorable returns distributions and high probability of losses. Lastly, in Chapter 7 we saw that these products are prone to biased information in their marketing material, leading investors to invest in something different than what they might expect. The results suggest that the benefits of these products are reaped by the issuer, facilitator, and distributor at the expense of the investor. This does not mean that the investor will not be able to earn returns, but it means that the products incur high costs putting the investor in an unfavourable position.

8.2 Strengths and weaknesses

The Monte Carlo method used to value the autocallable notes have been able to perform many iterations, leading the estimated values to converge to the true value. Thus, we are certain that the value estimates are precise. To check the validity of the valuation framework we valued another derivatives product, where we had a closed form solution. The value estimates from this test converged to the closed form solutions, suggesting that the numerical valuation model is valid. We also ensured that the historical correlation were incorporated into the model, by creating a correlation matrix based on simulated correlations between the underlying assets. This simulated correlation matrix were almost identical to the historical correlation matrix.

The major weakness of the analysis in Chapter 5 and Chapter 6 are the historical data which were used in the models. The parameters going into the model are only estimates of the correct values. Furthermore, the Black & Scholes assume that these parameters also are constant. This is seldom the case. However, the parameters provided to the model are our best guess of the correct values. In addition, we performed sensitivity analysis of the key parameters of both *Autocall Norske Selskaper* and *Autocall Telekom*. These sensitivity analyses revealed that for us to change our conclusion regarding the autocallable notes, the estimated parameters would have to be substantially wrong.

Lastly, We did only investigate two autocallable notes in this thesis. To provide a more generalized answer of the the research questions, we would preferably have analysed substantially more notes. Regarding the scope of the thesis, this were however not feasible.

9 Conclusion

We have in this thesis aimed to identify who reap the benefits of autocallable structured products. To examine this, we investigated two autocallable notes named *Autocall Norske Selskaper* and *Autocall Telekom*. Both of these notes were issued by Goldman Sachs, facilitated by Garantum Fondkommission, and distributed and sold by Garantum Norge. After conducting a numerical valuation of the notes, we found that both were extremely overpriced, suggesting a price premium of ca. 50%. We investigated further what investors can expect when investing in the two autocallable notes. We found that the probabilities of ending up with negative returns were larger then ending up with a surplus. This finding were present for both the *Autocall Norske Selskaper* and the *Autocall Telekom* notes. What is even more shocking is that if ending up with negative returns, these were most likely in the range of -5% to -30% annually. Thirdly, we investigated how the products are presented to potential investors. In this analysis we found a lack of vital information and supporting documents that seemed biased. These findings indicate that investors are not given sufficient information to be able to make well informed investment decisions. Ultimately, we believe that it is the issuer, the facilitator, and the distributor that reaps the benefits of the autocallable strucutred products. This on the behalf of the investors.

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