Predetermined Assembly Size and Equal Influence in MMP-Elections

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Abstract

In MMP-elections (Mixed Member Proportional representation), a QP-ballot contains a *first-vote* for party Q's candidate in a single-seat constituency and a *second-vote* for a list of candidates from party P in one common tally. In *split* ballots P≠Q.

Traditional accounting (e.g. in Bundestag elections) does not record a ballot's *combination* of first- and second-vote; collecting them in separate ballot boxes will not change the result. The assembly size is out of control (111 *extra-ordinary* list seats in 2017 (137 in 2021).

Faithful accounting uses these combinations to obtain a predetermined size (the law's Bundestag norm is 299 list seats), while still complying with MMP's *proportionality rule*.

The Federal Constitutional Court emphasizes the principle of all voters' *equal influence* on the result. In 2017 and 2021 many split QP-ballots gave full support to two winners, but QQ-ballots only to one (Q=CSU). Faithful accounting removes this and some other inequalities in voters' influence on the election outcome.

The 2017 election achieved a unique transparency by giving top priority to (strict) proportionality. As the main example, it allows the following exposition of MMP with faithful accounting. A broader discussion in a wider setting, with references, is found in *The Structure of MMP-elections*.

Key words: Mixed member proportional, equal influence, assembly size, split ballots, faithful accounting.

JEL classification D72.

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1. MMP-elections with traditional accounting

MMP (Mixed Member Proportional representation) is a family of methods designed for election of a legislature. Each voter casts a ballot with two votes. The first vote, *ErSt* (Erststimme) is for a single seat election in the voter's constituency C_k, $1 \le k \le c$; the winner is *directly* elected to the assembly.

The second vote, *ZwSt* (Zweitstimme), supports a *list* of candidates from a political *party*, P_j, $1 \le j \le p$; r of them qualify to contest for *list seats.*

MMP-elections started in W-Germany (1949). With changing rules it has been used to elect the *Bundestag* (federal legislature). Table 1 shows that r=7 parties P_j qualified, by winning 3 direct seats or by receiving 5% of the ZwSt.

BUNDESTAG ELECTION 2017									
J P_j	ErSt		direct	SEATS	list	ZwSt			
	%	$e(P_j)$	$\omega(j)$	total	$\alpha(J)$	$z(P_j)$	%	ZwSt/ total	
1 CDU	30.2	14,030751	185	200	15	12,447656	26.8	62238	
2 SPD	24.6	11,429231	59	153	94	9,539381	20.5	62349	
3 AfD	11.5	5,317499	3	94	91	5,878115	12.6	62533	
4 FDP	7.0	3,249238	0	80	80	4,999449	10.4	62493	
5 Linke	8.6	3,966637	5	69	64	4,297270	9.2	62279	
6 Grüne	8.0	3,717922	1	67	66	4,158400	8.9	62066	
7 CSU	7.0	3,255487	46	46	0	2,869688	6.2	62385	
	97–	44,966765	299	709	410	z=44,189959	95–	62327	

TABLE 1 The assembly got 709 seats due to the proportionality requirement: Before list seats were distributed, CSU already had ω(7)=46 seats and z(CSU) ZwSt; how many seats should the seven parties then get, with their z=44189959 ZwSt? The "theoretical" answer is the following *critical size*:

(1.1) $\omega(7) \times z/z(CSU) = 46 \times 44189959 / 2869688 = 708.348\ldots$ seats.

On average, 62327 ZwSt support each of the 709 seats; the right hand column of ratios illustrates the accuracy of the approximation algorithm.

If a party has received more seats than proportionality entitles it to, it has seats *"in overhang".* By law, a party is not allowed to have *direct seats* in overhang, and 410 list seats were distributed before the sum of totals passed the critical value: Only then were all CSU's 46 direct seats out of overhang. Although each ballot must combine one ErSt and one ZwSt, the tally is *as if* they were collected in different ballot boxes. In section 2, *"faithful accounting"* literally takes these ballot combinations into account.

The critical size is volatile, as shown in three consecutive elections:

	ω(7) ×	z /	z(CSU)	= critical size
2013	45	36,867417	3,243569	511.484…
2017	46	44,189959	2,869688	708.348…
2021	45	42,380698	2,402827	793.703…

TABLE 2 By 2017 rules, distribution of list seats stops when the assembly size has passed or reached both critical size and 598 seats. Thus,
with 2017 rules, the *assembly sizes* in 2013; 2017; 2021 are, respectively
max(512, 598)=598 ; max(709, 598)=709 ; max(794, 598)=794.
In fact however, they became, respectively, 631; 709 ; 736.

In 2013, 33 *extra-ordinary list seats* were distributed according to *complicated rules for the 2D allocation of list seats* to r=5 parties *and* 16 states.

The 2017 rules achieved a new *transparency*: everybody could check the proportionality (708.348…≈709, Table 1), but the transparency vanished again: In 2021, the main explanation is an emergency law *letting CSU keep 4 direct seats in overhang*; 2017 rules give 794–598=196 extra-ordinary list seats.

The ω(j) in Table 1 show both the *ErSt-success* of P_j and its *commitment in the final ZwSt tally*, since the proportionality rule encompasses both direct seats and list seats: All seats must be "paid" with ZwSt at the same price. A small ZwSt resource z(CSU) of CSU (=P_7) and a large success/commitment ω(7) give CSU its unique *pivotal* staus: According to (1.1), the critical size is determined by data *specific* for CSU, i.e. ω(7) and z/z(CSU).

The pivotal party has the highest of the ratios $\omega(j)/z(P_j)$.

By law, the W/S-L algorithm (Webster/Sainte-Laguë) distributes $\alpha(j)$ list seats (one-by-one) to the qualified P_j $(1 \le j \le r)$:

(1.2) Party P_j contests for its t^{th} ℓist seat, $P_j\ell t$, with the

$$\textit{contest number}\ \ z(P_j) \,/\, \{2\times[\omega(j)+t] - 1\}, \quad t\ge 1$$

Under *faithful accounting*, the $\omega(j)$ are replaced by the $\psi[\Lambda(P_j)]$, which are non-integers defined in (2.13).

The last nine of the 410 list seats in Table 1, with contest numbers, are:

402	403	404	405	406	407	408	409	410
CDUℓ13	SPDℓ93	FDPℓ80	AfDℓ89	Linℓ64	CDUℓ14	SPDℓ94	Grüℓ66	CDUℓ15
31513	31483	31443	31434	31367	31354	31277	31266	31197

TABLE 3 The overhang in CSU's 46 direct seats ends with CDUℓ15.

The mechanism behind the growth of the critical size is seen in (1.1) and in Table 2. One factor, $\omega(7)$, varies only between 100% and 97.8% of its maximal value 46. (CSU runs only in Bavaria, where there are 46 constituencies.)

But CSU's share z(CSU)/z of the ZwSt, drops from 8.8% (2013), to 6.5% (2017) and to 5.7% (2021); the factor z/z(CSU) is its *inverse*, and may be normalized to 100% (2013), 135% in 2017, and 154% in 2021.

A QP-ballot has ZwSt to party P and ErSt to (the candidate of) party Q; the ballot is *split* if Q≠P. This may be due to a voter's splitting of an intended PP-ballot when P is considered unable to win the direct seat.

But also an intended QQ-ballot may be split to help a coalition partner P to pass the 5%-threshold. This motivation was particularly strong in the 2017 election, with Q=CDU or CSU, and P=FDP: P had been a government partner of the union parties, CDU/CSU, but failed to pass the 5% threshold in 2013. In 2017, FDP got 5 million ZwSt and passed 10%, but got only 3.35 million ErSt.

CDU/CSU got 17.3 million ErSt and only 15.3 million ZwSt. But CSU was pivotal; the dramatic fall of z(CSU)/z raised *critical size* to 708.348…(Table 2). The urge to help FDP was smaller in 2021, but an experience from 2017 was that with its 2.9 million ZwSt, CSU got 0 list seats (Table 1). Rather than "wasting" their ZwSt in 2021 too, many wanted to make better use of it: z(CSU)/z dropped again, pushing the critical size to 793.326… seats (Table 2).

(1.3) Accounting matters In Table 1, P_j is "account owner"; ω(j) is P_j's ErSt success, but also its "commitment" which must be paid with ZwSt from its second account, z(P_j). The proportionality rule requires the seat distribution to go on until all direct seats have been paid; the last one belongs to the pivotal party (P_7=CSU in table 1), rescued from overhang by CDUℓ15; see Table 3.
The proportionality rule implies the existence of a pivotal party. Table 2 illustrates a consequence of the dwindling ZwSt supply z(CSU) for the pivotal CSU: The traditional accounting in Table 1 is *not compatible* with the idea that all z voters (44189959 in Table 1) should, through their ballot (ErSt *and* ZwSt together), have the same influence on the outcome.
For equality, the influence, that a QP-ballot gets through its ZwSt to P, must depend on what influence it already got through its ErSt to Q.
Then the ballot's *combination* of P and Q cannot be ignored. In *faithful accounting* the *set* Λ(P_j), *of voters* with ZwSt to P_j, replaces P_j as account owner. The *combination* of ErSt *and* ZwSt is used to replace ω(j) by ξ(j) *and* by ψ[Λ(P_j)], i.e. Λ(P_j)'s success *and* commitment, defined in (2.3) *and* (2.13).

2. Faithful accounting

(2.1) **Definitions** $\Lambda(P_j)$ is the set of $z(P_j)$ voters with ZwSt to party P_j. *Faithful accounting* records the total influence of $\Lambda(P_j)$ through its members' ErSt: Let $E(k)$ be the number of voters and ballots with *ErSt to the winner* of the direct seat in C_k. $N(j,k)$ members of $\Lambda(P_j)$ give ErSt to the winner; thus,

(2.2) $$E(k) = N(1,k) + N(2,k) + \ldots + N(p,k)$$

The fraction $1/E(k)$ measures the *ErSt-success* for each of these $E(k)$ voters.

Faithful accounting then deposits a seat fraction $N(j,k)/E(k)$ on $\Lambda(P_j)$'s *success account*. Imagine each of $N(j,k)$ ballots carrying to $\Lambda(P_j)$ one ZwSt for P_j, but also an ErSt-success $1/E(k)$ which *will reduce* the effect of the ballot's ZwSt.

$\xi(j)$ is the ErSt-success of $\Lambda(P_j)$; it is an aggregate over all C_k:

(2.3) $$\xi(j) = N(j,1)/E(1) + N(j,2)/E(2) + \ldots + N(j,c)/E(c) \text{ seat shares.}$$

All c direct seats are accounted for (sum over j in (2.3) and use (2.2)):

(2.4) $$\xi(1) + \xi(2) + \ldots + \xi(p) = c.$$

(2.5) EXAMPLE The $N(j,k)$-values in 2017 are unknown, but one lucky circumstance indicates that $\xi(7)$ is significantly smaller than $\omega(7)$: Since CSU ($=P_7$) runs only in Bavaria and got *all* 46 direct seats, *all* $e(7)=3255487$ ErSt (Table 1) supported the winner of a direct seat. *At most* $z(7)$ members of $\Lambda(\text{CSU})$ gave ErSt to a winner. Most likely, their ErSt-succes was *less than*

$$46 \times 2869688/3255487 \approx 40.55 \text{ direct seats, i.e. } \xi(7) \leq 40.55.$$

Aggregation of the $\xi(j)$ over those $\Lambda(P_j)$ that did not pass the threshold gives an ErSt-success f wich may be small, but still too large to be neglected:

(2.6) $$f = \xi(r+1) + \xi(r+2) + \ldots + \xi(p).$$

The precise value of f depends on the N(j,k),but they are ignored in traditional accounting. However, Table 1 allows a rough estimate of f:

There are *at least* 44966765–44189959 = 776806 split P_aP_b-ballots ($a\leq 7<b$); if their ErSt distribution is typical, their ErSt-success is at least

(2.7) $f \approx 299 \times 776806 / 44966765 \approx 5.2$ direct seats.

(2.8) **Preparation for W/S-L,** see (1.2), to distribute h list seats.

The r voter sets $\Lambda(P_1), \ldots, \Lambda(P_r)$, have ErSt-successes $\xi(1), \ldots, \xi(r)$; the supply of z ZwSt shall pay for $c-f$ direct seats and h list seats; thus

(2.9) at a price z/T ZwSt/seat, where $T = c-f+h$.

With 2017 data (Table 1) and estimate (2.7), $T \approx 299-5.2+299 \approx 592.8$, the task is to distribute 299 list seats to seven $\Lambda(P_j)$ with ErSt-success 299–5.2 direct seats.

In general, the task is to distribute h list seats so that the z ZwSt pays for $T = c-f+h$ seats. If a total of T seats are distributed proportionally, then

(2.10) there is a "price" $z/T = z/[c-f+h]$ ZwSt/seat, and, equivalently, a *"purchasing power"* T/z seats/ZwSt

With data from Table 1, a rough price estimate is based on (2.7):

$z/T \approx 44189959/592.8 \approx 74532$ ZwSt/seat, $T/z \approx 1/74532$ seats/ZwSt

A ballot which supports the direct winner in C_k carries to its voter set $\Lambda(P_j)$ a *success* 1/E(k), and a *purchasing power* T/z. *And thus, here is a snag:*

(2.11) If $\Lambda(P_j)$'s *commitment account* gets an *increment* $1/E(k)>T/z$, then *the ballot increases $\Lambda(P_j)$'s commitment more than its purchasing power.*

Thus, the ballot harms its voter set $\Lambda(P_j)$ and P_j. This is a case of *Negatives Stimmgewicht* (*negative vote weight*), which was discovered in earlier versions of Bundestag elections (but was due to a much more complicated mechanism); the federal constitutional court (2008) found it unconstitutional.

To avoid Negatives Stimmgewicht, some commitment will be *waived*:

(2.12) $\Lambda(P_j)$'s *commitment account* is *increased* by $\min[T/z, 1/E(k)]$.

$N(j,k)$ ballots carry this increase from C_k to $\Lambda(P_j)$: $\psi[\Lambda(P_j)]$ is $\Lambda(P_j)$'s *final (not waived) commitment* under faithful accounting, aggregated over all C_k:

(2.13) $$\psi[\Lambda(P_j)] = \sum_k N(j,k) \times \min[T/z, 1/E(k)], \quad 1 \le k \le c.$$

Thus, P_j's success/commitment $\omega(j)$ in Table 1 is replaced by *two quantities*:

(2.14) $\Lambda(Pj)$'s ErSt-success $\xi(j)$, see (2.3), *and* its commitment $\psi[\Lambda(P_j)]$, see (2.13).

The $\xi(j)$ give f, see(2.6) and (2.9); $\psi[\Lambda(P_j)]$ replaces $\omega(j)$ in the algorithm (1.2).

According to (2.13), there are *two types* of constituencies in the election:

Type 1, $E(k) \ge z/T$: Commitment is $1/E(k)$ per ballot with ErSt to the winner; each of them has *"surplus purchasing power"*, $T/z - 1/E(k)$, in total for C_k

(2.15) $$E(k) \times [T/z - 1/E(k)] = E(k) \times T/z - 1$$

Type 2, $E(k) < z/T$: Commitment is T/z per ballot with ErSt to the winner; each of them has *"waived commitment"* $1/E(k) - T/z$, in total for C_k

(2.16) $$E(k) \times [1/E(k) - T/z] = 1 - E(k) \times T/z$$

Each ballot with ErSt to the winner in C_k is accounted for in (2.15) or (2.16). Relatively few of them bring ZwSt to a $\Lambda(P_j)$ which does not participate in the list seat distribution ($r<j\le p$).

Very small $E(k)$ occur mainly in C_K where most ErSt are spread on three or more strong candidates. Other factors change less: Constituencies are designed to have *about equal population*, and *voter participation* is more stable than $E(k)$ which reflects just the winner's share of all ErSt. However, voters in C_k get higher ZwSt influence when reduced $E(k)$ increases C_k's waived commitment (2.16).

Moreover, let C_a and C_b, both of type 2, have $E(a)=40000$ and $E(b)=60000$. A voter with successful ErSt wins 1/40000 of a seat in C_a, and 1/60000 in C_b, while their ZwSt cannot influence the result (except, possibly, by helping a party

across the 5% threshold). A successful Erst in C_b gains a 1/60000 share, and one in C_a gains 50% more, i.e. 1/40000.
If all C_k were of type 1, these kinds of unequal influence could not occur.

In legislatures based entirely on single seat constituencies, winners are usually supported by a large plurality. This *Duvergerian mechanism* will, even in the MMP-context, let many direct seats go to parties that attract voters in the political center. However, Duverger's *incentive* is weakened when a "wasted" ErSt to P has a reliable *"fallback"* support in a ZwSt to the same P.

The Bundestag variation of MMP uses the common plurality method *"first-past-the-post"* in its ErSt elections. Some well known single-seat elections use the related *Two-Round method* which promotes the plurality winner (party W) and runner-Up (party U) to a *final election* in order to get a majority winner (e.g. the French presidential elections).

(2.17) **The *W-U method*** is an *instant runoff* version of the Two-Round method: Without delay, it allows distribution of list seats based on MMP-ballots as in Bundestag elections. It requires only three numbers of ErSt:
w for winner W, u for runner-Up U, and t for all others Together.
Each ballot with ErSt to a candidate *not* in {W,U} counts as half an ErSt for W and half an ErSt for U. Thus, W remains direct winner, while E(k) is raised from w to w+t/2 (i.e. a majority), and less commitment is waived, see (2.16).

(2.18) EXAMPLE (Bundestag election 2021, C_{153} (Leipzig II):
W=Linke, U=Grüne): (w, u, t) = (40938, 32995, 105526).

W-U changes this to (w+t/2, u+t/2) = (93701, 85758); thus E(k) increases from w to w+t/2. C_{153} becomes type 1. In effect, (w, u, t) voters, respectively,
carry commitments (1/93701, 0, 1/187402).
The u voters who support U are rewarded by carrying 0 commitment to their $\Lambda(P_j)$. The t voters keep a substantial ZwSt influence, but to avoid commitment completely with tactical ErSt to the *expected* runner-up is quite risky, since such action may instead create and support a new and unwanted winner.

3. Constitutional issues

Until 2013, the overhang concept was applied, partly to each state, partly to all 16 states together. Negatives Stimmgewicht was a consequence; it was declared unconstitutional by the Federal Constitutional Court, July 3rd 2008. *Obiter dictum*, there was, in para 92, also a statement on equal influence. [1]

With the rules of 2017, Negatives Stimmgewicht disappeared, but the sudden drop of z(CSU) from 3.2 million (Table 2) pushed the assembly size way above the legal norm of 598 seats. The drop was partly due to split QP-ballots with ErSt to Q=CSU and ZwSt to, e.g. P=FDP (intended to help FDP across the 5% threshold). It also highlighted the problem of unequal influence.

In 2017, 2.9 million ZwSt were not enough to give CSU any list seat. The experience that ZwSt to CSU were wasted, may be a reason for an even larger drop of z(CSU) to 2.4 million in 2021. The concomitant increase of the critical size (Table 2) cannot be blamed on voter behavior. Ballot splitting is a natural behavior, allowed from 1953, and stimulated by the name "Personalisiertes Verhältniswahl". Tally rules need not give voters' natural adaptation such an unnatural consequence as growing assembly size: Faithful accounting brings critical size below a predetermined assembly size c+h.

To avoid a new version of Negatives Stimmgewicht, a ballot carrying a large commitment 1/E(k) from C_k then gets some of it waived; (2.12) and (2.16) show the amount waived in the ballot and in C_k. Majority methods will reduce waiving; *"W-U"* works even without changing the present voting rules.

[1] Aus dem Grundsatz der Wahlgleichhhheit folgt für das Wahlgesetz, dass die Stimme eines jeden Wahlberechtigten grundsätzlich den gleichen Zählwert und die gleiche rechtliche Erfolgschance haben muss. Alle Wähler sollen mit der Stimme, die sie abgeben, den gleichen Einfluss auf das Wahlergebnis haben.

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