CONTRIBUTIONS TO THE METHODOLOGY AND PRACTICE OF OPTIMIZATION PROBLEMS WITH MULTIPLE PREFERENCES

BY **NAHID REZAEINIA**

Department of Business and Management Science
NHH Norwegian School of Economics

Acknowledgement

First of all, I would like to greatly thank my supervisor and teacher Mario Guajardo, for his exemplary support and guidance during my Ph.D. work. I am proud to have a scientific supervisor like him that was always available to encourage, teach and guide me from beginning to end. I was like a blind person in the optimization world, and he gave me modeling skills and taught me how to dream and achieve in the optimization world. I will be forever grateful to him for all he has done for me. I would also like to give special thanks to Julio C. Góez, who besides Mario, supported me and shared his knowledge with me in my Ph.D. journey. His discussions and comments gave me a deeper understanding of optimization and mathematical modeling. It has been a great honor and inspiration for me to work with both of them.

Also, my words are not sufficient to express my sincere thanks to Leif Kristoffer Sandel, who gave me the chance and allowed me to continue my research, and be a member of this department during my Ph.D. It was a valuable opportunity for me, and I will be forever grateful to him. I also greatly appreciate Stein Wallace for his attention to me as a member of the optimization and logistics group, and his support for writing Chapter 1. In addition, I would like to express my gratitude to Marie Van Beijeren, Senior Consultant of the CEMS at NHH, for her assistance in data acquisition and support in writing Chapter 2.

Furthermore, I would like to thank all my colleagues at the Department of Business and Management Science for a great and friendly working environment. I appreciate Charlotte, Kristin, Natalia, Stein, Torill, and Turid for their help during these years. I am thankful to my Ph.D. friends, Aija, Andreas, Atle, Andre, Ben, Benjamin, Coskucan, Christian, Christer, Evengelos, Henrik, Huynh, Ole, Paulo, Rabia, Simone, Tatevik, and Yue, among others, for making my Ph.D. time so amazing. Negar and Ahmad for encouraging me at the beginning of my journey. Narges, Masi and Adrian for their great friendship and support. Gabriel and Ondrej for helping and supporting me. Also, Ritvana and Mahnaz for taking good care of me and for their valuable friendship during these years.

Extraordinary thanks go to my family for their endless support. I will always be proud of having such a wonderful family. I do not have enough words to express my appreciation to my kind and devoted parents, who have always been very supportive and interested in my studies, gave me motivation, and taught me to be a strong person. I will be forever grateful to them for all the things that they did for me. My great appreciation also goes to my dear sister and sisters in law Neda, Zahra, Pedra, and also my brothers and brother in law Hamidreza, Saeed and Mehdi for everyday encouragement and kindness. Finally, I am very grateful to my kind, loyal and dear husband Mostafa for always being patient, giving me love, creating many opportunities for me, and supporting me in these years. I will never forget his kindness and I will be forever thankful to him.

Bergen, April 10, 2022
Nahid Rezaeinia

Contents

Introduction

Nowadays, making decisions and optimization problems incorporate several criteria, objectives, and preferences, which usually lead to conflicting conditions (Rosenhead and Mingers (2001); Marttunen et al. (2017)). To make reliable decisions, decision makers have to consider all these aspects in their decision-making (Ishizaka and Nemery, 2013). Conflicting objectives, criteria, and requirements, and also the inherent existence of a trade-off between factors such as social, environmental, and economic criteria, efficiency and fairness, costs and benefits, have made problems more complex in practice (Linkov et al. (2005); Guo et al. (2013)).

At institutions of higher education, for example, a number of decision-making problems arise on resource allocation involving multiple criteria (Nebel, 2020). These include situations where people must be grouped into collaborative research teams, lesson study groups, discussion groups, team meetings, and assigning rooms and time slots to activities (Nelson et al. (2010); Kauffeld and Lehmann-Willenbrock (2012)). In particular, allocating students to project teams and scheduling scientific meetings involve decisions with direct consequences on people and must be done purposely (Sleenhof et al., 2021). To build successful teamwork and meetings, decision makers should improve the quality of the allocation by taking an acceptable, transparent and fair decision. Hence, the users' preferences that are affected by the decisions have an increasing effect on the structure and interaction of the allocation process. Although considering the users' preferences helps in doing a well-substantiated and fair allocation that gives equal opportunities to the users and makes it acceptable for them, it may also increase the level of difficulty of the problem. In fact, decision makers' different perspectives on one side and users' preferences on the other side, may lead to a trade-off, for which it is harder to find a solution.

Different decision support tools and systematic approaches are developed to help meeting organizers, schools and universities' leaders to do objective allocation, evaluate the trade-off between criteria, find the potential solutions, and determine the optimal ones. The approaches are directed to determine an optimal solution for the problems (Eisenführ et al. (2009); Yatsalo et al. (2015); Marttunen et al. (2017)). Typically, there are two types of decision support tools for problems that contain multiple criteria and multiple conditions: multi-objective approaches and multi-attribute approaches (Hwang and Yoon (1981); Jahan et al. (2016)).

By increasing peoples' attention to cost minimization, output maximization, environmental protection, production management, efficient scheduling, efficient and fair assignment, etc., the use of multi-objective approaches or multi-objective optimization with large scale, non-linear functions, and with more constraints has become more popular (Cui et al., 2017). Addressing multi-objective problems is usually difficult due to the complex nature of the problem and satisfying conflicting objectives simultaneously (Li-Cheng et al. (2009); Cui et al. (2017)). Multi-objective approaches achieve the optimal goals by considering the interactions between the given conditions. These approaches have decision variables with either a continuous or an integer domain. When the domain of the decision variables is integer, the approach is defined as integer multi-objective optimization (Jahan et al., 2016).

Multi-attribute approaches are used when there is no one perfect solution to suit all the criteria. Also, they do not necessarily lead to the same solution for every decision maker. These approaches incorporate decision makers' preferences information and have the distinction of placing them in the decision-making process and helping them to find a compromise solution (Ishizaka and Nemery, 2013). Multi-attribute approaches explore the balance between the pros and cons of different alternatives and support decision makers to determine a specific goal. Generally, they define the structure of the decision problem, specify the performance of the alternatives with respect to the criteria, and determine a decision (Adem Esmail and Geneletti, 2018).

Various fields, including mathematics, engineering, social studies, economics, agriculture, energy saving, environmental protection, sustainable development, scheduling, and many other problems in everyday life, are characterized by inherent multiple conflicting objectives and criteria (Gunantara, 2018). Therefore, multi-objective and multi-attribute approaches are used in a broad range of research works, with a growing number of real-world applications either in public policy making or decisions for private corporations (Ishizaka and Nemery, 2013).

This thesis focuses on multi-objective decision-making and multi-attribute decision-making approaches, with particular attention to problems where a central decision maker must consider data on multiple preferences expressed by different persons. Here, the preferences may indicate wishes of persons who will be affected by the solution to the problem, or opinions from experts whose different judgement adds information to the decision maker.

The thesis is organized into four free-standing chapters, where the first and second chapters aim at implementing a solution in practice to two real-world problems, and the last two chapters aim at studying methods to solve optimization problems from a more methodological perspective. Hence, this thesis contributes to both the methodology and practice of optimization problems with multiple preferences.

With my co-authors Julio C. Góez and Mario Guajardo, in the first chapter, we study the conference scheduling problem by considering attendees' preferences. In the second chapter, we focus on an assignment problem where the preferences come from students, and the goal is to find an efficient and fair assignment of them to projects. Derived from that application, in the third chapter, we study different formulations of the trade-off between efficiency and fairness in integer assignment problems by considering the users' preferences. Lastly, in the fourth chapter, I introduce a multi-methodology approach to determine the location of a road among different alternatives based on experts' judgement. Integer programming, multi-objective programming, lexicographic goal programming, eigenvalue method, and utility additive theory are employed throughout the chapters. In the following, each chapter is described in more detail.

Chapter 1. Scheduling conferences using data on attendees' preferences

Co-authored with Julio C. Góez and Mario Guajardo

Scheduling is an important part of the organization of any scientific conference. Motivated by the actual context of three different conferences, this chapter introduces a conference scheduling problem based on the attendees' preferences. In this paper the main question is how to schedule the parallel talks of a conference, trying to address those preferences while also considering other requirements, such as limited time-slots, speakers availability, and thematic cohesion. We use integer programming to deal with this scheduling problem and arrive to the candidate schedule. The attendees' preferences are collected through a survey in two different utility levels, which then are used to build a collision cost function. The main contribution of the paper is the development of a decomposition approach, which first schedules sessions to time blocks minimizing the collision cost function at session level, and then it schedules talks within the sessions minimizing the collision cost function at talk level. This approach proves to find high quality solutions in considerably shorter time than other two approaches that we study in the paper. In addition, the contribution adds up to the literature on the practice of Operations Research, as the approach has been used for decision support in the implementation of the schedule of three real-world conferences.

Chapter 2. Efficiency and fairness criteria in the assignment of students to projects

Co-authored with Julio C. Góez and Mario Guajardo

A wide range of personnel assignment problems has focused on optimization problems, with the goal of maximizing a measure of efficiency. Lately, fairness has also gained importance in this problem, as an efficient solution might end up with some groups with more benefits than others. Motivated by an actual problem of allocation of students to business projects in a master's program in Norway, this chapter introduces an assignment problem and discusses modeling and solution approaches to incorporate both efficiency and fairness criteria. The problem takes as input the preferences of the students on the projects they most want to conduct, in addition to other conditions such as requirements from the companies that propose the projects and balance in the groups in terms of gender, nationality, languages, etc. The main question then is how to assign students to the projects, so that their preferences are addressed in a fair an efficient manner, while the other conditions are also satisfied. We develop a bi-objective approach for this problem, in which the main contribution is to capture trade-off between efficiency and fairness using quantitative measures. In particular, to measure fairness we adopt a non-linear function called Jain's index, and we also study a lexicographic approach based on a linear function. Furthermore, the solution approaches are discussed by using different sequences on the optimization of efficiency and fairness. The proposed approaches have been used in practice to support the decision of the administrative body in charge of the program during three consecutive years. The results show that the implemented solutions have been beneficial for students, companies, and the administrative staff.

Chapter 3. On efficiency and the Jain's fairness index in integer assignment problems

Co-authored with Julio C. Góez and Mario Guajardo

Derived from the previous chapter, Chapter 3 focuses on the methodological side of the trade-off between efficiency and Jain's fairness index in the integer unbalanced assignment problem. In fact, besides the particular problem of assigning students to projects, this type of assignment problem appears in many applications, and it has received large attention from the academia. Finding solutions that perform well in both efficiency and fairness simultaneously is, therefore, an important topic in assignment problems. In this paper the main question is how to assign resources to the users and maximize the total benefits of the users and increase the fairness of the assignment, where the number of resources and users are unequal and a same resource could not be share among multiple users. We propose a bi-objective approach based on integer programming, by considering efficiency and Jain's fairness function to obtain the trade-off in the assignment. Since the Jain's fairness index is a non-concave function, which increases the difficulty of solving the optimization problem, the paper's main contribution is developing reformulations to overcome this issue. We study two reformulations, where one is based on a convex quadratic objective function and the other one is based on mixed integer second order cone programming. The reformulations and the original formulation are tested using data from a real-world case and also experimental data from different scenarios, and their performance is analyzed in terms of solution quality and solving time. The results show that although all formulations conduce to high quality solutions, the convex quadratic reformulation outperforms the others in solving time.

Chapter 4. The eigenvalue-UTA approach for multi-criteria decision-making problems: A case study on a rural road selection in Iran

Rural road development has been identified by Picchio et al. (2018) as one of the critical challenges for forest sustainability in forest management. Despite the environmental consequences of rural road building, its consequences on development and the inhabitants' welfare, and its considerable construction costs, only a few studies have considered all of these criteria simultaneously in choosing the proper location for rural road building.

In this chapter, the focus is on a decision-making situation where all conflicting criteria need to be considered in the rural road location selection process. In the existing literature it is assumed that all the quantitative criteria can be evaluated and such evaluation is given as an input to the problem. However, in practice, it might be costly and time-consuming to evaluate these criteria. Therefore, it becomes important to address the question on how to evaluate the alternatives in lack of some data. To this aim, this paper contributes to developing a multi-methodology approach and evaluating the alternatives, combining previous methods from the literature.

Several approaches have been developed for multi-attribute decision-making problems. The approaches are divided into two main categories: those weighting the criteria and ranking the alternatives, and those determining only the rank of the alternatives where the weight of the criteria is computed beforehand. The multi-attribute approaches based on utility theory consider both weighting and ranking when the alternatives' evaluation regarding the criteria are available.

In this paper, the multi-methodology approach is based on the eigenvalue method and utility additive theory is proposed to take into account decision makers' preferences and to evaluate the alternatives. The eigenvalue method evaluates the importance of the alternatives respect to the criteria, and the UTA method determines the importance of the criteria and the priorities of the alternatives. The approach is illustrated in a rural road selection problem. To the best of my knowledge, no studies have considered this multi-methodology approach in addressing multi-attribute decision-making problems, especially where a part of the data is not available for the decision makers. In addition, the paper contributes to investigating the effect of cost, ecological, risk and opportunity aspects in the process of choosing the proper location for forest road building.

References

Adem Esmail, B. and Geneletti, D. (2018), ‘Multi-criteria decision analysis for nature conservation: A review of 20 years of applications’, *Methods in Ecology and Evolution* **9**(1), 42–53.

Cui, Y., Geng, Z., Zhu, Q. and Han, Y. (2017), ‘Multi-objective optimization methods and application in energy saving’, *Energy* **125**, 681–704.

Eisenführ, F., Weber, M. and Langer, T. (2009), *Rational decision making*, Springer.

Gunantara, N. (2018), ‘A review of multi-objective optimization: Methods and its applications’, *Cogent Engineering* **5**(1), 1502242.

Guo, C., Sheng, M., Zhang, Y. and Wang, X. (2013), ‘A Jain’s index perspective on α-fairness resource allocation over slow fading channels’, *IEEE communications letters* **17**(4), 705–708.

Hwang, C.-L. and Yoon, K. (1981), Methods for multiple attribute decision making, *in* ‘Multiple attribute decision making’, Springer, pp. 58–191.

Ishizaka, A. and Nemery, P. (2013), *Multi-criteria decision analysis: methods and software*, John Wiley & Sons.

Jahan, A., Edwards, K. L. and Bahraminasab, M. (2016), *Multi-criteria decision analysis for supporting the selection of engineering materials in product design*, Butterworth-Heinemann.

Kauffeld, S. and Lehmann-Willenbrock, N. (2012), ‘Meetings matter: Effects of team meetings on team and organizational success’, *Small Group Research* **43**(2), 130–158.

Li-Cheng, J., Dong-Dong, Y. and Wen-Ping, M. (2009), ‘Research on evolutionary multi-objective optimization algorithms’, *Journal of Software* **20**(2), 271–289.

Linkov, I., Sahay, S., Kiker, G., Bridges, T. and Seager, T. P. (2005), Multi-criteria decision analysis: A framework for managing contaminated sediments, *in* ‘Strategic management of marine ecosystems’, Springer, pp. 271–297.

Marttunen, M., Lienert, J. and Belton, V. (2017), ‘Structuring problems for multi-criteria decision analysis in practice: A literature review of method combinations’, *European Journal of Operational Research* **263**(1), 1–17.

Nebel, A. (2020), An Exploration of the Role of Political Skill on Decision-Making for Resource Allocation in Higher Education, PhD thesis, Creighton University.

Nelson, T. H., Deuel, A., Slavit, D. and Kennedy, A. (2010), ‘Leading deep conversations in collaborative inquiry groups’, *The Clearing House* **83**(5), 175–179.

Picchio, R., Tavankar, F., Venanzi, R., Lo Monaco, A. and Nikooy, M. (2018), 'Study of forest road effect on tree community and stand structure in three Italian and Iranian temperate forests', *Croatian Journal of Forest Engineering: Journal for Theory and Application of Forestry Engineering* **39**(1), 57–70.

Rosenhead, J. and Mingers, J. (2001), *Rational analysis for a problematic world revisited*, John Wiley & Sons.

Sleenhof, J. P., Thurlings, M. C., Koopman, M. and Beijaard, D. (2021), 'The role of structure and interaction in teachers' decision making during allocation meetings', *Teaching Education* pp. 1–23.

Yatsalo, B., Didenko, V., Gritsyuk, S. and Sullivan, T. (2015), 'Decerns: a framework for multi-criteria decision analysis', *International Journal of Computational Intelligence Systems* **8**(3), 467–489.

Chapter 1

Scheduling conferences using data on attendees' preferences*

Nahid Rezaeinia[a], Julio César Góez [a], Mario Guajardo [a]

[a]Department of Business and Management Science, NHH Norwegian School of Economics, 5045 Bergen, Norway

Abstract

Conferences organizers often face the challenge of scheduling a scientific program. This problem usually involves many talks that must be scheduled in parallel, subject to time and space limitations. This paper adopts an Attendee-Based-Perspective to the conference scheduling problem, in which we collect data on attendees' preferences and use these as a main driver to schedule the talks. We test three optimization approaches for this problem, based on integer programming formulations. The main approach divides the problem into two stages: the first stage schedules predefined thematic sessions and the second stage schedules talks within these sessions. We report results using real data instances of three conferences. The results show that our main approach can produce solutions swiftly, accommodating the requirements of the organizers while allowing attendees to attend most of their preferred talks without collisions. Our work has been used in practice to generate the actual schedule of these three conferences.

Keywords Conference Scheduling, Integer Programming, Optimization in Practice

*The current version of this paper is under first review in a scientific Journal.

1 Introduction

A large number of scientific meetings, such as conferences and workshops, are held annually in various fields. These meetings are important for the scientific progress, as they provide a venue for discussion of new ideas among researchers and facilitate new collaborations. Conferences usually require considerable budget expenses and effort. Particularly, when putting together a scientific programme, conference organizers face significant scheduling challenges and engage people working on the organization for several months ahead (Stidsen et al., 2018). Usually, the organizers need to schedule a large number of talks, often run in parallel, while respecting time and space limitations. Even in conferences held virtual, a recurrent format in the current pandemic times, scheduling remains as an important task for conference organizers.

Typically, conference scheduling is done manually (Manda et al., 2019). While a basic setup can perhaps be approached in this way, the consideration of several criteria may require the use of optimization tools. A poor schedule can have, for example, undesirable consequences in the satisfaction level of the attendees and presenters. As they usually incur high costs to take part of the conference (time consumption, registration fees, travel expenses), a rewarding goal of the scheduling is to provide attendees with the opportunity to attend all or most of their preferred talks. In achieving this, the organizers need first to know the preferences of the attendees and then to deploy a methodology to use these preferences appropriately in the generation of the schedule. In this paper, we report on the real-world implementation of a preference-driven approach to schedule conferences. Inspired in Vangerven et al. (2018), we implemented a survey asking attendees to elicit the talks they would like to attend, and then generated a schedule that attempts to minimize a collision cost function. This cost function is defined according to the attendees' preferences. Our approach is based on integer programming, and explores different variants to arrive at a candidate schedule. The approach has been adopted in practice to schedule three conferences so far.

The remainder of this paper is organized as follows. Section 2 reviews relevant literature. Section 3 explains the conference scheduling problem. Section 4 presents optimization models for this problem. Section 5 describes three application cases and reports results implemented in practice. Section 6 concludes with some final remarks.

2 Literature review

The scientific literature has paid increasing attention to the conference scheduling problem. A broad range of works have studied the problem, adopting a variety of approaches. In this section, we limit ourselves to review only papers that have used an Attendee-Based-Perspective (ABP) approach, which is the most relevant to our work (for other approaches, we refer the reader to works such as Nicholls (2007), Potthoff and Brams (2007), Edis and Edis (2013), Stidsen et al. (2018), Manda et al. (2019), Castaño et al. (2019), and Bulhoes et al. (2022)).

In an early attempt, Eglese and Rand (1987) study the conference scheduling problem with the ABP approach. They consider assigning sessions to time periods and rooms. They ask attendees to list their four most preferred sessions and also one reserve session. A set of weights is used to penalize scheduling alternatives that are not in the attendees' preferences list. The problem is addressed by an integer programming model, whose objective function is to minimize the sum of the weights. The authors use a simulated annealing algorithm to solve the problem. The approach is applied to schedule a conference with 15 different sessions, 4 time periods, and 7 rooms. In another problem, but of similar nature, Sampson and Weiss (1995) formulates an integer programming model to maximize the attendees' requests for sessions. Their problem consists of two parts. One is to assign sessions to periods, and the other one is to assign attendees to sections of the sessions. A heuristic procedure is developed to solve both parts simultaneously. They tested the procedure in randomly generated data instances. Le Page (1996) addresses the problem of assigning sessions to time-slots and rooms. In this problem, the rooms have different capacities. Sessions with the same topics must be in the same rooms, and the possibility of scheduling some consecutive sessions on the same day is considered. Each attendee provides a preferences list with the number of talks that they wish to attend. The preferences lists is used to build a conflict matrix that comprises the number of attendees who wish to attend each pair of sessions. A semi-automated heuristic in four steps is proposed to minimize the sum of the conflicts between the simultaneous sessions. The approach is applied to schedule a meeting with 35 sessions, 5 rooms, 7 time-slots, and 1100 attendees. Thompson (2002) proposes a heuristic algorithm to schedule sessions of a conference based on attendees' preferences. In this problem, the rooms have different capacities and are subject to limited availability. The author tests the proposed algorithm with randomly generated data and also with real data of a conference which consists of 47 sessions, 8 time-slots, 8 rooms, and 175 attendees. Sampson (2004) formulates a mixed integer programming model that maximizes a general attendees' utility function and uses it to schedule a conference with 213 sessions, 10 blocks, and 1086 attendees. They ask attendees to rank their preferences for talks, and then the resulting rankings are used to organize the sessions. A simulated annealing algorithm is developed to solve the model. To eliminate the sessions' hopping, they allow attendees to enroll only in scheduled sessions with room capacity constraints.

The references above considered the attendees' preferences for scheduling the sessions (that is, groups of talks) and not talks to specific time-slots within the sessions. In this respect, Ibrahim et al. (2008) use the combinatorial design theory and present a method for organizing three conferences. They assign talks to time-slots within some days and in three parallel sessions. Talks that belong to the same field must not be scheduled simultaneously. Also, talks that belong to the same pair of fields should not be scheduled in parallel more than once on the same day. Zulkipli et al. (2013) formulate a goal programming model for assigning talks to time-slots based on the attendees' preferences. In their study, attendees rank their preferred talks from 1 to 10. The resulting preferences list is used to generate weights associated to the talks. Then, the schedule is generated so as to have balanced weights over sessions scheduled in the same time-slots. The proposed model is applied to a case with 60 talks and 15 sessions. In a continuation of this work, Rahim et al. (2017) address a conference scheduling problem with attendees' preferences by a Domain Transformation Approach. The goal is to maximize the attendees' satisfaction, which is followed by the minimization of conflicts between sessions and time-slots. The approach attempts to avoid scheduling talks from the same author in parallel and also to avoid scheduling talks that are in the attendees' preferences list in parallel. They test the approach in a dataset with 60 talks and 26 respondents. Quesnelle and Steffy (2015) develop an integer programming model to assign talks to time-slots and rooms, based on a list of preferences of the attendees for the different talks. They consider rooms and presenters' availability, and also the possibility that some speakers should present more than one talk, and the possibility to offer some talks more than one time. They test the model in a data instance where the attendees' preferences are generated randomly.

In the paper closest to ours, Vangerven et al. (2018) address the scheduling of sessions and talks using a three-step approach. Each of these steps formulates an integer programming model. First, they schedule talks based on the attendees' preferences to maximize total attendance. The objective function associated with the schedule of talks minimizes missed attendance. In the second step, they reduce the number of session hopping or the overlap between the parallel sessions to allow more attendees to stay in the same room during a session. In the third step, they consider presenter availabilities in the scheduling. Their approach is applied to schedule four conferences and a positive impact is reported in practice. While our problem is similar, some features differ. First, we allow for preferences in two different utility levels, in contrast to the single level in their problems. As we report later, this has implications in the data profile of the preferences and in how they are used in the modeling and solution approach. Second, our approach is not fully driven by the data on the preferences, but it also incorporates a human-assisted grouping of the talks into sessions. Thus, instead of allowing any combination of talks within a same session as they do, our approach takes as input a partition of the talks into sessions made beforehand. This was motivated from practice, as the organizers of the conferences we worked with preferred to keep some thematic cohesion among talks of the same session. For testing purposes, we also formulate two other approaches, where one preserves the thematic cohesion and schedules talks into time-slots directly, and the other one schedules talks into time-slots relaxing the predefined sessions. Also, our problem considers presenter availabilities as side-constraints of the original problem, instead of postponing the satisfaction of these to a third goal. Lastly, our contribution adds up to the practice literature on conference scheduling, reporting the implementation of results in three real-world conferences.

3 The conference scheduling problem

As reviewed in the previous section, a variety of problem features may appear in different applications of conference scheduling. We follow Vangerven et al. (2018) to define some essential concepts. Typically, a conference format is comprised of several sessions. A session is a set of talks taking place consecutively in the same room. The number of consecutive talks in each session is defined as the length of a session. Consecutive sessions are separated by breaks. It is common to have several parallel sessions, that is, sessions taking place at the same time. The number of parallel sessions is usually limited by the number of available rooms. In the timetable of each conference, there are some predefined blocks. Each block is a set of time-slots with a specific length. In general, when putting together the scientific program, the organizers face a problem involving the assignment of talks to sessions, scheduling the sessions to the blocks, and scheduling talks of each session to specific time-slots within a block. When using the Attendee-Based-Perspective (ABP), the organizers need to know the preferences of the attendees with respect to the talks accepted for the conference. A list of preferences indicates the degree of interest of a conference attendee in each of the talks. Then, the scheduling task is performed attempting to maximize (or minimize) a measure of satisfaction (or cost) of the final schedule with respect to those preferences, while satisfying other conditions such as room and time limitations. We illustrate this with a simple example below.

Let us consider a conference with 16 talks that must be grouped into four sessions of four talks each, and scheduled over the course of two blocks with four time-slots each. To collect the preferences of the attendees, the organizers can run a survey where the talks are displayed and the attendees are asked to express their interest in each of the talk. The interest can be expressed by a scale of utility levels, where the highest interest is reflected in a higher number in the scale. In our example, let us focus on the preferences of only two attendees and a scale of three utility levels, where 0 means the attendee is not interested to attend the talk (*non-preferred*), 1 means the attendee is interested to attend the talk (*medium-preferred*), and 2 means the attendee is highly interested to attend the talk (*high-preferred*). Table 1 shows the preferences of two attendees for each of the 16 talks.

Table 1: Example of attendees' preferences list

Talk	**Attendee 1**	**Attendee 2**	**Talk**	**Attendee 1**	**Attendee 2**
A	2	2	*I*	0	0
B	0	2	*J*	2	0
C	0	2	*K*	0	2
D	2	2	*L*	2	0
E	0	2	*M*	0	2
F	2	2	*N*	1	0
G	1	2	*O*	0	0
H	2	0	*P*	2	2

Suppose that the talks have been grouped into sessions beforehand. The next problem is assigning sessions to the blocks and talks to the time-slots within each block. Figure 1 shows three possible schedules. In schedule 1, sessions 1 and 2 are scheduled in parallel, followed by sessions 3 and 4, also scheduled in parallel. If an attendee wants to attend two talks that are scheduled in parallel, we may assign a collision cost, since the attendee will not be able to attend these two talks simultaneously. In this paper, we compute this cost by adding up the multiplication of the attendee's utility level across every pair of talks scheduled in parallel. For example, for attendee 1 in Schedule 1, scheduling talks A and B in parallel contributes zero cost (2×0, using the data of Table 1), while scheduling talks D and J in parallel contributes with a cost of four (2×2). Likewise, talk F in parallel with L, and talk H in parallel with P contribute with eight more cost units. The pairs of parallel talks in sessions 3 and 4 of Schedule 1 do not add more costs. Thus, the total collision cost for attendee 1 in Schedule 1 is equal to 12. Another possible solution is Schedule 2, where the parallel sessions are 1 with 4, and 2 with 3. The total collision cost for attendee 1 in this schedule is 4, which improves considerably over the previous solution. Attendee 1 misses two medium-preferred talks (utility level 1) in Schedule 2, while in Schedule 1 the attendee misses three high-preferred talks (utility level 2). A third alternative is Schedule 3, which slightly modifies Schedule 2 by changing the order of the talks in session 2. The total collision costs for attendee 1 with this schedule is two, which again improves over the previous solution.

Even if the sessions are already scheduled to blocks, one may try improving a given solution by the specific scheduling of talks to time-slots within the block. An example is given in Figure

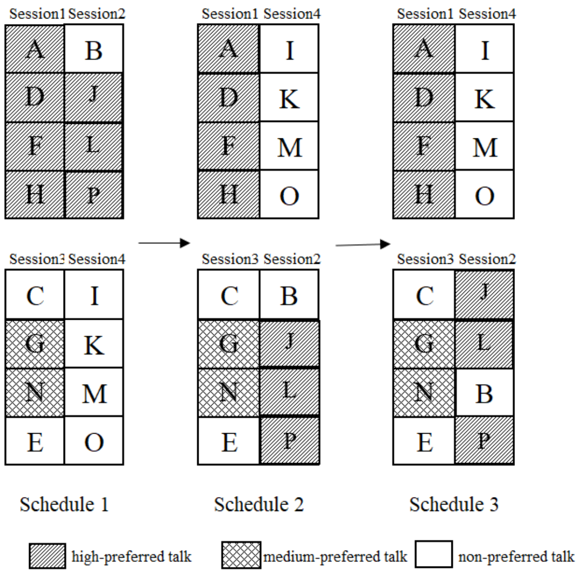


Figure 1: Illustrating the impact of scheduling on the collision costs for attendee 1 with predefined sessions

2, with two alternative schedules of talks for a same configuration of sessions and blocks. In this example, the collision costs for attendee 1 in Schedule 4 is seven. Schedule 5 modifies the schedule of talks to time-slots, while keeping the assignment of sessions to blocks, reducing the collision cost to one (equivalent to missing a medium-preferred talk).

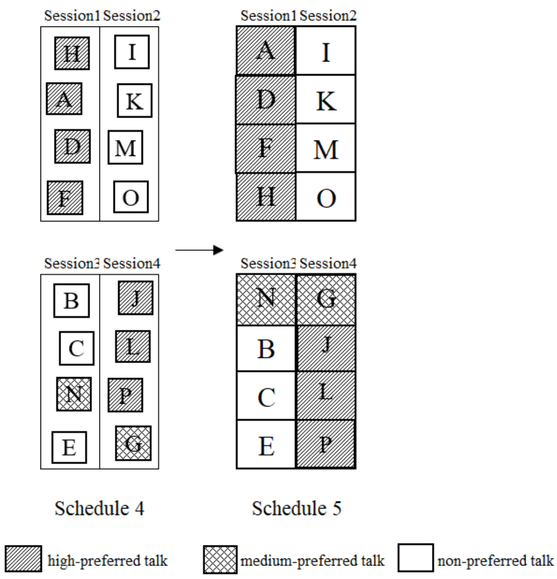


Figure 2: Illustrating the impact of scheduling on the collision costs for attendee 1 without predefined sessions

So far we have illustrated the problem only taking into account one attendee. The problem naturally becomes more difficult with several attendees, which we illustrate with the example in Figure 3. This example considers the preferences of both attendees displayed in Table 1. Note that talks A, D, F and P are high-preferred talks for both attendees. Furthermore, each attendee has some other preferences. With four sessions and four time-slots within a session, it becomes impossible to schedule the talks so that both attendees would be able to attend all of their preferred talks. In Schedule 6, Attendee 1 is allowed to attend all his/her preferred talks, while Attendee 2 misses three high-preferred talks.

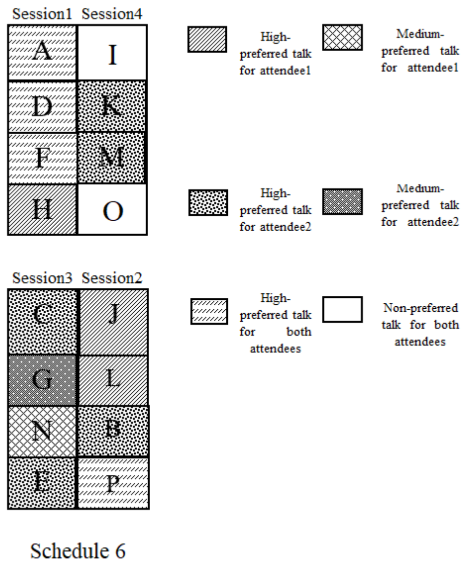


Figure 3: Illustrating the impact of scheduling on the collision costs for two attendees

In all the examples above, we have only focused on preferences and the basic structure of the conference. If on top of that, we add other conditions from the organizers and speakers, the problem becomes more difficult and practically impossible to approach manually. Motivated by the involvement in the organization of some conferences, we have studied several optimization modelling approaches based on the ABP perspective. These approaches are presented in the following section.

4 Optimization models

In this section, three Integer Linear Programming (ILP) approaches are formulated for the problem. In the first two, the talks are enforced to respect the predefined thematic sessions, while in the third one this condition is relaxed.

4.1 Enforced thematic cohesion

Scheduling a conference completely driven by the preferences of the attendees may lead to a schedule with poor thematic cohesion of the different talks assigned to a same session. In fact, although the preferences might naturally induce certain degree of cohesion (as each attendee would often prefer talks that belong to a same research area), some attendees might have interest in diverse topics. A fully preference-driven approach may well disregard such cohesion, but in the applications that triggered our work, the organizers preferred to assure thematic cohesion by grouping talks into sessions before we would make any attempt to minimize a collisions cost function. A simple barrier was that it would become impractical to coin an informative name for a session of talks grouped by a fully preference-driven approach without a common theme. Although non-automated, grouping talks into sessions beforehand allowed the organizers to secure a certain relationship among the talks and to overcome such a practical barrier. This motivated our first two approaches, which take as input a partition of the talks into sessions created beforehand by the scientific committee.

4.1.1 A single model with predefined sessions

Given a set of thematic sessions, we formulate a straightforward ILP model that assigns talks to time-slots, securing that talks within a same predefined session are scheduled to the same block. To this aim, we define the sets below.

- $\mathcal{P}$: set of conference attendees.
- $\mathcal{I}$: set of talks.
- $\mathcal{E}$: set of predefined sessions (that is, subsets of talks grouped beforehand).
- $\mathcal{K}$: set of time blocks.
- $\mathcal{S}_k$: set of time-slots within a block $k \in \mathcal{K}$.
- $\mathcal{S}$: set of time-slots, where $\mathcal{S} = \cup_{k \in \mathcal{K}} \mathcal{S}_k$.

We also define the parameters associated with the inputs and requirements made by the organizers below.

- c_{ij}: total collision cost over all attendees between talks $i, j \in \mathcal{I}$, where $i \neq j$. We calculate this cost using the input provided by the attendees as follows:

$$c_{ij} = \sum_{p \in \mathcal{P}} \phi_{ip}\phi_{jp} \quad \forall i, j \in \mathcal{I} : i \neq j \tag{1}$$

Here ϕ_{ip} is the utility attendee p stated it would obtain from attending to talk i. We restrict ϕ_{ip} to be in the set $\{0, \ldots, \Phi\}$, where Φ is the highest ranking allowed for an attendee to assign to a talk.

- α_{ie}: binary parameter equal to one if talk $i \in \mathcal{I}$ is assigned to session $e \in \mathcal{E}$, and zero otherwise.
- β_{ij}: binary parameter equal to one if talks i and j belong to the same session, and zero otherwise (it follows that $\beta_{ij} = \sum_{e \in \mathcal{E}} \alpha_{ie} \cdot \alpha_{je}\ \forall i, j \in I : i \neq j$).
- n: number of available rooms at the conference venue.

In the model we consider the two sets of binary variables defined below.

$$x_{is} = \begin{cases} 1 & \text{if talk } i \text{ is scheduled in time-slot } s \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$y_{ij} = \begin{cases} 1 & \text{if talks } i \text{ and } j \text{ are scheduled in the same time-slot} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

With the definitions above, we formulate the integer linear model below.

$$\min \sum_{i \in \mathcal{I}} \sum_{\substack{j \in \mathcal{I} \\ i \neq j}} c_{ij} y_{ij} \tag{4}$$

s.t.

$$\sum_{s \in \mathcal{S}} x_{is} = 1 \quad \forall i \in \mathcal{I} \tag{5}$$

$$\sum_{i \in \mathcal{I}} x_{is} \leq n \quad \forall s \in \mathcal{S} \tag{6}$$

$$x_{is} \leq \sum_{\substack{t \in \mathcal{S}_k \\ t \neq s}} x_{jt} \quad \forall i, j \in \mathcal{I}; s \in \mathcal{S}_k; k \in \mathcal{K} : \beta_{ij} = 1 \tag{7}$$

$$x_{is} + x_{js} \leq 1 + y_{ij} \quad \forall i, j \in \mathcal{I}; s \in \mathcal{S} : \beta_{ij} \neq 1 \tag{8}$$

$$x_{is}, y_{ij} \in \{0, 1\} \quad \forall i, j \in \mathcal{I}; s \in \mathcal{S} \tag{9}$$

The objective function (4) minimizes the collision costs between talks scheduled in the same time-slot. Constraints (5) ensure that each talk is assigned to one time-slot. Constraints (6) limit the number of talks in each time-slot by the number of available rooms. Constraints (7) impose that talks which belong to the same session are assigned to time-slots within the same block. Constraints (8) are logical relationships to identify the talks scheduled in the same time-slot. Constraints (9) define the domain of the variables.

4.1.2 A two-model decomposition approach with predefined sessions

As it will be reported in the results section, the previous formulation did not produce solutions so quickly. To speed up the process, an alternative approach was developed that preserves the predefined sessions but decomposes the problem in two steps. In the first step, the sessions are assigned to blocks, while minimizing a collisions cost function at sessions level. In the second step, the talks are scheduled to the specific time-slots within the corresponding blocks, while minimizing a collisions cost function at talks level. In each step, an integer programming model is formulated and solved. In these formulations, we maintain the definition of sets and parameters presented previously, and also add a few more definitions. First, we define parameter m_e as the length (number of time-slots) of session e. We also define a new parameter ζ_{ef} to calculate the total collision costs between session e and f as follows:

$$\zeta_{e,f} = \sum_{i \in \mathcal{I}} \sum_{\substack{j \in \mathcal{I} \\ i \neq j}} c_{ij} \alpha_{ie} \alpha_{jf} \quad \forall e, f \in \mathcal{E} \colon e \neq f \tag{10}$$

Note that in the expression (10), the collision cost is calculated at session level, as if attendees would not hop from one session to another.

Step 1: Assigning sessions to blocks

This step assigns sessions to blocks while minimizing the total collisions cost among sessions scheduled to the same block. For the optimization model in this step we define two sets of variables as follows:

$$w_{ek} = \begin{cases} 1 & \text{if session } e \text{ is assigned to block } k \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

$$z_{ef} = \begin{cases} 1 & \text{if sessions } e \text{ and } f \text{ are assigned to the same block} \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

The ILP model for assigning sessions to blocks is formulated in (13) – (17).

$$\min \sum_{e \in \mathcal{E}} \sum_{\substack{f \in \mathcal{E} \\ f \neq e}} \zeta_{ef} z_{ef} \tag{13}$$

s.t.

$$\sum_{k \in \mathcal{K}} w_{ek} = 1 \quad \forall e \in \mathcal{E} \tag{14}$$

$$\sum_{e \in \mathcal{E}} w_{ek} \leq n \quad \forall k \in \mathcal{K} \tag{15}$$

$$w_{ek} + w_{fk} \leq 1 + z_{ef} \quad \forall k \in \mathcal{K}; e, f \in \mathcal{E} \colon e \neq f \tag{16}$$

$$w_{ek}, z_{ef} \in \{0, 1\} \quad \forall e, f \in \mathcal{E}; k \in \mathcal{K} \tag{17}$$

The objective function (13) minimizes the total collision costs among sessions scheduled to the same block. Constraints (14) ensure that each session is assigned to one block. Constraints (15) ensure that the number of sessions assigned to each block does not exceed the number of available rooms. Constraints (16) are logical relationships to identify when two given sessions are scheduled in the same block. Constraints (17) state the binary domain of the variables.

Step 2: Scheduling talks to time-slots

In the previous step, the model assigns sessions to blocks and minimizes the number of conflicts using a collision cost function computed at sessions level. In practice, however, the attendees may hop from one session to another, to see talks that are not part of the same session but are scheduled at different time-slots. This is accounted for in the second step, where the optimal solution of the first step w^*_{ek} is used as an input that fixes the sessions scheduled to blocks. Given that, we attempt to minimize the conflicts between the parallel talks within each block, using a collision cost function computed at talks level. For this purpose, we formulate another ILP model that keeps the notation previously defined, and also incorporate some new definitions. In (18) a binary parameter γ_{ij} is defined to identify two different talks that belong to different sessions in the same block, as follows:

$$\gamma_{ij} = \sum_{k \in \mathcal{K}} \sum_{e \in \mathcal{E}} \sum_{\substack{f \in \mathcal{E} \\ f \neq e}} w^*_{ek} w^*_{fk} \alpha_{ie} \alpha_{jf} \quad \forall i, j \in \mathcal{I} : i \neq j \tag{18}$$

Two sets of decision variables are considered in the model. First, in (19) we define a set of binary variables to schedule the talks in the time-slots of their corresponding blocks.

$$u_{is} = \begin{cases} 1 & \text{if talk } i \text{ is scheduled in time-slot } s \text{ of its block} \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

Second, in (20) we define a set of binary variables to identify which pairs of talks are scheduled in the same time-slots.

$$v_{ij} = \begin{cases} 1 & \text{if talks } i \text{ and } j \text{ are scheduled in the same time-slot of the same block} \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

The ILP is formulated as follows.

$$\min \sum_{i \in \mathcal{I}} \sum_{\substack{j \in \mathcal{I} \\ \gamma_{ij}=1}} c_{ij} v_{ij} \tag{21}$$

s.t.

$$\sum_{s \in \mathcal{S}} u_{is} = 1 \quad \forall i \in \mathcal{I} \tag{22}$$

$$\sum_{\substack{i \in \mathcal{I} \\ \alpha_{ie}=1}} u_{is} = 1 \quad \forall e \in \mathcal{E}, s \in \mathcal{S} : s \leq m_e \tag{23}$$

$$u_{is} + u_{js} \leq 1 + v_{ij} \quad \forall i, j \in \mathcal{I}; s \in \mathcal{S} : \gamma_{ij} = 1 \tag{24}$$

$$u_{is}, v_{ij} \in \{0, 1\} \quad \forall i, j \in \mathcal{I}; s \in \mathcal{S} \tag{25}$$

The objective function (21) minimizes the collision costs between talks scheduled in the same time-slot. Constraints (22) ensure that each talk is assigned to one time-slot. Constraints (23) ensure that each time-slot is used. Constraints (24) are logical relationships to identify the talks scheduled in the same time slot of a block. Constraints (25) determine the nature of the variables as binary.

4.2 Relaxed thematic cohesion

In this section, we disregard the predetermined sessions and allow the talks to be scheduled directly to time-slots, driven only by the attendees' preferences. Although this approach does not necessarily meet the thematic cohesion wished by the organizers, it provides an idealistic basis for comparison of the solutions obtained by the two previous approaches.

The main decision in this third approach is whether a talk is assigned to a time-slot or not. To model this decision, we use the set of binary variables x_{is} and y_{ij} as defined in (2) and (3) previously. We then proceed to solve the following ILP model:

$$\min \sum_{i \in \mathcal{I}} \sum_{\substack{j \in \mathcal{I} \\ i \neq j}} c_{ij} y_{ij} \tag{26}$$

s.t.

$$\sum_{s \in \mathcal{S}} x_{is} = 1 \quad \forall i \in \mathcal{I} \tag{27}$$

$$\sum_{i \in \mathcal{I}} x_{is} = n \quad \forall s \in \mathcal{S} \tag{28}$$

$$x_{is} + x_{js} \leq 1 + y_{ij} \quad \forall i, j \in \mathcal{I}; s \in \mathcal{S} \tag{29}$$

$$x_{is}, y_{ij} \in \{0, 1\} \quad \forall i, j \in \mathcal{I}; s \in \mathcal{S} \tag{30}$$

The objective function (26) minimizes the collision costs between talks scheduled in the same time-slot. Constraints (27) impose each talk to be exactly in one time-slot. Constraints (28) limit the number of talks in the time-slots by the number of available rooms. Constraints (29) are logical relationships to identify if two given talks are assigned to the same time-slot. Constraints (30) state the binary nature of the variables.

4.3 Special requirements

The conference organizers may have some additional requirements such as unavailability of the authors at some hours, preventing scheduling talks with the same speaker in parallel sessions, assigning specific thematic sessions to a block or scheduling specific talks in a time-slot. Some different forms of these requirements are considered, which means the value of some of the variables becomes fixed.

For this purpose, some new definitions are introduced. The set $\mathcal{F}$ contains tuples (i, s) such that talk i must be scheduled in time-slot s. The set $\mathcal{B}$ contains tuples (e, k) such that session e must be assigned to block k. The set $\mathcal{T}$ contains tuples (e, k) such that session e cannot be assigned to block k. The set $\mathcal{A}$ contains tuples (e, f) such that session e cannot be scheduled in parallel with session f.

Then, the value of the corresponding variables is fixed by the following constraints:

$$u_{is} = 1 \qquad \forall (i,s) \in \mathcal{F} \tag{31}$$

$$w_{ek} = 1 \qquad \forall (e,k) \in \mathcal{B} \tag{32}$$

$$w_{ek} = 0 \qquad \forall (e,k) \in \mathcal{T} \tag{33}$$

$$z_{ef} = 0 \qquad \forall (e,f) \in \mathcal{A} \tag{34}$$

Constraints (31) impose the assignment of a specific talk to a specific time-slot. Constraints (32) impose the assignment of a specific session to a specific block. Constraints (33) forbid the assignment of a specific session to a specific block. Constraints (34) forbid a specific pair of sessions to be scheduled in parallel, thus preventing one author from having talks in parallel sessions. Note, as an alternative to incorporating the constraints (31) – (34), the definition of the variables could be expressed accordingly, but we prefer to formulate these constraints explicitly for the ease of explanation.

5 Numerical results and implementation

The models formulated in Section 4 have been used to support the organizers of three conferences: LOGMS (2017); INFORMS TSL Workshop (2018); and ICSP (2019). In this section, we first provide an overview of the data gathered for these conferences and then we summarize our numerical results.

5.1 Overview

The problem features and dimension vary from one conference to another. Table 2 provides an overview of the number of attendees that participated in the conference and the number of talks, sessions, time-slots within a session, available rooms, and blocks for each of the three conferences under study.

Table 2: Overview of condition of the conferences

	LOGMS	**TSL**	**ICSP**
Number of attendees	41	30	115
Number of talks	92	42	180
Number of sessions	23	14	60
Number of time-slots	4	3	3
Number of blocks	6	7	10
Number of available rooms	4	2	6

To collect the attendees' preferences, an online survey is sent out to all registered attendees, with some weeks of anticipation on the dates of the conferences. In the survey, attendees are given the possibility to rank their preferences on every talk, with two different utility degrees. To illustrate, Table 3 shows a partial overview of the results of the attendees' preferences survey for the LOGMS, where a rank equal to 2 means the attendee has a high preference to attend the talk and a rank equal to 1 means the attendee has a medium preference to attend the talk. An attendee p who ranked a talk i in the preferences list will have a utility ϕ_{ip} equal to the rank given to that talk. If a talk i was not ranked by the attendee p, the utility ϕ_{ip} is set equal to 0, and thus implies no collision cost.

Table 3: Partial overview of attendees' preferences in the LOGMS conference

	Attendees									
Talks	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
i_1	1								2	1
i_2	1		2	1		1		2		1
i_3	1	2			2					1
i_4	2		1			2			1	1
i_5	1						2		1	1
i_6	1	1		1		1		2		1
i_7	1		2							1
i_8	2			1	2		1		2	1

The number of preferences given per attendee and the number of preferences per talk are presented in Figures 4 and 5. Although the number of attendees and talks in the ICSP are higher than for the two other conferences, the attendees registered fewer preferences and also fewer choices per talk (a reason perhaps is that the ICSP covers a broad variety of topics, while LOGMS and TSL are conferences with more focused topics).

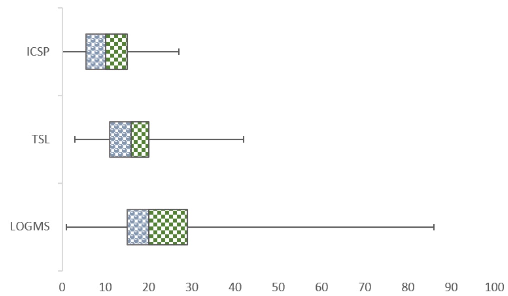


Figure 4: The number of preferences given per attendee

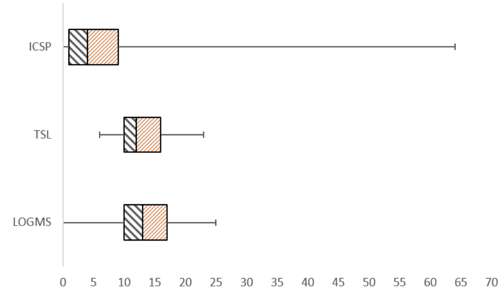


Figure 5: The number of preferences per talk

5.2 Numerical results

The optimization models proposed in Section 4 were implemented in AMPL and solved using Gurobi version 9.0. A time limit of three hours per run was set. To explore the effect of the different ranks of the preferences, we run the models first using an objective function that accounts for both levels of preferences (henceforth referred as *BP* runs), and then using an objective function that only takes into account the high-preferred talks (henceforth referred as *HP* runs).

The remainder of this section focuses on the numerical results for each conference.

5.2.1 LOGMS 2017

LOGMS is an international conference with specific topics on logistics and maritime systems. The seventh edition of the meeting was organized in Bergen (Norway) on August 23-26, 2017, with 92 talks. After the survey was sent out, we received answers from 41 attendees. In total, they indicated 1203 preferences, of which 485 are high-preferred talks and 718 are medium-preferred talks. On average, each attendee ranked 20 talks, and the number of talks rated per attendee varied between 1 and 86. According to the general programme of the conference and the time availability, the scientific programme had to be accommodated in 6 blocks. Considering the number of talks and available rooms, we organized five blocks with four parallel sessions and one block with three parallel sessions. Each session was composed of 4 time-slots. Also, we considered organizers' requirements as additional constraints in the scheduling of the conference. Table 4 summarizes the results obtained for the LOGMS conference.

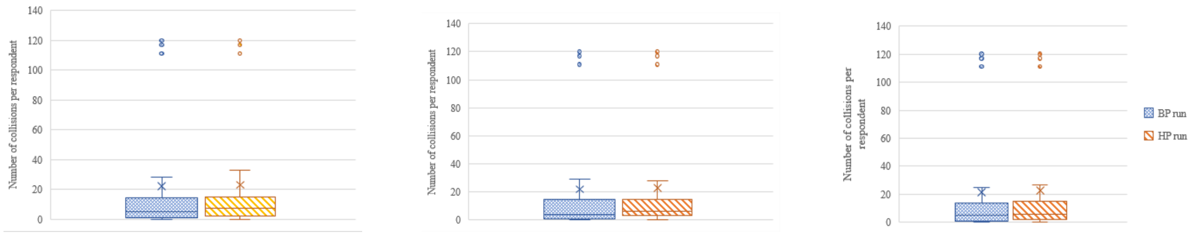
The rows preceded by the percentage intervals indicate the number of survey respondents that could attend such a percentage of their preference list (for example, in the solution to the BP run of the single model with enforced sessions, 28 survey respondents can attend between 90 and 100% of the talks that they ranked as highly preferred). The other rows present some

Table 4: Results for the LOGMS conference

Percentage of preferred talks	Single model				Decomposition approach				Relaxed model			
	BP run		**HP run**		**BP run**		**HP run**		**BP run**		**HP run**	
	Medium preferred	High preferred	Medium preferred	High preferred	Medium preferred	High preferred	Medium preferred	High preferred	Medium preferred	High preferred	Medium preferred	High preferred
90-100%	10	28	10	27	12	27	8	29	14	29	8	30
80-90%	11	6	13	10	14	9	12	8	8	7	12	8
70-80%	9	4	6	1	6	0	9	0	7	1	5	0
60-70%	3	1	3	1	1	2	1	2	3	2	8	1
50-60%	2	0	3	0	2	2	4	1	2	0	2	0
40-50%	0	2	1	2	1	1	1	1	2	2	1	2
30-40%	4	0	3	0	3	0	3	0	3	0	3	0
20-30%	0	0	0	0	0	0	1	0	0	0	0	0
10-20%	0	0	0	0	0	0	0	0	0	0	0	0
0-10%	0	0	0	0	0	0	0	0	0	0	0	0
Total collisions	906		938		901		944		878		931	
Average collisions	22.09		22.87		21.97		23.02		21.41		22.70	
Max. collision	120		120		120		120		120		120	
Min. collision	0		0		0		0		0		0	
St.dev.	39.85		39.60		39.95		39.42		40.04		39.70	
Time	3 hours		3 hours		13 seconds		4 seconds		3 hours		3 hours	

statistics on the number of collisions and solution time.

The most remarkable result is that the two-model decomposition approach is able to find high quality solutions in a few seconds, while the other approaches reach the time limit of three hours without an optimal solution. In particular, the BP run of the decomposition approach renders a solution with slightly less collisions than the single model, while their solution times are 13 seconds and 3 hour, respectively. The schedule found by the decomposition approach allows a great majority of the respondents to attend most of their preferred talks (e.g. 36 out of the 41 respondents can attend to between 80 and 100% of their high-preferred talks). The relaxed model reaches a lower collision cost solution within the 3 hours limit, but recall that this approach does not respect the sessions grouping predetermined by the conference organizers. Nevertheless, this solution gives some referential basis for comparison, revealing that the total number of collisions obtained by the BP run of the decomposition approach is only 2.6% higher (901 compared to 878). Note that this total number of collisions adds up the collisions over all the survey respondents, and some of them have expressed many preferences (for example, the maximum number of collision per respondent is equal to 120, which corresponds to a person who ranked all talks with either high or medium preference). The box plots in Figure 6 show the spread of the number of collisions per respondent. Without accounting for the outliers, these plots reveal that the obtained schedules are able to leave most respondents with a relatively low number of collisions. As a final remark of the results of the LOGMS conference, the BP run conduces to less collisions than the HP run in all three approaches. In particular, for the decomposition model, the solution time of the HP run is shorter (4 seconds vs 13 seconds), but conduces to 4.8% more collisions.



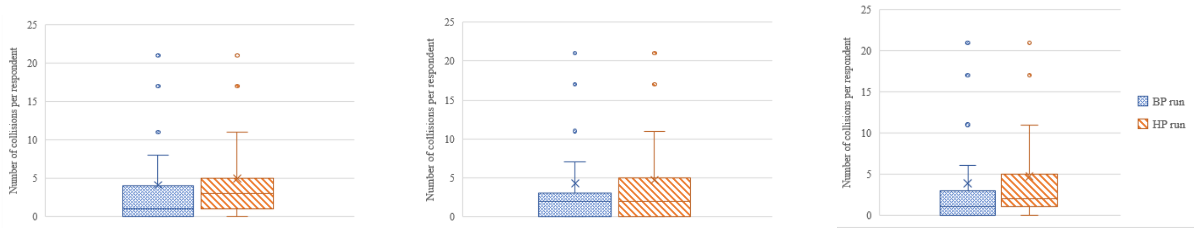
(a) Single model (b) Decomposition approach (c) Relaxed model

Figure 6: Box plots with the collisions per respondent in the solutions for the LOGMS conference

5.2.2 INFORMS TSL Workshop 2018

The Transportation Science and Logistics Society (TSL) of INFORMS organizes a workshop every one or two years, dedicated to specific topics. The sixth edition of the workshop was held on January 8-10, 2018, in Hong Kong (China), with focus on E-commerce and urban logistics. The workshop featured 42 talks. In connection with the organizers, we sent out the online survey and received responses from 31 attendees. They indicated 550 preferences, of which 253 were on high-preferred talks and 297 were on medium-preferred talks. On average, each attendee expressed preferences for 16 talks. The workshop featured 7 blocks over 3 days. We programmed two parallel sessions in each block, with each block composed of four time-slots, so as to accommodate the 42 talks. Again, the organizers of the conferences had some specific requirements that we considered as additional constraints in the models.

Table 5 summarizes the results obtained for this TSL Workshop and Figure 7 shows the spread in the number of collisions per respondent. The decomposition approach was again solved to optimality very quickly, in a matter of a second. In particular, the solution to the BP run of the decomposition approach is better in the number of collisions than the best solution found by the single model reached in the time limit of 3 hours. The HP run of the single model was solved to optimality in about 19 minutes. The solution of the decomposition approach stays competitive with respect to that solution, with only three more collisions (equivalent to about 2%). The relaxed model in this instance was solved to optimality in both the BP and the SP runs. It can also be observed that the lack of flexibility of the enforced sessions does not render many differences in terms of collisions. In fact, either 30 or 31 out of 31 respondents in the solutions with enforced sessions are left with the possibility to attend more than 80% of the talks they ranked. We find it interesting to observe that the solution time of the HP runs is lower than the solution time of the BP runs in all three approaches (resembling what we observed for the decomposition approach in the LOGMS conference). This suggests that the



(a) Single model (b) Decomposition approach (c) Relaxed model

Figure 7: Box plots with the collisions per respondent in the solutions for the TSL Workshop

incorporation of more than one level of preference as a choice in the online survey makes the problem harder to solve, which is perhaps related to the larger amount of non-zero coefficients in the objective function of the BP runs.

Table 5: Results for the TSL Workshop

Percentage of preferred talks	**Single model**				**Decomposition approach**				**Relaxed model**			
	BP run		**HP run**		**BP run**		**HP run**		**BP run**		**HP run**	
	Medium preferred	High preferred	Medium preferred	High preferred	Medium preferred	High preferred	Medium preferred	High preferred	Medium preferred	High preferred	Medium preferred	High preferred
90-100%	18	26	18	27	18	26	17	26	18	29	18	31
80-90%	6	4	5	4	6	4	6	5	8	2	7	0
70-80%	5	0	6	0	4	1	5	0	3	0	3	0
60-70%	1	1	1	0	2	0	2	0	1	0	2	0
50-60%	1	0	1	0	1	0	1	0	1	0	1	0
40-50%	0	0	0	0	0	0	0	0	0	0	0	0
30-40%	0	0	0	0	0	0	0	0	0	0	0	0
20-30%	0	0	0	0	0	0	0	0	0	0	0	0
10-20%	0	0	0	0	0	0	0	0	0	0	0	0
0-10%	0	0	0	0	0	0	0	0	0	0	0	0
Total collisions	132		151		128		154		120		147	
Average collisions	4.4		4.86		4.26		5.13		3.87		4.9	
Max. collision	21		21		21		21		21		21	
Min. collision	0		0		0		0		0		0	
St.dev.	6.62		6.53		6.72		6.46		6.76		6.51	
Time	3 hours		1136 seconds		1 seconds		0.8 seconds		274 seconds		10 seconds	

5.2.3 ICSP 2019

The ICSP is the premier conference of the Stochastic Programming Society and it serves as a meeting place for researchers working in stochastic programming, decisions under uncertainty, and related fields. ICSP 2019 took place on July 29 - August 2, 2019, in Trondheim (Norway). It was the fifteenth edition of the ICSP and attracted 180 talks. In the online survey of ICSP 2019, attendees were asked to express for each of the 180 talks whether or not they intended to attend. That is, a "yes" or "no" type of question for their preferred talks, which we then mapped to only one utility level instead of the two utility levels used for the previous conferences (given the much higher number of talks of the ICSP compared to the LOGMS and TSL events, the idea here was to simplify the survey, so more attendees would feel like answering it). We received responses from 115 attendees. In total, they indicated 1234 preferences in the online survey. On average, each attendee ranked 10 talks.

The conference featured 10 blocks over five days. To accommodate the 180 talks in this format, we organized six parallel sessions in blocks with length equal to three time-slots. Table 6 reports the results obtained for the ICSP event. The most remarkable outcome is that the decomposition approach found an optimal solution with zero collisions. This means that the conference schedule allowed the 115 attendees who responded the survey to attend all their preferred talks. Of course, this is facilitated by the data on the preferences, which in this case involved less conflicts (note the average of 10 preferences for the ICSP versus 20 and 16 for the LOGMS and TSL, respectively). The single model, in contrast, did not reach optimality and finished with a solution with 11 collisions after the 3 hour limit. The relaxed problem found a solution with zero collisions, equally good to the solution of the decomposition approach, but in a shorter time of 21 seconds. The decomposition approach took about 15 minutes, mostly spent in solving the model of the first step. Although this time was longer than the few seconds it took in the previous two conferences (probably because of the larger number of talks to schedule in this case), it was still acceptable for the purpose of planning the conference and publishing the scientific programme, which is done some weeks before the event. The box plots in Figure 8 eloquently illustrate the effectiveness of the solutions in addressing the preferences of the attendees to this conference.

Table 6: Results for the ICSP conference

Percentage of preferred talks	**Single model**	**Decomposition approach**	**Relaxed model**
90-100%	111	115	115
80-90%	4	0	0
70-80%	0	0	0
60-70%	0	0	0
50-60%	0	0	0
40-50%	0	0	0
30-40%	0	0	0
20-30%	0	0	0
10-20%	0	0	0
0-10%	0	0	0
Total collisions	11	0	0
Average collisions	0	0	0
Max. collision	2	0	0
Min. collision	0	0	0
St.dev.	0.36	0	0
Time	3 hours	898 seconds	21 seconds

Figure 8: Box plots with the collisions per respondent in the solutions for the ICSP conference

5.2.4 Implementation in practice

The solution approach adopted in practice for the three conferences was the decomposition approach, which showed to be sufficiently quick and effective in finding schedules with a low number of collisions. In the process of generating a solution, it was important to interact with the organizing committee, to secure that the grouping of sessions and all the requirements would be timely communicated and incorporated in the models. It is perhaps interesting to comment that in the beginning of the project (when we started scheduling the LOGMS conference in 2017), we proceeded with a fully preference-driven approach, based on the relaxed model. But after we computed some solutions and presented them to the organizing committee, they found that some of the talks grouped into the same session came up too disconnected from each other. That motivated the manual grouping of talks into sessions as a pre-processing step. While this was not hard to manage in these small to medium size conferences, when the number of talks is larger, it may become more challenging and a modelling approach to group talks into sessions based on keyword similarity such as in Castaño et al. (2019) could be used (still in large conferences though, a share of the sessions are often grouped beforehand, according to tracks or by invitations, which might help to implement this step). After the sessions were formed, the single model formulation was the most straightforward way of approaching the problem, but as it has been revealed in the numerical results above, it was time consuming and did not necessarily end up with the best solutions. This motivated the formulation of the decomposition approach, which allowed us to considerably speed up the solution process and to reach solutions that met the expectations of the organizers and attendees.

6 Concluding remarks

This paper has developed an optimization approach for conference scheduling, taking into account the preferences of the attendees. The data on preferences is collected by an online survey conducted some weeks before the conference. The main purpose of the approach then is to schedule parallel talks avoiding collisions among the talks that the attendees have expressed as their most preferred ones. We also consider special requirements from the organizers and speakers, and thematic cohesion among talks of the same session. Our decomposition approach was able to find high-quality solutions quickly, which were adopted in practice to schedule three conferences. Besides the specific context of these conferences, our mathematical formulations allow flexibility to consider a diverse set of requirements from organizers and speakers that may be used to schedule other conferences or meetings.

While the focus of this work was to provide decision support to conference organizers in practice, further research may focus on algorithmic development or model reformulation to speed up the solution process. Also, the pre-processing step of grouping talks into sessions can become more automated. The size of the conferences we scheduled ranged from 42 to 180 talks, which is rather on the small to medium size. When the number of talks is larger, further research may automate this step by adopting some methods such as text analysis on the abstracts or paper submissions. Collecting data on the preferences from the attendees is an essential input to this conference scheduling approach. In larger events this may also become more challenging, and the rate of responses might suffer when the menu of talks is too large. A predictive approach to model the preferences of those who did not respond to the survey, probably based on the preferences of those who responded and other data, may also be an interesting subject of future research.

Acknowledgements

We thank Stein W. Wallace for facilitating our engagement with the organizing committee of the conferences studied in this paper and for fostering the adoption of our approach to schedule these conferences.

References

Bulhoes, T., Correia, R. and Subramanian, A. (2022), 'Conference scheduling: a clustering-based approach', *European Journal of Operational Research* **297**(1), 15–26.

Castaño, F., Velasco, N. and Carvajal, J. (2019), 'Content-based conference scheduling optimization', *IEEE Latin America Transactions* **17**(04), 597–606.

Edis, E. and Edis, R. S. (2013), 'An integer programming model for the conference timetabling problem', *Celal Bayar University Journal of Science* **9**(2), 55–62.

Eglese, R. and Rand, G. (1987), 'Conference seminar timetabling', *Journal of the Operational Research Society* **38**(7), 591–598.

Ibrahim, H., Ramli, R. and Hassan, M. H. (2008), 'Combinatorial design for a conference: constructing a balanced three-parallel session schedule', *Journal of Discrete Mathematical Sciences and Cryptography* **11**(3), 305–317.

Le Page, Y. (1996), 'Optimized schedule for large crystallography meetings', *Journal of Applied Crystallography* **29**(3), 291–295.

Manda, P., Hahn, A., Beekman, K. and Vision, T. J. (2019), 'Avoiding "conflicts of interest": a computational approach to scheduling parallel conference tracks and its human evaluation', *PeerJ Computer Science* **5**, e234.

Nicholls, M. (2007), 'A small-to-medium-sized conference scheduling heuristic incorporating presenter and limited attendee preferences', *Journal of the Operational Research Society* **58**(3), 301–308.

Potthoff, R. F. and Brams, S. J. (2007), 'Scheduling of panels by integer programming: Results for the 2005 and 2006 new orleans meetings', *Public Choice* **131**(3-4), 465–468.

Quesnelle, J. and Steffy, D. (2015), Scheduling a conference to minimize attendee preference conflicts, *in* 'Proceedings of the 7th multidisciplinary international conference on scheduling: theory and applications (MISTA)', pp. 379–392.

Rahim, S. K. N. A., Jaafar, A. H., Bargiela, A. and Zulkipli, F. (2017), Solving the preference-based conference scheduling problem through domain transformation approach, *in* 'Proceedings of 6th International Conference of Computing Informatics', pp. 199–207.

Sampson, S. E. (2004), 'Practical implications of preference-based conference scheduling', *Production and Operations Management* **13**(3), 205–215.

Sampson, S. E. and Weiss, E. N. (1995), 'Increasing service levels in conference and educational scheduling: A heuristic approach', *Management Science* **41**(11), 1816–1825.

Stidsen, T., Pisinger, D. and Vigo, D. (2018), 'Scheduling euro-k conferences', *European Journal of Operational Research* **270**(3), 1138–1147.

Thompson, G. M. (2002), 'Improving conferences through session scheduling', *Cornell Hotel and Restaurant Administration Quarterly* **43**(3), 71–76.

Vangerven, B., Ficker, A. M., Goossens, D. R., Passchyn, W., Spieksma, F. C. and Woeginger, G. J. (2018), 'Conference scheduling—a personalized approach', *Omega* **81**, 38–47.

Zulkipli, F., Ibrahim, H. and Benjamin, A. M. (2013), Optimization capacity planning problem on conference scheduling, *in* '2013 IEEE Business Engineering and Industrial Applications Colloquium (BEIAC)', IEEE, pp. 911–915.

Chapter 2

Efficiency and fairness criteria in the assignment of students to projects*

Nahid Rezaeinia[a], Julio César Góez [a], Mario Guajardo [a]

[a]Department of Business and Management Science, NHH Norwegian School of Economics, 5045 Bergen, Norway

Abstract

Teamwork has increasingly become more popular in educational environments. With the also increasing mobility trends in the educational sector, internationalization and other diversity features have gained importance in the structure of teams. In this paper, we discuss an assignment problem arising in the allocation of students to business projects in a master program in Norway. Among other problem features, the students state their preferences on the projects they most want to conduct. There are also requirements from the companies that propose the projects and from the program administration. We develop a bi-objective approach to consider efficiency and fairness criteria in this assignment problem. We test the model using real data of 2017 and 2018, in joint collaboration with the administrative staff of the program. In light of the good results, our proposed solutions have been implemented in practice in 2019, 2020, and 2021. The implementation of these solutions have been beneficial for the administration, the students, and the companies.

Keywords Project assignment, Multiobjective optimization, Efficiency, Fairness, OR in Practice, OR in Education

*This paper has been presented in the EuroYoung Workshop (2019) and the ICSP XV Conference (2019), and it is published in *Annals of Operation Research*, 2021, DOI: 10.1007/s10479-021-04001-7.

1 Introduction

The educational sector is experiencing a substantial growth in the last years. The UNESCO Institute for Statistics has forecast an increase from 5.3 million students to 8 million between 2017 and 2025. Likewise, teamwork has increasingly become more popular in educational environments (Hansen, 2006). With the also increasing mobility trends in the educational sector, internationalization and other diversity features have gained importance in the structure of teams and teamwork (Kelly, 2008). International mobility and increasing demand for teamwork is particularly important in business environments, reflected in that many business schools have incorporated group projects and multicultural group working experiences into the curriculum, often in collaboration with companies and other organizations (Hansen, 2006; Huxham and Land, 2000). In general, collaboration in group projects is a good opportunity for students to learn teamwork, problem-solving, communication, leadership, and also to become more creative and more social in their future working places (Hansen (2006); Kelly (2008)). In fact, it has been documented that positive group experiences help them to be more productive in industry settings and help them to succeed in their careers (Henry, 2013). In this context, schools, universities and educational institutions play an important role in enabling teamworking experiences (Huxham and Land, 2000).

The decision process of allocating students to groups and matching these groups with projects is often in charge of administrative staff at schools, who do not necessarily have the background and tools to generate a solution with all the desirable features. Moreover, as some criteria in the problem might be in conflict with each other, reconciling all of them into a single solution poses a challenge that can barely be addressed by manual techniques. A poor solution can have undesirable consequences not only in the specific features of the groups (such as gender balance and international composition), but also in the students' perception about the fairness of the decision process.

This paper reports the real-world implementation of a bi-objective modelling approach, to address one of such problems arising in an international business master program in Norway. As a core activity of the program, the students have to complete a business project in collaboration with companies. These projects are posted in advance to the students, who then elicit a ranking of their preferred projects. The administration must form the groups and match them to the projects, taking into account the students' preferences, and specific requirements of the companies, among other aspects. The problem is a one-side preferences assignment while considering the other sides' requirements and conditions. Our bi-objective approach is based on integer programming, and incorporates concepts of efficiency and fairness regarding the preferences of the students. This approach has been implemented in practice during the last three years, replacing the traditional manual approach used before.

The remainder of this paper is organized as follows. Section 2 reviews relevant literature. Section 3 provides background on the applied setting that originates this work. Section 4 develops a series of mathematical modelling approaches for the problem, while Section 5 describes how these are used to compute a solution. Section 6 reports on the implementation in practice and numerical results. Concluding remarks are provided in Section 7.

2 Literature review

Random selection, self-selection and manually creating groups by a coordinator or instructor are common approaches to partition students into working teams. While these are easy to implement, they might fall into a number of shortcoming, such as lack of fit, isolation of some students, poor balance, and low level of satisfaction among students. These shortcomings have been addressed by a body of literature that finds its roots in classic assignment problems, including the celebrated work by Gale and Shapley (1962) on the admission of students to universities with two-sided preferences. In this section, we limit our review to papers in the education sector with the most similarities to our work, and to papers discussing efficiency and fairness criteria.

In the first stream, Krass and Ovchinnikov (2006) study the problem of assigning students to multiple non-overlapping groups considering diversity in skills, nationalities, genders, culture and academic backgrounds. They address the problem by an integer programming model, whose objective is to minimize the number of overlaps. The approach is applied to a case study arising at an MBA program in Canada. In another real-world case, Cutshall et al. (2007) introduces principles of equity and cohesion to form student teams in the core courses of a school of business in Indiana. Among other features, they attempt to match students with similar academic performance and also to avoid groups with lone female or lone international students (i.e., the number of female or international students in each group should be zero or at least two). They address the problem by an integer programming model, whose objective function is to minimize the maximum deviation of a team's academic performance.

Although both previous papers have successfully replaced manual methodologies easing the task of administrative staff and reporting positive results in practice, none of them considered the preferences of the students when forming the groups. In this respect, Lopes et al. (2008)

develop a mixed integer programming model for the allocation of students to design projects, considering students preferences' and project sponsors' requirements, among other conditions. The model uses a single objective function that maximizes a weighted sum of the number of projects staffed, minus penalties for not satisfying students' preferences and other desirable conditions. The model is successfully applied to an engineering program at the University of Arizona. In a different problem, but of similar nature, Krauss et al. (2013) assign students to classes at an elementary school in New York City. In the assignment process, they considered the students designated friends list and also recommendations from parents, teachers and school therapists. As solution approach, they first use an integer programming formulation to construct a feasible solution. Then, they use a genetic algorithm to improve along criteria on the recommendations of parents and teachers, while penalizing the violations of the constraints originally considered in the integer model.

Although the previous papers somehow incorporate multiple criteria, they do it by combining these criteria into a single objective function. In what is the closest article to ours, Magnanti and Natarajan (2018) address the problem of assigning students to projects using two optimization criteria sequentially: efficiency and fairness. In this context, efficiency is understood as the maximization of the total utility, that is, the sum of the utilities of the projects assigned to students. The utility associated to the assignment of a student to a specific project, is calculated according to the ranking of preferences elicited by the students before the optimization process. Among all the efficient solutions, Magnanti and Natarajan (2018) then proceed with a lexicographic max-min fairness criterion, which consists of minimizing the number of students assigned to their least preferred project and then repeat the process with the second-to-last preference and so on. They apply their approach in an undergraduate program at a university in Singapore and report positive impact in practice. While our problem and approach are similar, some important features differ. First, due to requirements of the coordinators and partner organizations, we incorporate some side constraints that differ from the side constraints in their application. In particular, we aim at incorporating balance on gender and nationality, and also specific requirements from the partners (such as wished skills and languages). Second, in addition to the first efficiency and then fairness approach to compute a solution, we also test the inverted sequence, that is, using the fairness criterion first and then efficiency. Previous literature in other contexts points to the importance of analysing the trade-off between fairness and efficiency. A solution based only on efficiency may become unacceptable for some agents (students in our case), while a solution based only on fairness may incur in a high efficiency loss or high price of fairness (Bertsimas et al. (2012); Nicosia et al. (2017)). Moreover, motivated by a literature stream using quantitative fairness measures, we are also interested in quantifying fairness. Besides avoiding the iterative process required to incorporate the lexicographic max-min fairness criterion, a fairness measure allows an easy and direct way to compare different solutions. For this purpose, we adopt the Jain's index (Jain et al. (1984)), a quantitative measure of fairness with advantageous features of being population size-independent, unaffected by scale, bounded and continuous. A broad range of works have used this index to measure fairness, including Sediq et al. (2012), Huaizhou et al. (2013), Guo et al. (2013), Huaizhou et al. (2013), Hoßfeld et al. (2016). Furthermore, our contribution reports results on a real-world case arising in a master program at the main business school in Norway. Our approach has been implemented in practice during the last three years, producing results that increase the utility of the students and improve fairness over the previous manual approach. The new approach does not only contribute to the satisfaction among the student community, but also eases the task of the administrative staff in terms of resource usage and also in terms of projecting transparency and objectiveness about the decision process.

3 Background

The Global Alliance in Management Education, also called CEMS (because of its former name Community of European Management School and International Companies), is a cooperation founded in 1988 involving business schools, universities, companies, and non-governmental organizations (NGOs). It currently consists of about 100 members from the five continents, with 33 of them in the educational sector. Normally, at most one school per country can have a partnership in the alliance. From Norway, NHH Norwegian School of Economics has been member of CEMS since 1992. The flagship program of the alliance is the CEMS MIM or CEMS Master's in International Management, a one-year program for students who are pursuing a Master's Degree at a CEMS member institution. Students who complete the program successfully receive one master's degree from CEMS, in addition to the master's degree from their home institution.

A central piece of the CEMS MIM curriculum requires students to complete a semester-long business project, in collaboration with one of the companies or NGOs that are partners of CEMS. The business project usually consists of a real problem faced by one of these partners, and the students are expected to address the problem and complete a final report that is submitted to the organization and to academic censors. Participating in a business project may play an important role in the future career of students, because it provides them with the chance to earn early experience as consultant or potentially to find a job position in the partner organization. Likewise, for these organizations it is valuable to get assigned a suitable team of students, who can address the challenges involved in the project and provide useful input. Therefore, the assignment of students to business projects is an important decision and is usually the responsibility of the administrative staff.

At NHH, the decision process consists of several stages. First, the companies prepare project proposals supported by the administrative staff and academic supervisors. Then, a full-day workshop is organized where the partner organizations present their projects to the students. After the workshop, each student sends a ranking with her/his five most preferred projects to the administrative staff. The staff also gathers other information about the students, such as gender, nationality, language skills, etc. Then, the staff attempts to create groups and assign them to projects, with the main goal of addressing the students' preferences. In addition, the staff considers other aspects, such as gender diversity, nationalities and language requirements. Until 2018, the assignment of students to projects at NHH was assigned manually. Although the staff tried to incorporate as much of these criteria as possible, the problem posed some challenges that could barely be addressed by hand. In particular, it turned practically impossible to not disappoint some of the students who ended up in projects far from their top preferences. In contrast, as some others were assigned to their most preferred project, it became hard to avoid perceptions of unfairness in the process. The difficulties of the problem are illustrated in Figure 1, which shows a real data instance on the preferences of the students. The example involves 35 students who ranked their top-five choices among 10 projects. The choices are labeled as *Utility 1*,…,*Utility 5*, where for a given student a value of 5 indicates that the project is the most preferred by the student, a value of 4 indicates the second most preferred project, and so on. Projects 1 and 5 are clearly the most popular, with seven and ten students ranking them as their top choice, respectively. With a maximum allowed of four students in each of these projects, it turned impossible to assign all these students to their top choice. On the other hand, only one student ranked project 8 as top choice and, moreover, none of the students ranked project 9 as top choice. As a fairness criterion, the administration will do their best to hopefully not assign students to their lowest ranked or non-ranked projects. However, depending on the number of students and available projects, it might be necessary to assign a group of students even to these less popular projects. Since it is within the interest of the program to keep the partnership with the companies that offer the projects, the administrative and academic staff will help them to make the project proposals attractive to the students. Nevertheless, avoiding situations like the one described in this example is not

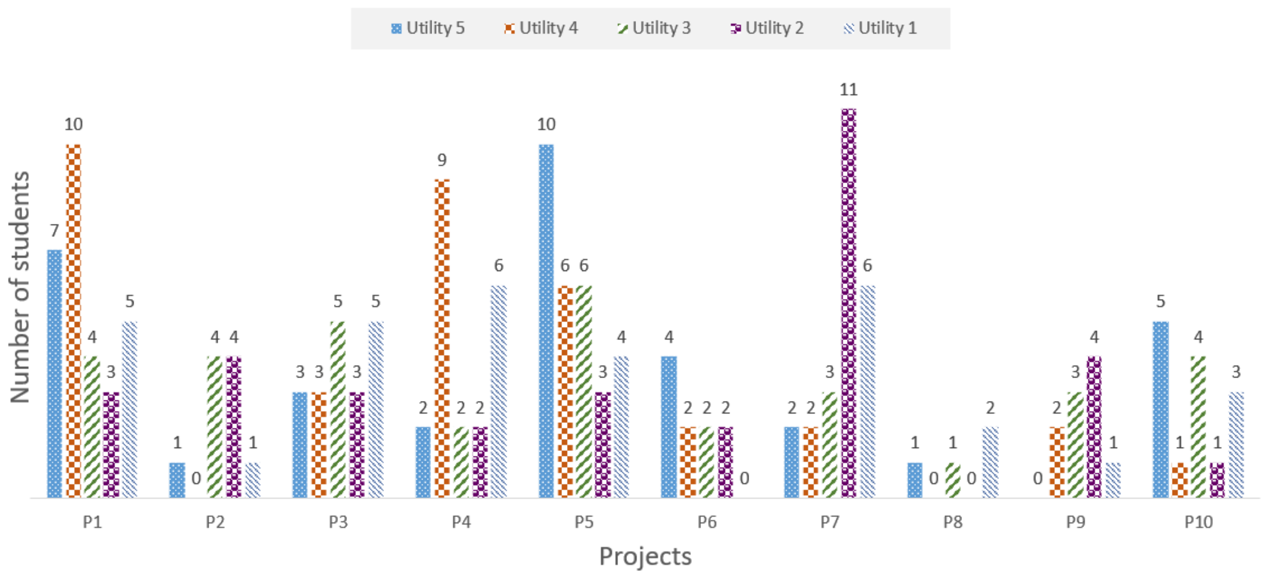


Figure 1: Distribution of the students' top-five preferences on 10 different projects

guaranteed, because the profile of preferences is realized according to the individual rankings of the students after the projects proposals have been presented to them.

To better illustrate the potential conflict between efficiency and fairness, we may consider alternative assignments of the 10 students who ranked project 5 as their first choice. In one possible assignment we can assign four students to their top choice, two students to their second choice, and four students to their third choice. The total utility for this assignment is 40. In another possible assignment, we can assign one student to her/his top choice, seven students to their second choice, and two students to their third choice. The total utility for this assignment is 39, which means a lower performance on the efficiency measure than the previous assignment. However, under a fairness criterion of assigning as few as possible students to a less preferred project, the second assignment is better because it assigns only two students to the third choice (in contrast to the four students assigned to the third choice by the first solution). Now consider a third alternative, in which we assign two students to their top choice, six students to their second choice, and two students to their third choice. The total utility for this assignment is 40, which is as good as the first solution according to the efficiency criterion. Likewise, the third assignment is equivalent to the second solution according to the fairness criterion, since both solutions assign two students to their third choice. Even though these measures of efficiency and fairness do not drive the solutions in completely opposed directions, this example illustrates that, in general, a more efficient assignment is not necessarily fairer neither the other way around. Also, the example illustrates that considering both criteria may conduce to solutions that are better than solutions constructed using a single criterion.

If on top of these two criteria, we add the other aspects considered by the administrative staff, it is understandable that computing a solution became a daunting task for them and the students would sometimes feel disappointed. This motivated us to undertake a collaboration with the administration as to support their decision-making process and to provide the study program with better solutions in practice. Our work involved testing several approaches, which we present in the following section.

4 Mathematical modelling

The foundation of the approaches developed in this paper is mathematical optimization. This section provides the details of the different components of the optimization formulations that will be used later to develop our multi-objective approaches.

To build the mathematical formulation, we denote the set of students participating in the program as $\mathcal{S} = \{s_1, \ldots, s_n\}$, the set of projects proposed by participating companies as $\mathcal{P} = \{p_1, \ldots, p_m\}$, and the set of required attributes as $\mathcal{C} = \{c_1, \ldots, c_l\}$. Here, the attributes are for example expertise fields, skills, nationality and gender, and they will be used to model the criteria for the assignment of students to different groups. Using these sets, we define the parameters and decision variables needed to build the mathematical optimization model for the projects assignment problem. For the sake of simplicity, in the reminder of this section we use the indices s for an arbitrary student in $\mathcal{S}$, p for an arbitrary project in $\mathcal{P}$, and c for an arbitrary attribute in $\mathcal{C}$.

First, we define the following parameters:

- u_{sp}: Defines the utility of a student s when assigned to a project p.
- a_{sc}: Defines the presence of an attribute c in a student s. This is a binary parameter, i.e., it is one if a student has an attribute c and zero otherwise.
- UB_p= Upper bound on the number of students that are needed for a project p.
- LB_p= Lower bound on the number of students that are needed for a project p.
- UB_{pc}= Upper bound on the number of students with attribute c that are needed for a project p.

- LB_{pc}= Lower bound on the number of students with attribute c that are needed for a project p.

The model has to decide which projects to assign and which students to match with those projects. We define two different sets of decision variables. First, in (1) we define a set of binary variables to decide the projects that will be assigned.

$$y_p = \begin{cases} 1, & \text{if project } p \text{ is selected,} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

The variable y_p allows to disregard some projects. This situation may happen due to the constraints on the upper and lower number of students needed for each projects. It may occur that none of the students in $\mathcal{S}$ are able to satisfy some projects' requirements, or the ranking of some projects may be consistently lower than other projects, which may rule it out when balancing efficiency and fairness. Second, in (2) we define a set of binary variables used to decide the assignment of students to the different projects.

$$x_{sp} = \begin{cases} 1, & \text{if student } s \text{ is allocated to project } p, \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

4.1 The assignment constraints

The initial problem of assigning the students to the different projects is defined by the assignment constraints (3) – (7).

$$\sum_p x_{sp} = 1 \qquad \forall s \in S \tag{3}$$

$$x_{sp} \leq y_p \qquad \forall s \in S, \forall p \in P \tag{4}$$

$$\sum_s x_{sp} \geq LB_p y_p \qquad \forall p \in P \tag{5}$$

$$\sum_s x_{sp} \leq UB_p y_p \qquad \forall p \in P \tag{6}$$

$$x_{sp}, y_p \in \{0,1\} \qquad \forall s \in S, \forall p \in P \tag{7}$$

Constraints (3) ensure that each student is assigned to a single project. Constraints (4) ensure that a project p is selected if at least one student is assigned to it. Constraints (5) and (6) enforce the upper and lower bounds on the number of students that are needed for each project. Finally, constraints (7) limit the decision variables to be binary.

4.2 Side constraints

One of the strengths of using mathematical optimization as the foundation of our approach is the possibility of including side constraints, as it is also highlighted in Magnanti and Natarajan (2018). Side constraints appear when one has some additional conditions beyond ensuring a proper assignment and enforcing the limits on the number of students per project. Side constraints may for example aim at obtaining balance on gender and nationality, as well as ensuring that specific requirements from the partners are met. Those requirements usually cover wished skills and languages. The constraints (8) and (9) are used here to model such requirements. However, this approach does not limit the form of the side constraints that may be considered. In general, the approach of this paper allows for side constraints of any form.

$$\sum_{s} a_{sc} x_{sp} \geq LB_{pc} y_p \qquad \forall p \in P, \forall c \in C \tag{8}$$

$$\sum_{s} a_{sc} x_{sp} \leq UB_{pc} y_p \qquad \forall p \in P, \forall c \in C \tag{9}$$

Constraints (8) and (9) ensure the lower and upper bounds on the number of students with specific attributes needed for each project are satisfied.

4.3 Measuring the quality of an assignment

The main aim of the approach of this paper is to achieve a balance between efficiency and fairness. For that purpose, two different goals are defined, which are formulated using linear functions as follows.

To optimize efficiency, the linear function defined in Equation (10) is used. Specifically, Equation (10) maximizes the overall utility of an assignment measured as the summation of the students utility obtained when assigned to a project times the assignment variables.

$$U = \max \sum_{s} \sum_{p} u_{sp} x_{sp} \tag{10}$$

To work towards fairness, the linear function defined in Equation (11) is used. Here, a utility level is fixed at $\hat{u}$ and then it minimizes the number of students that will receive such utility. In the context of this work, the utilities are assigned based on the rank of a project. For example, if a project was ranked as k by a student in the preferences survey, then it provides that student with a utility of u_k.

$$F_k = \min \sum_p \sum_{s:u_{sp}=\hat{u}} x_{sp} \tag{11}$$

Equation (11) is used later in a sequential process to minimize the number of students who are assigned to projects that are not their top choice.

4.3.1 Measuring fairness

A limitation when using (11) is that it does not provide an overall measure of the achieved fairness of the reached assignment. To overcome that limitation, the Jain's index (Jain et al., 1984) is used as an alternative to measure and optimize the assignment fairness. The Jain's index formulation is provided in (12).

$$f(w) = \frac{\left(\sum_{i=1}^{n} w_i\right)^2}{n \sum_{i=1}^{n} (w_i)^2} \qquad w_i \geq 0 \tag{12}$$

In (12) $w \in \mathbb{R}^n$, n is the number of participants, and w_i is the allocation given to the i-th user in a system. The Jain's index (12) has the desired properties of population size-independence, scale and metric independence, boundedness, and continuity. The index is broadly used to measure fairness in the assignment of resources in telecommunication networks, but it may also have applications in other areas. In particular, in this assignment problem the Jain's index is computed using the utilities and assignment decisions, as shown in (13).

$$f(x) = \max \ \frac{(\sum_s \sum_p u_{sp} x_{sp})^2}{n \sum_s (\sum_p u_{sp} x_{sp})^2} \tag{13}$$

In (13), given the constraint (3), the term $\sum_p u_{sp} x_{sp}$ provides the utility of the assignment for a student s to a project. Hence, given a total utility for an assignment, the index (13) provides the assignment fairness measure for that level of utility. In other words, this index provides a fairness measure for the utility achieved when the assignment efficiency is optimized.

The Jain's index is bounded by values that depend only on the number of participants. First, by definition the index is bounded above by 1, since $(\sum_s \sum_p u_{sp} x_{sp})^2 \leq n \sum_s (\sum_p u_{sp} x_{sp})^2$. Second, the lower bound for the index is $1/n$, which happens when only one participant is assigned the total utility. As an illustration of this lower bound, consider a case where only one student is assigned to one of the projects in her/his preference list and all others are assigned to projects outside their preference lists. In this solution, the former student receives the total utility obtained with the assignment while the others obtain zero of that utility. The fairness index evaluated at this solution achieves its lower bound $1/n$. Notice that for the index to take a value below $1/n$, one would need that $(\sum_s \sum_p u_{sp}x_{sp})^2 \leq \sum_s(\sum_p u_{sp}x_{sp})^2$, which is mathematically not possible. The dependence only on the number of participants provides bounds that are known beforehand and can potentially be used by optimization solvers to speed up the solution process.

5 Lexicographic solution approaches

To optimize the projects assignment problem, a lexicographic approach is used to prioritize the different goals. For this, the required optimization models are build using the elements introduced in Section 4. The assignment constraints in 4.1 and the side constraints in 4.2 ensure a valid assignment. Then, the different possible objective functions are prioritized in different orders with the aim to research the effect choosing one measure over the other.

5.1 Prioritizing efficiency

When priority is given to efficiency over fairness the following orders is used. First, the problem (14) is solved to optimize the overall efficiency of the assignment.

$$\begin{aligned} \max \sum_s \sum_p u_{sp}x_{sp} \\ s.t.\ (3) - (9) \end{aligned} \tag{14}$$

Second, using the optimal efficiency of the first step, denoted by U^*, fairness is improved. Optimizing the utility function may result in multiple optimal solutions with the *Pareto Efficient* property. A Pareto Efficient solution cannot be more efficient regarding someone's assignment unless fairness is reduced in someone's else assignment (Magnanti and Natarajan, 2018). Incorporating fairness aims to keep students' assignments as fair as possible, while keeping the same maximum utility. Two approaches are considered here to incorporate fairness.

5.1.1 Lexicographic fairness

First, a lexicographic approach is used to improve fairness while enforcing the same efficiency level obtained when solving (14). In this approach, the first step is to solve Problem (15) to minimize the number of students assigned to projects ranked 1. To ensure the utility is not worsening, the utility is constrained to be at least as good as the one obtained solving (14).

$$
\begin{aligned}
\min \ & \sum_{p} \sum_{s:u_{sp}=1} x_{sp} \\
s.t. \ & (3)-(9) \\
& \sum_{s} \sum_{p} u_{sp} x_{sp} \geq U^{*}
\end{aligned}
\tag{15}
$$

Using the solution of (15), the sequence of optimization problems (15) is solved. The sequence is obtained by changing the value of $k \in \{2, \ldots, K-1\}$. Here, K is the highest ranking a student can assign to a project. In other words, in each step the number of students assigned to a project with ranking k is minimized, starting with the lowest ranking and moving up one ranking at a time until the level before the maximum. That minimization is constrained to ensuring that at most F^{*}_{ℓ} students are assigned to a project with a rank $\ell \in \{1, \ldots, k-1\}$. Here, F^{*}_{ℓ} is the optimal value of the objective function at iteration ℓ in the sequence.

$$
\begin{aligned}
\min \ & \sum_{p} \sum_{s:u_{sp}=k} x_{sp} \\
s.t. \ & (3)-(9) \\
& \sum_{s} \sum_{p} u_{sp} x_{sp} \geq U^{*} \\
& \sum_{p} \sum_{s:u_{sp}=\ell} x_{sp} \leq F^{*}_{\ell} \quad \forall \ell \in \{1, \ldots, k-1\}
\end{aligned}
\tag{16}
$$

5.1.2 Jain's index

The second approach uses the Jain index to improve fairness. Here, the Problem (17) is solved to maximize the Jain's index while enforcing the same efficiency level found solving (14).

$$
\begin{aligned}
\max \ & \frac{(\sum_{s} \sum_{p} u_{sp} x_{sp})^{2}}{n \sum_{s} (\sum_{p} u_{sp} x_{sp})^{2}} \\
s.t. \ & (3)-(9) \\
& \sum_{s} \sum_{p} u_{sp} x_{sp} \geq U^{*}
\end{aligned}
\tag{17}
$$

5.2 Prioritizing fairness

When priority is given to fairness over efficiency the approaches discussed in Section 5.1 are inverted. This results in two different approaches.

5.2.1 Lexicographic fairness

The first approach uses a lexicographic approach to sequentially improve fairness. The aim is to minimize the number of students that are assigned to low ranked projects. First, the Problem (18) is solved to minimize the number of students assigned to projects ranked 1.

$$\begin{aligned} \min & \sum_{p} \sum_{s:u_{sp}=1} x_{sp} \\ s.t.\ & (3)-(9) \end{aligned} \tag{18}$$

Using the solution of (18), the sequence of optimization problems (19) is solved.

$$\begin{aligned} \min & \sum_{p} \sum_{s:u_{sp}=k} x_{sp} \\ s.t.\ & (3)-(9) \\ & \sum_{p} \sum_{s:u_{sp}=\ell} x_{sp} \leq F_{\ell}^{*} \quad \forall \ell \in \{1, \ldots, K-1\} \end{aligned} \tag{19}$$

Using the sequentially optimized fair assignment, the fairness level obtained is used as a lower bound to optimize efficiency in (20).

$$\begin{aligned} \max & \sum_{s} \sum_{p} u_{sp} x_{sp} \\ s.t.\ & (3)-(9) \\ & \sum_{p} \sum_{s:u_{sp}=\ell} x_{sp} \leq F_{\ell}^{*} \quad \forall \ell \in \{1, \ldots, K-1\} \end{aligned} \tag{20}$$

5.2.2 Jain's index

The second approach uses the Jain's index to optimize fairness first. The Jain's index is maximized solving Problem (21).

$$\begin{aligned} \max & \frac{(\sum_s \sum_p u_{sp} x_{sp})^2}{n \sum_s (\sum_p u_{sp} x_{sp})^2} \\ s.t.\ & (3)-(9) \end{aligned} \tag{21}$$

The optimal value of the Jain's index found, denoted as J^*, is used as a lower bound for fairness to optimize efficiency in (22).

$$\begin{aligned} \max\ & \sum_s \sum_p u_{sp} x_{sp} \\ s.t.\ & (3) - (9) \\ & \frac{(\sum_s \sum_p u_{sp} x_{sp})^2}{n \sum_s (\sum_p u_{sp} x_{sp})^2} \geq J^* \end{aligned} \tag{22}$$

6 Implementation and numerical results

The approaches proposed in Section 5 were implemented and tested with the data instances of 2017 and 2018, after the assignment through the old manual methodology had taken place. In light of the good results for those experimental instances, we supported the administration to conduct the assignment in 2019, 2020, and 2021 by putting this new methodology into practice. In this section we summarize our numerical results for the four years spanning from 2017 to 2020.

6.1 Overview

The input for the problem are the projects' descriptions provided by the companies, the administration's requests, and the students rankings. With the information in place, we identify the different attributes and categories. These may vary from one to another year, but in general, we identify five main categories. First we have *profile*, which refers to a student's interests or field. Second, we consider the *language skills*, because for some projects it is important to have at least one student able to speak a specific language. Third, we consider *nationality*. Some projects need students with specific nationalities to facilitate the collaboration with a partner company taking part in the project. Fourth, we consider *gender* balance, responding to the requirement of the administration. What balance means precisely, depends on the gender distribution of the class, but for purpose of our formulation this translates into lower and upper bounds on the number of students of a specific gender assigned to each of the allocated projects. Fifth, we include a category named *home requirement*, which requires assigning at least one student from NHH to each group.

Table 1 provides an overview of the number of students and projects that participated in the business project program and the number of requested attributes during the years 2017 to 2020.

Table 1: Overview of number of students, projects and required attributes

	2017	**2018**	**2019**	**2020**
Number of students	35	39	34	33
Number of projects	10	11	9	9
Number of requested attributes	10	21	3	9

As for the preference survey, at the beginning of the semester, students taking part at the business project semester at NHH are asked to rank up K projects. To illustrate, Table 2 shows a partial overview of the results of the students' preferences survey for the year 2017, where $K = 5$. A student s that ranked a project p in its list will have an utility u_{sp} equal to the rank given to that project. If a project p was not ranked by the student s, it will be assigned a utility $u_{sp} = 0$.

Table 2: Partial overview of students' preferences in 2017

	Projects									
Students	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
s_1	4				3	5	1		2	
s_2	3			4	5		2	1		
s_3		2	3	4	5					1
s_4	4			2	3	5	1			
s_5		2	5	3	4			1		
s_6	4					5	1	3	2	
s_7	5				2		1		4	3
s_8	1		2	4	5		3			
s_9	4	2		1	3					5
s_{10}	3		1	5	4		2			

6.2 Numerical results

The optimization models proposed in Section 5 were implemented in AMPL. The linear models were solved using CPLEX, version 12.8. For the non-linear models, we used BARON version 18.12.26. Additionally, to speed up BARON solution times, we provided the solver with the Jain's index bounds presented in Section 4.3.1. The computational runs are set to a time limit of one hour. Each of the instances from 2017 to 2020 were run using all the four proposed approaches. For the remaining of this section we use the following abbreviations to identify each of the approaches:

- PELF: first Prioritize Efficiency and then optimize using the Lexicographic Fairness;
- PEJI: first Prioritize Efficiency and then optimize fairness using the Jain's index;

- PLFE: first Prioritize fairness using the Lexicographic Fairness and then optimize Efficiency;
- PJIE: first Prioritize fairness using the Jain's index and then optimize Efficiency.

6.2.1 Experimental results

In Table 3 we summarize the projects selected each of the years 2017, 2018, 2019 and 2020. For each year, all the approaches returned the same set of projects. For the sake of clarity, note that each year has a list of projects that is different and independent from any other year. Hence, the list of projects for each year is unique and valid only for that year. Moreover, all approaches disregard some projects due to the low preferences and the few number of students that ranked them. For example, from the distribution of students' preferences in the year 2017 shown in Figure 1, the projects 8 and 9 were in the situation described. In particular, note that only one student ranked project 8 as first choice and no one did it for project 9, which resulted in those two projects being excluded from the final assignment.

Table 3: Summary of assigned projects

Year	**Assigned projects**
2017	$p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_{10}$
2018	$p_1, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}$
2019	$p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9$
2020	$p_1, p_2, p_3, p_4, p_5, p_7, p_8, p_9$

Table 4 summarizes the experimental results for 2017. The last column shows the assignment that was done manually in 2017 by the CEMS administration staff at NHH. The results for 2017 reached different assignments with small difference in total utilities and fairness. In all these assignments, 100% of the students are assigned to their top three choices. All the approaches reached an optimal solution within few seconds. Note that the approaches PELF and PLFE are based on integer linear models, while the approaches PEJI and PJIE are based on mixed integer non-linear models. Compared to the manual assignment, none of the solutions obtained have students assigned to their two bottom choices, while the manual assignment that was used in 2017 had 9% of students assigned to their two bottom choices.

The two best solutions for the 2017 instance were found by the approaches PELF and PLFE. First, recall that the approach PELF prioritize efficiency by maximizing the total sum of the student's utilities. The solution obtained maximizing utility assigns 25 students to their top choice, 7 to their second ranked choice, and 3 to their third ranked choice. Given that there are

Table 4: Efficiency and fairness results for 2017 using all the proposed approaches

	PELF	**PLFE**	**PEJI**	**PJIE**	**Manual assignment**
	(Percentage of students assigned)				
Utility 5	68 %	63 %	68 %	63 %	51 %
Utility 4	26 %	34 %	26 %	34 %	17 %
Utility 3	6 %	3 %	6 %	3 %	23 %
Utility 2	0	0	0	0	3 %
Utility 1	0	0	0	0	6 %
Utility 0	0	0	0	0	0
Total utility	162	161	162	161	142
Fairness	98.4%	98.61%	98.4%	98.61%	92.3%

not students ranked in the three bottom choices, the lexicographic fairness process is initialized constraining the lower ranked projects assignments to zero. Then, it starts minimizing the number of students assigned to their third ranked choice. After that process is finished, the solution found has 24 students assigned to their top choice, 9 assigned to their second ranked choice, and 2 assigned to their third ranked choice. To summarize, the consideration of fairness reduced the number of students assigned to the top choice and third ranked choices by one and two correspondingly, and increased the number of students assigned to their second ranked choice by 2. Notice that the PELF approach ensures that the two solutions will have the same utility of 162. Hence, fairness is optimized over the set of optimal solutions to the problem optimizing efficiency. Hence, the solution found is Pareto efficient. The details of these results are summarized in Table 5.

Table 5: Details of the PELF solutions for the 2017 instance

	Max efficiency	**Min assignments with utility 3**	**Min assignments with utility 4**
	(Number of students assigned)		
Utility 5	25	24	24
Utility 4	7	9	9
Utility 3	3	2	2
Utility 2	0	0	0
Utility 1	0	0	0
Utility 0	0	0	0
Total utility	162	162	162

Second, the PLFE approach is used to prioritize fairness. This approach starts minimizing the assignments to projects ranked below the top choice. In the first iteration, the assignments of students to projects with utility zero is minimized. After that iteration, the solution obtained has 8 students assigned to the their top choice, 4 to their second ranked choice, 13 to their third choice, and 10 to their fourth choice. There are no students assigned to their bottom choice or to a project that they did not rank. The process continues until it minimizes the number of students assigned to their second choice. In every iteration, the optimization continues to satisfy the assignment levels in the lower ranked projects found in previous iterations. The final solution has 22 students assigned to their top choice, 12 to their second ranked choice, and one to their third choice. There are no students assigned to their two bottom choices or to a project that they did not rank. After the last fairness iteration, the approach optimizes efficiency constrained to ensure the fairness level achieved in each iteration of the lexicographic fairness process. Notice that, in general, given the utility structure of our instances at the end of the lexicographic fairness, the overall utility is fixed and optimizing efficiency will not change the solution. This is reflected in the results summary presented in Table 6. Comparing the solutions found with the two approaches, PEFL and PLFE, there is a trade off between efficiency and fairness. As expected, PEFL yields a higher overall utility, while PLFE delivers a solution with less students assigned to their third ranked project. Here, the difference between the utility achieved when efficiency is prioritized and the utility achieved when fairness is prioritized provides the prize of fairness, which is equal to one utility unit in this instance.

Table 6: Details of the PLFE solutions for the 2017 instance

	Min assignments with utility 0	**Min assignments with utility 2**	**Min assignments with utility 3**	**Min assignments with utility 4**	**Max efficiency**
	(Number of students assigned)				
Utility 5	8	9	18	22	22
Utility 4	4	10	16	12	12
Utility 3	13	16	1	1	1
Utility 2	10	0	0	0	0
Utility 1	0	0	0	0	0
Utility 0	0	0	0	0	0
Total utility	115	133	157	161	161

The numerical results of the experiment using the 2018 instance are summarized in Table 7. All approaches found an optimal solution with the same level of utility and fairness. The optimal solutions found assigned all students to one of their two top choices. Also, the solutions found for the 2018 instance improves both utility and fairness when they are compared to the solution manually obtained by the staff. In addition, the solutions obtained with our approaches satisfied all the companies' requirements, while the manual assignment did not manage to satisfy all the requirements. That highlights the complexity of the problem, which was not trivial to handle manually and took many hours of work for the administration staff. However, with these optimization based approaches the solution obtained was compliant and it took only a few seconds to find it.

Table 7: Efficiency and fairness results for 2018 using all the proposed approaches

	PELF	**PLFE**	**PEJI**	**PJIE**	**Manual assignment**
	(Percentage of students assigned)				
Utility 5	71 %	71 %	71 %	71 %	38 %
Utility 4	29 %	29 %	29 %	29 %	38 %
Utility 3	0 %	0 %	0 %	0 %	21 %
Utility 2	0	0	0	0	3 %
Utility 1	0	0	0	0	0 %
Utility 0	0	0	0	0	0
Total utility	184	184	184	184	161
Fairness	99.09%	99.09%	99.09%	99.09%	96 %

6.2.2 Implementation results

For the years 2019 and 2020, the administrative staff has used the optimization based approach proposed in this paper. The results obtained are summarized in Table 8, which includes the performance of the assignments proposed for 2019 and 2020. In both years 2019 and 2020 all approaches were able to reach optimality within the same time limit. In particular, the PJIE approach took about 6 minutes to obtain an optimal solution, which was the longest time. From Table 1 we can see that the conditions for the CEMS assignment problem may change significantly from year to year. Indeed, the assignment problems faced in 2017 and 2018 were more demanding on the number of requested attributes by the projects. Note that in 2020, the students ranked up four projects and a project with rank 4 is the topmost preference.

Table 8: Implementation results for 2019 and 2020

	2019				**2020**			
	PELF	**PLFE**	**PEJI**	**PJIE**	**PELF**	**PLFE**	**PEJI**	**PJIE**
	(Percentage of students assigned)							
Utility 5	60 %	60 %	60 %	3 %	-	-	-	-
Utility 4	36 %	36 %	36 %	76 %	70 %	70 %	70 %	70 %
Utility 3	2 %	2 %	2 %	18 %	27 %	27 %	27 %	27 %
Utility 2	2 %	2 %	2 %	3 %	3 %	3 %	3 %	3 %
Utility 1	0	0	0	0	0	0	0	0
Utility 0	0	0	0	0	0	0	0	0
Total utility	153	153	153	129	121	121	121	121
Fairness	97.65%	97.65%	97.65%	98.08%	97.93%	97.93%	97.93%	97.93%

The results in Table 8 reveal that in the year 2019 all approaches except PJIE led to the same solution, while in 2020 all approaches led to the same solution. In both years all the students have been assigned to some of the projects stated in their top choices, and with the great majority assigned to one of their two most preferred choices. In practice, it has been positive for the administration being able to verify that the solution so obtained has such a performance along the different criteria and approaches. Being supported by optimization techniques has also improved the ability to conduct the process in a fast and objective way. In addition, both sides students and companies can rely now in that the decision process provides them with an assignment that take into account all the requirements and preferences.

As the numerical results of the implementation are affected by how the preferences of the students realized in practice, it is interesting to analyze other scenarios for comparison purposes. In particular, we may construct *best-case* and *worst-case* scenarios as a referential basis to compare the realized solution. In a best-case scenario, the preferences of the students would be such that everyone is allocated to her/his most preferred choice. It is easy to find a profile of preferences for this best-case scenario, by simply finding a feasible solution to the assignment problem with side-constraints. If a feasible solution exists, one can define the topmost preference of each student as the project to which she/he is assigned in this solution, to render an overall solution where the total utility is equal to the number of students multiplied by the highest utility. Note this solution also achieves the maximum of 100% in the fairness index (since everyone is assigned to a project ranked at the same level). In our case, we have verified that all approaches quickly reach such idealistic best solution, which in 2019 corresponds to a total utility equal to 170. Comparing to the results displayed in Table 8, we can see that the realized scenario of preferences allows to find a solution which reaches about 90% of the total utility of the best-case scenario. For the 2020 instance, the idealistic solution scores 132 in the total utility criterion, while the solution to the realized scenario reported in Table 8 scores 121, that is, about 92% in comparison to the best-case scenario. Note in the best-case scenario the preferences of the students are perfectly split among the assigned projects, which makes possible to provide all students with the highest utility. This involves some heterogeneity in the preferences of the students who are assigned to different projects (and homogeneity in the preferences of the students assigned to a same project). In contrast, we may think of a case where the preferences of all the students are fully homogeneous, that is, one of the projects is ranked as top-choice by all the students, another project is ranked as second choice by all the students, and so on. We could regard this as a worst-case scenario, in the sense that some few students will be assigned to their ranked projects, while all the others will be assigned to a non-ranked project. When running this scenario for our 2019 and 2020 data instances, the solution assigns between 12% and 15% of the students to a ranked project, and 40% of the students to a non-ranked project. The total utility is between 50 and 60, and the fairness between 48% and 50%. The large spread between these results and the results obtained for the best-case scenario reveals that the performance of the solutions may vary within a broad range. Moreover, note that an even worse scenario could be constructed by digging deeper into the constraints that enforce some students to be assigned to a project because of the required attributes (for example, when a project requires a German speaker and the only students who speak that language did not rank that project among their top choices). In this way, some data instances could eventually lead to solutions with total utility equal to zero. Fortunately, as shown by our results above, the scenarios realized in practice have allowed us to find solutions much closer to the best-case than the worst-case scenario.

7 Concluding remarks

This paper proposed a decision support tool for deciding the assignments of students to projects, taking into account the students' preferences and other problem requirements. Since, in general, it is not possible to assign all students to their most preferred project, we have developed a bi-objective integer optimization approach taking into account efficiency and fairness criteria. We have implemented the approaches to support the decision-making process of the administration at an international master program. Our solutions have been adopted in practice during the last three years and the plan is to continue with this application in the forthcoming years. The results are positive for the students and the companies, as well as the decision process turns easier to handle for the administration. Since the backgrounds of these stakeholders do not necessarily include optimization, our close collaboration with the administration has facilitated their understanding and the adoption of the new methodology.

Although our focus has been practice-oriented and in the specific case of a master program at NHH, our mathematical formulations allow flexibility to consider a diverse set of requirements and side constraints that may be needed in different setups, such as similar programs at other universities or institutions. Likewise, our work opens avenues to conduct future research involving more methodological aspects. For example, to the best of our knowledge, this is the first time that the Jain's index has been used to measure the fairness in an assignment problem involving people. While the index has appealing properties and provides a single measure of fairness (in contrast to the lexicographic order), it inherently involves a non-linear expression that affects the nature of the problem. A possible reformulation of the optimization models involving the Jain's index may lead to more efficient solution approaches. Also, studying different structures of preferences in a computational study remains of interest not only to investigate the computational performance of the approaches but also to analyze how they balance the trade-off between efficiency and fairness. Moreover, studying different utility functions and different measures of fairness are also topics for further research.

Acknowledgements

We would like to express our gratitude to Marie Van Beijeren, Senior Consultant of the CEMS at NHH, for her assistance in data acquisition and support in writing this article.

References

Bertsimas, D., Farias, V. F. and Trichakis, N. (2012), 'On the efficiency-fairness trade-off', *Management Science* **58**(12), 2234–2250.

Cutshall, R., Gavirneni, S. and Schultz, K. (2007), 'Indiana university's kelley school of business uses integer programming to form equitable, cohesive student teams', *Interfaces* **37**(3), 265–276.

Gale, D. and Shapley, L. S. (1962), 'College admissions and the stability of marriage', *The American Mathematical Monthly* **69**(1), 9–15.

Guo, C., Sheng, M., Zhang, Y. and Wang, X. (2013), 'A Jain's index perspective on α-fairness resource allocation over slow fading channels', *IEEE communications letters* **17**(4), 705–708.

Hansen, R. S. (2006), 'Benefits and problems with student teams: Suggestions for improving team projects', *Journal of Education for Business* **82**(1), 11–19.

Henry, T. R. (2013), Forming productive student groups using a massively parallel brute-force algorithm, *in* 'Proceedings of the World Congress on Engineering and Computer Science', Vol. 1, pp. 23–25.

Hoßfeld, T., Skorin-Kapov, L., Heegaard, P. E. and Varela, M. (2016), 'Definition of qoe fairness in shared systems', *IEEE Communications Letters* **21**(1), 184–187.

Huaizhou, S., Prasad, R. V., Onur, E. and Niemegeers, I. (2013), 'Fairness in wireless networks: Issues, measures and challenges', *IEEE Communications Surveys & Tutorials* **16**(1), 5–24.

Huxham, M. and Land, R. (2000), 'Assigning students in group work projects. can we do better than random?', *Innovations in Education and Training International* **37**(1), 17–22.

Jain, R. K., Chiu, D.-M. W. and Hawe, W. R. (1984), 'A quantitative measure of fairness and discrimination', *Eastern Research Laboratory, Digital Equipment Corporation, Hudson, MA* .

Kelly, P. (2008), 'Achieving desirable group-work outcomes through the group allocation process', *Team Performance Management: An International Journal* .

Krass, D. and Ovchinnikov, A. (2006), 'The University of Toronto's Rotman School of Management uses management science to create MBA study groups', *Interfaces* **36**(2), 126–137.

Krauss, B., Lee, J. and Newman, D. (2013), 'Optimizing the assignment of students to classes in an elementary school', *INFORMS Transactions on Education* **14**(1), 39–44.

Lopes, L., Aronson, M., Carstensen, G. and Smith, C. (2008), 'Optimization support for senior design project assignments', *Interfaces* **38**(6), 448–464.

Magnanti, T. L. and Natarajan, K. (2018), 'Allocating students to multidisciplinary capstone projects using discrete optimization', *Interfaces* **48**(3), 204–216.

Nicosia, G., Pacifici, A. and Pferschy, U. (2017), 'Price of fairness for allocating a bounded resource', *European Journal of Operational Research* **257**(3), 933–943.

Sediq, A. B., Gohary, R. H. and Yanikomeroglu, H. (2012), Optimal tradeoff between efficiency and Jain's fairness index in resource allocation, *in* '2012 IEEE 23rd International Symposium on Personal, Indoor and Mobile Radio Communications-(PIMRC)', IEEE, pp. 577–583.

Chapter 3

On efficiency and the Jain's fairness index in integer assignment problems

Nahid Rezaeinia[a], Julio César Góez [a], Mario Guajardo [a]

[a]Department of Business and Management Science, NHH Norwegian School of Economics, 5045 Bergen, Norway

Abstract

Given two sets of objects, the integer assignment problem consists of assigning objects of one set to objects in the other set. Traditionally, the goal of this problem is to find an assignment that minimizes or maximizes a measure of efficiency, such as maximization of utility or minimization of cost. Lately, the interest for incorporating a measure of fairness in addition to efficiency has gained importance. This paper studies the trade-off between these two criteria, using the Jain's index as a measure of fairness. The original formulation of the assignment problem with this index involves a non-concave function, which renders a non-linear non-convex problem, usually hard to solve. To this aim, we develop two reformulations, where one is based on a convex quadratic objective function and the other one is based on Mixed Integer Second-Order Cone Programming. We explore the performance of these reformulations in instances of real-world data derived from an application of assigning personnel to projects, and also in instances of randomly generated data. In terms of solution quality, all formulations prove to be effective in finding solutions capturing both efficiency and fairness criteria, with some slight differences depending on the type of instance. In terms of solving time, however, the performances of the formulations differ considerably. In particular, the convex quadratic approach proves to be much faster in finding optimal solutions.

Keywords Integer assignment, Multiobjective optimization, Efficiency, Fairness, Jain's index

1 Introduction

The assignment problem consists of allocating objects in one set to some other objects in another set. In one set, tasks such as jobs, courses, positions, projects, or resources are available to be done or used by the agents, individuals, or users of the other set. Operation managers are often faced with resource assignment problems, where quantitative decision models are useful to find a solution (Bertsimas et al., 2012). The optimal resource assignment and the trade-off between efficiency and fairness of the assignments have attracted increasing attention in several fields of academia and industry such as economics, computer science, wireless communications, and social fairness (Zabini et al., 2017). In dealing with the trade-off between these two criteria, research works have tried to develop methodologies to find solutions that perform well in both efficiency and fairness simultaneously (Guo et al., 2013). Often, the decision maker who performs the assignment is interested in maximizing an overall metric of efficiency, while the users affected by the assignment are interested in maximizing their own individual benefits. On the one hand, maximizing the assignment's performance on efficiency might lead to some users ending up in better conditions than other ones. On the other hand, maximizing a fairness metric among the users might potentially reduce the efficiency of the assignment (Zabini et al., 2017; Guo et al., 2014, 2013; Sediq et al., 2013). Therefore, studying the trade-off between efficiency and fairness in assignment problems is an important endeavour.

Motivated by a real-world problem of assigning students to projects (Rezaeinia et al., 2021), this paper studies the trade-off between efficiency and fairness in an integer unbalanced assignment problem. This type of assignment problems, where the number of resources and users are unequal, appear in several applications (see e.g. Rabbani et al. (2019) and Majumdar and Bhunia (2012)). In our problem, each resource must be assigned to only one user, unlike most of the literature studying the trade-off between fairness and efficiency in assignment problems, where a same resource could be shared among multiple users (see e.g. Schwarz et al. (2010), Sediq et al. (2012), Zhou et al. (2017), and Bui et al. (2019)). The different resources are characterized by a number of attributes. We assume that each user requires a limited number of resources and the benefits of each resource are not the same for all the users. Each user is then interested in maximizing its own benefit. However, from a central decision maker perspective, factors such as the limited number of available resources and the different valuation of the users on the different resources, make it practically impossible to assign the resources so that all users achieve a maximum benefit.

This paper proposes a bi-objective approach based on integer programming, incorporating the concepts of efficiency and fairness in the assignment. Here, an efficient assignment means an assignment that maximizes the total benefits. To measure fairness, we use the Jain's index (Jain et al., 1984). This index is defined as a ratio, where the numerator is the users' squared total benefits, and the denominator is the number of users times the total users' squared benefits. This ratio is a well-established measure of fairness and has some appealing properties, as emphasized in Guo et al. (2014); Sediq et al. (2013), and Lan et al. (2010). However, the Jain's index is a non-concave function (Guo et al., 2014; Sediq et al., 2013), which increases the difficulty of solving the trade-off problem. To overcome this issue, in this paper we study two reformulations tailored to the integer unbalanced assignment problem. One of these is based on a convex quadratic objective function and the other one is based on Mixed Integer Second-Order Cone Programming (MISOCP). Using data from a real-world case and also experimental data from different scenarios, we explore the performance of these reformulations in terms of solution quality and solving time.

The remainder of this paper is organized as follows. Section 2 reviews relevant literature. Section 3 develops mathematical modelling approaches for the trade-off problem, while Section 4 presents some reformulations. Section 5 discusses the numerical results. Concluding remarks are provided in Section 6.

2 Literature review

The scientific literature has paid increasing attention to the trade-off between efficiency and fairness in assignment problems. A wide range of works have studied the problem in various fields, especially in network assignment, wireless communication systems, digital transmission, telecommunications, and some other allocation problems with continuous resources. As a fairness measure, these works have usually adopted the Jain's index. This index was firstly defined by Jain et al. (1984), and since then it has been used in thousands of papers. In the following, we review papers that have used this index in assignment problems where, in addition to fairness, the solutions try to meet an efficiency criterion.

Schwarz et al. (2010) studied the trade-off between efficiency and fairness in a mobile communication system with multiple users. They address the problem using a non-linear integer program, where the objective is to maximize the total throughput of the users in the system. A linear approximation is proposed to simplify the non-linear problem, and a max-min model is proposed to maximize the minimum throughput and guarantee the minimum fairness level. Then, the Jain's index is used to quantify the obtained fairness. In another paper, Schwarz et al. (2011) used the Jains' index directly in the primary problem and studied the trade-off between efficiency and fairness in a wireless network. They proposed a utility maximization problem based on an α-fair utility function by considering the Jain's index as a constraint. Then, a Second Order Cone Programming (SOCP) adapted to the continuous multi-user scheduling problem was used to transform the problem from a non-convex to a convex optimization problem.

In a different problem, Sediq et al. (2012) studied the trade-off between the sum efficiency and Jain's fairness index in an assignment problem with continuous resources and multiple users. This work exploited some special conditions of the radio resource allocation problems to convert the non-convex problem to a convex optimization problem. Another paper that deals with trade-off between efficiency and fairness is Guo et al. (2013). They consider the increase of efficiency and fairness in the assignment of resources to users in a wireless communication system. They addressed the trade-off between efficiency and the Jain's index by two optimization models. In the first model, they considered maximizing efficiency subject to a Jain's index constraint. In the second model, they maximized the Jain's index by considering a constraint on the efficiency of the system. There is no reformulation for the Jain's index in this study, hence, an algorithm to find the optimal trade-off was proposed. The same authors studied a different problem but of similar nature in Guo et al. (2014). This problem considers the throughput maximization in a sub-channel and time slot allocation in downlink systems subject to a Jain's index constraint on both short-term and long-term fairness. First, a non-linear integer programming formulation was used to model the problem, and then the integer variables were relaxed into continuous ones and a SOCP formulation was used to convert the problem into a convex one. Song et al. (2016) used the Jain's index to measure the fairness of the obtained solution. The paper analyzes the trade-off problem among spectral efficiency, energy efficiency, and fairness in cooperative digital transmission systems. An α-fairness function was used in a multi-objective optimization model to represent the fairness rate. Then, an algorithm based on the Lagrangian dual decomposition method was used to obtain the solution set of the model. A heuristic resource allocation algorithm was also proposed to obtain a solution and maintain the trade-off between many users.

In another context, Kachroo et al. (2016) studied the trade-off between efficiency and fairness in radio resource allocation of maritime channels. Here, the problem consists of assigning the blocks of the radio resource to the users. To address it, a max-min integer linear programming model was formulated, where the objective function is to maximize the minimum total throughput from the resource, aimed at increasing fairness in the allocation system. Then, the obtained results were compared by using the Jain's fairness index. In another problem with continuous nature, Zhou et al. (2017) studied the load balancing problem in a cellular network, which aims at maximizing the system throughput and balancing the load of the network. They model the problem by a mixed-integer non-linear programming formulation with a non-convex objective function. A two-layer iterative algorithm is used to find a nearly optimal solution to the problem. Then, the Jain's index was used to measure the load balancing level in the network. Another paper using the Jain's index is Zabini et al. (2017), which studies the trade-off between throughput and fairness in a resource allocation problem in wireless communication systems. They address the trade-off problem by optimizing a model, whose objective function is to maximize the average throughput subject to a given specified value of fairness. Bui et al. (2019) studied the trade-off between throughput and fairness in a downlink non-orthogonal access network. They addressed the problem by a mixed-integer and non-convex optimization model. For practical implementation and overcoming the problem's complexity, the integrality of the variables is relaxed to a continuous formulation. Then, an approximation method is proposed to solve the relaxed problem, attempting to arrive at a locally optimal solution.

Most of the papers above deal with efficiency and fairness in continuous resource assignment problems, unlike our problem which has an integer nature. The only exception is Guo et al. (2014), which addresses an integer problem primarily but it can be converted to a continuous one. Among the reviewed papers, Zabini et al. (2017) is one of the closest to ours, as they approach a similar assignment problem. However, their problem has a continuous nature, in which each resource can be assigned to more than one user, while in our problem each resource should be assigned to only one user. Sediq et al. (2012) is also close to our article, in terms of exploiting the structure of the assignment problem and the Jain's fairness index to reformulate problem. However, some essential features differ. First, unlike the continuous nature of the resources in their problem, we deal with the discrete case, which has implications in how to reformulate the problem. Second, due to the specific requirements on the assignment of resources to users in our application, we consider some side-constraints that differ from their work. Moreover, we consider the different sequences in which the efficiency and fairness objective functions can be used in the optimization process, and we propose different reformulations to develop the corresponding approaches. Overall, our work contributes to study the trade-off between efficiency and Jain's fairness in assignment problems with discrete nature. On the methodological side, our reformulations aim at overcoming the difficulty that incorporating fairness imply on these problems. On the applied side, we illustrate our approaches in a real-world problem, and also explore their computational performance in several data instances.

3 Mathematical modelling

This section presents the different elements of the optimization formulations that will be used later to develop the main approaches.

To define parameters and decision variables in the mathematical formulations, we use two sets denoted by $\mathcal{U} = \{u_1, \ldots, u_n\}$, which represents the set of users, and $\mathcal{R} = \{r_1 \ldots, r_m\}$, which represents the set of resources. Additionally, we define $p_{ru} \in \mathbb{R}^+$ as the benefit obtained when a resource $r \in \mathcal{R}$ is assigned to a user $u \in \mathcal{U}$. Finally, $U_u \in \mathbb{Z}^+$ and $L_u \in \mathbb{Z}^+$ denote the upper and lower bounds respectively for the number of resources that a user u requires.

We use two sets of decision variables. First, the model has to decide to which users assign resources. We use the binary variable $y_u \in \{0, 1\}$, which is one if user u is selected and zero otherwise. This is required because given the condition of the unbalanced assignment problems and the number of resources needed for a user, all the users might not receive the resources they need. Second, the model has to assign resources to the users. For this purpose we use the variable $x_{ru} \in \{0, 1\}$, which is equal to one if resource r is allocated to user u, and zero otherwise.

3.1 Constraints

To model the conditions of the assignment problem under study, we define the following constraints:

$$\sum_{u\in\mathcal{U}} x_{ru} = 1 \qquad \forall r \in \mathcal{R}, \tag{1}$$

$$x_{ru} \leq y_u \qquad \forall r \in \mathcal{R}, \forall u \in \mathcal{U}, \tag{2}$$

$$\sum_{r\in\mathcal{R}} x_{ru} \geq L_u y_u \qquad \forall u \in \mathcal{U}, \tag{3}$$

$$\sum_{r\in\mathcal{R}} x_{ru} \leq U_u y_u \qquad \forall u \in \mathcal{U}, \tag{4}$$

$$x_{ru}, y_u \in \{0, 1\} \qquad \forall r \in \mathcal{R}, \forall u \in \mathcal{U}. \tag{5}$$

Constraint (1) ensures that all resources are assigned to only one user. Constraint (2) ensures that no resources will be assigned to users that were not selected. Constraints (3) and (4) impose the upper bounds and lower bounds on the number of resources that are needed for each user. Constraints (5) enforce the binary nature of the variables.

3.2 Efficiency and fairness functions

For the optimization models we consider two objectives: efficiency and fairness. We may define the efficiency and fairness of the assignment problem as a function of the vector x of decision variables. In (6), $B(x)$ measures the efficiency of an assignment x, which is the sum of the benefits obtained by the users. For the fairness we use the Jain's index $J(x)$ (Jain et al., 1984), which is formulated in (7) using the benefits of an assignment decision x.

$$B(x) = \sum_{u \in \mathcal{U}} \sum_{r \in \mathcal{R}} p_{ru} x_{ru} \tag{6}$$

$$J(x) = \max \quad \frac{\left(\sum_{u \in \mathcal{U}} \sum_{r \in \mathcal{R}} p_{ru} x_{ru}\right)^2}{n \sum_{u \in \mathcal{U}} \left(\sum_{r \in \mathcal{R}} p_{ru} x_{ru}\right)^2} \tag{7}$$

Note that equation (6) is a linear function on x. The Jain's index (7) is a non-linear continuous function with a bounded range in the closed interval $[\frac{1}{n}, 1]$. The Jain's index provides a fairness measure for an assignment and its two bounds represent two extreme situations. The lower bound $\frac{1}{n}$ corresponds to the least fair allocation. In that situation only one user benefits with the assignment and all the other users do not receive any resources or do not benefit from the assigned resources. The upper bound 1 corresponds to the fairest assignment in which all users receive the same benefit. Note that the fairness' bounds may be achieved without necessarily optimizing the users' benefits. Hence, when optimizing an assignment it is important to consider efficiency and fairness together.

3.3 General formulation

This paper aims to study the trade-off between efficiency and fairness. The main challenge when using the Jain's index is that full fairness would not necessarily lead to efficiency. Consider for example a two users case which is illustrated in Figure 1. The dots in the figure represent the set of feasible assignments C and the two axis represent the benefits obtained by each user with a given assignment. Note that full fairness is obtained when both users receive the same level of efficiency. However, that may happen when both users receive the same benefit, irrespective if it is the minimum or maximum possible, or something in between. To overcome that challenge, we propose a bi-objective approach where the pointed weakness of the Jain's index fairness function is balanced by maximizing the efficiency of the assignment for any given fairness level. Note that Figure 1 shows that efficiency and fairness are not necessarily conflicting. However, depending on how unbalanced the problem is, the optimization process

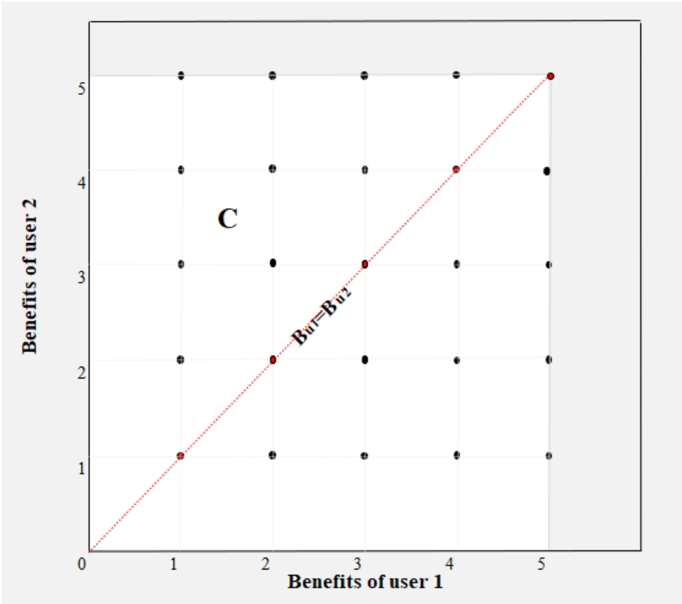


Figure 1: Efficiency and Jain's fairness comparison for feasible assignments in a set C

may demand more on the trade-off between efficiency and fairness.

The general formulation used for building our approach was proposed in Sediq et al. (2012) and Sediq et al. (2013) in the context of wireless communication assignments. Here, we define $\mathcal{C}$ as the set of vectors x, y satisfying the constraints (1) – (5). Then we obtain the first part of the formulation (8).

$$X_p \triangleq \{(x, y) \mid (x, y) = \underset{\substack{B(x) \geq p \\ (x,y) \in \mathcal{C}}}{\operatorname{argmax}} J(x)\} \tag{8}$$

In (8), X_p is the set of all assignment vectors $(x, y) \in \mathcal{C}$ that maximize the Jain's index subject to a minimum efficiency level of p. Note that (8) works with a prescribed level of efficiency. Hence, to optimize the trade-off, next we need to maximize the total efficiency $B(x)$ over the set X_p. In (9), $X_p^* \in X_p$ is one of the benefit vectors that maximizes the total efficiency. Note that X_p^* lays on the Pareto efficient frontier.

$$X_p^* \in \underset{(x,y) \in X_p}{\operatorname{argmax}} B(x) \tag{9}$$

The remainder of this section will focus on the optimization approaches analyzed in this paper, which are a variation of the general formulation presented in this section. Those approaches were previously proposed by Rezaeinia et al. (2021) for the trade-off problem. The models are based on a lexicographic method to prioritize one goal at a time.

3.3.1 Lexicographic Efficiency - Fairness

The first lexicographic approach prioritizes efficiency over fairness. Efficiency is computed using the utility function defined in (6). We formulate the first step of this approach in (10), where the aim is to obtain the maximum efficiency level for an assignment subject to constraints (1) - (5). Let B^* denote the optimal efficiency level obtained by solving the problem (10).

$$\begin{aligned} &\max\ B(x) \\ &s.t.\ (1)-(5) \end{aligned} \tag{10}$$

The second step of this approach is to optimize fairness while maintaining at least an efficiency level of B^*. We formulate the optimization problem of this second step in (11), where we maximize the Jain's index.

$$\begin{aligned} &\max\ J(x) \\ &s.t.\ (1)-(5) \\ &\quad\ B(x) \geq B^* \end{aligned} \tag{11}$$

Note that (11) is equivalent to solve the optimization problem considered in (8), where $p = B^*$. Hence, we are seeking an assignment that maximizes $J(x)$ and yields an efficiency at least equal to B^*.

Two remarks are important here. First, note that setting $p = B^*$ makes unnecessary to solve the optimization problem in (9). In other words, by construction we already have the best level of efficiency that can be achieved with just the assignment constraint. Henceforth, a more constrained problem will not improve on that efficiency level. Second, if the problem (10) is feasible, then we immediately have a solution that satisfies the feasible set of (11). In other words, we obtain a certificate of feasibility for both problems even though the second problem is more constrained. For later reference throughout the article, we formalize this second remark in Corollary 1 below.

Corollary 1. *The feasibility of Problem* (11) *follows from the feasibility of Problem* (10).

3.3.2 Lexicographic Fairness - Efficiency

The second lexicographic approach prioritizes fairness over efficiency. We formulate the first step of this approach in (12), where we optimize the fairness of the assignment subject to constraints (1) - (5). To optimize fairness, we use the Jain's index formulated in (7). Let J^*

be the optimal fairness level obtained by solving (12).

$$\begin{aligned} &\max\ J(x) \\ &s.t.\ (1) - (5) \end{aligned} \tag{12}$$

The second step of this approach is to optimize efficiency while maintaining at least a fairness level of J^*. We formulate the optimization problem of this second step in (13), where we maximize the efficiency utility function $B(x)$.

$$\begin{aligned} &\max\ B(x) \\ &s.t.\ (1) - (5) \\ &\quad J(x) \geq J^* \end{aligned} \tag{13}$$

Note that (12) is equivalent to solve the optimization problem considered in (8), where $p = 0$. Hence, since we are considering only non-negative utilities, we are seeking an assignment that maximizes $J(x)$ without imposing any lower bound constraint on the efficiency. Then, problem (13) solves the problem considered in (9) imposing J^* as the minimum level of fairness accepted.

Note that for this approach we also have that obtaining feasibility when solving problem (12) certifies the feasibility of problem (13). This is formalized in Corollary 2 below.

Corollary 2. *The feasibility of Problem* (12) *follows from the feasibility of Problem* (13)*.*

Corollaries 1 and 2 have more general implications. Notice that the problems in (10) and in (12) have the same feasible set. Hence, as a result we obtain that the feasibility of the problem in (12) is certified by the feasibility of the problem in (10), as stated in Corollary 3 below.

Corollary 3. *The feasibility of Problem* (12) *follows from the feasibility of Problem* (10)*.*

Even though the results in Corollaries (1), (2), and (3) follow easily from the definition formulation and construction, they have an important practical implication. First, we remark that the problem in (10) is a Mixed Integer Linear Problem (MILP), for which one can exploit the efficiency of the current state of the art of MILP solvers. However, the problems in (11), (12), and (13) are non-linear non-convex problems, for which the optimization process tend to be more challenging. To ease the challenge posed by the nature of those problems, if problem (10) is feasible, the solution found to it may be used to initialize the optimization of the problems (11), (12), and (13).

4 Reformulating the Jain's Index

One of the challenges when using the Jain's Index is that (7) is non-linear non-concave. This may lead to more computational effort and time to solve the problems in (11), (12), and (13). Additionally, it may not scale, i.e., when the dimension of X_p is large, it might become too difficult to solve the trade-off problem.

To show the non-concavity of the Jain's Index, we use a numerical example with the Jensen's concave inequality (Jensen, 1906). Note that Jain's index would be a concave function if and only if (14) is true.

$$J(\theta x + (1-\theta)x\prime) \geq \theta J(x) + (1-\theta)J(x\prime) \tag{14}$$

Let $\theta = 0.5$, $x = (4, 1, 5)$ and $x\prime = (2, 1, 3)$. Then, we have $J(x) = 0.7936$, $J(x\prime) = 0.8571$. The left and right-hand side of (14) are 0.8205 and 0.8253, respectively. From the result, $0.8205 \leq 0.8253$, which indicates that (14) is not true and, therefore, it shows that the Jain's Index is not a concave function.

The non-concavity of the Jain's Index affects the nature of the optimization problem and provides the motivation for looking at possible reformulations.

4.1 Reformulation for the Lexicographic Efficiency - Fairness approach

Here we show how $J(x)$ can be reformulated in problem (11). First, from Corollary 1 we know that if the problem in (10) is feasible, its optimal solution is also feasible for (11). Moreover, since the optimal value B^* found when optimizing (10) does not have any additional side constraints, we know that it is not possible to obtain a higher value for $B(x)$ within the set defined by the constraints (1) - (5). Hence, the constraint $B(x) \geq B^*$ will always be active for any feasible solution of (11), i.e., it will always be satisfied with equality. Therefore, we may write Problem (11) as shown in (15).

$$\begin{aligned} &\max\ J(x) \\ &\quad s.t.\ B(x) = B^* \\ &\quad\quad (1) - (5) \end{aligned} \tag{15}$$

Note that in this formulation we are fixing the value of $B(x) = B^*$. Now, recall the definition of the Jain's Index $J(x)$ in (7). Hence, as a result, we are fixing the numerator of $J(x)$. We may reformulate Problem (15) as follows:

$$\begin{aligned} &\max \frac{B^*}{n\sum_{u\in\mathcal{U}}\left(\sum_{r\in\mathcal{R}} p_{ru}x_{ru}\right)^2} \\ &s.t.\ B(x) = B^* \\ &\quad (1) - (5). \end{aligned} \tag{16}$$

Since B^* is obtained by (10), optimizing the problem in (16) is equivalent to optimizing the following problem:

$$\begin{aligned} &\max \sum_{u\in\mathcal{U}}\left(\sum_{r\in\mathcal{U}} p_{ru}x_{ru}\right)^2 \\ &s.t.\ B(x) = B^* \\ &\quad (1) - (5). \end{aligned} \tag{17}$$

The advantage of Problem (17) is that it has a convex quadratic objective function, which may help with the computational effort required to solve it.

4.2 Reformulation for the Lexicographic Fairness - Efficiency approach

Here we focus on the reformulation opportunity for the second approach considered in this paper. First, we have that the problem in (12) is maximizing the Jain's Index, which makes it a non-linear and non-convex problem. For that particular problem we have no reformulation to ease the challenge posed by $j(x)$. Hence, this will remain one of the more computationally demanding steps of our approaches.

Now, using Corollary 2, we know that if we find a feasible solution $(x^{'}, y^{'})$ to (12), then we have that $(x^{'}, y^{'})$ is feasible for the problem in (13). Note that $(x^{'}, y^{'})$ may be the optimal solution, but it is not required to be. This is relevant because it may happen that the Problem (12) may not be solved to optimality within a certain time limit, but a good enough feasible solution is available. Let J^* be the fairness value for that feasible solution $(x^{'}, y^{'})$. Using J^* we can reformulate (13) as a MISOCP. This is a mixed integer non-linear problem, whereby a linear objective function is optimized by considering some linear constraints and at least one quadratic cone constraint (Góez (2013)).

To reformulate the problem in (13) we focus on the constraint involving the Jain's Index. With that constraint we aim to ensure a level of fairness at least as good as J^*. Note that the use of $J(x)$ leads to a non-linear non-convex constraint. However, we can exploit the constant

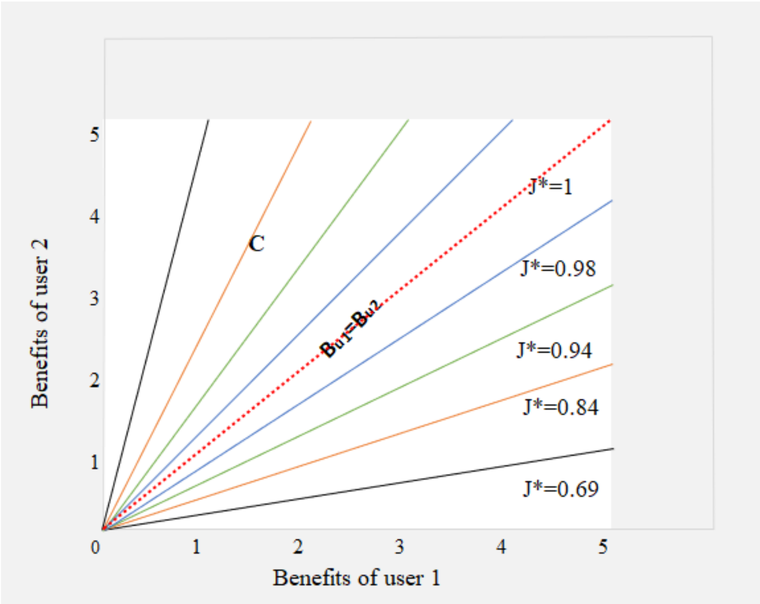


Figure 2: The affect of J^* on the trade-off between efficiency and fairness

right-hand side J^* to reformulate that constraint as a SOC as follows:

$$\sqrt{\sum_{u\in\mathcal{U}}(\sum_{r\in\mathcal{R}} p_{ru}x_{ru})^2} \leq \frac{\sum_{u\in\mathcal{U}}\sum_{r\in\mathcal{R}} p_{ru}x_{ru}}{\sqrt{nJ^*}}. \tag{18}$$

In (18) we obtained a second-order cone constraint in $\mathbb{R}^{n+1}$. Thus, the trade-off problem (13) can be written as (19).

$$\begin{aligned} &\max\ B(x) \\ &s.t.\ \sqrt{\sum_{u\in\mathcal{U}}(\sum_{r\in\mathcal{R}} p_{ru}x_{ru})^2} \leq \frac{\sum_{u\in\mathcal{U}}\sum_{r\in\mathcal{R}} p_{ru}x_{ru}}{\sqrt{nJ^*}} \\ &\qquad (1)-(5) \end{aligned} \tag{19}$$

The optimization problem in (19) is a MISOCP. Note the room for trade-off in Problem (19) is related to J^*. Figure 2 illustrates the effect of different J^* levels on the trade-off between efficiency and Jain's fairness. Problem (19) assigns resources to users and maximizes the total benefits of the users in the intersection of the possible region and the cone given by the fairness constraint. In the figure, by considering two users with a maximum of 5 units of benefit for each, the largest cone corresponds to $J^* \geq 0.69$. As J^* increases it approaches the value 1, which is the maximum possible level of fairness and is represented in the figure by the red dashed line.

5 Computational results

We test the approaches proposed in Section 4 in a personnel allocation problem, derived from a real-world application reported in Rezaeinia et al. (2021). This is an unbalanced integer assignment problem, where students must be assigned to projects in an educational programme. The projects are proposed by companies and presented to the students in a workshop, with the support of administrative staff and academic supervisors. After the workshop, the students are asked to fill out a preference survey and rank K projects by considering their skills, background, and projects description. The ranking is structured as follows: a project with rank K is the most beneficial project, and a project with rank 1 has the lowest benefit. Consequently, the remaining $K-2$ projects that the students are allowed to rank have benefits in the range $\{2,\ldots,K-1\}$. In this problem, the projects defined by the companies are equivalent to what in the models is the set of users and the students are equivalent to what in the model is the set of resources. Then, the students' benefits p_{ru} are computed based on the preference ranking obtained from the survey. Hence, $p_{ru} = k$ means that the benefit of student r from being assigned to project u is equal to k, where k is the given rank to the project by the student. Also, $p_{ru} = 0$ means that project u is not ranked by student r. In addition, the companies may have specific requirements on the team of students they will get assigned, on attributes such as educational background, language skills, and gender. These data are gathered in collaboration with the administrative staff, and is then used in an assignment model. For this purpose, we define $\mathcal{T} = \{t_1,\ldots,t_l\}$ as a set of attributes, and a_{rt} as a binary parameter equal to one if student r possesses attribute t, and zero otherwise. Then, the following side-constraints are used in the model:

$$\sum_{r\in\mathcal{R}} a_{rt}x_{ru} \geq L_{ut}y_u \qquad \forall u\in\mathcal{U}, \forall t\in\mathcal{T} \tag{20}$$

$$\sum_{r\in\mathcal{R}} a_{rt}x_{ru} \leq U_{ut}y_u \qquad \forall u\in\mathcal{U}, \forall t\in\mathcal{T} \tag{21}$$

In constraints (20)-(21), L_{ut} and U_{ut} specify upper and lower bounds on the number of students with attribute t that are needed by project u. Depending on these constraints, the number of projects, the number of students and their preferences, and other aspects in the problem, it is in general not possible to assign all students to their top choices. In consequence, different solutions might render different levels of efficiency and fairness and, therefore, it is important for the administration to find a solution considering both criteria. More details about the problem can be found in Rezaeinia et al. (2021). In what follows, we use the data instances of that paper, to study the performance of the reformulations we developed in Section 4. All computational codes have been implemented in AMPL. To solve the mathematical programming models, we use CPLEX version 12.10, except for model (12), which we solve by using BARON version 18.12.26. The computational runs are set to a time limit of two hours.

5.1 Real-world data instances

These instances of data correspond to five consecutive years, spanning from 2017 to 2021. Table 1 provides an overview on the number of students, the number of projects, and the number of requested attributes for each of these years.

Table 1: Overview of the real-world data instances

Year	Students	Projects	Attributes	Top rank
2017	35	10	10	5
2018	39	11	21	5
2019	34	9	3	5
2020	33	9	9	4
2021	36	9	9	5

Each of the instances from 2017 to 2021 was run using the Reformulation for the Lexicographic Efficiency-Fairness approach (RLEF) and the Reformulation for the Lexicographic Fairness-Efficiency approach (RLFE) We compare their results with results of the Lexicographic Efficiency-Fairness (LEF) approach described in Section 3, which provides a basis for comparison for the solutions obtained by the two other approaches.

For the instances of 2017, 2018, and 2020, all the approaches conduced to the same optimal solution within a few seconds. For 2019 and 2021 the results exhibited some differences, as shown in Table 2. In these years, the students had to rank 5 out of 9 projects, which determined the different levels of benefits detailed in the table. The RLEF and LEF approaches led to the same optimal solution, with a great majority of the students assigned to their first choice project. This translates into a high level of efficiency, reflected in a large amount of total benefits. The RLEF approach, in contrast, tends to assign less students to their first choice project and more to their second choice project, which renders more fairness but at the expense of less efficiency. This fact is especially notable in the 2019 instance. As for the solving time, the RLFE took about 10 minutes to obtain an optimal solution in the two instances, while the RLEF and LEF approaches took only few seconds.

Table 2: Students assignment for 2019 and 2021 using all the proposed approaches

	2019 RLEF	2019 RLFE	2019 LEF	2021 RLEF	2021 RLFE	2021 LEF
	(Number of students assigned)					
Benefit 5	20	1	20	24	22	24
Benefit 4	12	26	12	9	12	9
Benefit 3	1	6	1	2	1	2
Benefit 2	1	1	1	1	1	1
Benefit 1	0	0	0	0	0	0
Benefit 0	0	0	0	0	0	0
Total benefits	153	129	153	164	163	164
Fairness	97.65%	98.08%	97.65%	97.53%	97.64%	97.53%

5.2 Experimental data instances

For the purpose of testing the reformulated approaches in larger size problems, we use 240 experimental data instances of different sizes. We group these instances into four data sets, which differ in the number of students, projects, requested attributes, and the number of projects ranked by the students. Table 3 shows an overview of the generated data instances, according to these characteristics.

Table 3: Overview of the experimental data sets

	Students	Projects	Attributes	Ranked projects
Data sets A	150	35	20	5
Data sets B	260	56	20	10
Data sets C	360	80	25	15
Data sets D	500	110	25	20

Also, three different scenarios are constructed based on the students' preferences structure, and for each scenario we generate 20 instances (therefore, each of the four data sets consists of 60 instances). The scenarios are characterized as follows.

- **Random preferences scenario.** In a data instance based on the random scenario, students' preferences are split among the projects randomly, according to a discrete uniform distribution (that is, each project has the same probability of being chosen as the k-th most favourite by each student). This defines the benefit of assigning a student to each of the projects, where zero refers to non-beneficial preferences, and K refers to the top-beneficial preferences. There is no guarantee that all the students could be assigned to their most preferred choices in this scenario. If a feasible solution exists, some students might be assigned to their top-beneficial choices, some others to their second choices, and so on (some students might even be assigned to their non-ranked projects).

- **Semi-homogeneous preferences scenario.** In a semi-homogeneous data instance, the projects and the students are divided into groups of the same size. The first group of students rank the first group of projects, the second group of students rank the second part of the projects, and so on. For example, in an instance of data sets A, the first 30 students rank the first seven projects, the second 30 students rank from project 8 to project 14, and so on. Note in a semi-homogeneous scenario, the students are assigned to their top-beneficial projects when their preferences satisfy the requirements of the ranked project.

- **Homogeneous preferences scenario.** In a data instance based on the homogeneous scenario, one of the projects is ranked as top choice by all the students, another project is ranked as the second choice by all the students, and so on. In consequence, considering the limited number of students that can be assigned to each project, only a few students will be assigned to their best ranked projects in this scenario.

Table 4 summarizes the experimental results obtained by RLEF, RLFE, and LEF approaches for data sets A. The set includes 150 students, 35 projects, 20 requested attributes, and the students rank up to 5 projects.

Table 4: Results for data sets A using all the proposed approaches

	Random scenario			Semi-homogeneous scenario			Homogeneous scenario		
	RLEF	RLFE	LEF	RLEF	RLFE	LEF	RLEF	RLFE	LEF
	(Percentage of students assigned)								
Benefit 5	89	83	89	105	101	105	5	5	5
Benefit 4	40	46	40	30	31	30	5	5	5
Benefit 3	14	17	14	8	11	8	5	5	5
Benefit 2	5	1	5	1	2	1	5	5	5
Benefit 1	2	3	2	1	1	1	5	5	5
Benefit 0	0	0	0	5	4	5	125	125	125
Total benefits	659	655	659	672	667	672	75	75	75
Fairness	96.09 %	96.26 %	96.09 %	94.45 %	94.79 %	94.45 %	35 %	35 %	35 %
Assigned projects	31	31	31	31	31	31	31	31	31
Solved instances	20	0	20	20	0	20	20	0	20
Solving time (sec.)	0.264	7200	64.480	2.431	7200	493.780	1	7200	154.715

The table shows the average of the obtained results in different scenarios. Twenty different data instances were produced based on each scenario, and each data instance was run with the proposed approaches. Hence, each column summarizes the average results for the proposed approaches. The rows are preceded by the average number of the students that could obtain such a benefit level (for example, in the solution to the RLEF approach of the random scenario, after running twenty data instances, 89 students were assigned to their top-beneficial preferences). The other rows present some statistics on the total benefit and fairness level of the solutions on average across the 20 instances of each scenario. Also, the average number of projects selected by each approach is reported in the next row. Note that the number of selected projects depends on the distribution of students' preferences, thus it may happen that some projects are not assigned because they were not among the main preferences of the students. In addition, we report the number of data instances that reached the optimal solution in each approach. The most notable result is that the RLEF approach is able to find a high quality assignment in a few seconds, while the RLFE approach reaches the time limit of two hours with a feasible solution but without proven optimality. Also, it took longer for the LEF approach than the RLEF to reach an optimal solution. In the random and the semi-homogeneous scenarios, the RLEF and RLFE approaches reach the assignment with slight differences in the average of total benefits and fairness levels. In particular, in these scenarios, the RLFE approach renders a solution with a higher level of fairness than the RLEF and LEF approaches, while the total benefits level obtained by RLFE is less than in the two other approaches. Although the RLEF, RLFE, and LEF approaches reach the same assignment with the same average of total benefits and fairness levels in all the instances of the homogeneous scenario, their average solution times differ considerably, as they take 1 second, two hours, and 8 minutes, respectively. Also, the approaches differ in the number of instances solved to optimality. The RLEF and LEF approaches reach an optimal solution in all the 20 instances of each scenario. In contrast, the RLFE failed to reach optimality in all scenarios. In addition, there are significant differences between the students' assignments in different scenarios.

In the random scenario, a great majority of the students are assigned to their three first beneficial preferences, and none of the students are assigned to their non-beneficial projects. In the semi-homogeneous scenario, the vast majority of the students are assigned to some of their preferred projects, while a few students are assigned to non-beneficial projects. In the homogeneous scenario, only a fstudents were assigned to their beneficial projects, and all the rest were allocated to their non-beneficial projects.

In the data sets B, there are 260 students and 56 projects. Also, there are 20 requested attributes, and the students rank 10 projects as their beneficial projects. The numerical results of the experiment using the data sets B are summarized in Table 5.

The RLEF approach again solved the data instances very quickly, within a few seconds, and reached the optimal solution in all the instances of all scenarios. In contrast, the LEF approach did not reach the optimal solution in 5 of the 20 instances of the random scenario and failed in

Table 5: Results for data sets B using all the proposed approaches

	Random scenario			Semi-homogeneous scenario (Number of students assigned)			Homogeneous scenario		
	RLEF	RLFE	LEF	RLEF	RLFE	LEF	RLEF	RLFE	LEF
Benefit 10	157	156	157	160	157	161	5	5	5
Benefit 9	65	61	65	66	67	64	5	5	5
Benefit 8	23	29	23	20	23	19	5	5	5
Benefit 7	8	8	8	8	7	8	5	5	5
Benefit 6	4	4	4	3	3	4	5	5	5
Benefit 5	2	1	2	1	1	1	5	5	5
Benefit 4	1	1	1	1	1	1	5	5	5
Benefit 3	0	0	0	0	0	0	5	5	5
Benefit 2	0	0	0	0	0	0	5	5	5
Benefit 1	0	0	0	0	0	1	5	5	5
Benefit 0	0	0	0	1	1	1	210	210	210
Total benefits	2433	2430	2433	2437	2433	2433	275	275	275
Fairness	98.82%	98.86%	98.82%	98.59%	98.60%	98.45%	15.10%	15.10%	15.10%
Assigned projects	53	53	53	52	52	52	53	53	53
Solved instances	20	0	15	20	0	0	20	0	20
Solving time (sec.)	3.493	7200	3894.143	6.795	7200	7200	2.705	7200	3159.248

all instances of the semi-homogeneous scenario. Also, the RLFE approach was unable to reach optimality within the two hours time limit in all scenarios. Although the obtained assignments by the approaches are slightly different in terms of average total benefits and fairness levels in the random and semi-homogeneous scenarios, the RLFE renders assignments with fewer benefits and higher fairness than the other two approaches. In the homogeneous scenario, the RLEF, RLFE, and LEF approaches reached the same assignment, while there is a significant difference between their solution times. In the random scenario, there are no students assigned in their three bottom choices. In the semi-homogeneous scenario, a significant number of students were assigned to their three first beneficial choices, and a few of them were assigned to their last and even to non-beneficial projects. In the homogeneous scenario, all approaches assign five students to each benefit level, while the great majority of students is assigned to non-ranked projects.

Table 6 shows the obtained results for the data sets C. These instances consist of 360 students, 80 projects, and 25 required attributes. Also, we assume the students rank 15 projects, where a project with rank 15 is the top-beneficial preference.

The table shows that the RLEF approach obtained optimal solutions in all data instances within a few seconds, while in the semi-homogeneous scenario it took 10 minutes to reach optimality. However, the RLFE and LEF approaches conclude the time limit of two hours with a feasible solution, but without proven optimality. The approaches reached the same assignment in all instances of the homogeneous scenario, while in the other other two scenarios, there are slight differences between the assignments obtained by the approaches. In the random and semi-homogeneous scenarios, the assignments obtained by the RLFE approach render a

Table 6: Results for data sets C using all the proposed approaches

	Random scenario			Semi-homogeneous scenario			Homogeneous scenario		
	RLEF	RLFE	LEF	RLEF	RLFE	LEF	RLEF	RLFE	LEF
	(Number of students assigned)								
Benefit 15	211	211	215	215	215	218	5	5	5
Benefit 14	94	91	88	89	87	83	5	5	5
Benefit 13	32	33	32	32	32	32	5	5	5
Benefit 12	13	14	13	12	13	13	5	5	5
Benefit 11	4	8	6	6	6	7	5	5	5
Benefit 10	4	1	3	2	3	3	5	5	5
Benefit 9	2	2	2	1	1	1	5	5	5
Benefit 8	0	0	0	0	1	1	5	5	5
Benefit 7	0	0	0	0	0	0	5	5	5
Benefit 6	0	0	0	0	0	0	5	5	5
Benefit 5	0	0	0	0	1	0	5	5	5
Benefit 4	0	0	0	1	0	0	5	5	5
Benefit 3	0	0	0	0	0	0	5	5	5
Benefit 2	0	0	0	0	0	0	5	5	5
Benefit 1	0	0	0	0	0	0	5	5	5
Benefit 0	0	0	0	2	1	2	285	285	285
Total benefits	5155	5152	5143	5130	5128	5128	600	600	600
Fairness	99.45%	99.46%	99.16%	98.82%	98.86 %	98.85%	16.129%	16.129%	16.129%
Assigned projects	74	72	73	75	74	74	78	75	78
Solved instances	20	0	0	20	0	0	20	0	0
Solving time (sec.)	52.45	7200	7200	665	7200	7200	6.71	7200	7200

higher level of fairness and less total benefit than the two other approaches. Note that none of the students were assigned to their last eight preferences in the random scenario. In the homogeneous scenario, all approaches reached the same assignment. The students are assigned to projects with different benefit levels. The majority of the students are assigned to non-ranked projects.

In data sets D, we consider 500 students with 110 projects and 25 requested attributes. The students rank 20 projects, where a rank of 20 is the most preferred one. The results obtained are reported in Table 7.

The most remarkable outcome is that the RLEF approach found an optimal solution to all instances of each of the three scenarios, and there are considerable differences between its solution time and other approaches. The RLFE and LEF approaches reach the time limit of two hours with a feasible solution but not proven optimality. In the homogeneous scenario, all approaches reached the same assignment. In general, the approaches reached different assignments in each instance of the random scenario and also of the semi-homogeneous scenarios. There are slight differences in the total benefits and fairness levels, as the RLFE approach obtained assignments with less total benefits and higher fairness levels than the other approaches. Also, in the homogeneous scenario, a great number of students are assigned to non-beneficial projects, while in the two other approaches, the vast majority of the students are assigned to one of their three most beneficial projects.

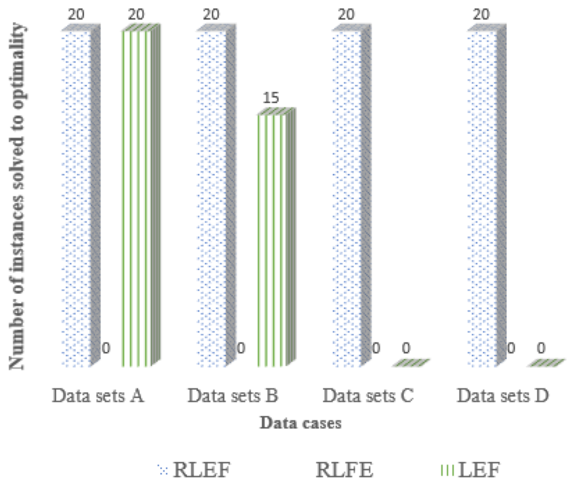
Table 7: Results for data sets D using all the proposed approaches

	Random scenario			Semi-homogeneous scenario (Number of students assigned)			Homogeneous scenario		
	RLEF	RLFE	LEF	RLEF	RLFE	LEF	RLEF	RLFE	LEF
Benefit 20	290	285	292	287	280	292	5	5	5
Benefit 19	129	132	127	129	119	120	5	5	5
Benefit 18	47	49	46	48	60	51	5	5	5
Benefit 17	19	19	19	21	28	20	5	5	5
Benefit 16	8	9	8	7	9	7	5	5	5
Benefit 15	5	5	5	5	3	5	5	5	5
Benefit 14	2	0	2	2	1	2	5	5	5
Benefit 13	0	0	1	1	0	1	5	5	5
Benefit 12	0	0	0	0	0	0	5	5	5
Benefit 11	0	1	0	0	0	0	5	5	5
Benefit 10	0	0	0	0	0	0	5	5	5
Benefit 9	0	0	0	0	0	0	5	5	5
Benefit 8	0	0	0	0	0	0	5	5	5
Benefit 7	0	0	0	0	0	0	5	5	5
Benefit 6	0	0	0	0	0	0	5	5	5
Benefit 5	0	0	0	0	0	0	5	5	5
Benefit 4	0	0	0	0	0	0	5	5	5
Benefit 3	0	0	0	0	0	0	5	5	5
Benefit 2	0	0	0	0	0	0	5	5	5
Benefit 1	0	0	0	0	0	0	5	5	5
Benefit 0	0	0	0	0	0	0	400	400	400
Total benefits	9651	9643	9648	9640	9620	9640	1050	1050	1050
Fairness	99.68%	99.68%	99.68%	99.67%	99.69%	99.66%	15.36%	15.36%	15.36%
Assigned projects	101	101	101	100	100	100	106	100	104
Solved instances	20	none	none	20	none	none	20	none	none
Solving time (sec.)	260.72	7200	7200	70.50	7200	7200	52.457	7200	7200

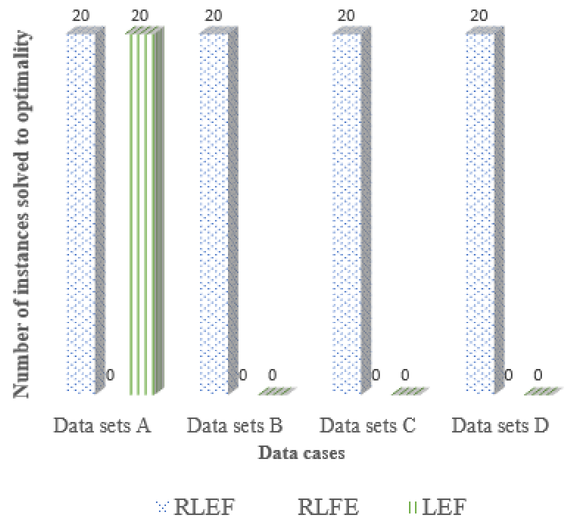
5.3 Discussion

The numerical results show that the size of the problem affects the performance of the approaches. All the proposed approaches reach optimality quickly in the small size instances, as illustrated in the runs with the real-world data. The trade-off problem becomes more difficult when the size of the problem increases, as shown in data sets A, B, C, and D. This also affects the performance of the approaches. For example, the RLFE reached the time limit of two hours without an optimal solution in all sets, while the LEF approach failed to reach an optimal solution in data sets C and D.

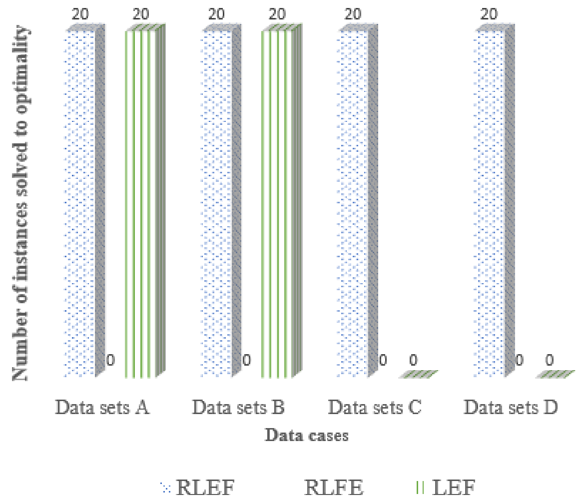
The results reveal that the users' preferences have significant influences on the difficulties of the trade-off problem and the performance of the approaches. The semi-homogeneous and the homogeneous scenarios are two examples in this case. The number of instances that failed to reach an optimal solution in these scenarios is more than in the random scenario. The students' preferences are not perfectly split among the projects. Hence, in the semi-homogeneous scenario, the students are assigned to their top-beneficial projects when their preferences satisfy the requirements of the ranked project. Also, depending on the conditions of the problem, some of the students may be assigned to their non-ranked projects. This is shown in data sets



(a) Random scenario



(b) Semi-homogeneous scenario



(c) Homogeneous scenario

Figure 3: Number of optimal solutions by the approaches in each scenario

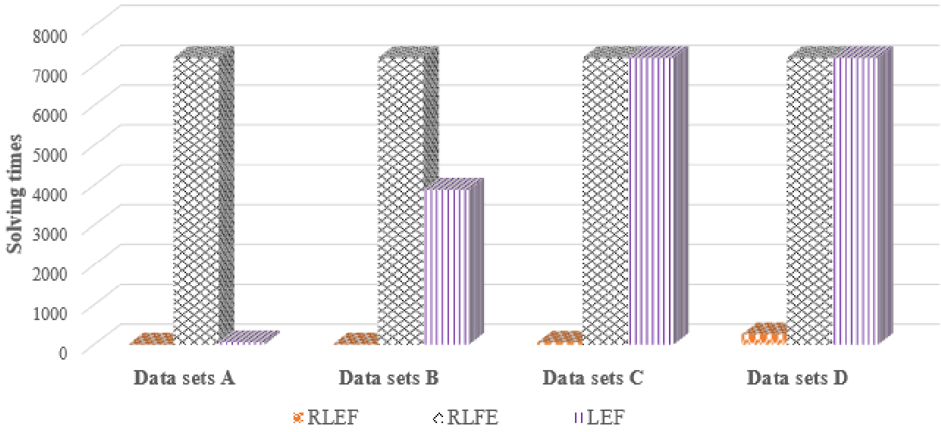


Figure 4: The solving time by the approaches in the random scenario

A, B, and C. Most of the students were assigned to non-beneficial projects in the homogeneous scenario. On average, the obtained solutions to the instances of the homogeneous scenario assigned 80% of the students to their non-ranked projects and 20% to the profitable project

The numerical results show the RLEF approach is more efficient in solving the trade-off problem. The number of solved instances by the approaches in each scenario is shown in Figure 3. In fact, the RLEF approach reached an optimal solution in all data instances of all the scenarios. Although the LEF failed to reach optimality in some data instances, its performance was slightly better than the RLFE approach, which leaves a more considerable number of instances unsolved.

Also, there are significant differences in the solving time used by the approaches. Figure 4 shows the solving times by the approaches in the random scenarios over the different data sets. The figure eloquently shows that the solution time by RLEF is less than the other two other approaches, while the RLFE approach reaches the time limit of two hours in all instances.

In addition, there are considerable differences among the solution time by the RLEF and LEF approaches when the problem's size increases. This effect is somewhat more prominent in data sets C and D. Note the trend of the solving times in the semi-homogeneous and the homogeneous scenarios are the same.

Furthermore, the results to the different data sets show that the RLFE obtained assignments with a higher level of fairness than the RLEF and LEF approaches. On the other hand, the RLEF approach allowed us to find more efficient solutions quickly, even when the size of the problem increased in the different scenarios.

6 Concluding remarks

This paper studied different formulations to address the trade-off between efficiency and fairness in the unbalanced integer assignment problem, where Jain's index is the fairness function. Since this index is defined by a non-concave function, it is challenging to solve the trade-off problem. The difficulty of the problem is highlighted more when the dimension of the problem is vast and many conditions are involved. To this aim, in addition to a basic formulation, we studied two reformulations, one based on a convex quadratic objective function and the other one based on Mixed Integer Second-Order Cone Programming. We analyzed the performance of these approaches in numerical experiments, carried out using both real-world data and randomly generated data. The results showed that, although all approaches may conduce to solutions that address well both efficiency and fairness measures, the convex quadratic formulation is much quicker than the other formulations.

To the best of our knowledge, this is the first time these types of reformulations of the Jain's index have been studied in assignment problems with integer nature. Further work could explore the performance of the proposed reformulations in optimization problems with different profit functions. Another avenue for future research is to study the problem when the users can share the non-continuous resources. Adapting the proposed approaches to study other integer problems requiring efficient and fair solutions also remains of interest.

References

Bertsimas, D., Farias, V. F. and Trichakis, N. (2012), 'On the efficiency-fairness trade-off', *Management Science* **58**(12), 2234–2250.

Bui, V.-P., Nguyen, P. X., Nguyen, H. V., Nguyen, V.-D. and Shin, O.-S. (2019), Optimal user pairing for achieving rate fairness in downlink noma networks, *in* '2019 International Conference on Artificial Intelligence in Information and Communication (ICAIIC)', IEEE, pp. 575–578.

Góez, J. C. (2013), Mixed integer second order cone optimization, disjunctive conic cuts: Theory and experiments, PhD thesis, Lehigh University.

Guo, C., Sheng, M., Wang, X. and Zhang, Y. (2014), 'Throughput maximization with short-term and long-term Jain's index constraints in downlink OFDMA systems', *IEEE Transactions on Communications* **62**(5), 1503–1517.

Guo, C., Sheng, M., Zhang, Y. and Wang, X. (2013), 'A Jain's index perspective on α-fairness resource allocation over slow fading channels', *IEEE Communications Letters* **17**(4), 705–708.

Jain, R. K., Chiu, D.-M. W. and Hawe, W. R. (1984), 'A quantitative measure of fairness and discrimination', *Eastern Research Laboratory, Digital Equipment Corporation, Hudson, MA* .

Jensen, J. L. W. V. (1906), 'Sur les fonctions convexes et les inégalités entre les valeurs moyennes', *Acta mathematica* **30**(1), 175–193.

Kachroo, A., Ozdemir, M. K. and Tekiner-Mogulkoc, H. (2016), Optimization of LTE radio resource block allocation for maritime channels, *in* '2016 IEEE 37th Sarnoff Symposium', IEEE, pp. 88–93.

Lan, T., Kao, D., Chiang, M. and Sabharwal, A. (2010), *An axiomatic theory of fairness in network resource allocation*, Proceedings IEEE INFOCOM.

Majumdar, J. and Bhunia, A. K. (2012), 'An alternative approach for unbalanced assignment problem via genetic algorithm', *Applied Mathematics and Computation* **218**(12), 6934–6941.

Rabbani, Q., Khan, A. and Quddoos, A. (2019), 'Modified Hungarian method for unbalanced assignment problem with multiple jobs', *Applied Mathematics and Computation* **361**, 493–498.

Rezaeinia, N., Góez, J. C. and Guajardo, M. (2021), 'Efficiency and fairness criteria in the assignment of students to projects', *Annals of Operations Research (In press)* .

Schwarz, S., Mehlführer, C. and Rupp, M. (2010), Calculation of the spatial preprocessing and link adaption feedback for 3GPP UMTS/LTE, *in* '2010 Wireless Advanced 2010', IEEE.

Schwarz, S., Mehlfuhrer, C. and Rupp, M. (2011), Throughput maximizing multiuser scheduling with adjustable fairness, *in* '2011 IEEE International Conference on Communications (ICC)', IEEE.

Sediq, A. B., Gohary, R. H., Schoenen, R. and Yanikomeroglu, H. (2013), 'Optimal tradeoff between sum-rate efficiency and jain's fairness index in resource allocation', *IEEE Transactions on Wireless Communications* **12**(7), 3496–3509.

Sediq, A. B., Gohary, R. H. and Yanikomeroglu, H. (2012), Optimal tradeoff between efficiency and Jain's fairness index in resource allocation, *in* '2012 IEEE 23rd International Symposium on Personal, Indoor and Mobile Radio Communications-(PIMRC)', IEEE, pp. 577–583.

Song, Z., Ni, Q., Navaie, K., Hou, S., Wu, S. and Sun, X. (2016), 'On the spectral-energy efficiency and rate fairness tradeoff in relay-aided cooperative OFDMA systems', *IEEE Transactions on Wireless Communications* **15**(9), 6342–6355.

Zabini, F., Bazzi, A., Masini, B. M. and Verdone, R. (2017), 'Optimal performance versus fairness tradeoff for resource allocation in wireless systems', *IEEE Transactions on Wireless Communications* **16**(4), 2587–2600.

Zhou, T., Liu, Z., Zhao, J., Li, C. and Yang, L. (2017), 'Joint user association and power control for load balancing in downlink heterogeneous cellular networks', *IEEE Transactions on Vehicular Technology* **67**(3), 2582–2593.

Chapter 4

The eigenvalue-UTA approach for multi-criteria decision-making problems: A case study on a rural road selection in Iran*

Nahid Rezaeinia[a]

[a]Department of Business and Management Science, NHH Norwegian School of Economics, 5045 Bergen, Norway

Abstract

Building roads in forest areas is experiencing a period of expansion. The purpose of these roads is not only to facilitate resource extraction and freight transportation, but also to enhance the welfare of inhabitants. This paper discusses the problem of evaluating a rural road network in the Hyrcanian forest in northern Iran. The evaluation must consider a number of aspects, such as economic and environmental criteria. In addition, opportunities and risks of forest road building are also considered. A multi-methodology decision-making approach is proposed to solve the problem, in which the evaluation of the alternatives is based on an eigenvalue method, and the importance of the criteria and the priorities of the alternatives are based on a utility additive method. The numerical results show that the proposed approach can structure and facilitate the decision-making process and it is flexible for use in similar cases.

Keywords Multi-criteria decision-making, Additive value model, Eigenvalue method, Road selection

*Part of this paper has been presented at 25th International Conference on Multiple Criteria Decision Making. A later version is under second review in a scientific journal.

1 Introduction

Road development is an important challenge in rural areas requiring forest sustainability (Picchio et al., 2018a). The primary purpose of these roads is to facilitate closeness to the forest area to ease transportation and optimize resources for extraction (Grigolato et al. (2013); Picchio et al. (2018b)). In recent decades, the importance of forests has added different functions and purposes for forest roads (Laschi et al., 2016). They help with fire control, damage reduction, and improving wildlife habitats (Esmaeili Sharif et al., 2016). In addition, efficient forest roads have a considerable effect on inhabitants' lifestyle, and they affect regional economic growth, providing social needs, water supply, and recreational activities (Demir (2007); Hayati et al. (2013)). Even in the current pandemic times, an efficient forest road can facilitate access to healthcare and play an essential role in saving lives of inhabitants in rural areas. However, forest road building and development activities pose heavy demands on the environment, which conflict with the principle of sustainable land use planning (Gumus et al., 2008). Forest roads increase the volume of traffic, as well as the levels of air and noise pollution. They also contribute to flora and fauna degradation (Akay et al., 2008), disturb the forest floor, damage soil structure, produce sediment (Rezaei, 2015), and heighten the risk of unwanted fires (Esmaeili Sharif et al., 2016). Moreover, in forest road building, it is crucial to address the safety of workers, transportation of products, and the comfort and economy of vehicle operations (Hayati et al., 2013). Since the forest road building and its maintenance activities are costly endeavors, an important decision should be taken before each step. As forest roads are important for different purposes and functions (Akay, 2006), multiple factors are involved in choosing the proper location for forest road building. These factors integrate most Cost, Ecological, Risk, and Opportunity (CERO) aspects into the decision-making process.

The decision-making process of forest road building, such as choosing the location of the road, is often left to forest managers, who are decision makers (DMs) and who do not necessarily have the required background and tools to make a proper decision. Moreover, because of the positive and negative effects of forest road building, some of the criteria in the decision-making process conflict with each other. Therefore, adopting all of them into a single solution is a challenge for DMs, and an inefficient decision might have undesirable economic and environmental consequences. This motivates the development of decision-making tools for selecting the location of forest roads among different alternatives and help managers to make a proper decision. This paper uses the eigenvalue method and UTilité Additive (UTA) approach and presents a multi-methodology decision support framework to address forest road location selection in the north of Iran. UTA is one of the Multi-Criteria Decision-Making (MCDM) methods designed to determine the importance of the criteria and prioritize the alternatives under multiple criteria. The eigenvalue method is a powerful approach used widely to obtain the priority weight vector of the criteria and alternatives. The proposed multi-methodology approach can incorporate different criteria to select the desirable road alternative.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 explains the forest road building problem as a MCDM problem, and Section 4 presents the methodology. Section 5 illustrates the use of this methodology in a case study of forest road selection in Iran. Concluding remarks are provided in Section 6.

2 Background

MCDM methods are used in a broad range of works such as environment management, water management, business, logistics, energy, and strategy planning (Toloie-Eshlaghy and Homayonfar, 2011). Since choosing an appropriate location for forest road building is a part of the decision-making science, a great body of literature has addressed the use of MCDM approaches for forest road location selection problems.

Hashemkhani Zolfani et al. (2011) studies the problem of forest road locating considering technical, transportation, environmental, social, and economic features as the evaluating criteria. The authors address the problem through Analytic Hierarchy Process (AHP) and Complex Proportional Assessment (COPRAS) methods for a case study that took place in Iran. In another similar case, Hayati et al. (2013) looked at the principle of forest road designing through the lens of efficiency and the environment and did so by considering slope, soil texture, landslide susceptibility, erosion susceptibility, distance from faults, lithology, geology, and distance from the stream. They address the problem by using the Delphi method for selecting the criteria, AHP for weighting the criteria, and a Geographic Information System (GIS) to identify the best road alternative.

Both of the previously mentioned papers helped forest managers to make decisions easily, but none of these papers considered cost criteria in forest management. In this respect Ghajar et al. (2013) developed functions to estimate the cost of forest road construction. They considered cleaning operation, embankment, pavement, grading, culverts, and ditch as the six main cost elements in the estimation by applying the functions in the optimization process. The functions are able to estimate the minimum construction cost. In another work by Jaafari et al. (2015a) the six main cost elements were used in the forest road alternative evaluation. Also, they evaluated the problem from the landslide perspective by considering slope, altitude, plan curvature, slope length, lithology, distance from streams, distance from faults, and rainfall. Their approach for finding a solution used PEEGER and GIS to design road alternatives and to extract the values of alternatives from the susceptibility map. In a different but related topic by Acar et al. (2017), cost criteria were considered differently in the evaluation. The most suitable roads were created by using the least-cost path analysis. The authors address the problem using the fuzzy logic approach. They evaluate the road alternatives by considering land slope, distance from mainstream, road density, and distance to other roads.

While this paper has some similarities with the applications reviewed in this section, some important features differ. First, due to the importance of economic and environmental criteria in forest road building and the importance of building an efficient forest road, CERO criteria are incorporated in the evaluation. A decision based only on economic criteria may have undesirable environmental consequences, while a decision based only on environmental criteria may produce an inefficient forest road without creating any opportunities for the area. In particular, the aim is to include all the CERO criteria and strike a balance in the decision-making process. Second, in the existing literature it is assumed that all the quantitative criteria can be evaluated and such evaluation is given as an input to the problem. In practice, however, evaluating these criteria is often costly and time-consuming. Hence, it might be impossible to access the quantitative data due to a lack of budget or other limitations. Thus, this paper contributes to developing a multi-methodology approach and evaluating the alternatives when access to the required data is costly. For this purpose, this paper proposes to jointly use the eigenvalue (Saaty, 1988) and the UTA (Siskos et al., 2016) methods. The eigenvalue method evaluates the importance of the alternatives with respect to the criteria, and the UTA method determines the importance of the criteria and the priorities of the alternatives. The use of the approach is illustrated in a case study on forest road selection in Iran.

3 The problem of forest road building using multi-criteria decision-making

This paper studies the evaluation of a forest network in the Hyrcanian forest. Hyrcanian or Caspian forest is a unique forested belt between the northern slopes of the Alborz mountains and the southern basin of the Caspian Sea. It covers 850 km of terrain across the south coast of the Caspian sea.[1] According to the World Wild Fund for nature (WWF), the Hyrcanian forest is one of 200 important ecoregions in the world, so its global significance is well established.[2] Also it is the only commercial forest in Iran, and it consists of 15 percent of the total forests of the country. The region includes a wide variety of plant communities. Specific plant species cover the Hyrcanian forest, which reflects the importance of environmental protection of this area for forest management in Iran. In addition, 60 percent of the region's trees (alive and dead) are used for timber production (Jaafari et al., 2015b), and the region has significant industrial importance.

In big countries like Iran and with a high population, a significant number of people live outside the urban areas. Hence, roads are critical facilities that deal with inhabitants' economics, health, and social welfare. Chelav is one of eight rural districts of Amol county in Mazandaran Province in the north of Iran. Chelav district consists of seven villages, and according to the last national census statistics in 2010, the area's population is 4327. Chelav forest is part of the Hyrcanian forest, and it is one of the most important forestry regions with high-quality commercial timber in northern Iran. There is one main road in Chelav district with two branches that is the only way for inhabitants to access the closest urban area (Amol) and meet their economic, educational, and health needs. Figure 1 shows Chelav district, its geographical location toward Amol city, and the main road for access to the area based on Google map satellite pictures.

As shown in Figure 1, most of the region is a forest area, and a significant number of the local population live there. The local inhabitants effectively protect the forest area from fire, illegal timber harvesting, and illegal hunting. The inhabitants of Chelav, in order to access the closest urban area (Amol town), have to get themselves to the main road. The forest road density in the region is very low, which causes accessibility difficulties. The low road density, challenges in access to health, education, and local market has led to the immigration of a large number of inhabitants to urban areas. Hence the forest managers decided to increase the forest road density in the region and provide inhabitants social welfare, improve the regional economic growth, and motivate inhabitants to stay in the area as well as environmental protection. In addition, the current pandemic situation and the importance of inhabitants' accessibility to hospitals and healthcare is the other motivation for building a road in the area. As shown in Figure 1, the region is divided into three forest areas. Twelve locations spread around these areas were selected as potential alternatives for forest road building. Considering the

[1] https://whc.unesco.org/en/list/1584/.

[2] https://www.iucn.org/news/world-heritage/201812/iucn-reviews-nine-new-world-heritage-nominations-2019.

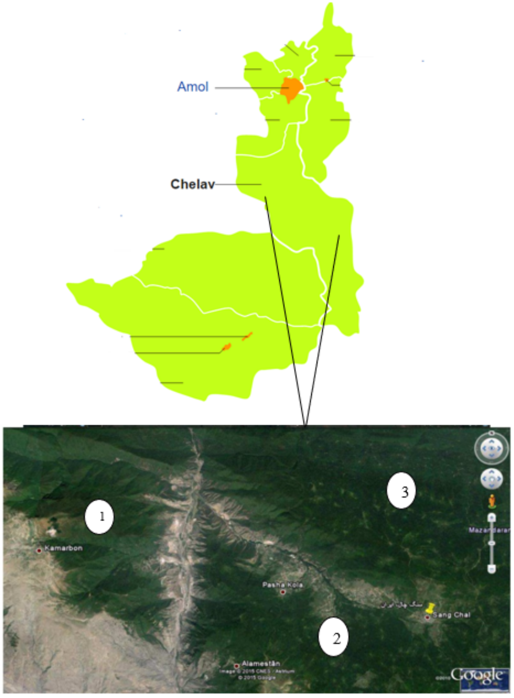


Figure 1: Chelav geographical location toward Amol and the main road of the area (Source: Google Map Satellite Pictures)

geographical features of the Chelav district, and due to the spread of the inhabitants in the forest area of the region, selecting a suitable location for forest road building is a complicated process. The selection should consider aspects on construction costs, environmental damage and inhabitants access to the main road.

Several criteria are identified to evaluate the forest roads alternatives, and they are defined based on the forest road studies in the literature. The identified criteria are categorized into four main groups: Cost, Ecological, Risk, and Opportunity (CERO). The cost criteria are considered to increase the benefits of forestry activities and minimize the construction cost. Six cost criteria are considered for forest road building, and they are defined based on Jaafari et al. (2015b). Clearing cost ($\mathcal{C}_1$) is defined based on the number of trees per hectare along each road. Embankment cost ($\mathcal{C}_2$) denotes excavation and cut-and-fill operation costs, which are needed for each road. Pavement cost ($\mathcal{C}_3$) denotes the cost of surfacing material based on the length of each road alternative. Grading cost ($\mathcal{C}_4$) denotes the cost of each road by considering the road's slope and surface. Drainage cost ($\mathcal{C}_5$) denotes the cost of excavation of ditches that are needed to control water along each road. The number of ditches depends on the length of each road alternative. Significant financial resources are required for maintenance activities

(Picchio et al. (2019); Mahanpoor et al. (2019); Akay et al. (2020)). Hence, maintenance cost ($\mathcal{C}_6$) denotes routine and periodic costs for rehabilitation of each road and makes it available throughout the entire year.

Due to the heavy effect of road building on the environment, the ecological criteria are considered to avoid spoiling the forest structure and to decrease the impact of construction. The altitude ($\mathcal{C}_7$) criterion denotes the height above sea level. Altitude is considered to reduce the landslide probability for the new forest road (Jaafari et al. (2015a); Pourghasemi et al. (2013)). Distance from faults ($\mathcal{C}_8$) is considered to avoid landslide occurrence. A road alternative with less fault density around and more distance from the fault line is preferred (Jaafari et al., 2015a). Distance from stream ($\mathcal{C}_9$) measures how far the road is from a stream. Since the probability of occurring landslide close to stream is high (Jaafari et al., 2015a), a road alternative with more distance from the stream is preferred. Ground condition ($\mathcal{C}_{10}$) is considered to ease the forest road building process. A road alternative with more sandy loam soil texture and fresh soil moisture is preferable (Jusoff, 2008). The slope ($\mathcal{C}_{11}$) criterion is considered to avoid mass movement along the forest road. The road with high slope is not suitable for road building (Mohammadi Samani et al. (2010); Jusoff (2008)).

Since the process of forest road building always entails with some risks, the environmental pollution criterion ($\mathcal{C}_{12}$) is considered to decrease the air, soil, and water pollution (Demir, 2007). A road alternative with less environmental pollution is preferred. Distance from wildlife habitats ($\mathcal{C}_{13}$) is considered to avoid partition or destruction of wildlife habitats (Gumus et al. (2008); Gumus (2015)). Progress to the mountain ($\mathcal{C}_{14}$) denotes the difficulties of access to it. Soil degradation criterion ($\mathcal{C}_{15}$) denotes the soil erosion and decrease of soil stability. The amount of soil erosion in the process of forest road building depends on the soil moisture and texture (Pourghasemi et al. (2013); Gumus et al. (2008)).

The forest road building, despite posing some risks, provides various opportunities. Accessibility for inhabitants ($\mathcal{C}_{16}$) denotes the ease of access for the local population. A road alternative that creates access for more number of population is preferred. Economic feature ($\mathcal{C}_{17}$) denotes the possibility of exploiting forestry productions (Gumus et al., 2008). Safety ($\mathcal{C}_{18}$) denotes the security usage of the road. Transportation ($\mathcal{C}_{19}$) is the most important function of forest roads, and here it denotes the safety of the vehicles, ease of transportation for forest products, and inhabitants' local products (Gumus, 2015). The description of the criteria are summarized in Table 1.

Forest road location selection is a critical and complicated decision for forest managers and DMs. The significance of the problem is indicated by multiple criteria, which are in direct or

Table 1: Description of the criteria

ID	Name of the criteria	Group's name	ID	Name of the criteria	Group's name
C1	Clearing cost	Cost	C11	Slope	Ecological
C2	Embankment cost	Cost	C12	Environmental pollution	Risk
C3	Pavement cost	Cost	C13	Distance from Wildlife	Risk
C4	Grading cost	Cost	C14	Progress to the mountain	Risk
C5	Drainage cost	Cost	C15	Soil digradation	Risk
C6	Maintenance cost	Cost	C16	Accessibility for inhabitants	Opportunities
C7	Altitude	Ecological	C17	Economic feature	Opportunities
C8	Distance from faults	Ecological	C18	Satety	Opportunities
C9	Distance from stream	Ecological	C19	Transportation	Opportunities
C10	Ground condition	Ecological			

indirect relationships with economic, environmental, and social welfare factors. Considering these quantitative and qualitative criteria, the conflict between them is not straightforward. To illustrate the difficulties of the problem, and the potential conflict between the criteria, consider cost and environmental impact. Forest road building and maintenance are costly activities in forestry, and forest managers try to reduce them. However, to have a forest road network with a good design and proper maintenance, and also low environmental impact, a significant financial investment is required (Picchio et al., 2018a). By adding risk and opportunity criteria to the problem, the difficult task of road evaluation for DMs becomes apparent. This is the motivation to propose a solution for the problem and support the forest road decision-making process.

By considering the CERO criteria, the forest road location selection is a problem with multiple criteria needing to prioritize the alternatives, and MCDM methods can be used for the problem. These decision-making tools are utilized frequently for complicated decision problems (Hashemkhani Zolfani et al., 2020) and facilitate the evaluation of alternatives when there are conflicts and interactions between the criteria (Montibeller et al., 2006). UTA (Jacquet-Lagreze and Siskos, 1982) is one of the MCDM methods, which is based on an additive value model. The method requires a given ranking for a set of alternatives and infers additive value functions to the alternative set. The method uses linear programming to access functions and to find weights for criteria so that the ranking obtained through the functions on alternative sets are as consistent as possible with the given ranking (Siskos et al. (2014); Siskos et al. (2016)). The UTA method uses the alternatives' evaluations with respect to the CERO criteria within the initial decision matrix. Evaluating alternatives based on the CERO criteria is a costly and time-consuming process and it requires estimating costs, ground conditions, ground slope, etc. When it is not possible for DMs to access all of these data, a reliable tool is needed to enable them to evaluate the alternatives. This paper proposes the use of the eigenvalue method (Saaty, 1988) when it is impossible to access all necessary data. The eigenvalue method is developed to obtain the priority weight vector of several criteria and alternatives in the decision-making process by synthesizing the pairwise comparison matrix (Saaty (1990); Sekitani and Yamaki (1999)). This paper presents a decision support framework using the eigenvalue-UTA approach and addresses the evaluation of forest roads in the Chelav forestry region by considering the CERO criteria.

4 Methodology

This section discusses the framework and the formulation of the eigenvalue-UTA approach. The eigenvalue method developed by Saaty (1988) is applied to determine the evaluation of the alternatives based on the CERO criteria. Then the UTA approach, which finds its roots in Jacquet-Lagreze and Siskos (1982) and is further developed in Siskos et al. (2016), is used to determine the importance of the criteria and prioritize the road alternatives. For the ease of presentation, this section contains a verbal explanation of the methods, while the mathematical details are presented in the appendices. First, Appendix A outlines the notation used for sets, parameters, and variables needed to build the mathematical formulation of the proposed approach.

Secondly, the mathematical formulation of the additive value system of the UTA method is provided in Appendix B, whose explanation in brief is as follows. First, the additive value function (B.1) obtains the global value of the alternatives, while (B.2) and (B.3) are the normalization constraints.
Note that the marginal and the global value functions have the monotonicity property. Eq. (B.4) illustrates the monotonicity property in the case of the global value function for two given roads a and b.

It is noteworthy that in the UTA method, the estimation of the marginal value functions is done according to a piecewise linear form (see Eqs. (B.5) and (B.6) for details).

The eigenvalue-UTA approach

The combination of the eigvenvalue method and the UTA method derives in an integrated decision support framework, whose two phases are illustrated in Figure 2. The first phase consists of three steps, and the second phase includes five steps, as outlined below.

Phase 1

Step 1:

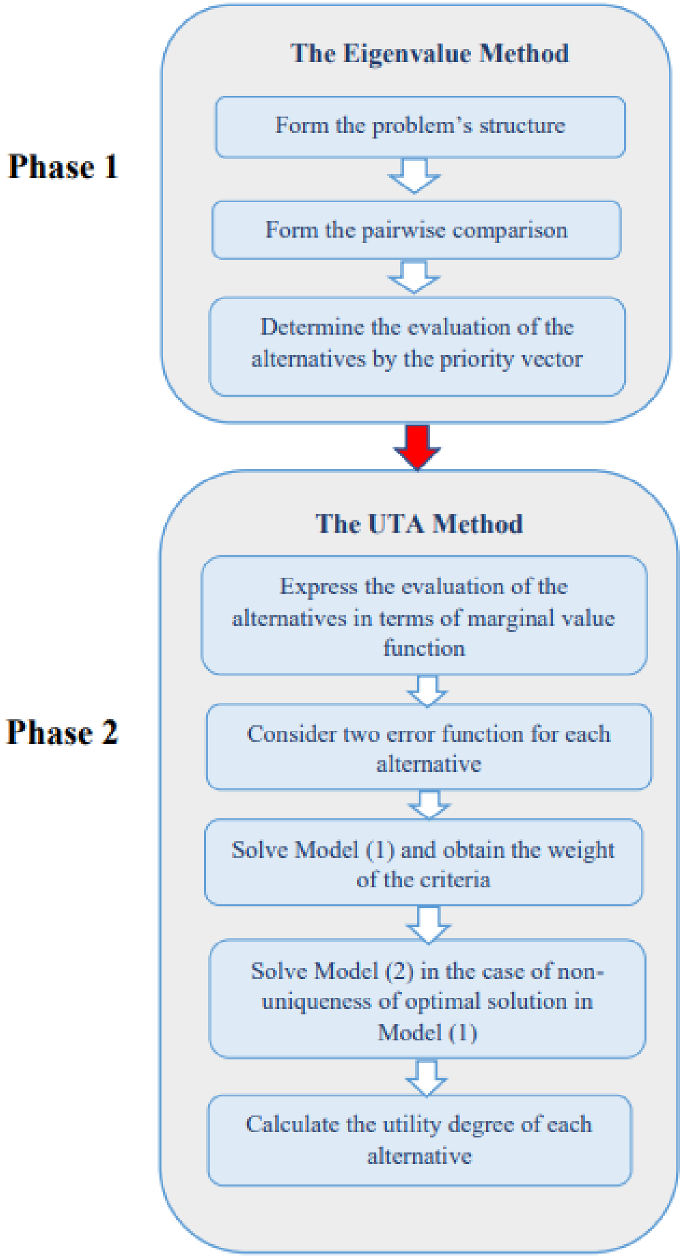


Figure 2: The framework of the multi-methodology approach

The criteria and alternatives are identified in this step, then the problem's structure is formed.

Step 2:

Pairwise comparison matrices are performed based on the problem's structure. In pairwise comparisons, DMs compare alternatives concerning each criterion on a scale of 1-9. On this scale, 1 represents two alternatives that contribute equally to the objective criterion, and 9 illustrates the DMs favoring one alternative over the other one with the highest possible validity.

Step 3:

The evaluations of the alternatives are determined by the eigenvalue solution method, which is detailed in Appendix C. The comparison matrices and the evaluation of the alternatives concerning each criterion are obtained using Eqs. (C.1)-(C.6). Also, the consistency of the DMs' judgement is measured by Eqs.(C.7)-(C.8).

Phase 2

Using the output of Phase 1, five steps are defined as follows based on the UTA method.

Step 4:

By using (C.9), the global values of the alternatives are expressed in terms of the marginal value function.

Step 5:

Underestimation and overestimation error functions are defined to minimize the difference between the global value of an alternative and the given ranking to that alternative by the DMs. The error functions are considered in Eq. (C.10) for each pair of consecutive alternatives in the ranking given by the DMs.

Step 6:

In this step, $Model(1)$ in Appendix C is solved to minimize the total deviations and to obtain the weights of the criteria under conditions (C.12)-(C.15). The objective function (C.11) adds up the error functions of the alternatives. Constraints (C.12)-(C.13) are based on the given rank by the DMs. Constraint (C.12) is formulated for a pair of consecutive alternatives, which are given unequal rank, and constraint (C.13) is for a pair of alternatives with equal rank. Constraint (C.14) ensures that the total weights of the criteria are equal to one. Constraint (C.15) states the nature of the decision variables.

Step 7:

$Model(2)$ aims at finding the mean additive value function of the optimal solutions, when the obtained solution of $Model(1)$ is non-unique.

Step 8:

The Eq. (C.9) is used to obtain the alternatives' global value and to determine their ranking. The weight of each criterion is obtained using Eq. (C.18).

To sum up, the proposed multi-methodology approach uses UTA methods in Phase 2 to provide the weight of the criteria and rank the alternatives. In particular, the eigenvalue-UTA approach assigns a marginal value function to the alternatives and uses optimization techniques to obtain the preferences objectively. This method takes the DMs' initial preferences, and minimizes the difference between the alternatives' global value function and the given preferences by the DMs. Therefore, the method provides the preferences in such a way that the ranking of alternatives is as consistent as possible with the DMs' preferences.

5 Case study

The deployment of the methodology explained in Section 4 is illustrated in a case study to evaluate road selection in a rural area of Iran. This section discusses the numerical results obtained for this case, followed by a sensitivity analysis.

5.1 Numerical results

The input of the problem is the DMs' preferences provided through questionnaire surveys, which I collected jointly with collaborators as part of a work detailed in Hashemkhani Zolfani et al. (2011). Twenty DMs who have the required knowledge and are aware of forest road problems participated in the study. Table 2 provides the DMs' information. Before distributing the final questionnaire between DMs, five experts with an average of 10 years experience in rural and forest transportation assessed the workability of the survey. They approved the CERO criteria and their classification through an interview.

In the first question of the questionnaire survey, the DMs were asked to determine the direction of the preferences of the criteria. The outcomes of this question are shown in Table 3.

To achieve the study's objective and to obtain the evaluation of the road alternatives, in the second question, the DMs were asked to indicate the level of importance of the alternatives by making a pairwise comparison with respect to each criterion. Each DM filled nineteen

Table 2: DMs' profile

Field	Frequency	Year of experience	Frequency
Forest management	4	Under 5 years	3
Geologist	4	5 to 10 years	4
Transportation engineer	3	11 to 15 years	4
Project management	2	15 to 20 years	5
Contractor	3	Above 20 years	4
Forest planner	4		

Table 3: Criteria's preferences direction

Criteria	$\mathcal{C}_1$	$\mathcal{C}_2$	$\mathcal{C}_3$	$\mathcal{C}_4$	$\mathcal{C}_5$	$\mathcal{C}_6$	$\mathcal{C}_7$	$\mathcal{C}_8$	$\mathcal{C}_9$	$\mathcal{C}_{10}$
Preferences direction	min	min	min	min	min	min	min	min	max	max
Criteria	$\mathcal{C}_{11}$	$\mathcal{C}_{12}$	$\mathcal{C}_{13}$	$\mathcal{C}_{14}$	$\mathcal{C}_{15}$	$\mathcal{C}_{16}$	$\mathcal{C}_{17}$	$\mathcal{C}_{18}$	$\mathcal{C}_{19}$	-
Preferences direction	max	min	max	min	min	max	max	max	max	-

pairwise comparison matrices using the 1-9 scale. In the last question, the DMs were asked to use the same scale and give an initial rank to the alternatives. After completing the data collection, the arithmetic mean was used to combine and aggregate the DMs' responses. Then, the comparison matrices ($S^i_{m\times m}$) and the alternatives' initial ranking were obtained. Also, equations (C.7)-(C.8) were used to check the consistency rate of the matrices. As an example, Table 4 illustrates the aggregated comparison matrix among road alternatives with respect to environmental pollution (S^{12}). The consistency rate of this matrix is 0.073, which is an acceptable rate. The comparison matrices were normalized using Eqs. (C.2)-(C.5). The normalized matrix with respect to environmental pollution (C^{12}) is shown in Table 5. The alternatives' initial ranking is shown in Table 6. Note the combined DMs' responses are rounded to the largest integer number. The rest of Phase 1 was implemented through the use of the software SuperDecision version 3.2[3].

Table 4: The alternatives comparison matrix with respect to environmental pollution (S^{12})

$\mathcal{C}_{12}$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
a_1	1	1/5	1/4	1/4	4	3	1/3	1/3	6	5	1/4	1/4
a_2	5	1	4	3	1/3	1/4	3	3	5	1/4	1/4	3
a_3	4	1/4	1	3	1/3	1/3	1/4	3	2	2	1/4	1/4
a_4	4	1/3	1/3	1	2	3	1/3	1/3	1/5	4	1/4	1/4
a_5	1/4	3	3	1/2	1	1/3	4	1/3	1/3	4	3	1/3
a_6	1/3	4	3	1/3	3	1	1/3	3	2	4	1/3	1/4
a_7	3	1/3	4	3	1/4	3	1	3	5	1/4	1/3	4
a_8	3	1/3	1/3	3	3	1/3	1/3	1	3	1/3	4	2
a_9	1/6	1/5	1/2	5	3	1/2	1/5	1/3	1	1/3	2	1/2
a_{10}	1/5	4	1/2	1/4	1/4	1/4	4	3	3	1	2	1/2
a_{11}	4	4	4	4	1/3	3	3	1/4	1/2	1/2	1	2
a_{12}	4	1/3	4	4	3	4	1/4	1/2	2	2	1/2	1

[3]https://www.superdecisions.com/.

Table 5: The normalized comparison matrix with respect to environmental pollution (C^{12})

$\mathcal{C}_{12}$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
a_1	0.03	0.01	0.01	0.01	0.21	0.16	0.02	0.02	0.20	0.21	0.02	0.02
a_2	0.17	0.06	0.16	0.11	0.02	0.01	0.18	0.17	0.17	0.01	0.02	0.21
a_3	0.14	0.01	0.04	0.11	0.02	0.02	0.01	0.17	0.07	0.08	0.02	0.02
a_4	0.14	0.02	0.01	0.04	0.05	0.16	0.02	0.02	0.01	0.17	0.02	0.02
a_5	0.01	0.17	0.12	0.02	0.05	0.02	0.23	0.02	0.01	0.17	0.21	0.02
a_6	0.01	0.22	0.12	0.01	0.15	0.05	0.02	0.17	0.07	0.17	0.02	0.02
a_7	0.10	0.02	0.16	0.11	0.01	0.16	0.06	0.17	0.17	0.01	0.02	0.28
a_8	0.10	0.02	0.01	0.11	0.15	0.02	0.02	0.06	0.10	0.01	0.28	0.14
a_9	0.01	0.01	0.02	0.18	0.15	0.03	0.01	0.02	0.03	0.01	0.14	0.03
a_{10}	0.01	0.22	0.02	0.01	0.01	0.01	0.23	0.17	0.10	0.04	0.14	0.03
a_{11}	0.14	0.22	0.16	0.15	0.02	0.16	0.18	0.01	0.02	0.02	0.07	0.14
a_{12}	0.14	0.02	0.16	0.15	0.15	0.21	0.01	0.03	0.07	0.08	0.04	0.07

Table 6: Alternatives' initial ranking

Alternatives	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
Initial rank	11	3	1	2	6	5	12	4	8	7	10	9

Then, Eq. (C.6) performs the evaluation of each alternative, and Eqs. (C.7)-(C.8) perform the evaluation of consistency measures. Table 7 illustrates the alternatives' evaluation with respect to environmental pollution (g_{12}). Note that the obtained evaluations were multiplied by 100.

Table 7: Alternatives' evaluation based on environmental pollution (g_{12})

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
g_{12}	14	7	8	8	10	6	5	9	10	6	9	8

The evaluations of the alternatives obtained by the eigenvalue method were used to complete the performance table, which is shown in Table 8.

In the second phase, the UTA method obtained the weight of the criteria and ranked the road alternatives. The number of segments for each criterion (α_i) was determined, and Eqs. (B.5)-(B.6) were used to estimate the marginal value function of the alternatives based on each criterion. Equations (1)-(2) illustrate the use of a piecewise linear function for a_2 based on the environmental pollution. Note $\mathcal{C}_{12}$ is a criterion with minimum preference direction. Here, α_{12} is equal to 3, and the worst and best evaluation level of the criterion is $[14, 5]$ (note that, following the convention in the relevant stream of literature, the upper bound of the interval appears first and the lower bound of the interval appears afterwards).

$$g_{12}^{j} = 14 + \frac{j-1}{3-1}(5-14), \qquad \forall j = 1, 2, 3 \tag{1}$$

By replacing j in Eq. (1) the interval $[14, 5]$ with three break-points was determined. Since the evaluation of a_2 based on $\mathcal{C}_{12}$ is between 9.5 and 5, the marginal value function of this evaluation is determined in Eq. (2).

Table 8: Performance table for the road location selection problem

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}	g_{13}	g_{14}	g_{15}	g_{16}	g_{17}	g_{18}	g_{19}
a_1	7	11	9	13	12	10	10	10	11	11	14	14	10	10	7	7	6	8	8
a_2	10	10	10	11	8	9	10	7	8	10	17	7	11	10	9	11	7	10	9
a_3	9	8	10	9	13	11	8	7	7	9	9	8	6	8	13	13	10	6	10
a_4	9	10	10	8	7	10	9	11	7	7	9	8	9	9	7	10	7	6	11
a_5	9	6	7	10	11	11	11	9	10	8	6	10	7	9	9	7	6	9	8
a_6	8	10	5	5	10	8	7	11	9	10	8	6	9	7	9	8	9	9	6
a_7	8	6	9	8	9	7	9	10	6	8	8	5	10	10	10	6	11	10	6
a_8	12	12	6	10	5	8	9	7	8	6	5	9	13	6	5	6	10	8	7
a_9	7	5	6	6	5	5	8	9	11	9	7	10	7	11	11	8	9	5	6
a_{10}	11	6	9	6	9	7	7	8	5	3	7	6	7	6	9	5	7	8	11
a_{11}	6	5	9	9	6	10	6	3	10	12	3	9	5	8	8	7	11	11	9
a_{12}	4	7	9	5	5	5	6	7	7	5	7	8	10	8	7	11	7	9	9

$$u(g_{12}^2) = u(5) + \frac{7-5}{9.5-5}(u(9.5) - u(5)) \tag{2}$$

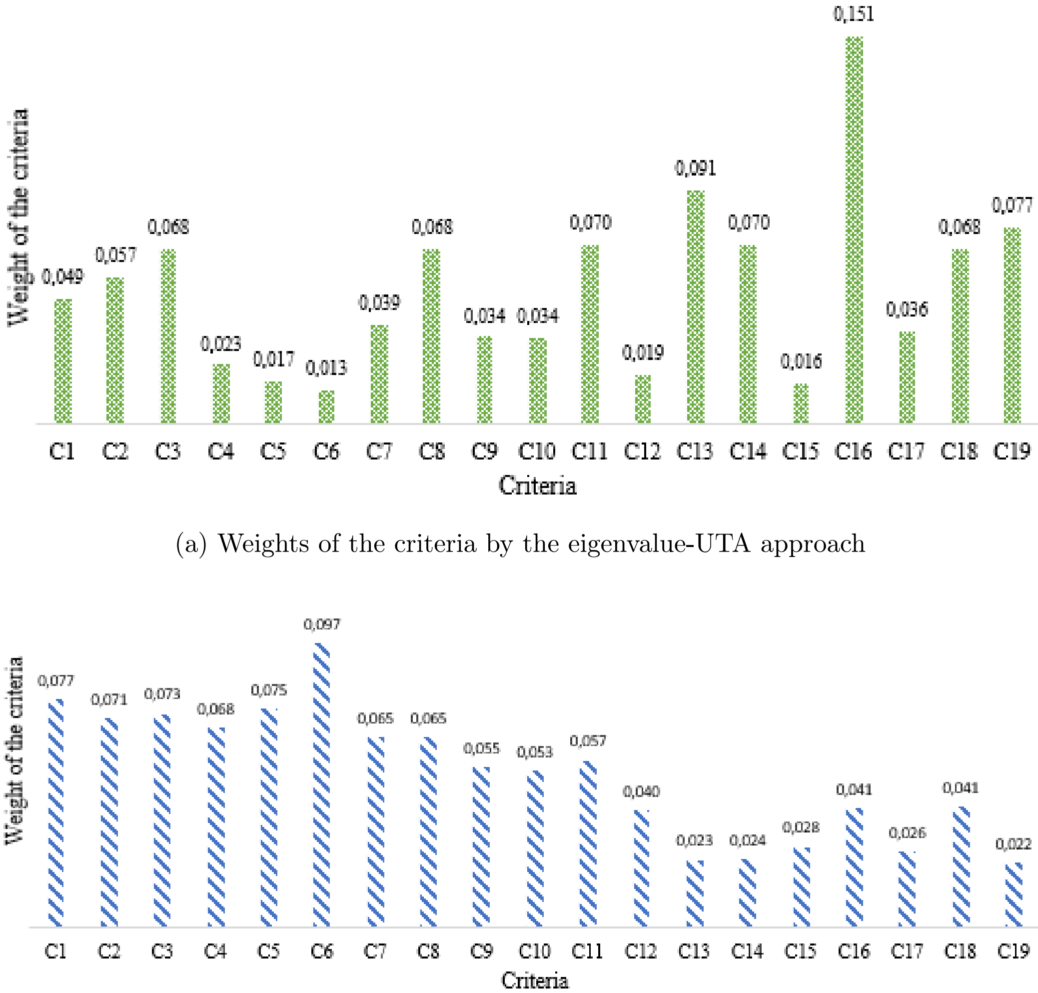
After estimating the marginal value function for all of the evaluations, Eq. (C.9) was used to express the global value of the alternatives. Eq. (3) illustrates the global value for alternative a_2. Note that a_2 is ranked as a seventh alternative ($k = 7$) in the initial ranking by DMs.

$$u(g(a_7)) = u(10) + \cdots + 0.75u(5) + 0.25u(13) + \cdots + u(9) \tag{3}$$

Computing all of these steps for all of the criteria and the alternatives is a time-consuming task. Hence, Phase 2 was implemented through the use of Diviz software version 19.1. [4] The software used performance table, preferences direction, number of segments, and alternatives' initial ranking as input data. Then, the UTA method obtained the weight of the criteria and ranked the alternatives. Figure 3a shows the relative importance of the CERO criteria.

The results show that accessibility for inhabitants ($\mathcal{C}_{16}$) from the opportunity group plays a vital role in forest road location selection. This criterion directly affects local populations' welfare, and it was considered as the most important criterion in the results. Easing inhabitants' accessibility to the closest urban area and facilitating their access to health, education, and local market is the main objective of this study. Distance from wildlife habitats ($\mathcal{C}_{13}$) is the second most important criterion to protect wildlife and avoid destroying it. According to the results, transportation ($\mathcal{C}_{19}$) from the opportunity group is the third most important criterion. The safety of the vehicles and ease of transportation are critical functions of any forest road. Progress to the mountains ($\mathcal{C}_{14}$) from the risk group is the fourth most important criterion. This criterion has an essential role in the time and cost of the road building. The slope of the ground ($\mathcal{C}_{11}$) from the ecological group is the fifth most important criterion. To avoid mass movement along the road and decrease maintenance costs, a road with less slope is preferred.

[4] http://www.diviz.org/.



(a) Weights of the criteria by the eigenvalue-UTA approach

(b) Weights of the criteria by the parsimonious AHP approach

Figure 3: Weights of the criteria

Table 9a summarizes the results on the obtained weights. Each column summarizes the results for each group. The criteria in each group appear sorted from higher to lower priority, and the final row shows the total weight of each group.

(a) The order of the criteria in CERO group by eigenvalue-UTA

	Cost	Ecological	Risk	Opportunity
Criteria	C_3	C_{11}	C_{13}	C_{16}
	C_2	C_8	C_{14}	C_{19}
	C_1	C_7	C_{12}	C_{18}
	C_4	C_9	C_{15}	C_{17}
	C_5	C_{10}		
	C_6			
Total weight	0.2270	0.2446	0.1962	0.3315

(b) The order of the criteria in CERO group by parsimonious AHP

	Cost	Ecological	Risk	Opportunity
Criteria	C6	C8	C12	C18
	C1	C7	C15	C16
	C5	C11	C14	C17
	C3	C9	C13	C19
	C2	C10		
	C4			
Total weight	0.460	0.294	0.114	0.129

Table 9: The order of the criteria in their group

By considering the total weight of the group, the opportunity is ranked as the most important one, and then ecological, cost, and risk are in the following priorities, respectively. This indicates that creating opportunities for inhabitants, providing their social needs, wildlife conservation, ease of transportation, and increasing safety in the study area are the main aims of the road building. Meanwhile, the managers are looking to reduce the environmental damage and reduce the risk of road building. Also, they prefer to allocate a more significant share of the budget to create more opportunities.

The obtained ranking for the alternatives is as consistent as possible with the given initial ranking in Table 6. The method proposed a_3 as the best location for forest road building. Based on the alternatives' evaluation in Table 8, this road is the best alternative, when it comes to increasing the accessibility for inhabitants. The road has an acceptable rate in terms of progress to the mountain and transportation. In addition, it has a moderate ground slope. Road a_4 is ranked as the second alternative. This road has a good position to facilitate accessibility for inhabitants, and it also has a reasonable distance from wildlife habitats. Although it is not easy to progress to the mountain from the location of a_4, it can ease transportation and safety of the vehicles. Road a_2 is the third alternative. It has a good location to create accessibility for inhabitants and transportation. It has a high ground slope, and it is not easy to progress to the mountain from its location. However, it is a far distance from wildlife habitats. Since the cost group is the third important group among the CERO criteria, the first three alternatives have a high total cost. The ranking of the other alternatives is according to the Table 6.
The solution obtained with the eigenvalue-UTA method considered all the criteria, all the environmental consequences and complexity of the problem, which would otherwise be challenging to handle manually. Using the proposed approach structures the decision-making process and

also improves the DMs' ability to address the problem. Also, they can rely on the decision process that provides them with solutions that take into account all the criteria and preferences.

5.2 Benchmarking

In this study, there are 12 alternatives. To compare the alternatives based on each criterion, 66 pairwise comparisons are asked. By considering 19 criteria, the number of pairwise comparisons increases to 1254 questions in total. Although the eigenvalue-UTA approach requires fewer preferences information than traditional AHP methods, recent AHP variants require fewer preferences information. For example, the Parsimonious AHP method (Abastante et al. (2018); Abastante et al. (2019)) is one of the developed methods based on the AHP that reduces the number of preferences and pairwise comparisons needed. The parsimonious AHP is structured in five steps (Abastante et al., 2019). The first step of the method is the DMs' direct ranking of the alternatives based on each criterion. In the second step, the DMs select some reference evaluations and use them in the third step to compare them with respect to each criterion. In the third step, the original version of the AHP method is used to obtain the relative importance of the criteria and reference evaluations. In the fourth step, the priorities of the reference evaluations are checked to be consistent with the given ranking by the DMs in the first step. In case of inconsistency, the DMs are asked to modify the ranking and the pairwise comparisons. Then, in the fifth step, the priority of the alternatives that are not referenced evaluations are obtained by piecewise linear interpolation. More details of the method are explained in Abastante et al. (2018) and Abastante et al. (2019). By considering four reference levels for the problem of this paper, the parsimonious AHP method reduces the number of pairwise comparisons from 1254 to 285. The significant difference between pairwise comparisons motivates to test the forest road building problem by the parsimonious AHP method and compare its results with the results of the eigenvalue-UTA approach.
The alternatives are evaluated on a $[3, 17]$ scale as shown in Table 8. These evaluations are considered as the DMs' direct ranking to the road alternatives. Then, four reference levels $\gamma_1 = 3, \gamma_2 = 8, \gamma_3 = 11, \gamma_4 = 17$ are determined, and 20 pairwise comparison matrices are provided. The first comparison matrix is the DMs' preferences on the 19 criteria. Table 10 shows the priorities of the criteria.

Table 11 illustrates the comparison of the reference levels with respect to environmental pollution C_{12}. By considering the references' priorities, their consistency is checked with the reference evaluations (note that the environmental pollution has minimum preference direction; see Table 3). Next, the piecewise linear interpolation is used to obtain the priorities of the

Table 10: Weights of the criteria by the parsimonious AHP

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
Weights	0.0774	0.0712	0.0729	0.068	0.0745	0.0969	0.0647	0.0649	0.0548	0.0532
Criteria	C11	C12	C13	C14	C15	C16	C17	C18	C19	
Weights	0.057	0.0402	0.0232	0.0235	0.0276	0.0405	0.0261	0.0412	0.022	

Table 11: The comparison of the reference levels respect to environmental pollution

C12	3	8	11	17	Priorities
3	1	7	8	9	0.610
8	1/7	1	7	8	0.247
11	1/8	1/7	1	6	0.104
17	1/9	1/8	1/6	1	0.036

road alternatives, which are not in the list of reference evaluations. Equation (4) illustrates the priority of road a_5 on C_{12}. Based on Table 8, the evaluation of a_5 on C_{12} is 10, which belongs to the interval $[11, 8]$. The priorities assigned to these references are 0.247 and 0.104, respectively. Hence, the priority of a_5 on C_{12} is obtained as follows:

$$u(10) = u(11) + \frac{u(8) - u(11)}{8 - 11}(10 - 11) = 0.151 \tag{4}$$

The final evaluation of the alternatives is obtained by multiplying their priorities by the rating of the relevant criterion in Table 10. For comparison purposes, the alternatives' final evaluation is shown in Table 12 for both the parsimonious AHP and the eigenvalue-UTA approaches.

Table 12: Alternatives' final evaluation by the parsimonious AHP and the eigenvalue-UTA approaches

Alternatives	Parsimonious AHP	Eigenvalue-UTA
a1	1	11
a2	2	3
a3	3	1
a4	4	2
a5	8	5
a6	5	5
a7	10	12
a8	9	4
a9	11	8
a10	12	7
a11	6	10
a12	7	9

The results show that the parsimonious AHP approach prioritizes roads a_1, a_2 and a_3 as the three top alternatives. Note that roads a_2 and a_3 are also ranked as top alternatives by the eigenvalue-UTA approach. In addition, road a_7, which is the worst alternative by the eigenvalue-UTA approach, is also prioritized among the three worst alternatives by the parsimonious AHP. Figure 3b shows the relative importance of the CERO criteria by the parsimonious AHP. The results show that all six criteria from the cost group are prioritized as the six top criteria. Also, the parsimonious AHP prioritizes all five criteria from the ecological group in the next five top criteria. Table 9b summarizes the priority of the criteria in their group and the total weight of each group by the parsimonious AHP. The total weight of the groups renders that the DMs rank cost as the most important group, followed by the ecological, opportunity, and risk groups, respectively.

Although there are some similarities between the ranking of the roads, some features of the methods differ. On the one hand, the parsimonious AHP compares more relevant objects, as shown in the case of this paper. The method requires less information on preferences and comparisons. Hence, applying this method implies time reduction, and it reduces the DMs' cognitive effort. On the other hand, the eigenvalue-UTA method ranks alternatives more consistently with the given initial ranking. The parsimonious AHP tries to keep the consistency of the priorities with respect to the reference evaluations, but in the case of inconsistency, it might be time-consuming to review the information that the DMs provide. Also, the eigenvalue-UTA method obtains the weight of the criteria as to keep the consistency in the alternatives' ranking.

5.3 Sensitivity analysis

Following Lahdelma and Salminen (2001) and Greco et al. (2010), the reliability of the proposed approach is checked by the sensitivity of the results to the alternatives' evaluation and varying the main points of the piecewise marginal value functions. This sensitivity analysis is limited to the alternatives' evaluation provided by the eigenvalue method only. Three experiments are defined to check the sensitivity of the results based on the evaluation of the alternatives. The evaluation of three top-ranked alternatives (a_3, a_4 and a_2) are changed concerning the two most important criteria (C_{16} and C_{13}). Then, the weights of the criteria and alternatives' ranking are assessed. The evaluation interval for these criteria is $[5, 13]$. In the first experiment, the evaluations of the alternatives are changed to the worst evaluation in the interval. Next, in the second experiment, the evaluations are changed to the middle evaluation of the interval. Then, the best evaluation in the interval is considered for the alternatives in the third experiment. Table 13 shows the sensitivity analysis and the results.

It can be seen that in all experiments, the weights of the criteria and their priorities change.

Table 13: Sensitivity analysis by considering alternatives' evaluation

Expt. No.		g13	g16	Alternatives' rank	Priority of the criteria
1	$a2$	5	5	$a3 > a4 > a2$	$C8 > C11 > C19 > C2 > C7 > C1 >$
	$a3$	5	5		$C14 > C18 > C10 > C16 > C4 > C17 >$
	$a4$	5	5		$C9 > C13 > C3 > C12 > C5 > C6 > C15$
2	$a2$	9	9	$a3 > a4 > a2$	$C16 > C13 > C14 > C19 > C8 > C2 >$
	$a3$	9	9		$C1 > C11 > C18 > C3 > C17 > C7 >$
	$a4$	9	9		$C10 > C9 > C4 > C12 > C5 > C15 > C6$
3	$a2$	13	13	$a3 > a4 > a2$	$C13 > C16 > C3 > C14 > C8 > C19 >$
	$a3$	13	13		$C18 > C9 > C17 > C2 > C1 > C7 >$
	$a4$	13	13		$C11 > C10 > C4 > C15 > C12 > C6 > C5$

In two out of three experiments, C_{16}, C_{13}, C_{19}, and C_{14} are prioritized as important criteria. The final ranking of the alternatives is equally consistent with the given initial ranking, although the evaluation of the alternatives change. Hence, obtaining the evaluation of the alternatives plays a significant role in weighting the criteria.

Also, four experiments are defined to investigate the variation of the results by modifying the main points of the piecewise marginal value functions. Since the main points of the piecewise marginal value function depend on the number of segments (α_i), the experiments are designed to vary this number in the selected criteria. The most important criteria in each group, which are C_3, C_{11}, C_{13}, and C_{16} are selected, and the number of segments is modified in different experiments. Table 14 summarizes the results of these experiments.

Table 14: Sensitivity analysis by considering the main poits of the piecewise marginal value function

Expt. No.	Criteria	Group	Initial segments	Expt. segments	Criteria priority in each group
1	C13	cost	3	8	$C3>C1>C2>C4>C5>C6$
	C11	ecological	3	8	$C11>C8>C7>C9>C10$
	C13	risk	9	14	$C15>C13>C14>C12$
	C16	opportunity	9	14	$C16>C19>C18>C17$
2	C13	cost	3	2	$C3>C2>C1>C4>C5>C6$
	C11	ecological	3	2	$C11>C8>C7>C9>C10$
	C13	risk	9	4	$C13>C14>C12>C15$
	C16	opportunity	9	4	$C16>C19>C18>C17$
3	C13	cost	3	10	$C4>C3>C2>C1>C6>C5$
	C11	ecological	3	10	$C8>C7>C9>C11>C10$
	C13	risk	9	3	$C15>C13>C14>C12$
	C16	opportunity	9	3	$C16>C19>C18>C17$
4	C13	cost	3	2	$C4>C1>C2>C3>C6>C5$
	C11	ecological	3	2	$C11>C8>C7>C9>C10$
	C13	risk	9	14	$C14>C15>C13>C12$
	C16	opportunity	9	14	$C16>C19>C18>C17$

For each experiment, the first and the second columns present the selected criteria and the group's name to which the criterion belongs, respectively. The third column shows the number of segments for each criterion that the DMs determined. The next column shows the number of segments for each criterion in the experiments. In addition, the last column summarizes the priority of the criteria in their group. In the first experiment, the number of segments increased for all selected criteria. In the second experiment, the number of segments decreased for all the selected criteria. In the third and fourth experiments, the number of segments increased for two criteria, and it decreased for two others. Varying the number of segments changed the weight of the criteria, which in some cases resulted in changes in the prioritization of the criteria. For example, comparing to the results displayed in Table 9a, in experiment 1, where the number of segments increased for all selected criteria, the weights of the criteria changed, and it led to varying the prioritization of the criteria in the group of cost and risk. Although the importance of the criteria changed in experiment 2, the priority of the criteria is the same as in Table 9a. Varying the number of segments also led to different priorities of the criteria in experiments 3 and 4. This illustrates that a different number of segments leads to different criteria weights, and it might change the priority of the criteria. However, the alternatives' ranking in all experiments is as consistent as possible with the given initial ranking, and it confirms the reliability of the ranking by the eigenvalue-UTA method.

6 Concluding remarks

This study proposed a combined use of the eigenvalue method and the UTA method to address MCDM problems and illustrated the resulting methodology in a case study of road building selection. The approach takes into account quantitative and qualitative criteria and evaluates the alternatives. In the case study, the purpose of building the road is mainly to increase the access for local inhabitants and provide easy access to forest products and their transportation. These objectives are costly and in conflict with sustainable environmental protection. Hence, selecting the right road is a difficult task. Most of the existing forest road evaluation models are limited to either cost or environmental criteria. In this paper, different qualitative and quantitative criteria on the Cost, Ecological, Risk, and Opportunity (CERO) have been considered in the alternatives' evaluations. Since access to quantitative data for this problem is costly and time-consuming, the eigenvalue method was used to assess the alternatives. Then, through the UTA method, optimization techniques were used to weight and rank the criteria and alternatives. The sensitivity analysis confirmed that the eigenvalue-UTA approach is an efficient and stable decision tool for the problem of selecting the location of a road. Also, the proposed model has structured and well-defined procedures with a straightforward computation process.

The multi-methodology approach and the CERO criteria defined in this paper can also be applied in other problems of similar nature, such as selecting the best area for harvesting wood or locating a factory around urban or rural areas. Moreover, using the eigenvalue method in UTA opens avenues to do future research in methodological aspects. This includes reformulating the proposed multi-methodology approach by using fuzzy set theory or interval data, and developing a new MCDM approach that can be used for similar cases.

Appendices

A Notations

The sets, parameters, and decision variables are denoted as follows.
$\mathcal{A}$ is the set of alternatives, and $\mathcal{I}$ is the set of criteria determined by DMs. These sets are used to define the parameters and decision variables needed to build the formulation of the eigenvalue-UTA method. For the sake of simplicity, in this paper, the indices a are used for an arbitrary alternative in $\mathcal{A}$ and i for an arbitrary criterion in $\mathcal{I}$.

The following parameters are defined:

- g_i: the evaluation of an alternative on criterion i.
- g_{i*}: the worse evaluation level of criterion i.
- g_i^*: the best evaluation level of the criterion i.
- g or $g(a)$: the evaluation vector of an alternative a on the n criteria as $(g_1, \ldots, g_n)$.
- α_i: number of segments, which is the number of points between the worse and the best evaluation level of criterion i.
- δ: small positive number.
- ϵ: arbitrary small number.

The multi-methodology approach has to decide the alternatives' evaluation regarding each criterion, weights of the criteria, and rank the alternatives. The decision variable w_{ij} is defined to decide the weight of criterion i in any break-point j of the interval $[g_{i*}, g_i^*]$. Also, two error functions σ^+ and σ^- are considered as decision variables. They correspond to underestimation and overestimation errors, respectively. Table A1 provides an overview of the sets, parameters, decision variables and the other notations that are used in this paper.

Table A1: Overview of sets, parameters and decision variables in the eigenvalue-UTA approach

	Notations	**Description**
Sets	$\mathcal{A}$	set of alternatives
	$\mathcal{I}$	set of criteria
Parameters	g_i	evaluation of an alternative on criterion i
	g_{i*}	worse evaluation level of criterion i
	g_i^*	best evaluation level of criterion i
	α_i	number of segments for criterion i
	k	given ranking by DMs to alternatives
	δ	small positive number
	ϵ	arbitrary small number
Decision variables	w_{ij}	weight of criterion i in break-point j
	$u_i(g_i)$	marginal value function or utility function of g_i
	p_i	relative weight of ith function $u_i(g_i)$
	W_i	weight of the criterion i
	σ^+	underestimation error
	σ^-	overestimation error
Vectors	g or $g(a)$	evaluation vector of an alternative on the criteria
	λ_{enb}	eigenvalue vector
	$\mathcal{G}$	evaluation vector of the alternative
Matrices	S^i	alternatives' comparison matrix respect to criterion i
	C^i	the normalized matrix of S^i
Indices	CI	consistency index
	RI	the average value of CI
	CR	consistency rate

B Additive value system

The additive value system of the UTA method as described in Siskos et al. (2016) is summarized as follows:

$$u(g) = \sum_{i=1}^{n} p_i . u_i(g_i) \tag{B.1}$$

s.t.

$$\sum_{i=1}^{n} p_i = 1 \tag{B.2}$$

$$u_i(g_{i*}) = 0, \; u_i(g_i^*) = 1 \qquad \forall i = 1, 2, \ldots, n \tag{B.3}$$

In the additive value system (B.1), $u_i(g_i)$ is a non-decreasing marginal value function or utility function of g_i, which is normalized between zero and one. Also, p_i is the relative weight of utility function $u_i(g_i)$. In addition, $u(g)$ or $u(g(a))$ indicates the global value of an alternative a.

Eq.(B.4) expresses the monotonicity property in the case of the global value function.

$$\begin{aligned} u(g(a)) > u(g(b)) \Leftrightarrow aPb \quad & \text{(preference)} \\ u(g(a)) = u(g(b)) \Leftrightarrow aIb \quad & \text{(indifference)} \end{aligned} \tag{B.4}$$

Eq. (B.5) below is the piecewise linear form for criterion i in the interval $[g_{i*}, g_i^*]$. Here, α_i is the number of segments for criterion i, and the interval is cut into $(\alpha_i - 1)$ equal intervals. The break-point g_i^j is given by the following:

$$g_i^j = g_{i*} + \frac{j-1}{\alpha_i - 1}(g_i^* - g_{i*}), \qquad \forall j = 1, 2, \ldots, \alpha_i \tag{B.5}$$

Note, for a criterion i in the interval $[g_{i*}, g_i^*]$, the breakpoints of the interval are obtained by changing the value of $j \in \{1, 2, \ldots, \alpha_i\}$. Hence, the marginal value of an alternative a in criterion i is defined based on Eq. (B.6) as follows:

$$u_i(g_i(a)) = u_i(g_i^j) + \frac{g_i(a) - g_i^j}{g_i^{j+1} - g_i^j}(u_i(g_i^{j+1}) - g_i^j), \forall i = 1, 2, \ldots, n; j = 1, 2, \ldots, \alpha_i : g_i(a) \in [g_i^j - g_i^{j+1}] \tag{B.6}$$

Eq. (B.7) is used to transform the variables expressing them in terms of w_{ij} as follows:

$$w_{ij} = u_i(g_i^{j+1}) - u_i(g_i^j) \geq 0, \; \forall i = 1, 2, \ldots, n; j = 1, 2, \ldots, \alpha_i - 1 \tag{B.7}$$

C The eigenvalue-UTA framework

Eigenvalue method

In the first phase of the framework, the eigenvalue method by Saaty (1990) is used as follows. First, in (C.1), $S^i_{m\times m}$ is the comparison matrix between alternatives with respect to criterion i, which is given by the DMs.

$$S^i = [s_{ab}]_{m\times m} \qquad a = 1, 2, \ldots, m; \; b = 1, 2, \ldots, m; \; i = 1, 2, \ldots, n \tag{C.1}$$

Eq.(C.2) represents the eigenvalue method. $\mathcal{G}$ is the goal and evaluation vector of the alternative, and λ_{enb} is the eigenvalue.

$$S\mathcal{G} = \lambda_{enb}\mathcal{G} \tag{C.2}$$

Eqs.(C.3)-(C.5) represent the normalization process of matrix S^i, and $C^i_{m\times m}$ is the normalized matrix.

$$E_a = [e_{ab}]_{m\times 1} \qquad a = 1, 2, \ldots, m \tag{C.3}$$

$$e_{ab} = \frac{s_{ab}}{\sum\limits_{a=1}^{m} s_{ab}} \tag{C.4}$$

$$C^i = [e_{ab}]_{m\times m} \qquad a = 1, 2, \ldots, m \; ; \; b = 1, 2, \ldots, m \tag{C.5}$$

In (C.6), the evaluation of each alternative on criterion i is obtained as follows:

$$g_i = \frac{\sum\limits_{b=1}^{m} e_{ab}}{m} \mathcal{G} = [g_i]_{m\times 1} \tag{C.6}$$

Eqs.(C.7)-(C.8) are used to measure the consistency of the DMs' judgement.

$$CI = \frac{\lambda_{enb} - m}{m - 1} \tag{C.7}$$

$$CR = \frac{CI}{RI} \tag{C.8}$$

CI is the consistency index. CR is the consistency rate. The denominator RI is known as the *average random consistency index*, whose value depends on the the size of the comparison matrix and it is tabulated in previous literature (see e.g. Xiaoxin et al. (2018)). It is commonly stated that pairwise comparisons of a matrix are acceptable if $CR \leq 0.1$.

UTA method

Following Appendix B, the framework now uses the UTA method taking as input the evaluation of the alternatives computed in the previous steps. Eq. (C.9) below expresses the global value of the alternatives $u(g(a_k))$, $k = 1, 2, \ldots, K$ in terms of the marginal value function $u_i(g_i)$, and the variables w_{ij}. Here, k is the given ranking by DMs to alternatives and K is the highest rank that the DMs can give to alternatives.

$$\begin{cases} u_i(g_i^1) = 0, \ \forall i = 1, 2, \ldots, n \\ u_i(g_i^j) = \sum\limits_{t=1}^{j-1} w_{it}, \ \forall i = 1, 2, \ldots, n; j = 2, 3, \ldots, \alpha_i \end{cases} \tag{C.9}$$

Eq.(C.10) below is defined for each pair of alternatives with consecutive ranking. The σ^+ and σ^- are underestimation and overestimation error, respectively.

$$\Delta(a_k, a_{k+1}) = u(g(a_k)) - \sigma^+(a_k) + \sigma^-(a_k) - u(g(a_{k+1})) + \sigma^+(a_{k+1}) - \sigma^-(a_{k+1}) \tag{C.10}$$

The following model minimizes the total deviations and obtains the weights of the criteria.

$Model(1)$:

$$\min \mathcal{Z} = \sum_{k=1}^{m}(\sigma^{+}(a_k) + \sigma^{-}(a_k)) \tag{C.11}$$

$$s.t.\ \Delta(a_k, a_{k+1}) \geq \delta,\ \forall k : a_k \succ a_{k+1} \tag{C.12}$$

$$\Delta(a_k, a_{k+1}) = 0,\ \forall k : a_k \approx a_{k+1} \tag{C.13}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{\alpha_{i-1}} w_{ij} = 1 \tag{C.14}$$

$$w_{ij} \geq 0,\ \sigma^{+}(a_k) \geq 0,\ \sigma^{-}(a_k) \geq 0,\ \forall i, j, k \tag{C.15}$$

The optimal objective value of $Model(1)$ is denoted as $\mathcal{Z}^*$. This value is used in the following model, which obtains the mean additive value function of the optimal solutions.

$Model(2)$:

$$\max u_i(g_i^*) = \sum_{t=1}^{j-1} w_{it},\ \forall i = 1, 2, .., n; j = 2, 3, .., \alpha_i \tag{C.16}$$

$$s.t.\ \text{(C.12)} - \text{(C.14)}$$

$$\sum_{k=1}^{m}(\sigma^{+}(a_k) + \sigma^{-}(a_k)) \leq \mathcal{Z}^* + \epsilon \tag{C.17}$$

The following Eq. (C.18) is solved to obtain the weight of the criteria W_i.

$$W_i = \sum_{t=1}^{j-1} w_{it} \qquad \forall i = 1, 2, \ldots, n; j = 2, 3, \ldots, \alpha_i \tag{C.18}$$

Acknowledgements

I want to express my sincere gratitude and appreciation to Mario Guajardo, professor in Operation Research at NHH Norwegian School of Economics, for supporting me in writing this article.

References

Abastante, F., Corrente, S., Greco, S., Ishizaka, A. and Lami, I. M. (2018), 'Choice architecture for architecture choices: Evaluating social housing initiatives putting together a parsimonious AHP methodology and the Choquet integral', *Land Use Policy* **78**, 748–762.

Abastante, F., Corrente, S., Greco, S., Ishizaka, A. and Lami, I. M. (2019), 'A new parsimonious

AHP methodology: Assigning priorities to many objects by comparing pairwise few reference objects', *Expert Systems with Applications* **127**, 109–120.

Acar, H. H., Dursun, E., Gulci, S. and Gumus, S. (2017), 'Assessment of road network planning by using GIS-based multi-criteria evaluation for conversion of coppice forest to high forest', *Fresenius Environmental Bulletin* **26**(3), 2380–2388.

Akay, A. E. (2006), 'Minimizing total costs of forest roads with computer-aided design model', *Sadhana* **31**(5), 621–633.

Akay, A. E., Erdas, O., Reis, M. and Yuksel, A. (2008), 'Estimating sediment yield from a forest road network by using a sediment prediction model and GIS techniques', *Building and Environment* **43**(5), 687–695.

Akay, A. O., Akgul, M., Demir, M. and Acar, H. H. (2020), 'Analysis of factors associated with the amount of forest road reconstruction activity in Turkey: Autoregressive distributed lag modelling approach', *Forest Ecology and Management* **458**, 117800.

Demir, M. (2007), 'Impacts, management and functional planning criterion of forest road network system in Turkey', *Transportation Research Part A: Policy and Practice* **41**(1), 56–68.

Esmaeili Sharif, M., Amoozad, M., Shirani, K., Gorgandipour, M. et al. (2016), 'The effect of forest road distance on forest fire severity (Case study: Fires in the Neka County Forestry)', *Ecopersia* **4**(2), 1331–1342.

Ghajar, I., Najafi, A., Karimimajd, A. M., Boston, K. and Ali Torabi, S. (2013), 'A program for cost estimation of forest road construction using engineer's method', *Forest Science and Technology* **9**(3), 111–117.

Greco, S., Słowiński, R., Figueira, J. R. and Mousseau, V. (2010), 'Robust ordinal regression', *Trends in Multiple Criteria Decision Analysis* pp. 241–283.

Grigolato, S., Pellegrini, M. and Cavalli, R. (2013), 'Temporal analysis of the traffic loads on forest road networks', *iForest-Biogeosciences and Forestry* **6**(5), 255.

Gumus, S. (2015), 'Evaluation of forest road networks located in Karadeniz Technical University Research and Practice Forest', *European Journal of Forest Engineering* **1**(1), 15–20.

Gumus, S., Acar, H. H. and Toksoy, D. (2008), 'Functional forest road network planning by consideration of environmental impact assessment for wood harvesting', *Environmental monitoring and assessment* **142**(1-3), 109–116.

Hashemkhani Zolfani, S., Rezaeiniya, N., Kazimieras Zavadskas, E. and Turskis, Z. (2011), 'Forest roads locating based on AHP and COPRAS-G methods: an empirical study based on Iran', *E+M. Ekonomie a Management* **14(4)**, 6–21.

Hashemkhani Zolfani, S., Yazdani, M., Ebadi Torkayesh, A. and Derakhti, A. (2020), 'Application of a gray-based decision support framework for location selection of a temporary hospital during COVID-19 pandemic', *Symmetry* **12**(6), 886.

Hayati, E., Majnounian, B., Abdi, E., Sessions, J. and Makhdoum, M. (2013), 'An expert-based approach to forest road network planning by combining Delphi and spatial multi-criteria evaluation', *Environmental monitoring and assessment* **185**(2), 1767–1776.

Jaafari, A., Najafi, A. and Melón, M. G. (2015b), 'Decision-making for the selection of a best wood extraction method: An analytic network process approach', *Forest Policy and Economics* **50**, 200–209.

Jaafari, A., Najafi, A., Rezaeian, J., Sattarian, A. and Ghajar, I. (2015a), 'Planning road networks in landslide-prone areas: A case study from the northern forests of Iran', *Land Use Policy* **47**, 198–208.

Jacquet-Lagreze, E. and Siskos, J. (1982), 'Assessing a set of additive utility functions for multicriteria decision-making, the UTA method', *European Journal of Operational Research* **10**(2), 151–164.

Jusoff, K. (2008), 'Construction of new forest roads in Malaysia using a GIS-Based decision support system.', *Computer and Information Science* **1**(3), 48–59.

Lahdelma, R. and Salminen, P. (2001), 'Smaa-2: Stochastic multicriteria acceptability analysis for group decision making', *Operations Research* **49**(3), 444–454.

Laschi, A., Neri, F., Brachetti Montorselli, N. and Marchi, E. (2016), 'A methodological approach exploiting modern techniques for forest road network planning', *Croatian Journal of Forest Engineering: Journal for Theory and Application of Forestry Engineering* **37**(2), 319–331.

Mahanpoor, M., Monajjem, S. and Balali, V. (2019), 'Sustainable highway maintenance: optimization of existing highway vertical alignment considering pavement condition', *Sustainability* **11**(6), 1659.

Mohammadi Samani, K., Hosseini, S., Lotfian, M. and Najafi, A. (2010), 'Planning road network in mountain forests using GIS and Analytic Hierarchical Process (AHP)', *Caspian Journal of environmental sciences* **8**(2), 151–162.

Montibeller, G., Gummer, H. and Tumidei, D. (2006), 'Combining scenario planning and multi-criteria decision analysis in practice', *Journal of Multi-Criteria Decision Analysis* **14**(1-3), 5–20.

Picchio, R., Latterini, F., Mederski, P. S., Venanzi, R., Karaszewski, Z., Bembenek, M. and Croce, M. (2019), 'Comparing accuracy of three methods based on the GIS environment for determining winching areas', *Electronics* **8**(1), 53.

Picchio, R., Pignatti, G., Marchi, E., Latterini, F., Benanchi, M., Foderi, C., Venanzi, R. and Verani, S. (2018a), 'The application of two approaches using GIS technology implementation in forest road network planning in an Italian mountain setting', *Forests* **9**(5), 277.

Picchio, R., Tavankar, F., Venanzi, R., Lo Monaco, A. and Nikooy, M. (2018b), 'Study of forest road effect on tree community and stand structure in three Italian and Iranian temperate forests', *Croatian Journal of Forest Engineering: Journal for Theory and Application of Forestry Engineering* **39**(1), 57–70.

Pourghasemi, H. R., Pradhan, B., Gokceoglu, C., Mohammadi, M. and Moradi, H. R. (2013), 'Application of weights-of-evidence and certainty factor models and their comparison in landslide susceptibility mapping at Haraz watershed, Iran', *Arabian Journal of Geosciences* **6**(7), 2351–2365.

Rezaei, J. (2015), 'Best-worst multi-criteria decision-making method', *Omega* **53**, 49–57.

Saaty, T. L. (1988), What is the Analytic Hierarchy Process?, *in* 'Mathematical models for decision support', Springer, pp. 109–121.

Saaty, T. L. (1990), 'Eigenvector and logarithmic least squares', *European Journal of Operational Research* **48**(1), 156–160.

Sekitani, K. and Yamaki, N. (1999), ‘A logical interpretation for the eigenvalue method in AHP’, *Journal of the Operations Research Society of Japan* **42**(2), 219–232.

Siskos, E., Askounis, D. and Psarras, J. (2014), ‘Multicriteria decision support for global e-government evaluation’, *Omega* **46**, 51–63.

Siskos, Y., Grigoroudis, E. and Matsatsinis, N. F. (2016), UTA methods, *in* ‘Multiple Criteria Decision Analysis’, Springer, pp. 315–362.

Toloie-Eshlaghy, A. and Homayonfar, M. (2011), ‘MCDM methodologies and applications: a literature review from 1999 to 2009’, *Research Journal of International Studies* **21**, 86–137.

Xiaoxin, Z., Jin, H., Ling, L., Yueping, W. and Xinheng, Z. (2018), Research of AHP/DEA evaluation model for operation performance of municipal wastewater treatment plants, *in* ‘E3S Web of Conferences’, Vol. 53, EDP Sciences, p. 04009.