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Macroeconomic Determinants of Long-Term Government Yields

A study of American and Norwegian yields using an ECM approach

Stig Torje Bjugn & Merethe Wangen

Supervisor: Jan Tore Klovland

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NORWEGIAN SCHOOL OF ECONOMICS

Abstract

This thesis examines the macroeconomic determinants of 10-year government bond yields in Norway and the US. We use Johansen cointegration testing and a VECM framework to identify long-run relationships between non-stationary variables. These relationships are further used in a more flexible ECM framework.

We find that US rates prior to 2007 had a stable long-run relationship with the US policy rate, 5-year inflation expectations and the current account. Furthermore, we find short-run effects from the policy rate, inflation expectations, VIX (expected volatility in financial markets) and PMI (business cycles), as well as some evidence of an effect of government debt.

We find that Norwegian rates over the entire period have a stable long-run relationship with the German 10-year rate and the Norwegian policy rate. We find short-run effects in the pre-2007 period for the policy rate, German 10-year rate, and the VIX index.

We find large changes in the post-2007 period for both countries. Neither rate react to any significant degree to deviations from long-run relationships, and the US long-run relationship breaks completely down. For both countries, most of the estimated short-run effects weaken, or disappear. In this period, the effect of government debt supply is clearer and we also find effects from increases in central bank reserves indicating that QE has had a large impact on long-term rates. In Norway, we only find a significant short-run effect from the German 10-year rate.

Overall, we find that the period since 2007 represent a large change in determination of long-term rates compared to the 16 years prior to that period. Even though there have been large changes, our models are still able to predict movements in the rates in recent years relatively well, with some exceptions which are discussed.

Preface

This thesis completes our MSc in Economics and Business Administration, with major in Economic Analysis (ECO) at NHH.

Working on the thesis has been challenging and we have learned a lot about doing empirical work in economics. In particular the data collection process and properly utilizing the relevant econometric methods proved to be interesting challenges. The choice of topic reflects our interest in empirical macroeconomics.

We would like to thank our supervisor, Jan Tore Klovland, for helpful input and encouraging comments. Furthermore, we would like to thank Yushu Li for input on econometric issues.

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Stig Torje Bjugn

Merethe Wangen

Contents

CONTENTS **4**

FIGURES **7**

TABLES **11**

1. INTRODUCTION **16**

1.1 GENERAL BACKGROUND 16

1.2 A SHORT STORY OF QUANTITATIVE EASING 17

1.3 OUTLINE OF THE THESIS 19

2. RELATED LITERATURE **20**

2.1 BERNANKE (2013) – LONG-TERM INTEREST RATES 20

2.2 AKRAM AND FRØYLAND (1997) – EMPIRISK MODELLERING AV NORSKE PENGEMARKEDS- OG OBLIGASJONSRENTER 24

2.3 HELLUM (2010) – HVA BESTEMMER UTVIKLINGEN I LANGSIKTIGE AMERIKANSKE STATSRENTER OVER TID? 28

2.4 ANG AND PIAZZESI (2003) – A NO-ARBITRAGE VECTOR AUTOREGRESSION OF TERM STRUCTURE DYNAMICS WITH MACROECONOMIC AND LATENT VARIABLES 29

2.5 EVANS AND MARSHALL (2007) – ECONOMIC DETERMINANTS OF THE NOMINAL TREASURY YIELD CURVE 29

2.6 KRISHNAMURTHY AND VISSING-JORGENSEN (2012) – THE AGGREGATE DEMAND FOR TREASURY DEBT 31

2.7 GAGNON, RASKIN, REMACHE AND SACK (2010) - LARGE-SCALE ASSET PURCHASES BY THE FEDERAL RESERVE: DID THEY WORK? 31

2.7.1 Time series analysis of the impact of the asset purchases 32

2.7.2 Event study of the possible announcement effect of the asset purchases 33

2.7.3 Main findings 33

2.8 KROGSTRUP, REYNARD AND SUTTER (2012) – LIQUIDITY EFFECTS OF QUANTITATIVE EASING ON LONG-TERM INTEREST RATES 34

3. PRESENTATION OF VARIABLES 37

3.1 10-YEAR GOVERNMENT BOND YIELDS 37

3.2 POLICY RATES 39

3.3 INFLATION EXPECTATIONS 41

3.4 PMI AND OUTPUT GAP 42

3.5 THE VIX INDEX 44

3.6 A SHORT DISCUSSION ON QE AND ITS CONSEQUENCES FOR CHOICE OF EXPLANATORY VARIABLES 45

3.7 GROSS GOVERNMENT DEBT 46

3.8 NON-BORROWED CENTRAL BANK RESERVES 47

3.9 CURRENT ACCOUNT 49

4. ECONOMETRIC METHODS 51

4.1 STATIONARY TIME SERIES 51

4.1.1 Stationarity restrictions 51

4.1.2 Testing for unit-root 53

4.2 COINTEGRATION 54

4.2.1 The Error Correction Model 56

4.3 THE VECTOR ERROR CORRECTION MODEL 57

4.3.1 The rank of the model 58

4.3.2 The Johansen test for cointegration 59

4.3.3 Restrictions on the cointegrating vector and deterministic terms 60

4.4 ESTIMATING THE VECM IN STATA 60

4.4.1 Deterministic term specification 60

4.4.2 Lag order selection 61

4.4.3 Testing for the rank of cointegration 62

4.4.4 Constraining the cointegrating vectors 62

4.4.5 Reviewing the estimated cointegrated equations 63

5. EMPIRICAL ANALYSIS 64

5.1 USA 64

5.1.1 Unit root analysis 64

5.1.2 Estimating the long-run relationship – Q1 1990-Q4 2006 74

5.1.3 Estimating the long-run relationship – Q1 1990-Q4 2012 83

5.1.4 ECM-modelling - Q1 1991-Q4 2006 88

5.1.5 ECM-modelling – Q1 1991-Q4 2012 101

5.2 NORWAY 119

5.2.1 Unit root analysis 119

5.2.2 Estimating the long-run relationship 124

5.2.3 ECM-modelling - Q1 1991-Q4 2006 130

5.2.4 ECM-modelling – Q1 1991-Q4 2012 138

6. DISCUSSION 148

6.1 WHAT DETERMINES THE 10-YEAR GOVERNMENT YIELD IN THE US AND NORWAY, AND HAVE THERE BEEN ANY CHANGES SINCE 2007? 148

6.2 ARE WE ABLE TO EXPLAIN THE DEVELOPMENTS IN THE YIELDS IN RECENT YEARS? 154

6.3 SOME WEAKNESSES OF OUR ANALYSIS 155

7. CONCLUSIONS 157

APPENDIX 160

UNIT ROOT TESTING 160

US variables 160

Norwegian variables 165

REFERENCES 170

Figures

Figure 2.1.1 – Decomposition of US 10-year Treasury yield presented by Bernanke (2013)21

Figure 2.1.2 - Forecasts of 10-year Treasury yield presented in Bernanke (2013) 23

Figure 3.1.1 - US, Norwegian and German 10-year government yields 38

Figure 3.1.2 - US 10-year rate, 2013-2014 39

Figure 3.2.1 - US and Norwegian policy rates 40

Figure 3.3.1 - US and Norwegian inflation, and US 5-year inflation expectations 42

Figure 3.4.1 - US and Norwegian output gap 43

Figure 3.4.2 - US PMI index 44

Figure 3.5.1 - The VIX index 45

Figure 3.7.1 - Log(Debt/GDP) for US 47

Figure 3.8.1 - Non-borrowed reserves (in % of GDP) for US 48

Figure 3.9.1 - Current account (in % of GDP) for US 50

Figure 5.1.1 – Levels and first differences of the 10-year rate with corresponding autocorrelations 65

Figure 5.1.2 - Levels and first differences of the policy rate with corresponding autocorrelations 66

Figure 5.1.3 - Levels and first differences of the expected inflation with corresponding autocorrelations 67

Figure 5.1.4 - Levels and first differences of the current account with corresponding autocorrelations 68

Figure 5.1.5 - Levels and first differences of log(Debt/GDP) with corresponding autocorrelations 69

Figure 5.1.6 - Levels and first differences of non-borrowed reserves with corresponding autocorrelations, Q1 1990-Q2 2008 70

Figure 5.1.7 - Levels and first differences of non-borrowed reserves with corresponding autocorrelations, Q3 2008-Q4 2012 71

Figure 5.1.8 - Levels and first differences of the VIX index with corresponding autocorrelations 72

Figure 5.1.9 - Levels and first differences of the PMI index with corresponding autocorrelations 73

Figure 5.1.10 - I(1)-variables included in long-run part of US model, Q1 1991-Q4 2006 74

Figure 5.1.11 - I(1)-variables included in long-run part of US model, Q1 1991-Q4 2014 75

Figure 5.1.12 – Disequilibrium and autocorrelation plot for the cointegrating equation from model 1 81

Figure 5.1.13 - Disequilibrium and autocorrelation plot for the cointegrating equation from model 2 82

Figure 5.1.14 - Comparison between the estimated cointegrated equations 82

Figure 5.1.15 - Disequilibrium and autocorrelation plot for the cointegrating equation 88

Figure 5.1.16 – Model-predicted and actual changes in the US 10-year rate, model including debt 96

Figure 5.1.17 – Model-predicted and actual changes in US 10-year rate, model excluding debt 96

Figure 5.1.18 - Model-predicted and actual US 10-year rate, model including debt 97

Figure 5.1.19 - Disequilibrium plot for the estimated cointegrated equation 98

Figure 5.1.20 - Model-predicted and actual US 10-year rate, model including debt 99

Figure 5.1.21 - A closer look at the out-of-sample period; model-predicted, actual and equilibrium 10-year rate 99

Figure 5.1.22 - A closer look at the out-of-sample period; explanatory variables in the cointegrating equation 100

Figure 5.1.23 - Model-predicted and actual changes in the US 10-year rate, estimation sample Q1 1990-Q4 2012........ 107

Figure 5.1.24 - Model-predicted and actual US 10-year rate, estimation sample Q1 1990-Q4 2012 108

Figure 5.1.25 - A closer look at recent years; model-predicted, actual and equilibrium 10-year rate 108

Figure 5.1.26 - A closer look at recent years; explanatory variables 109

Figure 5.1.27 - Model-predicted and actual changes in US 10-year rate 117

Figure 5.1.28 - Model-predicted and actual US 10-year rate 118

Figure 5.2.1 - Levels and first differences of the Norwegian 10-year rate with corresponding autocorrelations 119

Figure 5.2.2 - Levels and first differences of the German 10-year rate with corresponding autocorrelations 120

Figure 5.2.3 - Levels and first differences of the policy rate with corresponding autocorrelations 121

Figure 5.2.4 - Levels and first differences of the inflation rate with corresponding autocorrelations 122

Figure 5.2.5 - Levels and first differences of the output gap with corresponding autocorrelations 123

Figure 5.2.6 - I(1)-variables in the Norwegian model, Q1 1991-Q4 2006........ 124

Figure 5.2.7 - I(1)-variables in the Norwegian model, Q1 1991-Q4 2014........ 125

Figure 5.2.8 - Disequilibrium and autocorrelation plot for the cointegrating equation for Q1 1991-Q4 2006........ 129

Figure 5.2.9 - Disequilibrium and autocorrelation plot for the cointegrating equation for Q1 1991-Q4 2012 129

Figure 5.2.10 - Comparison between the estimated cointegrated equations 130

Figure 5.2.11 - Model-predicted and actual changes in the Norwegian 10-year rate 134

Figure 5.2.12 – Model-predicted and actual Norwegian 10-year rate 135

Figure 5.2.13 - Model-predicted and actual Norwegian 10-year rate 136

Figure 5.2.14 - A closer look at the out-of-sample period (2007-2014) 136

Figure 5.2.15 - Model-predicted and actual changes in the Norwegian 10-year rate, model without interaction terms 145

Figure 5.2.16 - Model-predicted and actual changes in the Norwegian 10-year rate, model with interaction terms 146

Figure 5.2.17 - Model-predicted and actual Norwegian 10-year rate 146

Figure 5.2.18 - A closer look at the period 2010-2014 147

Figure 6.1.1 – An illustration of the post-2007 period for Norway 152

Tables

Table 5.1.1 - Lag-order selection statistics for the underlying VAR model, with maximum lag length set to four 76

Table 5.1.2 – Lag-order selection statistics for the underlying VAR model, with maximum lag length set to eight. 76

Table 5.1.3 – Results from Johansen cointegration test, sample Q1 1990-Q4 2006. Trend specification: Case 4 – “Restricted constant” 76

Table 5.1.4 - Estimated cointegrating relationships and key information from the estimated VECM 78

Table 5.1.5 - Correlation matrix, sample Q1 1990 – Q4 2006 (67 observations) 84

Table 5.1.6 - Correlation matrix, sample Q1 2007 - Q4 2012 (24 observations) 84

Table 5.1.7 - Lag-order selection statistics for the underlying VAR model, with maximum lag length set to four 85

Table 5.1.8 - Lag-order selection statistics for the underlying VAR model, with maximum lag length set to eight 85

Table 5.1.9 - Results from Johansen cointegration test, sample Q1 1991-Q4 2012. Trend specification: Case 4 – “Restricted constant” 85

Table 5.1.10 - Estimated cointegrating relationship and key information from the estimated VECM 86

Table 5.1.11 - Estimated models for the US, sample Q1 1990-Q4 2006 89

Table 5.1.12 - Preferred models for the US, sample Q1 1990-Q4 2006, with and without debt 92

Table 5.1.13 - Preferred models for the US, sample Q1 1990-Q4 2006, with heteroskedasticity robust standard errors 95

Table 5.1.14 - Estimated models for the US, sample Q1 1990-Q4 2012 102

Table 5.1.15 - Preferred model for the US, sample Q1 1990-Q4 2012, with heteroskedasticity robust standard errors 106

Table 5.1.16 – Estimated models for the US with interaction terms, with heteroskedasticity robust standard errors 111

Table 5.2.1 - Lag-order selection statistics for the underlying VAR model, with maximum lag length set to four 126

Table 5.2.2 - Lag-order selection statistics for the underlying VAR model, with maximum lag length set to eight 126

Table 5.2.3 - Results from Johansen cointegration test, sample Q1 1991-Q4 2006. Trend specification: Case 4 – "Restricted constant" 126

Table 5.2.4 - Estimated cointegrating relationships and key information from the estimated VECM 127

Table 5.2.5 - Estimated models for Norway, sample Q1 1991-Q4 2006 131

Table 5.2.6 – Preferred model for Norway, sample Q1 1991-Q4 2006, with heteroskedasticity robust standard errors 133

Table 5.2.7 - Estimated models for Norway, sample Q1 1991-Q4 2012 139

Table 5.2.8 - Estimated models for Norway, with interaction terms 141

Table 5.2.9 - Preferred models for Norway, sample Q1 1991-Q4 2012, with heteroskedasticity robust standard errors 144

Table A. 1 - DF-GLS test for the 10-year rate in levels, with maximum lag length set to eight 160

Table A. 2 - DF-GLS test for the first-differenced 10-year rate, with maximum lag length set to zero 160

Table A. 3 - DF-GLS test for the policy rate in levels, with maximum lag length set to eight 160

Table A. 4 - DF-GLS test for the first-differenced policy rate, with maximum lag length set to four 161

Table A. 5 - DF-GLS test for the expected inflation in levels, with maximum lag length set to eight 161

Table A. 6 - DF-GLS test for the first-differenced expected inflation, with maximum lag length set to zero 161

Table A. 7 - DF-GLS test for the current account in levels, with maximum lag length set to eight 162

Table A. 8 - DF-GLS test for the first-differenced current account, with maximum lag length set to zero 162

Table A. 9 - DF-GLS test for log(Debt/GDP) in levels, with maximum lag length set to eight 162

Table A. 10 - DF-GLS test for first-differenced log(Debt/GDP), with maximum lag length set to eight 163

Table A. 11 - DF-GLS test for non-borrowed reserves in levels, Q1 1990-Q2 2008, with maximum lag length set to eight 163

Table A. 12 - DF-GLS test for first-differenced non-borrowed reserves, Q1 1990-Q2 2008, with maximum lag length set to zero 163

Table A. 13 - DF-GLS test for non-borrowed reserves in levels, Q3 2008-Q4 2012, with maximum lag length set to four 164

Table A. 14 - DF-GLS test for first-differenced non-borrowed reserves, Q3 2008-Q4 2012, with maximum lag length set to zero 164

Table A. 15 - DF-GLS test for the VIX index in levels, with maximum lag length set to four 164

Table A. 16 - DF-GLS test for the first-differenced VIX index, with maximum lag length set to four 164

Table A. 17 - DF-GLS test for the PMI index in levels, with maximum lag length set to four 165

Table A. 18 - DF-GLS test for the first-differenced PMI index, with maximum lag length set to four 165

Table A. 19 - DF-GLS test for the Norwegian 10-year rate in levels, with maximum lag length set to eight 165

Table A. 20 - DF-GLS test for the first-differenced Norwegian 10-year rate, with maximum lag length set to four 166

Table A. 21 - DF-GLS test for the German 10-year rate in levels, with maximum lag length set to eight 166

Table A. 22 - DF-GLS test for the first-differenced German 10-year rate, with maximum lag length set to four 166

Table A. 23 - DF-GLS test for the policy rate in levels, with maximum lag length set to four 167

Table A. 24 - DF-GLS test for the first-differenced policy rate, with maximum lag length set to four 167

Table A. 25 - DF-GLS test for the inflation rate in levels, with maximum lag length set to four 167

Table A. 26 - Augmented Dickey-Fuller test for the inflation rate in levels, with lag length set to four 168

Table A. 27 - DF-GLS test for the first-differenced inflation rate, with maximum lag length set to four 168

Table A. 28 - DF-GLS test for the output gap in levels, with maximum lag length set to four 168

Table A. 29 - Augmented Dickey-Fuller test for the output gap in levels, with lag length set to four 168

Table A. 30 - DF-GLS test for the first-differenced output gap, with maximum lag length set to four 169

1. Introduction

1.1 General background

Long-term interest rates have been at historically low levels in recent years. This has puzzled many, and made examining the underlying determinants of such rates extra interesting. In particular, we find it interesting to examine whether macroeconomic factors can explain the low levels experienced in recent years.

Not everyone find the low rates puzzling. Bernanke (2013) claims that "while the current constellation of long-term rates across many advanced countries has few precedents, it is not puzzling: It follows naturally from the economic circumstances of these countries and the implications of these circumstances for the policies of their central banks", pointing at low and stable expected inflation, low real interest rates that are expected to remain low for some time and most prominently a sharp fall in the term premium. Thus, there are natural explanations for the low rates. We still find it interesting to see if a model using macroeconomic factors can fully explain this, and furthermore, if the turbulent years since 2007 represent any major changes in the determination of long-term interest rates.

To examine this, we model 10-year government yields in an ECM-framework, using macroeconomic factors as explanatory variables. We wish to examine three questions. First, what are the determinants of long-term government bond yields? Second, have there been any changes in the determination of yields since 2007? Third, can our models explain the developments in the yields in recent years?

Our two rates of focus will be the 10-year yield on American and Norwegian government bonds. The US has been through a severe financial crisis and a period with QE; we are interested in seeing whether these events have affected the underlying dynamics of their long-term government bond yields. Furthermore, we are also interested in whether this has affected economies that have not been through the same economic turmoil and unconventional monetary policy. Naturally, Norway comes to mind as an example of such an economy.

We also considered including Japan, Germany, and the United Kingdom in our studies. The obvious reason is that we thus had a case for all the central banks that have recently applied

QE programs. The programs have differed, making each particular case interesting. The economic conditions have also differed, for instance with Europe also going through a sovereign debt crisis. Japan is of special interest because of their relatively long history of QE, and because of its renewed relevance through the more recent take at QE through “Abenomics”. However, due to some challenges with finding the needed data, Germany and the United Kingdom were excluded from the study. Japan was excluded mainly due to the dynamics in some of the data series demanding methods that were out of scope for this study.

We find that there have been some major changes since 2007 for both countries. For the US, we find that expected inflation, the current account and the VIX index all affect the 10-year rate less. Furthermore, we find that quantitative easing can to a large extent explain the low yields in recent years. For Norway, we find that the German 10-year rate explains most of the variation in the Norwegian rate. This is also the case after 2007, although the effect is weaker. We find significant effects from the Norwegian policy rate, which almost disappears in the post-2007 period, and also the VIX index, which completely disappear in the post-2007 period. For both countries, we identify long-run relationships between non-stationary variables. In both countries, the rates react relatively strongly to deviations from these relationships in the pre-2007 period and almost not at all in the post-2007 period. The period since 2007 thus seems to represent either a change or an extraordinary period in the determination of long-term interest rates.

1.2 A short story of quantitative easing

The low long-term Treasury yields in recent years have in part been credited the quantitative easing (QE) programs conducted by the Federal Reserve. The following four paragraphs give a short review of the programs based on Fawley and Neeley (2013).

The federal funds target rate reached 0.25% in December 2008, and have since effectively been between 0% and 0.25%. Thus, the policy rate was effectively at the zero level boundary (ZLB), and a further decrease was not possible. With the American economy still struggling with the financial crisis (and the aftermath of the subprime mortgage crisis), and the traditional monetary policy tool rendered useless, unconventional monetary policy was needed. This led to the Federal Reserve conducting a series of quantitative easing programs, in an attempt to further lower long-term interest rates and improve economic conditions.

The large scale asset purchase (LSAP) programs launched in November 2008 and March 2009 have been named “QE1”. The two programs focused on lowering the yields on mortgage-backed securities (MBS) and agency debt in MBS (while also purchasing Treasuries), but also led to a significant increase in reserves. In the fall of 2010, the financial crisis had passed, but the economy remained sluggish and inflation low. The Federal Reserve therefore announced that they would reinvest the principal payments from QE1 into Treasuries and that they also considered further purchases of Treasuries. This is often referred to as QE2.

The next round of asset purchases was announced in September 2011. The new program involved selling short-term assets to buy long-term assets, thus effectively extending the average maturity of the portfolio of assets held by the Federal Reserve. The program was therefore called the maturity extension program. No increase in reserves was involved. As with QE1, MBS and agency debt in MBS was again included. The program was originally scheduled to end in June 2012, but was later extended to last through 2012.

QE3 was announced in September 2012. Unlike the previous programs, the announcement did not entail any set amount, only a set pace. The previous programs had been criticized (see for instance Gagnon et. al. (2010)) for a lack of needed responsiveness - the Federal Reserve had to stick to their announced quantity of purchases throughout the program, even if economic conditions changed. Announcing a given pace of monthly purchases would provide increased flexibility as the pace could be decreased, or increased, as the situation developed. From December 2012, purchases were again financed through increased reserves. QE3 continued until October 2014 (Kearns, 2015).

This is not the first time the Federal Reserve has tried to use unconventional monetary policy to lower long-term interest rate. In fact, they tried something quite similar in the early 1960s, then named “Operation Twist”. (The maturity extension program is by some referred to as “Operation Twist II”.) The name “Operation Twist” stems from it being an attempt to “twist” the yield curve by lowering long yields and pushing up short yields. In contradiction to the quantitative easing programs of more recent years, however, Operation Twist was found to be less successful. Solow and Tobin noted in 1987 (as cited by Gagnon, et. al., 2010, p. 13) that the effect of the purchases of long-term Treasury securities by the Federal Reserve under Operation Twist was offset by the increased issuance of long-term debt, and no significant effect was found. Modigliani and Sutch (1966, p. 196) found that Operation

Twist were "most unlikely" to have reduced the spread between long and short rates on Treasuries by more than ten to twenty basis points. However, "Operation Twist" was a relatively small operation and e.g. Bernanke, Reinhart and Sack (2004) found evidence to suggest that a similar program on a larger scale could be successful, which is what has been attempted with the maturity extension program.

1.3 Outline of the thesis

The thesis consists of seven chapters. Since our thesis is empirical, we have not included a section with economic theory – this is instead included partly in the related literature chapter, and partly when we present the data. Chapter 2 gives an overview of some related literature. In Chapter 3, we present the variables used in the study, and discuss how they are believed to affect the 10-year rate. Chapter 4 reviews the econometric methods applied in the empirical analysis. Chapter 5 presents the empirical analysis. It is divided into two sections, one for each country. Our findings are then discussed in Chapter 6. Chapter 7 concludes.

2. Related Literature

In this chapter we will give an overview of some literature relevant to this thesis. There is a vast literature regarding the modelling of long-term yields or the yield curve as a whole, and we have chosen to focus on a thorough presentation of a handful relevant articles.

We first present a speech given by Bernanke (2013), which provides a good explanation of the fundamental ideas regarding long-term interest rates and also explains why they have been so low in recent years. We next present Akram and Frøyland (1997), a study of Norwegian interest rates, whose findings have influenced our model of Norwegian rates.

The modeling of the yield curve has historically either been based on a finance perspective, or a macroeconomic perspective. Modeling of interest rates based on a finance perspective resulted in models where interest rates depended on latent factors. Models typically included unobserved level and slope factors, sometimes also a curvature factor. Ang and Piazzesi (2003) were the first to combine these approaches, and they tried to show that interest rates were affected by macroeconomic shocks. They found that "models with macro factors forecast better than models with only unobservable factors" (p. 745). Their findings suggested that macroeconomic factors primarily affected short-term yields. However, Evans and Marshall (2007) built on the findings of Ang and Piazzesi (2003) and found stronger evidence of macroeconomic factors explaining a large part of variation in medium- and long-term interest rates. They further showed that different macroeconomic shocks had different effects. These articles thus show how the finance approach cannot exist independently from macroeconomic influence.

Our approach is a macroeconomic one. We therefore mainly present articles that apply this perspective. The articles look at how observable macroeconomic factors relate to the 10-year Treasury bond yield. The recent period of quantitative easing is of special interest, and we have included several articles that study the possible effects of the Federal Reserve's unconventional monetary policy.

2.1 Bernanke (2013) – Long-Term Interest Rates

This section covers a speech given by Ben Bernanke, then chairman of the Federal Reserve, at the "Annual Monetary/Macroeconomics Conference" March 1, 2013 in San Francisco, California. In it he addressed why long-term interest rates were so low in the United States and other major industrial countries, how long-term rates were likely to evolve, and the managing of risks associated with future developments in long-term rates.

Bernanke says that long-term yields can be decomposed into three components; expected inflation over the term of the security, the expected path of real short-term interest rates and a residual component, the term premium. Obviously, we cannot observe any of these components, but as Bernanke mentions, there are ways of estimating them. A decomposition used in this speech is shown in figure 2.1.1. In this thesis, we will try to find the effects of different macroeconomic factors on long-term rates, without decomposing the rate into these components. We will however, have these components in mind as we discuss potential effects and implications of our findings.

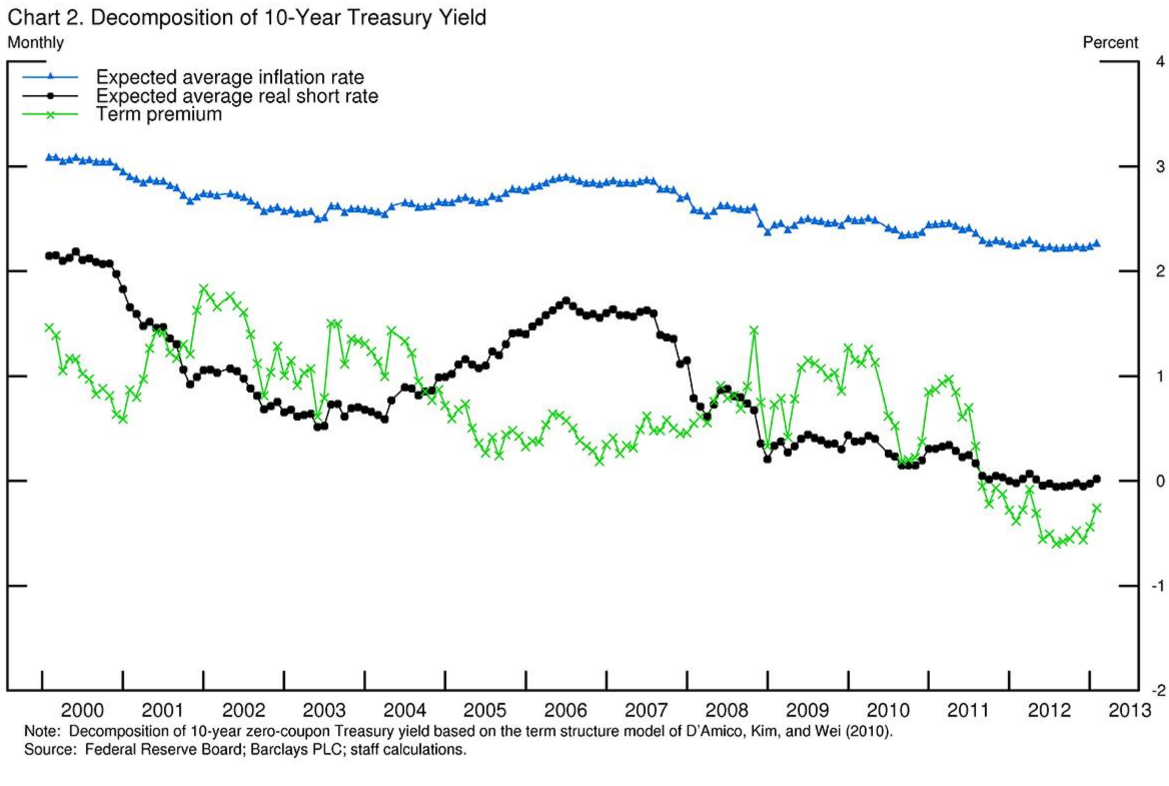

Figure 2.1.1 – Decomposition of US 10-year Treasury yield presented by Bernanke (2013)

Bernanke proceeds to talk about each of the three components and how they contribute to the low long-term yields seen in recent years.

The expected inflation component has been low and stable, which according to Bernanke reflects credibility of central bank commitment to price stability, “as well as considerable resource slack in the major industrial economies”. Bernanke remarks that as the 10-year rate has been below 2 percent for a period in recent years, and a decomposition of long-term rates used in his speech shows expected inflation components at about 2 percent, the net contribution of the final two components must be negative.

Expected average short-time rates over the coming 10-years had in recent years prior to Bernanke’s speech declined to near zero. In other words, real short-term interest rates were expected to remain low for some time, reflecting weak actual and expected recovery in advanced economies and need for continued accommodative monetary policy. Bernanke also said that this fall in addition possibly could reflect weaker long-term growth prospects.

The largest part of the fall from 2010 to 2013 seems to be a fall in the term premium. They have been low or negative, due to several factors, including effects from monetary policy. Bernanke explains that “the term premium is the extra return investors expect to obtain from holding long-term bonds, as opposed to holding and rolling over a sequence of short-term securities over the same period.” It partly compensates for the interest rate risk associated with holding long-term bonds. Bernanke points out two changes in the nature of the interest rate risk that may have contributed to the fall in the term premium. Reduced volatility of Treasury yields, in part because of short-term nominal interest rates being at the zero lower bound and expected to stay there for some time. Furthermore, increased negative correlation between stock and bond prices means that bonds are now more valuable as hedging instruments. Other factors Bernanke points at which may have reduced the term premium is increased so-called safe have demand for Treasuries (due to their safety and liquidity) and global demand for safe assets, including from foreign governments and central banks.

Bernanke continues to talk about how actions from the Federal Reserve, in particular through Large-Scale Asset Purchase (LSAP) programs, have affected term premiums. He stated that “to the extent that Treasury securities and agency-guaranteed securities are not perfect substitutes for other assets, Federal Reserve purchases of these assets should lower their term premiums, putting downward pressure on longer-term interest rates and easing financial conditions more broadly.” Bernanke further mentions, and we will also see, that research has found evidence that LSAPs have succeeded in bringing down term premiums.

With regards to how long-term rates are/were likely to evolve, Bernanke remarked that the FOMC anticipated long-term rates to gradually rise toward more normal levels given that recovery would continue at a moderate pace, with expected short-term real rates and term premiums returning to normal levels. The timing of the rise depends on developments in economic conditions. One important factor that can make rates rise is the market expecting that the date at which the Federal Reserve will tighten policy draws nearer and further rise will likely occur as monetary policy is actually tightened. In addition, there might also be some contribution to higher long-term rates from normalization of the term premium.

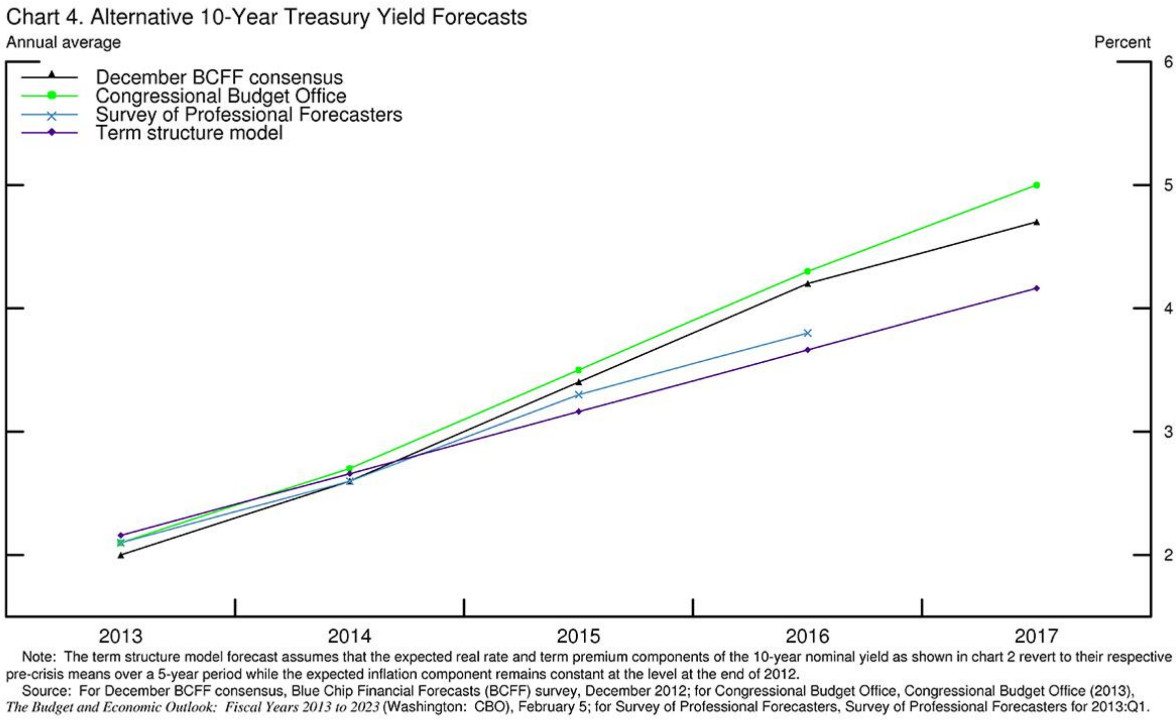

Figure 2.1.2 - Forecasts of 10-year Treasury yield presented in Bernanke (2013)

Figure 2.1.2 was used in Bernanke's speech and shows four different forecasts of the developments in the 10-year Treasury yields in the years following his speech. We will later refer back to this to compare it to actual rates in 2013 and 2014.

Finally, Bernanke related future developments in rates to risks concerning financial stability. He pointed out that some might argue that monetary policy should be tightened due to these risks, but that this might well be counterproductive. Furthermore, he talked in greater detail about how the Federal Reserve addresses financial stability concerns.

2.2 Akram and Frøyland (1997) – Empirisk modellering av norske pengemarkeds- og obligasjonsrenter

The title of this paper translates to "Empirical modelling of Norwegian money market and bond yields". Akram and Frøyland explain the developments in Norwegian 5-year government bond yields and 3-month money market yields during the 1990s using a dynamic system of equations, allowing them to study both factors that have only short-term effects and those that govern long-term developments. Furthermore, they account for the interaction between money market and bond yields.

The main finding from Akram and Frøyland's empirical analysis is that developments in both types of yields, both short-term and long-term, are mainly determined by developments in European yields.

Akram and Frøyland points to Norwegian monetary policy having an operative goal of stable foreign-exchange rates against European currencies, combined with high capital mobility, as reasons for why Norwegian interest rates to a large degree have to follow European rates. In other words, Norges Bank has limited freedom to set a level of interest rates which deviates from foreign rates, even if developments in the Norwegian economy warrant it. Their findings in this paper support this claim.

Akram and Frøyland next present some theoretical links between domestic and foreign nominal interest rates and between rates on securities with different duration. They start by presenting the following equation explaining development in domestic interest rates.

Equation 2.2.1:

$$R = R^* + e^e + \psi$$

This equation takes uncovered interest rate parity as a starting point. According to that theory, the domestic nominal interest rate, R, will be equal to foreign nominal interest rate, R^* plus expected depreciation against foreign currency, e^e. The term ψ is an exchange risk premium. For uncovered interest parity to hold, this premium must be zero. However, as the authors point out, most empirical research has rejected uncovered interest rate parity. This is most likely due to the risk premium not being zero and/or transaction costs.

As Akram and Frøyland notes, if the above equation is a correct representation of the determination of domestic interest rates, then macroeconomic variables only influence domestic interest rates to the degree in which they influence the risk premium or expected depreciation. Expectations about changes in foreign-exchange rates can depend on macroeconomic variables, like foreign-exchange reserves, inflation (both domestic and abroad), economic activity and growth in the money supply, among others. Akram and Frøyland do, however, refer to the fact that empirical research has found mixed results with regards to the link between macroeconomic variables and exchange rate expectations in the short-run.

Next, Akram and Frøyland presents the following equation linking yields on securities with different durations:

Equation 2.2.2

$$RL_t = \frac{1}{k}\left[\sum_{j=1}^{k} E_t\left(RK_{1,t+j-1}\right)\right] + E_t\phi_t$$

This equation expresses that the yield of a security with a duration of k periods at time t, RL_t, is the average expected yield of securities with a duration of 1 period, $RK_{1,t+j-1}$ being the 1 period yield at time $t+j-1$, plus expected extra return, $E_t\phi_t$. Expected extra return captures several premiums, such as the term premium and possible risk premia. The equation above expresses the same as the decomposition of long-term rates in Bernanke (2013). The first part is the expected part of short-term nominal interest rates, encompassing the expected short-term real rates and expected inflation components. The last part is the residual part, the term premium, which can include risk premia, such as the interest rate risk mentioned by Bernanke.

Akram and Frøyland refers to the much cited expectations hypothesis, according to which, the expected extra return will be zero. As they note, however, for this to hold fairly strict assumptions must hold and the hypothesis has been frequently rejected, usually explained by the presence of risk premia.

The two equations above represent two general equations for how interest rates are decided by the market. As Akram and Frøyland note, they do not necessarily represent alternative hypotheses. For example, short-run rates may be decided by equation 2.2.1, and the long-run rates by equation 2.2.2.

In the empirical analysis, they estimate an error correction model with the money market rate and 5-year government bond yield as dependent variables. We will focus on their findings relating to the 5-year government bond yield. Akram and Frøyland note that the Norwegian 5-year government bond yields seem to relate to both the ECU[1] yields and German yields. Norwegian yields seem to follow especially ECU yields closely, but also note that European rates (both money market and bond rates) seem to be mainly led by developments in German rates. Akram and Frøyland use ECU rates in their analysis, but note that given the strong focus on German rates in the market, this choice is not an obvious one. Their argument is that Norwegian rates had mostly been closer to ECU rates than German rates and that this choice was consistent with them using the ECU exchange rates in their analysis.

With regards to the money market rate, Akram and Frøyland find that uncovered interest parity holds in the long-run, i.e. in the long-run, Norwegian rates equal European rates, plus expected depreciation. In the short-run there can, however, be a risk premium present.

For Norwegian 5-year government bond yields, Akram and Frøyland find that their results do not support uncovered interest parity or the expectations hypothesis. They did, however, find that in the long-run, Norwegian long-term rates follow an approximate weighted average of both Norwegian short-term rates and European long-term rates, where European long-term rates had the greatest influence (coefficients were 0.126 and 0.87). In addition, they find a constant term in the long-run relationship that they interpret as a risk premium. The risk premium is 0.0013, relatively low, which can, according to Akram and Frøyland, be due to the solid finances of the Norwegian government. They find an adjustment parameter of -0.36, indicating a relatively quick adjustment in response to deviations from the long-run relationship (deviations are halved in less than two months).

Akram and Frøyland note that Isaachsen in a 1996 paper found similar evidence of a long-run relationship between long-term Norwegian rates, short-term Norwegian rates and international long-term rates.

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[1] ECU = European Currency Union

Furthermore, Akram and Frøyland find that long-term ECU-rates had the largest effect among variables that had an effect in the short-run. A change in long-term ECU-rates of 1 percentage point, were found to lead to a 0.7 percentage point increase in Norwegian 5-year rates in the same month. On a longer horizon, the effect was even larger. In the short-run, Akram and Frøyland find a weaker effect of the Norwegian money market rate than in the long-run. They found that a one percentage point change, lead to approximately a change of 0.07 percentage points in one month and 0.13 percentage points in the long-run. On the other hand, they found that a one percentage point increase in short-term ECU rates, led to an increase in the Norwegian 5-year rate of approximately 0.25 percentage points in two months. They did not find any effect of expected depreciation on the 5-year rate, except the effect that comes from the short-term rate. In the short-run, Akram and Frøyland also find that lagged changes in the 5-year rate have an impact, explaining this with slow adjustment of portfolios by market participants and/or lagged changes picking up effects from variables that are not present in the model.

Akram and Frøyland have additionally used several dummy variables to account for events where other variables were not able to explain the full development in rates.

They also included several macroeconomic variables in their analysis, including changes or levels in unemployment, the current account and Norwegian and foreign inflation rates. However, they did not find significant effects from unemployment, the current account or inflation in the EU and Germany. They found an insignificant positive effect from Norwegian inflation on both money market and government bond rates. The authors state that a possible reason for not finding significant effects from macroeconomic variables could be the fact that they use monthly data for a period of only seven years and that they would not exclude the possibility of finding significant effects if they used quarterly and/or yearly data for a longer period.

Finally, Akram and Frøyland find that their model make relatively good predictions out-of-sample.

2.3 Hellum (2010) – Hva bestemmer utviklingen i langsiktige amerikanske statsrenter over tid?

The title of this article translates to "What determines the development of long-term American government bonds over time?" Hellum (2010) looks at how the US 10-year government bond yield has developed over time, and estimates a model for the yield. The reason for his interest in the American yield is that it can act as a proxy for development in yields on a global basis. Yields incorporate cyclical movements, through the response of monetary policy, and since business cycles in most Western countries are related, at least to some extent, the yields will be related as well. Capital mobility can also lead to changes in the yield in one country affecting the yield in other countries as well.

Hellum uses the 3-month rate, long-term inflation expectations, the ISM index and the current account as explanatory variables. The ISM index acts as a proxy for cyclical movements, and the current account acts as a proxy for foreign demand for Treasuries.

The reason for the interest in the latter is the proposed "saving glut" hypothesis. The saving glut hypothesis was presented as a possible explanation to yields being low in the mid-2000s. The idea is that an increase in savings in Asia and oil producing countries led to increased foreign demand for US Treasuries, which lowered the yield.

Government debt was also included in a preliminary model, but there was not found any stable or significant relationship. Government debt increased after the financial crisis, something which should lead to investors demanding a higher yield. Although not noted by Hellum himself, we would like to add that the higher yield is due to the higher supply of government bonds leading to a lower liquidity premium on the price of these. Hellum notes that the reason for him not being able to estimate a positive and significant relationship is that the demand for government bonds did not decrease, due to special circumstances – demand from the Federal Reserve, and banks seeking to adjust to future countercyclical capital requirements, for instance.

In his estimation, he uses the sample Q1 1983 – Q4 2009. He finds that the 3-month rate, expected inflation, ISM index and current account all have a positive effect on the yield. Expected inflation has a coefficient above one, which he argues can be explained by the Taylor principle, i.e. that nominal rates increase more than the inflation expectations. He also argues that the estimated positive effect of the current account on the yield supports the "saving glut"-hypothesis – in a model excluding the current account, the predictions were quite similar except for in the mid-2000s (the period where the "saving glut" was pointed out to be relevant), where the new predictions were approximately one percentage point higher.

2.4 Ang and Piazzesi (2003) – A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables

This paper models joint dynamics of bond yields and macroeconomic variables using a vector autoregression (VAR). They use a term structure model that includes so-called latent variables, as well as inflation and economic growth factors to analyze how macroeconomic variables influence bond prices and the yield curve. Previous studies up to this point had usually used either latent factors or macroeconomic variables, making this a pioneering paper.

The paper uses zero coupon bond yields with maturities of 1 and 3 months from the Fama CRSP Treasury Bill files and of 12, 36 and 60 months from the Fama CRSP zero coupon files, for a sample ranging from June 1952 to December 2000. Furthermore they use several different measures of inflation and real activity, which they use to construct measures that are appropriate for use in the VAR. The authors state that their "list of variables includes most variables that have been used in monthly VARs in the macro literature" (p. 751).

They find that the model that includes macroeconomic factors forecast better than a model that only includes unobservable factors (i.e. the latent factors). Furthermore, they find that macroeconomic factors explain up to 85% of variation in bond yields, but primarily at the short-end and middle of the yield curve. At the long-end, they find that at a 1-month forecast horizon macro factors explain 60% of the variation. At very long forecast horizons, unobservable factors explain over 60% of the variation.

2.5 Evans and Marshall (2007) – Economic determinants of the nominal treasury yield curve

Evans and Marshall examine how different macroeconomic shocks affect the nominal Treasury yield curve. They look at technology shocks, believed to affect expected inflation and the term premium, and marginal-rate-of-substitution (MRS) shocks, believed to affect real rates and expected inflation. They also looked at fiscal shocks, but found no significant effect on the bond yields. In the study, they also account for monetary policy shocks, but these are not the focus of their paper and are therefore not discussed.

They argue that nominal Treasury yields should be affected by macroeconomic shocks because Treasury yields “are thought to assimilate vast amounts of information about the economy including information on the current stance of monetary and fiscal policy, as well as expectations of future economic activity, real interest rates, and inflation” (p. 1986).

They use a structural VAR framework for the empirical analysis, with the sample January 1959 – December 2000. When trying to look at the effect of macroeconomic shocks on the nominal yield curve, they include the federal funds rate, a range of nonfinancial macroeconomic variables, and short-, medium- and long-term zero-coupon Treasury yields (1 month, 1 year and 5 years, respectively). Whilst Ang and Piazzesi found most support for macroeconomic shocks affecting short- and medium-term interest rates, they find that 84% of the 5-year ahead variance of the 5-year yield is explained by nonfinancial macro factors. In comparison, Ang and Piazzesi only found that 48% is explained by macroeconomic factors. The discrepancy is due to Evans and Marshall’s assumption of interest rate smoothing (they include past values of the federal funds rate and bond yields in the structural VAR). By allowing for interest rate smoothing, they allow macroeconomic shocks to affect future interest rates, not only future macro variables. This allows the shocks to affect future interest rates through an additional channel, and the magnitude of the interest rate responses therefore increase.

The last part of the article tries to identify the macroeconomic shocks and their specific effect on the different yields. They find that short-, medium- and long-term yields all react similarly to an MRS shock, leading to a parallel shift upwards in the yield curve. They argue that the reason for the parallel shift is that the shock increases both the inflation and real rate. They find no evidence of an effect on the term premium from an MRS shock. The technology shock leads to parallel shift downwards in the yield curve. It increases real GDP and interest rates, but inflation falls. As the latter dominates, the overall effect is a downward shift. There seems to be an effect on the term premium from a technology shock. They find that the shocks also affect the federal funds rate through the Taylor principle, and that the long-term interest rates incorporate this in advance.

2.6 Krishnamurthy and Vissing-Jorgensen (2012) – The Aggregate Demand for Treasury Debt

In this paper, Krishnamurthy and Vissing-Jorgensen study the value investors put on the liquidity and safety of US Treasuries. They analyze the spread between assets with different liquidity (but similar safety) and the spread between assets with different safety (but similar liquidity). They find that both spreads are influenced by changes in Treasury supply, i.e. changes in Treasury supply influence the prices of both liquidity and safety.

They find that over their main sample period, 1926-2008, the average value investors have paid per year for the safety and liquidity of long-term Treasuries is 73 basis points, where at least 27 are for safety and up to 46 are for liquidity. The estimated results are fairly similar in magnitude for both long-term and short-term spreads. They find that the US government "has saved interest costs of about 0.25 percent of GDP per year" (p. 235) due to the demand for the liquidity and safety of Treasuries. They further argue that Treasury interest rates "are not appropriate a benchmark for "riskless" rates" (p. 235). They argue that "cost of capital computations using the capital asset pricing model should use a higher riskless rate than the Treasury rate: a company with a beta of zero cannot raise funds at the Treasury rate" (p. 235).

2.7 Gagnon, Raskin, Remache and Sack (2010) - Large-Scale Asset Purchases by the Federal Reserve: Did They Work?

The paper looks at the effect of the LSAP programs initiated by the Federal Reserve following the federal funds effective rate reaching the ZLB in December 2008. The idea was that purchasing substantial quantities of assets with medium and long maturities would reduce long-term yields for several asset classes. The article looks at whether the Federal Reserve succeeded in their attempt.

The article analyzes the effect of the program announced in November 2008, which focused on purchasing MBS and agency debt in MBS, and the program announced in March 2009, which also included long-term Treasury securities. The article looks at the effect the LSAP programs had on both agency-related securities and longer-term Treasury securities, but we will focus on the part regarding the latter as this is more relevant for our thesis.

2.7.1 Time series analysis of the impact of the asset purchases

They estimate a model that incorporates the business cycle, uncertainty and the net public sector supply of longer-term dollar-denominated debt securities. They believe that the effect of LSAPs on the yield is due to the “portfolio balance effect” – as the Federal Reserve purchases assets, this reduces the supply of these assets available to the public, leading to their yields decreasing. (This depends on the assumption that there exists a preferred habitat demand for those assets, i.e. imperfect asset substitutability.) To measure this effect, they use the net public sector supply of longer-term debt. The base variable is the publicly-held Treasury securities (not including certificates). They assume that private investors do not distinguish between public sector agencies. This imply that investors will be indifferent to, for instance, the government buying back government bonds and the Federal Reserve buying the government bonds – in both cases, the bonds are not available to them and are instead held by a public agency. Therefore, to measure the net supply, they subtract Treasuries held by public sector agencies, i.e. longer-term Treasuries held by the Federal Reserve and foreign official agencies, from the total supply.

They first assume all variables are stationary and estimate an OLS regression where the dependent variable is the 10-year term premium, and the explanatory variables that acts as proxies for the business cycle and uncertainty are the unemployment gap, core CPI inflation, inflation disagreement, and realized volatility. The sample is January 1985 to June 2008.

For robustness, they also account for the possibility of some of the variables not being stationary in the chosen sample, and estimate a DOLS as well, which includes a long-run relationship in levels and contemporaneous, lead and lagged first differences of all the explanatory variables. The speed of adjustment was found to be -0.15, implying that “deviations in the term premium from long-run equilibrium have a half-life of roughly five months” (p. 26). Interestingly, they find that the adjustment speed dropped significantly if the supply of debt was excluded from the long-run relationship and suggest that the supply of debt thus must be an important part of the long-run relationship.

They find that there is a positive and significant relationship between the explanatory control variables and the term premium when estimating an OLS regression. In the DOLS regression, they find that the coefficients in the long-run equation were similar to the ones obtained in the OLS regression. Finally, they find that there is a positive and significant relationship between the net supply of Treasuries and the term premium in both cases.

2.7.2 Event study of the possible announcement effect of the asset purchases

They also discuss the possible announcement effect of the LSAPs: Asset prices today should account for expectations regarding future returns on those assets. A future large asset purchase by the Federal Reserve would decrease future rates, and if investors know of such a large future asset purchase, it should already be incorporated in today's asset prices. A complete incorporation assumes that investors have perfect and complete information. An announcement regarding such a large asset purchase should therefore reduce the yields of those assets immediately.

They conduct an event study to confirm this effect. The study is conducted by measuring the cumulative changes in interest rates around announcements regarding the LSAP. Their response window is one day and the event set consists of eight days where there was official communication that contained new information regarding the LSAPs. They find that all three asset classes showed a decline – the 10-year rate declining 91 basis points. They get similar results when they expand the event set and response window. They find little evidence of a decline due to lower expectations of future short-term rates, the majority of the decline seems to be due to a decline in the term premium.

2.7.3 Main findings

With both approaches, they find that the asset purchases have had an effect on the 10-year term premium. The implied effect of the $1.725 trillion used in asset purchases on the yield was 38-82 basis points according to the time-series approach, and 50-100 basis points using the event study approach. One issue with the study is that the time-series analysis uses a sample that ends in June 2008, i.e. before the LSAPs were initiated. However, since the time-series study and the event study are based on different data, samples and methods, and the results still lie within the same range, their results seem robust. They thus conclude that the Federal Reserve have been successful in their attempt to lower the long-term term premium.

2.8 Krogstrup, Reynard and Sutter (2012) – Liquidity Effects of Quantitative Easing on Long-term Interest Rates

While Gagnon et. al. (2010) examined the portfolio balance effect of QE, Krogstrup, Reynard and Sutter examines the possible liquidity effect.

They describe the difference between the liquidity effect and the portfolio balance effect. The portfolio balance effect focuses on the fact that the supply available to the public of the purchased assets decreases, while the liquidity effect focuses on the fact that the supply of reserves increases. The latter should lead to the yield on reserves increasing. However, as the yield on reserves is fixed, the yield cannot increase in absolute terms, only in relative terms. Thus, an increase in reserves leads to downward pressure on yields for other liquid assets. In the ZLB period, this only applied to medium- and long-term liquid bonds, as these were the only liquid bonds with strictly positive yields.

Krogstrup et. al. thus argue that the portfolio balance effect will decrease the yield on the specific asset being purchased, while the liquidity effect will affect all medium- and long-term liquid assets. To test for the effect of both, they estimate a model used to capture the portfolio balance effect, similar to the model in Gagnon et. al. (2010), but add non-borrowed reserves to capture the liquidity effect as well. They do not take signaling effects into account by assuming that potential signaling effects will only be present at announcement times. They assume that investors do not have perfect and complete information, which mean that the effect of the LSAPs will not be fully accounted for at the time of the announcement, some of the effect will also occur when the purchases take place.

The supply effect is captured by the public supply of Treasury bonds, while the liquidity effect is captured by non-borrowed reserves. Ideally, one would use both as explanatory variables and thus distinguish between the two effects. However, the net supply of Treasury bonds available to the public depends on the purchases done by the Federal Reserve. They note that even though the supply decreases when the Federal Reserve purchases bonds, it also increases when the Treasury issues new bonds. Because it also varies with the net issuance of new bonds, its correlation with reserves is small. If reserves have an effect as well, this must be the liquidity effect.

They use the sample February 1990 – January 2011, with weekly frequency. The ZLB-period is defined to start in mid-December 2008. Their dependent variable is the average daily yield on 10-year US Treasury bonds over one week.

As the liquidity effect can vary between normal periods and the ZLB period, they include two variables for non-borrowed reserves, both multiplied with dummies. They also include a level dummy for the ZLB period, to account for possible factors that are not accounted for in the model. To capture the supply effect, they use the total supply of Treasury securities, adjusted for the holdings by the Federal Reserve.

In addition, they use largely the same control variables as Gagnon et. al. (2010) to account for the business cycle, and uncertainty (uncertainty in regards to expectations that may increase demand for safe assets and in regards to inflation expectations which may affect the term premium). The control variables are the federal funds target rate, the expected change in the 1-year rate one year ahead (to account for expected future monetary policy), the unemployment gap, the inflation rate of core CPI, 6-month realized volatility and the interquartile range of long-term inflation expectations. They mention that replacing 6-month realized volatility with the logarithm of the VIX index did not alter their findings.

They first estimate a model with a sample ending in June 2008, both with and without Treasury supply. The model with Treasury supply does better, but the predicted rate is still significantly above the actual 10-year rate during the ZLB-period. They then estimate a model with a sample ending in January 2011 instead, now with non-borrowed reserves included as well. They find that the reserves variable for normal times is not significant, while the reserves variable for the ZLB-period is negative and significant. The ZLB level dummy is negative, but not significant.

They find some support for a negative relationship between long-term yields and reserves and estimate that the 10-year Treasury yield fell 46-85 basis points between January 2009 and January 2011 due to liquidity effects. A further reduction of 20 basis points is credited the portfolio balance effect.

They note several possible issues with their analysis, the most important being the possibility that the 10-year rate and some of the explanatory variables contain a unit root. In case the dependent variable contains a unit root, they also regress on the 10-year term premium, and get similar results. They still admit that the risk of spurious correlations is not fully taken care of and note that a VAR in first difference could be a possible approach.

3. Presentation of Variables

As commented by Bernanke (2013), long-term interest rates consist of three components: the expected path of real short-term rates, expected inflation, and an unknown term premium. When modelling the term premium, most studies base it on movements in the business cycles and uncertainty with regards to inflation uncertainty and economic outlook. Common proxies for business cycle movements include the output gap, unemployment gap and various indexes, like the PMI index. Proxies with regards to uncertainty can be inflation uncertainty, for instance captured by the interquartile range of long-term inflation expectations like in Gagnon et. al. (2010) and Krogstrup et. al. (2012), and perception of market risk, like realized volatility or the VIX index. It is also believed that the supply of government debt have an effect on the term premium; this is also the case for the LSAPs conducted under QE. The latter may also have an effect on the expected path of real short-term rates through a signalling channel.

In this chapter, we present the variables we have chosen to include in our modelling. We will include variables that can explain all three components of the long-term rates. The policy rate is an obvious choice to try and capture the expected path of real short-term rates, and it can also reflect some of the cyclical element. Inflation, or expected inflation, will be used to capture the expected inflation over 10 years. Finally, we have included several variables that may help to explain the term premium, and in some cases in part have an influence on expectations for the future path of the other two components. For each variable, we will discuss its assumed relationship with our two variables of interest – the US and Norwegian 10-year rate. These will be presented first.

3.1 10-year government bond yields

The US and Norwegian 10-year government bond yields (for the US often termed Treasury yield) are the variables which we want to explain the variations in our models. In addition, we add the German 10-year government bond yield as an explanatory variable in the Norwegian model. This is based on arguments from Akram and Frøyland (1997), who found that long-term Norwegian government bond yields were primarily influenced by European long-term rates and in addition by Norwegian and European money market rates. Akram and Frøyland (1997) used ECU rates to explain Norwegian rates; they did however note that

German rates were possibly just as valid as an explanatory variable. As ECU rates have been discontinued, we choose to use German 10-year government bond yields as an explanatory variable in our analysis of Norwegian 10-year rates.

The link from Norwegian rates to international rates was based partly on uncovered interest parity, but allowing for a risk/term premium. Their findings suggest that European long-term rates should be used when modelling the Norwegian 10-year rate. It makes sense that interest rates in a small open economy like Norway are heavily influenced by foreign rates. In the United States, it is likely that a link to international rates are relevant, but we believe that the causality mainly runs from the US to other countries, meaning that it is valid to model US rates without taking into account foreign rates.

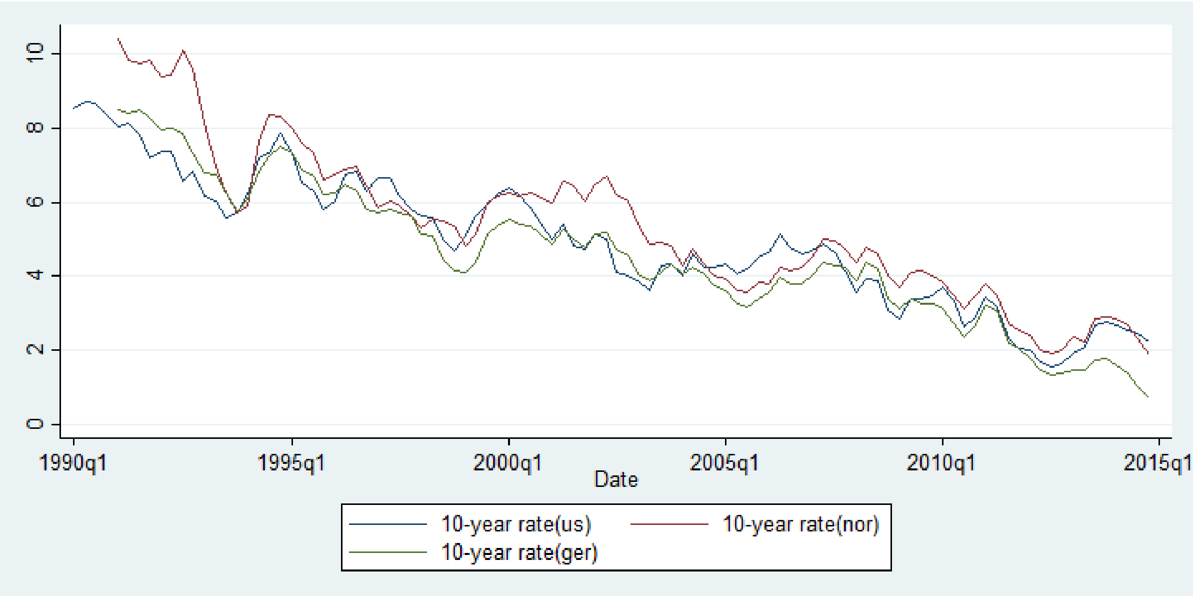

Figure 3.1.1 - US, Norwegian and German 10-year government yields

The estimation samples for our models will start in 1990 for the US (restricted by availability of data on the VIX index) and in 1991 for Norway (restricted by availability of data on the Norwegian policy rate, the folio rate). Figure 3.1.1 depicts the US, Norwegian and German 10-year rates. All three are taken from Macrobond[2] and the quarterly values are the average of end of month observations in that quarter. We see that all three have fallen, and follow each other closely, through our sample period.

[2] Primary names: 10-year rate(us): us10ygov; 10-year rate(nor): no10ygov; 10-year rate(ger): de10ygov

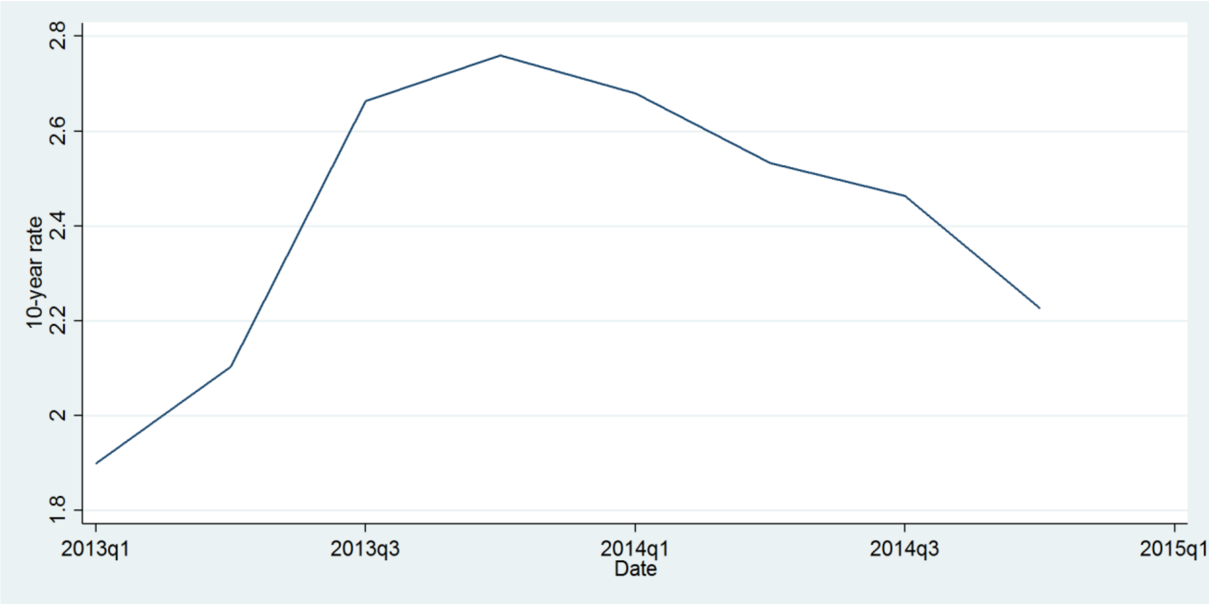

Figure 3.1.2 - US 10-year rate, 2013-2014

Bernanke (2013) presents forecasts for the development of the US 10-year rate from 2013 onwards (see figure 2.1.2). Bernanke stated: “The basic message is clear – long-term interest rates are expected to rise gradually over the next few years, rising (at least according to these forecasts) to around 3 percent at the end of 2014.” Figure 3.1.2 shows actual data for the first couple of years included in these forecasts (2013 and 2014). We see that the rise in the 10-year rate during 2013 was far higher than predicted by these forecasts, approaching the predicted level of the end of 2014. In 2014, however, the actual rate dropped rather than continuing the rise. This development will be examined further when we examine our estimated models.

3.2 Policy rates

Policy rates are the main instruments for conducting monetary policy for both the Federal Reserve and Norges Bank. Bernanke (2013) remarks that monetary policy does not, strictly, control real, but nominal short-term rates, but that this usually translates into having some control over real short-term rates over the short and medium term, due to inflation adjusting slowly. Over a longer term, however, “real interest rates are determined primarily by nonmonetary factors, such as the expected return to capital investments, which in turn is closely related to the underlying strength of the economy” (Bernanke, 2013).

As explained by Bernanke (2013), the expected path of short-term real interest rates is influenced by both the current stance of monetary policy and the expectations among participants in the market about future policy. The policy rate is also likely to influence expected inflation. Furthermore, the policy rate can have an influence on people's expectations of real activity, which can have an influence on the term premium.

Monetary policy, and thus the policy rate, is primarily driven by the economic outlook. As pointed out by Bernanke (2013), there are also other factors influencing monetary policy. One example he points at is nominal interest rates at the zero lower bound, which makes it prudent for central banks to employ other tools than the policy rate, as seen by e.g. quantitative easing programs.

Figure 3.2.1 depicts the US and Norwegian policy rates over our sample period. The data for these series were taken from Macrobond[3] and are computed as the average rates over the quarter. Figure 3.2.1 serves as an example to the US leading role in the world economy. Most movements in the US policy rate are followed by similar movements in the Norwegian policy rate, sometimes with a lag of as much as a couple of years.

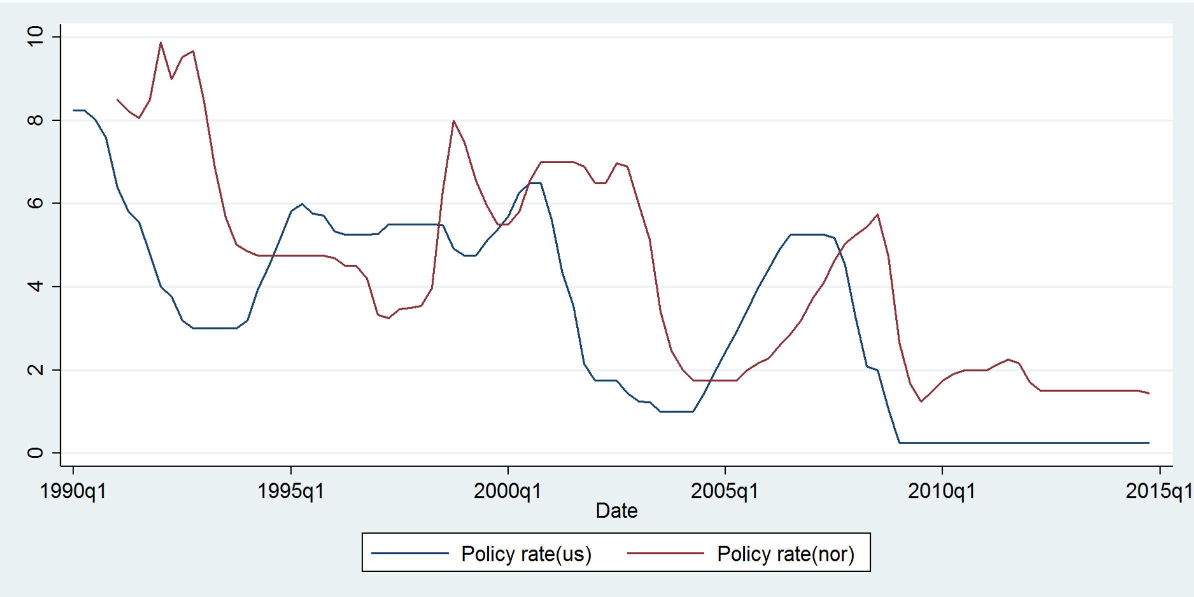

Figure 3.2.1 - US and Norwegian policy rates

[3] Primary names: Policy rate(us): usrate0001; Policy rate(nor): norate0001

3.3 Inflation expectations

Expected inflation over the term of the bond is a very important determinant of its yield. Furthermore, the uncertainty of this expectation will be important for the term/risk premium demanded by investors. Bernanke (2013) claims that “the downward trend and stabilization of expected inflation in the United States are products of the increasing credibility of the Federal Reserve’s commitment to price stability.” He points out that the commitment has been reaffirmed by the Federal Open Market Committee (FOMC) in recent years, including a long-run inflation target of 2 percent. He points at the anchoring of inflation expectations near 2 percent as a very important factor influencing long-term interest rates in recent years. In other words, more stable inflation expectations have significantly reduced the term premium.

Our inflation data include actual quarterly inflation data for both the US and Norway, taken from OECD[4], which can be included as a proxy for inflation expectations under a hypothesis of simple backward-looking expectations, as well as 5-year inflation expectations for the US from the University of Michigan Surveys of Consumers (the same expectations series used in Hellum, 2010), which were taken from the Datastream database[5] and where the value for a quarter is the average of monthly values in that quarter. We unfortunately did not find a good series for inflation expectations for Norway.

Our three series are all depicted in figure 3.3.1. We clearly see that the actual inflation series are far more volatile than long-term expectations. In our analysis, it turns out that we do not find significant effects when using actual inflation, in either model. The expected inflation series, however, do turn out to be significant in the US model. This reflects that actual inflation is not a good enough proxy for expectations in either country. The lack of a good inflation expectations variable in the Norwegian model need not necessarily be a large problem. Inflation expectations may very well be similarly stable in Norway as in the US, maybe even more stable, assuming Norges Bank’s inflation target is viewed as credible, which should be a relatively fair assumption to make. In the extreme case, where Norges

[4] Data found in General Statistics –> Key Short-Term Economic Indicators –> Consumer Prices – Annual Inflation

[5] Mnemonic: USUMINM5R

Bank's inflation target is viewed as perfectly credible, inflation expectations should be almost constant and thus would not explain variation in rates.

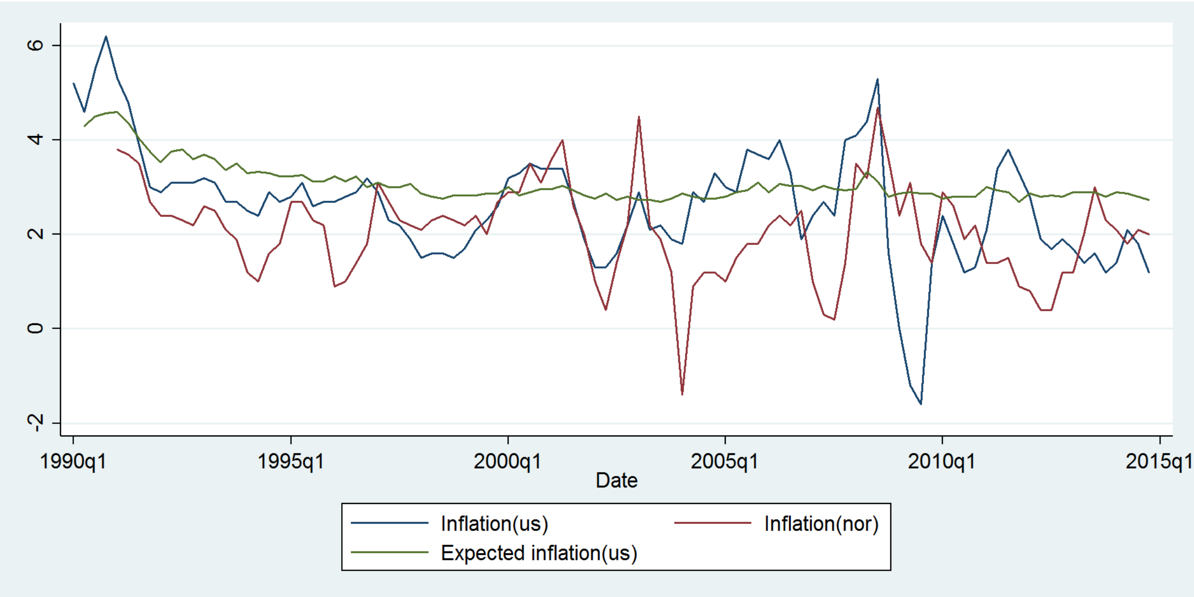

Figure 3.3.1 - US and Norwegian inflation, and US 5-year inflation expectations

3.4 PMI and output gap

PMI and output gap are included in the model as proxies for economic cycles/real activity. Yields, and in particular term premia, on low-risk bonds (such as Treasuries) tend to be higher during booms and lower during recessions. Investors are more willing to invest in risky assets during booms, making e.g. safety premia on Treasuries fall, which leads to higher Treasury yields. Thus, we expect to find that the PMI and output gap have a positive effect on the 10-year rate.

We prefer to use PMI, as PMI is survey-based, and therefore capture the element of how companies experience economic cycles. It thus captures beliefs and expectations regarding business cycles. It also turns out to perform better than the output gap in the US models. The PMI series for Norway starts in 2004, so we will make use of the output gap when estimating models for Norway.

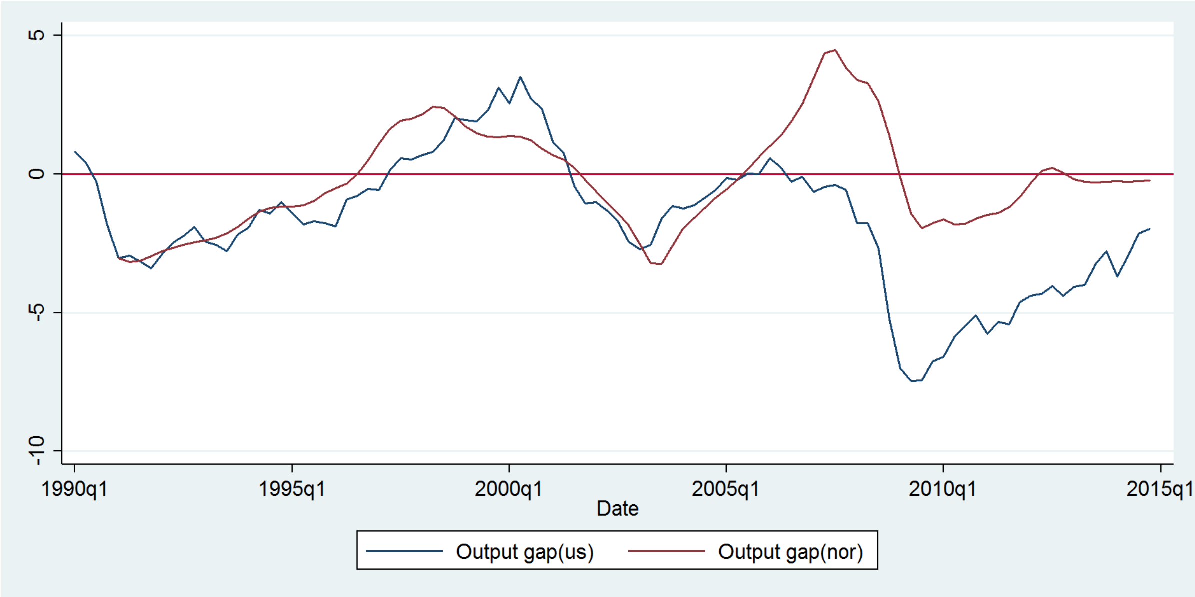

Figure 3.4.1 - US and Norwegian output gap

The output gap series are due to Oxford Economics and were taken from the Datastream database[6]. The US PMI series is the ISM PMI index and were taken from Macrobond[7]. The quarterly values we use are the average of monthly values. Figure 3.4.1 depicts the output gap series for both the US and Norway. The output gap series serve as a further attest to the globalization of the economy, as movements are roughly similar in both countries. The differences in level over the last nine or so years, highlight the role of Norway as a "different" country among advanced economies during and after the financial crisis, in that the Norwegian economy fared substantially better than many other advanced economies.

Figure 3.4.2 depicts the US PMI index. It is interesting to see how the expectation element of this index comes into play, in particular how it suggests very strong optimism in regard to the pace of recovery around 2010, shortly after the trough of the recession.

[6] Mnemonics: Output Gap(us): USXOGAP.R ; Output gap(nor): NWXOGAP.R

[7] Primary name: ussurv1055

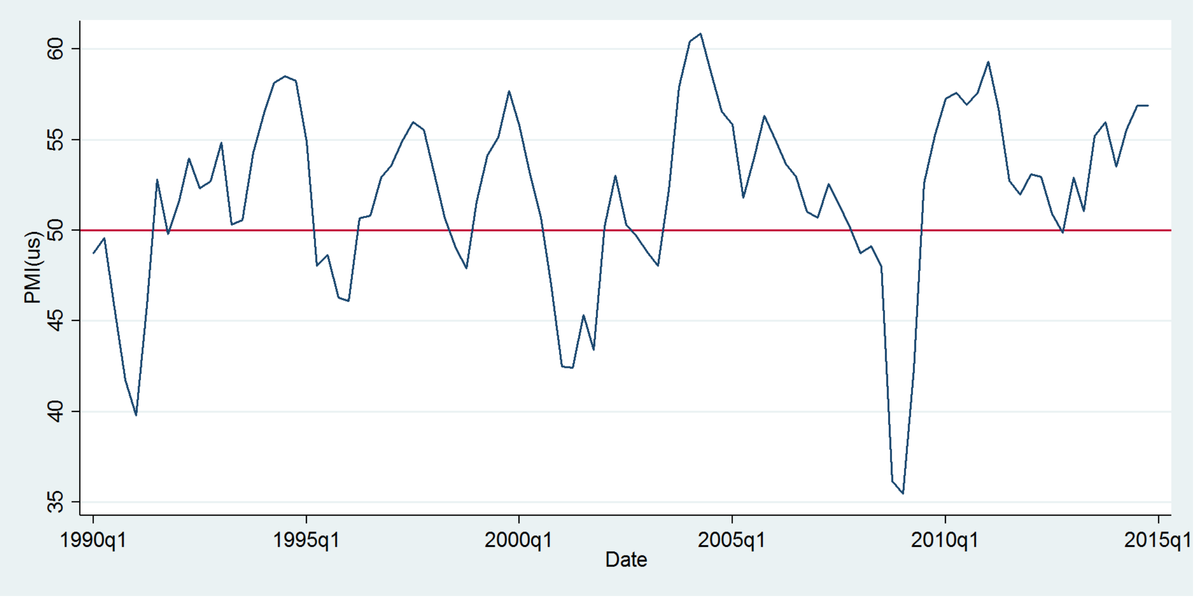

Figure 3.4.2 - US PMI index

3.5 The VIX index

The VIX index is an index of expected stock market volatility 30 days ahead. We include it as a proxy for how risky the financial market is perceived at a given time. Krishnamurthy and Vissing-Jorgensen (2012) focus on the value investors place on Treasuries for their safety and liquidity. In times where the market is perceived as less secure, investors will demand more of the assets perceived as more secure and liquid. The demand for long-term government bonds should then increase, which will increase their price and lower the yield. Given the degree of global integration in financial markets, we will include the VIX index in both the US and the Norwegian models. Figure 3.5.1 depicts the VIX index for our sample period. The data is taken from Macrobond[8] and quarterly values are the average value over the quarter.

[8] Primary name: vix

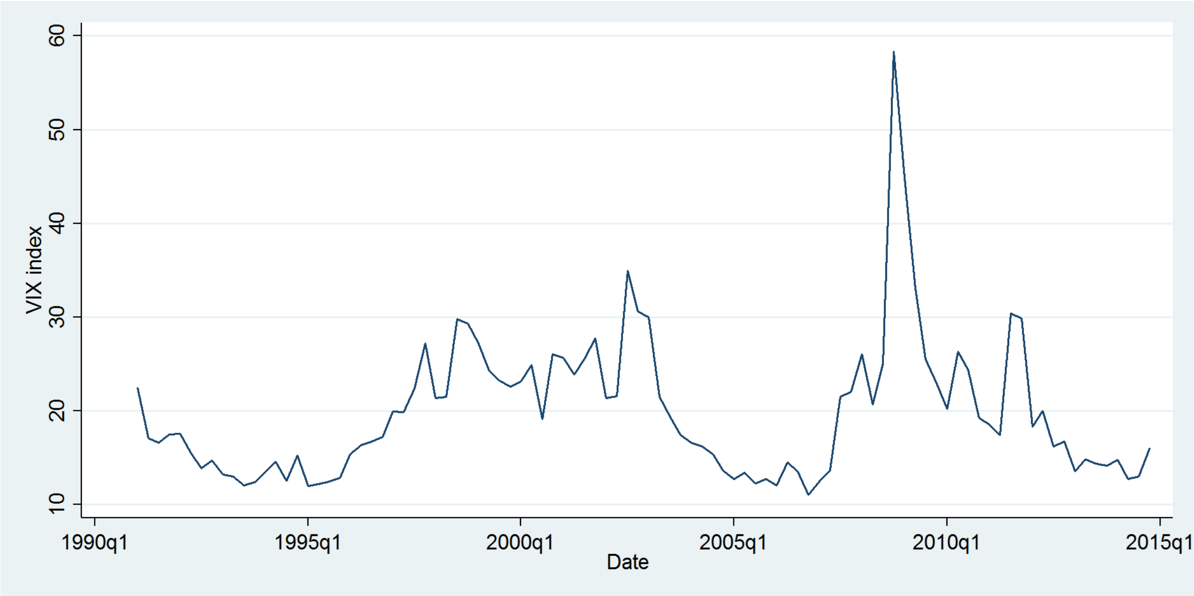

Figure 3.5.1 - The VIX index

3.6 A short discussion on QE and its consequences for choice of explanatory variables

Studies have suggested (mainly) three different effects from quantitative easing on long-term yields; a portfolio balance effect, a liquidity effect, and a signaling effect.

In 1958, Tobin (as cited in Gagnon et. al., 2010, p. 3) introduced the portfolio balance effect. Gagnon et. al. (2010) explain it as follows: When the Federal Reserve buys assets, this reduces the amount held by the private sector. It also increases short-term, risk-free bank reserves held by the private sector. As assets are not perfect substitutes, the price of the purchased assets should increase, and thus the yield decrease, as the available supply of the purchased assets decrease. They further argue that the decrease in yield mainly comes from a decrease in the term premium. They measured the effect by looking at the net supply of Treasuries, adjusted for the stock held by the Federal Reserve and official foreign agencies.

Gagnon et. al. (2010) also looks at the signaling effect. While the portfolio balance effect and the liquidity effect affect the term premium, the signaling effect affects the expected future short rate. The signaling effect is, simply put, the effect the LSAP programs has on the expectations of the future federal funds target rate. Gagnon et. al. assume that investors have perfect and complete knowledge, and thus the effect of the LSAPs should be incorporated by the market at the time of the announcement. In their event study, they find that the decline in the yield on announcement days is primarily attributed to a decline in the term premium, which they credit the portfolio balance effect. The signaling effect appears to have been small.

Krogstrup et. al. (2012) focus on the possible liquidity effect of quantitative easing. The liquidity effect is based on a hypothesis that an increase in the money supply should lead to a decrease in the short-term interest rates. As the short-term liquid assets' rates reach the zero lower bound, a later increase in the money supply cannot further lower the rates, because the two have become perfect substitutes. Krogstrup et. al. argue that the liquidity effect might still be active for liquid longer-terms bonds, as these still have a positive yield. As the Federal Reserve buys these bonds, reserves with zero yield increases, and this increase induces banks to seek higher returns. The demand for assets that still has a positive yield thus increases, making medium- to longer-term assets higher in demand, and lower in yield.

Thus, both QE-related research and research done on pre-QE data, such as Krishnamurthy and Vissing-Jorgensen (2012), have found that the 10-year yield, through its term premium, increase with the amount of government debt, i.e. the supply of government debt. Furthermore, it has been found that the increase in reserves during the QE-period has had a liquidity effect, reducing the term premium. It seems clear that both the supply of government debt and central bank reserves should be included in the US model.

3.7 Gross government debt

We only use the supply of government debt in our US model, not the Norwegian one. The market for Norwegian government debt is fairly small, the effects of changes in its supply are likely negligible. Furthermore, yields on Norwegian government bonds are likely heavily linked to European yields, with an extra liquidity premium due to being less liquid, and some influence from Norwegian factors.

Like Krishnamurthy and Vissing-Jorgensen (2012), we include debt held by the Federal Reserve in our measure of gross government debt. They found that their results did not change significantly if they exclude the holdings of the Federal Reserve. However, their data did not include the QE-period. As already mentioned, QE-related literature use government debt supply available to the public as their supply variable, and find that reductions in supply to the public are associated with lower yields. Unfortunately, we did not get the right data to construct this measure. However, as the Federal Reserve's purchases were to a large degree paid with reserves, we believe that increases in reserves will capture most of both the portfolio balance and liquidity effects referred to in the literature.

Figure 3.7.1 depicts our debt variable over time. The data is due to Oxford Economics, it is normalized on nominal GDP from OECD's Quarterly National Accounts reported quarterly in annualized numbers. Both series were taken from Datastream[9]. In line with previous studies, e.g. Krishnamurthy and Vissing-Jorgensen (2012), we use the natural logarithm of gross government debt in percent of GDP.

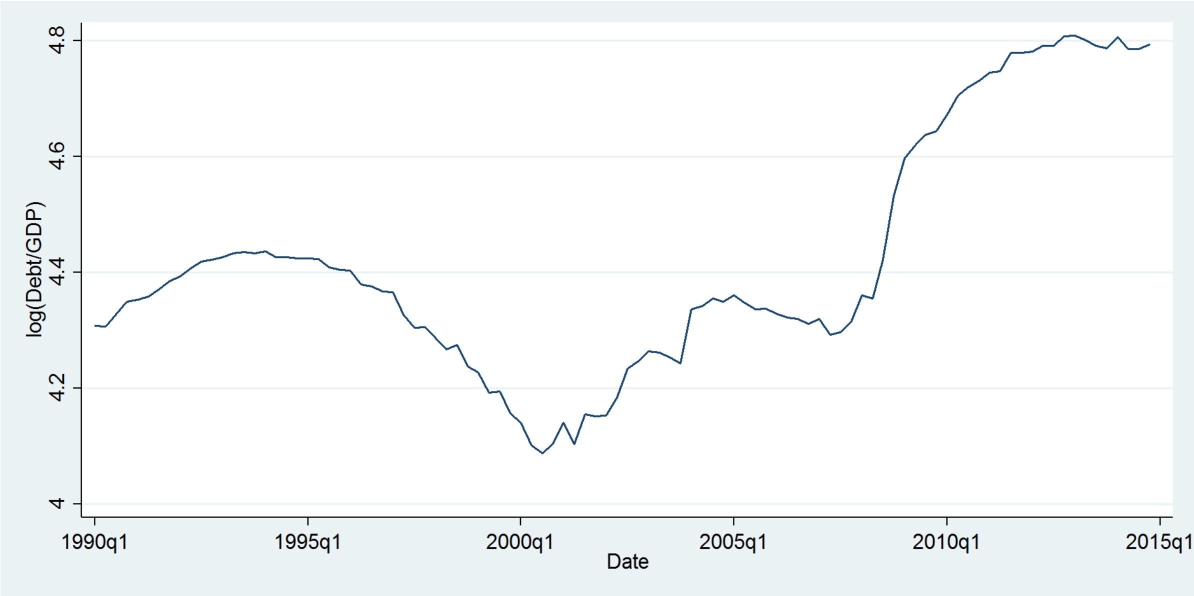

Figure 3.7.1 - Log(Debt/GDP) for US

3.8 Non-borrowed central bank reserves

Krogstrup et. al (2012) found liquidity effects from the increase in reserves during the LSAP programs. As mentioned in the previous section, we also believe that this increase will capture most of the portfolio balance effect. In line with Krogstrup et. al. (2012), we use non-borrowed reserves, i.e. we subtract total borrowings of depository institutions from the Federal Reserve from the total amount of reserves. For both these series the quarterly data are averages of daily values. The series were taken from the FRED database (Federal

[9] Mnemonics: General Government Debt, Gross: USXGGDB.A; Gross Domestic Product: USOEXA03B

Reserve Bank of St. Louis, 2015)[10] and normalized on the same nominal GDP series used for the government debt series.

Unlike for the debt series, we do not take the natural logarithm of our reserves series. There are some practical issues making this a natural choice. First, non-borrowed reserves where negative for two quarters in 2008, immediately prior to the first round of quantitative easing, we could thus not have included these observations if we used logarithms. Second, movements in reserves at very low levels are exaggerated when taking logs, making it possible that it could also exaggerate effects of reserves in the pre-QE period. Figure 3.8.1 depicts this series over time.

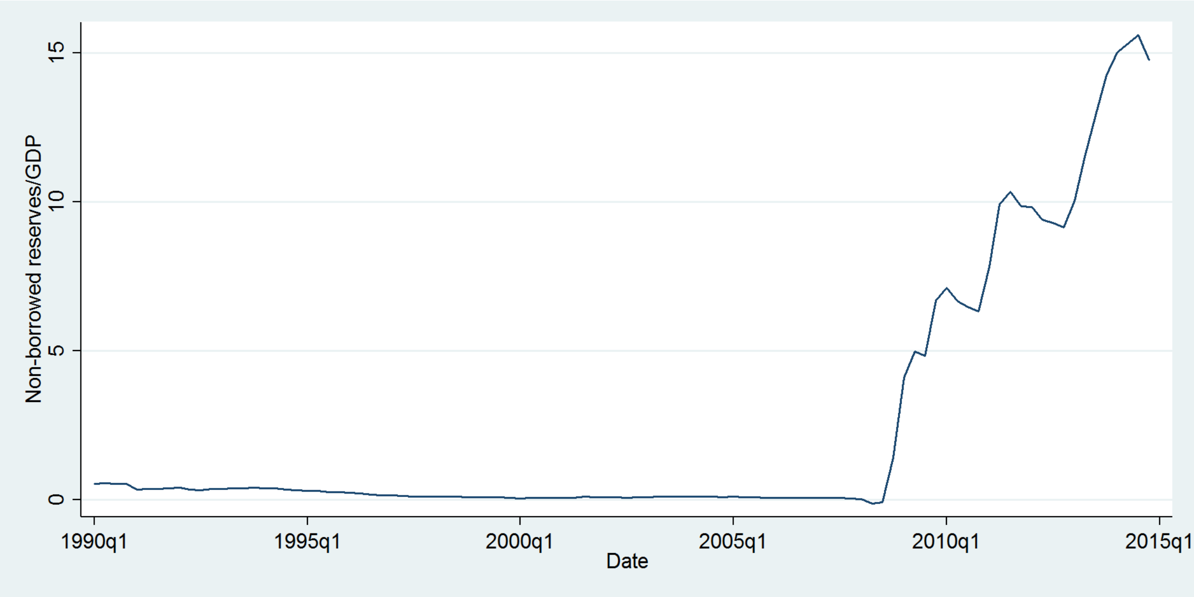

Figure 3.8.1 - Non-borrowed reserves (in % of GDP) for US

[10] Series IDs: Total Borrowings of Depository Institutions from the Federal Reserve: TOTBORR; Reserve Balances with Federal Reserve Banks: WRESBAL

3.9 Current account

There have been some papers that have investigated the possibility of an effect of the current account on long-term interest rates. Akram and Frøyland (1997) did not find any significant effect of the current account on Norwegian rates, and we do not believe that movements in the Norwegian current account in any significant way represent a difference for the demand for Norwegian government bonds.

For the US, things look a little different. Hellum (2010) notes that the current account can have differing effects on US long-term interest rates, depending on what the reason for the movements in the current account is. If the reason for a deficit in the current account is foreign demand for American treasuries, then the deficit can lead to a decrease in the yield as well. However, if the deficit is due to Treasuries being issued while American savings are low, then the price on government bonds will decrease, leading to higher yields. He claims that it seems like the deficit is mainly due to foreign demand for American treasuries from the early 1990s to the mid-2000s, which is supported by his findings. His views are supported by Bernanke (2013), who also states that global demands for safe assets, including from foreign governments and central banks, "particularly those with sustained current account surpluses" have contributed to lower term premia. Bernanke et. al. (2011) analyses the effect termed the "global saving glut" (GSG) hypothesis in the years 2003-2007. These were the years when the current account experienced an especially large deficit. He argues that the capital inflow in these years not only lead to low Treasury rates, but also might have had a depressing effect on the yield of mortgage-backed securities as well. This view is supported by Hellum's (2010) findings, where a model without the current account were unable to predict the low Treasury yields in this period.

Figure 3.9.1 - Current account (in % of GDP) for US

Figure 3.9.1 depicts the US current account for our sample period. The data is due to Oxford Economics and is taken from Datastream[11].

[11] Mnemonic: USXBCUS.Q

4. Econometric Methods

In this chapter we will explain the main econometric considerations and models we use in our analysis. Most concepts are fairly general econometric concepts, the presentations below are to a large extent inspired by Becketti (2013), Brooks (2014), Enders (2004) and Harris (1995).

4.1 Stationary time series

When analyzing time-series data, an important assumption is the assumption regarding stationarity. A stationary series will have a stable mean and variance, meaning that as long as we have a sufficiently long sample, the true mean and variance can be approximated using the sample. Enders (2004) describes the expectation, variance and covariance of a covariance stationary series as follows:

$$E(y_t) = E(y_{t-s}) = \mu \tag{1}$$

$$Var(y_t) = Var(y_{t-s}) = \sigma_y^2 \tag{2}$$

$$Cov(y_t, y_{t-s}) = Cov(y_{t-j}, y_{t-j-s}) = \gamma_s \tag{3}$$

where μ, σ_y^2 and γ_s are constants, and the equations hold for all *t* and *t-s*. This translates to having the same expected value, variance and covariance, independent of time, *t*. The last equation does not put any constraint on how fast covariance should decrease as *s* increases, only that the covariance between two observations that are *s* observations apart should stay constant. We will refer to covariance-stationary time-series as stationary time-series, for simplicity.

4.1.1 Stationarity restrictions

A stationary time-series must satisfy certain restrictions. An AR(1) process is stationary if we have:

$$y_t = a_0 + a_1 y_{t-1} + \epsilon_t \tag{4}$$

where $|a_1| < 1$, and ϵ_t is a white noise process. If $|a_1|$ is equal to 1, we say that the AR(1) process contains a unit root, i.e. that *y* contains a unit root. *y* is thus not a stationary process. The reason for assuming no unit root and error terms that are white noise, is that we do not want past shocks to *y* to affect future values of *y* permanently. If they did, then the series' first moment would not be stable. Let us say that a series containing a unit root suddenly experienced a large, positive shock. If the series previously had a mean of 25, a sample of the series starting after the shock would not have the same mean. The large, positive shock would affect all future observations, permanently affecting the mean. A series containing a unit root could experience several shocks, and thus the mean would vary with the sample, and thus our approximation as well. A sample of the series would thus not be enough to find a good approximation of *y*'s true mean.

If we use an OLS regression on variables containing a unit root, the regression may pick up on trends and means that are due to shocks and that do not describe the underlying series. Attributing such patterns to the variables may lead to regressions describing relationships that are not true – i.e. regressions with *spurious* results. We therefore need to test for unit roots in the variables. If we find that some of our variables contain a unit root, we need to transform the variables so that they become stationary. This is done by first-differencing.

A series containing one unit root will have stationary first differences. Let us say we have the following, general AR(1) process, with a possible mean and trend:

$$y_t = a_0 + a_1 y_{t-1} + \delta t + \epsilon_t \tag{5}$$

If we now subtract y_{t-1} from both sides of the equation, we get the following:

$$y_t - y_{t-1} = \Delta y_t = a_0 + (a_1 - 1)y_{t-1} + \delta t + \epsilon_t \tag{6}$$

If the process contained a unit root, i.e. $a_1 = 1$, the equation would be:

$$\Delta y_t = a_0 + \delta t + \epsilon_t \tag{7}$$

If the series contains one unit root, the first difference of the series does not. A process that is stationary is said to be integrated of order zero, or I(0). If the process' first difference is stationary, then the process is said to be integrated of order one, or I(1). Generally, if the process is only stationary after differencing *d* times, it is said to be integrated of order *d*, or I(*d*). Most economic time-series are either I(0) or I(1), i.e. they contain no more than one unit root.

Furthermore, if the series does not contain a unit root, but does contain a deterministic trend, i.e. the coefficient on the *t* in the equations above is significant, it is trend-stationary. In such a case, we would have to take into account the trend when using the series in regression analysis, either by detrending the series or including a trend as an explanatory variable. In the variables we use in our analysis, we will assume that there are no deterministic trends, i.e. that all trending behavior is stochastic.

4.1.2 Testing for unit-root

Two tests are generally applied for testing for unit roots – the Dickey-Fuller test (or augmented Dickey-Fuller (ADF) test) and the Dickey-Fuller Generalized Least Squares (DF-GLS) test. We will start by explaining the Dickey-Fuller test, as the DF-GLS test, which we will use, applies the same framework.

The Dickey-Fuller test tests whether, in equation 5, $a_1 = 1$. We cannot test the y_t equation, as this would require doing inference on a process that might not be stationary. The Dickey-Fuller test instead tests the first-differenced equation. Assuming the process is either I(0) or I(1), we would then test a stationary process. If we believed the process to be integrated of a higher order, i.e. contain more than one unit root, we would need to apply the Dickey-Fuller test on a process differenced more than once. However, most likely, all our processes are either I(0) or I(1), and we will therefore only consider the possibility of a single unit root.

The Dickey-Fuller test uses the first-differenced equation, equation 6, and has the following null and alternative hypothesis:

$$H_0: \theta = 0 \quad H_A: \theta \neq 0$$

where $\theta = a_1 - 1$. If the null hypothesis is correct, then θ equals zero, which means that a_1 equals one, i.e. *y* contains a unit root. The alternative hypothesis is thus that *y* does not contain a unit root, i.e. that the series is stationary.

The augmented Dickey-Fuller test takes into account the possibility of autocorrelation in the error terms. Autocorrelation in k lags can be augmented by testing the following equation instead of equation 6:

$$\Delta y_t = a_0 + \theta y_{t-1} + \delta t + \sigma_1 \Delta y_{t-1} + \cdots + \sigma_k \Delta y_{t-k} + \epsilon_t \qquad (8)$$

The null and alternative hypotheses remain unchanged.

The DF-GLS test, is a modification of the augmented Dickey-Fuller test, applied on GLS-detrended data. Becketti (2013, p. 384) reports that “studies have shown that this test has significantly greater power than the traditional augmented Dickey-Fuller test.” Therefore, we will primarily use the DF-GLS test when we test for unit roots, with some support from augmented Dickey-Fuller tests when needed.

4.2 Cointegration

Brooks (2014) notes that when the concept of stationarity was first introduced, issues with nonstationary variables were resolved by using differencing on the variables until they were stationary. A regression with first-differenced variables would show how one variable would react when another changed, i.e. it would show the short run relationship between the variables. However, there might be a long run relationship as well. Brooks gives an example of how ignoring a possible long run relationship could be a problem: If we have two variables that converge to equilibrium in the long run, then, when having converged to the equilibrium, the changes in the variables would be zero. A first difference equation would model how the variables react to changes in the other variables, but it has no value when the variables are in the equilibrium – all the first-differenced terms would be zero. The long run solution is not modeled. To model the long run relationship would be desirable as well, if there is one. This brings us to the concept of cointegration.

“A cointegrating relationship is a stationary linear combination of two or more non-stationary variables” (Becketti, 2013, p. 387). If such a relationship exists, then these variables are said to be cointegrated, and the linear combination is called the cointegrated vector.

If several variables constitute a cointegrating relationship, it means that the variables together constitute a relationship that has an equilibrium in the long run. When looking at the path of the variables over a longer period, you would notice that the distance between them seems deterministic (the distance remains constant, or keeps increasing, for example). A simple metaphor given by Murray (1995), is the story of the drunk and her dog. The drunk and her dog are walking home. When looking at their paths independently, they are two random walks. However, looking at the two paths together, you would notice that they never stray far apart. Together, they are thus cointegrated, and the distance between them will have a long run equilibrium. How fast the drunk and the dog will adjust when deviating from this equilibrium distance, can be referred to as the speed of adjustment. The cointegrating relationship can be described as follows:

$$y_t = \beta_1 x_t + \epsilon_t \tag{9}$$

Or rearranging:

$$\epsilon_t = y_t - \beta_1 x_t \tag{10}$$

We have chosen to normalize on y_t. More generally, the equation would be:

$$\epsilon_t = \beta_0 y_t - \beta_1 x_t \tag{11}$$

Where the vector,

$$\beta = (\beta_0, \beta_1) \tag{12}$$

is the cointegrating vector that describes the cointegrating relationship between the two variables. Since both $\beta_0 y_t - \beta_1 x_t$ and $\gamma\beta_0 y_t - \gamma\beta_1 x_t$ (where γ is a constant) would describe the cointegrating relationship, it is generally preferred to normalize the equation by setting the coefficient of y_t equal to one. The cointegrating vector will thus have a unique solution as long as we impose a restriction of unity on y_t.

When the two variables are in equilibrium, $y_t = \beta_1 x_t$. Thus, $\epsilon_t = 0$ in the equilibrium. We call this the equilibrium error. If we assume that y and x are cointegrated with the cointegrating vector β, then the RHS of equation 11 is stationary. It thus follows that the LHS must be stationary as well. In other words, we assume that the equilibrium error is stationary. It has an expected value of zero, since the deviation from equilibrium is zero in the long run, or else it would not be the long run equilibrium. This result is important, since it is the estimated equilibrium errors that are usually used in practice.

4.2.1 The Error Correction Model

If there is a cointegrating equation, then the equation is stationary and can be included in an equation with the first-differenced variables:

$$\Delta y_t = \alpha(y_{t-1} - \beta_1 x_{t-1}) + \gamma \Delta x_t + u_t \qquad (13)$$

u_t is assumed to be independent and identically distributed (i.i.d.) with zero mean, i.e. the error terms are assumed to be white noise. In practice, the long run relationship would be estimated and the estimated equilibrium errors would be included instead. Since the long run relationship is represented by an error correction term, the model is named the Error Correction Model (ECM). Substituting the long run relationship with the estimated error term from equation 10, we thus get the ECM:

$$\Delta y_t = \alpha(\hat{\epsilon}_{t-1}) + \gamma \Delta x_t + u_t \qquad (14)$$

The cointegrating relationship is included by using last period's observations. This is because the variables can only react to a deviation after the deviation has taken place. As in the metaphor: The drunk and the dog both wanders aimlessly. Then, they notice that they have strayed too far apart. With their next step, they thus try to reduce the distance between them. How fast the drunk and her dog try to reduce the deviation from the preferred distance is referred to as the speed of adjustment. The speed of adjustment, α in equation 14, says how fast the variables adjust to any deviation. If the dog uses its first step after the deviation to immediately reach the preferred distance, then $\alpha_2 = 1$. The complete system of equations would be as follows:

$$\Delta y_t = \alpha_{1(\hat{\epsilon}_{t-1})} + \gamma_1 \Delta x + u_{1,t} \qquad (15)$$

$$\Delta x_t = \alpha_2(\hat{\epsilon}_{t-1}) + \gamma_2 \Delta y + u_{2,t} \qquad (16)$$

The cointegrating relationship is included in both equations, but the two variables can adjust to deviations differently. If the drunk is y and the dog is x, then it is possible to have $\alpha_1 = 0$ and $\alpha_2 = 0.9$, indicating that the drunk does not notice the dog, while the dog tries hard to maintain the long run equilibrium distance.

Our main models will be relatively simple error correction models, using contemporaneous and lagged differences of the explanatory variables, allowing for lagged differences of the 10-year rate and including an error correction term. To estimate the long-run relationships that will constitute the error correction terms, we employ a more advanced, but less flexible, framework, a vector error correction model (VECM). We thus turn to discussing the estimation of such models in the next section.

4.3 The Vector Error Correction Model

We will now explain the vector error correction model (VECM) in several steps. A VECM looks much like equation 15 and 16 and the intuition is the same. However, the VECM is an extension of a vector autoregression model (VAR) for first differenced non-stationary variables that are cointegrated. A VAR assumes that the variables are autoregressive, and thus each variable in the system depends on the past values of all the variables in the system. A VAR with first differences is thus a system of first differenced dependent variables, where all are assumed to depend on past first differences of all variables. The VECM is the VAR model, including error correction terms. So, when generalizing equation 15 and 16 into a VECM, we allow for several endogenous variables and use lagged first differences of these as explanatory variables:

$$\Delta y_{1,t} = \alpha_{1,1}\left(\hat{\epsilon}_{1,t-1}\right) + \cdots + \alpha_{1,r}(\hat{\epsilon}_{r,t-1}) + \gamma\Delta y_{1,t-1} + \cdots + \gamma\Delta y_{k,t-1-n} + u_{1,t} \tag{17}$$

$$\vdots$$

$$\Delta \mathrm{y}_{k,t} = \alpha_{k,1}\left(\hat{\epsilon}_{1,t-1}\right) + \cdots + \alpha_{1,r}\left(\hat{\epsilon}_{r,t-1}\right) + \gamma_{k,1,1}\Delta y_{1,t-1} + \cdots + \gamma_{k,k,n}\Delta y_{k,t-1-n} + u_{k,t} \tag{18}$$

The VECM above assumes k variables, $n+1$ lagged differences and r error correction terms. The coefficients for the error correction terms uses the notation $\alpha_{i,j}$, where i denotes the dependent variable, and where j denotes the respective error correction term.

Furthermore, the coefficients for the first differences uses the notation $\gamma_{i,j,l}$, where i denotes the dependent variable, j denotes the respective independent variable, and l denotes the respective lag. As with the ECM, u is assumed to be white noise. To simplify the notation, it is most often written in vector notation:

$$\Delta y_t = \alpha\hat{\epsilon}_{t-1} + \gamma_1 \Delta y_{t-1} + \cdots + \gamma_{\,n} \Delta y_{t-1-n} + u_t \tag{19}$$

$$\downarrow$$

$$\Delta y_t = \alpha\beta' y_{t-1} + \gamma_1 \Delta y_{t-1} + \cdots + \gamma \Delta y_{t-1-n} + u_t \tag{20}$$

4.3.1 The rank of the model

If we have several variables, we can also have several cointegrating relationships. In the VECM above, we have allowed for r error correction terms, which mean that we have allowed for r cointegration relationships. With r error correction terms in the model, we say that the model has a rank of r.

With rank, we mean the number of independently linear combinations in the equation system, which corresponds to the number of cointegrating relationships. If the rank is zero, this means that none of the variables are cointegrated and we cannot identify a stable long-run relationship between our non-stationary variables. With k variables, we cannot have more than $k-1$ in rank, or else the cointegrated equations would not be independent. If $k = r$, we have one independent linear combination for each variable. If all these are stationary, then the variables themselves are all stationary and there are no cointegrating relationships. It is worth noting that if m of the k variables included in the VECM are I(0), then these will be independent linear combinations by themselves and thus increase the rank with m. If we find that $r = R$, then the number of cointegrated relationships will in fact be $R - m$. Furthermore, including stationary variables when trying to estimate cointegration relationships between non-stationary variables, will distort the estimations, possibly rendering them spurious. This is one reason why it is important to test for whether our variables are I(0) or I(1). The fact that the VECM is only appropriate for non-stationary variables and that it does not estimate contemporaneous effects, means that the VECM is not appropriate as our final model setup. We will thus only use VECM as a means to estimating long-run relationships between non-stationary variables, which will then be included in our final model.

4.3.2 The Johansen test for cointegration

To find the rank of a VECM, the most used approach is the Johansen cointegration test. Johansen (1995) derived two test statistics for testing the rank of a model, the trace and max statistics. When testing for the rank using the trace statistic, the general hypotheses are:

$H_0: r \leq R$ and $H_A: r > R$

When conducting this test, we start with $R = 0$. If we cannot reject the null hypothesis, then we conclude that the rank is zero. If we can reject, we continue with a new test, now with $R = 1$. We continue testing, each time increasing R by one, until we can no longer reject the null hypothesis. The max statistic works in the same fashion, only with slightly different null and alternative hypotheses:

$H_0: r = R$ and $H_A: r = R + 1$

Again, we continue to increase R until the null hypothesis is not rejected (see e.g. Enders (2004) for an explanation of the Johansen test).

Lütkepohl, Saikkonen and Trenkler (2001) compare the two tests, they conclude that "no major differences between corresponding maximum eigenvalue and trace tests are detected. (…) Based on our simulations, we have a preference for the trace tests. This result justifies the common practice in empirical work of using either both types of tests simultaneously or applying the trace tests exclusively (p. 305)." Based on this conclusion, we will focus on the trace statistic when conducting cointegration testing.

4.3.3 Restrictions on the cointegrating vector and deterministic terms

If we had three variables, y, x and z, and found the rank to be one, with a cointegrating relationship between y and x, then the corresponding cointegrating vector would be $\beta = (\beta_1, \beta_2, \beta_3) = (1, \beta_2, 0)$, i.e. with restrictions on the cointegrating coefficients on y and z. Generally, we can only estimate the cointegrating vector if we place at least $r * r$ constraints on the cointegrating vectors, and at least r constraints in each vector (Harris, 1995).

When writing the VECM above, we implicitly assumed no deterministic terms in the model. This is not always appropriate, and when estimating a VECM it is necessary to specify which deterministic terms should be included. This is also the case when conducting the Johansen test for cointegration, as the distribution of the test statistics vary with the trend specification. We will come back to this in the next section.

4.4 Estimating the VECM in Stata

When specifying a VECM, we need to specify the appropriate lag order, the cointegration rank and the deterministic trend specification. The test statistics in the Johansen cointegration test depends on the deterministic trend restrictions and the chosen lag length and we therefore need to decide these matters first. We will start by evaluating the possible trend specification and decide the lag order, before testing the rank of the model using the Johansen cointegration test. After deciding on the rank, we will try to restrict the cointegrating vector. We can then estimate the VECM.

4.4.1 Deterministic term specification

When estimating a VECM, we have to specify which deterministic components to include. It is possible to allow for quadratic and/or linear time trends in the levels of the variables as well as a linear trend and/or a constant mean in the cointegrating relationships. Stata allows for five different specifications, described below.

Case	Description
Case 1: Unrestricted trend	Allowing for quadratic trends in the levels of the variables and that the cointegrating relationships are trend stationary.
Case 2: Restricted trend	Allowing for linear trends in the levels of the variables and that the cointegrating relationships are trend stationary.
Case 3: Unrestricted constant	Allowing for linear trends in the levels of the variables and cointegrating relationships which are stationary around a constant mean.
Case 4: Restricted constant	Allowing no trends in the levels of the variables and cointegrating relationships which are stationary around a constant mean.
Case 5: No trend	Allowing no nonzero means or trends in the model.

To decide which specification is most appropriate, we evaluate how the variables in a proposed model develop over time and what we believe is appropriate from an economic standpoint. In the cases were we are not certain, several specifications will be considered.

4.4.2 Lag order selection

The second step is to decide how many lagged first differences that should be included in the model. The lag length chosen will be the applicable lag length for the underlying VAR of the VECM, while the VECM will contain one less lag. Several information criteria can be applied when deciding the lag length. Stata reports the final prediction error (FPE), Akaike information criterion (AIC), Schwarz’s Bayesian information criterion (SBIC), the Hannan-

Quinn information criterion (HQIC) and a likelihood-ratio test. Becketti (2013) notes that the FPE and AIC tend to overestimate the lag length, while the HQIC and SBIC provide consistent estimates.

Becketti (2013) notes that the tests are sensitive to the maximum lag length considered. Since we are dealing with quarterly data, a lag length of four, eight or twelve, i.e. one, two or three years, seems natural. However, since we are dealing with a relatively small sample, we will not consider a higher lag length than eight. Furthermore, it does not seem likely that something which happened more than two years ago, or maybe even more than one year ago, has an impact on the changes in the interest rate today.

4.4.3 Testing for the rank of cointegration

After deciding on the trend specification and the lag length, the next step is to identify the rank of the VECM. To find the rank of a model, we use the Johansen cointegration test. When running the test in Stata, we ask for both the trace and max statistic, and reject the null hypothesis when the test statistic is higher than the critical value at the five percent level. When we reject a null hypothesis for a rank of R, we continue with considering the rank $R + 1$. We stop testing when we can no longer reject the null hypothesis at the five percent level.

4.4.4 Constraining the cointegrating vectors

After deciding on the rank of the model, we will construct the VECM. When constructing the VECM, we must specify the number of lags in the underlying VAR, the trend specification and the rank. Stata will then estimate the first difference equations for all the variables in the model, in addition to the cointegrating vectors for the cointegrating equations. If we do not specify any constraints for the cointegrating vector(s), the Johansen normalization procedure will construct a set of standard constraints. In most cases, we will have some beliefs regarding the vectors, and will try to define these and test for their validity.

If we place more restrictions than we have to on the cointegration vectors, Stata will automatically test for the validity of the imposed constraints, using a Likelihood-ratio (LR) test. The null hypothesis of the test is that the imposed constraints are valid, against the alternative hypothesis that they are not. Even though the estimated coefficients are superconsistent, their standard errors are not (Enders, 2004). Thus, the coefficients are not asymptotic t-distributed and inference is invalid. We may still let the t-statistics guide us in restricting the cointegrating vector.

4.4.5 Reviewing the estimated cointegrated equations

After finding cointegrating equations that looks reasonable, the final test is to see whether the combination of variables and cointegrating vector actually forms a cointegrating relationship – i.e. whether the estimated linear combination of the variables is stationary. Becketti (2013) states that it is not clear whether we can make use of formal unit root tests for this purpose and that we must make due with more informal tests. We therefore look at the time-series plot of the residuals of the equations within the sample period, and the corresponding autocorrelation plots. For the estimated cointegrating relation to be stationary, the residuals should vary around zero, have stable variance and show little persistence in the autocorrelation plot.

5. Empirical Analysis

This chapter presents the modelling of the long-term government yield for the two countries. We will start by presenting our US model. The analysis starts with a brief unit root analysis on each variable, as this is essential for the subsequent steps. Only variables found to be I(1) can be included in the cointegration analysis. We will test both levels and first differences of all variables, even those found to be I(0), as we only want to include first differences in the short-run part of the model to more easily be able to interpret the estimated coefficients. After the unit root analysis, we model the long-run relationships, before we include these in the final ECMs. The same structure applies for Norway.

For both countries, we have chosen to divide our full sample in two periods. We use the pre-2007 period (Q1990-Q4 2006 for the US and Q1 1991-Q4 2006 for Norway) as the base for “normal” times. The starting point is based on the availability of all variables, as data for the VIX index starts in Q1 1990, and data for the Norwegian folio rate starts in Q1 1991. We have chosen Q4 2006 as the cut-off point to avoid including as much disturbance as possible also from the last several quarters before the outbreak of the financial crisis. In the post-2007 period the part that is included in model estimations starts in Q1 2007 and ends in Q4 2012, since we wish to keep two years left for out-of-sample analysis.

The estimated models and its implications will be discussed as they are presented in this chapter, while chapter 6 will give a larger overview as well as comparing the different model estimates.

5.1 USA

5.1.1 Unit root analysis

For all variables, we assume no deterministic trend. Several of our variables seem to exhibit a trend, but we believe this to be stochastic. When conducting the DF-GLS tests, we need to specify a maximum lag length. A maximum lag length of zero, four or eight seems natural since we are dealing with quarterly data. We will use the number of significant lags found in the autocorrelation plots to determine which maximum lag length we choose for each variable. After deciding on the maximum lag length, the DF-GLS test shows three different lag length tests. We will base our conclusion mainly on the test statistics for the lag lengths suggested by these tests. The output from the DF-GLS tests are presented in the Appendix.

10-year rate

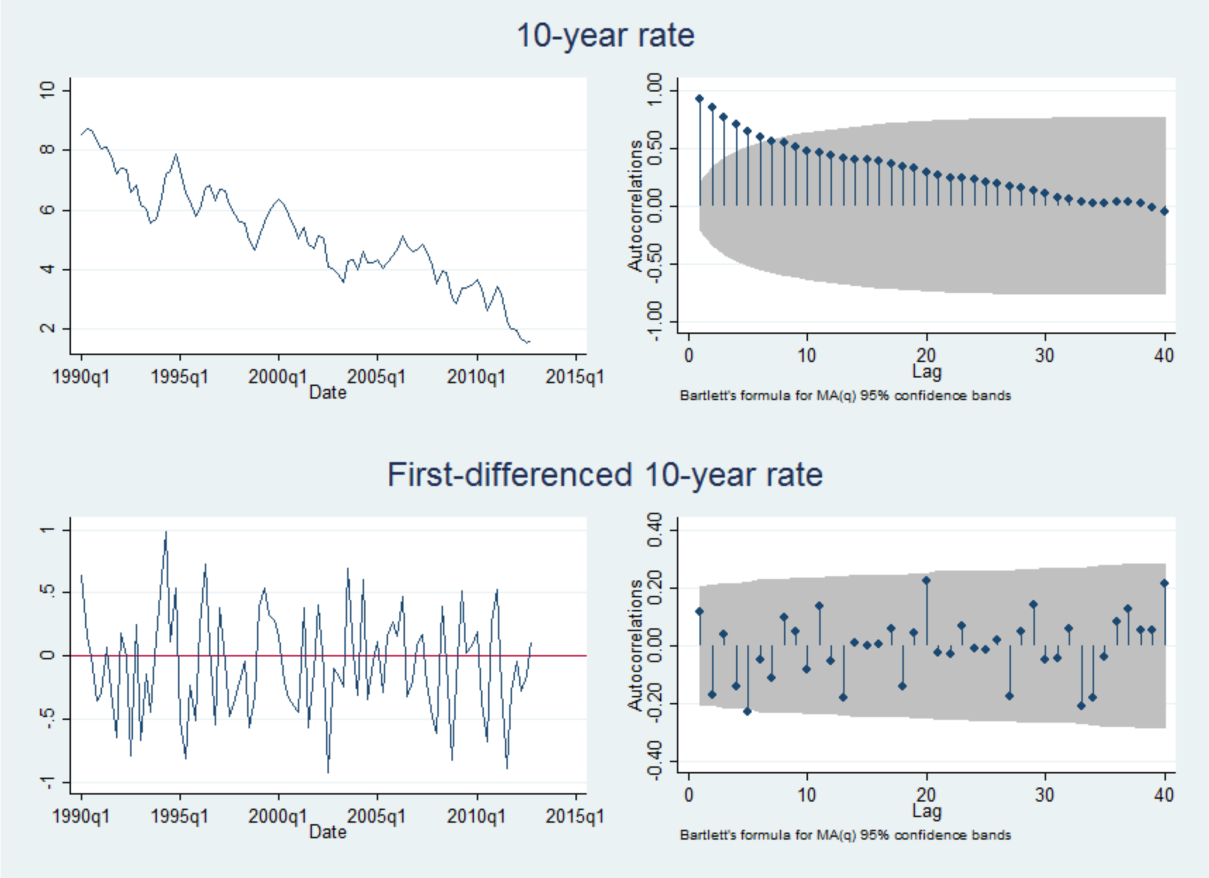

Figure 5.1.1 – Levels and first differences of the 10-year rate with corresponding autocorrelations

Based on the autocorrelation plot in figure 5.1.1 for the 10-year rate in levels, we test for a maximum lag length of eight, which gives us the output shown in table A.1. The lag length tests suggest a lag length of four and one lags. None of the corresponding test statistics are lower than the critical values and we therefore accept the null hypothesis of a unit root.

We then test the first-differenced 10-year rate. Here, the autocorrelation plot shows a negative autocorrelation at lag five that may be significant. However, it is only at lag five and is only barely outside the 95% confidence bands. We therefore test for a maximum lag length of zero, and can conclude that null hypothesis can be rejected (see table A.2) – the 10-year rate is I(1), as expected.

Policy rate

Figure 5.1.2 - Levels and first differences of the policy rate with corresponding autocorrelations

The autocorrelation plot for the federal funds target rate is shown in the upper right panel in figure 5.1.2. The plot shows five lags outside the confidence band, leading to us testing with a maximum lag length of eight. From the DF-GLS test (table A.3), we find that we cannot reject the null hypothesis of a unit root.

The autocorrelation plot for the first-differenced series shows three lags outside the confidence band. We test for a maximum lag length of four and find that we can reject the null hypothesis at the 1% level for all suggested lag lengths (see table A.4). The policy rate thus seems to be I(1).

Expected inflation

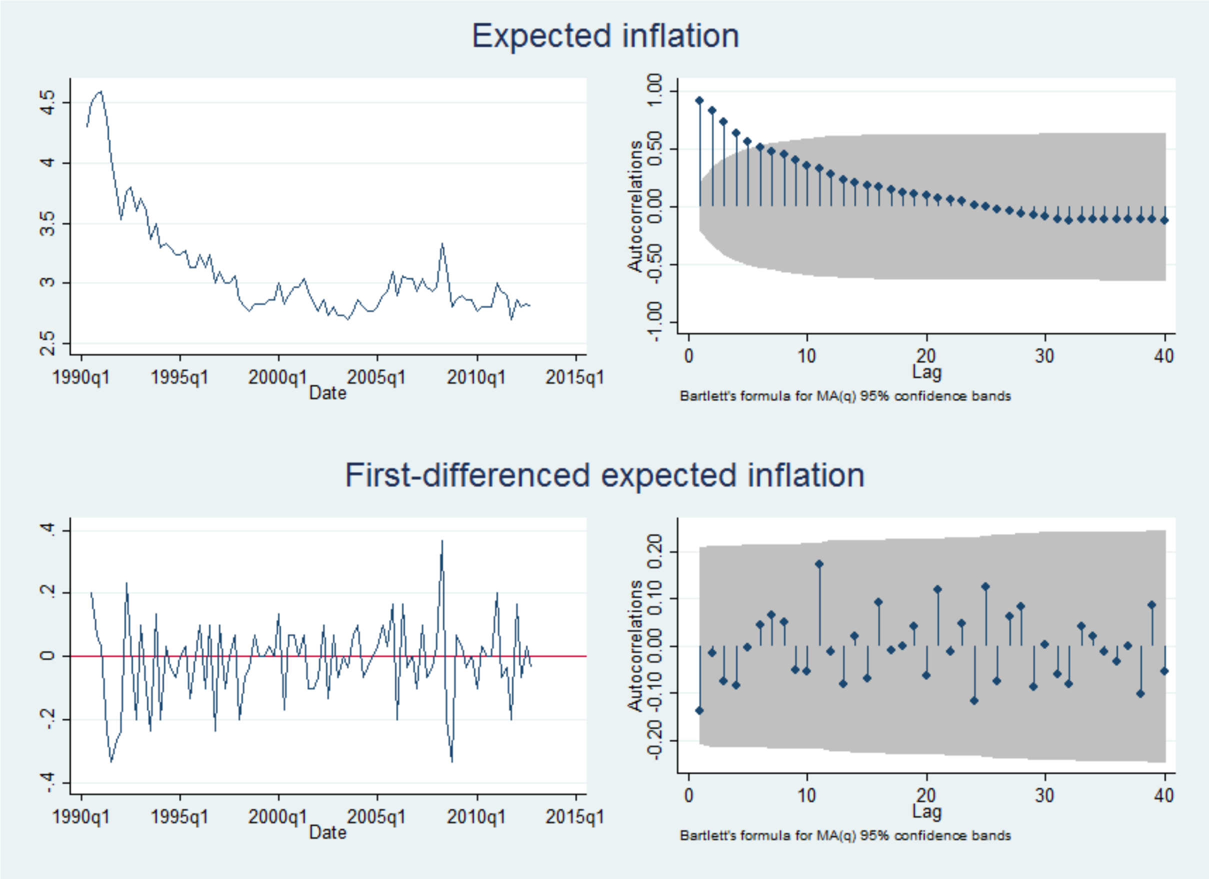

Figure 5.1.3 - Levels and first differences of the expected inflation with corresponding autocorrelations

Based on the autocorrelation plot shown in figure 5.1.3, we conduct the DF-GLS test with the maximum lag length set to eight. The output of the test is presented in table A.5. We cannot reject the null hypothesis of a unit root.

The autocorrelation plot for the first-differenced series shows no lags with significant autocorrelation. We therefore test the first differenced series assuming a maximum lag length of zero (see table A.6). We can strongly reject the null hypothesis and can conclude that expected inflation is integrated of order one.

Current account

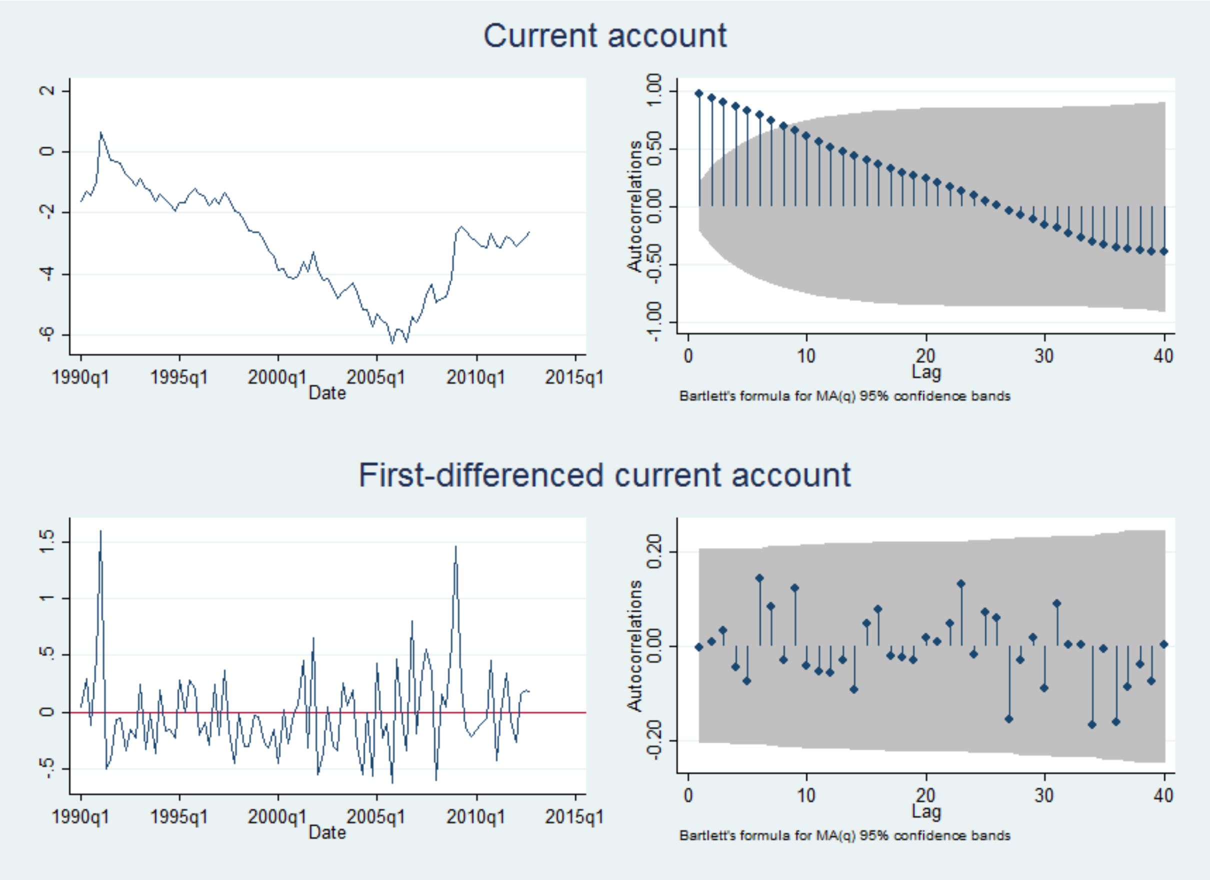

Figure 5.1.4 - Levels and first differences of the current account with corresponding autocorrelations

The current account has seven lags of autocorrelation, according to the autocorrelation plot in figure 5.1.4. We thus allow for a maximum lag length of eight when conducting the DF-GLS test, presented in table A.7. We cannot reject the null hypothesis of a unit root at the suggested lag lengths.

Table A.8 shows the output for the DF-GLS test conducted on the first-differenced series. Maximum lag length were set to zero, as the autocorrelation plot indicated no autocorrelated lags. We see that we can clearly reject the null hypothesis and conclude that the current account is I(1).

Gross government debt

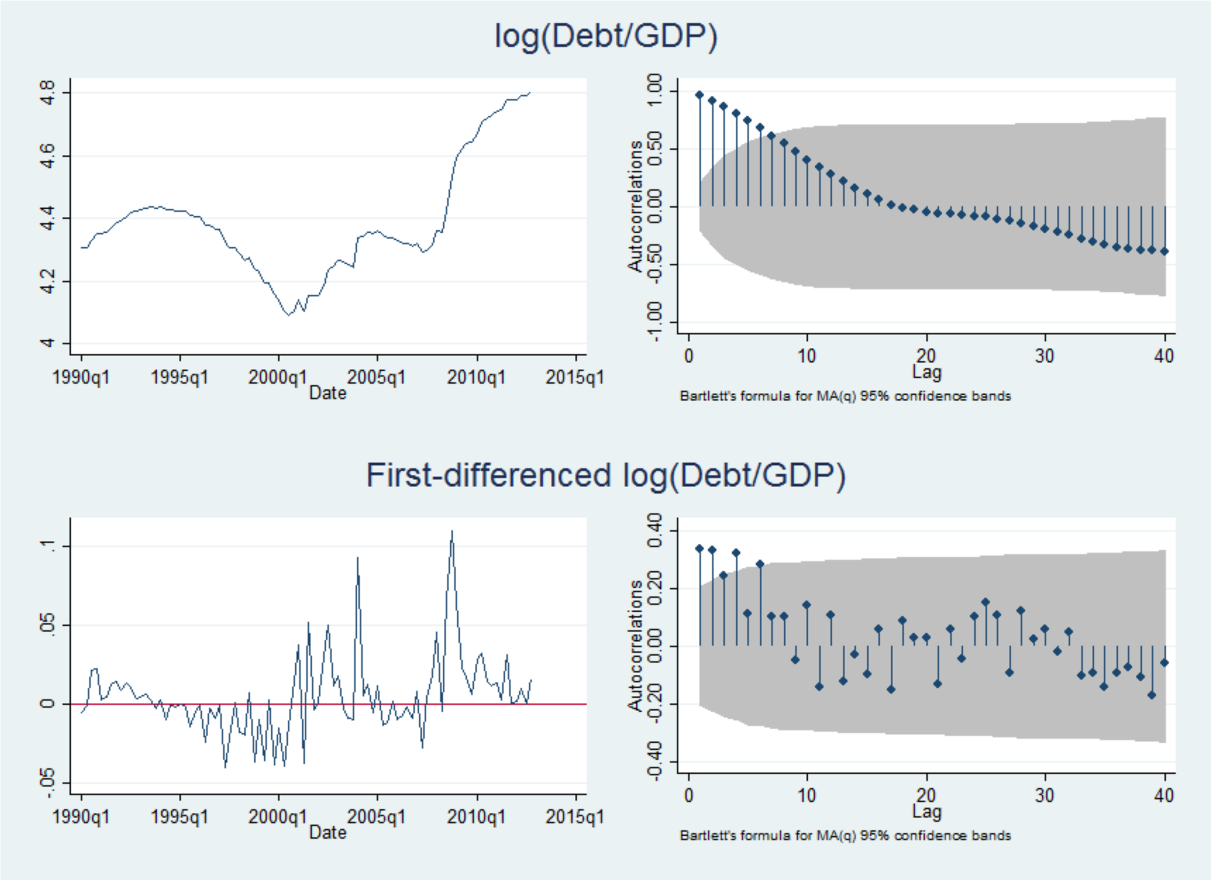

Figure 5.1.5 - Levels and first differences of log(Debt/GDP) with corresponding autocorrelations

Figure 5.1.5 shows the relevant plots for log(Debt/GDP). There are six lags outside the confidence band in the autocorrelation plot in the upper right panel, so we conduct the DF-GLS test with a maximum lag length of eight. The test output is shown in table A.9. Lag lengths two and six are suggested – we cannot reject the null hypothesis at either lag length.

The autocorrelation plot for the first-differenced series shows two, perhaps even four or six, significant lags. We start by testing with a maximum lag length of eight, shown in table A.10. The lag length tests suggest a lag length of one or five, for which we can reject the null hypothesis at the 1% or 10% level respectively. For all lag lengths shorter than five, we can reject on the 1% or 5% level.

Non-borrowed reserves

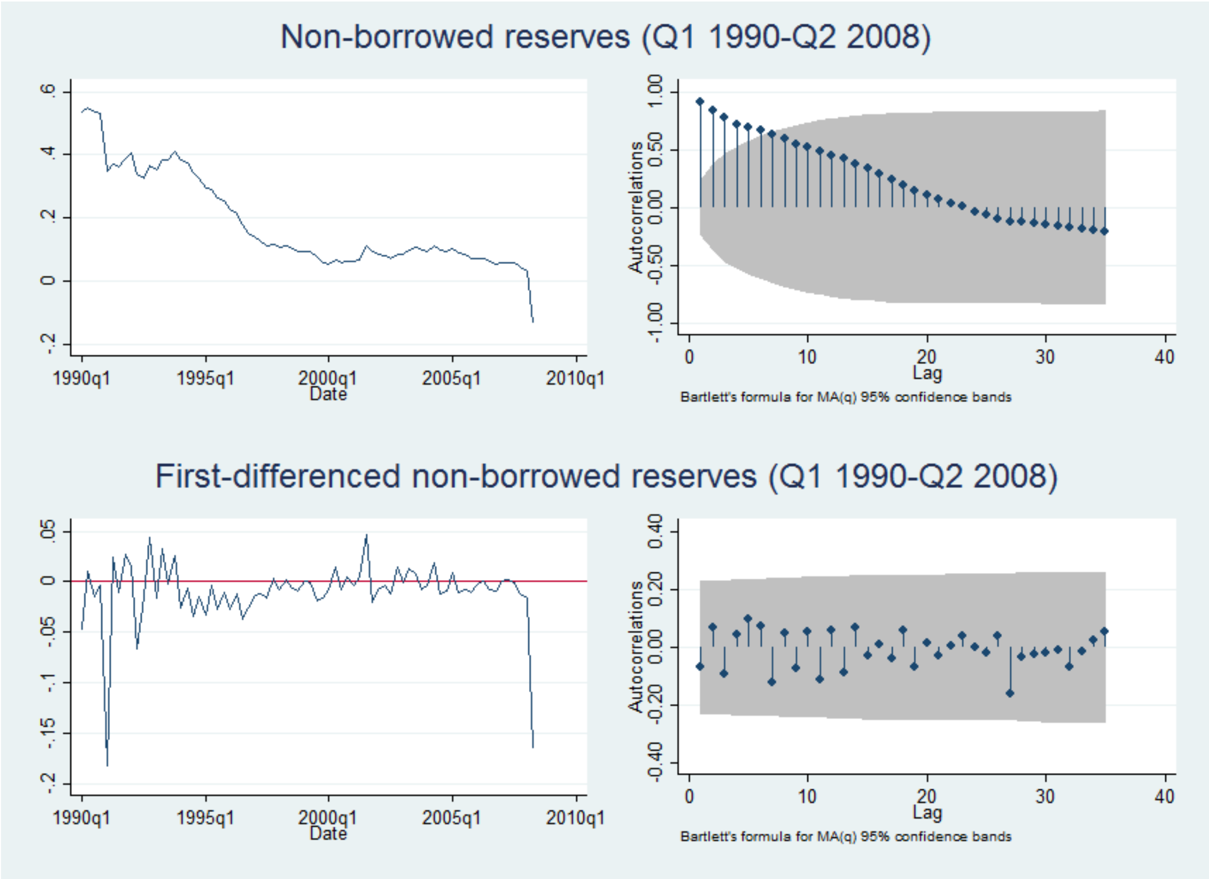

Figure 5.1.6 - Levels and first differences of non-borrowed reserves with corresponding autocorrelations, Q1 1990-Q2 2008

Because reserves have a structural break between Q2 and Q3 2008, the period before and after the structural break must either be tested separately, or in a test that accounts for the structural break. We choose to test the two periods separately.

We first test for the period before the structural break. Based on the autocorrelation plot in figure 5.1.6, we conduct the DF-GLS test with a maximum lag length of eight. The test statistics are all positive, and we cannot reject the null hypothesis (see table A.11). The test for the first-differenced series is shown in table A.12. We can strongly reject the null hypothesis of a unit root and conclude that reserves before the structural break are integrated of order one.

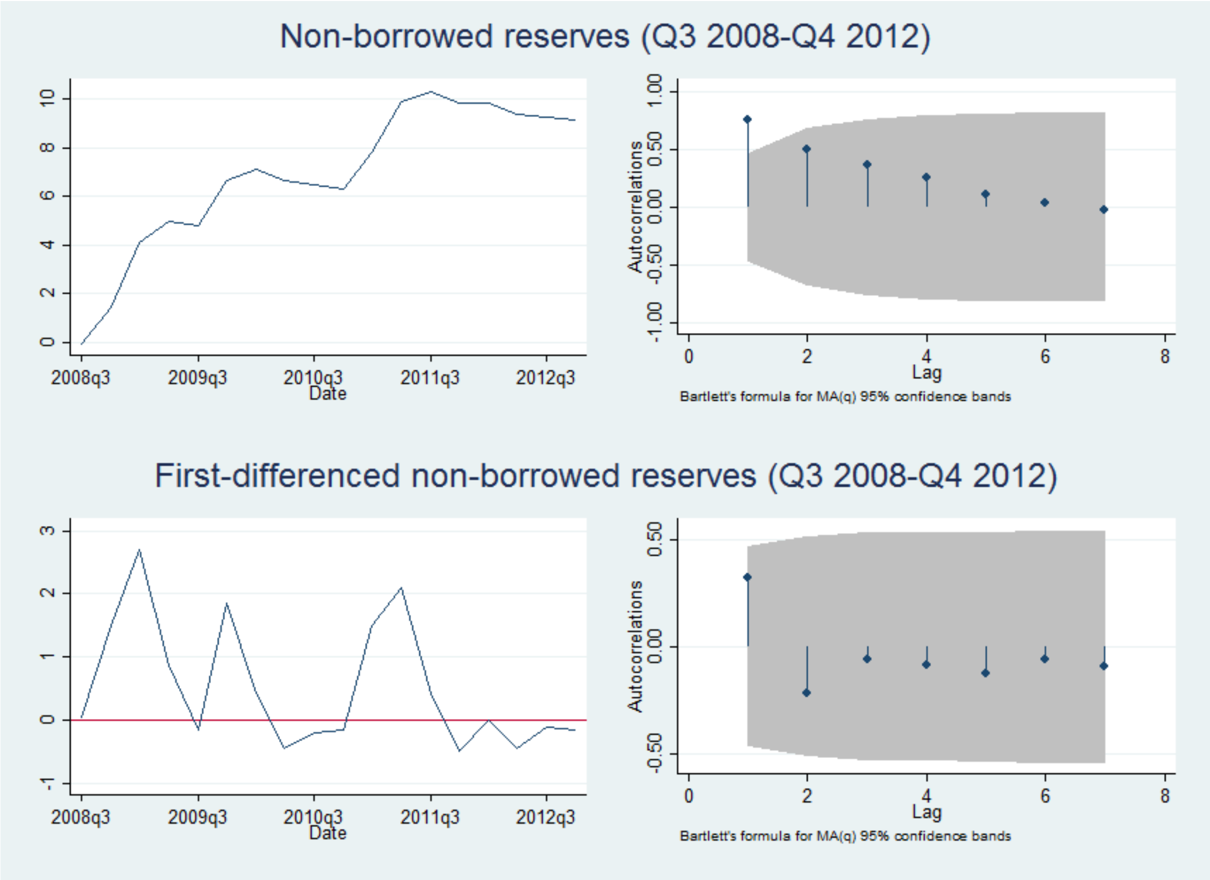

Figure 5.1.7 - Levels and first differences of non-borrowed reserves with corresponding autocorrelations, Q3 2008-Q4 2012

For reserves after the structural break, the autocorrelation plot (shown in figure 5.1.7) for the levels suggests one lag. We thus test with a maximum lag length of four, and find that we cannot reject the null hypothesis (see table A.13). We move on to test the first-differenced series, which has no significant lags. The test output is shown in table A.14 and shows that we can strongly reject the null hypothesis. As can be seen from figure 5.1.7, the sample after the structural break is very small, only containing 18 observations. This reduces the power of the tests, making the conclusions less certain. However, the conclusions from the tests correspond to our beliefs, i.e. that reserves are I(1), also after the structural break.

VIX index

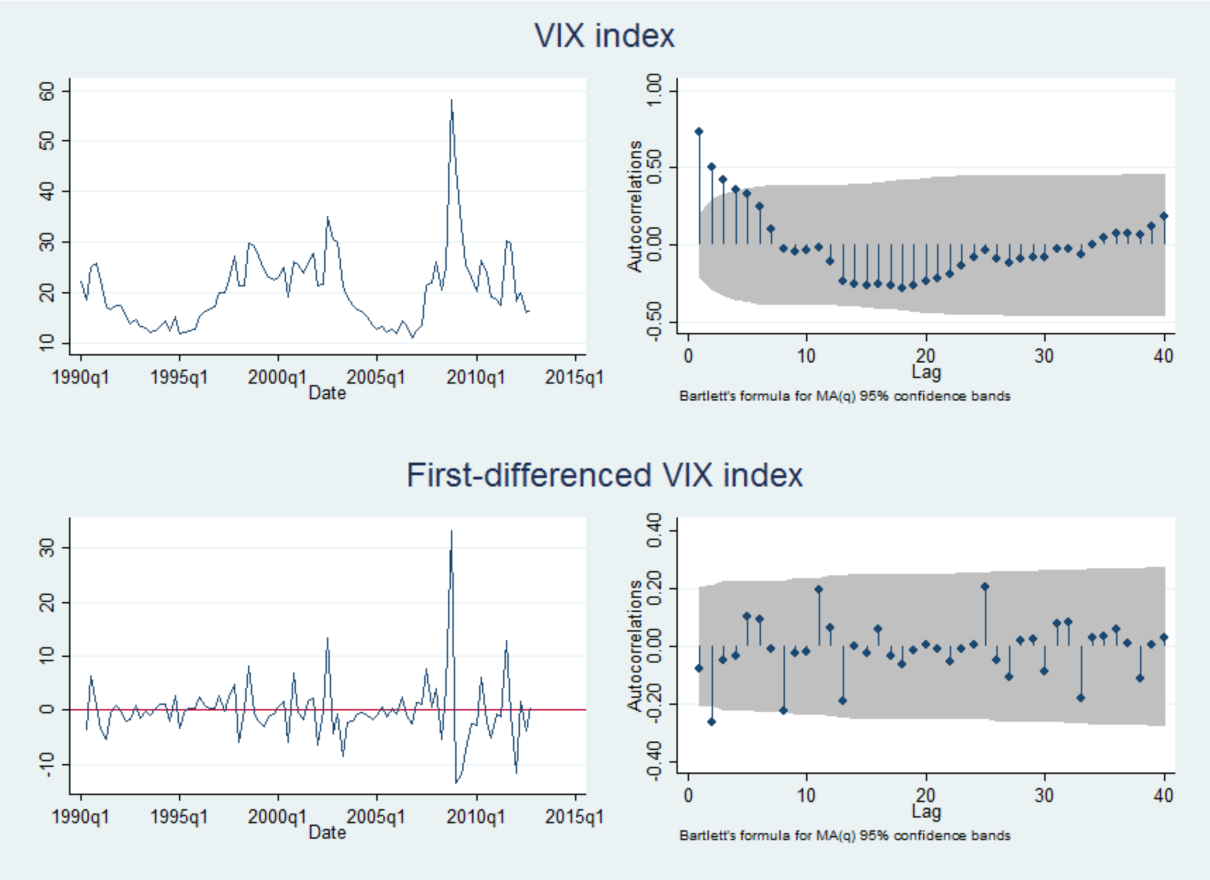

Figure 5.1.8 - Levels and first differences of the VIX index with corresponding autocorrelations

The plots of the VIX index, displayed in figure 5.1.8, indicate that it is stationary in both levels and first differences. DF-GLS tests for both are conducted with a maximum lag length of four, and we can reject the null hypothesis of a unit root in both cases (see tables A.15 and A.16). We thus conclude that VIX is stationary in both levels and first differences.

PMI index

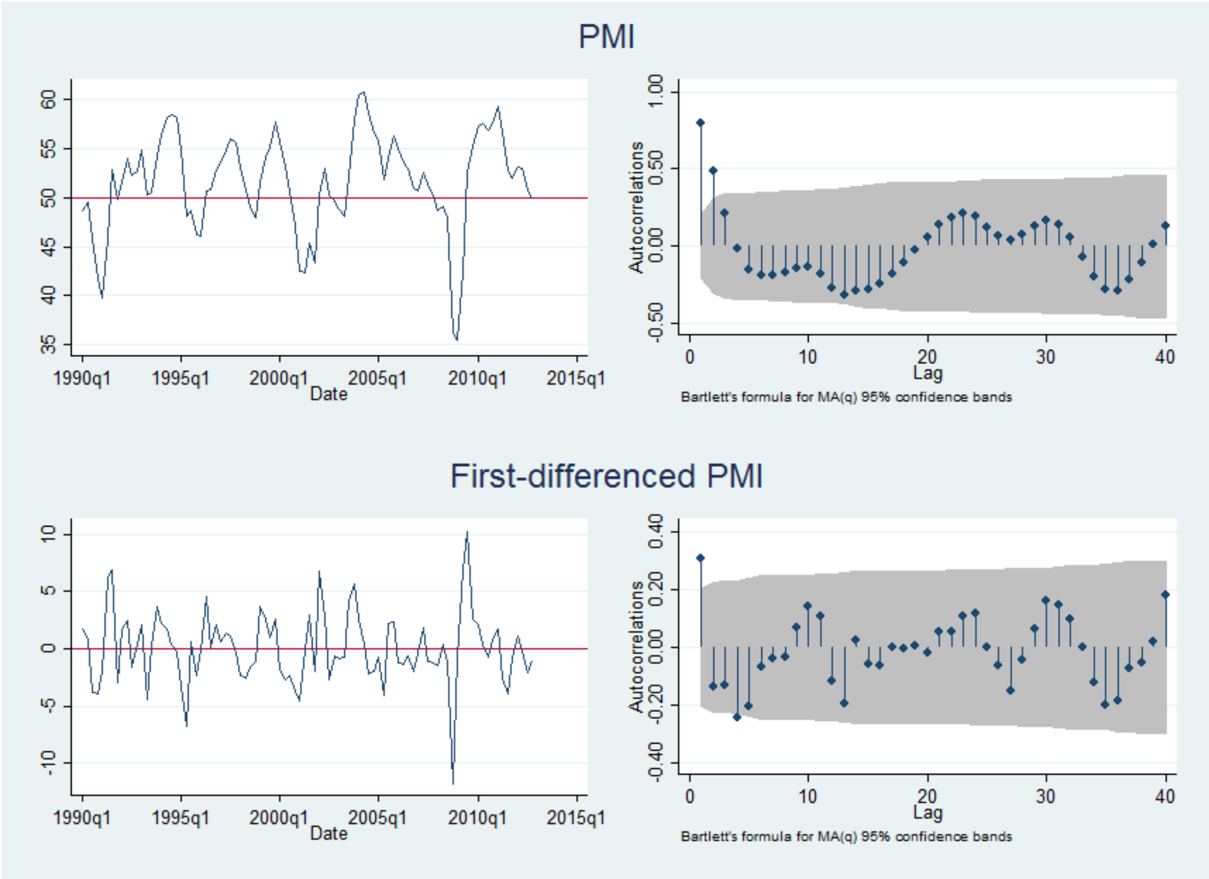

Figure 5.1.9 - Levels and first differences of the PMI index with corresponding autocorrelations

The PMI index also looks stationary, judging from the plot shown in figure 5.1.9. DF-GLS tests are conducted for the PMI index in levels and first differences, and the output for the two tests is presented in tables A.17 and A.18. We can conclude that the PMI index is stationary both in levels and first differences.

5.1.2 Estimating the long-run relationship – Q1 1990-Q4 2006

Evaluating the time-series

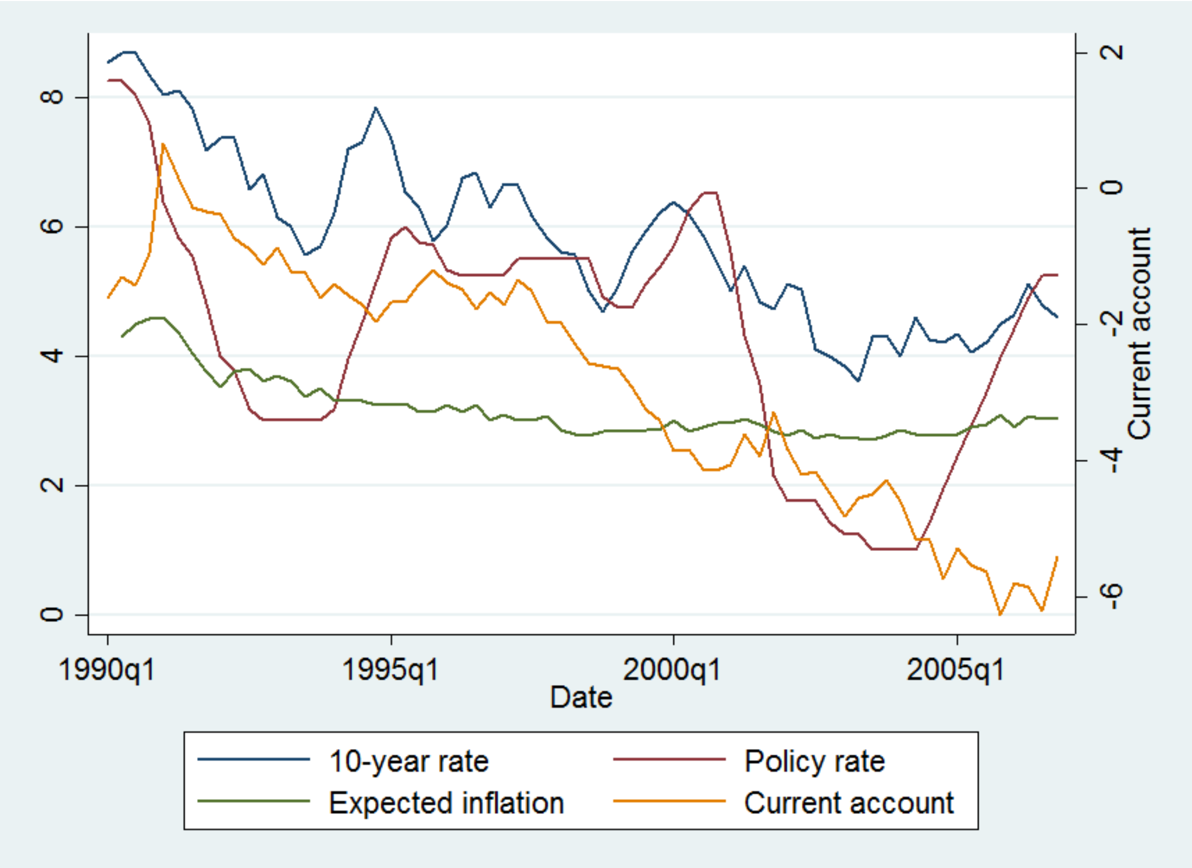

Figure 5.1.10 - I(1)-variables included in long-run part of US model, Q1 1991-Q4 2006

The US variables that are integrated of order one, and thus could be included in a Johansen test for cointegration are the 10-year rate, the policy rate, expected inflation, the current account, gross government debt and reserves. We do not believe that neither debt nor reserves are likely to be a part of a stable long-run relationship with the 10-year rate. We did experiment with including them, but this did not yield fruitful results.

In the sample ending in 2006 (see figure 5.1.10), we see from the plot that the 10-year rate, policy rate, current account and expected inflation, may have some sort of common long-run trend. If we look at the plot of the series for the entire period for which we have data (see figure 5.1.11), the common developments seem much less clear. In particular, the current account have experienced a total turnaround since 2006. We conduct separate analyses of the long-run relationship in Q1 1990-Q4 2006 and Q1 1990-Q4 2012, with the former being analyzed in this section and the latter being analyzed in chapter 5.1.3.

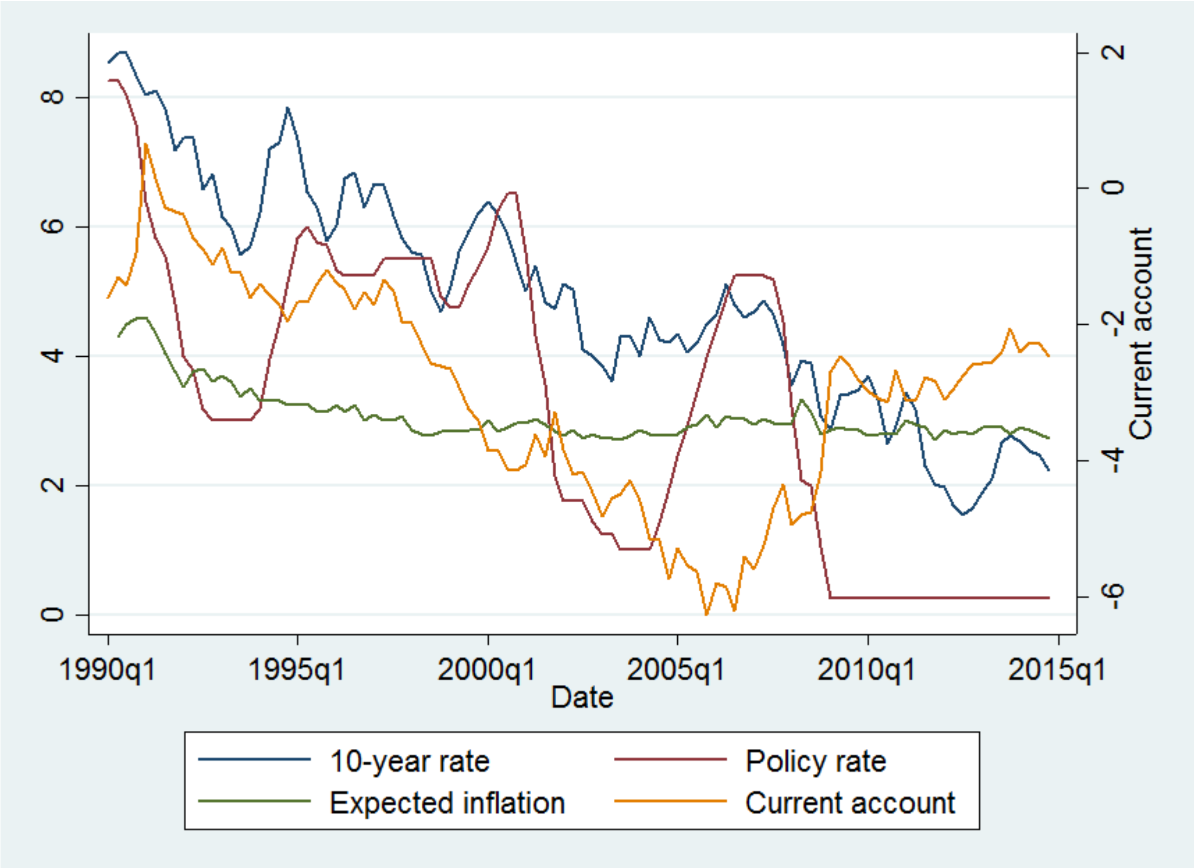

Figure 5.1.11 - I(1)-variables included in long-run part of US model, Q1 1991-Q4 2014

Checking appropriate lag length

We will in this section present our results from the lag testing on the underlying VAR model of the VECM. The conclusion of the criteria depends on how many lags they evaluate. We will therefore test both when specifying a maximum lag length of four, and a maximum lag length of eight. Stata output for both tests are shown in tables 5.1.1 and 5.1.2. We have tested for a VAR model for the 10-year rate, expected inflation rate, policy rate and current account. We find support for lag length two, three, four and eight, and will test for these lag lengths in the following rank testing.

Table 5.1.1 - Lag-order selection statistics for the underlying VAR model, with maximum lag length set to four

Selection-order criteria
Sample: 1991q2 - 2006q4 Number of obs = 63

lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-285.769				.116196	9.19902	9.25254	9.33509
1	-6.56941	558.4	16	0.000	.000027	.843473	1.11106	1.52383
2	39.5806	92.3	16	0.000	.000011*	-.113669*	.367991*	1.11098*
3	55.455	31.749	16	0.011	.000011	-.109684	.586047	1.65925
4	70.0331	29.156*	16	0.023	.000012	-.064544	.845259	2.24868

Table 5.1.2 – Lag-order selection statistics for the underlying VAR model, with maximum lag length set to eight.

Selection-order criteria
Sample: 1992q2 - 2006q4 Number of obs = 59

lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-256.219				.079619	8.821	8.87598	8.96185
1	-3.14377	506.15	16	0.000	.000026	.784534	1.05945	1.48878
2	46.4937	99.275	16	0.000	8.3e-06	-.355717	.139122*	.911933*
3	63.0926	33.198	16	0.007	8.3e-06*	-.37602*	.338747	1.45503
4	73.6318	21.078	16	0.176	.00001	-.190908	.743788	2.20354
5	92.8661	38.469	16	0.001	9.9e-06	-.300547	.854078	2.6573
6	105.77	25.807	16	0.057	.000012	-.195579	1.17897	3.32567
7	116.749	21.959	16	0.145	.000017	-.025385	1.5691	4.05926
8	138.76	44.022*	16	0.000	.000017	-.229143	1.58527	4.41891

Johansen cointegration test

Table 5.1.3 – Results from Johansen cointegration test, sample Q1 1990-Q4 2006. Trend specification: Case 4 – "Restricted constant"

Maximum rank	**Trace statistic**				**5% critical value**
	Lags = 2	Lags = 3	Lags = 4	Lags = 8	
0	53.6252	72.5581	86.6663	68.3765	53.12
1	**29.4864***	43.6605	50.0159	35.5425	34.91
2	14.5953	21.4765	20.0616	**14.9531***	19.96
3	3.9601	**5.9692***	**8.5659***	5.8600	9.42
4					
Suggested rank	1	3	3	2	

* Cannot reject the null hypothesis

When conducting the Johansen cointegration test, we find support for a maximum rank of one for lag length two, a rank of three for lag length three and four, and a rank of two for lag length eight (see table 5.1.3). We therefore consider a rank of one, two or three when modelling the VECM.

Estimating cointegrating relationships

We tried estimating the VECM with rank two or three using various constraints. For example, with an assumed rank of three, we tested for the possibility of the 10-year rate having a separate cointegrating relationship with each of the other variables. However, the assumption that yielded the most fruitful results were a rank of one. Therefore the model we present in the following is a model with rank one.

After testing with various lag lengths, we found that a lag length of four and five gave the most reasonable results. Other lag lengths resulted in relationships that did not seem sensible, and/or a magnitude on the coefficients that did not seem reasonable.

We generally believe that there should be a constant term in the cointegrating equation, but no constant in the short-run part of the model, as this would imply a deterministic trend in the levels of our variables, which seems unreasonable. However, we have additionally examined the possibility of allowing for trends in the variables, to make up for in-sample trending behavior that is not explained in this relatively restricted model, to see whether this can aid us in identifying the long-run relationship(s).

Under both assumptions, the equations did not change much when we went from four lags to five. However, only the two models with four lags are presented, as the models with five lags looked less stationary (when looking at the plots of the cointegrating equations, both exhibited a clear, increasing trend).

Table 5.1.4 presents the two models with our preferred cointegrating equations. Remember that the VECM is on the form as given in equation 19 and 20 in chapter 4.3, and the output given by Stata can therefore be quite extensive, depending on the number of variables, cointegrating equations and lagged first differences that are included in the model. Since we are only estimating these models to estimate the cointegrating equation, we will only present information we find relevant. A lot of the output is therefore not presented in the table. Most importantly, this is the case for the estimated short-run effects, i.e. the estimated coefficients for the first differences. These are not included as they are not important for our purpose, and because excluding them greatly simplifies the table.

The two first rows specify the lag length in the underlying VAR model, and the trend specification. We then move on to present the AIC for the entire model, and the R-squared for all equations in the equation system. The VECMs we estimate only include the variables necessary to estimate the long-run relationship. The R-squared are thus not important information, but they give a rudimentary idea of how well the estimated cointegrated equation will perform in later models – if the R-squared for the 10-year rate is close to zero, the estimated cointegrated equation will probably not help explain the variation in the 10-year rate in later estimations.

The rows titled “Alpha” refer to the adjustment parameters. For instance, the alpha for the 10-year rate refers to the adjustment parameter for the 10-year rate, i.e. it says how strongly the 10-year rate reacts to deviations from the estimated cointegrated equation, according to the estimated model. The last rows, titled CE, present the estimated cointegrated equations. The equations are presented on the same form as in equation 10 in chapter 4.2. All adjustment parameters refer to the equation presented in the CE-part of the table. These adjustment parameters will not be used later, but serve as a rudimentary test of whether the estimated cointegrated equations are reasonable. For instance, if the alpha for the policy rate is -38, the estimated model is most likely incorrect.

Table 5.1.4 - Estimated cointegrating relationships and key information from the estimated VECM

	Variable	**Model 1**	**Model 2**
Lags		4	4
Trend specification		Restricted constant	Unrestricted constant
AIC		0.3484	0.1743
R-sq	10-year rate	0.3845***	0.4914***
	Expected inflation	0.3780***	0.3736***
	Policy rate	0.7384***	0.7408***
	Current account	0.2170	0.2991

Alpha	10-year rate	-0.5768*** (0.1511)	-0.7849*** (0.1516)
	Expected inflation	0.1105** (0.449)	0.0616 (0.0498)
	Policy rate	-0.0833 (0.1058)	-0.0529 (0.1162)
	Current account	0.2262* (0.1309)	0.1566 (0.1367)
CE	10-year rate	1	1
	Expected inflation	-1.3179*** (0.1986)	-1.1631*** (0.1832)
	Policy rate	-0.2038*** (0.0432)	-0.2015*** (0.0399)
	Current account	-0.3479*** (0.0536)	-0.3400*** (0.0494)
	Constant term	-1.8980 *** (0.7870)	-2.2341

*Significant at 10% level **Significant at 5% level ***Significant at 1% level   Standard errors in parentheses
Stars in R-sq row reports whether the equations are significant according to a chi^2-test.

Looking at the information in table 5.1.4, we find that the R-squared is higher for the 10-year rate in model 2, with more than a 0.1 increase. While the R-squared is not very high for any of the models, we do note that the chi^2-test can reject the null hypothesis of no joint significance of all the estimated parameters at the 1% level.

We have also reported the AIC of the two models. There is a sizeable decrease in the AIC when moving from model 1 to model 2. This means that both the R-squared and the AIC favors model 2.

When evaluating cointegrating relationships, we have primarily looked at whether the sign of the coefficients is as expected, based on economic beliefs, and whether the magnitude seems reasonable. Looking at the cointegrating equations for the models, we see that the magnitude seems sensible. Both models imply that there is a positive relationship between the 10-year rate and the other variables, which is as expected. We also note that the coefficient of the inflation rate is above one. This implies that the 10-year rate will increase more than the expected rate of inflation itself, which means that the so-called Taylor principle is met (as mentioned in e.g. Hellum (2010)). We see that our coefficient for inflation expectations is smaller than the one found in Hellum (2010), ours is 1.16, and his is

1.71. Our coefficient for the policy rate is also smaller than his coefficient on the 3-month government bill rate, 0.20 and 0.36 respectively, while our coefficient on the current account is larger than his, 0.34 and 0.18 respectively.

We have also looked at the speed of adjustment parameters, i.e. the alphas. We have normalized on the 10-year rate, meaning that a positive residual means that the 10-year rate is above its long-run equilibrium (or the other variables below). For the variables to return to the equilibrium, they must adjust towards it. The most common case is that the chosen normalized variable, for us the 10-year rate, adjusts downwards following a positive disequilibrium, while the other variables should adjust upwards. An adjustment coefficient with an absolute value of one means that the variable immediately moves to the equilibrium. In most practical cases, variables will slowly move towards the equilibrium. We therefore expect to find coefficients with an absolute value less than one. Since we are estimating the VECM mainly to estimate the cointegrating equation, which later will be used in an ECM for the 10-year rate, we will mainly focus on the alpha for the 10-year rate.

In the estimated VECM, the 10-year rate has a negative alpha as expected, with a relatively large magnitude. The 10-year rate has a higher speed of adjustment in model 2. The other alphas look as expected, expect for the alphas for the policy rate, which is negative. However, the coefficients are close to zero and not significant. Arguably, the policy rate will depend on much more than long-term interest rates and the estimated alphas therefore does not give reason for concern.

Judging by the cointegrating equations and the alphas, there is little difference between the two models. We know that model 2 has the best fit, which should imply that we should apply model 2. However, we are also dependent on the estimated cointegrated equation to be I(0). We will therefore evaluate both of them.

Reviewing the estimated long-run relationships

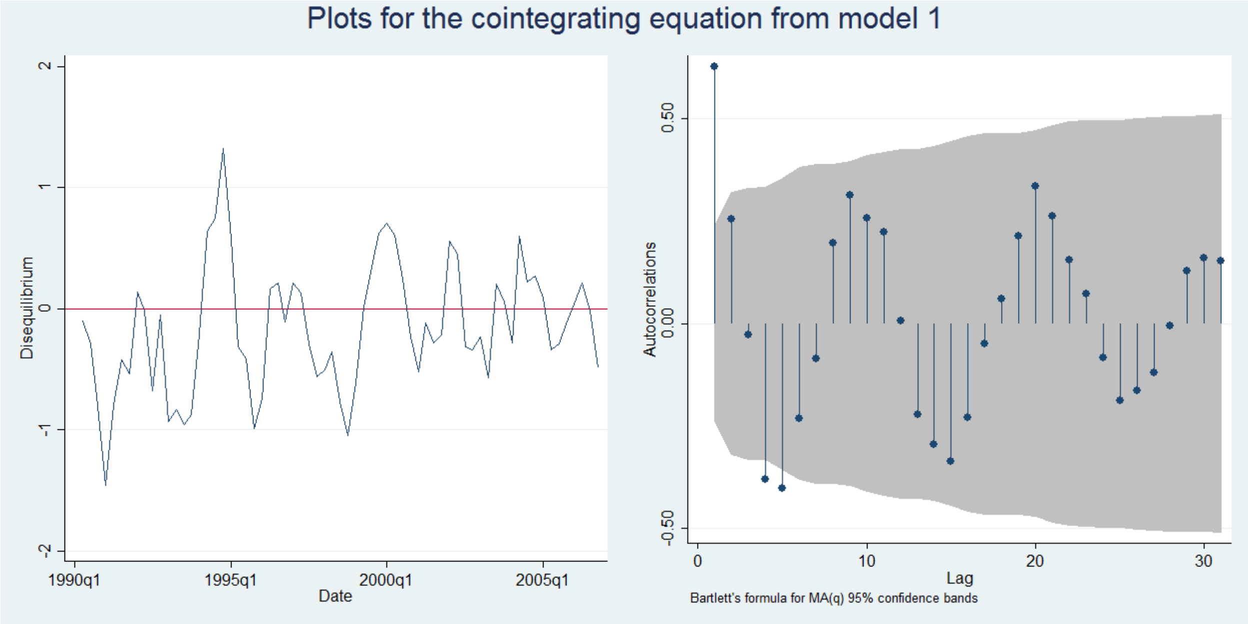

Figure 5.1.12 – Disequilibrium and autocorrelation plot for the cointegrating equation from model 1

In figure 5.1.12, we see that the cointegrating equation vary around zero, but there seems to be a small, increasing trend, or a nonzero mean, which is unfortunate. The autocorrelation plot looks good, with only one significant lag. The series seem to be stationary, although the time-series plot indicates that the cointegrating equation is missing something.

Since there seemed to be something missing from the estimated cointegrated equation above, we tried estimating the model allowing for constant terms in the first-differenced equations, which gave us model 2 above. Allowing for a constant term in the first-differenced equation implies that there is a deterministic linear time trend in the levels of the variables. We do not believe this is the case. However, we are also aware of the limitations of our estimated model. We have tried to include variables that might have a long-run relationship with the 10-year rate, and have neglected variables that might be of importance to the other variables. Especially inflation and the current account both exhibit a decreasing trend throughout the sample period, which we do not believe can be fully explained by the variables in the model. Allowing for a constant term in the first differences might solve this misspecification.

When adding a constant term in the first-differenced equations, we find that both the inflation rate and current account has a negative and significant constant term. Furthermore, the constant terms for the 10-year rate and policy rate both have high p-values (i.e. they are clearly not significant) and are so small that they are negligible. It thus seem like the assumption of a linear trend in the levels does not alter the estimation of the 10-year rate significantly.

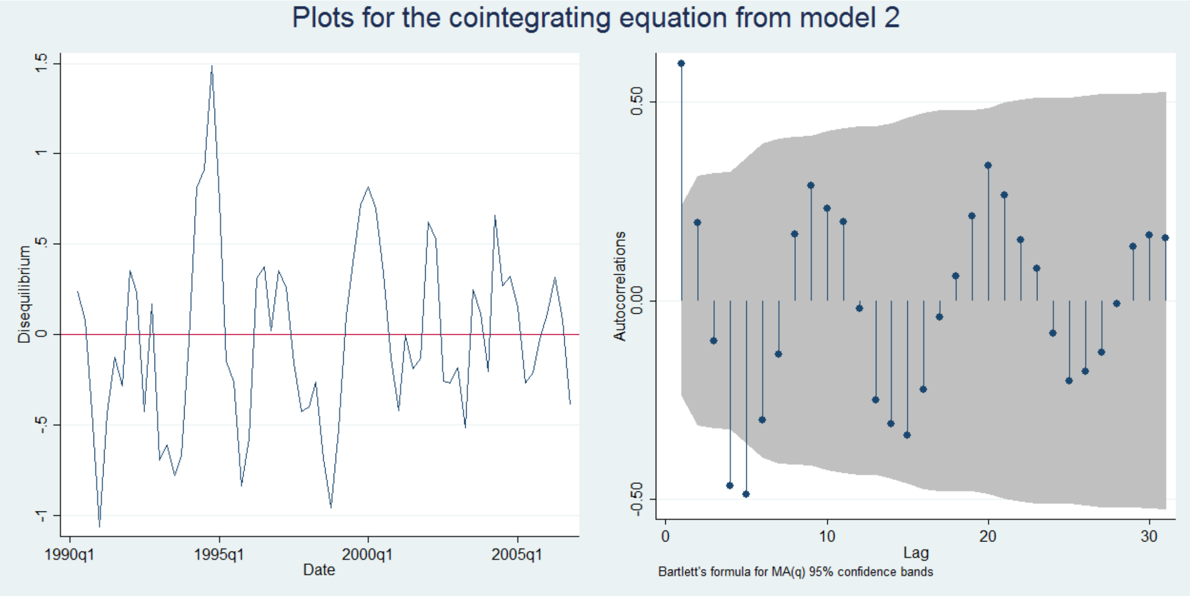

Figure 5.1.13 - Disequilibrium and autocorrelation plot for the cointegrating equation from model 2

Figure 5.1.13 shows the time-series plot and autocorrelation plot of the cointegrating equation from model 2. While the autocorrelation plot shows little change, there seem to be some change in the time-series plot.

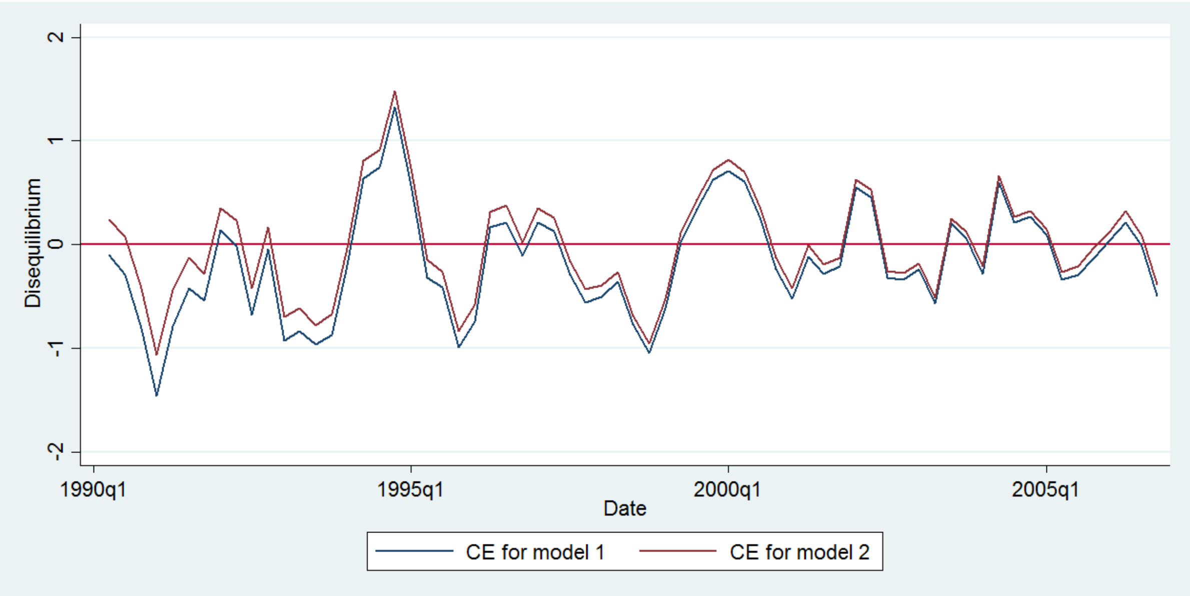

Figure 5.1.14 - Comparison between the estimated cointegrated equations

Figure 5.1.14 shows a plot of both cointegrating equations. While the difference is not large, the cointegrated equation from model 2 has a somewhat less pronounced trend. Since the assumption of a linear trend does not alter the estimated first-differenced 10-year rate, and this model seems to perform somewhat better, we will use the estimated cointegrated equation from model 2 in the ECM.

5.1.3 Estimating the long-run relationship – Q1 1990-Q4 2012

When we now prolong our estimation sample, we tried to re-estimate the cointegrating equation between the 10-year rate, the expected inflation rate, the federal funds target rate, and the current account. However, now we did not get any fruitful results with this combination of variables; it seems the estimated relationship either was wrongfully specified or has broken down during the crisis years, like we suggested when discussing figure 5.1.11. We therefore tried to identify some other stable long-run relationship.

Since we would like to examine whether anything changed from our previous sample to this, we looked at the correlation matrix for the pre-2007 period (table 5.1.5) and for the post-2007 period (table 5.1.6) as a first approach. Only the expected inflation rate and the policy rate seem to maintain the same relation to the 10-year rate in both periods. Both retain a positive and high correlation with the 10-year rate, although the fall in correlation between the 10-year rate and expected inflation of 0.25 is noteworthy.

We did test for a cointegrating relationship between all three variables, but this did not yield sensible results. It seems like a cointegrating relationship between the 10-year rate and the policy rate has the most support, both based on preliminary cointegration testing and VECM-modeling, and the sample correlations. The estimation of a long-run relationship between the 10-year rate and policy rate is presented below.

Table 5.1.5 - Correlation matrix, sample Q1 1990 – Q4 2006 (67 observations)

	10-year rate	Policy rate	Expected inflation	Current account
10-year rate	1.0000			
Policy rate	0.6846	1.0000		
Expected inflation	0.8104	0.4685	1.0000	
Current account	0.8135	0.4379	0.6892	1.0000

Table 5.1.6 - Correlation matrix, sample Q1 2007 - Q4 2012 (24 observations)

	10-year rate	Policy rate	Expected inflation	Current account
10-year rate	1.0000			
Policy rate	0.7569	1.0000		
Expected inflation	0.5555	0.4614	1.0000	
Current account	-0.7245	-0.8929	-0.6204	1.0000

Checking appropriate lag length

Table 5.1.7 and 5.1.8 show tests for the appropriate lag length for the underlying VAR model between the 10-year rate and policy rate. The lag test assuming a maximum lag length of four mostly supports three lags, except for the more parsimonious SBIC criterion which suggests only two lags. When the test allowed for eight lags, only the LR statistic changed its conclusion – from three to eight lags.

Table 5.1.7 - Lag-order selection statistics for the underlying VAR model, with maximum lag length set to four

Selection-order criteria
Sample: 1990q1 - 2012q4 Number of obs = 92

lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-334.53				5.1558	7.31587	7.338	7.3707
1	-98.496	472.07	4	0.000	.033238	2.27165	2.33803	2.43612
2	-45.7703	105.45	4	0.000	.011526	1.2124	1.32303	1.4865*
3	-38.8484	13.844*	4	0.008	.010821*	1.14888*	1.30376*	1.53263
4	-37.952	1.7928	4	0.774	.011584	1.21635	1.41548	1.70974

Table 5.1.8 - Lag-order selection statistics for the underlying VAR model, with maximum lag length set to eight

Selection-order criteria
Sample: 1990q1 - 2012q4 Number of obs = 92

lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-334.53				5.1558	7.31587	7.338	7.3707
1	-98.496	472.07	4	0.000	.033238	2.27165	2.33803	2.43612
2	-45.7703	105.45	4	0.000	.011526	1.2124	1.32303	1.4865*
3	-38.8484	13.844	4	0.008	.010821*	1.14888*	1.30376*	1.53263
4	-37.952	1.7928	4	0.774	.011584	1.21635	1.41548	1.70974
5	-35.8942	4.1155	4	0.391	.012096	1.25857	1.50196	1.86161
6	-31.8864	8.0156	4	0.091	.012112	1.2584	1.54604	1.97108
7	-30.9655	1.8418	4	0.765	.012977	1.32534	1.65723	2.14766
8	-20.7725	20.386*	4	0.000	.011374	1.19071	1.56685	2.12267

Johansen cointegration test

Table 5.1.9 - Results from Johansen cointegration test, sample Q1 1991-Q4 2012. Trend specification: Case 4 – "Restricted constant"

Maximum rank	**Trace statistic**			**5% critical value**
	Lags = 2	Lags = 3	Lags = 8	
0	**18.3048***	22.2656	23.4853	19.96
1	3.6126	**2.8280***	10.3657	9.42
2				
Suggested rank	0	1	2	

* Cannot reject the null hypothesis

We conduct the Johansen cointegration test for two, three and eight lags, which are the lag length suggested by the lag testing. The results are shown in table 5.1.9. We see that the test yields different conclusions for each of the lag lengths we test for. The unit root testing concluded that both variables were I(1), so we believe that the true rank is either one or zero, as a rank of two would suggest that both variables are actually I(0). The lag test gave overwhelming support to a lag length of three, for which the test supports cointegration between the two variables. As we now have some support for a cointegrating relationship between the 10-year rate and the policy rate, the next step is to see whether we can estimate a sensible cointegrating equation between them.

Estimating cointegrating relationships

We have found support for three lags in the underlying VAR model, and the Johansen test concluded that there was a cointegrating relationship when applied on that particular model. We have only allowed for a constant term in the cointegrating equation when estimating the VECM. Although testing primarily supported three lags in the underlying VAR model, i.e. two lags in the VECM, we have also estimated the VECM for other lag lengths. The estimated cointegrated equation seemed quite robust for differing lag lengths. The model presented below is for the model with three lags in the underlying VAR, i.e. the model supported by the preliminary testing. This model had the cointegrating equation that performed best in the informal stationarity testing, as well as the lowest AIC and one of the highest R-squared.

Table 5.1.10 - Estimated cointegrating relationship and key information from the estimated VECM

	Variable	**Model 1**
Lags		3
Trend specification		Restricted constant
AIC		1.1361
R-sq	10-year rate	0.0867
	Policy rate	0.7034***
alpha	10-year rate	-0.0137 (0.0336)
	Policy rate	0.0860*** (0.0215)
CE	10-year rate	1

	Policy rate	-0.9993*** (0.1244)
	Constant term	-1.3662** (0.5523)

*Significant at 10% level **Significant at 5% level ***Significant at 1% level
Standard errors in parentheses

In table 5.1.10, we see that the R-squared for the first difference equation of the 10-year rate is lower in this model than the model for period 1. We need to keep in mind that this model only includes the 10-year rate and the policy rate, so a low R-squared is not surprising. Both alphas have the expected sign, although only the alpha for the policy rate is significant. Note that neither the chi^2-test nor the alpha are significant at the 10% level for the 10-year rate. However, while the chi^2-test had a p-value of 0.1472, the alpha has a p-value of 0.683. This indicates that the 10-year rate might not react significantly to disequilibria from this estimated cointegration relationship when we add it in an ECM.

The estimated relationship indicates that there is a positive relationship between the two variables. We can compare the estimated relationship with the estimated relationship for period 1. We see that now that the relationship only includes the 10-year rate and the policy rate, the policy rate has a much larger coefficient, likely making up for missing variables.

Reviewing the estimated long-run relationships

Figure 5.1.15 shows the time-series plot and autocorrelation plot for the estimated cointegrated equation. The time-series plot looks quite volatile, but it varies around zero and the variance seems stable. The autocorrelation plot show signs of some persistence - there are four significant lags. The autocorrelation plot does not look as good as it did for the last period. Since the alpha is not significant at the 10% level and the autocorrelation plot shows signs of persistence, we are not sure how well the error correction term will perform in the ECM. However, the estimated relationship looks sensible, and performs well in the time-series plot, so we will include the error correction term in the initial ECM-modelling to see how it performs.

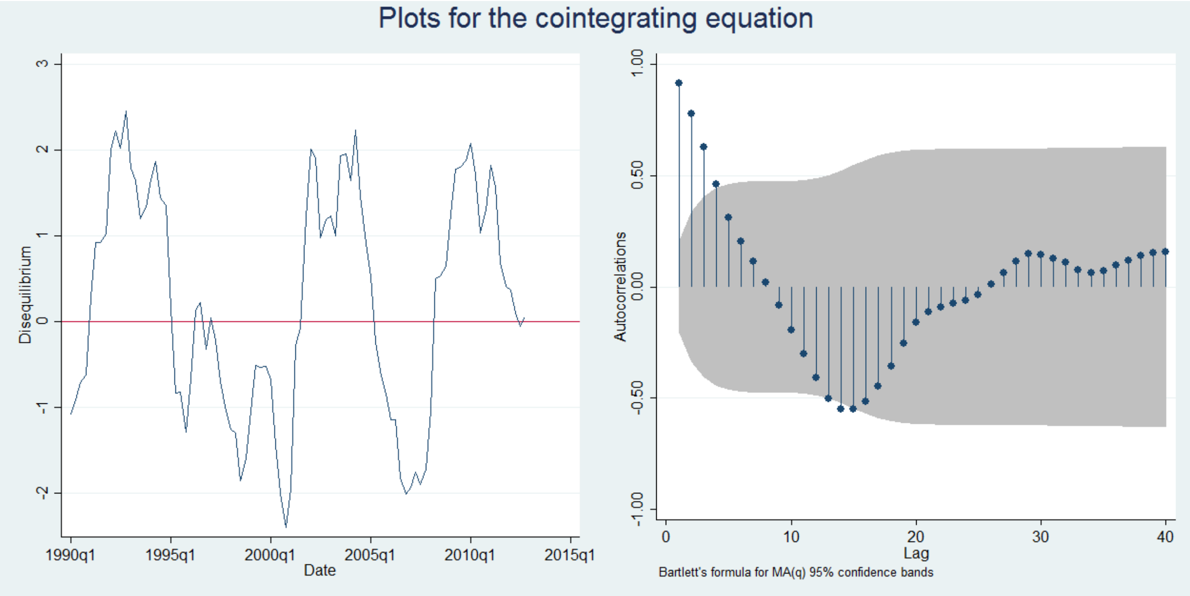

Figure 5.1.15 - Disequilibrium and autocorrelation plot for the cointegrating equation

5.1.4 ECM-modelling - Q1 1991-Q4 2006

In chapter 5.1.2 we estimated a long-run relationship between the US 10-year rate, policy rate, expected inflation and current account, based on the period Q1 1991-Q4 2006. We use this equation as an error correction term (ECT) when we now include additional variables to estimate an ECM with short- and long-run components. The ECT is as follows:

Equation 5.1.1 – Error correction term based on Q1 1991-Q4 2006:

$$ECT = 10y\ rate - 1.163 * Exp.inflation - 0.202 * Policy\ rate - 0.340 * CA - 2.234$$

When we tested the lag length of the variables considered for the cointegrating equation, we got most support for an underlying VAR model with two lags. In a first-differenced VAR model, this corresponds to only one lag. However, in our ECM, where several variables are added, we will allow for two lags initially, to see whether any of them are significant.

Table 5.1.11 shows some of the model specifications we estimated. Since the table is presented to illustrate considerations leading to the final model, we have chosen to break with convention and present the p-values in the parentheses. The standard errors will be presented in a later table for the preferred models. We have estimated models both with our output gap series and with the PMI index as the proxy for the business cycles. Since the PMI index turned out to be the more significant of the two, we present only model estimations using PMI as the proxy for the business cycles.

The first column shows the most general model, with all variables and two lags of first differences included. In the second column, all second lags are removed. Our preferred model, where only significant terms are included is shown in the fourth and final column. The third column is the same as the fourth, except that the second lagged differences of the policy rate is removed.

Table 5.1.11 - Estimated models for the US, sample Q1 1990-Q4 2006

	(1) D.10-year rate	(2) D.10-year rate	(3) D.10-year rate	(4) D.10-year rate
LD.10-year rate	0.221	0.0303	0.113	0.200^{*}
	(0.203)	(0.849)	(0.311)	(0.097)
L2D.10-year rate	0.255		0.276^{**}	0.330^{***}
	(0.195)		(0.030)	(0.009)
D.Policy rate	0.286	0.374^{*}	0.446^{***}	0.396^{***}
	(0.187)	(0.066)	(0.006)	(0.009)
LD.Policy rate	-0.450	-0.0622	-0.238	-0.425^{**}
	(0.112)	(0.700)	(0.129)	(0.027)
L2D.Policy rate	0.445^{**}			0.292^{**}
	(0.043)			(0.030)
D.Expected inflation	0.901^{*}	0.705^{*}	0.872^{**}	0.850^{***}
	(0.094)	(0.088)	(0.011)	(0.008)
LD.Expected inflation	-0.0946	0.180		
	(0.808)	(0.683)		
L2D.Expected inflation	0.151			
	(0.714)			
D.log(Debt/GDP)	-4.600^{*}	-6.352^{***}	-6.005^{***}	-5.241^{***}
	(0.092)	(0.001)	(0.000)	(0.000)
LD.log(Debt/GDP)	5.759^{***}	5.315^{***}	4.795^{***}	5.491^{***}
	(0.008)	(0.006)	(0.002)	(0.000)
L2D.log(Debt/GDP)	-1.739			
	(0.403)			
D.Current account	-0.0234	0.0371		
	(0.884)	(0.779)		
LD.Current account	0.0123	0.178		
	(0.942)	(0.236)		
L2D.Current account	-0.0938			
	(0.605)			

D.VIX index	-0.00710	0.00126		
	(0.636)	(0.927)		
LD.VIX index	-0.0169	-0.00922		
	(0.203)	(0.390)		
L2D.VIX index	0.0203			
	(0.170)			
D.Non-borrowed reserves/GDP	0.628	0.872		
	(0.626)	(0.461)		
LD.Non-borrowed reserves/GDP	2.174	1.495		
	(0.211)	(0.409)		
L2D.Non-borrowed reserves/GDP	-0.795			
	(0.679)			
D.PMI	0.0572^{**}	0.0543^{***}	0.0462^{**}	0.0505^{***}
	(0.037)	(0.010)	(0.012)	(0.006)
LDPMI	0.0107	0.00902		
	(0.663)	(0.617)		
L2D.PMI	0.0259			
	(0.217)			
L.Error correction term	-0.378^{***}	-0.239^{*}	-0.388^{***}	-0.394^{***}
	(0.008)	(0.065)	(0.000)	(0.000)
Observations	64	65	66	66
R^2	0.670	0.532	0.549	0.588
AIC	45.90	52.27	35.22	31.24

p-values in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Perhaps the most striking thing from the model estimates is the effect of government debt. Not only do we get a negative contemporaneous effect and a positive lagged effect, but whether the total effect is positive or negative depends on which other variables are included in the model. It does have the expected positive total effect in our preferred model in column 4, but we see that if we were to remove e.g. the second lagged differences of the policy rate, the total effect would become negative, meaning that our findings in regards to the effect of debt are not very robust. Therefore we will estimate a model without debt included later in the chapter to examine whether this makes any difference for the estimated effects of other variables.

Interpreting the contemporaneous and lagged effect of debt individually, we see that a positive shock to debt will initially decrease the yield, before the yield adjusts back in the subsequent period. This reaction could imply that we see a liquidity effect in periods were new treasuries are issued; the demand for treasuries and the liquidity of them might be much higher in these periods, depressing the yield, while in the next period the yield bounces back.

We see that the variables we find to have a significant effect, in addition to government debt and the long-run relationship, are the policy rate, expected inflation and the PMI index. All of these have the expected positive effect on the 10-year rate.

Reserves are not significant, and the estimated effect is positive, which is contrary to our belief. Furthermore, we do not believe it should be included as long as we keep the current sample, as we do not believe that reserves had a significant effect on the 10-year rate before the short rate reached the zero level boundary, in line with Krogstrup et. al. (2012). We believe that the model is picking up correlation rather than causality – both the 10-year rate and reserves declined during the sample. We believe that including reserves in this model might lead to a spurious relation and we choose to remove reserves completely from this model.

We also note that neither the contemporaneous nor the lagged first difference of the current account is significant, they did not become significant after other insignificant terms were removed either, so we remove current account from the short-run part of the model. In a previously estimated model where we only included a cointegrated relation between the 10-year rate and the policy rate, we found that the current account was significant and had a positive effect on the 10-year rate. It might be the case that this earlier model picked up on the long-run relation between the two variables. If the two variables only have a long-run relation, then the short-run variables of the current account should not be included in the model. It is plausible that the 10-year rate might depend on the level and major movements of capital inflow, which is reflected by a deficit on the current account, rather than minor fluctuations. The long-run effect identified from the current account indicates that the increasingly negative current account in the period to some degree reflects savings from abroad flowing into the US, among other things being invested in Treasuries, depressing their yield. This is the effect that Hellum (2010) also found.

Finally, we find no significant effect from the VIX index, which is somewhat surprising, as we believe the market's perception of risk should affect the 10-year rate.

Table 5.1.12 shows our preferred model from table 5.1.11 and the best model estimate we find when we remove debt. No new variables were found to be significant, but we now ended up including two lagged differences of the PMI index, while removing the corresponding terms of the 10-year rate and policy rate. The findings on PMI in this model might suggest that the effect of PMI differs depending on whether it is just a temporary change or adjustment or if we get continuous movement, signaling a boom or recession.

Table 5.1.12 - Preferred models for the US, sample Q1 1990-Q4 2006, with and without debt

	(1) D.10-year rate	(2) D.10-year rate
LD.10-year rate	0.200*	
	(0.097)	
L2D.10-year rate	0.330***	
	(0.009)	
D.Policy rate	0.396***	0.306**
	(0.009)	(0.012)
LD.Policy rate	-0.425**	
	(0.027)	
L2D.Policy rate	0.292**	
	(0.030)	
D.Expected inflation	0.850***	1.006***
	(0.008)	(0.005)
D.log(Debt/GDP)	-5.241***	
	(0.000)	
LD.log(Debt/GDP)	5.491***	
	(0.000)	
D.PMI	0.0505***	0.0605***
	(0.006)	(0.000)
LD.PMI		-0.0118
		(0.401)
L2D.PMI		0.0378**
		(0.015)
L.Error correction term	-0.394***	-0.353***
	(0.000)	(0.000)
Observations	66	66
R^2	0.588	0.433
AIC	31.24	44.34

p-values in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The total short-run effect of the policy rate is 0.560 in the model which includes debt, i.e. a one percentage point increase in the policy rate leads to a 0.560 percentage point increase in the 10-year rate.[12] The policy rate represents the effects of monetary policy and short-term interest rates, which makes the basis for the long-term rates. It is also strongly linked to the business cycle. A relatively strong effect is therefore not surprising. We see that this effect is almost three times the coefficient in the long-run relationship, indicating the extra effect the policy rate might have when changes in it is announced. The relatively low long-run coefficient also reflects the high variability in it compared to the long-run rate over the sample (as can be seen in figure 5.1.11). In the model excluding debt, however, the short-run effect of the policy rate is lower, 0.306. It is worth noting that the estimated effect would be higher if we were more lenient on the significance level, including lagged differences of the policy rate and/or the 10-year rate. Furthermore, in this model, the PMI index explains more of the cyclical element that previously likely was picked up by the policy rate. Finally, there might of course be some bias caused by the exclusion of government debt.

We also find that the expected inflation has a positive relation to the 10-year rate, with a total short-run effect in the model which includes debt of 1.809.[13] If long-term inflation expectations increase with one percentage point, the model predicts a 1.809 percentage point increase in the 10-year rate following the increase. Just as in the case of the policy rate, we find that short-run reactions are stronger than the long-run relationship. It is also worth noting that this effect is closer to the coefficient on expected inflation in Hellum (2010). In the model without debt, the short-run effect of expected inflation is estimated to be 1.006, i.e. smaller than the long-run effect.

The total effect of log(debt/GDP) is 0.532[14], i.e. a one percent increase in gross government debt, increases the 10-year rate by 0.5 basis points, so the total effect is relatively small, while the temporary fall suggested as new debt is issued is much larger (the contemporaneous effect of a one percent increase in debt is estimated to be a fall in the 10-year rate of 5.2 basis points).

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[12] Sum of the short-run coefficients = 0.396 - 0.425 + 0.292 = 0.263. Denominator = 1 - 0.200 - 0.330 = 0.470. Total short-run effect = 0.263/0.470 = 0.560. The two last steps are needed to adjust for the effect of variables that are implied through the coefficients on lags of the dependent variable.

[13] Total short-run effect = 0.850/0.470=1.809.

[14] Sum of the short-run coefficients = -5.241 + 5.491 = 0.25. Total effect = 0.25/0.470 = 0.532.

The total effect of PMI is 0.107[15] in the model which includes debt and 0.087[16] in the model excluding debt. This implies that if the PMI index increase by one point, the 10-year rate would increase by approximately 8.7-10.7 basis points. The business cycles thus have an economically significant and expected effect in this model.

The error correction term has an adjustment parameter of -0.394 in the model including debt and -0.353 in the model which excludes debt, indicating that 10-year rate moves relatively quickly towards the long-term equilibrium.

[15] Total effect = 0.0505/0.470 = 0.107.

[16] Sum of short-run coefficients = 0.0605 – 0.0118 + 0.0378 = 0.0865.

Examining the fit of the model

Table 5.1.13 - Preferred models for the US, sample Q1 1990-Q4 2006, with heteroskedasticity robust standard errors

	(1) D.10-year rate	(2) D.10-year rate
LD.10-year rate	0.200^{*}	
	(0.118)	
L2D.10-year rate	0.330^{***}	
	(0.121)	
D.Policy rate	0.396^{***}	0.306^{**}
	(0.146)	(0.118)
LD.Policy rate	-0.425^{**}	
	(0.187)	
L2D.Policy rate	0.292^{**}	
	(0.131)	
D.Expected inflation	0.850^{***}	1.006^{***}
	(0.307)	(0.348)
D.log(Debt/GDP)	-5.241^{***}	
	(1.412)	
LD.log(Debt/GDP)	5.491^{***}	
	(1.413)	
D.PMI	0.0505^{***}	0.0605^{***}
	(0.0176)	(0.0123)
LD.PMI		-0.0118
		(0.0139)
L2D.PMI		0.0378^{**}
		(0.0150)
L.Error correction term	-0.394^{***}	-0.353^{***}
	(0.101)	(0.0934)
Observations	66	66
R^2	0.588	0.433
AIC	31.24	44.34

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5.1.13 displays our two preferred models from this estimation sample, this time with heteroskedasticity robust standard errors in parentheses, rather than p-values. We will now examine how well these models predict the actual movements in the 10-year rate.

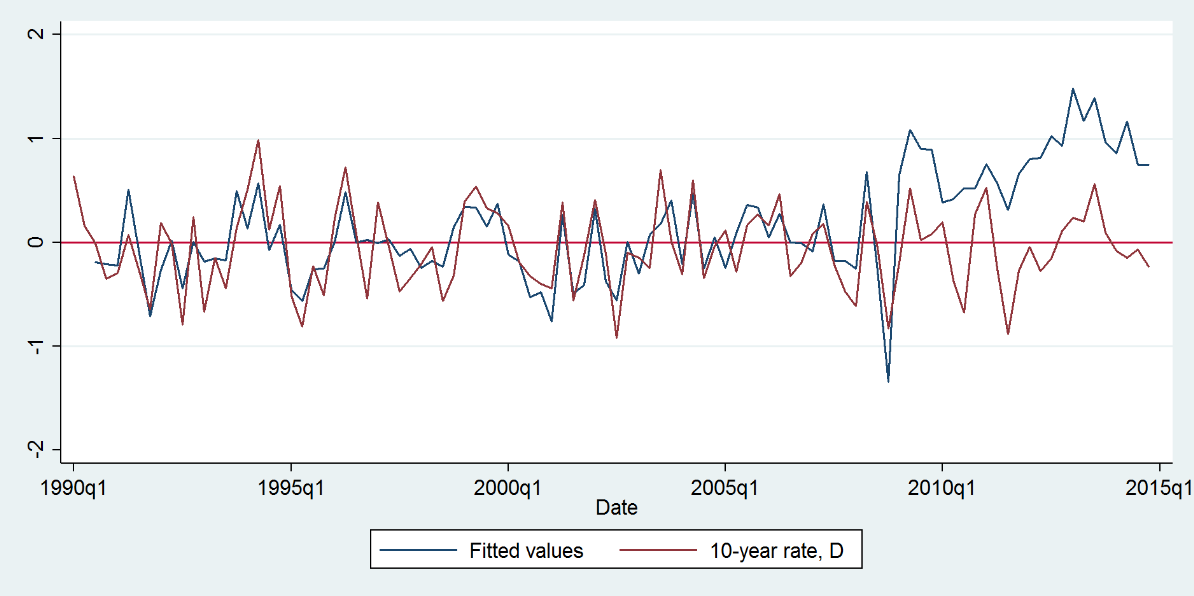

Figure 5.1.16 – Model-predicted and actual changes in the US 10-year rate, model including debt

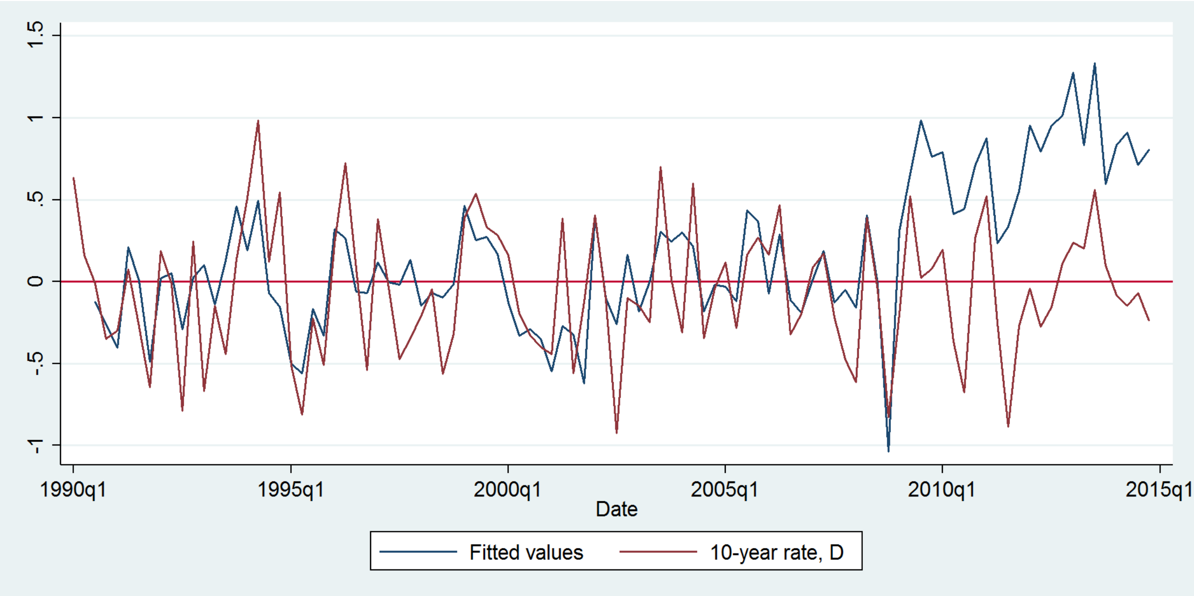

Figure 5.1.17 – Model-predicted and actual changes in US 10-year rate, model excluding debt

Figure 5.1.16 and 5.1.17 shows the changes in the 10-year rate predicted by our two models, both in- and out-of-sample. Both models perform relatively well in our estimation sample, and also early in the out-of-sample period. The model with debt included performs a bit better, not surprisingly as it has a superior R^2 and an extra explanatory variable. From around 2009 and onwards, though, both models continuously predict large increases in the rate which has not occurred.

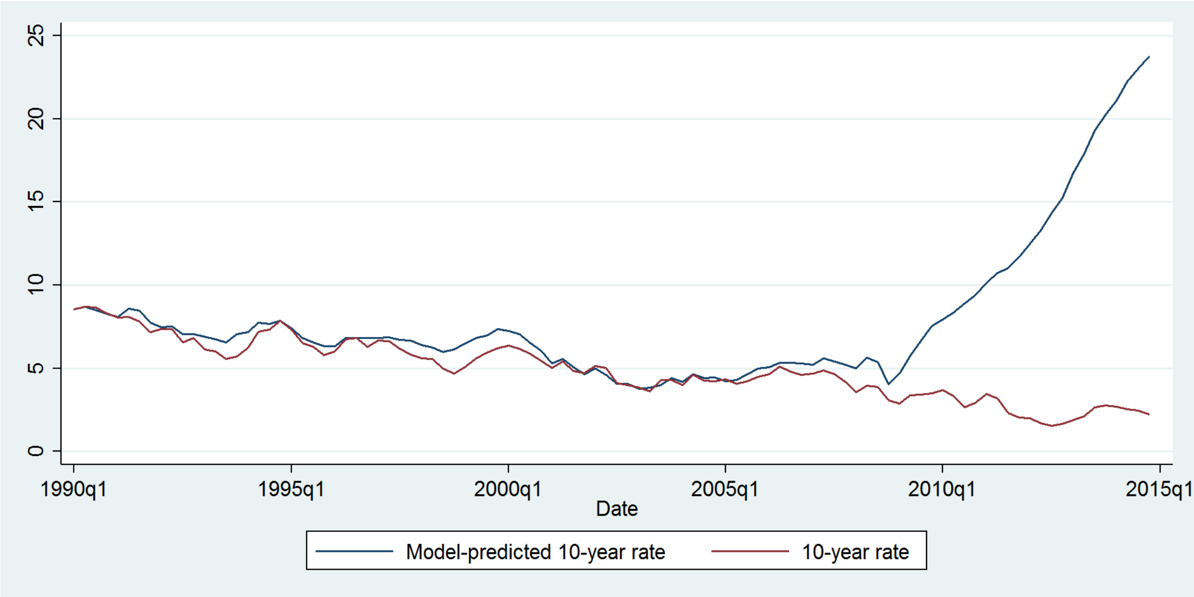

Figure 5.1.18 - Model-predicted and actual US 10-year rate, model including debt

Figure 5.1.18 shows the model-predicted 10-year rate, using the model that includes debt. We use the actual value in Q3 1990 as a starting point, and then successively adds the predicted changes from the model. We see that the estimated model does okay in the sample period, except predicting somewhat higher rates approximately around 1994 and 1999-2000. However, we see that the estimated model has a terrible out-of-sample fit, indicating that some of the relations between the variables must have changed in the recent years.

The significant divergence we see in the predictions are due to the error correction term. A large, negative disequilibrium over a longer period causes the model to predict excessively high estimates. Figure 5.1.19 shows that there has been a negative disequilibrium in the cointegrating relationship since 2006. It seems that the previously estimated long-term relationship between the 10-year rate, federal funds target rate, expected inflation rate and the current account has changed in the recent years, which also explains why we were only able to identify a long-run relationship between the 10-year rate and the policy rate when prolonging our estimation sample. Figure 3.9.1 shows that the current account was falling through the entire estimation sample, but has been increasing since 2007, to a large degree reflecting the disequilibrium in the cointegrating equation. It seems that a change in the effect and possibly composition of the current account may be responsible for the breakdown of the long-run relationship.

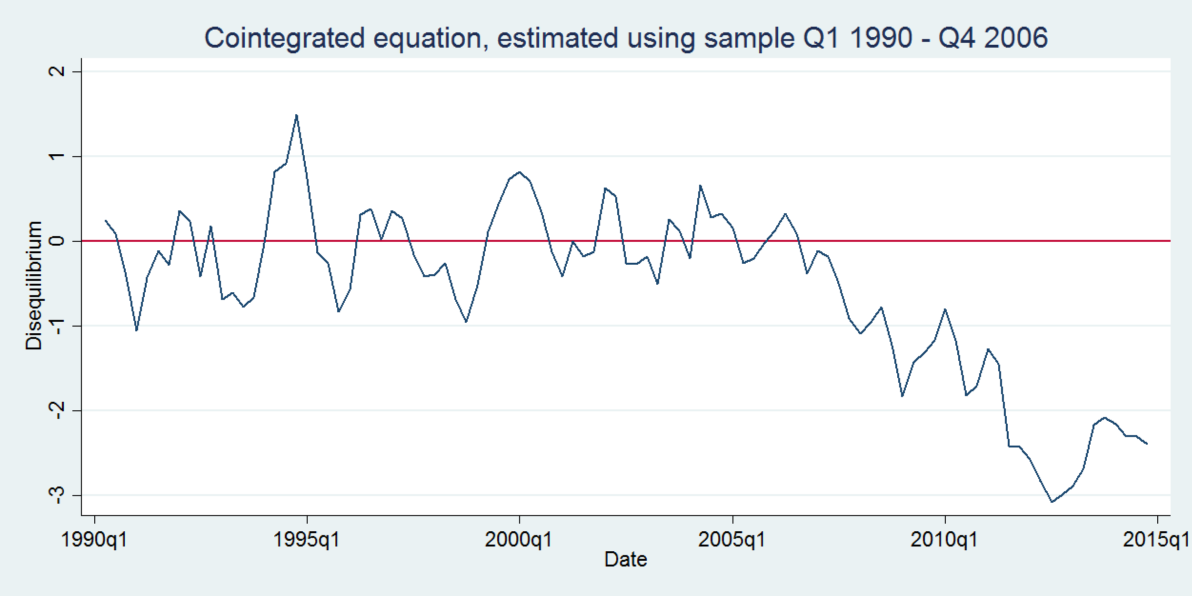

Figure 5.1.19 - Disequilibrium plot for the estimated cointegrated equation

When the error correction term used in predictions is based on the actual value of the 10-year rate, and the 10-year rate is below its equilibrium value, it will contribute to increases in the predicted rate, even if the predicted rate is actually above equilibrium. In other words, this form of prediction can hugely over-predict movements in the rate, especially out-of-sample, when the 10-year rate consistently deviates from the long-run relationship. To correct for this issue, we replace the actual lagged value of the 10-year rate with the lagged predicted value in the error correction term when we make predictions. Starting from the actual rate in Q2 1990 and then successively adding predicted changes using this methodology, we get the model-predicted values of the 10-year rate displayed in figure 5.1.20.

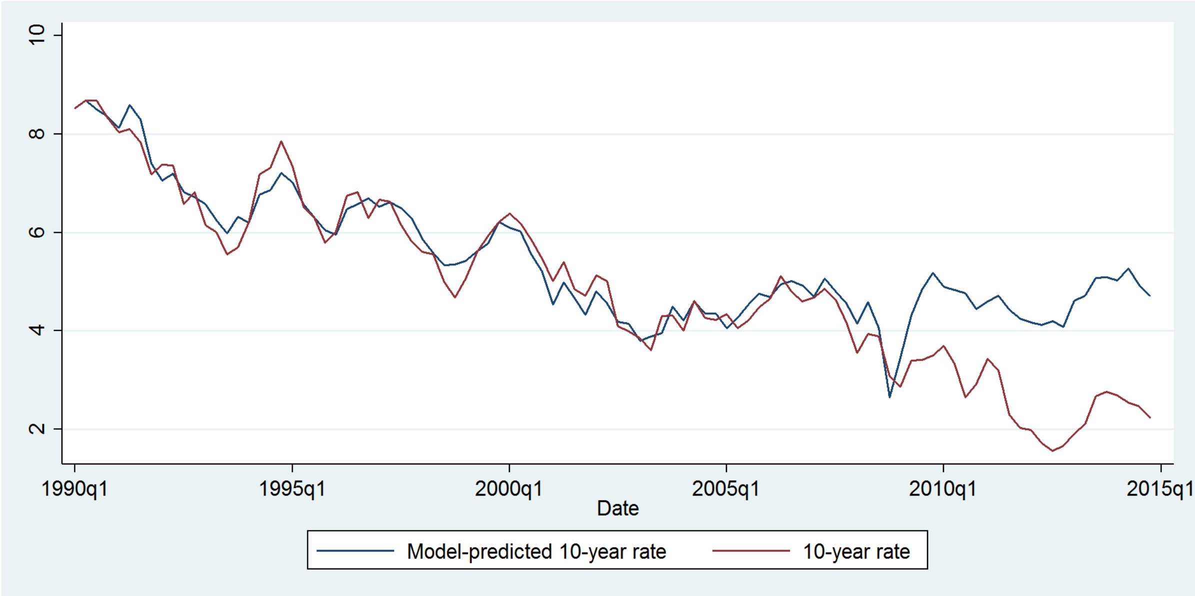

Figure 5.1.20 - Model-predicted and actual US 10-year rate, model including debt

We see that using the prediction methodology where we adjust the error correction term for our predictions, we get predictions that perform better both in- and out-of-sample. The deviation from the long-run equilibrium still results in the predictions lying far above the actual values since 2009, but now at much more reasonable levels than in figure 5.1.18.

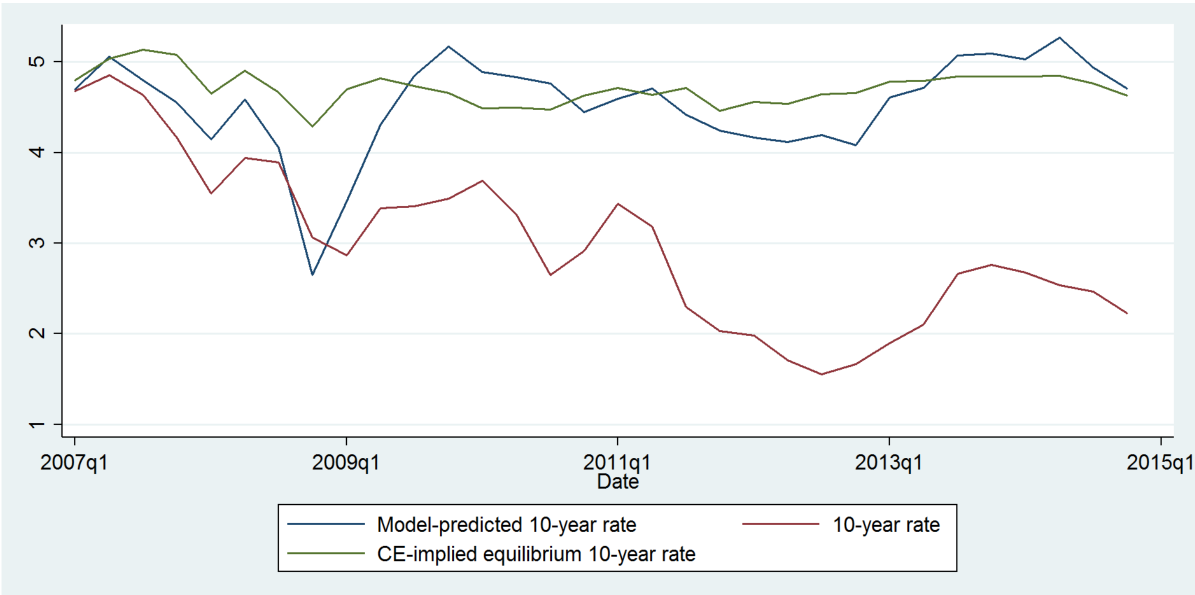

Figure 5.1.21 - A closer look at the out-of-sample period; model-predicted, actual and equilibrium 10-year rate

Figure 5.1.21 provides a closer look at the out-of-sample period for the actual 10-year rate, the rate predicted by our model and the equilibrium 10-year rate suggested by the cointegrating equation. We see that all three start the period at approximately the same level and that the model predicts the actual rate relatively well for the first two years, while the equilibrium rate does not follow the other two downwards. From 2009 and onwards though, the predicted values start moving around the equilibrium value suggested by the cointegrating equation at a level that is approximately twice the actual level of the 10-year rate. In figure 5.1.22, we see how the three variables that make up the cointegrating equation with the 10-year rate have developed in the out-of-sample period. Expected inflation have been almost constant, while the policy rate fell in 2007 and 2008, pulling the equilibrium rate downwards, then it has been constant ever since. It is clear that the factor pushing the equilibrium rate up is the current account. Our findings for the fit of our first model are thus that the effect from current account has changed or disappeared in recent years. We have already found that the long-run relationship we were able to identify when using a longer sample only included the 10-year rate and the policy rate, it will be interesting to see what other changes we find in our coming model estimations.

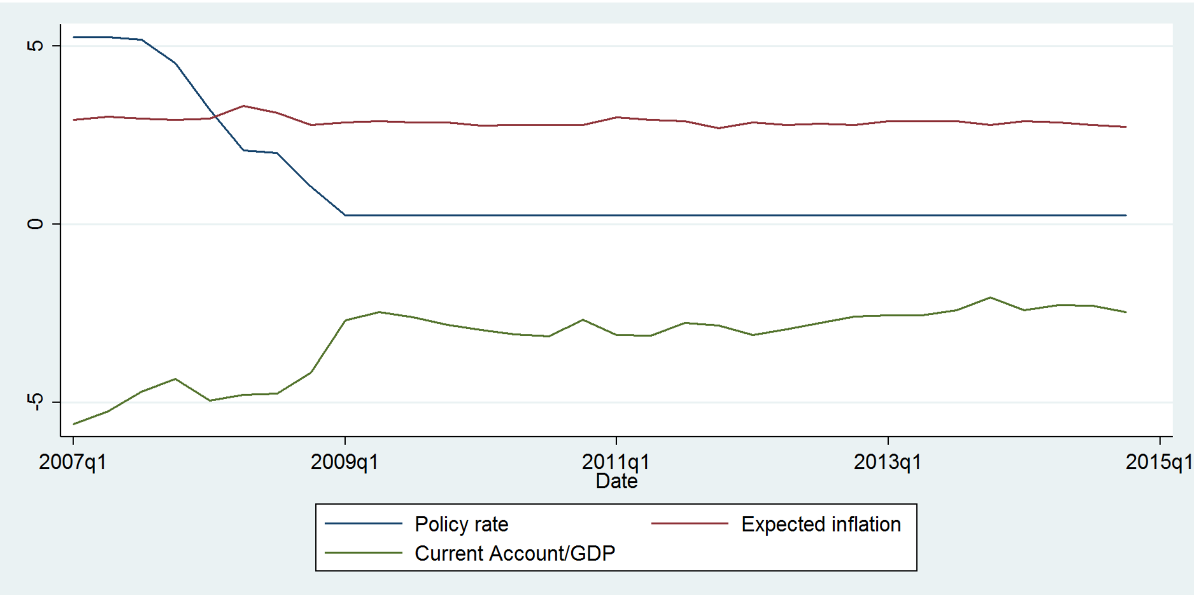

Figure 5.1.22 - A closer look at the out-of-sample period; explanatory variables in the cointegrating equation

5.1.5 ECM-modelling – Q1 1991-Q4 2012

When estimating the ECM for the period Q1 1991-Q4 2006 equation 5.1.1 was used as an error correction term. When estimating the long-run relationship, we saw that when we prolonged the sample period to Q1 1991-Q4 2012, we could no longer include expected inflation and the current account in the long-run relationship. When we now estimate the ECM for this period, we will therefore primarily use equation 5.1.2 as the error correction term. Equation 5.1.1 is also used in one estimation in the section where we allow for differing effects across periods.

Equation 5.1.1 – Error correction term based on Q1 1991-Q4 2006:

$$ECT = 10y\ rate - 1.163 * Exp.inflation - 0.202 * Policy\ rate - 0.340 * CA - 2.234$$

Equation 5.1.2 – Error correction term based on Q1 1991-Q4 2012:

$$ECT = 10y\ rate - 0.999 * Policy\ rate - 1.366$$

The model in this section will use a sample starting in Q1 1990 and ending in Q4 2012. Reserves has a structural break occurring in Q2 2008 – Q3 2008, due to the start of QE1, so the period before and after need to be modelled separately. To allow for this we include an interaction term between reserves and a dummy which takes the value 1 starting in Q3 2008.

Table 5.1.14 shows some estimated models for this period. Column 1 shows the general model including both contemporaneous and two lags of the first differences of all variables. Column 2 shows the same, but with only one lag of the first differences. In column 3, our preferred model is shown.

Table 5.1.14 - Estimated models for the US, sample Q1 1990-Q4 2012

	(1) D.10-year rate	(2) D.10-year rate	(3) D.10-year rate
LD.10-year rate	-0.0156 (0.903)	-0.0458 (0.707)	
L2D.10-year rate	-0.0438 (0.763)		
D.Policy rate	0.413** (0.012)	0.513*** (0.003)	0.493*** (0.000)
LD.Policy rate	-0.175 (0.361)	-0.286* (0.051)	-0.265** (0.014)
L2D.Policy rate	0.0313 (0.829)		
D.Expected inflation	0.313 (0.459)	0.430 (0.191)	0.608** (0.027)
LD.Expected inflation	0.147 (0.622)	0.106 (0.745)	
L2D.Expected inflation	0.559* (0.083)		
D.log(Debt/GDP)	-3.580** (0.022)	-4.663*** (0.003)	-3.845*** (0.003)
LD.log(Debt/GDP)	7.605*** (0.000)	6.060*** (0.002)	7.309*** (0.000)
L2D.log(Debt/GDP)	-0.487 (0.811)		
D.Current account	-0.0502 (0.584)	-0.0151 (0.868)	
LD.Current account	0.0781 (0.459)	0.0880 (0.368)	
L2D.Current account	-0.182 (0.144)		
D.VIX index	0.00814 (0.406)	0.00701 (0.378)	0.00651 (0.408)
LD.VIX index	-0.0112 (0.226)	-0.0106 (0.147)	-0.0163** (0.032)
L2D.VIX index	0.0132* (0.060)		
D.Non-borrowed reserves/GDP	-0.444 (0.730)	-0.0659 (0.952)	
D.Non-borrowed reserves/GDP x (Q3 2008-)	0.566 (0.665)	0.208 (0.852)	0.0819 (0.202)

LD.Non-borrowed reserves/GDP	0.956	0.760	
	(0.415)	(0.564)	
LD.Non-borrowed reserves/GDP x (Q3 2008-)	-1.023	-0.917	-0.0435
	(0.387)	(0.496)	(0.475)
L2D.Non-borrowed reserves/GDP	0.543		
	(0.745)		
L2D.Non-borrowed reserves/GDP x (Q3 2008-)	-0.718		-0.179**
	(0.670)		(0.021)
D.PMI	0.0840***	0.0553***	0.0676***
	(0.000)	(0.001)	(0.000)
LD.PMI	0.00688	0.00188	-0.00843
	(0.720)	(0.905)	(0.509)
L2D.PMI	0.0224		0.0235*
	(0.164)		(0.050)
L.Error correction term	-0.0962**	-0.0840**	-0.108***
	(0.025)	(0.039)	(0.004)
Observations	88	89	90
R^2	0.630	0.520	0.570
AIC	59.67	64.61	45.92

p-values in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Several considerations had to be made before we ended up with the model in column 3. First, the current account's drastic change in behavior changes its impact in the model. We see that if we include two lags of it, we find that it has a total negative effect, while it was positive in our previous model (and in the study done by Hellum (2010)). If we include only one lag, however, the total effect is positive and not significant. Furthermore, we see that allowing current account in the model seems to make it more difficult to pin down the effects of other variables. We believe that the composition of and effect from the current account has been so drastically altered since 2007, that including it in this model will only be a disturbance to the model estimation. Therefore we remove it completely.

Second, we find no significant effect from reserves in the pre-QE period, as we expected. We do, however, find the expected total negative effect in the QE-period. We see that allowing reserves to be included in the period where its effect was not significant (it is furthermore estimated to be positive in this period, contrary to what would be expected), and we believe in reality non-existent, only harms the estimations of the rest of the model. Therefore, reserves are only included from the start of quantitative easing onwards.

Finally, we see that allowing for two lagged differences of the VIX index makes us find a total positive effect, which is not what we would expect. Furthermore, we are very skeptical of believing that the main effect from VIX would be at its second lag. Allowing it in the model also disturbs the estimation of other effects. We thus believe that the effect we find from the second lagged differences of the VIX index is wrongfully attributed to it. We therefore remove this lag from the model.

After these considerations and removing further insignificant terms, we end up with the model in column 3. The main difference from the model in chapter 5.1.4 is that we find a significant negative effect from reserves in the QE-period, indicating that quantitative easing has had the desired effect of reducing interest rates. In addition, we now find a significant total negative effect from the VIX index, although with a lag. Furthermore, we find a much stronger positive effect from debt. Finally, the error correction term included in this model does only include the 10-year rate and the policy rate, and it is perhaps not surprising that the speed of adjustment is drastically lower than what we saw in the model in chapter 5.1.4.

The total short-run effect of the policy rate in this model is 0.228[17], less than half what we found for the sample ending in 2006. It is worth noting, however, that the policy rate now plays a larger role in the included error correction term, and that the inclusion of lagged differences of the 10-year rate in the earlier model increased the estimated effects of all variables in that model. The effect of expected inflation is 0.608, only about a third of that estimated in the earlier model and no longer consistent with the Taylor principle. This reduction is dramatic - remember that expected inflation is also gone from the error correction term. Reduced estimated effect of expected inflation can, however, be a sign of drastically more stable expectations. More on this when we discuss our model with interaction terms.

The estimated total effect of log(debt/GDP) is 3.464[18]in this model. That indicates that a one percent increase in the supply of debt, increases the 10-year rate by 3.5 basis points, thus we find much stronger evidence than previously of a supply effect on the 10-year rate.

[17] Sum of short-run coefficients = 0.493 - 0.265 = 0.228.

[18] Sum of coefficients = - 3.845 + 7.309 = 3.464.

The estimated total effect of a one percent of GDP increase in non-borrowed reserves, from the inception of QE onwards, is a 0.141 percentage point decrease in the 10-year rate[19]. This indicates a significant effect of QE. A one percent increase in non-borrowed reserves decrease the yield (primarily through the term premium) by 14.1 basis points.

In this model we find that an increase of one in the VIX index, decrease the 10-year rate by 1 basis point[20], we thus find only a slight effect of expected volatility in the financial market on the 10-year rate. The estimated total effect of an increase of 1 in the PMI index is 0.083[21], roughly the same effect as estimated in our previous model. Finally, the adjustment parameter in this model is -0.108, suggesting a much slower reaction to disequilibria than our previous model. This is not very surprising though, considering that we only could include the 10-year rate and the policy rate in the long-run relationship for our longer sample.

[19] Sum of coefficients = 0.0819 – 0.0435 – 0.179 = -0.1406.

[20] Sum of coefficients = 0.00651 – 0.0163 = -0.00979.

[21] Sum of coefficients = 0.0676 – 0.00843 + 0.0235 = 0.08267.

Examining the fit of the model

Table 5.1.15 - Preferred model for the US, sample Q1 1990-Q4 2012, with heteroskedasticity robust standard errors

	(1) D.10-year rate
D.Policy rate	0.493***
	(0.125)
LD.Policy rate	-0.265**
	(0.106)
D.Expected inflation	0.608**
	(0.269)
D.log(Debt/GDP)	-3.845***
	(1.230)
LD.log(Debt/GDP)	7.309***
	(1.457)
D.VIX index	0.00651
	(0.00782)
LD.VIX index	-0.0163**
	(0.00746)
D.Non-borrowed reserves/GDP x (Q3 2008-)	0.0819
	(0.0637)
LD.Non-borrowed reserves/GDP x (Q3 2008-)	-0.0435
	(0.0606)
L2D.Non-borrowed reserves/GDP x (Q3 2008-)	-0.179**
	(0.0760)
D.PMI	0.0676***
	(0.0146)
LD.PMI	-0.00843
	(0.0127)
L2D.PMI	0.0235*
	(0.0118)
L.Error correction term	-0.108***
	(0.0363)
Observations	90
R^2	0.570
AIC	45.92

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

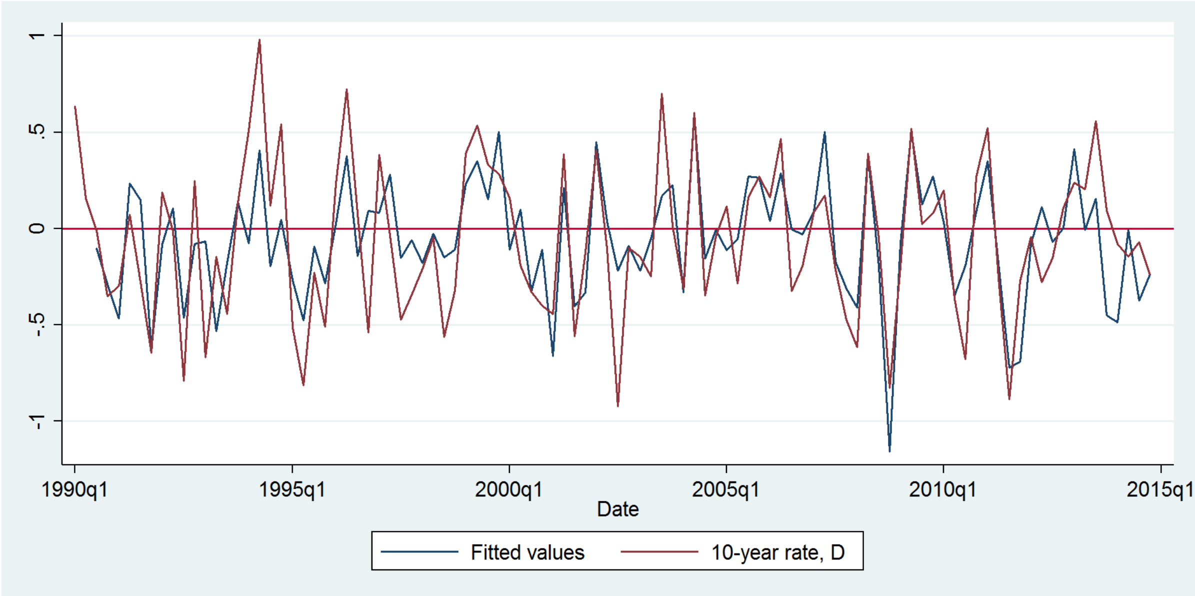

Figure 5.1.23 - Model-predicted and actual changes in the US 10-year rate, estimation sample Q1 1990-Q4 2012

Table 5.1.15 shows our preferred model for the sample Q1 1990-Q4 2012, when we do not allow for changing effects from variables other than the central bank reserves. In figure 5.1.23 we see the predicted changes in the 10-year rate from this model, compared to the actual changes. The model predicts the changes before 2007 a little bit, but not terribly, worse than our previous model, but performs drastically better in later years. Both of these observations were expected as we have had to make relatively drastic changes to the model when prolonging the sample. Furthermore, it seems like the model struggles somewhat in our two out-of-sample years, 2013 and 2014.

Figure 5.1.24 shows the actual 10-year rate and the 10-year rate predicted by our model, using the methodology where we adjust the error correction term for our prediction. It performs significantly worse than our previous model before 2007, but much better after. We see that it predicts higher rates than what has actually occurred for most of the period, not just the post-financial crisis years. The model does a relatively good job of explaining the extremely low 10-year rate seen from late 2011 to early 2013, meaning that it seems the low policy rate and effects from quantitative easing is (almost) sufficient to explain the low rate. In fact, we see that the model predicts that the rate would go even lower in 2014, not being able to fully pick up the increase in the 10-year rate in our two out-of-sample years.

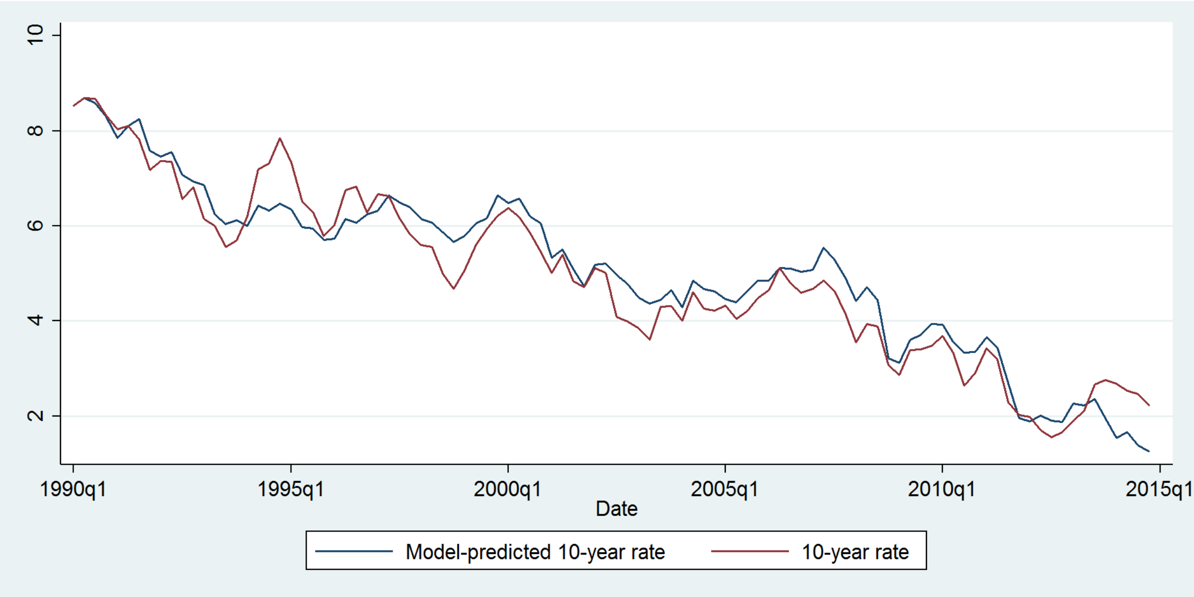

Figure 5.1.24 - Model-predicted and actual US 10-year rate, estimation sample Q1 1990-Q4 2012

Figure 5.1.25 shows the actual 10-year rate, the rate predicted by our model and the equilibrium 10-year rate suggested by the error correction term for the period 2010-2014 (i.e. the last three in-sample years, which includes the lowest rate, and our two out-of-sample years). We see that the equilibrium rate suggested by the cointegrating equation is constant throughout this period. This is due to it only consisting of the 10-year rate and the policy rate and the policy rate being unchanged throughout the period.

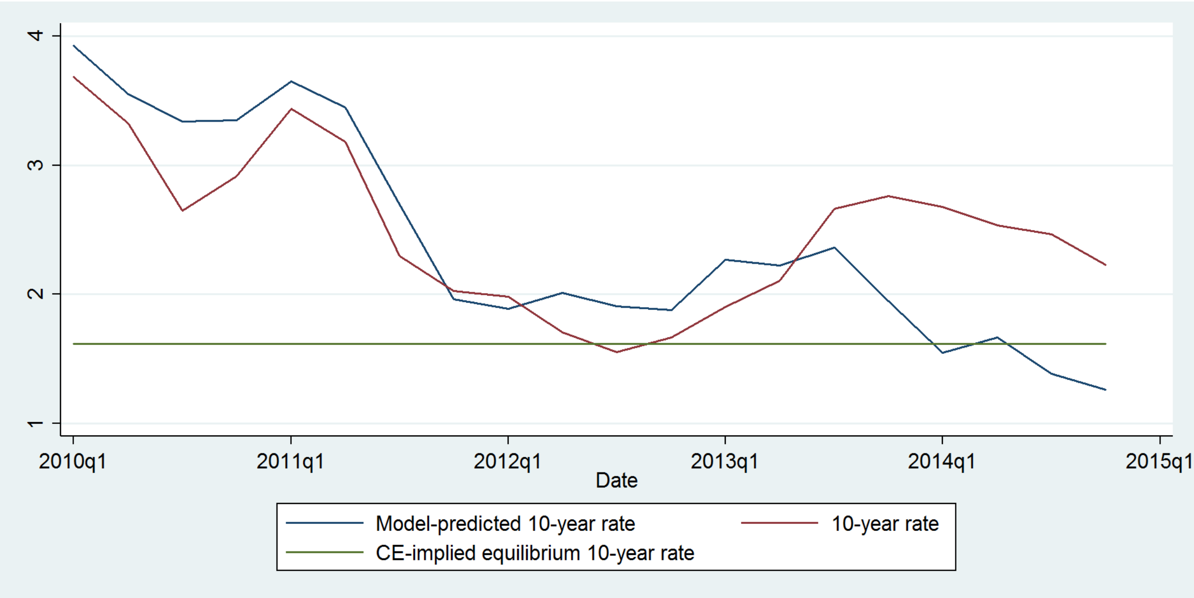

Figure 5.1.25 - A closer look at recent years; model-predicted, actual and equilibrium 10-year rate

In figure 5.1.26 we see how the explanatory variables in our model have developed during the recent years. We see that in addition to the policy rate being unchanged throughout, expected inflation, and for the most part also the PMI index and the VIX index are very stable. The largest movements are by far seen in the reserves. Looking at the movements in the actual and model-predicted 10-year rate and considering that we found that the largest effect of the reserves came at the second lag, it seems that much of the fall in 2011 can be attributed to QE2, which started in late 2010. This is further helped somewhat by a peak in the VIX index, a fall in the PMI index and a downward pull from the policy rate.

We see, however, that we are not able to explain the trough in the rate in 2012. There does not seem to be any obvious candidates to explain this among our variables either. One possibility is that we do not capture the full effect of quantitative easing. In particular, when we use increases in reserves to capture QE-effects, we will not capture effects of the maturity extension program, if there are any. This program entailed purchasing long-term assets and selling short-term assets, and thus did not result in increasing reserves. It was announced in September 2011 and lasted through 2012. QE3 was announced in September 2012, and it was further announced in December 2012 that the Federal Reserve would again finance purchases using reserves. Thus, QE conducted in 2012, i.e. the maturity extension program, did not entail any increases in reserves and can therefore not be captured by our model. We believe that this is the reason for the unexplained trough in 2012.

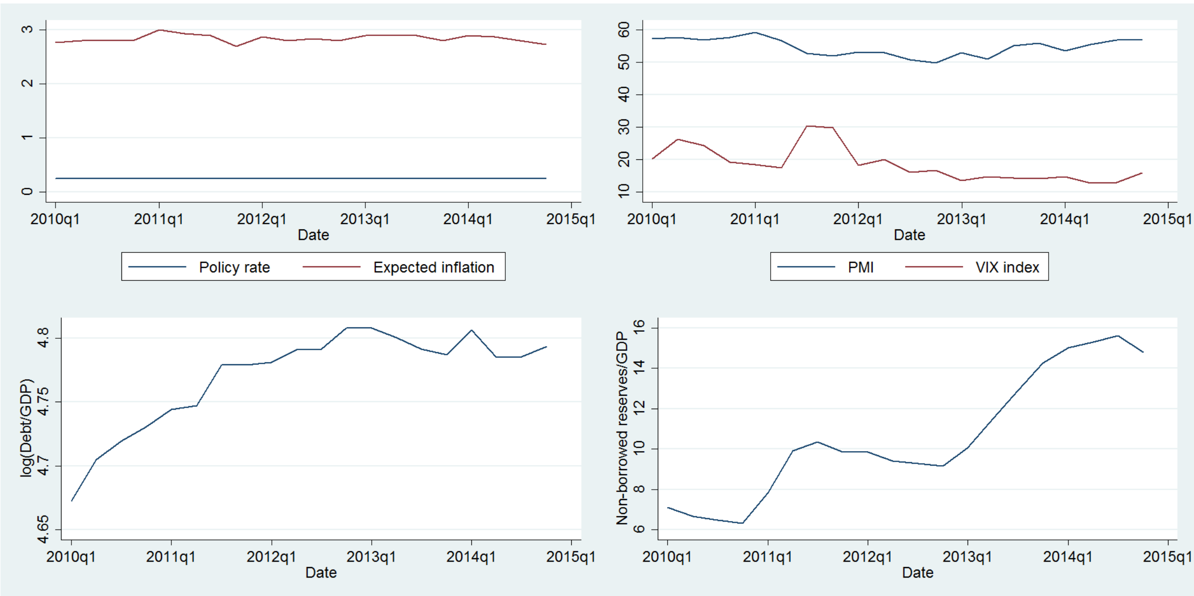

Figure 5.1.26 - A closer look at recent years; explanatory variables

We see that our model does not fully explain the increase in the 10-year rate in 2013. Among our variables, the only one explaining an increase in that year is the PMI index, which sees a small increase. Meanwhile, both the increase in reserves and the decrease in debt suggest a decrease and explain why the model-predicted rate continues to fall. The large increase in the actual rate seen here, might be a result of increased optimism about the future, to a larger degree than what the PMI index is able to pick up, and expectations about tightening monetary policy through reduced asset purchases and/or increased policy rate. Meanwhile the estimated effects of the fall in debt and increase in reserves might overestimate the effect of QE3. It is possible that the true effects would have been captured better if we to a larger extent took into consideration the differences in different programs. Furthermore, the marginal effect of QE might be smaller during QE3 than it was when QE was a "new" phenomenon. Finally, the model-predictions might be unnecessarily "held back" by the presence of the error correction term.

Allowing for changing effects between periods

This model will use a sample starting in Q1 1990 and ending in Q4 2012. We are interested in testing whether any of the variables have had a different effect on the yield in the later years. We will do this by using a dummy, taking the value 1 from Q1 2007 onwards, and creating interaction terms between it and the other variables. The exception is non-borrowed reserves, which continues with its own dummy, which takes the value 1 starting in Q3 2008. As with the previous models, we will only consider including two lags or less.

We have previously estimated two different error correction terms, based on two different periods. It does not seem like a good idea to include both simultaneously, as both include the 10-year rate's relationship with the policy rate, but with two more variables included in the relationship estimated for the sample ending in 2006. We have therefore considered three approaches. The first is estimate a model with only the error correction term from the estimated cointegrating relationship based on Q1 1990-Q4 2006, "error correction term 1" (Equation 5.1.1). The second is to only include the error correction term from the estimated cointegrating relationship based on Q1 1990-Q4 2012, "error correction term 2" (equation 5.1.2). The third and final approach is to include error correction term 1 for Q1 1990-Q4 2006 and error correction term 2 for Q1 2007-Q4 2012. The latter did not yield fruitful results and will therefore not be presented.

Column 1 in table 5.1.16 shows the best model when we included error correction term 1, in the following referred to as model 1. Column 2 shows the best model when we included error correction term 2, in the following referred to as model 2.

Table 5.1.16 – Estimated models for the US with interaction terms, with heteroskedasticity robust standard errors

	(1) D.10-year rate	(2) D.10-year rate
D.Policy rate	0.282**	0.261**
	(0.107)	(0.102)
D.Policy rate x (2007-)	-0.0574	-0.0480
	(0.238)	(0.309)
D.Expected inflation	1.108***	0.903**
	(0.336)	(0.403)
D.Expected inflation x (2007-)	-0.734	-0.612
	(0.670)	(0.940)
D.log(Debt/GDP)	-5.762***	-4.131**
	(1.795)	(1.898)
D.log(Debt/GDP) x (2007-)	4.643	1.115
	(3.224)	(4.774)
LD.log(Debt/GDP)	3.187*	7.198***
	(1.753)	(1.857)
LD.log(Debt/GDP) x (2007-)	-0.149	-3.039
	(4.691)	(3.932)
D.VIX index		-0.0102
		(0.0147)
D.VIX index x (2007-)		0.0218
		(0.0169)
LD.VIX index		-0.0260**
		(0.0116)
LD.VIX index x (2007-)		0.0146
		(0.0278)
D.Non-borrowed reserves/GDP x (Q3 2008-)	0.0207	0.0879
	(0.1000)	(0.151)
LD.Non-borrowed reserves/GDP x (Q3 2008-)	0.000712	-0.0572
	(0.0723)	(0.105)
L2D.Non-borrowed reserves/GDP x (Q3 2008-)	-0.249***	-0.223
	(0.0871)	(0.141)
D.PMI	0.0617***	0.0883***

	(0.0132)	(0.0166)
D.PMI x (2007-)	0.00175	0.00190
	(0.0229)	(0.0268)
LD.PMI	-0.000518	-0.00260
	(0.0123)	(0.0141)
LD.PMI x (2007-)	0.0149	-0.0214
	(0.0244)	(0.0630)
L2D.PMI	0.0346^{**}	0.0385^{**}
	(0.0137)	(0.0160)
L2D.PMI x (2007-)	-0.0533^{***}	-0.0412
	(0.0186)	(0.0313)
L.Error correction term 1 x (-2007)	-0.311^{***}	
	(0.0945)	
L.Error correction term 2		-0.118^{**}
		(0.0504)
L.Error correction term 2 x (2007-)		0.106
		(0.0925)
Observations	90	90
R^2	0.581	0.589
AIC	51.56	60.01

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Discussion on model 1

For the model with error correction term 1, we were unable to estimate a model where debt had a positive effect on the yield in both periods. We are skeptical of a negative effect of debt on the yield in the first period. However, since the cointegrating relationship is only included for the pre-2007 period, it might be the case that debt tries to adjust for some of the effect from the cointegrating equation, or that it does not interact well with one of the variables included in the cointegrating equation, e.g. the current account. However, since debt was significant, and the model overall seemed to do quite well, we chose to present this model as well.

Reserves are still only included in the post-2007 period. Furthermore, the error correction term are only included in the first period. If we do not believe that a variable has an effect on the 10-year rate in a particular period, including the variable in that period might cause unnecessary noise. This is why the error correction term and reserves is only included in the period we believe they have a causal relationship with the yield. When we included the error correction term for the second period as well, the coefficient was near zero.

Model 1 thus includes debt, with one lag, where log(debt/GDP) has a negative effect in the pre-2007 period of -2.575 and a positive effect in the post-2007 period of 1.919[22]. The negative coefficient in the pre-2007 period are likely picking up falls in the rate that should have been attributed to other variables, while there possibly were no significant supply effect on the yield in that period. The estimated effect in the post-2007 period is roughly half of what we found in the model without interaction terms, we believe the estimated effects in that model are more representative of the actual effect, and that this estimated effect is too low due to the, most likely wrongful, negative effect found in the pre-2007 period.

Reserves, with two lags, is only included in the second period, where it has the expected negative effect on the yield. The total effect of a one percent of GDP increase in non-borrowed reserves is a reduction in the 10-year rate of 0.228 percentage points.[23] The estimated effect is thus a bit stronger than in the model without interaction terms.

PMI is included with two lags, and the effect is positive for both periods, the effect of an increase of 1 in the PMI index is 0.096 and 0.059, for the pre-2007 period and the post-2007 period respectively[24]. The former is roughly equivalent with our previously estimated effects for the PMI index. The reduction in magnitude comes from the second lag of the interaction term, which has a coefficient of -0.0533 and is significant on the 1% level. The model thus finds a significant reduction in the effect of PMI between the two periods.

Policy rate has a positive effect in both periods, the estimated effect of a one percentage point increase in the policy rate is a 0.282 percentage point and a 0.225 percentage point increase in the 10-year rate for the pre-2007 and post-2007 periods respectively[25]. However, the reduction in the post-2007 period is not statistically significant. The policy rate has been stable at 0.25 (or more precisely the target has been that the federal funds rate should lie between 0 and 0.25) since Q1 2009, which means that for 16 of the 24 observations from the post-2007 period there has been no movement. It is therefore not surprising that the model has trouble finding a change in the effect from the policy rate to be significant.

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[22] Sum of coefficients, pre-07 = -5.762 + 3.187 = -2.575. Sum of coefficients, post-07 = -2.575 + 4.643 – 0.149 = 1.919.

[23] Sum of coefficients = 0.0207 + 0.000712 – 0.249 = - 0.228.

[24] Sum of coefficients, pre-07 = 0.0617 – 0.000518 + 0.0346 = 0.095782. Sum of coefficient, post-07 = 0.095782 + 0.00175 + 0.0149 – 0.0533 = 0.059132.

[25] Sum of coefficients, post-07 = 0.282 - 0.0574 = 0.2246.

Expected inflation has a positive effect on the yield, with a not statistically significant, albeit large, reduction in the post-2007 period. The estimated effect of a one percentage point increase in long-term inflation expectations is a 1.108 percentage point and a 0.374 percentage point increase in the 10-year rate in the pre-2007 period and post-2007 period respectively[26].

Does this mean that inflation expectations have been of less importance in the recent years? Not necessarily. Keep in mind that the post-2007 period constitutes a smaller sample, making the standard errors of the interaction terms large. With such a small sample, we can only say that the model finds weak evidence to suggest that inflation expectations have less effect. This is likely to a large degree also due to inflation expectations being stable in this period. In fact Bernanke (2013) stated that there has been an increased credibility to the Federal Reserve's commitment to price stability, and that "the anchoring of long-term inflation expectations near 2 percent has been a key factor influencing long-term interest rates over recent years. It almost certainly helped mitigate the strong disinflationary pressures immediately following the [financial] crisis." Thus, stable inflation expectations may have had a key role in long-term interest rates not falling to even lower levels.

Furthermore, when the Federal Reserve has a long-term inflation target at two percent, which is viewed as credible by market participants, it is not unlikely that small changes in expected inflation will have less impact on the 10-year rate than what was previously the case, due to less uncertainty surrounding the expectations. Our beliefs are further supported by the findings of Krishnamurthy and Vissing-Jorgensen (2011), who in a study on the possible channels for QE found that implied volatility on inflation swaptions decreased following both QE1 and QE2, by 38 and 3 basis points respectively, implying a significant fall in inflation uncertainty following QE1 and a slight fall following QE2.

The adjustment parameter on the error correction term (for the pre-2007 period) is significant at the 1% level, with a negative coefficient of -0.311. Thus, the speed of adjustment is estimated to be slightly slower than suggested in the models in chapter 5.1.4 As previously mentioned, when the error correction term was included for the post-2007 period as well, the coefficients were nearly identical, only with opposite signs. This

[26] Sum of coefficients, post-07 = 1.108 – 0.734 = 0.374.

suggested that this cointegrating relationship had no influence in the post-2007 period, which is why we removed it from that period.

Discussion on model 2

Also in model 2, we find a significant negative effect from reserves. A one percent of GDP increase in non-borrowed reserves is in this model estimated to decrease the 10-year rate by 0.192 percentage points.[27]

Debt now has a positive effect in both periods, albeit the estimated effect is smaller in the post-2007 period, while we saw a clear increase in the estimated effect between the two models without interaction terms when we prolonged the sample. This model suggest that a one percent increase in debt in the pre-2007 period led to an increase in the 10-year rate of approximately 3.1 basis points, while the effect in the post-2007 period was approximately 1.1 basis points.[28] One explanation for this might be the fact that we do not adjust our debt measure to only include the supply of government debt to the public, and thus our debt measure increase more in the post-2007 period than a more appropriate measure would do.

PMI is positive for both periods, but the effect is smaller in the post-2007 period. The effects are 0.124 and 0.064, respectively.[29] While the reduction was statistically significant in model 1, this is not the case in model 2.

The policy rate is positive in both periods, again with a (not significant) reduction in the post-2007 period. A percentage point increase in the policy rate increases the yield by 0.261 and 0.213 for the pre-2007 period and the post-2007 period, respectively.[30] The estimated effects are thus roughly similar to the estimated effects in model 1.

Expected inflation has a positive coefficient in the pre-2007 period, and again we see a large, but not significant, reduction in magnitude in the post-2007 period. The estimated effects of a one percentage point increase in expected inflation increases the 10-year rate by 0.903 and

---

[27] Sum of coefficients = 0.0879 – 0.0572 – 0.223 = -0.1923.

[28] Sum of coefficients, pre-07 = -4.131 + 7.198 = 3.067. Sum of coefficients, post-07 = 3.067 + 1.115 – 3.039 = 1.143.

[29] Sum of coefficients, pre-07 = 0.0883 – 0.00260 + 0.0385 = 0.1242. Sum of coefficients, post-07 = 0.1242 + 0.00190 – 0.0214 – 0.0412 = 0.0635.

[30] Sum of coefficients, post-07 = 0.261 – 0.048 = 0.213.

0.291, respectively.[31] The estimated effects are thus somewhat lower in model 2 than they were in model 1. If we also take into account that the error correction term in model 1 includes expected inflation, it is clear that model 1 suggests a larger effect from changes in inflation expectations.

Model 2 includes VIX as well. The estimates suggest that an increase of 1 in the VIX index in the pre-2007 period reduced the yield by 3.6 basis points. In the post-2007 period, however, the sum of the coefficients is 0.0002, i.e. the effect is basically zero in this period.[32]

The adjustment parameter on the error correction term is negative in both periods, -0.118 and -0.012, respectively.[33] We thus see that there is virtually no error correction in the post-2007 period. The coefficient for the interaction term is positive, nearly cancelling out the coefficient for the pre-2007 period. However, the interaction term is not significant (it has a p-value of 0.256). It might be the case that we cannot find a significant change due to the small sample size in the post-2007 period - the standard error is nearly twice as large as the standard error for the coefficient for the pre-2007 period.

Model 2 includes mostly the same variables as model 1, but includes error correction term 2 instead of 1, and also includes the VIX index. Model 2 has a higher R-squared than model 1, but since it estimates more coefficients, this is as expected. AIC is lower for model 1. We cannot say that one of the models is significantly better than the other, so we are interested in examining the fit of both. Since the two models include some differing variables, they might behave differently.

[31] Sum of coefficients, post-07 = 0.903 – 0.612 = 0.291.

[32] Sum of coefficients, pre-07 = -0.0102 – 0.0260 = -0.0362. Sum of coefficients, post-07 = -0.0362 + 0.0218 + 0.0146 = 0.0002.

[33] Sum of coefficients, post-07 = -0.118 + 0.106 = -0.012.

Examining the fit of the models

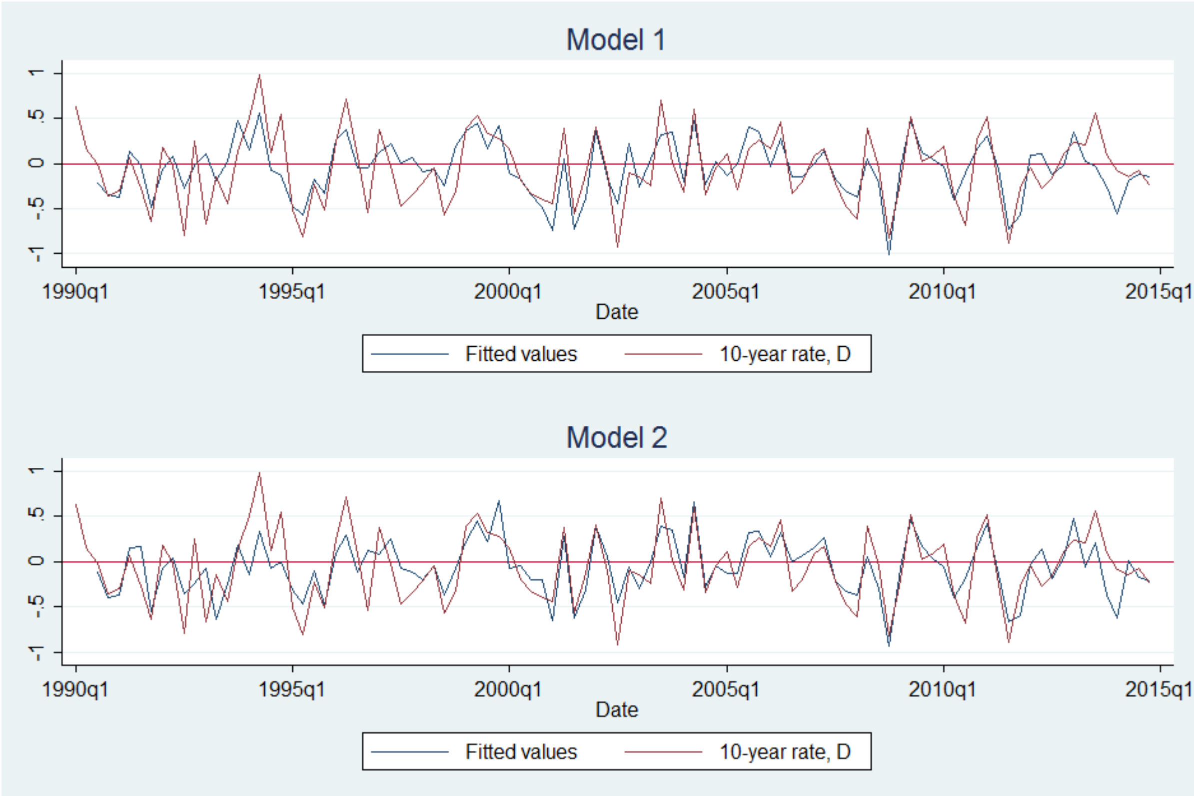

Figure 5.1.27 - Model-predicted and actual changes in US 10-year rate

Figure 5.1.27 shows the actual and model-predicted changes in the 10-year rate for both model 1 and 2. It seems like model 2 follows the changes better in the first five years, and is then outperformed by model 1. Both models seem to predict the changes quite well, except that they sometimes predict less magnitude than what is observed.

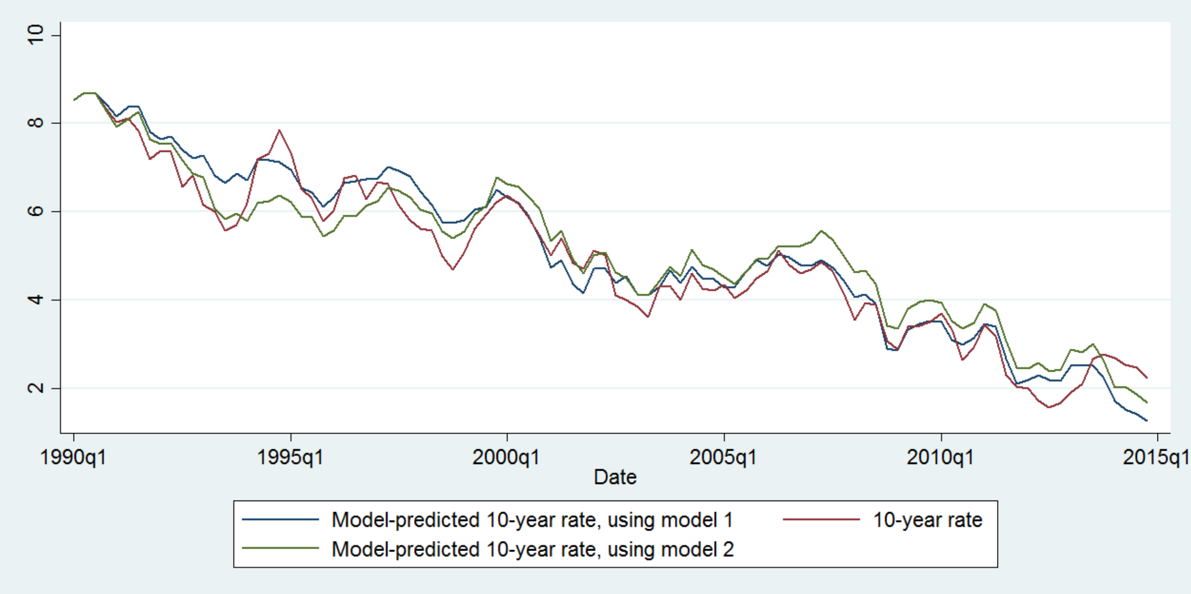

Figure 5.1.28 - Model-predicted and actual US 10-year rate

Figure 5.1.28 shows the model predictions in levels, together with the 10-year rate. As we saw in the plots of the changes, model 2 seems to follow the movements better in the first years, although model 1 is closer to the peak in 1994-1995. It looks like model 1 does consistently better after this.

The previous section estimated a model for the whole period, without interaction dummies. It therefore did not allow for a change in the relationship between the yield and the error correction term. We saw that the best model in that section managed to estimate the drop in yield in 2011-2013 better than the two models in this section, even though the estimated effect of increases in reserves were smaller in that model and in the models with interaction terms. The failure of the models with interaction terms to explain the drop as well as the model without is probably due to the error correction term, which estimated a positive disequilibrium in the period, pulling the predictions in the previous section downwards, while not being present in the models in this section. We can therefore ask ourselves whether the cointegration relationship between the 10-year rate and policy rate has gotten renewed importance in later years. Anyway, as seen in the previous section it does not seem to be sufficient to fully explain the large drop seen in 2011-2013.

5.2 Norway

When estimating a model for the Norwegian 10-year government bond yield, we start from the basic model with the policy rate representing the role of monetary policy and short-term interest rates, the inflation rate as a proxy for inflation expectations (based on a basic hypothesis of backward-looking expectations) and the output gap, representing the role of business cycles. In addition we add the VIX index, just as in the US case, to represent the effect of expected volatility in stock markets. Furthermore, Akram & Frøyland (1997) found that European, and in particular German, money market and bond rates were the main drivers behind the development of Norwegian interest rates. Therefore, we add the German 10-year government bond yield as an explanatory variable in the model.

5.2.1 Unit root analysis

10-year rates

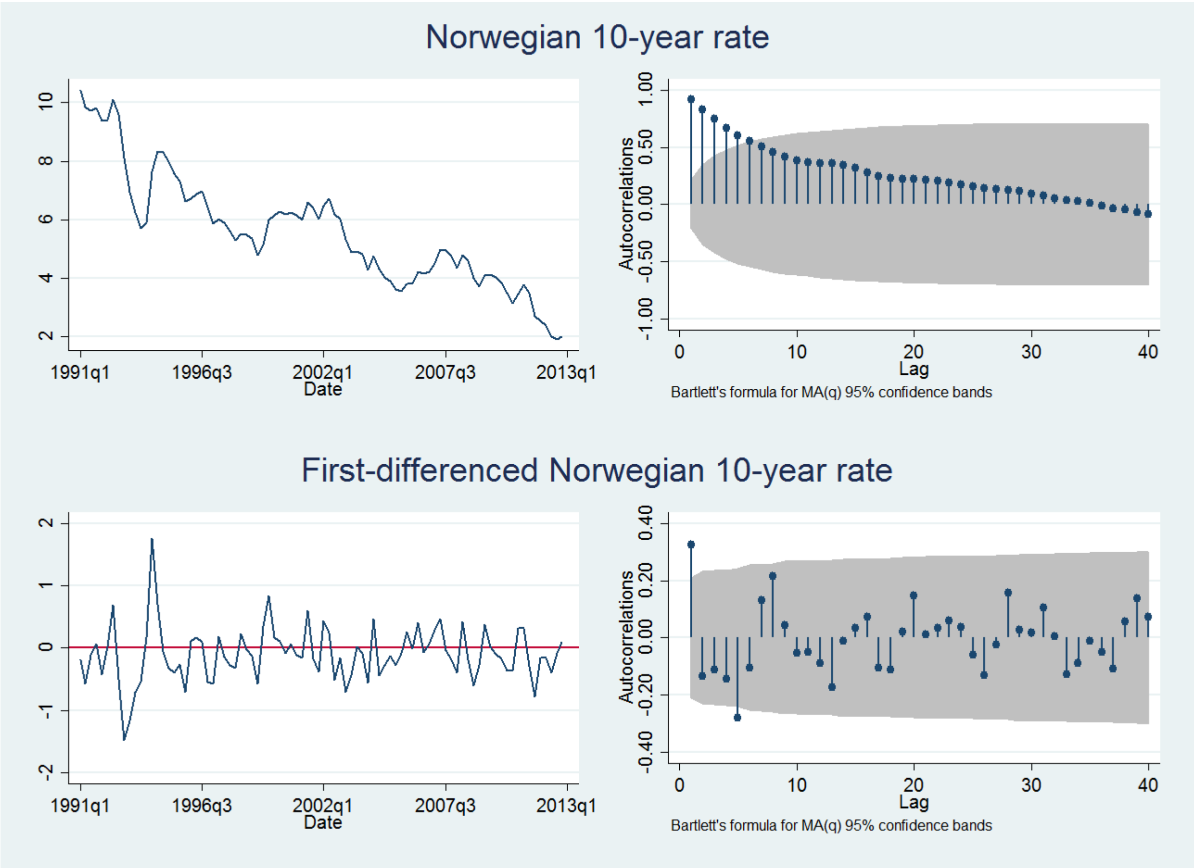

Figure 5.2.1 - Levels and first differences of the Norwegian 10-year rate with corresponding autocorrelations

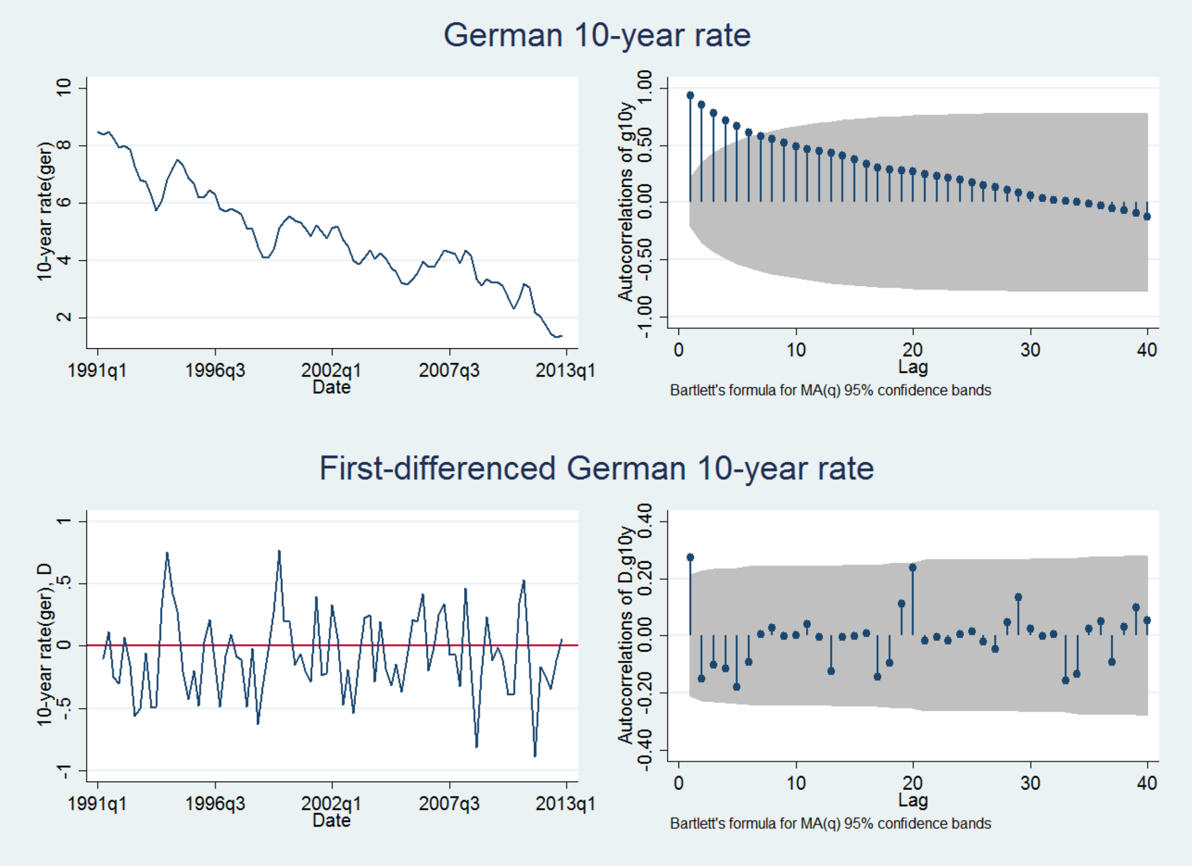

Figure 5.2.2 - Levels and first differences of the German 10-year rate with corresponding autocorrelations

Figures 5.2.1 and 5.2.2 show the levels and first differences of both the Norwegian and German 10-year rate, as well as their autocorrelations. We see that both 10-year rates have fallen over time, and are clearly not stationary. The plot of the autocorrelations shows some persistence in both cases. The first-differenced rates look stationary. They vary around zero and do not show signs of persistence. Tables A.19-A.22 show output from DF-GLS testing. They show that we can accept the null hypothesis of a unit root in the levels and reject it in the first differences of the 10-year rates, as expected.

Policy rate

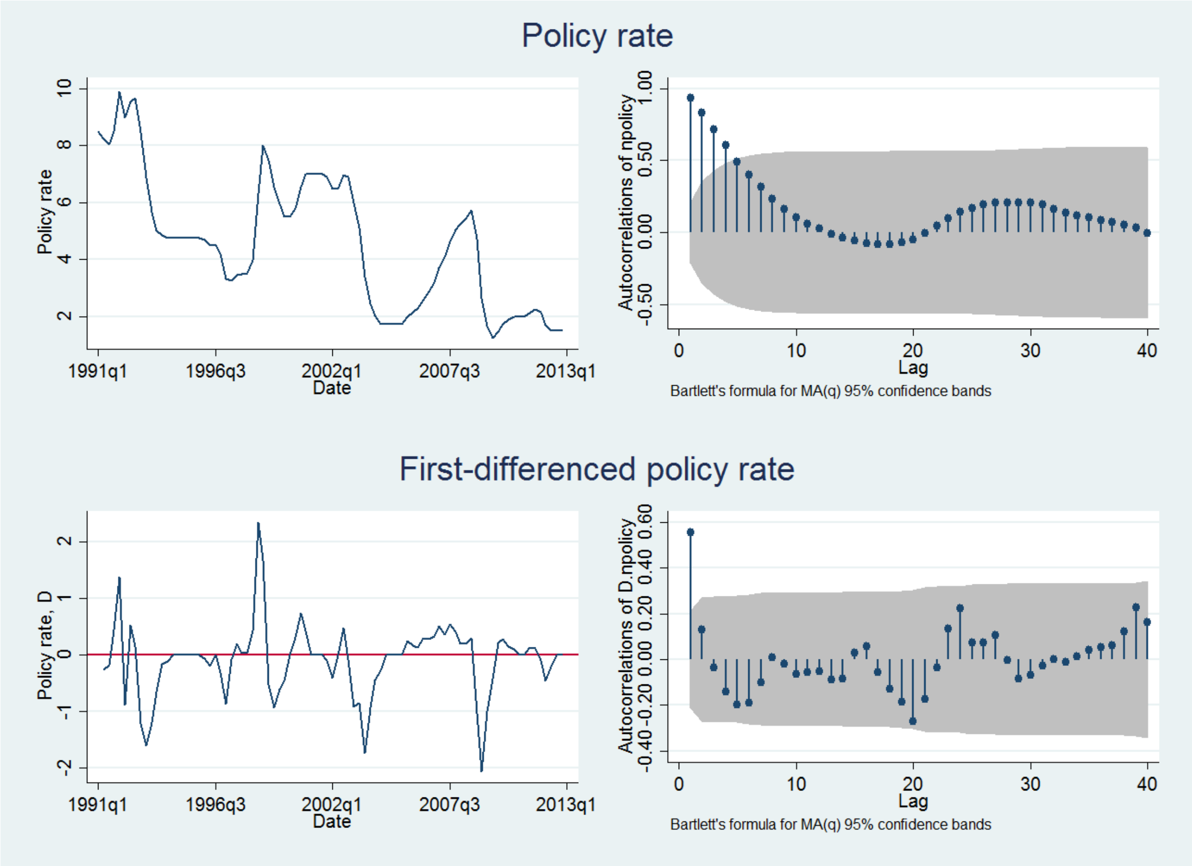

Figure 5.2.3 - Levels and first differences of the policy rate with corresponding autocorrelations

In figure 5.2.3, we see that the Norwegian policy rate seems to exhibit a similar decrease over time as the 10-year rates, although less clear. Although, we suspect it is non-stationary and shares a common stochastic trend with the long-term rates, it looks less clearly non-stationary than the 10-year rates. This is supported by the autocorrelation plot which shows less persistence than the case was for the 10-year rates. The first-differenced policy rate looks stationary and does not exhibit persistence. Tables A.23 and A.24 show output from DF-GLS testing. We cannot reject the null hypothesis of a unit root in the levels of the policy rate, while we reject it for the first differences.

Inflation rate

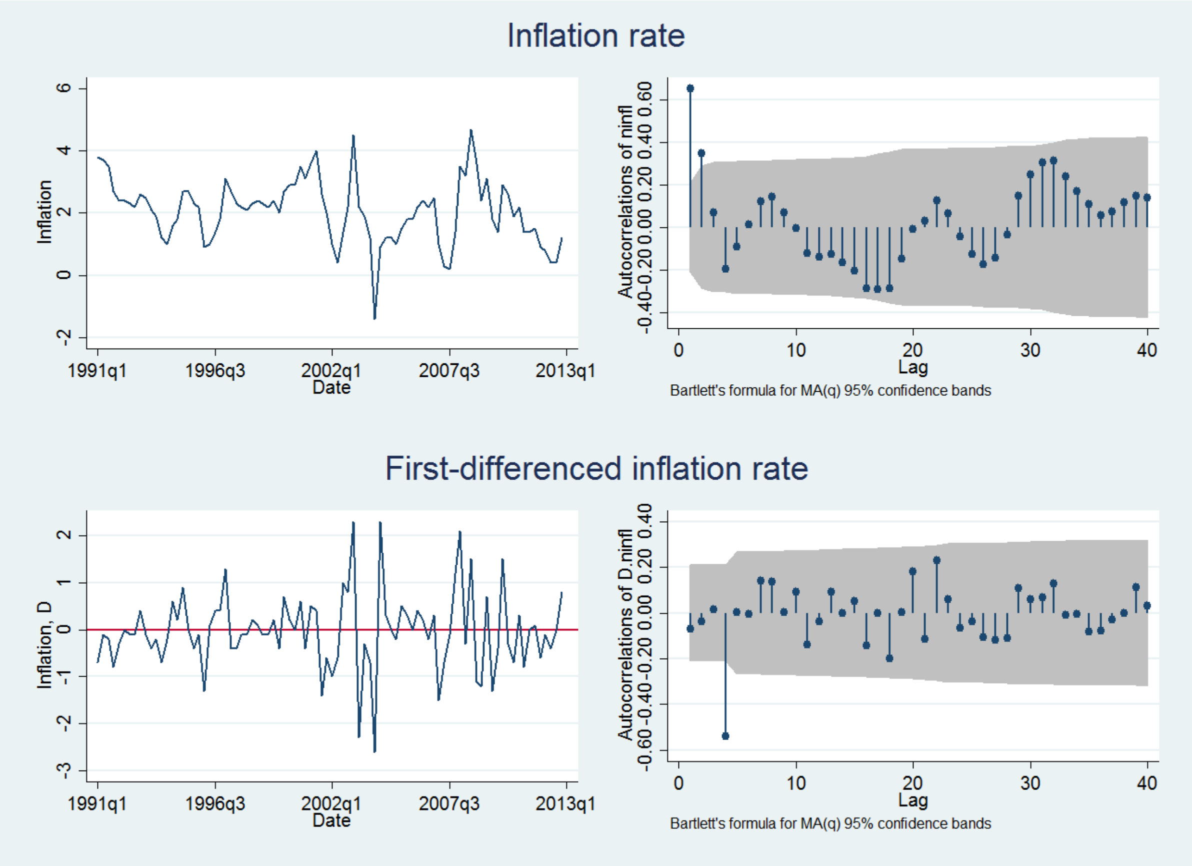

Figure 5.2.4 - Levels and first differences of the inflation rate with corresponding autocorrelations

Figure 5.2.4 displays the Norwegian inflation rate in levels and first differences, as well as their autocorrelations. None of the series exhibit persistence, and looking at the plots of their developments over time, both look like they may be I(0). Since the levels have two significant lags of autocorrelation, our standard when conducting DF-GLS testing is using a maximum lag length of four (see table A.25). All three lag length tests suggest that the appropriate lag length is four, for which we cannot reject the null hypothesis of a unit root.

Still, given the plot of the series and the lack of persistence, we are not entirely comfortable with just accepting this conclusion. This is supported by observing that had we restricted the maximum lag length to three, we would reject the null hypothesis of a unit root at the 5% level. Since we are uncertain, we turn to an ADF-test with four lagged differences included to see what it concludes (see table A.26). The ADF-test rejects unit root at the 5% level. Thus, unit root testing on the levels of inflation is inconclusive, but we are leaning toward believing that it is I(0), thus we will not include it when we estimate a long-run relationship between I(1)-variables. From table A.27, we see that we reject the null hypothesis of a unit root in the first-differenced inflation rate on the 1% level, so we can use them in the short-run part of the model.

Output gap

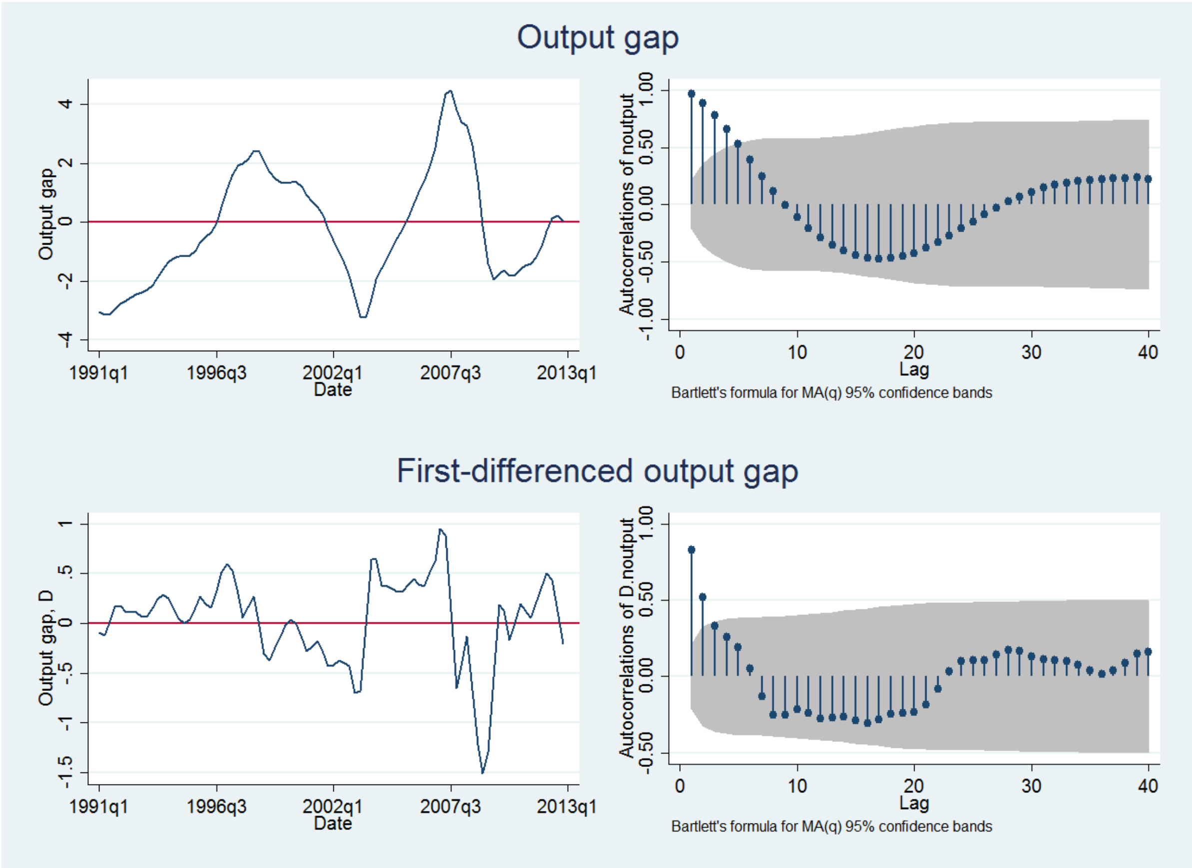

Figure 5.2.5 - Levels and first differences of the output gap with corresponding autocorrelations

Figure 5.2.5 shows the Norwegian output gap in levels and first differences, with corresponding autocorrelation plots. Our a priori expectation is that both the levels and differences would be I(0). The output gap by definition has a constant mean of zero, and it is natural to expect that the variation should not change much over time. The plots mostly support this, although the levels exhibit a bit of persistence, perhaps not that surprising as booms and recessions usually last for several years, and the variation in the first differences may seem to change somewhat.

Table A.28 shows output for DF-GLS testing on the levels. All lag length tests suggest four lags, for which we cannot reject the null hypothesis of unit root, although we are not far away from the 10% critical value. It is worth noting though, that the tests using 3 and 1 lags suggest rejecting unit root at the 1% level. This, combined with our expectation that the output gap should be I(0) means that we further examine the levels of the output gap with an ADF-test. The ADF-test with four lagged differences (see table A.29) rejects unit root at the 1% level. We thus conclude that the levels are most likely I(0). The test for the first differences clearly rejects unit root at the 1% level (see table A.30).

5.2.2 Estimating the long-run relationship

Evaluating the time-series

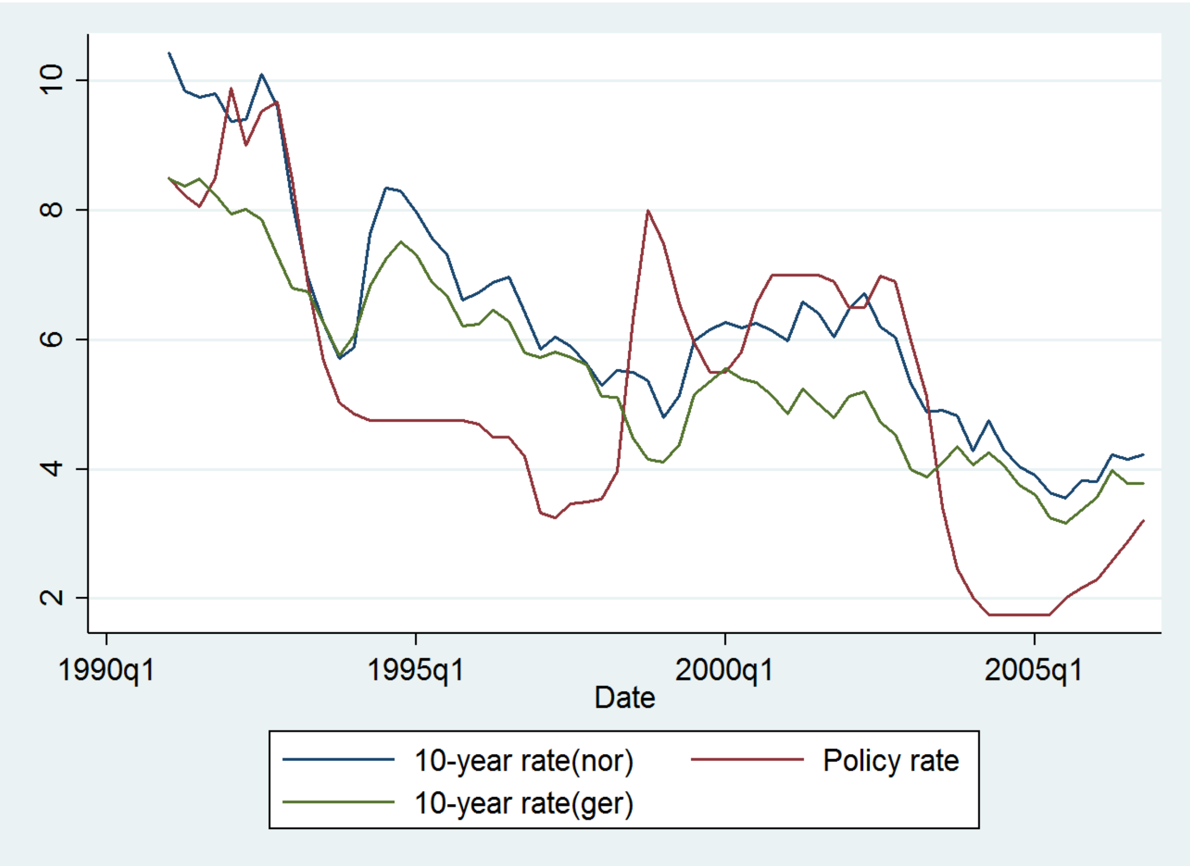

Figure 5.2.6 - I(1)-variables in the Norwegian model, Q1 1991-Q4 2006

The variables in our Norwegian models that are integrated of order one, and thus appropriate to use in a Johansen test for cointegration are the Norwegian and German 10-year rates and the Norwegian policy rate. In the sample ending in 2006 (see figure 5.2.6), we see that the 10-year rates follow each other closely, with the Norwegian rate almost always being a little bit higher than the German rate, likely due to the higher liquidity of German bonds leading to a liquidity premium in their price. The policy rate does not follow the others to the same degree, but seems to have some sort of relationship with them. All three rates seem to follow a negative stochastic trend in this period. If we look at the plot of the series for the entire period for which we have data (see figure 5.2.7), there is not the same tendencies suggesting a change in the long-run relationship that we saw in the US case.

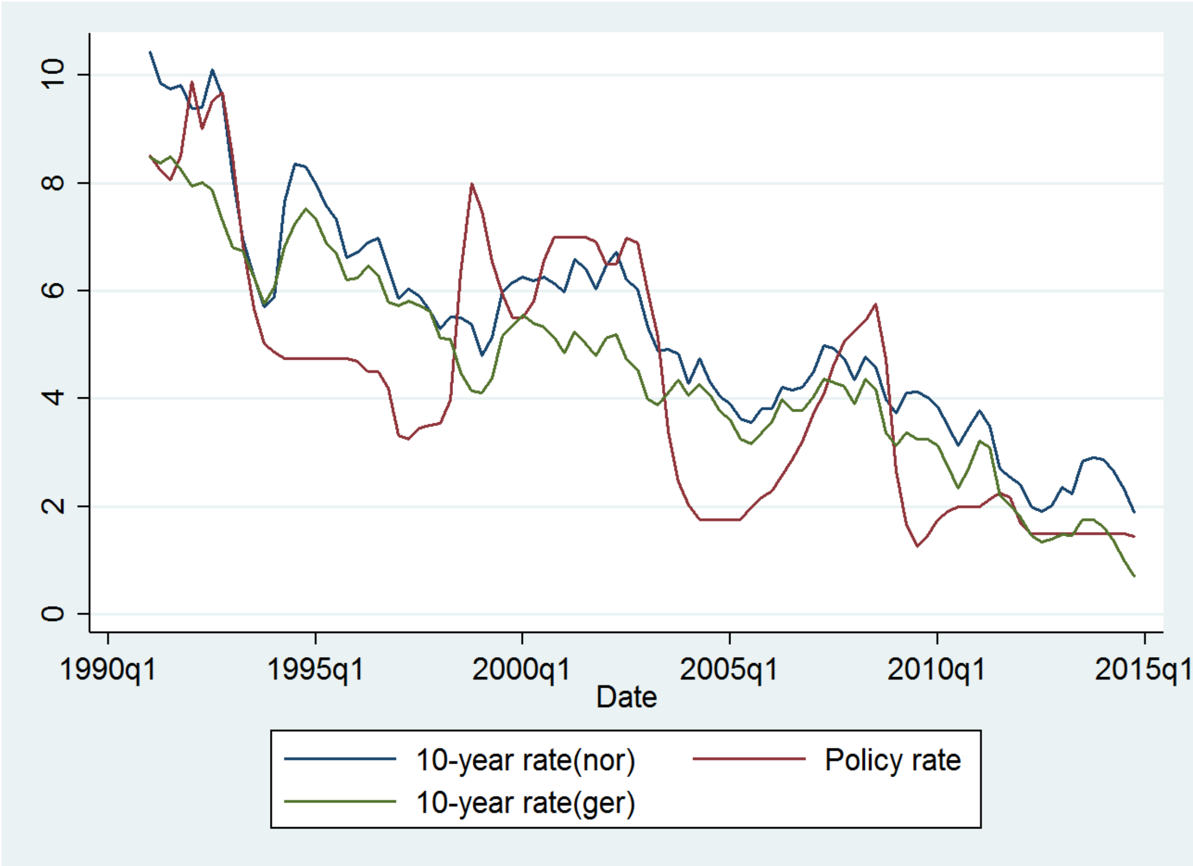

Figure 5.2.7 - I(1)-variables in the Norwegian model, Q1 1991-Q4 2014

Checking appropriate lag length

When testing for the appropriate lag length in the underlying VAR model (see tables 5.2.1 and 5.2.2), based on the Norwegian and German 10-year rates and the Norwegian policy rate, we almost exclusively get support for two lags. The only exception is that the LR-statistic suggests six lags when we allow for a maximum lag length of eight. Furthermore, running the same tests using a sample ending in 2012, we get the same results except that the LR-statistic now suggests eight lags when allowing for a maximum lag length of eight.

Table 5.2.1 - Lag-order selection statistics for the underlying VAR model, with maximum lag length set to four

Selection-order criteria
Sample: 1992q1 - 2006q4 Number of obs = 60

lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-239.097				.641636	8.06989	8.11085	8.1746
1	-66.854	344.49	9	0.000	.002782	2.62847	2.79231	3.04734
2	-39.319	55.07*	9	0.000	.001504*	2.01063*	2.29736*	2.74365*
3	-33.5828	11.472	9	0.245	.001687	2.11943	2.52903	3.1666
4	-31.7074	3.7508	9	0.927	.002164	2.35691	2.8894	3.71824

Table 5.2.2 - Lag-order selection statistics for the underlying VAR model, with maximum lag length set to eight

Selection-order criteria
Sample: 1993q1 - 2006q4 Number of obs = 56

lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	-208.875				.388037	7.56697	7.60903	7.67547
1	-53.8467	310.06	9	0.000	.00211	2.35167	2.51993	2.78567
2	-21.366	64.961	9	0.000	.000915*	1.51307*	1.80753*	2.27258*
3	-14.2411	14.25	9	0.114	.000986	1.58004	2.0007	2.66505
4	-6.57113	15.34	9	0.082	.001049	1.62754	2.17439	3.03805
5	3.2651	19.672	9	0.020	.001042	1.59767	2.27072	3.33369
6	13.0774	19.625*	9	0.020	.001048	1.56866	2.36791	3.63018
7	15.5531	4.9515	9	0.839	.001391	1.80167	2.72712	4.1887
8	20.3732	9.6403	9	0.380	.00173	1.95096	3.0026	4.66348

Johansen cointegration test

Table 5.2.3 - Results from Johansen cointegration test, sample Q1 1991-Q4 2006. Trend specification: Case 4 – "Restricted constant"

Maximum rank	**Trace statistic**			**5% critical value**
	Lags = 2	Lags = 6	Lags = 8	
0	37.4573	39.8526	35.9610	34.91
1	**15.1334***	**13.7600***	**12.3657***	19.96
2	5.2507	5.0001	3.0248	9.42
3				
Suggested rank	1	1	1	

We run the Johansen cointegration test for two, six and eight lags in the underlying VAR. Table 5.2.3 displays the results we get when using a sample ending in 2006. We see that the trace statistic indicates that the rank (and thus the number of stable long-run relationships) is one. If we use a sample ending in 2012, we get almost identical results, the only exception being that the trace statistic was unable to reject a rank of 0 when specifying six lags in the underlying VAR.

Estimating cointegrating relationships

Table 5.2.4 - Estimated cointegrating relationships and key information from the estimated VECM

	Variable	**Sample ending Q4 2006**	**Sample ending Q4 2012**
Lags		2	2
Trend specification		Restricted constant	Restricted constant
AIC		2.0630	1.9574
R-sq	10-year rate (Nor)	0.4163***	0.3015***
	Policy rate	0.3860***	0.4007***
	10-year rate (Ger)	0.2193***	0.1573***
Alpha	10-year rate (Nor)	-0.7727*** (0.1628)	-0.5413*** (0.1288)
	Policy rate	-0.3006 (0.2277)	0.0269 (0.1698)
	10-year rate (Ger)	-0.2988** (0.1235)	-0.2148** (0.1038)
CE	10-year rate (Nor)	1	1
	Policy rate	-0.2039*** (0.0352)	-0.2206*** (0.0410)
	10-year rate (Ger)	-0.9055*** (0.0543)	-0.8487*** (0.0561)
	Constant term	-0.2108 (0.2476)	-0.4187** (0.2010)

*Significant at 10% level **Significant at 5% level ***Significant at 1% level Standard errors in parentheses

Stars in R-sq row reports whether the equations are significant according to a chi^2-test.

To estimate the long-run relationship between the three I(1)-variables in the Norwegian model, we use Stata to estimate a VECM. Following the results from the Johansen Cointegration test, we specify the rank to one. From the lag length testing, a lag length of two clearly seems most appropriate. We did experiment with a longer lag length, but this did more to distort the estimated model than to improve it.

In table 5.2.4 we display key information for the estimated VECMs using a sample ending in 2006 and a sample ending in 2012. We see that the cointegrating equation does not show any major changes between the two model estimations. We see that we find that the Norwegian 10-year rate has a long-run positive relationship with both the German 10-year rate and the Norwegian policy rate. As expected the coefficient of the German rate is very high. Furthermore, we see that, considering how many relevant parameters are missing, the model explains impressively much of the variation in the Norwegian 10-year rate and policy rate, especially in the sample ending in 2006.

We see that the adjustment parameter for the Norwegian 10-year rate suggests that it adjusts quickly towards equilibrium. The adjustment parameter for the German 10-year rate (in both periods) and the policy rate (in the sample ending in 2006) are both negative (note that the alpha of the policy rate is not significant on the 10% level). A negative sign on these alphas suggests that the German rate and the policy rate move away from equilibrium. However, the large negative alpha on the Norwegian 10-year rate means that in total there is movement towards equilibrium. In addition, both the German rate and the policy rate depend heavily on factors that are not included in this model, which is only meant to provide us with an error correction term to use in our further modelling.

Reviewing the estimated long-run relationships

Plotting the estimated cointegrated equations and their autocorrelation plots for the periods for which they are estimated (figure 5.2.8 and 5.2.9), we see that it looks like we have successfully found a stationary relationship between the three I(1)-variables.

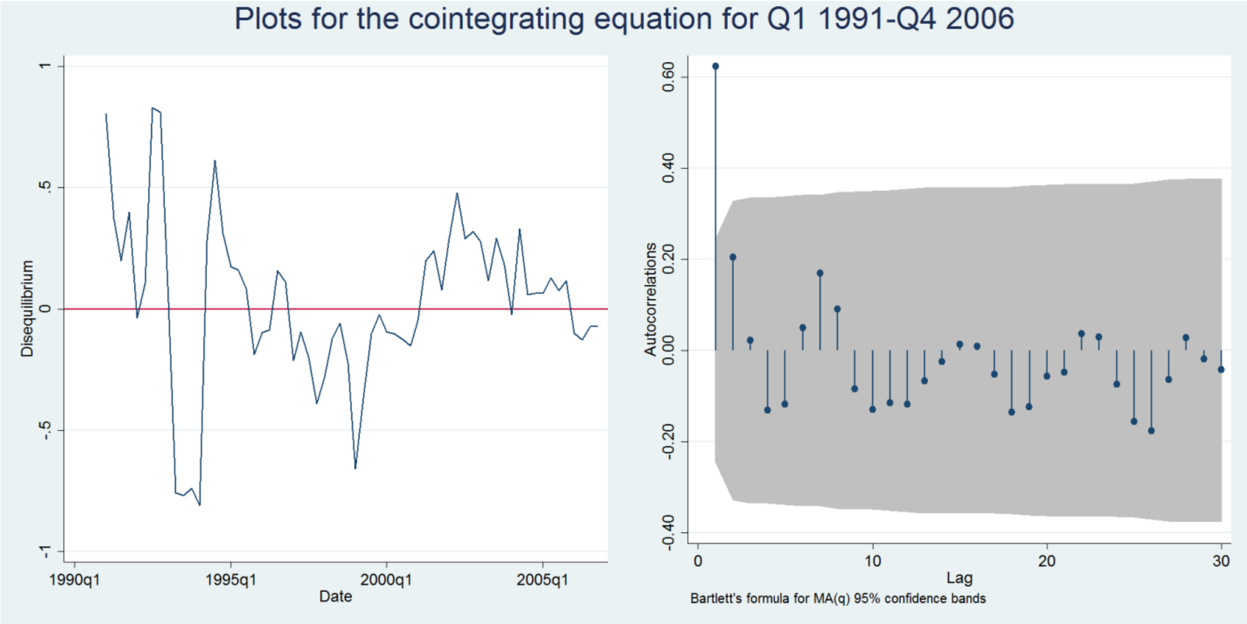

Figure 5.2.8 - Disequilibrium and autocorrelation plot for the cointegrating equation for Q1 1991-Q4 2006

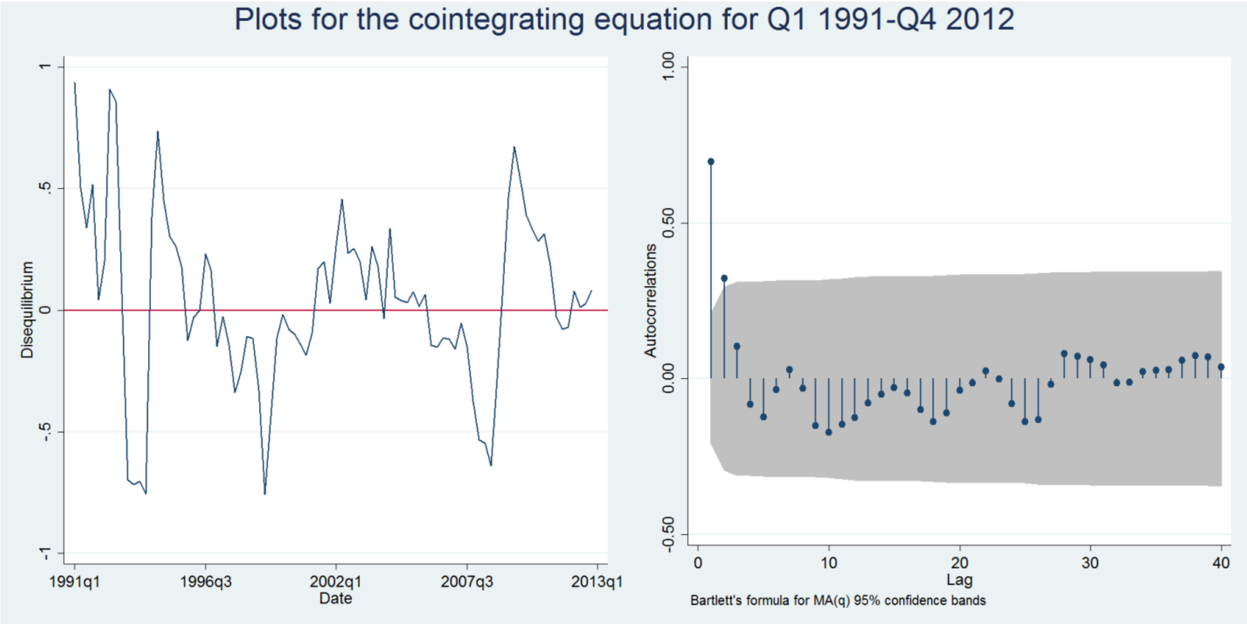

Figure 5.2.9 - Disequilibrium and autocorrelation plot for the cointegrating equation for Q1 1991-Q4 2012

Figure 5.2.10 shows how the two estimated equations compare when plotted for the entire period for which we have data. We see that there are no major differences between the two, except that the equation estimated for a sample ending in 2012, not surprisingly, is closer to equilibrium in recent years. We see that the relationship we have found suggests a positive disequilibrium in recent years, reflecting that Norwegian interest rates have not fallen as much as German rates.

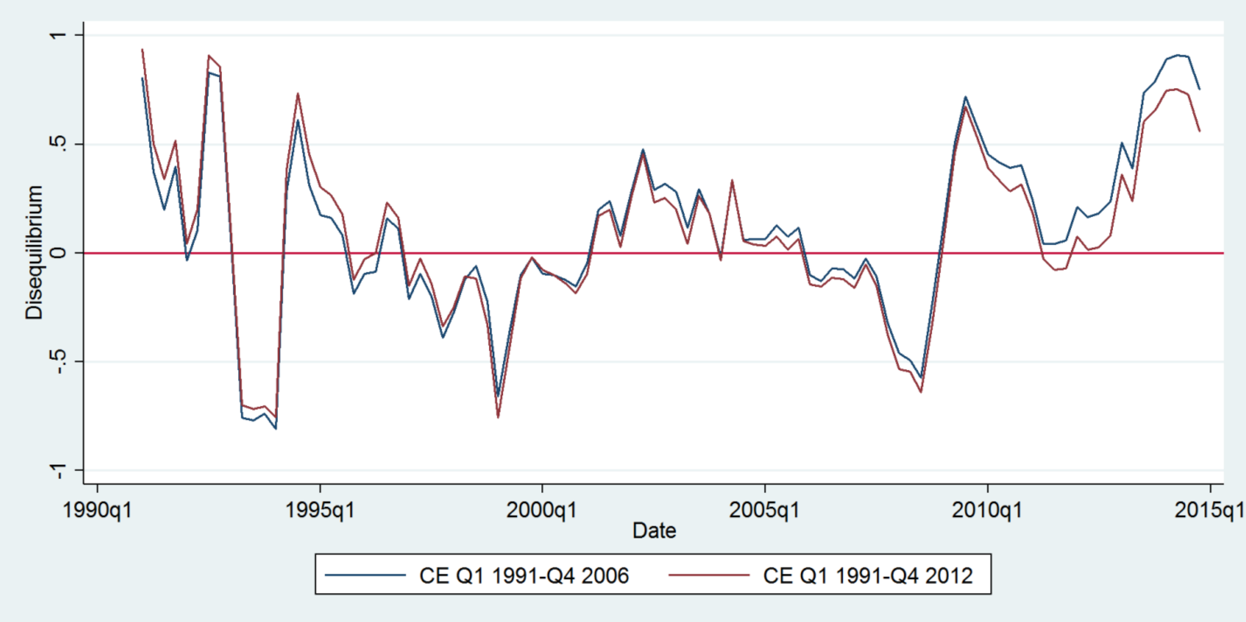

Figure 5.2.10 - Comparison between the estimated cointegrated equations

5.2.3 ECM-modelling - Q1 1991-Q4 2006

In the previous section we estimated a long-run relationship between the Norwegian 10-year rate, the Norwegian policy rate and the German 10-year rate. We use the equation estimated for the period Q1 1991-Q4 2006 as an error correction term when we now include I(0)-variables to estimate an ECM with short- and long-run components. The ECT is as follows:

Equation 5.2.1 – Error correction term based on Q1 1991-Q4 2006

$$ECT = 10y\ rate\ (nor) - 0.2039 * Policy\ rate - 0.9055 * 10y\ rate\ (ger) - 0.2108$$

We start the estimation of the ECM by estimating it with two, one and none lagged differences for all included variables, as well as the lagged error correction term. We find that there were no second lags that should be included in the model. The estimated models with one and none lags are displayed in column 1 and 2 in table 5.2.5. Column 3 is the same specification as in column 2, except that we include the lagged first difference of the Norwegian 10-year rate. In column 4, only statistically significant terms are included. In column 5, we additionally include the first-differenced VIX index, which is not statistically significant on the 5% or 10% level in this specification, but has a relatively low p-value (0.126) and is significant with approximately the same total effect in column 1. When considering that its effect is economically interesting, we choose to add it in column 5, which is our preferred model. Finally, column 6 displays the best model we find if we do not include the error correction term.

Table 5.2.5 - Estimated models for Norway, sample Q1 1991-Q4 2006

	(1) D.10-year rate(nor)	(2) D.10-year rate(nor)	(3) D.10-year rate(nor)	(4) D.10-year rate(nor)	(5) D.10-year rate(nor)	(6) D.10-year rate(nor)
LD.10-year rate(nor)	0.156		0.168^{**}	0.157^{**}	0.173^{**}	
	(0.190)		(0.023)	(0.032)	(0.017)	
D.Policy rate	0.339^{***}	0.307^{***}	0.254^{***}	0.238^{***}	0.253^{***}	0.298^{***}
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
LD.Policy rate	-0.0957					
	(0.303)					
D.Inflation	0.0414	0.0302	0.0236			
	(0.186)	(0.316)	(0.484)			
LD.Inflation	-0.0617^{*}					
	(0.050)					
D.Output gap	-0.000493	-0.0215	-0.0149			
	(0.998)	(0.759)	(0.830)			
LD.Output gap	-0.0894					
	(0.636)					
D.10-year rate(ger)	1.038^{***}	1.180^{***}	1.074^{***}	1.096^{***}	1.072^{***}	1.199^{***}
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
LD.10-year rate(ger)	0.00546					0.163^{*}
	(0.971)					(0.063)
D.VIX index	-0.0204^{**}	-0.00971	-0.0131		-0.0124	-0.0128
	(0.049)	(0.226)	(0.112)		(0.126)	(0.126)
LD.VIX index	-0.00653					
	(0.531)					
L.Error correction term	-0.373^{***}	-0.322^{***}	-0.407^{***}	-0.387^{***}	-0.403^{***}	
	(0.002)	(0.001)	(0.000)	(0.000)	(0.000)	
Observations	62	63	63	63	63	62
R^2	0.826	0.787	0.804	0.797	0.803	0.755
AIC	3.124	3.918	0.584	-3.182	-2.949	8.301

p-values in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Let us take a closer look at the estimated model, with focus on our preferred model in column 5. When looking at the estimated coefficients, it is striking how big of a role the German 10-year rate plays. It has a short-run effect of 1.296 and a long-run effect of 0.906, we thus find an almost one-to-one relationship between the Norwegian and German rates, reinforcing the findings of Akram & Frøyland (1997).[34] We actually find even stronger influence than they did; their estimated coefficients for the ECU-rate were 0.87 and 0.7 for the long- and short-run respectively.

Furthermore, we find the expected positive influence from the policy rate. The policy rate has a short-run effect of 0.306, while the estimated long-run effect was 0.204.[35] It thus seem like the 10-year rate reacts more strongly in the short-run than the estimated long-run relationship would suggest.

The inflation rate, however, turns out not to be significant. This can mean either that the inflation rate is not a good proxy for inflation expectations in Norway, or that inflation and/or inflation expectations are so stable in Norway that we are not able to pick up a significant effect. We expect that the latter is the case. In that case we can see this as an indication that Norges Bank's 2.5 percent inflation target is viewed as credible. It could of course also be the case that the heavy influence from European rates leads to Norwegian inflation not playing any large role as long as it is not extremely different from what is experienced in the rest of Europe.

We see that the output gap is not significant. Furthermore, it has a negative sign, while we would expect that the sign would be positive, since better economic times normally would mean that investors to a lesser degree would seek the safety of government bonds and also demand a higher term premium. Since it is very far from statistically significant, we should not speculate too much on why there is a negative coefficient here. However, one possible effect that could explain the output gap having a negative effect, is that strong economic times in Norway potentially has made investing in Norway more attractive to international investors. This could particularly be the case if Norway is doing better than other advanced economies. In such a case we could see investors, who would have invested in e.g. German government bonds choose to invest in Norwegian ones instead, thereby pushing up the price and down the yield. If this is true, then this may offset the expected effect, thus leading to the output gap not being significant in our model.

[34] Denominator = 1 – 0.173 = 0.827. Total short-run effect = 1.072/0.827 = 1.296.

[35] Total short-run effect = 0.253/0.827 = 0.306.

We find some evidence of an effect of the VIX index on the Norwegian 10-year rate, although it is not significant in most specifications. In our preferred model, we find that increasing the VIX index by one decreases the 10-year rate by 1.5 basis points, meaning that the Norwegian 10-year rate decreases when more volatility is expected in stock markets.[36]

Finally, we find that the 10-year rate reacts relatively strongly to deviations from the long-run relationship with the German rate and the policy rate. The coefficient on the lagged error correction term is -0.403 in our preferred model, indicating that the 10-year rate closes forty percent of the disequilibrium in a quarter.

Examining the fit of the model

Table 5.2.6 – Preferred model for Norway, sample Q1 1991-Q4 2006, with heteroskedasticity robust standard errors

	(1) D.10-year rate(nor)
LD.10-year rate(nor)	0.173^{**}
	(0.0705)
D.Policy rate	0.253^{***}
	(0.0533)
D.10-year rate(ger)	1.072^{***}
	(0.0976)
D.VIX index	-0.0124
	(0.00802)
L.Error correction term	-0.403^{***}
	(0.0940)
Observations	63
R^2	0.803
AIC	-2.949

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

[36] Total short-run effect = -0.0124/0.827 = -0.0150.

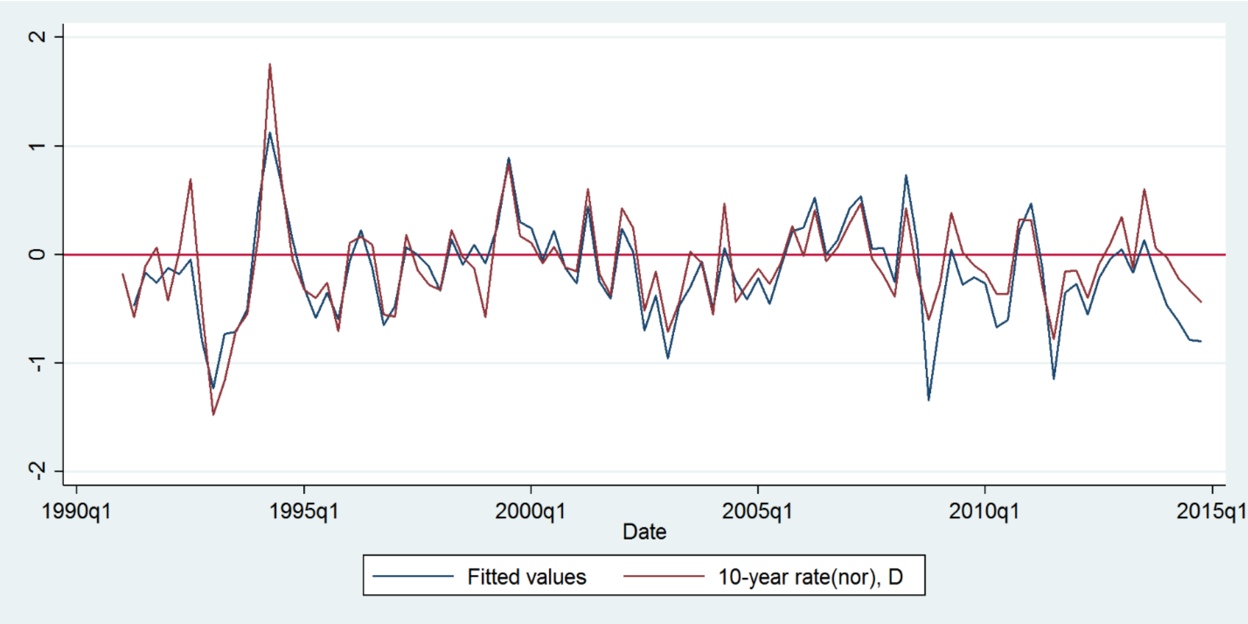

Figure 5.2.11 - Model-predicted and actual changes in the Norwegian 10-year rate

Let us now examine how well our preferred model (table 5.2.6) predicts the actual movements in the Norwegian 10-year rate. Figure 5.2.11 shows the actual and model-predicted changes in the Norwegian 10-year rate, both in- and out-of-sample. We see that the model predicts the movements pretty well. In-sample we see that the model is not able to fully explain the most extreme changes in the 1990s, while out-of-sample, we see that the model generally predicts larger changes than what has actually happened, in particular larger falls in the rate. This difference reflects that the Norwegian rate has not fallen as much as the German rate over recent years.

Figure 5.2.12 depicts the actual Norwegian 10-year rate, compared to the rate predicted by our model if we start with the actual value in Q1 1991 and then successively add the predicted changes from the model. We see that although it consequently predicts a somewhat lower rate, the model performs relatively well until the early 2000s. Looking at recent years we see that the model predicts that the rate would plummet, even approaching -5% towards the end of 2014.

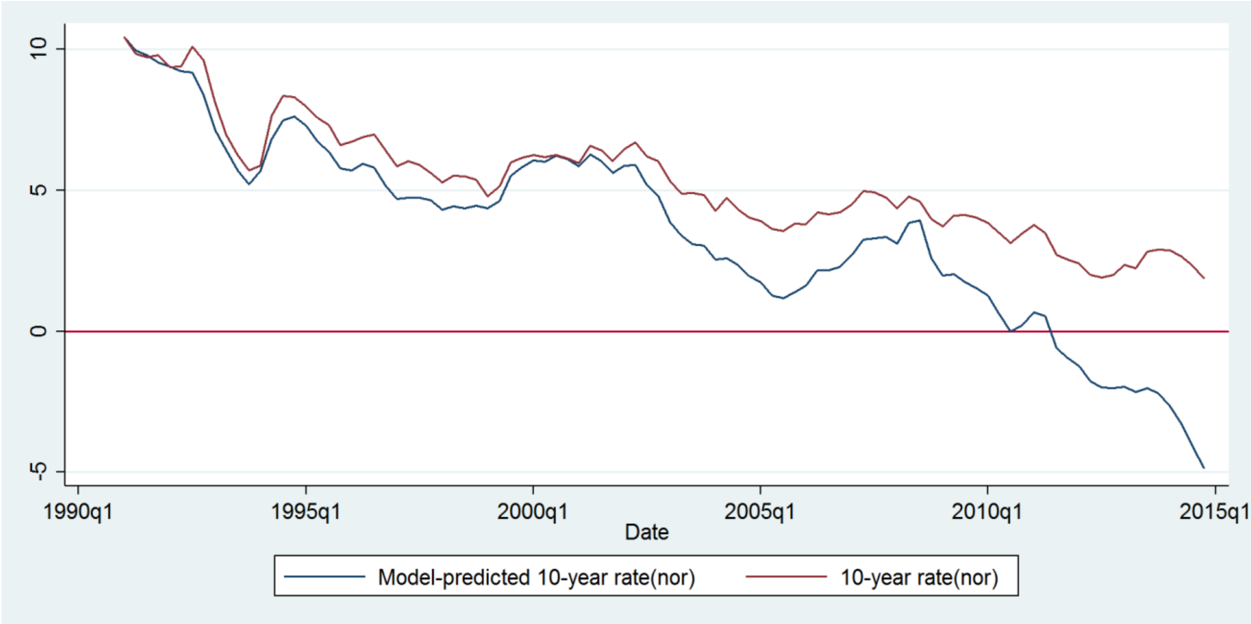

Figure 5.2.12 – Model-predicted and actual Norwegian 10-year rate

What can explain the predicted fall is the deviation of the 10-year rate from its estimated long-run relationship with the German rate and the policy rate. The 10-year rate has been above (at times far above) its equilibrium value according to the long-run relationship (see figure 5.2.10). These predictions thus suffer from the same problems we experienced in the US model. This form of prediction can hugely over-predict movements in the rate, especially out-of-sample, when the 10-year rate consistently deviates from the long-run relationship.

To correct for this issue, we replace the actual lagged value of the 10-year rate with the lagged predicted value in the error correction term when we make predictions. Starting from the actual rate in Q1 1991 and then successively adding predicted changes using this methodology, we get the model-predicted values of the Norwegian 10-year rate displayed in figure 5.2.13.

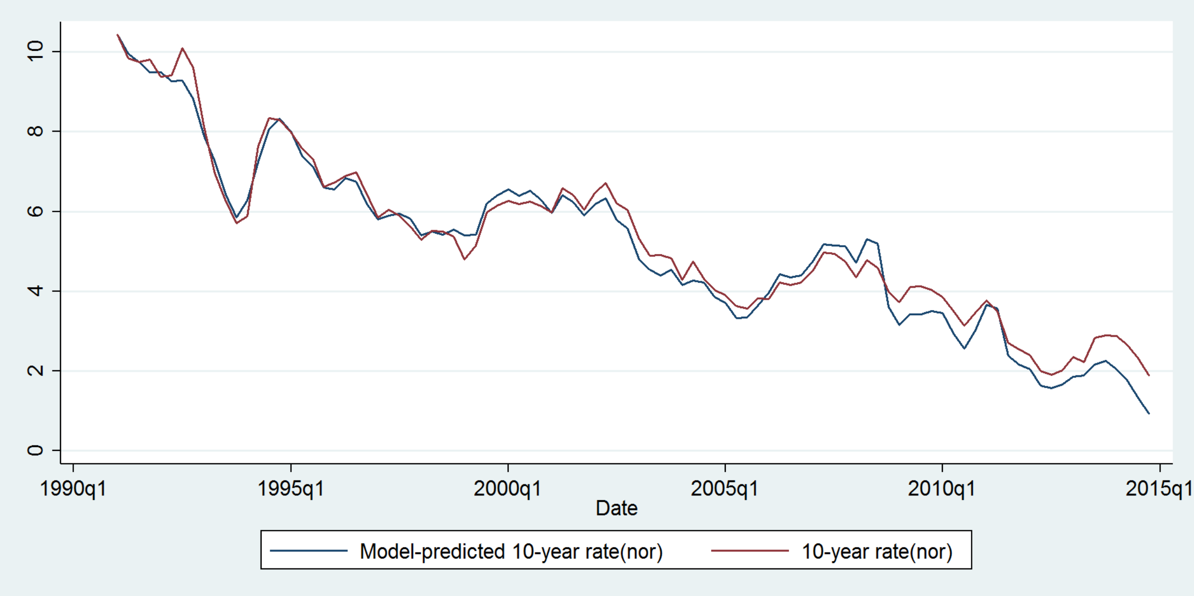

Figure 5.2.13 - Model-predicted and actual Norwegian 10-year rate

We see that this prediction was much better. The model predicts the actual 10-year rate pretty well in-sample. Out-of-sample it still predicts the correct direction of movements, but does not hit the level as well as in-sample. In particular, it predicts a lower rate than has actually occurred for almost the entire out-of-sample period.

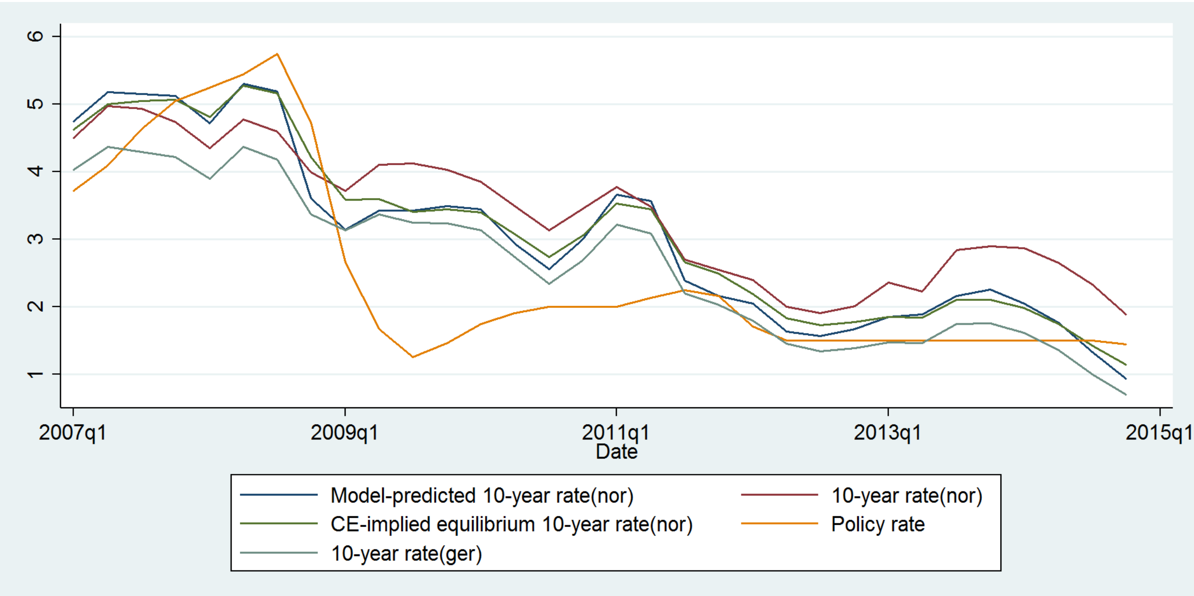

Figure 5.2.14 - A closer look at the out-of-sample period (2007-2014)

Figure 5.2.14 depicts, for the out-of-sample period, the actual Norwegian 10-year rate, the rate predicted by the model, the equilibrium rate suggested by the estimated long-term relationship, as well as the two most important explanatory variables, the German 10-year rate and the policy rate. We see that both the suggested equilibrium rate and the rate predicted by the model starts the period by rising above the actual rate, likely due to the high level of the policy rate in that period. For the rest of the period though, both lies between the German and the actual Norwegian rates, much of the time lying closer to the German rate than the Norwegian rate.

To sum up, the Norwegian 10-year rate has been higher in recent years than our model predicts, and it seems to not follow its estimated long-term relationship with the policy rate and the German rate as closely as before. It seems like the Norwegian rate does not respond as strongly to the German rate as previously, likely due to the fact that Norway has been less affected by the financial crisis and the sovereign debt crisis than Germany and the EU. We have already seen that when we estimated the long-run relationship using a sample ending in 2012 rather than 2006, we found a slightly smaller coefficient on the German rate, as well as a larger constant term. We expect that when we move on to estimate the full model for this sample we will find more evidence of less German influence on the Norwegian rate, as well as perhaps a significant positive effect from the output gap.

5.2.4 ECM-modelling – Q1 1991-Q4 2012

When estimating the ECM for the period Q1 1991-Q4 2006 equation 5.2.1 was used as an error correction term. When we now move on to estimating a model for Q1 1991-Q4 2012 we will primarily use the long-run relationship estimated using a sample from that period (equation 5.2.2), but we will also consider the implications of using equation 5.2.1, as this is our baseline long-run relationship.

Equation 5.2.1 – Error correction term based on Q1 1991-Q4 2006

$$ECT = 10y\ rate\ (nor) - 0.2039 * Policy\ rate - 0.9055 * 10y\ rate\ (ger) - 0.2108$$

Equation 5.2.2 – Error correction term based on Q1 1991-Q4 2012

$$ECT = 10y\ rate(nor) - 0.2206 * Policy\ rate - 0.8487 * 10y\ rate(ger) - 0.4187$$

Again, we start the estimation of the ECM by estimating it with two, one and none lagged differences for all included variables, as well as the lagged error correction term (the one estimated for this sample). Again, no second lags are necessary. The model estimates with one and none lags are displayed in column 1 and 2 in table 5.2.7. Removing terms that are not significant at the 10% level gives us column 3 which is the best model for this sample which does not allow for a change in the coefficients from 2007 onwards. In column 4 we display the same model, but with the "old" error correction term. This does not lead to any major differences in the estimated coefficients. Finally, column 5 displays the best model estimation for this sample when we do not include an error correction term. The most interesting implication of this last model is that the negative impact of the output gap we briefly discussed in chapter 5.2.3 is statistically significant.

Table 5.2.7 - Estimated models for Norway, sample Q1 1991-Q4 2012

	(1) D.10-year rate(nor)	(2) D.10-year rate(nor)	(3) D.10-year rate(nor)	(4) D.10-year rate(nor)	(5) D.10-year rate(nor)
LD.10-year rate(nor)	0.236**		0.289**	0.301***	
	(0.034)		(0.012)	(0.009)	
D.Policy rate	0.260***	0.234***	0.231***	0.224***	0.278***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
LD.Policy rate	-0.120		-0.136**	-0.133**	-0.103*
	(0.131)		(0.050)	(0.047)	(0.077)
D.Inflation	-0.00612	-0.0171			
	(0.811)	(0.542)			
LD.Inflation	-0.0499*				
	(0.060)				
D.Output gap	0.0331	-0.0841			-0.121**
	(0.720)	(0.128)			(0.037)
LD.Output gap	-0.124				
	(0.208)				
D.10-year rate(ger)	0.988***	1.063***	0.996***	1.005***	1.110***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
LD.10-year rate(ger)	-0.151		-0.226*	-0.253**	
	(0.213)		(0.061)	(0.042)	
D.VIX index	0.000571	-0.00215			
	(0.924)	(0.682)			
LD.VIX index	0.00269				
	(0.483)				
L.Error correction term	-0.325***	-0.264***	-0.333***		
	(0.001)	(0.001)	(0.001)		
L.Old error correction term				-0.340***	
				(0.001)	
Observations	86	87	86	86	86
R^2	0.799	0.758	0.785	0.785	0.737
AIC	-6.287	-2.015	-12.74	-12.47	0.868

p-values in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Let us take a look at the model estimates, with focus on our preferred model in column 3.

Firstly, we find that the 10-year rate still reacts relatively strongly to deviations from the long-run relationship with the German rate and the policy rate, although the speed of adjustment has fallen slightly, from -0.403 to -0.333.

We still see that the German 10-year rate plays a major part in explaining the developments in the Norwegian 10-year rate. In this model we find that the German rate has a positive short-run effect of 1.083 and long-run effect of 0.849.[37] Even if there is still a strong German influence on the Norwegian 10-year rate, we see that the short-run effect, the long-run effect, and the speed of adjustment are all smaller in this model than in the previously estimated model.

For the policy rate, the total short-run effect is estimated to be 0.134, while the long-run effect was 0.221.[38] The estimated short-run effect is lower, while the long-run effect is slightly higher, than in the previous model. If we also consider the slower speed of adjustment in this model, we see that the influence of the policy rate has become considerably weaker.

Inflation and output gap are still insignificant and our arguments from chapter 5.2.3 about these variables are still valid. Furthermore, when we work with this longer sample there is no longer strong enough evidence of an effect from the VIX index to include it in our preferred model.

To sum up, when we extend our sample we only find significant effects from the German 10-year rate and the Norwegian policy rate on the Norwegian 10-year rate. However, for both of these the identified effects are considerably weaker than when we only used a sample that stopped in Q4 2006. It seems that the determination of Norwegian long-term rates has changed somewhat in the years during and following the financial crisis and sovereign debt crisis. Therefore, we next estimate a model where we allow the coefficients to be different in the two periods Q1 1991-Q4 2006 and Q1 2007-Q4 2012.

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[37] Denominator = 1 – 0.289 = 0.711. Sum of short-run coefficient = 0.996 – 0.226 = 0.770. Total short-run effect = 0.770/0.711 = 1.083.

[38] Sum of short-run coefficients = 0.231 – 0.136 = 0.095. Total short-run effect = 0.095/0.711 = 0.134.

Table 5.2.8 - Estimated models for Norway, with interaction terms

	(1) D.10-year rate(nor)	(2) D.10-year rate(nor)	(3) D.10-year rate(nor)	(4) D.10-year rate(nor)
LD.10-year rate(nor)	0.173**	0.159**	0.148	0.161**
	(0.018)	(0.034)	(0.258)	(0.038)
LD.10-year rate(nor) x (2007-)	-0.169*	-0.151*	0.0342	-0.179*
	(0.063)	(0.095)	(0.891)	(0.053)
D.Policy rate	0.253***	0.258***	0.286***	0.308***
	(0.000)	(0.000)	(0.000)	(0.000)
D.Policy rate x (2007-)	-0.303***	-0.309***	-0.251**	-0.289***
	(0.000)	(0.000)	(0.011)	(0.002)
LD.Policy rate			-0.0854	-0.0928
			(0.312)	(0.224)
LD.Policy rate x (2007-)			0.0136	-0.00108
			(0.889)	(0.990)
D.10-year rate(ger)	1.071***	1.074***	1.090***	1.065***
	(0.000)	(0.000)	(0.000)	(0.000)
D.10-year rate(ger) x (2007-)	-0.118	-0.121	-0.206*	-0.112
	(0.369)	(0.373)	(0.082)	(0.408)
LD.10-year rate(ger)			-0.00108	
			(0.995)	
LD.10-year rate(ger) x (2007-)			-0.172	
			(0.529)	
D.VIX index	-0.0124	-0.0114		-0.0140*
	(0.132)	(0.145)		(0.094)
D.VIX index x (2007-)	0.0148*	0.0137*		0.0183**
	(0.093)	(0.099)		(0.043)
L.Error correction term		-0.355***	-0.324***	-0.329***
		(0.000)	(0.004)	(0.002)
L.Error correction term x (2007-)		0.330***	0.224*	0.264**
		(0.003)	(0.074)	(0.031)
L.Old error correction term	-0.403***			
	(0.000)			
L.Old error correction term x (2007-)	0.378***			
	(0.001)			
Observations	87	87	86	86
R^2	0.822	0.816	0.817	0.824
AIC	-20.81	-17.62	-14.48	-17.58

p-values in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

In table 5.2.8 we see model estimates where we regress the changes in the Norwegian 10-year rate on the explanatory variables and interaction terms between the explanatory variables and a dummy which takes the value one for all observations from Q1 2007 onwards. Thus, we can examine whether there has been significant differences in the effects of different variables in the two periods Q1 1991-Q4 2006 and Q1 2007-Q4 2012. The first column in the table displays our preferred model from chapter 5.2.3 with the estimation sample prolonged to Q4 2012 and interaction terms added. Column 2 shows the same model, but with the "new" error correction term in place of the "old" one. Column 3 displays how our preferred model from table 5.2.7 would look like if we included interaction terms. Column 4 displays the model when we only include significant regressors (focusing on a 10% significance level). The lagged difference of the policy rate and its interaction term are jointly significant on the 10% level according to an F-test. This final model is our overall preferred model for explaining the developments in the Norwegian rate since 1991.

Looking at our preferred model, we find that the speed of adjustment has fallen dramatically. The coefficient is -0.329 before 2007 and -0.065 from Q1 2007 onwards, indicating only a slight correction of a disequilibrium each quarter.[39]

Furthermore, the model suggests that also the short-run effect of the German rate has fallen somewhat, from 1.269 in the pre-2007 period (roughly the same as in our preferred model from chapter 5.2.3) to 0.936 in the post-2007 period.[40] However, even though the change is significant in magnitude, it is not statistically significant in our preferred model. In the model in column 3, however, we see that the estimated change in effect is larger and also significant at a 10% level.

What is significant though is the change in the effect of the policy rate. In the pre-2007 period the estimated effect of a one percentage point increase in the policy rate is a 0.256

[39] Sum of coefficients, post-07 = -0.329 + 0.264 = -0.065.

[40] Denominator, pre-07 = 1 – 0.161 = 0.839. Total short-run effect, pre-07 = 1.065/0.839 = 1.269. Denominator, post-07 = 0.839 + 0.179 = 1.018. Sum of short-run coefficients, post-07 = 1.065 – 0.112 = 0.953. Total short-run effect, post-07 = 0.953/1.018 = 0.936.

percentage point increase in the 10-year rate, while in the post-2007 period we find that the total effect is -0.074, i.e. as good as zero.[41]

Finally, we see that when we allow for interaction terms in the model, VIX can again be included. The estimated effect of a one percentage point increase in VIX decreases the 10-year rate by 1.4 basis points in the pre-07 period, and actually increases the 10-year rate by 0.4 basis points in the post-2007 period.[42] The positive effect of VIX on the yield in the post-2007 period seems puzzling. However, while the change is significant, the effect is also virtually zero. Since we found that VIX could be included in our pre-2007 model, but not when we estimated a model for the full sample, a model where VIX has an effect only in the pre-2007 period seems sensible. The reason for this could again be the fact that Norway was not very severely hit by the crises in the post-2007 period, and thus it might not be surprising that turmoil in the financial markets has less impact on Norwegian rates.

To sum up, we find that both monetary policy and international impulses have had less impact in the years during and following the financial crisis and during the sovereign debt crisis. Why do we get this result? The answer probably lies in the fact that the mentioned crises have had considerably less impact in Norway than in most other developed economies, thus it is only natural that Norwegian long-term rates would not follow European rates as closely as before. More precisely, both the expected path of short-run interest rates, the expected inflation over coming years and the term premium has probably seen considerably less reductions in Norway than in e.g. Germany.

The fact that we see almost no effect from the Norwegian policy rate in this period could possibly be a result of reductions in it being seen less as a signal of a worse outlook for the Norwegian economy than previously, and more as necessary reductions due to lower interest rates abroad. As e.g. Akram and Frøyland (1997) discusses, Norges Bank have to largely take into account European rates when setting its rate. Furthermore, the Norwegian policy rate has been very stable over this period, meaning that it is difficult to pick up an effect from it.

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[41] Sum of short-run coefficients, pre-07 = 0.308 – 0.0928 = 0.2152. Total short-run effect, pre-07 = 0.2152/0.839 = 0.256. Sum of short-run coefficients, post-07 = 0.2152 – 0.289 – 0.00108 = -0.07488. Total short-run effect, pre-07 = -0.07488/1.018 = -0.07356.

[42] Sum of short-run coefficients, post-07 = -0.0140 + 0.0183 = 0.0043.

Examining the fit of the model

Table 5.2.9 - Preferred models for Norway, sample Q1 1991-Q4 2012, with heteroskedasticity robust standard errors

	(1) D.10-year rate(nor)	(2) D.10-year rate(nor)
LD.10-year rate(nor)	0.161^{**}	0.289^{**}
	(0.0762)	(0.113)
LD.10-year rate(nor) x (2007-)	-0.179^{*}	
	(0.0909)	
D.Policy rate	0.308^{***}	0.231^{***}
	(0.0790)	(0.0616)
D.Policy rate x (2007-)	-0.289^{***}	
	(0.0897)	
LD.Policy rate	-0.0928	-0.136^{**}
	(0.0756)	(0.0682)
LD.Policy rate x (2007-)	-0.00108	
	(0.0841)	
D.10-year rate(ger)	1.065^{***}	0.996^{***}
	(0.107)	(0.0821)
D.10-year rate(ger) x (2007-)	-0.112	
	(0.135)	
LD.10-year rate(ger)		-0.226^{*}
		(0.119)
D.VIX index	-0.0140^{*}	
	(0.00827)	
D.VIX index x (2007-)	0.0183^{**}	
	(0.00887)	
L.Error correction term	-0.329^{***}	-0.333^{***}
	(0.105)	(0.0953)
L.Error correction term x (2007-)	0.264^{**}	
	(0.120)	
Observations	86	86
R^2	0.824	0.785
AIC	-17.58	-12.74

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5.2.9 displays our preferred models with and without interaction terms, now with heteroskedasticity robust standard errors in parentheses rather than p-values. We will now examine how well these models predict the actual movements in the Norwegian 10-year rate.

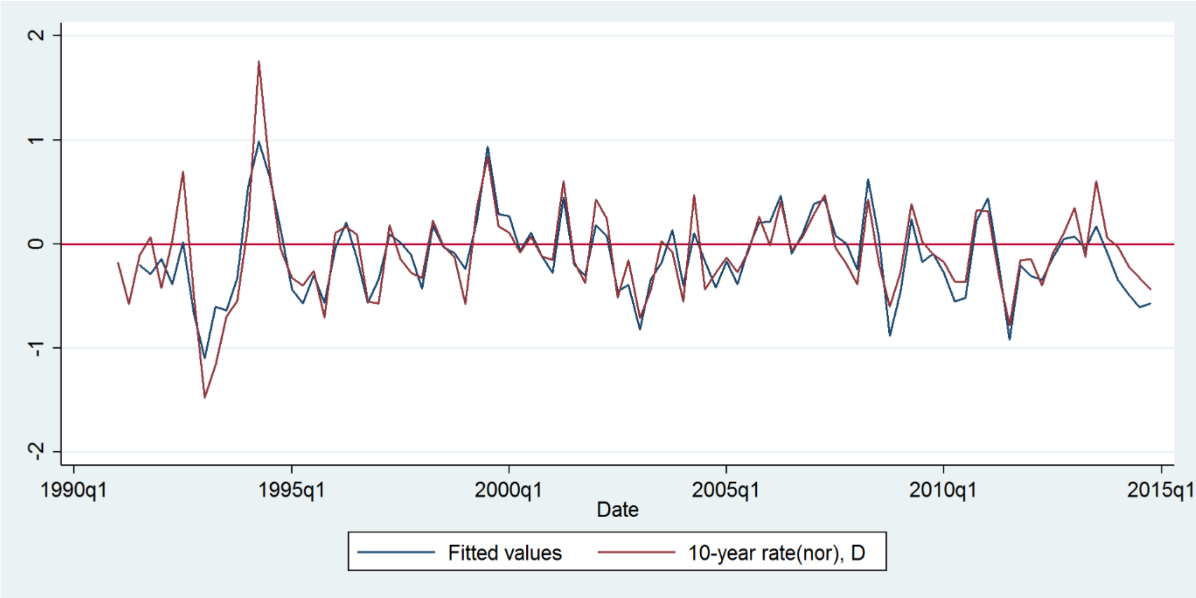

Figure 5.2.15 - Model-predicted and actual changes in the Norwegian 10-year rate, model without interaction terms

Figure 5.2.15 shows actual changes in the Norwegian 10-year rate and changes predicted by the model from table 5.2.9 without interaction terms. We see that this model predicts the movements in the rate well, and that it over-predicts falls in the rate in the years following the outbreak of the financial crisis to a lesser degree than the preferred model from chapter 5.2.3 (see figure 5.2.11). Not surprisingly though, the model including interaction terms (see figure 5.2.16) performs even better, predicting the changes in the rate almost perfectly after 2007.

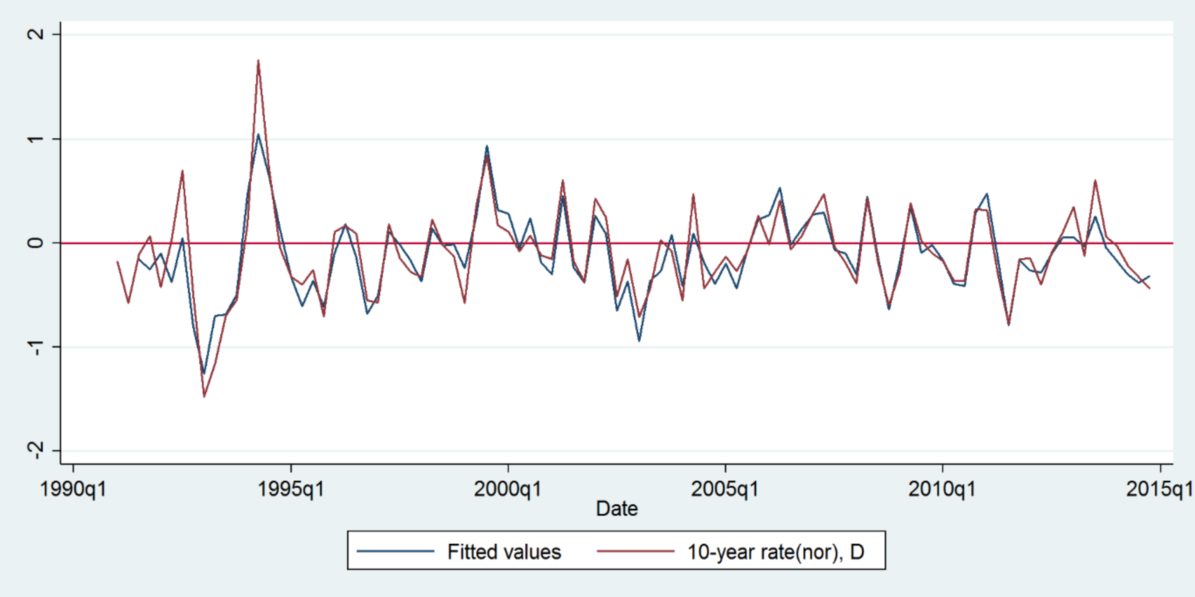

Figure 5.2.16 - Model-predicted and actual changes in the Norwegian 10-year rate, model with interaction terms

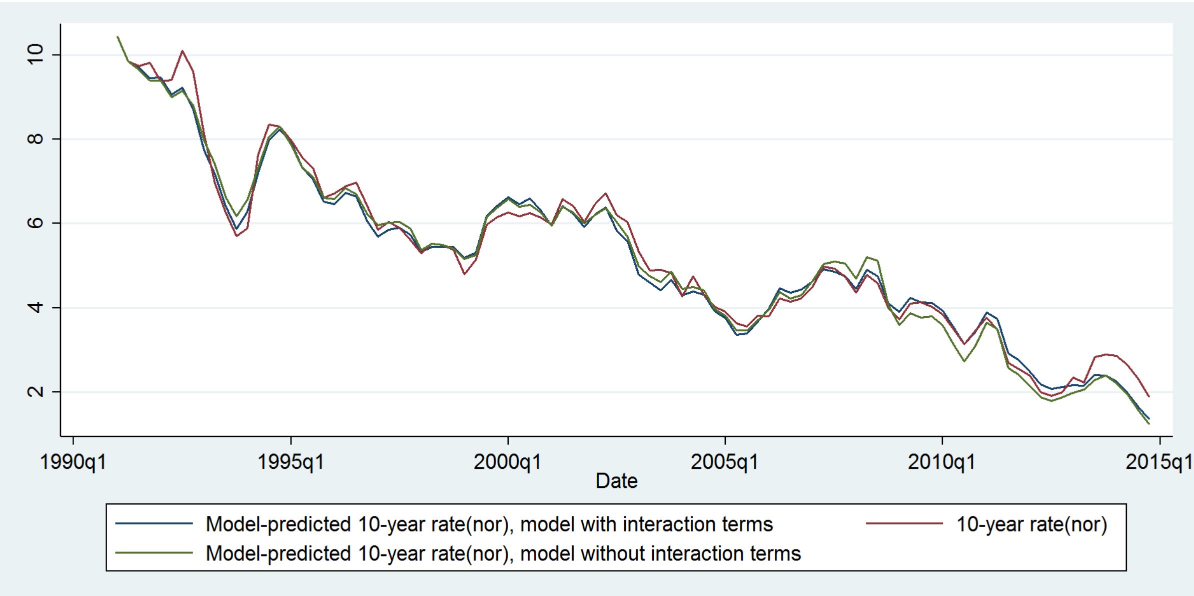

Figure 5.2.17 - Model-predicted and actual Norwegian 10-year rate

Figure 5.2.17 displays the actual and predicted Norwegian 10-year rate, where the predicted rate is based on the actual rate in Q2 1991 and added predicted changes. The predicted changes are again based on a model with a corrected error correction term, i.e. where the lagged actual value of the 10-year rate is replaced by the lagged predicted value. We see that both the model with and the model without interaction terms perform well. Not surprisingly, the model with interaction terms performs somewhat better, in particular from 2007 onwards. In the out-of-sample period (2013 and 2014), both models agree on the level of the rate and predicts a lower level of the Norwegian 10-year rate than was actually the case, as they were not able to fully predict the increases in the rate during 2013. It is interesting to see though, that with the exception of the largest increase in 2013, the models predict the out-of-sample movements in the rate well. The actual rate was in fact closing in on the predictions during 2014.

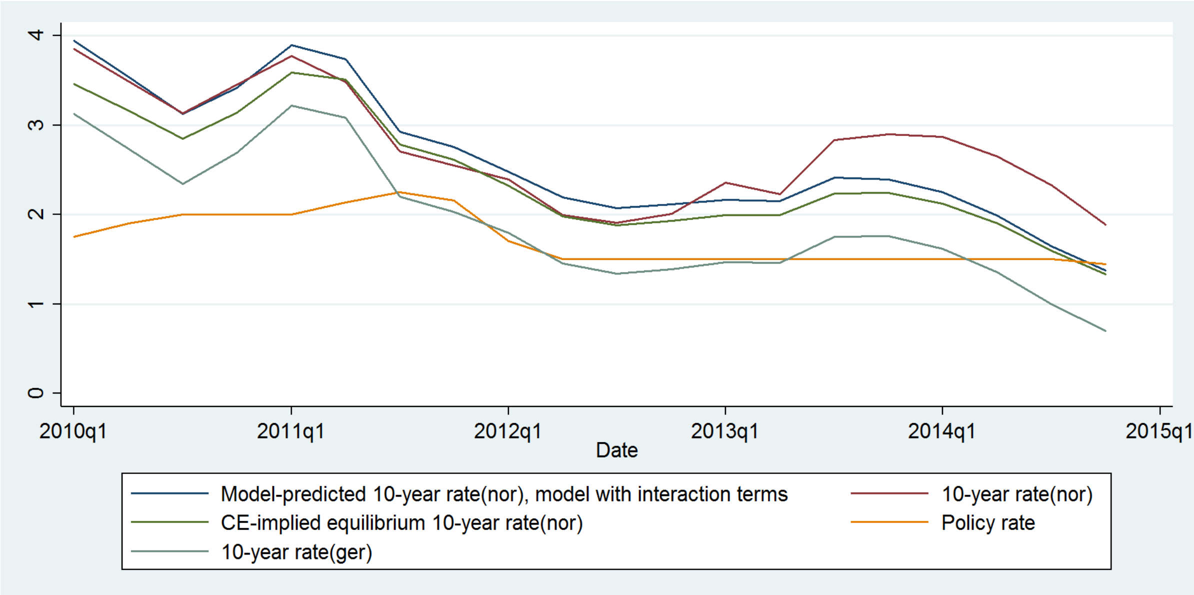

Figure 5.2.18 - A closer look at the period 2010-2014

Figure 5.2.18 depicts for the years 2010-2014, the model-predicted Norwegian 10-year rate (still with the base period Q2 1991), the equilibrium 10-year rate implied by the cointegrating equation (equation 5.2.2), the actual 10-year rate and the two most important explanatory variables (the German 10-year rate and the policy rate). We see that both the predictions from the ECM and the predictions from the long-run relationship follow the actual 10-year rate quite well while we are still in our estimation sample, alternating at being closest to the actual rate. We see, however, that the model is not able to fully explain increases in the rate in late 2012 and from the second to the third quarter of 2013. Especially this last movement looks like an exaggerated reaction to the increase in the German rate in the same period. Possibly, this is a result of Norway having fared reasonably well during the crises experienced much harder elsewhere, so that an indication of better economic times in the rest of Europe lead to reduced fear of a Norwegian recession, creating a stronger reaction in Norwegian long-term rates than the model would suggest.

6. Discussion

6.1 What determines the 10-year government yield in the US and Norway, and have there been any changes since 2007?

Most estimated effects have been relatively thoroughly discussed in chapter 5. Thus, this part serves as a summary of the effects we have found, reiterating some interpretations briefly, as well as comparing effects across model specifications, periods and countries, and discussing any differences. We start by discussing the long-run relationships briefly. Next, we discuss each variable individually, starting with the variables that have effects in both countries. We then move on to look closer on effects that are exclusive to one country, starting with Norway.

We have estimated several error correction models for both the US and Norway, finding that several macroeconomic variables have long- and/or short-run effects on the 10-year rates. For Norway, we find that the 10-year rate has a long-run relationship with the policy rate and the German 10-year rate. When we estimate this relationship for our long sample, we find a somewhat lower coefficient for the German rate and a larger constant term. This may reflect how Norway were less severely hit by the financial crisis, thus there were less “need” for the Norwegian rate to follow the German rate downwards. Furthermore, the larger constant term might reflect higher expected future short-term rates and higher term premium than Germany, due to expectations that the recession would be more short-lived in Norway.

The fact that the Norwegian rate has reacted less to developments abroad is also reflected in the speed of adjustment. We find an adjustment parameter of -0.403 or -0.329 in the pre-2007 period, for the cointegrating equation estimated for the short sample and the one estimated for the long sample respectively. When we use the full long sample and the equation estimated for the same sample, the adjustment parameter is -0.333, while the speed of adjustment for the post-2007 period is estimated to only -0.065.

For the US, we estimate a long-run relationship between the 10-year rate, the policy rate, expected inflation and the current account in the pre-2007 period. This relationship seems to completely fall apart in the later period, which means that we could only include the 10-year rate and the policy rate in the cointegrating equation for the full sample. The current account was included in the first equation on the basis of it reflecting savings from abroad flowing into the US market (the "global saving glut"). The current account has experienced a total turnaround in recent years, making it difficult to include in the model in any sensible way. The reason why expected inflation could not be included in the relationship for the long sample is unclear, but as previously discussed, it has become increasingly stable, and it seems to have less of a common trend with the policy rate and 10-year rate than previously.

We see that in the pre-2007 period, the US 10-year rate adjusts relatively strongly to deviations from the long-run relationship estimated for that period, having an adjustment parameter of between -0.311 and -0.394. In the post-2007 period the 10-year rate were found to not adjust in response to deviations in this relationship. When using the cointegrating equation estimated for the long sample, we find an adjustment parameter of -0.118 for the pre-2007 period, -0.108 for the full long sample and -0.012 in the post-2007 period. It thus seems like the US 10-year rate does not react significantly to any long-run relationship in the post-2007 period.

We find that the policy rates have both long-run and short-run effects in both countries. We find that the estimated long-run effect of a one percentage point increase in the policy rate is an increase in the 10-year rate of between 20.4 and 22.1 basis points in Norway and of 20.2 basis points in the US. The long-run coefficient in the cointegrating equation estimated for the longer sample in the US is 0.999, but is exaggerated due to that relationship only consisting of the 10-year rate and the policy rate. It is interesting to note though, that it suggests a one-to-one reaction in the 10-year rate.

The short-run effect of a one percentage point increase in the policy rate is estimated to be an increase in the 10-year rate of between 25.6 and 30.6 basis points in Norway and between 26.1 and 30.6 basis points in the US in the pre-2007 period (the model estimated for the shorter sample that includes debt suggests an effect of 56 basis points, but this deviates so much from the rest of the estimates that we consider it unrealistic). What we can conclude is that the policy rates in both the US and Norway had approximately the same both short-run and long-run effects in the pre-2007 period, and that the short-run effects are estimated to be a little bit stronger than the long-run effects.

In Norway, we find that the effect of policy rate has fallen significantly in the post-2007 period. When estimating the model for the long sample we find an effect of 13.4 basis points, and when allowing the effect to change between periods, we see that the estimated effect in the post-2007 period is basically zero. In the US we see a slight, not statistically significant, tendency for the policy rate to have less effect in the post-2007 period. It is estimated to be 22.8 basis points when estimating over the entire long sample and the effect estimated for the post-2007 period is between 21.3 and 22.5 basis points.

The only other variable that is included in both US and Norwegian models is the VIX index, although it was not significant in all specifications. We find some evidence that it is significant in the pre-2007 period in both countries. For Norway, the estimated effect of increasing VIX by one is a decrease in the 10-year rate of about 1.4-1.5 basis points in this period. The same effect for the US is estimated to be 3.6 basis points in the model with interaction terms that uses the long-run relationship that only includes the 10-year rate and the policy rate. In the US model estimated for the long sample, the estimated effect is 1 basis point, while VIX was not significant in the preferred models where the cointegrating equation based on the short sample were included. The estimated effect in the US in the period where we find an effect is thus more than twice as large as in Norway. It is not surprising that the US rate reacts more strongly to volatility in the US. It is, however, interesting to find this effect in Norway, as we would expect that the effect from volatility would already be included in the influence from the German rate.

The fact that we find no effect from VIX in the post-2007 period is interesting. Intuitively, we would think that a period with such an amount of financial turmoil would lead to this index having a larger effect. However, there are three possible reasons for not finding an effect. First, the sample is short, making it difficult to find statistic support for an effect, especially when we consider that the effect found in the earlier period is not very large and not significant in all specifications. Second, the Federal Reserve has conducted quantitative easing programs in the US, these explain some of the variation in the rate there (and indirectly in Norway through international influences), making it harder to identify the effect of VIX. Finally, market participants might have a permanent higher demand for safe assets such as Treasuries, making minor variations in the VIX index obsolete as an explanatory factor for the demand for safe assets. Remember that our first period corresponds with “the Great Moderation”; Clarida (2010) notes that in this period, roughly 1987-2007, market volatility was low, business cycles modest and monetary policy fairly predictable. Thus, minor fluctuations in the market perception of risk might have had a larger effect in that period, making investors temporarily move funds to safe assets.

We find, as we expected based on the arguments of Akram and Frøyland (2007), that the German 10-year rate has a very strong influence on the Norwegian rate. In the long-run relationship, we find that a one percentage point increase in the German rate increases the Norwegian rate by 90.6 basis points according to the equation based on the short sample, and by 84.9 basis points according to the equation based on the long sample. The short-run effect of a one percentage point increase in the German rate is estimated to be an increase in the Norwegian rate of about 126.9-129.6 basis points in the pre-2007 period. When estimating the effect for the full sample, it is estimated to be 108.3 and the estimated effect in the post-2007 period is 93.6 basis points. We thus see that the short-run effect is estimated to be stronger than the long-run effect and that both have become smaller in the post-2007 period.

We see that all the explanatory variables in the Norwegian model have been estimated to have less effect in the post-2007 period. In fact, only the German rate has a short-run effect in this period, while the policy rate only has a very slight influence through the error correction term. We believe that the main reason for these observations is the much mentioned fact that the crises in this period were experienced less severely in Norway than in most other advanced economies. Therefore, there was less reason for the Norwegian rate to follow the German rate closely as it dropped, and reductions in the Norwegian policy rate might have been viewed as purely necessary due to lower rates abroad, reinforced by the fact that Norges Bank over much of the period have hesitated in bringing the policy rate down, even starting to raise it during 2010 and 2011 (as we can see in figure 6.1.1).

Additionally, as we can see from figure 6.1.1, the policy rate also follows the output gap to some extent. Thus, the policy rate might partly reflect the lower rates abroad and partly reflect the weakening output gap. Since the fall in the output gap following the financial crisis was relatively small (see figure 3.4.1 for a comparison of the output gap for the US and Norway), the low rates and slow economy might have been viewed as transitory, such that the long-term rate incorporates expected future short rates at a higher level. The subsequent increase in the Norwegian policy rate might of course also make it difficult to identify an effect from the policy rate as we here see the policy rate being raised while the 10-year rate fall in response to lower rates abroad.

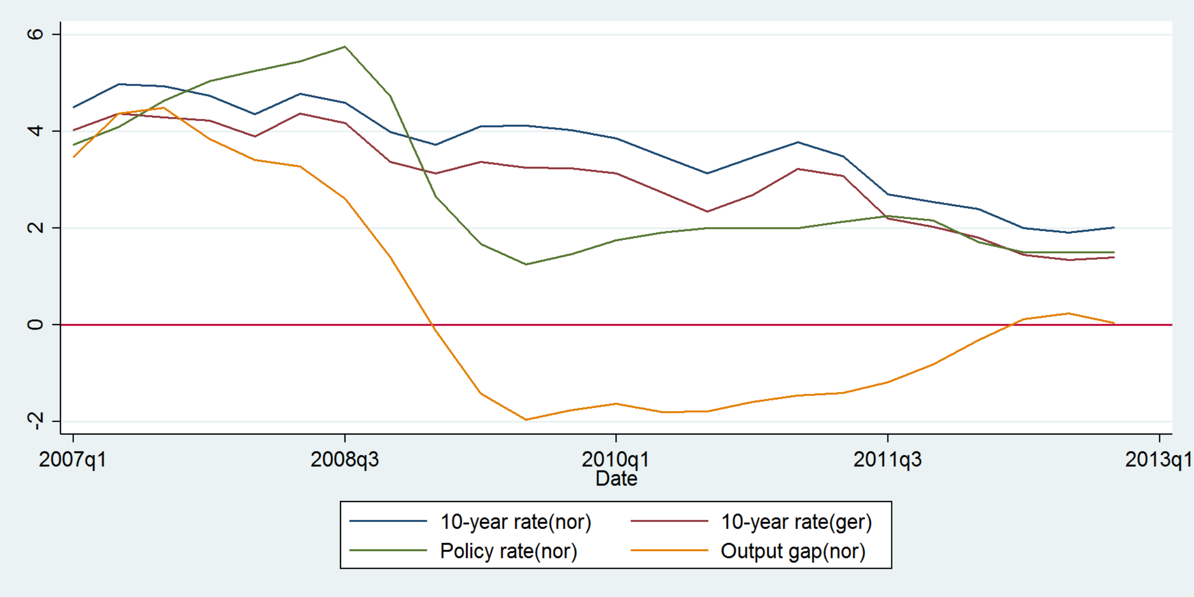

Figure 6.1.1 – An illustration of the post-2007 period for Norway

Let us now move on to discuss the variables that are only included in the US model, either because they were only included there from the start or because they were not significant in the Norwegian model.

We find that expected inflation has a relatively strong influence on the US 10-year rate. In the pre-2007 period, it is included in the long-run relationship. The estimated long-run effect of a one percentage point increase in inflation expectations is a 116.3 basis point increase in the 10-year rate. The estimated short-run effect of a one percentage point increase in inflation expectations is an increase in the 10-year rate of between 90.3 and 110.8 basis points, depending on model specification. In addition, we estimate this effect to be 180.9 basis points in the model for the short sample which includes debt. This is extreme compared to the other estimates, but it is relatively close to the estimated long-run effect in Hellum (2010). In the model where we estimate a coefficient for the entire long sample, the estimated effect is 60.8 basis points. In the post-2007 period, the estimated effect is between 29.1 and 37.4 basis points. As discussed in chapter 5.1.5, this large reduction most likely mainly reflects inflation expectations being much more stable in recent years.

For Norway, we tried to use actual inflation data as a proxy for inflation expectations, based on a hypothesis of simple backward-looking expectations. As discussed in chapter 5.2.3, the inflation series was not significant in the Norwegian model, due to it being a bad proxy and/or the Norwegian inflation expectations being so stable that they do not explain much of the variation in the 10-year rate.

We find that an increase of one in the PMI index is estimated to lead to an increase in the US 10-year rate of 8.7-12.4 basis points in the pre-2007 model, depending on model specification. When estimating a coefficient for the long sample, the estimated effect is 8.3 basis points. The estimated effect for the post-2007 period is 5.9-6.4 basis points. Thus, we find some evidence that the business cycles have somewhat lower effect in recent years.

For Norway, we tried to include the output gap. However, the effect was found to be not significant and negative, so it was dropped from the model. In chapter 5.2.3, this finding was discussed shortly.

The estimated effect of US government debt supply has varied considerably depending on how the model has been specified. In particular, we find an unexpected negative effect in some specifications. Two possible explanations come to mind. First, there may be no effect from changes in supply of government debt in the pre-2007 period, and the estimated effects in that period are only correlations. Second, our debt measure is not completely appropriate for its purpose. Ideally, the measure should be adjusted to reflect the supply of Treasuries available to the public.

The effect of a one percent increase in government debt is estimated to be anything from a fall of 2.6 basis points to an increase of 3.1 basis points in the first period. The effect estimated for the full long sample is an increase of 3.5 basis points, while the estimated effect for the post-2007 period is an increase of between 1.1 and 1.9 basis points.

To capture effects from quantitative easing, we included non-borrowed reserves in the US model. The estimated effect of a one percent of GDP increase in non-borrowed reserves during the QE-period is a decrease in the 10-year rate of between 14.1 and 22.8 basis points. From Q3 2008 to Q3 2014 the total increase in non-borrowed reserves was 15.67 percent of GDP, indicating a total decrease in the 10-year rate of between 221 and 357 basis points. Our estimate for the effect of the increase in debt in the same period is an increase in the 10-year rate of between 40 and 69 basis points. This gives an estimated net decrease in the 10-year rate in the range 181-288 basis points. In the same period the 10-year rate fell by 143 basis points. The fact that we predict lower than actual rates in 2014 can thus be due to overestimating the effect of QE (or at least of QE3).

If we look at the period from the start of QE in Q3 2008 to the point where the 10-year rate was at its lowest, Q3 2012, the total increase in non-borrowed reserves was 9.37 percent of

GDP, indicating a decrease in the 10-year rate of between 132 and 214 basis points. The estimate for the effect of increase in debt is an increase in the 10-year rate of between 41 and 70 basis points, giving a range for the net effect of a fall of between 91 and 143 basis points. In the same period, the actual 10-year rate fell by 234 basis points. Of course, these calculations are very rudimentary.

Finally, an interesting observation from this discussion is that almost all estimated effects when using the entire sample Q1 1990-Q4 2012, and not allowing for changing effects, are weighted averages of the estimated pre- and post-2007 effects. This is of course not surprising, but considering how large some of the changes are, it illustrates that not allowing for changing effects might yield estimated effects that are not representable for either period.

6.2 Are we able to explain the developments in the yields in recent years?

This question has been thoroughly examined in chapters 5.1.4-5 and 5.2.3-4. Here we thus only shortly discuss the findings.

Our models for the US predict the movements and levels of the rate reasonably well in recent years until early 2012. At that point, none of the model specifications are able to predict the trough in the rate that happens during 2012. In chapter 5.1.4, we pointed to not capturing the effects of the maturity extension program as a possible explanation for this. Following that trough, the models are not able to fully predict the increase in the rate in 2013. Three possible explanations for this come to mind. First, an increase in the PMI might have been met with more optimism than the model predicts. Second, and related to the first, the increase can reflect expectations about a tightening of monetary policy. Third, we may overestimate the effect of QE3.

We see signs that we may overestimate effects of QE at some points, underestimate at other points and hitting them well at yet other points. This reflects the rather simplistic way QE effects have been included in the model. To more accurately estimate the effects of QE, it is necessary to take into account the differences between the different programs, if possible use a more suitable measure of the supply of government debt and possibly also allow for decreasing marginal effects.

Our models for Norway explain the movements and levels of the rate very well. Perhaps, surprisingly well. This of course reflects the large degree to which the Norwegian 10-year rate follows the German 10-year rate. The only major movement in the rate our models miss is the increase from Q2 to Q3 2013. This increase seems to be a stronger reaction to increases in rates abroad than the model suggests. We have previously mentioned that this may be related to the fact that Norway had fared well through the hard times and that increases in foreign rates were seen as a sign that the danger had passed and that the global economy was recovering.

We see that a common denominator for our two countries is that our models are not able to predict increases in long-term rates in 2013. This might be due to exaggerated optimism with regards to expected recovery. These increases are followed by falling rates in 2014, highlighting that optimism may have been exaggerated. Our models predict the falls reasonably well, although a bit exaggerated in the US. The increases in 2013 might also reflect a normalization, in the sense that the estimated effects of PMI in the US and the German rate in Norway were higher in the “normal” pre-2007 period, compared to the post-2007 period, and both of these contributed to increases in 2013.

6.3 Some weaknesses of our analysis

In our analysis, as is always the case with empirical analysis, there are some weaknesses. There are especially three weaknesses we would like to further discuss. The first and second regards capturing the effect of QE and supply of debt in the model; the third relates to general robustness.

A weakness with our analysis is that we do not distinguish between the different effects of QE, and the different programs conducted under QE. Arguably, only the programs that entailed increasing reserves will have a liquidity effect, the other programs will only have a portfolio balance effect. Even if we assume that there exists preferred habit demand for assets with certain properties, i.e. being safe, liquid and having a long maturity, the effect on the yield will be largest on the assets being purchased. Therefore, the programs focusing on Treasuries will have a larger effect on Treasury yields.

Using this line of thought, we have reason to believe that QE1 had an effect on the Treasury yield both through the portfolio balance effect and the liquidity effect, as assets were financed through increases in reserve. However, all asset purchases were not Treasuries: a large part was MBS, and agency-debt in MBS. In comparison, QE2 only consisted of Treasury purchases, and were financed through increased reserves. The maturity extension program again included MBS and agency debt in MBS. In addition, the purchases were not financed by reserves, so there should have been no liquidity effect. QE3 also included all three asset classes, and in the end entailed an increase in reserves. In conclusion, the programs' effect have probably varied, with QE2 having the strongest effect on long-term Treasury yields.

Distinguishing between the different programs would have been problematic in our time-series framework. Using quarterly frequency, dummies and interaction terms could not have been applied for each program as the sample for each would have been too small. Capturing the fact that the effect of a program could be observed when the purchases took place, when the purchases was announced, with a lag or as expectations of future announcements become more certain, would make setting the dates for the different dummies difficult as well.

We also have problems capturing the effect of government debt supply. The estimated effect of debt in many cases depends on which other variables are included. As mentioned earlier, this is probably due to our measure for debt not properly accounting for the supply of debt that actually affects the yield. Ideally, we would like to have a measure of debt that reflects the supply of Treasuries available to the public, in line with most literature that have studied effects of QE-programs. We were unfortunately not able to find the right data to construct this measure.

Finally, some of our results do not seem very robust. The estimated effect of several variables, for instance the VIX index, depend on how many lags were included (allowing for two lags in some cases gave a positive net effect from VIX). The VECM estimation also gave differing results depending on how the cointegrating vector was defined and how many lags that was included in the model. This means that the results depend a great deal on our assumptions throughout the study. We do note that all our assumptions have been mentioned, and argued for, in this thesis. We believe that they are both economically sensible and econometrically appropriate.

7. Conclusions

In this thesis, we have attempted to model 10-year government yields using macroeconomic variables in an ECM-framework. We wished to examine which variables determined the yield, if there had been any changes in this regard since the outbreak of the financial crisis in 2007, and to what degree we were able to explain developments in recent years, in particular the record-low rates experienced during this period.

Most prior research have modelled the yield curve as a whole, and the macroeconomic factors included have been primarily measures of real activity and inflation. In recent years, more focus has been on other possible determinants, including supply effects with regards to supply of government bonds and different effects from quantitative easing programs. Furthermore, e.g. Akram and Frøyland (1997) have found that Norwegian interest rates are heavily influenced by European rates, leading us to conclude that we had to include the German 10-year government bond yield as an explanatory variable in the Norwegian model. Some other variables such as the current account and the VIX index were included based on suggestions in prior research, to examine whether they were significant determinants for the 10-year rates.

While prior research have primarily looked on effects in given periods, we are additionally interested in examining whether the effects change across periods. More specifically, whether the years following the outbreak of the financial crisis represent a period were the usual determinants of the 10-year rate did not apply in the same way. In other words, have there been any changes to effects and which variables that are relevant?

We find that there are significant changes in the effects of several macroeconomic determinants in the US. For instance, we find that current account no longer seem to have a relationship with the 10-year rate, and that inflation expectations have less effect on the yield. Furthermore, we find that the QE programs were successful and led to a decrease in long-term rates. We could not determine a robust effect of debt in the pre-2007 period, but we did find a clear positive relation with the 10-year rate in the post-2007 period. The VIX index has a weak, negative relation to the 10-year rate in the pre-2007 period, while we could not find a significant effect in the post-2007 period. The policy rate is thus the only determinant in the US model for which the effect seems to be largely unchanged after 2007, although the short-run effect was estimated to be a little bit lower.

We thus find that there have been some major changes going from the pre-2007 period to the post-2007 period. One is that inflation has become more stable, leading to it having a weaker relation to the yield. Furthermore, our findings with regards to the VIX index indicate that investors' perception of risk might have changed following the end of the Great Moderation. Since the policy rate still have largely the same effect in both periods, at the same time as QE proved to be successful, we believe that in the post-2007 period when unconventional monetary policy were conducted, conventional monetary policy still remained credible. Finally, one of our more interesting findings is the role current account no longer seems to have in explaining long-term government yields. As mentioned in chapter 2.3, Hellum (2010) notes that it was an important determinant in explaining low rates in the mid-2000s, and we found that it did have a long-run relationship with the 10-year rate in the pre-2007 period. Hellum (2010) also noted that the current account can have differing effects on the yield, dependent on what constitutes the reason for the deficit. Our finding indicates that the composition of the current account has changed, making it difficult to identify an effect.

In the Norwegian model, we also find some major changes from the pre-2007 period to the post-2007 period. Just like in the US case, we find that VIX has a significant effect in the first period, but not in the last period. Furthermore, the policy rate, which has roughly the same effect in Norway as in the US in the pre-2007 period, has no short-run effect on the 10-year rate in the post-2007 period. It is still included in the long-run relationship, but the rate adjusts only slightly in response to disequilibria in this relationship. This leaves the German 10-year rate as the only major determinant of the Norwegian 10-year rate in the post-2007 period. This highlights the importance of global developments for interest rates in a small, open economy such as Norway. However, even the effects of the German rate are lower in the post-2007 period.

The changes are likely due to Norway having been less severely hit by the crises in the post-2007 period. Expectations that the setbacks actually experienced in Norway would be transitory and that the economic outlook would be good might have led to the rate responding less to factors that would suggest it should fall. Lower policy rate may have been viewed as transitory and the German rate has lost some of its influence. In other words, the Norwegian 10-year rate incorporates higher expected short-term rates and a higher term premium than the German rate, due to expectations that the Norwegian economy would continue to do well.

Future research

We have found several interesting changes between the “normal” pre-2007 period and the turbulent post-2007 period. The changes are so sizeable, that they should be taken into account. Estimating one effect for the entire period yields a weighted average of the effects in the two periods which is not entirely representative for either period. It would be interesting if future research had put more light on the changes we have identified. Furthermore, it is clear that our measures to capture effects of supply of government bonds and of quantitative easing are a bit simplistic. Ideally, future research should strive for getting more appropriate measures to capture these effects. This would entail taking into account the differences in different LSAP programs, and would likely require a higher data frequency than quarterly data.

Appendix

Unit root testing

US variables

Table A. 1 - DF-GLS test for the 10-year rate in levels, with maximum lag length set to eight

DF-GLS for us10y   Number of obs = 83

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
8	1.193	-2.603	-2.035	-1.736
7	1.544	-2.603	-2.052	-1.752
6	1.316	-2.603	-2.068	-1.767
5	1.288	-2.603	-2.083	-1.782
4	0.975	-2.603	-2.098	-1.796
3	0.569	-2.603	-2.112	-1.809
2	0.807	-2.603	-2.126	-1.821
1	0.439	-2.603	-2.137	-1.832

Opt Lag (Ng-Perron seq t) = 4 with RMSE .3927194
Min SC = -1.679853 at lag 1 with RMSE .4093579
Min MAIC = -1.757159 at lag 1 with RMSE .4093579

Table A. 2 - DF-GLS test for the first-differenced 10-year rate, with maximum lag length set to zero

DF-GLS for D.us10y   Number of obs = 91

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
0	-4.770	-2.603	-2.131	-1.824

Table A. 3 - DF-GLS test for the policy rate in levels, with maximum lag length set to eight

DF-GLS for ustarget   Number of obs = 83

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
8	-0.445	-2.603	-2.035	-1.736
7	-0.394	-2.603	-2.052	-1.752
6	-0.313	-2.603	-2.068	-1.767
5	-0.372	-2.603	-2.083	-1.782
4	-0.557	-2.603	-2.098	-1.796
3	-1.044	-2.603	-2.112	-1.809
2	-0.989	-2.603	-2.126	-1.821
1	-0.996	-2.603	-2.137	-1.832

Opt Lag (Ng-Perron seq t) = 4 with RMSE .2990239
Min SC = -2.249248 at lag 1 with RMSE .3079361
Min MAIC = -2.308512 at lag 4 with RMSE .2990239

Table A. 4 - DF-GLS test for the first-differenced policy rate, with maximum lag length set to four

DF-GLS for D.ustarget Number of obs = 87

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
4	-3.472	-2.603	-2.092	-1.789
3	-3.339	-2.603	-2.106	-1.802
2	-2.865	-2.603	-2.118	-1.813
1	-3.399	-2.603	-2.129	-1.823

Opt Lag (Ng-Perron seq t) = 3 with RMSE .3092523
Min SC = -2.190535 at lag 1 with RMSE .3177152
Min MAIC = -2.005559 at lag 2 with RMSE .3158512

Table A. 5 - DF-GLS test for the expected inflation in levels, with maximum lag length set to eight

DF-GLS for useinfl5 Number of obs = 82

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
8	0.311	-2.604	-2.036	-1.737
7	0.456	-2.604	-2.053	-1.753
6	0.530	-2.604	-2.069	-1.769
5	0.619	-2.604	-2.085	-1.783
4	0.676	-2.604	-2.100	-1.798
3	0.687	-2.604	-2.114	-1.811
2	0.585	-2.604	-2.127	-1.823
1	0.352	-2.604	-2.139	-1.834

Opt Lag (Ng-Perron seq t) = 2 with RMSE .1056155
Min SC = -4.352665 at lag 1 with RMSE .1075206
Min MAIC = -4.438155 at lag 2 with RMSE .1056155

Table A. 6 - DF-GLS test for the first-differenced expected inflation, with maximum lag length set to zero

DF-GLS for D.useinfl5 Number of obs = 89

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
0	-5.741	-2.604	-2.135	-1.828

Table A. 7 - DF-GLS test for the current account in levels, with maximum lag length set to eight

DF-GLS for usca   Number of obs = 83

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
8	-1.308	-2.603	-2.035	-1.736
7	-1.380	-2.603	-2.052	-1.752
6	-1.241	-2.603	-2.068	-1.767
5	-1.061	-2.603	-2.083	-1.782
4	-1.123	-2.603	-2.098	-1.796
3	-1.151	-2.603	-2.112	-1.809
2	-1.143	-2.603	-2.126	-1.821
1	-1.078	-2.603	-2.137	-1.832

Opt Lag (Ng-Perron seq t) = 6 with RMSE .3400683
Min SC = -1.990717 at lag 1 with RMSE .3504289
Min MAIC = -2.043951 at lag 1 with RMSE .3504289

Table A. 8 - DF-GLS test for the first-differenced current account, with maximum lag length set to zero

DF-GLS for D.usca   Number of obs = 91

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
0	-9.407	-2.603	-2.131	-1.824

Table A. 9 - DF-GLS test for log(Debt/GDP) in levels, with maximum lag length set to eight

DF-GLS for lusdebt   Number of obs = 83

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
8	-0.800	-2.603	-2.035	-1.736
7	-0.896	-2.603	-2.052	-1.752
6	-1.052	-2.603	-2.068	-1.767
5	-0.519	-2.603	-2.083	-1.782
4	-0.769	-2.603	-2.098	-1.796
3	-0.242	-2.603	-2.112	-1.809
2	0.003	-2.603	-2.126	-1.821
1	0.675	-2.603	-2.137	-1.832

Opt Lag (Ng-Perron seq t) = 6 with RMSE .0225927
Min SC = -7.309442 at lag 2 with RMSE .0238832
Min MAIC = -7.420966 at lag 2 with RMSE .0238832

Table A. 10 - DF-GLS test for first-differenced log(Debt/GDP), with maximum lag length set to eight

DF-GLS for D.lusdebt Number of obs = 83

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
8	-2.312	-2.603	-2.035	-1.736
7	-2.113	-2.603	-2.052	-1.752
6	-2.087	-2.603	-2.068	-1.767
5	-2.012	-2.603	-2.083	-1.782
4	-2.578	-2.603	-2.098	-1.796
3	-2.441	-2.603	-2.112	-1.809
2	-3.167	-2.603	-2.126	-1.821
1	-3.827	-2.603	-2.137	-1.832

Opt Lag (Ng-Perron seq t) = 5 with RMSE .0227702
Min SC = -7.360042 at lag 1 with RMSE .0239148
Min MAIC = -7.206287 at lag 5 with RMSE .0227702

Table A. 11 - DF-GLS test for non-borrowed reserves in levels, Q1 1990-Q2 2008, with maximum lag length set to eight

DF-GLS for usnbreserves Number of obs = 65

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
8	1.049	-2.610	-2.049	-1.753
7	1.143	-2.610	-2.070	-1.773
6	0.960	-2.610	-2.091	-1.794
5	1.084	-2.610	-2.112	-1.814
4	1.321	-2.610	-2.132	-1.833
3	1.355	-2.610	-2.151	-1.850
2	1.484	-2.610	-2.168	-1.867
1	1.696	-2.610	-2.184	-1.881

Opt Lag (Ng-Perron seq t) = 7 with RMSE .0248918
Min SC = -7.117125 at lag 1 with RMSE .0267082
Min MAIC = -7.118327 at lag 1 with RMSE .0267082

Table A. 12 - DF-GLS test for first-differenced non-borrowed reserves, Q1 1990-Q2 2008, with maximum lag length set to zero

DF-GLS for D.usnbreserves Number of obs = 73

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
0	-4.690	-2.610	-2.173	-1.867

Table A. 13 - DF-GLS test for non-borrowed reserves in levels, Q3 2008-Q4 2012, with maximum lag length set to four

DF-GLS for usnbreserves   Number of obs = 13

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
4	-0.512	-2.660	-2.465	-1.966
3	-0.640	-2.660	-2.449	-1.977
2	-0.382	-2.660	-2.516	-2.055
1	-0.546	-2.660	-2.618	-2.156

Opt Lag (Ng-Perron seq t) = 0 [use maxlag(0)]
Min SC   = .0722787 at lag 1 with RMSE .8511521
Min MAIC = -.1072275 at lag 1 with RMSE .8511521

Table A. 14 - DF-GLS test for first-differenced non-borrowed reserves, Q3 2008-Q4 2012, with maximum lag length set to zero

DF-GLS for D.usnbreserves   Number of obs = 17

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
0	-2.775	-2.660	-2.637	-2.257

Table A. 15 - DF-GLS test for the VIX index in levels, with maximum lag length set to four

DF-GLS for usvix   Number of obs = 87

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
4	-2.289	-2.603	-2.092	-1.789
3	-2.587	-2.603	-2.106	-1.802
2	-2.788	-2.603	-2.118	-1.813
1	-3.533	-2.603	-2.129	-1.823

Opt Lag (Ng-Perron seq t) = 0 [use maxlag(0)]
Min SC   = 3.388067 at lag 1 with RMSE 5.169113
Min MAIC = 3.545665 at lag 4 with RMSE 5.085576

Table A. 16 - DF-GLS test for the first-differenced VIX index, with maximum lag length set to four

DF-GLS for D.usvix   Number of obs = 86

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
4	-2.949	-2.604	-2.094	-1.791
3	-4.141	-2.604	-2.107	-1.804
2	-5.089	-2.604	-2.120	-1.815
1	-7.126	-2.604	-2.131	-1.825

Opt Lag (Ng-Perron seq t) = 4 with RMSE 5.676187
Min SC   = 3.646348 at lag 1 with RMSE 5.878957
Min MAIC =   4.6103 at lag 4 with RMSE 5.676187

Table A. 17 - DF-GLS test for the PMI index in levels, with maximum lag length set to four

DF-GLS for uspmi Number of obs = 87

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
4	-3.369	-2.603	-2.092	-1.789
3	-4.160	-2.603	-2.106	-1.802
2	-3.840	-2.603	-2.118	-1.813
1	-4.560	-2.603	-2.129	-1.823

```
Opt Lag (Ng-Perron seq t) = 1 with RMSE  2.699951
Min SC   = 2.089132 at lag  1 with RMSE  2.699951
Min MAIC = 2.500661 at lag  2 with RMSE  2.692891
```

Table A. 18 - DF-GLS test for the first-differenced PMI index, with maximum lag length set to four

DF-GLS for D.uspmi Number of obs = 87

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
4	-4.524	-2.603	-2.092	-1.789
3	-5.376	-2.603	-2.106	-1.802
2	-5.042	-2.603	-2.118	-1.813
1	-6.475	-2.603	-2.129	-1.823

```
Opt Lag (Ng-Perron seq t) = 3 with RMSE  2.947064
Min SC   = 2.307805 at lag  1 with RMSE  3.011898
Min MAIC = 3.568448 at lag  2 with RMSE  3.006069
```

Norwegian variables

Table A. 19 - DF-GLS test for the Norwegian 10-year rate in levels, with maximum lag length set to eight

DF-GLS for n10y Number of obs = 79

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
8	0.793	-2.605	-2.038	-1.740
7	1.011	-2.605	-2.055	-1.756
6	1.148	-2.605	-2.072	-1.772
5	1.210	-2.605	-2.089	-1.788
4	0.855	-2.605	-2.105	-1.803
3	0.587	-2.605	-2.120	-1.817
2	0.574	-2.605	-2.134	-1.830
1	0.225	-2.605	-2.146	-1.841

```
Opt Lag (Ng-Perron seq t) =  5 with RMSE  .3749811
Min SC   = -1.741124 at lag  1 with RMSE   .396186
Min MAIC = -1.832167 at lag  2 with RMSE  .3882303
```

Table A. 20 - DF-GLS test for the first-differenced Norwegian 10-year rate, with maximum lag length set to four

```
DF-GLS for D.n10y                                     Number of obs =    83

               DF-GLS mu       1% Critical      5% Critical     10% Critical
  [lags]     Test Statistic       Value            Value            Value

    4            -5.668           -2.605           -2.098           -1.796
    3            -5.313           -2.605           -2.112           -1.809
    2            -5.221           -2.605           -2.126           -1.821
    1            -6.787           -2.605           -2.137           -1.832

Opt Lag (Ng-Perron seq t) =  4 with RMSE  .3942939
Min SC   = -1.675969 at lag  1 with RMSE  .4101535
Min MAIC = -.1305862 at lag  2 with RMSE  .4099503
```

Table A. 21 - DF-GLS test for the German 10-year rate in levels, with maximum lag length set to eight

```
DF-GLS for g10y                                       Number of obs =    79

               DF-GLS mu       1% Critical      5% Critical     10% Critical
  [lags]     Test Statistic       Value            Value            Value

    8             1.459           -2.605           -2.038           -1.740
    7             1.519           -2.605           -2.055           -1.756
    6             1.530           -2.605           -2.072           -1.772
    5             1.525           -2.605           -2.089           -1.788
    4             1.312           -2.605           -2.105           -1.803
    3             1.061           -2.605           -2.120           -1.817
    2             1.108           -2.605           -2.134           -1.830
    1             0.644           -2.605           -2.146           -1.841

Opt Lag (Ng-Perron seq t) =  2 with RMSE  .3077561
Min SC   = -2.192618 at lag  1 with RMSE  .3161247
Min MAIC = -2.269409 at lag  2 with RMSE  .3077561
```

Table A. 22 - DF-GLS test for the first-differenced German 10-year rate, with maximum lag length set to four

```
DF-GLS for D.g10y                                     Number of obs =    82

               DF-GLS mu       1% Critical      5% Critical     10% Critical
  [lags]     Test Statistic       Value            Value            Value

    4            -5.181           -2.605           -2.100           -1.798
    3            -5.240           -2.605           -2.114           -1.811
    2            -5.356           -2.605           -2.127           -1.823
    1            -6.861           -2.605           -2.139           -1.834

Opt Lag (Ng-Perron seq t) =  1 with RMSE  .3046924
Min SC   = -2.269424 at lag  1 with RMSE  .3046924
Min MAIC = -.5145772 at lag  2 with RMSE  .3046781
```

Table A. 23 - DF-GLS test for the policy rate in levels, with maximum lag length set to four

DF-GLS for npolicy — Number of obs = 83

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
4	-0.992	-2.605	-2.098	-1.796
3	-1.222	-2.605	-2.112	-1.809
2	-1.152	-2.605	-2.126	-1.821
1	-1.617	-2.605	-2.137	-1.832

```
Opt Lag (Ng-Perron seq t) =  2 with RMSE  .5049761
Min SC   = -1.218544 at lag  1 with RMSE  .5155552
Min MAIC = -1.281876 at lag  2 with RMSE  .5049761
```

Table A. 24 - DF-GLS test for the first-differenced policy rate, with maximum lag length set to four

DF-GLS for D.npolicy — Number of obs = 82

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
4	-4.175	-2.605	-2.100	-1.798
3	-4.529	-2.605	-2.114	-1.811
2	-4.359	-2.605	-2.127	-1.823
1	-5.205	-2.605	-2.139	-1.834

```
Opt Lag (Ng-Perron seq t) =  1 with RMSE  .4751259
Min SC   =  -1.38087 at lag  1 with RMSE  .4751259
Min MAIC = -.5854646 at lag  2 with RMSE  .4749758
```

Table A. 25 - DF-GLS test for the inflation rate in levels, with maximum lag length set to four

DF-GLS for ninfl — Number of obs = 83

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
4	-1.045	-2.605	-2.098	-1.796
3	-2.592	-2.605	-2.112	-1.809
2	-2.459	-2.605	-2.126	-1.821
1	-2.479	-2.605	-2.137	-1.832

```
Opt Lag (Ng-Perron seq t) =  4 with RMSE  .6858963
Min SC   = -.4878626 at lag  4 with RMSE  .6858963
Min MAIC = -.6135722 at lag  4 with RMSE  .6858963
```

Table A. 26 - Augmented Dickey-Fuller test for the inflation rate in levels, with lag length set to four

Augmented Dickey-Fuller test for unit root Number of obs = 88

	Test Statistic	Interpolated Dickey-Fuller: 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-3.281	-3.527	-2.900	-2.585

MacKinnon approximate p-value for Z(t) = 0.0158

Table A. 27 - DF-GLS test for the first-differenced inflation rate, with maximum lag length set to four

DF-GLS for D.ninfl Number of obs = 83

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
4	-3.504	-2.605	-2.098	-1.796
3	-5.147	-2.605	-2.112	-1.809
2	-3.872	-2.605	-2.126	-1.821
1	-5.285	-2.605	-2.137	-1.832

Opt Lag (Ng-Perron seq t) = 4 with RMSE .8167741
Min SC = -.1415783 at lag 3 with RMSE .8375559
Min MAIC = .7662006 at lag 2 with RMSE .8967685

Table A. 28 - DF-GLS test for the output gap in levels, with maximum lag length set to four

DF-GLS for noutput Number of obs = 83

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
4	-1.726	-2.605	-2.098	-1.796
3	-2.681	-2.605	-2.112	-1.809
2	-1.401	-2.605	-2.126	-1.821
1	-2.648	-2.605	-2.137	-1.832

Opt Lag (Ng-Perron seq t) = 4 with RMSE .149306
Min SC = -3.537319 at lag 4 with RMSE .149306
Min MAIC = -3.613185 at lag 4 with RMSE .149306

Table A. 29 - Augmented Dickey-Fuller test for the output gap in levels, with lag length set to four

Augmented Dickey-Fuller test for unit root Number of obs = 88

	Test Statistic	Interpolated Dickey-Fuller: 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-2.907	-2.605	-1.950	-1.610

Table A. 30 - DF-GLS test for the first-differenced output gap, with maximum lag length set to four

DF-GLS for D.noutput Number of obs = 83

[lags]	DF-GLS mu Test Statistic	1% Critical Value	5% Critical Value	10% Critical Value
4	-2.747	-2.605	-2.098	-1.796
3	-3.434	-2.605	-2.112	-1.809
2	-2.259	-2.605	-2.126	-1.821
1	-4.808	-2.605	-2.137	-1.832

```
Opt Lag (Ng-Perron seq t) =  3 with RMSE  .1525283
Min SC   = -3.547854 at lag  3 with RMSE  .1525283
Min MAIC = -3.351988 at lag  4 with RMSE  .1509542
```

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