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Portfolio Risk Management in Shipping

A Multi-factor Approach

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NORWEGIAN SCHOOL OF ECONOMICS

"Believe me, no: I thank my fortune for it,

My ventures are not in one bottom trusted,

Nor to one place; nor is my whole estate

Upon the fortune of this present year:

Therefore my merchandise makes me not sad"

- *Antonio, Merchant of Venice*
  (Act 1, Scene 1)

Abstract

This paper applies modern portfolio theory to manage portfolio risk for real shipping investments. The aim is to propose a multi-factor model for explaining vessel return variation. In doing so, we seek to improve extant research on shipping markets and clarify the underlying risk factors in the industry. To corroborate the necessity of a more comprehensive model, we include other well-established methods for comparison.

The study comprises the three main shipping segments, i.e. wet bulk, dry bulk and container. By including different segments, an investor can diversify segment-specific risks connected to the particular trade flows and seasonal demand patterns. Additionally, we include vessel size and age to enrich the investment universe. Different sized vessels are exposed to different trades due to characteristics of cargo and physical limitations, e.g. stowage factor and canal dimensions. Larger vessels are therefore exposed to more operational risk in terms of flexibility than smaller vessels. The age factor also represents a differing risk profile of the investment, as newer vessels with favorable cost structures are theoretically less risky albeit more expensive.

We find clear differences in risk-return characteristics regarding all three aspects between the various vessels. This indicates great diversification potential. We show the superiority of the multi-factor model compared to the naïve and single-index optimization frameworks. Our model highlights five main risk factors and vastly improves the explanatory power of return variation.

From a statistical point of view, a diversified portfolio outperforms segment-specific portfolios within dry bulk and container vessels. This is not the case for wet bulk, following the strong performance of recent years. Economically, we find recent trends in shipping to accommodate diversification following a decomposition of the value chain, e.g. Ship Finance International Ltd. This enables diversifying across all segments, in turn reducing cash-flow volatility and possibly adding value for shareholders.

Preface

This paper is written as part of the MSc. Program in Economics and Business Administration at the Norwegian School of Economics (NHH). The timeframe of this thesis has been one semester, and its workload represents 30 ECTS.

We have found the process highly educational as it has expanded our knowledge on both financial and shipping related subjects. Combining our interest and areas of expertise has been beneficial, as it provides a more detailed approach regarding financial modelling and maritime economics.

With this paper, we wish to encourage more research on a neglected subject in maritime research, namely diversification and asset allocation. In capital-intensive industries, e.g. shipping, the importance of asset allocation is magnified. The topic is important for all stakeholders holding a share in a vessel or a fleet.

We wish to thank our supervisor, Roar Adland, for encouraging us to be as independent as possible throughout this paper. Through his comments and suggestions, he has guided us towards expanding the research frontier on diversification in real investments in shipping.

Additionally, we would like to thank Tor Wergeland for insightful comments throughout the process of writing this paper.

Last but not least, we are grateful for the support provided from the Norwegian Shipowners Association through their research fund at The Norwegian School of Economics.

Bergen, June 2015

Jørgen Lian

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1. Introduction

In 1979, the former president and chairman of Wilhelm Wilhelmsen Leif T. Løddesøl wrote an article on why some shipping firms fail and others succeed (1979). According to Løddesøl, spreading risk through a diversified fleet seems to be one of five reasons some shipowners survive in such a volatile industry. Peter Lorange states, "several of the disasters involving Scandinavian shipping companies in the early 1980's can be traced back to confusion about their risk exposure" (Lorange, 2009, p. 187). According to the author, the co-variation between shipping segments seems to have declined over time. As a consequence, the gains from diversification could be more evident than before. Thus, it becomes important for all stakeholders in shipping to enhance their understanding of today's risk exposure.

Encouraged by the abovementioned words of Lorange and Løddesøl, our objective becomes twofold. First and foremost, we seek to formulate a more accurate risk relationship among various vessel types, both across and within segments. By including several factors, we approximate the real exposure to certain risk factors, adding to previous research. Secondly, we wish to investigate whether or not a diversified portfolio of ships outperforms a more segment specialized portfolio, given our research approach. To our knowledge, this is the first paper with this particular intention.

In our paper, we apply modern portfolio theory to vessel returns and propose a multi-factor framework to manage portfolio risk. We compare the multi-factor model to other models and examine diversification opportunities in three dimensions; segment, size and age.

Taking the perspective of a shipping oriented investor, viewing vessels as financial assets, we do not handle the issue of operational decisions, e.g. spot or time charter. We also restrict our paper to the unlevered returns of the real assets. It is important to mention such a constraints, as shipping is known for its high debt-to-equity ratios due to high asset tangibility.

This paper should be of interest to several stakeholders in the shipping industry, e.g. hedge fund managers, shipowners, shipping banks and others with a share of real investments in shipping.

After a literature review on relevant shipping and financial theory, we present and discuss the methods used in our paper. Following this, a chapter is devoted to data selection. Finally, results are presented and discussed followed by a brief concluding remark.

2. Literature review

The proceeding sections will cover literature on shipping and maritime economics crucial to understanding the challenges encountered while writing this paper. Last but not least, a section is devoted to a brief literature review of relevant financial theory.

2.1 Previous papers

Previous studies on diversification in shipping by Magirou et al. (1997) and KôseogĞlu & Karagûlle (2013) have concentrated their efforts on dry bulk and tanker or simply dry bulk markets respectively. Both research papers focus on earnings obtained from one-year time charter contracts (TC). In addition, a master's thesis from MIT in Ocean Systems Management has touched upon the subject (Patitsas, 2004), and Melbø (2013) has written a short paper on the topic. Common for all the aforementioned works is their limitation in the application of modern financial theory, particularly the restricted use of a single-factor framework.

Referring to the study by Magirou et al. (1997), the main assumption is that the shortest holding period possible is one year. Making such an assumption smoothens the relative volatility of freight markets and increases the perceived correlation (Albertijn, et al., 2011). Reason being that one-year time charter data fail to capture monthly seasonal patterns in seaborne trade. It is common knowledge in the industry that certain trades are more active during certain months than others, e.g. crude oil during the winter months. As Kavussanos & Alizadeh (2001) confirm, seasonal patterns exist due to the underlying seasonal demand for commodities transported. Since portfolio optimization is very sensitive to its inputs, smoothing might make a large difference towards the result.

Theoretically, one expects the risk-adjusted return from one-year time charter to equal one-year expected continuous spot operations (Adland, 2002). However, portfolio optimization using variance minimization would not yield the same results due to differing volatility in spot and TC-markets (Glen & Martin, 1998). Consequently, we suggest using monthly time charter equivalent data obtained from voyage charter earnings, cf. chapter 4.

In the case of KôseogĞlu & Karagûlle (2013) , one-year TC rates are sampled in weekly observations. While this might increase data availability, it also presents a problem from a practical point of view. A shipowner cannot possibly fix his vessel on a new one-year TC each week. Obviously, this has implications concerning the practical validity of the results. Furthermore, KôseogĞlu & Karagûlle adress the issue of diversification from a co-integration perspective.

For Patitsas (2004), quarterly earnings based on spot rates provide a starting point for portfolio optimization. Despite being closer to our study, quarterly earnings would be less volatile compared to monthly earnings, all else equal.

In addition to the abovementioned papers, a graduate paper ("høyere avdeling") from 1986 has applied portfolio optimization to shipping (Koch, 1987). In Koch's study, Wallenius Willhelmsen Logistics' (WWL) capital allocation across shipping segments was analyzed with regards to risk minimization. At the time, the paper made an important contribution to risk management at WWL. Koch's finding was the fact that benefits of diversification, i.e. risk minimization, are not obtained simply by spreading capital across several segments, one must also consider the interdependency and common exposure to risk factors. In WWL's case, what was once considered as a well-diversified portfolio seemed quite the contrary. Koch's contribution is particularly important with respect to our paper, as we seek to capture the risk factors by using a more advanced model, cf. chapter 3.

Cullinane (1995) examines hedging strategies in shipping as an optimization problem using freight forward derivatives. The study applies Markowitz portfolio selection methodology to dry bulk shipping markets. Despite being slightly different to our problem, covering the trade-off between spot and time charter, Cullinane's article provides an insight into early approaches to portfolio optimization in shipping. In addition, Berg-Andreassen (1998) examines modern portfolio theory and its implications for optimal chartering policies. Albeit slightly different, the article is an example of the usefulness and application of portfolio theory to shipping. Lastly, Norman (1981) adopts a portfolio-based approach for chartering rules in bulk shipping. Being a pioneer in the field, Norman illustrates the gains of portfolio optimization.

Whilst the aforementioned studies cover portfolio optimization from a theoretical point of view, some work has been conducted towards an applied perspective. A program for optimizing fleet composition, SHIPMIX (Schilbred, 1992), was developed at the Norwegian School of Economics during the early 90's. Using the Markowitz method for optimizing the portfolio, Schilbred was able to show diversification opportunities in real investments.

Our paper differs from previous research concerning both modeling inputs for portfolio optimization (i.e. covariance modeling) and data selection. Besides capturing risk more accurately through a multi-factor approach, studying a longer time series with a higher frequency (i.e. more observations) is particularly important.

2.2 Shipping cycles

Freight markets are known for their volatility. As explained by Koopmans (1939), the demand and supply fundamentals drive this volatility. In the short run, supply of vessels is fixed at an upper limit. As a result, the supply of transport changes from elastic to inelastic depending on fleet utilization. When demand is low, supply is elastic and vessels are slow steaming or being laid up. As demand increases, the freight rate steadily increases until all vessels are actively trading. At this point, supply becomes inelastic and freight rates soar. Consequently, the prevailing freight rate is determined by demand alone. This relationship between supply and demand is known as the "hockey stick graph" (figure 1).

As a side note, the concept of fleet utilization can be particularly difficult to measure in shipping. Due to slow steaming, i.e. sailing at low speeds, measuring total fleet capacity becomes complicated. Multiplying deadweight tonne (DWT) capacity with trade distance, i.e. tonne miles, one can better understand freight demand, as both volume *and* distance determine the demand for freight. Consequently, measuring supply and demand in shipping is often done by tonne miles, as the graph below illustrates.

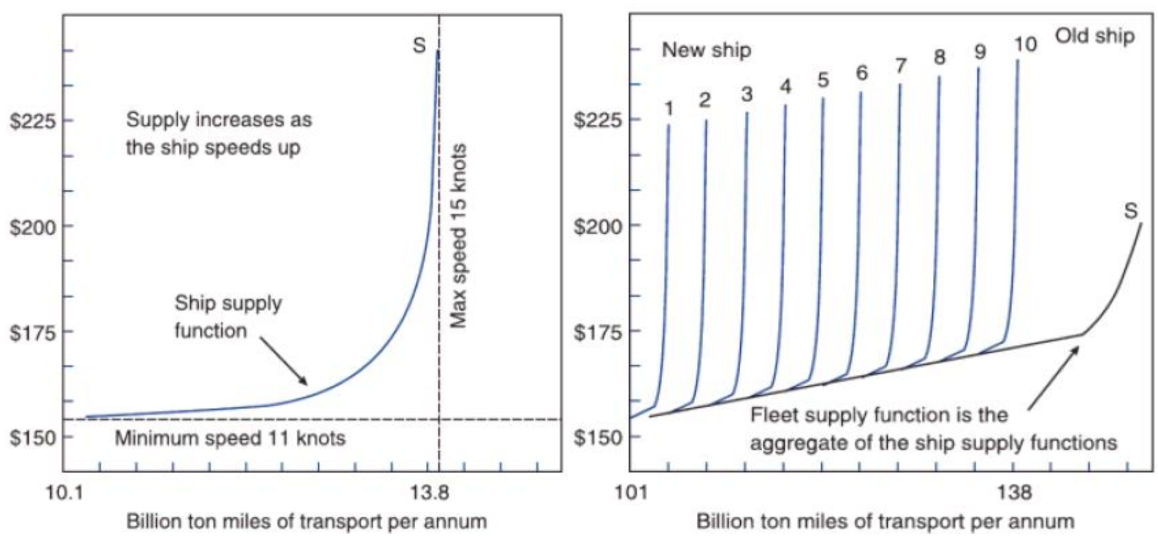

Figure 1 - The "hockey stick" supply curve for each vessel (left) and on aggregate (right) (Stopford, 2009, p. 161)

Because of high rates, shipowners decide to order new vessels increasing the capacity in the medium to long-run perspective. Needless to say, this is a gamble on future freight rates. More often than not, future rates will fall and new vessels only increase the oversupply in the market. Understanding shipping market cycles is important for any shipowner, as positioning relative to your competitors is key.

When discussing cycles in shipping, it is often in terms of the short-term business cycles (5-10 years). According to Stopford (2009), a typical shipping cycle lasts for 7 years from peak to peak. It can be divided into four parts; trough, recovery, plateau and peak. However, cycles may be shorter or longer depending on shipowners' behavior. Following this line of reasoning, it is important to study a sufficient time-period when working with investments in shipping.

Even though different vessels are employed in different trades, the boom-bust cycle mentioned above is similar to them all. A common factor for all seaborne trade is the dependency on general economic activity. However, different segments will experience different cyclical patterns, causing some vessels to switch into other trades. This leakage effect, as described by Strandenes (2012), forces freight rates down in the entered segment and up in the exited segment as capacity is increased or reduced. This effect exists within segments, e.g. within wet or dry bulk, and across segments, e.g. combination carriers or OBOs. The existence of leakage effects, as described by Strandenes, does provide some context for the origin of this paper. By capturing the different trade patterns, one could seek to utilize the discrepancies between different segments. In addition, a reduction of combination carriers in the market does perhaps create further diversification potential. A plausible theory could be that the reduction of such vessels has led the integration, i.e. leakage effect, between segments to decrease.

On the other hand, a paper written by Sødal et al. (2008) finds empirical evidence underlining a possible comeback of combination carriers. Combination carriers became unpopular due to their relatively expensive construction and unprofitable operations. However, if freight markets become less integrated on a short-term basis, and the real price of a new combo carrier does not exceed the quoted secondhand price, new combo carriers could enter the market. Perhaps more importantly for our paper, Sødal et al. (2008) argue the possibility of triangulation to be the most significant force behind the revival of combo carriers. Triangulation is essentially the ability to carry different loads on different legs of a single voyage, avoiding empty ballast legs, thus maximizing vessel utilization. If the trade flow for seaborne goods are structured in a way that makes such an arrangement economically viable, combination carriers may very well be profitable. In our paper, we seek to utilize the varying seasonal patterns of trade to minimize the risk for a diversified shipowner. Increased combination carrier trade would, all else equal, work against our objective, as this would lead to markets being more co-integrated due to the leakage effect.

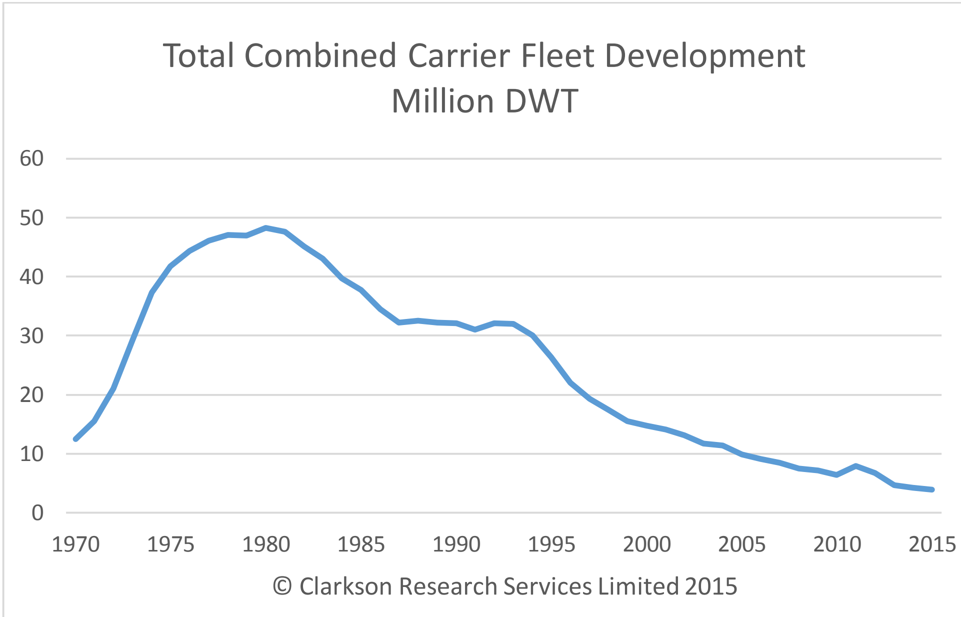

Figure 2 - Development of the combined carrier fleet

At time of writing, there is little evidence supporting the revival of combination carriers. Figure 2 shows the fleet of combination carriers serving both dry and wet bulk markets has steadily declined. The current combined carrier fleet accounts for approximately 3.79 million DWT, a small fraction of the total tanker fleet exceeding 500 million DWT (Clarkson Research Services Ltd., 2015).

2.3 The market for freight

The market for transport of seaborne goods is divided between spot and time charter markets. To fully understand the difference, one must understand the properties of the two markets.

As the table below illustrates, the uncertainty surrounding the shipowners' cash flow depends on the chosen contract.

Spot market	**Time charter market**
Voyage Hire - Operational expenses - Voyage costs (Incl. Bunker fuel) = **Operating Earnings**	TC – hire - Operational expenses = **Operating Earnings**

Table 1 - Shipowners cash flow (Kavussanos, 2010)

In shipping, freight rate volatility is often the main concern. However, fluctuations in costs is also an important aspect. A shipowner trading in the time charter market does not need to worry about the price of bunker fuel, which accounts for a significant proportion of the costs. Consequently, the volatility of bunker prices is one of the reasons why spot is considered more risky than time charter. Apart from bunker price volatility, the perceived excess risk of trading in the spot market is due to the uncertainty of fixing your vessel in the future, i.e. unemployment risk.

Consequently, choosing which market to operate in is equivalent to choosing preferred level of operating risk exposure. A shipowner with an optimistic expectation of the future might enter into a voyage charter (spot) in order to be eligible for higher freight rates in the future. On the other hand, a shipowner with pessimistic expectations might enter into a long-term TC in order to secure future earnings. Thus, the TC market allows shipowners, and cargo owners, to allocate risk.

Considering the potential unemployment risk in spot markets, one expects that the spot rate trades at a premium compared to time charter rates. The premium merely reflect the risk of not being able to fix the vessel in future periods. This rationale is confirmed by Adland (2002) in his Ph.D. thesis concluding that the risk-adjusted returns of both chartering strategies must be the same. By studying the spot market, we can conveniently derive monthly earnings and assume that the results are also valid for a shipowner operating primarily in time charter markets. The validity of the results in this paper rest, to some extent, on this theory.

Despite not being subject to unemployment risk, shipowners operating in time charter markets often face a charter default risk, i.e. counterparty risk. The risk of the charterer defaulting on the contract will vary with freight market conditions, duration and financial situation of the charterer. Unfortunately for the shipowner, the risk of default increases as the spot rate decreases. A decline in spot rates will shift the term structure of freight rates downwards (Adland & Jia, 2008). Subsequently, the charterer could default on his contract and seek to enter into a new contract with a lower TC rate. The shipowner runs the risk of having his vessel re-delivered and must find a new employment at a short notice. Thus, contrary to intuition, time charter contracts are definitely not exempt from risk.

In line with maritime economic theory, Kavussanos (1996b) proves that spot rates are indeed more volatile than time charter rates, accounting for vessel size. The main reason, touched upon above, is the difference in shipowner cash flow and inherent uncertainty (Kavussanos & Alizadeh, 2002).

2.3.1 Freight market co-integration

For the purpose of diversification, co-integration[1] between markets is important. For instance, if Capesize and Panamax vessels were co-integrated, one would expect the difference in earnings to fluctuate within a certain interval. If not, the difference between earnings could become very large, with no tendency to revert. Following economic theory, we would expect freight rates within segments to be co-integrated, simply due to an arbitrage-pricing theorem. For instance, freight rates for a Capesize vessel cannot possibly sky rocket, as charterers eventually would consider parcel splitting into several Panamax vessels. Following this, one would expect markets to adjust such that the risk adjusted return from operating the different vessels is equal. There has been many attempts to prove that freight rates are indeed co-integrated, i.e. that a long-run relationship exists. Relevant literature being Kavussanos (1996b) and Veenstra & Franses (1997). The findings of Veenstra & Franses indicate that freight rates are in fact co-integrated. Such findings are also the result of Berg-Andreassen (1996). This is to be expected due to the economic reasoning above. Although co-integration does not exclude diversification per se, it does limit the diversification potential due to the mean reversion process.

Contrary to the results above, research done by Koekebakker et al. (2006) implies that freight rates are in fact stationary, albeit non-linear, in line with maritime economic theory. If freight rates were non-stationary, there would be no theoretical ceiling nor floor for freight rates. Referring to Koopmans (1939), there must be a lower bound for freight rates where vessels leave the market, i.e. scrapping. According to Tvedt (1996), there must also be a theoretical upper boundary where alternative sources of transportation becomes economically viable. Thus, for such boundaries to exist, freight rates must be stationary. The study of Koekebakker et al. on spot freight rates, excluded the endogenous effect of bunker prices. Additionally, changes in standard vessel specification in a time series may lead to a higher probability of wrongly rejecting the stationary behavior of freight rates (Koekkebakker, et al., 2006).

Stationary freight rates imply that the markets are not co-integrated. However, such results do not seem to be in line with economic reasoning, particularly the arbitrage-pricing theorem mentioned above. On the other hand, if freight rates are indeed stationary, there might be more gains from diversification than otherwise assumed. If freight rates are non-stationary (co-integrated), then diversification potential is de facto limited. To summarize, there seems to be ambiguous results towards the actual diversification potential in shipping concerning co-integrated freight rates.

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[1] For an introduction to the concepts of stationarity and co-integration, cf. Hill et al. “Principles of Econometrics 4th.ed”, chapter 12.

Regarding segment diversification, Jia & Adland (2002) provides valuable insight towards the potential across segments from a shipowner's perspective. As pointed out by the authors of the paper, correlations between freight markets seem to be time varying. Unfortunately, there is stronger evidence of freight market correlation in depressed markets than booming markets (Jia & Adland, 2002). This result highlights the importance of studying a sufficiently long time series when looking at returns in shipping markets. The findings of Jia & Adland are clearly a concern for shipowners running a diversified fleet, as they conclude the gains from diversification against negative investment returns to be negligible.

A glance at prevailing tanker and dry bulk freight rates illustrate the findings of Jia & Adland (2002). Freight rates across segments seem to be positively correlated during depressed markets. However, at time of writing, tanker freight rates are at a six-year high and climbing. This is a major difference compared to dry bulk freight rates, as the following graph illustrates:

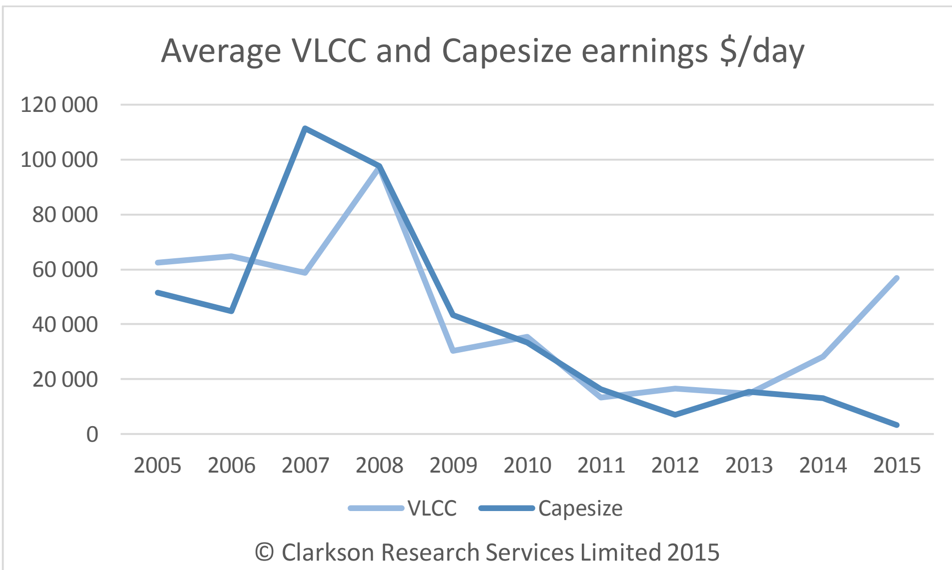

Figure 3 - Freight rates for VLCC vs. Capesize

The current difference in tanker and dry bulk earnings, as illustrated above, could serve as an argument in favor of a portfolio strategy in shipping, or at least spark an argument towards the benefit gained from such a strategy. In contrast to the conclusion of Jia & Adland (2002), operating a diversified fleet could provide benefits if the decline of one segment is not due to a common factor. Alternatively, the factor behind a boost in freight rates in one segment is segment specific.

2.4 The market for vessels

The market for buying and selling vessels can, similar to freight markets, be divided into a primary and secondary (auxiliary) market (Wijnolst & Wergeland, 1996). Although both markets deal with supply and demand of vessels, it is important to distinguish between their specific properties.

While the newbuilding market offers more capacity to the overall fleet, the secondhand market offers a transfer of risk. In many ways, the secondhand market displays many of the same characteristics as any traded asset. However, it is important to understand the characteristics of both secondhand and newbuilding markets, as they are to some extent co-dependent.

2.4.1 Newbuilding market

Similar to the prices of secondhand vessels, newbuilding prices are determined by demand and supply fundamentals. However, the supply of new vessels differs from the supply of secondhand vessels. Berth capacity at shipyards, production costs and the size of the orderbook are important factors of supply. On the demand side, freight rates, price of secondhand vessels, excess liquidity and expectations are important aspects (Stopford, 2009, pp. 202-212). In times of high freight rates, prices for newbuildings may skyrocket as shipowners compete for available berths. Increased demand for new vessels lead to a longer delivery time as capacity is pushed to its limits. Ironically, the relationship between delivery time of new vessels and freight rates is positively correlated, much to the shipowners' dismay.

Since delivery of new vessels include a 2-3 year time lag, supply of vessels is fixed upwards in the short run (Koopmans, 1939). In the short run, shipowners can only reduce supply by putting their vessels into lay-up, slow steaming or scrapping, thus the industry supply curve is the aggregated marginal cost curve, cf. figure 1. Looking at supply from a medium- to long-term perspective, the supply of transport could be increased by adding more vessels. Consequently, it is important to distinguish between short-run and long-run supply of vessels.

The volatility of newbuilding prices is partly explained by the sentimental investment behavior of shipowners. Newbuildings are highly correlated with secondhand market prices (Beenstock, 1985). However, they are less volatile than secondhand prices. This "stickiness", or lack of volatility, could be simply because newbuilding prices can be thought of as a kind of futures contracts on a vessel (Adland & Jia, 2014), and are therefore encumbered by risk. However, one would perhaps expect newbuilding prices to be more volatile than they seemingly are. The phenomenon can possibly be explained by the underlying delivery lag and the fact that newbuilding prices are not directly comparable across time. Another possibility, as mentioned by the authors, is the fact that in distressed markets, lack of liquidity leads to a bias due to the use of "last done deal" quotation. This bias is also present in secondhand markets, as we will discuss later on in this chapter.

A good explanation of shipping investment on an industry level is the cobweb theorem (Kaldor, 1934) as described by Stopford (2009, pp. 335-337). In the long run, the supply curve expands when new ships are delivered. Increased supply causes freight rates to plummet. Low freight rates lead to slow steaming, lay-ups and eventually scrapping of vessels. Due to less supply of tonnage, the market contracts. The contraction eventually leads to increased freight rates, filling the owner's bank accounts, and motivating additional ordering of new vessels. Underlined by Greenwood & Hansen (2014), pro cyclical behavior in shipping markets dramatically amplify economic fluctuations. Seeing that the pro cyclical behavior is common across all shipping segments, diversification may provide some benefits as the timing of the cycles could vary from segment to segment. However, one could argue that vessel newbuildings are predominantly co-integrated as the capacity at shipyards is fixed regardless of vessel type. All vessels compete for the same berths and the same resources, thus it is economically unlikely that newbuilding prices deviate too much from each other. If this is the case, diversification potential with regards to this cyclicality could be diminished by the co-integration.

In line with Greenwood & Hansen's article (2014), one might expect high prices to be followed by low prices and vice versa. Such trends would imply a pattern of stationarity in newbuilding prices. On the other hand, there are studies indicating that vessel markets are non-stationary (Hale & Vanags, 1992) and that vessel prices are co-integrated (Glen, 1997). These findings imply that a period of high prices not necessarily must be followed by low prices, and that vessel prices (and returns) move together in the long-run (co-integration). Non-stationary behavior is to be expected, as shipbuilding capacity is limited. Again, if the markets are in fact co-integrated, this may diminish the efficiency and potential of diversification. However, as pointed out by Tvedt (2003), these results invalidate the two fundamental assumptions in classic business cycle theory of shipbuilding (Tinbergen, 1931):

1) *The downward trend in prices is due to increased efficiency.*
2) *The prices in the market has a cyclical or mean reversion pattern (i.e. stationarity) due to a mismatch between demand for new vessels and delivery of new vessels (delivery lag).*

Certainly, one would expect there to be some degree of positive correlation across shipping segments as they are affected by common factors such as world economic growth. On the other hand, the findings of Tvedt (2003) imply that vessel prices and freight rates are indeed stationary. As mentioned above, this was also the result of a more recent study by Koekkebakker et al. (2006) on freight rates. Once again, previous studies on shipping markets provide ambiguous results.

2.4.2 Secondhand market

The main factors behind vessels prices in both the primary and auxiliary markets are present freight rates and expectations of future freight rates. There is a close correlation between the freight rate obtained by a particular vessel and its market value. Again, this correlation is stronger in the secondhand market compared to the newbuilding market, since there is no delay in delivery. Secondly, the price of a secondhand vessel is influenced by age. Normal scrapping age of a standard vessel is 25-30 years. Accordingly, a vessel is said to depreciate by approximately 5% per annum (Stopford, 2009, p. 237). Naturally, the scrapping age of a vessel varies with market sentiment. In depressed markets, a vessel could in fact risk being scrapped as early as 15 years, depending on the scrap metal prices and future expectations in freight markets.

Mutual to other asset markets, the market for vessels has its speculative investors earning profits from "asset play" strategies. The volatility of secondhand markets enable investors the opportunity to buy low and sell high. Due to the instant availability of vessels, the secondhand market is more volatile than the newbuilding market (Kavussanos, 2010). In very strong freight markets, the price of a secondhand vessel may exceed the newbuilding price by several million dollars (Adland, et al., 2006). In contrast to the newbuilding market, the price adjustment to freight rates is instantaneous. Following this, one might expect the secondhand market for vessels to be perfectly liquid, at least for standardized vessels as studied in our paper.

However, there is evidence supporting a sentiment varying liquidity, violating the assumption that the secondhand market is perfectly liquid (Albertijn, et al., 2011). The introduction of stricter bank lending policy (Basel II & III) might reduce the liquidity even further and increase volatility in vessel prices (Kashyap & Stein, 2004). In combination with bank lending policy, fair value accounting (IFRS) of vessels may amplify the cyclicality of vessel values. This is a problem for shipowners and banks alike. If the value of a vessel decreases due to mark-to-market accounting, the observable collateral value on the loan decreases, possibly violating debt covenants. If a bank forecloses on a vessel or a fleet, they may in fact *increase* their losses due to the "collateral channel".

In short, this is the result of negative externalities from one foreclosure causing a run on collateral throughout the industry. Albeit unintentional, the introduction of new accounting standards and lending policies might actually destabilize markets in recession, contrary to its purpose (Merrill, et al., 2012). However, we will assume liquid vessel markets for the purpose of this paper. Since the period at hand includes the 2008 financial crisis, the discussion above is important to include. The fact that certain shipping banks chose not to act on violations of debt covenants during the crisis highlights the issue (Albertijn, et al., 2011).

A good example of the illiquidity in the secondhand market is the occurrence of “sticky” prices. Similar to newbuilding prices, the industry’s reliance on last done deal often introduces “sticky” prices even at times of large freight rate movements (Adland & Koekebakker, 2007). This has implications for our study, as the “stickiness” might induce a bias in volatility of vessel prices. In distressed markets, one would expect vessel value to be even lower than the quoted prices, i.e. the quoted price of a vessel exceeds its fundamental value. If this is in fact the case, perceived volatility is reduced due to the “stickiness” bias.

The above discussion and assumption of liquid markets and volatility is important for two reasons. First, as our financial models rely on the assumption of perfect markets, we need secondhand values to be liquid at all times. Secondly, as discussed later on, our financial models are very sensitive to the parameters used, including asset volatility. Note that the stickiness described here is fundamentally different from the case of sticky newbuilding prices. In the newbuilding case, stickiness is thought to be explained by economic fundamentals. For secondhand prices, the problem is due to the psychological reliance of last quoted deal. This represents a form of market inefficiency related to the availability of information.

Similar to the freight market, the prices of secondhand vessels are closely correlated to each other, especially as different size vessels often serve as near perfect substitutes within segments. In periods of general booms, all vessels prices tend to move in a similar direction. Economic rationale dictate that we would expect vessels in the secondhand market to be co-integrated, limiting the possible gains from diversification. The reasoning is also in line with previous research on the subject according to Hale & Vanags (1992), Glen (1997) and Kavussanos (1997). However, as with freight rates, some segments may enjoy a boom while others experience a bust. The substitute argument above is only valid within segments, i.e. among vessel size. The possibility of markets moving in opposite directions could indicate some diversification opportunities, at least across segments (for instance dry bulk and tanker). This discussion simply mirrors our previous discussions of freight rate correlation and co-integration.

To conclude, the inherent properties of newbuilding and secondhand markets described above are important for the work done in this paper. Financial theory rests on certain assumptions regarding liquidity, information and competitive markets. Therefore, we have chosen to only study the secondhand market in our paper. The main argument to only include secondhand vessels is the fact that secondhand markets are naturally more liquid than newbuilding markets, despite the inherent "stickiness" bias discussed above. Furthermore, a newbuilding contract cannot be chartered out to earn operating revenues, complicating comparing returns of different vessels. Additionally, as stated above, newbuilding prices are not directly comparable across time (Adland & Jia, 2014).

2.5 Diversification in shipping

The following subsections will cover the three dimensions, i.e. segment, size and age, of diversification potential studied in this paper.

2.5.1 Vessel segment

The typical routes for each vessel type is determined by the trade flow for various commodities. For instance, VLCCs are predominantely used for freight out of the Arabian Gulf to developed countries whilst Capesizes normally trade out of Brazil or Australia to developing countries, e.g. iron ore to China. This is important, as idiosyncratic factors affecting each particular trade may imply some diversification potential. This line of reasoning is underlined by the work of Kavussanos (2010), stating that the volatility of a vessel is due to common *and* trade-specific risk factors. A closure of the Suez Canal might for instance affect wet bulk more than dry bulk, due to the major oil trades being exported out of the Middle East (Stopford, 2009, p. 438). Conversely, a closure of Chinese steel mills affect dry bulk more than wet bulk vessels. By diversifying, the exposure to trade-specific shocks are minimized. Particularly considering the freight rate co-integration discussed above. Since parcel splitting within segments cap freight rates in bulk shipping, a shock to one particular trade or vessel type has ramifications for other vessels within the same segment.

Worth mentioning, wet bulk vessels rely on a completely different set of loading and unloading facilities than dry bulk vessels. For product tankers, even more complicated cargo handling is required (Stopford, 2009, p. 445). Consequently, the barrier of entry is slightly higher in chemicals and product tanker trade than other bulk trades. This could have implications for our paper, since we must assume that an investor in shipping can enter any business regardless of such entry barriers. In addition to market characteristics, lack of observations in specialized shipping makes inclusion of such segments difficult, e.g. LNG markets.

To conclude, the markets for dry and wet bulk transport have several important characteristics. First and foremost, there are generally low barriers to entry. Secondly, the concentration of ownership is weak (Wijnolst & Wergeland, 1996), indicating a competitive market which is important for the validity of our results. Finally, the exposure to trade-specific risk factors highlight some diversification potential across segments.

In contrast to the abovementioned bulk segments, container shipping has been subject to widespread cooperation among competitors. Previously, the liner industry was organized in conferences cooperating on providing transport (Wijnolst & Wergeland, 1996). However, in 2008, container conferences were banned by antitrust regulations. Despite being more open on pricing of capacity, several leading liner companies were under investigation by the European Commission in 2011 (ECSA, 2015).

Furthermore, an important aspect of container trade are the significant economies of scale (Strandenes, 2012). The importance of running on schedule means that a liner operator must, in addition to having a large fleet, be able to organize it efficiently. Consequently, chartering space on each other's container lines became a possibility during the 90's. This separation between owning transport and operating transport has led to increased flexibility for container owners.

The barriers to entry affect the degree of competition in the business. As UNCTAD (2010) points out, the twenty largest firms controlled 69 % of capacity in 2009. Obviously, this has consequences for our study as competitive markets is an important assumption in our models. With such significant barriers to entry, one could question the practical ability to diversify into container trade. On the other hand, a growing fraction of liner shipping is being done on the open market (Lorange, 2009, p. 21). This should enable investors the opportunity to partake in the container segment.

In practice, liner companies differentiate by offering levels of service depending on the importance of punctuality and tailored requirements (Wijnolst & Wergeland, 1996). As cargo increases in value, it is normal with higher freight rates and speed of delivery often increases as well. Since trade patterns are normally fixed (Strandenes, 2012), vessels are seldom fixed on the spot market. This provides some challenges to our study regarding the nature of the observations, which we will discuss in chapter 3.

For the purpose of this paper, we consider investments in liner shipping to be vessel investments only, i.e. one assumes that the market is sufficiently competitive for chartering out vessels. At least, the option to sell container slots provides some practical investment opportunities. As pointed out by Lorange & Norman (1973), liner trade has always been considered a safe trade, and could thus be important in a portfolio optimization context in addition to bulk vessels. The question remains whether liner trade is practically suitable for diversification, which we will consider towards the end of this paper.

2.5.2 Vessel size

An important aspect of bulk shipping is the economies of scale on cargo unit level. Naturally, unit cost declines by employing larger vessels. Since the commodities transported by these vessels are in bulk, there is a constant pressure to increase vessel size (table 2). However, as vessel size increases, operational flexibility decreases leading to more volatility in freight earnings (Kavussanos, 1996b).

	Mid-1980's (DWT)	**Mid-2000's (DWT)**
Handysize	25,000	30,000
Handymax	45,000	*
Panamax	50-80,000	60-80,000
Capesize	>80,000	>100,000
VLOC (VALEMAX*)	>300,00	400,000*

Table 2 - Dry bulk vessel evolution (Gratsos, et al., 2012)

Due to the economies of scale and characteristics of various commodities (for instance stowage factor), bulk vessels operate in more or less separate trades. The low unit costs of large bulk vessels, i.e. Capesize and VLCC, are ideal for long hauls. Since natural resources often are extracted far from their intended use, it is desirable to transport as much as possible in one go. As a consequence, the vessels are designed almost exclusively for certain trades, offering limited flexibility.

Medium sized vessels such as Panamax and Suezmax vessels are less volatile, as they can pass through their respective canals increasing their operational opportunities. Last, the smaller vessels such as Aframax, Handymax and Handysize are even less volatile, as they can dock at an increasing number of ports. As more trade routes open, the vessel is less likely to be off-hire, as it is less exposed to trade-specific shocks. Correspondingly, small vessels are less volatile, making them less risky investments (Kavussanos, 1996b). This trait is common to wet and dry bulk vessels alike. It also holds for container vessels, as small vessels are able to dock at more ports than larger vessels.

Container trade is often set in a fixed trade pattern, regardless of short-term variations in demand (Strandenes, 2012). The fixed trade pattern implies that container owners must be able to provide for incremental demand, limiting the possibility of doing other trades. In contrast to the trades above, container shipping exhibits economies of scale on a firm level posing a barrier to entry (Wijnolst & Wergeland, 1996). As vessel size increases, cargo handling and operations become increasingly important considering the variable costs in container shipping. Since frequency between ports is predetermined, the proportion of *actual* short-term variable costs limits itself to cargo handling costs. Thus, it becomes increasingly important to optimize cargo handling as vessel size increases. Lastly, small container vessels are more likely to be chartered by a competitor to cover marginal demand surplus than a large vessel, underlining the off-hire argument from preceding paragraphs.

The findings of Kavussanos (1996b) are demonstrated by comparing the volatility in terms of earnings fluctuations among Capesize and Handymax vessels in the graph below:

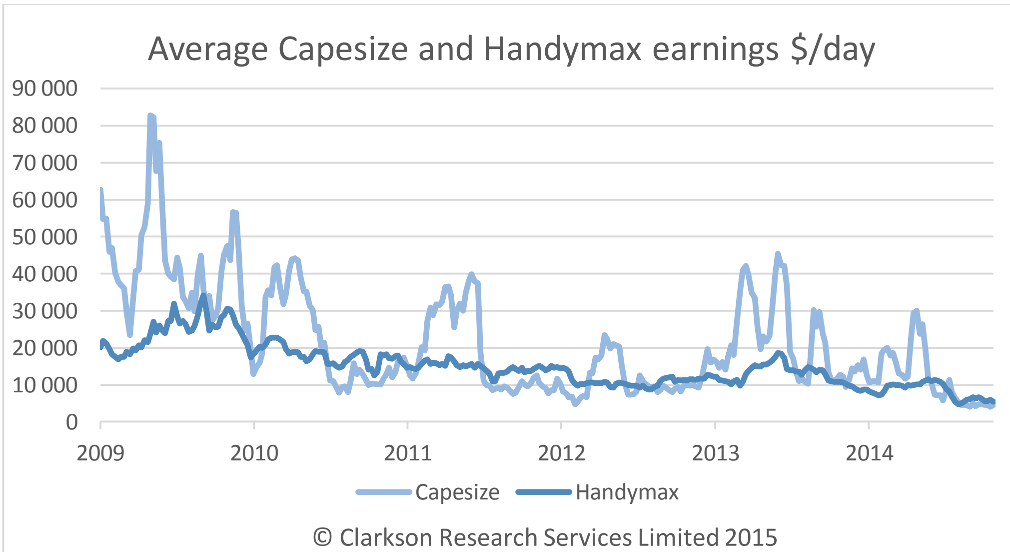

Figure 4 - Average earnings Capesize vs. Handymax

According to theory, a risk-averse investor should invest in Handymax vessels rather than Capesize vessels, and thus be less subject to operational risk, i.e. unemployment (Kavussanos, 2010).

Finally, since the technology employed is quite homogenous in bulk shipping, most vessels are interchangeable within segments capping demand for relatively higher freight rates (Lyridis & Zacharioudakis, 2012). Reverting to freight market co-integration, a cargo owner's decision to split cargo regulates the relative freight rate between different sized vessels. Particularly in the tanker segment, freight rates are linked by "chain" reactions from one size group to the adjacent size group (Strandenes, 1999). Contrary to the size diversification argument raised above, the cargo splitting argument implies an upper boundary for diversification gains. The chain reaction described by Strandenes states that diversification within segments is somewhat mitigated, in contrast to the idea of fleet composition presented by Kavussanos (2010).

2.5.3 Vessel age

In addition to utilizing vessel size to diversify investments, an investor could also allocate his funds across vessels differing in age. Old vessels have a relatively higher proportion of variable to total costs compared to new vessels, thus they are more flexible with regards to short-term lay-up decisions (Lorange & Norman, 1973). In other words, the alternative cost due to lay-up is less for an old vessel compared to a new vessel.

For our paper, issues of financing are disregarded when focusing solely on the asset and its unlevered returns. Therefore, the discussion of operational leverage is less apparent. All else equal, an old vessel will be laid up or scrapped earlier than a new vessel due to the cost structure discussed above. An old vessel has higher daily operating costs, larger crew, more routine maintenance and lower fuel efficiency. This cost differential between new and old vessels determines the short-run supply curve, as mentioned introductorily. For an old vessel, the lay-up point occurs at a higher freight rate than for a newer vessel (cf. figure 1). In shipping, this can be described as a “cash flow race”, i.e. modern vessels can survive lower freight rates longer. Thus, from an operational point of view and excluding capital costs, newer vessels are theoretically less risky, in contrast to the argument of Lorange & Norman.

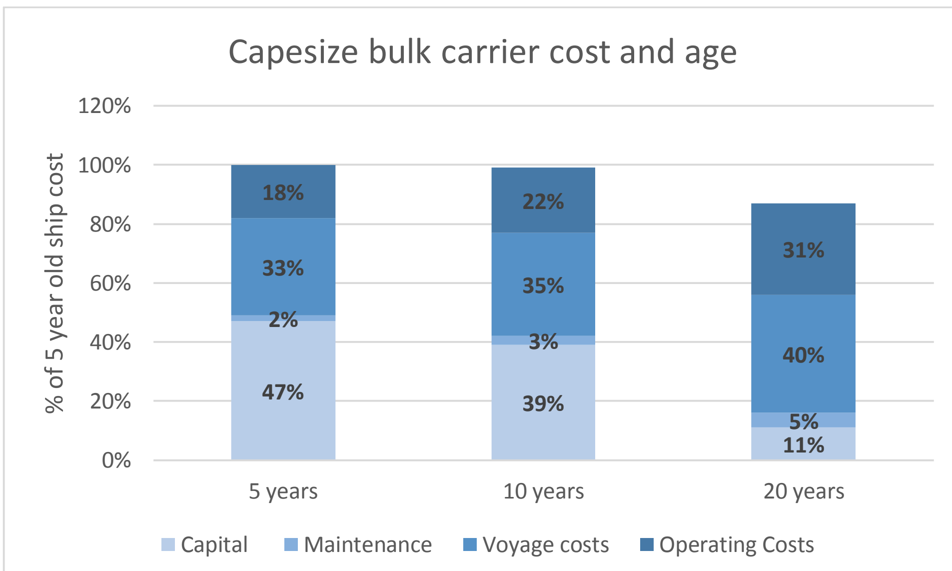

Figure 5 - Vessel costs (Stopford, 2009, p. 222)

Since freight rates obtained by a vessel are considered unaffected by age, old vessels should be perfect substitutes for new vessels. Given this substitutability, operational leverage can easily be altered to some degree by choosing the desired age of a vessel or fleet.

By including an age dimension to our paper, we are able to study the optimal fleet allocation regarding segment, size *and* age, investigating the risk-return trade-off in a three dimensional space. However, it is worth mentioning that empirical results indicate that fleet age does not seem to affect pricing of shipping company risk (Grammenos & Arkoulis, 2003). On the other hand, choosing a particular fleet age profile has always been a vital part of shipping strategy. In the words of Stopford, “owners of new and old ships are in very different businesses” (2009, p. 222).

2.6 Modern portfolio theory

The intuitive understanding of diversification is no recent phenomenon. The phrase “*don’t put all your eggs in one basket”* can be traced back to early 17th century in Cervantes’ *Don Quixote* and Shakespeare’s *Merchant of Venice*. However, an adequate theory of diversification when risks are correlated was missing until the 1950s (Markowitz, 1999).

Harry Max Markowitz’s article on portfolio selection (1952), and the following book *Portfolio Selection: Efficient Diversification of Investments* (1959), crowned him the father of modern portfolio theory (MPT). He displayed investor’s trade-off between risk and return in the mean-variance space and gave statistical meaning to the term “diversification” (Benninga, 2008).

As Markowitz (1952) states, the portfolio selection problem is divided into two stages. The first is estimating input parameters and the second is the application of optimization theory on these inputs. The second stage is the topic of Markowitz’s article that would become the foundation of future financial micro analysis, besides earn him a Nobel Prize in 1990. At a time when the prevailing rule was the law of large numbers and maximizing expected return, Markowitz presented the importance of correlation. A rational investor should seek to minimize portfolio risk and maximize return by combining less than perfectly correlated asset. Applying this theory to a universe of risky assets, one can derive an efficient frontier of dominant portfolios in mean-variance space. The observant reader may have noticed that the abovementioned work of Koch (1987) illustrates this in practice for WWL, i.e. the importance of correlation when diversifying a shipping portfolio.

Combining risky assets with a risk-free asset, James Tobin (1958) presents his separation theorem. When an investor is risk-averse and has propensity to hold some of the risk-free asset (cash in the case of Tobin), then all risky assets can be considered as one single well-diversified risky portfolio. The choice of the investor is therefore concerning the allocation of funds between the risk-free asset and risky portfolio. For a given set of expected returns, variances and covariances, the proportions among risky assets will always be the same.

Extending Tobin’s model by including both borrowing and lending at the same rate, William Sharpe (1964) showed with the capital asset pricing model (CAPM) that in equilibrium, the market portfolio is the only efficient portfolio. Lintner (1965) and Mossin (1966) independently developed similar models. Moreover, the CAPM aimed to explain pricing of all capital assets where Tobin had described simply monetary assets. However, the most remarkable conclusion of the CAPM is that the expected return of each asset is linearly related to its beta and only its beta. This beta is the coefficient reflecting sensitivity of an asset to general market risk, and this risk factor is assumed to identify all correlation between risky assets. The underlying economic reasoning is that investors should be compensated only for so-called systematic risk, and not the excess *idiosyncratic* risk that can be easily mitigated by diversification (Sharpe, 1964, p. 436).

Already with Markowitz (1959, pp. 96-101) the possibility of simplifying the necessary input computations by using a single-index model (SIM) was mentioned. The total number of inputs to Markowitz's optimization of N assets can be expressed as N(N+3)/2 (Elton, et al., 2014, p. 128). For 50 and 100 assets, this amounts to 1,325 and 5,150 estimates respectively. Estimates can be found by three main methods (King, 1966, p. 165). The first is the traditional estimator, namely the sample covariance matrix. Using the historical data inevitably assumes that history will repeat itself. Therefore, this can be called the *naïve method*. Besides, the method contains vast amounts of compounded noise (Jobson & Korkie, 1980). The second is a subjective estimation by an expert analyst. This is not only a large amount of data for analysts to predict, the correlation between assets are also difficult to vindicate. Third and finally, there is the derived estimation by modelling the covariance structure discussed in the following paragraphs. Developing such a model specifically for shipping investments is the exact motivation for this paper, cf. chapter 3. Methods.

The form of the CAPM is precisely that of a SIM. The framework rests on the assumption that assets are only related through common responses to the underlying index, which provides the model with a desirable quality. Sharpe (1963) first introduced such a framework intended to model the covariance structure, thereby simplifying the portfolio selection. The vast number of inputs for the optimization procedure were drastically reduced as the covariance between assets is given by the product of asset betas and the common index's variance (Elton, et al., 2014, p. 133). This totals 3N+2 estimates for N assets and for the 50 and 100 assets discussed above estimates are now 152 (1,325) and 302 (5,150). Given that the assumptions of the SIM holds, this clearly simplifies the portfolio optimization procedure. However, the question is at what cost?

Early work from Benjamin King (1966) studied 63 stocks on New York Stock Exchange (NYSE) over the period 1927-1960, and was able to show that the market-index model on average could explain 50% of stock variance. Adding further explanatory industry factors to the model, King managed to explain on average a further 10% of common stock movement. The study indicated that the structure imposed by the SIM might not be the best to replicate reality.

Since King's study, attempts have been made to implement multi-factor models to account for several common factors on asset pricing and the covariance structure to bring theory closer to reality. Especially the models of Fama & French (1993) and Chen, Roll and Ross (1986) have received considerable attention as descriptive multi-factor models. The latter type of fundamental risk model has gained support and been employed by the financial industry, e.g. Salomon Brothers seven-factor model (Sorensen, et al., 1989). The models of Chen et al. and Sorensen et al. focus on utilizing macroeconomic factors to explain stock returns. Consequently, they will provide us with a starting point for modeling the covariance structure among shipping assets.

Lately, multi-factor models have been the industry standard (Ledoit & Wolf, 2003), but the question remains. What is the ideal amount of structure to impose on our model? Moreover, what factors should be included? It becomes a discussion regarding the cost of simplification. Ledoit (2000) explains the dilemma; the two extremes are the single-factor model and the sample covariance matrix, which essentially resembles an N-factor model for the N number of assets studied. By adding factors to the single-factor framework, we lose structure and therefore increase error with hopes of adding information. Elton et al. (2014, pp. 168-169) states that "simple seems to be better than complex" when constructing models and adding more factors "tend to pick up more random noise than information". Furthermore, when deciding upon which factors to include, there is no universal consensus and the best factors will vary for different sets of data. Clearly, the exercise of choosing factors for a given data set is, in the words of Ledoit, an art. This is why multi-factor modeling in many ways becomes an exercise in "fishing for factors", an issue we seek to solve later on in our paper.

Previous studies of multiple risk factors in shipping have concentrated on the returns of shipping stocks. Articles of Grammenos & Arkoulis (2002) and Drobetz et al. (2010) both find relatively low market betas in the single-index model of the CAPM framework. While the variance of shipping stocks generally is greater than the market, the model shows defensive betas of below unity signalizing large proportions of risk not explained by the market model. They propose multi-factor models to reflect the true risks in shipping stocks and the full value of the industry's diversification potential to investors. As stated by Drobetz et al., improving the SIM is important in three ways. First, a better understanding of risk factors can improve fundamental analysis concerning the economic determinants of return volatility. Secondly, a multi-factor model enhances the factor risk profile of each investment, which is particularly useful for diversification purposes. Finally, the differences in returns are better explained by a multi-factor model, enabling valuations that are more accurate.

Being relatively close to our paper's modeling approach, the results from Drobetz et al. (2010) are astonishingly poor. The explanatory power of multi-factor models on shipping stock returns ranges from 25-38%. In addition, Westgaard et al. (2007) specifically study the tanker market and propose a multifactor framework for explaining stock returns. The final model explains 27% of return variation, proving difficulties in determining good explanatory factors. Finally, Kavussanos et al. (2002) study macroeconomic factor models and their explanatory power on an industry level, not exclusive to shipping. In our paper, the works of Drobetz et al. and Westgaard et al. will provide us with a benchmark regarding explanatory power of our models. It is however, important to note that both Drobetz et al. and Westgaard et al. study stock returns, slightly different from our paper. Last but not least, Kavussanos & Marcoulis (2005) draw the conclusion that both micro (i.e. firm level) and macroeconomic factors contribute to explain return on shipping stocks. This is perhaps even more true for individual investors in direct investments, as for instance the degree of leverage in asset play is crucial to the potential profit or loss. However, in this paper we focus entirely on the asset itself and exclude such specific investment decisions of investors, e.g. debt financing etc.

Concerning shipping as an alternative investment, a paper by Grelck et al. (2009) studies the gains from investing in shipping stocks in a portfolio compared to more traditional stocks and bonds. According to the authors, shipping stocks could add attractive risk/return properties, increasing the efficiency of the portfolio. Since the study is done on a stock-level, it is not directly comparable. However, similar use of efficient portfolios and comparison of portfolios will benefit our paper.

Whilst the market structure of various segments has been discussed introductorily, an assumption regarding investing in vessels need to be commented. A traditional manner of investing in vessels has been to form limited partnerships to finance shipbuilding and operations. Especially the Norwegian *Kommandittselskap* (KS) coastline and the German equivalent *Kommanditgesellschaft* (KG) have been prominent in shipping. These organizations, among others, provide the necessary divisibility of investments for practical application of portfolio theory, i.e. the option to invest in smaller portions of vessels.

3. Methods

As mentioned during the review of Markowitz (1952; 1959), the optimization procedure requires expectations of future returns, variance and covariance. The procedure itself is widely accepted by practitioners and academics, provided investors seek to maximize expected utility of their wealth and show risk-aversion. Additionally, the returns should be reasonably symmetrically distributed to provide valid results (Bertsimas, et al., 2004), cf. appendix 5. Theoretically, the uncertainty regarding optimization results is only connected to the estimation of necessary inputs.

The method consists of constructing portfolios of risky assets with the following characteristics for portfolio return, variance and standard deviation:

$$E(r_P) = \sum_{i=1}^{K} X_i E(r_i) \tag{1}$$

$$\sigma_P^2 = \sum_{i=1}^{K} (X_i^2 \sigma_i^2) + \sum_{i=1}^{K} \sum_{\substack{j=1 \\ j \neq i}}^{K} (X_i X_j \sigma_{ij}) \tag{2}$$

$$\sigma_P = \sqrt{\sigma_P^2} \tag{3}$$

The *E(r)* denotes expected return of assets *i* (and *j*) or portfolio *P*. *X* are the weights of the *K* assets in the portfolio. σ^2 is the variance risk measure and σ the standard deviation. The importance of the covariance term σ_{ij} in (2) becomes apparent in the process of minimizing portfolio risk.

Generally, there are no limits to short selling, i.e. negative positions in assets (X_i). This might enable favorable hedging opportunities for investors. Real-world investors however, are often faced with regulations regarding short positions, such as the Securities and Exchange Commission (SEC) regulations of the US. In the case of real investments, feasible short positions are less clear than in stock and derivatives markets. Although such derivatives have emerged for real asset markets as well, we assume no short sales throughout our paper. Furthermore, the usual assumption of efficient markets (Fama, 1970) states that the total composition of the market should resemble the *true market portfolio*. Accordingly, the net effects of short selling are cancelled out on aggregate (Benninga, 2008, p. 329), making negative positions unnecessary to optimize the portfolio.

The market portfolio is derived under the assumptions first presented in Sharpe's CAPM of unlimited borrowing and lending at a constant *pure interest rate*. This resembles the risk-free rate r_f below and otherwise in our paper, which is further discussed under chapter 4. Data. A neat procedure of finding this portfolio is maximizing the Sharpe ratio:

$$S = \frac{E(r_P) - r_f}{\sigma_P} \tag{4}$$

Maximizing the Sharpe ratio ensures that the investor receives the greatest return per unit of risk in the mean-variance space, in line with standard rationality assumptions. Using this procedure, we are able to give recommendations of optimal portfolios for investors.

3.1 Modelling returns on shipping investments

To determine the portfolio return of equation (1), it is necessary to find estimates on expected returns for the potential assets. In general, returns on an investment consist of both capital appreciation and net revenues, i.e. dividends. Vessel returns will therefore depend on earnings less operating expenses (OPEX) in addition to resale value at the end of the holding period. For period *t*, vessel *i*'s return is calculated by the following formula:

$$r_{i,t} = \frac{\left(Sale\ price_{i,t+1} * (1-\alpha)\right) + Freight\ earnings_{i,t} - OPEX_{i,t}}{Purchase\ price_{i,t}} - 1 \tag{5}$$

Equation (5) illustrates the return on shipping investments as a function of vessel net earnings and capital appreciation. α is an adjustment factor to reflect wear and tear on the vessel. The formula clarifies what data is necessary for our paper, which is discussed and quantified in the following chapter 4.

3.2 Modelling the covariance structure

This paper intends to explore various methods of deriving inputs of the correlation between assets. Our main focus is on deriving covariance estimates between vessels. We will obtain these estimates using the following methods. However, it is first necessary to emphasize that we do not seek to find valid estimates of future expected returns per se. Such estimates can be easily accounted for by practitioners in our models. The chosen inputs for returns are therefore simply the historical mean of the sample, based on period returns determined in equation (5):

$$E(r_i) = \bar{r}_i = \frac{1}{N}\sum_{t=1}^{N} r_{i,t} \tag{6}$$

For N observations of return r for vessel i at time t. This general method of calculation is used throughout this paper for return inputs in the models below.

3.2.1 Naïve estimation

Naïve estimation is equivalent to taking the historical average of the data. As this is the most comprehensive approach to modelling returns, the method contains a lot of “noise”. However, an upside to this approach is the intuitive calculation of standard statistical sample variance and covariance’s. The inputs to this naïve approach to portfolio optimization is calculated in the following manner:

$$\sigma_i^2 = \frac{1}{N-1}\sum_{t=1}^{N}(r_{i,t} - \bar{r}_i) \tag{7}$$

$$\sigma_{ij} = \frac{1}{N-1}\sum_{t=1}^{N}(r_{i,t} - \bar{r}_i)(r_{j,t} - \bar{r}_j) \tag{8}$$

Using the historic equally weighted average implicitly assume a constant volatility both throughout the period at hand *and* ahead. This is an important assumption for the variance and covariance estimates. Other approaches would be to use moving averages, exponentially moving averages (EWMA) or GARCH models. Such models make up for time varying volatility in different ways. This could yield interesting insight, as there is some evidence towards volatility clustering in freight markets (Kavussanos, 1996a). However, as Suganuma (2000) concludes, it is difficult to find a consistently outperforming model when considering the abovementioned options. This is why practitioners often prefer the use of such intuitive constant volatility models, i.e. sample mean.

Interpreting the covariance measure in itself does not yield information as to the scope of co-movement between assets. To quantify the measure, practitioners instead make use of the correlation coefficient:

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \tag{9}$$

The coefficient ρ_{ij} indicates the proportion of co-movement (σ_{ij}) of the total variation ($\sigma_i\sigma_j$) for assets *i* and *j*. As opposed to the incomparable magnitudes of covariance measures, this number presents the proportion as an absolute value between 1 and 0, and is either negative or positive depending on the relation of the variables, i.e. co-movement in the same or opposite directions. This measure makes the figures easily understandable and comparable and will be used for displaying our data in the appendix.

Note that the method in equation (7) is used throughout this paper when estimation of variance has been necessary, for instance as the input of estimated market variance below. This is analogous to the use of equation (6) for returns.

3.2.2 Single-factor model

The single-factor approach utilizes a simple market model by regressing returns of each vessel on a market index. The model assumes that the rate of return on any given asset can be explained by its correlation with the market portfolio, i.e. the market index. Compared to the naïve method above, the single-factor model explains returns only by the exposure to one factor, usually the stock market. The greatest challenge for this particular model is deciding upon which index is most appropriate.

The general market model can explain the return for each vessel using the following equation:

$$r_i = \alpha_i + \beta_i r_m + \varepsilon_i \tag{10}$$

Graphically, the α is the intercept of the linear function above origo when the market return r_m is zero. Adjusted for the risk-free rate, α represents an excess return on the market premium in line with CAPM (cf. Sharpe (1964)). β is the sensitivity of asset return on the expected market return, i.e. the model's linear coefficient. The final term ε represents the unexplained error.

Using the model in (10), we are able to apply statistical software to the vessel returns and, with ordinary least squares (OLS) regressions, determine the asset's betas. By certain assumptions (Elton, et al., 2014, p. 130), we can model the covariance between any two assets (*i* and *j*):

$$\sigma_{ij} = \beta_i \beta_j \sigma_m^2 \tag{11}$$

This simplifies the calculation of the covariance matrix as we assume that the only reason assets move together is a common response to market movements. The necessary inputs are the betas of assets *i* and *j* besides the market variance estimated according to equation (7).

An important drawback with the single-factor model is the assumption that the covariance between the asset and the market remains constant throughout time. There have been several studies towards the fact that sensitivity to the market factor, popularly called beta (β), is time varying (e.g. Bollerslev et al. (1988)). This implicitly affects our mean-variance optimization, as the covariance between assets is a product of their betas. When estimating market exposure, practitioners often restrict the time series employed for such reasons. In our case, this becomes a trade-off due to the cyclicality of shipping. By only using a short time series, we risk omitting business cycle fluctuations. On the other hand, we also run the risk of producing a misrepresentative estimate for future market exposure.

3.2.3 Multi-factor model

We can easily expand the framework of the single-factor model above to include more explanatory factors. This may be necessary if there seems to be additional sources of correlation than just a single market factor (cf. King (1966) and Fama & French (1993)). The general equation for asset return in an L-factor model is as follows:

$$r_i = a_i + \sum_{w=1}^{L} b_{i,w} F_w + \varepsilon_i \tag{12}$$

The single-factor model ignores any relation between assets other than the market factor. In the multi-factor framework, more of the residual correlation is explained by including additional factors *F* where factor sensitivities are represented by *b*. Still there will be an error term *ε* that is not explained and assumed to be uncorrelated between assets.

Examining the model, two questions arise. How many factors to include, and which factors are most appropriate. This will be the main topic during our discussion under chapter 4. Data and reflects the decision on market index for the single-factor model.

Once the factor selection is completed, we use the model of equation (12) with factors for an OLS regression of vessel returns. The factor betas can then be used in the following. As with single-factor models, studying a certain period of data will affect our loadings on factors, i.e. betas. One could expect that the exposure towards different factors is time varying. The discussion on appropriate length of period for analysis is therefore analogous to the discussion for single-factor models.

Correlation and covariance properties of multi-index models

Under similar assumptions as for the single-factor framework, and if the factor variables in our multi-factor model are perfectly uncorrelated, the process of calculating an optimal portfolio is vastly simplified (Elton, et al., 2014, p. 157). Ensuring that our factors are uncorrelated (i.e. orthogonal), the exercise of calculating the covariance between assets can be illustrated by the following equation:

$$\sigma_{i,j} = \sum_{w=1}^{L} b_{i,w} b_{j,w} \sigma_{F_w}^2 \tag{13}$$

Covariance in the single-factor model (11) is a special case of the general equation in (13) when the only factor *F* is the market return r_m. In general, the necessary inputs to covariance calculation are each asset's sensitivity *b* to each factor and the factor's variance σ^2.

Orthogonalizing the factors is important due to the common risk premium among factors (Klein & Chow, 2010). By following this procedure, we deduce a more precise estimate of the covariance among factors, eliminating the noise caused by multicollinearity. However, the process of regression orthogonalization is easier said than done. One must order the variables from most to least important. Naturally, such a process leads to a "selection bias" in deciding the order of importance (Klein & Chow, 2010). In our paper, we have drawn on mathematical properties to calculate uncorrelated, orthogonalized, covariance estimates from the multi-factor model.

The covariance of a multi-factor model, written on matrix form, is described below (Hsu, 2015).

General model in equation (12):

$$r_i = a_i + \sum_{w=1}^{L} b_{i,w} F_w + \varepsilon_i$$

Assuming:

$$E[\varepsilon_i] = 0 \qquad E[\varepsilon_i F_w] = 0 \qquad E[\varepsilon_i \varepsilon_j] = 0$$

The covariance is given by:

$$\hat{\Sigma} = \beta \Sigma_F \beta' + D_\varepsilon \tag{14}$$

β is the K-vessel by L-factor matrix of betas, Σ_F the L by L covariance matrix estimate of factors and D_ε the diagonal matrix of $\sigma_{\varepsilon,i}^2$. The major benefit of equation (14) is that it allows us to use the factor loadings obtained without orthogonalizing the variables prior to regressing, in contrast to the general model in equation (13). Additionally, it is a straightforward procedure for obtaining “correct” estimates of covariance for multi-factor models.

3.3 Optimizing the portfolios

Using computational software in STATA and Excel , each method above results in estimates of covariance between assets. These are organized in variance-covariance matrices for inputs to the portfolio optimization procedure of minimizing portfolio risk. An overview of the matrices are provided in the appendix, both for the covariances and correlation coefficients.

Mathematically, our optimization problem is expressed in the following way:

$$Minimize: \quad \sigma_P^2 = \sum_{i=1}^{K}(X_i^2\sigma_i^2) + \sum_{i=1}^{K}\sum_{\substack{j=1 \\ j\neq i}}^{K}(X_i X_j \sigma_{ij})$$

$$Subject\ to: \quad \sum_{i=1}^{K} X_i E(r_i) = r_p^*$$

$$\sum_{i=1}^{K} X_i = 1$$

$$X_i \geq 0 \ \ for \ \ i \in 1, \dots, K$$

We minimize portfolio variance from equation (2), for a given set of returns r_P* and constrain the portfolio weights to sum to unity, not allowing negative weights (i.e. short positions).

The specific programming of subroutines using *Visual Basic for Applications* (VBA) in excel is provided in appendix 10. Such programming easily executes the tedious iterative calculations necessary to minimize the portfolio variance at each given level of portfolio return.

4. Data

4.1 Vessel return data

Referring to equation (5), this section will discuss the required data used as the basis for our analysis. The frequency of observations is connected to a predetermined holding period of one month to best reflect the risks of shipping investments, as mentioned in chapter 2 above. Firstly, we discuss the selection of various vessels and their prices, including capital depreciation, i.e. α from equation (5). Thereafter, the components of earnings and OPEX are highlighted and determined.

4.1.1 Vessels

There is a need for transparency in order to analyze the different shipping segments. Consequently, earlier research has been restricted to the main bulk trades, i.e. wet and dry, besides container trade. Other segments of interest might be the smaller specialized trades, e.g. LNG, or certain offshore services, e.g. drill ships. To gain a thorough understanding of the shipping industry as a whole, one could argue the need to include most segments. However, more specialized services are clearly less flexible with regards to operations and ships tend to be on long-term contracts, even specifically constructed. As mentioned under the literature review, such characteristics represent an imperfection from the assumption of perfect markets and might undermine our results. Finding sufficient data on specialized and minor trades also poses an issue. Furthermore, including the three most liquid shipping segments will cover approximately 85% of registered tonnage (Clarkson Research Services Ltd., 2015). This is in line with the conclusion of Lorange (2009, p. 16), stating that a comprehensive analysis of bulk, tanker and container shipping would cover the majority of the industry. In light of this, we have limited this paper to examine the following main segments.

Wet Bulk

Selected wet bulk data, i.e. tanker market, consist of the following vessels; *VLCC*, *Suezmax*, *Aframax* and *Handymax* clean products tanker. Note that only the Handymax tanker is consistent with regards to vessel assumptions (i.e. size and hull design) in the period of study. It is important to be consistent with the standard vessel used to calculate returns and secondhand value. Besides the issue of vessels increasing in size, tankers also evolve from single-hull to double-hull constructions following the IMO MARPOL convention (International Martime Organization, 2015).

Since we are studying returns, the imperfect overlap of vessel standards regarding hull design is less of an issue. The solution is consistency when calculating returns for a vessel and its corresponding earnings, purchase price, sales price and deprecation. Differences in size is accounted for by adjusting data for each vessel to a per deadweight tonne basis. This is equally important for the other vessel segments in this study. For instance, we assume the vessel price per DWT obtained within a segment (e.g. Capesize, VLCC, etc.) is constant across small differences in size, i.e. DWT. By following this procedure, we can handle the issue of time series overlap of different sized vessels and varying standard vessel assumptions within each segment.

Dry Bulk

For dry bulk, data includes *Capesize*, *Panamax* and *Handymax* vessels. As with tankers, there are issues regarding vessel standards. A natural increase of size within vessel classes has caused a shift in the standard vessel assumptions over time, cf. table 2. As with tankers, prices and returns are calculated on a per DWT basis to adjust for this issue.

Container

The container market, as mentioned in chapter 2. Literature review, does not display the same features as dry bulk and wet bulk markets concerning information, liquidity and market structure. Despite these flaws and discrepancies, we believe including containers in our study is important for the following reasons. Firstly, the container trade has increased its market share in shipping over the past decade, making it an important segment to include in our analysis. Additionally, it is likely to be a less volatile industry due to its differing market structure, the predetermined demand of transport and the vast range of cargo carried (Strandenes, 2012). By predetermined, we imply that container freight is less volatile with regards to demand than dry bulk and wet bulk trade (Lorange & Norman, 1973). This aspect should yield interesting results when included in the portfolio optimization procedure.

Chosen vessels to include are *Panamax* and *Handymax* sized container vessels, which are approximately 3400 twenty-foot equivalent unit (TEU) and 1050 TEU respectively. As opposed to the bulk segments above, container freight is based on TEU size measure and not DWT. Our calculations have therefore been made on a per TEU basis.

Wet bulk		**Dry bulk**		**Container**	
Type	Size (DWT)	Type	Size (DWT)	Type	Size (TEU)
VLCC	310	Capesize	180	Panamax	3400
Suezmax	160	Panamax	76	Handysize	1050
Aframax	105	Handymax	56		
Handymax	37				

Table 3 - Standard vessels (DWT in thousand)

Data availability and "selection bias"

The initial starting point for gathering vessel prices is the secondhand price of five-year old vessels. For the most part, we were able to retrieve data from 1990 until today. Unfortunately, Clarkson's does not report on the container industry until late 1996. This poses a problem concerning data selection and missing data. Although there exists ways of replacing monotone missing data in time series (Horton & Kleinman, 2007), it is always a discussion whether such a procedure does more harm than good. Truncating (i.e. shortening) data as a response to missingness could lead to biased results. This is a reoccurring problem in applied finance (Stambaugh, 1997).

The Markowitz portfolio selection model is sensitive to model inputs, particularly covariance estimates. Truncating returns implies discarding crucial information concerning correlational patterns among assets (Peterson & Grier, 2006). However, recent methods to solve this issue place strict assumptions regarding the correlation among assets. For instance, it could be useful to use the history of one stock to backfill missing observations of another, assuming they are exposed to the same underlying factors. In our case, we fear this is too strong of an assumption to make. Even though all seaborne trade is affected by global economic growth, segment specific factors may vary. This view on shipping and diversification is shared by Kavussanos (2010). If there were no idiosyncratic factors, diversification would be impossible. Preforming a backfilling procedure to eliminate missingness could yield spurious results due to the omitted idiosyncratic factors among vessels. Overall, it becomes a dilemma of "out of the fire, into the frying pan".

For the purpose of compatibility, we have chosen to restrict our time series from October 1996 to December 2014 for all vessels. Among finance practitioners, the rule of thumb is to include five to seven years of monthly data in time series analysis. Therefore, truncating returns from 25 to just over 18 years of data should not be critical in our study. Additionally, 18 years should be an appropriate length to capture several shipping cycles. Referring to the abovementioned discussion of single- and multi-factor models, period length has an implicit effect on the estimated betas. Given the fact that betas seem to be time varying, one needs to find the right balance both with regards to capturing business cycles and producing valid estimates. However, we believe the period of 18 years to be an adequate starting point. Furthermore, we will investigate the effects of shortening the dataset and explore sub-periods in our following analysis.

Referring to the literature review on ship prices, our study entails the possibility of a bias in reported sales prices, i.e. "sticky prices". This bias is particularly inherent during the financial crisis of 2008. The "stickiness", or lack of accurate prices, is mentioned by Clarkson as a cautionary note to its time series. At times of great distress, the deals done in the market are often below reported prices, leading to a potential upward bias.

To further expand the scope of our paper, we aim to include vessels of different ages in our analysis. Hopefully, this can capture the different risk-reward profiles connected to investors decision to invest in an expensive modern fleet or less expensive dated vessels, i.e. “sweating the assets”. Clarkson’s database includes secondhand prices for various vessel types at five-year intervals. However, less data is provided the older the vessels and the time series’ standard vessels deviate substantially when increasing vessel age. To produce valid time series for every asset over the entire period, we have deemed it sufficient to include only five- and ten-year old vessels. Expanding the analysis any further, would in our opinion compromise the validity of our results when largely based on adjusted estimates. Including two different ages should still provide valuable insight for our analysis.

We still encounter some missing data regarding the value of ten-year old wet bulk vessels. Post November 2001, Clarkson’s report values on both five- and ten-year old vessels. In order to backfill values prior to this, we have used the average ratio of five-year to ten-year values. As there is some co-integration regarding the value of vessels and age, this should be an appropriate procedure. Assuming there is no “two-tier” market with regards to vessel quality (Strandenes, 1999), the only difference between a five- and ten-year old vessels should be the difference in the length of the discounted future cash flows. In theory, the current value of a vessel (*P*) can be expressed by the expected future price, expected future operating earnings (π_{t+1}) and expected future rate of return (R_{t+1}) (Alizadeh & Nomikos, 2007):

$$P_t = \left(\frac{E(P_{t+1}) + E(\pi_{t+1})}{1 + E(R_{t+1})}\right) \tag{15}$$

However, a potential pitfall of using the ratio between five- and ten-year old is the difference in standard vessel assumptions between two time series. Considering the fact that vessel value per DWT increases as size decreases, calculating the abovementioned ratio when the underlying vessel sizes do not match could be a problem. On the other hand, we find that this procedure yields more accurate in-sample results than both simple linear depreciation and a regression-based approach. The results from alternative approaches were particularly poor in depressed markets, as the value of a ten-year old vessel at times became negative. Intuitively, this cannot be the case as the scrap value in the corresponding period should be a lower bound for the value of a vessel. We believe the ratio is better suited to capture the interdependency mentioned above regarding age. Notice that this procedure was only necessary for wet bulk vessels other than Handymax for the period prior to November 2001.

Capital depreciation

According to Wijnolst & Wergeland (1996) the capital expenses depend on a number of factors. In this paper, we do not include any capital costs other than depreciation of vessel value. According to the discussion of chapter 2, the average scrapping age for a bulker can vary from 15 to 30 years depending on market sentiment. In our paper, depreciation is done in equal proportions over the lifetime of the vessel and allocated monthly. We assume the lifetime of a vessel to be 25 years, a normal assumption for ship depreciation. Consequently, we will employ an appropriate depreciation factor of 4 % p.a. This factor is in line with estimates found in the literature and earlier studies, cf. Stopford (2009, p. 237) and Magirou, et al. (1997, p. 30).

4.1.2 Earnings

The income generated from operating a vessel is calculated using monthly average spot earnings quoted in $/day for all vessels reported by Clarkson's Shipping Intelligence Network. In contrast to earlier studies, monthly earnings allow a more flexible investment strategy. Even though one month may be too short of a holding period in reality, at least it enables the option of flexibility. More importantly, in contrast to extant literature, monthly data should better reflect the true variation. Earnings are calculated by subtracting bunker costs, port charges, canal dues and brokerage commissions from total voyage revenue (Clarksons Research Services Limited, 2015). For consistency's sake, it is necessary to account for changes in the standard vessel assumptions. This is done in a similar fashion as for vessel prices by operating on per DWT basis.

As with vessel prices, we have encountered some difficulty obtaining data for all vessels. Due to lack of spot freight observations for container ships, we have based our calculations on average 6-12 month time-charter figures. Being a moving average and relatively short term TCs, this figure should be reasonably representative for expected theoretical spot rates. Considering the nature of liner trade discussed in chapter 2, operating in pure spot markets is rather uncommon. Thus, shorter TCs might be the appropriate rate for an investor in the segment. We believe such assumptions to be well founded and appropriate for our analysis.

In our calculations, we have assumed that each vessel is on-hire 30 days per month. This assumption is very close to assuming full employment, which might be unlikely especially in depressed markets. Furthermore, we do not model any particular cargo utilization factor. This is mainly due to the different stowage factors for each trade depending on commodity carried. Stowage factor is perhaps more important for dry than wet bulk, as it varies a lot in dry bulk shipping. For the cost components described below, we have assumed that each month lasts for 30 days on average, i.e. 360 days a year.

4.1.3 Costs

Voyage costs

As mentioned, the quoted earnings discussed above are adjusted for typical voyage costs. For a detailed explanation on the calculation of voyage costs, revise Clarkson's Shipping Intelligence Weekly: Sources and Methods (Clarksons Research Services Limited, 2015). When calculating returns from shipping investments, we have chosen to omit other voyage costs than those stipulated by Clarkson's. Besides, commissions, bunker and port costs account for the majority of voyage specific costs (Stopford, 2009).

Operational expenses

In maritime economics research, operational expenses (OPEX) is often assumed exogenous or independent across time. Little empirical work has been done on OPEX alone. Researchers are often more interested in freight rates or vessel prices as such. According to Koehn (2008), the inattention to OPEX may be due to the lack of sufficient data and relatively little variation over time. However, as Koehn states, a different approach to OPEX may significantly change the results of previous research, as it is an important component to the returns from shipping operations.

Adland & Strandenes (2006) argue that OPEX changes with market sentiment. When freight rates are high, maintenance is delayed to increase supply of freight. In this paper, freight rate-variant OPEX is not accounted for as such. However, including some variability in OPEX is important, as the marginal supply of vessels is determined by the industry cost structure (Koopmans, 1939).

It is important to bear in mind that the components of OPEX do not increase proportionately with vessel size (Strandenes, 2014), yielding limited insight from comparison across vessels and segments. In line with Adland (2002), we assume that OPEX is independent of time or spot charter. This also seems to be the conclusion of empirical studies on the subject (Koehn, 2008). In reality though, time charter may in fact lead to higher OPEX, i.e. more maintenance (Adland, 2002).

Illustrated by Stopford (2009), the cost profile changes with age, all else equal. When comparing direct cash cost, a five-year old vessel has lower operational expenses than a 15- or 20-year old ship. However, the proportional development of OPEX to total costs is exponentially diminishing. After an initial delivery period of 2 to 3 years, the shipowner must start spending money on maintenance etc. Near scrapping age, that is in excess of 20 years, the vessel owner may decide not to spend any additional money on repairs and maintenance. Being near scrapping, the shipowner may simply run the vessel as long as it makes a profit. If freight rates suddenly increase to a point of exceeding the cost of "catching up" the neglected maintenance, the shipowner may start spending money again. A natural response to expectations of increased economic lifetime for the vessel (Koehn, 2008).

The findings of Koehn underline that OPEX increases in a non-linear way with a diminishing effect of age. This is not unexpected, as this is in line with maritime economic theory. However, an important result from Koehn is that between the ages of 4 and 20, the OPEX increase seems to be linearly constant. This allows us to incorporate the difference between a five- and ten-year old vessel assuming a constant relationship. Unfortunately, Koehn does not provide any accurate absolute figures on OPEX development, only relative percentages. Based on a case study by Stopford (2009), we assume that a five-year old vessel has 20% less operational expenses than a ten-year old vessel. Resting on Koehn's research, we assume that this relationship is valid among all our included vessels and vessel segments, i.e. container, dry bulk and wet bulk.

OPEX estimates are extracted from a report by Drewry Maritime Research (2012), supplied by Marsoft International AS (cf. appendix 3 and 4). Even though this may be a rough estimate, it gives a good indication on the overall cost level of different vessels. In addition, it provides considerably more meaningful figures than a general cost increase. From 2008 – 2012 exact figures are reported by Drewry, and by looking at the historical development of total OPEX, we have been able to estimate costs prior to 2008. For the two following years (2013 – 2014), Drewry's forecasts from 2012 have been used. The numbers are based on average OPEX for the period, during which the average vessel is in excess of nine years. Therefore, the numbers resemble that of ten-year old vessels in our analysis. For five-year old vessels an adjustment of 20% has been used, cf. the previous paragraph. When studying the report, it is worth noticing that Drewry does not report costs conditional on age, only size and segment. Hence, the need to make a simplified assumption regarding age as discussed above.

Due to the underlying components of OPEX, a general cost increase would be too rough an estimate. Bearing in mind that OPEX includes staffing costs, stores & lubricants, repairs & maintenance, insurance and other general costs, one might expect the different cost components to evolve slightly different, albeit in an upwards trending line. Our only implicit assumption when using the general cost development prior to 2008 is that this development is common across vessel types.

4.2 Model inputs

Having decided on vessel data for analysis, we need to find appropriate data for the parameters in our framework. Mainly, the selection will involve deciding among various indexes and factors for our single- and multi-factor models.

4.2.1 Risk-free rate

Firstly, to optimize the Sharpe ratio in equation (4), we need to find a general estimate of the risk-free rate. Sharpe's theories of asset pricing include a true risk-free asset with a corresponding pure risk-free rate. However, in practice such an investment is difficult to find. The investment universe covers a wide specter of possible investments, from very risky to *nearly* risk-free.

Usual estimates of the risk-free rate are rates on solid government securities and interbank rates, specifically London Interbank Offered Rate (LIBOR) within shipping. The main issue with these rates is the included risk premium to compensate investors for risk, however minute. This risk premium is mainly due to an inevitable default risk. Excluding governments in financial distress, the default risk in interbank markets is expected to be higher than for government securities. According to this reasoning, we have chosen to use US Treasury securities as an estimate of risk-free rate. Kavussanos & Marcoulis (2005), among others, use 3-month US Treasury Bills in their analysis of shipping and transport industries.

Besides deciding on a risk-free investment, it is important that the duration of the security matches the theoretical investment horizon of one month in this paper. Therefore, we have chosen to use the 1-month Treasury bill provided by The U.S. Department of the Treasury. Data is collected from the Macrobond database.

4.2.2 Stock market index

As discussed in chapter 3.2.2 *Single-factor model*, it is necessary to decide on a market index for the model. Considering the international nature of shipping, a general index seems appropriate. Shipping services have no apparent deep-rooted connection to specific geographic regions. Over time, demand patterns will change and the services are offered where they are needed. Consequently, the chosen index should have similar qualities. Following what seems to be a conventional choice of index, we propose using the MSCI All World (MSCIAW) index by Morgan Stanley. This is in line with previous research by Grammenos & Arkoulis (2002) and Kavussanos & Marcoulis (2005). The MSCIAW index provides an equity-weighted proxy for the entire world's stock markets. The composition of the index is particularly desirable as it covers approximately 1600 securities in 22 countries, excluding stocks with little liquidity. It should reflect economic activity as a whole and capture the mentioned shifts in trading patterns and demand.

4.2.3 Factors for the multi-factor model

Our approach to factor selection is based on economic rationale regarding fundamental supply and demand of underlying commodities. Models of this manner are known as fundamental macroeconomic factor models, opposed to microeconomic models concerned with firm-specific factors (e.g. debt ratios).

The perspective of this paper is aimed towards the real investments in shipping. Previous studies on macroeconomic multi-factor models in shipping, including Grammenos & Arkoulis (2002), Drobetz et al. (2010) and Westgaard et al. (2007), have concentrated on explaining stock returns. Such deviating perspectives may culminate in deferring factor selection for the final model. At the operational firm level (which is reflected in stock prices), one would expect factors more closely related to operations and microeconomics to be of higher significance. This is important to bear in mind during later comparisons.

It is also important to be certain of causality when including variables in our model. Essentially, the independent variable x must affect y, and not the other way around. Including for instance orderbook activity would yield causality issues, as it is unclear in which direction causality runs. Do freight rates affect the orderbook, or does the orderbook affect freight rates? Both are likely to be true, cf. chapter 2.4.1.

Furthermore, it would be desirable for the included factors to have different, optimally inverse, correlations with vessel returns across segments. In order to obtain such variables, we studied commodities with plausible substitute ability or variables with implied dependency on such commodities. By studying factors that affect the demand for a commodity, preferably a substitutable commodity, one might expect it to be significant across segments. This is particularly interesting for a commodity that primarily is transported in dry bulk, while its substitute is transported in wet bulk. Such a factor could possibly segregate the shipping sectors in our model. A good example of such commodities are oil and coal, both being suitable for heating purposes with a common seasonal trend. Additionally, we searched for common variables capturing general economic trends, as it affects all seaborne trade.

The process of specifying a final model is a general-to-specific model specification procedure, i.e. a kitchen-sink approach. Since we do not know in advance which factors that are significant in explaining vessel returns, we include several plausible explanatory variables. Starting with the most general model, we eliminate variables with little or no explanatory power, one at a time, until we arrive at the most parsimonious model possible. A reasonable argument for this model process is to arrive at a final set of variables, where the final solution is possible to replicate. If one should try to follow a specific-to-general approach, i.e. adding one variable at a time, there would be a bias regarding which variable to add next. Consequently, one might end up with several models depending on which variable was added and in what order. Thus, the general-to-specific approach is deemed logically superior to the specific-to-general approach.

A description of the statistical methods and characteristics of our factors can be found in appendix 1.

4.2.4 Model variables

The decision to apply theory to monthly variables has its limitations when searching for fundamental macroeconomic factors. Such data is often quoted on a yearly basis, sometimes quarterly, and in retrospect due to the amount of data necessary to process. The opposing data from financial markets does not encounter these issues and is normally readily available. We seek to include variables with full time series for the entire period and have essentially achieved this with the following variables. However, as you will notice in the regressions for the final model in part 5 below, the series are based on 215 observations opposed to 218 for the single-factor model. This is due to lacking observations of some variables for the final three months of 2014 at the time of writing.

The 10 initial variables included in the most general model are listed in the following table:

- Total OECD industrial production growth
- Change in world steel production
- Change in USD exchange rate
- Change in world demand for oil products
- Change in average price of Australian coal
- Change in price (unit value)
- Change in wheat price
- Change in bond spread
- Return on MSCIAW
- Change in gold price
- Change in world oil stock

Table 4 - Variables for the multi-factor model

For our final model, we have chosen to include the five following variables; total OECD industrial production growth (month-on-month) ***(OECDTOT)***, change in average monthly price on Australian Coal ***(dAUSCOAL)***, monthly change in world oil stock ***(dWOS)***, change in world steel production ***(dWSP)*** and change in price (unit value) ***(dPUV)***.

The proceeding section provides the economic reasoning and detailed descriptions for each of the final variables. All data is accessed through the Macrobond database except the OECD industrial production, which can be accessed through OECD Stats website.

Total OECD industrial production growth (month-on-month)

Including OECD industrial production growth as a variable should capture major economic trends and consumption of commodities across the world. Being a worldwide index, it is not influenced by changes in consumption and production patterns the same way as for instance a European production index. As a worldwide indicator on aggregate production, we expect this factor to have statistical significance independent of vessel characteristics, i.e. shipping sector, age and size. Intuitively, an increase in world production would positively affect all trade in general. Therefore, we expect positive correlation and do not expect this variable to differentiate substantially between vessels. Furthermore, the original time series is already stationary being a monthly change index.

Change in average monthly price on Australian coal

For heating and industries' electrical purposes, coal is a substitute for oil. If the price of oil sharply increases relative to coal, consumers might switch to coal power. As households consume more coal, the demand for coal freight increases and oil freight decreases. Consequently, we would expect this variable to affect both dry and wet bulk trade. As coal and oil is transported by different vessels, we suspect the factor to be differently correlated across segments. Specifically, the impact on wet and dry bulk may be inverse. The World Bank provides data on coal prices. To meet the requirements of stationary time series, the variable is constructed as the change in oil price month-on-month, i.e. first difference.

Change in monthly world oil stock

Oil is important, as it is a key driver of economic growth and consumption. The wet shipping trades mainly center on transportation of oil, including crude and refined products. At first thought, there is intuitive meaning to including the price of oil as a variable to our model in line with the discussion of coal price above. However, certain properties connected specifically to oil price obscure the effect on shipping markets.

Firstly, the presence of a cartel pricing mechanism in OPEC, and particularly Saudi Arabia as "swing producer", clouds the insight gained from studying solely oil prices. It seems that the price equilibrium in crude oil markets is determined and controlled by the supply side rather than demand mechanics. This results in market inefficiencies and makes the prevailing market price an insufficient indicator on the underlying market conditions. Especially when considering the shipping doctrine that freight is affected by volume and not prices. Higher prices may reflect either constrictions in OPEC supply, or generally increased demand. This clearly demonstrates that a positive price shift can be caused by increasing or decreasing volumes and thereby freight demand.

Secondly, the explanatory significance of an oil price variable is further clouded since the price itself accounts for an average of 66% of voyage costs in shipping (Stopford, 2009). Bunker costs being directly connected to the oil price. If we still believe the variable to be an indicator of trade volumes despite the arguments above, a positive shift in price resembles increased demand for freight and should be positively correlated to especially wet vessel returns. However, the increased costs in bunkers will likely offset a large part of the positive effect.

The ambiguity of the oil price effect has been commented in earlier research, particularly by Westgaard et al. (2007) and Drobetz et al. (2010). Both papers find oil price statistically insignificant on itself. Since we believe that oil trade is an important factor for shipping, we must incorporate a different variable than oil price. Articles by Ahlsalawi (1998) and more recently Ghouri (2006) explain the role of oil inventories as a supply-demand equalizer. If the demand for oil increases, prices increase and more oil is taken from inventories and offered to the market. In a similar way, when the oil price decreases, inventory managers increase their stock levels in order to reach a point of equilibrium between supply and demand (Westgaard, et al., 2007).

Therefore, including a variable for world oil stock levels will likely be a better estimate on oil market activity than price. The International Energy Association (IEA) is the primary source of observations. As with the variable for coal price, world oil stock levels is constructed as a change variable by using the monthly first difference of the data.

Change in monthly world steel production

The demand for steel in industrialized countries is thought to decline in the future. Reason being the ability to recycle steel. This implies that steel demand has a natural limit, at least production of "new" steel products. Nonetheless, steel is tied to economic activity, as it is an input in construction and growth. By including steel as a variable, we try to gain "exposure" to the increased economic growth of developing markets, such as Brazil, China and India. In addition, small vessels may be more sensitive towards steel, being a minor bulk trade. Some steel products may even affect container freight when shipped as general cargo. In addition, it may also capture some of the demand for iron ore, which often is shipped on large vessels. The World Steel Association registers world steel production figures on a monthly basis. Again, the first difference is applied to create the change variable.

Change in monthly price (unit value)

Price (unit value) (PUV) tracks the development in the price of traded goods, measured as *unit value*. In essence, it captures two main developments. Firstly, it is affected by the general price development of both manufactured goods and primary commodities. This is a development likely to affect all trades and vessels. Secondly, the index depends on the composition of world trade (von der Lippe, 2007). For instance, a switch from cheaper to more expensive products, would lead to the index increasing. This would more often than not demerit the index. However, as an explanatory factor to our model, this is the most interesting aspect of including PUV.

The nature of traded goods has evolved due to the globalization of production processes. Outsourcing of labor-intensive production to developing countries entails several stages of shipping with increasing cargo value. This trade in valuable finished goods accounts for an increasing part of world trade, and especially when considering value, not volume. Thus, we can utilize this to our advantage since we are particularly interested in the composition of traded products with regards to value.

As mentioned earlier, demand for freight in the container or specialized segments increase when the value, and therefore the importance of reliable transport, increases. We would therefore expect this variable to explain container returns, as the products shipped by containers generally are more valuable than bulk products. Furthermore, the minor bulk trades operate further down the value chain handling more valuable goods than large bulk trades. Thus, such an index will likely capture the relative activity between large and minor bulk trades. The data is provided by CPB Netherlands Bureau for Economic Policy Analysis and accessed through Macrobond.

4.2.5 The multi-factor model

From the general equation (12), our final regression model can be written as follows, the lower case *i* represents each vessel and *b* is the sensitivity factor:

$$r_i = a_i + b_{i,1}\boldsymbol{OECDTOT} + b_{i,2}\boldsymbol{dAUSCOAL} + b_{i,3}\boldsymbol{dWOS} + b_{i,4}\boldsymbol{dWSP} + b_{i,5}\boldsymbol{dPUV} \qquad (16)$$

An interesting observation is that our factor selection procedure excludes the stock market index. A usual starting point for the multi-factor model is expanding the single-index framework. Therefore, our model may seem rather unorthodox. However, we are certain that the remaining variables are the most significant for a general model covering all three segments.

The outset for factor selection is explained in above. Following this procedure, the more specific variables prove to work better together in explaining the movements also explained by the stock index. These findings might be due to a form of omitted variable bias, with MSCIAW mainly capturing the effect of other variables (Wooldridge, 2013). A bias of this kind is often a problem when working with single-factor models. The problem arises because we are underspecifying the reality in the model of shipping returns.

5. Empirical results

This section is dedicated to a thorough discussion on our analysis and findings. Firstly, we provide the statistics of our data, laying the foundation for further analysis. Thereafter, we utilize the methods presented in chapter 3 to model vessel returns, before showing the implications of our models. A section is dedicated to discussing the practical validity of our results, before briefly examining the sensitivity of our model and reviewing a short case study to illustrate its practical application.

5.1 Data statistics

By using equation (5) we calculated vessel returns for each vessel presented in table 3. Analyzing the returns for our sample period from 1996 to 2014 yields the following statistics:

		Wet bulk				Dry bulk			Container	
		VLCC	Suezmax	Aframax	Handymax	Capesize	Panamax	Handymax	Panamax	Handysize
5 year old	$\bar{r}_i$	1.14 %	1.37 %	1.13 %	1.10 %	1.61 %	1.13 %	1.55 %	0.33 %	0.51 %
	σ_i^2	0.002073	0.001917	0.002403	0.0028162	0.00529	0.005774	0.004494	0.003967	0.002687
	σ_i	4.553 %	4.378 %	4.902 %	5.307 %	7.273 %	7.599 %	6.704 %	6.298 %	5.184 %
10 year old	$\bar{r}_i$	1.44 %	1.96 %	1.37 %	1.53 %	2.38 %	1.42 %	1.85 %	0.50 %	0.77 %
	σ_i^2	0.003363	0.004998	0.003258	0.003316	0.007383	0.007102	0.006205	0.004654	0.003586
	σ_i	5.799 %	7.069 %	5.708 %	5.758 %	8.593 %	8.428 %	7.877 %	6.822 %	5.988 %

Table 5 - Descriptive statistics

The risk and return for each asset is given by the estimators discussed in chapter 3, namely the sample mean (6), variance (7) and standard deviation (3). Bear in mind that these are monthly statistics and therefore seem rather attractive when compared to for instance stock market returns over the same period. The MSCI All World index, our proxy for the world stock market, earned an average monthly return of 0.46% with standard deviation of 4.67% during the same period. 1-month US Treasury bills show an average monthly return of 0.20%, i.e. our risk-free rate. Another observation is that age certainly seems to be an essential risk factor. For all vessel types, the older vessel is associated with increased risk, i.e. standard deviation. Such differences in risk should theoretically be rewarded by a premium in the returns, which also seems to be the case in our empirical data. The discussions of chapter 2 regarding the cost structure of older vessels can be an explanation for these findings. Although our assumptions imply no off-hire risk, the underlying risk factor would still be present in the secondhand price fluctuations.

To illustrate further, the results in table 5 can be plotted graphically in the mean-variance space. This is done in the figure below and the results show promising features with respect to diversification. In addition to including the various vessels, we have also plotted the stock market index as a reference. The figure illustrates what seems to be a clear difference between the three shipping segments. However, the difference in segments become even clearer when removing the effects of age in our analysis, i.e. identifying the triangular points from the circular. Similar figures for each age group are included in appendix 6 for your convenience.

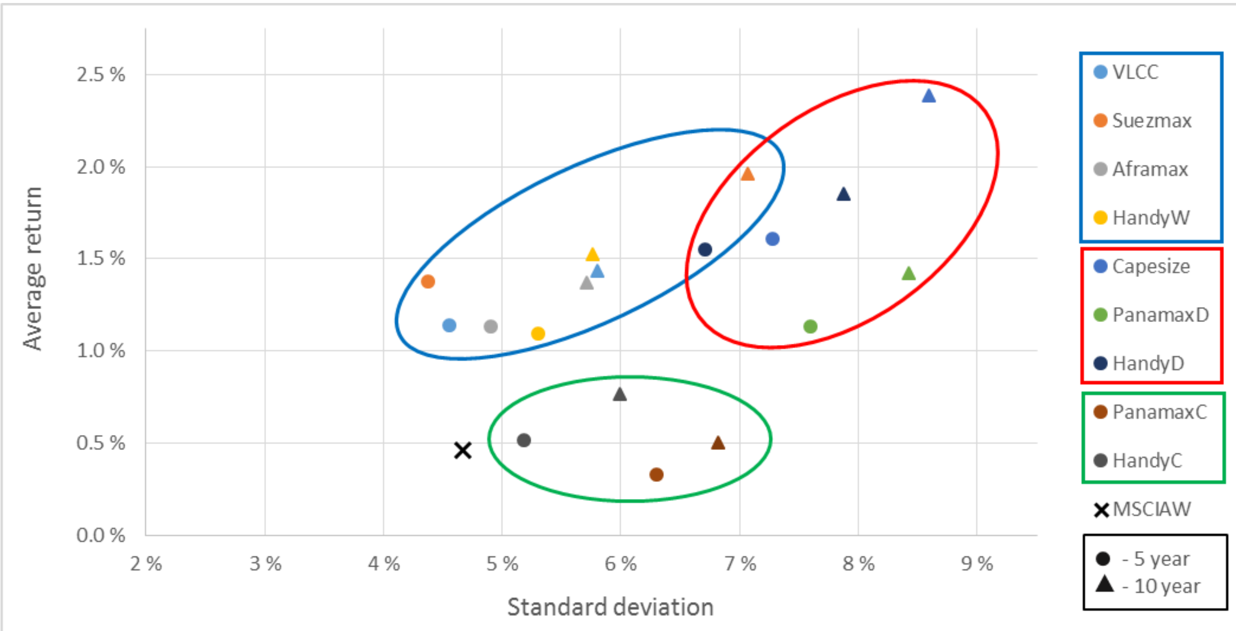

Figure 6 - Risk-return characteristics

The wet bulk vessels are encased by the blue circle and seem to offer the better trade-off between risk and reward. Dry bulk (red) show high returns coupled with the highest risk, whilst the container segment (green) has low average returns and relatively high risk. In fact, when comparing container vessels with the black cross symbolizing a well-diversified stock portfolio, they seem to provide inferior investments. However, as the essential part of building portfolios is the way asset returns are correlated, this alone is not grounds for excluding investments in container vessels from a final portfolio.

Figure 6 displays some overlap between the illustrative circles for wet and dry bulk to include ten-year Suezmax (orange triangle) and five-year dry bulk Handymax (dark blue dot) in their respective groupings. As mentioned above, the differences between segments are somewhat reduced when including the two age segments in the same figure. In table 5 we found both return and risk to increase with age of the vessels. These findings are reflected in the right and upward shift of the triangular points for ten-year old vessels relative to the circular points marking five-year old vessels.

When compared to the findings of Patitsas (2004, p. 35), Magirou et al. (1997, p. 38) and Schilbred (1992, p. 13), this clear segregation is very interesting. The mentioned papers display clusters of various vessel types mixed together, and the differences should be discussed. Patitsas, perhaps being closest to our study, uses quarterly returns in his analysis, likely masking some of the unique variability of the vessels. Magirou et al. use a somewhat deferring methodology, besides studying yearly returns, when basing the earnings on time charter rates. Furthermore, there is inconsistency with regards to age of vessels. Specifically, wet bulk returns are based on older ships than for dry bulk returns. From our analysis above, such age difference might cause higher risk and return for the wet bulk segment in the comparative analysis of Magirou et al. Finally, Schilbred limits his study to four years of data. As discussed above under literary review, this may not reflect the true variations in a full shipping business cycle, thus affecting the results. Schilbred also shows some inconsistency concerning vessel age.

The underlying differences between the abovementioned studies highlight some possible explanations for the newfound results in figure 6. However, it is important to note the time between the mentioned studies, the most recent being more than ten-years old. It may therefore very well be that reasons for the differences rather are connected to fundamental changes in the shipping markets. For instance, we suspect the reduction in tonnage of combination carriers since the 1980s (cf. figure 2) to play an important role in this development through the leakage effect mentioned in chapter 2.2.

Our interesting observations form the backdrop for this paper. The differences regarding all three aspects, i.e. size, segment and age, lead us to believe there is substantial potential in clarifying the underlying mechanics of shipping returns for the purpose of portfolio optimization. Besides an interesting theoretical aspect, investigating the possibilities for diversification can provide valuable information for several stakeholders to the shipping industry, e.g. shipping companies, banks or fund managers.

For the continuation of this paper, future expected returns and variance of the vessels are based on the historical sample values presented in table 5 above. This is necessary to provide recommendations on relevant future portfolio allocations. The implications of such an assumption have been discussed in the preceding parts of this paper. As mentioned, the values for these inputs can easily be adjusted to account for individual views on the future conditions for each vessel type. Therefore, this imposes no limitations to our underlying model, albeit it affects the results. The concept of predicting returns is especially familiar for analysts, and the variance, or standard deviation, is also a perceptible feature for analysis and prediction. However, the vast number of estimates necessary, and the complex cross-sectional interdependencies of covariances, supports the need for a simplified model of future covariance predictions. Such a model will in turn be implicitly dependent on future estimates of variance for input variables, i.e. the explanatory factors, in the model. Hence, we also base future expectations of variance for our chosen factors on their historical samples, cf. equation (7) with following remarks in chapter 3.

5.2 Modeling the covariance structure

To better map the interdependency of shipping returns to common factors, this section presents the results of the prescribed factor models laid out in chapters 3 and 4. The results from regressions with both single- and multi-factor models are portrayed in the tables below.

5.2.1 Single-factor model

The regression results with beta values and corresponding t-statistics for all vessels:

Results from Single-Factor 5yr old vessels

	(1) VLCC	(2) Suezmax	(3) Aframax	(4) HandyW	(5) Capesize	(6) PanamaxD	(7) HandyD	(8) PanamaxC	(9) HandyC
MSCIAW	0.102	0.112^{*}	0.0941	0.175^{**}	0.449^{***}	0.430^{***}	0.312^{***}	0.0475	0.0943
	(1.54)	(1.76)	(1.32)	(2.29)	(4.43)	(4.03)	(3.27)	(0.52)	(1.25)
_cons	0.0109^{***}	0.0132^{***}	0.0109^{***}	0.0102^{***}	0.0140^{***}	0.00933^{*}	0.0141^{***}	0.00309	0.00470
	(3.53)	(4.46)	(3.27)	(2.84)	(2.95)	(1.87)	(3.15)	(0.72)	(1.34)
N	218	218	218	218	218	218	218	218	218
R^2	0.011	0.014	0.008	0.024	0.083	0.070	0.047	0.001	0.007

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Results from Single-Factor 10yr old vessels

	(1) VLCC	(2) Suezmax	(3) Aframax	(4) HandyW	(5) Capesize	(6) PanamaxD	(7) HandyD	(8) PanamaxC	(9) HandyC
MSCIAW	0.134	0.168	0.175^{**}	0.160^{*}	0.392^{***}	0.518^{***}	0.347^{***}	0.0427	0.121
	(1.60)	(1.64)	(2.13)	(1.93)	(3.21)	(4.40)	(3.09)	(0.43)	(1.39)
_cons	0.0137^{***}	0.0188^{***}	0.0129^{***}	0.0145^{***}	0.0220^{***}	0.0118^{**}	0.0169^{***}	0.00482	0.00713^{*}
	(3.49)	(3.93)	(3.34)	(3.73)	(3.85)	(2.14)	(3.22)	(1.04)	(1.75)
N	218	218	218	218	218	218	218	218	218
R^2	0.012	0.012	0.021	0.017	0.045	0.082	0.042	0.001	0.009

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 6 - Single-factor regressions

For wet bulk vessels, (1) through (4), the world stock index generally seems to be a poor explanatory factor. When statistically significant, it tends to be for smaller vessels and with low coefficients of determination, i.e. R-squared, near 2%. Handymax product tankers are the only vessels to be significantly explained independent of age. The difference in betas regarding age between five- and ten-year vessels is minor when accounting for the levels of explanatory power in the model. A reasonable theory for the poor results is that the stock index is closely related to production and its drivers, e.g. the oil-price. Referring to the discussion of chapter 4.2.4 Model variables, oil-price is ambiguous in explaining the demand and thus the profitability of wet bulk trades. This is especially true for the major trades of crude oil serviced by the larger vessels. Minor wet bulk trades are more diverse, e.g. value-added refined oil products. Hence, this may be the reason that smaller vessels can be explained slightly better within the confines of this model.

For dry bulk, (5) to (7), the stock index seems to be a far better explanatory variable for returns. As displayed above, the variable is statistically significant on a 1% significance level for all vessels. Having tested for serial correlation (cf. appendix 1.1), a plausible theory behind such high t-values is that dry bulk commodities are more tied to the stock markets than wet bulk. On the other hand, the results might be due to the omitted variable bias as discussed in conclusion of chapter 4 above. However, the explanatory power presented by R-squared remains fairly low with values reaching just above 8%. The loadings are positive, but well below unity.

Keeping the mechanics of dry bulk freight markets in mind, we can justify the statistical results from an economic perspective. As for wet bulk, stock markets are thought to reflect levels of production and associated expectations. In turn, this factor should reflect the need for industrial inputs such as materials or electricity. For dry bulk, this should affect the demand for ores and coal which is a major part of the trade. A relative increase in oil price should also motivate a switch towards coal, further enhancing the effect. BP's statistical review (2014) shows an evident increase in the use of coal energy driven by the Asia Pacific region during our period of analysis.

Finally, the global stock index is a very poor explanatory variable for container vessels, (8) and (9), being insignificant for all vessels and ages. Referring to the earlier discussions of the market for container freight, this finding should not be too surprising. The demand for freight, from a shipowner's perspective, is more or less predetermined in the short run. The market structure of logistics operators hiring freight capacity on time charters presumably leaves a great deal of the market risk on the charterers. Their risk may in turn depend on typical container good's sensitivity to the general economic climate. Nevertheless, the results indicate that we are not able to capture the demand for liner trade sufficiently with such a general factor.

Truth be told, we expected the explanatory power of a stock index in the single-factor framework to be quite low. To some extent, such an index captures general economic activity, but it does not seem to capture seasonal demand patterns in commodity trade or general demand for freight. This is concurrent with earlier studies on seaborne trade, especially the findings of Kavussanos & Marcoulis (1998). Additionally, we expect the index to capture some degree of microeconomic fluctuations inherent in stock prices. For our unlevered returns, such factors should not play a substantial role in vessel returns. Somewhat comparable to our study, Melbø's paper (2013) attempts to use the Clarksea index as an explanatory variable. Despite choosing a pure shipping related index, the model fails at explaining any large part of vessel returns. As mentioned earlier in this paper, research points towards the need for a more complex model suited to better explain shipping returns.

5.2.2 Multi-factor model

Using the final multi-factor model in equation (16) for regression, we arrive at the following factor loadings explaining vessel returns:

Results from Multi-Factor 5yr old vessels

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC
OECDTOT	0.425	0.540	0.640*	0.164	0.124	0.564	0.759	1.082**	0.855**
	(1.26)	(1.64)	(1.71)	(0.40)	(0.24)	(1.04)	(1.60)	(2.30)	(2.24)
dAUSCOAL	0.180***	0.108**	0.115**	0.124**	0.183**	0.225***	0.230***	0.110	0.0722
	(3.74)	(2.29)	(2.16)	(2.13)	(2.49)	(2.90)	(3.40)	(1.64)	(1.33)
dWOS	0.661*	0.473	-0.112	0.752*	0.155	0.169	0.430	0.459	0.499
	(1.86)	(1.35)	(-0.28)	(1.75)	(0.28)	(0.29)	(0.86)	(0.92)	(1.24)
dWSP	0.0217	-0.0494	-0.166**	0.0658	0.265**	0.0576	0.147	0.111	0.135*
	(0.31)	(-0.72)	(-2.12)	(0.78)	(2.47)	(0.51)	(1.49)	(1.13)	(1.70)
dPUV	0.442**	0.410*	0.418*	0.579**	1.485***	1.486***	1.158***	0.664**	0.685***
	(2.04)	(1.93)	(1.73)	(2.21)	(4.48)	(4.24)	(3.80)	(2.19)	(2.79)
_cons	0.00859***	0.0112***	0.0102***	0.00841**	0.0121***	0.00740	0.0114***	0.000441	0.00234
	(2.88)	(3.84)	(3.06)	(2.33)	(2.66)	(1.54)	(2.71)	(0.11)	(0.69)
N	215	215	215	215	215	215	215	215	215
R^2	0.150	0.097	0.093	0.087	0.213	0.196	0.219	0.109	0.127

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Results from Multi-Factor 10 yr old vessels

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC
OECDTOT	0.285	-0.123	0.579	0.570	1.632***	0.998*	1.842***	1.040*	1.041**
	(0.65)	(-0.22)	(1.31)	(1.30)	(2.65)	(1.67)	(3.38)	(1.97)	(2.36)
dAUSCOAL	0.187***	0.125	0.169***	0.178***	0.194**	0.270***	0.172**	0.101	0.0883
	(3.00)	(1.58)	(2.67)	(2.84)	(2.21)	(3.16)	(2.21)	(1.34)	(1.40)
dWOS	0.567	1.083*	-0.00433	0.756	0.421	-0.0856	0.576	0.150	0.611
	(1.23)	(1.85)	(-0.01)	(1.63)	(0.65)	(-0.14)	(1.00)	(0.27)	(1.31)
dWSP	0.0750	-0.0263	-0.0907	0.0709	0.0730	0.0910	0.0918	0.0573	0.104
	(0.82)	(-0.23)	(-0.98)	(0.77)	(0.57)	(0.73)	(0.81)	(0.52)	(1.12)
dPUV	0.556**	0.393	0.169	0.169	1.481***	1.477***	1.602***	0.478	0.905***
	(1.98)	(1.10)	(0.59)	(0.60)	(3.74)	(3.83)	(4.56)	(1.40)	(3.18)
_cons	0.0111***	0.0175***	0.0123***	0.0126***	0.0194***	0.00969*	0.0131***	0.00260	0.00415
	(2.86)	(3.56)	(3.13)	(3.24)	(3.56)	(1.83)	(2.72)	(0.56)	(1.06)
N	215	215	215	215	215	215	215	215	215
R^2	0.109	0.042	0.065	0.080	0.187	0.207	0.244	0.064	0.138

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 7 - Multi-factor regressions

Wet bulk vessels, (1) to (4)

For wet bulk vessels, the most important variables are dAUSCOAL, dWOS, dWSP and dPUV.

The price of coal (***dAUSCOAL)*** is statistically significant for all vessels except ten-year Suezmax. This is an interesting observation. Since coal is a substitute for oil, e.g. for heating and electrical purposes, such a relationship clearly has economic founding. When the price of coal increases relatively enough compared to oil, consumers will switch from coal to oil and vice versa. Subsequently, the price of coal has a positive effect, i.e. positive betas, on demand for oil and its transport. Where previous research has failed to find statistically significant variables regarding oil trade by using price (Westgaard, et al., (2007) and Drobetz, et al., (2010)), using a substitute for oil might have solved the problem of causality and ambiguous results. However, attempting to explain the inconsistency for Suezmax vessels proves difficult.

Coupled with the variable of changes in oil stock (***dWOS***), we might find grounds for an explanation. Admittedly, this variable is barely statistically significant and only for some vessels. We included the oil stock as an attempt to find appropriate alternatives to oil price as explanation for wet bulk vessel returns. With the coal price proving to be a good variable to capture the substitution effect, we can imagine that the two variables might be interfering with one another somewhat concerning wet bulk freight demand. This could be the reason for dWOS being slightly significant for ten-year Suezmax. Especially if one considers the possibility of this variable capturing more of the demand for floating storage capacity, not affected by the IMO convention regarding double hull standards for tankers built from 1992 onward. However, these conclusions should be read with caution when regarding the variable's t-statistics.

World steel production (***dWSP***) loads negatively on five-year Aframax returns. The variable is statistically significant on the 5% level, and the value of the t-statistic is just above the rule of thumb for t-statistics, $t > |2|$. Seeking an explanation for the negative loading, we found that this vessel type accounted for a majority of new vessel deliveries over the period of analysis. Clarkson's report more than 50 vessels per year for the period 2003 to 2012. Increasing the supply of vessels will naturally force rates down. Thus, if steel production can be connected to shipbuilding activity, this could explain the result.

Furthermore, the price index (***dPUV***) seems to be significant for both VLCC vessels and all other five-year wet bulk vessels. Firstly, the effect of increasing commodity prices will influence the index. If the oil price increase over the period has facilitated the possibility of increased margins in wet bulk, but freight costs still proportionately decreasing, one could expect larger vessels to reap most of this benefit. Secondly, our basic theory regarding cargo value stipulates increasing importance of speedy delivery. This should generally translate to dPUV being a stronger explanatory variable for modern vessels, since it reflects changes in the composition of traded goods. However, if this is the case for bulk segments is clearly debatable referring to research on two-tier markets for bulk commodities (Strandenes, 1999). Still, the trend seems clearer for the Handymax vessels, which coincidently transport products that are more specialized and therefore somewhat applicable to this cargo value theory.

Dry bulk vessels, (5) to (7)

As for dry bulk, the important factors are OECDTOT, dAUSCOAL, dWSP and dPUV.

The major commodities for dry bulk transport is coal, iron ore and grain. Opposed to the main producers of wet bulk commodities, major producers of dry bulk commodities are members of the OECD. Australia is good example of a major exporter of coal and ore. Since these exports are directly a part of OECD production (***OECDTOT***), it is natural that this factor is significant for explaining dry bulk vessel returns. Accordingly, the loadings should be positive, since an increase in production implies an increase of freight. This is also the result for all ten-year vessels showing statistical significance, although Panamax is very close to the rejection range. For the five-year vessels we astonishingly do not achieve the same results.

The price of coal (***dAUSCOAL***) is positively significant across all vessels. Compared to the discussions on oil price as a factor, we believe the market for coal to be closer to perfectly competitive. Therefore, the price better reflects the demand for coal freight. A coal price increase will reflect increased demand for dry bulk freight, as we see positive betas for this factor.

Although there seems to be a slight tendency of decreased significance as vessel size increases, it is still significant for large vessels. Due to the stowage factor of coal, this might be reasonable to expect. Other dry bulk commodities, for instance iron ore, have a much lower stowage factor and heavier weight. Following this, a theory could be that coal has primarily been transported with Panamax vessels and iron ore with Capesize vessels.

For five-year Capesize vessels, this reasoning seems to be reflected in the significant variable for world steel production (***dWSP***). Iron ore, as well as coking coal, are essential to production of new steel. An increase in production will entail an increase in freight demand. Following the suggestion for this variable on wet bulk, if steel production can be tied to shipbuilding activity there will be an ambiguity in the explanation provided by the variable. This may be the reason for world steel production not being as good an explanatory variable for dry bulk as we initially hoped.

Finally, the price index (***dPUV***) loads positively on all dry bulk vessels. A striking result from table 7 above is the high t-statistics of dPUV on every dry bulk vessel. Consequently, symbolizing that price development of goods is a very important factor in explaining returns from dry bulk vessels. Compared to wet bulk, the higher statistical significance could be because dry bulk commodities are, compared to oil, a larger part of valuable mass marketed consumables. Price increases in the end products will be strongly reflected in demand for such dry bulk commodities, e.g. grain is an input to food (bread etc.), timber is an input to construction etc. This is an economically viable explanation for the results.

Container vessels, (8) and (9)

For container vessels, two particular explanatory variables seem to be good indicators of returns, OECDTOT and dPUV.

OECD production growth (***OECDTOT***) is significant for all the analyzed container vessels. This is not surprising, as a lot of the container trade comprises OECD countries. Typically, the final leg of a globalized production process happens in an OECD country, e.g. branding, packaging, quality control etc., before the end product is sold and shipped. As anticipated, the betas are positive and pretty close to unity, meaning that it is tightly connected to the development of OECD production.

The price index (***dPUV***) is a significant factor for all container vessels except the ten-year Panamax. Our expectations are that higher value goods should imply more demand for container freight since delivery and logistics often become more important. This essentially also seems to be the case.

Lastly, world steel production (***dWSP***) is barely significant on a 10% level with regards to five-year Handysize container vessels. This might reflect the abovementioned possibility of transporting steel products as container goods. However, a variable with such low t-values should be interpreted with caution.

Earlier research on multi-factor models in shipping, have been focused on stock returns. However, some similarities are expected. The main issue is the differing choice of explanatory variables. Referring to Drobetz et al. (2010), we find similar effects of industrial production on explaining container returns. Their use of G7 industrial production should be largely in line with our use of OECD industrial production. The main explanatory variable in their paper is the world stock index and a currency basket. The first is excluded from our model and the second seems to be tightly related to microeconomic decisions, cf. the discussions in preceding parts of this paper.

Whilst Drobetz et al. focus on all segments in line with our analysis, Westgaard et al. (2007) solely studies the tanker market using a multi-factor framework. The results show a relationship with world return and exchange rate fluctuations, which we suspect is affected by the fact that stock returns were the basis for the study. Furthermore, there is explanatory power in the oil inventory levels coinciding with our choice of world oil stock as a variable. Even with a specialized tanker market model the authors have not managed explaining more than 27% of return variation, clearly highlighting the difficulties involved.

5.2.3 Model comparison

The final multi-factor model above results from the necessity to find an applicable framework for analyzing return variation. From a theoretical point of view, as discussed in the end of chapter 2, the two extremes of the single-factor model and naïve input estimation both suffer from deficiencies. The aim is therefore to increase the explanatory power, whilst not sacrificing applicability.

In line with previous research, the explanatory power of a multi-factor model considerably outperforms the single-factor model. Including macroeconomic variables that presumably affect seaborne trade vastly improves the explanatory power. The models are comparable in adjusted R-square measure, as they are both in the same functional form (Wooldridge, 2013):

Adjusted R-Squared		Wet bulk				Dry bulk			Container	
		VLCC	Suezmax	Aframax	Handymax	Capesize	Panamax	Handymax	Panamax	Handysize
5 year old	Single	0.63 %	0.96 %	0.34 %	1.92 %	7.89 %	6.55 %	4.28 %	-0.34 %	0.26 %
	Multi	12.94 %	7.49 %	7.11 %	6.53 %	19.39 %	17.71 %	20.05 %	8.76 %	10.65 %
10 year old	Single	0.72 %	0.78 %	1.60 %	1.24 %	4.11 %	7.81 %	3.78 %	-0.38 %	0.43 %
	Multi	8.82 %	1.88 %	4.27 %	5.81 %	16.75 %	18.81 %	22.56 %	4.12 %	11.78 %

Table 8 - Model comparisons using adjusted R-squared

A quick glance on table 8 reveals a striking difference between the two models. It clearly shows the importance and magnitude of increasing the number of explanatory factors for explaining vessel returns, thereby highlighting the aim of our research.

5.3 Practical implications

At last, having determined all the inputs to our optimization problem, we apply the method from the final section of chapter 3. We will investigate the practical implications of applying the different frameworks to our vessel statistics, namely the naïve, single- and multi-factor approach. As mentioned, the covariance and respective correlation matrices from each method can be found in appendix 7.

When plotted graphically, the three methods clearly yield different estimates for the investment universe of efficiently diversified portfolios:

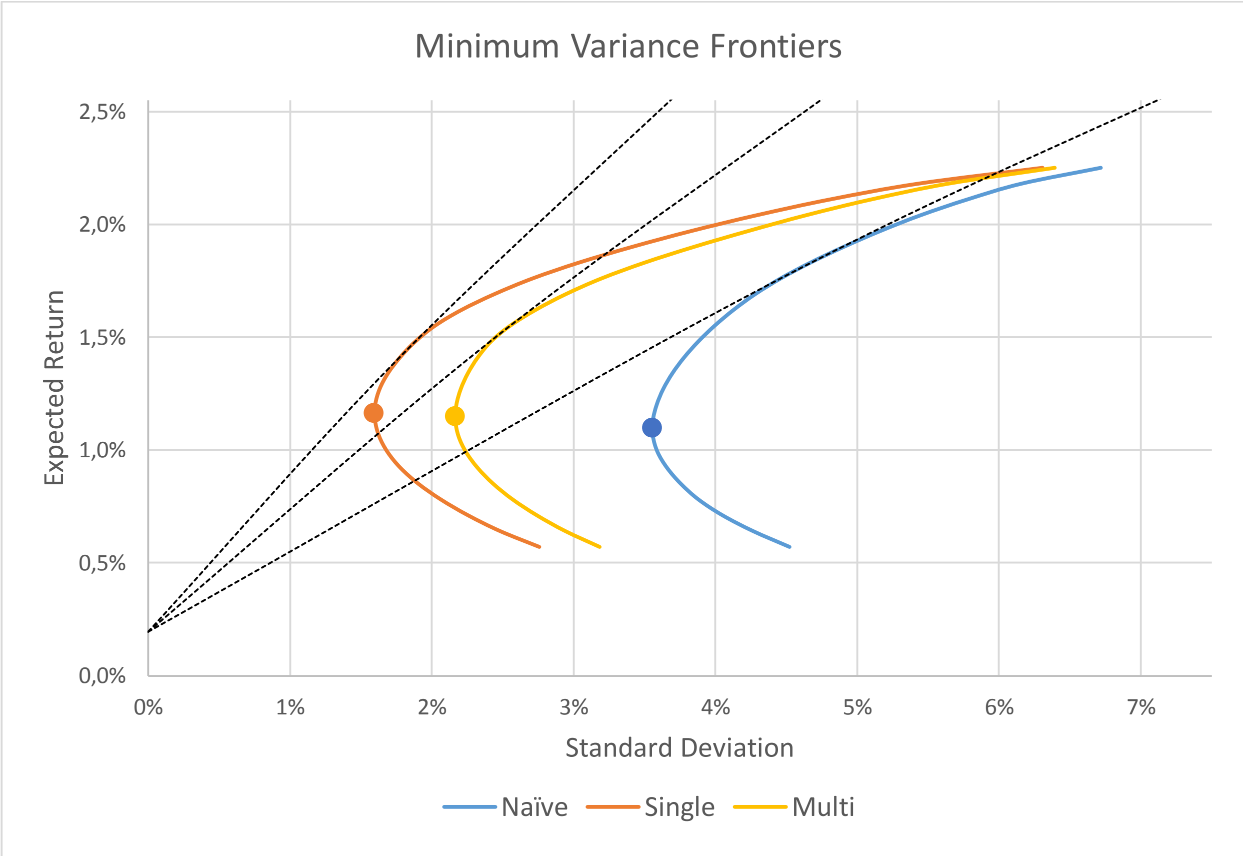

Figure 7 - Minimum variance frontiers

The frontiers represent the portfolios with lowest risk for predetermined levels of return. The line section above the marked global minimum variance portfolios (GMVP) shows the *efficient frontier*. All rational risk averse investors would select a portfolio on this line based on their individual utility function. However, as first proposed by Tobin (1958) an investor can allocate between a portfolio of risky assets and the risk-free asset. The dashed line, representing the capital allocation line (CAL), illustrates this fact assuming unlimited borrowing and lending at the risk-free rate (Sharpe (1964), Lintner (1965) and Mossin (1966)). In the point of tangency between the efficient frontier and CAL, we find the optimal portfolio for any investor. Under the assumptions that all investors have similar expectations about the future (Fama, 1970), this line will be the *capital market line* (CML) from the CAPM.

It is important to explain the differences in figure 7. Given the initial constant estimates of return and variance for each vessel independent of method, all discrepancies are due to the imposed correlation structures. Without imposing structure, the naïve method implicitly assumes any co-movement of asset returns is due to a fundamental source of correlation. The fact that some of this co-movement simply is contributable to random noise is not accounted for. On the other hand, the single-factor model offers the highest possible amount of structure. All correlation is tied to one single factor and any other co-movement is disregarded. The covariance term in equation (2) is considerably reduced, thereby showing increased potential for diversification and resulting in the minimum variance frontier with lowest risk. Naturally, including additional factors in the multi-factor model explains more co-movement and in turn decreases theoretical benefits of diversification. This positions the feasible set of efficient portfolios from the multi-factor model between the two extremes of structural impositions, i.e. naïve and single-factor.

Bearing in mind that choice of method directly affects the characteristics of optimal portfolios, one cannot determine the best model by simply choosing the one with best performance measures in terms of Sharpe ratio, Treynor measure or Jensen's alpha. This misconception obscures the true objective of such studies, which is to determine the most fitting model for future predictions. Several papers have addressed the issue of modelling covariance. According to Elton et al. (2014), simple seems to be better. However, the research has mainly been focused on stock markets where the single-index model has contributed greatly to the explanation of variation. The poor results of the single-factor model shown in table 6 underpin the necessity of increasing the explanatory power in our model for the real investment market of shipping. The relative reward of added information to increased noise is high when initially increasing the number of factors. In doing so, we clearly manage to increase the explanatory power in our multi-factor model compared to that of the SIM, cf. table 8. However, any empirical foundation for assessment of such information-to-noise trade-off will have to be done ex post and can be subject for future research.

Optimizing the Sharpe ratio of portfolios for each method, we are able to extract the optimal risky portfolios presented below:

Optimal		Wet bulk				Dry bulk			Container		(Per age)
portfolio		VLCC	Suezmax	Aframax	Handymax	Capesize	Panamax	Handymax	Panamax	Handysize	Sum
5 year	Naïve	0.00 %	27.46 %	0.00 %	0.00 %	9.14 %	0.00 %	0.00 %	0.00 %	0.00 %	36.60 %
	Single	10.02 %	13.96 %	8.69 %	5.99 %	3.84 %	1.27 %	5.31 %	0.40 %	1.68 %	51.17 %
	Multi	7.46 %	18.26 %	9.52 %	5.97 %	3.69 %	0.00 %	2.94 %	0.00 %	0.00 %	47.84 %
10 year	Naïve	0.00 %	19.02 %	0.00 %	21.81 %	22.57 %	0.00 %	0.00 %	0.00 %	0.00 %	63.40 %
	Single	8.13 %	7.99 %	7.39 %	8.69 %	5.86 %	1.65 %	4.92 %	1.36 %	2.83 %	48.83 %
	Multi	8.26 %	12.78 %	10.24 %	11.46 %	6.64 %	0.00 %	2.78 %	0.00 %	0.00 %	52.16 %

Table 9 – Sharpe optimal vessel portfolios for each method

Between the models, there are clear differences. The structured models of single- and multi-factor display a broader allocation among assets compared to the naïve approach. As claimed by Ledoit and Wolf (2003), the naïve model shows tendencies of weighting the extremes in the covariance estimation. This results in a proposed investment in just 5 out of the 18 assets. Intuitively, this may seem unrealistic regarding the true possibilities of diversification among assets. The implicit factors of the naïve optimization attribute too much of idiosyncratic risk to economically unjustifiable common factors, i.e. random noise from lacking structure. By controlling for excessive idiosyncratic factors, a multifactor model expands the optimal fleet composition to 12 out of 18 possible vessels. Utilizing a multi-factor framework to model covariance is an acknowledgment of Kavussanos's (2010) argument of certain factors affecting specific shipping markets. Limiting the correlation between vessels to just one factor, the single-factor model proposes the most diverse portfolio including all 18 assets.

Another interesting observation is the allocation between the two ages of vessels. For the Sharpe optimal portfolios, the total allocation of the investment should be spread rather equally between ages. Furthermore, there seems to be no clear tendency towards investing more in either larger or smaller vessels on a general basis. We shall investigate the aggregate numbers of table 9 further in the following section by decomposing and optimizing the portfolios for each segment.

However, it is also of interest to examine the global minimum variance portfolios (GMVP) for each of the methods in order to increase our understanding of levels of risk for different assets. Considering the possibility of not being able to split an investment between a risky vessel portfolio and the risk-free asset, the portfolio decision depends on the investors risk profile and utility curve. The optimal portfolio will in this case lie somewhere on the efficient frontier, i.e. the curve segment of the minimum variance frontier above the dot marking GMVP in figure 7. The allocation of the GMVPs are provided in the following table:

Min Var		Wet bulk				Dry bulk			Container		(Per age)
portfolio		VLCC	Suezmax	Aframax	Handymax	Capesize	Panamax	Handymax	Panamax	Handysize	Sum
5 year	Naïve	5.40 %	30.30 %	11.08 %	14.00 %	8.24 %	0.00 %	0.00 %	0.00 %	26.25 %	95.26 %
	Single	10.75 %	11.49 %	9.35 %	7.13 %	2.16 %	2.07 %	3.52 %	6.02 %	8.35 %	60.84 %
	Multi	11.25 %	15.21 %	12.27 %	9.16 %	0.64 %	0.00 %	0.05 %	4.69 %	8.86 %	62.14 %
10 year	Naïve	0.00 %	0.00 %	0.00 %	4.74 %	0.00 %	0.00 %	0.00 %	0.00 %	0.00 %	4.74 %
	Single	6.30 %	4.02 %	6.14 %	6.16 %	1.76 %	1.26 %	2.34 %	5.16 %	6.02 %	39.16 %
	Multi	6.00 %	6.04 %	8.62 %	7.55 %	0.00 %	0.00 %	0.00 %	5.01 %	4.65 %	37.86 %

Table 10 - Global minimum variance portfolios for each method

The most striking difference when compared to the findings in table 9 is the shift towards newer vessels. This trend applies to all models, naturally least extreme for the single-factor model. We find this to be in line with the examination of vessel returns from the introduction of this chapter. Referring again to chapter 2, the cost structure of older vessels result in increased risk. Consequently, a high degree of risk aversion for a shipowner could be reflected in lower average age of the fleet. Another observation is that container vessels are more attractive when the aim is minimizing the risk of the portfolio. In general, one might expect a more widespread diversification for the minimum variance portfolio in line with the traditional doctrine of diversification, i.e. the more assets the better. This also seems to be the case among vessel types with investments spread more evenly than for the Sharpe-optimal portfolios.

5.3.1 Segment specific diversification

For shipping, as in any other industry, specialization is far from uncommon. There exist several economically rational explanations for a focused business orientation. These will be discussed thoroughly later in this chapter. Accordingly, it is of interest to examine the allocation between vessels within the boundaries of each shipping segment. We will examine the Sharpe-optimal portfolios below, and refer to appendix 8 for each segment's minimum variance portfolios. The differences are in line with the general results above when minimizing risk (standard deviation), i.e. higher proportions of five-year vessels and more evenly distributed among the assets.

Wet bulk

For wet bulk vessels only, we arrive at the following optimal portfolio allocations:

Optimal		Wet bulk				(Per age)
portfolio		VLCC	Suezmax	Aframax	Handymax	Sum
5 year	Naïve	0.00 %	39.78 %	0.00 %	0.00 %	39.78 %
	Single	14.01 %	19.29 %	12.06 %	9.17 %	54.52 %
	Multi	10.64 %	20.90 %	11.49 %	8.02 %	51.06 %
10 year	Naïve	3.18 %	22.62 %	0.00 %	34.41 %	60.22 %
	Single	11.38 %	11.05 %	10.76 %	12.29 %	45.48 %
	Multi	10.50 %	13.59 %	11.69 %	13.17 %	48.94 %

Table 11 – Sharpe optimal pure wet bulk portfolios

Firstly, we note a reasonably similar allocation between ages as we found in table 9. This supports a theory that invested capital in shipping should be evenly divided among vessel age for the best risk-reward trade-off. Secondly, we might be tempted to conclude that the best investment strategy seems to be an overweight in five-year Suezmax, with corresponding underweight in five-year Handymax vessels.

Dry bulk

The optimal allocation within the dry bulk sector is as follows:

Optimal portfolio		Dry bulk Capesize	Panamax	Handymax	(Per age) Sum
5 year	Naïve	33.71 %	0.00 %	0.00 %	33.71 %
	Single	17.80 %	8.53 %	21.69 %	48.02 %
	Multi	19.94 %	2.73 %	22.13 %	44.80 %
10 year	Naïve	60.91 %	0.00 %	5.38 %	66.29 %
	Single	22.62 %	9.76 %	19.60 %	51.98 %
	Multi	28.78 %	6.33 %	20.10 %	55.20 %

Table 12 – Sharpe optimal pure dry bulk portfolios

As with wet bulk, there is seemingly a near 50/50 allocation between five- and ten-year vessels. Perhaps a slight tendency towards older vessels. However, there is a clear underweight in the mid-segment represented by Panamax vessels. Ten-year Capesize also seems to be a slightly more favorable investment than the Handymaxes or younger five-year Capes.

Container

Finally, the Sharpe optimal portfolio for a pure container fleet:

Optimal portfolio		Container Panamax	Handysize	(Per age) Sum
5 year	Naïve	0.00 %	0.00 %	0.00 %
	Single	8.90 %	31.23 %	40.13 %
	Multi	1.69 %	31.57 %	33.26 %
10 year	Naïve	0.00 %	100.00 %	100.00 %
	Single	17.46 %	42.41 %	59.87 %
	Multi	16.73 %	50.01 %	66.74 %

Table 13 - Sharpe optimal pure container portfolios

For container trade, ten-year vessels are somewhat preferred to five-year vessels. Especially, in the case of the naïve optimization. With fewer assets, the drawbacks of the naïve method become clear. The other methods are far from as extreme, but still have an overweight in older vessels. This might reflect the degree of service differentiation in liner shipping that does not exist in major bulks. Investing in new and modern vessels is perhaps a strategy to gain market share. In turn, this may have put upward pressure on prices of modern vessels making investments in older vessels more profitable. Furthermore, there seems to be near a three-to-one ratio of Handysize over Panamax vessels. Drawing on our literature review, the size factor is likely to increase the risk due to flexibility issues. Smaller vessels are likely preferred when hired by a container operator to cover the marginal demand surplus for container freight, resulting in upward pressure on rates for these vessels.

5.3.2 General remarks

It is important to note that the results above are based on historical data. Hence, our results indicate the optimal portfolio compositions over the past period. To provide recommendations for shipping investments, it is necessary to make assumptions regarding the future. As discussed in previous chapters, and following the norm of financial forecasting, our starting point has been to base future predictions on the historical findings. Such assumptions are well founded for asset variation, which is the main topic of our research. Therefore, it is again necessary to clarify that forecasting future returns is not the intention per se. Essentially, we seek to identify the co-movement in return variation as to enable more efficient risk management in vessel investments.

Following this reasoning, one can argue that the minimum variance portfolios are the most relevant for analysis because it is derived independent of forecasted returns. However, a Sharpe-optimal portfolio exemplifies the effect of the separation theorem, albeit also dependent on a risk-free rate. It is unlikely that a real world investor would end up investing in a minimum variance portfolio if such a risk-free investment exists. Thus, a Sharpe-optimal allocation is likely to be closer to reality in offering a useful recommendation for an investor. Furthermore, a comparison of the two sets of portfolios provide valuable information to understanding the investment universe.

The optimal fleet compositions we propose depend on the risk-free rate. If rates were different, the slope of the capital allocation line in figure 7 could change, and therefore the point of tangency. This relationship may be problematic when studying an industry with a large degree of cyclicality, such as shipping, if the interest rate markets and shipping cycles are not fully integrated. Because of the lag between ordering and delivery of ships, adapting to changes in investment environment is not straightforward. In good times, the pro-cyclical investment behavior under prevailing expectations will likely affect the world fleet composition. The situation is often enhanced by banks and investors offering capital at low rates. As the sentiment changes, the decisions from good times are not easily reversed. The effect of pro-cyclical investment behavior is underlined by Greenwood & Hansen (2014), and their findings strengthen this statement. Shipowners more often than not neglect competition and realistic future expectations when deciding to invest in new vessels. By overinvesting, shipowners depress future values of secondhand vessels and freight rates. Higher interest rates of depressed markets will again encourage too risky investments by lowering the slope of the CAL. To conclude, optimal fleet composition in high rate environments may very well differ from optimal composition in low rate conditions.

The results of the optimizations above are derived under the assumption that investors are rational in terms of risk aversion within the mean-variance space. In the shipping industry, major operators have been known to act intuitively rather than base decisions on strict models. Lorange and Norman (1973) conducted a survey of 17 Scandinavian shipowners and found their preferences to vary according to their liquidity conditions. In good periods, the shipowners had tendencies of risk-proneness rather than aversion. This attitude might be reason for the many glorious tales of successful shipowners' "gut feeling". Several devastating crises and bankruptcies being the other side of the coin. Although it was claimed that the personal sentiment of the shipowner might not be fully reflected in the final decisions of large companies, the findings are an interesting fact for our discussion. Especially considering the weak concentration of ownership in many shipping segments.

An interesting finding is the rather consistent allocation between ages, indicating that the secondhand vessel market seems very near perfectly competitive. Such a conclusion can be drawn since there is an approximate 50/50 weight in the recommended portfolios. If this was not the case, arbitrageurs would invest in the relatively cheaper investment thereby exerting pressure and forcing prices back to equilibrium. In other words, it seems that shipping investors are well aware of vessel prices and orient their investments towards the underpriced assets. This finding also shows that shipping investors adapt according to a Sharpe-optimal allocation as discussed above, since the allocation in the minimum variance portfolios clearly overweight newer vessels (cf. table 10).

When comparing tables 9 and 10, we saw a shift towards newer vessels to minimize the portfolio risk. Referring to the literature review in chapter 2, we discussed the operational risks connected to age *and* size. Therefore, one would expect lower risk for smaller vessels to represent their trading flexibility. Due to our assumptions regarding vessel hire 360 days per year, the true variability of the inherent unemployment risk is not incorporated in our model. Especially for depressed markets, the de facto downside risk might not be reflected because of this assumption. Any necessary adjustments to account for this fact need to be made outside the model.

Until now in this paper, the pros and cons of each method for modelling asset correlation have been reviewed and exemplified. It is an ongoing debate whether to choose a naïve or, perhaps, an over-simplified approach to explaining vessel return variation. Our aim is to propose a model in line with the gold mean philosophy, thereby minimizing the shortcomings of the two extreme methods. By choosing the multi-factor model, we can see from the resulting portfolio allocation that we more often than not end between the two extremes. Often quite close to the single-factor model, but still far from the same.

For some vessels the multi-factor model even goes beyond both alternative methods. Referring to table 10, our model weights five-year VLCC and Aframax tankers heavier than the other frameworks. The same for ten-year Suezmax, Aframax and Handymax tankers. On the other end, it proposes the lowest weight of all in five-year Capesize bulkers. This shows the fact that the model is capable of going beyond that of typical averaging models, deemed by some as the best alternative (Elton, et al., 2014). It all comes down to the model specification and choice of factors. In such a way, the multi-factor model can pick up interdependencies where other methods fall short and, by doing so, giving valuable insight to shipping investors. In our humble opinion, the proposed model may indeed be the superior choice.

The remainder of this paper is solely focused on the results from the proposed multi-factor model. We now turn to applying and testing the model in the following discussions.

5.4 Practical validity of results

Naturally, diversification within a segment is more common than diversification across segments. Such specialization focus with a clear-cut risk profile is common for any industry, leaving the responsibility of diversification to the investors. This concept is well-known in corporate finance for guiding the endeavors of firms. However, for investors in real assets such as vessels, there might be grounds to conclude that diversification should be obtained already on the initial investment level. In order to determine the potential superiority of diversifying vessel investments from a practical point of view, the following subsections cover both statistical and economic reasoning for cross-segment diversification.

The starting point for the discussion are the segment-specific portfolios from tables 11 to 13 for the multi-factor model. Comparing the investment universe within the confines of each segment with full diversification, we can see the differences in minimum variance frontiers illustrated in figure 8. Referring to figure 6, the frontiers are located according to the coinciding risk-return characteristics for each segment and encompassed by the frontier for the full investment universe. Immediately, we can see the differences between segments. The wet bulk portfolio seems to be the major contributor to the favorable characteristics of the fully diversified portfolios. Imagining the capital allocation line between levels of risk-free rate and the corresponding Sharpe-optimal portfolios, cf. figure 7, the optimal portfolio of wet bulk should be closest to the fully diversified portfolio. Hence, we can conclude that the wet bulk portfolio seems superior to the other segment specific portfolios from the perspective of a mean-variance optimizing investor. On the other hand, a risk-prone investor seeking the highest expected returns might prefer investing in a pure dry bulk fleet if splitting between vessels and a risk-free asset is not possible.

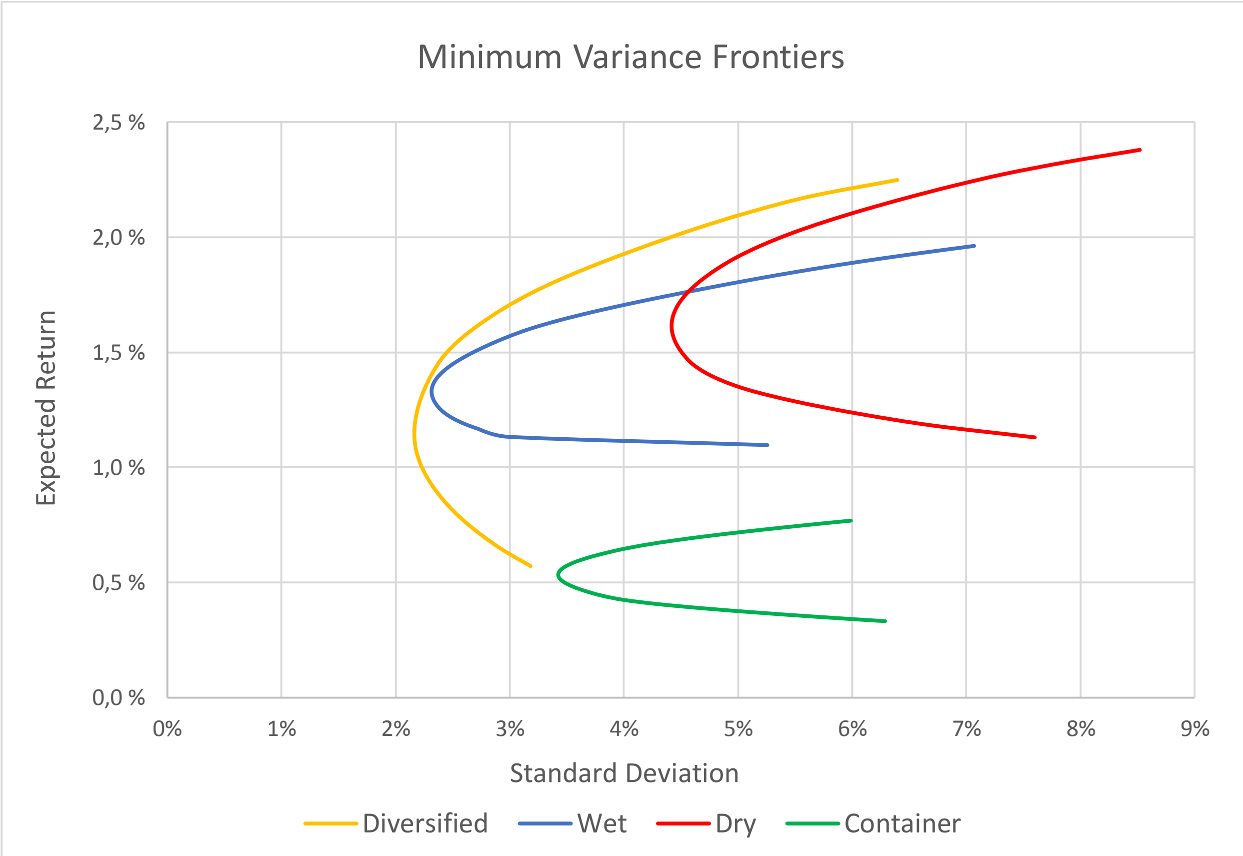

Figure 8 - The minimum variance frontiers for segment specific portfolios

5.4.1 Statistical evaluation

First and foremost, for proposed diversification to be deemed useful it is worth discussing the gains of diversification from a statistical point of view. In previous literature on the diversification properties of assets to a portfolio, the Gibbons, Ross, Shanken (GRS) test to determine statistical significance is often employed (Gibbons, et al., 2007). By applying the GRS-test (cf. appendix 2), we can prove whether the diversification benefits are actually statistically significant or not. Utilizing the fact that the Sharpe ratio measures the return and risk of each portfolio in a single number, we are able to statistically reject whether or not the increase in portfolio performance is significant. We test each segment-specific portfolio from figure 8 against the performance of the fully diversified fleet. The test is straightforward as long as the Sharpe ratio of the enhanced, i.e. diversified, portfolio outperforms the base portfolio. The results of the test are presented below:

Portfolio	*Sharpe(multi)*	GRS test W	*Test Stat.*	Signifiance
Diversified	0.533			
Dry	0.346	0.0707	*3.745*	***
Wet	0.506	0.0109	*0.579*	
Container	0.113	0.1259	*6.674*	***

*Table 14 - GRS-test on the significance of diversification benefits across segments (significance levels on the 1% level is marked ***)*

When comparing the gains of holding a cross-segment diversified portfolio with a pure dry bulk portfolio, the gains from diversification prove to be statistically significant on a 1% significance level. Accordingly, combining the risk-return characteristics of all segments clearly seems to be more beneficial than simply optimizing your portfolio within dry bulk. This is not surprising given the 0.186 increase in Sharpe ratio when comparing dry bulk to a diversified composition. As a consequence, the gains from diversification seem to be both economically and statistically significant based on Sharpe ratio.

Compared to an optimal wet bulk portfolio however, the diversified portfolio across segments does not prove to be statistically significant. From the statistician's point of view, the Sharpe ratio achieved by diversifying within wet bulk vessels seems to be high enough to discourage including other segments. In other words, the added risk-adjusted return from other segments is not high enough. In terms of Sharpe ratio, a diversified portfolio achieves a mere 0.027 increase over the pure wet bulk. Considering the fact that a higher Sharpe ratio always is preferable, this comparison seem to be economic but not statistically significant

Thirdly, holding a diversified portfolio proves more beneficial than only a diversified container portfolio. This hardly comes as a surprise, since container vessels offer the lowest Sharpe ratio in our sample. The difference in Sharpe ratio between the enhanced portfolio and base portfolio is the largest in our sample, 0.420 to be precise. Such a large increase must be said to be economically significant, as it offers a large gain in the risk-adjusted return.

Based on the findings of this paper, holding a diversified portfolio only makes "statistical sense" if you are in dry bulk or container shipping. Since previous studies on diversification in shipping, mainly Magirou et al. (1997), Melbø (2013) and Patitsas (2004) do not discuss the statistical relevance of their results, comparison to earlier research is difficult. However, statistically rejecting results that imply economic significance is not unheard of when employing the GRS-test (Rubens, et al., 1998). Furthermore, the full employment assumption and "sticky prices" bias might remove some of the possible diversification gains, and consequently reduce the power of the test. To conclude, only dry bulk or container shipping might enhance returns by diversifying across segments. However, combining the three segments could be more difficult and costly than it may seem regarding operations and management.

5.4.2 Economic evaluation

Although our focus in this paper does not include operations and management as such, shipping has historically been an integrated business. Being a shipowner has traditionally entailed brokerage, manning, operations etc. From this vantage point, diversification across segments has been impeded.

First, while there are no considerable economies of scale on the firm level in bulk shipping, this is not true in container freight. Mentioned in the literature review, container shipping requires a lean organization in addition to simply having the capital available to purchase a vessel. Cargo handling and efficient scheduling are perhaps the two most important aspects of the container business, since reliant and on-time service is essential.

Secondly, combining dry and wet bulk shipping could prove difficult, as the two rely on quite different cargo handling facilities. Additionally, operating in both dry and wet bulk markets necessitates a thorough understanding of two fundamentally different markets, with specific demand patterns and trade flows. Such expertise represents a cost for the firm under the assumption that diversification between segments must include operations and management. Knowledge of the market and relations to cargo owners seems to be even more important for smaller product tankers than in bulk in general.

On the other hand, if the managerial expertise required in different segments is somewhat equal, diversification may add value for investors. For instance, if a company diversifies to secure less volatile earnings, it could increase financial leverage yielding value to its investors in form of extra tax shields. The relationship between reduced volatility in earnings and debt in shipping, referred to as the Shipping Corporate Risk Trade-off hypothesis has been studied empirically. According to Merikas et al. (2011), the inverse relationship between market and financial risk is stronger after the 2008 financial crisis. An important result of Merikas et al. is the conclusion that market risk in shipping is actually accounted for in firm capital structure. The result of the study proves that the shipping industry is not foreign to the concept of risk management through operational or strategic decisions.

Choosing which segment to operate in is analogous to deciding firm strategy. In the mid 1980's, a project co-operation between McKinsey & Co. and the Centre for International Economics and Shipping at the Norwegian School of Economics developed a framework for potential shipping strategies. Albeit designed for industry analysis, the matrix is also a good starting point for understanding the nature of shipping segments.

Economies of scale	Insignificant	Significant
Significant	**Contract Shipping** - Concentrated industry - Positive scale effect of fleet size - Fairly homogenous service - Liquid secondhand market - Close customer relations	**Industry Shipping** - Concentrated industry - Positive scale effects of fleet size - Specialized services - Difficult secondhand market - Tailor-made customer service
Insignificant	**Commodity Shipping** - Fragmented industry - No scale effect in fleet size - Homogenous service - Liquid secondhand market - Little direct customer contact	**Specialty Shipping** - Local monopolies - Limited scale effects of fleet size - Specialized services - Difficult secondhand market - Direct customer contact

Differentiation

Figure 9 – Strategic types of shipping (Wijnolst & Wergeland, 2009, p. 108)

Considering the matrix above, the characteristics of commodity shipping (dry and wet bulk) and industry shipping (container) are seemingly opposite. In contrast to bulk shipping, economies of scale are significant in liner trade. Since the proportion of actual short-term variable costs in container shipping is limited to cargo handling, the importance of having a sufficiently large fleet cannot be understated. Elaborated by Wijnolst & Wergeland, “the existence of economies of scale will inevitably lead to consolidation pressures in the pursuit of scale advantages” (Wijnolst & Wergeland, 2009, p. 108). However, it is worth mentioning that the pursuit of such advantages in global container trade is not equally present in container feeder, i.e. short-haul, trade. As an increasing share of container trade is done on the open market, this segment is slowly evolving into what may resemble a commodity shipping industry from a shipowner’s perspective. This is an apparent trend in other niche and specialized segments as well, particularly for LNG (Wang & Notteboom, 2011).

Regarding managerial complexity, container shipping differs substantially from bulk shipping. Primarily, container shipping offers a specialized service whilst bulk does not. Providing specialized services can be both time and capital consuming, putting unnecessary strain on the shipping firm. Additionally, the firm must develop a sufficient customer support service if it enters into container shipping, as contact with customers is an important part of container trade. In contrast, most customer interaction in bulk shipping is done through an intermediary broker. Thus, in order to provide both bulk and container shipping, the firm must use it additional resources eventually requiring even higher returns for it to be profitable. The added complexity of cross-segment diversification may very well prove to be too costly. However, in recent years, alternatives to physical investing in shipping has emerged.

Highlighted by Drobetz et al. (2012), hedging business risk with operational or strategic decisions may, as mentioned above, prove too costly, complicated and time consuming. On the other hand, the development of freight forward agreements (FFAs) may enable a shipowner to seek exposure to a segment without engaging in operations. Furthermore, the design of FFAs allow investors to easier purchase smaller units than buying a physical vessel. In Cullinane's article (1995), FFAs are added as possible investment assets to improve the efficiency of a portfolio. However, it is unclear to what extent such investment strategies are being pursued in practice. For the most part, FFAs are mainly done to either lock in a freight rate or manage default risk.

In addition to FFAs, the extensive use of outsourcing ship management and operations imply that diversification across segments is quite possible, i.e. from a practical perspective. Traditionally, being a shipowner often entailed manning, brokerage, maintenance, financing vessels etc. However, recent trends in shipping following deregulation and lowered transaction costs point towards specialization in all aspects of the value chain (Lorange, 2009, p. 82). Moving away from the integrated shipping firm, more and more companies are identifying one particular aspect to focus on. Note that this must not be confused with earlier specialization trends. Previously, firms have specialized within a segment but still been integrated within the value chain. Thus, the new trend is focusing on core competences (e.g. chartering, S&P, manning etc.), outsourcing non-essential business units.

A good example of this is Frontline Ltd. Previously, Frontline both owned and operated their tanker fleet as an integrated firm. Following the establishment of Ship Finance International Ltd. (SFI), Frontline could sell its vessels to SFI, re-charter the vessels on bare-boat charter contracts, and eventually put the vessels back on spot contracts. By doing this, Frontline specializes in chartering and trading (brokerage) whilst SFI specializes in owning and financing. More importantly, SFI is able to raise capital at a substantially lower cost compared to what Frontline was able to earlier. Another example of a pure shipowning firm is Seaspan Ltd., specializing in "leasing" out their container fleet on long time charter contracts. According to Lorange, "historically, many shipping companies have specialized, but without this aim of decomposing the value chain to determine which core activities they should choose to focus on" (Lorange, 2009, p. 83). In 2008, it was reported that the percentage of container liners outsourcing ownership of vessels rose from 23 percent to 52 percent over a ten-year period (Lorange, 2009, p. 91). Expanding on this trend, a viable option could perhaps be to specialize in owning vessels across segments in order to reduce their overall cash flow volatility even further. This seems to be the case for SFI, holding an apparently well-diversified across all three dimensions, i.e. segment, size and age (Ship Finance International Ltd., 2015). However, it is beyond the scope of this paper to discuss shipping strategy to a great length. To conclude, recent trends in shipping enable diversification across segments without hefty management costs.

5.5 Sensitivity analysis

As explained in chapter 4, our period of analysis spans over 18 years. Referring to finance literature on the subject of portfolio optimization, the usual period of study for monthly returns is from 5 to 7 years. The reasons for these limits are slowly evolving fundamental conditions over longer periods that are seemingly irrelevant for future investment decisions. Regarding shipping as an industry with long business cycles and relatively stable market fundamentals, we have decided to go far beyond this prescribed rule of thumb for time series. As a response to this fact, and a curiosity as to time varying betas (Bollerslev, et al., 1988), we will in this section split our time series. The truncated series will be used in the same regressions as above, and differences are briefly commentated.

Splitting our time series in the middle give the regression results for the multi-factor model in appendix 9. When comparing loadings and t-statistics with the results for our entire series in table 7, we find many of the same variables to be statistically interesting and relatively close values for the majority of factor loadings. Admittedly, the regression results are not exactly the same. Some of the important differences are as follows.

For wet bulk, the coal price variable seems to have been a more important factor during the first half than the second, with significant and higher loadings for the first period. The same can be observed for the price index. There is some evidence for the world oil stock variable becoming a more important factor towards the end of the period. Overall, the differences are far from deterrent.

For dry bulk, the differences are even less clear when comparing to the full series. Coal price and the price index remain the dominant variables with values well within reason. A tendency however, is that OECD industrial production has perhaps become more important the last half of the period. The same can be said for world steel production in explaining five-year Capesize returns.

Finally, container returns do show some differences during the period. The explanatory power of the price index was dominant during the first half. This has changed to some degree in favor of OECD production.

Based on the split data regressions, there is some reason to believe there exist sources of error and especially for the container returns. However, we are reasonably satisfied with the comparison. In establishing such factor models, we are aware of the risk of including noise and uncertainty. Still, compared to the explanatory power of a single-factor model and the portfolios recommended using the naïve method, there is reason to believe that the risks are by far outweighed by the benefits.

5.6 Case - Investing USD 1 billion in vessels

Our portfolio recommendations indicate the proportion of capital to invest in each asset, not proportions of vessels, e.g. in terms of DWT. To demonstrate the implications of our findings we will construct a fictional portfolio of the standard vessels from table 3. Following the underlying simplifications of our model, the investment universe is limited to three vessel segments. For each segment there is the predetermined number of vessel sizes, i.e. 4 in wet bulk, 3 in dry and 2 in container. Furthermore, each vessel can either be five or ten years of age.

The starting point will be the proposed capital allocation following the multi-factor model from table 9:

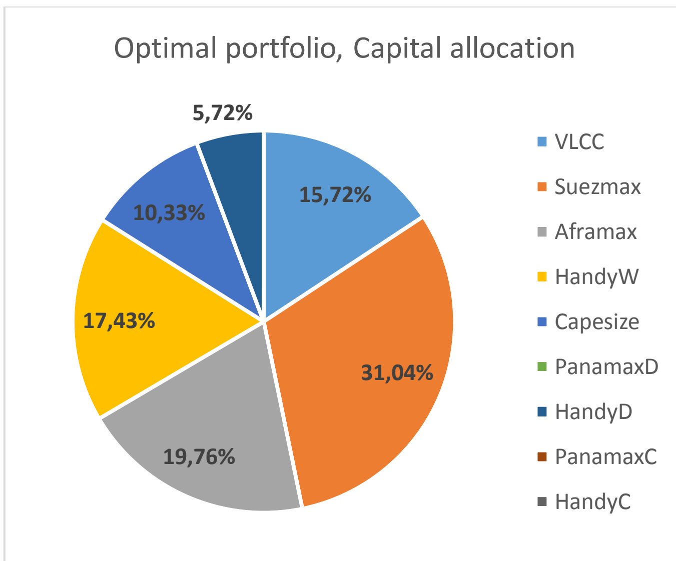

Figure 10 - The optimal capital allocation between segments and vessels

Note that for a fully diversified portfolio among all segments the optimal fleet excludes any investments in dry bulk Panamax or any of the two container vessels. This is of course in line with the optimization results above. Since no investments are made in container vessels, we have no issues with the TEU size measure for container and will be able to include a figure for the DWT allocation as well as percentage of number of vessels. We find the optimal portfolio using vessel prices from year end 2014, i.e. the last observation in our sample:

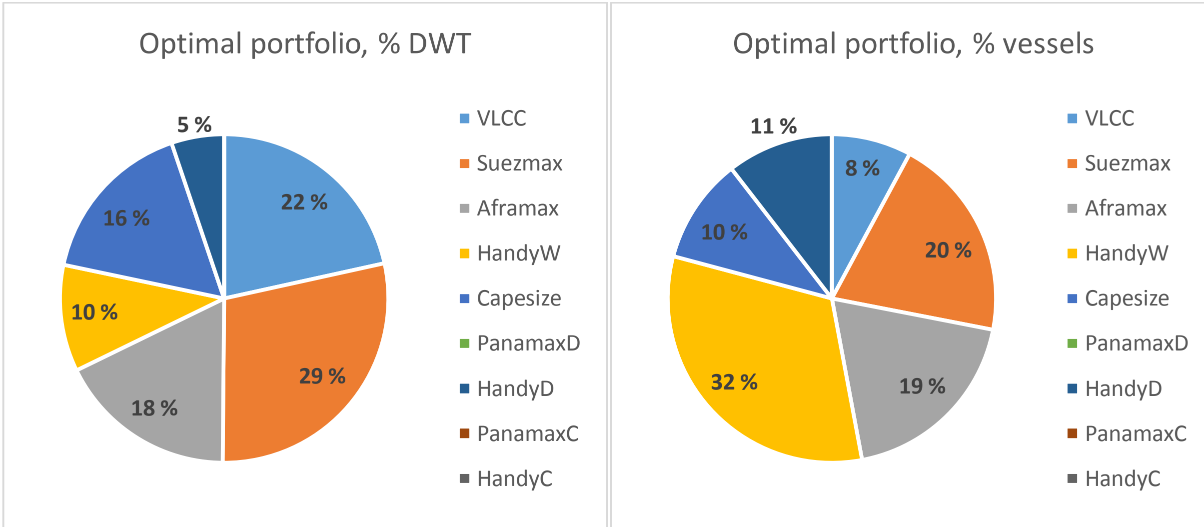

Figure 11 - Optimal vessel portfolio in percentage of DWT (left) and number of vessels (right)

The proportions are altered when translating capital into tonnage and number of vessels. The changes are easily explained by the fact that larger vessels are cheaper per DWT, but definitely more expensive as a whole. Dividing the investment capital of $1 billion according to our proposal would approximately (rounded numbers) translate to the following fleet of vessels:

Vessel	Wet bulk				Dry bulk		(Per age)
fleet	VLCC	Suezmax	Aframax	Handymax	Capesize	Handymax	Sum
5 year	1	3	2	3	1	1	11
10 year	2	3	4	7	2	2	20
Total	3	6	6	10	3	3	31

Table 15 - The approximate number of vessels in the optimal fleet for a fictional $1 billion shipping fund

If there were to be restrictions to invest within each segment, the same procedure provides the following figures:

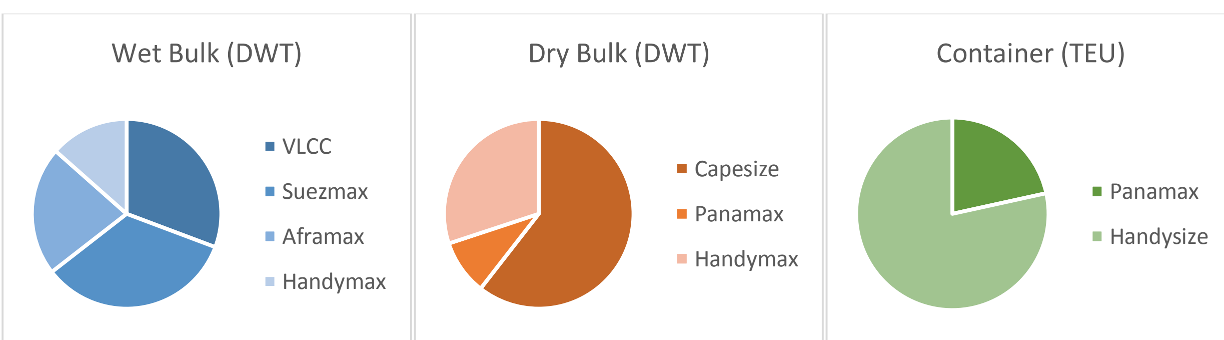

Figure 12 - Optimal portfolios for each segment based on capacity (DWT or TEU)

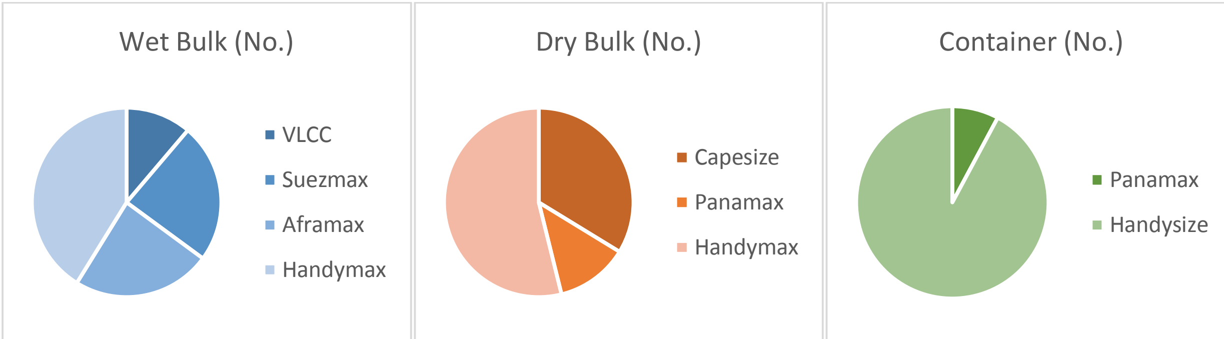

Figure 13 - Optimal portfolios for each segment based on number of vessels

The specific numbers for all figures are provided in appendix 8, for convenience.

The demonstrated results above show the recommended allocation in vessels for a rational investor to maximize expected returns per unit of risk. As mentioned, the return and variance inputs are solely base on the historical sample. These might be subject to change if adjusted for subjective opinions of shipping analysts. However, under the assumption that the historic data is somewhat representative for future expectations, our recommendations are a reasonable starting point.

6. Concluding remarks

At the outset of this paper, we argued that diversification in shipping may have more potential than first assumed. To our knowledge, this is the most comprehensive empirical study to date exploring this potential. In contrast to the single-index approach of earlier research (Magirou et al. (1997), Patitsas (2004), Schilbred (1992) and Melbø (2013)), we propose a more advanced model to explain vessel return variation.

First, we find the risk-return characteristics of each vessel type to be clustered within each segment as shown in figure 6. In extant literature, such relationships had yet to be proved empirically. This highlights, now more than ever, the necessity of assessing financial theory on risk management in the perspective of shipping.

Secondly, our findings show important factors for explaining variations in returns from shipping operations across the three segments. Including macroeconomic factors instead of a single market index as explanatory variables increased the explanatory power of the model vastly. Previous literature employing multi-factor to shipping has been limited to studying stocks. For the first time, we are able to study the relationship among real assets using such a model. Furthermore, we construct a model for the general shipping industry as a whole, where earlier attempts have been made somewhat sector specific. This is even more important concerning diversification when held against the aforementioned finding.

Lastly, we evaluate the statistical and economic significance of applying a portfolio approach to real investments in shipping. Holding a diversified portfolio statistically outperforms a segment specific dry bulk or container portfolio. The same cannot be inferred for a pure wet bulk portfolio, due to its strong historical performance. From an economic perspective however, there are no apparent obstructions for diversifying across all three segments. Recent trends in shipping enable holding a diversified portfolio following the decomposition of the value chain.

Our results are important for shipping strategy and business risk management. The findings of our paper should be interesting for several stakeholders in the shipping industry, including shipowners, hedge funds, shipping banks etc. A good example could be a hedge fund manager seeking the optimal risk-return trade-off in his portfolio or a shipping bank enhancing their understanding of risk in their loan portfolio.

The perspective of this paper has been aimed towards constructing a comprehensible model with forward looking application. In this respect, the variables should be general and observable, thus enabling reasonably accurate financial prediction. To further increase the explanatory power of a multi-factor model, a suggested future topic would be to examine microeconomic variables. In addition, one might apply a model for unexpected changes to macroeconomic variables instead. This is a common approach in macroeconomic models explaining stock returns under the assumptions of efficient markets.

Due to missing data, it has been necessary to make certain assumptions regarding our time series. Available data will increase for prospective studies, as more observations are added to the database, enabling more detailed future research by increasing the sample length and scope. This is a response to more trade being done on the open market, a trend perhaps most obvious in container and specialized shipping. Another simplification is the capital depreciation factor, α, assumed to be constant, disregarding market sentiment. An alternative procedure is the linear depreciation between market value and scrap value for each period (Patitsas, 2004).

We are satisfied with the findings of this paper, especially as a foundation for future research to build upon. Hopefully, it will spark an interest for multi-factor frameworks used in the assessment of real asset investing, specifically explaining the volatile investment environment present in shipping markets. Regardless of our results, this paper might pave the way for exiting future research.

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Appendix

1. Statistical properties of time series data

In order to use the ordinary least squares regression method (OLS) in this paper, one must assume certain assumptions concerning the properties of the data (Wooldridge, 2013).

1. The first assumption states that the time series process follows a model linear in its parameters.
2. In the sample, no independent variable is constant nor a perfect linear combination of the others.
3. For each t, the expected value of the error u_t, given the explanatory variables for all time periods, is zero (i.e. Zero Conditional Mean assumption).
4. Conditional on **X**, the variance of u_t is the same for all t (Homoscedasticity assumption).
5. Conditional on **X**, the errors in two different time periods are uncorrelated (No serial correlation assumption).
6. The errors are independent of **X** and are independently and identically distributed as Normal$(0, \sigma^2)$.

1.1 Testing for serial correlation

In our static model, we can assume that strict exogeneity holds, i.e. that assumption 3 above is not violated in its strictest form. In order to test for autocorrelation, we use the Durbin Watson test statistic (Wooldridge, 2013) on the residuals (û) from the regressions.

$$DW = \frac{\sum_{t=2}^{n}(\hat{u}_t - \hat{u}_{t-2})^2}{\sum_{t=1}^{n}\hat{u}_t^2}$$

Our null hypothesis is no serial correlation in the error terms. Fortunately, all our regressions are outside the rejection range of the test, either below the lower bound or above the upper bound. Inferring that we cannot reject our null hypothesis of no serial correlation. Consequently, serial correlation does not seem to be of an issue in our specified models.

If serial correlation were present, the standard errors and t-statistics from our regression model above would not be valid. Thus, we would be prone to making wrong conclusions regarding the statistical significance of the factors.

1.2 Random walk and spurious regressions (unit root)

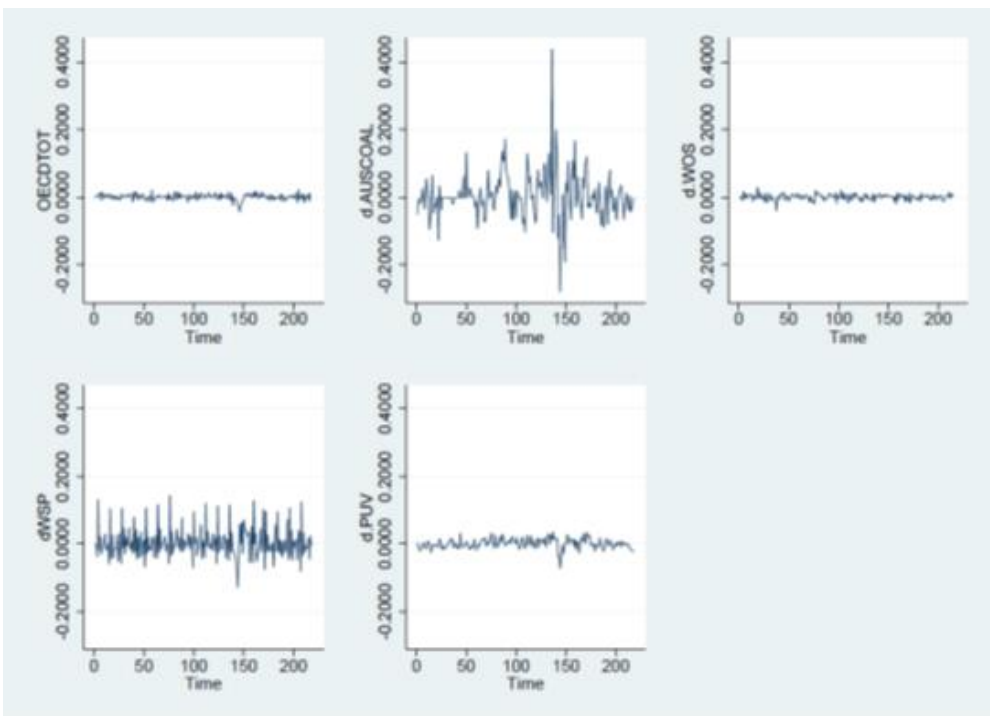
When studying time series data, the concept of unit root and stationarity are central. For our paper, static regression models are crucial tools in creating covariance matrices and studying correlations among vessels. In classical single-index model, one does not need to worry about spurious regression results since the variables at hand are returns. For example, asset returns, contrary to asset prices, are said to be stationary. However, when using a multi-factor model, this simplicity is no longer present. Spurious regression results simply imply finding correlations between variables that, in reality, do not exist. A stochastic time series is said to be stationary if the following conditions are met (Hill, et al., 2011):

$$E(y_t) = \mu \; constant \; mean$$

$$Var(y_t) = \sigma^2 \; constant \; variance$$

$$Cov(y_t, y_{t+s}) = Cov(y_t, y_{t-s}) = \gamma_s \; covariance \; depends \; only \; on \; s, not \; on \; t$$

As mentioned above, in this paper, we have introduced models where other variables than returns are being included. Such time series often display random walk, with or without trend. In order to correct for this, we generate the first difference of the variables (i.e. change). If the generated variables are stationary, the variables are said to be integrated of order 1, or I (1). To test for this, we conduct a Dickey-Fuller test for unit root (Hill, et al., 2011) on our "new" variables. The graphs below display the first differenced variables used in our multi-factor model.



If a dickey-fuller test for unit root (including drift) on the generated variables leads to rejection of the null hypothesis, the new variables can be included as explanatory variables for our static models. Fortunately, we can keep the null hypothesis of no unit root for all our first differenced variables. The conclusions above could be derived simply by looking at a graphical illustration of the variables. Clearly, all the variables oscillate around a constant mean with somewhat constant variance.

2. Relative efficiency of a portfolio

In the recent literature on the diversification properties of efficient portfolios, a procedure has emerged that allows the application of a significance test to analyze whether the addition of a further component to a portfolio enhances diversification (Gibbons, et al., 2007). The test utilizes the fact that the Sharpe ratio measures the return *and* the risk characteristic of an asset or a portfolio in a single number. The test has been extensively used for investigating the properties of real estate investments as alternative investments, in addition to more traditional stocks and bonds.

The test compares the Sharpe ratio of a base portfolio to that of a diversified enhanced portfolio.

The following null hypothesis is testable:

H_0: The Sharpe ratio of portfolio *B* (S_B) does not differ significantly from the Sharpe ratio of portfolio *E* (S_E)

H_A: The Sharpe ratio of portfolio *B* (S_B) does differ significantly from the Sharpe ratio of portfolio *E* (S_E)

Clearly, we would hope to reject our null hypothesis inferring a statistical gain from diversification.

The test is a two-sided Wishart-distributed test. Furthermore, it can be proved that the Wishart-distributed test can be transformed into an F-distribution (Gibbons, et al., 2007).

The test statistic *W* can be expressed by the following equation:

$$W = \left[\frac{\sqrt{1+S_E^2}}{\sqrt{1+S_B^2}}\right] - 1$$

The test is straightforward as long as the fraction above is positive, i.e. enhanced portfolio outperforms base portfolio. Under certain assumptions, the F distribution can be approximated:

$$\frac{T(T-N-1)}{N(T-2)} W \sim F_{\%sig,N,(T-N-1)}$$

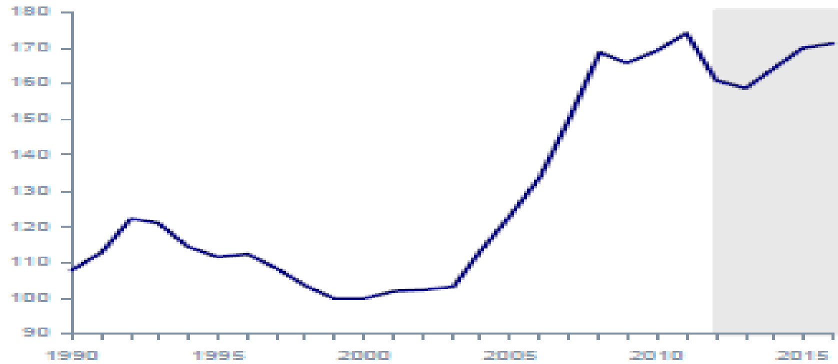
Where *T* is the amount of observations (218) and *N* the number of assets/portfolios (4).

Consequently, one should reject H_0 if our "modified" test statistic is above the given F-value.

The power of the test depends on the relation between *T* (observations) and *N* (assets/portfolios). According to Gibbons et al., there is a threshold of $\frac{T}{N} \geq 3$ .In smaller samples, the test may prove inconclusive or lead to wrong conclusions. In our case, we can simply state that our data set is way beyond this threshold value.

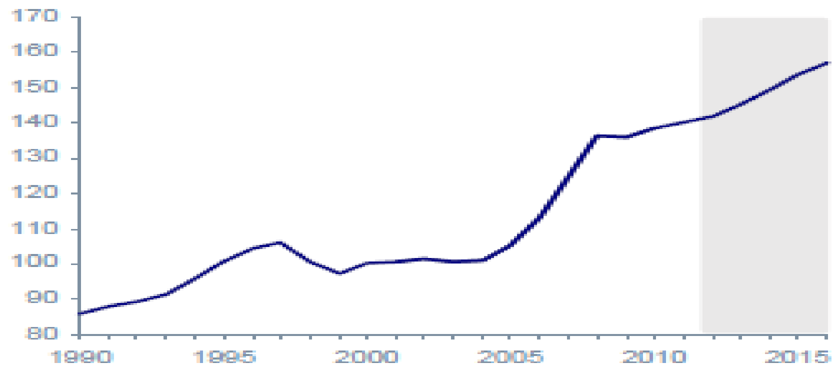
3. OPEX figures from Drewry (Marsoft AS) – 2000 = index base year

Figure 1.1 **Evolution of operating costs (Index: 2000=100): total operating costs**



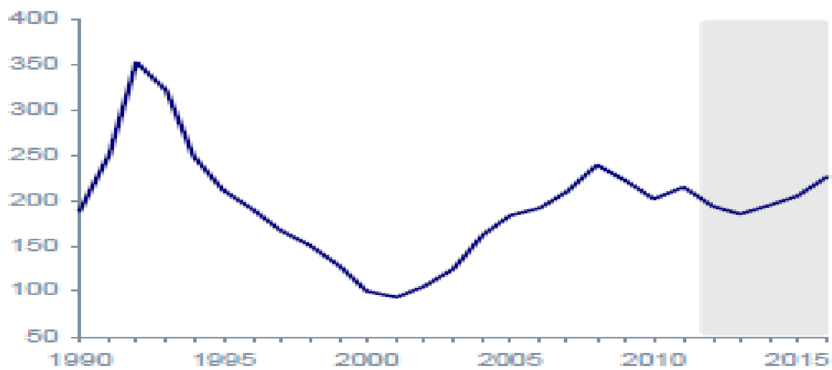
Source: Drewry Maritime Research

Figure 1.2 **Evolution of operating costs (Index: 2000=100): manning**



Source: Drewry Maritime Research

Figure 1.3 **Evolution of operating costs (Index: 2000=100): insurance**



Source: Drewry Maritime Research

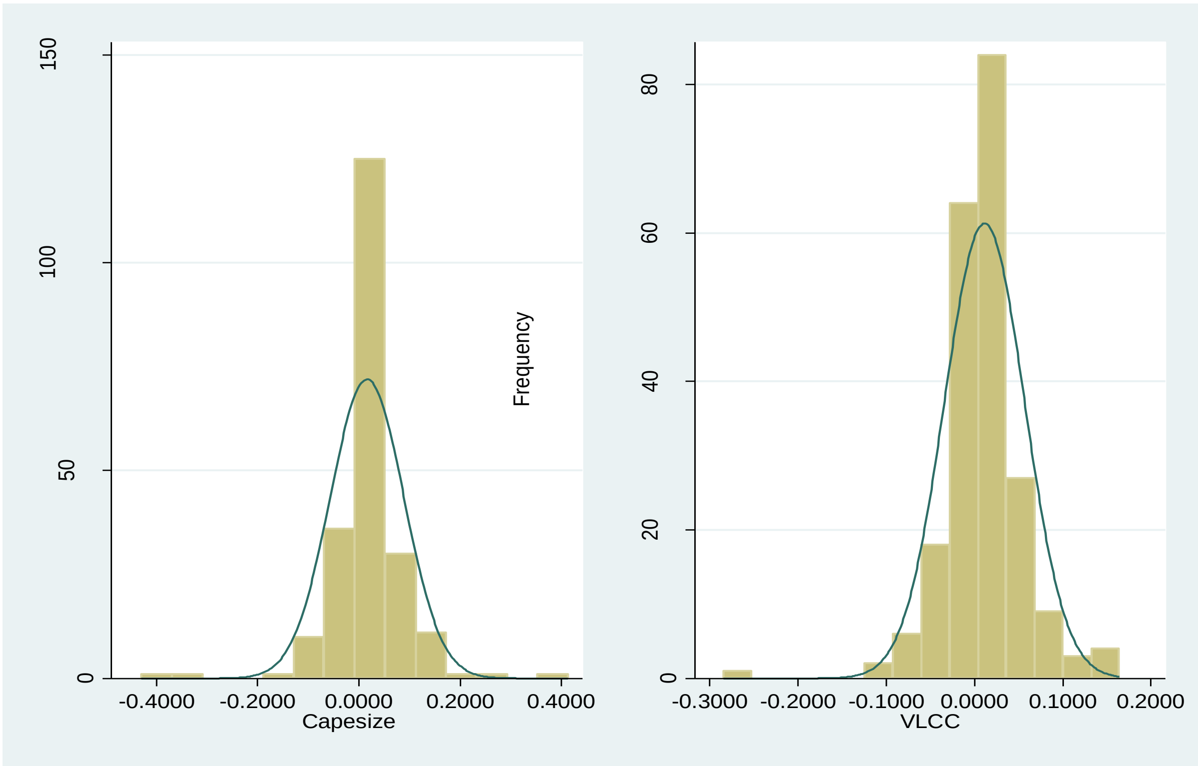
4. Actual and forecasted OPEX figures by Drewry (Marsoft AS)

Table 1.2 **Total operating costs – 2008-2016** (US$ per day)

Sector	Vessel	2008	2009	2010	2011	2012	2013	2014	2015	2016
Oil Tankers (dwt)	MR1	7,860	8,270	8,185	8,440	8,096	8,045	8,319	8,602	8,880
	LR1	8,650	8,609	8,520	8,880	8,528	8,473	8,765	9,067	9,360
	LR2	9,090	9,010	9,045	9,465	8,820	8,761	9,064	9,378	9,681
	Aframax	9,899	9,793	10,245	10,475	9,796	9,714	10,051	10,401	10,738
	Suezmax	11,165	11,111	11,436	11,815	10,704	10,606	10,977	11,362	11,731
	VLCC	12,670	12,290	12,807	13,280	12,410	12,288	12,719	13,166	13,594
Chemical Tankers (dwt)	5-6,000	5,596	5,560	5,678	5,865	5,645	5,611	5,803	6,001	6,207
	8-9,000	6,263	6,226	6,381	6,588	6,279	6,273	6,486	6,706	6,933
	10-12,000	6,901	6,866	6,972	7,190	6,890	6,893	7,125	7,364	7,613
	18-20,000	7,395	7,353	7,459	7,700	7,346	7,342	7,591	7,849	8,117
	22-24,000	7,935	7,882	8,000	8,263	7,851	7,838	8,105	8,381	8,667
	35-37,000	8,683	8,620	8,748	9,034	8,724	8,711	9,011	9,321	9,642
	40-45,000	9,058	8,987	9,121	9,420	9,057	9,036	9,347	9,669	10,003
LPG (cbm)	3-5,000	4,729	4,617	4,680	4,783	4,748	4,725	4,883	5,046	5,215
	6-8,000	5,210	5,099	5,182	5,345	5,257	5,228	5,403	5,584	5,771
	12-15,000	6,246	6,103	6,280	6,306	6,217	6,166	6,373	6,587	6,808
	30-35,000	7,783	7,665	7,810	7,979	7,887	7,851	8,114	8,387	8,668
	50-55,000	8,426	8,221	8,374	8,570	8,407	8,359	8,641	8,932	9,233
	75-80,000	9,314	9,133	9,295	9,489	9,313	9,253	9,566	9,889	10,224
LNG (cbm)	140-150,000	14,105	13,833	14,064	14,510	15,510	15,430	15,965	16,518	17,091
Dry Bulk (dwt)	Handysize	5,278	5,204	5,284	5,474	4,966	4,933	5,100	5,272	5,451
	Handymax	5,433	5,327	5,409	5,569	5,061	5,028	5,198	5,375	5,558
	Supramax	6,222	6,115	6,204	6,401	5,830	5,799	5,995	6,197	6,407
	Panamax	6,825	6,692	6,762	6,966	6,483	6,445	6,664	6,890	7,125
	Post Panamax	7,255	7,035	7,099	7,356	6,791	6,745	6,975	7,212	7,457
	Capesize	8,117	7,891	8,012	8,283	7,522	7,472	7,726	7,989	8,260
	VLOC	8,452	8,315	8,460	8,882	7,834	7,776	8,041	8,315	8,599
Containers (teu)	500-750	4,434	4,262	4,350	4,560	4,299	4,285	4,427	4,575	4,727
	1-2,000	5,238	4,985	5,088	5,295	4,985	4,961	5,117	5,289	5,467
	2-3,000	6,761	6,355	6,498	6,756	6,295	6,234	6,444	6,661	6,886
	3-4,000	8,747	8,396	8,586	9,120	8,423	8,318	8,602	8,895	9,198
	5-6,000	10,619	10,063	10,300	9,890	9,377	9,268	9,583	9,910	10,247
	8-9,000	11,612	11,255	11,523	11,835	11,179	11,026	11,405	11,798	12,205
	10-12,000	13,132	12,950	12,897	13,420	12,606	12,436	12,868	13,315	13,778
General Cargo (dwt)	5-10,000	4,090	4,128	4,181	4,299	4,302	4,279	4,426	4,578	4,736
	15-20,000	4,776	4,807	4,908	5,100	5,098	5,073	5,249	5,432	5,621
Reefer (cft)	550,000	6,173	6,253	6,317	6,495	6,434	6,410	6,637	6,872	7,116
Ro-Ro (dwt)	10,000	5,563	5,650	5,915	6,115	6,001	5,972	6,178	6,391	6,613

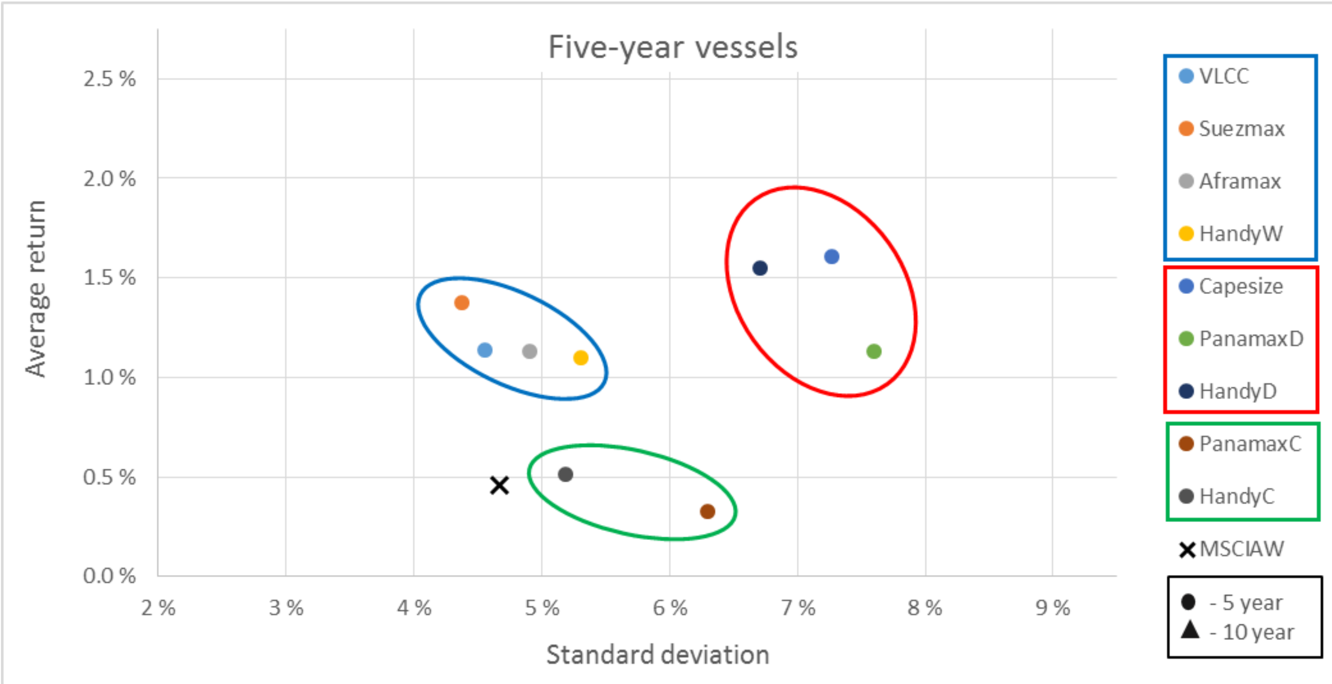
Source: Drewry Maritime Research

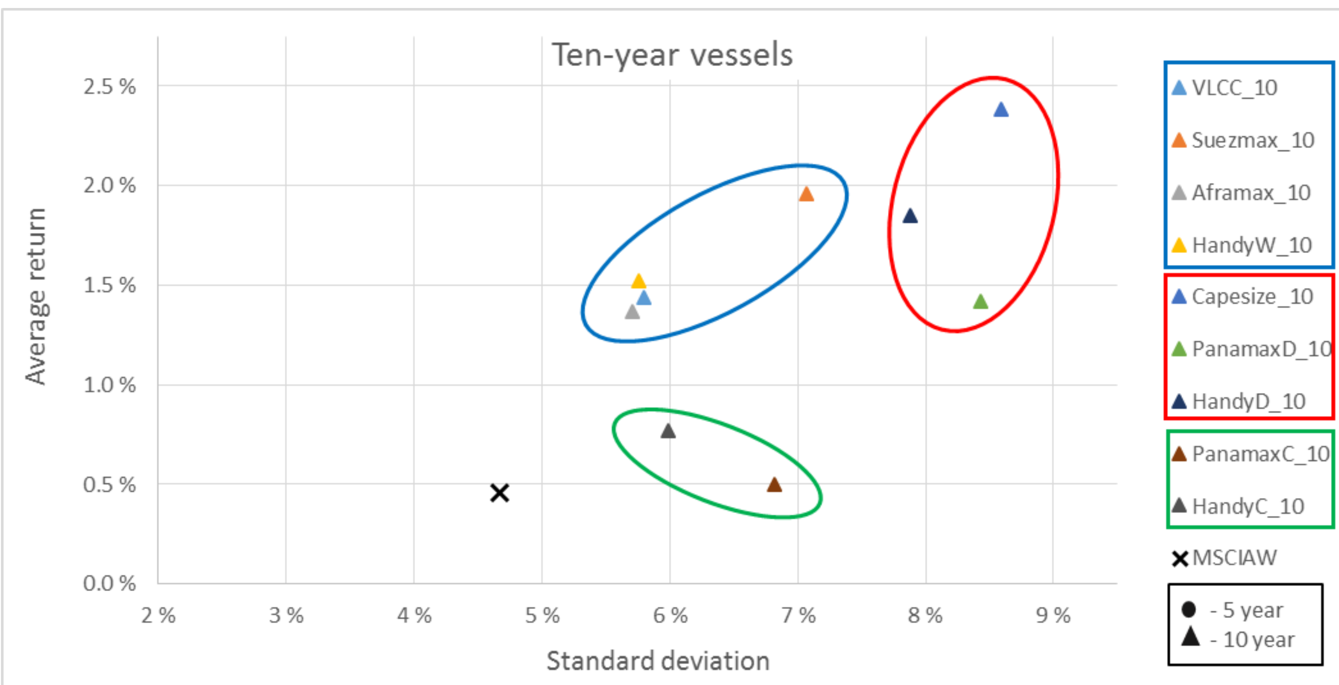
5. Distribution of vessel returns, exemplified by Capesize and VLCC returns



Studying the distribution of vessel returns, we find them to be relatively close to normal and symmetric. This strengthens the validity of results using the optimizing techniques of this paper, cf. chapter 3. The figure above illustrates this fact for two of our vessel types, i.e. Capesize and VLCC. The other vessels show similar characteristics.

6. Mean-return characteristics isolated for five- and ten-year vessels





7. Variance-Covariance (VCV) and correlation matrix for each method

7.1 Naïve method

VCV:

			5 year old vessels									10 year old vessels								
			Wet bulk				Dry bulk			Container		Wet bulk				Dry bulk			Container	
			VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC	VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC
5 year old vessels	Wet bulk	VLCC	0.002073	0.001589	0.00155	0.001056	0.001055	0.001417	0.001325	0.001323	0.000797	0.002306	0.001935	0.001712	0.001166	0.001407	0.001783	0.001451	0.001305	0.000966
		Suezmax	0.001589	0.001917	0.001512	0.001026	0.000898	0.001117	0.000991	0.001212	0.000613	0.001781	0.002232	0.001686	0.00109	0.001054	0.001393	0.001102	0.001266	0.000721
		Aframax	0.00155	0.001512	0.002403	0.00111	0.000895	0.001332	0.001247	0.00105	0.000639	0.001708	0.002035	0.002222	0.001254	0.001444	0.001819	0.001389	0.001132	0.000682
		HandyW	0.001056	0.001026	0.00111	0.002816	0.000674	0.001224	0.001024	0.001155	0.000929	0.001155	0.001858	0.001246	0.001684	0.001111	0.001518	0.001041	0.001159	0.00104
	Dry bulk	Capesize	0.001055	0.000898	0.000895	0.000674	0.00529	0.002961	0.0027	0.001004	0.000996	0.00139	0.001188	0.001152	0.000942	0.00287	0.00367	0.003123	0.000986	0.001145
		PanamaxD	0.001417	0.001117	0.001332	0.001224	0.002961	0.005774	0.004032	0.001693	0.001747	0.001752	0.001556	0.001459	0.001619	0.004949	0.005524	0.004429	0.001722	0.001852
		HandyD	0.001325	0.000991	0.001247	0.001024	0.0027	0.004032	0.004494	0.001609	0.001578	0.001675	0.00137	0.001417	0.001501	0.004097	0.00464	0.004437	0.001616	0.00159
	Conta	PanamaxC	0.001323	0.001212	0.00105	0.001155	0.001004	0.001693	0.001609	0.003967	0.0021	0.001279	0.001337	0.001071	0.001192	0.000893	0.001832	0.001683	0.003815	0.002165
		HandyC	0.000797	0.000613	0.000639	0.000929	0.000996	0.001747	0.001578	0.0021	0.002687	0.000831	0.000861	0.000647	0.000997	0.001457	0.00186	0.001829	0.002247	0.002688
10 year old vessels	Wet bulk	VLCC	0.002306	0.001781	0.001708	0.001155	0.00139	0.001752	0.001675	0.001279	0.000831	0.003363	0.002247	0.002037	0.001345	0.001948	0.002309	0.001686	0.001246	0.001046
		Suezmax	0.001935	0.002232	0.002035	0.001858	0.001188	0.001556	0.00137	0.001337	0.000861	0.002247	0.004998	0.002327	0.001319	0.001323	0.001991	0.001493	0.001548	0.000966
		Aframax	0.001712	0.001686	0.002222	0.001246	0.001152	0.001459	0.001417	0.001071	0.000647	0.002037	0.002327	0.003258	0.001483	0.001529	0.002007	0.001516	0.001151	0.000829
		HandyW	0.001166	0.00109	0.001254	0.001684	0.000942	0.001619	0.001501	0.001192	0.000997	0.001345	0.001319	0.001483	0.003316	0.001514	0.001959	0.001564	0.001142	0.001172
	Dry bulk	Capesize	0.001407	0.001054	0.001444	0.001111	0.00287	0.004949	0.004097	0.000893	0.001457	0.001948	0.001323	0.001529	0.001514	0.007383	0.005277	0.004841	0.000868	0.00153
		PanamaxD	0.001783	0.001393	0.001819	0.001518	0.00367	0.005524	0.00464	0.001832	0.00186	0.002309	0.001991	0.002007	0.001959	0.005277	0.007102	0.005209	0.001853	0.002148
		HandyD	0.001451	0.001102	0.001389	0.001041	0.003123	0.004429	0.004437	0.001683	0.001829	0.001686	0.001493	0.001516	0.001564	0.004841	0.005209	0.006205	0.001744	0.001992
	Conta	PanamaxC	0.001305	0.001266	0.001132	0.001159	0.000986	0.001722	0.001616	0.003815	0.002247	0.001246	0.001548	0.001151	0.001142	0.000868	0.001853	0.001744	0.004654	0.002438
		HandyC	0.000966	0.000721	0.000682	0.00104	0.001145	0.001852	0.00159	0.002165	0.002688	0.001046	0.000966	0.000829	0.001172	0.00153	0.002148	0.001992	0.002438	0.003586

Correlation:

		5 year old vessels									10 year old vessels								
		Wet bulk				Dry bulk			Container		Wet bulk				Dry bulk			Container	
		VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC	VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC
Wet bulk	VLCC	1	0.797179	0.694248	0.437064	0.318525	0.409443	0.4342	0.461184	0.337489	0.873118	0.601147	0.658617	0.44468	0.359682	0.46458	0.404556	0.420255	0.354418
	Suezmax	0.797179	1	0.70422	0.441453	0.281943	0.335729	0.337754	0.439323	0.269984	0.701416	0.721055	0.674659	0.432466	0.280155	0.377488	0.31958	0.423974	0.275033
	Aframax	0.694248	0.70422	1	0.426835	0.250872	0.357514	0.379566	0.340162	0.251458	0.600614	0.58712	0.794041	0.44424	0.342856	0.44022	0.359799	0.338346	0.232362
	HandyW	0.437064	0.441453	0.426835	1	0.174706	0.30342	0.287738	0.345674	0.337678	0.375176	0.495215	0.411247	0.550908	0.243563	0.339341	0.249111	0.320069	0.327195
Dry bulk	Capesize	0.318525	0.281943	0.250872	0.174706	1	0.535787	0.553645	0.219206	0.264038	0.329513	0.231072	0.277381	0.224797	0.459253	0.598759	0.544989	0.198775	0.262764
	PanamaxD	0.409443	0.335729	0.357514	0.30342	0.535787	1	0.791598	0.353722	0.443394	0.397494	0.289685	0.336265	0.369973	0.757919	0.862655	0.739849	0.332149	0.407094
	HandyD	0.4342	0.337754	0.379566	0.287738	0.553645	0.791598	1	0.381061	0.454008	0.430908	0.289029	0.370199	0.38893	0.711149	0.821355	0.840268	0.353301	0.395984
Conta	PanamaxC	0.461184	0.439323	0.340162	0.345674	0.219206	0.353722	0.381061	1	0.643253	0.350148	0.300285	0.297888	0.328778	0.16494	0.345221	0.339113	0.88796	0.574091
	HandyC	0.337489	0.269984	0.251458	0.337678	0.264038	0.443394	0.454008	0.643253	1	0.276281	0.235014	0.218553	0.333923	0.327134	0.425652	0.447944	0.635269	0.865764
Wet bulk	VLCC	0.873118	0.701416	0.600614	0.375176	0.329513	0.397494	0.430908	0.350148	0.276281	1	0.547963	0.615403	0.402754	0.390863	0.472527	0.369048	0.31493	0.301063
	Suezmax	0.601147	0.721055	0.58712	0.495215	0.231072	0.289685	0.289029	0.300285	0.235014	0.547963	1	0.576651	0.324025	0.217754	0.334145	0.26805	0.320958	0.228293
	Aframax	0.658617	0.674659	0.794041	0.411247	0.277381	0.336265	0.370199	0.297888	0.218553	0.615403	0.576651	1	0.451034	0.311829	0.417256	0.337096	0.29562	0.242391
	HandyW	0.44468	0.432466	0.44424	0.550908	0.224797	0.369973	0.38893	0.328778	0.333923	0.402754	0.324025	0.451034	1	0.305904	0.403576	0.344893	0.290587	0.339818
Dry bulk	Capesize	0.359682	0.280155	0.342856	0.243563	0.459253	0.757919	0.711149	0.16494	0.327134	0.390863	0.217754	0.311829	0.305904	1	0.728667	0.715126	0.147999	0.297247
	PanamaxD	0.46458	0.377488	0.44022	0.339341	0.598759	0.862655	0.821355	0.345221	0.425652	0.472527	0.334145	0.417256	0.403576	0.728667	1	0.784567	0.322329	0.425701
	HandyD	0.404556	0.31958	0.359799	0.249111	0.544989	0.739849	0.840268	0.339113	0.447944	0.369048	0.26805	0.337096	0.344893	0.715126	0.784567	1	0.324444	0.422333
Conta	PanamaxC	0.420255	0.423974	0.338346	0.320069	0.198775	0.332149	0.353301	0.88796	0.635269	0.31493	0.320958	0.29562	0.290587	0.147999	0.322329	0.324444	1	0.596864
	HandyC	0.354418	0.275033	0.232362	0.327195	0.262764	0.407094	0.395984	0.574091	0.865764	0.301063	0.228293	0.242391	0.339818	0.297247	0.425701	0.422333	0.596864	1

7.2 Single-factor model

VCV:

			5 year old vessels									10 year old vessels								
			Wet bulk				Dry bulk			Container		Wet bulk				Dry bulk			Container	
			VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC	VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC
5 year old vessels	Wet bulk	VLCC	0.002073	2.48E-05	2.09E-05	3.88E-05	9.97E-05	9.54E-05	6.93E-05	1.05E-05	2.09E-05	2.99E-05	3.73E-05	3.89E-05	3.56E-05	8.71E-05	0.000115	7.7E-05	9.49E-06	2.68E-05
		Suezmax	2.48E-05	0.001917	2.29E-05	4.26E-05	0.000109	0.000105	7.59E-05	1.16E-05	2.3E-05	3.27E-05	4.09E-05	4.27E-05	3.9E-05	9.55E-05	0.000126	8.44E-05	1.04E-05	2.94E-05
		Aframax	2.09E-05	2.29E-05	0.002403	3.59E-05	9.22E-05	8.82E-05	6.4E-05	9.75E-06	1.94E-05	2.76E-05	3.45E-05	3.6E-05	3.29E-05	8.05E-05	0.000106	7.12E-05	8.78E-06	2.48E-05
		HandyW	3.88E-05	4.26E-05	3.59E-05	0.002816	0.000171	0.000164	0.000119	1.81E-05	3.6E-05	5.13E-05	6.42E-05	6.69E-05	6.12E-05	0.00015	0.000198	0.000132	1.63E-05	4.61E-05
	Dry bulk	Capesize	9.97E-05	0.000109	9.22E-05	0.000171	0.00529	0.000421	0.000306	4.65E-05	9.24E-05	0.000132	0.000165	0.000172	0.000157	0.000384	0.000507	0.00034	4.19E-05	0.000118
		PanamaxD	9.54E-05	0.000105	8.82E-05	0.000164	0.000421	0.005774	0.000293	4.45E-05	8.85E-05	0.000126	0.000158	0.000164	0.00015	0.000368	0.000486	0.000325	4.01E-05	0.000113
		HandyD	6.93E-05	7.59E-05	6.4E-05	0.000119	0.000306	0.000293	0.004494	3.23E-05	6.42E-05	9.15E-05	0.000114	0.000119	0.000109	0.000267	0.000352	0.000236	2.91E-05	8.23E-05
	Conta	PanamaxC	1.05E-05	1.16E-05	9.75E-06	1.81E-05	4.65E-05	4.45E-05	3.23E-05	0.003967	9.77E-06	1.39E-05	1.74E-05	1.82E-05	1.66E-05	4.06E-05	5.36E-05	3.59E-05	4.43E-06	1.25E-05
		HandyC	2.09E-05	2.3E-05	1.94E-05	3.6E-05	9.24E-05	8.85E-05	6.42E-05	9.77E-06	0.002687	2.77E-05	3.46E-05	3.61E-05	3.3E-05	8.08E-05	0.000107	7.14E-05	8.8E-06	2.49E-05
10 year old vessels	Wet bulk	VLCC	2.99E-05	3.27E-05	2.76E-05	5.13E-05	0.000132	0.000126	9.15E-05	1.39E-05	2.77E-05	0.003363	4.93E-05	5.14E-05	4.71E-05	0.000115	0.000152	0.000102	1.25E-05	3.55E-05
		Suezmax	3.73E-05	4.09E-05	3.45E-05	6.42E-05	0.000165	0.000158	0.000114	1.74E-05	3.46E-05	4.93E-05	0.004998	6.43E-05	5.89E-05	0.000144	0.00019	0.000127	1.57E-05	4.43E-05
		Aframax	3.89E-05	4.27E-05	3.6E-05	6.69E-05	0.000172	0.000164	0.000119	1.82E-05	3.61E-05	5.14E-05	6.43E-05	0.003258	6.14E-05	0.00015	0.000198	0.000133	1.64E-05	4.62E-05
		HandyW	3.56E-05	3.9E-05	3.29E-05	6.12E-05	0.000157	0.00015	0.000109	1.66E-05	3.3E-05	4.71E-05	5.89E-05	6.14E-05	0.003316	0.000137	0.000181	0.000121	1.5E-05	4.23E-05
	Dry bulk	Capesize	8.71E-05	9.55E-05	8.05E-05	0.00015	0.000384	0.000368	0.000267	4.06E-05	8.08E-05	0.000115	0.000144	0.00015	0.000137	0.007383	0.000443	0.000297	3.66E-05	0.000103
		PanamaxD	0.000115	0.000126	0.000106	0.000198	0.000507	0.000486	0.000352	5.36E-05	0.000107	0.000152	0.00019	0.000198	0.000181	0.000443	0.007102	0.000392	4.83E-05	0.000137
		HandyD	7.7E-05	8.44E-05	7.12E-05	0.000132	0.00034	0.000325	0.000236	3.59E-05	7.14E-05	0.000102	0.000127	0.000133	0.000121	0.000297	0.000392	0.006205	3.23E-05	9.14E-05
	Conta	PanamaxC	9.49E-06	1.04E-05	8.78E-06	1.63E-05	4.19E-05	4.01E-05	2.91E-05	4.43E-06	8.8E-06	1.25E-05	1.57E-05	1.64E-05	1.5E-05	3.66E-05	4.83E-05	3.23E-05	0.004654	1.13E-05
		HandyC	2.68E-05	2.94E-05	2.48E-05	4.61E-05	0.000118	0.000113	8.23E-05	1.25E-05	2.49E-05	3.55E-05	4.43E-05	4.62E-05	4.23E-05	0.000103	0.000137	9.14E-05	1.13E-05	0.003586

Correlation:

			5 year old vessels									10 year old vessels								
			Wet bulk				Dry bulk			Container		Wet bulk				Dry bulk			Container	
			VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC	VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC
5 year old vessels	Wet bulk	VLCC	1	0.012423	0.009358	0.016074	0.030106	0.027586	0.022692	0.003676	0.008875	0.011307	0.011598	0.014975	0.013587	0.022263	0.029954	0.021461	0.003056	0.009842
		Suezmax	0.012423	1	0.010667	0.018321	0.034314	0.031443	0.025864	0.00419	0.010116	0.012888	0.013219	0.017068	0.015486	0.025376	0.034141	0.024462	0.003483	0.011217
		Aframax	0.009358	0.010667	1	0.013802	0.02585	0.023686	0.019484	0.003157	0.007621	0.009709	0.009958	0.012858	0.011666	0.019116	0.025719	0.018428	0.002624	0.00845
		HandyW	0.016074	0.018321	0.013802	1	0.044399	0.040683	0.033466	0.005422	0.013089	0.016676	0.017104	0.022084	0.020037	0.032833	0.044175	0.031651	0.004507	0.014514
	Dry bulk	Capesize	0.030106	0.034314	0.02585	0.044399	1	0.076197	0.062679	0.010154	0.024515	0.031233	0.032035	0.041363	0.037529	0.061495	0.082737	0.05928	0.008442	0.027184
		PanamaxD	0.027586	0.031443	0.023686	0.040683	0.076197	1	0.057433	0.009305	0.022463	0.028619	0.029354	0.037901	0.034388	0.056348	0.075812	0.054319	0.007735	0.024909
		HandyD	0.022692	0.025864	0.019484	0.033466	0.062679	0.057433	1	0.007654	0.018478	0.023542	0.024146	0.031177	0.028287	0.046351	0.062362	0.044682	0.006363	0.02049
	Conta	PanamaxC	0.003676	0.00419	0.003157	0.005422	0.010154	0.009305	0.007654	1	0.002994	0.003814	0.003912	0.005051	0.004583	0.007509	0.010103	0.007239	0.001031	0.003319
		HandyC	0.008875	0.010116	0.007621	0.013089	0.024515	0.022463	0.018478	0.002994	1	0.009207	0.009444	0.012194	0.011063	0.018129	0.024391	0.017476	0.002489	0.008014
10 year old vessels	Wet bulk	VLCC	0.011307	0.012888	0.009709	0.016676	0.031233	0.028619	0.023542	0.003814	0.009207	1	0.012032	0.015535	0.014095	0.023097	0.031075	0.022265	0.003171	0.01021
		Suezmax	0.011598	0.013219	0.009958	0.017104	0.032035	0.029354	0.024146	0.003912	0.009444	0.012032	1	0.015935	0.014458	0.02369	0.031874	0.022837	0.003252	0.010472
		Aframax	0.014975	0.017068	0.012858	0.022084	0.041363	0.037901	0.031177	0.005051	0.012194	0.015535	0.015935	1	0.018667	0.030588	0.041154	0.029486	0.004199	0.013521
		HandyW	0.013587	0.015486	0.011666	0.020037	0.037529	0.034388	0.028287	0.004583	0.011063	0.014095	0.014458	0.018667	1	0.027752	0.037339	0.026753	0.00381	0.012268
	Dry bulk	Capesize	0.022263	0.025376	0.019116	0.032833	0.061495	0.056348	0.046351	0.007509	0.018129	0.023097	0.02369	0.030588	0.027752	1	0.061184	0.043838	0.006243	0.020103
		PanamaxD	0.029954	0.034141	0.025719	0.044175	0.082737	0.075812	0.062362	0.010103	0.024391	0.031075	0.031874	0.041154	0.037339	0.061184	1	0.058981	0.008399	0.027047
		HandyD	0.021461	0.024462	0.018428	0.031651	0.05928	0.054319	0.044682	0.007239	0.017476	0.022265	0.022837	0.029486	0.026753	0.043838	0.058981	1	0.006018	0.019379
	Conta	PanamaxC	0.003056	0.003483	0.002624	0.004507	0.008442	0.007735	0.006363	0.001031	0.002489	0.003171	0.003252	0.004199	0.00381	0.006243	0.008399	0.006018	1	0.00276
		HandyC	0.009842	0.011217	0.00845	0.014514	0.027184	0.024909	0.02049	0.003319	0.008014	0.01021	0.010472	0.013521	0.012268	0.020103	0.027047	0.019379	0.00276	1

7.3 Multi-factor model

VCV ($\hat{\Sigma}$):

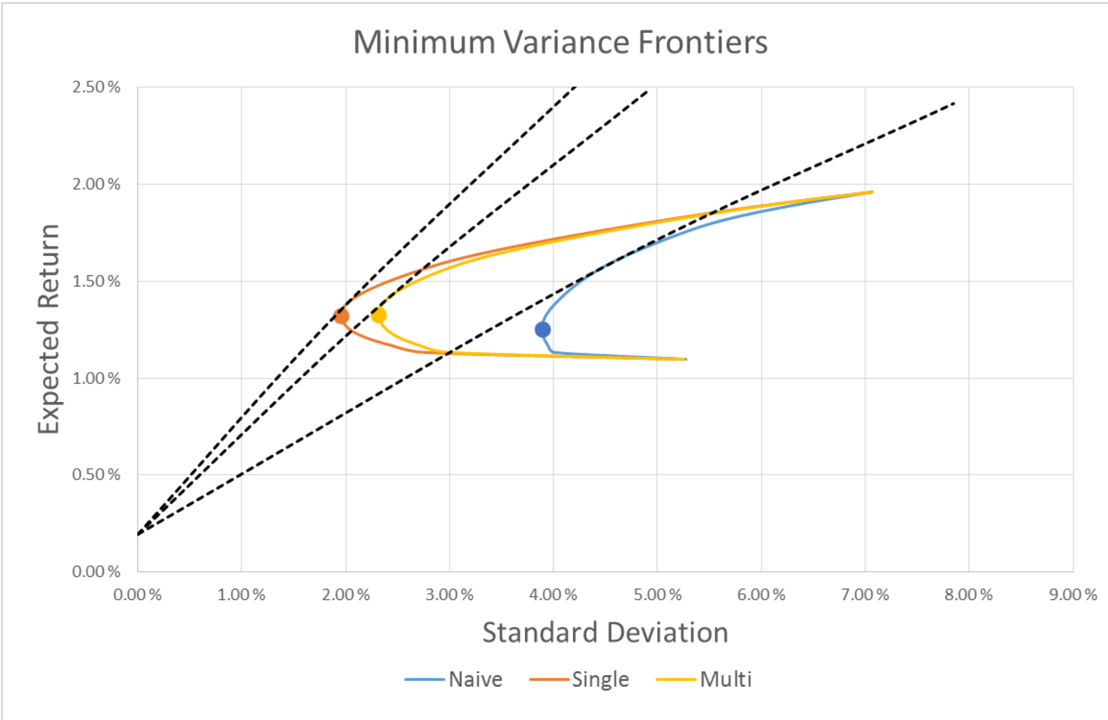
			5 year old vessels									10 year old vessels								
			Wet bulk				Dry bulk			Container		Wet bulk				Dry bulk			Container	
			VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC	VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC
5 year old vessels	Wet bulk	VLCC	0.002073	0.00023	0.000219	0.000271	0.000497	0.00055	0.000524	0.00033	0.000283	0.000336	0.000219	0.000235	0.000279	0.000598	0.00062	0.000617	0.000271	0.000352
		Suezmax	0.00023	0.001917	0.000186	0.000195	0.000337	0.000406	0.000376	0.000248	0.00021	0.000237	0.000165	0.000181	0.000199	0.00046	0.000455	0.000479	0.000206	0.00027
		Aframax	0.000219	0.000186	0.002403	0.000171	0.000308	0.000412	0.000362	0.000234	0.000188	0.000221	0.00014	0.000207	0.000177	0.000464	0.000469	0.000478	0.000209	0.000252
		HandyW	0.000271	0.000195	0.000171	0.002816	0.000471	0.000495	0.000468	0.000291	0.000259	0.000296	0.0002	0.00018	0.000236	0.000527	0.000545	0.000548	0.000229	0.00032
	Dry bulk	Capesize	0.000497	0.000337	0.000308	0.000471	0.00529	0.001083	0.001018	0.000615	0.000558	0.000589	0.000283	0.000329	0.000419	0.001125	0.001227	0.001166	0.000493	0.000667
		PanamaxD	0.00055	0.000406	0.000412	0.000495	0.001083	0.005774	0.001047	0.000645	0.000567	0.000618	0.000338	0.000409	0.000457	0.001211	0.001285	0.001254	0.000531	0.000702
		HandyD	0.000524	0.000376	0.000362	0.000468	0.001018	0.001047	0.004494	0.000624	0.00055	0.00059	0.000312	0.000381	0.000456	0.001134	0.001197	0.001177	0.000513	0.000669
	Conta	PanamaxC	0.00033	0.000248	0.000234	0.000291	0.000615	0.000645	0.000624	0.003967	0.000374	0.000364	0.000184	0.000242	0.000298	0.000751	0.00075	0.00079	0.000353	0.000453
		HandyC	0.000283	0.00021	0.000188	0.000259	0.000558	0.000567	0.00055	0.000374	0.002687	0.000316	0.000161	0.000194	0.000254	0.000657	0.000654	0.000694	0.000304	0.000403
10 year old vessels	Wet bulk	VLCC	0.000336	0.000237	0.000221	0.000296	0.000589	0.000618	0.00059	0.000364	0.000316	0.003363	0.000224	0.000242	0.000297	0.000657	0.000699	0.000677	0.000295	0.000387
		Suezmax	0.000219	0.000165	0.00014	0.0002	0.000283	0.000338	0.000312	0.000184	0.000161	0.000224	0.004998	0.000142	0.000187	0.000345	0.000346	0.000357	0.000138	0.000212
		Aframax	0.000235	0.000181	0.000207	0.00018	0.000329	0.000409	0.000381	0.000242	0.000194	0.000242	0.000142	0.003258	0.000207	0.000454	0.000478	0.000462	0.000215	0.000248
		HandyW	0.000279	0.000199	0.000177	0.000236	0.000419	0.000457	0.000456	0.000298	0.000254	0.000297	0.000187	0.000207	0.003316	0.000514	0.000527	0.000531	0.000245	0.000309
	Dry bulk	Capesize	0.000598	0.00046	0.000464	0.000527	0.001125	0.001211	0.001134	0.000751	0.000657	0.000657	0.000345	0.000454	0.000514	0.007383	0.001389	0.001438	0.000626	0.000811
		PanamaxD	0.00062	0.000455	0.000469	0.000545	0.001227	0.001285	0.001197	0.00075	0.000654	0.000699	0.000346	0.000478	0.000527	0.001389	0.007102	0.001437	0.000627	0.000802
		HandyD	0.000617	0.000479	0.000478	0.000548	0.001166	0.001254	0.001177	0.00079	0.000694	0.000677	0.000357	0.000462	0.000531	0.001438	0.001437	0.006205	0.000656	0.000857
	Conta	PanamaxC	0.000271	0.000206	0.000209	0.000229	0.000493	0.000531	0.000513	0.000353	0.000304	0.000295	0.000138	0.000215	0.000245	0.000626	0.000627	0.000656	0.004654	0.00037
		HandyC	0.000352	0.00027	0.000252	0.00032	0.000667	0.000702	0.000669	0.000453	0.000403	0.000387	0.000212	0.000248	0.000309	0.000811	0.000802	0.000857	0.00037	0.003586

Correlation:

			5 year old vessels									10 year old vessels								
			Wet bulk				Dry bulk			Container		Wet bulk				Dry bulk			Container	
			VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC	VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC
5 year old vessels	Wet bulk	VLCC	1	0.115435	0.098066	0.112041	0.149951	0.158985	0.171675	0.115223	0.119964	0.127128	0.068032	0.090254	0.106579	0.152822	0.161643	0.17216	0.087167	0.129266
		Suezmax	0.115435	1	0.086854	0.084053	0.10578	0.12199	0.128225	0.0898	0.092451	0.093443	0.053165	0.072587	0.078783	0.122197	0.123194	0.138925	0.069085	0.102824
		Aframax	0.098066	0.086854	1	0.065751	0.086259	0.110614	0.11012	0.075778	0.073999	0.077603	0.040474	0.073935	0.062814	0.110263	0.113567	0.123785	0.062352	0.085932
		HandyW	0.112041	0.084053	0.065751	1	0.121949	0.122711	0.131662	0.086983	0.093973	0.096293	0.053426	0.059485	0.077094	0.115527	0.121907	0.131186	0.063305	0.100559
	Dry bulk	Capesize	0.149951	0.10578	0.086259	0.121949	1	0.196025	0.208699	0.134199	0.148033	0.139646	0.055109	0.079169	0.100094	0.179949	0.200202	0.203467	0.099425	0.153096
		PanamaxD	0.158985	0.12199	0.110614	0.122711	0.196025	1	0.205616	0.134714	0.143972	0.140328	0.062876	0.094339	0.104361	0.185439	0.200686	0.209464	0.102484	0.1543
		HandyD	0.171675	0.128225	0.11012	0.131662	0.208699	0.205616	1	0.147894	0.158165	0.151687	0.065744	0.099578	0.118069	0.196934	0.211808	0.222882	0.112124	0.166558
	Conta	PanamaxC	0.115223	0.0898	0.075778	0.086983	0.134199	0.134714	0.147894	1	0.114517	0.099519	0.041343	0.067407	0.08227	0.138752	0.141246	0.159161	0.08207	0.120179
		HandyC	0.119964	0.092451	0.073999	0.093973	0.148033	0.143972	0.158165	0.114517	1	0.105277	0.043909	0.065411	0.085073	0.147459	0.149632	0.169955	0.086047	0.129806
10 year old vessels	Wet bulk	VLCC	0.127128	0.093443	0.077603	0.096293	0.139646	0.140328	0.151687	0.099519	0.105277	1	0.054603	0.073209	0.088986	0.1318	0.143013	0.148183	0.074675	0.111558
		Suezmax	0.068032	0.053165	0.040474	0.053426	0.055109	0.062876	0.065744	0.041343	0.043909	0.054603	1	0.035279	0.045942	0.056763	0.058154	0.064181	0.028553	0.050074
		Aframax	0.090254	0.072587	0.073935	0.059485	0.079169	0.094339	0.099578	0.067407	0.065411	0.073209	0.035279	1	0.063061	0.092647	0.099368	0.102766	0.055108	0.072615
		HandyW	0.106579	0.078783	0.062814	0.077094	0.100094	0.104361	0.118069	0.08227	0.085073	0.088986	0.045942	0.063061	1	0.103836	0.108629	0.11711	0.062372	0.08953
	Dry bulk	Capesize	0.152822	0.122197	0.110263	0.115527	0.179949	0.185439	0.196934	0.138752	0.147459	0.1318	0.056763	0.092647	0.103836	1	0.191835	0.212502	0.106783	0.157656
		PanamaxD	0.161643	0.123194	0.113567	0.121907	0.200202	0.200686	0.211808	0.141246	0.149632	0.143013	0.058154	0.099368	0.108629	0.191835	1	0.216465	0.109074	0.158823
		HandyD	0.17216	0.138925	0.123785	0.131186	0.203467	0.209464	0.222882	0.159161	0.169955	0.148183	0.064181	0.102766	0.11711	0.212502	0.216465	1	0.122032	0.181639
	Conta	PanamaxC	0.087167	0.069085	0.062352	0.063305	0.099425	0.102484	0.112124	0.08207	0.086047	0.074675	0.028553	0.055108	0.062372	0.106783	0.109074	0.122032	1	0.090649
		HandyC	0.129266	0.102824	0.085932	0.100559	0.153096	0.1543	0.166558	0.120179	0.129806	0.111558	0.050074	0.072615	0.08953	0.157656	0.158823	0.181639	0.090649	1

8. Segment-specific analysis (P=Sharpe optimal portfolios and Pmin=minimum variance)*

8.1 Wet bulk



P*	Naive	Single	Multi
Exp. R	1.561 %	1.376 %	1.404 %
Variance	0.001968	0.000398	0.000571
St. Dev	4.436 %	1.995 %	2.389 %
rf rate:	0.1949 %		
Sharpe	0.308018	0.5922	0.506219

Wet bulk

	5yrs				10yrs					P*	
P*	VLCC	Suezmax	Aframax	HandyW	VLCC	Suezmax	Aframax	HandyW	*SUM*	5yr	10yr
Naive	0.00 %	39.78 %	0.00 %	0.00 %	3.18 %	22.62 %	0.00 %	34.41 %	*100.00 %*	39.78 %	60.22 %
Single	14.01 %	19.29 %	12.06 %	9.17 %	11.38 %	11.05 %	10.76 %	12.29 %	*100.00 %*	54.52 %	45.48 %
Multi	10.64 %	20.90 %	11.49 %	8.02 %	10.50 %	13.59 %	11.69 %	13.17 %	*100.00 %*	51.06 %	48.94 %

MVP	Naive	Single	Multi
Exp. R	1.255 %	1.325 %	1.329 %
Variance	0.001512	0.000381	0.000535
St. Dev	3.888 %	1.951 %	2.314 %

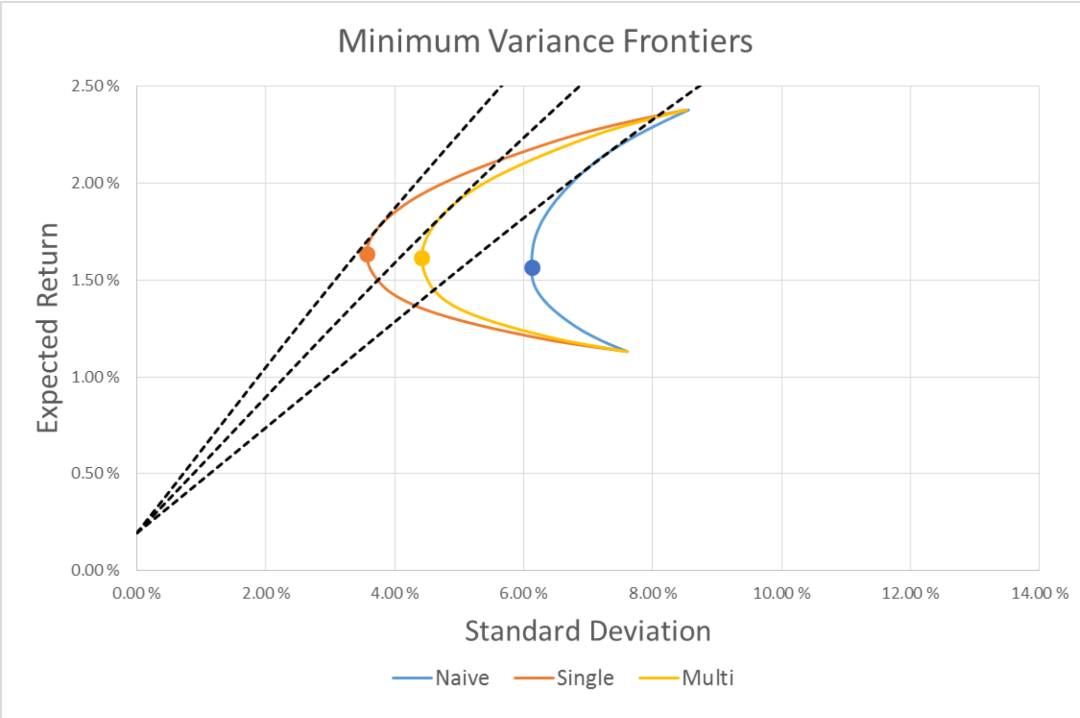
Wet bulk

	5yrs				10yrs					Pmin	
Pmin	VLCC	Suezmax	Aframax	HandyW	VLCC	Suezmax	Aframax	HandyW	*SUM*	5yr	10yr
Naive	19.36 %	34.69 %	12.43 %	22.02 %	0.00 %	0.00 %	0.00 %	11.50 %	*100.00 %*	88.50 %	11.50 %
Single	17.13 %	18.43 %	14.83 %	12.00 %	10.28 %	6.73 %	10.34 %	10.25 %	*100.00 %*	62.40 %	37.60 %
Multi	15.63 %	19.49 %	15.55 %	12.22 %	8.71 %	7.38 %	10.94 %	10.08 %	*100.00 %*	62.89 %	37.11 %

P* (%DWT)		Wet bulk				(Per age)	(Total)
		VLCC	Suezmax	Aframax	Handymax	Sum	Sum
5 year	Naïve	0.00 %	36.42 %	0.00 %	0.00 %	36.42 %	100.00 %
	Single	16.69 %	16.01 %	8.92 %	4.36 %	45.99 %	100.00 %
	Multi	12.72 %	17.43 %	8.53 %	3.83 %	42.51 %	100.00 %
10 year	Naïve	5.99 %	29.91 %	0.00 %	27.68 %	63.58 %	
	Single	19.41 %	13.25 %	12.38 %	8.97 %	54.01 %	
	Multi	17.99 %	16.36 %	13.50 %	9.64 %	57.49 %	

P* (%vessels)		Wet bulk				(Per age)	(Total)
		VLCC	Suezmax	Aframax	Handymax	Sum	Sum
5 year	Naïve	0.00 %	19.25 %	0.00 %	0.00 %	19.25 %	100.00 %
	Single	6.24 %	11.60 %	9.85 %	13.67 %	41.36 %	100.00 %
	Multi	4.64 %	12.32 %	9.19 %	11.71 %	37.86 %	100.00 %
10 year	Naïve	1.64 %	15.81 %	0.00 %	63.30 %	80.75 %	
	Single	7.26 %	9.60 %	13.67 %	28.11 %	58.64 %	
	Multi	6.56 %	11.56 %	14.54 %	29.48 %	62.14 %	

8.2 Dry bulk



P*	Naive	Single	Multi
Exp. R	2.095 %	1.761 %	1.844 %
Variance	0.004968	0.00138	0.002266
St. Dev	7.048 %	3.714 %	4.761 %
rf rate:	0.1949 %		
Sharpe	0.269579	0.421563	0.346333

Dry bulk

	5yrs			10yrs				P*	
P*	Capesize	PanamaxD	HandyD	Capesize	PanamaxD	HandyD	*SUM*	5yr	10yr
Naive	33.71 %	0.00 %	0.00 %	60.91 %	0.00 %	5.38 %	*100.00 %*	33.71 %	66.29 %
Single	17.80 %	8.53 %	21.69 %	22.62 %	9.76 %	19.60 %	*100.00 %*	48.02 %	51.98 %
Multi	19.94 %	2.73 %	22.13 %	28.78 %	6.33 %	20.10 %	*100.00 %*	44.80 %	55.20 %

MVP	Naive	Single	Multi
Exp. R	1.567 %	1.634 %	1.615 %
Variance	0.003745	0.001268	0.001952
St. Dev	6.120 %	3.561 %	4.419 %

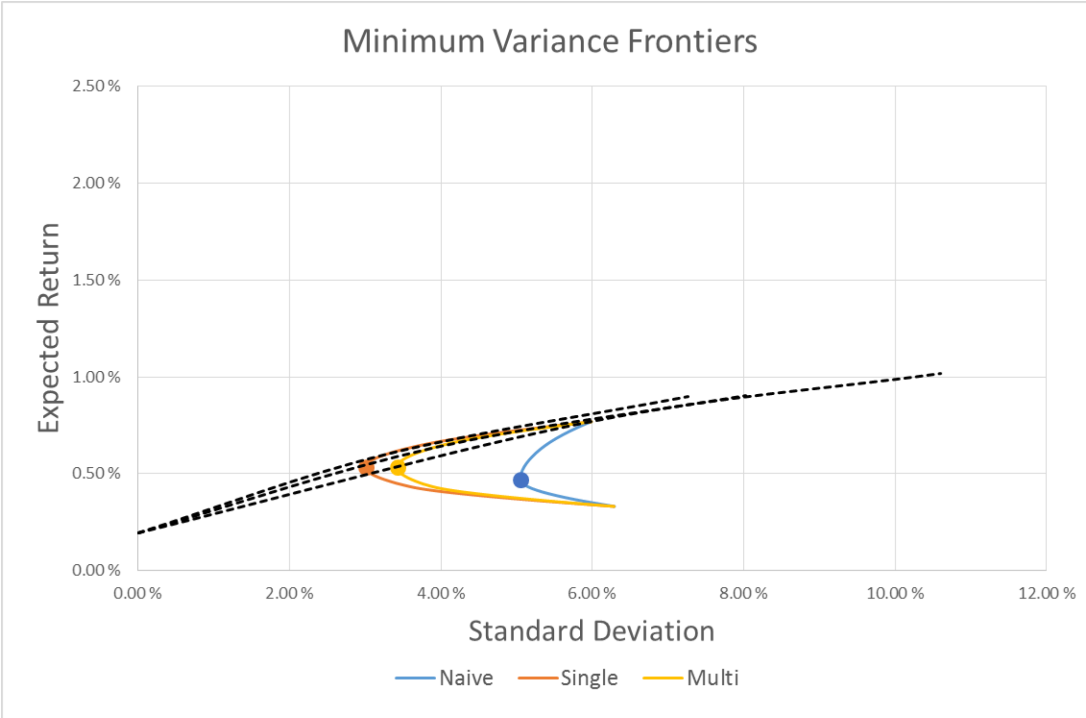
Dry bulk

	5yrs			10yrs				Pmin	
Pmin	Capesize	PanamaxD	HandyD	Capesize	PanamaxD	HandyD	*SUM*	5yr	10yr
Naive	39.53 %	6.57 %	51.54 %	2.35 %	0.00 %	0.00 %	*100.00 %*	97.65 %	2.35 %
Single	18.10 %	16.65 %	23.28 %	13.15 %	12.55 %	16.27 %	*100.00 %*	58.03 %	41.97 %
Multi	20.27 %	17.39 %	25.11 %	11.76 %	11.48 %	13.99 %	*100.00 %*	62.77 %	37.23 %

P* (%DWT)		Dry bulk			(Per age)	(Total)
		Capesize	Panamax	Handymax	Sum	Sum
5 year	Naïve	29.79 %	0.00 %	0.00 %	29.79 %	100.00 %
	Single	19.93 %	7.26 %	13.27 %	40.46 %	100.00 %
	Multi	21.74 %	2.26 %	13.19 %	37.19 %	100.00 %
10 year	Naïve	66.55 %	0.00 %	3.67 %	70.21 %	
	Single	31.32 %	11.31 %	16.91 %	59.54 %	
	Multi	38.80 %	7.13 %	16.88 %	62.81 %	

P* (%vessels)		Dry bulk			(Per age)	(Total)
		Capesize	Panamax	Handymax	Sum	Sum
5 year	Naïve	27.55 %	0.00 %	0.00 %	27.55 %	100.00 %
	Single	10.37 %	8.94 %	22.19 %	41.50 %	100.00 %
	Multi	12.11 %	2.99 %	23.62 %	38.72 %	100.00 %
10 year	Naïve	61.55 %	0.00 %	10.90 %	72.45 %	
	Single	16.29 %	13.93 %	28.27 %	58.50 %	
	Multi	21.62 %	9.42 %	30.24 %	61.28 %	

8.3 Container



P*	Naive	Single	Multi
Exp. R	0.768 %	0.603 %	0.636 %
Variance	0.003586	0.001091	0.001531
St. Dev	5.988 %	3.303 %	3.913 %
rf rate:	0.1949 %		
Sharpe	0.095758	0.123667	0.112745

Container

	5yrs		10yrs		
P*	PanamaxC	HandyC	PanamaxC	HandyC	*SUM*
Naive	0.00 %	0.00 %	0.00 %	100.00 %	*100.00 %*
Single	8.90 %	31.23 %	17.46 %	42.41 %	*100.00 %*
Multi	1.69 %	31.57 %	16.73 %	50.01 %	*100.00 %*

P*	
5yr	10yr
0.00 %	100.00 %
40.13 %	59.87 %
33.26 %	66.74 %

MVP	Naive	Single	Multi
Exp. R	0.470 %	0.533 %	0.533 %
Variance	0.002547	0.000904	0.001173
St. Dev	5.047 %	3.007 %	3.424 %

Container

	5yrs		10yrs		
Pmin	PanamaxC	HandyC	PanamaxC	HandyC	*SUM*
Naive	23.92 %	76.08 %	0.00 %	0.00 %	*100.00 %*
Single	22.61 %	33.27 %	19.28 %	24.84 %	*100.00 %*
Multi	21.81 %	34.80 %	19.35 %	24.03 %	*100.00 %*

Pmin	
5yr	10yr
100.00 %	0.00 %
55.88 %	44.12 %
56.61 %	43.39 %

P* (%TEU)		Container		(Per age)	(Total)
		Panamax	Handysize	Sum	Sum
5 year	Naïve	0.00 %	0.00 %	0.00 %	100.00 %
	Single	7.79 %	21.47 %	29.26 %	100.00 %
	Multi	1.45 %	21.31 %	22.76 %	100.00 %
10 year	Naïve	0.00 %	100.00 %	100.00 %	
	Single	21.38 %	49.36 %	70.74 %	
	Multi	20.12 %	57.12 %	77.24 %	

P* (%vessels)		Container		(Per age)	(Total)
		Panamax	Handysize	Sum	Sum
5 year	Naïve	0.00 %	0.00 %	0.00 %	100.00 %
	Single	3.01 %	26.90 %	29.91 %	100.00 %
	Multi	0.53 %	25.04 %	25.57 %	100.00 %
10 year	Naïve	0.00 %	100.00 %	100.00 %	
	Single	8.27 %	61.82 %	70.09 %	
	Multi	7.30 %	67.13 %	74.43 %	

9. Regressions for sensitivity analysis

9.1 Five-year vessels

Multi-Factor 5yr old vessels pre mid-way

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC
OECDTOT	0.0690	0.165	0.533	-1.158*	0.133	0.206	0.0824	0.0908	-0.556
	(0.15)	(0.35)	(0.95)	(-1.76)	(0.17)	(0.26)	(0.12)	(0.16)	(-1.14)
dAUSCOAL	0.236***	0.196**	0.278***	0.206*	0.223*	0.246*	0.273**	0.137	0.0987
	(3.19)	(2.60)	(3.11)	(1.97)	(1.81)	(1.96)	(2.42)	(1.51)	(1.27)
dWOS	0.231	0.362	-0.435	0.749	-0.127	-0.191	0.107	0.510	0.767*
	(0.61)	(0.95)	(-0.96)	(1.41)	(-0.20)	(-0.30)	(0.19)	(1.11)	(1.94)
dWSP	0.0204	-0.0204	-0.108	-0.0484	0.166	-0.0541	0.110	0.117	0.228**
	(0.23)	(-0.22)	(-1.00)	(-0.38)	(1.12)	(-0.36)	(0.80)	(1.06)	(2.42)
dPUV	0.695**	0.905***	0.962***	0.821*	1.568***	1.660***	1.512***	1.066***	0.807**
	(2.36)	(3.01)	(2.70)	(1.97)	(3.19)	(3.31)	(3.36)	(2.95)	(2.60)
_cons	0.0192***	0.0204***	0.0197***	0.0230***	0.0194***	0.0146**	0.0169***	0.0119***	0.0123***
	(5.28)	(5.52)	(4.48)	(4.48)	(3.21)	(2.36)	(3.05)	(2.67)	(3.23)
N	110	110	110	110	110	110	110	110	110
R^2	0.154	0.158	0.193	0.122	0.149	0.152	0.171	0.116	0.139

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Multi-Factor 5yr old vessels post mid-way

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	VLCC	Suezmax	Aframax	HandyW	Capesize	PanamaxD	HandyD	PanamaxC	HandyC
OECDTOT	0.563	0.788*	0.770	0.823	0.0205	0.756	1.283*	1.629**	1.622***
	(1.16)	(1.72)	(1.56)	(1.65)	(0.03)	(0.95)	(1.90)	(2.22)	(2.81)
dAUSCOAL	0.183***	0.104*	0.0931	0.0939	0.182*	0.237**	0.241***	0.121	0.0738
	(2.81)	(1.68)	(1.40)	(1.40)	(1.85)	(2.21)	(2.66)	(1.22)	(0.95)
dWOS	1.328**	0.789	0.709	0.582	0.893	1.173	1.487	0.438	0.335
	(2.03)	(1.27)	(1.06)	(0.86)	(0.90)	(1.09)	(1.63)	(0.44)	(0.43)
dWSP	0.0386	-0.0461	-0.160	0.152	0.376**	0.189	0.229	0.113	0.0678
	(0.37)	(-0.46)	(-1.50)	(1.41)	(2.37)	(1.10)	(1.57)	(0.71)	(0.54)
dPUV	0.302	0.143	0.171	0.369	1.404***	1.245**	0.771*	0.363	0.512
	(0.95)	(0.47)	(0.53)	(1.13)	(2.91)	(2.38)	(1.74)	(0.75)	(1.35)
_cons	-0.00113	0.00263	0.000480	-0.00340	0.00389	-0.000108	0.00627	-0.00934	-0.00634
	(-0.24)	(0.60)	(0.10)	(-0.72)	(0.55)	(-0.01)	(0.97)	(-1.33)	(-1.15)
N	106	106	106	106	106	106	106	106	106
R^2	0.191	0.109	0.102	0.152	0.269	0.239	0.279	0.132	0.181

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

9.2 Ten-year vessels

Multi-Factor 10 yr old vessels pre mid-way

	(1) VLCC	(2) Suezmax	(3) Aframax	(4) HandyW	(5) Capesize	(6) PanamaxD	(7) HandyD	(8) PanamaxC	(9) HandyC
OECDTOT	0.101	-1.236	0.480	-0.0892	0.619	0.652	1.501*	-0.421	-0.493
	(0.18)	(-1.27)	(0.83)	(-0.14)	(0.68)	(0.71)	(1.68)	(-0.71)	(-0.71)
dAUSCOAL	0.268***	0.405**	0.247***	0.284***	0.309**	0.424***	0.184	0.124	0.0384
	(3.00)	(2.62)	(2.68)	(2.76)	(2.14)	(2.92)	(1.29)	(1.32)	(0.35)
dWOS	0.00929	1.286	-0.482	0.334	-0.344	-0.474	0.271	0.257	0.706
	(0.02)	(1.64)	(-1.03)	(0.64)	(-0.47)	(-0.64)	(0.37)	(0.53)	(1.25)
dWSP	0.00226	0.0373	-0.136	0.0747	-0.105	-0.0389	0.0786	0.0844	0.182
	(0.02)	(0.20)	(-1.22)	(0.60)	(-0.60)	(-0.22)	(0.46)	(0.74)	(1.36)
dPUV	0.711**	0.553	0.631*	0.428	1.434**	1.685***	1.797***	0.710*	1.174***
	(2.00)	(0.90)	(1.72)	(1.04)	(2.50)	(2.92)	(3.17)	(1.88)	(2.66)
_cons	0.0242***	0.0349***	0.0263***	0.0263***	0.0305***	0.0191***	0.0192***	0.0180***	0.0176***
	(5.51)	(4.61)	(5.80)	(5.20)	(4.31)	(2.69)	(2.75)	(3.88)	(3.22)
N	110	110	110	110	110	110	110	110	110
R^2	0.132	0.105	0.135	0.090	0.125	0.182	0.136	0.064	0.088

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Multi-Factor 10 yr old vessels post mid-way

	(1) VLCC	(2) Suezmax	(3) Aframax	(4) HandyW	(5) Capesize	(6) PanamaxD	(7) HandyD	(8) PanamaxC	(9) HandyC
OECDTOT	0.228	0.351	0.538	0.866	2.180**	1.195	2.057***	1.763**	1.864***
	(0.35)	(0.58)	(0.83)	(1.44)	(2.49)	(1.42)	(2.81)	(2.08)	(3.15)
dAUSCOAL	0.178**	0.0211	0.175**	0.163**	0.157	0.231**	0.197**	0.114	0.142*
	(2.01)	(0.26)	(2.00)	(2.01)	(1.33)	(2.05)	(2.00)	(1.00)	(1.78)
dWOS	1.402	0.586	0.798	1.476*	1.877	1.099	1.673*	0.300	1.096
	(1.57)	(0.71)	(0.90)	(1.81)	(1.58)	(0.97)	(1.69)	(0.26)	(1.37)
dWSP	0.151	-0.0721	-0.0263	0.0911	0.257	0.260	0.153	0.0512	0.0706
	(1.07)	(-0.55)	(-0.19)	(0.70)	(1.36)	(1.43)	(0.97)	(0.28)	(0.55)
dPUV	0.502	0.493	-0.0550	0.0234	1.324**	1.281**	1.353***	0.217	0.524
	(1.16)	(1.23)	(-0.13)	(0.06)	(2.30)	(2.32)	(2.81)	(0.39)	(1.35)
_cons	-0.00157	0.00184	-0.00148	0.0000654	0.00997	-0.000438	0.00616	-0.0116	-0.00874
	(-0.25)	(0.32)	(-0.24)	(0.01)	(1.19)	(-0.05)	(0.88)	(-1.44)	(-1.54)
N	106	106	106	106	106	106	106	106	106
R^2	0.134	0.043	0.067	0.115	0.262	0.253	0.336	0.091	0.243

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

10. Visual Basic for Applications (VBA) programming (general program)

```
Sub Optimizer()
'Clear area
Range("A2:E26").ClearContents
'Naive optimizing
    'Reset SOLVER
    SolverReset
    'Set constraint weights must sum to 100%
    SolverAdd CellRef:="$Y$47", Relation:=2, FormulaText:="100%"
    'Set expected return equal to target
    SolverAdd CellRef:="$B$46", Relation:=2, FormulaText:="$B$50"
    'Set constraint for short sales
    SolverOptions AssumeNonNeg:=True
    'Repeat for 25 different values of the expected return
    For Counter = 1 To 25
        'Increment the target expected return by 0.07%, starting at 0.500%
        Range("B50") = 0.005 + Counter * 0.0007
        'Minimize variance by changing portfolio weights
        SolverOk SetCell:=Range("$B$47"), MaxMinVal:=2, ValueOf:=0, ByChange:=Range("$G$47:$X$47"), _
                Engine:=1, EngineDesc:="GRG Nonlinear"
        'Click Solve
        SolverSolve UserFinish:=True
        'Plot numbers
        Range("A2:E26").Cells(Counter, 1) = Counter
        Range("A2:E26").Cells(Counter, 2) = Range("$B$46")
        Range("A2:E26").Cells(Counter, 3) = Range("$B$48")
    'Next expected return
    Next Counter
'Single optimizing
    'Reset SOLVER
    SolverReset
    'Set constraint weights must sum to 100%
    SolverAdd CellRef:="$Y$48", Relation:=2, FormulaText:="100%"
    'Set expected return equal to target
    SolverAdd CellRef:="$C$46", Relation:=2, FormulaText:="$B$50"
    'Set constraint for short sales
    SolverOptions AssumeNonNeg:=True
    'Repeat for 25 different values of the expected return
    For Counter = 1 To 25
        'Increment the target expected return by 0.07%, starting at 0.500%
        Range("B50") = 0.005 + Counter * 0.0007
        'Minimize variance by changing portfolio weights
        SolverOk SetCell:=Range("$C$47"), MaxMinVal:=2, ValueOf:=0, ByChange:=Range("$G$48:$X$48"), _
                Engine:=1, EngineDesc:="GRG Nonlinear"
        'Click Solve
        SolverSolve UserFinish:=True
        'Plot st.dev.
        Range("A2:E26").Cells(Counter, 4) = Range("$C$48")
    'Next expected return
    Next Counter
'Multi optimizing
    'Reset SOLVER
    SolverReset
    'Set constraint weights must sum to 100%
    SolverAdd CellRef:="$Y$49", Relation:=2, FormulaText:="100%"
    'Set expected return equal to target
    SolverAdd CellRef:="$D$46", Relation:=2, FormulaText:="$B$50"
    'Set constraint for short sales
    SolverOptions AssumeNonNeg:=True
    'Repeat for 25 different values of the expected return
    For Counter = 1 To 25
        'Increment the target expected return by 0.07%, starting at 0.500%
        Range("B50") = 0.005 + Counter * 0.0007
        'Minimize variance by changing portfolio weights
        SolverOk SetCell:=Range("$D$47"), MaxMinVal:=2, ValueOf:=0, ByChange:=Range("$G$49:$X$49"), _
                Engine:=1, EngineDesc:="GRG Nonlinear"
        'Click Solve
        SolverSolve UserFinish:=True
        'Plot st.dev.
        Range("A2:E26").Cells(Counter, 5) = Range("$D$48")
    'Next expected return
    Next Counter
```

```vba
'Minimum variance portfolios:
'Naive
    'Reset SOLVER
    SolverReset
    'Set constraint weights must sum to 100%
    SolverAdd CellRef:="$Y$40", Relation:=2, FormulaText:="100%"
    'Set constraint for short sales
    SolverOptions AssumeNonNeg:=True
    'Minimize variance by changing portfolio weights
    SolverOk SetCell:=Range("$B$41"), MaxMinVal:=2, ValueOf:=0, ByChange:=Range("$G$40:$X$40"), _
        Engine:=1, EngineDesc:="GRG Nonlinear"
    'Click Solve
    SolverSolve UserFinish:=True
'Single
    'Reset SOLVER
    SolverReset
    'Set constraint weights must sum to 100%
    SolverAdd CellRef:="$Y$41", Relation:=2, FormulaText:="100%"
    'Set constraint for short sales
    SolverOptions AssumeNonNeg:=True
    'Minimize variance by changing portfolio weights
    SolverOk SetCell:=Range("$C$41"), MaxMinVal:=2, ValueOf:=0, ByChange:=Range("$G$41:$X$41"), _
        Engine:=1, EngineDesc:="GRG Nonlinear"
    'Click Solve
    SolverSolve UserFinish:=True
'Multi
    'Reset SOLVER
    SolverReset
    'Set constraint weights must sum to 100%
    SolverAdd CellRef:="$Y$42", Relation:=2, FormulaText:="100%"
    'Set constraint for short sales
    SolverOptions AssumeNonNeg:=True
    'Minimize variance by changing portfolio weights
    SolverOk SetCell:=Range("$D$41"), MaxMinVal:=2, ValueOf:=0, ByChange:=Range("$G$42:$X$42"), _
        Engine:=1, EngineDesc:="GRG Nonlinear"
    'Click Solve
    SolverSolve UserFinish:=True
'Sharpe optimal portfolios
'Naive
    'Reset SOLVER
    SolverReset
    'Set constraint weights must sum to 100%
    SolverAdd CellRef:="$Y$33", Relation:=2, FormulaText:="100%"
    'Set constraint for short sales
    SolverOptions AssumeNonNeg:=True
    'Maximize Sharpe by changing portfolio weights
    SolverOk SetCell:=Range("$B$33"), MaxMinVal:=1, ValueOf:=0, ByChange:=Range("$G$33:$X$33"), _
        Engine:=1, EngineDesc:="GRG Nonlinear"
    'Click Solve
    SolverSolve UserFinish:=True
'Single
    'Reset SOLVER
    SolverReset
    'Set constraint weights must sum to 100%
    SolverAdd CellRef:="$Y$34", Relation:=2, FormulaText:="100%"
    'Set constraint for short sales
    SolverOptions AssumeNonNeg:=True
    'Maximize Sharpe by changing portfolio weights
    SolverOk SetCell:=Range("$C$33"), MaxMinVal:=1, ValueOf:=0, ByChange:=Range("$G$34:$X$34"), _
        Engine:=1, EngineDesc:="GRG Nonlinear"
    'Click Solve
    SolverSolve UserFinish:=True
'Multi
    'Reset SOLVER
    SolverReset
    'Set constraint weights must sum to 100%
    SolverAdd CellRef:="$Y$35", Relation:=2, FormulaText:="100%"
    'Set constraint for short sales
    SolverOptions AssumeNonNeg:=True
    'Maximize Sharpe by changing portfolio weights
    SolverOk SetCell:=Range("$D$33"), MaxMinVal:=1, ValueOf:=0, ByChange:=Range("$G$35:$X$35"), _
        Engine:=1, EngineDesc:="GRG Nonlinear"
    'Click Solve
    SolverSolve UserFinish:=True
'End subroutine
End Sub
```