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A Different Perspective On Volatility?

An Empirical Analysis of the Effects of Volatility-Management in a Norwegian Context

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Abstract

To raise awareness of volatility-management — that is, improving portfolio performance by adjusting exposure according to volatility information, this thesis aims to provide empirical evidence on the effects of volatility-management in a Norwegian context. We find that volatility-managed multifactor portfolios that are rebalanced monthly outperform its nonmanaged counterparts. Specifically, our strategy generates an annualized alpha of up to 5.56% and an appraisal ratio of 0.72 before transaction costs. In economic terms, this implies that an investor who manages volatility increases the Sharpe ratio by 0.72 annually compared to an investor who ignores volatility timing. We also find that the benefits are not limited to short-term investors, but remain modest at a rebalancing frequency of up to 12 months. These results may originate from some investors reacting slowly to changes in market volatility, which leads to an unfavorable risk-return trade-off.

Our results suggest that participants investing in the Norwegian market may capitalize on prior volatility information, which challenges the weak form of the efficient market hypothesis. This provides an incentive to pay attention to volatility fluctuations.

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1 Introduction

Volatility-management strategies adjust factor exposure based on prior volatility information to improve performance. These strategies are characterized by reduced positions in the underlying factors following periods of high volatility and more aggressively levered positions following periods of low volatility. Although volatility-management has proven beneficial in large, established markets, there are conflicting views on the applicability of this strategy. There is also a general lack of research on the potential benefits in smaller equity markets. In this thesis, we therefore seek to provide empirical evidence of the effects of volatility-management in a Norwegian context. The strategy may give rise to higher risk-adjusted returns and is therefore of interest to participants who invest in the Norwegian market both in the long-term and in the short-term.

A positive risk-return relation is eminently considered the backbone of financial theory and can be traced back to the introduction of Modern Portfolio Theory by Markowitz (1952) and the Capital Asset Pricing Model by Sharpe (1964). However, recent studies on volatility-management — that is, enhancing portfolio performance by adjusting exposure according to volatility information, show that this relationship may not hold empirically. Investors could therefore generate profits by exploiting an unfavorable risk-return trade-off following volatility fluctuations. The research is extensive and shows strong empirical performance for volatility-managed versions of well-established trading strategies such as the momentum factor (Barroso and Santa-Clara, 2015; Daniel and Moskowitz, 2016), a wide range of asset pricing factors (Moreira and Muir, 2017b; 2019), and industry portfolios (Grobys and Kolari, 2020). These results contradict the widely held notion that higher volatility constitutes an opportunity to purchase at the bottom as the strategy adjusts positions downward when volatility is high. It also suggest insufficient compensation for excessive risk-taking, which has led scholars to reevaluate the classic risk-return trade-off. The results are therefore interesting as they pose a challenge to the reputedly positive risk-return relationship in time series of returns. However, there are conflicting views on whether the volatility-management strategy is beneficial when considering the practicality of the method (Cederburg et al., 2020) and transaction costs (Barroso and Detzel, 2021).

Previous research has traditionally focused on the implications of volatility-management in large, established markets, with an emphasis on the United States. Markets of a smaller nature, such as the Norwegian equity market, have therefore rarely been considered. As a result, the volatility-management strategy may only be representative of markets of a certain size and character, and less applicable to smaller equity markets where significant size differences affect outcomes. In addition, the existing literature focuses almost exclusively on a monthly rebalancing frequency to adjust factor exposure and neglects analysis of other frequencies. Therefore, by responding to market changes on a monthly basis, the method may be less suitable for short-term investors with an investment horizon of less than one month.

We believe that it is interesting and necessary to fill the gap in the existing literature by being, to the best of our knowledge, the first across all markets to study the effects of volatility-management at a biweekly rebalancing frequency. A shorter rebalancing frequency could lead to more accurate volatility estimates and may contribute to a better approach by responding rapidly to changes in market volatility. We are also among the few to provide empirical evidence on the effects of volatility-management in smaller equity markets, particularly in a Norwegian context. Since we want to study the benefits in their entirety, we will analyze the effects of a monthly and a biweekly rebalancing frequency. Our research question is therefore as follows:

Is volatility-management advantageous for participants investing in the Norwegian market?

To answer this question, we aim to replicate Moreira and Muir's (2017b) methodology and construct volatility-managed portfolios. In these portfolios, factor exposure is adjusted downward after a period of high volatility and upward after a period of low volatility. It is highly interesting to compare our results with international research as our thesis addresses the current shortage of research on volatility-management in smaller markets and aims to provide real value for participants operating in the Norwegian market.

The remainder of this thesis is organized as follows. In Chapter 2, we discuss the empirical literature on volatility-management and our contribution to the existing literature. In Chapter 3, we present the data used and the descriptive statistics of the nonmanaged factors included in the study. In Chapter 4, we explain in detail the methodology applied in our analysis. In Chapter 5, we present our main empirical findings from the analysis and compare them with the results of existing research. In Chapter 6, we assess the robustness of our results and examine the volatility-management strategy from various perspectives, namely, business cycle risks, a risk parity strategy, a long-term perspective, leverage constraints, and transaction costs. Finally, Chapter 7 provides our concluding remarks as well as limitations and suggestions for future research.

2 Literature Review

2.1 Empirical Literature

The empirical literature on volatility timing began with Fleming et al. (2001). Driven by the notion that volatility is predictable to some extent, they systematically consider the value of volatility timing for a risk-averse investor in the U.S. market. This is achieved through a daily allocation to four asset classes, with asset class weights decreasing as volatility increases and increasing as volatility decreases. To measure the impact of volatility timing, they calculate the fee a risk-averse investor would be willing to pay to switch from the static nonmanaged portfolio to the volatility-managed portfolio. They find that volatility timing strategies outperform their static counterparts and emphasize that transaction costs must exceed the estimated annual fee of 3.28% by a factor of six to outweigh the benefits of volatility timing. Continuing their 2001 study, Fleming et al. (2003) examine the economic benefits of volatility timing by adopting a realized volatility approach. In particular, they suggest that intraday returns are a more reliable measure in calculating daily return volatility, as opposed to the daily returns applied in Fleming et al. (2001). Fleming et al. (2003) demonstrate the economic value of a change in methodology, where a risk-averse investor would be willing to pay among 50 to 200 basis points annually to change. Additionally, they provide evidence that volatility timing remains profitable at time horizons of up to one year. This suggests that volatility timing is not limited to short-term investors, but is also suitable for investors with a long-term perspective. These early contributions laid the foundation for further research on the effects of volatility timing.

Among more recent research in the field of volatility-management, Barroso and Santa-Clara (2015) propose a risk-management method for the pervasive momentum factor in the U.S. market. The method is based on adjusting the factor exposure monthly according to the realized variance of the previous six months, while applying a constant target volatility. By ensuring a constant target volatility, the volatility-managed momentum factor and the nonmanaged counterpart contain similar risk. They report a drastic increase in the Sharpe ratio from 0.53 for the nonmanaged momentum strategy to 0.97 for its volatility-managed counterpart. They also emphasize the potentially large crash risk of the original momentum factor, which acts unappealing to investors, and document a substantial risk reduction when volatility is managed. The results are robust across subsamples and are also evident in an international context, namely Japan, France, the UK, and Germany. This may indicate a valuable strategy for markets outside the U.S.

Daniel and Moskowitz (2016) construct a dynamic strategy that adjusts risk exposure to the momentum factor on a monthly basis to maximize the unconditional Sharpe ratio. Thus, the dynamic strategy increases or decreases weights according to volatility information to maximize the Sharpe ratio. In contrast, the constant volatility strategy adjusts risk exposure such that the target volatility is met. Daniel and Moskowitz (2016) find that the dynamic volatility-management strategy nearly doubles the Sharpe ratio compared to a static buy-and-hold strategy, while outperforming the constant volatility approach by Barroso and Santa-Clara (2015). Moreover, the results show remarkable consistency across sample periods, asset classes, and equity markets. Furthermore, Grobys and Kolari (2020) extends the empirical research on the momentum factor by addressing the considerable gap in the asset pricing literature on industry portfolios. Their results suggest that a risk-managed industry momentum strategy generates between 33 and 36 basis points per month relative to the nonmanaged strategy.

Our thesis is directly related to Moreira and Muir (2017b), who study the effect of volatility-managing aggregated pricing factors in the U.S. market over the period from 1926 to 2015. Their research is based on the observation that volatility changes are not offset by corresponding changes in expected returns. This implies potential benefits of volatility timing due to an unfavorable risk-return trade-off. Therefore, they construct a strategy that capitalizes on the volatility persistence property by scaling factor exposure according to the previous month's realized variance. They document alphas of significant size across multiple factors and utility gains of up to 65% for a mean-variance investor through volatility-management. Moreover, they argue that the volatility-managed market portfolio remains partially robust for up to 18 months. These results contradict traditional financial theory, as the strategy implies a willingness to reduce risk during periods of high volatility, as opposed to increased or constant risk-taking. The strategy is also of particular interest as they state real-time implementation is feasible. Moreira and Muir (2019) add to the existing literature, which emphasize short-term volatility timing (Fleming et al., 2001; 2003; Moreira and Muir, 2017b), by examining the benefits of volatility-management from the perspective of a long-term investor. Specifically, they consider a long-term investor who allocates funds between a riskless and a risky asset, with expected returns and volatility varying over time. Their results suggest that a long-term investor with a 20-year time horizon foregoes the equivalent of 2.4% of annual wealth by neglecting the variation in volatility. In conclusion, these results may suggest that volatility-management is irrespective of investment horizon to a large extent and ignorance could lead to large utility losses.

Following the findings of Moreira and Muir (2017b), numerous papers challenge the impact of volatility-management. Liu et al. (2019) claim that volatility-management strategies applied to the stock market experience a look-ahead bias. They state the bias occurs when calculating the weight parameter according to the unconditional volatility for the entire sample period and argue that the results may be greatly affected by the parameter. In general, they conclude that adopting an out-of-sample approach to avoid a look-ahead bias drastically weakens the performance of volatility-management. They also point out the difficulties of implementing the strategy in practice due to extensive drawdowns. However, the volatility-management strategy outperforms the market in times of financial adversity and the authors cannot ignore the possibility of outperforming the market when trading individual stocks using prior volatility information.

Cederburg et al. (2020) challenge the practicality of Moreira and Muir's (2017b) method and state that a strategy construction using ex-post data prevents real-time implementation and may overstate the value of volatility-management in practice. By applying an out-of-sample approximation to an extensive set of 103 equity trading strategies, they find limited support that volatility-managed portfolios consistently achieve a higher Sharpe ratio relative to its nonmanaged counterparts. However, similar to Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016), implementing the strategy in real-time remains beneficial for the momentum factor. Nevertheless, the authors do not reject the likelihood of investors possessing ways of estimating portfolio weights that exploit alphas, such as robust optimization methods (Kirby and Ostdiek, 2012). Using an out-of-sample approach requires parameter estimation based on historical data, which may also be subject to estimation risk.

Barroso and Detzel (2021) further analyze whether limits to arbitrage imposed by transaction costs, arbitrage risk, and short-selling restrictions provide a reasonable explanation for the benefits achieved by volatility-management in the U.S. context. Contrary to Moreira and Muir (2017b), who attribute transaction costs only to the market factor, Barroso and Detzel (2021) argue that nonmarket factors generate sizable transaction costs due to large positions in expensive small-cap stocks. Accounting for estimated transaction costs, they find that volatility-managed nonmarket factors, with the exception of the momentum factor, fail to generate significantly positive alphas across subsamples. These results persist when cost-mitigation strategies are employed. However, consistent with Moreira and Muir (2017b), they document robust abnormal returns for the market factor after imposing transaction costs, suggesting plausible benefits of volatility-management.

2.2 Our Contribution to Existing Literature

This thesis contributes to the existing literature by, to our knowledge, being the first to conduct extensive research on the potential benefits of volatility-management in smaller equity markets, which have been neglected by other researchers. Our contribution reach well beyond the implications of volatility-management by providing a thorough analysis of the different perspectives that may explain the phenomenon. We therefore believe that we can contribute by offering real value to participants in the Norwegian market. In addition, we offer a new approach to the existing literature on volatility-management by proposing a biweekly rebalancing frequency that could be suitable for investors with a short-term horizon of less than one month.

3 Data

3.1 Data Description

To conduct this study, we obtain daily and monthly data from Bernt Arne Ødegaard's (2022) website from 1980:01 to 2020:11. Using company data from the Oslo Stock Exchange, Ødegaard constructs two indices, one value-weighted and one equally weighted, where we use the value-weighted index as we intend to invest in the entire market (Ødegaard, 2021). The index represents the market return factor (MKT).

We also obtain daily and monthly data for well-known pricing factors in the Norwegian market such as the Fama-French size factor (SMB), the value factor (HML), and the momentum factor (UMD). Ødegaard (2021) constructs the three factors following the procedure of Fama and French (1996). The size factor is based on a strategy in which a market participant buys small stocks, defined by a low market capitalization, and finances the position by short-selling big stocks, defined by a high market capitalization. Firms are classified as large if their market capitalization is above the median, while small firms are below the median market capitalization. The value factor is based on buying stocks with a high book-to-market ratio and financing the position by short-selling stocks with a low book-to-market ratio. Stocks with a high book-to-market ratio are above the 70^{th} percentile, while stocks with a low book-to-market ratio are below the 30^{th} percentile. Moreover, the momentum factor is based on buying stocks that have performed particularly well over the past *t-12* to *t-2* months and funding the position by short-selling stocks that have performed poorly over the same period. The companies in the low prior return portfolios are below the 30^{th} percentile, while the companies in the high prior return portfolio are above the 70^{th} percentile.

In addition, we consider a liquidity factor (LIQ). The liquidity factor is based on a strategy in which a market participant buys the least liquid stocks and finances the position by short-selling the most liquid stocks. Næs et al. (2009) sort the stocks into three portfolios based on the relative spread as a proxy for liquidity. All factors are reported from 1980:01 to 2020:11. However, we lack observations from the first six months of 1981 for several factors. Thus, to obtain a balanced data set, we exclude the initial six months and obtain observations from 1981:07 onward. Similarly, for MKT, we exclude data points up to 1981:07 and obtain a total of 473 months for each factor. Table 1 provides statistical properties of the nonmanaged factors.

Furthermore, it is worth noting that we do not include the profitability factor (RMW) and the investment factor (CMA) due to the lack of accounting data for the Norwegian market. The data are only available from 2013 and would result in considerably fewer data points compared to the other variables. The lack of data could also potentially diminish the validity of the volatility-managed strategy. Additionally, we do not consider the Carhart Momentum factor due to its high correlation with UMD of 0.78, as shown in the correlation matrix in Table 15. By including both variables, we risk imperfect multicollinearity in a multivariate analysis (Stock and Watson, 2019, p.230), which affects the precision of the estimates. Therefore, similar to Moreira and Muir (2017b), we only consider UMD. However, consistent with Næs et al. (2009), who document a reasonably well explanatory power of returns on OSE and a portfolio containing a market factor, a liquidity factor, and a size factor, we are confident that the included variables are sufficient.

Table 1: Statistical Summary Nonmanaged Factors

The table shows descriptive statistics for each nonmanaged factor. The mean represents the mean factor return, std.dev measures the dispersion of factor returns, and min and max represent the highest and lowest factor returns, respectively. Skewness represents the symmetry of the distribution, kurtosis measures how the distribution is scattered between the extremes, and MDD measures the maximum drawdown in percent. The sample ranges from 1981:07 to 2020:11 and all values are per month.

	MKT	SMB	HML	UMD	LIQ
N	473	473	473	473	473
MEAN	1.88	0.73	0.35	0.75	0.15
STD. DEV	5.94	4.40	5.25	5.66	4.54
MIN	-23.79	-16.63	-19.64	-24.27	-17.66
MAX	19.72	21.08	22.16	25.48	16.42
SKEW	-0.47	0.34	0.19	-0.17	0.12
KURTOSIS	1.72	2.07	1.44	1.69	0.91
MDD	-46.12	-34.26	-78.79	-66.78	-74.28

4 Methodology

4.1 Portfolio Formation

We replicate the methodology of Moreira and Muir (2017b) and construct the volatility-managed portfolios by scaling the factor exposure according to the inverse realized variance of the previous month. That is, the portfolio scales in and out of the factor each month according to the previous conditional variance. The managed portfolio is then

$$f_{t+1}^{\sigma} = \frac{c}{\hat{\sigma}_t^2(f)} f_{t+1} \tag{1}$$

where f_{t+1} denotes the nonmanaged risk factor corresponding to a buy-and-hold strategy and $\hat{\sigma}_t^2(f)$ denotes the realized variance of daily returns in the month prior to the portfolio formation date. Moreover, c is a constant term denoting the target level of volatility such that $\sigma(f_{t+1}^{\sigma}) = \sigma(f_{t+1})$ for the entire sample period. Moreira and Muir (2017b) emphasize that the Sharpe ratio is invariant to the specification of c. Therefore, using the entire sample to calculate the constant term does not affect the results of the strategy.

The strategy either leverage or deleverage its position in the nonmanaged risk factor according to the scaling factor, denoted by S, at time t.

$$S_t = \frac{c}{\hat{\sigma}_t^2(f)} \tag{2}$$

When current market volatility is low, the scaling factor is above one, resulting in a leveraged position, while episodes of high volatility entail a deleveraged position and a scaling factor below one. The realized variance, $\hat{\sigma}_t^2(f)$, is calculated as

$$\hat{\sigma}_t^2(f) = RV_t^2(f) = \sum_{d=1/D_m}^{1} \left(f_{t+d} - \frac{\sum_{d=1/D_m}^{1} f_{t+d}}{D_m} \right)^2 \tag{3}$$

where f_{t+d} denotes the nonmanaged risk factor return on index day d in month t, and $(\frac{\Sigma_{d=1/D_m}^{1} f_{t+d}}{D_m})$ is the sum of all daily factor observations in month t divided by the exact number of trading days D_m[1] in month t. It is worth mentioning that Moreira and Muir (2017b) includes the observations of the 22 most recent trading days in the conditional

[1] The notation is similar to that of Grobys and Kolari (2020).

variance calculation[2], while we consider all trading days within a given month. Nevertheless, the average number of trading days in a month in our data set is 22. The realized variance for month t is constructed as follows. For each preceding month of the formation date, we calculate the average return for factor f using the daily observations and then subtract the average return from each daily observation of factor f in month t. Thereafter, we square the daily excess factor return and sum the number of squares corresponding to the number of trading days D_m in that particular month.

Figure 1: Rebalancing Timeline

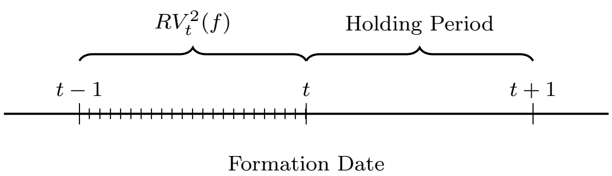


Figure 1 shows an excerpt from the monthly time series. The period from *t-1* to t represents the daily returns used to calculate the realized variance that determines the factor exposure at formation month t. We remain in the position for one month, from t to *t+1*, before rebalancing.

Furthermore, we extend our analysis by increasing the rebalancing frequency to biweekly. We construct the volatility-managed portfolios by scaling the risk exposure according to the inverse realized variance of the previous two weeks. The managed portfolio is then

$$f_{t+1}^{BW,\sigma} = \frac{c}{\hat{\sigma}_t^2(f^{BW})} f_{t+1}^{BW} \tag{4}$$

where f_{t+1}^{BW} denotes the nonmanaged risk factor and $\hat{\sigma}_t^2(f^{BW})$ denotes the realized variance of daily returns during the two weeks prior to the portfolio formation date. Moreover, c denotes the constant scaling factor such that $\sigma(f_{t+1}^{BW,\sigma}) = \sigma(f_{t+1}^{BW})$ for the entire sample period. Note that *t+1* in equation (4) corresponds to biweek *t+1*. We construct the biweekly returns as follows. For each factor, we assign the days of the week to their respective week numbers and sum the daily returns to obtain the weekly returns. Consequently, we construct 2058 weeks for the entire sample. Thereafter, by aggregating two and two adjacent weeks in a month, we obtain 1029 biweekly nonmanaged returns.

[2]See Moreira and Muir (2017b) equation (2).

4.2 Empirical Methodology

We assess the performance of the strategy from the perspective of a mean-variance investor who allocates funds in a risky portfolio according to the mean-variance trade-off

$$\frac{E_t[f_{t+1}]}{\sigma_t^2(f)} \tag{5}$$

where $E_t[f_{t+1}]$ denotes the expected factor return at time t. This implies that the optimal weight in a volatility-managed factor depends on how appealing the mean-variance trade-off appears to be. To evaluate performance, we run a univariate time series regression for each volatility-managed factor on the nonmanaged factor

$$f_{t+1}^{\sigma} = \alpha + \beta f_{t+1} + \epsilon_{t+1} \tag{6}$$

where α denotes the intercept, β denotes the slope, and ϵ denotes the error term. The objective is to analyze whether the volatility-management strategy generates excess payoffs relative to the standard nonmanaged factor. A positive significant intercept implies that the managed strategy expands the mean-variance frontier and therefore increases the Sharpe ratio relative to the standard factor. Hence, for each given unit of risk, the mean-variance investor earns a higher return for the optimal portfolio. Additionally, the slope defines the movement of the volatility-managed strategy with respect to the movement of the original factor by one unit.

For each volatility-managed factor, we also report the root mean squared error (RMSE) obtained from the error term in equation (6). This allows us to calculate the annualized appraisal ratio (AR) based on monthly data[3].

$$AR = \sqrt{12}\frac{\alpha}{RMSE} \tag{7}$$

The appraisal ratio quantifies the extent to which the volatility-managed portfolio, deployed by a mean-variance investor, increases the Sharpe ratio relative to an investor constrained to the original factor. In other words, it represents the excess Sharpe ratio.

[3]Note that we multiply the biweekly appraisal ratio by $\sqrt{24}$ to retrieve annual numbers.

5 Analysis

In the following section, we evaluate the performance of the volatility-managed strategy relative to its nonmanaged counterpart. First, we apply the strategy in a monthly single factor environment and compare it to a biweekly rebalancing frequency. We then extend our analysis to a multifactor environment for both rebalancing frequencies.

5.1 Monthly Single Factor Portfolio Results

Before reporting the main results, we refer to our replication of Moreira and Muir (2017b) for the U.S. market in Table 14 in Appendix A2. Panel A shows our volatility-managed replication of MKT, SMB, HML, Mom, RMW, and CMA, while Panel B shows the original results of Moreira and Muir (2017b). We obtain similar results with only minor deviations and can therefore conclude that we adhere to the methodology. Moreover, we note that the results for the Norwegian market may differ compared to the U.S. market, especially due to size differences. Ødegaard (2021) constructs the Norwegian size factor (SMB) and value factor (HML) following the procedure of Fama and French (1996) by dividing firms into size classes and value classes. Given the exceptional size difference across the markets, reflected in the relative difference between small and big firms and firms with high and low book-to-market ratios, it is reasonable to expect a difference in outcomes. The size difference also affects market liquidity, as U.S. companies are likely to be more liquid relative to Norwegian companies.

Table 2 Panel A shows the results for the Norwegian market obtained by regressing each volatility-managed factor on the corresponding nonmanaged factor at a monthly rebalancing frequency. We observe positively significant alphas for the market factor (MKT), the Fama-French momentum factor (UMD), and the liquidity factor (LIQ). This may indicate that the volatility-managed factors generate excess returns relative to the nonmanaged factor, thus increasing the Sharpe ratio. The most prominent result is shown for UMD with an annualized alpha of 9.21%. This is consistent with Moreira and Muir (2017b), who document superior momentum results relative to the other factors [4].

[4] See results in Moreira and Muir (2017b) Table I

Furthermore, we document a positive and marginally[5] significant size factor (SMB) in addition to a negatively significant value factor (HML). This is contrary to Moreira and Muir (2017b), who reports a negative SMB and a positive HML for the U.S. market. This could be due to the size difference between the Norwegian and U.S. market, which consequently affect factor construction. Nonetheless, this is an interesting result as it implies that it would be beneficial to manage the Norwegian HML factor opposite to the volatility-management strategy. That is, it would be beneficial to increase risk exposure following periods of high volatility and vice versa, which is contrary to the strategy. This abnormality is further discussed in Section 6.1.

Additionally, for each factor we report the appraisal ratio, denoted AR, which specifies the extent to which volatility-management can be applied to expand the mean-variance frontier and thus increase utility gains. We report significant utility gains across multiple factors by appraisal ratios ranging from 0.29 to 0.71 for positively significant factors, with the momentum factor providing the most noticeable gains for the mean-variance investor.

In Panel B, we extend to a multivariate analysis by regressing each volatility-managed factor on the three nonmanaged Fama and French (1993) factors, UMD and LIQ. The regression equation is then:

$$f^{\sigma}_{t+1} = \alpha + \beta MKT_{t+1} + \beta SMB_{t+1} + \beta HML_{t+1} + \beta UMD_{t+1} + \beta LIQ_{t+1} + \epsilon_{t+1} \quad (8)$$

Panel B shows robust results across a range of asset pricing factors where UMD and LIQ remain statistically significant at the 1% level. Most noticeable, LIQ and SMB drastically increase the magnitude of annualized alpha to 5.76% and 3.35%, respectively, although SMB remains marginally significant at the 10% level. On the other hand, although still statistically significant, the annualized alpha of UMD decrease which corresponds to the findings of Moreira and Muir (2017b). In addition, we find that the single-factor volatility timing strategy of MKT and HML may be priced by other aggregate factors, as the variables become statistically insignificant in the multivariate analysis.

[5]Defined by p - value < 0.1

By including subsamples, we can assess whether volatility-management is beneficial over shorter horizons and is not conditional on rare events or specific time periods. Therefore, in Panel C, we form two independent subsamples that span the entire sample: 1981:07 to 2001:12 and 2002:01 to 2020:11, and analyze each factor separately. In addition, we include a subsample that contains both the 1987 crash and the 2008 crash to examine whether the strategy performs better in a period with considerable fluctuations. In the 1981-2001 subsample, UMD is statistically significant with an annualized alpha of 7.79%, while HML remains marginally negatively significant with an annualized alpha of -5.84%. This implies a profitable period for the volatility-managed UMD factor and an unprofitable period for the volatility-managed HML factor. In the 2002-2020 subsample, UMD, MKT, and LIQ are statistically significant with annualized alphas of 9.95%, 6.97%, and 5.13%, respectively. The higher reported alphas in 2002-2020 may be due to greater volatility fluctuations, consistent with the theoretical assumption that periods of higher volatility variation should be the most profitable for the strategy. This is even more evident in the 1987-2008 subsample, where MKT, SMB, and UMD show statistically significant alphas of considerable size. In addition, HML's alpha decreases slightly compared to the 1981-2001 subsample, but its significance improves. Nevertheless, the overall results are less consistent across subsamples than in the 40-year total sample. This could be due to the size of the subsamples, as the statistical power may be weaker compared to the 30-year subsamples of Moreira and Muir (2017b).

Table 2: Volatility-Managed Monthly Single Factors

Panel A shows the results of the bivariate time series regressions of each monthly volatility-managed factor on the nonmanaged factor, $f^{\sigma}_{t+1} = \alpha + \beta f_{t+1} + \epsilon_{t+1}$. The managed factor, f^{σ}_{t+1}, scales risk exposure according to the inverse realized variance of the previous month, $f^{\sigma}_{t+1} = \frac{c}{\hat{\sigma}^2_t(f)} f_{t+1}$. In Panel B, we expand to a multivariate regression by including all factors as control variables. Panel C shows the bivariate regression alphas in subsamples from 1981-2001, 2002-2020, and 1987-2008. Alphas and RMSE are annualized in percent by multiplying monthly factors by 12, and the appraisal ratio is calculated as $\frac{\alpha}{RMSE}\sqrt{12}$. The sample ranges from 1981:07-2020:11 and is based on monthly data. Standard errors are adjusted for heteroskedasticity.

Panel A: Bivariate Regressions

	(1)	(2)	(3)	(4)	(5)
	MKTσ	SMBσ	HMLσ	UMDσ	LIQσ
MKT	0.77				
	t = 15.14				
SMB		0.76			
		t = 14.04			
HML			0.75		
			t = 14.52		
UMD				0.75	
				t = 14.88	
LIQ					0.78
					t = 15.74
Alpha (α)	4.85	2.83	−3.83	9.21	3.78
	t = 2.16	t = 1.77	t = −1.97	t = 4.65	t = 2.41
N	472	472	472	472	472
R^2	0.58	0.58	0.56	0.56	0.61
RMSE	46.20	34.40	41.57	45.20	33.81
AR	0.36	0.29	−0.32	0.71	0.39

Panel B: Multivariate Regression Alphas

Alpha (α)	3.32	3.35	−2.13	6.43	5.76
	t = 1.36	t = 1.94	t = −1.18	t = 3.17	t = 3.06

Panel C: Bivariate Subsample Regressions

	MKTσ	SMBσ	HMLσ	UMDσ	LIQσ
α : 1981 − 2001	4.25	3.29	−5.84	7.79	2.99
	t = 1.27	t = 1.41	t = −1.79	t = 2.55	t = 1.21
α : 2002 − 2020	6.97	2.45	−0.37	9.95	5.13
	t = 2.35	t = 1.15	t = −0.17	t = 4.10	t = 2.49
α : 1987 − 2008	7.07	4.16	−5.81	9.86	1.07
	t = 2.11	t = 1.95	t = −2.00	t = 3.38	t = 0.46

Figure 2 illustrates the accumulated monthly returns for the volatility-managed market factor compared to the nonmanaged market factor. In general, we observe that the managed strategy outperforms the nonmanaged strategy, and there is a positive gap that persists over most of the 1987-2008 period. Hence, the strategy generates a significant portion of its excess returns for the 1987-2008 subsample, which is also evident in Panel C of Table 2. On the contrary, the strategy underperformed from 2012 to approximately 2015, which may be explained by the low market volatility during this period.

Figure 2: Accumulated Returns

The figure plots the accumulated monthly returns for the volatility-managed market factor and the nonmanaged market factor for the entire sample period.

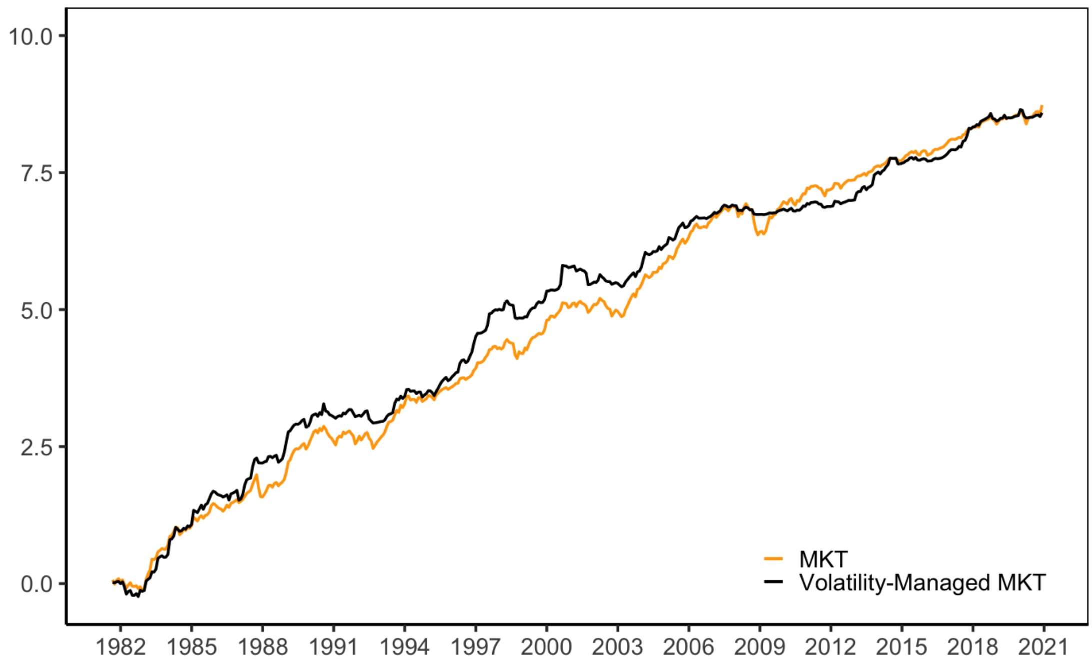
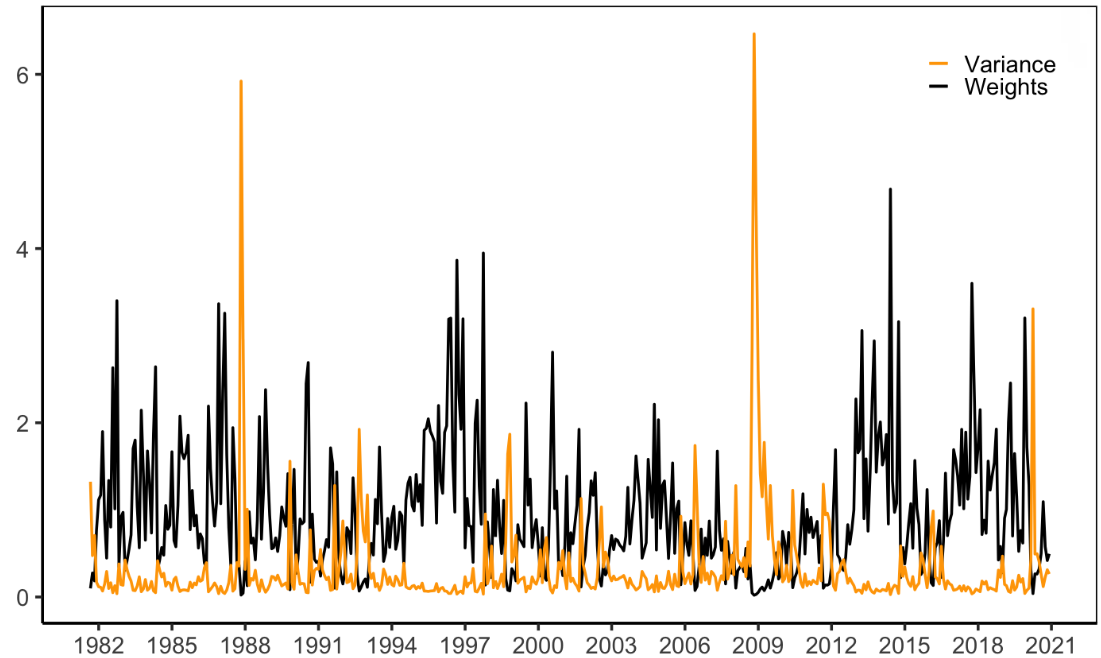


Figure 3 shows the monthly time series evolution of the scaling factor for the volatility-managed market factor and the realized variance. We observe a downward adjustment of the factor exposure when the market faces adversity and an upward adjustment following periods of low volatility. For instance, in October 1987, the Norwegian market experienced volatility of approximately 24%, which caused the market factor weight to drop to 4.42% the following month. It is also particularly noteworthy that from 2012 to 2015, when the volatility-managed market factor underperformed relative to its nonmanaged counterpart, as shown in Figure 2, was a period of low market volatility. It is therefore evident that the largest performance losses of volatility-management are to be expected in times of low volatility due to increased risk exposure.

Figure 3: Market Factor Weights Versus Variance

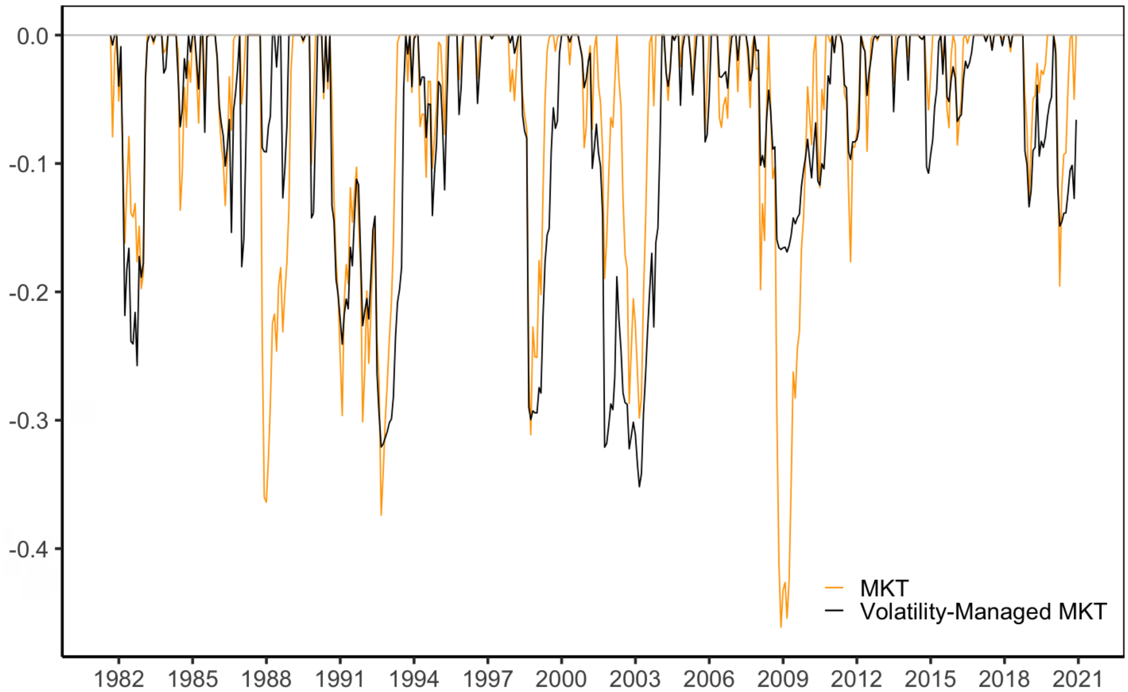
The figure shows the time series evolution of the monthly scaling factor for the volatility-managed market factor and the monthly realized variance. Note that the variance shown has been magnified by a factor of one hundred for ease of visualization.



On the other hand, periods of significant declines in returns are often associated with high volatility, and since the strategy reduces risk following periods of high volatility, abrupt reductions in returns are avoided. This is evident in Figure 4, which illustrates the monthly percentage drawdowns for the volatility-managed market factor and its nonmanaged counterpart. It is apparent that the strategy prevents the drastic losses that occurred in the 1987 and 2008 financial crises, as shown by the depth of the drawdowns. Specifically, the Norwegian market reached a maximum drawdown of 46% in 2008 compared to a volatility-managed maximum drawdown of 17% over the same period, which is appealing to investors. However, the strategy takes higher risk when volatility is low, such as in 1982 and 2003, which consequently leads to the largest portfolio losses.

Figure 4: Market Factor Drawdown

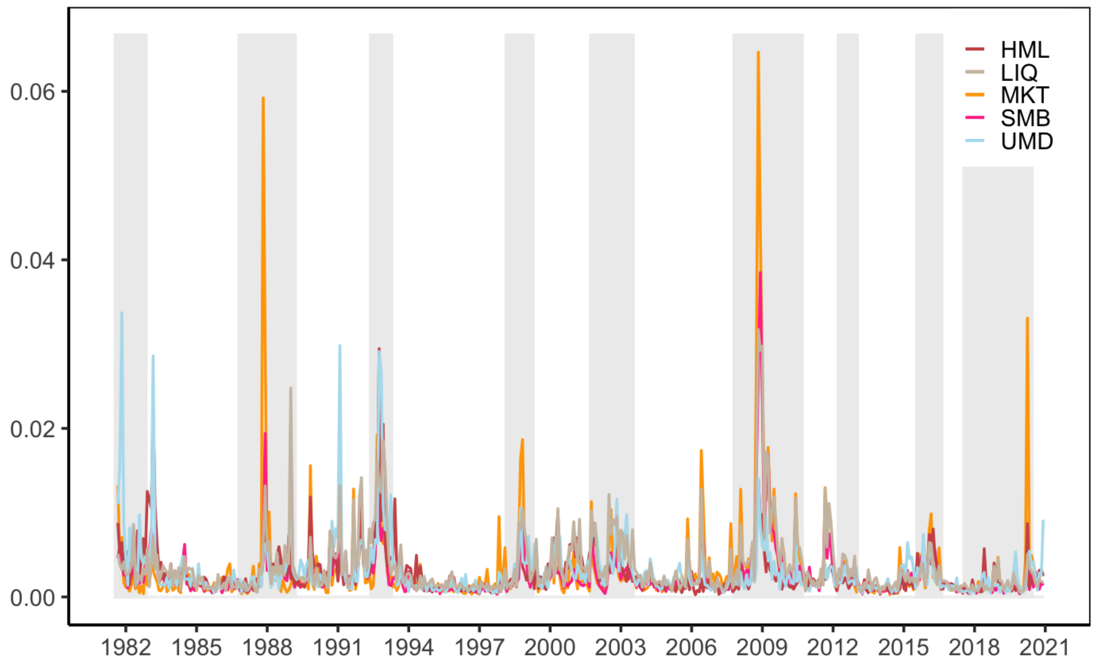
The figure plots the monthly percentage drawdowns for the volatility-managed market factor and the nonmanaged market factor.



Finally, in Figure 5 we plot the realized variance across all factors considered in this study and OECD (2022) recessions for the Norwegian market, as indicated by the grey shaded bars. There is a clear tendency for volatility to increase during recessions, which is consistent with the empirical evidence that volatility tends to be high during recessions (Schwert, 1989). We also observe a clear volatility comovement. The volatility fluctuates significantly over time, peaking during the instability caused by the 1987 and 2008 financial crises and during the escalation of the Covid-19 pandemic in 2020.

Figure 5: Variance By Factors

The figure plots the time series of the monthly variance of each factor for the entire sample period. Grey shaded bars indicate OECD (2022) recessions for the Norwegian market.



5.2 Biweekly Single Factor Portfolio Results

In this section, we evaluate the performance of volatility-management at a biweekly rebalancing frequency. Previous literature has focused on volatility timing at a daily rebalancing frequency (Fleming et al., 2001; 2003) and a monthly rebalancing frequency (Barroso and Santa-Clara, 2015; Daniel and Moskowitz, 2016; Moreira and Muir, 2017b; Cederburg et al., 2020; Barroso and Detzel, 2021). Thus, the existing literature covers both ends of a spectrum where daily rebalancing dramatically increases transactions and monthly rebalancing decreases transactions, but neglects to analyze frequencies in between. As a result, we believe it is interesting and necessary to fill the gap in the existing literature regarding rebalancing frequency and contribute by, to our knowledge, being the first across all markets to study the impact of a biweekly rebalancing frequency.

Panel A of Table 3 shows the results obtained by regressing each volatility-managed factor on the corresponding nonmanaged factor at a biweekly rebalancing frequency. Compared to monthly rebalancing in Table 2, we observe an increase in the annualized alpha of MKT and LIQ of 2.04 and 0.5 percentage points, respectively. MKT shows increased utility gains indicated by the appraisal ratio, albeit modest due to the increased RMSE. This may suggest that there are extreme values with a low probability of occurrence that affect the factor returns of MKT. On the other hand, LIQ reports that the benefit to the mean-variance investor does not increase compared to monthly rebalancing as the increased alpha is offset by a considerable increase in RMSE. Moreover, UMD remains statistically significant even though annualized alpha decreases relative to monthly rebalancing, while SMB and HML become statistically insignificant.

Panel B shows the results of a multivariate analysis that includes all factors as control variables. Most strikingly, UMD and LIQ increase the size of the annualized alpha by 1.27 and 0.46 percentage points, respectively, compared to the multivariate analysis at a monthly rebalancing in Table 2. This could be due to a reduction in downside losses. In fact, the maximum drawdown of UMD and LIQ decreases by about 14 and 15.75 percentage points, respectively, when rebalancing occurs biweekly rather than monthly. This may suggest that the increased rebalancing frequency avoids significant losses in adverse market conditions, as we respond more quickly to changes in volatility. In contrast, MKT is not significant, similar to monthly rebalancing, suggesting that the nonmarket factor's consists of price information related to the scaled market portfolio.

In Panel C, we create two independent subsamples: 1981:07 to 2001:12 and 2002:01 to 2020:11. In the 1981-2001 subsample, UMD outperforms the monthly frequency by 0.61 percentage points. This may be due to a reduction in drawdowns as the maximum drawdown is 32% at a biweekly frequency, compared to 46% for the corresponding monthly subsample. On the contrary, in the 2002-2020 biweekly subsample, UMD experiences longer periods of declines, where the maximum drawdown is 24%, compared to 16% for the monthly subsample. Moreover, LIQ is not statistically significant in either subsample, and therefore we cannot rule out the possibility that the results are influenced by specific time periods or events. On the other hand, MKT has marginally significant high alphas for both subsamples. Thus, compared to the results from MKT in the monthly subsamples presented in Table 2, the results from the biweekly subsamples appear to be more consistent over shorter time periods.

In summary, for single factors rebalanced monthly in Table 2, we find evidence that volatility-management can be beneficial for several factors in a Norwegian context. Most notably, UMD shows remarkable consistency across subsamples and in a multivariate analysis. This is consistent with the strong performance of momentum in the previous literature (Barroso and Santa-Clara, 2015; Moreira and Muir, 2017b; Barroso and Detzel, 2021). Moreover, we find that pricing factors may reduce losses, resulting from maximum drawdowns, when rebalancing biweekly. This may be due to investors' ability to respond more rapidly to fluctuations in market volatility, as opposed to a monthly rebalancing frequency.

Table 3: Volatility-Managed Biweekly Single Factors

Panel A shows the results of the bivariate time series regressions of each biweekly volatility-managed factor on the nonmanaged factor, $f_{t+1}^{BW,\sigma} = \alpha + \beta f_{t+1}^{BW} + \epsilon_{t+1}$. The managed factor, $f_{t+1}^{BW,\sigma}$, scales risk exposure according to the inverse realized variance of the previous two weeks, $f_{t+1}^{BW,\sigma} = \frac{c}{\hat{\sigma}_t^2(f)} f_{t+1}^{BW}$. In Panel B, we include all factors as control variables. Panel C reports the bivariate regression alphas in subsamples from 1981-2001 and 2002-2020. Alphas and RMSE are annualized in percent by multiplying the biweekly factors by 24, and the appraisal ratio is calculated as $\frac{\alpha}{RMSE}\sqrt{24}$. The sample ranges from 1981:07-2020:11 and is based on biweekly data. Standard errors are adjusted for heteroskedasticity.

Panel A: Bivariate Regressions

	(1) MKTσ	(2) SMBσ	(3) HMLσ	(4) UMDσ	(5) LIQσ
MKT	0.56				
	t = 10.73				
SMB		0.63			
		t = 10.98			
HML			0.64		
			t = 15.32		
UMD				0.66	
				t = 15.60	
LIQ					0.65
					t = 13.70
Alpha (α)	6.89	1.68	−1.09	8.30	4.28
	t = 2.48	t = 0.98	t = −0.55	t = 4.01	t = 2.14
N	1,028	1,028	1,028	1,028	1,028
R^2	0.31	0.40	0.41	0.43	0.42
RMSE	79.88	56.28	61.48	67.32	63.98
AR	0.42	0.15	−0.09	0.60	0.33

Panel B: Multivariate Regression Alphas

Alpha (α)	4.78	2.00	0.68	7.70	6.22
	t = 1.54	t = 1.01	t = 0.33	t = 3.07	t = 2.66

Panel C: Bivariate Subsample Regressions

	MKTσ	SMBσ	HMLσ	UMDσ	LIQσ
α : 1981 – 2001	5.96	0.55	−3.92	8.40	4.42
	t = 1.65	t = 0.24	t = −1.24	t = 2.61	t = 1.54
α : 2002 – 2020	7.78	1.98	1.91	7.32	4.15
	t = 1.85	t = 0.77	t = 0.85	t = 3.06	t = 1.49

5.3 Multifactor Portfolio Results

We extend our analysis to a multifactor environment by creating portfolios consisting of all our factors. We apply portfolio mathematics and construct each portfolio as follows. Using the data for the entire time series, we apply the covariance equation (17) in Panel A in the Appendix B1 to construct a covariance matrix similar to that in Panel B. This provides the static weights for each factor such that the Sharpe ratio of the portfolio is maximized. Thereafter, we multiply the optimal static weights, denoted b, by the monthly returns of each factor and sum the returns to obtain a vector of multifactor returns, denoted F_{t+1}. Consequently, we define the mean-variance efficient, hereafter "MVE", portfolio as $f_{t+1}^{MVE} = b^{`}F_{t+1}$, where $b^{`}$ is a vector containing the static weights. The volatility-managed MVE portfolio is then

$$f_{t+1}^{MVE,\sigma} = \frac{c}{\hat{\sigma}_t^2(f^{MVE})} f_{t+1}^{MVE} \tag{9}$$

where $f_{t+1}^{MVE,\sigma}$ denotes the volatility-managed multifactor portfolio, f_{t+1}^{MVE} denotes the nonmanaged counterpart, and c denotes the constant scaling factor such that $\sigma(f_{t+1}^{MVE,\sigma}) = \sigma(f_{t+1}^{MVE})$ for the entire sample period. We calculate the realized variance of the volatility-managed multifactor portfolio using the daily multifactor returns, which is calculated by multiplying the static factor weights by the daily return data. Note also that the weight of each factor remains constant in both the managed and nonmanaged portfolio. Therefore, the scaling factor determines whether the risk exposure, given by the weight in the volatility-managed MVE portfolio, is increased or decreased as a function of the previous month's realized variance.

It is worth noting that the calculation of static weights is based on ex-post data, which prevents the implementation of the strategy in real-time. However, the construction of MVE portfolios allows us to assess whether volatility-management is beneficial for investors currently invested in multiple factors. As Moreira and Muir (2017a) notes, for an investor who has access to all available factors, the appropriate benchmark is the MVE portfolio, as this is the portfolio that an investor with full access would hold.

We construct MVE portfolios that include the three Fama and French (1993) factors, denoted FF3, and the three Fama-French factors plus the momentum factor, denoted FF4. We also construct a portfolio containing the four factors plus a liquidity factor, denoted FF4 + LIQ, and a portfolio containing the value factor and the momentum factor, denoted HML + UMD. The value and momentum portfolio is based on the results of Asness et al. (2013), which report consistent return premia of value and momentum strategies by examining them jointly. Similar to Asness et al. (2013), we find a negative correlation between the momentum and value factor, as shown in Table 15 in Appendix A3. Although our negative correlation is modest, it may imply that a combination strategy is closer to the efficient frontier than an isolated strategy. In contrast to the equal factor exposure applied by Asness et al. (2013), we optimize the combination by making the portfolio mean-variance efficient. In doing so, we challenge the existing literature since, to our knowledge, we are the first to study the benefits of volatility-management for the combination portfolio.

Table 4: Monthly MVE Portfolios

Panel A shows the results of the bivariate time series regressions of each monthly volatility-managed MVE portfolio on the nonmanaged portfolio, $f_{t+1}^{MVE,\sigma} = \alpha + \beta f_{t+1}^{MVE} + \epsilon_{t+1}$. The managed factor, $f^{MVE,\sigma}$, scales risk exposure according to the inverse realized variance of the MVE portfolio from the previous month, $f_{t+1}^{MVE,\sigma} = \frac{c}{\hat{\sigma}_t^2(f^{MVE})} f_{t+1}^{MVE}$. Panel B shows the bivariate regression alphas for each MVE portfolio in subsamples from 1981-2001 and 2002-2020. Alphas and RMSE are annualized in percent by multiplying the monthly factors by 12, and the appraisal ratio is calculated as $\frac{\alpha}{RMSE}\sqrt{12}$. The sample ranges from 1981:07-2020:11 and is based on monthly data. Standard errors are adjusted for heteroskedasticity.

Panel A: MVE Portfolios

	(1) FF3	(2) FF4	(3) FF4 + LIQ	(4) HML + UMD
Alpha (α)	4.76	5.07	5.56	3.73
	t = 4.82	t = 5.51	t = 7.08	t = 2.44
N	472	472	472	472
R^2	0.51	0.52	0.52	0.55
RMSE	21.38	19.54	26.75	32.39
AR	0.77	0.89	0.72	0.40

Panel B: MVE Portfolios - Subsamples

	FF3	FF4	FF4 + LIQ	HML + UMD
$\alpha : 1981 - 2001$	4.94	5.09	5.68	2.38
	t = 3.42	t = 3.49	t = 4.79	t = 1.04
$\alpha : 2002 - 2020$	0.95	3.27	3.33	4.54
	t = 0.78	t = 3.58	t = 4.64	t = 2.38

Panel A of Table 4 shows the results obtained by regressing each volatility-managed MVE portfolio on its nonmanaged counterpart. We find that for each factor combination, the managed MVE portfolios exhibit positive significant alphas of considerable size relative to the static MVE portfolios. The most distinguished result is shown for FF4 + LIQ with an annualized alpha of 5.56% and an appraisal ratio of 0.72. In economic terms, this entails that an investor who manages the volatility of a multifactor portfolio containing all factors increases the Sharpe ratio by 0.72 annually compared to an investor who is limited to the nonmanaged portfolio. Similar to Moreira and Muir (2017b), the multifactor portfolios generally provide larger utility gains for the mean-variance investor through a higher appraisal ratio compared to the single factors shown in Table 2. These results are robust to a multivariate analysis since the nonmanaged portfolio is a product of multiple factors. Therefore, we should obtain similar results irrespective of whether we regress on the nonmanaged MVE portfolio as a whole or on each single factor that makes up the portfolio[6].

Moreover, we find that while the pure combination of HML and UMD yields a lower alpha compared to the other combinations, a volatility-managed combination still achieves an annualized alpha of 3.73%. This suggests that a volatility-managed approach may be beneficial to apply to the nonmanaged approach of Asness et al. (2013) and would be an interesting topic for further research. However, when comparing the alphas with the single factors in Table 2, the combination strategy does not achieve a higher alpha than the isolated UMD. This is despite the fact that the combination strategy achieves a significantly lower RMSE than the isolated UMD, suggesting that the combination is more stable as it is less driven by extreme observations.

In Panel B, we examine the performance of the MVE portfolio in two subsamples: 1981:07 to 2001:12 and 2002:01 to 2020:11. In the 1981-2001 subsample, all specifications except the combination of HML and UMD achieve significant alphas of considerable size and improve results relative to the full sample. This is contrary to the corresponding monthly single factor subsample in Table 2, which shows weak results across multiple factors. This may therefore suggest that MVE portfolios are more robust to periods of lower volatility fluctuations and less affected by specific events. In the 2002-2020 subsample, FF4 and FF4 + LIQ remains statistically significant with annualized alphas of 3.27% and 3.33%, respectively. A noticeable finding is also the large significant alpha of HML + UMD which may be driven by UMD experiencing a profitable period.

Our results suggest that a participant investing in a multifactor portfolio in the Norwegian market may benefit from volatility-management. Since the investor thus capitalizes on the use of past volatility information, our results challenge the weak form of the efficient market hypothesis, which states that excess profits should not be obtainable from historical price information (Fama, 1970).

[6]We also attempted to regress on each factor that makes up the portfolio, which did not affect the results.

Table 5: Biweekly MVE Portfolios

Panel A shows the results of the bivariate time series regressions of each biweekly volatility-managed MVE portfolio on the nonmanaged portfolio, $f_{t+1}^{BW,MVE,\sigma} = \alpha + \beta f_{t+1}^{BW,MVE} + \epsilon_{t+1}$. The managed factor, $f_{t+1}^{BW,MVE,\sigma}$, scales risk exposure according to the inverse realized variance of the MVE portfolio from the previous two weeks, $f_{t+1}^{BW,MVE,\sigma} = \frac{c}{\hat{\sigma}_t^2(f^{BW,MVE})} f_{t+1}^{BW,MVE}$. Panel B shows the bivariate regression alphas for each MVE portfolio in subsamples from 1981-2001 and 2002-2020. Alphas and RMSE are annualized in percent by multiplying the biweekly factors by 24, and the appraisal ratio is calculated as $\frac{\alpha}{RMSE}\sqrt{24}$. The sample ranges from 1981:07-2020:11 and is based on biweekly data. Standard errors are adjusted for heteroskedasticity.

Panel A: MVE Portfolios

	(1) FF3	(2) FF4	(3) FF4 + LIQ	(4) HML + UMD
Alpha (α)	3.05 t = 3.03	3.27 t = 3.82	3.17 t = 3.73	4.41 t = 3.05
N	1028	1028	1028	1028
R^2	0.30	0.30	0.32	0.42
RMSE	27.54	25.88	23.37	45.83
AR	0.54	0.62	0.67	0.47

Panel B: MVE Portfolios - Subsamples

	FF3	FF4	FF4 + LIQ	HML + UMD
$\alpha : 1981 - 2001$	4.79 t = 3.81	4.21 t = 3.83	4.60 t = 4.45	3.08 t = 1.35
$\alpha : 2002 - 2020$	1.36 t = 1.06	1.91 t = 1.73	0.74 t = 0.70	4.66 t = 2.81

Panel A of Table 5 shows the results obtained by regressing each volatility-managed MVE portfolio on the corresponding nonmanaged portfolio at a biweekly rebalancing frequency. We find that all MVE portfolios, although statistically significant, perform worse at a higher frequency relative to monthly. The exception is HML + UMD, which improves the magnitude of the annualized alpha by 0.68 percentage points. In addition, the appraisal ratio increases, which may indicate that the combination strategy is more beneficial at a higher frequency of rebalancing.

Panel B shows the corresponding subsample analysis at a biweekly frequency. The 1981-2001 subsample shows a similar pattern to the monthly MVE portfolios in Table 4, as all portfolios except the HML + UMD strategy are statistically significant. In contrast, an interesting result for the 2002-2020 subsample is that statistical significance deteriorates for all portfolios, while HML + UMD remains statistically significant. Thus, HML + UMD trends in the opposite direction as the other MVE portfolios.

In summary, we generally observe larger utility gains in the multifactor portfolios for the mean-variance investor compared to the single factor analysis in Section 5.1 and 5.2. Results for the MVE portfolios show more consistency across time periods and are robust to multivariate analysis. These results suggest potential benefits of volatility-management in the Norwegian context. Moreover, while we find that volatility-management is beneficial for multifactor portfolios with biweekly frequency, we find no evidence that it is systematically advantageous compared to a monthly frequency.

6 Discussion

In the following section, we examine the volatility-management strategy from different perspectives and try to uncover what might explain the results we obtain. First, we examine the strategy in terms of business cycle risk and in comparison to a risk parity strategy. Then, we examine how outcomes change with the investor's investment horizon, impose strict leverage constraints, and evaluate whether the strategy is robust to transaction costs. Finally, we propose a probable explanation for our results.

6.1 Business Cycle Risk

The OECD (2022) has identified 9 business cycles of different lengths for the Norwegian market over the period 1980 - 2020, as shown in Figure 5. The business cycles are periods characterized by strong volatility fluctuations. It is therefore reasonable to assume that our results may be explained by business cycle risk, since empirical studies have shown that the average excess equity returns are higher in recessions than in expansionary periods (Lustig and Verdelhan, 2012). To further explore the impact of business cycle risk on volatility-management, we present the relative beta estimate of the volatility-managed factors in Table 6. The relative beta represents the beta in a recession relative to the beta in a nonrecession period. Specifically, we regress each monthly volatility-managed single factor on the nonmanaged factor and add a dummy variable indicating whether the Norwegian market is in an OECD (2022) recession. The regression equation is then

$$f_t^\sigma = \alpha_0 + \alpha_1 1_{rec,t} + \beta_0 f_t + \beta_1 1_{rec,t} \times f_t + \epsilon_t \tag{10}$$

where α_0 and $\alpha_1 1_{rec,t}$ denote the nonrecession and recession alpha, respectively. Moreover, $\beta_0 f_t$ denotes the nonrecession beta, while $\beta_1 1_{rec,t} \times f_t$ denotes the interaction term that defines the beta of the volatility-managed factor in a recession relative to a nonrecession estimate. Thus, the interaction term provides the beta difference. A negative recession beta coefficient implies that the volatility-management strategy takes less risk during recessions and therefore has a lower beta than during nonrecession periods. Note that in Table 6 we only report the nonrecession beta of the volatility-managed factor and the coefficient of the interaction term to emphasize the change in risk loading.

Table 6: Recession Betas by Factor

In this table, we regress each volatility-managed single factor on the original factor and add dummies, $1_{rec,t}$, indicating OECD (2022) recessions that is interacted with the orginal factors, $f_t^\sigma = \alpha_0 + \alpha_1 1_{rec,t} + \beta_0 f_t + \beta_1 1_{rec,t} \times f_t + \epsilon_t$. This provides the relative beta of the volatility-managed factor in a recession relative to a nonrecession estimate. The sample ranges from 1981:07-2020:11 and is based on monthly data. Standard errors are adjusted for heteroskedasticity.

	(1) MKTσ	(2) SMBσ	(3) HMLσ	(4) UMDσ	(5) LIQσ
MKT	0.89 t = 13.81				
MKT$\times 1_{rec}$	−0.26 t = −2.75				
SMB		0.91 t = 12.29			
SMB$\times 1_{rec}$		−0.34 t = −3.63			
HML			0.81 t = 10.48		
HML$\times 1_{rec}$			−0.15 t = −1.54		
UMD				0.88 t = 10.61	
UMD$\times 1_{rec}$				−0.30 t = −2.95	
LIQ					0.91 t = 15.80
LIQ$\times 1_{rec}$					−0.26 t = −2.68
N	472	472	472	472	472
R^2	0.60	0.60	0.57	0.58	0.63

Table 6 shows statistically significant negative recession beta coefficients for a wide range of factors, indicating lower risk during recessions. For instance, the nonrecession beta for the volatility-managed market factor is 0.89, while the recession beta coefficient is -0.26. This implies that the volatility-managed market beta during a recession is 0.63, which is lower than the nonrecession beta. Similar to Moreira and Muir (2017b), we find evidence of lower risk exposure of volatility-managed Norwegian factors during recessions, as indicated by the negative recession beta coefficients. This may suggest that our results are not driven by business cycle risk. The lower beta during recessions also contradicts conventional wisdom, as the volatility-management strategy suggests a willingness to reduce risk in periods of high volatility, as opposed to an increased or constant risk-taking. For instance, after the adverse market conditions in 2008, there was a widespread belief that those who reduced their equity positions would forego an exceptional buying opportunity (Buffett, 2008; Cochrane, 2008). Nonetheless, we find that the volatility-managed market factor avoided the abrupt declines in returns during the 2008 financial crisis by adjusting its positions downward when volatility was high, as shown in Figure 4. In contrast, the nonmanaged market factor suffered significant losses by remaining in its position.

A peculiar discovery in the monthly single factor analysis of the Norwegian market, as shown in Table 2, is the consistently negative volatility-managed HML factor. This implies that the HML factor would be more profitable if the exposure was increased when volatility is high, which is contrary to the volatility-management strategy. Although the recession beta coefficient in Table 6 is negative, HML is not statistically significant. Consequently, we cannot state whether volatility-managed HML takes relatively less risk in recessions or whether it is affected by business cycle risk. Moreover, Table 13 in Appendix A1 presents descriptive statistics for the nonmanaged factors, shown in Panel A, and the volatility-managed factors, shown in Panel B. We find that the volatility-managed HML differ from the other factors by generating a negative mean factor return. In addition, the managed HML produce a substantial maximum drawdown and exhibit negative skewness, further highlighting the inadequate performance.

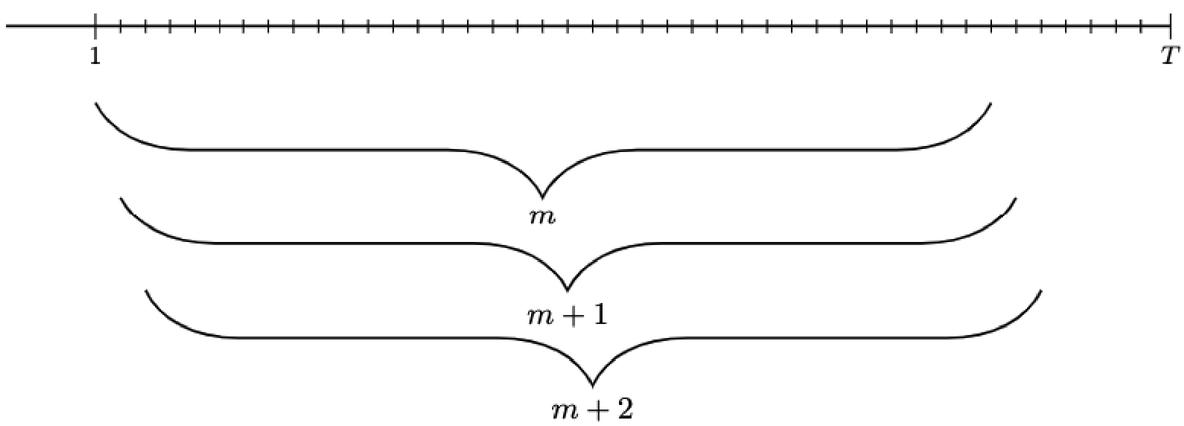
6.2 Contrasting with a Risk Parity Strategy

Similar to volatility-management, which exploits a weak risk-return trade-off when volatility increases, a risk parity strategy exploits a weak risk-return relation in the cross-section and may therefore explain our results. Risk parity is a portfolio optimization strategy that focuses on diversifying risk rather than capital. The optimal asset class weights are calculated such that the risk contribution of each asset to the total portfolio risk remains equal (Asness et al., 2012). We replicate the methodology of Moreira and Muir (2017b), derived from Asness et al. (2012), and construct risk parity portfolios as follows. For each month, we calculate the estimated volatility, denoted $\hat{\sigma}_t^i$, for each factor i within a given portfolio. We then adjust the factor weights according to

$$b_{i,t} = \frac{1/\hat{\sigma}_t^i}{\sum_i 1/\hat{\sigma}_t^i} \tag{11}$$

where $b_{i,t}$ denotes the weight in factor i at time t and $\hat{\sigma}_t^i$ denotes the three-year rolling volatility of monthly returns up to time *t-1*. Figure 6 shows an excerpt of the rolling window, where m denotes the size of the rolling window and T denotes the total sample size. The first rolling window consists of observations from period 1 to m, which in our case is 1981:07 and 36 months, respectively. The second rolling window consists of observations from period 2 to *m+1* and so on. Note that due to the size of the rolling window and the lagged volatility estimate, we obtain observations for 436 months, with the first observation being 1984:08.

Figure 6: Rolling Window



We obtain risk parity factor portfolios for each MVE portfolio considered in this study

$$RP_{t+1} = b_t' f_{t+1} \tag{12}$$

where RP_{t+1} denotes the risk parity return for a given portfolio, which is a product of a vector of factor weights, b_t', and a vector of factor returns, f_{t+1}. The vector of factor returns is the sum of each factor weight at time t multiplied by each factor return at time *t+1*, which also lags the expression. This implies that the risk parity strategy reallocates between high and low volatility factors on a monthly basis such that the individual risk contribution remains equal. In contrast, the constructed MVE portfolios in Section 5.3 keep the weighting of individual factors constant and therefore adjust the exposure to the risk of the entire portfolio. Hence, volatility-managing MVE portfolios in the Norwegian market is conceptually different from the construction of a risk parity portfolio.

In Table 7, we examine the difference empirically by regressing the monthly volatility-managed MVE portfolios on both the nonmanaged counterpart and the risk parity portfolio containing the corresponding factors. It is worth noting that we reconstruct the original volatility-managed MVE portfolio to span the same 436 months as the risk parity portfolio to provide a similar basis for comparison. Consistent with Moreira and Muir (2017b), we observe minimal changes in alpha across all MVE portfolios when controlling for the constructed risk parity portfolios. The results remain statistically robust, suggesting that a risk parity strategy is unlikely to explain the returns generated by volatility-managing multifactor portfolios in a Norwegian context.

An interesting finding is the exceptional improvement in all portfolios except FF4 + LIQ, when the rolling three-year window is used as the basis of estimation rather than the realized variance of the previous month. This approach excludes the first three years of the data set. Compared to the monthly results for the entire sample in Table 4, FF3, FF4, and HML + UMD improve by 2.09, 2.15, and 3.52 percentage points, respectively. It is particularly striking that the combination portfolio of HML + UMD, derived from Asness et al. (2013), but modified to exhibit mean-variance efficiency, suddenly achieves the highest alpha. This is all the more interesting since UMD has large volatility fluctuations in the initial three years, as can be seen in Figure 5, which should theoretically be a profitable period for the strategy. Thus, if we exclude the first three years, it is reasonable to believe that we are foregoing potential profits. Nonetheless, HML is also subject to notable fluctuations in the first three years. This could negate UMD's potential gains, as we find evidence in Section 6.1 that volatility-managed HML might thrive if exposure is increased when volatility is high. Intuitively, this implies that volatility-managed HML becomes less profitable during periods of high volatility and could therefore explain the performance improvement of HML + UMD if we omit the initial three years.

Table 7: Time Series Alphas Controlling for Risk Parity Factors

Panel A shows the results of the multivariate time series regressions of each monthly volatility-managed MVE portfolio on the nonmanaged portfolio and a risk parity factor, $RP_{t+1} = b_t^{'} f_{t+1}$, where $b_t^{'}$ is the sum of asset class weights, $b_{i,t} = \frac{1/\hat{\sigma}_t^i}{\sum_i 1/\hat{\sigma}_t^i}$ and f_{t+1} is a vector of pricing factors. The volatility is measured on a rolling three-year basis of monthly returns up to time *t-1* and the methodology is derived from Asness et al. (2012). Panel B shows the results of the original time series regressions of each monthly volatility-managed MVE portfolio on the nonmanaged counterpart when reconstructed to contain 436 observations as the risk parity portfolio. The sample ranges from 1984:08-2020:11 and is based on monthly data. Standard errors are adjusted for heteroskedasticity.

Panel A: Controlled for Risk Parity

	(1) FF3	(2) FF4	(3) FF4 + LIQ	(4) HML + UMD
Alpha (α)	6.85 t = 7.33	7.22 t = 9.29	5.57 t = 8.49	7.25 t = 4.73
N	436	436	436	436
R^2	0.53	0.52	0.55	0.58
RMSE	19.97	18.06	15.32	31.39

Panel B: Original Time Series Regression

	(1) FF3	(2) FF4	(3) FF4 + LIQ	(4) HML + UMD
Alpha (α)	6.75 t = 7.26	7.27 t = 9.38	5.61 t = 8.52	7.25 t = 4.74
N	436	436	436	436
R^2	0.52	0.52	0.55	0.58
RMSE	20.15	18.08	15.32	31.39

6.3 Rebalancing at Lower Frequencies

Thus far, we have assessed the volatility-management strategy from the perspective of a short-term investor through monthly and biweekly rebalancing and found evidence that it is beneficial in a multifactor environment. An interesting question is therefore how the multifactor strategy behaves from the vantage point of a long-term investor as rebalancing is done at a lower frequency. By examining a lower frequency, we can gain better insight into risk behavior while analyzing whether volatility-management in the Norwegian context is limited to short-term investors. Specifically, we use the monthly constructed MVE portfolios from equation (9) in Section 5.3, applying the static single factor weights similar to the monthly analysis, but remain in the position for T months before rebalancing. Note that we preserve the essence of the volatility-managed MVE portfolio by keeping the single factor weights static, while adjusting the exposure to the entire portfolio when rebalancing. We run bivariate time series regressions

$$\frac{c}{\hat{\sigma}_t^2(f)} f_{t\to t+T} = \alpha + \beta f_{t\to t+T} + \epsilon_{t+T} \tag{13}$$

where $f_{t\to t+T}$ denotes the cumulative factor returns from buying at the end of month T and remaining in the position until the end of month $t + T$. Moreover, $\hat{\sigma}_t^2(f)$ denotes the realized variance of daily multifactor returns for the previous month. The procedure is applied to each MVE portfolio and the rebalancing frequency ranges from the baseline strategy of one month to twelve months. Due to our limited data of 40 years compared to Moreira and Muir's (2017b) 89 years, we end the rebalancing frequency at twelve months.

Table 8 shows the alphas for the Norwegian market obtained by regressing the volatility-managed MVE portfolio on its nonmanaged counterpart at a rebalancing frequency of up to twelve months. We observe positive significant alphas of considerable size across a range of multifactor portfolios for long-term time horizons, but a gradual decrease in alpha size with increasing holding period. A notable result is that all portfolios except FF4 + LIQ are statistically weak in the three-month interval and recover after six months before steadily declining through the horizon. This pattern is consistent with Moreira and Muir (2017b), which observe a rapid decline in multifactor portfolio alphas in the 2-3 month interval, followed by a rebound and a gradual decline[7].

[7]See illustration in Moreira and Muir (2017b) Figure 6.

Another interesting finding is the underperformance of HML + UMD in the long run, where the portfolio deteriorates after six months. This may be due to the erratic performance of HML in the Norwegian context.

Table 8: Rebalancing at a Lower Frequency

The table shows the alphas and respective t-stat of the bivariate time series regressions of each volatility-managed MVE portfolio on the nonmanaged portfolio, $\frac{c}{\hat{\sigma}_t^2(f)} f_{t \to t+T} = \alpha + \beta f_{t \to t+T} + \epsilon_{t+T}$. The rebalancing frequency range from one to twelve months. The managed factor, $\frac{c}{\hat{\sigma}_t^2(f)} f_{t \to t+T}$, scales risk exposure according to the inverse realized variance of the month prior to the formation date and $f_{t \to t+T}$ denotes the cumulative factor returns from buying at the end of month T and remaining in position until the end of month $t + T$. The sample ranges from 1981:07-2020:11 and is based on monthly data. Standard errors are adjusted for heteroskedasticity.

	FF3		FF4		FF4 +LIQ		HML +UMD	
N	α	t-stat	α	t-stat	α	t-stat	α	t-stat
1	4.76	4.82	5.07	5.51	5.56	7.08	3.73	2.44
3	2.07	1.44	1.71	1.30	2.40	1.95	2.42	1.60
6	3.74	1.87	3.11	2.27	3.70	2.05	3.16	3.16
9	3.12	2.25	2.88	2.35	3.30	2.38	0.21	0.80
12	1.75	2.09	2.17	1.82	2.42	1.88	0.03	0.17

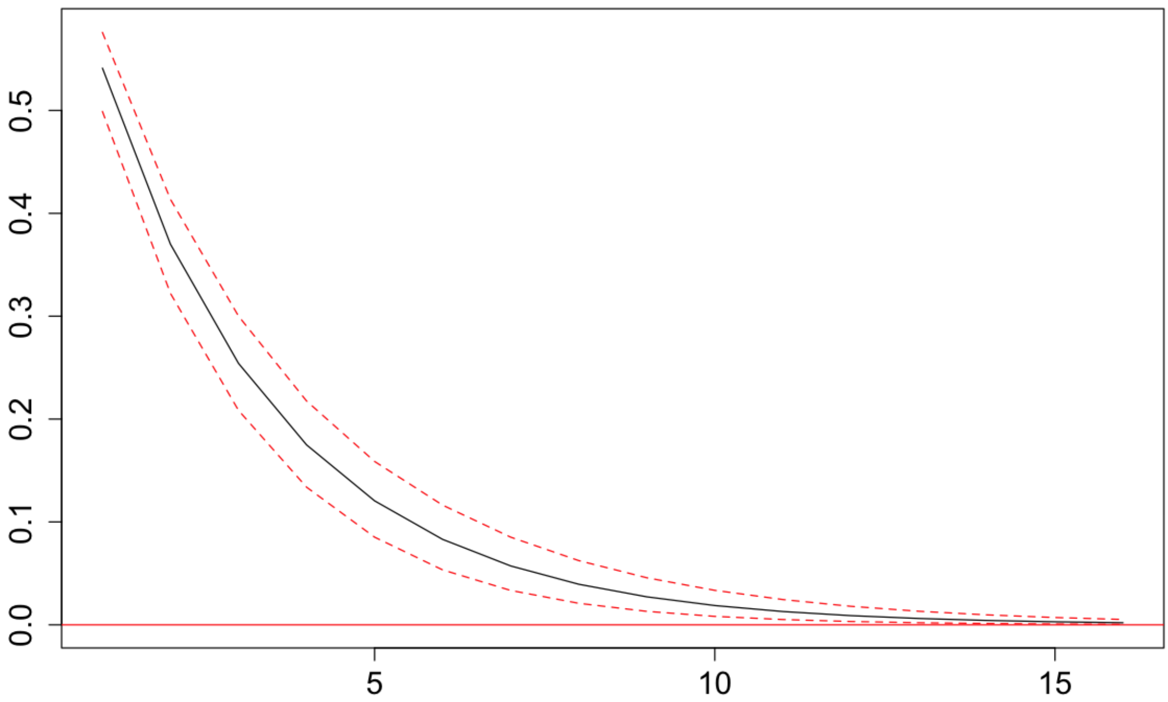
To further examine the behavior of risk, we include information on how volatility changes over time. In particular, we document how the expected variance of FF4 + LIQ responds to a one standard deviation shock of the realized variance. To do this, we run a vector autoregression (VAR), similar to Moreira and Muir (2017b), using one lag of log realized variance to predict the response of future values (Stock and Watson, 2019, p.649). We run VAR at the monthly frequency and plot the impulse response function in Figure 7 to examine the effects of a variance shock.

Figure 7 shows the impulse response function, represented by the solid line, of the expected variance of FF4 + LIQ to a shock of the realized variance. Bootstrapped 95% confidence intervals are given by the dashed lines. We find that when a shock occurs, the expected variance responds significantly positively to the shock and initially increases. The effect then gradually decreases and converges to zero about 15 months after the initial shock, consistent with the fact that the variance is greatly mean-reverting. This implies that the mean-variance investor should reduce portfolio exposure on impact to avert an unfavorable risk-return trade-off and then increase risk exposure when the shock subsides. Moreover, the gradual decline in variance in response to the shock is also consistent with the gradual decline in alphas in Table 8. This is because we would expect the volatility-management strategy to generate the largest alphas at the time of the shock due to lower risk exposure and, conversely, smaller alphas when the shock decreases.

Overall, we find that the benefits of volatility-managing multifactor portfolios in the Norwegian context are not limited to short-term investors, but are also suitable for long-term investors with a time horizon of up to twelve months. These benefits apply both to investors who limit themselves to a few factors, such as FF3 , and to those who use multiple factors, such as FF4 + LIQ.

Figure 7: Response of the Expected Variance to a Realized Variance Shock

The figure shows the impulse response function, represented by the solid line, of the expected variance of FF4 + LIQ to a shock of the realized variance. The x-axis is in months and we calculate impulse responses using a VAR of one lag of log realized variance. The y-axis represents the beta coefficient. The dashed lines indicate the bootstrapped 95% confidence intervals.

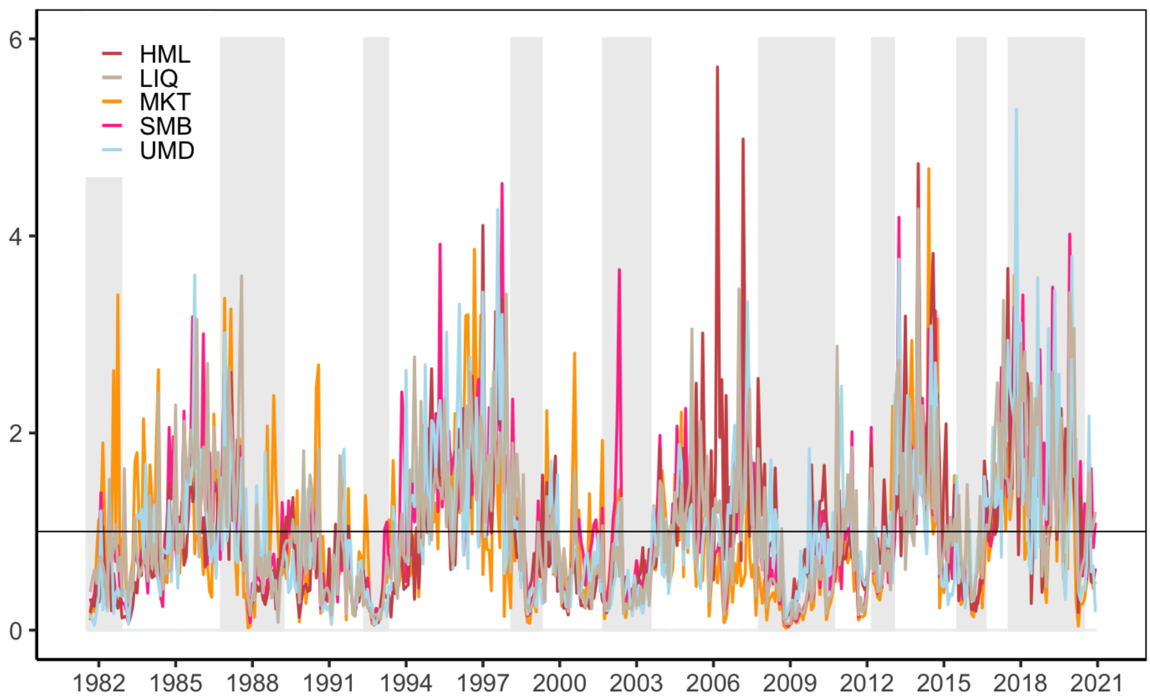


6.4 Leverage Constraints

In this section, we examine the importance of leverage for volatility-management in a Norwegian context. Volatility-managed portfolios inherently require the use of significant leverage to achieve sufficient factor exposure following periods of low volatility. This is illustrated graphically in Figure 8, which plots the time series evolution of the monthly scaling factor for each volatility-managed single factor and OECD (2022) recessions for the Norwegian market, indicated by the grey shaded bars. The scaling factor, given in equation (2) in Section 4.1, is a measure that defines the leverage required to invest in the volatility-managed factor in month t. The solid line in Figure 8 represents a scaling factor of one. A factor below one indicates a period of high volatility and hence lower exposure to the managed factor, while a factor above one indicates a period of low volatility and hence a leveraged position. We observe a pronounced comovement in scaling across risk factors, with the strategy drastically downscaling risk exposure during OECD (2022) recessions while levering substantially during periods of low volatility.

Figure 8: Scaling Factors

The figure plots the time series evolution of the monthly scaling factor for each volatility-managed factor over the entire sample period. The scaling factor at time t is denoted by $S_t = \frac{c}{\hat{\sigma}_t^2(f)}$ and the grey shaded bars indicate OECD (2022) recessions for the Norwegian market.



Furthermore, Table 9 shows the factor weights at the 75^{th}, 90^{th}, and 99.9^{th} percentiles for the monthly baseline strategy without leverage constraints, shown in Panel A, and the corresponding biweekly strategy, shown in Panel B. The percentiles highlight the importance of leverage in the upper distribution. Although the average scaling for each of the monthly volatility-managed factors is approximately one, the 99.9^{th} percentile weight surpass 400% in each monthly case. Further, the importance of leverage increases for all factors with biweekly rebalancing, reaching 936% for the HML factor. This suggests that biweekly rebalancing is more dependent on leverage relative to monthly rebalancing.

Table 9: Percentile Weights

Panel A reports the factor weights at the 75^{th}, 90^{th}, and 99.9^{th} percentile for the monthly baseline strategy without leverage. Panel B reports weights of the corresponding biweekly returns. The sample ranges from 1981:07-2020:11.

Panel A: Monthly Rebalancing

Percentile	(1) MKTσ	(2) SMBσ	(3) HMLσ	(4) UMDσ	(5) LIQσ
75	1.29	1.37	1.37	1.30	1.42
90	1.91	2.03	1.90	1.96	2.03
99.9	4.34	4.37	5.37	4.81	3.95

Panel B: Biweekly Rebalancing

Percentile	(1) MKTσ	(2) SMBσ	(3) HMLσ	(4) UMDσ	(5) LIQσ
75	1.04	1.23	1.09	1.19	1.28
90	1.79	1.98	1.86	1.93	2.11
99.9	7.81	5.99	9.36	7.99	8.10

Consequently, it is important to assess how the strategy behaves when facing strict leverage constraints. By imposing constraints, we prevent our results from being driven by severe outliers. Extensive leverage may also become costly or is not feasible. Therefore, limiting leverage can be viewed as a cost-mitigation strategy as it reduces time series variation in factor leverage (Barroso and Detzel, 2021). We construct the leverage constrained volatility-managed MVE portfolios according to

$$f_{t+1}^{MVE,\sigma} = min(\frac{c}{\hat{\sigma}_t^2(f^{MVE})}, \bar{L}) f_{t+1}^{MVE} \tag{14}$$

where $\bar{L}$[8] denotes the imposed leverage constraint and $min(\frac{c}{\hat{\sigma}_t^2(f^{MVE})}, \bar{L})$ denotes the constraint function that scales the risk exposure according to the minimum value of the scaling factor or the imposed leverage constraint. Similar to Moreira and Muir (2017b), we obtain the constraint by imposing an upper bound of $\bar{L} \leq 1 \mid 1.5$ on the factor weights, implying no leverage or 50% leverage, respectively. Although the investment constraint is strict, it depends on the investor, with some investors being unable to apply leverage due to borrowing restrictions, as Black (1972) assumes. Imposing strict constraints therefore makes the strategy more available to constrained investors. In addition, we impose a constraint of $\bar{L} \leq 2$ to preserve the essence of volatility-managed strategies.

[8]The leverage constraint notation is similar to Grossman and Vila (1992).

Table 10: Volatility-Managing and Leverage Constraints

Panel A shows the alphas of the monthly bivariate time series regressions of each volatility-managed MVE portfolio on the nonmanaged portfolio for the baseline strategy without constraints and when leverage constraints of $\bar{L} \leq 1 \mid 1.5 \mid 2$ are imposed. The managed factor, $f_{t+1}^{MVE,\sigma}$, scales risk exposure according to the minimum value of the scaling factor or the leverage constraint, $f_{t+1}^{MVE,\sigma} = min(\frac{c}{\hat{\sigma}_t^2(f^{MVE})}, \bar{L}) f_{t+1}^{MVE}$. Panel B shows the alphas of the corresponding biweekly portfolios. The sample ranges from 1981:07-2020:11 and is based on monthly data. Standard errors are adjusted for heteroskedasticity.

Panel A: Monthly Rebalancing

	(1) FF3	(2) FF4	(3) FF4 + LIQ	(4) HML + UMD
$\frac{c}{\hat{\sigma}_t^2(f^{MVE})}$	4.76	5.07	5.56	3.73
	t = 4.82	t = 5.51	t = 7.08	t = 2.44
min($\frac{c}{\hat{\sigma}_t^2(f^{MVE})}$,1)	2.16	2.26	2.49	1.68
	t = 4.34	t = 4.56	t = 5.79	t = 2.21
min($\frac{c}{\hat{\sigma}_t^2(f^{MVE})}$,1.5)	3.36	3.48	3.73	2.42
	t = 4.97	t = 5.28	t = 6.58	t = 2.16
min($\frac{c}{\hat{\sigma}_t^2(f^{MVE})}$,2)	4.08	4.38	4.57	2.97
	t = 5.08	t = 5.79	t = 7.13	t = 2.28

Panel B: Biweekly Rebalancing

	(1) FF3	(2) FF4	(3) FF4 + LIQ	(4) HML + UMD
$\frac{c}{\hat{\sigma}_t^2(f^{MVE})}$	3.05	3.27	3.17	4.41
	t = 3.03	t = 3.82	t = 3.73	t = 3.05
min($\frac{c}{\hat{\sigma}_t^2(f^{MVE})}$,1)	2.02	2.04	2.15	1.42
	t = 4.25	t = 4.43	t = 4.92	t = 2.01
min($\frac{c}{\hat{\sigma}_t^2(f^{MVE})}$,1.5)	2.47	2.49	2.64	2.29
	t = 4.19	t = 4.40	t = 4.69	t = 2.49
min($\frac{c}{\hat{\sigma}_t^2(f^{MVE})}$,2)	2.64	2.72	2.98	3.02
	t = 3.83	t = 4.33	t = 4.70	t = 2.80

In Panel A of Table 10, we study the impact of leverage by regressing the monthly volatility-managed MVE portfolio on its nonmanaged counterpart, imposing leverage constraints. We find that all multifactor portfolios have sizable alpha values that remain statistically robust across constraints. This suggests that volatility-management is beneficial even in the presence of stringent leverage constraints. In particular, FF4 + LIQ achieve an alpha of 2.49% when a leverage constraint of one is imposed. Thus, one can apply the strategy without leverage, which reduces transaction costs, and still achieve an annual gain of 2.49%. However, comparing the changes in alpha from the baseline strategy without constraints to imposing constraints, we find that FF4 + LIQ is the most affected portfolio when the constraint is one, as alpha decreases by 3.07 percentage points. In contrast, HML + UMD is the least affected multifactor portfolio with a decrease of 2.05 percentage points. This shows that although HML and UMD are subject to periods of substantial leverage according to Figure 8, volatility-management is beneficial even when leverage is not an option. We also observe a gradual increase in alpha for all portfolios when constraints are relaxed. This is not surprising since the portfolios contain factors that are leveraged drastically during periods of low volatility. It is therefore reasonable to believe that less intrusive constraints are beneficial to the strategy.

Panel B of Table 10 shows the impact of leverage constraints for the corresponding biweekly outcomes. Similar to monthly rebalancing, we find significant alphas across portfolios and leverage constraints, implying that volatility-management is beneficial even at a biweekly rebalancing frequency. The higher percentiles in the biweekly analysis in Table 9 suggested that a higher rebalancing frequency is more sensitive to leverage. However, looking closely at the results in Table 10, we find that the changes in alpha are drastically smaller at a biweekly rebalancing than monthly for all portfolios except HML + UMD. This may contradict the notion of leverage dependence. For instance, FF4 + LIQ at a biweekly frequency and a leverage constraint of one decrease by 1.02 percentage points compared to the strategy without constraints. On the other hand, the corresponding monthly portfolio decreases by 3.07 percentage points. In fact, the average biweekly single factor scaling is approximately 0.94, which is lower than the monthly average of about 1, suggesting that leverage constraints have less impact on a biweekly rebalancing frequency.

Another notable result is that, unlike the other portfolios, HML + UMD is significantly more sensitive to leverage constraints on a biweekly basis, decreasing by 2.99 percentage points when a leverage constraint of one is introduced. Consequently, the biweekly results of HML + UMD, which were originally more pronounced in Table 5 than the monthly results, become less so. This is an interesting finding since the corresponding portfolio is the least affected by leverage constraints when rebalanced monthly. A possible explanation could be that the average absolute change in weights from rebalancing period to rebalancing period is higher for HML + UMD at a biweekly frequency than monthly[9]. A higher average absolute change in weights implies that the portfolio leverage positions to a greater extent and is therefore more sensitive to leverage constraints. Consequently, the imposed constraints distort the excess alpha that HML + UMD achieved on a biweekly basis.

Extending Black's (1972) model, Frazzini and Pedersen (2014) show empirically that funding constraints alter returns for the betting-against-beta factor by examining the effects of financing constraints on asset returns in the cross-section and in time series. Thus, theoretically, leverage could explain our results. However, we find that investors with access to few factors, as in FF3, and to multiple factors, as in FF4 + LIQ, can benefit from volatility-management with a monthly rebalancing frequency, even when subject to strict leverage constraints. Consequently, leverage constraints do not seem to be a plausible explanation for our results.

[9]In fact, the average absolute change in weights for HML + UMD is 0.77 at a biweekly frequency and 0.55 at a monthly frequency.

6.5 Incorporating Transaction Costs

In this section, we study the impact of transaction costs on the considered monthly volatility-managed MVE portfolios. Recent research by Barroso and Detzel (2021) questions the results of Moreira and Muir (2017b) due to the increased transaction costs associated with rebalancing a volatility-managed portfolio. Therefore, it is necessary to examine the impact of trading costs in a Norwegian context. Note that we do not perform a biweekly transaction cost analysis as the biweekly rebalancing frequency performs worse than the monthly when leverage constraints are imposed in Section 6.4. Monthly volatility-management appears to be more attractive, and it is likely to remain so when transaction costs are incurred due to less frequent rebalancing.

Due to the lack of empirically realistic transaction costs for price factors in the Norwegian market, we must take a conservative approach in our estimation. Similar to Moreira and Muir (2017b), we therefore apply three cost measures, where 1 basis point comes from Fleming et al. (2003) and 10 basis points from Frazzini and Pedersen (2014). We also add an additional 4 basis points to account for increased costs in periods of high volatility[10]. We then add a sizable premium for trading in the Norwegian market compared to the U.S. market, based on the difference in relative spread given by Klova and Ødegaard (2019). As Klova and Ødegaard (2019) shows by comparing the Norwegian and U.S. market from 1993 to 2017, the relative spread of transactions in the Norwegian market is higher. It is reasonable to assume that the difference is due to company size, as Norwegian companies tend to be smaller. Consequently, they are also associated with higher transaction costs, which is consistent with Ødegaard's (2008) assertion that there is a clear relationship between size and transaction costs, with large firms having the lowest costs.

In particular, Klova and Ødegaard (2019) define four size categories ranging from small to large firms. We consider the difference in the relative spread of medium-sized firms to compensate for possible size differences between Norwegian and U.S. firms. The difference in relative spread is 182%[11], which represents our trading premium in the Norwegian market. This is added to the existing transaction costs of 1, 10 and 14 basis points. Thus,

[10]The additional 4 basis points are from Moreira and Muir (2017b), who consider an increase in the VIX from 20% to 40%, which corresponds to the 98^{th} percentile.

[11]See Klova and Ødegaard (2019) Table 1.

the cost per transaction in the Norwegian market is 1.82, 18.2, and 25.48 basis points, respectively. Similar to Fleming et al. (2003), we incorporate transaction costs by subtracting the given rate from each monthly return of the volatility-managed portfolio, since rebalancing occurs once a month. We do not subtract the transaction costs from the nonmanaged factor since it is a static portfolio and thus represents a zero-cost portfolio. The net return after costs for the volatility-managed MVE portfolio f at time t is then

$$f^{MVE,\sigma}_{net,t} = f^{MVE,\sigma}_{gross,t} - (\Delta \mathrm{w} * \mathrm{TC}) \tag{15}$$

where $f^{MVE,\sigma}_{gross,t}$ denotes the volatility-managed returns when ignoring costs, Δw denotes the absolute change in weights from month *t-1* to *t*, and TC denotes the transaction cost estimate. It is important to note that the rates originally used by Moreira and Muir (2017b) are for the market factor, which implies that the transaction costs we use are intended for the market factor only. Thus, the transaction costs for the remaining factors in the multifactor portfolio are not accounted for in the absence of realistic costs for the Norwegian market. We acknowledge that this is a rather simplistic approach, but emphasize the importance of including transaction costs in some form to realistically examine the returns generated by volatility-management.

Table 11: Transaction Costs and Leverage Constraints

Panel A shows the results of the bivariate time series regression of the monthly volatility-managed FF3 portfolio on the nonmanaged portfolio when transaction costs are imposed. Panel B and C shows the corresponding results for FF4 and FF4 + LIQ, respectively. The net return after costs for the volatility-managed portfolio is $f_{net,t}^{MVE,\sigma} = f_{gross,t}^{MVE,\sigma} - (\Delta\text{w} * \text{TC})$. We consider the baseline strategy without leverage constraints and cost-mitigation strategies that reduce trading activity, such as leverage constraints of $\bar{L} \leq 1 \mid 1.5 \mid 2$ and using the inverse volatility, $\frac{c}{\hat{\sigma}_t(f)}$, instead of realized variance in calculating the scaling factor. Specifically, we report the average absolute change in monthly weights, Δw, and the respective alpha of each strategy excluding costs. We then report the alphas when introducing transaction costs of 1.82, 18.2, and 25.48 basis points per rebalancing frequency. Finally, we report the necessary costs in basis points that reduce the alpha to zero.

Panel A: FF3 with Transaction Costs

	Δw	α	1.82bps	18.2bps	25.48bps	Break Even
$\frac{c}{\hat{\sigma}_t^2(f)}$	0.69	4.76	4.60	3.18	2.55	55bps
		t = 4.82	t = 4.65	t = 3.17	t = 2.20	
$\frac{c}{\hat{\sigma}_t(f)}$	0.36	3.64	2.88	2.15	1.82	67bps
		t = 3.88	t = 4.66	t = 3.44	t = 2.90	
$\min(\frac{c}{\hat{\sigma}_t^2(f)},1)$	0.19	2.16	2.12	1.73	1.56	92bps
		t = 4.34	t = 4.25	t = 3.43	t = 3.07	
$\min(\frac{c}{\hat{\sigma}_t^2(f)},1.5)$	0.35	3.36	3.28	2.58	2.27	79bps
		t = 4.97	t = 4.85	t = 3.74	t = 3.27	
$\min(\frac{c}{\hat{\sigma}_t^2(f)},2)$	0.48	4.08	3.97	3.01	2.58	70bps
		t = 5.08	t = 4.94	t = 3.77	t = 3.13	

Panel B: FF4 with Transaction Costs

	Δw	α	1.82bps	18.2bps	25.48bps	Break Even
$\frac{c}{\hat{\sigma}_t^2(f)}$	0.67	5.07	4.92	3.54	2.92	61bps
		t = 5.51	t = 5.33	t = 3.76	t = 3.07	
$\frac{c}{\hat{\sigma}_t(f)}$	0.35	3.53	3.04	2.32	2.00	72bps
		t = 4.64	t = 4.96	t = 3.54	t = 3.20	
$\min(\frac{c}{\hat{\sigma}_t^2(f)},1)$	0.19	2.26	2.22	1.83	1.65	95bps
		t = 4.56	t = 4.37	t = 3.61	t = 3.24	
$\min(\frac{c}{\hat{\sigma}_t^2(f)},1.5)$	0.36	3.48	3.40	2.68	2.36	80bps
		t = 5.28	t = 5.14	t = 3.97	t = 3.46	
$\min(\frac{c}{\hat{\sigma}_t^2(f)},2)$	0.47	4.38	4.27	3.32	2.90	76bps
		t = 5.79	t = 5.63	t = 4.29	t = 3.71	

Panel C: FF4 + LIQ with Transaction Costs

	Δw	α	1.82bps	18.2bps	25.48bps	Break Even
$\frac{c}{\hat{\sigma}_t^2(f)}$	0.63	5.56	5.39	4.14	3.58	73bps
		t = 7.08	t = 7.05	t = 5.37	t = 4.62	
$\frac{c}{\hat{\sigma}_t(f)}$	0.32	3.36	3.27	2.62	2.34	85bps
		t = 5.71	t = 6.36	t = 5.06	t = 4.49	
$\min(\frac{c}{\hat{\sigma}_t^2(f)},1)$	0.17	2.49	2.45	2.11	1.96	122bps
		t = 5.79	t = 5.70	t = 4.87	t = 4.50	
$\min(\frac{c}{\hat{\sigma}_t^2(f)},1.5)$	0.35	3.73	3.66	3.03	2.75	97bps
		t = 6.58	t = 6.52	t = 5.24	t = 4.71	
$\min(\frac{c}{\hat{\sigma}_t^2(f)},2)$	0.48	4.57	4.48	3.63	3.25	88bps
		t = 7.13	t = 6.97	t = 5.54	t = 4.92	

Table 11 shows the results for the Norwegian market obtained by regressing the monthly volatility-managed MVE portfolio on its nonmanaged counterpart while imposing transaction costs of 1.82, 18.2, and 25.48 basis points. We examine the baseline strategy without transaction costs and also consider cost-mitigation strategies that slow down the time series variation in leverage applied to the factor. In other words, strategies that reduce trading activity, such as leverage constraints of $\bar{L} \leq 1 \mid 1.5 \mid 2$ and using the standard deviation instead of the realized variance in calculating the scaling factor.

Panel A of Table 11 shows the results for the monthly volatility-managed FF3 portfolio. We find that the baseline strategy without leverage constraints endures transaction costs even in periods of high volatility, as shown by the cost estimate of 25.48. The strategy also withstands transaction costs when subject to cost-mitigation strategies. This could be due to the sharpe reduction in trading activity, as evidenced by the absolute change in monthly weights, which reduces the cost per rebalancing. The reduced trading activity has a positive impact on transaction costs and could be related to the fact that the leverage restricted strategy is not exposed to high leverage during certain periods. This is contrary to the baseline strategy without constraints. Similar to Moreira and Muir (2017b), the smallest absolute change in weights is found at a strict leverage of one [12], which is reasonable as it reduces trading activity the most. On the contrary, the absolute change in monthly weights with a leverage constraint of 1.5 is significantly higher than that of Moreira and Muir (2017b). This suggests that Norwegian factors are more sensitive to leverage compared to the US.

Moreover, investors who are not subject to leverage constraints and wish to minimize their costs may benefit from using realized volatility instead of realized variance in portfolio construction. This reduces trading activity by about 48%[13] while the annualized alpha remains modest when transaction costs are taken into account. The break even value for an investor restricted to no leverage options is also 92 basis points, implying that the strategy can tolerate higher costs than those we use. This lays the foundation for further research to assign empirically realistic costs for both the market factor and nonmarket factors in a Norwegian context.

[12] See results in Moreira and Muir (2017b) Table IV

[13] This is similar to Moreira and Muir (2017b) documenting a 48% decrease in trading activity for the market factor when the methodology is changed.

Panel B shows the results for the monthly volatility-managed FF4 portfolio. We observe minor differences in the absolute change in weights compared to FF3 and therefore the allocated costs are quite similar between the two portfolios. However, since FF4 initially has a higher annualized alpha, the alpha is higher across all constraints net of transaction costs. The portfolio is also slightly more robust, as shown by the marginal increase in the break even point.

Finally, Panel C shows the results for the monthly volatility-managed FF4 + LIQ portfolio. We find that FF4 + LIQ, which includes all the factors considered in the study, mitigates costs to a greater extent by the smallest changes in absolute weight. This pattern is evident for the baseline strategy without constraints and for all cost-mitigation strategies except for a leverage constraint of 2. Interestingly, we find that while the strategy achieves higher alphas when leverage constraints are relaxed, the break even value is highest at a constraint of one. This may suggest that the strategy is most robust when no leverage is applied. This is also evident across all the MVE portfolios and underscores the significant costs associated with applying leverage. Compared to Moreira and Muir (2017b), which finds a higher break even when constraints are relaxed, we find the opposite. This is most likely due to the severe change in trading activity when constraints are relaxed from 1 to 1.5, effectively doubling trading activity for FF4 + LIQ. Moreira and Muir (2017b), on the other hand, finds that trading activity for the market factor remains the same[14].

In summary, we find that volatility-management in the Norwegian context resists transaction costs even when subjected to strict cost-mitigation strategies. Thus, the strategy is not limited to investors who are not subject to leverage constraints. This reinforces the notion of a realistic implementation of the strategy. However, since we only include an estimate of transaction costs, we emphasize the importance of further research and leave the calculation and implementation of empirically realistic transaction costs in the Norwegian context to future work.

[14]See results in Moreira and Muir (2017b) Table IV

6.6 A Plausible Explanation of Our Results

Thus far, we find evidence that volatility-managing a multifactor portfolio at a monthly frequency in a Norwegian context is neither driven by business cycle risks nor explained by a risk parity strategy. We also find that it is robust to rigorous leverage constraints, that it withstands a conservative transaction cost estimate and that it is not only beneficial for short-term investors but also suitable for long-term investors up to twelve months. It is therefore pertinent to ask what might explain our results.

An unequivocal explanation for our results is the subject of future work. However, an intuitive explanation could be that some investors react slowly to market changes and are therefore exposed to a weaker risk-return trade-off when volatility increases. Table 12 shows the results obtained by regressing future returns on the realized variance for each factor considered in this study. The regression equation is then

$$f_{t+1} = \alpha + \beta \hat{\sigma}_t^2(f) + \epsilon_t \tag{16}$$

where f_{t+1} denotes returns one month ahead and $\beta \hat{\sigma}_t^2(f)$ denotes the beta coefficient that explains the relation between variance and returns. In general, we find that variance has a weak predictive power for the returns of a number of factors, with the exception of HML, which is statistically positive. This implies that there is a positive relation between variance and returns for HML, which is consistent with the fact that HML improves performance when risk is increased in the presence of high volatility, as shown in Section 6.1. This is contrary to the volatility-management strategy. The positive relation and the magnitude of the beta coefficient of HML also suggest that the returns increase sharply when the market experiences a volatility shock. This is graphically illustrated in Figure 9, where we run a VAR at the monthly frequency on one lag of log realized variance and expected returns for HML. This allows us to examine the impulse response of HML's expected returns to a shock in realized variance. Although the magnitude is modest, we observe an immediate increase in expected returns following a volatility shock, suggesting that investors in HML respond quickly. On the contrary, we find no statistical significance for the remaining factors in Table 12, suggesting a weak relation between variance and returns. It is therefore reasonable to assume that we do not experience a sharp increase in expected returns following a volatility shock [15]. Thus, as investors do not react instantaneously, the relation between volatility and expected returns becomes disproportionate. This results in an unfavorable risk-return trade-off as expected variance react sharply to a volatility shock, as shown in Figure 7. Accordingly, similar to Moreira and Muir (2017b), this suggests that volatility-management is beneficial in a multifactor environment in the Norwegian context and investors can prevent the unfavorable trade-off by initially reducing risk exposure.

Table 12: Relation Between Variance and Future Returns

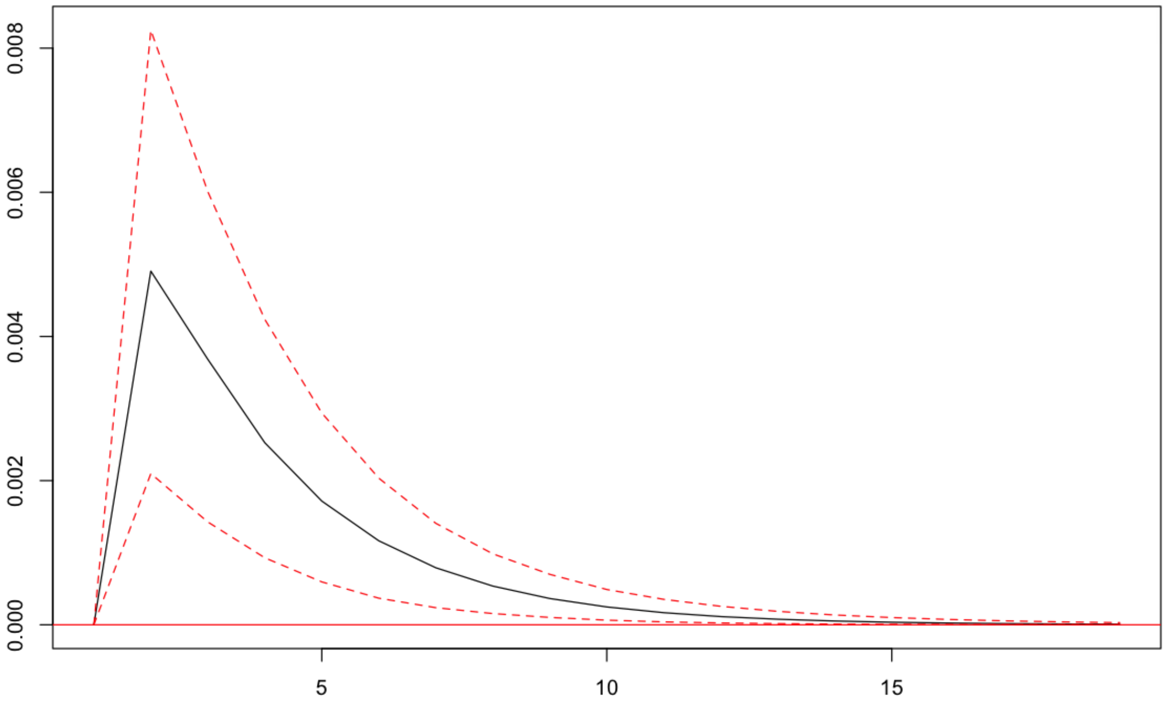
In this table, we regress each expected nonmanaged factor return on realized variance, $f_{t+1} = \alpha + \beta\hat{\sigma}_t^2(f) + \epsilon_t$. The beta coefficient represents the relation between variance and future returns. The sample ranges from 1981:07-2020:11 and is based on monthly data. Standard errors are adjusted for heteroskedasticity.

	(1)	(2)	(3)	(4)	(5)
	f_{t+1}^{MKT}	f_{t+1}^{SMB}	f_{t+1}^{HML}	f_{t+1}^{UMD}	f_{t+1}^{LIQ}
$\hat{\sigma}_t^2(f^{MKT})$	−1.04				
	t = −1.36				
$\hat{\sigma}_t^2(f^{SMB})$		−1.39			
		t = −1.19			
$\hat{\sigma}_t^2(f^{HML})$			2.87		
			t = 2.84		
$\hat{\sigma}_t^2(f^{UMD})$				−2.30	
				t = −1.81	
$\hat{\sigma}_t^2(f^{LIQ})$					−1.19
					t = −1.51
N	472	472	472	472	472
R^2	0.01	0.01	0.02	0.02	0.01

[15] We also examined the impulse response function of expected returns to a shock in realized variance for the remaining factors and find evidence to support this claim.

Figure 9: Response of the Expected Returns to a Realized Variance Shock

The figure shows the impulse response function, represented by the solid line, of the expected returns of HML to a shock in realized variance. The x-axis is in months and we calculate impulse responses using a VAR of one lag of log realized variance. The y-axis represents the beta coefficient. The dashed lines indicate the bootstrapped 95% confidence intervals.



7 Conclusion

In this chapter, we will conclude our study by summarizing the main research findings in terms of the research objectives and the value and contribution thereof. We will also identify the limitations of the study and make suggestions for future research.

7.1 Concluding remarks

The aim of this thesis was to provide empirical evidence on the effect of volatility-management in a Norwegian context and assess whether it is advantageous. To answer our research question, we replicate the methodology of Moreira and Muir (2017b) and apply the simple trading rule of reducing exposure after periods of high volatility and vice versa, based on volatility information. The results indicate that volatility-managed multifactor portfolios that are rebalanced monthly outperform their nonmanaged counterparts with the corresponding target volatility. Specifically, for a multifactor portfolio that includes all the factors considered in this study, our strategy generates an annualized alpha of 5.56% and an appraisal ratio of 0.72. In economic terms, this implies that an investor who manages volatility increases the Sharpe ratio by 0.72 annually compared to an investor confined to a nonmanaged portfolio. The improved results are consistent with Moreira and Muir (2017b), and suggest that an investor can capitalize on prior volatility information, which challenges the weak form of the efficient market hypothesis.

We also fill the gap in the existing literature regarding volatility-management at a higher rebalancing frequency. While we find that volatility-managing multifactor portfolios is beneficial at a biweekly rebalancing frequency, we find no evidence that it is systematically advantageous compared to monthly. This is because the pronounced results we find with a biweekly rebalancing frequency are offset by leverage constraints.

We conduct numerous extensive tests to assess the robustness of our multifactor results at a monthly rebalancing frequency while attempting to explain the phenomenon. We show empirically that volatility-managed factors in the Norwegian market reduce risk during OECD (2022) recessions, implying that our results are not driven by business cycle risk. This is contrary to traditional financial strategies that suggest increased risk-taking or remaining in position during recessions. In addition, we provide evidence that the strategy is conceptually different from a risk parity strategy and cannot be explained empirically by this matter. We also show that volatility-management remains beneficial even when subject to rigorous leverage constraints and that it withstands a conservative transaction cost estimate. Consequently, none of these explanations can fully justify our results.

We also examine how outcomes change with the investment horizon. We find that the gains from volatility-managing a multifactor portfolio remain modest for a rebalancing frequency of up to 12 months and that alpha gradually decreases with time horizon. We explain this fact by running a vector autoregression to examine the impulse response of expected variance to a shock in realized variance. We find a sharp increase in variance followed by a gradual convergence to zero about 15 months after the shock. Thus, since the volatility-managed investor reduces risk on impact, we expect the largest alpha at the time of the shock and a gradual decline in alpha as poor market conditions ameliorate, and the shock subsides. We therefore provide an incentive to reduce risk during market adversity and the implications of our results affects not only short-term investors but also long-term investors. Finally, we link our results to the relationship between lagged volatility, future volatility, and expected returns. We show empirically that a plausible explanation is that some investors are slow to respond to changes in market volatility. Consequently, due to the disproportionate relationship between volatility and expected returns created by a volatility shock, slow traders face an unfavorable risk-return trade-off.

In this thesis we find evidence that participants in the Norwegian market may benefit from volatility-management — that is, adjusting their positions according to volatility information. We also find that the strategy is not restricted to short-term investors, but is also transferable to investors with a long-term perspective. These results may originate from some investors reacting slowly to changes in market volatility, resulting in an unfavorable risk-return trade-off. With this thesis, we aim to raise awareness of the potential benefits of volatility-management in the Norwegian context and believe that this strategy has an important implication for traditional finance theory.

7.2 Limitations and Suggestions for Further Research

Our study lays the foundation for further interesting research on volatility-management. First of all, our focus on assessing the value of volatility-management to investors in real-time necessitates the use of out-of-sample tests. From a practical perspective, as argued by Cederburg et al. (2020), volatility-management may be challenging to implement in real-time as the foundation of the scaling factor relies on ex-post data. Thus, it would be interesting to examine whether out-of-sample versions of our results perform similarly well. Secondly, a limitation of our study is the lack of empirically realistic transaction costs for both the market factor and nonmarket factors in the Norwegian market. This led to a conservative transaction cost estimate. Although we apply a considerable premium for trading in the Norwegian market, an improvement in our study could be to calculate and apply realistic transaction costs. In turn, similar to Barroso and Detzel (2021), this would clearly define whether transaction costs can explain our findings.

Future research may also assess whether the combination portfolio of momentum and value proposed by Asness et al. (2013) improves outcomes through volatility-management. This is relevant as we find evidence that the HML + UMD portfolio in a Norwegian context is beneficial to volatility-manage. Finally, due to the lack of data compared to Moreira and Muir (2017b), we only examine the benefits of volatility-management for long-term investors up to 12 months. To better understand the implications of our results, future studies may conduct similar research for the Norwegian market to clarify whether volatility-management is beneficial beyond the 12-month horizon.

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Appendix

A1 Statistical Summary

Table 13: Statistical Summary

Panel A shows descriptive statistics for each nonmanaged factor and Panel B shows corresponding statistics for each volatility-managed factor. The mean represents the mean factor return, std.dev measures the dispersion of factor returns, and min and max represent the highest and lowest factor returns, respectively. Skewness represents the symmetry of the distribution, kurtosis measures how the distribution is scattered between the extremes, and MDD measures the maximum drawdown in percent. The sample ranges from 1981:07 to 2020:11 and all values are per month.

Panel A: Nonmanaged Factors

	MKT	SMB	HML	UMD	LIQ
N	473	473	473	473	473
MEAN	1.88	0.73	0.35	0.75	0.15
STD. DEV	5.94	4.40	5.25	5.66	4.54
MIN	-23.79	-16.63	-19.64	-24.27	-17.66
MAX	19.72	21.08	22.16	25.48	16.42
SKEW	-0.47	0.34	0.19	-0.17	0.12
KURTOSIS	1.72	2.07	1.44	1.69	0.91
MDD	-46.12	-34.26	-78.79	-66.78	-74.28

Panel B: Volatility-Managed Factors

	MKTσ	SMBσ	HMLσ	UMDσ	LIQσ
N	473	473	473	473	473
MEAN	1.82	0.79	-0.04	1.32	0.43
STD. DEV	5.94	4.40	5.25	5.66	4.54
MIN	-22.79	-24.03	-34.01	-15.60	-25.51
MAX	34.93	28.59	30.33	32.62	19.17
SKEW	0.73	0.72	-0.40	1.24	0.57
KURTOSIS	4.79	7.27	7.55	4.87	4.42
MDD	-35.18	-36.29	-88.67	-46.41	-44.10

A2 Replication of Moreira and Muir

Table 14: Replication of Moreira and Muir

Panel A shows the results of replicating Moreira and Muir (2017b) through bivariate time series regressions of each monthly volatility-managed factor on the nonmanged factor, $f^{\sigma}_{t+1} = \alpha + \beta f_{t+1} + \varepsilon_{t+1}$. The managed factor, f^{σ}_{t}, scales risk exposure according to the inverse realized variance of the previous month, $f^{\sigma}_{t} = \frac{c}{\hat{\sigma}^2_{t-1}} f_t$. The sample ranges from 1926 to 2015 and consists of the three Fama and French (1993) factors, RMW, and CMA. The data are obtained from Kenneth French's (2022) website. Panel B shows the original results by Moreira and Muir. Standard errors are in parentheses and adjusted for heteroskedasticity.

Panel A: Replication

	MKTσ	SMBσ	HMLσ	Momσ	RMWσ	CMAσ
MKT	0.60					
	(0.05)					
SMB		0.60				
		(0.09)				
HML			0.54			
			(0.08)			
Mom				0.45		
				(0.07)		
RMW					0.58	
					(0.09)	
CMA						0.68
						(0.05)
Alpha (α)	4.89	−0.52	1.68	12.88	2.70	0.33
	(1.56)	(0.93)	(1.04)	(1.75)	(0.90)	(0.67)
N	1,073	1,073	1,073	1,068	629	629
R^2	0.36	0.36	0.30	0.20	0.34	0.46
RMSE	51.66	30.77	35.43	50.83	21.73	17.48

Panel B: Original

	MKTσ	SMBσ	HMLσ	Momσ	RMWσ	CMAσ
MKT	0.61					
	(0.05)					
SMB		0.62				
		(0.08)				
HML			0.57			
			(0.07)			
Mom				0.47		
				(0.07)		
RMW					0.62	
					(0.08)	
CMA						0.68
						(0.05)
Alpha (α)	4.86	-0.58	1.97	12.51	2.44	0.38
	(1.56)	(0.91)	(1.02)	(1.71)	(0.83)	(0.67)
N	1,065	1,065	1,065	1,060	621	621
R^2	0.37	0.38	0.32	0.22	0.38	0.46
RMSE	51.39	30.44	34.92	50.37	20.16	17.55

A3 Correlation Matrix

Table 15: Correlation matrix

In this table, we calculate the correlation of monthly nonmanaged factors.

Correlation Matrix

	MKT	SMB	HML	UMD	PR1YR	LIQ
MKT	1.00	-0.40	0.07	-0.10	-0.14	-0.57
SMB	-0.40	1.00	-0.19	0.11	0.12	0.56
HML	0.07	-0.19	1.00	-0.03	-0.00	0.02
UMD	-0.10	0.11	-0.03	1.00	0.78	-0.05
PR1YR	-0.14	0.12	-0.00	0.78	1.00	-0.03
LIQ	-0.57	0.56	0.02	-0.05	-0.03	1.00

B1 Covariance

Figure 10: Covariance

Panel A shows the covariance formula used to calculate the covariance of two factors. By applying the formula, we construct the covariance matrix in Panel B, which is the basis for computing the static weights of the MVE portfolio. Note that the covariance matrix corresponds to a portfolio containing all the factors considered in this study.

Panel A: Covariance Equation

$$COV(x, y) = \frac{\sum(xi - \bar{x}) \times (yi - \bar{y})}{N} \tag{17}$$

Panel B: Covariance Matrix

$$\begin{array}{cc}
 & \begin{array}{ccccc} \text{MKT} & \text{SMB} & \text{HML} & \text{UMD} & \text{LIQ} \end{array} \\
\begin{array}{c} \text{MKT} \\ \text{SMB} \\ \text{HML} \\ \text{UMD} \\ \text{LIQ} \end{array} &
\begin{bmatrix}
VAR(MKT) & COV(MKT, SMB) & COV(MKT, HML) & COV(MKT, UMD) & COV(MKT, LIQ) \\
COV(SMB, MKT) & VAR(SMB) & COV(SMB, HML) & COV(SMB, UMD) & COV(SMB, LIQ) \\
COV(HML, MKT) & COV(HML, SMB) & VAR(HML) & COV(HML, UMD) & COV(HML, LIQ) \\
COV(UMD, MKT) & COV(UMD, SMB) & COV(UMD, HML) & VAR(UMD) & COV(UMD, LIQ) \\
COV(LIQ, MKT) & COV(LIQ, SMB) & COV(LIQ, HML) & COV(LIQ, UMD) & VAR(LIQ)
\end{bmatrix}
\end{array}$$