Norwegian School of Economics
Bergen, Autumn 2022

Trying to Beat the Market

An Empirical Analysis of the Historical and Potential Active Returns of the Government Pension Fund Global

Hanna Vetle Olsen & Ole Kristian Reiming

Supervisor: Gernot Peter Doppelhofer

Master Thesis in Economics and Business Administration

Major: Financial Economics

NORWEGIAN SCHOOL OF ECONOMICS

"Owning the Stock Market Over the Long Term is a Winner's Game, but Attempting to Beat the Market is a Loser's Game"

John C. Bogle, 2007

Abstract

The Government Pension Fund Global (hereafter the GPFG) helps finance the Norwegian welfare state and aims to be managed in such a way that it benefits both current and future generations. Today, the fund is managed closely to a benchmark index based on a mandate determined by the Ministry of Finance, but it is also managed actively to generate excess returns. The active management of the fund is a heated topic and there have been frequent debates related to the management model of the fund. This thesis aims to contribute to the discussion and investigates the historical and potential active management and returns, through our research question: "*How has the active management and accompanying active returns of the GPFG been historically, and how could increased active management impact active returns?*"

Our thesis rests on three supportive analyses: a historical analysis evaluating fund performance and active management, a scenario-analysis investigating potential active returns, and lastly a qualitative study validating our findings.

We first analyse the historical active returns and management of the fund. We find that active returns predominantly have been significant throughout the investigated time periods, and that active management has created additional returns for the fund, both in terms of benchmark risk-adjusted alpha and factor risk-adjusted alpha. We further establish the historical degree of active management and find an average active share of 18.92% from 2015 to 2020 and an annual tracking error of 0.63% since inception, essentially defining the GPFG as an index fund.

Furthermore, we construct three synthetic portfolios combining the GPFG with the New Zealand Superannuation Fund, to analyse active returns of portfolios with higher degrees of active management. All three synthetic portfolios outperform the GPFG's historical active returns both in-sample and out-of-sample, clearly indicating that there exists an opportunity for the GPFG to increase its active returns by increasing active management. Our initial findings are further evaluated in light of existing empirical research in the field of active versus passive management, where the broad consensus contradicts our quantitative findings.

After having emphasized empirical research, we still find that active management and its accompanying returns have created significant value historically and that the fund could increase its active returns by increasing active management. Additionally, we question why the tracking error limit set by the Ministry of Finance is not exploited, and further recommend that this should be considered.

Acknowledgements

This thesis is written as part of our Master of Science in Economics and Business Administration at NHH, majoring in Financial Economics. The process of writing the thesis has been challenging and rewarding, yet also very interesting. The in-depth exploration of one of the central questions regarding Norway's financial economy has been a memorable journey, and we will cherish the experience on this topic in the future, both as employees and citizens.

We want to use this opportunity to thank our supervisor, Gernot Doppelhofer, who has contributed with good advice, constructive input and guided us through this assignment. We could not have undertaken this journey without the generously provided knowledge from our supervisor. We also want to recognize our families and friends for their support throughout the process.

Norwegian School of Economics, December 2022

Hanna Vetle Olsen

Ole Kristian Reiming

Contents

1 Introduction

The Government Pension Fund Global (hereafter the GPFG) is a part of laying the foundation for the Norwegian welfare state for generations to come, and naturally, the management model of the fund is of essence for the Norwegian economy. At the time of writing, the GPFG has a value of NOK 12,700 billion (approximately USD 1,270 billion) and is managed to benefit both current and future generations. Even though luck and random coincidence of events have played a large role in the substantial growth of the fund, democratic decisions have ensured that the enormous petroleum wealth accrue to the community and is managed beneficially.

Since the inception of the GPFG, there have been frequent debates related to the management model of the fund, and whether this should be based on active or passive management. Where passive portfolio management aims to replicate a benchmark return, active portfolio management aims to outperform a selected benchmark by continuously making investment decisions regarding the portfolio's holdings. The broad literature on the topic can be summarized by John C. Bogle:

"Owning the Stock Market Over the Long Term is a Winner's Game, but Attempting to Beat the Market is a Loser's Game"

The quote presented above essentially sets the framework for our master's thesis, the final work of our master's degree in Financial Economics. With this thesis, we want to investigate how historical active management and accompanying active returns have been for the equity portfolio of the GPFG, and whether there is scope for increasing active returns by increasing active management.

1.1 Research Question

In this thesis, we analyze the active returns of the GPFG by both looking at the active returns historically and how different scenarios for active management of the equity portfolio can impact returns. The goal of the thesis is to provide a nuanced analysis to answer the following research question:

How has the active management and accompanying active returns of the GPFG been historically, and how could increased active management impact its active returns?

We structure the thesis into three parts to answer our research question.

1. **Historical Evaluation of the GPFG:** Performance evaluation, significance of historical active returns and establishing the historical degree of active management
2. **Scenario-Analysis: Potential Active Returns for the GPFG:** Fund selection for synthetic portfolio construction and respective synthetic portfolio comparison
3. **Discussion Leveraging Existing Research:** Evaluating findings in light of existing empirical research

In the first part of the thesis, we analyze the historical performance of the GPFG and the significance of its active returns. This is respectively conducted through a selection of key performance measures and regression analyses using Jensen's Alpha and the Fama-French Five-Factor Model. We also take a glance at the active management of the fund, by establishing the historical degree of active management using tracking error and active share. In the Historical Analysis of the GPFG, we strive to outline the historical degree of active management and its active performance, to create a basis of comparison between the GPFG and presented funds and synthetic portfolios in the Scenario-Analysis.

Furthermore, in the second part of the thesis, we aim to understand how increased active management can impact active returns, through an ex-post experiment. In the ex-post experiment, we leverage historical return data to deliver an after-the-fact study. We construct three synthetic portfolios as a combination of (1) the GPFG and (2) a selected fund that has a higher degree of active management in terms of tracking error and active share. Thus, by construction, our three synthetic portfolios will have a higher degree of active management relative to the GPFG and create a suitable foundation to investigate our research question. We also include a 6-month prediction of active returns for the synthetic portfolios, the GPFG, and the selected fund. The reasoning behind this is to provide a basis for discussion of future active returns.

Lastly, we evaluate our results in light of existing empirical research in the field of active versus passive portfolio management. Our Scenario-Analysis is based on a set of assumptions and simplifications, and followingly, it will be important to include empirical research to provide a nuanced answer to our research question.

1.2 Structure of the Thesis

The thesis is structured according to the presented research question and its outlined subparts. In Chapter 2, we present the context of our thesis, including an introduction to the GPFG, essential financial concepts and information about relevant Sovereign Wealth Funds. Followingly, we outline the methodology for answering our research question in Chapter 3 and Chapter 4, respectively for the Historical Analysis and the Scenario-Analysis. Further, in Chapter 5, the raw data, following data treatment and finalized datasets for our analyses are presented. The analyses for the thesis are presented in Chapter 6 and Chapter 7, namely for the Historical Analysis and the Scenario-Analysis respectively. Followingly, Chapter 8 outlines the limitations of our findings. The analyses are then extended with a discussion of our findings related to existing empirical research in Chapter 9. Finally, the Concluding Remarks of the thesis are presented in Chapter 10.

2 Context

In this chapter, we present the relevant context for our thesis. The chapter aims at providing the necessary background information for the thesis, to be able to answer our research question. Initially, we introduce the GPFG, including its investment strategy and equity benchmark. Further, we outline fundamental financial concepts that are essential throughout the thesis, before lastly presenting information on other Sovereign Wealth Funds we use in our analyses.

2.1 The Government Pension Fund Global

The GPFG was established in 1990 by Norway's Parliament, to regularly transfer surplus from the government's petroleum revenue, with its first injection in 1996. The fund's purpose was to provide the government flexibility in fiscal policy if the oil prices were to fall or the mainland economy were to stagnate. Additionally, the fund contributes to laying the foundation of the future welfare state, by representing a tool for solving challenges related to an ageing population and declining income from the petroleum industry. The Ministry of Finance is responsible for the management of the fund while the Central Bank of Norway, more specifically Norges Bank Investment Management (NBIM) is responsible for the fund's operations (NBIM, 2022).

Since its first injection in 1996, the fund has grown significantly in terms of market value, due to capital injections and high returns on investments (Ødegaard & Dahlquist, 2018). For maintaining the long-term value of the GPFG, in 2001, the rule of action was introduced, a fiscal policy limiting the capital withdrawals to the GPFG's expected real returns (NBIM, 2022).

Since the establishment of the fund, multiple significant changes have been made. Initially, the fund was invested similarly to the Central Bank of Norway's foreign exchange reserves, thus, only in assets invested outside of Norway and in government bonds (NBIM, 2022). In 1997 the Ministry of Finance decided that 40% of the assets were to be invested in equities, and the further composition of assets in the fund has changed multiple times since then. For instance, the GPFG aims at having an equity share of 70%, which has entailed higher returns and fluctuations in market value. Emerging markets were further added to the equity benchmark index in 2000, and in 2004 ethical guidelines were introduced for the fund. Established changes to the fund's investment strategy have generally been based on thorough investigations, often through expert reports (Andreassen, et al., 2022).

2.1.1 Investment Strategy

The objective of the GPFG's investments is to achieve the highest possible return at an acceptable risk level and the fund further needs to be responsibly managed within this objective (The Ministry of Finance, 2021) & (Andreassen, et al., 2022).

The investment strategy of the fund is designed with a basis in the presented objective above, its unique characteristics, advantages of the asset manager and beliefs regarding the financial markets (The Ministry of Finance, 2021). The Ministry of Finance has developed an investment strategy with some main features, among them (1) a wide spread of investments, (2) harvesting risk-premiums, (3) limited deviation from the benchmark index, (4) responsible management, (5) cost-effective management and (6) transparency (The Ministry of Finance, 2021). The investment strategy is communicated through restrictions for certain investments, a benchmark index, a risk budget and other requirements established by the Ministry of Finance. However, the limit for the allowed standard deviation of active returns (tracking error) at 1.25% is the restriction of most importance in our thesis. This limit implies that the active risk of the fund should not exceed 1.25%.

The GPFG has a high-risk-bearing ability, where the amount of risk taken depends on the risk tolerance of its asset owner, namely the Norwegian people, represented by political authorities (The Ministry of Finance, 2021). The main determinant of the overall risk of the fund is determined through the selected equity share.

2.1.2 Benchmark

The fund's investments are compared to a benchmark index that is defined in the management mandate from the Ministry of Finance. As of today, the strategic benchmark consists of 70% listed equities and 30% fixed income. Nevertheless, the fund may invest in a broader set of assets where the board has expressed the intention of public listing (Norges Bank Investment Management, 2019).

The equity benchmark index is based on the FTSE Global All Cap Index, provided by FTSE Russell. The index represented 8,921 constituents in 49 countries in 2020. However, the equity benchmark for the fund deviates from the FTSE Global All Cap Index in two dimensions; (1)

geographical distribution and (2) ethical exclusions. First, the benchmark index possesses a larger weight in European developed markets, and lower weights in US and Canadian markets, compared to the FTSE Global All Cap Index. The fund is also not allowed to invest in Norway or securities denominated in NOK (Norges Bank Investment Management, 2019).

Second, the fund is restricted from investing in securities issued by companies that have been excluded by The Central Bank of Norway (Norges Bank Investment Management, 2019). The Ministry of Finance has issued guidelines for the exclusion of companies from the fund and established an independent Council on Ethics to conduct ethical assessments of companies. This assessment is based on two criteria: product-based exclusions and conduct-based exclusions. The first criterion entails that the fund cannot invest in companies that produce certain types of weapons, base their operations on coal or produce tobacco. The latter criteria entail that the fund may exclude companies where there is an unacceptable risk of conduct in terms of violation of ethical norms (Norges Bank Investment Management, 2019).

2.2 Financial Concepts

After introducing the GPFG, we now want to present some fundamental financial concepts that will be essential throughout our thesis. For an experienced reader, this section could be viewed as redundant. Nevertheless, we see the necessity of presenting these concepts at the beginning of the thesis to make important distinctions and ensure consistency in the academic terms used throughout the analyses. We emphasize the concept of portfolio management, including active and passive management, as well as provide a definition of the term alpha and a derivation of the Efficient Market Hypothesis.

2.2.1 Portfolio Management

Portfolio management is defined as the activity of trading and investing capital to generate returns and increase the capital base. The goal of asset management is to maximize the value of the portfolio while mitigating risk to an acceptable level for the asset owner. Portfolio management is mainly divided into two different strategies, namely passive and active management (Chen, 2022). The definition of each strategy is presented below.

Passive portfolio management is an investment strategy aiming to mirror an established index or benchmark. The goal of this strategy is to achieve the same return as the chosen index over time. The index or benchmark chosen resembles a recommendation of holdings, including recommended weights to hold in each asset based on the portfolio's market value (Lioudis, 2021).

Active portfolio management is defined by an investor who actively tracks the performance of an investment portfolio and makes continuous investment decisions regarding the portfolio's holdings. The goal of this strategy is to outperform a selected benchmark. Thus, active management may require investment analysis, research, forecasts, and personal experience to make well-considered investment decisions to outperform the market. Generally, active portfolio management is quite resources intensive as the managers of such funds aim to identify assets that are wrongly priced in the market, and thus gain profit from this mispricing (Chen, 2022).

2.2.2 Alpha

As emphasized in both the Introduction and Research Question, the focus of this thesis will mainly revolve around the active returns of the GPFG. Throughout the thesis, we will also include the term alpha (α). In the following, we aim to provide a distinction between the terms active return and alpha.

Both the term active return and alpha refer to the excess return that a portfolio generates above the benchmark. Throughout the thesis, the term **active return** will refer to the difference between the portfolio return and the benchmark return, measuring the contribution of active management to the portfolio's return. Further, the term **alpha** will refer to the risk-adjusted contribution of active management and is represented as the intercept in a regression of active returns on the benchmark or other risk factors (Chen, 2022).

2.2.3 Efficient Market Hypothesis

The Efficient Market Hypothesis is a central concept in financial economics. As presented in Sharpe (1970), the Efficient Market Hypothesis states that markets are efficient when all asset prices reflect all relevant information for the particular asset.

There are three distinctions of the Efficient Market Hypothesis: the strong form, the semi-strong form and the weak form. The strong form efficiency implies that all information, both available public information and private information (inside information), are reflected in share prices, while the semi-strong form implies that share prices at all times will reflect all publicly available information. The weak form, on the other hand, claims that today's share price only reflects data for historical prices and that technical analysis cannot be used to make beneficial investment decisions (Santos, 2021). In the following, when evaluating the Efficient Market Hypothesis, we leverage the semi-strong form.

When the Efficient Market Hypothesis holds, the alpha term emphasized in the previous section does not exist, as all assets are correctly priced, and investors cannot exploit mispricing to generate excess return. Sharpe (1991) therefore defines a passive investor as someone who believes in the Efficient Market Hypothesis and holds the market portfolio, as the investor does not believe that there is a possibility to generate excess returns. The hypothesis, which is a cornerstone of modern financial theory, is supported by academic evidence (Downey, 2022). However, the hypothesis is often disputed and there are oppositions arguing that investors have outperformed the market over a long time and generated significant returns from an active investment strategy. That should be impossible if the Efficient Market Hypothesis holds.

This further facilitates a modification of the Efficient Market Hypothesis. Studies demonstrate that the degree of market efficiency varies, both between markets and over time. The Modified Efficient Market Hypothesis claims that the financial markets are close to efficient the majority of the time, while active investments contribute to eliminating mispricing and making markets more efficient (NBIM, 2009). In markets that are not characterized as efficient, asset prices can then deviate from their fair value, and investment managers can exploit patterns that might occur. In such a market, alpha exists and there is an opportunity to generate excess returns from active management in certain periods of time.

2.3 Introduction to Other Sovereign Wealth Funds

The last part of this chapter provides a summary of different Sovereign Wealth Funds that we present and further use in our thesis. We deliver a brief overview of the respective funds' objectives and mandates. Throughout the thesis, we aim to find comparable peers to the GPFG, and the funds presented are therefore chosen due to a high degree of transparency (Døskeland, 2022).

2.3.1 New Zealand Superannuation Fund

The New Zealand Superannuation Fund was established in 2001 with the purpose of sustainable investment that delivers strong returns for the people of New Zealand (New Zealand Superannuation Fund, Purpose and Mandate, 2022). The fund aims at improving the ability of the government to pay superannuation and reduce the tax burden for future generations. At the time of writing, the fund has a market value of USD 35 billion.

The fund gains contributions from the New Zealand Government in line with a constructed Treasury Contribution Rate Model. From 2035, the Government of New Zealand will start by withdrawing capital. However, the fund aims to grow until the 2070s and can be considered a long-term global investment fund (New Zealand Superannuation Fund, 2022).

The New Zealand Superannuation Fund reports that the majority of the fund is managed passively, where two-thirds are invested in line with a reference portfolio. The reference portfolio consists of 80% equities and 20% fixed income. However, the fund emphasizes that active investment has some substantial benefits for the fund, allowing them to increase diversification and its natural advantages, providing significant returns with small additional risks (New Zealand Superannuation Fund, 2022). The fund's key characteristics include its long time horizon, known liquidity profile, operational independence from the government and sovereign status.

2.3.2 Alaska Permanent Fund

The Alaska Permanent Fund was established in 1976 by the Alaskan people with the purpose to preserve and convert non-renewable oil and mineral wealth into a financial resource for future generations (Alaska Permanent Fund, 2022). Their diversified portfolio of public and private investments aims at providing a long-term risk-adjusted return of 5% above the consumer price index. At the time of writing, the fund has a market value of approximately USD 76 billion.

The Alaska Permanent Fund reports that equities represent most of the asset allocation, and the fund is invested in more than 3000 companies worldwide (Alaska Permanent Fund, 2022). All equities are managed externally by financial management firms, selected based on specific knowledge either in terms of investment styles or company size. Furthermore, most of the portfolio is actively managed, with some equities passively managed. This allows investment managers to actively sell and buy equities based on their expertise (Frank, 2017).

2.3.3 Korea Investment Corporation

Korea Investment Corporation (KIC) is a Sovereign Wealth Fund in South Korea founded in 2005, with the purpose of managing public funds that are entrusted by the government and the Bank of Korea (Korea Investment Corporation, 2022). The objective of the fund is to generate consistent and excess returns compared to relevant benchmarks, with an acceptable risk level. Under the fund's mandate, KIC can invest in public equities, bonds, commodities, private equity, real estate, and hedge funds (Korea Investment Corporation, 2022). At the time of writing, the fund has a market value of approximately USD 200 billion.

2.3.4 Caisse de dépôt et placement du Québec

Caisse de dépôt et placement du Québec (CDPQ) is an institutional investor established in 1965 for managing public pension plans and insurance programs (CDPQ, 2022). Today, the fund has a long-term perspective with a total portfolio including several types of assets, where each asset class has specified a risk-return profile. At the time of writing, the fund has a market value of approximately USD 392 billion.

3 Methodology for Historical Analysis of the GPFG

This chapter is dedicated to explaining the empirical methods we use to conduct our historical analysis of the GPFG in Chapter 6. There is a well-established approach for fund evaluation in empirical literature, including several expert reports evaluating the GPFG's historical performance (Bauer, Christiansen, & Døskeland, 2022) & (Ødegaard & Dahlquist, 2018). We leverage similar methods with some alterations in our historical analysis, which are presented in detail in the following sections.

Initially, we outline the overall methodology used for the historical analysis. The goal is to provide an overview of Chapter 6, its subparts, and how they are connected, before presenting the specific methods used to answer the first part of our research question on how active management and accompanying active returns have been historically. We illustrate the methodology for Chapter 6 in Figure 1:

Figure 1 - Methodology for Historical Analysis of GPFG

Firstly, we investigate return and risk performance measures for the GPFG. This includes arithmetical averages of return, including portfolio return, benchmark return, and active return excluding and including costs. We also calculate the standard deviation and the risk-adjusted return measure Sharpe Ratio.

Followingly, we aim to provide a more comprehensive evaluation of the active returns for the GPFG. We investigate the risk-adjusted active returns, through different regression analyses, namely Jensen's Alpha and the Fama French Five-Factor model. This is to understand how active management has contributed to adding significant value to the fund's portfolio returns. Further, we investigate whether the generated active returns are a consequence of random luck in the financial markets or well-considered investment decisions.

Lastly, we analyse the historical active management of the GPFG through tracking error and active share. To be able to answer our research question, we must establish the current degree of active management and create a basis of comparison for further analysis.

The goal of the approach presented above is to gain a comprehensive understanding of the GPFG's historical performance and active management to answer the first part of our research question. In the following sections, we describe the methodology used in each step of the outlined approach for Chapter 6 presented in Figure 1, in terms of performance measures, evaluation of active returns, and degree of active management.

3.1 Performance Measures

Our point of departure for the Historical Analysis of the GPFG is, as emphasized, the calculation of risk and return performance measures in terms of arithmetical average and standard deviation[1]. We assume that the methodology behind such measures is well-known, and we will therefore not elaborate on this any further. Furthermore, we rely on the reward-to-volatility measures Sharpe Ratio and Information Ratio. In the following, these measures will briefly be listed.

3.1.1 Sharpe Ratio

Sharpe Ratio is a reward-to-volatility measure widely in use for evaluating investment performance and represents a portfolio's average excess return compared to the risk-free rate, adjusted for the average standard deviation of return (Santos, 2021). A higher sharpe ratio implies a higher expected return per unit of risk (Santos, 2021). The formula of sharpe ratio is denoted by Equation 1.

Equation 1

$$Sharpe\ Ratio = \frac{E[r_p] - r_f}{\sigma_p}$$

[1] The arithmetical average and standard deviation are calculated by using elementary excel formulas (=AVERAGE and =STDEV) on historical data.

3.1.2 Information Ratio

The **Information Ratio** (IR) measures the excess return generated from excess risk taken compared to a benchmark. The IR divides the mean active return R_A by active risk, denoted by its standard deviation $\sigma(R_A)$. The formula of IR is denoted by Equation 2.

Equation 2

$$Information\ Ratio = \frac{R_A}{\sigma(R_A)}$$

A passive manager strives for an IR approximately equal to zero. This implies that the manager performs as well as the benchmark, and no active return is achieved. However, an active manager strives to achieve a higher information ratio, as this indicates a higher risk-adjusted performance of the portfolio (Ang, 2014).

From the IR, one can further assess if the portfolio outperformed the benchmark, but the measure does not give any insight into how the portfolio outperformed the benchmark. For instance, the portfolio can be outperformed through smaller persistent gains versus more extreme events, or due to well-considered investment decisions versus luck.

3.2 Evaluation of Historical Active Returns

Followingly, to provide a more comprehensive evaluation of the active returns for the GPFG, we analyse the significance of historical active returns through benchmark and factor risk-adjusted alphas and investigate whether the active returns are a consequence of skill or luck. The benchmark risk-adjusted alpha is evaluated through Jensen's Alpha, while the factor risk-adjusted alpha is evaluated by using the Fama French Five-Factor model. The following sections will outline the Jensen's Alpha estimation, the Fama French Five-Factor model, and the t-test used to test for skill versus luck.

3.2.1 Jensen's Alpha Estimation

As emphasized, we find the benchmark risk-adjusted alpha through a **Jensen's Alpha estimation**. This evaluates the average return on a portfolio subtracted the return predicted by using the Capital Asset Pricing Model (CAPM).

CAPM was introduced in the 1960s by Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966), and gives a prediction of the relationship we should observe between risk and expected return of an asset (Santos, 2021). CAPM is expressed in Equation 3:

Equation 3

$$E[r_p] = r_f + \beta(r_m - r_f)$$

According to CAPM, the return of an asset is a linear function of the asset's systematic risk, and thus, the market solely prices systematic risk. From Equation 3, the expected return of a portfolio consists of the risk-free rate and a risk premium.

Jensen (1968) compares a portfolio's average return to the predicted return from CAPM, given the portfolio's beta and the average return of the market. Here, over- or underperformance compared to CAPM is expressed as alpha and measures the absolute performance. The formula of Jensen's Alpha is denoted by Equation 4, assuming CAPM holds:

Equation 4

$$Jensen's\ Alpha\colon\ R_P - R_f = \alpha + \beta\left(R_B - R_f\right) + \epsilon_t$$

In Equation 4, R_P is the realized return on a portfolio, R_f is the risk-free return, R_B is the realized return on the appropriate benchmark index and β is the systematic risk with respect to the chosen benchmark index.

In Chapter 6, we use Equation 4 to perform a regression of portfolio return on benchmark return, both excess of the risk-free rate. The intercept α represents Jensen's Alpha which is the active return after having adjusted for difference in beta-risk between the fund and the benchmark.

Appraisal Ratio

A limitation of Jensen's Alpha is that the unsystematic risk a fund has taken for achieving alpha is not considered. **Appraisal Ratio** (AR) takes this risk into account, representing the alpha per unit of unsystematic risk (Treynor & Black, 1973). Thus, the measure is similar to Jensen's Alpha but divided with the unsystematic risk of a portfolio (Bauer, Christiansen, & Døskeland, 2022). The unsystematic risk of a portfolio is the standard deviation of the residual obtained from the CAPM regression. The formula for the appraisal ratio is denoted by Equation 5:

Equation 5

$$Appraisal\ Ratio = \frac{\alpha_p}{\sigma_{p,\varepsilon}}$$

3.2.2 Fama French Five-Factor Model

Further, the factor risk-adjusted alpha is evaluated by using the Fama and French (2015) Five-Factor model, as recommended by the GPFG's factor-model expert group (Dahlquist, Polk, Priestley, & Ødegaard, 2015). When conducting our regressions, the dependent variable is the active return. We interpret the estimated slope coefficients of the regressions as active exposures to each systematic factor, and alpha as the active value creation of the GPFG above the exposure to the risk factors within market, size, value, profitability, and investment (Bauer, Christiansen, & Døskeland, 2022).

Fama and French (2015) established the five-factor model, enhancing a previous three-factor model (1993). The model adds four explanatory variables to the times series regression approach of Black, Jensen, and Scholes (1972) for explaining the variation of returns of stocks and portfolios (Fama & French, 1993). The regression is denoted by Equation 6. The risk factors will be briefly outlined in the following.

Equation 6

$$R_P^a = \alpha + \beta_{MKT}MKT_t + \beta_{SMB}SMB_t + \beta_{HML}HML_t + \beta_{RMW}RMW_t + \beta_{CMA}CMA_t + \epsilon_t$$

From Equation 6, the **MKT-factor** is an overall market factor representing the market excess return over the risk-free rate (Fama & French, 1993).

The **SMB-factor** (small minus big) takes into consideration that small firms with low market capitalization on average outperform large firms with high market capitalization. The SMB-factor therefore considers the difference between a portfolio consisting of small firms and a portfolio consisting of big firms (Fama & French, 1993).

Fama & French's (1993) **HML-factor** (high minus low) compares firms with high book-to-market ratio (value stocks) and firms with low book-to-market ratio (growth stocks). The findings of Fama & French (1993) indicates that on average, portfolios with high book-to-market ratio outperform portfolios with low book-to-market ratio, thus value-stock portfolios outperform growth-stock portfolios (Santos, 2021). This is due to firms with a high book-to-market ratio are more exposed to financial distress and need more compensation for risk (Santos, 2021).

The **RMW-factor** (Robust minus weak) considers the difference between the return on portfolios of stocks with robust profitability and the ones with weak profitability (Santos, 2021).

The last factor, the **CMA-factor** (conservative minus aggressive), takes into account the difference between returns on a diversified portfolio of so-called conservative firms to a diversified portfolio of aggressive firms, in terms of investment activity. A firm would be characterized as aggressive if assets grow fast, and conservative if not (Santos, 2021).

3.2.3 Separating Skill and Luck

After analysing the active return and alpha for the GPFG, we aim to investigate whether the returns originate from well-considered investment decisions (knowledge and skill) or merely luck in financial markets. To evaluate this, we leverage a t-test with the null hypothesis that the true average active return is equal to zero (Døskeland, 2022). A t-test is a statistical hypothesis test used to investigate whether an average value is significantly different from a null hypothesis. The hypotheses of the test are presented below.

$H_0: Excess\ return\ is\ related\ to\ coincidences\ (\overline{r_A} = 0)$
$H_A: Excess\ return\ is\ related\ to\ skill\ (\overline{r_A} \neq 0)$

If the t-test returns a t-value above 1.96 or a p-value below 5%, we find evidence to reject the null hypothesis, indicating that the active return is indeed related to knowledgeable investment-decisions.

3.3 Establishing Historical Degree of Active Management

The last part of the Historical Analysis aims to investigate the active management of the GPFG through tracking error and active share. Measures such as active return and R-squared do not provide significant value to the evaluation of active management, and we therefore select to proceed with only tracking error and active share in our analysis (Bjerksund & Døskeland, 2015). We rely on these two measures to establish how actively the fund has been managed.

3.3.1 Tracking Error

Tracking error volatility is the classic way of evaluating active management and is defined as the time series standard deviation of active returns. This is further an indicator of how actively the fund is managed and the fund's corresponding risk level (Cremers & Petajisto, 2006). A high tracking error could entail an increased possibility for higher active returns, provided skilled portfolio management (Bjerksund & Døskeland, 2015). This implies that a fund needs to allow a certain amount of tracking error to generate active returns. Tracking error is denoted by Equation 7:

Equation 7

$$Tracking\ Error = \sigma(R_A) = \sigma(R_P - R_B)$$

3.3.2 Active Share

Active share is defined by Cremers & Petajisto (2009) as the percentage of a fund's portfolio that is deviating from the benchmark at a certain point in time. In contrast to tracking error, the fluctuation and correlation in the equity returns in the market are not affecting this metric (Bjerksund & Døskeland, 2015). For funds that solely invest in equities without short positions, active share will be between 0% and 100%. The active share is then the fraction of the portfolio that is deviating from the benchmark (Cremers & Petajisto, 2006).

A high active share is not a guarantee for active returns; however, it is a necessary condition to achieve it (Bjerksund & Døskeland, 2015). Cremers & Petajisto (2009) further classified funds according to their degree of active share. Funds with an active share below 20% are considered *index funds*, while funds with an active share between 20% and 60% are *closet indexers* and funds with an active share above 60% are considered *active funds* (Bjerksund & Døskeland, 2015).

Active share can be calculated as denoted by Equation 8, where W_{P_i} and W_{B_i} are the weights of asset i in the portfolio and the benchmark. Further, the sum of the difference between W_{P_i} and W_{B_i} for all assets in the fund is calculated.

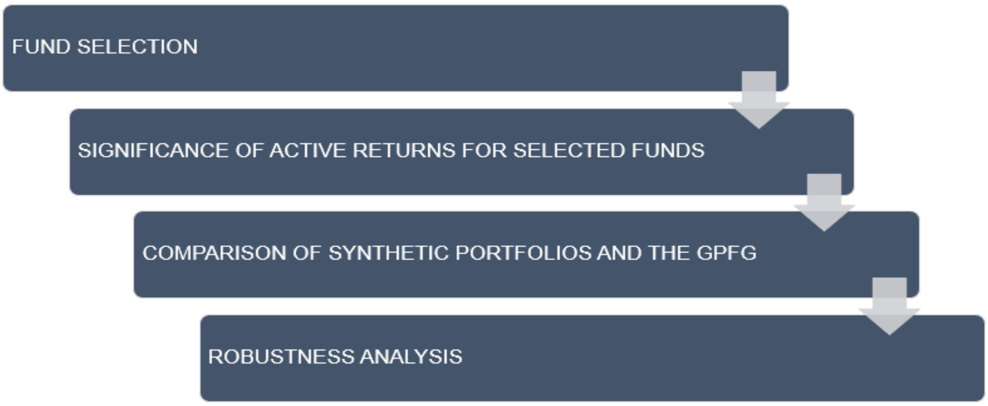
Equation 8

$$Active\ Share = \frac{1}{2}\sum_{i=1}^{N} |W_{P_i} - W_{B_i}|$$

4 Methodology for Scenario-Analysis of Potential Active Returns

This chapter is dedicated to explaining the methodology for analyzing potential active returns of the GPFG, in terms of comparing active returns for the GPFG with synthetically constructed portfolios. The analysis rests on several supporting parts enabling a comparison between the GPFG and the synthetic portfolios. We illustrate the overall methodology for the Scenario-Analysis in Chapter 7 in Figure 2:

Figure 2 - Methodology for Chapter 7: Scenario-Analysis



Firstly, in the Fund Selection, we compare the GPFG with other Sovereign Wealth Funds in terms of performance and active management. We use already presented performance and active management measures from Chapter 3 for the comparison, mainly active return, standard deviation, sharpe ratio, tracking error, and active share. All measures presented above will be included in a holistic evaluation of the funds, with the main emphasis on the degree of active management. The Fund Selection aims to select two funds for synthetic portfolio construction, one for the main analysis and one for robustness analysis. Thus, we create portfolios that are a combination of the historical active returns of the GPFG and the historical active returns of the selected funds.

After performing the Fund Selection, we examine if the funds have significant historical active risk-adjusted returns using the presented Five-Factor Model by Fama & French (2015) in Chapter 3. This step is included for ensuring the reliability of our constructed portfolios.

Our next step is comparing our constructed portfolios, denoted Portfolio 1, Portfolio 2, and Portfolio 3, with the GPFG. This comparison aims to investigate potential active returns and answer the second part of our research question on how increased active management can impact the GPFG's active returns. We then extend the analysis to predict active returns for the constructed portfolios. We end the Scenario-Analysis with a robustness analysis of our findings, conducting a brief equivalent analysis with another fund from the Fund Selection, to substantiate the findings from the main analysis.

In the following sections, we will present the methodology necessary for each step in the Scenario-Analysis, presented in Figure 2. As previously stated, the approach for the Fund Selection and Significance of Active Returns for Selected Funds is already presented in Chapter 3 and will not be emphasized any further. Thus, we present the remaining methodology for the Scenario-Analysis in this chapter, namely the synthetic portfolio construction and introduction of ARIMA models.

4.1 Synthetic Portfolio Construction

We rely on three different approaches to construct synthetic portfolios. The three portfolios are constructed as a combination of return data from the GPFG and a selected fund. The general formula for the active returns of the three constructed portfolios is presented in Equation 9. This formula is inspired by the ridge regression presented in Stock & Watson (1999).

Equation 9

$$r_{A,t}^{Synthetic\ Portfolio} = \lambda_t r_{A,t}^{Selected\ Fund} + (1 - \lambda_t) r_{A,t}^{GPFG}$$

The weight of each fund is denoted λ_t. The most prominent modification from Stock & Watson's (1999) ridge regression is a time-variant λ_t. Equation 9 will be the initial model for all three synthetic portfolios, however, the approach to calculating λ_t will differ between the three.

We estimate the weights λ_t based on active return, risk-adjusted return, and predictive quality, respectively for Portfolio 1, Portfolio 2, and Portfolio 3. Portfolio 1 and Portfolio 2 will therefore be based on performance, with the highest weight distributed to the best-performing fund in each period t. Thus, we hypothesize that both Portfolio 1 and Portfolio 2 will deliver increased active returns as the portfolios consistently favor the best-performing fund. Portfolio 3 will, however, be based on predictive quality, with the highest weight distributed to the fund with the best predictiveness. Thus, Portfolio 3 resembles an experiment where a hypothesis of the outcome is elusive. The construction of each portfolio will be more thoroughly outlined below.

4.1.1 Portfolio 1 – Active Return

Our first synthetic portfolio will be constructed based on the weighted average of active returns between the selected fund and the GPFG. We aim to create a synthetic portfolio with a higher degree of active management than the GPFG, to analyze what returns such a portfolio can deliver. Intuitively, the fund delivering the highest active return will have the highest weight in Equation 9 for each period t.

We base our calculation of λ_t^{AR} on a weighted average. For the purpose of our thesis, we chose to limit λ_t^{AR} between 0% and 100%. The calculation is presented below in Equation 10:

Equation 10

$$\lambda_t^{AR} = \frac{\bar{r}_A^{\,Selected\ Fund}}{\bar{r}_A^{\,Selected\ Fund} + \bar{r}_A^{\,NBIM}} \text{ , } where\ \lambda_t^{AR} \in (0,1)$$

4.1.2 Portfolio 2 – Risk-Adjusted Return

Our second portfolio leverage risk-adjusted return and the weights are created based on the weighted average of active return per unit risk. Unlike Portfolio 1, Portfolio 2 will emphasize the possible risk of increased active returns. Thus, the portfolio with the highest risk-adjusted return will be allocated the highest weight for each period t.

Similarly to Portfolio 1, we limit λ_t^{RA} between 0% and 100%. We use the following formula presented in Equation 11:

Equation 11

$$\lambda_t^{RA} = \frac{\bar{r}_A^{\,Selected\ Fund} / \sigma^{Selected\ Fund}}{\bar{r}_A^{\,Selected\ Fund} / \sigma^{Selected\ Fund} + \bar{r}_A^{\,NBIM} / \sigma^{NBIM}} \text{ , } where\ \lambda_t^{RA} \in (0,1)$$

4.1.3 Portfolio 3 – Predictive Quality

Our third constructed portfolio is based on the predictiveness of each fund, denoted by the mean squared prediction error (MSPE). We estimate predictive models and compare their values to the actual realized returns throughout the investigated time period. Our first step will be to perform a pseudo-out-of-sample analysis, using ARIMAs, which are introduced in the subsequent section. The MSPE is calculated as the difference between the predicted value of the ARIMA and the realized value, denoted by Equation 12:

Equation 12

$$MSPE_{i,t} = (y_i - \widehat{y_i})^2$$

Furthermore, we base the calculation of weight λ_t^{PQ} on the inverse MSPE, emphasizing that the fund with the highest predictive quality has a larger relative weight in each period t. The formula for calculating λ_t^{PQ} is presented below. Again, we limit λ_t^{PQ} between 0% and 100%.

Equation 13

$$\lambda_t^{PQ} = \frac{MSPE_{i,t}^{-1}}{\sum_{i=1}^{N} MSPE_{i,t}^{-1}}, where\ \lambda_t^{PQ} \in (0,1)$$

We evaluate the appropriate ARIMA every second year, thus for every 24 observations in the dataset, given the extensive process of scoping the models.

4.2 Introduction to ARIMA Forecasting

A part of our Scenario-Analysis contains the use of Autoregressive Integrated Moving Average models (ARIMAs). We mainly use ARIMAs to create Portfolio 3 through a pseudo-out-of-sample analysis, while also leveraging the models to forecast future active returns for the GPFG, the selected funds, and the constructed portfolios. The identification of optimal ARIMAs for prediction in our analysis is based on the Box-Jenkins methodology. Even though the use of ARIMAs is present and facilitates our analyses, it has limited value in directly answering our research question. Additionally, the derivation of the models and the process of scoping them are extensive and we therefore refer to the Appendix for the complete derivation.

Furthermore, throughout our thesis, we also rely on testing whether a time series follows a random walk process. For this purpose, we leverage Wald-Wolfowitz Runs tests. The methodology of random walk, its implications, and the Wald-Wolfowitz Runs test are further elaborated on in the Appendix.

5 Data Treatment

This chapter is dedicated to presenting the collected data and following data treatment conducted for answering our research question. Intuitively, this chapter follows a similar structure as the presented structure for the analyses. Thus, our point of departure is presenting the data obtained for performing the Historical Analysis of the GPFG, respectively through evaluating fund performance and active management. Followingly, a description of the data and data treatment enabling the Scenario-Analysis will be presented.

5.1 Data for Historical Analysis of the GPFG

5.1.1 Evaluating Fund Performance

Return Data

For evaluating the performance of the GPFG in terms of return and risk, we have downloaded monthly return data of the equity portfolio from the fund (NBIM, 2022). The dataset contains monthly observations in US dollars over the actual portfolio return of the fund, the return of the benchmark, and the active return. The return data consists of 287 observations from January 1998 to December 2021.

Throughout our thesis, we often depend on adjusting the dataset of monthly observations into annual values. To adjust the dataset, we multiply the average monthly return[2] by 12 and the monthly standard deviation[3] by $\sqrt{12}$. This data adjustment relies on the time series being independent and identically distributed (IID), which is further elaborated on in the Appendix.

Costs

In evaluating the performance of the GPFG, we use monthly returns both before and after costs. We will consider operating costs, which consist of transactional costs and management costs. The raw data collected from the GPFG is returns net of transaction costs. Thus, we consider the management costs separately. To collect data on management fees, we reviewed annual reports from 1998 until 2021, which all report management fees as a percent of assets under management (NBIM, 2022). The GPFG's management fees consist of base fees and performance-based fees related to the active return generated. We convert the annual data into

[2] Computed by leveraging the (=AVERAGE) formula in Excel.
[3] Computed by leveraging the (=ST.DEV) formula in Excel.

monthly observations by dividing the annual management fees by 12. Arguably, it's feasible to assume that the base fee is equally distributed during the year, while the performance-based fee can fluctuate between months. For the purpose of this thesis, we assume that all management fees are distributed equally during the year.

When evaluating returns before and after management costs, denoted "Excluding Costs" and "Including Costs", we subtract management fees in percentage points from the active return for each monthly observation.

Furthermore, we also need to consider benchmark costs. Benchmark costs imply the transaction costs related to (1) inflows and extraordinary benchmark changes and (2) replication of the benchmark (Bauer, Christiansen, & Døskeland, 2022). Preferably, these costs should have been subtracted from the benchmark return. However, due to scarcity of data we are not able to estimate the benchmark costs. Thus, the returns after management costs presented in the thesis, are slightly conservative. We therefore also chose to report on returns before costs for all analyses conducted in the thesis, as the true active return presumably will lie between these two values.

The Fama French Factor Data

We have downloaded monthly global factor returns for Fama & French's (2015) Five-Factor model on Kenneth R. French's Data Library (Kenneth R. French, 2022). These data also include a risk-free rate based on a one-month Treasury Bill, which we use throughout our thesis.

5.1.2 Evaluating Active Management

Holdings Data

For analysing the historical active management of the GPFG, data on the fund's holdings in equities were downloaded from the GPFG's website from 31.12 each year from 2015 to 2020 (NBIM, 2022). This data contained information about holdings in terms of company name, market value (in NOK and USD), region, country, industry, ownership, and incorporation country.

Benchmark Data

The GPFG's equity benchmark is based on the FTSE Global All Cap Index, by FTSE Russell. We have obtained benchmark data from 2015 to 2020. During the period working with this thesis, we have on multiple occasions requested data for 2021 as well, however, we have not received this data from FTSE Russell. The data obtained consists of information on the equities in the benchmark, including a country code, constituent name, market capitalization before and after investibility match, weight in the benchmark index, and specific weights under different geographical exclusions.

Benchmark Adjustment

The benchmark received from FTSE Russell needs adjustments for our analysis of active management, given that the actual benchmark used by the GPFG is not identical to the FTSE Global All Cap Index. We adjust for geographical affiliation, excluded companies, and different equity classes in our dataset for securing a reliable dataset from 2015 until 2020.

First, all Norwegian companies are removed from the benchmark data and their associated weights are distributed equally between the benchmark's remaining holdings. We further conduct a geographical adjustment of the benchmark in line with The Ministry of Finance's established adjustment factors for different regions, illustrated in Table 1 below:

Table 1 - Regional Factors for Adjusting Benchmark

Region	Factor
Developed Markets in Europe	2.5
Developed Markets in North America	1
New Countries in the FTSE Index from 2015	0
Other Developed and Emerging Markets	1.5

We multiply these factors with the corresponding regional classification of the country of each holding, as emphasized in Table 12 - Regional Classification by FTSE Russell in the Appendix. We rely on these factors and assign new weights to the equities in the benchmark data, using the formula presented in Equation 14 by the Ministry of Finance:

Equation 14

$$\frac{Market\ Capitalization_i * factor_i}{\sum_i Market\ Capitalization * factor_i}$$

There is further a discrepancy in terms of equity classes between the holdings data and the benchmark data from FTSE Russell. In the FTSE Global All Cap Index, several equity classes of companies are included. However, in the holdings data, there is no distinction between different equities for the same company. Consequently, the company is only listed once in the holdings data and this needs to be adjusted to merge the holdings with the benchmark data. We assume that the fund holds the same equity classes as the benchmark, and we consistently choose the equity class with the largest weight in the benchmark when matching the holdings data with the benchmark data. This assumption has implications for the calculation of active share, given that the benchmark weights of the equities at hand may be different, potentially leading to deviations in our active share calculations.

Data Merging of the Holdings- and Benchmark Data

To calculate the active share, we need to merge the datasets containing the GPFG's holdings and the adjusted benchmark data. To find the corresponding benchmark data for each holding, we utilize the company name for linking the two datasets. The merging is conducted in Excel through the add-in Fuzzy Lookup. Observing the datasets, we find discrepancies in the formulation of company names between the two datasets, making it challenging to merge the two datasets directly with traditional Excel tools. To illustrate, the position in Kia Motors Corporation is denoted "Kia Motors Corp" in the holdings data and "Kia Motors" in the benchmark data, making them unable to match directly. To solve this, we use the Fuzzy Lookup Add-in, which merges variables based on a similarity percentage. We use a similarity match of 85% throughout our analysis. After conducting the merging, we faced two obstacles; (1) some observations are wrongfully matched and (2) some observations did not match, even though there is a corresponding position in the benchmark. To solve this, we went through the dataset manually, first checking for wrongful matches and then adding matches that were not caught by the similarity match. This resulted in the addition of hundreds of companies manually.

After having successfully merged the two datasets, we still had some non-matched observations. For these observations, we assume active positions denoted by a benchmark weight of 0%. As presented in Table 2, the number of observations in the holdings data and the benchmark data are different, indicating that several positions by structure will remain non-matched. This substantiates our assumption that non-matched observations indeed can be assumed to be active positions. We use Index & Match to import the corresponding weights for holdings and benchmark respectively, into the merged dataset. Then, we calculate the active share by using Equation 8.

Table 2 presents a summary of the datasets used in the active share calculations. The Table illustrates the number of holdings in the GPFG, amount of benchmark positions, the number of positions we matched through Fuzzy Lookup, and the non-matched observations, which we assume to be active positions.

Table 2 - Summary of Dataset for Active Share Calculations

	2015	2016	2017	2018	2019	2020
Holdings GPFG	9051	8986	9147	9159	9203	9124
Benchmark Positions	7704	7668	7774	7817	8823	8921
Matched with Fuzzy Lookup	**6639**	**6654**	**6730**	**6832**	**7267**	**6993**
Non-matched*	2412	2332	2417	2327	1936	2131

*Assumed to be active positions with a corresponding benchmark weight of 0%.

5.2 Data for Scenario-Analysis: Potential Active Returns for the GPFG

5.2.1 Data for Fund Selection

To be able to conduct the Fund Selection, we obtained return data from different Sovereign Wealth Funds. We rely on a transparency list of several Sovereign Wealth Funds internationally, presented by Døskeland (2022). After reviewing these funds' annual reports, we were able to create datasets of active returns from four different funds, namely Korea Investment Corporation, Alaska Permanent Fund, New Zealand Superannuation Fund, and Caisse de dépôt et placement du Québec (CDPQ). Characteristics of the data are presented below in Table 3:

Table 3 - Fund Summary

	Korea Investment Corporation	Alaska Permanent Fund	New Zealand Superannuation Fund	Caisse de dépôt et placement du Québec
Time of measurement	Annual	Quarterly	Monthly	Annual
Reporting period	2017-2021	2008-2021	2003-2021	2004-2021
Number of observations	5	56	225	18

These datasets vary in terms of before/after costs reporting, different fiscal year endings, reporting periods, and number of observations. We will amplify the retrieval of data and the difference for each fund below.

New Zealand Superannuation Fund Data

To evaluate the performance of the New Zealand Superannuation Fund, we have downloaded monthly return data published by the fund (New Zealand Superannuation Fund, 2022). The dataset contains 225 monthly observations from September 2003 to December 2021. The return data also contains the monthly benchmark return and the monthly active return of the fund.

The return dataset for the New Zealand Superannuation Fund has one limitation, as the fund only publishes data for the entire portfolio and does not present separate data for just equities. Thus, we cannot distinguish equity return data from overall return data published for the entire fund. However, the New Zealand Superannuation Fund states that the majority of the fund's investments are global equities, constituting the majority of the total return (New Zealand Superannuation Fund, 2022). Based on this, we believe that the active return presented in the dataset mainly comes from equity holdings, and we, therefore, continue to rely on this dataset throughout the thesis.

To adjust monthly data, we multiply the mean of monthly observations by 12 and the monthly standard deviation by $\sqrt{12}$ to annualize the data where needed. This data adjustment relies on the time series being independent and identically distributed (IID), which is further elaborated on in the Appendix.

Alaska Permanent Fund Data

To evaluate the performance of the Alaska Permanent Fund, we have collected quarterly data published in the fund's annual reports and created a dataset with the quarterly return, benchmark return, and active return data for the equity portfolio of the fund (Alaska Permanent Fund, 2022). This dataset contains 56 observations from Q3 2008 to Q2 2022.

To annualize the quarterly dataset, we multiply the average quarterly data by 4 and the quarterly standard deviation by $\sqrt{4}$. This data adjustment relies on the time series being independent and identically distributed (IID), which is further elaborated on in the Appendix.

Korea Investment Corporation Data

To evaluate the performance of the Korea Investment Corporation, we have collected annual data from 2017 to 2021 published by the fund (Korea Investment Corporation, 2022). The dataset contains annual return, annual benchmark return, and annual active return for the fund with 5 observations.

Caisse de dépôt et placement du Quebec Data

To evaluate the performance of the CDPQ, we have collected annual data from 2004 to 2021 published by the fund (CDPQ, 2022). The dataset contains annual return, annual benchmark return, and annual active return for the fund with 18 observations.

5.2.2 Data Treatment for Synthetic Portfolio Construction

As presented in Chapter 4, we construct three synthetic portfolios each demanding specific data treatment. This data treatment is presented in the following sections below.

Portfolio 1 – Active Return

To calculate the lambda values presented in Equation 9 for our first synthetic portfolio, we rely on the active return of the respective funds. Our dataset contains monthly observations of active returns both for the GPFG and the selected fund. As presented in the Methodology, we calculate the weight in the selected fund denoted λ_t^{AR}, by leveraging the weighted average of the funds' respective active return. Our objective with this portfolio construction is to allocate the highest weight λ_t^{AR} to the fund with the highest active return for each period t.

Our first step in the data treatment for Portfolio 1 is to calculate the average active returns for both funds through a 24-month moving average window. This is to create less volatile values λ_t^{AR} that are more feasible in practice. Further, we make a weighted average of the moving-average active returns calculated for both funds using Equation 10.

In line with our presented methodology, we limit λ_t^{AR} to the range between 0% and 100%. A weighted average calculation will deliver lambda values outside this limited interval, because of observations with a negative active return. We solve this by using the absolute value of the active returns in our dataset when calculating the weighted average. However, for the observations with a negative active return, the lambda calculation using absolute values will reward a fund with a great negative value compared to a fund with a lower positive value. This contradicts our objective of allocating the highest weight to the best-performing fund and must be addressed. We therefore manually evaluate all observations that have negative values of active return and assess whether the respective weights calculated by absolute value for the two funds should be shifted.

To ensure clarity in the approach of our weight calculation, we illustrate with a concrete example from our dataset. For a given observation, the average active return for the selected fund and the GPFG is -0.17% and 0.003% respectively. By leveraging absolute values, the weight allocated to the selected fund and the GPFG is 98.13% and 1.87% respectively. This contradicts our objective to allocate the highest weight to the best-performing fund. Therefore, we shift the respective weights between the two funds, so that the selected fund is denoted a weight of 1.87% and the GPFG is denoted a weight of 98.13%.

By leveraging the approach presented above, we ensure that we allocate the highest weight to the fund with the highest active return for all observations in the dataset. We perform this manual adjustment for 36 of a total of 225 observations (~16%) in our dataset.

When the weights have been calculated, we use Equation 9 and calculate the active return of Portfolio 1 for each observation t. Figure 3 presents an extract of our dataset for Portfolio 1.

Figure 3 – Portfolio 1: Extract from Dataset

Month	Active Return Selected Fund	Active Return GPFG	24-Month Moving-Average Return Selected Fund	24-Month Moving-Average Return GPFG	Lambda	1-Lambda	Active Return Portfolio 1
30.09.2003	0,00 %	0,07 %	0,02 %	0,07 %	20,27 %	79,73 %	0,05 %
31.10.2003	-0,35 %	0,15 %	0,03 %	0,06 %	33,85 %	66,15 %	-0,02 %
30.11.2003	0,49 %	-0,06 %	0,05 %	0,05 %	47,91 %	52,09 %	0,20 %
31.12.2003	-0,19 %	-0,12 %	0,03 %	0,08 %	24,75 %	75,25 %	-0,14 %
31.01.2004	-0,17 %	0,13 %	0,03 %	0,11 %	19,18 %	80,82 %	0,07 %
29.02.2004	-0,03 %	0,05 %	0,05 %	0,12 %	28,19 %	71,81 %	0,03 %
31.03.2004	0,08 %	0,27 %	0,04 %	0,11 %	25,25 %	74,75 %	0,22 %
30.04.2004	-0,01 %	0,00 %	0,04 %	0,10 %	26,41 %	73,59 %	0,00 %
31.05.2004	-0,08 %	-0,08 %	0,07 %	0,10 %	41,59 %	58,41 %	-0,08 %
30.06.2004	-0,10 %	0,11 %	0,07 %	0,10 %	42,86 %	57,14 %	0,02 %
31.07.2004	-0,04 %	-0,24 %	0,07 %	0,09 %	44,72 %	55,28 %	-0,15 %
31.08.2004	-0,01 %	-0,10 %	0,07 %	0,07 %	47,10 %	52,90 %	-0,06 %
30.09.2004	0,73 %	0,21 %	0,05 %	0,08 %	36,37 %	63,63 %	0,40 %
31.10.2004	-0,20 %	-0,03 %	0,00 %	0,07 %	5,79 %	94,21 %	-0,04 %
30.11.2004	0,27 %	0,20 %	0,00 %	0,08 %	1,55 %	98,45 %	0,20 %
31.12.2004	0,06 %	0,22 %	-0,02 %	0,08 %	18,55 %	81,45 %	0,19 %
31.01.2005	0,27 %	0,26 %	0,00 %	0,08 %	0,00 %	100,00 %	0,26 %
28.02.2005	0,42 %	-0,02 %	-0,02 %	0,07 %	22,74 %	77,26 %	0,08 %
31.03.2005	0,01 %	-0,18 %	-0,04 %	0,07 %	33,00 %	67,00 %	-0,12 %
30.04.2005	-0,96 %	-0,17 %	-0,02 %	0,08 %	19,80 %	80,20 %	-0,33 %
31.05.2005	0,08 %	0,30 %	0,03 %	0,09 %	25,48 %	74,52 %	0,24 %
30.06.2005	-0,36 %	0,28 %	0,03 %	0,09 %	27,31 %	72,69 %	0,10 %
31.07.2005	-0,06 %	0,18 %	0,11 %	0,09 %	55,97 %	44,03 %	0,05 %
31.08.2005	0,55 %	0,15 %	0,12 %	0,10 %	55,99 %	44,01 %	0,38 %
30.09.2005	0,37 %	0,00 %	0,04 %	0,07 %	33,70 %	66,30 %	0,12 %
31.10.2005	0,03 %	-0,11 %	0,00 %	0,09 %	4,66 %	95,34 %	-0,10 %
30.11.2005	-0,05 %	0,54 %	0,00 %	0,10 %	4,36 %	95,64 %	0,52 %
31.12.2005	-0,19 %	0,60 %	0,00 %	0,07 %	0,60 %	99,40 %	0,59 %

Portfolio 2 – Risk-Adjusted Return

For our second constructed portfolio, we leverage risk-adjusted return when calculating the weights presented in Equation 9. We have a dataset with monthly observations of active returns for both the GPFG and the selected fund, and again, we rely on a 24-month window to calculate a moving average for active returns. Additionally, we calculate the standard deviation of active returns in the corresponding window. Then, we calculate the active return per unit risk by dividing the average active return by the standard deviation for the moving-average window. Our objective with this portfolio construction is to allocate the highest weight λ_t^{RA} to the fund with the highest risk-adjusted active return.

Furthermore, our second step will be to calculate the weighted average between the risk-adjusted return for each fund. Similarly to Portfolio 1, the challenge with negative values appears. We solve this in the same manner as for Portfolio 1. We leverage the absolute values in the calculation of the weighted average, and then manually evaluate all observations with a negative value of active return. For these observations, we shift the weights so that the highest weight will be allocated to the fund with the highest active return per unit risk. We perform this manual adjustment for 34 of a total of 225 observations (~15%) in our dataset.

After the calculation of weights have been performed, we use Equation 9 and calculate the active return of Portfolio 2 for each observation t. Figure 4 provides an extract of the dataset for Portfolio 2.

Figure 4 - Portfolio 2: Extract from Dataset

Month	Active Return Selected Fund	Active Return GPFG	24-Month Moving-Average Return Selected Fund	24-Month Moving-Average Return GPFG	Standard Deviation Selected Fund	Standard Deviation GPFG	Risk-Adjusted Return Selected Fund	Risk-Adjusted Return GPFG	Lambda	1-Lambda	Active Return Portfolio 2
30.09.2003	0,00 %	0,07 %	0,02 %	0,07 %	0,34 %	0,16 %	0,05	0,40	10,75 %	89,25 %	0,06 %
31.10.2003	-0,35 %	0,15 %	0,03 %	0,06 %	0,35 %	0,16 %	0,09	0,38	19,23 %	80,77 %	0,05 %
30.11.2003	0,49 %	-0,06 %	0,05 %	0,05 %	0,34 %	0,17 %	0,14	0,31	30,87 %	69,13 %	0,11 %
31.12.2003	-0,19 %	-0,12 %	0,03 %	0,08 %	0,33 %	0,19 %	0,08	0,40	16,10 %	83,90 %	-0,13 %
31.01.2004	-0,17 %	0,13 %	0,03 %	0,11 %	0,33 %	0,21 %	0,08	0,50	13,38 %	86,62 %	0,09 %
29.02.2004	-0,03 %	0,05 %	0,05 %	0,12 %	0,33 %	0,22 %	0,14	0,53	20,86 %	79,14 %	0,03 %
31.03.2004	0,08 %	0,27 %	0,04 %	0,11 %	0,34 %	0,24 %	0,11	0,45	19,02 %	80,98 %	0,23 %
30.04.2004	-0,01 %	0,00 %	0,04 %	0,10 %	0,34 %	0,23 %	0,11	0,43	19,80 %	80,20 %	0,00 %
31.05.2004	-0,08 %	-0,08 %	0,07 %	0,10 %	0,38 %	0,23 %	0,19	0,45	30,28 %	69,72 %	-0,08 %
30.06.2004	-0,10 %	0,11 %	0,07 %	0,10 %	0,38 %	0,24 %	0,19	0,40	32,23 %	67,77 %	0,04 %
31.07.2004	-0,04 %	-0,24 %	0,07 %	0,09 %	0,38 %	0,25 %	0,18	0,35	34,20 %	65,80 %	-0,17 %
31.08.2004	-0,01 %	-0,10 %	0,07 %	0,07 %	0,38 %	0,27 %	0,17	0,27	38,47 %	61,53 %	-0,07 %
30.09.2004	0,73 %	0,21 %	0,05 %	0,08 %	0,40 %	0,27 %	0,11	0,30	27,63 %	72,37 %	0,35 %
31.10.2004	-0,20 %	-0,03 %	0,00 %	0,07 %	0,38 %	0,27 %	0,01	0,28	4,16 %	95,84 %	-0,04 %
30.11.2004	0,27 %	0,20 %	0,00 %	0,08 %	0,38 %	0,26 %	0,00	0,30	1,09 %	98,91 %	0,20 %
31.12.2004	0,06 %	0,22 %	-0,02 %	0,08 %	0,38 %	0,27 %	-0,05	0,31	13,93 %	86,07 %	0,20 %
31.01.2005	0,27 %	0,26 %	0,00 %	0,08 %	0,39 %	0,27 %	0,00	0,28	0,00 %	100,00 %	0,26 %
28.02.2005	0,42 %	-0,02 %	-0,02 %	0,07 %	0,39 %	0,26 %	-0,05	0,26	16,59 %	83,41 %	0,06 %
31.03.2005	0,01 %	-0,18 %	-0,04 %	0,07 %	0,38 %	0,26 %	-0,09	0,27	25,46 %	74,54 %	-0,13 %
30.04.2005	-0,96 %	-0,17 %	-0,02 %	0,08 %	0,39 %	0,26 %	-0,05	0,30	14,08 %	85,92 %	-0,28 %
31.05.2005	0,08 %	0,30 %	0,03 %	0,09 %	0,34 %	0,25 %	0,09	0,37	20,50 %	79,50 %	0,25 %
30.06.2005	-0,36 %	0,28 %	0,03 %	0,09 %	0,34 %	0,25 %	0,10	0,36	21,95 %	78,05 %	0,14 %
31.07.2005	-0,06 %	0,18 %	0,11 %	0,09 %	0,44 %	0,25 %	0,25	0,35	41,91 %	58,09 %	0,08 %
31.08.2005	0,55 %	0,15 %	0,12 %	0,10 %	0,44 %	0,26 %	0,28	0,38	42,69 %	57,31 %	0,32 %
30.09.2005	0,37 %	0,00 %	0,04 %	0,07 %	0,55 %	0,28 %	0,07	0,26	20,70 %	79,30 %	0,08 %
31.10.2005	0,03 %	-0,11 %	0,00 %	0,09 %	0,55 %	0,29 %	0,01	0,30	2,47 %	97,53 %	-0,10 %
30.11.2005	-0,05 %	0,54 %	0,00 %	0,10 %	0,55 %	0,28 %	0,01	0,35	2,30 %	97,70 %	0,53 %
31.12.2005	-0,19 %	0,60 %	0,00 %	0,07 %	0,55 %	0,28 %	0,00	0,25	0,30 %	99,70 %	0,59 %

Portfolio 3 – Predictive Quality

To calculate the weights for Portfolio 3, we rely on the predictive quality of each fund. As presented in the Methodology, the calculation λ_t^{PQ} will be based on a weighted average of the inverted MSPE. Our objective is to allocate the highest weight λ_t^{PQ} to the fund with the best predictive quality.

Our first step will be to perform a pseudo-out-of-sample analysis, using ARIMAs. We reserve an adequate number of observations to create satisfactory ARIMAs, and further predict the subsequent observations. We compare the predicted return to the realized active return and calculate the MSPE denoted in Equation 12. We conduct these predictions for each monthly observation in the dataset, evaluating the optimal ARIMA through the Box-Jenkins Method every second year (every 24 observations).

We then calculate the weights by using a weighted average between the funds' respective inverted MSPEs. By using the inverted MSPE, we will not encounter the challenge of negative values. Figure 5 presents an extraction of the dataset for Portfolio 3.

Figure 5 - Portfolio 3: Extract from Dataset

Month	Active Return Selected Fund	Active Return GPFG	MSPE Selected Fund	MSPE GPFG	24-Month Moving-Average MSPE Selected Fund	24-Month Moving-Average GPFG	Inverted 24-Month Moving-Average MSPE Selected Fund	Inverted 24-Month Moving-Average MSPE GPFG	Lambda	1-Lambda	Active Return Portfolio 3
31.01.2008	-0,26 %	-0,02 %	0,0035 %	0,0000 %	0,0047 %	0,0013 %	21 258,9	78 954,5	21,21 %	78,79 %	-0,07 %
28.02.2008	0,44 %	0,06 %	0,0021 %	0,0000 %	0,0046 %	0,0013 %	21 740,0	78 714,6	21,64 %	78,36 %	0,14 %
31.03.2008	-0,33 %	-0,31 %	0,0021 %	0,0015 %	0,0045 %	0,0013 %	22 156,7	78 647,9	21,98 %	78,02 %	-0,32 %
31.04.2008	-0,23 %	0,24 %	0,0001 %	0,0003 %	0,0045 %	0,0012 %	22 368,5	82 530,4	21,32 %	78,68 %	0,14 %
31.05.2008	0,36 %	0,20 %	0,0005 %	0,0002 %	0,0045 %	0,0012 %	22 140,8	83 470,9	20,96 %	79,04 %	0,23 %
30.06.2008	1,30 %	-0,13 %	0,0144 %	0,0004 %	0,0045 %	0,0012 %	22 234,8	83 711,2	20,99 %	79,01 %	0,17 %
31.07.2008	-0,17 %	-0,15 %	0,0002 %	0,0005 %	0,0040 %	0,0012 %	24 802,0	84 451,6	22,70 %	77,30 %	-0,15 %
31.08.2008	-0,76 %	-0,05 %	0,0021 %	0,0001 %	0,0040 %	0,0012 %	24 694,1	85 741,8	22,36 %	77,64 %	-0,21 %
30.09.2008	-0,87 %	-1,39 %	0,0053 %	0,0209 %	0,0041 %	0,0012 %	24 346,8	85 917,4	22,08 %	77,92 %	-1,27 %
31.10.2008	0,39 %	-0,45 %	0,0013 %	0,0023 %	0,0039 %	0,0003 %	25 563,2	331 494,4	7,16 %	92,84 %	-0,39 %
30.11.2008	-0,46 %	0,01 %	0,0032 %	0,0000 %	0,0039 %	0,0002 %	25 638,1	486 990,1	5,00 %	95,00 %	-0,02 %
31.12.2008	-0,61 %	0,47 %	0,0039 %	0,0022 %	0,0038 %	0,0002 %	26 533,0	487 837,9	5,16 %	94,84 %	0,41 %
31.01.2009	1,06 %	0,23 %	0,0038 %	0,0004 %	0,0036 %	0,0001 %	27 540,4	860 565,4	3,10 %	96,90 %	0,25 %
28.02.2009	0,32 %	-0,03 %	0,0001 %	0,0001 %	0,0035 %	0,0001 %	28 358,4	986 340,7	2,79 %	97,21 %	-0,02 %
31.03.2009	-1,98 %	0,15 %	0,0220 %	0,0001 %	0,0037 %	0,0001 %	26 806,7	1 015 591,9	2,57 %	97,43 %	0,10 %
30.04.2009	-2,46 %	0,26 %	0,0348 %	0,0005 %	0,0029 %	0,0001 %	34 901,9	1 065 979,2	3,17 %	96,83 %	0,17 %
31.05.2009	0,17 %	0,30 %	0,0001 %	0,0006 %	0,0014 %	0,0001 %	70 255,8	1 216 220,6	5,46 %	94,54 %	0,29 %
30.06.2009	0,22 %	0,08 %	0,0022 %	0,0000 %	0,0022 %	0,0001 %	46 096,9	1 800 518,6	2,50 %	97,50 %	0,09 %
31.07.2009	0,24 %	0,02 %	0,0004 %	0,0000 %	0,0021 %	0,0001 %	47 872,5	1 618 460,2	2,87 %	97,13 %	0,03 %
31.08.2009	-0,75 %	0,12 %	0,0062 %	0,0000 %	0,0023 %	0,0001 %	43 222,1	1 573 354,0	2,67 %	97,33 %	0,09 %
30.09.2009	-0,13 %	0,08 %	0,0001 %	0,0000 %	0,0022 %	0,0001 %	46 435,8	1 200 653,7	3,72 %	96,28 %	0,07 %
31.10.2009	0,86 %	-0,02 %	0,0044 %	0,0000 %	0,0024 %	0,0001 %	41 972,8	723 411,1	5,48 %	94,52 %	0,03 %
30.11.2009	0,24 %	0,09 %	0,0000 %	0,0000 %	0,0022 %	0,0001 %	45 486,8	690 117,0	6,18 %	93,82 %	0,10 %
31.12.2009	-0,23 %	0,18 %	0,0001 %	0,0002 %	0,0027 %	0,0001 %	36 790,6	675 539,4	5,16 %	94,84 %	0,16 %

6 Historical Analysis of the GPFG: Fund Performance and Active Management

This chapter is dedicated to performing the Historical Analysis of the GPFG for answering the first part of our research question on how the active management and accompanying active returns of the GPFG have been historical. Our point of departure is a historical performance evaluation, intending to evaluate the fund's historical performance through several key performance measures for return and risk. We rely on arithmetically calculated returns, standard deviation, and sharpe ratio for both the portfolio and the benchmark of the GPFG.

Furthermore, we extend the analysis to include risk-adjusted active returns, through Jensen's Alpha and Five-Factor regressions. We also include the information and appraisal ratio. We will conduct the regressions and calculate both performance measures for three different periods, respectively 1998-2021, 2007-2021, and 2015-2021, excluding and including management costs. By including several periods in our analysis, we create nuances both related to market fluctuations and differences in the number of observations.

Lastly, we investigate the active management of the GPFG through the measures tracking error and active share. This is to establish how the active management has been historically and to create a basis of comparison for further analysis.

6.1 Return and Risk Performance Measures

In this section, we aim to present an overview of the GPFG's equity portfolio performance. We calculate the arithmetical average, standard deviation, and sharpe ratio, for the portfolio return, the benchmark return, and the active return, both excluding and including management costs. As emphasized, these measures are presented for three different time periods, namely 1998-2021, 2007-2021, and 2015-2021.

In Table 4, we present summary statistics for the equity portfolio's performance, with the performance measures outlined above.

Table 4 - Total Risk and Return

Full Sample (1998-2021)

	Portfolio	Benchmark	Active	
			Excluding Costs	Including Costs
Arit. mean	8.70%	8.26%	0.44%	0.36%
St. Deviation	16.58%	16.30%	0.73%	0.73%
Sharpe Ratio	0.42	0.40		
N	287	287	287	287

Second Sample (2007-2021)

	Portfolio	Benchmark	Active	
			Excluding Costs	Including Costs
Arit. mean	8.18%	7.88%	0.29%	0.22%
St. Deviation	17.53%	17.20%	0.62%	0.62%
Sharpe Ratio	0.42	0.41		
N	180	180	180	180

Last Sample (2015-2021)

	Portfolio	Benchmark	Active	
			Excluding Costs	Including Costs
Arit. mean	11.07%	10.73%	0.33%	0.28%
St. Deviation	14.65%	14.46%	0.41%	0.41%
Sharpe Ratio	0.70	0.69		
N	84	84	84	84

Note: All numbers are annualized. The columns “excluding costs” and “including costs” represent the active return before and after management costs are included, respectively.

We find that the annualized arithmetical portfolio return is 8.70%, 8.18%, and 11.07% for the respective periods. After subtracting the benchmark, we find active returns of 0.44%, 0.29%, and 0.33% excluding management costs, and 0.36%, 0.22%, and 0.28% including management costs. Furthermore, we find that the risk-adjusted sharpe ratio for the portfolio is higher compared to the benchmark.

From Table 4, we further find that the last sample period (2015-2021) delivers the highest portfolio and benchmark return, with the lowest corresponding standard deviation. This is also reflected in the high sharpe ratio calculated for this period. Furthermore, the last sample period has a lower active return compared to the full sample period, while delivering higher active returns than the second sample period. Thus, the 1998-2021 time period possesses the highest active returns of the evaluated samples.

The findings presented in this section provide an overview of the GPFG's performance, both in terms of portfolio, benchmark, and active returns. We find that the annualized active return for the full sample period (since inception) is 0.44% and 0.36% respectively excluding and including costs. As initially stated, our research question revolves around the active returns of the GPFG, and the subsequent sections will therefore provide a more in-depth exploration of the active returns presented above.

6.2 Evaluation of Active Returns

In this section, our focus lies on the evaluation of the active returns presented above in 6.1 Return and Risk Performance Measures. We aim to understand the significance of active returns and how these returns are affected by risk factors. The first step conducted is estimating Jensen's Alpha, before further emphasizing risk factors in the financial markets that could affect active returns, using the Fama French Five-Factor model. These two analyses will be complementary in assessing the active returns of the GPFG. Additionally, we extend the analysis to investigate whether the active returns are a consequence of skill or luck.

6.2.1 Jensen's Alpha Estimation

We estimate Jensen's Alpha by utilizing CAPM, to assess the active return adjusted for systematic risk. We regress the excess return[4] of the portfolio on the excess return of the benchmark to find an estimate for α and β. The estimated intercept α is the average contribution of active management after adjusting for beta risk.

Table 5 – Beta-Adjusted Active Returns

	1998-2021		2007-2021		2015-2021	
	Excluding Costs	Including Costs	Excluding Costs	Including Costs	Excluding Costs	Including Costs
Constant α	0.0034**	0.0026*	0.0016	0.00091	0.0021	0.0016
	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)	(0.0004)
p-value	0.017	0.068	0.24	0.51	0.142	0.267
β	1.016***	1.016***	1.018***	1.018***	1.012***	1.012***
	(0.0086)	(0.0086)	(0.008)	(0.008)	(0.0096)	(0.0096)
p-value	0.000	0.000	0.000	0.000	0.000	0.000
N	287		180		84	
R-squared	0.998	0.998	0.999	0.999	0.999	0.998
AR	0.47	0.36	0.21	0.12	0.43	0.33
IR	0.60	0.50	0.47	0.36	0.82	0.69

Standard errors are in parentheses

**** p<.01, ** p<.05, * p<.1*

Notes: The table shows annualized α and β using the regression in Equation 1. Returns are expressed in decimal numbers. The columns "excluding costs" and "including costs" represent the active return before and after management costs are included, respectively.

Table 5 describes the results of regressing the excess return of the portfolio on the excess return of the benchmark, to estimate α and β. We find that the constant α for the full sample period excluding management costs at 0.34% is significantly different from zero on the 5% level. The active return including management costs for the full sample period at 0.26% is significant on the 10% level. Both risk-adjusted active returns are lower than the average active return we computed in 6.1 Return and Risk Performance Measures. As the β is higher than 1 and the

[4] As emphasized in the Methodology, the excess return refers to the portfolio return above the risk-free rate.

benchmark return is higher than the risk-free rate, it is to be expected that α is below the initial estimation.

Furthermore, the AR is positive with an estimated value of 0.47 being lower than the estimated IR value of 0.60 for the full sample period. The calculated IR indicates that the GPFG is to some extent actively managed, as the IR has a value above zero.

6.2.2 Five-Factor Model Regressions

To further evaluate the historical alpha and account for risk factors, we use the Fama French Five-Factor model and perform a regression analysis with active return as the dependent variable and the five factors denoted in the model as explanatory variables. The Fama French Factors capture structural trends in the market, and by including these factors we can be more confident in the results of the estimated constant α. We are using the Five Factor model on the reduced form without any income factors.

Table 6 presents the results from the regression analysis. We perform three regressions for different periods to understand if the α is more or less prominent across different periods and with a different number of observations.

Table 6 - Regression Analysis of Active Return

	1998-2021		**2007-2021**		**2015-2021**	
	Excluding Costs	Including Costs	Excluding Costs	Including Costs	Excluding Costs	Including Costs
Constant α	0.0034***	0.0026**	0.0023*	0.0016	0.0028**	0.0023*
	(0.00376)	(0.00376)	(0.00392)	(0.00392)	(0.00037)	(0.00037)
p-value	0.009	0.043	0.093	0.246	0.029	0.071
MKT-RF	0.1531***	0.1534***	0.1575***	0.1579***	0.0824**	0.0825**
	(0.0096)	(0.0096)	(0.0093)	(0.0093)	(0.0094)	(0.0094)
p-value	0.000	0.000	0.000	0.000	0.013	0.013
SMB	0.4992***	0.4971***	0.3323***	0.3291***	0.3043***	0.3036***
	(0.0193)	(0.0193)	(0.0261)	(0.0261)	(0.0259)	(0.0259)
p-value	0.000	0.000	0.000	0.000	0.001	0.001
HML	-0.0458	-0.0472	0.1702**	0.1693**	0.2782***	0.2786***

	(0.0205)	(0.0205)	(0.0246)	(0.0246)	(0.0243)	(0.0243)
p-value	0.52	0.506	0.047	0.048	0.001	0.001
RMW	0.0514	0.0513	0.0337	0.0314	0.0779	0.0784
	(0.0255)	(0.0255)	(0.0364)	(0.0364)	(0.0331)	(0.0331)
p-value	0.561	0.562	0.789	0.804	0.499	0.496
CMA	-0.3352***	-0.3346***	-0.5096***	-0.510***	-0.4165***	- 0.4162***
	(0.0305)	(0.0305)	(0.0348)	(0.0348)	(0.4049)	(0.4049)
p-value	0.002	0.002	0.000	0.000	0.004	0.004
Observations	287		180		84	
R-squared	0.3613	0.3615	0.4010	0.4012	0.4373	0.4374

Standard errors are in parentheses

****p<.01, **p<.05, *p<.1*

Notes: The table shows annualized α and β using the regression in Equation 6. Returns are expressed in decimal numbers. The columns "excluding costs" and "including costs" represent the active return before and after management costs are included, respectively.

From the regression analyses, we find that the alpha excluding and including costs are significantly different from zero with a value of 0.34% and 0.26% respectively for the full sample period. These are the same values of alpha presented in Table 5, indicating that adding risk factors does not impact the value of active returns beyond the benchmark for this period. However, we do find that the MKT-RF, SMB, and CMA factors all have a significant impact on active returns when only including risk factors in our regression.

For the second period (2007-2021), the alpha excluding costs is significant on the 10% level with a value of 0.23%, while the alpha including costs is not statistically different from zero. Both values differ from those presented in Table 5, indicating that adding risk factors have an impact on active returns and strengthen the alpha estimations. For the last period (2015-2021), we find an alpha of 0.28% excluding costs and 0.23% including costs. The alphas are significant on the 5% and 10% levels respectively. Also, these values differ from the beta-adjusted values, meaning that the risk-factors account for some effect on the active returns. For the last two time periods, the MKT-RF, SMB, HML, and CMA risk factors all have a significant impact on active returns.

6.2.3 Separating Skill and Luck

As presented in the Methodology, the last step in evaluating active returns is to emphasize whether the calculated returns derive from well-considered investment decisions or merely luck in the financial markets. To assess whether active returns are due to skill or coincidence/luck, we use a regular t-test testing for whether the true average active returns are equal to or different from zero.

Table 7 - One Sample T-Test Results

	1998-2021		**2007-2021**		**2015-2021**	
	Excluding Costs	Including Costs	Excluding Costs	Including Costs	Excluding Costs	Including Costs
Mean	0.0044***	0.0036**	0.0029*	0.0022	0.0033**	0.0028*
Standard error	(0.0004)	(0.0004)	(0.0005)	(0.0005)	(0.0004)	(0.0004)
t-value	2.9586	2.426	1.8395	1.39	2.163	1.829
p-value	0.0035	0.016	0.0675	0.166	0.0335	0.071
N	287		180		84	

**** p<.01, ** p<.05, * p<.1*

Notes: The columns "excluding costs" and "including costs" represent the active return before and after management costs are included, respectively.

Table 7 presents the results of the t-test. We find that the mean of active returns both excluding and including costs are significant on the 1% and 5% level for the full sample period with values of 0.44% and 0.36%. These results indicate that the active returns obtained are related to knowledge and skill, rather than luck. For the second sample period, we find that only the mean excluding costs at 0.29% is significant on the 10% level. For the last sample period, we find active return values of 0.33% and 0.28% excluding and including costs. The return before costs is significant on the 5% level, indicating a relation to knowledge and skill. Our t-test, therefore, finds that active returns predominantly have been a consequence of well-considered investment decisions.

6.3 Establishing Historical Degree of Active Management

After evaluating the historical performance of the GPFG, we will now take a glance at the degree of active management historically. This is essential to answer the first part of our research question on how active management and accompanying returns for the GPFG have been historically. For this purpose, we calculate the tracking error and active share through the presented methodology and aim to establish the historical degree of active management.

6.3.1 Tracking Error

To evaluate the degree of historical active management of the GPFG's equities, we have analyzed the historical tracking error of the fund, as outlined in Equation 7, by calculating the standard deviation of the active returns. Figure 6 illustrates the tracking error for the equity portfolio from 1998 to 2021 based on our calculations and the tracking error limit set by the Ministry of Finance[5].

Figure 6 - Historical Tracking Error of the fund

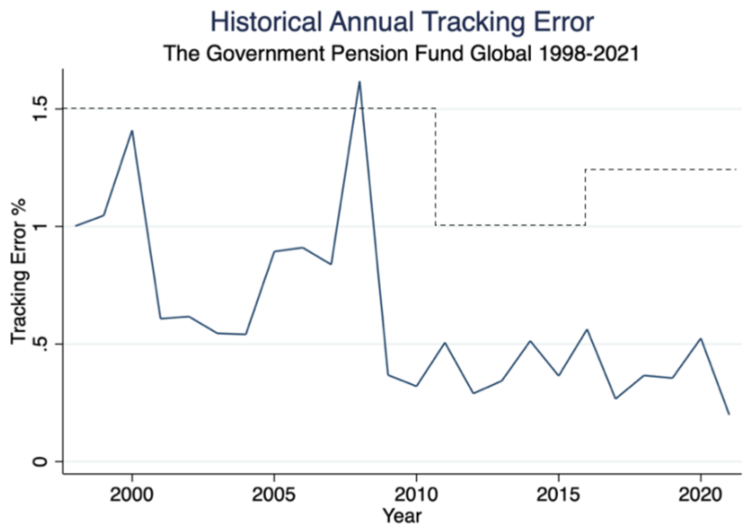


Figure 6 demonstrates a fluctuating tracking error since inception in 1998, and a general decrease in the volatilities during the latest years. The equity portfolio of the fund does not exploit the tracking error limit, except for 2008. Generally, the fund has operated with an annual tracking error between 0.20% and 1.62%. The average annual tracking error since inception is 0.63%. A low tracking error during the years at scope implies limited deviations from the benchmark returns and that the fund on average has not exploited its tracking error limit.

[5] As illustrated in Figure 6, the tracking error limit has ranged from 1% to 1.5% since inception.

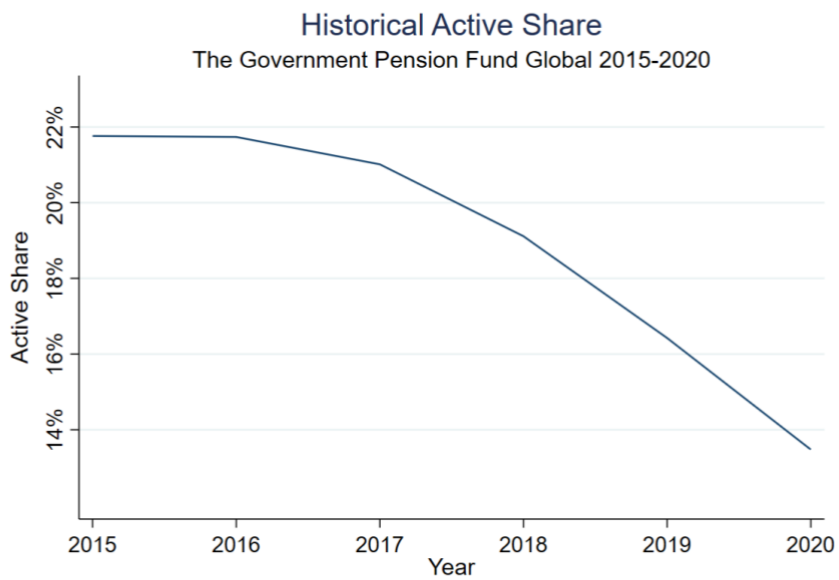
The reduced volatilities in recent years can according to Ødegaard & Dahlquist (2018) be explained by the reduction in the world's equity markets, which reduces tracking error for any given benchmark deviation. Further, equity portfolios have in recent years also moved closer to their benchmarks and the lower tracking error might be explained by a lower active risk-taking of the GPFG (Ødegaard & Dahlquist, 2018) & (Døskeland, Bauer & Christiansen, 2022).

From Figure 6, one can further observe a spike in the tracking error of the equity portfolio in 2008. This can be linked to the financial crisis in 2008, where the GPFG bought equities for 160 billion euros between 2007 and 2009, the majority during the crisis. At the same time, the Ministry of Finance decided to change the equity share of the fund from 40% to 60% (Sparre, 2012).

6.3.2 Active Share

To further evaluate the degree of historical active management of the GPFG's equities, we have calculated the historical active share of the equity portfolio. For this purpose, we use Equation 8 presented in the Methodology. Figure 7 plots the active share for the equity portfolio from 2015 to 2020 based on our data treatment and further calculations.

Figure 7 - Historical Active Share of the Fund



The calculation of weights for each equity holding in the benchmark are as emphasized in 3.3 Establishing Historical Degree of Active management adjusted in line with the strategic benchmark changes of The Ministry of Finance, thus, adjusted for geographical affiliation and exclusions. When using our calculated weights and the formula of active share presented in Equation 8, we find that the GPFG's active share has been in the range of 13.48% to 21.76% during the years investigated, with an average active share of 18.92%. Additionally, our data suggest a decrease in active share since 2015, indicating that the fund has been less active in the time period investigated. The reduction in active share is especially clear in 2019 and 2020, respectively with an active share of 16.43% and 13.48% of the equity portfolio.

To compare our findings, the GPFG reports the degree of overlap with the benchmark based on individual stocks. In their reporting, a 100% overlap would indicate that the portfolio of the fund is identical to the benchmark, thus a 0% active share. For the equity portfolio, the overlap of the GPFG has been in the range of 80-85%, which implies an active share between 15-20%. Thus, we can observe that our data demonstrate a similar range, with minor deviation. This deviation can be a result of our assumptions in the analysis of active share, our constructed benchmark, or unmatched holdings which we assume to be active positions.

6.4 Summary of the Historical Analysis of the GPFG

In the Historical Analysis of the GPFG, we have analysed the fund's historical performance. We provide an overview of the GPFG's performance, both in terms of portfolio, benchmark, and active returns. Initially, we find that the annualized active return for the full sample period (since inception) is 0.44% and 0.36% respectively when excluding and including costs.

In the Evaluation of Active Returns, we further investigate these returns and find that the benchmark risk-adjusted alpha is only significant for the full sample period with a value of 0.34% and 0.26% excluding and including costs respectively. When evaluating the factor risk-adjusted alpha, these values still hold with increased significance. The factor risk-adjusted alpha for the 2007-2021 period is only significant excluding costs with a value of 0.23%, while the risk-adjusted alpha for the 2015-2021 period is significant both when excluding and including costs. These results indicate that active returns predominantly have been significant throughout the investigated periods and that active management has created additional return for the fund. Nevertheless, all estimated returns for active management are relatively small compared to the fund's total equity value creation.

In Establishing Historical Degree of Active Management, we find a fluctuating tracking error during the years investigated, however, with a declining tracking error since inception. The average tracking error since inception is 0.63%, thus being substantially lower than the current tracking error limit of 1.25%. Our data further suggest a reduction in active share since 2015, with an average active share of 18.92% from 2015 to 2020, indicating that the GPFG since 2015 has invested less actively. Our calculated active share further implies that the fund is close to being defined as an index fund during all years investigated, by Cremers & Petajisto's (2009) definition.

As initially presented, the Historical Analysis of the GPFG aims to answer the first part of our research question regarding how active management and accompanying active returns have been historical. In summary, this chapter establishes the historical degree of active management and provides evidence that returns yielded from active management have been significant. These results lay the foundation for the subsequent Scenario-Analysis, where we aim to create synthetic portfolios with a higher degree of active management than the GPFG.

7 Scenario-Analysis: Potential Active Returns of the GPFG

This chapter of the thesis is dedicated to analyzing potential active returns for the GPFG, through creating synthetic portfolios with a higher degree of active management. More specifically, we rely on historical active return data from other funds and create three synthetic portfolios. As emphasized in Chapter 5, we create portfolios based on active return, risk-adjusted return, and predictive quality. The analysis represents an ex-post experiment for answering the second part of our research question of how increased active management can impact the GPFG's active returns.

Our point of departure is a Fund Selection, where we examine other Sovereign Wealth Funds and compare their performance and active management with the GPFG. The aim is to select funds for constructing synthetic portfolios in combination with the GPFG. The extent of this performance and active management analysis will therefore be less comprehensive compared to the performance analysis conducted in Chapter 6. We will select two funds with a higher degree of active management for constructing the synthetic portfolios, one used for our main analysis, and one used for a robustness analysis.

Furthermore, we evaluate the significance of the historical active returns for the selected funds. We leverage the same approach as in Chapter 6 and use the Fama French Five-Factor model. We require significant active returns for the selected funds for a reliable analysis.

Lastly, we compare the synthetic portfolios with the GPFG's past and predicted active returns. A robustness analysis will be included to substantiate our findings.

7.1 Fund Selection

7.1.1 Performance & Active Management Comparison

Our first step is a comparison of the initially presented Sovereign Wealth Funds. We report portfolio return, active return, standard deviation, sharpe ratio, annual tracking error as well as the number of observations for the following funds: New Zealand Superannuation Fund, Alaska Permanent Fund, Korea Investment Corporation, and Caisse de dépôt et placement du Québec (CDPQ). Due to insufficient and inconsistent active return data for the funds, costs are excluded from the analysis.

Table 8 presents the mentioned measures above for all funds. The GPFG is included for comparison.

Table 8 - Fund Selection: Performance Measures

	GPFG	NZ SF	APF	KIC	CDPQ
Portfolio Return	8.70%	9.97%	8.28%	16.25%	9.95%
Active Return	0.44%	1.27%	0.90%	0.45%	0.29%
Standard Deviation	16.58%	9.81%	19.22%	14.94%	14.93%
Tracking Error	0.63%	1.87%	1.61%	1.47%	1.87%
Sharpe Ratio	0.42	0.9	0.41	1.02	0.59
N	287	225	56	5	18

Due to differences in reporting between funds, several delimitations must be considered before evaluating the results in Table 8. These delimitations include benchmark consideration, difference in transparency, different fiscal year endings, and reporting periods. The latter is leading to a difference in the number of observations between the funds. To provide an example, the GPFG reports monthly returns from 1998-2021, while the Alaska Permanent Fund reports quarterly data from 2008-2021. KIC and CDPQ further only report annual returns from 2017-2021 and 2003-2021 respectively. Thus, the data available for KIC and CDPQ is narrow and comparison with other funds based on a higher number of observations can be called into question. The inclusion of these funds will therefore be suboptimal, and we chose to proceed with the New Zealand Superannuation Fund and the Alaska Permanent Fund.

From Table 8 we find that the New Zealand Superannuation Fund provides the highest annual active return of 1.27%. We also discover that the fund has a relatively low standard deviation, indicating that the fund manages to create a high active return without taking much additional risk. The fund has a sharpe ratio of 0.9, which is approximately twice as high as the GPFG. With these characteristics, in addition to a high number of monthly observations, the New Zealand Superannuation Fund might be an interesting basis for comparison to answer the second part of our research question.

Furthermore, we observe that the Alaska Permanent Fund delivers a 0.90% annual active return and has a slightly higher risk profile than the GPFG, with a standard deviation of 19.22%. Even so, they report an active return almost twice as high as the GPFG. The fund also reports an almost matching sharpe ratio. We thus find that the GPFG and the Alaska Permanent Fund deliver a similar risk profile and relatively similar performances in the reporting period.

Additionally, when evaluating the tracking error, we observe a value of 1.87% and 1.61% for the New Zealand Superannuation Fund and the Alaska Permanent Fund respectively. To compare, the GPFG has an annual tracking error of 0.63% as calculated in Chapter 6. Given that both the New Zealand Superannuation Fund and the Alaska Permanent Fund have a higher tracking error compared to the GPFG, this could indicate that the funds are more actively managed.

However, the active share also needs to be addressed when analysing active management. As mentioned in the Context, New Zealand Superannuation Fund reports that most of the fund is managed passively and that two-thirds are invested in line with a reference portfolio. Therefore, we assume that one-third of the fund is actively managed. This assumption implies that the New Zealand Superannuation Fund is more actively managed than the GPFG. Further, when evaluating the Alaska Permanent Fund, the fund emphasizes that most of its equity portfolio is indeed actively managed. Hence, like the New Zealand Superannuation Fund, it is plausible to assume that the active share of the Alaska Permanent Fund is higher than the GPFG.

In terms of active management, both funds therefore provide value for answering the second part of our research question on how increased active management can impact active returns. Based on the difference in the number of observations between the funds, we select to proceed with the New Zealand Superannuation Fund as our main analysis. Given the less satisfactory number of observations for the Alaska Permanent Fund, this fund will represent a robustness analysis in evaluating our results.

7.1.2 Significance of Active Returns for Selected Funds

At this point, we have selected the New Zealand Superannuation Fund and the Alaska Permanent Fund for constructing synthetic portfolios to answer the second part of our research question. Before proceeding, we want to establish the factor risk-adjusted alpha for the funds and whether these are significant. The evaluation will be in line with our presented methodology, and we will perform the regression analysis both for the New Zealand Superannuation Fund and the Alaska Permanent Fund. As emphasized, we aim for significant alphas to ensure reliability in our synthetically constructed portfolios.

We leverage the Fama French Five-Factor model and perform a regression analysis. The regression results are presented in Table 9.

Table 9 - Factor Risk-Adjusted Alphas for New Zealand Superannuation Fund and Alaska Permanent Fund

	(1) New Zealand Superannuation Fund	(2) Alaska Permanent Fund
Constant α	0.012***	0.012**
	(0.000)	(0.002)
MKT-RF	-0.6***	-0.048
	(0.035)	(0.028)
SMB	0.468	0.724***
	(0.094)	(0.084)
HML	0.864***	0.112
	(0.09)	(0.07)
RMW	0.18	-0.004
	(0.132)	(0.1)

CMA	-0.216	0.148
	(0.132)	(0.096)
Observations	225	56
R-squared	.154	.389

Standard errors are in parentheses

**** p<.01, ** p<.05, * p<.1*

Note: We use monthly return data for the New Zealand Superannuation Fund and quarterly data for the Alaska Permanent Fund for the regression. The coefficients in this table are annualized.

From the regression analysis, we find that the factor risk-adjusted alpha without cost considerations for the New Zealand Superannuation Fund is significantly different from zero on the 1% significance level, with a value of 1.2%. We also find that the MKT-RF and HML factors have a significant impact on active returns. For the Alaska Permanent Fund, we find that the factor risk-adjusted alpha is significantly different from zero on the 5% level, with a similar value of 1.2%[6]. For the Alaska Permanent Fund, we find that the SMB factor has a significant impact on active returns.

Therefore, the regression analyses find that both funds have significant active returns in terms of factor risk-adjusted alpha, and we proceed with constructing synthetic portfolios with these two funds.

[6] Given that we have annualized the numbers, the alpha values are not identical, but they appear to be in the same range.

7.2 Comparison of Synthetic Portfolios and the GPFG

In this section, we aim to compare active returns from our synthetically constructed portfolios with active returns from the GPFG, to answer the second part of our research question on how increased active management can impact the GPFG's active returns. Followingly, we will present the constructed portfolios and compare them to the active returns of the GPFG. As earlier emphasized, we also include a prediction of future active returns in our analysis for all synthetically constructed portfolios. Hence, to establish a basis for comparison for future active returns between the constructed portfolios and the two funds, we need to select appropriate predictive models through ARIMA for both the GPFG and the New Zealand Superannuation Fund. We refer to the Appendix for these derivations.

7.2.1 Presentation of the Synthetic Portfolios

In the following, we investigate the returns of the three synthetically constructed portfolios denoted by Portfolio 1, Portfolio 2, and Portfolio 3. We present the cumulative active returns for each portfolio to evaluate its historical performance and compare it to the GPFG and the New Zealand Superannuation Fund. Additionally, we present the weighting λ_t used to create the portfolios and we further aim to predict the future active returns of each portfolio, to provide a basis for discussion of the GPFG's future active returns.

Portfolio 1 - Active Return

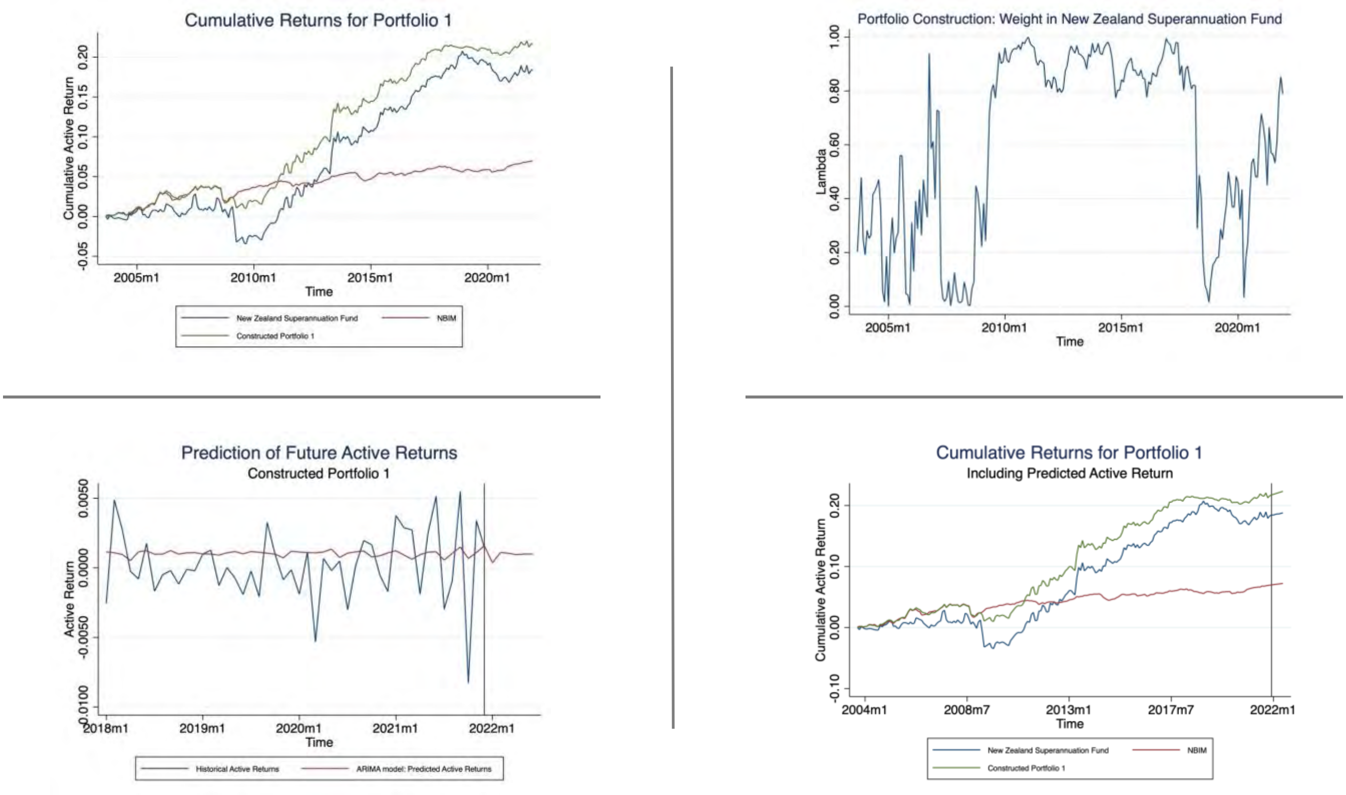
The first synthetic portfolio is created by leveraging the weighted average between the funds' respective active returns for each observation available in the dataset. We find that Portfolio 1 nearly continuously outperforms the New Zealand Superannuation Fund. We also discover that Portfolio 1 is closely related to the GPFG until 2010 while outperforming the GPFG in the following years, as hypothesized in Chapter 4.

Figure 8 also illustrates the weighting λ_t^{AR} used to create Portfolio 1. We observe that the portfolio is based on less volatile weights, due to the 24-month moving average window used to create the weighting. This ensures that the weights are more realistic in practice and make the estimations for Portfolio 1 more reliable.

Furthermore, we also want to predict the future performance of Portfolio 1 with a 6-month prediction horizon. In line with the Box-Jenkins methodology, we investigate whether the historical values of the synthetic portfolio are stationary, through an Augmented Dickey-Fuller test. The statistical test is outlined in the Appendix. The test finds evidence that the data is stationary, indicating that the statistical properties of the time series do not change, and followingly can be used to predict future returns of Portfolio 1. We leverage the Box-Jenkins Method to find the optimal ARIMA for future predictions and refer to the Appendix for this estimation.

When evaluating the Prediction of Future Active Returns in Figure 8 graphically, we observe that the estimated ARIMA has some similar fluctuations as the historical values in our dataset, however, the fit to historical values is limited. Even though we observe a somewhat limited fit of our ARIMA and the historical active return values, the selected ARIMA does not follow a random walk[7] and can therefore be used for investigating future active returns in our 6-month prediction horizon. Our prediction presented in Figure 8 indicates that the active returns for Portfolio 1 will continue a declining trend before stabilizing in the subsequent months.

Figure 8 - Portfolio 1



[7]The Wald-Wolfowitz Runs test is outlined in the Appendix and provides evidence that the ARIMA does not follow a random walk.

Portfolio 2 - Risk-Adjusted Return

Our second synthetic portfolio is constructed based on risk-adjusted return. We utilize the weighted average of each fund's active return per unit risk to construct Portfolio 2. Again, we present the cumulative historical active returns, portfolio weights, and future prediction for Portfolio 2 below in Figure 9.

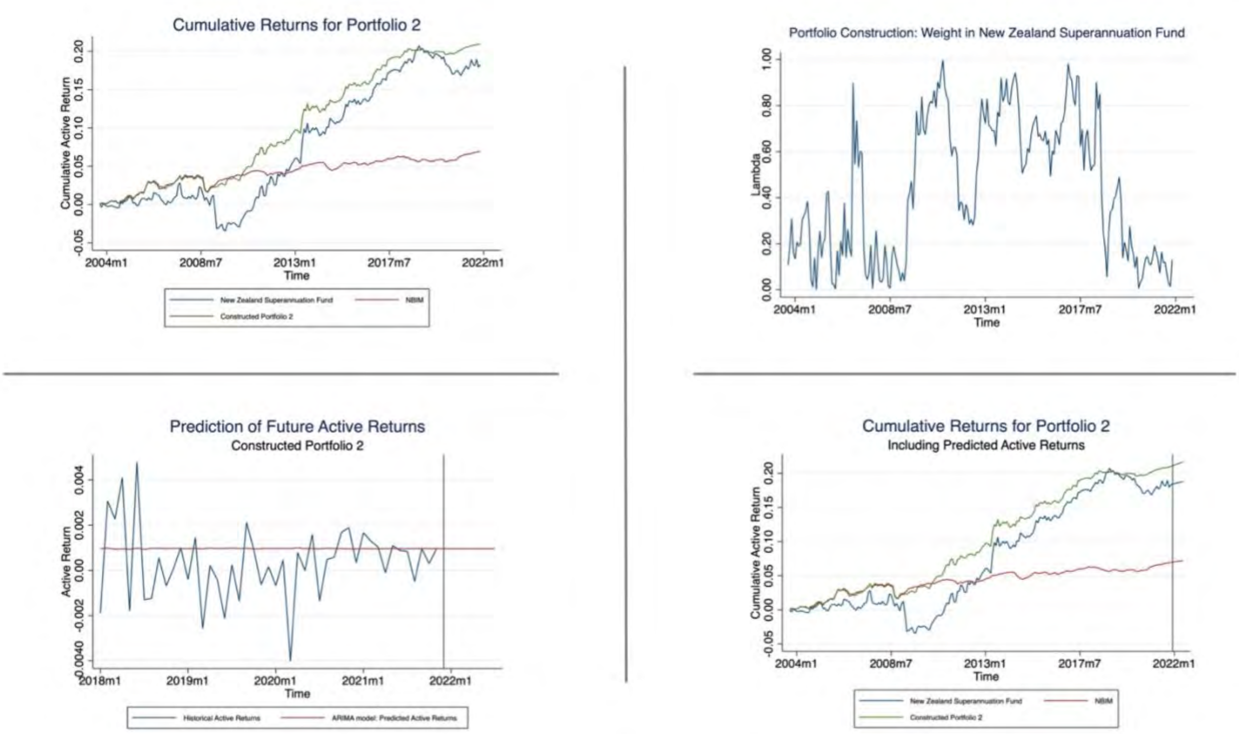
We find that Portfolio 2 is closely related to the GPFG until 2008 while demonstrating similar fluctuations as the New Zealand Superannuation Fund in the following years. Portfolio 2 roughly continuously outperforms the New Zealand Superannuation Fund, with an exception in 2018q1. After 2009, Portfolio 2 also continuously outperforms the GPFG.

Figure 9 illustrates the weighting λ_t^{RA} used to create Portfolio 2. As emphasized, the 24-month moving average is included when estimating the weights, to ensure smoothened lambda values. However, when comparing the lambda values of Portfolio 2 with the lambda values of Portfolio 1, we discover that the weights used to construct Portfolio 2 are more volatile.

Additionally, we aim to predict the future active returns of Portfolio 2. Similarly to Portfolio 1, we find evidence of stationarity in the historical data, indicating that these values can be used to predict future active returns for Portfolio 2[8]. When evaluating the estimated ARIMA[9] in Figure 9, we observe that the ARIMA resembles a straight line, indicating that the model is poorly fitted to historical values. This indicates that the scoped ARIMA does not provide substantial value for investigating future active returns in our 6-month prediction horizon.

[8] The Augmented Dickey Fuller test for Portfolio 2 is outlined in the Appendix.
[9] We find an optimal ARIMA (1,0,0) through the Box-Jenkins methodology.

Figure 9 - Portfolio 2



Portfolio 3 - Predictive Quality

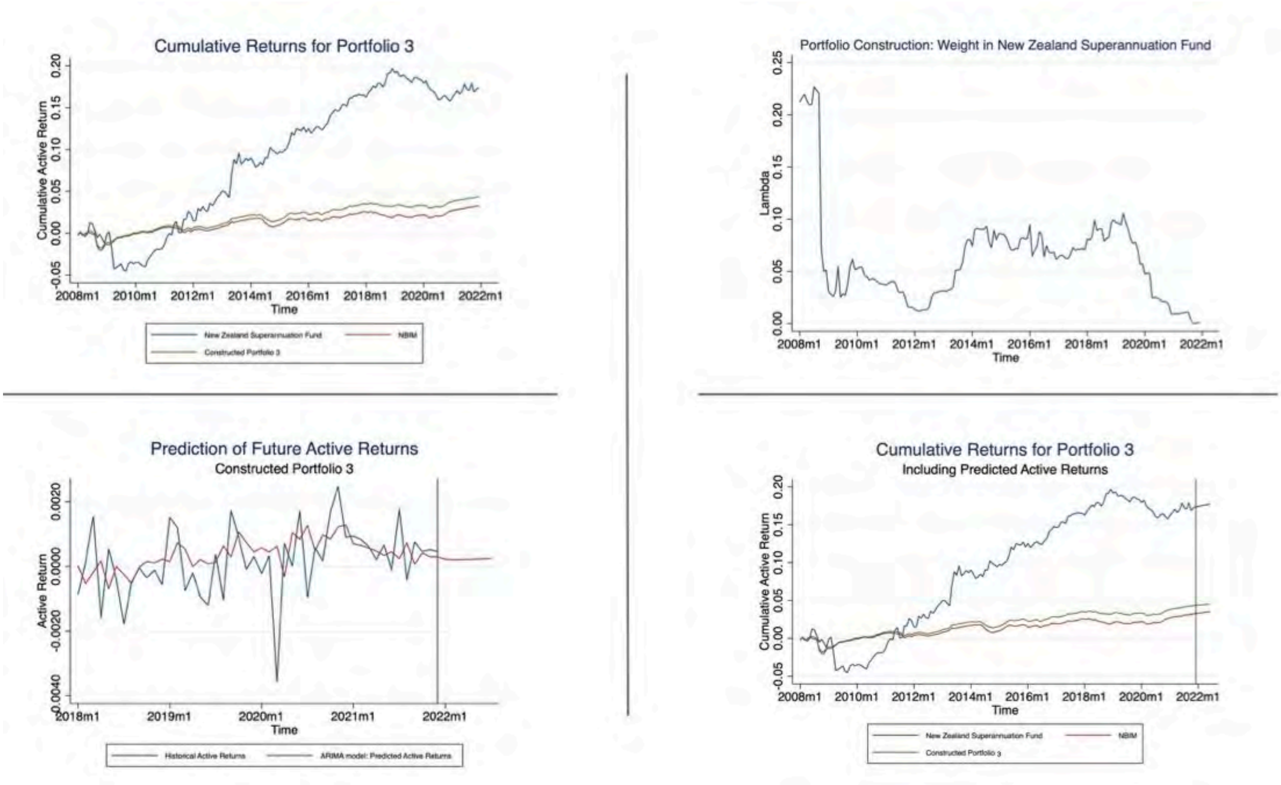
The third synthetic portfolio is constructed based on the predictive quality of the active returns for the GPFG and the New Zealand Superannuation Fund. We leverage the weighted average of the inverted MSPE for each fund. We aim to investigate if a portfolio based on predictive quality can outperform the GPFG.

Figure 10 presents the findings from the evaluation of Portfolio 3. We find that the GPFG and Portfolio 3 are closely related until 2012 while outperforming the GPFG in subsequent years. However, compared to Portfolio 1 and Portfolio 2, this portfolio delivers active returns close to the GPFG.

Furthermore, the weighting λ_t^{PQ} used to create Portfolio 3 is also included in Figure 10. The lambda values used to construct Portfolio 3 are fairly stable compared to the lambda values of Portfolio 1 and Portfolio 2. This further suggests that our findings from Portfolio 3 are more reliable, as the weights are more realistic.

Lastly, we also aim to predict the future active returns of Portfolio 3. Again, we commence with testing whether our historical values are stationary. The Augmented Dickey-Fuller test demonstrates that the historical values have the same statistical properties throughout the time series, which further can be used to predict future active returns. From Figure 10, we observe that the ARIMA[10] has a fairly suitable fit to historical values, making our predictions more reliable. The ARIMA does not follow a random walk process[11] and we can further use the model for investigating future active returns in our 6-month prediction horizon. Our prediction presented in Figure 10 indicates that the active returns for Portfolio 3 will follow a stable trend in the following months.

Figure 10 - Portfolio 3



[10] The scoping of an optimal ARIMA for Portfolio 3 is elaborated on in the Appendix.
[11] A Wald-Wolfowitz runs test provides evidence that the ARIMA does not follow a random walk.

7.2.2 Discussion of Findings from the Synthetic Portfolios

Through the Presentation of Synthetic Portfolios, we have set the foundation to answer the second part of our research question on how increased active management can impact active returns for the GPFG. We will now discuss and compare our findings from each synthetic portfolio by summarizing performance measures, cumulative returns, and the weights used to construct the portfolios, before providing an initial conclusion of our Scenario-Analysis.

Performance Measures

Table 10 presents annualized active return and tracking error for the constructed portfolios, in addition to the GPFG and the New Zealand Superannuation Fund. We present the active return and tracking error both from the respective inceptions and from 2008 and onwards[12]. We also include the risk-adjusted return.

Table 10 - Comparison of Constructed Portfolios

	GPFG		NZ SF		Portfolio 1		Portfolio 2		Portfolio 3
	Since Inception	Since 2008	Since Inception	Since 2008	Since Inception	Since 2008	Since Inception	Since 2008	Since Inception (2008)
Active Return	0.44%	0.23%	1.01%	1.24%	1.19%	1.29%	1.15%	1.25%	0.31%
Tracking Error	0.73%	0.60%	2.06%	2.20%	1.53%	1.70%	1.17%	1.26%	0.56%
IR*	0.60	0.38	0.49	0.56	0.78	0.76	0.98	0.99	0.55
N	287	169	225	169	225	169	225	169	169

*IR represents the risk-adjusted return by calculating active return per unit active risk

From Table 10, we find that Portfolio 1 delivers an active return of 1.19% with a tracking error of 1.53%. Thus, this portfolio delivers three times the current active return of the GPFG, with twice as high tracking error. Even though this implies a higher degree of risk, the tracking error of 1.53% is in the proximity of the tracking error limit of 1.25%. Additionally, Portfolio 2 generates an active return of 1.15%, again more than twice as high as the GPFG, while being within the tracking error limit set by the Ministry of Finance. The findings from the analysis of Portfolio 1 and Portfolio 2, therefore clearly indicate that an increased degree of active management could generate higher active returns for these constructed portfolios. When

[12] The reasoning behind this is to provide a neutral basis for comparison, as Portfolio 3 is based on 169 observations from 2008 and onwards.

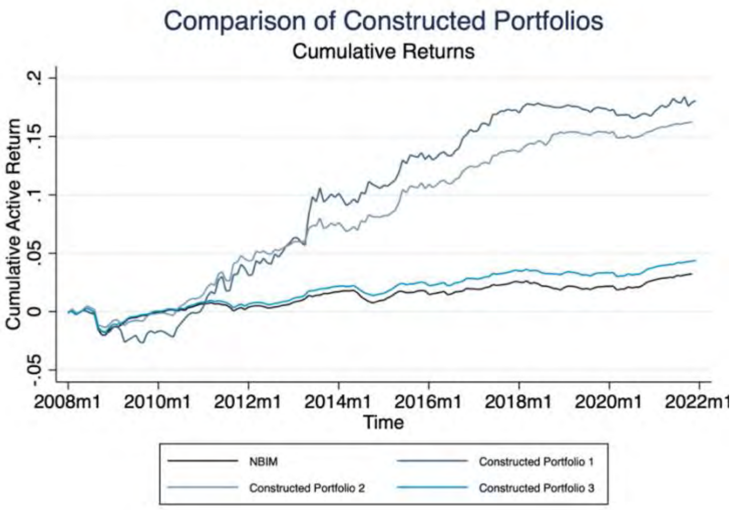
analyzing Portfolio 3, we also discover that this portfolio delivers a higher active return of 0.31%, with a lower tracking error of 0.56% compared to the GPFG. This implies that higher active returns are achievable without increasing active management in terms of tracking error.

Even though the analysis of Portfolio 1 and Portfolio 2 distinctly suggests that increased active returns can be achieved by increased active management, Portfolio 3 opposes this to some extent. Followingly, there is an opportunity for increased active returns by pursuing other strategies that necessarily do not increase risk. However, as presented in Table 10, all portfolios deliver a higher risk-adjusted return than the GPFG, indicating that any additional risk taken on by increased active management will be compensated for. Thus, the performance measures for all constructed portfolios presented in Table 10 indicate that increased active management could yield increased active returns.

Cumulative Active Returns

Additionally, we present the cumulative active returns of the synthetic portfolios and the GPFG in Figure 11. We observe that all portfolios outperform the GPFG, where Portfolio 1 and Portfolio 2 deliver substantially higher cumulative active returns compared to Portfolio 3. The presentation of cumulative active returns illustrates the historical development of each portfolio, as well as the GPFG, and provides support to our findings that the GPFG could increase its active returns by increasing active management.

Figure 11 - Comparison of Cumulative Active Returns for All Constructed Portfolios

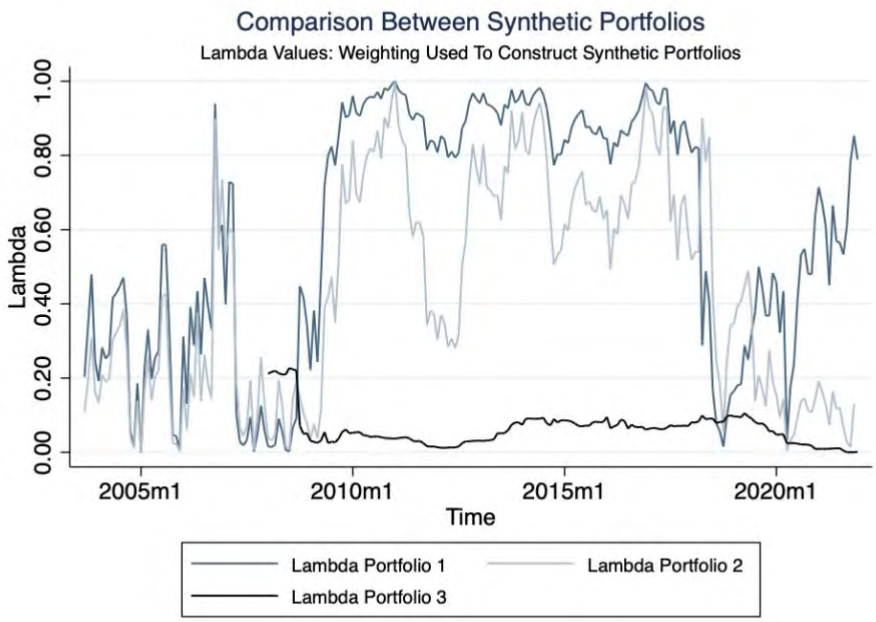


Note: We calculate the cumulative return from 2008 for all funds and onwards. The true cumulative return for Portfolio 1 and Portfolio 2 from their respective inceptions have been presented earlier.

Lambda

We also present the lambda values used to construct each of the synthetic portfolios, to observe how they deviate from one another. As mentioned, the weights used to create Portfolio 1 and Portfolio 2 follow a similar pattern and are more volatile compared to the weights of Portfolio 3. Generally, Portfolio 1 and Portfolio 2 have higher lambda values, indicating a higher weight in the New Zealand Superannuation Fund, while Portfolio 3 demonstrates a closer relation to the GPFG. The inclusion of lambda values in this discussion emphasizes the reliability of our analysis, where we discover that the weights of Portfolio 3 are more realistic compared to the other two.

Figure 12 - Comparison of Lambda Values



Initial Conclusion

Our analysis discovers that all our constructed portfolios outperform the GPFG for the investigated time periods, both in terms of return, risk-adjusted return, and cumulative return. Furthermore, we discover that the tracking error of Portfolio 2 and Portfolio 3 is within the tracking error limit of 1.25%, implying that increased active return is theoretically achievable within the current restrictions set by the Ministry of Finance.

The estimated ARIMAs and predicted future returns of Portfolio 1 and Portfolio 3 both indicate fairly stable active returns within our 6-month prediction horizon. This suggests that the results of the historical analysis will not change within the subsequent 6 months. Additionally, the predicted active returns for all constructed portfolios outperform the predicted active returns of the GPFG.

In summary, our Scenario-Analysis finds that all our constructed portfolios outperform the GPFG's past active returns. Therefore, the analysis demonstrates that the GPFG can achieve increased active returns by increasing active management. This finding will further be substantiated with a robustness analysis in the following section.

7.3 Robustness Analysis

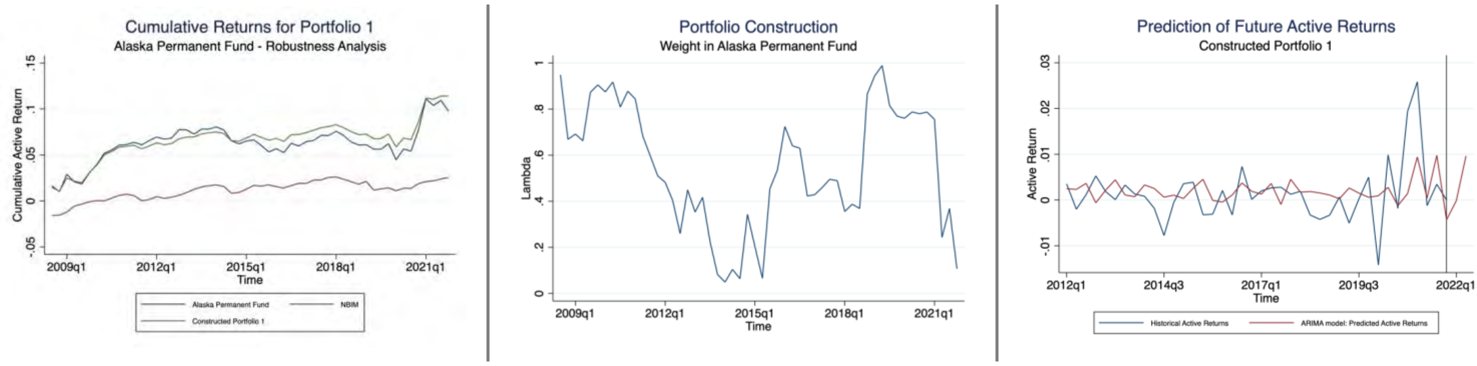
In this section, we aim at conducting a robustness analysis for evaluating the results in section 7.2 Comparison of Synthetic Portfolios and the GPFG. We conduct a coinciding analysis with the Alaska Permanent Fund, for evaluating the active returns of similarly constructed portfolios using another fund.

Below, we present the synthetically constructed portfolios as a combination of the GPFG and the Alaska Permanent Fund. We do not construct Portfolio 3 based on predictive quality in the robustness analysis. The quarterly data available for the Alaska Permanent Fund reduces the number of observations, which prevents us from pursuing a meaningful analysis for Portfolio 3. The goal of the robustness analysis remains to substantiate our main findings; that there exists an opportunity to increase active returns by increasing active management for the GPFG.

7.3.1 Portfolio 1 - Active Return

Figure 13 presents the findings of the first synthetic portfolio constructed as a combination of the GPFG and the Alaska Permanent Fund. In similarity with our main analysis, we discover that Portfolio 1 is closely related to the GPFG in the first time-period, before following the same fluctuations as the Alaska Permanent Fund in the subsequent quarters. We find that Portfolio 1 outperforms the GPFG, with fairly stable lambda values throughout the time period. The stable lambda values can be a consequence of quarterly data and fewer observations in our dataset. Our robustness analysis for Portfolio 1 indicates that the findings from our main analysis still hold, given continuously higher cumulative returns compared to the GPFG.

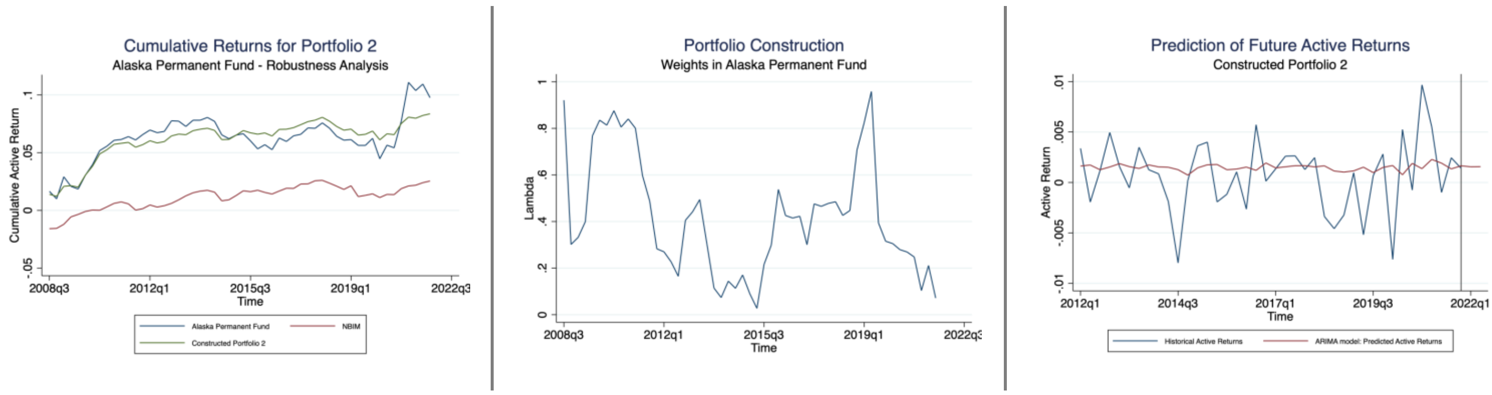
Figure 13 - Portfolio 1 with the Alaska Permanent Fund



7.3.2 Portfolio 2 - Risk-Adjusted Return

The findings from the robustness analysis of our second constructed portfolio are presented in Figure 14. We find that Portfolio 2 is closely related to the Alaska Permanent Fund towards the latter half of the time period, while the last quarters are more related to the GPFG in terms of performance and fluctuations. Portfolio 2 also presents more stable weights across the time period compared to our main analysis. Again, the robustness analysis substantiates our initial findings, given continuously higher cumulative returns compared to the GPFG.

Figure 14 - Portfolio 2 with Alaska Permanent Fund



7.3.3 Discussion of Findings from the Robustness Analysis

Table 14 presented below, summarizes the findings of the robustness analysis. We include the GPFG and the Alaska Permanent Fund in the table for comparison. For the GPFG, we again present figures calculated both since inception in 1998 and since 2008[13].

We find that both Portfolio 1 and Portfolio 2 deliver higher active returns than the GPFG. Furthermore, the tracking error of Portfolio 1 and Portfolio 2 is higher than the GPFG, indicating a higher degree of active management. Additionally, the tracking error for each portfolio is in close proximity of the limit set by the Ministry of Finance. The risk-adjusted return of both synthetic portfolios is also higher than the risk-adjusted return calculated for the GPFG in the same time period, indicating that any additional risk taken on will be compensated for.

The findings of our robustness analysis substantiate our initial findings, namely that there exists an opportunity for the GPFG to increase its active returns by increasing active management.

Table 14 - Findings from Robustness Analysis

	GPFG		**Alaska Permanent Fund**	**Portfolio 1**	**Portfolio 2**
	Since 2008	Since Inception			
Active Return	0.19%	0.44%	0.90%	0.85%	0.62%
Tracking Error	0.74%	0.63%	1.61%	1.32%	0.87%
IR*	0.26	0.70	0.56	0.64	0.71
N	54	287**	56***	54	54

* IR represents the risk-adjusted return by calculating active return per unit active risk
**Annualized values calculated since inception 1998 to 2021
*** Annualized values calculated based on all available data for Alaska Permanent Fund

[13] We provide a neutral basis for comparison as we only have return data for the Alaska Permanent Fund and the two synthetic portfolio from 2008 until 2021.

7.4 Summary of the Scenario-Analysis: Potential Active Returns of the GPFG

In Chapter 7, we have analyzed the potential active returns of the GPFG, by creating synthetic portfolios to answer the second part of our research question on how increased active management can impact active returns.

Firstly, through a comparison of different Sovereign Wealth Funds, we selected the New Zealand Superannuation Fund and the Alaska Permanent Fund to construct synthetic portfolios, respectively for a main- and robustness analysis. We found significant factor-risk-adjusted alphas for both funds, which allowed us to continue our analysis.

The three synthetic portfolios were constructed based on active return, risk-adjusted return, and predictive quality. All synthetic portfolios deliver higher active returns and risk-adjusted returns compared to the GPFG. For Portfolio 1 and Portfolio 2, these returns entail increased tracking error, however, Portfolio 3 delivers higher active returns with a lower tracking error compared to the GPFG. The main analysis therefore clearly indicates that there is an opportunity for increased active returns by increasing active management when evaluating the constructed portfolios.

Furthermore, we included a robustness analysis to substantiate our initial findings. Due to scarcity of data, we were not able to construct Portfolio 3 in our robustness analysis. However, the results of Portfolio 1 and Portfolio 2 demonstrate similar findings as our main analysis.

The Scenario-Analysis finds that the answer to the second part of our research question on how increased active management can impact the GPFG's returns is; there exists an opportunity to increase active returns by increasing active management. However, limitations of the analysis and other empirical research must be included to provide a well-nuanced answer to our research question. This will be emphasized in the following sections, and the final conclusion of our thesis is presented in the Concluding Remarks.

8 Limitations of the Analyses

The analyses presented in this thesis rest on several assumptions and simplifications that need to be addressed. This section aims to nuance our findings and addresses the main limitations of both the Historical Analysis presented in Chapter 6 and the Scenario-Analysis presented in Chapter 7. Naturally, there are many limitations in an exploratory academic study, such as this master's thesis. However, this chapter will only emphasize relevant limitations with a direct impact on the answer to our research question.

8.1 Historical Analysis

First, in the Historical Analysis, we calculate the active share of the GPFG. When conducting the calculation, we rely on a replicated benchmark based on the FTSE Global All Cap Index, as we do not possess the actual benchmark provided by the Ministry of Finance. As presented, our methodology aims to ensure that the replicated benchmark is as close to the benchmark provided by the Ministry of Finance as possible. However, errors in our data treatment approach may lead to deviations in the calculation of active share.

Furthermore, we did not receive data for the equity benchmark from FTSE Russell for 2021 and the first half of 2022. As presented in our calculation, we have found a decreasing trend in active share since 2015. However, we do not possess information to evaluate whether this trend has continued or not.

Additionally, we establish a significant factor risk-adjusted alpha through the Five-Factor model, without adjusting the factor analysis for emerging markets. If the GPFG deviates from its benchmark and has systematically overweighted emerging markets, this could naturally be the reason for strong excess returns compared to the benchmark, as less efficient markets are characterized by higher risk premiums (Hoddevik & Priestley, 2022). The inability to adjust for this entails that we cannot state that the historical factor-adjusted excess return has been truly significant.

8.2 Scenario-Analysis

Further, it is important to stress that our Scenario-Analysis is conducted in an ex-post environment and that our constructed portfolios therefore are not strategies that the GPFG could realistically follow. However, the portfolios are meant to provide reasonable indications for active returns. Even though we have ensured smoothened lambda-values, the weights of our constructed portfolios are volatile between the two funds and are not feasible for the GPFG to follow in practice.

It also needs emphasizing that the Scenario-Analysis arguably is narrow, given that we use one fund in our main analysis, and therefore, our conclusion rests on solely combining the GPFG with the New Zealand Superannuation Fund. Our results may therefore be a consequence of this fund's unique characteristics, implying that our findings may not hold in general. However, the analysis is for this reason extended with a robustness analysis to strengthen the findings.

Moreover, there are some limitations of the construction of Portfolio 1 and Portfolio 2 affecting the findings of the Scenario-Analysis. These two portfolios are based on active return and risk-adjusted return respectively. Structurally, these portfolios favor the best-performing fund, indicating that the portfolios most likely will have a higher active return compared to the GPFG, as hypothesized. For this reason, we include Portfolio 3 in our analysis, given that it resembles an experiment with an elusive outcome.

Due to the emphasized limitations of our analyses and to further substantiate our findings from an empirical perspective, we extend the analysis to investigate broad empirical research on active management. By including this in our thesis, we strive to provide a well-nuanced answer to our research question of how the active management and accompanying active returns of the GPFG have been historically and how increased active management could impact its active returns.

9 Active Management of the GPFG: Discussion Leveraging Existing Research

This chapter is dedicated to discussing our findings presented in 6.4 Summary of the Historical Analysis and 7.4 Summary of the Scenario-Analysis in light of empirical research conducted on similar topics. Our point of departure is presenting empirical research on the matter of active versus passive portfolio management. Further, we aim to discuss our findings in light of the discovered empirical research and provide additional substance to the answer to our research question. We supplement the discussion to include other risk factors that could be of importance when evaluating portfolio management of the GPFG from a macro perspective.

9.1 Empirical Research on Active versus Passive Management

In the following subsection, we aim to present some general arguments on the topic of active versus passive portfolio management from existing empirical research. Our goal is to investigate what empirical research deems important when evaluating the different strategies of portfolio management, mainly in terms of performance, management costs, and the theory of efficient markets.

9.1.1 Performance

The performance of actively managed funds and managers' ability to generate excess returns are frequently discussed in literature, and a myriad of research suggests that actively managed funds do not outperform passively managed funds. To initiate, Blake (1993), Malkiel (1995), and Gruber (1996) investigated returns on actively managed portfolios and found that on average, actively managed mutual funds do not generate higher returns than market indices or index funds. This finding is further substantiated by Malkiel (2003), who finds that investors are likely to receive higher returns with a passive investment strategy compared to active portfolio management. The author also illustrates that approximately 70% of active managers are outperformed by the S&P 500. Therefore, the presented research favors passive portfolio management, in line with the broad literature in the field.

Fortin & Michelson (2002) find similar results as indicated above; that index funds on average outperform funds that are actively managed. Nevertheless, their study finds that funds investing in international stocks and small company equity[14] significantly outperform their respective indices. Their research suggests that fund managers of these funds were to a greater extent able to leverage mispricing in these financial markets that presumably were less efficient (Fortin & Michelson, 2002).

Additionally, Kremnitzer (2012) investigated whether active management in emerging markets is correlated with superior returns. Emerging markets are often characterized as less efficient and thus, with a higher opportunity to leverage mispricing and generate excess returns (Hoddevik & Priestley, 2022). The study find a strong relationship between active management and higher risk-adjusted return, and that actively managed funds outperformed their passive counterparts on average, with 3-year net costs excess returns of 2.87% (Kremnitzer, 2012). The study used data on all existing US mutual funds and Exchange-Traded Funds (ETFs)[15] that were dedicated to emerging markets. The findings, therefore, indicate a possibility to generate excess returns in less efficient markets, which further favors active management as an investment strategy.

Furthermore, Petajisto (2018) finds indications that the most active mutual funds outperformed their respective indices after costs, while so-called closet indexers with a lower degree of active management consistently underperformed their indices. According to Petajisto (2018), active managers are therefore not all equal, and even though on average, actively managed funds underperform compared to passively managed funds, investment performance will depend on the degree of active management. More specifically, Petajisto (2018) suggests that the most active stock pickers have outperformed their indices by 1.26% after costs. This relation between a high degree of active management and active performance is even more significant for funds of larger size.

9.1.2 Management Costs

Management costs are further an important difference between active and passive management. Generally, active management is quite resource intensive as the managers of such funds aim to identify assets that are wrongly priced in the market, and thus gain profit from this mispricing (Chen, 2022). Management costs will naturally be incurred for both an active and passive management strategy, but these costs will be higher for active management as this strategy is

[14] Fortin & Michelson (2002) define eight broad investment categories, among them International Stock and Small Company Equity

[15] ETFs are a group of securities that mirror sectors of indexes, representing passive portfolio management.

more resource-intensive and requires more frequent transactions. Essentially, passive management can therefore have a higher risk-adjusted expected return after costs compared to active management (Thorburn, 2017).

Accordingly, the cost perspective of portfolio management is an imperative argument when evaluating active versus passive management. Bogle (1996) states that the case for selecting an index fund is compelling due to the index fund's fundamental cost advantage. Essentially, active investors must outperform passive investors by the costs of their management fees to make the active investment strategy advantageous. Research also suggests that the difference between returns of actively and passively managed funds approximately equals the difference in management fees between the two (Dale & Miller, 2018). Furthermore, Blake (1993), Malkiel (1995), and Gruber (1996) further suggest a correlation between the inability to outperform passive management and the increased costs of active management. We, therefore, find that the management costs of an actively managed fund can lead to underperformance compared to passive management, net of costs, which in turn serves as an argument against such a strategy.

Nevertheless, one also needs to emphasize the presence of economies of scale in asset management (NBIM, 2009). A large fund has easier access to information and the ability to conduct thorough analyses. Thus, a fund would be able to operate with lower costs as a proportion of the assets under management. Research finds that many funds have a declining rate structure of their management fees where their fees decrease when assets increase in size. The declining structure of management fees reflects that the fund expects that economies of scale will be realized, both in management and operations when the fund size increases (Rea, Reid, & Millar, 1999). This implies that excess return from active management can be easier to achieve for a bigger fund, with lower additional costs.

9.1.3 Efficient Market Hypothesis

Initially in the thesis, we presented the Efficient Market Hypothesis, which states that financial markets are characterized as efficient, where asset prices reflect all relevant information about the particular asset. These markets often consist of many investors and analysts, reducing the opportunity of identifying mispricing and generating excess returns from active management. Arguably, such a market would favor passive management as generating excess returns is challenging. The majority of the research presented above finds that actively managed funds do not outperform passively managed funds, net of costs. This endorses the Efficient Market

Hypothesis, as active investors on average are not generating excess returns above their respective benchmarks.

On the contrary, market efficiency varies, indicating that the Efficient Market Hypothesis does not necessarily hold. Market frictions, costs of extracting information, and restrictions connected to the capital structure are important reasons contradicting market efficiency in financial markets (NBIM, 2009). The degree of market efficiency varies, both between markets and over time, which lays the foundation for a modification of the Efficient Market Hypothesis. Thus, active management can create excess returns, and risk-taking investors present in multiple financial markets can leverage deviations in market efficiency to create profit (NBIM, 2009). Research finds that even though actively managed funds on average do not outperform passively managed funds, a significant minority of active managers do add value and contribute to ensuring and maintaining efficiency in the capital markets (Jones & Wermers, 2011). The Modified Efficient Market Hypothesis, therefore, facilitates that active management can be profitable, providing a favorable argument for this investment strategy in less efficient markets.

9.2 Our Findings in the Context of Empirical Research

In the previous section, we have outlined general arguments on active versus passive management, mainly in terms of performance, management costs, and the theory of efficient markets. The purpose of the following section is to cointegrate our findings from Chapter 6 and Chapter 7 with the general research on active versus passive management presented above, to provide a well-nuanced answer to our research question.

9.2.1 Performance

To initiate the discussion, our findings suggest that both the benchmark risk-adjusted alpha and the factor risk-adjusted alpha are significant for the full sample period investigated for the GPFG. Our findings, therefore, indicate that active returns predominantly have been significant throughout the investigated time periods and that active management has created additional return for the GPFG. Our research suggests similar findings when evaluating the New Zealand Superannuation Fund and the Alaska Permanent Fund. Both funds have factor risk-adjusted alphas of approximately 1.2%. The significance of the presented alphas above clearly implies that active management creates additional returns beyond the benchmark value creation for all funds investigated. Arguably, this points in the direction that the GPFG should not be passively managed.

Our findings in the Scenario-Analysis support the arguments presented above, as it clearly indicates an opportunity for increased active returns for the GPFG. When evaluating the synthetically constructed portfolios, all portfolios outperform the GPFG both in terms of active return and risk-adjusted return in-sample and out-of-sample. Portfolio 1 and Portfolio 2 deliver substantially higher active returns, with a tracking error in close proximity to the limit set by the Ministry of Finance. Portfolio 3 also delivers a higher active return with a lower tracking error than the GPFG. From Portfolio 3, we, therefore, discover that higher active returns can be achieved without this entailing increased risk. However, as presented in Chapter 7, the Scenario-Analysis finds that increased active returns are feasible by increasing active management.

As presented in 9.1, we find that the majority of research clearly states that on average, actively managed funds are unable to outperform passively managed funds. Our findings, therefore, contradict the broad existing research on the topic, as we find that active management predominantly provides significant excess returns for the three funds investigated. However, the presented research also emphasizes that there are exceptions to this consensus, especially related to (1) investment in international shares, (2) funds with a particularly high degree of active management, and (3) funds that are heavily invested in emerging markets. These exceptions may contribute to explaining the discrepancies between our findings and the broad empirical research.

Firstly, the international exposure of the three funds investigated, namely the GPFG, the New Zealand Superannuation Fund, and the Alaska Permanent Fund, is high and can therefore be seen in the context of the research from Fortin & Michelson (2002). As previously stated, the authors find that funds with the majority of assets invested in international stocks have a better ability to leverage mispricing compared to other funds. This will be relevant for all three funds mentioned above given their mandates, but particularly relevant for the GPFG, whose management model is entirely based on holding international positions.

The correlation between the degree of active management and active returns can also be inferred from our findings, even if vaguely so. From our calculations in Chapter 6, we find that the GPFG has an average active share of 18.92% from 2015 to 2020, while the New Zealand Superannuation Fund has an active share of approximately 33%. Furthermore, we find that the

New Zealand Superannuation Fund has delivered higher annualized active returns since inception compared to the GPFG. Even though the relation between the two findings might be weak, it could point in the direction of the findings in Petajisto (2018), where the author establishes a strong link between the most actively managed fund and superior active performance.

Lastly, the exception of being heavily invested in emerging markets must be addressed. As elaborated in 9.1, Kremnitzer (2012) finds a strong relationship between active management and higher returns compared to passive counterparts in emerging markets. If the GPFG deviates from its benchmark and has systematically overweighted emerging markets, this could naturally be the reason for strong excess returns compared to the benchmark, as less efficient markets are characterized by higher risk premiums. We have not performed our factor analyses with an adjustment for emerging markets, due to scarcity of data, and this could imply that the factor-adjusted excess returns found are not truly significant. Followingly, it can be argued that the alphas found do not represent true significant performance.

The preceding paragraphs attempt to explain the differences between the results we find in this master's thesis and other existing research, by assessing the exceptions we have found to the established conclusion that active funds on average are not able to outperform passive funds. The findings from our analyses imply that active management would be an advantageous investment strategy for the GPFG, which is further supported by the exceptions listed above. Nevertheless, it seems that our findings related to performance go against the broad consensus in the literature.

9.2.2 Management Costs

As emphasized in 9.1, the management costs perspective of active management is an imperative argument when evaluating active versus passive management. In our analyses, we also emphasize the importance of costs, and we perform all factor analyses both excluding and including management costs. When evaluating beta-adjusted active returns for the GPFG, we find a significant alpha on the 5% level and the 10% level respectively when excluding and including management costs for the full sample period. We find similar results when evaluating the factor risk-adjusted alphas. Additionally, for the second sample period (2007-2015) we find that the factor risk-adjusted alpha is significant on the 10% level excluding costs while losing its significance when including costs in the regression analysis. This supports the importance of costs when evaluating if active management contributes to generating excess returns. Our findings further suggest that when introducing management costs to the analysis, the possible gain in terms of excess returns of active management decreases, which aligns with the presented research in 9.1.

However, it is important to underline that we still find significant active returns for several time periods after we include management costs in the analysis. Our findings, therefore, indicate that management costs are of great importance in the assessment of active versus passive management, but that active management can generate significant active returns compared to passive funds after costs are considered. This is thus both supportive and contradictory compared to the broad research in the field which states that costs are an important lever when evaluating active versus passive management and that the difference between the two essentially equals the high management costs of active management.

Furthermore, it must be emphasized that large funds also have the advantage of economies of scale. A large fund like the GPFG can more easily utilize its resources and has easier access to both analyses and information that can be used to generate profit from active management. Arguably, a large fund such as the GPFG can therefore create active returns with lower additional costs. When evaluating the management of the GPFG, economies of scale can therefore be a favoring argument for increased active management.

9.2.3 Efficient Market Hypothesis

As already elaborated, our analyses find significant alphas for all evaluated funds, namely the GPFG, the New Zealand Superannuation Fund, and the Alaska Permanent Fund. For the GPFG we investigate the beta-risk adjusted alphas and the factor-risk adjusted alphas for three different time periods. Our findings show that both risk-adjusted alphas have been significant for the full sample period, while the significance varies across the other time periods. The significant alphas contradict the Efficient Market Hypothesis in the given time period, as it should not be possible to generate active returns from active management in efficient markets. This can further endorse the Modified Efficient Market Hypothesis, as these funds have generated significant excess returns above their respective benchmarks.

Nevertheless, we must also emphasize the time periods where the risk-adjusted alphas are not significant. The beta risk-adjusted alpha is not significant for the second (2007-2021) and third (2015-2021) time period investigated, supporting the Efficient Market Hypothesis, as alpha is not statistically different from zero in the given time periods. However, the factor risk-adjusted alpha excluding and including costs are both significant in the third time period. Thus, the risk-adjusted alphas have predominantly been significant in all the time periods investigated. The significant past active returns of the GPFG, therefore, indicate that the fund has potentially leveraged deviations in market efficiency to create profit. This is further an argument that the GPFG has an opportunity to increase active returns through active management since (1) there exist inefficiencies in financial markets and (2) the GPFG is an investor with international exposure present in multiple markets that can exploit inefficiencies to generate profit.

9.2.4 Summary: Implications of Empirical Research on Our Findings

In the former sections, we have aimed to evaluate the findings from our analyses in the context of empirical research conducted on the topic. We have mainly investigated arguments related to the performance of active versus passive management, the associated management costs, and further how this relates to the theory of efficient markets.

When we assess our findings against other research, the most prominent argument throughout the discussion has still been that increased active management can provide some benefits for the GPFG. This is due to the predominant arguments from our quantitative analyses, as well as the fact that existing research also emphasizes some advantages of active management and provides certain exceptions to the broad consensus that active management on average does not outperform passive management. Nevertheless, it must be pointed out that this conclusion goes against the consensus in the literature, and of course that our analysis has limitations that may affect the findings we use as arguments for active management in the previous discussion. However, our conclusion points in the direction that we believe that the GPFG can reap benefits from increasing active management.

Furthermore, there is an important argument in financial theory that has not been addressed through the preceding discussion, namely the assets owner's risk tolerance. The owner's risk tolerance lays the foundation for a fund's investment strategy. This implies that a change in the management of a fund must require (1) that the change is within the owner's risk tolerance and (2) that clear and unambiguous information about what this change entails is provided to the owner (Andreassen, et al., 2022).

An increase in active management could entail increased risk and increased costs. Generally, in a market consisting of both actively managed funds and index funds, an investor can choose the degree of risk and costs they find acceptable. However, the extensive Norwegian petroleum wealth is only managed by NBIM and thus the active management of the fund will impose both risks and costs on the asset owner, which ultimately is the Norwegian people. Increased active management can therefore be deemed imprudent as one cannot guarantee that the asset owner, namely the Norwegian people, tolerates the associated risk and cost increase of active management. However, according to NBIM (2009), the additional risk of their active management is not to be considered substantial in the risk evaluation of the entire fund. This could imply that increased active management of the GPFG can generate active returns, without increasing the risk to the asset owner considerably. If this is to be believed, we can say that our conclusion, which points in the direction of increased active management, will be feasible as long as it does not impose substantially increased risk on the Norwegian people.

9.3 Additional Risk-Factors From A Macro Perspective

Additionally, to the discussion above, other factors could be evaluated when investigating whether the GPFG should increase its active management. This section will emphasize some of the additional risk factors the fund faces from a macro perspective, and how this can affect the management of the fund in terms of the degree of active management. We are living in exceptional times, and observe several political and economic developments, that could affect the risk of the fund and consequently what investment strategy the GPFG should pursue. We will mainly address both climate risk and geopolitical risk, the latter including both security policy and general political trends we see from a macro perspective. The goal of this section is to highlight that there are other dimensions to the discussion regarding the management of the GPFG, and we will briefly present some of them.

Initially, it's reasonable to assume that the fund will face significant climate risk in the future. The green shift is ongoing, and we observe that increasing awareness of climate change, as well as direct climate measures at political levels, indicate that society, economy, and industry now must be transformed into more climate-friendly operations. One can expect that such measures will continue to a greater extent in the foreseeable future, and thus there is uncertainty related to how this will affect the economy and in consequence, the investments of the GPFG. Additionally, the increased climate awareness coerces investors to make different assessments than before when it comes to investment decisions. The climate perspective includes an unpredictable horizon, ethical dilemmas, and, in general, the risk of rapid changes in climate policy and management of operations (Andreassen, et al., 2022).

The GPFG is already addressing climate risk, both through expert reports on the topic, as well as ethical exclusions of investments. At the same time, there is no reason to believe that the trend will change, and thus the GPFG should be prepared for changes that affect their investments and leverage this trend to their advantage.

Furthermore, we also see major developments in the political situation, both internationally and regionally. Globalization has historically been an important development factor. However, now we observe an opposite effect, where trends of more national political governance and increased regulation are on the rise (Andreassen, et al., 2022). This will be particularly relevant for the GPFG, as the management model is largely based on holding positions with international exposure, where this is most profitable. A greater degree of regulation could prevent several investments for the GPFG.

Generally, politics and economy are linked to a greater extent, which has consequences in today's geopolitical situation with some of its challenges presented above. This became especially clear after Russia invaded Ukraine. As a result, the GPFG is to divest all the fund's investments in Russia, which in December 2021 had a value of NOK 27 billion (Bache & Tangen, 2022). One could therefore argue that the potential of divesting holdings as a result of political decisions, should be evaluated when making investments in certain regions. Furthermore, the general instability in the financial markets after the invasion suggests that it might be advantageous for the GPFG to have the flexibility to actively invest and limit potential losses in the event of crises (Fortin & Michelson, 2002).

As deduced above, several other factors can affect the risk of the investments for the GPFG and thus how the management should be in terms of the degree of active management. With this section, we want to illustrate that several dimensions should be taken into account when reviewing the management model of the GPFG, even though we do not include them directly in our analysis or the answer to our research question.

10 Concluding Remarks

Throughout this master thesis, we have examined the active management and accompanying active returns of the GPFG. The thesis has been structured according to our presented research question; *How has the active management and accompanying active returns of the GPFG been historically, and how could increased active management impact its active returns?*

The answer to our research question rests on three supportive analyses: a historical analysis evaluating fund performance and active management, a scenario-analysis investigating potential active returns, and lastly a qualitative study validating our findings.

Initially, the Historical Analysis finds that active returns predominantly have been significant throughout all investigated periods and that active management has created additional returns for the fund. Further, we have established the historical degree of active management, through an average active share of 18.92% from 2015 to 2020 and an annual tracking error of 0.63% since inception, evidently determining that the fund can be characterized as an index fund.

The Scenario-Analysis indicates that there exists an opportunity for the GPFG to increase its active returns by increasing active management. The three synthetic portfolios, all based on a fund with a higher degree of active management, evidently outperform the GPFG's active returns both in- and out-of-sample. Additionally, the three portfolios have a tracking error nearby of the limit set by the Ministry of Finance. However, empirical research casts doubt on our findings, as the established consensus is that active management does not outperform passive management as summarized by the initially presented quote of John C. Bogle.

After considering empirical research, the initial conclusion to the second part of our research question from the quantitative analyses remains; there exists an opportunity for the GPFG to increase its active returns by increasing active management. The empirical research highlights several key aspects that must be considered if increasing the degree of active management. However, we believe that the GPFG, with its perpetual investment horizon and considerable size, is in a unique position to reap the benefits of increased active management.

The thesis concludes that active management and its accompanying returns have created significant value historically and that the fund can increase its active returns by increasing active management. Therefore, it is our point of view that the GPFG should consider taking advantage of its full tracking error limit and increasing its active management in the future.

11 Final Reflections

Lastly, after presenting the Concluding Remarks of our thesis, we want to provide a short and subjective reflection on the methodology and approach chosen to answer our research question. The methodology used to answer the first part of our research question (Historical Analysis) is well-established in previous research, while the methodology used to answer the second part of our research question (Scenario-Analysis) has been more exploratory and elusive. Throughout the last 4 months, we have strived to create a methodology for the Scenario-Analysis, which is intuitive, sophisticated, and directly answers our research question. It is our opinion that our analysis meets these criteria.

However, we must emphasize that our presented methodology has been the result of adjusting empirical approaches, trying and failing, and making our own decisions and assumptions with the means available to us. As presented in Chapter 8, the methodology therefore has a set of concrete limitations that could affect our conclusion. Nevertheless, we believe that our methodology is suitable and provides interesting findings for active management and active returns of the GPFG. It is our belief that further research on the topic can provide substantial value in the evaluation of active management for the GPFG, and further for discussing the financing of the Norwegian welfare state in the future. Additionally, today's geopolitical situation makes the question of increased active management even more pressing.

12 Bibliography

Alaska Permanent Fund. (2022, November 15). Retrieved from Fund Management: https://apfc.org/fund-management/

Alaska Permanent Fund. (2022). *Alaska Permanent Fund* . Retrieved from Report Archive: https://apfc.org/report-archive/

Andreassen, H. M., Pedersen, J., Svarva, O., Thorburn, K., Sverdrup, U., Seland, H., . . . Undheim, A. (2022). *Fondet i en brytningstid.* Oslo: Norges offentlige utredninger.

Ang, A. (2014). *Asset Management: A Systematic Approach to Factor Investing.* Oxford University Press.

Bache, I. W., & Tangen, N. (2022, Mars 15). *Statens Pensjonsfond Utlands Investeringer i Russland.* Retrieved from Norges Bank Investment Management: https://www.nbim.no/no/publikasjoner/brev-til-finansdepartementet/2022/statens-pensjonsfond-utlands-investeringer-i-russland/

Bauer, R., Christiansen, C., & Døskeland, T. (2022). *A Review of the Active Management of Norway's Government Pension Fund Global.*

Bjerksund, P., & Døskeland, T. (2015). *Mål på aktiv forvaltning av aksjefond.* Bergen.

Blake, E. (1993). The Performance of Bond Mutual Funds. *Journal of Business*, 371-403.

Bogle, J. (1996, May 8). Be Not the First... Nor Yet the Last. The Vanguard Group, from speech presented May 8, 1996, at the AIMR annual conference in Atlanta, Georgia.

Box, G. E., & Pierce, D. A. (1970). Distribution of Residual Autocorrelations in Autoregressive-Integrated Moving Average Time Series Models. *Journal of the American Statistical Association*.

Box, G., & Jenkins, G. (1976). *Time Series Analysis: Forecasting and Control* . Prentice Hall.

Bradley, J. V. (1968). *Distribution-Free Statistical Tests.* Prentice-Hall.

Burnham, K. P., & Anderson, D. (2002). *Model Selection and Multimodel Inference.* New York: Springer.

CDPQ. (2022, November 15). *CDPQ*. Retrieved from Our History: https://www.cdpq.com/en/about-us/history

CDPQ. (2022). *CDPQ*. Retrieved from Annual Reports: https://www.cdpq.com/en/performance/annual-reports

Chen, J. (2022, June 11). *Active Management*. Retrieved from Investopedia: https://www.investopedia.com/terms/a/activemanagement.asp

Chen, J. (2022, March 19). *Alpha: What It Means In Investing, With Examples*. Retrieved from Investopedia: https://www.investopedia.com/terms/a/alpha.asp

Cremers , M., & Petajisto, A. (2006). *How Active Is Your Fund Manager? A New Measure That Predicts Performance.*

Cremers, M., & Petajisto, A. (2009, March 31). How Active is Your Fund Manager? A New Measure That Predicts Performance. *The Review of Financial Studies*.

Dahlquist, M., Polk, C., Priestley, R., & Ødegaard, B. (2015). *Norges Bank's Expert Group on Principles for Risk Adjustment of Performance Figures.* Norwegian Ministry of Finance .

Dale, P., & Miller, M. (2018, August). Active versus Passive Investing: Evidence from the 2009-2017 Market. *Journal of Accounting and Finance* , pp. 119-144.

D'Amico, J. (2021, May 18). ARIMA models in Stata - Part 1: Identification. Canada.

D'Amico, J. (2021, May 21). ARIMA models in Stata - Part 2: Estimation. Canada.

Døskeland, T. (2022, March 7). FIE 426 Kapitalforvaltning: Evaluation. Bergen, Norway.

Downey, L. (2022, June 27). *Efficient Market Hypothesis: Definition and Critique*. Retrieved from Investopedia: https://www.investopedia.com/terms/e/efficientmarkethypothesis.asp

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 3-56.

Fama, E. F., & French, K. R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, 1-22.

Fortin, R., & Michelson, S. (2002). Indexing Versus Active Mutual Fund Management. *Journal of Financial Planning*, 1-11.

Fortin, R., & Michelson, S. (2002, January). Indexing Versus Active Mutual Fund Management. *FPA Journal*.

Frank, S. (2017). *An Alaskan's Guide to the Permanent Fund.* Alaska Permanent Fund .

Ganti, A. (2022, May 31). *What Is Asset Management, and What Do Asset Managers Do?* Retrieved from Investopedia: https://www.investopedia.com/terms/a/assetmanagement.asp

Gruber, M. J. (1996). Another Puzzle: The Growth in Actively Managed Mutual Funds. *Journal of Finance* , 783-810.

Hoddevik, H., & Priestley, R. (2022, March 29). Spaltister om oljefondsrapport: En faglig fallitterklæring. *Dagens Næringsliv*.

Hyndman, R. J., & Athanasopoulos, G. (2018). *Forecasting: Principles and Practise.* OTexts.

Jensen, M. (1968, May). The Performance of Mutual Funds in the Period 1945-1964. *The Journal of Finance*.

Jensen, M. C. (1968). The Performance of Mutal Funds in the Period 1945-1964. *The Journal of Finance*.

Jensen, M., Black, F., & Scholes, M. S. (1972). The Capital Asset Pricing Model: Some Empirical Tests. *Praeger Publishers* .

Jones, R., & Wermers, R. (2011, September 1). Active Management in Mostly Efficient Markets. *Financial Analysts Journal*.

Kenneth R. French. (2022). Retrieved from Data Library: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Kidd, D. (2011). *The Sharpe Ratio and the Information Ratio.* Retrieved from CFA Institute: https://deborahkidd.com/wp-content/uploads/The-Sharpe-Ratio-and-the-Information-Ratio-1.pdf

Korea Investment Corporation. (2022, November 2022). *Korea Investment Corporation*. Retrieved from Background: http://www.kic.kr/en/01/01/01.jsp

Korea Investment Corporation. (2022). *Korea Investment Corporation*. Retrieved from Investment: https://www.kic.kr/en/02/03/01.jsp

Kremnitzer, K. (2012). Comparing Active and Passive Fund - Management in Emerging Markets. *Univeristy of California, Berkeley*, 1-43.

Lioudis, N. (2021, December 14). *Passive vs. Active Portfolio Management: What's the difference?* Retrieved from Investopedia: https://www.investopedia.com/ask/answers/040315/what-difference-between-passive-and-active-portfolio-management.asp

Malkiel, B. G. (1995). Returns from Investing in Equity Mutual Funds 1971 to 1991. *Journal of Finance*, 549-572.

Malkiel, B. G. (2003). Passive Investment Strategies and Efficient Markets. *European Financial Management*, 1-10.

Mcquarrie, A., & Tsai, C.-L. (1998). *Regression and Time Series Model Selection.* Singapore: World Scientific Publishining.

NBIM. (2009). *Årsrapport 2009: Den aktive forvaltningen av Statens Pensjonsfond Utland.* NBIM.

NBIM. (2022). *NBIM*. Retrieved from Returns: https://www.nbim.no/en/the-fund/returns/

NBIM. (2022). *NBIM*. Retrieved from Reports: https://www.nbim.no/en/the-fund/returns/

NBIM. (2022). *NBIM*. Retrieved from Investments: https://www.nbim.no/en/the-fund/investments/#/

NBIM. (2022). *Norges Bank Investment Management*. Retrieved from The History: https://www.nbim.no/en/the-fund/the-history/

New Zealand Superannuation Fund. (2022). *New Zealand Superannuation Fund*. Retrieved from Monthly Returns: https://www.nzsuperfund.nz/performance/investment/monthly-returns/

New Zealand Superannuation Fund. (2022). *Purpose and Mandate*. Retrieved from https://www.nzsuperfund.nz/about-the-guardians/purpose-and-mandate/

Norges Bank Investment Management. (2019). *Risk and Return.* Oslo: Norges Bank Investment Management.

Petajisto, A. (2018). Active Share and Mutual Fund Performance. *Financial Analysts Journal*, 73-93.

Pierce, R. G. (u.d). Financial Econometrics Lecture 2: ARIMA models. (p. 8). Oxford: Oxford University.

Rea, J., Reid, B., & Millar, K. (1999, December 5). Operating Expense Ratios, Assets, and Economies of Scale in Equity Mutual Funds. *Investment Company Institute Perspective*, pp. 1-16.

Rompotis, G. G. (2009). Active vs. Passive Management: New Evidence from Exchange Traded Funds. *National and Kapodistrian University of Athens, Greece* , 1-18.

Santos. (2021, August). Investments - Fundamental Consepts. Bergen, Norway.

Santos. (2021, September 21). Multifactor Models and Market Efficiency. Bergen, Norway.

Sharpe, W. F. (1970). Efficient capital markets: a review of theory and empirical work. *The Journal Of Finance*, 418-420.

Sharpe, W. F. (1991). The arithmetic of active management. *Financial Analysts Journal*, pp. 7-9.

Sharpe, W. F. (1994). The Sharp Ratio. *Journal of Portfolio Management*, 49-58.

Smith, T., Scott, G., & Munichiello, K. (2020, November 19). *Investopedia*. Retrieved from Random Walk Theory: Definition, How It's Used, and Example: https://www.investopedia.com/terms/r/randomwalktheory.asp

Sparre, M. R. (2012, November 29). *Finanskrisen kunne blitt enda dypere uten oljefondet*. Retrieved from Dagens Næringsliv: https://www.dn.no/-finanskrisen-kunne-blitt-enda-dypere-uten-oljefondet/1-1-1918045

Spurgin, R. (2001). How to Game Your Sharpe Ratio. *The Journal of Alternative Investments* , 38-46.

Stock, J. H., & Watson, M. W. (1999). Forecasting Inflation. *Journal of Monetary Economics*, pp. 293-335.

Svetunkov, I. (2022). *Forecasting and Analytics with ADAM.*

The Ministry of Finance. (2021, May 11). *Investeringsstrategi*. Retrieved from https://www.regjeringen.no/no/tema/okonomi-og-budsjett/statens-pensjonsfond/statens-pensjonsfond-utland-spu1/investeringsstrategi/id696849/

Thorburn, K. S. (2017, April 3). *NHH Bulletin* . Retrieved from https://www.nhh.no/nhh-bulletin/artikkelarkiv/2017/april/oljefondet-bor-forvaltes-passivt/

Treynor, J. L., & Black, F. (1973). How to Use Security Analysis to Improve Portfolio Selection. *The Journal of Business*, 66-86.

Wooldridge, J. M. (2015). *Introductory econometrics: A modern approach.* Cengage Learning.

Ødegaard, B. A., & Dahlquist, M. (2018). *A Review of Norges Bank's Active Management.*

Appendix

The Appendix chapter will follow the outlined structure of the thesis.

4 Methodology for Scenario-Analysis

4.2 ARIMAs for Forecasting

As briefly mentioned in Chapter we leverage ARIMAs to predict future active returns. The following section will explain the in-depth forecasting methodology used in this thesis to create a basis for comparison to discuss future active returns. For this matter, we are using autoregressive integrated moving average models (ARIMAs). The identification of ARIMAs in our analysis is based on Box-Jenkins methodology. In this section, firstly, an introduction to ARIMA is conducted before the Box-Jenkins Method for model identification is outlined.

An ARIMA provides forecasts based on historical variation in an individual time series (Wooldridge, 2015). The model class is a combination of an autoregressive (AR) and moving average model (MA), and thus consists of an autoregressive and a moving average element. When using ARIMAs, a factor for differencing is required, since stationarity in the time series is needed.

Equation 15 outline a general ARIMA for a given stochastic time series y_t:

Equation 15

$$y'_t = c + \emptyset_1 y'_{t-1} + \cdots + \emptyset_p y'_{t-p} + \theta_1 \varepsilon_{(t-1)} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t$$

Given the requirement of stationarity:

Equation 16

$$y'_t = \Delta^d y_t$$

In Equation 15, y'_t is the differenced time series where d is the number of differentiations needed to achieve a stationarity series. The elements on the right-hand side of the equation include both lagged values and errors of y_t (Hyndman & Athanasopoulos, 2018). The model is referred to as an ARIMA (p,d,q) model as explained below.

Table 11 - Elements of an ARIMA

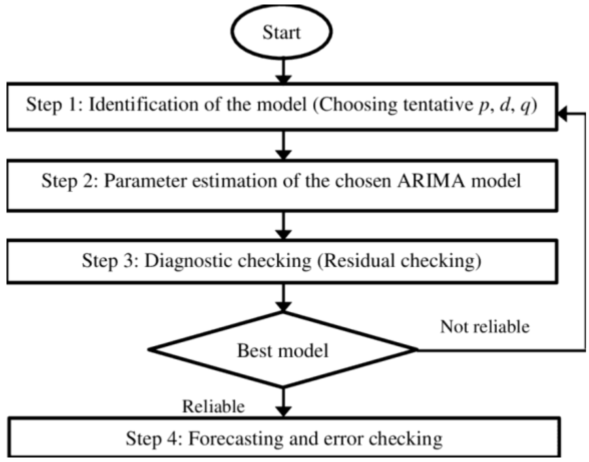
Element	Meaning
p	Value of the autoregressive order, thus number of historical values of y_t with associated coefficient ϕ.
d	Number of differentiations needed to achieve stationarity

q	Value of the moving average order, thus the number of past error terms, with associated coefficient θ

4.2.1 The Box-Jenkins Method for Model Selection

We will use The Box-Jenkins Methodology for model selection to determine the appropriate ARIMAs to conduct our forecast and consequently answering our research question. The Box-Jenkins method is a systematic iterative method for identifying, fitting, checking and using ARIMAs, thus forecasting time series based on lagged values (Box & Jenkins, 1976). The framework consists of different phases, demonstrated in Figure 10:

Figure 10 - Box-Jenkins Method for selecting ARIMA Model



In the following, the steps of The Box-Jenkins framework will be outlined for our analysis. Before commencing on the selection of p and q, in line with the Box-Jenkins framework, stationarity needs to be tested. Thus, testing for stationarity will be the first step.

4.2.2 Testing for Stationarity

A stationary time-series is a time-series where the mean and the standard deviation is constant, and the time-series does not contain any trend component (Hyndman & Athanasopoulos, 2018). If the data is not stationary, inference on the predictive ability of the ARIMA cannot be drawn, and the forecasting can entail spurious results (Wooldridge, 2015). If the time-series are not stationary, the next step would be to difference the time-series. If the time-series is stationary, the component (d) will be zero.

Our approach for securing stationarity is by graphical observation of the time series, correlogram and the augmented Dickey Fuller test. We will briefly explain the Dickey-Fuller test before commencing on the ARIMA selection.

Augmented Dickey-Fuller test

The Augmented Dickey-Fuller (ADF) test is a statistical test testing for stationarity. More specifically, we are testing if our data has a unit root, thus a stochastic trend in the time series (Wooldridge, 2015). If a time series possesses a unit root, it would demonstrate a systematic and unpredictable pattern, and can lead to spurious results. The null hypothesis is that there is a unit root in the data (the data is non-stationary), and the alternative hypothesis states that the data is stationary. The ADF test is rejected if the test statistic is less than the critical value (Wooldridge, 2015).

4.2.3 Step 1: Identification

When stationarity has been determined, the next step of the Box-Jenkins framework is to identify the appropriate autoregressive order (p) and moving average order (q). This is conducted through an analysis of the partial autocorrelation function and the autocorrelation function.

The respective partial autocorrelation and autocorrelation function determines the correlation between lags of a time series. The partial autocorrelation is determined by calculating the partial correlation between the values of two time periods and adjusting out influence of intermediate lags. To specifically determine a reasonable autoregressive order (p), one takes the last lag of the last significant partial autocorrelation that can be observed in the respective function (Hyndman & Athanasopoulos, 2018). To further find the value of the moving average order (q), one uses the autocorrelation function and takes the last lag of the last significant autocorrelation that one can observe in the respective function.

However, as there is no formal test for identifying (p) and (q), multiple models with $p > 0$ and $q > 0$ can be plausible for the forecasting. However, Box & Jenkins (1976) stress the importance of *parsimony* when comparing plausible alternative models, involving selecting the model with the lowest amount of model parameters that provides a sufficient statistical fit. Throughout the thesis, we delimit (p) and (q) to be less than five, given the importance of parsimony.

4.2.4 Step 2: Model Estimation

To find the best statistical fit for forecasting, this thesis relies on five commonly used criterions: (1) the significance of the regression coefficients, (2) sigma squared, (3) log likelihood, (4) the Akaike (AIC) information criterion and (5) the Bayesian (BIC) information criterion. We argue that these five commonly used criterions yield an adequate best fit selection of ARIMAs.

When evaluating these criterions, we firstly aim for significant regression coefficients. Secondly, we investigate sigma squared, which we want to minimize. Further, we want to maximize the log likelihood, while minimizing the AIC and BIC information criterions (D'Amico, 2021). AIC and BIC will be further elaborated on below.

The AIC and BIC are two information criterions proposed in literature (Pierce, u.d). Both criterions share similar objective and is closely related, where the criterions aim to yield the most parsimonious model. Thus, the criterions adjust the Mean Square Error (MSE) by a multiplicative penalty for estimated number of parameters k (Pierce, u.d). The criterions estimate prediction error, where the aim is to minimize in-sample residual sum of squares.

The AIC of Akaike (1973) and BIC of Schwarz (1978) is outlined in Equation 17 and Equation 18:

Equation 17

$$AIC = -2\log(L) + 2(p + q + k + 1)$$

Equation 18

$$BIC = AIC + [\log(T) - 2](p + q + k + 1)$$

From the Equations, BIC contains a higher penalty compared to AIC. Thus, the BIC will more frequently select a more parsimonious model than AIC. In the broad literature, there seems to be discrepancies in the which of the estimations recommended, given their distinctions. It can be shown that BIC is consistent, meaning that when the "correct" model is among the compared models, the probability of choosing this model approached towards 1 in light of increased sample size (Pierce, u.d). However, the AIC is defined as asymptotically efficient, implying that, when the sample size increases, it will choose a sequence of models approaching the "correct" model in the same pace as any comparable criterion (Pierce, u.d). To sum up, none of the two criterions can be considered superior, and it's recommended to use both as complements, instead of substitutes.

In line with the broad literature in the field, we rely on using both criterions as complements when identifying appropriate ARIMAs, to ensure more robust models for conducting our forecasts.

4.2.5 Step 3: Model Diagnostics

The final step before commencing the forecasting is to ensure that our selected model fulfil the requirements for a stable univariate process. For this stage, we need to test if the residuals of the model can be inferred as white noise. If our selected model is an adequate fit, we should not observe any significant residual autocorrelation. In this thesis, we rely on using a portmanteau test for assessing if the residuals can be inferred as white noise.

The classical portmanteau test is introduced by Box & Pierce (1970) and applies to the residuals of a time series when an ARIMA is selected. The test investigates if the group of autocorrelations of a time series are different than zero or not. The null hypothesis is that the time series is white noise thus the selected model does not demonstrate a lack of fit. The alternative hypothesis is that we have serial correlation, thus the model exhibit lack of fit (Box & Pierce, 1970).

The test statistic Q of the portmanteau test is presented in Equation 19:

Equation 19

$$Q = n(n+2)\sum_{j=1}^{m}\left(\frac{r_j^2}{n-j}\right)$$

Here, m is autocorrelations of the residuals (lags included), n is number of observations and r_j is accumulated autocorrelations of residuals. The null hypothesis if rejected if the condition in Equation 20 holds:

Equation 20

$$Q > X_{1-\alpha,h}^2$$

Here, $X_{1-\alpha,h}^2$ is the chi-square distribution for a significance level α and h degree of freedom. Given that the portmanteau test applies to residuals, the degrees of freedom must take into account the parameters from the ARIMA, thus subtracting the number of parameters in the ARIMA model, m, p and q.

4.2 Random Walk

A random walk can be defined as a process where a current value of a variable of interest is equal to the past value and an error term characterised as white noise. A random walk process is denoted in Equation 21:

Equation 21

$$y_t = y_{t-1} + \epsilon_t$$

The implications of the random walk process denoted in Equation 21 is that the best prediction for the variable in next period is the same as the past value, and the change in y is therefore random. The mean of a random walk process is constant, and the variance is not, thus, a random walk process is not stationary. Given a time series random movements, random walk theory emphasizes that it is futile to predict future movements (Smith, Scott, & Munichiello, 2020).

4.2.1 Wald-Wolfowitz Runs test

Throughout this thesis, we will on some occasions test whether a presented time series follows a random walk. To examine this, we use a Wald-Wolfowitz Runs test. Such a runs test is a non-parametric test that tests a random walk hypothesis.

To do a run test, we first divide the dataset into positive and negative monthly return data. We define runs as the number of months in which the given return holds the same negative or positive value. Our null hypothesis is that the monthly data investigated follows a random process. Thus, by rejecting the null hypothesis we find evidence that the data does not follow a random walk.

We perform the runs in Stata and report the test results accordingly.

5. Data Treatment

5.1.2 Evaluating Active Management

When evaluating the active management of the GPFG, we rely on the FTSE Russel All Cap Index. However, some modifications to this benchmark must be done to resemble the benchmark presented to the GPFG from the Ministry of Finance. The specific regional classification used in the benchmark adjustment is elaborated on below.

Benchmark Adjustment

Table 12 - Regional Classification by FTSE Russell elaborates on the regional classification set by FTSE Russel. The outline below, indicates which factors to use for the corresponding country to adjust the FTSE All Cap Index benchmark.

Table 12 - Regional Classification by FTSE Russell

Developed Markets in Europe	Developed Markets in North America	New Countries in the FTSE Index from 2015	Other Developed and Emerging Markets
Austria	Canada	Romania	Brazil
Belgium	United States	Saudi Arabia	Czech Republic
Luxembourg			Greece
Denmark			Hungary
Finland			Malaysia
France			Mexico
Germany			South Africa
Ireland			Taiwan
Italy			Thailand
Netherlands			Turkey
Poland			Chile
Spain			China
Sweden			Colombia
Switzerland			Egypt
United Kingdom			Iceland
			India
			Indonesia
			Kuwait
			Pakistan
			Philippines
			Qatar
			UAE
			Australia
			Hong Kong
			Israel
			Japan
			South-Korea
			New-Zealand
			Singapore

5.1.2 Data for Fund Selection

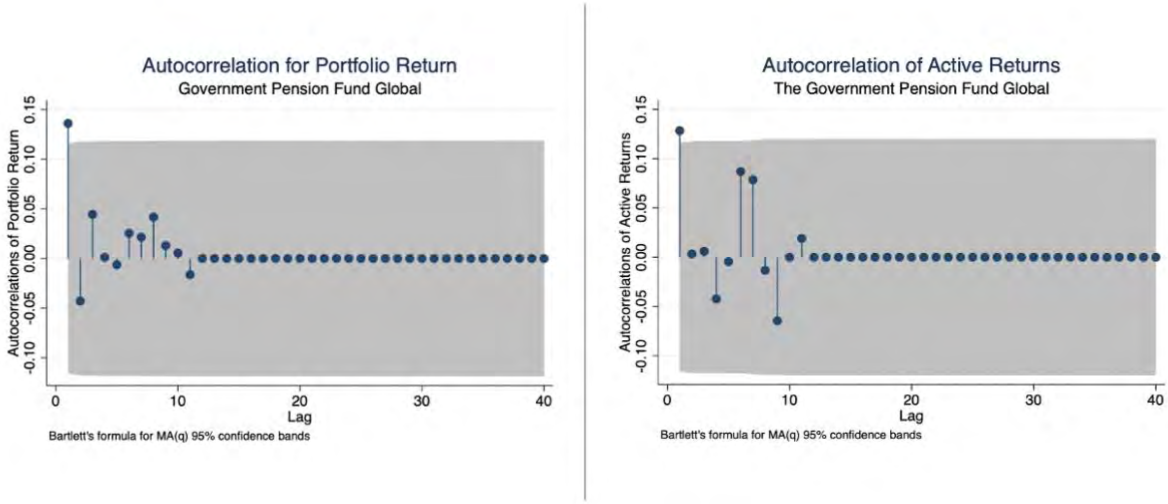
Testing for Independent and Identically Distributed Values

Throughout our thesis, we rely on converting monthly data to either quarterly or annual data. We can perform such a conversion by calculating the product of monthly returns for the desired time period, to find the quarterly and annualized returns. However, a simpler approach is to leverage the monthly average or standard deviation in the desired time period. To convert return data, we multiply by either 4 or 12, respectively for quarterly and annual data. In a similar manner, we multiply by either $\sqrt{4}$ or $\sqrt{12}$ respectively to convert volatility data. However, in order to use this method, the return data must be independent and identically distributed (IID) and we test this by investigating the autocorrelation of the obtained return data.

The Government Pension Fund Global

We present the autocorrelation for both portfolio returns and active returns for the GPFG below. We find one lag outside the 95% confidence band, indicating that we observe some autocorrelation in the data investigate. However, due to the limited extent and for simplicity, we continue with the assumption of IID.

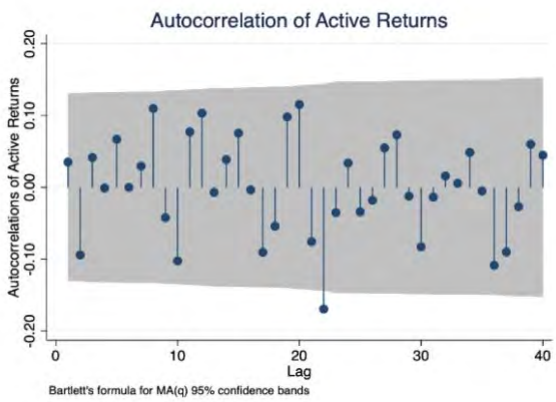
Figure 11 - Autocorrelation for the GPFG

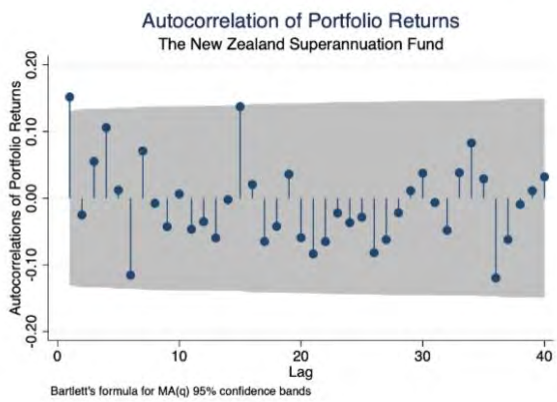


The New Zealand Superannuation Fund

We present the autocorrelation for both portfolio returns and active returns for the New Zealand Superannuation Fund below. We find one lag outside the 95% confidence band, indicating that we observe some autocorrelation in the data investigate. However, due to the limited extent and for simplicity, we continue with the assumption of IID.

Figure 12 - Autocorrelation of the New Zealand Superannuation Fund

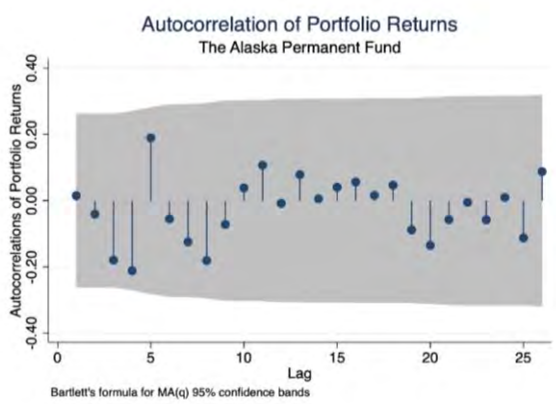


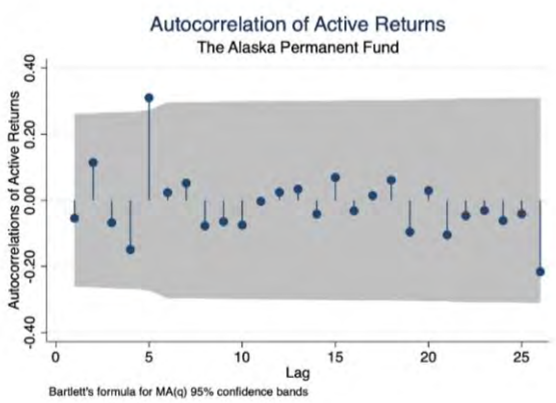


The Alaska Permanent Fund

We present the autocorrelation for both portfolio returns and active returns for the Alaska Permanent Fund below. We find no lags and one lag outside the 95% confidence band respectively, indicating that we observe some autocorrelation in the data investigate. However, due to the limited extent and for simplicity, we continue with the assumption of IID.

Figure 13 - Autocorrelation of the Alaska Permanent Fund





5.2.2 Data Treatment for Synthetic Portfolio Construction

In our main thesis, we leverage extensive data treatment to construct our three synthetic portfolios. The majority of data treatment conducted is related to Portfolio 3 and is further elaborated on below.

Portfolio 3: Based on Predictive Quality

Our third constructed portfolio is based on the predictive quality of each fund. We evaluate each ARIMA used to derive the predicted return every second year for both funds in the analysis, namely the GPFG and the New Zealand Superannuation Fund. We reexamine the current dataset and select the ARIMA model with best fit for the historical values. We limit our analysis to investigate a maximum of 5 components for the p and q each time period, due to parsimony.

The selected ARIMAs used to predict future active returns (which are further used to determine the MSPE, and create the lambda-values for Portfolio 3) are based on different sets of historical values reexamined every second year. The ARIMAs scoped for each time period are elaborated on below, respectively for the GPFG and the New Zealand Superannuation Fund.

The Government Pension Fund Global

Time period: 2003-2007

When evaluating autocorrelation and partial autocorrelation for active return for GPFG in the time period 2003-2007, we find zero lags on autocorrelation and 6 lags on partial autocorrelation. As mentioned above, our selection analysis will be limited to 5 potential models.

Table 13 - Selecting Optimal ARIMA Based on 2003-2007 Historical Values

	A	**B**	**C**	**D**	**E**	
	(1,0,0)	(2,0,0)	(3,0,0)	(4,0,0)	(5,0,0)	**Most satisfactory**
C, AR, MA	1/2	1/3	¼	1/5	1/6	A
SigmaSQ	0.0022303	0.0022255	0.002225	0.0021737	0.0021467	E
Log likelihood	243.7031	243.8188	243.8212	244.9499	245.5235	E
Akaike	-481.4062	-479.6376	-477.6423	-477.8999	-477.0471	A
Bayesian	-475.5525	-471.8327	-467.8861	-466.1924	-463.3884	A
Most satisfactory						**A**

Time period: 2003-2009

When evaluating autocorrelation and partial autocorrelation for active return for GPFG in the time period 2003-2009, we find 1 lag on autocorrelation and 13 lags on partial autocorrelation. As mentioned above, our selection analysis will be limited to 5 potential models.

Table 14 - Selecting Optimal ARIMA Based on 2003-2009 Historical Values

	A (1,0,1)	**B** (2,0,1)	**C** (3,0,1)	**D** (4,0,1)	**Most satisfactory**
C, AR, MA	1/3	2/4	2/5	0/6	B
SigmaSQ	0.0026397	0.0025589	0.002559	0.002622	B
Log likelihood	343.3565	344.8295	344.8297	343.8435	C
Akaike	-678.7131	-679.659	-679.6594	-673.687	C
Bayesian	-669.3901	-668.0054	-668.0057	-657.3719	A
Most satisfactory					**=B,C**

Time period: 2003-2011

When evaluating autocorrelation and partial autocorrelation for active return for GPFG in the time period 2003-2011, we find 1 lag on autocorrelation and 4 lags on partial autocorrelation. As mentioned above, our selection analysis will be limited to 5 potential models.

Table 15 - Selecting Optimal ARIMA Based on 2003-2011 Historical Values

	A (1,0,0)	**B** (1,0,1)	**C** (2,0,1)	**D** (3,0,1)	**E** (4,0,1)	**Most satisfactory**
C, AR, MA	1/2	0/3	1/4	1/5	0/6	A
SigmaSQ	0.002393	0.0023896	0.0023516	0.0023513	0.0023776	D
Log likelihood	461.6038	461.7405	462.6152	462.6479	462.2246	D
Akaike	-917.2077	-915.4809	-917.2304	-913.2957	-910.4492	C
Bayesian	-909.3922	-905.0603	-906.8097	-897.6647	-892.213	A
Most satisfactory						**D**

Time period; 2003-2013

When evaluating autocorrelation and partial autocorrelation for active return for GPFG in the time period 2003-2013, we find 1 lag on autocorrelation and 2 lags on partial autocorrelation. As mentioned above, our selection analysis will be limited to 5 potential models.

Table 16 - Selecting Optimal ARIMA Based on 2003-2013 Historical Values

	A	B	C	D	
	(1,0,0)	(1,0,1)	(2,0,1)	(0,0,1)	**Most satisfactory**
C, AR, MA	1/2	0/3	1/4	½	=A,D
SigmaSQ	0.0021926	0.002190 8	0.002164 6	0.002190 9	B
Log likelihood	583.2431	583.3391	584.0915	583.3385	B
Akaike	-1160.486	-1158.678	-1158.183	-1160.677	D
Bayesian	-1152.025	-1147.397	-1144.082	-1152.216	D
Most satisfactory					**D**

Time period: 2003-2015

When evaluating autocorrelation and partial autocorrelation for active return for GPFG in the time period 2003-2015, we find 1 lag on autocorrelation and 3 lags on partial autocorrelation. As mentioned above, our selection analysis will be limited to 5 potential models.

Table 17 - Selecting Optimal ARIMA Based on 2003-2015 Historical Values

	A	B	C	D	E	
	(1,0,0)	(1,0,1)	(2,0,1)	(3,0,1)	(0,0,1)	**Most satisfactory**
C, AR, MA	1/2	0/3	3/4	1/5	½	C
SigmaSQ	0.0020689	0.0020682	0.0020197	0.002042	0.0020689	C
Log likelihood	704.7167	704.7689	707.2191	705.8659	704.7105	C
Akaike	-1403.433	-1401.538	-1406.438	-1401.732	-1403.421	C
Bayesian	-1394.442	-1389.549	-1394.449	-1386.746	-1394.429	C
Most satisfactory						**C**

Time period: 2003-2017

When evaluating autocorrelation and partial autocorrelation for active return for GPFG in the time period 2003-2017, we find 1 lag on autocorrelation and 3 lags on partial autocorrelation. As mentioned above, our selection analysis will be limited to 5 potential models.

Table 18 - Selecting Optimal ARIMA Based on 2003-2017 Historical Values

	A	**B**	**C**	**D**	**E**	
	(1,0,0)	(1,0,1)	(2,0,1)	(3,0,1)	(0,0,1)	**Most satisfactory**
C, AR, MA	1/2	0/3	2/4	1/5	½	=A,C,E
SigmaSQ	0.0019787	0.001978	0.0019502	0.0019501	0.0019784	D
Log likelihood	826.6668	826.7297	828.2719	828.2719	826.6942	=C,D
Akaike	-1647.334	-1645.459	-1646.544	-1644.544	-1647.388	E
Bayesian	-1637.334	-1632.869	-1630.806	-1625.659	-1637.946	E
Most satisfactory						**E**

Time period: 2003-2019

When evaluating autocorrelation and partial autocorrelation for active return for GPFG in the time period 2003-2019, we find 1 lag on autocorrelation and 3 lags on partial autocorrelation. As mentioned above, our selection analysis will be limited to 5 potential models.

Table 19 - Selecting Optimal ARIMA Based on 2003-2019 Historical Values

	A	**B**	**C**	**D**	**E**	
	(1,0,0)	(1,0,1)	(2,0,1)	(3,0,1)	(0,0,1)	**Most satisfactory**
C, AR, MA	1/2	0/3	3/4	3/5	½	C
SigmaSQ	0.0019024	0.0019021	0.0018736	0.0018877	0.0019028	C
Log likelihood	949.7365	949.7601	951.7021	950.9475	949.6962	C
Akaike	-1893.473	-1891.52	-1893.404	-1889.895	-1893.392	A
Bayesian	-1883.639	-1878.408	-1877.014	-1870.226	-1883.558	A
Most satisfactory						**C**

The New Zealand Superannuation Fund

Time period: 2003-2007

When evaluating autocorrelation and partial autocorrelation for active return for NZ in the time period 2003-2007, we find zero lags on autocorrelation and 8 lags on partial autocorrelation.

Table 20 - Selecting Optimal ARIMA Based on 2003-2007 Historical Values

	A (1,0,0)	**B** (2,0,0)	**C** (3,0,0)	**D** (4,0,0)	E (5,0,0)	**Most satisfactory**
C, AR, MA	0/2	1/3	¼	1/5	1/6	B
SigmaSQ	.004362	.0042486	.0042414	.0040498	.0039639	E
Log likelihood	208.8243	210.1431	210.227	212.4523	213.4315	E
Akaike	-411.6486	-412.2862	-410.4539	-412.9045	-412.8629	D
Bayesian	-405.7948	-404.4812	-400.6977	-401.197	-399.2042	A
Most satisfactory						**E**

Time period: 2003-2009

When evaluating autocorrelation and partial autocorrelation for active return for NZ in the time period 2003-2009, we find two lags on autocorrelation and 14 lags on partial autocorrelation.

Table 21 - Selecting Optimal ARIMA Based on 2003-2009 Historical Values

	A (1,0,1)	**B** (2,0,1)	**C** (3,0,1)	**D** (4,0,1)	**E** (5,0,1)	**F** (1,0,2)	**G** (2,0,2)	**H** (3,0,2)	**I** (4,0,2)	**J** (5,0,2)	**Most satisfactory**
C, AR, MA	1/3	1/4	2/5	2/6	0/7	1/4	0/5	3/6	4/7	3/8	I
SigmaSQ	0.005666	0.0054383	0.0054103	0.0053631	0.0053972	0.005507	0.0054322	0.0054004	0.0051097	0.0050483	J
Log likelihood	285.248	288.2837	288.3933	288.7567	288.8151	286.4217	288.3679	288.7704	290.875	291.3882	J
Akaike	-562.496	-566.5675	-564.7866	-565.5135	-561.6302	-562.8434	-564.7358	-563.5407	-567.75	-566.7765	I
Bayesian	-553.173	-554.9138	-550.8022	-551.5291	-542.9844	-551.1897	-550.7514	-547.2256	-551.4349	-548.1306	B
Most satisfactory											**I**

Time period: 2003-2011

When evaluating autocorrelation and partial autocorrelation for active return for NZ in the time period 2003-2011, we find 3 lags on autocorrelation and 13 lags on partial autocorrelation.

Table 22 - Selecting Optimal ARIMA Based on 2003-2011 Historical Values

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	K	L	M	
	(1,0,1)	(2,0,1)	(3,0,1)	(4,0,1)	(5,0,1)	(1,0,2)	(2,0,2)	(3,0,2)	(4,0,2)	(5,0,2)	(1,0,3)	(2,0,3)	(5,0,3)	**Most satisfactory**
C, AR, MA	1/3	1/4	0/5	0/6	0/7	1/4	0/5	2/6	3/7	3/8	3/5	2/6	1/7	K
SigmaSQ	.0058168	.0056579	.0056564	.0056563	.0056209	.0057064	.0056548	.0052882	.0052554	.0052536	.005522	.0052693	.005649	K
Log likelihood	372.7439	375.4475	375.4765	375.4783	376.0554	374.6251	375.4985	379.5243	380.4104	380.4193	377.4633	379.853	375.5943	J
Akaike	-737.4877	-740.895	-738.9531	-736.9566	-736.1107	-739.2502	-738.997	-745.0486	-746.8208	-744.8386	-742.9265	-745.706	-735.1886	I
Bayesian	-727.0671	-727.8692	-723.3221	-718.7205	-715.2693	-726.2243	-723.3659	-726.8124	-728.5846	-723.9972	-727.2955	-727.4699	-714.3472	I
Most satisfactory														**I**

Time period: 2003-2013

When evaluating autocorrelation and partial autocorrelation for active return for NZ in the time period 2003-2013, we find 2 lags on autocorrelation and 8 lags on partial autocorrelation.

Table 23 - Selecting Optimal ARIMA Based on 2003-2013 Historical Values

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	
	(1,0,1)	(2,0,1)	(3,0,1)	(4,0,1)	(5,0,1)	(1,0,2)	(2,0,2)	(3,0,2)	(4,0,2)	(5,0,2)	**Most satisfactory**
C, AR, MA	2/3	1/4	0/5	0/6	4/7	1/4	0/5	4/6	5/7	0/8	I
SigmaSQ	.006601	.0065274	.0065224	.006512	.006445	.0065209	.006519	.0065068	.0064088	.0064444	I
Log likelihood	446.5565	447.9292	448.0159	448.2054	449.4101	448.0465	448.0685	448.2704	449.9481	449.4194	I
Akaike	-885.1131	-885.8584	-884.0317	-882.4108	-882.8202	-886.093	-884.137	-882.5408	-883.8963	-880.8388	F
Bayesian	-873.8319	-871.757	-867.11	-862.6688	-860.258	-871.9916	-867.2154	-862.7988	-861.334	-855.4562	F
Most satisfactory											**I**

Time period: 2003-2015

When evaluating autocorrelation and partial autocorrelation for active return for NZ in the time period 2003-2015, we find 3 lags on autocorrelation and 4 lags on partial autocorrelation.

Table 24 - Selecting Optimal ARIMA Based on 2003-2015 Historical Values

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	
	(1,0,1)	(2,0,1)	(3,0,1)	(4,0,1)	(1,0,2)	(2,0,2)	(3,0,2)	(4,0,2)	(1,0,3)	(2,0,3)	(4,0,3)	
C, AR, MA	2/3	0/4	2/5	0/6	1/3	0/4	3/6	5/7	2/5	0/6	0/8	H
SigmaSQ	.0064678	.0064302	.0061775	.0064115	.0064211	.0064208	.0064134	.0062934	.0064108	.0064105	.0061783	C
Log likelihood	536.0207	536.8947	537.1875	537.3047	537.082	537.0924	537.236	539.7237	537.2923	537.2941	540.9029	K
Akaike	-1064.041	-1063.789	-1062.375	-1060.609	-1064.164	-1062.185	-1060.472	-1063.447	-1062.585	-1060.588	-1063.806	E
Bayesian	-1052.052	-1048.803	-1044.392	-1039.629	-1049.178	-1044.202	-1039.491	-1039.47	-1044.601	-1039.608	-1036.831	A
Most satisfactory												**A**

Time period: 2003-2017

When evaluating autocorrelation and partial autocorrelation for active return for NZ in the time period 2003-2017, we find 3 lags on autocorrelation and 2 lags on partial autocorrelation.

Table 25 - Selecting Optimal ARIMA Based on 2003-2017 Historical Values

	A	**B**	**C**	**D**	**E**	**F**	
	(1,0,1)	(2,0,1)	(1,0,2)	(2,0,2)	(1,0,3)	(2,0,3)	**Most satisfactory**
C, AR, MA	3/3	¼	2/4	1/5	4/5	1/6	A
SigmaSQ	.0061724	.0061462	.006139	.0061376	.0061139	.0061236	E
Log likelihood	630.9919	631.7286	631.9241	631.9505	632.5953	632.3038	E
Akaike	-1253.984	-1253.457	-1253.848	-1251.901	-1253.191	-1250.608	A
Bayesian	-1241.394	-1237.72	-1238.111	-1233.016	-1234.306	-1228.575	A
Most satisfactory							**A**

Time period: 2003-2019

When evaluating autocorrelation and partial autocorrelation for active return for NZ in the time period 2003-2019, we find 2 lags on autocorrelation and 2 lags on partial autocorrelation.

Table 26 - Selecting Optimal ARIMA Based on 2003-2019 Historical Values

	A	**B**	**C**	**D**	
	(1,0,1)	(2,0,1)	(1,0,2)	(2,0,2)	**Most satisfactory**
C, AR, MA	1/3	2/4	2/4	0/5	B,C
SigmaSQ	.0059905	.0059585	.005956	.0059558	A
Log likelihood	724.924	725.9571	726.0408	726.045	B
Akaike	-1441.848	-1441.914	-1442.082	-1440.09	C
Bayesian	-1428.736	-1425.524	-1425.691	-1420.421	A
Most satisfactory					**A**

7 Scenario-Analysis of Potential Active Returns for GPFG

7.2 Synthetic Portfolios

As emphasized in our main analysis, we aim to predict the future active returns of all synthetic portfolios to create a basis for comparison of future active returns. The first step will therefore be to predict the future active returns of the GPFG and the New Zealand Superannuation Fund. The overview of selecting ARIMAs and the more in-depth scoping of optimal models is elaborated on in 7.2.1 and 7.2.2 respectively.

Secondly, we aim to predict the future returns of the synthetic portfolios. The scoping of optimal ARIMAs for the synthetic portfolios is presented in 7.2.3.

7.2.1 General Overivew of Selecting ARIMAs for the GPFG and the New Zealand Superannuation Fund

As presented in 4 Methodology for Scenario-Analysis, we leverage the Box-Jenkins Method in selecting the appropriate ARIMAs. Given the extent of the Box-Jenkins Method, we delimit the steps into two brief tables. For the thorough process of scoping the ARIMAs, we refer to the Appendix.

Table 27 presents a summary of the identification step of the Box-Jenkins Method, for finding the most suitable ARIMA(p,d,q). We find that the time-series are stationary and leverage the partial autocorrelation and autocorrelation functions respectively to determine p and q. In the Appendix, we present the approach of testing for stationarity, as well as the related partial autocorrelation and autocorrelation graphs.

Table 27 - Model Identification

	(1) GPFG	(2) NZ
Stationary Time Series*	Yes	Yes
Autocorrelation Function (lags)**	3	2
Partial Autocorrelation Function (lags)***	3	2
Number of potential ARIMA models	9	4

**Tested through graphical interpretation, correlogram and Dickey-Fuller test.*
***Lags outside the 95% confidence band for the autocorrelation function, determining q*
****Lags outside the 95% confidence band for the partial autocorrelation function, determining p*

Table 28 presents the model specifications for the selected ARIMAs. The five criterions emphasized in the methodology (significance of the coefficients, sigma squared, log likelihood, Akaike and Bayesian criterions) are used for comparing the relevant models found through the lags in the autocorrelation and partial autocorrelation function. Table 28 also presents the results from the Portmanteau test, in line with our presented methodology, for testing for a stable univariate process, required for selecting an ARIMA. We refer to the Appendix for the Portmanteau test.

Table 28 - Model Specifications for Selected ARIMAs

	(1) GPFG	(2) NZ
Selected ARIMA	**(2,0,3)**	**(1,0,2)**
Significant coefficients*	2/6	2/4
SigmaSQ	0.002001	0.0058779
Log Likelihood	1373.597	817.8356
AIC	-2733.194	-1625.671
BIC	-2707.577	-1608.703
Portmanteau test, Prob>Chi 2(40)	0.8254**	0.5985

**Number of significant coefficients at 5% level*
***Cannot reject the null hypothesis that the residuals of the model can be inferred as white noise*

From Table 28 we pursue with an ARIMA (2,0,3) for GPFG and an ARIMA (1,0,2) New Zealand Superannuation Fund. These models will serve as basis of comparison when discussing future implications of increased active management.

7.2.2 Detailed Approach for Selecting ARIMAs for GPFG and New Zealand Superannuation

To estimate a model for GPFG and New Zealand Superannuation Fund for forecasting future active return, we use Out-of-Sample estimation in STATA. We aim to select appropriate ARIMAs that fits the time-series and the past return values. Then, we can utilize the ARIMA model to predict future active returns for the funds based on its past values.

The Government Pension Fund Global

Model Selection

As emphasized in Chapter 4; Methodology we rely on the Box-Jenkins Method to find and select the adequate ARIMA-model that best fit the past values of the time series in question. The Box-Jenkins method provide three steps towards finding the optimal time-series model: (1) Identification, (2) Model Estimation and (3) Model Diagnostics. We will in the following use these steps for finding the optimal ARIMA model for forecasting active returns of GPFG.

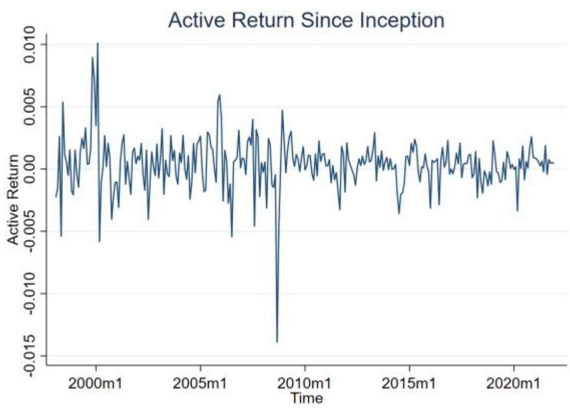
Step 1: Identification

Firstly, we are testing if the time series of the active return of the Fund is stationary or not, to be able to determine if we need to take the difference of our time series to achieve stationarity.

When testing for stationarity we use three different approaches as outlined in Chapter 4: (1) Graphical observation, (2) Correlogram and (3) Formal Tests.

Figure 14 demonstrates the active return of the Fund from February 1998 until December 2021. As illustrated, there is no clear trend in the active return of the fund during the years at scope, where the active returns fluctuate. This indicates a stationary time series.

Figure 14 - Graphical Interpretation GPFG



When supplementing the graphical observation with a correlogram (a summary of the correlation at different periods of time (autocorrelation)), we can observe values tending to degrade to zero, indicating a stationary time series. We also supplement with a Dickey-Fuller test for unit root, illustrated in Table 29.

Table 29 - Dickey Fuller test

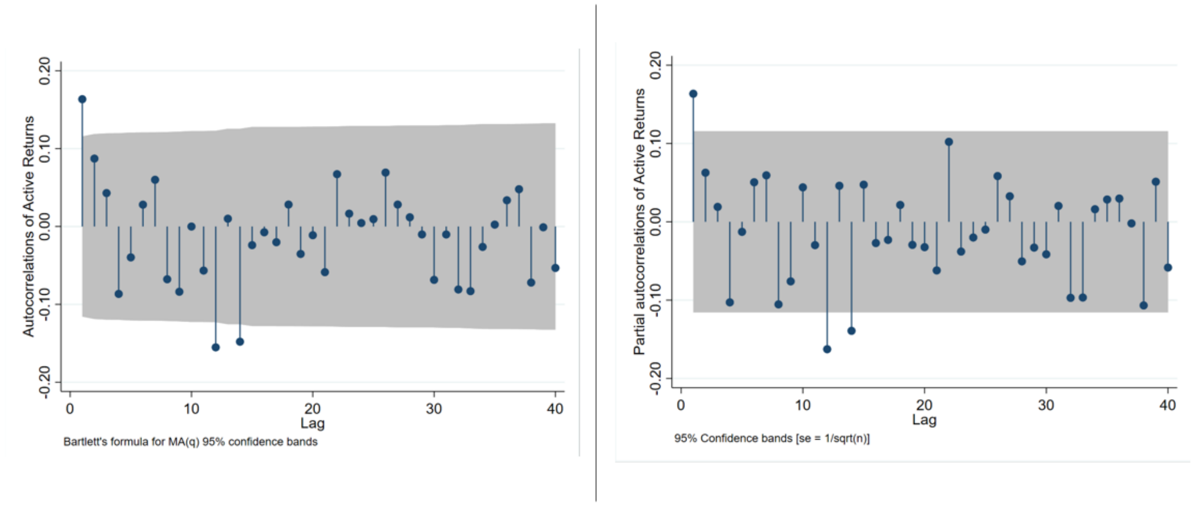
Dickey-Fuller test for unit root H0: Random walk without drift, d = 0				Number of obs = 219 Number of lags = 0
	Test Statistic	----------------------Critical Value-----------------		
Dickey Fuller		1%	5%	10%
Z(t)	-14.330	-3.457	-2.879	-2.57

MacKinnon approximate p-value for Z(t) = 0.0000.

The p-value of the Dickey-Fuller test is equal to zero. We can reject the null hypothesis that the time series has a unit root. Therefore, our time series of active returns is stationary. Given stationarity, we do not need to difference our time series to achieve stationarity, and the (d) component of our ARIMA-model is zero. The next step is to determine the order of the AR (p) and the MA component (q) of the model.

For determining (p) and (q) we use the autocorrelation and the partial autocorrelation function, illustrated in Figure 15. To determine the order of (q), we use the lags exceeding the 95% confidence band of the autocorrelation function, and for determining (q), we use the lags exceeding the 95% confidence band of the partial autocorrelation function. There are three lags outside the confidence bands of 95%, indicating that (q) and (p) could be either 1, 2 or 3.

Figure 15 – Autocorrelation & Partial Autocorrelation Function



To sum up the identification part of the Box-Jenkins Method, we have 9 potential ARIMA models that can fit our dataset.

Step 2: Model Estimation

As emphasized in the Methodology, we rely on five different parameters when evaluating which model is most satisfactory: the significance of the regression coefficients, sigma squared, log likelihood and the Akaike and Bayesian criterions.

We plot the results for every parameter for each of the potential ARIMAs. We denote them model A-I and can see the results in the left column of the table below. For the sigma squared parameter and the Akaike and Bayesian criterions, a low value is desirable. For the log likelihood parameter, a high value is desirable.

We find that model F performs best across the estimation parameters. This indicates that and ARIMA model with $p = 2, d = 0, q = 3$ is the best fit model for our active return dataset.

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	
	(1,0,1)	(1,0,2)	(1,0,3)	(2,0,1)	(2,0,2)	(2,0,3)	(3,0,1)	(3,0,3)	**Most satisfactory**
C, AR, MA	2/3	3/4	2/5	1/4	3/4	2/6	3/4	4/7	=B, E, G
SigmaSQ	0.0020801	0.0020595	0.0020532	0.0020793	0.0020595	0.002001	0.0020548	0.001993	I
Log likelihood	1365.07	1366.894	1367.606	1365.182	1366.894	1373.597	1367.437	1373.522	F
Akaike	-2722.14	-2723.788	-2723.212	-2720.364	-2723.788	-2733.194	-2724.873	-2731.043	F
Bayesian	-2707.502	-2705.491	-2701.256	-2702.066	-2705.491	-2707.577	-2706.576	-2701.768	F
Most satisfactory									**F**

Step 3: Model diagnostic

In line with the presented methodology in chapter 4, we need to test the requirements for a stable univariate process to continue with the selected ARIMA. Table 30 illustrates the results from the Portmanteau test.

Table 30 - Portmanteau Test for White Noise

Portmanteau test for white noise H0: Residuals are white noise	
Portmanteau (Q) statistic	31.6182
Prob>Chi2(40)	0.8254

With a p-value $> 0{,}05$, we cannot reject the null hypothesis of the residuals being white noise. We therefore move forward with the $ARIMA$ $(2{,}0{,}3)$.

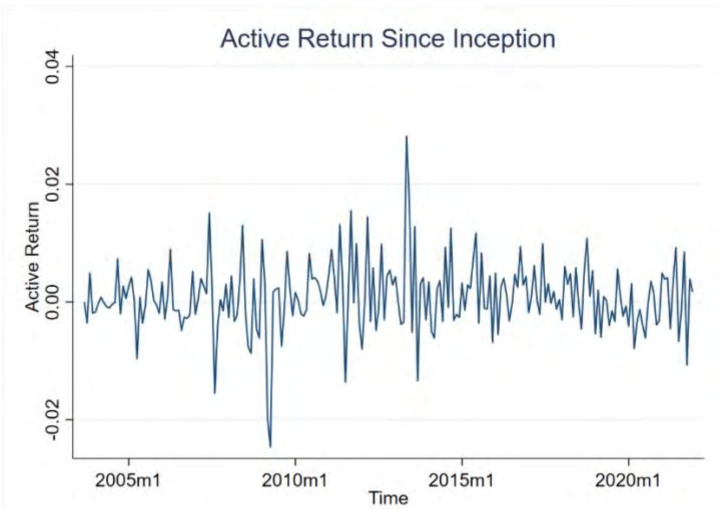
The New Zealand Superannuation Fund

We will in the following select an appropriate ARIMA for forecasting active returns of New Zealand Superannuation Fund.

Step 1: Identification

Figure 16 demonstrates the active return of the fund from September 2003 until December 2021. As illustrated, there is no clear trend in the active return of the fund during the years at scope, where the active returns fluctuate. This indicates a stationary time series.

Figure 16- Graphical Interpretation NZ



When supplementing the graphical observation with a correlogram (a summary of the correlation at different periods of time (autocorrelation)), we can observe values tending to degrade to zero, indicating a stationary time series. We also supplement with a Dickey-Fuller test for unit root, illustrated in Table 31.

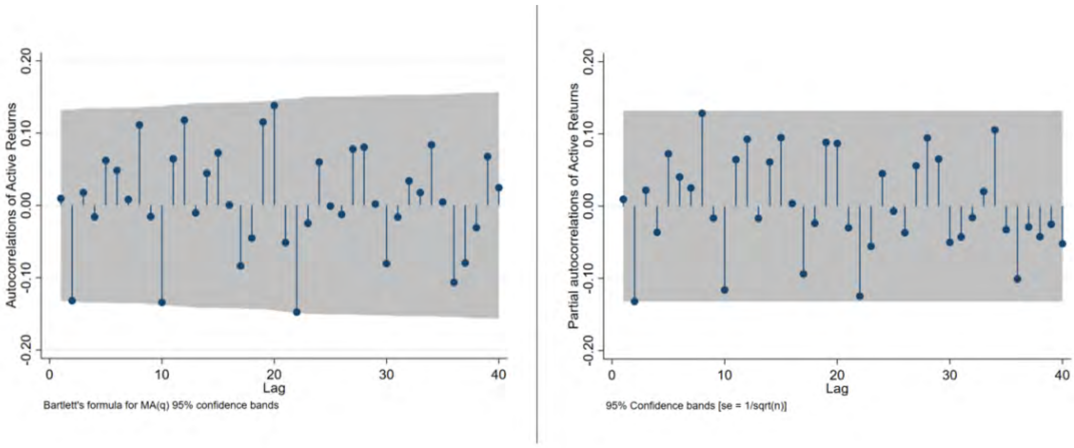
Table 31 - Dickey Fuller Test

Dickey-Fuller test for unit root H0: Random walk without drift, d = 0				Number of obs = 219 Number of lags = 0
	Test Statistic	----------------------Critical Value-----------------		
Dickey Fuller		1%	5%	10%
Z(t)	-14.592	-3.470	-2.882	--2.572
MacKinnon approximate p-value for Z(t) = 0.0000.				

The p-value of the Dickey-Fuller test is equal to zero. We can reject the null hypothesis that the time series has a unit root. Therefore, our time series of active returns is stationary. Given stationarity, we do not need to difference our time series to achieve stationarity, and the (d) component of our ARIMA-model is zero. The next step is to determine the order of the AR (p) and the MA component (q) of the model.

For determining (p) and (q) we use the autocorrelation and the partial autocorrelation function, illustrated in Figure 17. To determine the order of (q), we use the lags exceeding the 95% confidence band of the autocorrelation function, and for determining (q), we use the lags exceeding the 95% confidence band of the partial autocorrelation function. There are three lags outside the confidence bands of 95%, indicating that (q) and (p) could be either 1, 2 or 3.

Figure 17 - Autocorrelation & Partial Autocorrelation Function of NZ



To sum up the identification part of the Box-Jenkins Method, we have 4 potential ARIMAs that can fit our dataset.

Step 2: Model Estimation

We denote them relevant models from A-D and can see the results in the table below. We find that model B performs best across the estimation parameters. This indicates that an ARIMA model with $p = 1, d = 0, q = 2$ is the best fit model for our active return dataset.

	A (1,0,1)	**B** (1,0,2)	**C** (2,0,1)	**D** (2,0,2)	**Most satisfactory**
C, AR, MA	1/3	2/4	1/4	1/4	B
SigmaSQ	0.0059318	0.0058779	0.0058809	0.0058786	B
Log likelihood	815.8675	817.8356	817.7473	817.8378	D
Akaike	-1623.735	-1625.671	-1625.495	-1623.676	B
Bayesian	-1610.16	-1608.703	-1608.526	-1603.314	A
Most satisfactory					**B**

Step 3: Model diagnostic

Table 32 illustrates the results from the Portmanteau test. With a p-value $> 0{,}05$, we cannot reject the null hypothesis of the residuals being white noise. We therefore move forward with the $ARIMA$ (1,0,2).

Table 32 - Portmanteau Test for White Noise

Portmanteau test for white noise H0: Residuals are white noise	
Portmanteau (Q) statistic	37.1666
Prob>Chi2(40)	0.5985

7.2.2 Comparison of Synthetic Portfolios and GPFG

Portfolio 1: Finding the optimal ARIMA

After constructing a synthetic portfolio based on excess return, Portfolio 1, we aim to find the optimal ARIMA model for future prediction. As we test for stationarity, partial autocorrelation and autocorrelation, we find the possible components of p=3, d=0 and q=0. That provides us with three possible ARIMA models for Portfolio 1. We find the optimal model by leveraging the Box-Jenkins method.

Table 33 - Portfolio 1: Selecting Optimal ARIMA

	A	**B**	**C**	
	(1,0,0)	(2,0,0)	(3,0,0)	**Most satisfactory**
C, AR, MA	1/2	1/3	¼	A
SigmaSQ	0.0044129	0.0044058	0.0044004	C
Log likelihood	880.9416	881.2809	881.5576	C
Akaike	-1755.883	-1754.562	-1753.115	A
Bayesian	-1745.702	-1740.987	-1736.147	A
Most satisfactory				**A**

As presented in the table above, we find that model A(1,0,0) is the most satisfactory model for the active return of Portfolio 1.

Testing for Stationarity and Random Walk

Furthermore, as presented in the Methodology, we aim to test whether both the historical values and the scoped ARIMA for the synthetic portfolio follows a random walk process. We leverage a Wald-Wolfowitz test and present the findings below.

Table 34 - The Augmented Dickey Fuller Test for Portfolio 1

Dickey-Fuller test for unit root H0: Random walk without drift, d = 0				Number of obs = 219 Number of lags = 0
	Test Statistic	-----------------------Critical Value------------------		
Dickey Fuller		1%	5%	10%
Z(t)	-15.466	-3.470	- 2.882	-2.572

MacKinnon approximate p-value for Z(t) = 0.0000.

Table 35 - Wald-Wolfowitz Runs Test for Portfolio 1

	ARIMA (1,0,0)
Number of Runs	2
Z(t)	10.58
p-value	0

Portfolio 2: Finding the optimal ARIMA

The autocorrelation and partial autocorrelation function only provides us with p=2 and p=1 respectively for the 12-month moving average and 24-month moving average model. Further, d=0 and q=0. Thus, there is no necessity to provide a comprehensive comparison of the selected model and the optimal model is presented in the thesis.

Testing for Stationarity and Random Walk

Table 36 - The Augmented Dickey Fuller Test for Portfolio 2

Dickey-Fuller test for unit root
H0: Random walk without drift,
d = 0

Number of obs = 219

Number of lags = 0

	Test Statistic	-----------------------Critical Value------------------		
Dickey Fuller		1%	5%	10%
Z(t)	-14.813	-3.471	- 2.882	-2.572

MacKinnon approximate p-value for Z(t) = 0.0000.

Portfolio 3: Finding the optimal ARIMA

Furthermore, we aim to find the optimal ARIMA model for the third constructed portfolio. As before, we leverage the Box-Jenkins method and test for stationarity, autocorrelation and partial autocorrelation. We find the components p=4, d=0 and q=1, which provide us with six possible ARIMA models. As presented in the Table below, we find that the optimal ARIMA is C(2,0,1).

Table 37 - Portfolio 3: Selection Optimal ARIMA

	A (1,0,0)	B (1,0,1)	C (2,0,1)	D (3,0,1)	E (4,0,1)	F (0,0,1)	Most satisfactory
C, AR, MA	1/2	0/3	¾	1/5	3/6	½	C
SigmaSQ	0.0015668	0.0015667	0.0015165	0.0015442	0.0015299	.0015754	C
Log likelihood	846.6474	846.6597	850.8421	849.062	849.8596	845.742	C
Akaike	-1687.295	-1685.319	-1693.684	-1686.124	-1687.719	-1685.484	C
Bayesian	-1677.923	-1672.824	-1681.188	-1667.38	-1668.975	-1676.112	C
Most satisfactory							**C**

Testing for Stationarity and Random Walk

Table 38 - The Augmented Dickey Fuller Test for Portfolio 3

Dickey-Fuller test for unit root — Number of obs = 219
H0: Random walk without drift, d = 0 — Number of lags = 0

	Test Statistic	----Critical Value----		
Dickey Fuller		1%	5%	10%
Z(t)	-9.601	-3.488	-2.886	-2.576

MacKinnon approximate p-value for Z(t) = 0.0000.

Table 39 - Wald-Wolfowitz Runs Test for Portfolio 3

	ARIMA (2,0,1) **With 0**
Number of Runs	35
Z(t)	-6.72
p-value	0

7.4 Robustness Analysis

7.4.1 Alaska Permanent Fund

In the robustness analysis we aim to substantiate the findings from our main analysis. Thus, we also need to scope the optimal ARIMAs for the portfolios constructed in the robustness analysis, namely as a combination between the GPFG and the New Zealand Superannuation Fund. The scoping of the ARIMAs is presented below.

Portfolio 1: Finding the optimal ARIMA

Table 40 – Portfolio 1: Selecting Optimal ARIMA

	A (1,0,0)	**B** (2,0,0)	**C** (3,0,0)	**D** (4,0,0)	**E** (5,0,0)	**Most satisfactory**
C, AR, MA	1/2	1/3	0/4	1/5	1/6	A
SigmaSQ	0.0065342	0.0064353	0.0064349	0.0061763	0.0060119	E
Log likelihood	195.0347	195.829	195.8312	197.8158	199.0787	E
Akaike	-384.0694	-383.658	-381.6624	-383.6316	-384.1574	E
Bayesian	-378.1025	-375.7021	-371.7175	-371.6977	-379.2345	E
Most satisfactory						**E**

Portfolio 2: Finding the optimal ARIMA

Table 41 – Selecting Optimal ARIMA

	A (1,0,0)	**B** (2,0,0)	**C** (3,0,0)	**Most satisfactory**
C, AR, MA	1/2	1/3	0/4	A
SigmaSQ	0.0043051	0.0042003	0.0041921	C
Log likelihood	217.5623	218.8402	218.9403	C
Akaike	-429.1245	-429.6804	-427.8806	B
Bayesian	-426.1576	-421.7244	-417.9375	A
Most satisfactory				**A**