Norwegian School of Economics
Bergen, Fall 2022

The Smart Way to Heat

Analysing the Performance of Smart Heating Compared to Established Heating Practices in Norwegian Homes

Ingvild Kvaal & Hanne Grethe Tysnes

Supervisor: Mario Guajardo

Master thesis, Economics and Business Administration

Major: Business Analytics

NORWEGIAN SCHOOL OF ECONOMICS

This thesis was written as a part of the Master of Science in Economics and Business Administration at NHH. Please note that neither the institution nor the examiners are responsible – through the approval of this thesis – for the theories and methods used, or results and conclusions drawn in this work.

Acknowledgements

This thesis was written as a part of our Master of Science in Economics and Business Administration, with a major in Business Analytics, at the Norwegian School of Economics (NHH). We would like to express our sincere gratitude to everyone who has helped and supported us during the process of writing this master's thesis.

First, we would like to thank our supervisor Mario Guajardo for his guidance, encouragement, and valuable feedback throughout the research and writing process. We especially thank him for inspiring us to create something ourselves.

We would also like to acknowledge professionals within the field of building physics for their helpful discussions and insights. In addition, we would like to thank our friends and family for their support and encouragement while writing this thesis.

Thank you all for your support.

Norwegian School of Economics

Bergen, December 2022

Ingvild Kvaal

Hanne Grethe Tysnes

Abstract

Europe's rapid shift towards renewable energy and electrification has led to a global energy crisis with accelerating power prices. This development has raised our curiosity about whether smart heating can reduce consumer costs and prevent grid congestion.

This thesis explores the performance of smart heating compared to non-smart heating practices in Norwegian homes. A case study of a demo house in Bergen is conducted using a mixed integer linear programming approach aiming to minimise cost. The interplay between technical building standards, climate, and electricity price fluctuations is considered. Furthermore, the performance of heating practices is evaluated based on total cost and electricity consumption. The study also considers two scenario analyses, which investigate the impact of building standards and price volatility on the performance of heating practices.

Findings from the base case show that there is room for improvement in the heating behaviours in Norwegian homes. Smart heating reduces the total electricity cost and avoids grid congestion by utilising hours of low demand and the building's heat-storing capacity. Although some of the non-smart heating behaviours have lower total electricity consumption, they impose an extensive load on the electricity grid at certain hours.

Findings from the first scenario analysis show that a house's construction standard is crucial for smart heating's ability to heat efficiently. The higher heat loss of a TEK 97 house makes the smart behaviour less effective, indicated by the increased cost per kWh from NOK 1.84 to NOK 1.97. In addition, the second scenario analysis reveals that smart heating is superior during high price volatility, yet maintains a sustainable grid load distribution.

This study conclusively reveals that smart heating is superior to non-smart heating in terms of cost efficiency and societal benefit. Findings show that implementing smart heating in Norwegian homes can save costs for householders while reducing the risk of grid congestion.

Keywords – House Heating, Power Prices, Cost Minimisation, Smart Heating

Abbreviations

AMPL	A Mathematical Programming Language
BRA	Utility floor space, refers to the area within the surrounding walls of the dwelling, in Norwegian named “bruksareal”
DiBK	Norwegian Building Authority
DUT	Designed outdoor temperature
GHI	Global horisontal irradiance
GTI	Global tilted irradiance
IEA	International Energy Agency
K	Kelvin
kWh	Kilowatt per hour
NCCS	Norwegian Centre for Climate Services
NS-EN 12831	Norwegian Standard - Energy performance of buildings - Method for calculation of the design heat load
NS-EN 15251	Norwegian Standard - Indoor environmental input parameters for design and assessment of energy performance of buildings addressing indoor air quality, thermal environment, lighting and acoustics
NS3031	Norwegian Standard - Calculation of Energy Performance of Buildings
NVE	The Norwegian Water Resources and Energy Directorate
OED	Ministry of Petroleum and Energy
Pa	Pascal
SSB	Statistics Norway
TEK	Technical requirements for construction works
W	Watt
Wh	Watt per hour

Contents

List of Figures

List of Tables

1 Introduction

1.1 Scope of Research

The increasing energy consumption and large-scale transformation towards green energy production in Europe have placed great emphasis on the effectiveness of electricity consumption. This is because a reliable supply of electricity is crucial for modern society. Furthermore, an energy crisis is emerging as renewable power production is unpredictable and geopolitical uncertainty rages in the scenery, causing power prices to accelerate. Consequently, the global energy crisis has postponed the goal of achieving universal and affordable access to energy (IEA, 2022). This development has raised our curiosity about whether energy consumption in Norwegian homes can be carried out in a way that is beneficial for consumers' private economy but also for society.

The global energy crisis has intensified the need for effective energy consumption. One way to address the issue is by managing electricity demand through consumption adaptation, which involves avoiding power consumption during peak hours and shifting consumption to off-peak hours. Large consumption peaks can lead to power deficits, expensive grid upgrades and reliance on expensive imports (Buvik et al., 2022). Therefore, an important question is whether heating habits in Norwegian homes utilise their full potential in terms of cost while simultaneously avoiding the socio-economic consequences of grid congestion. To investigate the issue, this thesis seeks to analyse the performance and potential of smart heating compared to established heating behaviours.

Lars Myhre, technical manager at Boligprodusentenes Forening, thinks the potential for smart heating in modern buildings is large.

> *"In modern, well-insulated homes, the potential to push heating should be large. Earlier, we talked about night-lowering of the temperature, now we are talking more about raising the temperature at night when electricity is cheaper and living on the stored heat the following day"*
>
> - Lars Myhre, technical manager in Boligprodusentenes Forening

The largest share of electricity consumed in Norway is consumed by private households (SSB, 2022c), whereas heating accounts for 67 percent of this consumption (NVE, 2022a). Historically, people have heated their homes without much reflection on cost because of the affordable electricity prices (Wolff et al., 2017). However, with the recent record-high prices, it is of great interest to study if the price adaptation of a smart heating strategy can resolve the information imbalance between the non-smart strategies and grid operators, as the non-smart strategies are unable to process price information. By comparing the performance of the smart heating strategy to typical non-smart heating strategies, we can gain insights into whether smart heating can benefit consumers and society. To obtain comparable results, we have utilised methods within business analytics to formulate an optimisation model.

We intend to study the performance of an optimised smart heating strategy based on cost minimisation and compare it to established non-smart strategies with respect to cost and electricity consumption - can smart heating benefit consumers and society?

A case study has been constructed to account for the significant variations in climate, power prices, and building standards within Norway. The case study consists of a retrospective analysis using historical data from the past year to calculate and optimise the heating strategies. The results for each strategy will be compared in terms of total cost and electricity consumption. The model is formulated using mathematical programming and essential concepts within building physics.

1.2 Structure of the Thesis

The outline of the thesis is divided into eight sections. Section 2 provides background information on the Norwegian electricity grid, heating demand patterns, and building regulations. Section 3 describes the problem and the different heating strategies we will study. Section 4 explains the methodology used to explore the issue, and section 5 covers the computation and implementation of the data in the optimisation model. In section 6, we summarise and discuss our findings, and section 7 discusses the validity of the results. Finally, our concluding remarks are presented in section 8.

2 Background

This section is divided into three parts. The first part provides an overview of the Norwegian electricity grid, including electricity production and infrastructure, market structure, bidding zones, and regulations. The second part involves an overview of heating demand and regulations in private Norwegian households, including demand patterns, energy efficiency, regulations on technical requirements and building regulation history. Lastly, the literature review summarises previous research in the field of smart heating in homes.

2.1 The Norwegian Electricity Grid

2.1.1 Grid Infrastructure and Electricity Production

The Norwegian electricity grid is operated and managed by Statnett, which is responsible for reliable and well-functioning power transmission (Statnett, 2022). Maintaining balance in the power grid is crucial to avoid electricity surplus or deficit as it is difficult to store electricity over time (Ministry of Petroleum and Energy, 2021b). The balance is maintained by an electricity grid connecting consumers and producers through high-voltage power lines. Accordingly, this network is a necessary infrastructure and must handle large fluctuations in production and consumption (Ministry of Petroleum and Energy, 2019b).

The Norwegian electricity grid is divided into five price zones, or bidding zones, because the weather heavily influences the balance of supply and demand between different parts of the country. The variation in climate causes variable power production, and the current grid capacity cannot align differences in supply and demand between the bidding zones, causing a difference in spot price (Statnett, 2022).

Norway stands out from other European countries as the resource base for energy production depends on the annual precipitation and hydropower production capacity. Other countries are secured through fossil fuels bought in the energy market and used in thermal power plants (Ministry of Petroleum and Energy, 2021a). Hydropower accounts for 85.7 percent of all power production in Norway, while the remaining 14.3 percent is generated from thermal, solar, and wind power (SSB, 2022b). Storing water in large reservoirs facilitates flexible and stable hydropower production, making this resource the most reliable renewable power source.

2.1.2 The Power Market

Norway is part of a large power market in Europe, Nord Pool, which safeguards the supply capacity and provides an efficient and secure day-ahead and intraday market (Nord Pool, 2022a). In other commodity markets, prices are determined by the microeconomic concept of supply and demand. However, as the flow of electricity requires instant balance, the microeconomic concepts will only hold partially.

To ensure a balance between production and consumption, Nord Pool facilitates power traders to participate in a bidding process where bids, offers, and the capacity of each bidding area determine the price per hour on the following day (Nord Pool, 2022a). The prices are divided into area-specific prices and a system price applicable to the Nordic region. The area-specific prices are affected by the transmission capacity between the areas, whereas congestion in transmission will lead to differences in price. On the other hand, the Nordic system price is an unconstrained reference price determined after all area prices are established (Nord Pool, 2022b).

Despite not having fossil fuels in the power production mix, Norwegian power prices are influenced by European gas and coal prices. This is because the market is connected through international oversea power lines, and Europe depends on expensive gas-fired power generation to balance out variability in weather-dependent renewables (Volue, 2022). In other words, European electricity prices will reflect the oil and gas prices when renewables are insufficient to satisfy power demand.

2.1.3 Future Power Market

The future power market in Europe is characterized by electrified consumption and fossil energy sources being replaced by renewables (Buvik et al., 2022). Accordingly, Buvik et al. (2022) states that power prices are expected to become more volatile in the short term. In addition, climate policies and technology will be decisive for the future power market in the longer term (Birkelund et al., 2021).

We are moving towards an increasingly weather-dependent power system with a tighter power balance in northern Europe (Buvik et al., 2022). The significant growth in power consumption in Norway is expected to be driven by electrification, particularly in power-intensive industries and transport. As a result, power demand is expected to increase more than the available production. Thus, a power deficit and grid congestion can occur when it is cold with little wind and a generally high-power demand over several days. Grid congestion refers to a situation in which the capacity of the electrical grid is exceeded by the demand for electricity. From being an energy-dimensioned system with a significant excess of power, Norway's power system is gradually moving towards becoming an effect-dimensioned system like the rest of Europe. Better use of the electricity grid and flexibility in existing and new consumption can be decisive in ensuring the national power balance (Buvik et al., 2022).

Historically, power prices in Norway have changed following variations in the water influx to hydropower plants (Birkelund et al., 2021). However, the prospect of power prices suggests increased volatility due to the expectation that renewable, weather-dependent sources will replace nuclear and coal power plants. Especially the share of power generation from solar and wind is expected to rise significantly because of the high cost of fossil power production (IEA, 2021). The repercussion of such change in the production mix is high power prices when renewable energy is insufficient and low during appropriate weather conditions (Birkelund et al., 2021).

Technological development is crucial to adopt cheaper and more efficient ways of producing renewable energy and improving batteries for energy storage (Birkelund et al., 2021). Furthermore, such development is essential to reach the EU's net zero 2050 goal, which calls for a massive expansion of power production to meet the needs of the growing global economy. Such an expansion includes the electrification of fossil-fuel consumption and hydrogen production from electrolysis (IEA, 2021). This will require better utilisation or further development of the power grid within and between European countries (Birkelund et al., 2021).

2.1.4 Regulations

2.1.4.1 Monopolistic Grid Operation

In Norway, the electricity grid is operated in a monopolistic nature as it is not socio-economic effective to build competing power networks. Consequently, the sector is subject to wide-ranging regulations to prevent power firms from capitalizing on their monopoly position (Ministry of Petroleum and Energy, 2019a). For example, if a firm intend to build, own, or operate a power production site, it must be granted a license by the authorities. Such a license imposes a responsibility to ensure sufficient capacity, development, and maintenance. In addition, the grid operators have revenue limits to incentivise an effective operation. Each consumer is bound to their local grid operator and must pay the monthly network tariff for connecting to the network (Ministry of Petroleum and Energy, 2019a).

2.1.4.2 Network Tariff

The cost associated with having electricity transferred to a house and the operation and improvements of the electricity network is financed by a network tariff. The tariff design is determined by the authorities and reflects the cost of efficient operation, development, and maintenance of the grid. However, the local network company decides the exact charges according to the cost associated with operating the local grid (NVE, 2022c).

The current design of the network tariff was introduced 1st of July 2022 and consists of a fixed and a variable component. The fixed component, called the capacity link, is determined by the maximum power consumed simultaneously (BKK, 2022). Therefore, the more a consumer uses simultaneously, the more must be paid in tariff charges. As of 2022, the monthly fixed tariff rates for private household ranges from NOK NOK 125 to NOK 781 based on their maximum kWh consumption. Essentially, the fixed component motivates consumers to spread their consumption throughout the hours of the day.

The variable component of the network tariff is called the energy link and is a time-differentiated cost added to the price per kWh (BKK, 2022). The variable tariff rates are higher during the day than during the night and weekends, rewarding people who use electricity during off-peak hours. Overall, the current tariff model aims to incentivize consumers to utilize the capacity of the electricity grid throughout the day.

2.2 Heating Demand and Building Regulation

In the following section, we will look at the Norwegian heating demand patterns, energy efficiency and technical building regulations.

2.2.1 Demand patterns

Norway is one of the top consumers of electricity in the world per capita (Statista, 2022). The largest share of electricity is consumed by private households, followed by power-intensive manufacturing and construction (SSB, 2022c). Historically, electrical heating has been the most affordable as it is associated with low cost and low maintenance. The low electricity cost has formed a habit of ample use of space heaters. Causing electric heating to serve as much as 80 percent of the demand for heating in private households. The remainder is mainly covered by biofuel, which includes wood firing (NVE, 2022d).

Natural fluctuations in demand for heating throughout the year can impose a significant impact on grid load. In Norway, the maximum grid load usually occurs in connection with colder periods as a large share of the heating is based on electricity (Europower, 2020). The colder periods typically happen during the heating season, which is a general term for the period of the year when the need for heating is prominent, and the risk of grid congestion is significant. In Norway, this season stretches from October to April, which is considerably longer than in most European countries (ECMWF, 2022).

Consumption of electricity for heating also varies throughout the day as a result of weather conditions and occupancy patterns. Demand typically decreases during midday when sun irradiance and outdoor temperature are at their highest. Contrarily, demand increases in the afternoon when residents are home and sun irradiance and outdoor temperature are low (Sæle,hanne, 2021). Changes in desired indoor temperature throughout the day also affect the daily demand patterns. A survey of Norwegian households' electricity habits by SINTEF and CICERO Sæle et al. (2022) shows that 4 out of 10 Norwegian homes have automatic power control. The most common forms of governance are control systems for day/night lowering of room temperature (20.9%), technology for control of certain appliances (13.6%) and smart houses (6.2%).

2.2.2 Energy Efficiency

Energy efficiency is about utilizing the available energy in the best possible way, thereby helping reduce the need for power (NVE, 2022b). Moreover, reducing power peaks through energy efficiency is a measure of improving the power balance (Buvik et al., 2022). New technologies, such as applications and smart devices, can contribute to energy efficiency when planning energy use according to fluctuations in power consumption and prices (Johannessen, 2019). There is often a significant cost associated with installing new energy-efficient solutions. However, it can be a long-term investment contributing to a reduction in electricity costs and use. NVE estimates that the potential for increased energy efficiency in buildings corresponds to a 10 percent reduction in Norway's electricity consumption. The most cost-effective measures recommended by NVE are lowering the temperature at night, re-insulating cold ceilings, and measures on ventilation and energy-efficient lighting equipment (NVE, 2022b).

The Norwegian government is increasing the attention towards energy efficiency for private households (Regjeringen, 2022). The aim is to facilitate for more people to reduce their energy use and costs. Espen Barth Eide, the Minister of climate and environment, stated, "we must use the energy we have as efficiently as possible. Therefore, energy efficiency is an important priority in work to achieve our climate goals" (Regjeringen, 2022). One of the measures implemented is economic support through Enova for several energy efficiency measures in private households, such as smart power management, upgrading the building envelope and installing solar panels (Enova, 2022b). A smart power control system uses price information to push electricity use to times of the day with lower electricity prices without compromising comfort (Enova, 2022a). Such systems are often utilised in conjunction with, for example, the hot water tank, charging of vehicles and other flexible consumption sources. Upgrading the building envelope refers to stricter and newer building regulations to reduce heat loss, which reduces the need for power.

2.2.3 Regulations on Technical Requirements for Construction

The Norwegian regulations on technical requirements for construction specify minimum requirements, energy efficiency measures and overall energy requirements for new and older buildings in Norway (Geving, 2021). In addition, the national building regulation ensures that projects are planned, designed, and executed concerning good visual quality, universal design, and technical standards for safety, environment, health, and energy (DiBK, 2017). TEK 17 is the most recent regulation published in 2017, and updates are made frequently. Older regulations are TEK 87, TEK 97, TEK 07 and TEK 10. The date when a building application is submitted determines which regulation must be followed, and, thus, what technical requirements apply to the building. In addition, the law has guidance attached and refers directly to standards or measures specifying more detailed requirements for material and execution.

The building regulations are the most crucial obligation for energy efficiency in buildings and set requirements for the total energy demand of the entire building (Geving, 2021). All buildings must either satisfy the complete net energy requirements in kWh per m^2 heated utility floor space or follow a set of energy-saving measures. Nevertheless, the building regulations provide minimum requirements for leakage figures and thermal insulation of the various building parts that must be met to ensure a minimum quality of the building construction. The required insulation is specified by U-values, or thermal transmittances, which describes how well a structural component of the building transmits heat (Thue, 2019).

2.2.4 Building Regulation History

Since the 1970s, Norway's focus on the environment and environmental protection has increased (Bugge, 2011). This section will consider the development concerning energy efficiency requirements from TEK 87 to TEK 17.

TEK 87 did not focus largely on energy efficiency but stated that a house should be built to promote a good energy economy. Accordingly, average U-value requirements for the different building envelope parts were introduced (DiBK, 1987). In TEK 97, the focus on energy efficiency shifted, appearing more precise and stricter by implementing maximum U-values in addition to the average U-values from TEK 87. Also, energy-saving measures as an alternative for calculating the energy efficiency of a house were introduced (DiBK, 1997).

The TEK 07 regulation wanted to reduce the energy demand for new and refurbished houses by 25 percent (Sintef, 2007). Consequently, this resulted in stricter requirements for U-values, air quality, ventilation systems and temperature regulation. Another factor that affected TEK 07 was the introduction of the EU directive about energy efficiency in buildings which implied that the Norwegian building requirement followed an international development (IEA, 2019). The next regulation, TEK 10, was mainly a re-organisation of the building requirements from 2007, accompanied by stricter standards for energy-saving measures (DiBK, 2016). The newest regulation, TEK 17, makes requirements more precise and relaxed (DiBK, 2017). The relaxation of specific requirements provides more significant opportunities for individual adaptations. Overall, the building requirements are moving towards passive house standards, significantly reducing the use of heating as part of the total energy use.

2.3 Literature Review

This section will provide an overview of the current state of knowledge on smart heating to identify gaps in the available research. We have conducted a comprehensive and critical evaluation of existing research while gaining great inspiration for our thesis. Automated smart home technology has recently increased, with many homeowners adopting these systems to improve their home's energy efficiency and comfort. However, there is limited research on the performance of smart heating compared to established heating patterns.

One of the key themes emerging from the research is the potential for smart heating to improve energy efficiency. Many studies have found that these systems can help homeowners to manage their energy consumption better and reduce overall energy use. For example, a study by Bozchalui et. al Bozchalui et al. (2012) found that a house located in Ontario, Canada saved up to 20 percent on energy costs and a 50 percent reduction in peak demand by using automated decision-making technologies in smart grids at residential energy hubs. The researchers presented a mathematical mixed integer linear programming problem aiming to minimise energy consumption, the total cost of electricity and gas, emissions, peak load on the grid, or any combination of these objectives while considering end-user preferences. The study also conducted several case studies to evaluate the model's performance. This study provides evidence of smart heating's ability to decrease cost and peak demand. Still, the study is based on houses with private energy sources, such as solar panels. It is possible that a smart heating system may not have the same effect on energy efficiency in homes without access to private electricity production.

Another study by de Oliveira et al. (2013) proposes a method for optimising a house heating system in a scenario with fluctuating energy prices. The study suggests a dynamic optimisation model based on energy and mass balances, whereas the objective is to minimise energy costs over an infinite horizon. Findings from the study revealed that in a scenario with fluctuating energy prices, the economic benefit of using real-time dynamic optimisation schemes is considerable. The method uses a moving horizon approach aiming to capture important trends adding predictions of temperature and power prices as noise variables. Consequently, the model is optimised based on an algorithm created with estimates of unknown variables of future power prices and outdoor temperature. However, the study was conducted in 2013, and the recent enlarged power price fluctuations might make future power prices more challenging to model.

Ali et al. (2014) propose a linear programming approach to optimise the demand response of electrical heating with partial storage technology. The researchers aim to minimise consumers' total energy costs without compromising comfort by combining demand response control of direct electrical heating and partial thermal storage. The model optimises according to dynamic electricity prices by shifting power consumption from peak periods to low-peak hours. The optimisation model depends on predictable electricity prices, such as the day-ahead prices in the Nordic power markets. The researchers verified their model with simulations and found that the linear programming model reduced the simulated house's total energy cost. Moreover, the study also found that utilising the thermal inertia of a house's mass is advantageous even with relatively small heat storage abilities.

One key issue for the smart heating models introduced by Bozchalui et al. (2012), de Oliveira et al. (2013) and Ali et al. (2014) is the lack of standardisation, which makes it difficult to compare the energy performance of different homes. In addition, many homeowners do not fully utilise the energy-saving capabilities of their automated smart heating systems, often due to a lack of understanding or awareness.

Another interesting aspect of smart heating is the combination of smart home systems, big data and machine learning. Machorro-Cano et al. (2020) presents a big data and machine learning-based home automation system to achieve home comfort and energy efficiency

(HEMS-IoT). The researchers use a machine learning algorithm to learn about occupants' home and away and energy consumption patterns and classify houses in relation to energy consumption. A case study, where a smart home was supervised to ensure comfort and reduced energy consumption, was constructed to validate the methods. The study tackles the standardisation issues by aiming to provide personal energy-saving recommendations. However, exact numbers on savings compared to other established heating methods are not provided. In addition, there are concerns about the security and privacy of these systems, as they rely on data transmission and storage, which can be vulnerable to cyberattacks.

This section's reviewed literature and methods have inspired us to write this thesis. However, we recognise that few studies combine crucial components in building energy modelling, standardised consumer behaviour, power grid regulations and power prices to find how an optimised smart heating strategy performs compared to typical heating practices. With this thesis, we will attempt to fill in the gaps in this area of research by optimising a smart heating strategy and minimising total energy costs, aiming to understand whether such technology can benefit consumers and add socio-economic value. Consequently, a mathematical optimisation model is formulated to compare the performance of a flexible heating strategy with typical heating strategies in terms of cost, kWh consumption and the effect on grid load.

3 Problem Description

3.1 About the Problem

As energy prices in Europe continue to rise due to the shift towards renewable energy sources, there is a growing need for efficient electricity use in Norwegian homes. The problem to be studied in this thesis is whether smart heating can benefit consumers and society by managing the increasing and unpredictable cost of home heating (Botnen, 2022).

Home heating can be complex and depends on factors such as building standards, local climate, heating sources, and occupant preferences. To facilitate a sufficient and valid scientific framework to address this issue, we have created a case study of a price-conscious consumer in Bergen, Norway, using a demo house to account for critical and local variations in input data. The case study will compare the performance of smart heating to established heating practices, using historic hourly electricity prices and local climate data to analyse each heating strategy. The optimisation model is designed to be as realistic as possible, incorporating Norwegian building standards and other relevant input values. However, certain assumptions are made to keep the model simple and suitable for future scenarios.

3.2 Heating in Small Family Houses in Norway

In a cold Nordic climate, maintaining a comfortable indoor temperature is essential. About 48 percent of all houses in Norway are single-family homes, requiring a sizeable amount of power to keep the preferred indoor temperature (SSB, 2022a). This case study is limited to only focus on electrical heating as electricity accounts for 80 percent of the energy used for heating in Norwegian private households (NVE, 2022d). Electric heaters, heating floor cables, radiant heating, and heat pumps are all systems of electric heating (Rosvold and Aksdal, 2018). Even though the ratio between added and delivered heat varies somewhat between the systems, this case study looks at the total added kWh with a one-to-one ratio.

Heating in private households is individual and shaped by the occupants' personal preferences and daily routines. Numbers provided by Standard Norge (2020) and the variation in electricity prices retrieved from Nord Pool indicate that people are typically home and awake between 7 a.m. and 9 a.m. and between 4 p.m. and 11 p.m. The optimisation model is formulated to reflect the objectives of the case study and considers this behaviour. The patterns remain whether it is a weekday or weekend, meaning that the case study does not account for any abnormal behaviour during weekends. This is substantiated by a study of heating patterns in English homes, which found that weekend days and weekdays are far more similar in their heating pattern and duration than commonly assumed (Huebner et al., 2013). The described perception of typical Norwegian household behaviour sets the base of the model's schedule for occupancy.

A monthly network tariff fee must be paid to cover the cost of delivering electricity to the demo house. However, the fixed component of the tariff model is excluded from the case study because it corresponds to the maximum total average kWh consumed by a household, and the case study concerns only power consumed for heating. However, the variable component is included and will vary with the time of the day, charging more when electricity is used during the day and less during the night.

A house is delimited by being unable to store heat for extended periods, and requirements in the technical regulations, TEK 17, determine the demo house's energy efficiency. Regardless of the demo house's technical standard, there will be continuous heat loss through the building envelope. If heat is not provided, the indoor temperature will align with the outdoor temperature, violating the indoor temperature requirement. Consequently, the dimension of the house's electrical heating system is an important parameter determined by the dimensional power requirement explicitly calculated for the demo house.

Natural heat is supplied to the demo house by sun irradiance, people, and lighting, while any remaining heat deficiency is covered by electrical heating. According to NS 3031, heated water and equipment should not be included when calculating internal heat gain in private houses and are therefore excluded from the case study. This is because the impact of heated water and equipment on indoor temperature is assumed to be small and difficult to measure, providing unnecessary complexity to the problem (Myhre et al., 2012).

The cold climate in Norway leads to low demand for cooling. This is reflected in the Norwegian technical building requirements, which do not require specific measurements for electrical cooling in private houses. Consequently, the case study is not formulated to provide any cooling to the demo house. However, there are physical measures like sun shading in place to reduce heat gain from solar irradiances when solar radiation is above a certain level.

3.3 Strategies

How occupants manage house heating is diverse. Accordingly, the four different heating strategies to be analysed and compared in the case study will be presented in this section:

1. Strategy constant
2. Strategy night
3. Strategy night/day
4. Strategy flex

The first three strategies represent conventional, non-smart heating practices commonly found in Norwegian households. The fourth strategy, the "flex" strategy, represents smart heating. We aim to compare the strategies in terms of total cost and kWh consumption, first in a base case and then in the scenario analysis. The fixed occupancy schedule described in section 3.2 will be used for all strategies. The strategies are ranged after the degree of complexity to reflect potential investment cost or need for involvement from occupants.

3.3.1 Strategy Constant

The constant strategy sets the thermostat to a target temperature and leaves it there, even if there are no people home for extended periods. This implies that the heater keeps the same desired indoor temperature throughout the day. Accordingly, the amount of kWh used every hour is determined by the difference between the heat loss and the heat gain required to maintain a steady temperature. This strategy requires approximately no involvement from the residents of the demo house and is regarded as a strategy with low complexity.
Degree of complexity: Low

3.3.2 Strategy Night

The night strategy tolerates a lower temperature when people are sleeping. This implies keeping a desired temperature during daytime and allowing the temperature to drop during the night from 10 p.m. until 6 a.m., which is the same time slot as the variable network tariff. The temperature requirement for day temperature is set one hour before residents wake up as it takes some time to raise the temperature in the house. The night strategy suggests a moderate degree of complexity because the occupants either need to invest in heating solutions with the ability of time control or manually lower the temperature at night.
Degree of complexity: Moderate

3.3.3 Strategy Night/day

The night/day strategy involves lowering the temperature during the night, from 10 p.m. until 6 a.m., and during the day while occupants are away, from 9 a.m. until 3 p.m. The temperature is set to the desired level one hour before occupants wake up or return home, as it takes some time to raise the temperature in the house. Like strategy night, strategy night/day suggests moderate complexity.
Degree of complexity: Moderate

3.3.4 Strategy Flex

Strategy flex performs smart house heating by pushing heating to periods when the electricity price is low, without compromising the indoor temperature requirement when occupants are home and awake. This is the only strategy that executes smart behaviour. A high degree of complexity is suggested for this strategy as it requires investing in a smart heating system that controls the heat by considering future electricity prices. Tibber is an example of a Norwegian provider that offers smart heating panels and thermostats (Tibber, 2022).
Degree of complexity: High

4 Methodology

This part of the thesis will present the methodology used to study the problem described in section 3. A mathematical optimisation model enables us to include building physics-related constraints and other specific strategy requirements. This approach allows us to find the optimal heating schedule by minimising the cost of the flex strategy. However, for the non-smart heating strategies, the strict requirements for heating result in only one feasible solution, which makes the models independent of the electricity price. This implies that these methods behave more like calculation problems rather than optimisation problems. Despite this, the same methodology and data program are used for practical reasons and to ensure an equal basis for comparison.

First, relevant methodology from building physics is detailed. Furthermore, mathematical programming with a focus on linear programming is introduced. Finally, the case study problem is formulated as a mathematical optimisation model.

4.1 Building Physics

In the following section, relevant building physics and associated equations are presented. This covers the concept of heat balance, which includes heat loss and heat gain, calculating dimensional power requirements, and heat capacity.

The heat balance states that heat supply should equal heat loss to maintain the desired balance temperature (Enova et al., 2011). Thus, with knowledge of a building's total heat loss, subtracting all additional heat from the sun, lighting and equipment, the energy demand for the house's heating system can be calculated (Geving, 2021). In warmer climates or during the summer season, electric power can cool down the building to keep the heat balance under control.

Table 4.1 summarises the factors in the heat balance and how they are affected. The calculation for the different heat losses and supplies will be elaborated in the coming subsections.

Factors in the heat balance:	**Is particularly affected by:**
Heat loss:	
Transmission	U-value, building shape
Infiltration	Airtightness
Ventilation	Ventilation requirement/system, heat recovery
Heat supply	
Lightning	Amount and effect
Persons	Number of persons and their activity level
Equipment	Amount and effect
Sun irradiance	Orientation, shading

4.1.1 Heat Loss

The heat loss from a building occurs in the air exchange through walls, roofs, windows, slabs, and thermal bridges (El Saied et al., 2021). The overall heat loss consists of three components: transmission heat loss, ventilation heat loss and infiltration heat loss. The total heat loss can be calculated by these formulas:

$$H = H^{trans} + H^{vent} + H^{inf} \tag{4.1}$$

$$Heat\ loss = H * \triangle T * t \tag{4.2}$$

Geving (2021)

The heat transfer coefficient H $[W/m2K]$ is the sum of heat loss due to transmission, infiltration, and ventilation loss for every degree Kelvin difference between indoor and outdoor temperature. To find the total heat loss for a house for a given period, the heat transfer coefficient is multiplied by the temperature difference $\triangle T$ and time t (Geving, 2021).

4.1.1.1 Transmission Heat Loss

Transmission heat loss describes the heat loss through structural parts of the building, such as walls, windows, doors, roofs, ground, and thermal bridges (Geving, 2021). Hence, better house insulation will contribute to lower heat loss. The heat transfer coefficient for transmission describes the heat loss through all surfaces for every degree Kelvin and can be calculated by the following formula:

$$H^{trans} = \sum_{n \in N} U_n * A_n + \psi \tag{4.3}$$

Geving (2021)

where U_n $[W/m^2K]$ represents the thermal transmittance, also called “U-value”, and describes the amount of energy lost through a square meter of that material for every degree Kelvin difference in temperature between inside and outside. The area A_n $[m^2]$ relates to the internal surface of the external structures it applies. ψ represents the normalised cold bridge value $[W/K]$.

A cold bridge, also named a thermal bridge, is a part of a heat-insulated building with significantly poorer insulation than the rest of the building, thus contributing to increased heat loss (Thue, 2019). The thermal bridge loss used to be included in the U-values, but regulations after TEK 07 specify normalised cold bridge values per square meters of heated area (Enova et al., 2011).

Temperature difference against earth for walls and floor to the ground should be modified as the ground temperature is more stable than the air temperature (Geving, 2021). Consequently, transmission heat loss to the ground, H^{ground}, can be separated from H^{trans} to account for the correct temperature difference when calculating total heat loss. The remaining transmission heat loss to air and cold bridges can be denoted as H^{out}.

$$H^{trans} = H^{ground} + H^{out} \tag{4.4}$$

Geving (2021)

4.1.1.2 Ventilation Heat Loss

Ventilation heat loss describes the controlled heat loss that occurs in renewing the indoor air through a ventilation system (Enova, et al., 2011). The loss depends on the air change in the building and the efficiency of heat recovery and can be calculated by this formula:

$$H^{vent} = (1 - \beta/100) * C * Q \tag{4.5}$$

Geving (2021)

where β describes the heat recovery efficiency [%], C is the heat capacity of air $[W/m^3K]$, and Q is the air volume flow per hour $[m^3/h]$. The heat capacity of air (C) is constant for relevant temperatures and set to 0.33 kWh/m^3K.

4.1.1.3 Infiltration Heat Loss

Infiltration heat loss refers to a building's heat loss due to air exchange other than air through the ventilation system (Geving, 2021). Heat loss occurs due to uncontrolled air leakages through joints and cracks around windows and doors. The infiltration heat loss can be calculated by this formula:

$$H^{inf} = C * R * V \tag{4.6}$$

where C is the heat capacity of air $[kWh/m^3K]$, R is the number of air shifts per hour, and V is the volume of the house $[m^3]$.

$$R = e * n_{50}(h^{-1}) \tag{4.7}$$

Geving (2021)

The number of air shifts through infiltration, R, depends on the building's airtightness and external wind effects (Geving, 2021). The airtightness can be expressed in terms of the leakage airflow through the building's envelope per hour at a pressure of 50 Pascals $[n_{50}(h^{-1})]$. We add a terrain shielding coefficient e as the real pressure difference due to wind is much lower than 50 Pascal. In documenting energy efficiency concerning TEK 17, it is assumed "moderate" shielding and the associated shielding coefficient is set to 0.07 (Geving, 2021).

4.1.2 Heat Supply

To accurately calculate the electricity required for heating, it is essential to consider the heat supplied naturally by sun irradiance and internal heating sources. The overall heat supply consists of sun irradiance and heat from internal sources such as lights and people.

4.1.2.1 Solar Gain

Direct and diffuse solar irradiance through windows may provide houses with considerable heat. The amount of heat supplied by the sun depends on the strength of the direct and diffuse radiation at any instant and the orientation and angle of the window surface (Barakat, 2008). Global horizontal irradiance (GHI) is the sum of the direct and diffuse radiation and is usually measured on a horizontal plane (Smidsrød et al., 2008). For a vertically positioned window, the global tilted irradiance (GTI) with a 90-degree slope is used to evaluate the accurate irradiance (Gueymard et al., 2008). Diffuse radiation is partly firmament radiation and partly reflection from the surroundings (Smidsrød et al., 2008). Accordingly, snow, sea, clouds, or other reflective surfaces increase the amount of diffuse radiation. The solar gain can be calculated by equation (4.8)

$$Solar\ gain = \sum_{s \in S} I_s * A_s^w * g_s^{tot} \tag{4.8}$$

Larsen (1982)

where I_s is the global tilted irradiance $[W/m^2]$, which includes cloud coverage and air pollution. A_s^w is the area of the windows $[m2]$ and g_s^{tot} is the total solar transmittance factor, abbreviated as “g-value”. These values are computed and summarised for all sky directions s.

$$g_s^{tot} = g_s^{glass} * g_s^{shading} \tag{4.9}$$

Tekna (2021)

An important aspect when calculating solar gains is the absorptivity (g^{glass}) and sun-shading of the glass ($g^{shading}$), denoted by the g-value, g^{tot} (Smidsrød et al., 2008). The absorptivity depends on the number of glass panes, the angle of the solar radiation and the degree of polarization of the incident sunbeam (Barakat, 2008). Solar protection helps reduce the proportion of solar radiation that hits the outside pane and passes to the inside. Outdoor sun shading gives a lower g-value and is preferred to minimise heat gain from the sun effectively. TEK 10 requires the total solar transmittance factor to be less than 0.15 for solar-exposed facades (DiBK, 2016). Conversely, TEK 17 has no minimum requirement but demands measures to prevent overheating, such as external sun shading (DiBK, 2017).

4.1.2.2 Internal Heat Load

Internal heat gain, or internal load, are heat generated from people, lights, and equipment in a house. Heat supply generated from people varies with activity level, age, and gender. Similarly, lightning and equipment provide heat when they are in use. Standard values for internal heat load are measured in W/m^2.

4.1.3 Dimensional Power Requirement for Heating

A house's heating system must provide appropriate heat to maintain the desired indoor temperature regardless of outdoor temperature. This is known as the dimensional power requirement, and it can be calculated using methods from NS-EN 12831 (Standard Norge, 2017). This method uses a winter outdoor design temperature (DUT) to determine the heating system's minimum effect to maintain a comfortable indoor temperature on freezing days. A winter design temperature is the lowest average temperature over three days for a specific area in the last 30-year period. This parameter is included to ensure that the heating system can adequately heat the house on the coldest days (Stene and Øiene Smedegård, 2013).

The dimensional power requirement is calculated by equation 4.10.

$$El^{design} = H * (T^{in} - DUT) \tag{4.10}$$

Enova et al. (2011)

where H is the heat transfer coefficient, T^{in} is the desired indoor temperature, and DUT is the outdoor design temperature for the specific location.

This calculation excludes any sources of natural heat, as the heating system should be able to maintain the desired indoor temperature independently. Some studies recommend adding a safety margin to the calculated power dimension. However, as the winter design temperature is an infrequent observation, this is unnecessary (Hansen, 2016).

For heating behaviours with temperature lowering, the power system must be dimensioned to increase the temperature even on extremely cold days. Thus, the dimensional power requirement should calculate with additional effect, depending on the resident's patience with the temperature rise.

4.1.4 Heat Capacity

When the indoor temperature changes, the heat capacity of the building materials and their impact on the thermal conditions must be considered (Skari, 2016). Heat capacity tells how much energy must be provided to an object to cause a one-unit change in its temperature (Halliday et al., 2021). An essential factor for heat capacity is thermal mass that mass enables different parts of the house to store heat and create internal heat flows that help balances the heat demand (Skari, 2016). Heavier objects have more significant heat-storing properties than lighter objects (Myhre et al., 2012). In periods with heat deficiency, the stored heat will be discharged from the objects and provide heat to the surrounding environment until equilibrium reaches. Consequently, low outdoor temperatures in short periods will not cause a significant shift in power demand because stored heat will provide heat while the room temperature decreases.

Normalised heat capacity expresses the energy $[Wh]$ stored in the construction per square meter of floor area $[m^2]$ per degree Kelvin of temperature change (Myhre et al., 2012). The heat capacity of air is low and will cause significant fluctuations in room temperature. However, adding the heat capacity of furniture, walls, floors, and roof when calculating heat efficiency will provide inertia against substantial changes in temperature and make calculations more realistic.

4.2 Mathematical Programming

Mathematical programming, also known as mathematical optimisation, is one of the best-developed models in operational research and management science (Williams, 2013). Bradley Bradley (1977) defines *mathematical programming* as a mathematical representation aimed at programming or planning the best possible allocation of scarce resources. There must be something variable in the problem that can be controlled or affected by the decision-maker to use optimisation as a tool (Lundgren et al., 2010). Non-linear, integer and linear programming are examples of mathematical programming forms, where the latter is the most widely applied one.

Lundgren Lundgren et al. (2010) defines *optimisation* as the science of making the best possible decision for a specified target with restrictions on the type of decision that can be made. The objective function expresses the objective to be minimised or maximised and depends on the decision variable(s). Furthermore, a set of constraints describes the restriction on the values of the decision variables. Optimisation generates large models and large groups of input data where computer support is required to find the best possible solution.

A particular working approach is preferred when optimisation models analyse a given problem scenario and solve a decision problem for a given application (Lundgren et al., 2010).

1. The optimisation problem is identified and simplified as some aspects of an issue are often too complicated to be included in the optimisation model.
2. The problem is formulated mathematically as an optimisation model with decision variables, objective function, and constraints.
3. The model is solved using an appropriate solution algorithm. Examples of commercial solvers are CPLEX and OSL.
4. Finally, the results are evaluated.

4.2.1 Linear Programming

Linear Programming (LP) is concerned with maximising or minimising a linear expression subject to linear constraints (Vajda, 2009). George B. Dantzig developed the simplex method for solving the general linear-programming problem (Bradley, 1977). This method's computational efficiency and robustness and the availability of digital computers have made linear programming widely applied in the business environment. Following is an LP problem written in general form (Lundgren et al., 2010):

$$\textit{minimise} \;\; Z = \sum_{j=1}^{n} c_j x_j \tag{4.11}$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, \qquad i = 1, \ldots, m \tag{4.12}$$

$$x_j \geq 0, \qquad j = 1, \ldots, n \tag{4.13}$$

The objective function aims to be minimised and includes the coefficient c_j and the decision variable x_j. a_{ij} is the coefficient to x_j in the first constraint, while b_i represents the coefficient on the right-hand side of the same constraint.

A mixed integer linear programming (MILP) problem is a type of LP problem that allows some of the variables to be integer-valued, whereas the objective function and constraints are still in linear form (Chinneck, John W, 2015). This includes when one or more variables are restricted as binary variables that only take values 0 and 1.

4.3 Optimisation Model

The optimisation model is formulated mathematically in the following section with associated sets, parameters, decision variables, objective functions, and constraints. Lastly, we will introduce some unique adjustments for the different strategies.

4.3.1 Sets

First, the sets in the model are defined.

D: Set of all days
H: Set of all hours
S: Set of all directions

The set of days includes all days in a year, while the group of hours consists of all hours in a day, including hour 0. The set of directions includes the four directions in the sky.

4.3.2 Parameters

Parameters for heat loss, heat supply, temperature and electric use are defined.

Heat loss parameters

H^{out}	Heat transfer coefficient to outdoors $[W/K]$
H^{ground}	Heat transfer coefficient to the ground $[W/K]$
H^{inf}	Heat transfer coefficient to infiltration $[W/K]$
H^{vent}	Heat transfer coefficient to ventilation $[W/K]$

Heat gain parameters

g^{glass}	g-value for glass only
$g^{shading}$	g-value for shading only
$Sun_{s,d,h}$	Solar radiation for direction s on day d in hour h [W/m^2]
A^w_s	Window area for direction s [m^2]

G^{light}_h	Heat gain from lights in hour h [kWh]
G^{peop}_h	Heat gain from people in hour h [kWh]
A^{BRA}	Heated utility floor space $[m^2]$

Temperature Parameters

T^{start}	Initial indoor temperature [℃]
T^{goal}	Indoor temperature goal [℃]
$T^{out}_{d,h}$	The outdoor temperature on day d in hour h [℃]
T^{ground}_d	The ground temperature on day d [℃]
θ	Normalised heat capacity $[kWh/K]$

Electric Use Parameters

$El^{price}_{d,h}$	Price of electricity on day d in hour h [NOK]
$Tariff^{var}_h$	Variable network tariff in hour h [NOK]
T^{design}	Dimensional power requirement [NOK]

Big M

M	Big M

4.3.3 Decision Variables

Heat Loss Variables

$Loss^{tot}_{d,h}$	Total heat loss on day d in hour h [kWh]

Heat Supply Variables

$g^{tot}_{s,d,h}$	Total g-value for windows for direction s on day d in hour h
$z_{s,d,h}$	1 if sun shading is on for direction s on day d in hour h, 0 otherwise
$G^{sun}_{s,d,h}$	Heat gain from the sun for direction s on day d in hour h $[kWh]$
$Supply^{tot}_{d,h}$	Total heat gain on day d in hour h $[kWh]$

Temperature Variables

$\triangle T^{air}_{d,h}$	The temperature difference between indoor and outdoors on day d in hour h [℃]
$\triangle T^{ground}_{d,h}$	The temperature difference between indoor and ground on day d in hour h [℃]
$T^{in}_{d,h}$	The indoor temperature on day d in hour h [℃]

Electric use variables

$El^{use}_{d,h}$	Electric use on day d in hour h $[kWh]$

4.3.4 Objective Function

$$min \sum_{d \in D} \sum_{h \in H : h > 0} el^{use}_{d,h} * (el^{price}_{d,h} + Tariff^{var}_{h}) \tag{4.14}$$

The objective function aims to minimise the total cost of electricity, taking both the el-price and the variable network tariff into account.

4.3.5 Constraints

Non-negativity Condition

$$Loss^{tot}_{d,h},\ g^{tot}_{s,d,h},\ G^{sun}_{s,d,h},\ Supply^{tot}_{d,h},\ El^{use}_{d,h} \geq 0, \qquad \forall\ s \in S, d \in D, h \in H \tag{4.15}$$

All the above variables must be greater or equal to 0.

Heat Loss Constraint

$$Loss^{tot}_{d,h} = ((H^{out} + H^{inf} + H^{vent}) * \triangle T^{air}_{d,h} + H^{ground} * \triangle T^{ground}_{d,h})/1000, \forall\ d \in D, h \in H : h > 0 \tag{4.16}$$

4.16 ensures that total heat loss is adjusted according to the difference between indoor and outdoor temperatures. The same adjustment applies to heat loss to the ground.

Heat Supply Constraints

Sun gain

$$M * z_{s,d,h} \geq sun_{s,d,h} - 150, \qquad \forall\ s \in S, d \in D, h \in H : h > 0 \tag{4.17}$$

$$150 * z_{s,d,h} \leq sun_{s,d,h}, \qquad \forall\ s \in S, d \in D, h \in H : h > 0 \tag{4.18}$$

$$g^{tot}_{s,d,h} = g^{glass} - (g^{glass} - g^{glass} * g^{shading}) * z_{s,d,h}, \quad \forall\, s \in S, d \in D, h \in H : h > 0 \quad (4.19)$$

4.17, 4.18 and 4.19 ensure that the total g-value of the windows includes extra shading if the solar irradiance for a specific direction is more than 150W/m2 per hour. Otherwise, the total g-value will only include the regular g-value for glass.

$$G^{sun}_{d,h} = \sum_{s \in S} g^{tot}_{s,d,h} * A^{w}_{s} * sun_{s,d,h}/1000, \qquad \forall\, d \in D, h \in H : h > 0 \qquad (4.20)$$

4.20 ensures that the right amount of sun gain is calculated for each hour of the day by adjusting for the g-value and the area of the windows.

Total Heat Supply

$$Supply^{tot}_{d,h} = G^{sun}_{d,h} + G^{light}_{h} + G^{peop}_{h}, \qquad \forall\, d \in D, h \in H : h > 0 \qquad (4.21)$$

The total heat supply is the sum of heat gain from sun, lighting, and people

Electric Use Constraint

$$El^{use}_{d,h} \leq El^{design}, \qquad \forall\, d \in D, h \in H : h > 0 \qquad (4.22)$$

4.22 ensures that kWh consumption per hour is less or equal to the designed power requirement.

Temperature Constraints

$$\triangle T^{air}_{d,h} = T^{in}_{d,h-1} - T^{out}_{d,h-1}, \qquad \forall\, d \in D, h \in H : h > 0 \qquad (4.23)$$

4.23 ensures that the difference between indoor and outdoor temperature equals the difference between the indoor and outdoor temperature for the last hour. This constraint is necessary to calculate heat loss through walls and roofs correctly.

$$\triangle T^{ground}_{d,h} = T^{in}_{d,h-1} - T^{ground}_{d}, \qquad \forall\, d \in D, h \in H : h > 0 \qquad (4.24)$$

4.24 ensures that the difference between indoor and ground temperature equals the difference between the indoor and ground temperature for the last hour. This constraint is necessary to calculate heat loss through the ground correctly.

$$T^{in}_{d,h} = T^{in}_{d-1,h+24}, \qquad \forall\, d \in D, h \in H : d > 1 \wedge h = 0 \tag{4.25}$$

4.25 ensures that the indoor temperature for hour 0 equals the indoor temperature for hour 24 the day before. This constraint is necessary to ensure the right indoor temperature is transferred from one day to the next.

$$T^{in}_{d,h} = T^{start}, \qquad \forall\, d \in D, h \in H : d = 1 \wedge h = 0 \tag{4.26}$$

4.26 ensures that the initial indoor temperature on the first hour of the first day of simulation equals the start temperature.

$$T^{in}_{d,h} = T^{in}_{d,h-1} + (Supply^{tot}_{d,h} - Loss^{tot}_{d,h} + El^{use}_{d,h})/\,\theta, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.27}$$

4.27 ensures that the indoor temperature is continuously adjusted following heat loss and heat supply. The temperature fluctuation is accounted for by dividing the heat supply and loss by the normalised heat capacity.

4.3.6 Strategy Constant

Variables

$x_{d,h}$	1 if electricity can be used on day d in hour h, 0 otherwise
$Net^{kWh}_{d,h}$	kWh that must be added to reach the desired temperature on day d in hour h

Constraints

$$Net^{kwh}_{d,h} = Loss^{tot}_{d,h} - Supply^{tot}_{d,h} + (T^{goal} - T^{in}_{d,h-1}) * \theta, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.28}$$

4.28 calculates the kWh surplus or deficit to reach the goal temperature before any electricity is added.

$$Net^{kwh}_{d,h} \geq -M * (1 - x_{d,h}), \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.29}$$

$$Net^{kwh}_{d,h} \leq M * x_{d,h}, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.30}$$

4.29 and 4.30 solve the issue of knowing when to provide electrical heat to ensure that the desired indoor temperature is maintained. The binary variable will serve as a switch, turning the heater on or off.

$$El^{use}_{d,h} \leq x_{d,h} * M, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.31}$$

$$El^{use}_{d,h} \geq -x_{d,h} * M, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.32}$$

$$El^{use}_{d,h} \leq Net^{kWh}_{d,h} + (1 - x_{d,h}) * M, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.33}$$

$$El^{use}_{d,h} \geq Net^{kWh}_{d,h} - (1 - x_{d,h}) * M, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.34}$$

4.31, 4.32, 4.33 and 4.34 ensure that electric use equals the exact kWh needed to reach the desired indoor temperature.

4.3.7 Strategy Night

Paramteres

T^{night}	The minimum indoor temperature during night [°C]

Variables

$x_{d,h}$	1 if electricity can be used on day d in hour h, 0 otherwise
$Net^{kWh}_{d,h}$	kWh that must be added to reach the desired temperature on day d in hour h
$l_{d,h}$	1 if the designed power requirement is exceeded to reach the desired temperature on day d in hour h, 0 otherwise

Constraints

$$Net^{kwh}_{d,h} = Loss^{tot}_{d,h} - Supply^{tot}_{d,h} + (T^{night} - T^{in}_{d,h-1}) * \theta, \quad \forall\, d \in D, h \in H : h > 0 \wedge h < 6 \vee h > 22 \tag{4.35}$$

$$Net^{kwh}_{d,h} = Loss^{tot}_{d,h} - Supply^{tot}_{d,h} + (T^{goal} - T^{in}_{d,h-1}) * \theta, \quad \forall\, d \in D, h \in H : h > 5 \wedge h < 23 \tag{4.36}$$

4.35 and 4.36 calculate the kWh surplus or deficit to reach the goal temperature before adding electricity. T^{night} is used as the goal temperature until 6 a.m. and from 11 p.m. as indicated in 4.35, while T^{goal} is used as the desired temperature from 6 a.m. until 11 p.m. as indicated by 4.36.

$$Net^{kwh}_{d,h} \geq -M * (1 - x_{d,h}), \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.37}$$

$$Net^{kwh}_{d,h} \leq M * x_{d,h}, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.38}$$

4.37 and 4.38 solve the issue of knowing when to provide electrical heat to ensure that the desired indoor temperature is maintained. The binary variable will serve as a switch, turning the heater on or off.

$$Net^{kwh}_{d,h} - El^{design} \geq -M * (1 - l_{d,h}), \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.39}$$

$$Net^{kwh}_{d,h} - El^{design} \leq M * l_{d,h}, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.40}$$

4.39 and 4.40 calculate whether the dimensional power requirement of the house is exceeded when aiming to reach the goal temperature.

$$El^{use}_{d,h} \leq x_{d,h} * M, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.41}$$

$$El^{use}_{d,h} \geq -x_{d,h} * M, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.42}$$

$$El^{use}_{d,h} \leq El^{design} + (1 - l_{d,h}) * M, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.43}$$

$$El^{use}_{d,h} \geq El^{design} - (1 - l_{d,h}) * M, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.44}$$

$$El^{use}_{d,h} \leq Net^{kWh}_{d,h} + (1 - x_{d,h}) * M + El^{design} * l_{d,h} * M, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.45}$$

$$El^{use}_{d,h} \geq Net^{kWh}_{d,h} - (1 - x_{d,h}) * M + El^{design} * l_{d,h} * M, \qquad \forall\, d \in D, h \in H : h > 0 \tag{4.46}$$

4.41, 4.42, 4.43, 4.44, 4.45 and 4.46 ensure that that power consumption equals the kWh needed to reach the desired indoor temperature without violating the designed power requirement.

4.3.8 Strategy Night/Day

Strategy Night/day has all the same definitions as strategy Night. The only difference is that constraint 4.35 is defined for all $d \in D, h \in H : h > 0 \wedge h < 6 \vee h > 9 \wedge h < 15 \vee h > 22$ and constraint 4.36 is defined for all $d \in D, h \in H : h > 6 \wedge h < 9 \vee h > 14 \wedge h < 23$

4.3.9 Strategy Flex

$$T^{in}_{d,h} \geq T^{goal}, \qquad \forall d \in D, h \in H : h > 5 \land h < 9 \lor h > 14 \land h < 23 \tag{4.47}$$

The indoor temperature must be equal to or higher than the goal temperature when occupants are home and awake.

5 Data Description

This section will describe the data used to solve the optimisation problem. The model will optimise each heating strategy individually and rely on input data from different sources. Some of the data needs additional pre-processing and computation before implementation. Accordingly, this section is divided into two subsections: data processing and implementation.

5.1 Data Processing

Large parts of the retrieved data are raw and require pre-processing and computation before being introduced to the model. This sub-section will describe and cover the pre-processing performed on the data.

5.1.1 Hourly Day-ahead Power Prices

The hourly historical power prices are retrieved from Nord Pool's data portal (Nord Pool, 2022b). Nord Pool runs the leading power market in Europe and has established a unique portal with historical power market data behind a paywall. Fortunately, Nord Pool has granted us a limited period of free access to support our research.

The retrieved power prices stretch from 1. October 2021, 1 a.m. to 30. September 2022 at midnight. Each observation applies to one hour, making 8 760 observations in total. As AMPL cannot read dates meaningfully, we replaced the dates with numbers ranging from 1 to 365 to represent each day throughout the year. The raw data is measured in NOK/MWh but transformed to øre/kWh by dividing the observations by 10. We did this transformation because the hourly electricity cost is usually measured in øre/kWh. Moreover, as the problem focuses on private households in Bergen, we retrieved the power data from price zone NO5, which relates to the western part of Norway.

5.1.2 Solar Irradiance

The solar irradiance data is collected from the interactive tool of the European photovoltaic geographical information system, particularly from the satellite-based PVGIS-SARAH2 database. The database has an hourly time resolution with a temporal range from 2005 – 2020 (European Comission, 2022). However, the database only provides long-term averages, unable to capture exact daily variance in cloud coverage within each month.

The retrieved irradiance data is global tilted irradiance (GTI) measured on a 90-degree tilted plane in Florida, Bergen. The data is retrieved as daily average irradiance and retrieved for planes facing in the azimuth directions: north, south, east, and west to capture irradiance through windows placed in all celestial directions. The SARAH2 satellite captures cloud coverage and air pollution affecting sun irradiance through albedo and cloud opacity. Albedo is a measure of diffuse reflection of solar irradiance and helps the satellite to separate land surface and cloud coverage. Accordingly, cloud opacity measures the thickness of the detected clouds (Honsberg, 2019). In this way, we consider cloud coverage in the solar irradiance data. An illustration of the hourly data for the sum of all directions for each month is illustrated in appendix A1. The data is downloaded for each month separately and assembled in Excel. Data from different azimuth directions are implemented in a three-dimensional matter in AMPL.

5.1.3 Temperaure Data

Hourly air temperature in Florida, Bergen from 1. October 2021 to 30. September 2022 is retrieved from the Norwegian Centre for Climate Services (NCCS). NCCS provides relevant information about climate change, including weather data and statistics (Norwegian Centre for Climate Services, 2022). Ground temperature is included in the model to capture the difference in transmission loss from the building envelope to the ground. The ground temperature is more stable than the air temperature and fluctuates around the average air temperature for a specific location (Blom, 2006). Accordingly, we use the monthly average air temperature as the outside temperature for the floor against the ground. The air and ground temperature are collected from the weather station SN50540 in Florida, Bergen and assembled in Excel by replacing dates with numbers ranging from 1-365.

5.2 Data Implementation

This section will outline and justify the different input values for the model.

5.2.1 Dimensions of the House

The dimensions of the house are essential input values as it determines the heat loss and heat gain and, thus, how much energy must be supplied to maintain a comfortable indoor temperature.

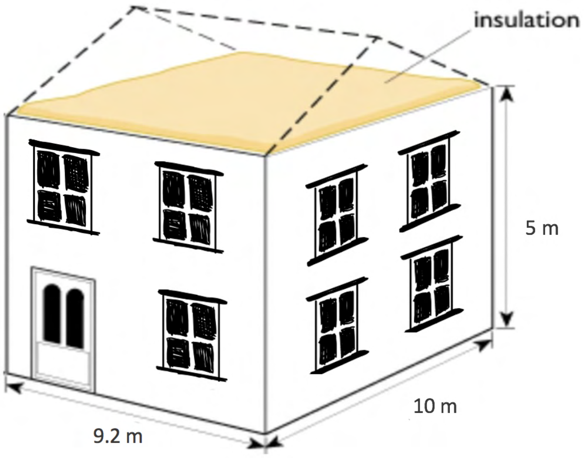


Figure 5.1: Illustration of the demo house

The demo house for this optimisation problem is a single-family house with a utility floor space of 184 m^2 divided between two floors in accordance with the typology of Norwegian residential buildings built after 2011 (Brattebø et al., 2016). The demo house is illustrated in Figure 5.1. Windows and doors are set to less than 20 percent of heated utility floor space, although the requirement is less than 25 percent in TEK 17. The reason for this is to enable us to compare results with TEK 97 standards in the scenario analysis without changing the dimensions of the demo house. In a real case, temperature variations between different rooms will occur. However, the model will simplify this aspect by looking at the demo house as one unit, assuming the same indoor temperature in all rooms. Table 5.1 summarises all relevant dimension input values. The demo house is thought to be located in Florida, Bergen.

Input	***Value***
Heated utility floor space	184 m^2
Length	10 m
Width	9.2 m
Height	5 m
Windows and doors	37 m^2
Ceiling	92 m^2
Floor to ground	92 m^2
Wall surface*	155 m^2
Volume**	414 m^3
Floors	2
Location	Bergen, Florida

* *Wall surface is equal to* $2 * 5 * 10 + 2 * 5 * 9.2 - 37$ m^2 *(windows and doors)* $= 155$ m^2
** *We assume that partition walls and floor dividers make up 10 % of the gross internal volume. Thus, total volume equals* $5 * 10 * 9.2 * 0.9 = 414$ m^3

Table 5.1: Summary of dimensions of the demo house

5.2.2 Building Physics

Minimum energy efficient measures from TEK 17 are used to meet the overall net energy requirements without calculating the overall energy usage per m^2. All U-values, leakage numbers, cold bridge values and temperature efficiency for heat recovery are determined according to these requirements. Table 5.2 summarises these input values.

Input	***Value***
Outer walls	0.18 [U-value]
Roof	0.13 [U-value]
Floor	0.10 [U-value]
Windows and doors	0.80 [U-value]
Heat recovery in ventilation systems	80 %
Air leakage at 50 pa/h	0.6
Normalised cold bridge value	0.05 W/m^2K
Air volume flow	1.2 m^3/hm^2

Table 5.2: Energy efficiency measures TEK 17

Windows

To meet the requirement of the U-value for windows, three-layer panes with gas filling and coating are the most common (Geving, 2021). These windows have a g-value of about 0.53 (Geving, 2021), while outdoor sun shading has a g-value of approximately 0.18 (Tekna, 2021). The window area is assumed to be equally divided on each side of the house. The input values for windows are summarised in Table 5.3.

Input	***Value***
g-value glass	0.53
g-value shading	0.18

Table 5.3: Summary of g-values

Heat Transfer Coefficients

The heat transfer coefficients can be calculated based on the dimensions of the house and energy efficiency requirements in TEK 17. Table 5.4 summarises these calculations.

Heat Transfer Coefficients TEK 17

	U-value	**Area**	**Heat transfer coefficient**			
Outer walls	0.18	155	0.18 * 155 = 27.90	78.66	H^{out}	25.79 %
Roof	0.13	92	0.13 * 92 = 11.96			11.06 %
Windows and doors	0.80	37	0.80 * 37 = 29.60			27.36 %
Cold bridges	-	-	0.05 * 184 = 9.20			8.51 %
Floor to ground	0.10	92	0.10 * 92 = 9.20	9.20	H^{ground}	8.51 %
Ventilation*	-	-	-	14.57	H^{vent}	13.47 %
Infiltration**	-	-	-	5.74	H^{inf}	5.31 %
Total				**108.17**	**H**	**100 %**

* $H^{vent} = (1 - /100) * C * Q = (1 - 0.8) * 0.33 * (1.2 * 184) = 14.57$

** $R = e * n_{50}(h^{(-1)}) = 0.07 * 0.6 = 0.042$, $H^{inf} = C * R * V = 0.33 * 0.042 * 414 = 5.74$

See methodology for details about the calculation

Table 5.4: Heat transfer coefficient calculation TEK 17

Normalised Heat Capacity

Myhre et al. (2012) calculate a suitable normalised heat capacity of 28 Wh/m^2K for an average small family house with two floors. Accordingly, we use this value to describe the thermal inertia of the demo house, which gives a total of 28 $Wh/m^2K*184\ m^2 = 5\ 152\ Wh$. This means that the demo house requires 5 152 Wh to raise the temperature in the house by one Kelvin if there is no heat loss.

5.2.3 Internal Load

The normalised input data for internal load from NS 3031 for Norwegian private houses are displayed in Table 5.5 (Standard Norge, 2020).

Persons	1.5 W/m2
Lightning	1.7 W/m2

Table 5.5: Summary of internal load data

5.2.4 Temperature

It is recommended that the room temperature is set to a minimum of 19 ℃ and kept below 22℃ when electrical heating is required (DiBK, 2017). Thus, we have chosen an indoor temperature of 21 as the optimal indoor temperature for this optimisation model. Moreover, for buildings without cooling, the Norwegian Standard NS-EN 15251 specifies an adaptive temperature model that allows for higher indoor temperatures than 26 ℃ to be accepted when warm outside (Myhre et al., 2012). Bergen's designed outdoor temperature for winter is -10 (NemiTek, 2019). Table 5.6 summarises all chosen temperature values.

Input	***Value***
Optimal indoor temperature	21℃
Designed outdoor temperature, Bergen	−10℃

Table 5.6: Temperature values

5.2.5 Dimensional Power Requirement

We use the temperature data to calculate the dimensional power requirement.

$$EL^{design} = H * (T^{in} - DUT) = 108.17 * (21 - (-10)) = 3\ 353\ W$$

For the night and night/day strategies, the heating system must be dimensioned to maintain the desired temperature and increase the temperature in the morning and afternoon. For this problem, we assume that the heating system must be able to raise the temperature by at least one degree within one hour, which means that 5 152 Wh must be added to the dimensional power requirement.

$$EL^{design} = 3\ 353W + 5\ 152W = 8\ 505\ W$$

5.2.6 Variable Network Tariff

BKK, the local grid operator, decides the network tariff for Bergen, and the current variable costs are displayed in Table 5.7.

Day	***Night/weekend***
49.90	39.90

Table 5.7: Variable network tariff costs.

6 Results

The analysis will present, compare, and discuss the results and findings obtained for each heating strategy. Subsequently, we will conduct two scenario analyses to investigate how the heating strategies will perform in a house built with TEK 97 standards and how the future change in electricity price volatility may affect the behaviour and performance of the strategies. The fixed network tariff is not included in the annual total cost and optimal solution because the rate also depends on other sources of power consumption.

6.1 Analysis of the Heating Strategies

6.1.1 Presentation of The Results

The first part of the analysis will present the results for all strategies individually before they are compared and discussed. The optimisation model is run in AMPL using the CPLEX solver. January is chosen for illustrations, as this month is associated with a significant heating demand.

6.1.1.1 Strategy Constant

The optimal objective function is NOK 12 042, which represents the cost of the constant strategy from October 2021 to October 2022. This corresponds to a total annual electricity use of 6 121 kWh. Figure 6.1 displays the average electrical cost, electrical usage, heat loss, heat gain and indoor and outdoor temperature for January 2022. The grey shadows illustrate the times of the day when occupants are home and awake.

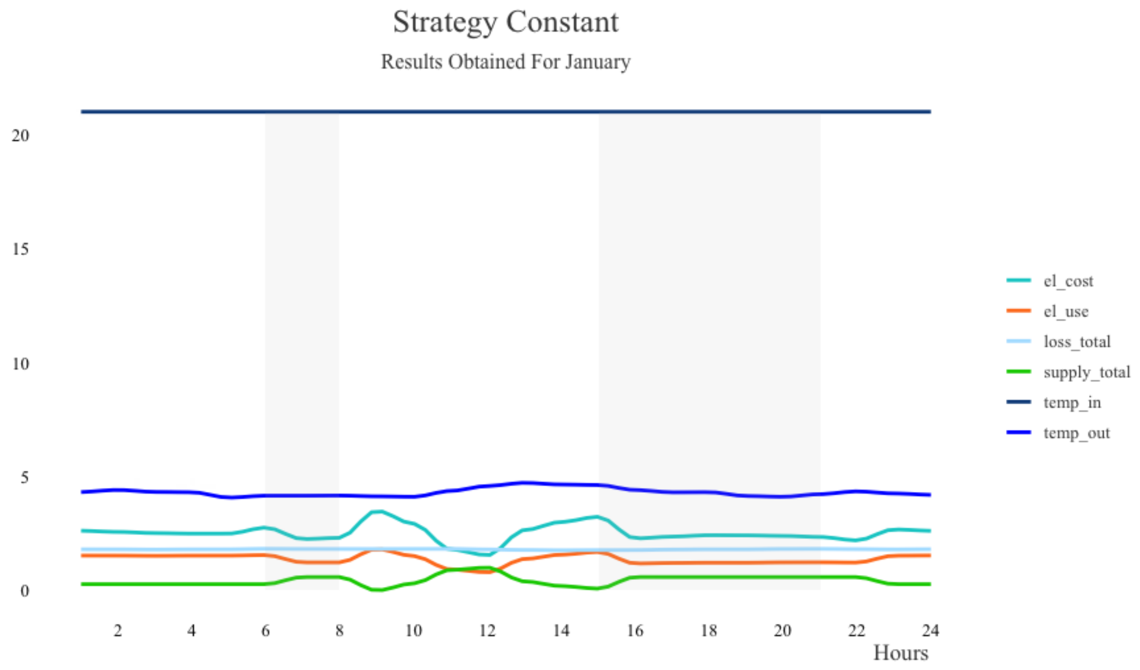


Figure 6.1: Electric cost, electric use, heat loss, heat supply and indoor and outdoor temperature for strategy constant, January 2022

We observe that the electrical use and cost for the constant strategy follow the variation in heat supply closely, indicating that the strategy does not execute any smart behaviour but only responds to the external effects on the indoor temperature. Furthermore, in terms of grid load, the strategy has a relatively uniform consumption pattern accompanied by variations in outdoor temperature and solar gain.

6.1.1.2 Strategy Night

The optimal objective function of the night strategy is NOK 11 936, which represents the cost of the night heating strategy from October 2021 to October 2022. This corresponds to a total annual electricity use of 5 923 kWh. Figure 6.2 displays the average electrical cost, electrical usage, heat loss, heat gain and indoor and outdoor temperature for January 2022. The grey shadows illustrate the times of the day when occupants are home and awake.

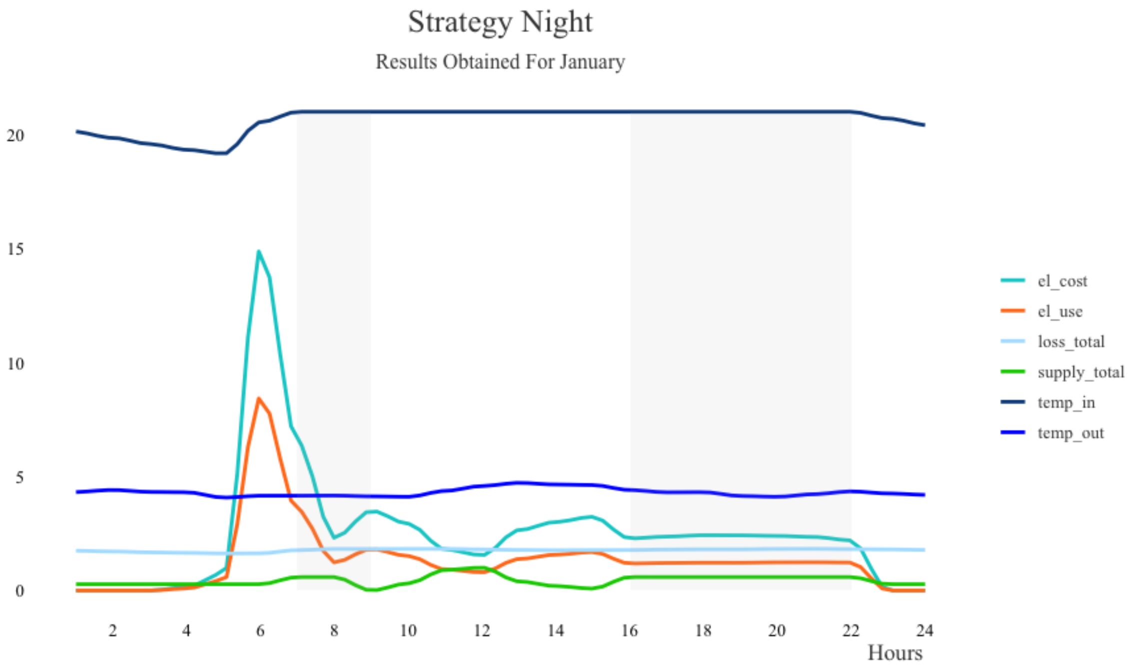


Figure 6.2: Electric cost, electric use, heat loss, heat supply and indoor and outdoor temperature for strategy night, January 2022

We observe that the night strategy's electricity use and cost follow the natural heat supply variation during the daytime. However, the strategy has very little electricity usage during the night but extensive use in the morning when heating is necessary to raise the temperature after night lowering. Like strategy constant, it does not execute any smart behaviour. Still, it responds to the extra heat demand in the mornings and other external effects on the indoor temperature throughout the day. Despite the relatively low annual use of kWh, the night strategy inflicts extra load on the electricity grid in the morning, which does not comply with the network tariffs incentive to spread power consumption.

6.1.1.3 Strategy Night/day

The optimal objective function is NOK 11 620, which represents the cost of the night/day heating strategy from October 2021 to October 2022. This corresponds to a total annual electricity use of 5 848 kWh. Figure 6.3 below displays the average electricity cost, electricity usage, heat loss, heat gain and indoor and outdoor temperature for January 2022. The grey shadows illustrate the times of the day when occupants are home and awake.

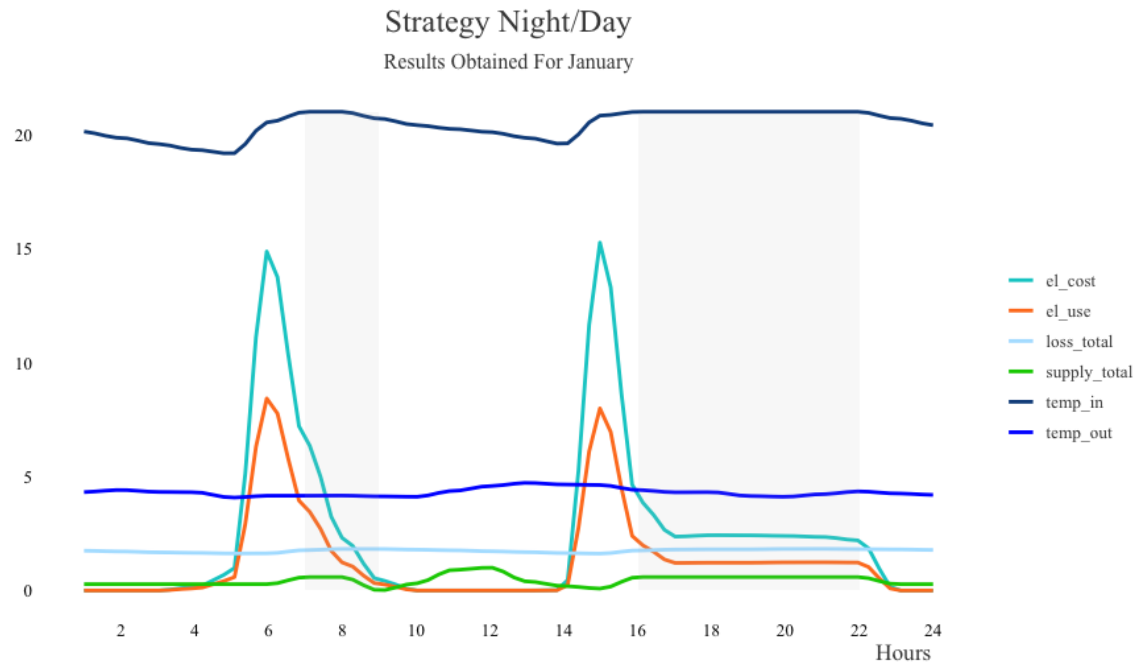


Figure 6.3: Electric cost, electric use, heat loss, heat supply and indoor and outdoor temperature for strategy night/day, January 2022

Unlike the constant and night strategy, we observe that the electricity use and cost of the night/day strategy follow the night and day lowering patterns decided by the occupancy patterns rather than the daily variation in heat supply and outdoor temperature. This pattern is evident through the high morning and afternoon consumption peaks. Despite the low annual use of kWh, the night/day strategy inflicts a considerable load on the electricity grid in the morning and the afternoon when the overall grid load is usually on top. This consumption pattern does not comply with the network tariff's incentive for smarter load distribution.

6.1.1.4 Strategy Flex

The optimal objective function is NOK 11 141, which represents the cost of the flex heating strategy from October 2021 to October 2022. This corresponds to a total annual electricity use of 6 075 kWh. Figure 6.4 below displays the average electricity cost, electricity usage, heat loss, heat gain and indoor and outdoor temperature for January 2022. The grey shadows illustrate the times of the day when occupants are home and awake.

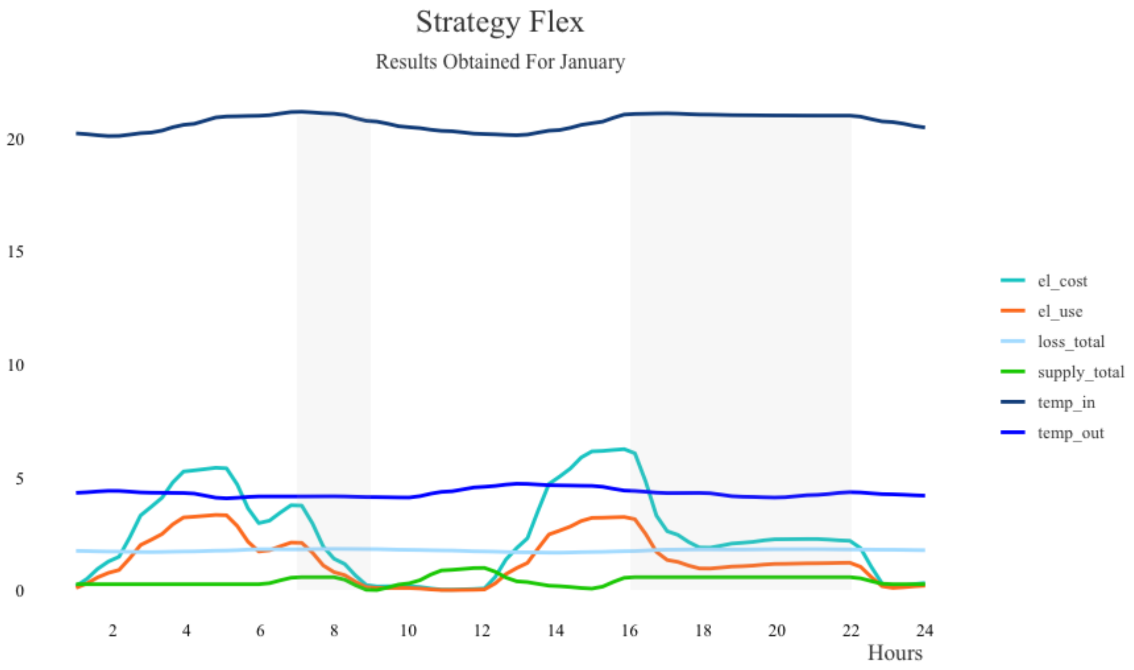


Figure 6.4: Electric cost, electric use, heat loss, heat supply and indoor and outdoor temperature for strategy flex, January 2022

We observe that the flex strategy's cost and electrical use follow the variation in occupancy patterns to ensure desired room temperature when occupants are home and awake. Unlike the night and night/day strategy, it does not use extensive power in the morning and afternoon to heat the house, despite lowering the temperature during night and day. This strategy executes smart behaviour by adapting to the variation in power prices and grid load, which implies that it is likely to start the heating process at night and earlier in the afternoon, or whenever the prices are low.

The strategy utilises the house's heat-storing abilities to keep the house at a comfortable temperature at a low cost during the price peak by providing more heat than necessary in advance when prices are lower. However, the excess heat supplied causes a higher heat loss as temperature differences increase between indoors and outdoors. Even though heat is not needed before 7 a.m., figure 6.4 shows that the flex strategy heats more during the night and less in the morning when prices rise. It is profitable to shift consumption as long as the savings of doing so are greater than the costs of the additional kilowatts that must be supplied to compensate for an increased heat loss. As a result, the strategy has high annual electricity consumption but low cost. However, most of the kWh consumption occurs when the grid load is low, which complies with the incentives of the tariff model.

6.1.2 Comparison of the strategies

In this part of the thesis, we will compare the results obtained for each heating strategy based on cost, annual kWh consumption, and its impact on the power grid. We will consider each strategy's specific requirements and heating schedules to make a fair comparison. For instance, the constant strategy is likely to be more expensive due to its strict constraints and need for a consistently high and stable indoor temperature. On the other hand, the flex and night/day strategies have similar temperature requirements, allowing a direct comparison of their costs and consumption.

Despite their differences, studying the variations in cost, kWh consumption, heating patterns, and impact on the grid load between the flex and the non-smart strategies is interesting. By doing so, we can gain valuable insights into the potential of smart heating and whether it complies with government objectives.

NOK/kWh

Table 6.1 summarises the average cost per kWh consumed by the four heating strategies. The flex strategy has the lowest price per kWh, indicating that heating is most cost-effective when electricity prices are low. The constant strategy also achieves a lower price per kWh than the night and night/day strategy, indicating that the latter two strategies consume more electricity during hours with higher prices. The average cost is calculated by dividing it by its total kWh usage. Even though the flex strategy performs well in terms of cost per kWh, it is also essential to consider its total cost and kWh usage to evaluate its overall performance.

Constant	*Night*	*Night/day*	*Flex*
1.97	2.01	1.99	1.83

Table 6.1: Average NOK/kWh for TEK 17

Total cost and kWh consumption

Figure 6.5 shows each strategy's total cost and kWh consumption from October 2021 to October 2022. The cost difference between the strategies ranges from NOK 11 141 for the flex strategy to NOK 12 042 for the constant strategy. This means that the constant, night, and night/day strategies cost 8.08 percent, 7.13 percent, and 4.29 percent more than the flex strategy. The kWh consumption also varies among the strategies, with the night/day strategy consuming the least at 5 848 kWh and the constant strategy consuming the most at 6 121 kWh.

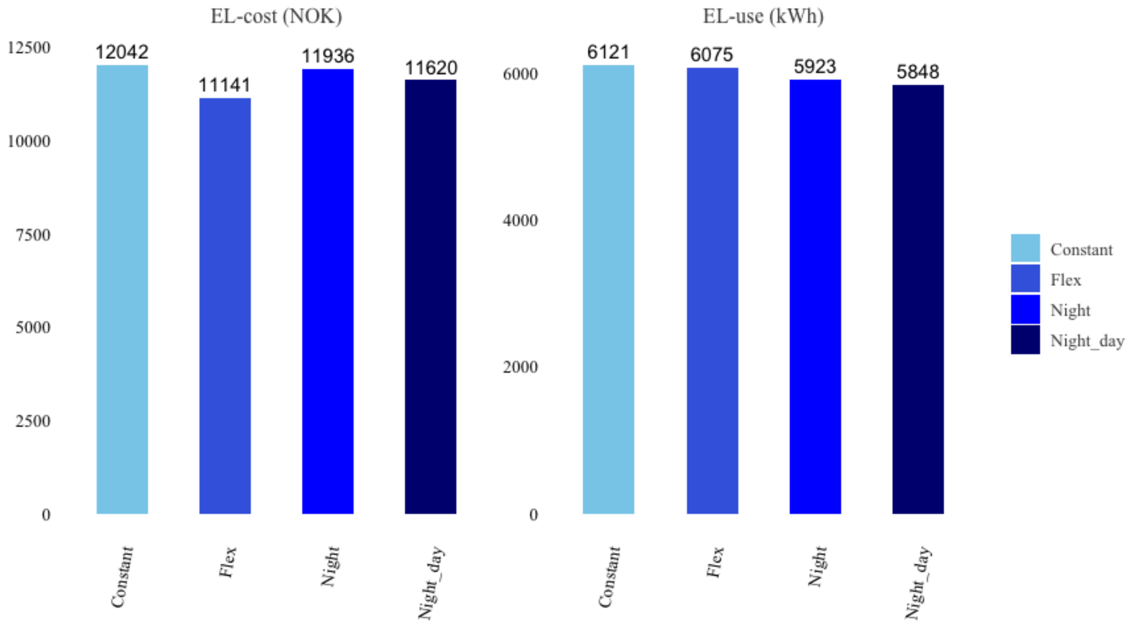


Figure 6.5: Total cost and kWh consumption for the four strategies

Despite their low kWh consumption, the night and night/day strategies are more expensive than the flex strategy. This is because the night and night/day strategies do not consider electricity prices when heating but use the kWh necessary at every instant to maintain the desired indoor temperature. As a result, these strategies require a heating system with a higher power dimension to provide appropriate amounts of heat in the morning and afternoon. The high consumption after temperature lowering can also lead to a high fixed and variable tariff cost, as the maximum power determines the network tariff consumed simultaneously and whether consumption occurs during peak hours. The fixed tariff for these two strategies may make them less attractive than the flex strategy.

The Constant strategy is the only strategy that does not allow temperature lowering. Its constant kWh usage and inflexible consumption pattern lead to a higher kWh consumption and high annual cost than the flex strategy. However, its low degree of complexity and stable electricity consumption may appeal to individuals who value simplicity and consistency in their heating systems, as it does not require frequent adjustments or complex smart heating systems. The high cost and kWh usage associated with the constant strategy can be seen in Figure 6.5.

Overall, the flex strategy appears to be the most efficient heating strategy regarding cost and kWh consumption compared to the non-smart strategies in the case study.

Distribution of consumption

In addition to total kWh consumption, the distribution of consumption is crucial for avoiding grid congestion and ensuring reliable and affordable electricity. Figure 6.6 shows each strategy's average power consumption pattern, which reflects the nature of the behavioural constraints imposed on them. Although the night and night/day strategies have the lowest total kWh consumption, they have high peaks in the mornings and afternoons, which increases the grid load during these times. This is illustrated by the pink and dark blue graphs in Figure 6.6. Although lowering the temperature can be energy efficient because it reduces total energy usage, this consumption pattern may not align with the new tariff model implemented by the Norwegian grid operator, which aims to encourage consumers to evenly distribute their energy consumption throughout the day in order to avoid the need for costly grid upgrades. As Buvik et al. (2022) suggest, Norway is moving towards an effect-dimensioned power system, meaning that consumption must adapt to the amount of available electricity. Consequently, distributing the grid load evenly will be even more important in the coming years.

Regardless of the high kWh usage and cost, the constant strategy distributes consumption relatively evenly compared to the other strategies, as shown by the turquoise graph in Figure 6.6. While the non-smart strategies do not adapt to prices and changes in grid load, the flex strategy utilises price peaks and troughs by minimising consumption when prices and grid load are high and supplying excess heat when prices and grid load are low, as illustrated by the green line in Figure 6.6. Therefore, the flex strategy is most effective for balancing the grid load.

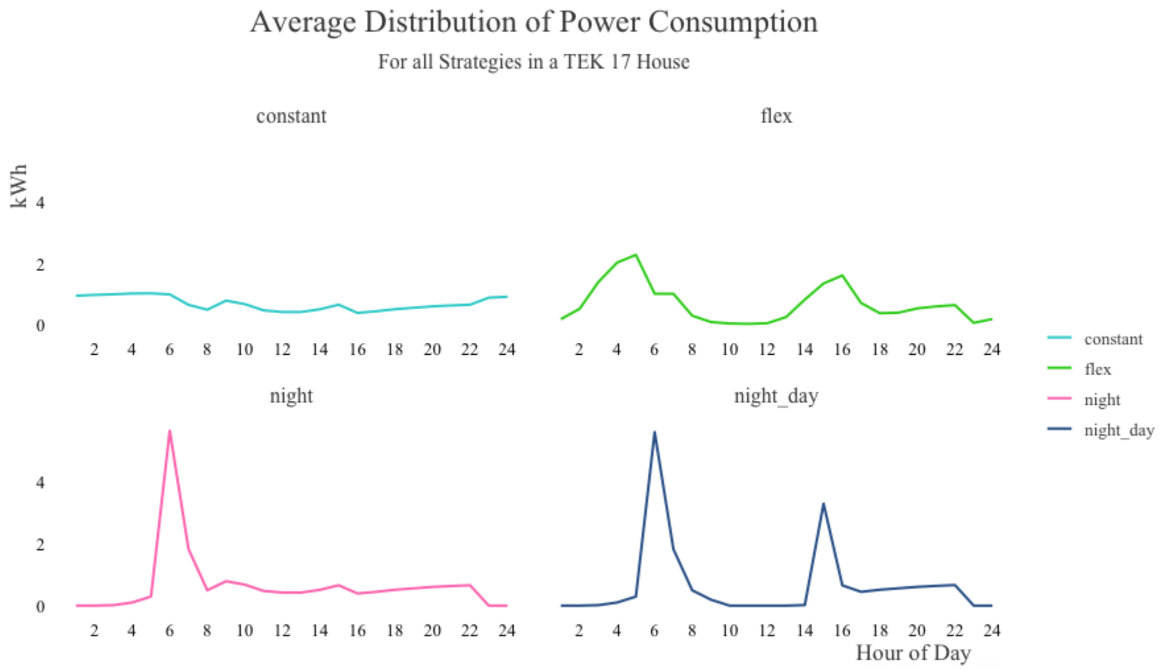


Figure 6.6: Average heating pattern for all strategies

To summarise, the smart flex strategy shows an improvement from the non-smart heating behaviours regarding cost per kWh and total cost. Despite the higher kWh usage for the flex strategy, it sustainably distributes its consumption load in contrast to the night and night/day strategies.

6.2 Scenario Analysis

The findings from analysing the base case have raised our curiosity about whether smart heating by the flex strategy will be beneficial if the house is built in compliance with older technical requirements and if power price volatility were to change in the future. Therefore, two scenario analyses will be performed in the following subsections.

6.2.1 Scenario 1: House Built According to TEK 97 Standards

Around 85 percent of single-family houses in Bergen were built before 2000 (SSB, 2022). While many of these homes have been renovated or upgraded since their construction, many still have poor insulation and do not meet the TEK 17 standard for building envelope performance. The higher heat loss in older buildings might limit the ability to shift heating to periods with lower prices. We use the TEK 97 regulation to investigate this issue, as this standard was the first to specify energy-saving measures as an alternative for calculating total energy efficiency. We expect that total energy usage and cost will increase for all strategies, but we will analyse their relative performance and trends to understand how they are affected.

6.2.1.1 Data Adjustments

The only adjustment made to the model is the input data for building requirements which is changed from TEK 17 to TEK 97. By keeping everything else constant, we can study the impact of the change in isolation. Table 6.2 summarises the adjustment made for the input data, and table 6.3 shows the new calculation of the heat transfer coefficients based on these values. In addition, the new dimensional power requirement is calculated as TEK 97 requires a more comprehensive heating system.

Input	***Value TEK 97***	***Value TEK 17***
Outer walls	0.22 [U-value]	0.18 [U-value]
Roof	0.15 [U-value]	0.13 [U-value]
Floor	0.15 [U-value]	0.10 [U-value]
Windows and doors*	1.60 [U-value]	0.80 [U-value]
Heat recovery in ventilation systems	60 %	80 %
Air leakage at $50 Pa/h$	4	0.6
Normalised cold bridge value	-**	0.05 W/m^2K
Air volume flow	1.2 $m^3/h\ m^2$ ***	1.2 $m^3/h\ m^2$

** Two-layer panes with a g-value of 0.65 is chosen to meet the required U-value for TEK 97 (Geving, 2021).*
*** For TEK 97, the normalised cold bridge values are incorporated in the U-values.*
**** TEK 97 does not have any specific requirement for air volume flow. Thus, the value is set equal to the requirement in TEK 17.*

Table 6.2: Building Requirements TEK 17 and TEK 97

Heat Transfer Coefficients TEK 17

	U-value	**Area**	**Heat transfer coefficient**			
Outer walls	0.22	155	0.22 * 155 = 34.10	107.10	H^{out}	18.11 %
Roof	0.15	92	0.15 * 92 = 13.80			7.33 %
Windows and doors	1.60	37	1.60 * 37 = 59.20			31.44 %
Cold bridges	-	-	-			-
Floor to ground	0.15	92	0.15 * 92 = 13.80	13.80	H^{ground}	7.33 %
Ventilation*	-	-	-	29.15	H^{vent}	15.48 %
Infiltration**	-	-	-	38.25	H^{inf}	20.31 %
Total				**188.30**	**H**	**100 %**

* $H^{vent} = (1 - \beta/100) * C * Q = (1 - 0.6) * 0.33 * (1.2 * 184) = 29.15$

** $R = e * n_{50}(h^{(-1)}) = 0.07 * 4 = 0.28$, $H^{inf} = C * R * V = 0.33 * 0.28 * 414 = 38.25$

See the methodology section for details about the calculation

Table 6.3: Calculation of heat transfer coefficients TEK 97

Dimensional power requirement – TEK 97:

The dimensional power requirements must be adjusted as a new heat transfer coefficient is calculated. For constant and flex strategy:

$$El^{design} = H * (T^{in} - DUT) = 188.3 * (21 - (-10)) = 5\ 837\ W$$

For night and night/day strategy:

$$El^{design} = 5\ 837\ W + 5\ 152\ W = 10\ 989\ W$$

6.2.1.2 Results

Table 6.4 summarises the results from scenario 1 with TEK 97 standards and the base case scenario with TEK 17 standards. As anticipated, the TEK 97 house has a significantly higher kWh usage and cost per year than the TEK 17 house. The results will be studied in detail in the coming subsections.

		Constant	*Night*	*Night/day*	*Flex*
Total cost	**TEK 17**	12 042	11 936	11 620	11 141
	TEK 97	28 853	27 921	26 876	26 225
kWh usage	**TEK 17**	6 121	5 923	5 848	6 075
	TEK 97	14 169	13 554	13 217	13 497

Table 6.4: Total cost and kWh usage for TEK 17 and TEK 97

NOK/kWh

Table 6.5 shows the average electricity price per kWh for the various strategies in the base case (TEK 17) and scenario 1 (TEK 97). The flex strategy consistently achieves the lowest average cost, while the night strategy consistently achieves the highest average cost. In scenario 1, the constant and night/day strategies have switched positions compared to the base case, but the difference between the two strategies is minimal. Despite an overall increase in cost from TEK 17 to TEK 97, the difference for the flex strategy is the most significant, where its average price per kWh increases from NOK 1.83 for TEK 17 to NOK 1.94 for TEK 97. This suggests that it is more difficult to achieve the same efficiency level when using smart heating in a TEK 97 house compared to a TEK 17 house.

		Constant	*Night*	*Night/day*	*Flex*
NOK/kWh	**TEK 17**	1.97	2.01	1.99	1.83
	TEK 97	2.04	2.06	2.03	1.94

Table 6.5: NOK/kWh for TEK 17 and TEK 97

Relative Savings in Cost

The results show that it is more beneficial for a modern, well-insulated house to shift electricity consumption to periods with lower prices than it is for an older house with poorer insulation. Table 6.6 displays the relative savings between all strategies for TEK 17 and TEK 97. While the relative savings between flex and night/day is 4.12 percent for the TEK17 demo house, it decreases to 2.42 percent for the TEK 97 demo house. This indicates that the two strategies align towards the same level of effectiveness. Since the flex strategy is more complex and costly to implement than the night/day strategy, the savings from a flex strategy may be perceived as minimal for a TEK 97 house.

The savings for the night, night/day and flex relative to constant increases significantly from TEK 17 to TEK 97. This suggests that lowering the temperature during certain periods can effectively save energy costs in older homes with higher heat loss. However, with the new variable tariff cost and better house insulation, the savings of night lowering relative to constant temperature in a TEK 17 house are negligible at 0.88 percent.

TEK 17	*Constant*	*Night*	*Night/day*	*Flex*
Constant	-	0.89%	3.63%	8.08%
Night	-0.88%	-	2.72%	7.13%
Night/day	-3.50%	-2.65%	-	4.29%
Flex	-7.48%	-6.65%	-4.12%	-

TEK 97	*Constant*	*Night*	*Night/day*	*Flex*
Constant	-	3.34%	7.36%	10.02%
Night	-3.23%	-	3.89%	6.47%
Night/day	-6.85%	-3.74%	-	2.48%
Flex	-9.11%	-6.07%	-2.42%	-

Table 6.6: Relative savings between the vertical and horizontal strategy for TEK 17 and TEK 97

When studying the monthly relative savings between the flex and the night/day strategy, it appears that the savings vary with temperature. This is confirmed by running the model with the same electricity price for all days to exclude the potential effect of price fluctuations throughout the year. Figure 6.7 illustrates the relative savings between the two strategies for TEK 17 and TEK 97, along with the monthly average temperature. The performance of the flex strategy relative to night/day is significantly better during the summer season and minimal during the heating season for TEK 17 and TEK 97. This is because the heat loss is higher during the heating season, making it less profitable to store heat and shift consumption. Additionally, the monthly savings for the flex strategy are significantly higher for TEK 17 compared to TEK 97. Overall, the smart behaviour of the flex strategy appears to be more effective when the heat loss is low, whether it is due to a warmer climate or better insulation.

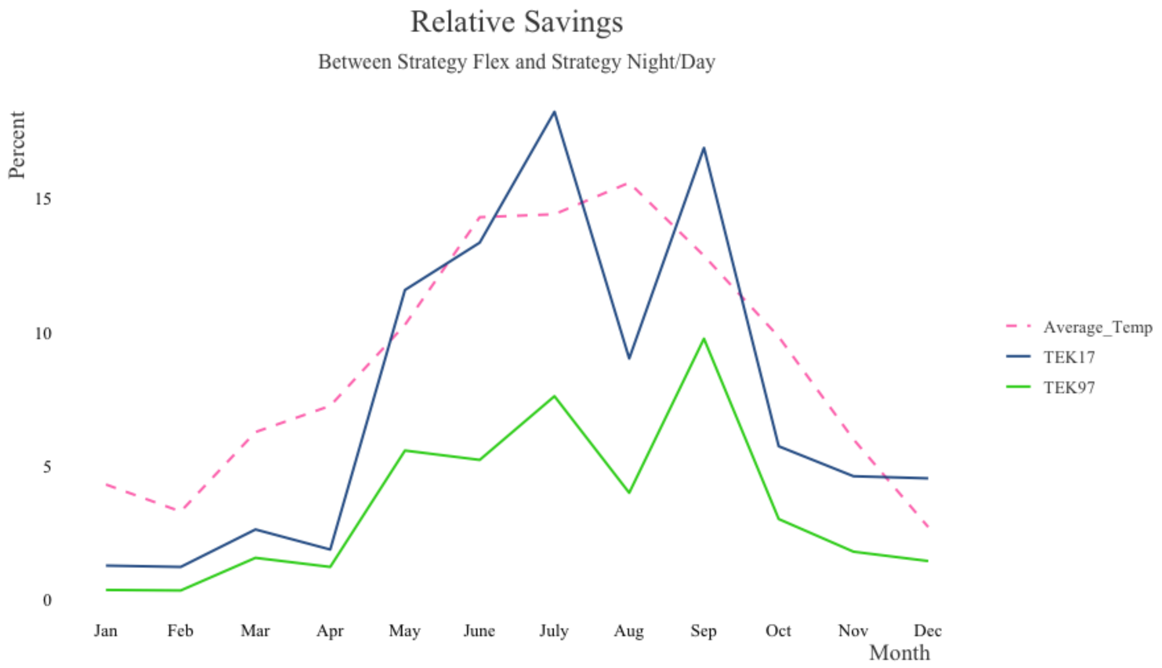


Figure 6.7: Relative monthly savings between strategy flex and strategy night/day for TEK 17 and TEK 97

Absolute Savings in Cost

When looking at the absolute savings in cost, the results differ from the relative savings because the TEK 97 demo house has a significantly higher kWh usage. Table 6.7 displays the absolute savings for TEK 17 and TEK 97. Although the percentage relative saving between flex and night/day is better for a TEK 17 house, the potential saving in absolute terms is higher for a TEK 97 house. This is because the energy consumption is higher for a house with poorer insulation, which increases the kWh usage and absolute savings. For temperature-lowering strategies, the absolute values enhance the effect of temperature lowering for an older house, leading to a considerable decrease in cost compared to the constant strategy.

TEK 17	*Constant*	*Night*	*Night/day*	*Flex*
Constant	-	195	405	933
Night	(106)	-	210	738
Night/day	(422)	(316)	-	528
Flex	(900)	(794)	(478)	-

TEK 97	*Constant*	*Night*	*Night/day*	*Flex*
Constant	-	932	1,977	2,628
Night	(932)	-	1,045	1,696
Night/day	(1,977)	(1,045)	-	651
Flex	(2,628)	(1,696)	(651)	-

Table 6.7: Absolute savings in cost for TEK 17 and TEK 97

kWh usage

In the same way as the performance in terms of cost for the flex strategy relative to the other strategies decreases from the TEK 17 to the TEK 97 house, the relative use of kWh also decreases. Table 6.8 summarise the relative savings in kWh consumption between the different strategies for TEK17 and TEK97. The poor ability of the TEK 97 house to store heat prevents the flex strategy from fully utilising its potential, resulting in reduced kWh consumption relative to the other strategies compared to the TEK 17 house.

Flex, night and night/day, all including temperature lowering, use less kWh relative to the constant strategy for the TEK 17 and the TEK 97 house. This corresponds with NVE's recommendation of night lowering as an energy efficiency measure. As the quality of a building's envelope decreases, the heat loss per degree Kelvin difference between indoors and outdoors increases. Therefore, the savings from lowering the temperature are greater in older houses with higher heat loss, leading to a significant relative reduction of kWh for the temperature-lowering strategies from TEK 17 to TEK 97. These findings suggest that temperature lowering is a particularly relevant energy efficiency measure for older houses with poor insulation.

TEK 17	*Constant*	*Night*	*Night/day*	*Flex*
Constant	-	3.23%	4.68%	0.76%
Night	-3.13%	-	1.40%	-2.39%
Night/day	-4.47%	-1.38%	-	-3.74%
Flex	-0.76%	2.45%	3.88%	-

TEK 97	*Constant*	*Night*	*Night/day*	*Flex*
Constant	-	4.54%	7.21%	4.98%
Night	-4.34%	-	2.55%	0.42%
Night/day	-6.72%	-2.49%	-	-2.08%
Flex	-4.74%	-0.42%	2.12%	-

Table 6.8: Relative kWh savings for TEK 17 and TEK 97

Temperature lowering as a measure of energy efficiency does not consider grid load. When looking at the hourly average total power consumption for the different strategies, which is illustrated in Figure 6.8, it is clear that the night and night/day strategies impose a significant load on the grid in the morning and afternoon. This applies particularly to TEK 97, which requires a larger dimensioned heating system. Subsequently, the recommended measure of night lowering should be updated as the building envelope for houses improves and the focus on evenly distributed grid load remains. Furthermore, Figure 6.8 illustrates how the behaviour of the flex strategy moves towards the night/day strategy for the TEK 97 house when imposing a significant load on the grid in the morning and afternoon.

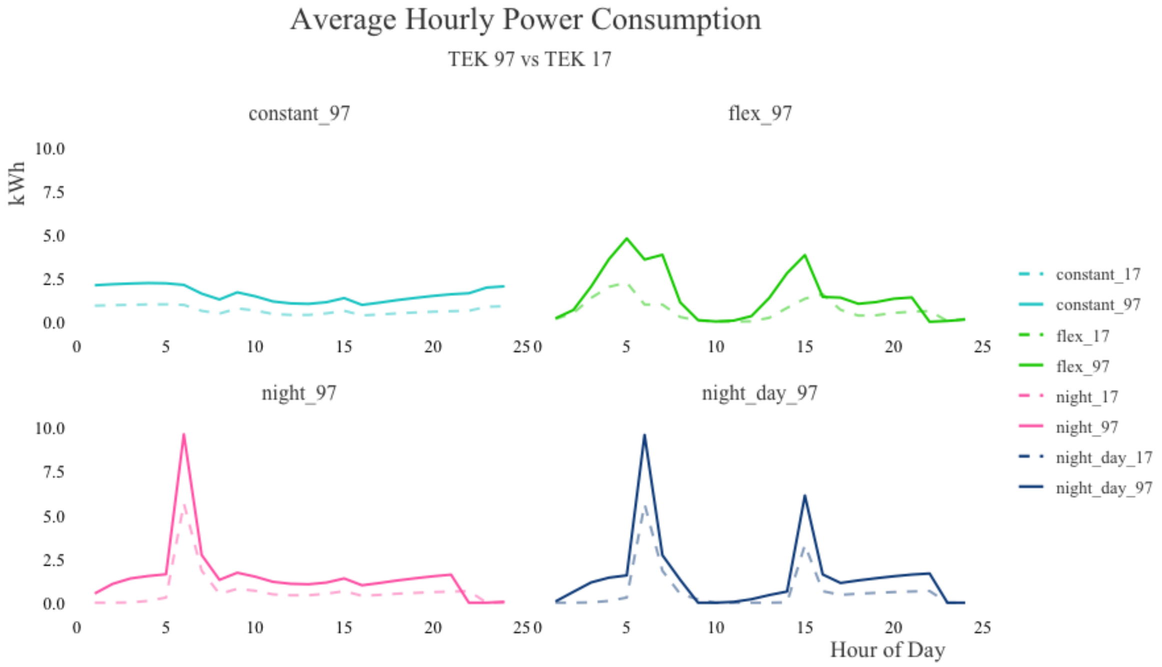


Figure 6.8: Average heating patterns for all strategies TEK 97 vs TEK 17

Conclusively, the scenario analysis shows that the flex strategy is less effective in an older house with a larger heat loss than in a TEK 17 house. However, still more effective than the non-smart strategies. On the other hand, the night-lowering strategies are more beneficial in a TEK 97 house than in a TEK 17 house, but the kWh consumption is not distributed sustainably, with high consumption in peak hours. Despite the higher kWh consumption of the flex strategy and its heating behaviour moving towards the night/day strategy behaviour, the imposed grid load is still relatively well distributed in a TEK 97 house.

6.2.2 Scenario 2: Future change in volatility

The fluctuation of energy prices is crucial in determining the profitability of shifting electricity consumption. The degree of price fluctuation, or volatility, is expected to increase as the electricity grid becomes more reliant on renewable energy sources (Birkelund et al., 2021). In this scenario, we will investigate how changes in price volatility throughout the day will impact the performance of different heating strategies. Because predicting future electricity prices is difficult, we will examine three situations retrospectively with historical price and consumption patterns.

1. Low volatile market - consumer flexibility and improved technology for storage can help reduce price volatility within a day.
2. High volatile market – increased fluctuations within a day.
3. Extreme volatile market - even more extreme fluctuations within a day, pushing prices to record highs and lows. On some days, prices can even become close to 0 or negative.

6.2.2.1 Data Adjustments

The only adjustment made to the model is the price of electricity, while all other data is kept the same as in the base case to study the effect of the price fluctuation in insulation.

The price of electricity has the following adjustments for future markets compared to the base case market:

1. Low volatile market – all fluctuations are decreased by 10 %
2. High volatile market – all fluctuations are increased by 20 %
3. Extreme volatile market – all fluctuations are increased by 40 %

This means that all prices above that day's average are increased by the respective percentage rate, while the same rate reduces all values below the average. The price variation for an arbitrarily selected day is illustrated in Figure 6.9. The price fluctuates around the average price according to its level of volatility.

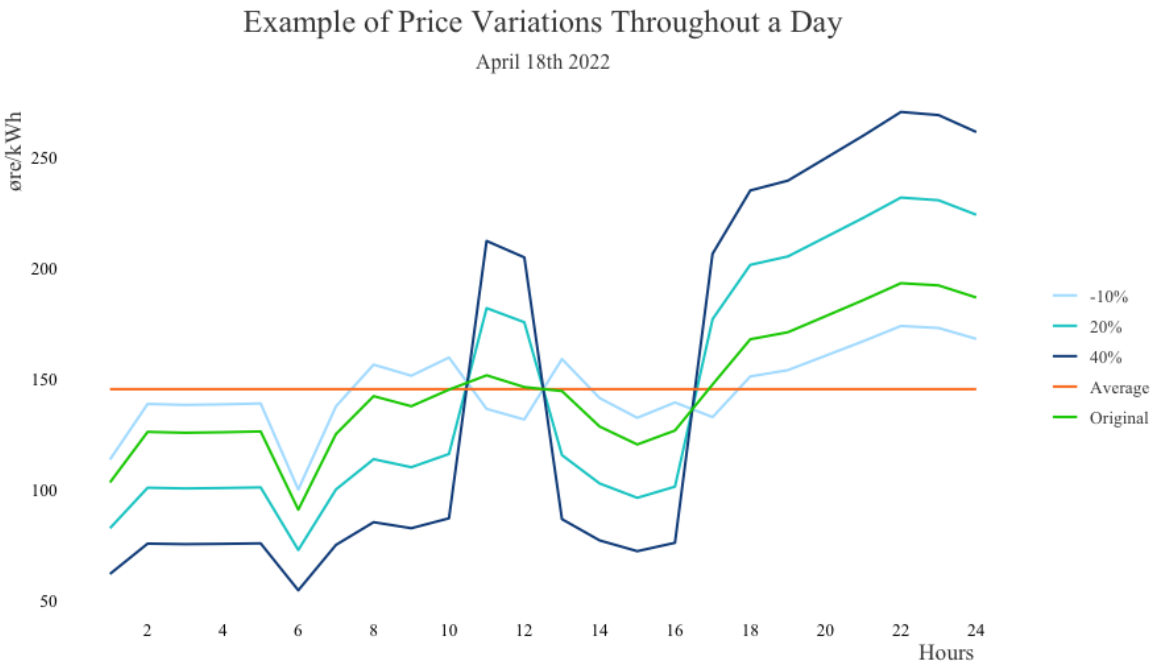


Figure 6.9: Price variations for an arbitrarily selected day

6.2.2.2 Results

Total cost

The smart behaviour of the flex strategy appears to be an effective way to reduce electricity costs in more volatile market conditions. This is because it significantly decreases annual costs by adapting consumption to times when electricity prices are lower. The total annual costs for the various strategies in the different volatility markets are summarised in Table 6.9. The reduction in annual cost for the flex strategy is significant, with a 10 percent decrease in electricity cost from the base case to the more volatile market and a further 11 percent decrease in cost to the extremely volatile market. This is in contrast to the non-smart strategies, which do not show significant changes in cost and stay around the same cost level, regardless of the price volatility.

	Constant	*Night*	*Night/day*	*Flex*
Low volatile	12 026	11 845	11 694	11 195
Base case	12 042	11 847	11 637	11 109
High volatile	12 073	11 849	11 522	10 068
Extreme volatile	12 105	11 852	11 407	8 647

Table 6.9: Total cost for the different volatility markets

The flex strategy is designed to adapt to market changes and fluctuating electricity prices. This means that, even as prices and price volatility vary over time, the flex strategy will always strive to optimise heating by minimising costs. While the exact savings achieved by the flex strategy may vary depending on market conditions, it is designed to adapt and provide the greatest benefits regardless of the level of volatility. Overall, the flex strategy offers a flexible and effective way to save electricity costs in volatile market conditions.

kWh usage

The increased electricity consumption of the flex strategy in times of higher price volatility is a necessary trade-off for its significant cost savings. The total electricity consumption for each volatility market is shown in Table 6.10. By storing heat during times of low prices and releasing it during times of high prices, the flex strategy is able to take advantage of the price difference and save costs. However, heat loss and kilowatt consumption increase with increased volatility for the smart flex strategy. In contrast, the non-smart strategies have strict heating constraints and cannot adapt to price fluctuations and therefore do not show changes in electricity consumption with varying volatility. Overall, the smart behaviour of the flex strategy is an effective way to minimise electricity costs in volatile market conditions.

	Constant	*Night*	*Night/day*	*Flex*
Low volatile	6121.33	5895.65	5831.24	6089.58
Base case	6121.33	5895.65	5831.24	6062.75
High volatile	6121.33	5895.65	5831.24	6346.39
Extreme volatile	6121.33	5895.65	5831.24	6466.82

Table 6.10: Total electricity consumption for the different volatility markets

The increased kWh usage of the flex strategy in times of higher volatility is not only beneficial in terms of cost savings, but also in terms of grid load management. By consuming more electricity when prices are low, and the grid load is minimal, the flex strategy helps to distribute its grid demand more evenly and avoid imposing extra load in peak demand periods. Figure 6.10 shows the average hourly consumption pattern for the flex strategy in different volatility markets, with the constant strategy added as a reference point. It shows that the flex strategy adapts its consumption to the market's price fluctuations and grid load, consuming even more during low-demand periods and less during high-demand periods as volatility increases. This helps to optimise the use of the grid and improve its overall efficiency.

The similarity in behaviour between the high and the extremely high volatility market scenarios suggests that the flex strategy already fully utilises the cheapest hours of the day in the high volatility market. This can be explained by the limitations of the dimensional power requirement, limiting the strategies' electricity consumption. Thus, the difference in cost savings between the high and extreme volatility markets is mainly due to the even lower prices in the extreme volatility market, allowing for even greater savings.

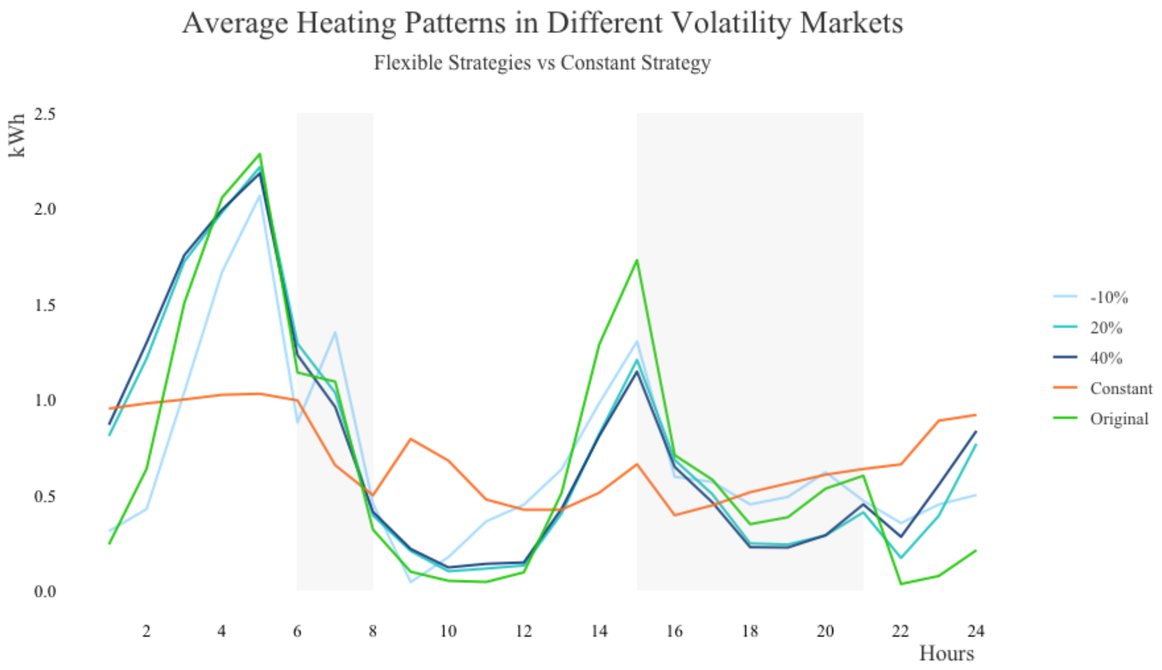


Figure 6.10: Average heating patterns for different volatility markets

While the flex strategy can provide significant cost savings in more volatile electricity markets, it is important to consider its potential negative impact on indoor climate and comfort. The flex strategy involves heating up the ovens explosively in specific periods in order to avoid heating later in the day when prices are higher. This can result in temperatures above the preferred indoor temperature, which can be uncomfortable for occupants. Additionally, as price fluctuations intensify, the flex strategy may require more electricity during times when the indoor temperature is already above the preferred level, causing further discomfort. Therefore, it is important to carefully consider the potential trade-offs between cost savings and indoor climate comfort when implementing the flex strategy.

6.3 Summary of the Analysis

All things considered, the base case analysis found that the flex strategy's smart behaviour benefits consumers and society. Results from the retrospective analysis reveal that smart heating has the lowest cost, as it can instantly adapt to power price variations while respecting all strategy constraints. Despite its relatively high kWh usage, it distributes its consumption evenly, thereby contributing to sustainable utilisation of the electricity grid.

The base case analysis also found that the night and night/day strategies have lower kWh usage than the flex strategy, but can negatively impact the electricity grid with its high consumption peaks. Because these two strategies involve lowering the temperature in the morning and afternoon, they require a high effect to raise the temperature again, which imposes an additional strain on the local electricity grid. This increases the risk of high investment costs for the grid operator, who must upgrade the power grid to handle higher demand. Also, the new network tariff can result in higher fixed tariff costs for these strategies, potentially making the flex and the constant strategies more cost-effective as they do not require larger dimensional power requirements of the house. According to the base case analysis, implementing night and day lowering strategies in modern, well-insulated buildings does not offer any significant benefits compared to the smart flex strategy.

Findings from the first scenario analysis revealed that the construction standard of a house is important for the smart flex strategy to utilise its potential. Homes with higher heat loss are less able to store energy for later use, making it less profitable to use a smart heating strategy. Additionally, the benefits of temperature lowering in terms of cost and kWh usage are more significant for older homes with higher heat loss. This strengthens the argument that temperature lowering may be less relevant for new, well-insulated homes than smart heating, as new houses are more suited to retain heat.

The analysis of the second scenario shows that greater volatility in electricity prices leads to more extreme behaviour of the flex strategy, with larger cost savings and higher kWh consumption. In contrast, the non-smart heating strategies are almost unaffected by changes in price volatility. While the flex strategy's ability to distribute consumption and reduce grid load can offset its high kWh usage, extreme behaviour is a factor that also may negatively impact the indoor climate. As prices fluctuate and the flex strategy adapts its consumption accordingly, temperatures can rise too high in specific periods, impacting the comfort of occupants. Therefore, it is important to consider the potential trade-offs between cost savings and indoor climate comfort when implementing a smart flex strategy in extremely volatile market conditions.

7 Discussion

This part of the thesis discusses the limitations of the data, assumptions, and the validity of the results. Furthermore, we will propose and discuss future work related to this field of study.

7.1 Limitations and validity of results

The validity of the results obtained in this thesis must be carefully considered in light of the limitations and assumptions of the data and model used. In order to provide a more thorough understanding of these limitations and assumptions, a detailed discussion with justification for each assumption and its potential impact on the results is included in this section. In addition, we will discuss how the model could be refined and expanded to include more complex components and factors to provide a more accurate and comprehensive representation of the optimisation problem.

7.1.1 Limitations of the Formulated Heating Strategies

The heating strategies discussed are designed to reflect typical heating patterns, but a few simplifications are made to avoid excessive complexity. Firstly, the strategies assume fixed home and away patterns and do not allow for variation in occupancy patterns. Secondly, temperature requirements are fixed, so the model does not account for various personal preferences for indoor temperature. Thirdly, wood-firing or other alternative heating sources are not included. Lastly, all strategies rely on human interaction or technology to accomplish their objectives. For instance, the flex strategy uses smart technology to optimise heating based on hourly power prices and automatic thermostats.

Regarding the assumption of fixed home and away patterns, the Constant Strategy is required to keep the indoor temperature at 21 degrees, regardless of occupants being away. Thus, in real life, they may wish to adjust or turn off indoor temperature when going away for longer periods. This simplification is added to remove the complexity imposed by the individual behaviour of different households. Moreover, the night, night/day and flex strategies are formulated around the fixed occupancy patterns where residents are away every day between 9 a.m. and 4 p.m. This means that the model cannot capture heat demand variations based on real-life occupancy patterns. These assumptions are based on Standard Norway's NS 3031 (Standard Norge, 2020).

The model follows power prices and climate data for the western part of Norway, making the obtained results only valid for this region. To broaden the validity of the results, it would be better to be able to include more data from a wider range of regions and climates. This would allow the model to be tested and validated in various conditions, making it more applicable to a wider range of locations simultaneously. However, the model can be easily customised with different input data, so users can test it for different regions. By incorporating these changes, the results obtained could become more robust, and be used to achieve more evidence to support the conclusion.

7.1.2 Justification of Important Parameters and Simplifications

The demo house has been modelled to reflect a typical Norwegian private family house. Even though the geometry of the demo house is determined based on standardisations, the results obtained are only valid for this specific house. Accordingly, any adjustments made to the house will cause different results, limiting the types of houses for which the results will be valid. However, obtained results will provide insight into interesting aspects of each heating strategy in terms of cost and kWh consumption.

The optimal indoor temperature of 21 degrees is a fixed parameter and is only subject to change at certain hours when running temperature-lowering strategies. This implies that results are only valid for houses keeping the same indoor temperature. Even a one-degree change in indoor temperature will affect heat loss and change the demand for heat. However, this parameter is chosen based on reliable Norwegian standardised values, only limited by not capturing variations in different households' preferred indoor temperatures.

Parameters concerning the indoor heat supply are also subject to variation, and the optimisation model is limited by being unable to capture this. The amount of heat supplied by lighting and people varies with the effect of light bulbs and residents' body weight and metabolism. These variations are complicated to capture accurately, thus, the respective parameters have been simplified and set to reliable, standardised values used by construction engineers. Equipment and heated water generally do not provide enough heat to private households to be considered relevant in an optimisation problem. As a result, they are not included in the model.

The model assumes that heat supplied by either natural or electrical sources equally affects the temperature of all surfaces in the demo house. This simplification implies that there is no temperature differentiation between rooms. However, in reality, an equal temperature and heat supply distribution are unlikely due to variations in sun exposure on the house surfaces and internal heat supply. Even though the model accounts for variations in sun irradiance from different directions outside the house, it assumes that all heat provided is instantly and equally distributed throughout the house. Thus, the model sees the house as a unit to avoid excessive complexity.

The network tariff rates are determined locally, meaning that the model's tariff rates are only valid for Bergen and its surrounding municipalities. The fixed component of the tariff is based on the total electricity consumption consumed simultaneously. As a result, there is a high level of uncertainty regarding electricity consumption unrelated to heating. To address this, the fixed component of the network tariff is omitted when calculating the electricity cost for the different strategies. However, possible impacts on the fixed tariff are discussed in section 6.

Despite formulating the model with some assumptions and simplifications, the model provides insight and interesting aspects of the different heating strategies. Furthermore, with the scenario analysis, the model also provides us with insights into how the objectives of each strategy may change when adjusting different parameters.

7.2 Further Work

While writing this thesis, we encountered many interesting topics related to smart heating, heat production, consumption, and energy efficiency. However, as we could not cover everything in this thesis, we would like to present our proposals for further research.

Firstly, an interesting problem to study is the potential cost savings and grid preservation benefits of incorporating other heating sources into the heating strategies. These could include wood firing, solar panels, heat pumps, waterborne underfloor heating, or district heating. In more detail, it would be interesting to study how the performance of the flex strategy is affected when including other heating sources in the energy mix. In addition, it would be interesting to look at future scenarios of cost development for new heating methods.

Another interesting problem to explore is how the power demand in houses will change when the new technical building requirement TEK 20 is officially introduced. It is reasonable to expect the new regulations to require more energy-efficient measures in new buildings. Consequently, an interesting question to study is how the improvements will affect the impact on the electricity grid and the heating cost. Furthermore, another exciting aspect is how the TEK 20 regulation will affect the performance of smart heating compared to non-smart heating strategies. Overall, these proposals are areas that caught our interest while working on this thesis and could be relevant topics for further work.

8 Conclusion

This study aims to analyse the performance of smart heating compared to traditional heating practices in terms of costs and electricity consumption to gain insight into whether smart technology can benefit consumers and society. In addition, two different scenario analyses are carried out to study how other building requirements and changes in future price volatility will affect the performance of the heating strategies.

Findings from the base case analysis reveal that smart heating benefits Norwegian consumers and society in terms of cost. The flex strategy is the most cost-effective, with a total cost of NOK 11 141, due to its ability to adapt to price variations and utilise the house's heat storage capabilities. The non-smart night and night/day strategies have lower total electricity consumption at 5 848 kWh and 5 923 kWh compared to 6 075 kWh for the flex strategy. However, the night and night/day strategies increase the risk of grid congestion with an unsustainable distribution of grid load. In contrast, the flex strategy avoids this by distributing its consumption when the grid load is low. Findings from the first scenario analysis reveal that the smart behaviour of the flex strategy will be less effective in a TEK 97 house because of large heat loss, reducing its ability to push heating. Moreover, the second scenario analysis indicates that the flex strategy is superior when price volatility increases.

Overall, smart house heating performs better than non-smart strategies in terms of cost and is, therefore, beneficial for consumers and society. Despite the larger electricity consumption of the flex strategy, its constant adaption to power prices enables it to distribute the consumption sustainably. Thus, by combining strict building requirements and smart heating technology, Norwegian households can save costs and contribute to sustainable electricity utilisation through their heating behaviour.

Smart heating - the smart way to heat

References

Ali, M., Jokisalo, J., Siren, K., and Lehtonen, M. (2014). Combining the demand response of direct electric space heating and partial thermal storage using lp optimization. *Electric Power Systems Research*, 106:160–167.

Barakat, S. A. (2008). Solar heat gains through windows in canada. Technical report, National Research Council Canada, Ottawa,Canada.

Birkelund, H., Arnesen, F., Hole, J., Spilde, D., Jelsness, S., Aulie, F. H., and Haukeli, I. E. (2021). Langsiktig kraftmarkedsanalyse. Technical report, NVE.

BKK (2022). Ny modell vil premierer smart bruk av strømnettet. https://nett.bkk.no/artikkel/b6e3df43-e183-41bd-8f43-88701cfd99e4.

Blom, P. (2006). Konstruksjoner mot grunnen. Technical report, Byggforsk.

Botnen, O. J. (2022). Electricity prices in europe: What is next. https://www.volue.com/news/electricity-prices-europe-what-is-next.

Bozchalui, M. C., Hashmi, S. A., Hassen, H., Canizares, C. A., and Bhattacharya, K. (2012). Optimal operation of residential energy hubs in smart grids. *IEEE Transactions on Smart Grid*, 3(4):1755–1766.

Bradley, Stephen P.and Hax, A. C. M. T. L. (1977). *Applied Mathematical Programming*. Addison-Wesley.

Brattebø, H., O'Born, R. J., Sartori, I., Klinski, M., and Nørstebø, B. (2016). Typologier for norske boligbygg-eksempler på tiltak for energieffektivisering.

Bugge, H. C. (2011). *Lærebok i miljøforvaltningsrett*. Universitetsforlaget.

Buvik, M., Cabrol, J., Spilde, D., Skaansar, E., Roos, A., and Grytli, (2022). Norsk og nordisk effektbalanse fram mot 2030. Technical report, NVE.

Chinneck, John W (2015). Practical optimization: a gentle introduction. http://www.sce.carleton.ca/faculty/chinneck/po.html.

de Oliveira, V., Jäschke, J., and Skogestad, S. (2013). Dynamic online optimization of a house heating system in a fluctuating energy price scenario. *IFAC Proceedings Volumes*, 46(32):463–468.

DiBK (1987). *Byggeforskrift 1987*. Norwegian Building Authority, Oslo,Norway.

DiBK (1997). *Forskrift om krav til byggverk*. Norwegian Building Authority, Oslo,Norway.

DiBK (2016). *Byggteknisk forskrift (TEK10)*. Norwegian Building Authority, Oslo,Norway.

DiBK (2017). *Byggteknisk forskrift (TEK17) med veiledning.* Norwegian Building Authority, Oslo,Norway.

ECMWF (2022). Catalogue of archive products. https://www.ecmwf.int/en/forecasts/datasets.

El Saied, A., Maalouf, C., Bejat, T., and Wurtz, E. (2021). Slab-on-grade thermal bridges: A thermal behavior and solution review. *Energy and Buildings*, pages 111–770.

Enova (2022a). Smart strømstyring for boliger. https://www.enova.no/privat/alle-energitiltak/smart-stromstyring.

Enova (2022b). Smarte energi- og klimatiltak. https://www.enova.no/privat/alle-energitiltak/.

Enova, Bioenergiforening, N., Landsforening, N. R., Forening, N. V., Varmepumpeforening, N., og Miljøteknisk Forening, N. E., Fjernvarme, N., and Solenergiforening, N. (2011). Kompetansekompendium for varmeanlegg 2011. Technical report, Enova, Trondheim,Norway.

European Comission (2022). Photovoltaic geographical information system. https://re.jrc.ec.europa.eu/pvg_tools/en/.

Europower (2020). Fleksibel oppvarming kan dempe nettbehovet. https://www.fjernvarme.no/fleksibel-oppvarming-kan-dempe-nettbehovet.

Geving, s. (2021). *Praktisk bygningsfysikk.* Fagbokforlaget.

Gueymard, C. A. et al. (2008). From global horizontal to global tilted irradiance: how accurate are solar energy engineering predictions in practice? In *Solar 2008 Conf., San Diego, CA, American Solar Energy Society.*

Halliday, D., Resnick, R., and Walker, J. (2021). *Fundamentals of Physics.* Wiley.

Honsberg, C.B; Bowden, S. (2019). Satellite derived irradiance estimates. www.pveducation.org.

Huebner, G. M., McMichael, M., Shipworth, D., Shipworth, M., Durand-Daubin, M., and Summerfield, A. (2013). Heating patterns in english homes: Comparing results from a national survey against common model assumptions. *Building and Environment*, 70:298–305.

IEA (2019). Energy performance of buildings directive. https://www.iea.org/policies/868-energy-performance-of-buildings-directive-epbd-200291ec.

IEA (2021). Net zero by 2050. https://www.iea.org/reports/net-zero-by-2050.

IEA (2022). Global energy crisis. https://www.iea.org/topics/global-energy-crisis.

Johannessen, K. (2019). Hva er enØk? https://nemitek.no/energisparing-energiokonomisering-enova/hva-er-enok/103681.

Larsen, B. T. (1982). *Manuell metode for beregning av energiforbruk i boliger.* Norges byggforsknings institutt, Oslo,Norway.

Lundgren, J., Rnnqvist, M., and Värbrand, P. (2010). *Optimization.* Studentlitteratur.

Machorro-Cano, I., Alor-Hernández, G., Paredes-Valverde, M. A., Rodríguez-Mazahua, L., Sánchez-Cervantes, J. L., and Olmedo-Aguirre, J. O. (2020). Hems-iot: A big data and machine learning-based smart home system for energy saving. *Energies*, 13(5):1097.

Ministry of Petroleum and Energy (2019a). Regulering av nettvirksomheten. https://energifaktanorge.no/regulering-av-energisektoren/regulering-av-nettvirksomhet/.

Ministry of Petroleum and Energy (2019b). Strømnettet. https://energifaktanorge.no/norsk-energiforsyning/kraftnett/.

Ministry of Petroleum and Energy (2021a). Electricity production. https://energifaktanorge.no/en/norsk-energiforsyning/kraftproduksjon/.

Ministry of Petroleum and Energy (2021b). Strømforsyning og strømnettet. https://www.regjeringen.no/no/tema/energi/stromnettet/stromforsyning-og-stromnettet/id2353792/.

Myhre, L., Schild, P. G., Pettersen, T. D., Blom, P., and Gullbrekken, L. (2012). Veileder for prosjektering av passivhus - småhus. Technical report, Sintef, Oslo, Norway.

NemiTek (2019). Dimensjonerende utetilstand. https://stensaasventtek.nemitek.no/dimensjonering-av-klimaanlegg-kapittel-62-stensaas-leif-i-ventilasjonsteknikk-del-i-grunnlaget-og-systemer-1998/dimensjonerende-utetilstand/161575.

Nord Pool (2022a). About us. https://www.nordpoolgroup.com/en/About-us/.

Nord Pool (2022b). Price calculation. https://www.nordpoolgroup.com/en/trading/Day-ahead-trading/Price-calculation/.

Norwegian Centre for Climate Services (2022). Observations and weather statistics. https://klimaservicesenter.no.

NVE (2022a). Energibruk i bygg: Effektiv og fossilfri energibruk i norske bygninger. https://www.nve.no/energi/energisystem/energibruk-effektivisering-og-teknologier/energibruk-i-bygg/.

NVE (2022b). Energieffektivisering. https://www.nve.no/energi/energisystem/energibruk-effektivisering-og-teknologier/energieffektivisering/.

NVE (2022c). Nettleie. https://www.nve.no/reguleringsmyndigheten/kunde/nett/nettleie/.

NVE (2022d). Varme. https://www.nve.no/energi/energisystem/termisk-energi/varme/.

Regjeringen (2022). Styrker innsatsen for økt energieffektivisering. https://www.regjeringen.no/no/aktuelt/milliardsatsing-til-energieffektivisering/id2932296/.

Rosvold, K. A. and Aksdal, A. (2018). elektrisk oppvarming. https://snl.no/elektrisk_oppvarming.

Sintef (2007). Småhus som tilfredsstiller energikravene i tek-2007. Technical report, Sintef.

Skari, I. T. (2016). Reduksjon av energibruk i bygninger ved aktiv bruk av bygningens termiske masser og av et intelligent ikt-verktøy for tilbakemelding til bruker. Master's thesis, NTNU.

Smidsrød, M., Erichsen, I. B., and AS, H. (2008). Veiledning for utforming av glassfasader. Technical report, NTNU and Statens bygningstekniske etat and Statsbygg and Enova.

SSB (2022a). Boliger. https://www.ssb.no/bygg-bolig-og-eiendom/bolig-og-boforhold/statistikk/boliger.

SSB (2022b). Elektrisitet.

SSB (2022c). Elektrisitet, 08311: Nettoforbruk av elektrisk kraft (gwh), etter forbrukergruppe, statistikkvariabel og år. https://www.ssb.no/statbank/table/08311/tableViewLayout1/.

Standard Norge (2017). *Energy performance of buildings - Method for calculation of the design heat load - Part 1: Space heating load, Module M3-3.* Standard Norge, Oslo,Norway.

Standard Norge (2020). Inndata for normert energiberegning. https://www.standard.no/ns3031.

Statista (2022). Electricity consumption per capita worldwide in 2021, by selected country. https://www.statista.com/statistics/383633/worldwide-consumption-of-electricity-by-country/.

Statnett (2022). Derfor har vi prisområder. https://www.statnett.no/om-statnett/bli-bedre-kjent-med-statnett/om-strompriser/fakta-om-prisomrader/.

Stene, J. and Øiene Smedegård, O. (2013). Hensiktsmessige varme- og kjøleløsninger i bygninger. Technical report, COWI.

Sæle, H., Aasen, M., and Christensen, I. (2022). Ny nettleiemodell for norske husholdninger. Technical report, SINTEF and CICERO.

Sæle,hanne (2021). Bruk strømmen smart for å redusere strømregninga. https://blogg.sintef.no/sintefenergy-nb/bruk-strommen-smart-for-a-redusere-stromregninga/.

Tekna (2021). Solskjerming og energiberegninger. https://www.tekna.no/fag-og-nettverk/bygg-og-anlegg/byggbloggen/solskjerming-og-energiberegninger/.

Thue, J. V. (2019). Kuldebro. https://snl.no/kuldebro_bygningsteknikk.

Tibber (2022). Varm opp hjemmet ditt smartere. https://tibber.com/no/smart-styring/varmestyring.

Vajda, S. (2009). *Mathematical programming*. Dover Publications, New York.

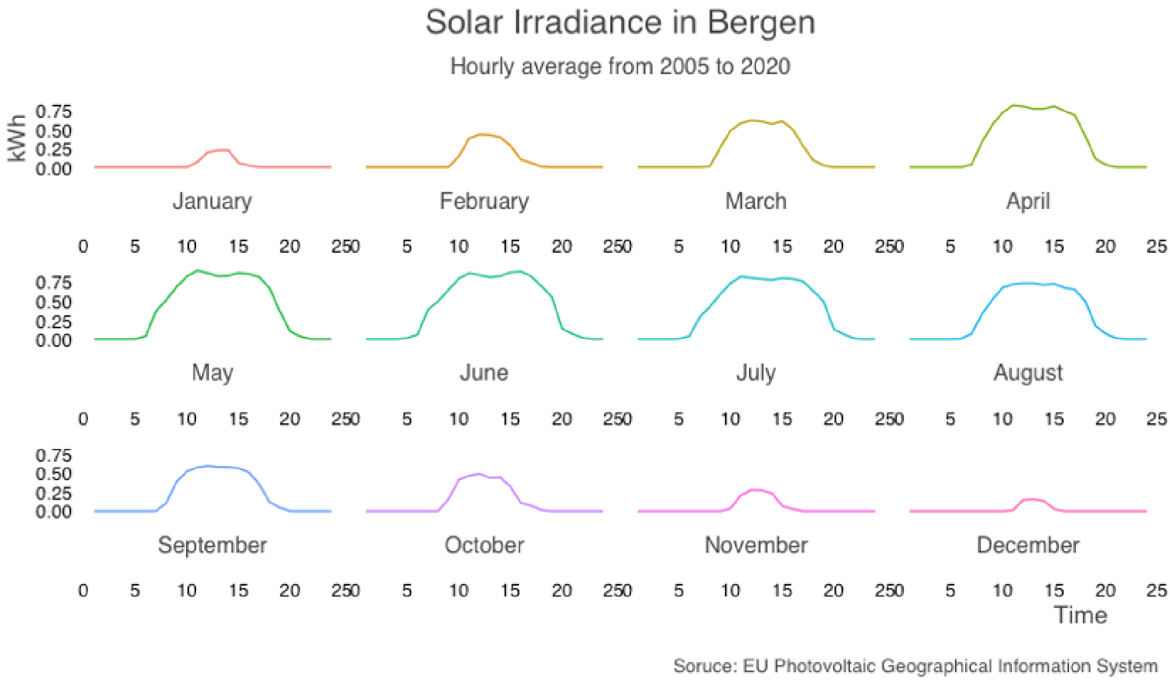
Volue (2022). Falling prices ahead in long-term forecast for electric power in central europe. https://www.volue.com/news/falling-prices-long-term-forecast-electric-power.

Williams, P. H. (2013). *Model Building in Mathematical Programming*. John Wiley Sons Inc.

Wolff, A., Weber, I., Gill, B., Schubert, J., and Schneider, M. (2017). Tackling the interplay of occupants' heating practices and building physics: Insights from a german mixed methods study. *Energy research & social science*, 32:65–75.

Appendix

A1 Solar irradiance in Bergen



Solar Irradiance in Bergen

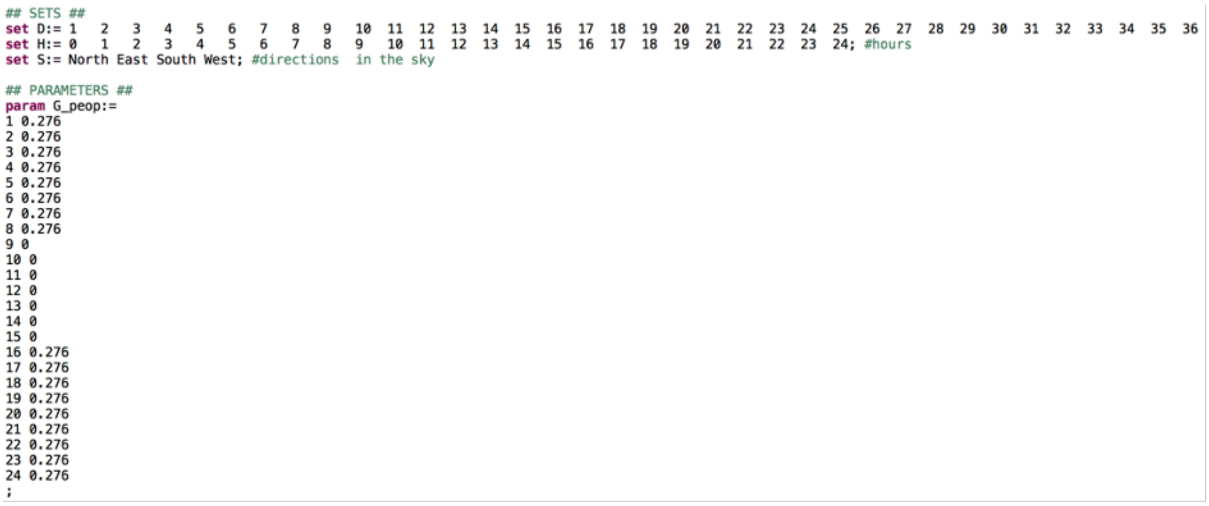
Hourly average from 2005 to 2020

Soruce: EU Photovoltaic Geographical Information System

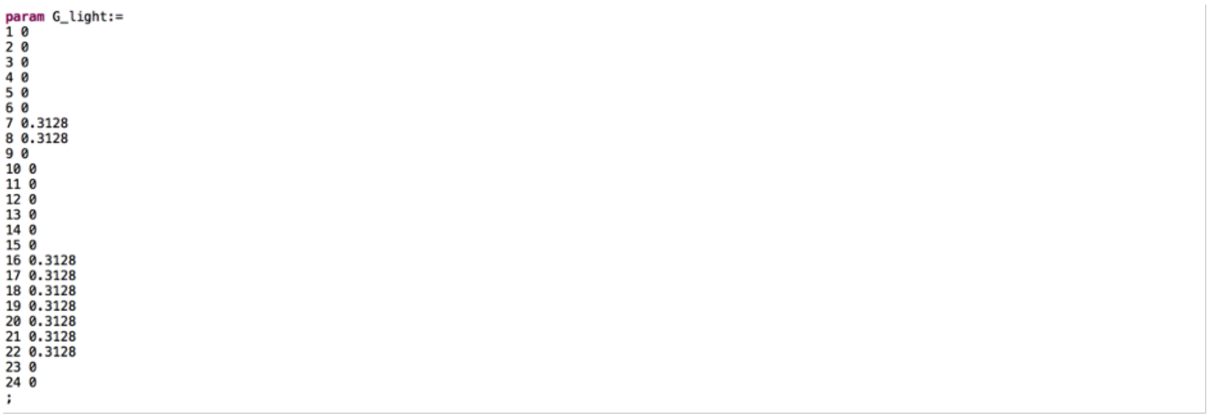
Figure A1.1: Average hourly solar irradiance in Bergen

A2 AMPL Data file

Due to the amount of data, only an extract of the AMPL data file is shown below.

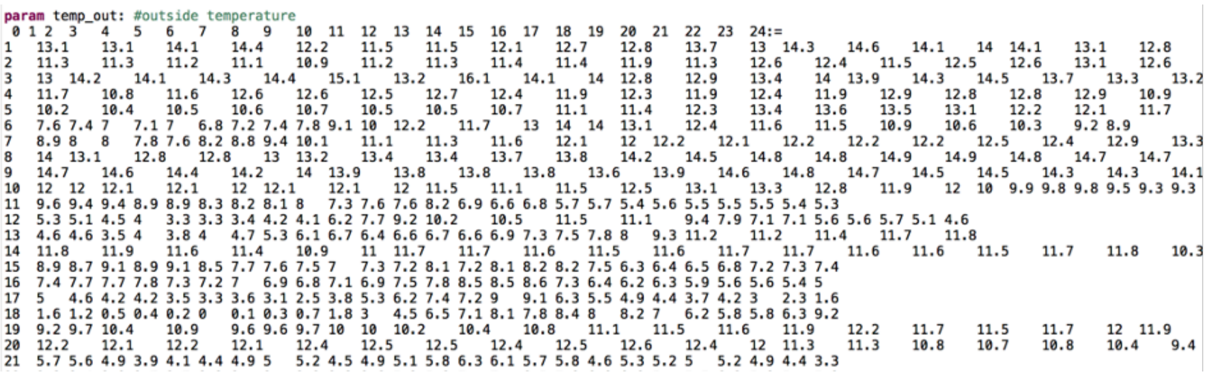


```
## SETS ##
set D:= 1   2   3   4   5   6   7   8   9   10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25  26  27  28  29  30  31  32  33  34  35  36
set H:= 0   1   2   3   4   5   6   7   8   9   10  11  12  13  14  15  16  17  18  19  20  21  22  23  24; #hours
set S:= North East South West; #directions  in the sky

## PARAMETERS ##
param G_peop:=
1 0.276
2 0.276
3 0.276
4 0.276
5 0.276
6 0.276
7 0.276
8 0.276
9 0
10 0
11 0
12 0
13 0
14 0
15 0
16 0.276
17 0.276
18 0.276
19 0.276
20 0.276
21 0.276
22 0.276
23 0.276
24 0.276
;
```

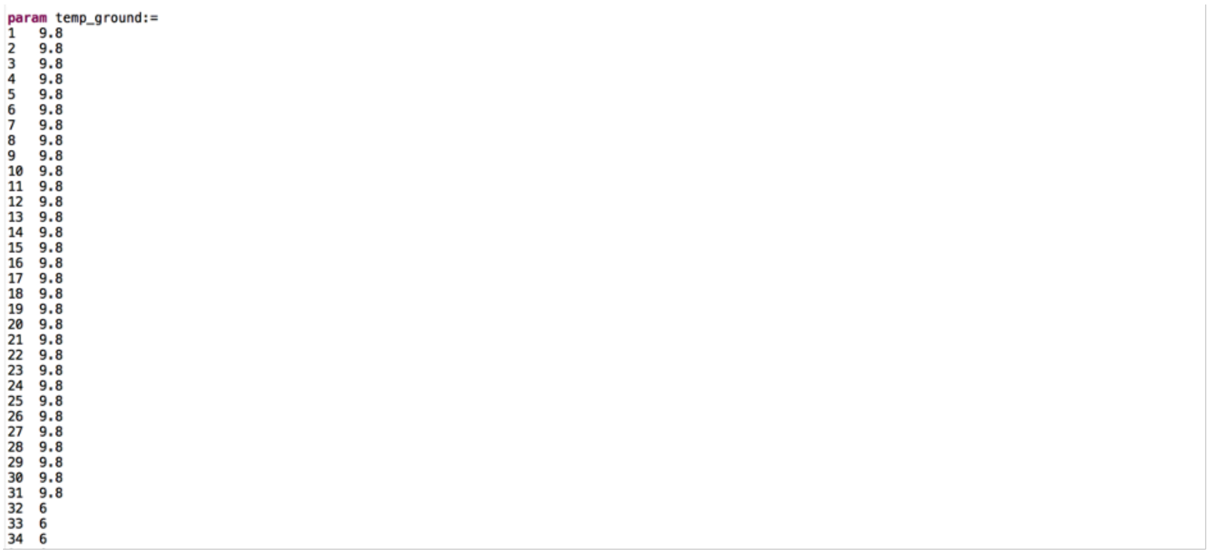


```
param G_light:=
1 0
2 0
3 0
4 0
5 0
6 0
7 0.3128
8 0.3128
9 0
10 0
11 0
12 0
13 0
14 0
15 0
16 0.3128
17 0.3128
18 0.3128
19 0.3128
20 0.3128
21 0.3128
22 0.3128
23 0
24 0
;
```

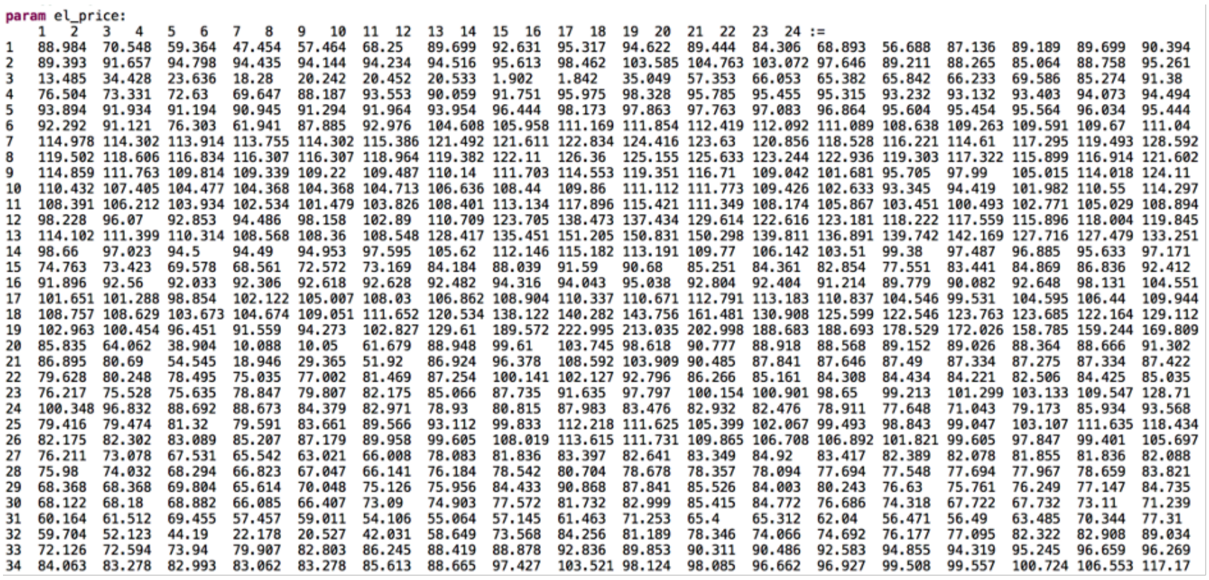
```
param tariff_var:=
1   39.90
2   39.90
3   39.90
4   39.90
5   39.90
6   49.90
7   49.90
8   49.90
9   49.90
10  49.90
11  49.90
12  49.90
13  49.90
14  49.90
15  49.90
16  49.90
17  49.90
18  49.90
19  49.90
20  49.90
21  49.90
22  39.90
23  39.90
24  39.90
;


param area_w:=
North 9.25
East 9.25
South 9.25
West 9.25
;
```



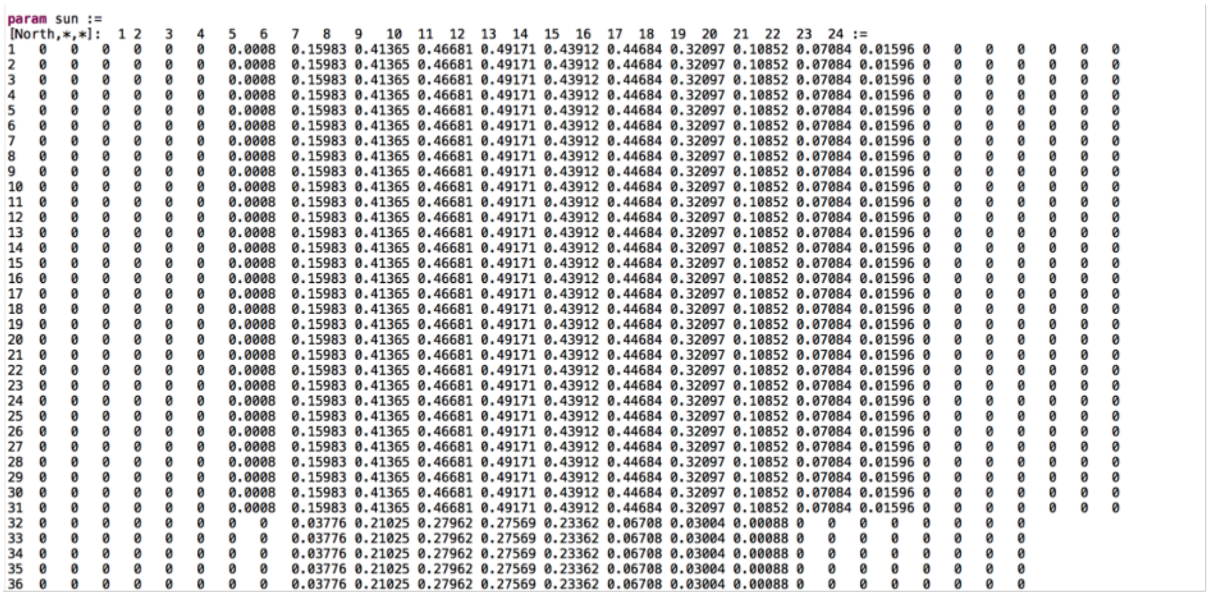
```
param temp_out: #outside temperature
 0 1 2  3   4   5   6   7   8   9   10  11  12  13  14  15  16  17  18  19  20  21  22  23  24:=
1   13.1    13.1    14.1    14.4    12.2    11.5    11.5    12.1    12.7    12.8    13.7    13  14.3    14.6    14.1    14  14.1    13.1    12.8
2   11.3    11.3    11.2    11.1    10.9    11.2    11.3    11.4    11.4    11.9    11.3    12.6    12.4    11.5    12.5    12.6    13.1    12.6
3   13  14.2    14.1    14.3    14.4    15.1    13.2    16.1    14.1    14  12.8    12.9    13.4    14  13.9    14.3    14.5    13.7    13.3    13.2
4   11.7    10.8    11.6    12.6    12.6    12.5    12.7    12.4    11.9    12.3    11.9    12.4    11.9    12.9    12.8    12.8    12.9    10.9
5   10.2    10.4    10.5    10.6    10.7    10.5    10.5    10.7    11.1    11.4    12.3    13.4    13.6    13.5    13.1    12.2    12.1    11.7
6   7.6 7.4 7   7.1 7   6.8 7.2 7.4 7.8 9.1 10  12.2    11.7    13  14  14  13.1    12.4    11.6    11.5    10.9    10.6    10.3    9.2 8.9
7   8.9 8   8   7.8 7.6 8.2 8.8 9.4 10.1    11.1    11.3    11.6    12.1    12  12.2    12.1    12.2    12.2    12.2    12.5    12.4    12.9    13.3
8   14  13.1    12.8    12.8    13  13.2    13.4    13.4    13.7    13.8    14.2    14.5    14.8    14.8    14.9    14.9    14.8    14.7    14.7
9   14.7    14.6    14.4    14.2    14  13.9    13.8    13.8    13.8    13.6    13.9    14.6    14.8    14.7    14.5    14.5    14.3    14.3    14.1
10  12  12  12.1    12.1    12  12.1    12.1    12  11.5    11.1    11.5    12.5    13.1    13.3    12.8    11.9    12  10  9.9 9.8 9.8 9.5 9.3 9.3
11  9.6 9.4 9.4 8.9 8.9 8.3 8.2 8.1 8   7.3 7.6 7.6 8.2 6.9 6.6 6.8 5.7 5.7 5.4 5.6 5.5 5.5 5.5 5.4 5.3
12  5.3 5.1 4.5 4   3.3 3.3 3.4 4.2 4.1 6.2 7.7 9.2 10.2    10.5    11.5    11.1    9.4 7.9 7.1 7.1 5.6 5.6 5.7 5.1 4.6
13  4.6 4.6 3.5 4   3.8 4   4.7 5.3 6.1 6.7 6.4 6.6 6.7 6.6 6.9 7.3 7.5 7.8 8   9.3 11.2    11.2    11.4    11.7    11.8
14  11.8    11.9    11.6    11.4    10.9    11  11.7    11.7    11.6    11.5    11.6    11.7    11.7    11.6    11.6    11.5    11.7    11.8    10.3
15  8.9 8.7 9.1 8.9 9.1 8.5 7.7 7.6 7.5 7   7.3 7.2 8.1 7.2 8.1 8.2 8.2 7.5 6.3 6.4 6.5 6.8 7.2 7.3 7.4
16  7.4 7.7 7.7 7.8 7.3 7.2 7   6.9 6.8 7.1 6.9 7.5 7.8 8.5 8.5 8.6 7.3 6.4 6.2 6.3 5.9 5.6 5.6 5.4 5
17  5   4.6 4.2 4.2 3.5 3.3 3.6 3.1 2.5 3.8 5.3 6.2 7.4 7.2 9   9.1 6.3 5.5 4.9 4.4 3.7 4.2 3   2.3 1.6
18  1.6 1.2 0.5 0.4 0.2 0   0.1 0.3 0.7 1.8 3   4.5 6.5 7.1 8.1 7.8 8.4 8   8.2 7   6.2 5.8 5.8 6.3 9.2
19  9.2 9.7 10.4    10.9    9.6 9.6 9.7 10  10  10.2    10.4    10.8    11.1    11.5    11.6    11.9    12.2    11.7    11.5    11.7    12  11.9
20  12.2    12.1    12.2    12.1    12.4    12.5    12.5    12.4    12.5    12.6    12.4    12  11.3    11.3    10.8    10.7    10.8    10.4    9.4
21  5.7 5.6 4.9 3.9 4.1 4.4 4.9 5   5.2 4.5 4.9 5.1 5.8 6.3 6.1 5.7 5.8 4.6 5.3 5.2 5   5.2 4.9 4.4 3.3
```



```
param temp_ground:=
1   9.8
2   9.8
3   9.8
4   9.8
5   9.8
6   9.8
7   9.8
8   9.8
9   9.8
10  9.8
11  9.8
12  9.8
13  9.8
14  9.8
15  9.8
16  9.8
17  9.8
18  9.8
19  9.8
20  9.8
21  9.8
22  9.8
23  9.8
24  9.8
25  9.8
26  9.8
27  9.8
28  9.8
29  9.8
30  9.8
31  9.8
32  6
33  6
34  6
```



```
param el_price:
    1   2   3   4   5   6   7   8   9   10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  :=
1   88.984  70.548  59.364  47.454  57.464  68.25   89.699  92.631  95.317  94.622  89.444  84.306  68.893  56.688  87.136  89.189  89.699  90.394
2   89.393  91.657  94.798  94.435  94.144  94.234  94.516  95.613  98.462  103.585 104.763 103.072 97.646  89.211  88.265  85.064  88.758  95.261
3   13.485  34.428  23.636  18.28   20.242  20.452  20.533  1.902   1.842   35.049  57.353  66.053  65.382  65.842  66.233  69.586  85.274  91.38
4   76.504  73.331  72.63   69.647  88.187  93.553  90.059  91.751  95.975  98.328  95.785  95.455  95.315  93.232  93.132  93.403  94.073  94.494
5   93.894  91.934  91.194  90.945  91.294  91.964  93.954  96.444  98.173  97.863  97.763  97.083  96.864  95.604  95.454  95.564  96.034  95.444
6   92.292  91.121  76.303  61.941  87.885  92.976  104.608 105.958 111.169 111.854 112.419 112.092 111.089 108.638 109.263 109.591 109.67  111.04
7   114.978 114.302 113.914 113.755 114.302 115.386 121.492 121.611 122.834 124.416 123.63  120.856 118.528 116.221 114.61  117.295 119.493 128.592
8   119.502 118.606 116.834 116.307 116.307 118.964 119.382 122.11  126.36  125.155 125.633 123.244 122.936 119.303 117.322 115.899 116.914 121.602
9   114.859 111.763 109.814 109.339 109.22  109.487 110.14  111.703 114.553 119.351 116.71  109.042 101.681 95.705  97.99   105.015 114.018 124.11
10  110.432 107.405 104.477 104.368 104.368 104.713 106.636 108.44  109.86  111.112 111.773 109.426 102.633 93.345  94.419  101.982 110.55  114.297
11  108.391 106.212 103.934 102.534 101.479 103.826 108.401 113.134 117.896 115.421 111.349 108.174 105.867 103.451 100.493 102.771 105.029 108.894
12  98.228  96.07   92.853  94.486  98.158  102.89  110.709 123.705 138.473 137.434 129.614 122.616 123.181 118.222 117.559 115.896 118.004 119.845
13  114.102 111.399 110.314 108.568 108.36  108.548 128.417 135.451 151.205 150.831 150.298 139.811 136.891 139.742 142.169 127.716 127.479 133.251
14  98.66   97.023  94.5    94.49   94.953  97.595  105.62  112.146 115.182 113.191 109.77  106.142 103.51  99.38   97.487  96.885  95.633  97.171
15  74.763  73.423  69.578  68.561  72.572  73.169  84.184  88.039  91.59   90.68   85.251  84.361  82.854  77.551  83.441  84.869  86.836  92.412
16  91.896  92.56   92.033  92.306  92.618  92.628  92.482  94.316  94.043  95.038  92.804  92.404  91.214  89.779  90.082  92.648  98.131  104.551
17  101.651 101.288 98.854  102.122 105.007 108.03  106.862 108.904 110.337 110.671 112.791 113.183 110.837 104.546 99.531  104.595 106.44  109.944
18  108.757 108.629 103.673 104.674 109.051 111.652 120.534 138.122 140.282 143.756 161.481 130.908 125.599 122.546 123.763 123.685 122.164 129.112
19  102.963 100.454 96.451  91.559  94.273  102.827 129.61  189.572 222.995 213.035 202.998 188.683 188.693 178.529 172.026 158.785 159.244 169.809
20  85.835  64.062  38.904  10.088  10.05   61.679  88.948  99.61   103.745 98.618  90.777  88.918  88.568  89.152  89.026  88.364  88.666  91.302
21  86.895  80.69   54.545  18.946  29.365  51.92   86.924  96.378  108.592 103.909 90.485  87.841  87.646  87.49   87.334  87.275  87.334  87.422
22  79.628  80.248  78.495  75.035  77.002  81.469  87.254  100.141 102.127 92.796  86.266  85.161  84.308  84.434  84.221  82.506  84.425  85.035
23  76.217  75.528  75.635  78.847  79.807  82.175  85.066  87.735  91.635  97.797  100.154 100.901 98.65   99.213  101.299 103.133 109.547 128.71
24  100.348 96.832  88.692  88.673  84.379  82.971  78.93   80.815  87.983  83.476  82.932  82.476  78.911  77.648  71.043  79.173  85.934  93.568
25  79.416  79.474  81.32   79.591  83.661  89.566  93.112  99.833  112.218 111.625 105.399 102.067 99.493  98.843  99.047  103.107 111.635 118.434
26  82.175  82.302  83.089  85.207  87.179  89.958  99.605  108.019 113.615 111.731 109.865 106.708 106.892 101.821 99.605  97.847  99.401  105.697
27  76.211  73.078  67.531  65.542  63.021  66.008  78.083  81.836  83.397  82.641  83.349  84.92   83.417  82.389  82.078  81.855  81.836  82.088
28  75.98   74.032  68.294  66.823  67.047  66.141  76.184  78.542  80.704  78.678  78.357  78.094  77.694  77.548  77.694  77.967  78.659  83.821
29  68.368  68.368  69.804  65.614  70.048  75.126  75.956  84.433  90.868  87.841  85.526  84.003  80.243  76.63   75.761  76.249  77.147  84.735
30  68.122  68.18   68.882  66.085  66.407  73.09   74.903  77.572  81.732  82.999  85.415  84.772  76.686  74.318  67.722  67.732  73.11   71.239
31  60.164  61.512  69.455  57.457  59.011  54.106  55.064  57.145  61.463  71.253  65.4    65.312  62.04   56.471  56.49   63.485  70.344  77.31
32  59.704  52.123  44.19   22.178  20.527  42.031  58.649  73.568  84.256  81.189  78.346  74.066  74.692  76.177  77.095  82.322  82.908  89.034
33  72.126  72.594  73.94   79.907  82.803  86.245  88.419  88.878  92.836  89.853  90.311  90.486  92.583  94.855  94.319  95.245  96.659  96.269
34  84.063  83.278  82.993  83.062  83.278  85.613  88.665  97.427  103.521 98.124  98.085  96.662  96.927  99.508  99.557  100.724 106.553 117.17
```



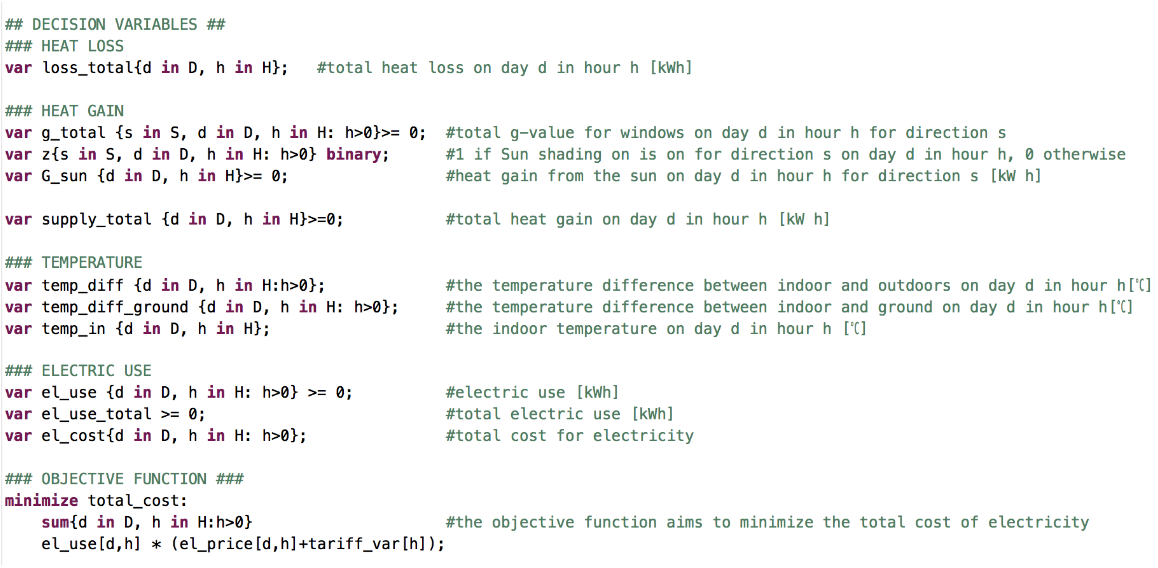
```
param sun :=
[North,*,*]:  1 2   3   4   5   6   7   8   9   10  11  12  13  14  15  16  17  18  19  20  21  22  23  24 :=
1   0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
2   0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
3   0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
4   0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
5   0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
6   0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
7   0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
8   0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
9   0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
10  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
11  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
12  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
13  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
14  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
15  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
16  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
17  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
18  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
19  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
20  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
21  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
22  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
23  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
24  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
25  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
26  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
27  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
28  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
29  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
30  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
31  0   0   0   0   0   0   0.0008  0.15983 0.41365 0.46681 0.49171 0.43912 0.44684 0.32097 0.10852 0.07084 0.01596 0   0   0   0   0   0   0
32  0   0   0   0   0   0   0   0   0.03776 0.21025 0.27962 0.27569 0.23362 0.06708 0.03004 0.00088 0   0   0   0   0   0   0   0
33  0   0   0   0   0   0   0   0   0.03776 0.21025 0.27962 0.27569 0.23362 0.06708 0.03004 0.00088 0   0   0   0   0   0   0   0
34  0   0   0   0   0   0   0   0   0.03776 0.21025 0.27962 0.27569 0.23362 0.06708 0.03004 0.00088 0   0   0   0   0   0   0   0
35  0   0   0   0   0   0   0   0   0.03776 0.21025 0.27962 0.27569 0.23362 0.06708 0.03004 0.00088 0   0   0   0   0   0   0   0
36  0   0   0   0   0   0   0   0   0.03776 0.21025 0.27962 0.27569 0.23362 0.06708 0.03004 0.00088 0   0   0   0   0   0   0   0
```

Figure A2.1: Extract of the AMPL data file

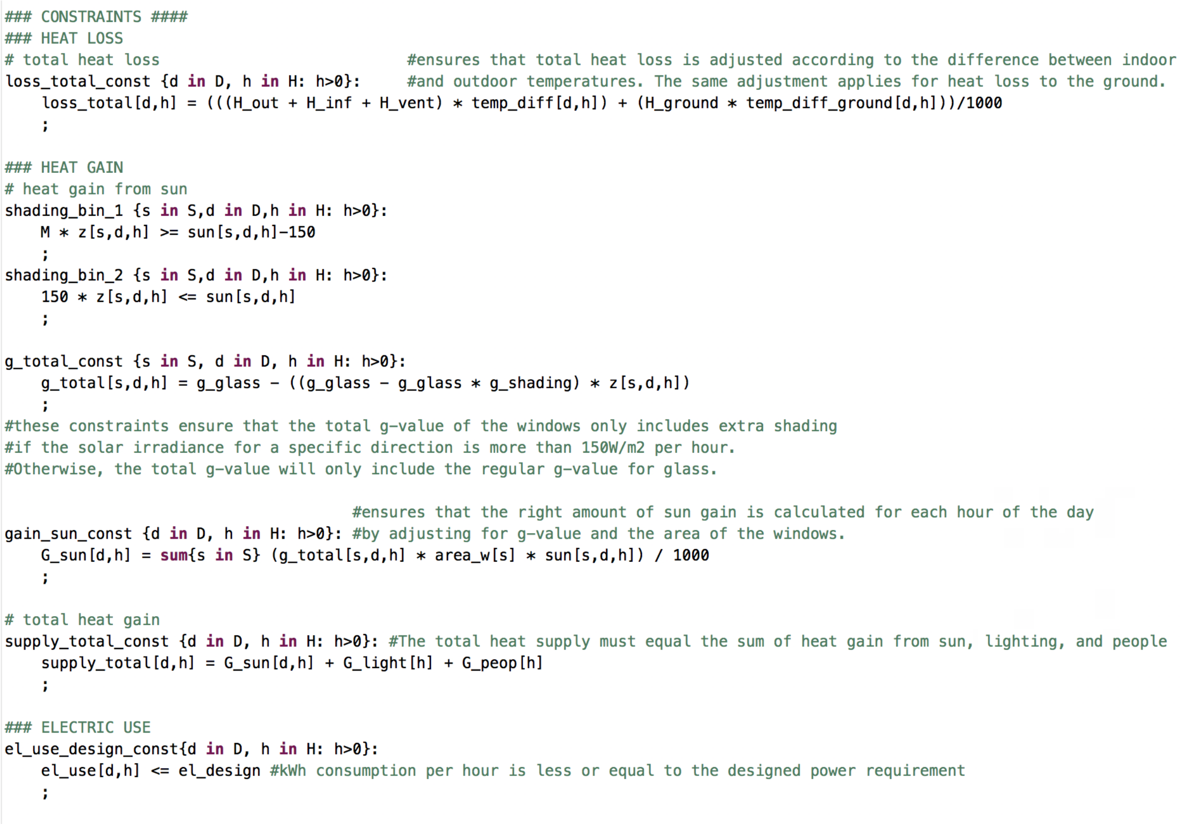
A3 AMPL Model file

A3.1 General model

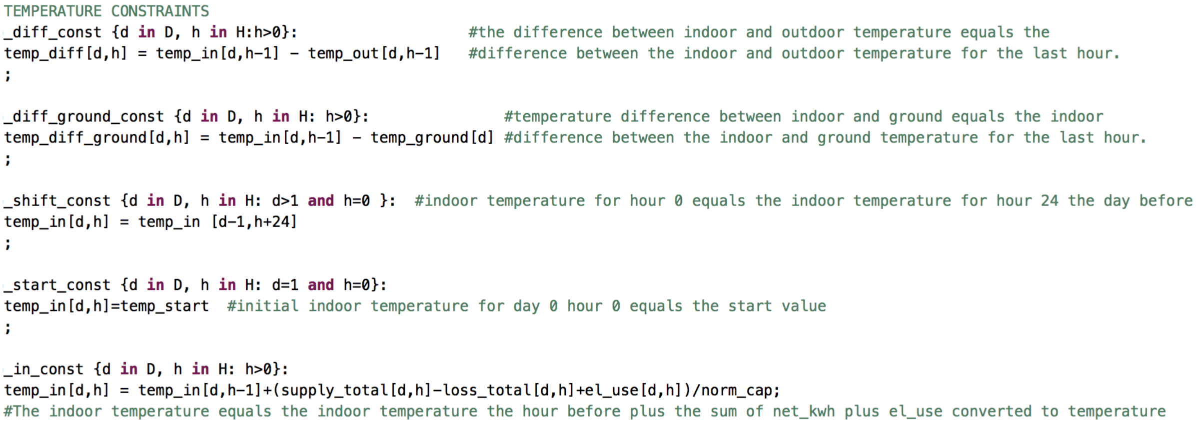
```
## SETS ##
set D; #set of days
set H; #set of hours
set S; #set of directions in the sky

## PARAMETERS ##
### HEAT LOSS
#TEK17
param H_out = 78.66;         #heat transfer coefficient to outdoors [W/K]
param H_ground = 9.2;        #heat transfer coefficient to ground [W/K]
param H_inf = 5.74;          #heat transfer coefficient to infiltration [W/K]
param H_vent = 14.586;       #heat transfer coefficient to ventilation [W/K]

### HEAT GAIN
#sun
param g_glass = 0.53;        #g-value for glass only TEK 17
param g_shading = 0.18;      #g-value for shading only
param sun{S,D,H}>= 0;        #solar radiation on day d in hour h for direction s [W/m2]
param area_w{S}>= 0;         #window area per direction s [m2]
#internal load
param G_light {h in H: h>0}>= 0;      #heat gain from lights in hour h [kWh]
param G_peop {h in H: h>0}>= 0;       #heat gain from people in hour h [kWh]

### TEMPERATURE
param temp_start = 21;       #initial indoor temperature [℃]
param temp_goal = 21;        #indoor temperature goal [℃]
param temp_out {D,H};        #outdoor temperature on day d in hour h [℃]
param temp_ground {D};       #ground temperature on day d [℃]
param norm_cap = 5.152;      #normalised heat capacity [kWh/K]

### ELECTRIC USE
param el_price {D,H};           #price of electricity on day d in hour h [NOK]
param tariff_var{H} >= 0;       #variable network tariff in hour h [NOK]
param el_design = 3.353;        #dimensional power requirement TEK 17 [kWh/h]

## BIG M
param M = 10000;             #Big M
```



```
## DECISION VARIABLES ##
### HEAT LOSS
var loss_total{d in D, h in H};   #total heat loss on day d in hour h [kWh]

### HEAT GAIN
var g_total {s in S, d in D, h in H: h>0}>= 0;  #total g-value for windows on day d in hour h for direction s
var z{s in S, d in D, h in H: h>0} binary;      #1 if Sun shading on is on for direction s on day d in hour h, 0 otherwise
var G_sun {d in D, h in H}>= 0;                 #heat gain from the sun on day d in hour h for direction s [kW h]

var supply_total {d in D, h in H}>=0;           #total heat gain on day d in hour h [kW h]

### TEMPERATURE
var temp_diff {d in D, h in H:h>0};             #the temperature difference between indoor and outdoors on day d in hour h[℃]
var temp_diff_ground {d in D, h in H: h>0};     #the temperature difference between indoor and ground on day d in hour h[℃]
var temp_in {d in D, h in H};                   #the indoor temperature on day d in hour h [℃]

### ELECTRIC USE
var el_use {d in D, h in H: h>0} >= 0;          #electric use [kWh]
var el_use_total >= 0;                          #total electric use [kWh]
var el_cost{d in D, h in H: h>0};               #total cost for electricity

### OBJECTIVE FUNCTION ###
minimize total_cost:
    sum{d in D, h in H:h>0}                     #the objective function aims to minimize the total cost of electricity
    el_use[d,h] * (el_price[d,h]+tariff_var[h]);
```



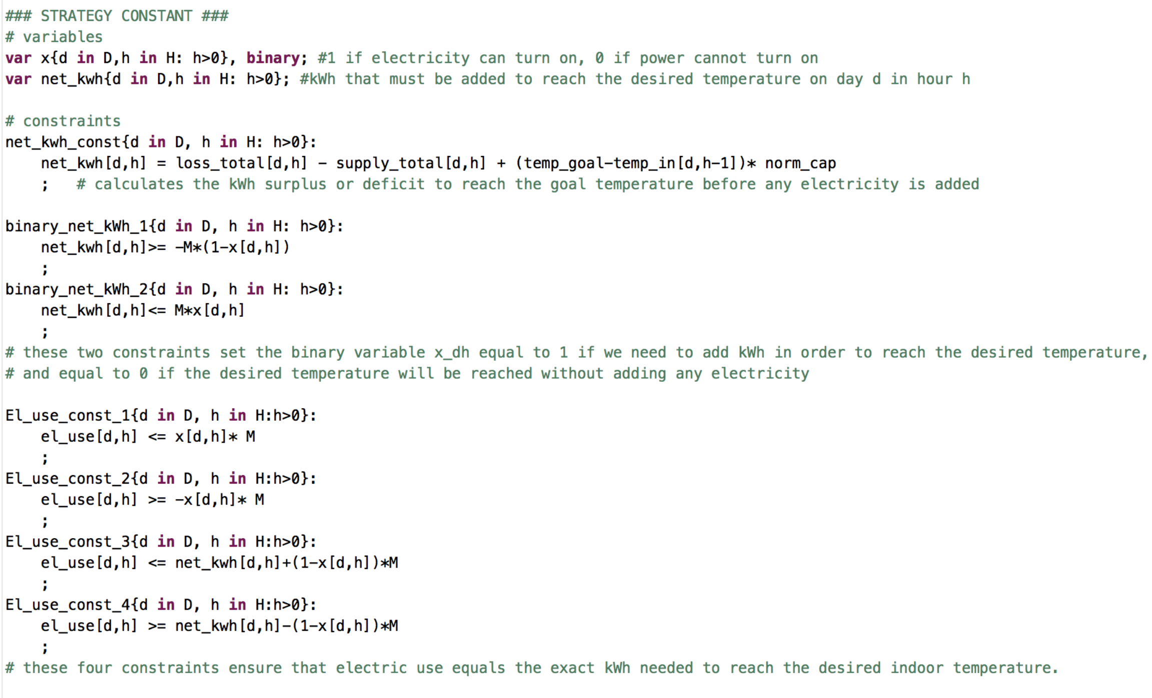
```
### CONSTRAINTS ####
### HEAT LOSS
# total heat loss                           #ensures that total heat loss is adjusted according to the difference between indoor
loss_total_const {d in D, h in H: h>0}:     #and outdoor temperatures. The same adjustment applies for heat loss to the ground.
    loss_total[d,h] = (((H_out + H_inf + H_vent) * temp_diff[d,h]) + (H_ground * temp_diff_ground[d,h]))/1000
    ;

### HEAT GAIN
# heat gain from sun
shading_bin_1 {s in S,d in D,h in H: h>0}:
    M * z[s,d,h] >= sun[s,d,h]-150
    ;
shading_bin_2 {s in S,d in D,h in H: h>0}:
    150 * z[s,d,h] <= sun[s,d,h]
    ;

g_total_const {s in S, d in D, h in H: h>0}:
    g_total[s,d,h] = g_glass - ((g_glass - g_glass * g_shading) * z[s,d,h])
    ;
#these constraints ensure that the total g-value of the windows only includes extra shading
#if the solar irradiance for a specific direction is more than 150W/m2 per hour.
#Otherwise, the total g-value will only include the regular g-value for glass.

                                      #ensures that the right amount of sun gain is calculated for each hour of the day
gain_sun_const {d in D, h in H: h>0}: #by adjusting for g-value and the area of the windows.
    G_sun[d,h] = sum{s in S} (g_total[s,d,h] * area_w[s] * sun[s,d,h]) / 1000
    ;

# total heat gain
supply_total_const {d in D, h in H: h>0}: #The total heat supply must equal the sum of heat gain from sun, lighting, and people
    supply_total[d,h] = G_sun[d,h] + G_light[h] + G_peop[h]
    ;

### ELECTRIC USE
el_use_design_const{d in D, h in H: h>0}:
    el_use[d,h] <= el_design #kWh consumption per hour is less or equal to the designed power requirement
    ;
```



```
TEMPERATURE CONSTRAINTS
_diff_const {d in D, h in H:h>0}:                    #the difference between indoor and outdoor temperature equals the
temp_diff[d,h] = temp_in[d,h-1] - temp_out[d,h-1]   #difference between the indoor and outdoor temperature for the last hour.
;

_diff_ground_const {d in D, h in H: h>0}:              #temperature difference between indoor and ground equals the indoor
temp_diff_ground[d,h] = temp_in[d,h-1] - temp_ground[d] #difference between the indoor and ground temperature for the last hour.
;

_shift_const {d in D, h in H: d>1 and h=0 }:  #indoor temperature for hour 0 equals the indoor temperature for hour 24 the day before
temp_in[d,h] = temp_in [d-1,h+24]
;

_start_const {d in D, h in H: d=1 and h=0}:
temp_in[d,h]=temp_start  #initial indoor temperature for day 0 hour 0 equals the start value
;

_in_const {d in D, h in H: h>0}:
temp_in[d,h] = temp_in[d,h-1]+(supply_total[d,h]-loss_total[d,h]+el_use[d,h])/norm_cap;
#The indoor temperature equals the indoor temperature the hour before plus the sum of net_kwh plus el_use converted to temperature
```

Figure A3.1: General model

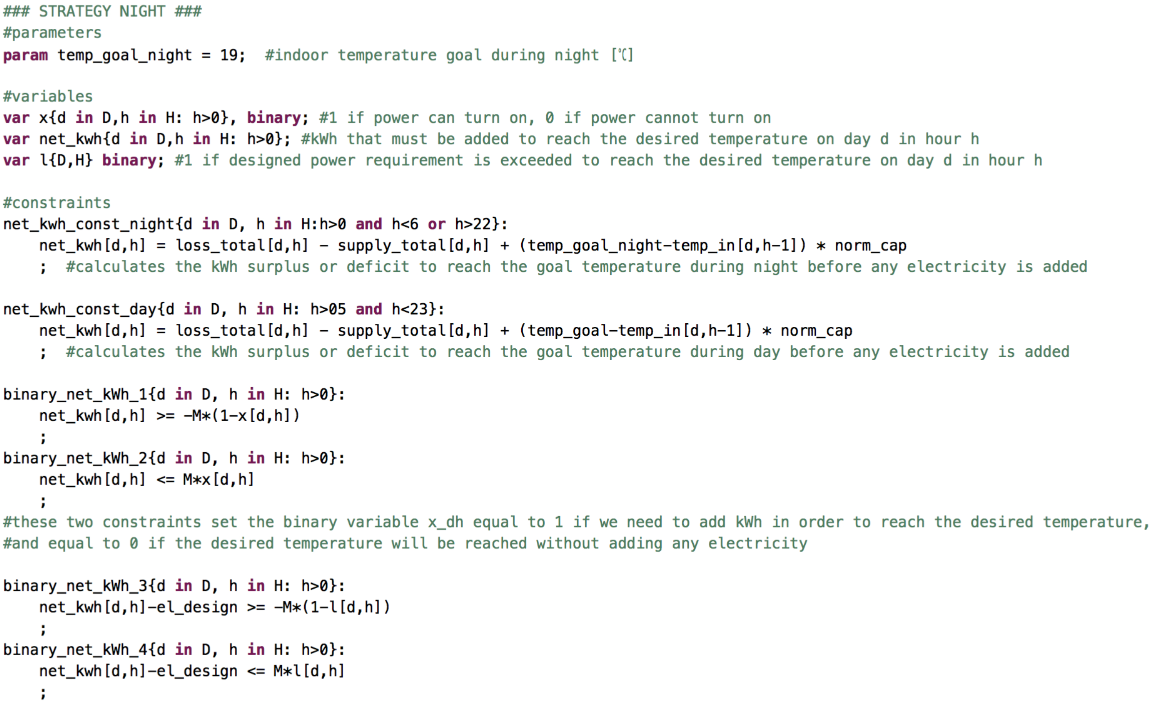
A3.2 Strategy Constant



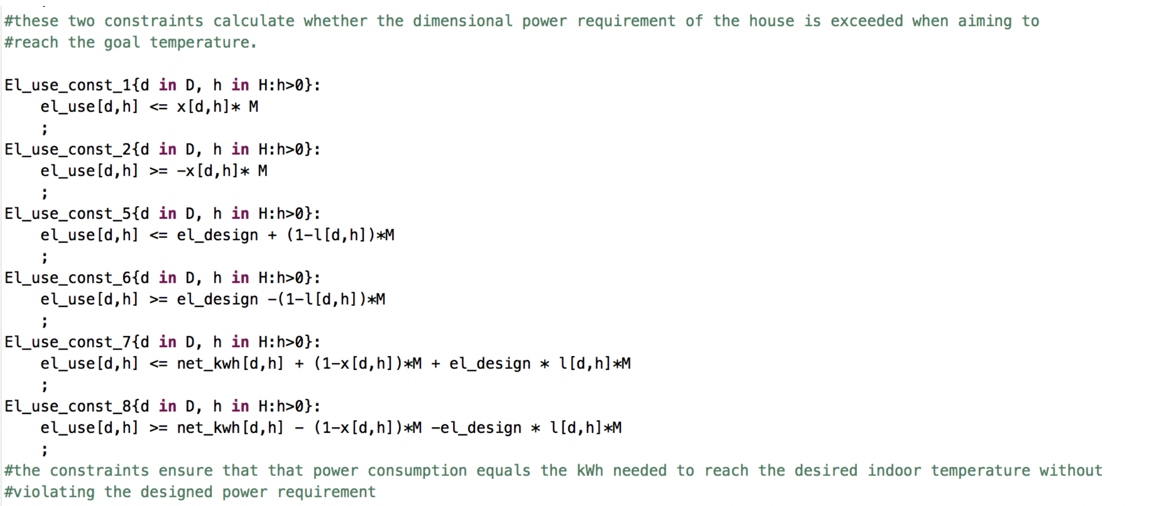
```
### STRATEGY CONSTANT ###
# variables
var x{d in D,h in H: h>0}, binary; #1 if electricity can turn on, 0 if power cannot turn on
var net_kwh{d in D,h in H: h>0}; #kWh that must be added to reach the desired temperature on day d in hour h

# constraints
net_kwh_const{d in D, h in H: h>0}:
    net_kwh[d,h] = loss_total[d,h] - supply_total[d,h] + (temp_goal-temp_in[d,h-1])* norm_cap
    ;   # calculates the kWh surplus or deficit to reach the goal temperature before any electricity is added

binary_net_kWh_1{d in D, h in H: h>0}:
    net_kwh[d,h]>= -M*(1-x[d,h])
    ;
binary_net_kWh_2{d in D, h in H: h>0}:
    net_kwh[d,h]<= M*x[d,h]
    ;
# these two constraints set the binary variable x_dh equal to 1 if we need to add kWh in order to reach the desired temperature,
# and equal to 0 if the desired temperature will be reached without adding any electricity

El_use_const_1{d in D, h in H:h>0}:
    el_use[d,h] <= x[d,h]* M
    ;
El_use_const_2{d in D, h in H:h>0}:
    el_use[d,h] >= -x[d,h]* M
    ;
El_use_const_3{d in D, h in H:h>0}:
    el_use[d,h] <= net_kwh[d,h]+(1-x[d,h])*M
    ;
El_use_const_4{d in D, h in H:h>0}:
    el_use[d,h] >= net_kwh[d,h]-(1-x[d,h])*M
    ;
# these four constraints ensure that electric use equals the exact kWh needed to reach the desired indoor temperature.
```

Figure A3.2: Mod file stratgey constant

A3.3 Strategy Night



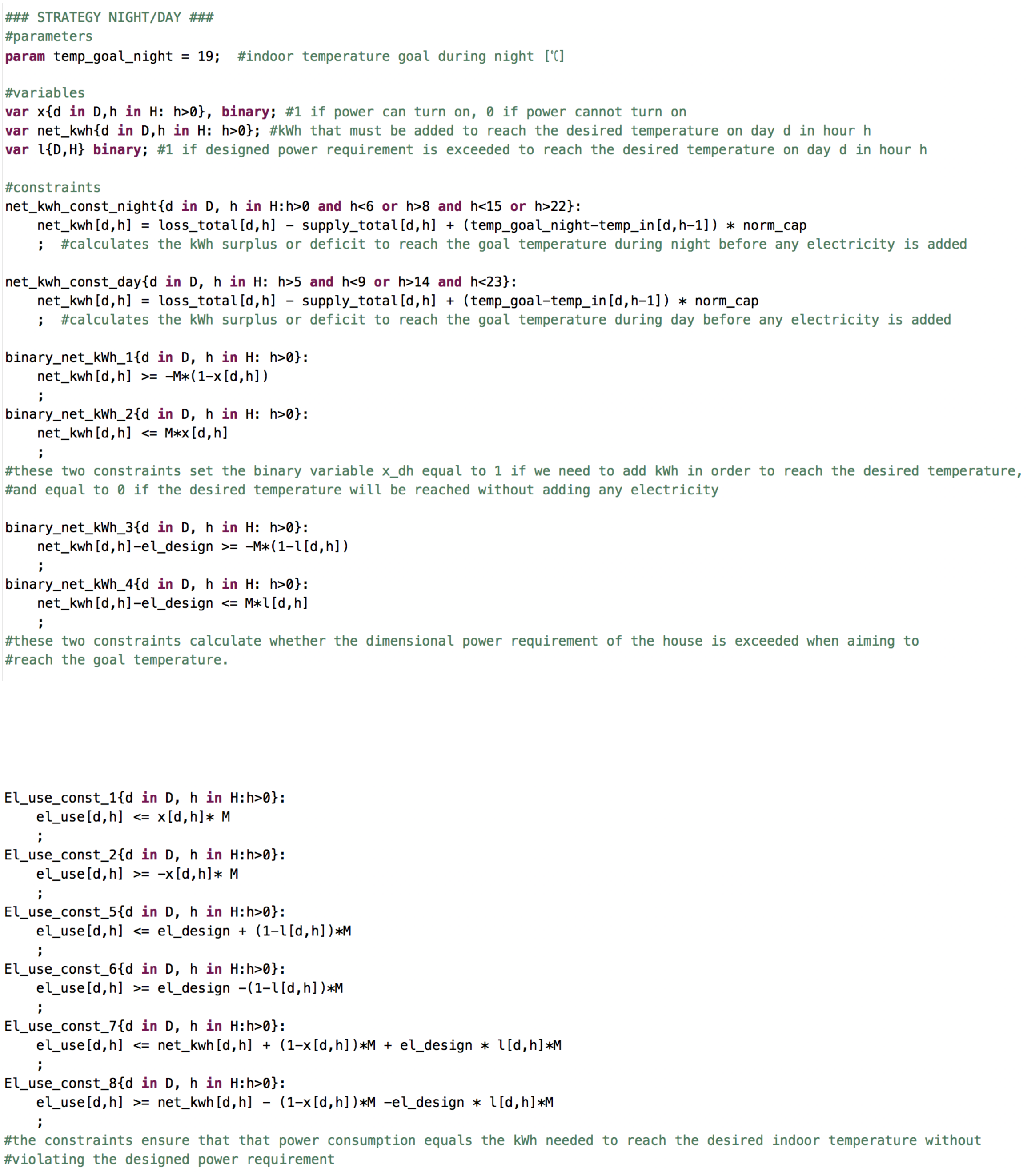
```
### STRATEGY NIGHT ###
#parameters
param temp_goal_night = 19;  #indoor temperature goal during night [℃]

#variables
var x{d in D,h in H: h>0}, binary; #1 if power can turn on, 0 if power cannot turn on
var net_kwh{d in D,h in H: h>0}; #kWh that must be added to reach the desired temperature on day d in hour h
var l{D,H} binary; #1 if designed power requirement is exceeded to reach the desired temperature on day d in hour h

#constraints
net_kwh_const_night{d in D, h in H:h>0 and h<6 or h>22}:
    net_kwh[d,h] = loss_total[d,h] - supply_total[d,h] + (temp_goal_night-temp_in[d,h-1]) * norm_cap
    ;  #calculates the kWh surplus or deficit to reach the goal temperature during night before any electricity is added

net_kwh_const_day{d in D, h in H: h>05 and h<23}:
    net_kwh[d,h] = loss_total[d,h] - supply_total[d,h] + (temp_goal-temp_in[d,h-1]) * norm_cap
    ;  #calculates the kWh surplus or deficit to reach the goal temperature during day before any electricity is added

binary_net_kWh_1{d in D, h in H: h>0}:
    net_kwh[d,h] >= -M*(1-x[d,h])
    ;
binary_net_kWh_2{d in D, h in H: h>0}:
    net_kwh[d,h] <= M*x[d,h]
    ;
#these two constraints set the binary variable x_dh equal to 1 if we need to add kWh in order to reach the desired temperature,
#and equal to 0 if the desired temperature will be reached without adding any electricity

binary_net_kWh_3{d in D, h in H: h>0}:
    net_kwh[d,h]-el_design >= -M*(1-l[d,h])
    ;
binary_net_kWh_4{d in D, h in H: h>0}:
    net_kwh[d,h]-el_design <= M*l[d,h]
    ;
```



```
#these two constraints calculate whether the dimensional power requirement of the house is exceeded when aiming to
#reach the goal temperature.

El_use_const_1{d in D, h in H:h>0}:
    el_use[d,h] <= x[d,h]* M
    ;
El_use_const_2{d in D, h in H:h>0}:
    el_use[d,h] >= -x[d,h]* M
    ;
El_use_const_5{d in D, h in H:h>0}:
    el_use[d,h] <= el_design + (1-l[d,h])*M
    ;
El_use_const_6{d in D, h in H:h>0}:
    el_use[d,h] >= el_design -(1-l[d,h])*M
    ;
El_use_const_7{d in D, h in H:h>0}:
    el_use[d,h] <= net_kwh[d,h] + (1-x[d,h])*M + el_design * l[d,h]*M
    ;
El_use_const_8{d in D, h in H:h>0}:
    el_use[d,h] >= net_kwh[d,h] - (1-x[d,h])*M -el_design * l[d,h]*M
    ;
#the constraints ensure that that power consumption equals the kWh needed to reach the desired indoor temperature without
#violating the designed power requirement
```

Figure A3.3: Mod file strategy night

A3.4 Strategy Night/Day

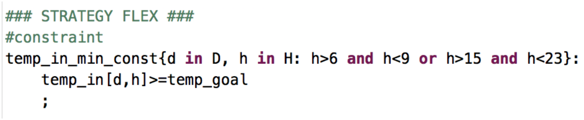


```
### STRATEGY NIGHT/DAY ###
#parameters
param temp_goal_night = 19;  #indoor temperature goal during night [℃]

#variables
var x{d in D,h in H: h>0}, binary; #1 if power can turn on, 0 if power cannot turn on
var net_kwh{d in D,h in H: h>0}; #kWh that must be added to reach the desired temperature on day d in hour h
var l{D,H} binary; #1 if designed power requirement is exceeded to reach the desired temperature on day d in hour h

#constraints
net_kwh_const_night{d in D, h in H:h>0 and h<6 or h>8 and h<15 or h>22}:
    net_kwh[d,h] = loss_total[d,h] - supply_total[d,h] + (temp_goal_night-temp_in[d,h-1]) * norm_cap
    ;  #calculates the kWh surplus or deficit to reach the goal temperature during night before any electricity is added

net_kwh_const_day{d in D, h in H: h>5 and h<9 or h>14 and h<23}:
    net_kwh[d,h] = loss_total[d,h] - supply_total[d,h] + (temp_goal-temp_in[d,h-1]) * norm_cap
    ;  #calculates the kWh surplus or deficit to reach the goal temperature during day before any electricity is added

binary_net_kWh_1{d in D, h in H: h>0}:
    net_kwh[d,h] >= -M*(1-x[d,h])
    ;
binary_net_kWh_2{d in D, h in H: h>0}:
    net_kwh[d,h] <= M*x[d,h]
    ;
#these two constraints set the binary variable x_dh equal to 1 if we need to add kWh in order to reach the desired temperature,
#and equal to 0 if the desired temperature will be reached without adding any electricity

binary_net_kWh_3{d in D, h in H: h>0}:
    net_kwh[d,h]-el_design >= -M*(1-l[d,h])
    ;
binary_net_kWh_4{d in D, h in H: h>0}:
    net_kwh[d,h]-el_design <= M*l[d,h]
    ;
#these two constraints calculate whether the dimensional power requirement of the house is exceeded when aiming to
#reach the goal temperature.
```

```
El_use_const_1{d in D, h in H:h>0}:
    el_use[d,h] <= x[d,h]* M
    ;
El_use_const_2{d in D, h in H:h>0}:
    el_use[d,h] >= -x[d,h]* M
    ;
El_use_const_5{d in D, h in H:h>0}:
    el_use[d,h] <= el_design + (1-l[d,h])*M
    ;
El_use_const_6{d in D, h in H:h>0}:
    el_use[d,h] >= el_design -(1-l[d,h])*M
    ;
El_use_const_7{d in D, h in H:h>0}:
    el_use[d,h] <= net_kwh[d,h] + (1-x[d,h])*M + el_design * l[d,h]*M
    ;
El_use_const_8{d in D, h in H:h>0}:
    el_use[d,h] >= net_kwh[d,h] - (1-x[d,h])*M -el_design * l[d,h]*M
    ;
#the constraints ensure that that power consumption equals the kWh needed to reach the desired indoor temperature without
#violating the designed power requirement
```

Figure A3.4: Mod file strategy night/day

A3.5 Strategy Flex



```
### STRATEGY FLEX ###
#constraint
temp_in_min_const{d in D, h in H: h>6 and h<9 or h>15 and h<23}:
    temp_in[d,h]>=temp_goal
    ;
```

Figure A3.5: Mod file strategy flex

A4 Output values for strategy Flex

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
93	0.00	0.00	0.81	3.35	3.35	3.35	3.35	1.63	0.00	0.00	0.00	0.00
94	0.00	0.00	0.00	3.35	3.35	2.14	0.00	0.81	0.00	0.00	0.00	0.00
95	0.00	0.00	0.00	0.22	3.35	0.00	3.35	0.89	0.00	0.00	0.00	0.00
96	0.00	0.00	3.35	3.35	3.35	2.49	0.00	0.00	0.00	0.00	0.00	0.00
97	0.00	0.00	3.35	3.35	3.35	3.35	0.00	0.00	0.00	0.00	0.00	0.00
98	3.35	3.35	3.35	3.35	3.35	1.34	0.00	1.87	0.00	0.00	0.00	0.70
99	0.00	1.75	3.35	3.35	3.35	3.35	0.00	0.00	0.00	0.00	0.00	0.00
100	0.00	0.00	3.21	3.35	3.35	0.00	3.35	1.36	0.00	0.00	0.00	0.00
101	0.00	0.00	3.35	3.35	3.35	3.35	3.35	3.35	3.35	3.35	0.55	0.00
102	0.28	3.35	3.35	3.35	3.35	3.35	0.00	0.00	0.00	0.00	0.00	0.00
103	0.00	0.00	3.35	3.35	3.35	0.00	3.07	0.00	0.00	0.00	0.00	0.00
104	0.00	0.00	0.78	3.35	3.35	0.00	3.35	0.96	0.00	0.00	0.00	0.00
105	0.00	0.00	0.00	3.35	3.35	0.27	3.35	0.00	0.00	0.00	0.00	0.00
106	0.00	2.59	3.35	3.35	3.35	0.00	0.00	0.00	0.00	0.00	0.00	0.00
107	0.00	0.00	3.35	3.35	3.35	1.84	3.35	1.34	0.00	0.00	0.00	0.00
108	0.00	0.00	0.00	3.35	3.35	1.84	3.35	1.14	0.00	0.00	0.00	0.00
109	0.00	0.00	3.35	3.35	3.35	1.06	3.35	1.47	0.00	0.00	0.00	0.00
110	0.00	1.50	3.35	3.35	3.35	3.35	0.00	0.00	0.00	0.00	0.00	0.00
111	0.00	0.00	0.00	3.35	3.35	0.67	3.35	0.96	0.00	0.00	0.00	0.00
112	0.00	0.16	3.35	3.35	3.35	3.35	3.35	1.75	0.00	0.00	0.00	0.32
113	0.00	0.59	3.35	3.35	3.35	3.35	3.35	0.00	0.00	0.00	0.00	0.00
114	0.00	0.00	3.35	3.35	3.35	0.06	3.35	0.00	0.00	0.00	0.00	0.00
115	0.00	0.00	0.00	3.35	3.35	0.29	3.35	0.92	0.00	0.00	0.00	0.00
116	0.00	0.00	0.94	3.35	3.35	0.00	3.35	0.97	0.00	0.00	0.00	0.00
117	0.00	1.60	3.35	3.35	3.35	3.35	3.35	0.00	0.00	0.00	0.00	0.00
118	0.00	0.00	0.00	3.35	3.35	1.48	3.35	1.12	0.00	0.00	0.00	0.00
119	0.00	0.00	3.35	3.35	3.35	2.71	0.00	0.00	0.00	0.00	0.00	0.00
120	0.00	3.35	3.35	3.35	3.35	0.00	2.26	0.00	0.00	0.00	0.00	0.00
121	0.00	0.00	0.00	3.35	3.35	3.35	3.35	3.35	0.00	0.00	0.00	0.00
122	0.00	3.24	3.35	3.35	3.35	0.00	0.00	1.24	0.00	0.00	0.00	0.00
123	0.00	3.35	3.35	3.35	3.35	3.35	1.09	0.00	0.00	0.00	0.00	0.00

	13	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
93	3.35	3.35	2.29	3.35	0.00	0.00	0.00	0.74	0.78	0.76	0.00	0.00
94	0.00	3.35	3.35	3.35	0.46	0.00	0.00	0.89	0.92	0.91	3.35	0.00
95	3.35	3.35	3.35	0.41	2.95	0.00	1.46	1.51	1.51	1.48	0.00	3.35
96	1.17	3.35	3.35	3.35	2.67	0.00	1.29	1.38	1.41	1.41	0.00	3.27
97	1.73	3.35	3.35	3.35	2.83	0.00	1.40	1.55	1.63	1.50	0.00	0.00
98	3.35	3.35	3.35	3.35	1.50	1.48	1.41	1.39	1.35	1.34	0.00	0.00
99	0.13	3.35	3.35	3.35	1.18	1.18	1.21	1.14	1.13	1.11	0.00	0.00
100	1.70	3.35	3.35	3.35	1.37	1.36	1.41	1.34	1.28	1.32	0.00	0.00
101	0.00	0.00	3.35	3.35	0.00	0.00	0.00	0.00	1.32	1.34	0.00	0.00
102	0.72	3.35	3.35	3.35	1.14	1.07	1.04	2.14	0.00	1.07	0.00	0.00
103	0.00	2.73	3.35	3.35	2.11	0.00	1.05	1.07	1.07	1.08	0.00	0.00
104	0.00	0.53	3.35	3.35	0.78	0.77	0.76	0.78	0.81	0.82	0.00	0.00
105	0.00	0.62	3.35	3.35	0.78	0.78	0.75	0.72	0.71	0.69	0.00	0.00
106	0.03	3.35	3.35	3.35	1.24	1.34	1.35	1.41	1.49	1.50	0.00	0.00
107	1.38	3.35	3.35	3.35	1.25	1.25	1.23	1.21	1.18	1.17	0.00	0.00
108	0.13	3.35	3.35	3.35	1.34	1.25	1.10	1.15	1.27	1.21	0.00	0.00
109	3.35	3.35	3.35	3.35	0.80	0.00	1.34	1.39	1.39	1.38	0.00	0.00
110	0.00	2.46	3.35	3.35	1.02	1.04	1.00	1.05	1.04	1.05	0.00	0.00
111	0.00	3.33	3.35	3.35	1.51	1.62	1.73	1.80	1.70	1.82	0.00	0.00
112	3.35	3.35	3.35	3.35	1.58	1.62	1.59	1.62	1.61	1.62	0.00	0.00
113	1.24	3.35	3.35	3.35	1.27	1.24	1.22	1.22	1.21	1.20	0.00	0.00
114	0.00	2.47	3.35	3.35	0.99	1.00	0.99	0.98	0.96	0.95	0.00	0.00
115	0.00	1.30	3.35	3.35	0.89	0.96	0.98	0.99	0.98	0.97	0.00	0.00
116	0.00	1.99	3.35	3.35	1.12	1.14	1.16	1.17	1.18	1.18	0.00	0.00
117	0.00	0.00	3.35	3.35	1.18	1.17	1.09	1.05	0.97	0.95	0.00	0.00
118	0.00	2.75	3.35	3.35	1.10	1.12	1.10	1.13	1.07	1.13	0.00	0.00
119	0.00	3.03	3.35	3.35	1.11	1.18	1.24	1.25	1.26	1.30	0.00	0.00
120	1.65	3.35	3.35	3.35	1.45	1.45	1.39	1.34	1.33	1.21	0.00	0.00
121	0.00	0.00	0.00	3.17	1.23	1.24	1.34	1.28	1.39	1.23	0.00	0.00
122	1.29	3.35	3.35	3.35	3.35	2.88	0.00	0.00	1.60	1.55	0.00	0.00
123	3.16	3.35	3.35	3.35	1.62	1.65	1.66	1.77	1.74	1.74	0.00	0.00

Table A4.1: Mod file strategy night