MMP-Elections: Equal Influence and Controlled Assembly Size

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Highlights

- A volatile Bundestag size is due to ”traditional accounting” of ballot data.
- A market model uses “faithful accounting” on essential ballot data now wasted.
- It is shown that voters’ equal influence and constant assembly size are obtained.

Abstract

In an MMP election (Mixed Member Proportional) of a legislature, a QP-ballot supports party Q in a single-seat constituency and a list of candidates from P. With $\omega(j)$ constituency seats won and list support in $z(P_j)$ ballots, *party* P_j

wins $\alpha(j)$ list seats, so that $\omega(j)+\alpha(j)$ becomes *proportional* to $z(P_j)$.

The *pivotal* party, P_{j^*}, has the highest of all ratios $\omega(j)/z(P_j)$. Proportionality implies, for all P_j passing some threshold, that $[\omega(j)+\alpha(j)]/z(P_j) \geq \omega(j^*)/z(P_{j^*})$.

In the smallest proportional assembly, all ≥ are equalities and $\alpha(j^*)=0$.

The pivotal party's list support, $z(P_{j^*})$, is naturally volatile. An election with $\alpha(j^*)=0$ tells that $z(P_{j^*})$ list votes were wasted, and many voters learn it. Thus, between Bundestag elections 2017 and 2021, $z(P_{j^*})$ dropped significantly. The smallest possible size of a proportional assembly rose from 709 to 794 seats, while the legal norm is 598.

But an ad-hoc law of 2020 abandoned the proportionality rule, shrinking the assembly from 794 to 736 seats.

$\omega(j)$ measures and records the success of party P_j in the single-seat tallies; it also records how much $\alpha(j)$ is reduced by P_j's constituency success. The paper compares this *"traditional accounting"* and *"faithful accounting"*: The latter records a Q_iP_j-ballot, with Q_i as *constituency winner*, with a tiny seat fraction that reduces $\alpha(j)$. Traditional accounting treats party P_j as a basic entity. Faithful accounting replaces it by the set $\Lambda(P_j)$ of *voters* with list vote for P_j.

This is a paradigm shift: Traditional accounting works even if constituency votes and list votes are collected in separate ballot boxes. But in faithful accounting, each ballot's *combination* of Q_i and P_j is essential.

Main results: The change from traditional to faithful accounting brings the assembly size under control. A large inequality in voters influence is substantially reduced.

Introduction

In MMP-elections (Mixed Member Proportional representation), a QP-ballot contains a *first-vote* for party Q's candidate in a single-seat constituency and a *second-vote* for a list of candidates from party P in one common tally. In *split ballots*, P≠Q. Usually, t he first-vote tallies use the *first-past-the-post* method.

Traditional accounting neglects all ballot *combinations* of first- and second-vote: Collecting them in separate ballot boxes would not change the result. As explained in Section 1, the assembly size is out of control: 111 *extra-ordinary* list seats in Germany's Bundestag election 2017, and much more in 2021. Presently there are 299 constituencies, and the law's *norm* is 299 list seats.

Faithful accounting, as shown in Section 2, makes use of these combinations in order to obtain a predetermined assembly size, while also complying with MMP's *proportionality rule*.

Our mathematical framework is a market model: Second-votes are "currency" to pay a "market price" for list seats and (fractions of) direct seats.

The Federal Constitutional Court (2008) emphasizes the principle of all *voters' equal influence* on the result.

Under *traditional accounting,* casting split QP-ballots (instead of QQ or PP) is a strategy that possibly may increase a ballot's influence. *Faithful accounting* thoroughly follows each single ballot and counteracts this possibility.

However, if constituency C's direct seat is won with too small plurality, then voters in C get too high influence. Section 2 also describes the *"W-U method",* which gives the first-past-the-post winner support from a majority, see (2.17).

The 2017 election achieved a unique transparency by giving top priority to strict proportionality. Its key data are used throughout this paper (Table 1).

1. MMP-elections with traditional accounting

MMP is a family of methods designed for election of a legislature. Each voter casts a ballot with two votes.

The first-vote, *ErSt* (Erststimme) is for a single seat election in the voter's constituency C_k, $1 \le k \le c$: The winner is elected *directly* to the assembly.

The second-vote, *ZwSt* (Zweitstimme), supports a *list* of candidates from a political *party*, P_j, $1 \le j \le p$; r of them *qualify* to contest for *list seats* ($j \le r$).

MMP started in W-Germany (1949). With changing rules it has been used to elect the *Bundestag* (federal legislature). Table 1 shows that r=7 parties P_j qualified, by winning 3 direct seats or by receiving 5% of the ZwSt:

BUNDESTAG ELECTION 2017									
J	P_j	ErSt		direct	SEATS	list	ZwSt		
		%	$e(P_j)$	$\omega(j)$	total	$\alpha(J)$	$z(P_j)$	%	ZwSt/ total
1	CDU	30.2	14,030751	185	200	15	12,447656	26.8	62238
2	SPD	24.6	11,429231	59	153	94	9,539381	20.5	62349
3	AfD	11.5	5,317499	3	94	91	5,878115	12.6	62533
4	FDP	7.0	3,249238	0	80	80	4,999449	10.4	62493
5	Linke	8.6	3,966637	5	69	64	4,297270	9.2	62279
6	Grüne	8.0	3,717922	1	67	66	4,158400	8.9	62066
7	CSU	7.0	3,255487	46	46	0	2,869688	6.2	62385
		97–	44,966765	299	709	410	z=44,189959	95–	62327

TABLE 1 The assembly got 709 seats due to the proportionality requirement: Before list seats were distributed, CSU already had $\omega(7)=46$ seats and z(CSU) ZwSt; the "theoretic" total is the following *critical size*::

(1.1) $\omega(7) \times z/z(CSU) = 46 \times 44189959 / 2869688 = 708.348\ldots$ seats.

On average, 62327 ZwSt support each of the 709 seats; the right hand column of ratios illustrates the accuracy of the approximation algorithm.

If a party has received more seats than proportionality entitles it to, it has seats *"in overhang"*. By law, a party was not allowed to have *direct seats* in overhang, and 410 list seats were distributed when the sum of totals passed the critical value: Only then were all CSU's 46 direct seats out of overhang.

Each ballot must combine one ErSt and one ZwSt, but the tally is *as if* they were collected in different ballot boxes. In Section 2, *"faithful accounting"* takes these ballot combinations into account.

The critical size is highly volatile. Three consecutive elections illustrate this:

	ω(7) ×	z /	z(CSU) =	critical size
2013	45	36,867417	3,243569	511.484…
2017	46	44,189959	2,869688	708.348…
2021	45	42,380698	2,402827	793.703…

TABLE 2 By 2017 rules, distribution of list seats stops when the assembly size has passed or reached both critical size and 598 seats. Thus,

with 2017 rules, the *assembly sizes* in 2013; 2017; 2021 are, respectively

max(512, 598)=598 ; max(709, 598)=709 ; max(794, 598)=794.

In fact however, they became, respectively, 631 ; 709 ; 736.

In 2013, 33 *extra-ordinary list seats* were distributed according to *complicated rules for the 2D allocation of list seats* to r=5 parties *and* 16 states.

In 2017, a new *transparency* was obtained: Everybody could check the proportionality: 708.348…≈709. Then, new ad-hoc complications destroyed it:

In 2021, the main explanation is an emergency law to curb an assembly growth. It abandoned proportionality, letting CSU keep 4 direct seats in overhang: At assembly size 736, CSU was, by Table 2, entitled to only

$$736 \times 2402827 \,/\, 42380698 \;=\; 41.728\ldots \text{ seats.}$$

With the transparent 2017 rules, there would have been 794–598=196 extra-ordinary list seats.

Behnke (2020) doubted (based on simulations) the efficiency of the new ad-hoc rules, and also their constitutionality.

The $\omega(j)$ in Table 1 show both the *ErSt-success* of P_j and its *commitment in the final ZwSt tally*, since the proportionality rule encompasses both direct seats and list seats: All seats must be "paid" with ZwSt at the same price. A small ZwSt resource z(CSU) of CSU (=P_7) and a large success/commitment $\omega(7)$ give CSU its unique *pivotal status* : According to (1.1), the critical size is determined by data that are *specific* for CSU, i.e. $\omega(7)$ and z(CSU)/z.
The pivotal party has the highest ratio $\omega(j)/z(P_j)$.

By law, the W/S-L algorithm [Webster (1832)/Sainte-Laguë (1910)] distributes $\alpha(j)$ list seats (one-by-one) to the qualified P_j ($1 \leq j \leq r=7$):

(1.2) Party P_j contests for its t^{th} ℓist seat, $P_j\,\ell t$, with its

$$\textit{contest number}\ \ z(P_j) \,/\, \{2\times[\omega(j) + t] - 1\}, \quad t \geq 1$$

Under *faithful accounting*, the $\omega(j)$ are replaced by the $\psi[\Lambda(P_j)]$, which are non-integers defined in (2.13).

The last nine of the 410 list seats in Table 1, with contest numbers, are:

402	403	404	405	406	407	408	409	410
CDUℓ13	SPDℓ93	FDPℓ80	AfDℓ89	Linℓ64	CDUℓ14	SPDℓ94	Grüℓ66	CDUℓ15
31513	31483	31443	31434	31367	31354	31277	31266	31197

TABLE 3 The overhang in CSU's 46 direct seats ends with CDUℓ15.

The mechanism behind the growth of the critical size is seen in (1.1) and Table 2. One factor, $\omega(7)$, varies only between 100% and 97.8% of its maximal value 46. (CSU runs only in Bavaria, with 46 constituencies.) But CSU's share z(CSU)/z of the ZwSt, drops from 8.8% (2013), to 6.2% (2017) and to 5.7% (2021); the factor z/z(CSU) in (1,1) may be normalized to

100% (2013), 135% in 2017, and 154% in 2021.

A QP-ballot has ZwSt to party P and ErSt to the candidate of party Q; the ballot is *split* if Q≠P. This may be due to a voter splitting an intended PP-ballot when P's candidate is deemed unable to win the constituency's direct seat.
An intended QQ-ballot may be split in order to help a coalition partner P to pass the 5%-threshold. This motivation was particularly strong in the 2017 election, with Q=CDU or CSU, and P=FDP: P had been a government partner of the union parties, CDU/CSU, but failed to pass the 5% threshold in 2013. In 2017, FDP got 5 million ZwSt and passed 10%, but got only 3.35 million ErSt. But CSU was pivotal; the dramatic fall of z(CSU)/z raised *critical size* to 708.348... (Table 2).

The urge to help FDP got smaller in 2021, but an experience from 2017 was that its 2.9 million ZwSt gave CSU $\alpha(7)=0$ list seats (Table 1). Rather than "wasting" their ZwSt in 2021 too, many wanted to make better use of it: z(CSU)/z dropped again, pushing critical size well above 793 seats (Table 2).

(1.3) **Accounting matters** In Table 1, P_j is "account owner"; $\omega(j)$ is P_j's ErSt success, but also its "commitment" which it must pay with ZwSt from another account, i.e. $z(P_j)$. The proportionality rule requires seat distribution to go on until the price has dropped enough, i.e. all direct seats are paid. The last of them belongs to the pivotal CSU; see Table 1. It finally got rescued from overhang by CDUℓ15 at assembly size 709; see Table 3.

The proportionality rule implies the existence of a pivotal party. Table 2 illustrates a consequence of the dwindling ZwSt supply z(CSU) of the pivotal CSU: The traditional accounting in Table 1 is *not compatible* with the idea that all z voters (44189959 in Table 1) should, through their ballot (ErSt *and* ZwSt together), have the same influence on the outcome:

(1.4) For equality, the influence that a QP-ballot gets through its ZwSt to P, *must* depend on what influence it already got through its ErSt to Q. Therefore, the ballot's *combination* of P and Q cannot be ignored.

In *faithful accounting*, the *set* $\Lambda(P_j)$ *of voters* with ZwSt to P_j (Λ for Leute, i.e. *real* persons), replaces *party* P_j (just a *legal* entity) as account owner. Thus,

(1.5) $\omega(j)$ in Table 1, i.e. P_j's ErSt success *and* ZwSt commitment, will be replaced by $\xi(j)$, see (2.3) *and* by $\psi[\Lambda(P_j)]$, see (2.13), i.e. respectively $\Lambda(P_j)$'s ErSt success *and* $\Lambda(P_j)$'s ZwSt commitment.

2. Faithful accounting

(2.1) **Definitions** $\Lambda(P_j)$ is the set of $z(P_j)$ voters with ZwSt to party P_j.

Faithful accounting records the total influence of $\Lambda(P_j)$ through its members' ErSt: Let E(k) be the number of voters and ballots with *ErSt to the winner* of the direct seat in C_k. N(j,k) members of $\Lambda(P_j)$ give ErSt to this winner; thus,

(2.2) $$E(k) = N(1,k) + N(2,k) + \ldots + N(p,k)$$

The fraction 1/E(k) measures the *ErSt-success* for each of these E(k) voters.

Faithful accounting then deposits a seat fraction N(j,k)/E(k) on $\Lambda(P_j)$'s *success account*: Imagine each of N(j,k) ballots carrying to $\Lambda(P_j)$ one ZwSt for P_j, but also an ErSt-success 1/E(k) which *must reduce* the effect of the ballot's ZwSt. $\xi(j)$ is the ErSt-success of $\Lambda(P_j)$; it is an aggregate over all C_k:

(2.3) $$\xi(j) = N(j,1)/E(1) + N(j,2)/E(2) + \ldots + N(j,c)/E(c) \text{ seat shares.}$$

All c direct seats are accounted for [sum over j in (2.3) and use (2.2)]:

(2.4) $$\xi(1) + \xi(2) + \ldots + \xi(p) = c.$$

(2.5) **EXAMPLE** The N(j,k)-values in 2017 are unknown, but one lucky circumstance in Table 1 indicates that ξ(7) is significantly smaller than **ω**(7): Since CSU (=P_7) runs only in Bavaria and won *all* 46 direct seats, *all* e(CSU)=3255487 ErSt (Table 1) support the winner of a direct seat. Of them, only z(CSU)–x, with unknown x≥0, belong to Λ(CSU).

Assume they are distributed over the 46 Bavarian constituencies like all e(CSU) voters with ErSt to CSU; tthen *their ErSt-success is,*

at most, $46 \times 2869688/3255487 \approx 40.55$ direct seats, i.e. $\xi(7) \leq 40.55$.

Aggregation of the ξ(j) over those $\Lambda(P_j)$ that *did not pass the threshold*, gives an ErSt-success f which may be small, but still too large to be ignored:

(2.6) $$f = \xi(r+1) + \xi(r+2) + \ldots + \xi(p).$$

The value of f depends on the N(j,k), and they are ignored in traditional accounting. However, Table 1 allows a rough estimate of f:

There are *at least* 44966765 – 44189959 = 776806 split P_aP_b-ballots ($a \leq 7 < b$);

if their ErSt distribution is typical, *their ErSt-success is*

(2.7) *at least* $f \approx 299 \times 776806 / 44966765 \approx 5.2$ direct seats.

(2.8) **Preparation for W/S-L to distribute h list seats,** see (1.2).

The r voter sets $\Lambda(P_1), \ldots, \Lambda(P_r)$, have ErSt-succes $\xi(1) + \ldots + \xi(r) = c - f$;

a supply of z ZwSt shall pay for $c-f$ direct seats and h list seats; thus

(2.9) at a price z/T ZwSt/seat, where $T = c - f + h$.

With 2017 data (Table 1) and estimate (2.7), T ≈299–5.2+299 =592.8, the task is to distribute 299 list seats to seven $\Lambda(P_j)$ with ErSt-success 299–5.2 direct seats.

In general, the task is to distribute h list seats so that the z ZwSt pay

for $T = c - f + h$ seats. A total of T seats get distributed proportionally:

(2.10) **Market price** There is a "price" $z/T = z/[c-f+h]$ ZwSt/seat, and, equivalently, a *"purchasing power"* T/z seats/ZwSt.

With 2017-data from Table 1, a rough price estimate is based on (2.7):

$$z/T \approx 44189959/592.8 \approx 74532 \text{ ZwSt/seat}, \quad T/z \approx 1/74532 \text{ seats/ZwSt}$$

Negatives Stimmgewicht A ballot which supports the direct winner in C_k carries to its voter set $\Lambda(P_j)$ a *success* $1/E(k)$, and a *purchasing power* T/z. *Thus, here is a* ***snag****:*

(2.11) If $\Lambda(P_j)$'s *commitment account* gets an *increment* $1/E(k)>T/z$, then *the ballot increases $\Lambda(P_j)$'s commitment more than its purchasing power.*

Thus, the ballot harms its voter set $\Lambda(P_j)$ and P_j. This is a case of *Negatives Stimmgewicht* (*negative vote weight*), which was discovered in earlier versions of Bundestag elections (but was due to a much more complicated mechanism); the federal constitutional court (2008) found it unconstitutional.

Waiving To avoid Negatives Stimmgewicht, some commitment is *waived*:

(2.12) $\Lambda(P_j)$'s *commitment account* is *increased* by $\min[T/z, 1/E(k)]$.

Every ballot carries to its $\Lambda(P_j)$ purchasing power which, at least, pays for its *remaining* commitment (2.12), in total

(2.13) $$\psi[\Lambda(P_j)] = \textstyle\sum_k N(j,k) \times \min[T/z, 1/E(k)], \quad 1 \le k \le c.$$

When the supply, z ZwSt, has been spent, no direct seats are in overhang.

Party P_j's success/commitment $\boldsymbol{\omega}(j)$ in Table 1 is replaced by *two quantities*:

(2.14) $\Lambda(Pj)$'s ErSt-success = $\xi(j)$, see (2.3), *and*
$\Lambda(Pj)$'s (remaining) commitment = $\psi[\Lambda(P_j)]$, see (2.13).

The $\xi(j)$ give f, see (2.6) and (2.9);
$\psi[\Lambda(P_j)]$ replaces $\boldsymbol{\omega}(j)$ in the algorithm (1.2).

Amounts of surplus and waiving According to (2.13), there are *two types* of constituencies in the election:

Type 1, $E(k) \geq z/T$: Commitment is $1/E(k)$ per ballot with ErSt to the winner; each one of them has *"surplus purchasing power"*, $T/z - 1/E(k)$; in total for C_k,

(2.15) $$E(k) \times [T/z - 1/E(k)] = E(k) \times T/z - 1$$

Type 2, $E(k) < z/T$: Commitment before waiving is $1/E(k)$ per ballot with ErSt to the winner; each *"waived commitment"* is $1/E(k) - T/z$; in total for C_k,

(2.16) $$E(k) \times [1/E(k) - T/z] = 1 - E(k) \times T/z$$

Each ballot with ErSt to the winner in C_k is accounted for in (2.15) or (2.16). Relatively few of them bring ZwSt to a $\Lambda(P_j)$ which does not participate in the list seat distribution ($r<j\leq p$).

Unequal influence Very small E(k) occur mainly in C_K where most ErSt are spread on three or more strong candidates. Other factors change less: Constituencies have *about equal population*, and *voter participation* is more stable than the number E(k) of just those with successful ErSt.

By (2.16), more waiving means less ErSt spent on winning the direct seat, i.e. more ZwSt with 0 commitment, and more influence of the constituency's electorate on the assembly's list seats. Although it seems unlikely that this mechanism can give rise to tactical voting, it is a flaw. ErSt elections that require a majority to support the winner, should be considered.

In legislatures based entirely on single seat constituencies, winners are usually supported by a large plurality: This *Duvergerian mechanism* will, even in the MMP-context, let many direct seats go to parties that attract support in split ballots from voters near the political center. However, Duverger's *incentive* is weaker when a "wasted" ErSt has a *"fallback"* influence in the ballot's ZwSt.

The Bundestag variation of MMP uses the common plurality method *"first-past-the-post"* in its ErSt elections. Some single-seat elections use the related *Two-Round method* which promotes the plurality winner (party W) and runner-up (party U) to a *final* in order to get a majority winner (e.g. the French presidential elections); (2.17) describes an *instant runoff version*:

(2.17) **The *W-U method*** Without delay, W-U allows distribution of list seats based on MMP-ballots as in Bundestag elections. It requires only three numbers of ErSt:

w for winner W, u for runner-up U, and t for all others Together.

Each ballot with ErSt to a candidate *not* in {W,U} counts as half an ErSt for W and half an ErSt for U. Thus, W remains direct winner, while E(k) is raised from w to w+t/2 (i.e. a majority), and less commitment is waived, see (2.16).

(2.18) **EXAMPLE** (Bundestag election 2021, C_{153} =Leipzig II):

W=Linke, U=Grüne: (w, u, t) = (40938, 32995, 105526).

W-U changes this to (w+t/2, u+t/2) = (93701, 85758); thus E(k) increases from w to w+t/2. C_{153} becomes type 1. In effect, (w, u, t) voters, respectively, carry commitments (1/93701, 0, 1/187402) to their $\Lambda(P_j)$. The u voters who support U are rewarded by carrying 0 commitment to their $\Lambda(P_j)$. The t voters keep a substantial ZwSt influence. To avoid commitment completely with tactical

ErSt to the *expected* runner-up is quite risky, since such action may instead create and support a new and unwanted winner.

(2.19) Disambiguation If all ballots contain a complete strict ranking of the constituency candidates, then RCV (Ranked Choice Voting) in each single seat tally may unambiguously count each ballot as supporting a winner or a runner-up. Best known is IRV (Instant Runoff Voting), used for more than 100 years in Australia. A noteworthy alternative, also Australian, is Baldwin's elimination method; it picks the Condorcet winner when one exists.

3. Constitutionality, legality, and legitimacy

(3.1) Assembly size and equal influence Before 2017, Germany applied the overhang concept, partly to each state, partly to all 16 states together. *Negatives Stimmgewicht* was a consequence, which was declared unconstitutional by the Federal Constitutional Court, July 3rd 2008.

Obiter dictum, there was, in para 92, also a statement on equal influence. [1]

With the rules of 2017, Negatives Stimmgewicht disappeared, but the sudden drop of z(CSU) from 3.2 million (Table 2) pushed the assembly size way beyond the legal norm of 598 seats. The drop was partly due to split QP-ballots with

[1] **Aus dem Grundsatz der Wahlgleichheit folgt für das Wahlgesetz, dass die Stimme eines jeden Wahlberechtigten grundsätzlich den gleichen Zählwert und die gleiche rechtliche Erfolgschance haben muss. Alle Wähler sollen mit der Stimme, die sie abgeben, den gleichen Einfluss auf das Wahlergebnis haben.**

ErSt to Q=CSU and ZwSt to, e.g. P=FDP (intended to help FDP across the 5% threshold). It also highlighted the problem of unequal influence.

(3.2) Ballot splitting and accounting method In 2017, 2.9 million ZwSt were not enough to give CSU any list seat. The experience that ZwSt to CSU were wasted, may be a reason for an even larger drop of z(CSU) to 2.4 million in 2021. Tally rules, not voter behavior, should be blamed for the concomitant rise of critical size (Table 2): Split ballots are natural, a choice allowed from 1953, stimulated by the name *"Personalisiertes Verhältniswahl"*.

(3.3) Who shall own the account $z(P_j)$? Tally rules should never give voters' natural adaptation the unnatural consequence of excessive assembly size: *Faithful accounting* brings critical size below a predetermined assembly size c+h. More thoroughly than it may seem, the change affects the *tally* but not the *voting* rules. Faithful accounting records ballot information which traditional accounting ignores, by transferring the success/commitment account from Pj to $\Lambda(P_j)$, i.e. from a political entity to voters. Thereby it also eliminates troublesome anomalies.

(3.4) Waived commitment The new thoroughness includes paying attention to the pluralities E(k). A ballot carrying big ErSt success 1/E(k) from C_k then gets some commitment waived, (2.12): Otherwise the voters would get influence even through (a new form of) Negatives Stimmgewicht. The waived fraction of C_k's direct seat through all ballots is in (2.16).

(3.5) A majority winner of first-past-the-post If E(k) is very small, waiving increases the influence of C_k's electorate. W-U is an elimination method, suggested here to increase E(k): First-past-the-post winner and runner-up remain and share the ErSt to all other candidates, according to the *"symmetrizing principle"* for handling indifference in incomplete ballots.

(3.6) Transparency

[redacted] ψ[Λ(CSU)], see (2.13). This is [redacted] Λ(CSU): In 2017 they seem to inherit CSU's pivotality. A new Table 1 will be as transparent as the old one: Everybody can calculate the new critical size; by design, it cannot be above the legal norm c+h [redacted].

(3.7) Predetermined size and traditional accounting

[redacted]

Naturally, "daughter" parties A_x and B_x appeared in Albania's political arena. Party strategists urged supporters to cast a split ballot, respectively AA_x or BB_x. The ZwSt would then help to pay for list seats to the daughter party, rather than just to pay down on the mother party's heavy commitment for direct seats; those would be kept anyway (with just an "overhang label"). But MMP was new to many supporters, who naturally received some coaching.

OSCE - Odihr (Organization for Security and Cooperation in Europe - Office for Democratic Institutions and Human Rights) monitors many elections in its member countries. Its final report from the Albanian election (2005) explains well the "*daughter party technique*". But those who make the arena rules should be held responsible, not the voters and party coaches who are confined in the arena and adapt to its rules.
The daughter party technique was also used in Italy 2001, and media made this widely known (Mudambi and Navarra, 2004).

(3.8) MMP, perceptions and attitudes Some voters think that the very notion of candidacy hinges on an *existing vacant* seat. "Candidacy" for other kinds of seats, e.g. 111 *extra-ordinary* ones in Table 1, raises problems of motivation, conceptuality and legitimacy. In the words of Hettlage (2018): *Ohne Kandidat, kein Mandat.*

How voters and politicians perceive the workings of their own election method is itself a topic for investigation, e.g. Jankowski & al (2020), Behnke (2015), and Linhart and Bahnsen (2020). With traditional accounting, MMP's structure has its pitfalls: When predetermined size is also required, traditional accounting may even allow the *daughter party technique,* described in (3.7).

Sister parties Under traditional accounting, a simple remedy to curb the Bundestag size, is a fusion of the *"sister parties*": CSU runs only in Bavaria and CDU only in the 15 other states; CDU/CSU is one group in the assembly. If also tallied as one party, they reduce the critical size in 2017:

$$[\omega(1) + \omega(7)] \times z / [z(CDU) + z(CSU] = 666.434\ldots \text{ seats; see Table 1.}$$

This remedy makes CDU/CSU pivotal, with 231 seats of 667, instead of the real outcome, 246 of 709. What were the perceptions and attitudes among union party leaders and their 246 seat winners concerning this remedy?

Table 4 indicates that critical size gets much less volatile with a nation-wide pivotal party; see Table 2 for the first line. In 2017, z(CDU/CSU) would have caused 667–598=69 extra-ordinary list seats, instead of 111.

	2013	2017	2021
CSU pivotal	511.484...	708.348...	793.326...
CDU/CSU pivotal	478.970...	666.426...	542.161...

TABLE 4 Tallied as one party, CDU/CSU is pivotal in all three elections. Coalition partner FDP didn't pass the 5% threshold in 2013, an obvious incentive for later moves from Λ(CDU) and Λ(CSU) to Λ(FDP). In 2021, CDU went from 15 to 53 list seats, but also from 185 to 98 direct seats; reduced commitment for CDU/CSU brought the critical size below the norm 598.

Waived commitment (2.16) avoids negative vote weight in C_k of type 2, i.e. with too few ErSt for the direct seat winner. *W-U* is a first-past-the-post method, it curbs waiving, increases E(k), and promotes equal influence. Weinmann and Grotz (2020) study how to curb the assembly size without "invasive" changes, but stay within the frame of traditional accounting. By recording ballot data (*now ignored*), faithful accounting is a noninvasive and potent remedy, even without a unified union CDU/CSU.

(3.9) Ideologies and pragmatism A chosen norm c=h has given the Bundestag MMP an important flexibility, even under traditional accounting: Despite flaws, it shows a balance between concentration towards a political center of "*pragmatism*" and a spread-out in a wider landscape of *ideologies*; "Right", "Left", "Green", "Progressive",… (Linhart & al, 2019).

Winners of a direct seat have an electoral basis where thousands of voters support them in a *split ballot*. This is the *compromise* strategy, an ErSt intended for the most liked *feasible* candidate. Dowding and Van Hees (2008) expressly praise such "manipulation" for use in single-seat elections.

Sometimes it is difficult to compose a government basis with participation from the outer landscape. An alternative with basis in the pragmatic center tends to be viable: A large number of representatives are widely accepted in their own constituency; this legitimizes their support of such alternative. Thus, four elections (1965, 2005, 2013, 2017) have given a Bundestag providing a basis for the *Grand Coalition*, i.e. CDU/CSU & SPD.

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