Multi-Periodic Distributional-Robust Stackelberg Game with Price-History-Dependent Demand and Environmental Corrective Actions

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DISCUSSION PAPER

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Abstract

The paper investigates a multi-period supply channel facing uncertain and price-history-dependent demands and environmental regulations. The knowledge about the demands is limited to its mean and standard deviation in each period, i.e., there is incomplete information on the actual distribution. A distributional robust approach is conducted to address incompleteness. The chain is incorporating environmental policies such as pollution constraints and (optimal) corrective taxes. A single contract covers all periods. Numerical examples highlight the benefits of a single contract.

JEL: C61, C62, C63, C72, C73, D81, Q52.

Keywords: Dynamic Games, Single Contract, Distributional-Robust Demand, Price-History-Dependent Demand, Pollution Reduction, Sustainability.

1. Introduction

With rapid global economic development, environmental challenges have been deteriorating constantly (Yang, Zheng, & Xu, 2014). The major source of this problem has been regarded as greenhouse gas emitted by production, services, and consumption (Song & Leng, 2012). This problem has attracted more countries' attention since the 1980s (Ma, Zheng, Zhang, Li, & Ma, 2022). Nowadays, sustainability is a key subject for environmentalists, economists, industrialists, consumers, academia, and governments (Manupati, Jedidah, Gupta, Bhandari, & Ramkumar, 2019; Yang, Zheng, & Xu, 2014). With the aim of environmental protection and reduction of pollution, many governments have agreed to contribute to the goal of emissions reduction by at least 50% by 2050 as reported by the International Energy Agency (Liu, Holmbom, Segerstedt, & Chen, 2015; Song & Leng, 2012). Furthermore, because of consumer awareness development, many governments and companies have implemented pollution reduction policies and displayed their attempts to reduce their footprint by pasting a tag on their products, like Tesco and Boots. The actions that increase consumer awareness of environmental concerns, encourage them to opt for a product with a lower environmental footprint if its price is affordable. However, actions with lower pollution may result in a higher cost for the channel players (Yang, Zheng, & Xu, 2014).

Among all environmental pollution reduction policies, two types of policies, emissions capacity constraint, and emissions tax regulation have been analyzed by many countries or regions, such as the European Union (EU), Canada, China, and the IMO (Bai, Xu, Gong, & Chauhan, 2022). Emissions capacity constraints involve setting a maximum limit, or carbon cap, on the level of emissions allowed within the supply channel. In contrast, emissions tax regulation imposes a cost on each unit of production or pollution, requiring the channel to pay a tax fee for each unit of pollution generated through production or consumption. According to Luo, et al., emissions tax is considered one of the most effective market-based mechanisms and enjoys widespread acceptance worldwide. More than 20 countries, including Canada, Australia, the United Kingdom, and the United States, have already implemented emissions tax policies (Luo, Zhou, Song, & Fan, 2022). For instance, the Dutch government has planned to impose a CO_2 emissions tax on industrial companies starting in 2021, initially set at 30 euros per ton of CO_2

emitted. This amount would increase to 125-150 euros by 2030, ensuring the sustainability of industrial firms (Blomberg[1]).

As economic globalization has deepened, the world economy has evolved into a complex and interconnected system. However, a significant challenge remains in establishing crucial pollution reduction targets within such a complex economic framework (Jiang, 2022) and encouraging industries to collaborate. This underscores the importance of developing an algorithm capable of identifying optimal solutions while considering environmental pollution reduction constraints. Supply channels have increasingly prioritized ecological sustainability in alignment with the United Nations’ Sustainable Development goals (Kannan, Solanki, Kaul, & Jha, 2022). Therefore, it is vital to examine the impact of pollution reduction activities on economic players (Yang, Zheng, & Xu, 2014).

Kannan et al. analyzed the barriers to implementing pollution reduction policies in India. They employed the Best Worst method to determine the relative importance of the barriers, focusing on regulatory policies. They establish interrelationships among the barriers of pollution reduction policies, using the Decision-Making Trial and Evaluation Laboratory. Regarding their categories, the economic category was found to carry the highest weight, followed by the organizational and environmental categories. Their finding highlights several key observations, including the lack of initial funding, hidden costs, uncertain carbon market price, lack of research and development, lack of support from the authorities, lack of alternative energy sources, unaccountability of production waste, fear to shift to a new system, lack of in-house reverse logistics, irrational current taxes, unaccountability of supply chain actors and lack of social demand (Kannan, Solanki, Kaul, & Jha, 2022).

Wu et al. review the progress made in carbon neutrality efforts. They note that 120 countries worldwide have proposed carbon-neutral goals, such as China, accompanied by national development, different cities and industries have actively included carbon neutrality in their development plans (Wu, Tian, & Guo, 2022). In a study by Xu et al., the performance of emissions tax policy in a supply chain composed of one supplier and two financially asymmetric manufacturers was investigated under Cournot competition. The researchers argue that emissions tax plays a crucial role in restricting carbon emissions and improving environmental performance for climate change mitigation (Xu, Fang, & Govindan, 2022).

[1] https://www.bloomberg.com/news/articles/2019-06-28/dutch-government-plans-co2-emissions-levy-for-industrial-firms

Luo et al. developed the Stackelberg game to evaluate the impact of the emissions tax on (re)manufacturing decisions within a closed-looped supply chain. They examined both scenarios with no investment in pollution reduction technology, as well as with investment in centralized and decentralized closed-loop supply chains. Their finding suggests that emissions tax encourages manufacturers to invest in pollution reduction technology or engage in remanufacturing. Moreover, when the tax is low, the pollution level in the centralized closed-loop supply chain exceeds that of the decentralized model (Luo, Zhou, Song, & Fan, 2022). Choi and Cai discuss the environmental challenges arising from shorter lead times in the production process, which can result in inadequate control of chemical and material processing operations. To address this issue, they propose the imposition of an environmental tax on suppliers to incentivize investment in green technologies (Choi & Cai, 2020). Zhang et al. explore a single-period Stackelberg model considering three regulatory approaches, tax, subsidy, and tax-subsidy policies. Their study aims to assess the effects of each method on channel profit and environmental pollution (Zhang, Hong, Chen, & Glock, 2020).

Hong et al. explore the integration of tax regulations and green product design strategies, where the decision variables are the degree of product greenness and retail price being made by the manufacturer and the retailer respectively (Hong, Wang, & Gong, 2019). Manupati et al. investigate different production-distribution and inventory problems in a multi-echelon supply chain, considering three pollution reduction policies viz tax, strict capacity caping, and a cap-and-trade system. They also incorporate lead-time considerations using a non-linear mixed integer programming model (Manupati, Jedidah, Gupta, Bhandari, & Ramkumar, 2019). In a study by Song et al., a stochastic production capacity problem is expanded to incorporate cap-and-trade and pollution tax regulations. The researchers found that firms increase their capacity when capacity investment is low enough, leading to higher unit profit (Song, Govindan, Xu, Du, & Qiao, 2017). The manufacturer in Chen et al.'s model employs two different techniques, standard and green technology using cap-and-trade and capacity constraints to reduce pollution. They demonstrate that emissions trade yields higher profits compared to capacity constraints (Chen, Chan, & Lee, 2016). Choi incorporates pollution tax policy into a fashion apparel problem and examines the implementation of a quick response system through reduced lead time, faster delivery mode, and local sourcing instead of offshore sourcing (Choi T. M., 2013).

Zhang and Xu study a newsvendor multi-item production plan with stochastic, but constant demand[2], considering both cap-and-trade and pollution tax regulations (Zhang & Xu, 2013).

In our model, a Stackelberg game composed of a manufacturer and a retailer is considered, wherein the manufacturer leads the channel in a multi-period setting. The retailer faces a demand that is time and price-history dependent, i.e., the current and prior prices determine the demand in each period. Moreover, the incomplete information on demand distribution leads to opting for a distributional-robust (DR) approach. The proposed algorithm operates under a single contract, wherein the players consider their decisions and the resulting consequences across all periods simultaneously. The primary objective is to ensure the attainment of the highest possible value. In the following, we study two environmental pollution reduction policies: pollution tax and capacity constraint and propose optimal algorithms satisfying the environmental policies. The decision variables are the prices. However, in the pollution tax model, the tax fee is also a decision made by the regulator. He addresses the chain problem by endogenizing the pollution externalities. The contributions of this paper are hence

- to solve multi-period Stackelberg distributional-robust game,
- with a dynamic and price-history-dependent demand,
- under a single contract, compared to periodic contracts, which makes significantly different results by utilizing more information and freedom to decide,
- propose an algorithm with environmental constraints, viz capacity constraints and pollution tax,
- acquiring the corrective tax that the regulator puts on the leader to fully compensate for the pollution produced.

2. Model Framework

In our model, demand is dynamic and price history-dependent for a perishable commodity produced by the manufacturer and sold by the retailer. The manufacturer leads and the retailer follows him, while both aim at maximizing their expected values by making pricing policies, leading to order quantity decisions under a single contract. With a single contract, the players can improve their expected value by regulating their decisions when they can observe the connected decisions' reactions. The players have a certain number of periods and decide on all their variables simultaneously. Unlike periodic contracts, a single contract is not subgame

[2] A stochastic constant demand can be formed as $D = \mu + \sigma\varepsilon$, where μ and σ are constant values representing the mean and standard deviation of the demand and ε is a distribution with mean zero and variance 1.

perfect, but if the players cling to the contract, both may obtain higher values. This can be exploited in DR settings as well as in more unrealistic situations with complete demand information.

The channel produces an externality in the form of pollution such that the order quantity may be obliged to follow environmental protection policies to reduce the environmental footprint. We employ two policies: capacity constraints and environmental taxes. In the capacity constraint system, the pollution produced in period k by manufacturing the ordered quantity q_k is $e_k q_k$ which cannot exceed a certain cap M_k, i.e., $0 \leq e_k q_k \leq M_k$. The tax can be split into two subcases: Any given tax on a unit of production or order quantity (environmental or not) and a corrective tax that exactly endogenizes the cost of eliminating pollution flow in the chain. The latter tends to depend on quantity or production volume. It depends on the damage function or the flow of externality costs. We consider a corrective tax where either the manufacturer or the retailer incorporates this decision in their formulation and pays the tax. All taxes and pollution or production caps are allowed to be dynamic.

The next subsection deals with the price-history-dependent demand structure. The DR model under a single contract is introduced in subsection 2.2. It is formed for the DR model, but the algorithm is applicable in the case with complete information by replacing the profits and quantity functions corresponding to the actual demand distribution. In the following, subsections 2.3 and 2.4 organize the single contract model for capacity constraints and corrective taxes. Each proposed model is provided with a non-trivial example in section 3.

2.1. Demand Structure

The demand in period $k \in \{1, \dots, n\}$ as a dynamic function of prices is modeled as

$$D_k(\vec{\boldsymbol{r}}_k) = \mu_k(\vec{\boldsymbol{r}}_k) + \sigma_k(\vec{\boldsymbol{r}}_k)\, \varepsilon_k, \qquad \vec{\boldsymbol{r}}_k = (r_1, \cdots r_k) \tag{1}$$

where μ_k and σ_k are deterministic functions of time and retail prices and represent the mean and standard deviation of demand at period k. The epsilons are uncorrelated random variables independent of prices with mean and standard deviation equal to 0 and 1 respectively. Incomplete information in the present setting means that the distributions for the ε_k are unknown. This is typically the situation in most real-world cases, either the complete information is not accessible, or it is too costly to obtain it. Hence, it is worthwhile to implement an approach that does not rely on the specificities of the ε_k-distributions, i.e., a distributional-robust (DR) approach is the way forward. It implies replacing the retailer's expected value/profit with a tight lower bound, i.e., at least one set of distributions results in an expected value equal to this bound and no other distribution implies a lower expected value. That is, no fully informed situation will have a lower expected profit for the same set of means and variances.

2.2. Model Formulation

The retailer orders q_k from the manufacturer with the wholesale price w_k considering the demand D_k and sells the amount of $\min(D_k, q_k)$ to the customers at the price r_k in period k. The time scope is divided into n discrete intervals referred to as periods. If q_k exceeds the demand, $(q_k - D_k)^+$ can be salvaged (discarded) at a price (cost) of s_k. The retailer's profit function in period k is[3]

$$\pi_k^{r}(\vec{\boldsymbol{r}}_k, w_k, q_k) = r_k \min(D_k, q_k) + s_k (q_k - D_k)^+ - w_k q_k + B_k^{r}(q_k), \tag{2}$$

where the first term represents the revenue of sold items, the second indicates the revenue (or cost) of the leftovers, the third term is the cost of purchase, and the last term addresses any other gains or costs by acquiring q_k. The function B_k^{r} may be non-linear. An example of this non-linear part can be a damage function. The retailer's objective function is then the expected values of profits,

$$\begin{aligned} E[\pi_k^{r}] &= r_k (\mu_k - E(D_k - q_k)^+) + s_k(q_k - \mu_k + E(D_k - q_k)^+) - w_k q_k \\ &\quad + B_k^{r}(q_k) \\ &= (r_k - s_k)\mu_k - (w_k - s_k) q_k - (r_k - s_k) E[D_k - q_k]^+ + B_k^{r}(q_k). \end{aligned} \tag{3}$$

The demand distribution, if known, provides a solution for the term $E[D_k - q_k]^+$. However, Cauchy- Schwartz inequality (Fakhrabadi & Sandal, 2023)[4] assists with

$$E[D_k - q_k]^+ \leq \frac{1}{2}\left(\sqrt{\sigma_k^2 + (q_k - \mu_k)^2} - q_k + \mu_k\right). \tag{4}$$

By replacing $E[D_k - q_k]^+$ in Eq. (3) by the right-hand side of Eq. (4), the expected DR approach for the retailer is obtained as a lower bound for the model with known distribution as follows

$$\begin{aligned} \Pi_k^{r}(\vec{\boldsymbol{r}}_k, w_k) &\equiv E[\pi_k^{r}]_{DR} \\ &= (r_k - s_k)\mu_k - (w_k - s_k) q_k - (r_k - s_k)\frac{\sqrt{\sigma_k^2 + (q_k - \mu_k)^2} - q_k + \mu_k}{2} \\ &\quad + B_k^{r}(q_k) \leq E[\pi_k^{r}]_D, \end{aligned} \tag{5}$$

[3] See list of notations list in **Appendix 1**

[4] See **Appendix 2**.

where $E[\pi_k^r]_D$ represents the profit of the same model with known distribution. From now on we use the term 'profit' for this bound (Π_k^r).

The manufacturer's expected profit is calculated as

$$\Pi_k^m(q_k, w_k) = E[\pi_k^m] = (w_k - c_k^m)\, q_k + B_k^m(q_k), \tag{6}$$

where the first term represents the manufacturer revenue, and the second term addresses any other linear or non-linear gains or losses associated with q_k. In the Stackelberg game, the manufacturer declares his price first, conditioned on the retailer's optimal reaction (r_k, q_k). In the multi-period problem, both the retailer and manufacturer aim to optimize their values given by

$$\mathrm{J}_1^{\mathrm{x}} = \alpha_1\, \Pi_1^{\mathrm{x}} + \alpha_2\, \Pi_2^{\mathrm{x}} + \alpha_3 \Pi_3^{\mathrm{x}} + \cdots + \alpha_{\mathrm{n}}\, \Pi_{\mathrm{n}}^{\mathrm{x}} \quad \text{for } \mathrm{x} \in \{m, r\}, \tag{7}$$

where n is the number of periods and $\{m, r\}$ indicates the manufacturer (m) and the retailer (r). The parameter α represents

$$\alpha_k = \beta_1 \cdot \beta_2 \cdot \cdots \cdot \beta_k, \tag{8}$$

where β_k is discounting factor for period k, and $\alpha_1 = \beta_1 = 1$. In a single contract, $w^* = [w_1^*, \dots, w_n^*]$ is revealed and then $r^* = [r_1^*, \dots, r_n^*]$ and $q^* = [q_1^*, \dots, q_n^*]$ are declared. The players can observe the consequences of their decisions simultaneously and change their decisions, if necessary, before finalizing the optimization process and signing a contract. The price history-dependent demand may allow for strategic decisions by manipulating future demand to improve optimal return. The manufacturer knows exactly his profit when the single contract is written. The retailer has all the risk by knowing a lower bound on his expected total return. The key findings are summarized in the following propositions.

Proposition 1 Optimal Order Quantity

The optimal order quantity q_k for any pair (w_k, r_k) is given by

$$(w_k - s_k) - \frac{1}{2}(r_k - s_k)\left[\frac{q_k - \mu_k}{\sqrt{\sigma_k^2 + (q_k - \mu_k)^2}} - 1\right] + \frac{\partial B_k^r(q_k)}{\partial q_k} = 0. \tag{9}$$

The special case $B_k^r(q_k) \equiv 0$ yields

$$q_k(\vec{\boldsymbol{r}}_k, w_k) = \mu_k(\vec{\boldsymbol{r}}_k) + \frac{\sigma_k(\vec{\boldsymbol{r}}_k)}{2} \frac{2\eta_k - 1}{\sqrt{\eta_k\,(1 - \eta_k)}} \;\; and \;\; \eta_k = \frac{r_k - w_k}{r_k - s_k}. \tag{10}$$

Proposition 2 Single Contract Key Feature

The manufacturer gains at least a payoff equal to the subgame perfect total payoff.

Proof.

A single-contract model benefits from taking into consideration all decisions simultaneously. The subgame perfectness restricts the choice of decisions. Hence, the decision space for a single contract covers the subgame perfect choices.

A single contract may create significantly different results by utilizing this freedom to allow for strategic pricing when all periods are considered simultaneously, i.e., optimizing without a fixed term structure. This leads to the manufacturer's benefits $\mathrm{J}_{\mathrm{SC}}^{m} \geq \mathrm{J}_{\mathrm{PC}}^{m}$, where SC and PC represent the single and periodic contracts respectively.

2.3. The Model Formulation Under Capacity Constraints

If the regulator's strategy to reduce pollution generated by the manufacturer is defined as a capacity constraint, $e_k q_k \leq M_k$ must hold, where e_k represents the pollution from producing one unit of product, and M_k is the maximum pollution permitted in period k. Hence, the manufacturer has to constrain his optimization by $q_k \leq q_k^c = M_k/e_k$. Hence utilizing Eq. (5) and Eq. (6) as the players' profits and Eq. (7) as their payoffs, the game is

$$\max_{w \in \mathcal{W}} J_1^m \quad \text{s.t. } (r,q) = \arg\max_{(r,q)\in\mathcal{R}} J_1^r. \tag{11}$$

$\mathcal{W}$ and $\mathcal{R}$ represent constraints on the manufacturer and retailer optimization, where $\{q_k \leq q_k^c\} \in \mathcal{W}$. The outputs of this optimization are w^*, r^*, q^* and the periodic returns for the chain members.

2.4. The Model Formulation Under the Pollution Tax Policy

If the government decides to implement a corrective tax policy, the channel is required to pay tax for each unit of pollution or spend a cost to clean the pollution it has caused. Since our channel consists of a manufacturer and a retailer, we address the problem of each player when facing the pollution tax.

2.4.1 Manufacturer as Tax Collector

The regulator, cognizant of the problem formulations faced by the players, imposes a Pigouvian tax (Corrective tax). Notice that a damage function can be internalized by the manufacturer, by issuing the quantity-dependent tax $\tau_k = \tau_k(q_k)$

$$\Pi_k^m(w_k, q_k) = (w_k - c_k^m - \tau_k) \cdot q_k = (w_k - c_k^m)q_k + B_k^m(q_k), \tag{12}$$

and the retailer's problem stays unchanged. Here $B_k^m(q_k)$ are the damage functions implied by the tax issued. The Corrective tax is

$$\tau_k(q_k) = -\frac{B_k^m(q_k)}{q_k}. \tag{13}$$

Hence, this tax is issued as a non-fixed tax that depends on the actual production. It automatically generates a cost that exactly pays for the damage and the manufacturer considers it when he makes his decisions. Therefore, the manufacturer internalizes the pollution damage (B_k^m) and his optimization gives the optimal quantity (q_k^*) and thereby the tax $\tau_k^*(q_k^*)$ that is imposed on the manufacturer to mitigate their environmental footprint or the cost they would incur to remove the pollution.

2.4.2 Retailer as Tax Collector

According to the retailer profit function in Eq. (5), any given periodic tax (τ_k) can be accommodated by setting $B_k^r(q_k) = -\tau_k q_k$, resulting in

$$\Pi_k^r(\vec{\boldsymbol{r}}_k, w_k, q_k) = (r_k - s_k)\,\mu_k - (w_k + \tau_k - s_k)\,q_k - (r_k - s_k)\,\frac{\sqrt{\sigma_k^2 + (q_k - \mu_k)^2} - q_k + \mu_k}{2}. \tag{14}$$

This is equal to the case without B_k^r, but with w_k replaced by $w_k + \tau_k$. The best order quantity q_k for any given set of parameters (r_k, s_k, τ_k, w_k), is then given by Eq. (10) where w_k is replaced by $w_k + \tau_k$. If the tax is only on sold items, it is equivalent with $B_k^r = 0$ and r_k replace with $r_k - \tau_k$ in Eq. (10).

A Pigouvian tax will endogenize a damage cost function $B_k^r(q_k)$, and Eq. (10) determines the best order quantities for any given set of (r_k, s_k, w_k). The damage function is revealed by the quantity-dependent tax $\tau_k(q_k)$ issued by the regulator as

$$B_k^r(q_k) = -q_k \cdot \tau_k(q_k). \tag{15}$$

In this case, there are no shortcuts to determine the best order quantities. The full version of Eq. (9) must be applied.

Both sections 2.3 and 2.4 are solved under a single contract. The periodic backward induction algorithm, which is commonly utilized to solve such problems, cannot solve the price history-dependent problems under the ordering/production constraints.

3. Numerical Implementation

We begin the numerical illustration by comparing an unconstraint single contract with a periodic contract. Later, in section 3.2 we move on to the unconstraint single contract model with a short memory and its extension to the models with capacity constraint and tax policies.

As mentioned before, the demand may be affected by previous periods' price decisions. Indeed, an increase in the price today may bring about a decrease (increase) in the customer base tomorrow. Hence, price settings in one period may change the future customer base, and therefore change the future demand and modify the supply channel's values. Although, the effect of each period's price might fade out over time. This effect can be labeled as memory and denoted by $\Phi_{\mathrm{k}}(\vec{\boldsymbol{r}}_{k-1})$, where $\Phi_1 = 1$ and $\vec{\boldsymbol{r}}_k = (r_1, \dots, r_k)$ and $k \in \{2, \dots, n\}$.

In the rest of this paper, we deal with a demand scaled by the price history such that the coefficient of variation only depends on the current price, i.e.,

$$D_k(\vec{\boldsymbol{r}}_k) = \Phi_{\mathrm{k}}(\vec{\boldsymbol{r}}_{k-1})\, d_k(r_k)$$
$$\text{where } d_k(r_k) = \tilde{\mu}_k(r_k) + \tilde{\sigma}_k(r_k)\varepsilon_k. \tag{16}$$

So, from Eq. (1)

$$\mu_k(\vec{\boldsymbol{r}}_k) = \Phi_{\mathrm{k}}(\vec{\boldsymbol{r}}_{k-1})\, \tilde{\mu}_k(r_k)$$
$$\sigma_k(\vec{\boldsymbol{r}}_k) = \Phi_{\mathrm{k}}(\vec{\boldsymbol{r}}_{k-1})\, \tilde{\sigma}_k(r_k). \tag{17}$$

For the numerical examples, we apply the following scaled demand terms

$$\tilde{\mu}_k(r_k) = \frac{100\left(10+\frac{1}{1+k}\right)}{r_k^2}, \quad \tilde{\sigma}_k(r_k) = \frac{\tilde{\mu}_k(r_k)}{2\sqrt{3}}. \tag{18}$$

The parameters are set to constant over time by $c_k^m = 2, s_k = 1, \beta_k = 0.96$ for all $k \in \{1, \dots, 12\}$ in all examples. We utilize two different kinds of scaling factors representing long-term (section 3.1) and short-term (the rest of the examples) memory. We have limited the scaling factor to perform in the range $[0.7, 2]$, meaning $\Phi_{\mathrm{k}} = \max(\min(2, \Phi_{\mathrm{k}}), 0.7)$ at each arbitrary period k.

3.1. Case 1, Single Contract vs. Periodic Contract

We have structured this paper on a single contract in section 2.2, but it is worth comparing the same problem set with a periodic contract where the algorithm commences from the last period and steps back to the first. We exemplify these cases by the scaling factor

$$\Phi_k(\vec{\boldsymbol{r}}_{k-1}) = \prod_{i=2}^{k} g_i(r_{i-1}), \qquad \text{and} \quad g_k = e^{\gamma_k(K_k - r_{k-1})}. \tag{19}$$

K_k is the market price preference and γ_k represents the strength of a current deviation on the future demand (marginal log scale). The parameters are set to $K_k = 6, \gamma_k = 0.04$.

For a DR periodic problem, the players' total value,

$$J_k^x = \pi_k^x + \beta_{k+1} \cdot g_{k+1} \cdot J_{k+1}^x \qquad x \in \{m, r\} \tag{20}$$

is optimized in each period k, i.e., the players optimize their current situation in the game, ensuring a subgame-perfect solution by starting at the end (Fakhrabadi & Sandal, 2023; Gholami, Sandal, & Ubøe, 2021). In Eq. (20), J^m and J^r address the manufacturer and retailer values respectively. Utilizing Eq. (7) for the single contract and Eq. (20) for the periodic contract, Figure 1. illustrates players' profits.

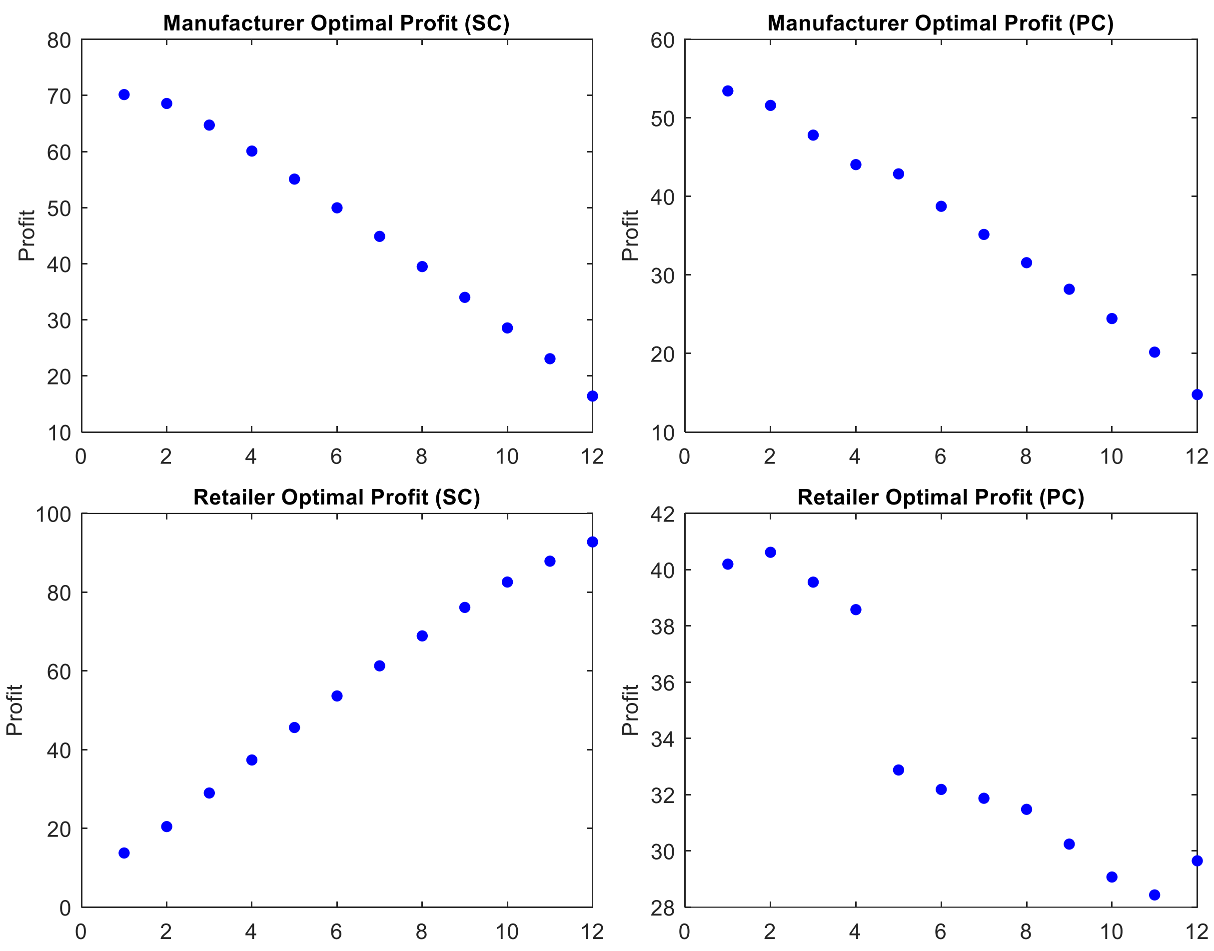

Figure 1: Optimal profits, single contract (SC) vs. periodic contract (PC)

SC and PC are the models with single and periodic contracts respectively. The players' total value was revealed as

$$J_{SC}^m = 555 > J_{PC}^m = 433, \quad J_{SC}^r = 670 > J_{PC}^r = 408$$

This is equivalent to a relative increase of 28 % and 64% in the total returns of the manufacturer and the retailer by utilizing a single contract instead of periodic ones. It is observed that this single contract is beneficial for both the manufacturer and retailer which is compatible with the statement in 2.4.

Embedding the optimal SC wholesale prices (w^*) into the periodic contract algorithm and solving the problem for retail price yields $\hat{J}^m = 471$ and $\hat{J}^r = 491$. This outcome highlights a significant finding: the periodic framework fails to recognize the superior values identified by the SC, even when the optimal wholesale prices w^* are provided.

The optimal prices are plotted in Figure 2. The lower prices obtained by the single contract (SC) are accompanied by higher quantities leading to a larger market (Figure 3) and higher returns. The leader collects more profit in the beginning and the follower in the end in the SC case.

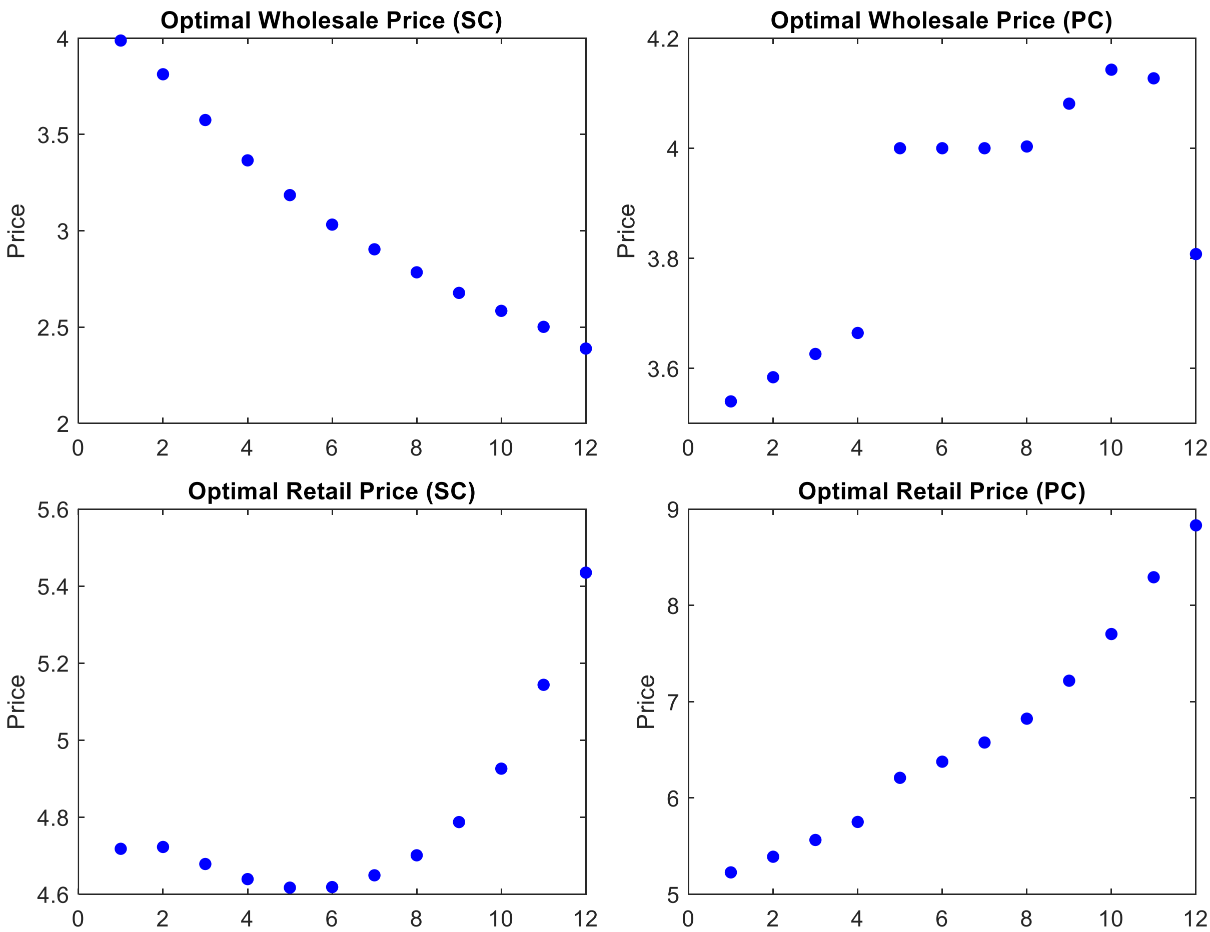

Figure 2: Optimal prices, single contract (SC) vs. periodic contract (PC)

Figure 3 mirrors the effectiveness of each contract form in stimulating market growth. The results reveal that the single contract model consistently bolsters the market, while the periodic contract deviates from this trend from the third period and begins to contract the market at $k = 6$. The red line represents the threshold that separates the market stimulation and market contraction phases based on the price-history effects.

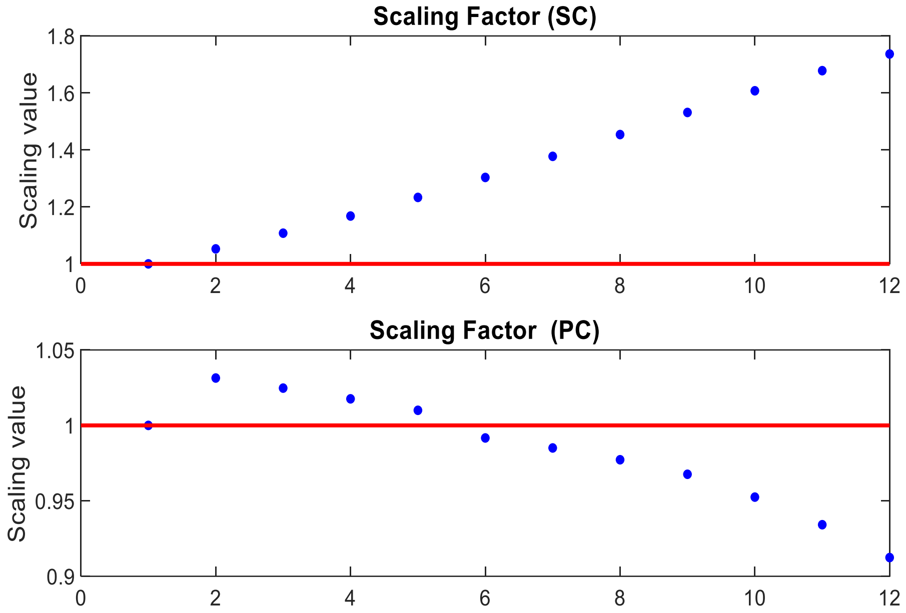

Figure 3: Scaling factor, single contract (SC) vs. periodic contract (PC)

3.2. Case 2, Unconstraint Single Contract Problem with a Short Memory

In this example, we only implement a single contract with scaling factors

$$\Phi_k = e^{\gamma_k (r_{k-2} - r_{k-1})}. \tag{21}$$

The explicit memory effect of each price only lasts for two periods. Therefore,

$$\Phi = \{1, e^{\gamma_2 (R - r_1)}, e^{\gamma_3 (r_1 - r_2)}, e^{\gamma_4 (r_2 - r_3)}, \dots\}. \tag{22}$$

Φ is the effective scaling factor and R is a given reference price (model parameter) for the second period. The parameters are set to $\gamma_k = 0.04$, and $R = 10$ for $k \in \{1, \dots, 12\}$. Figure 4 illustrates the optimal profits for the manufacturer and retailer.

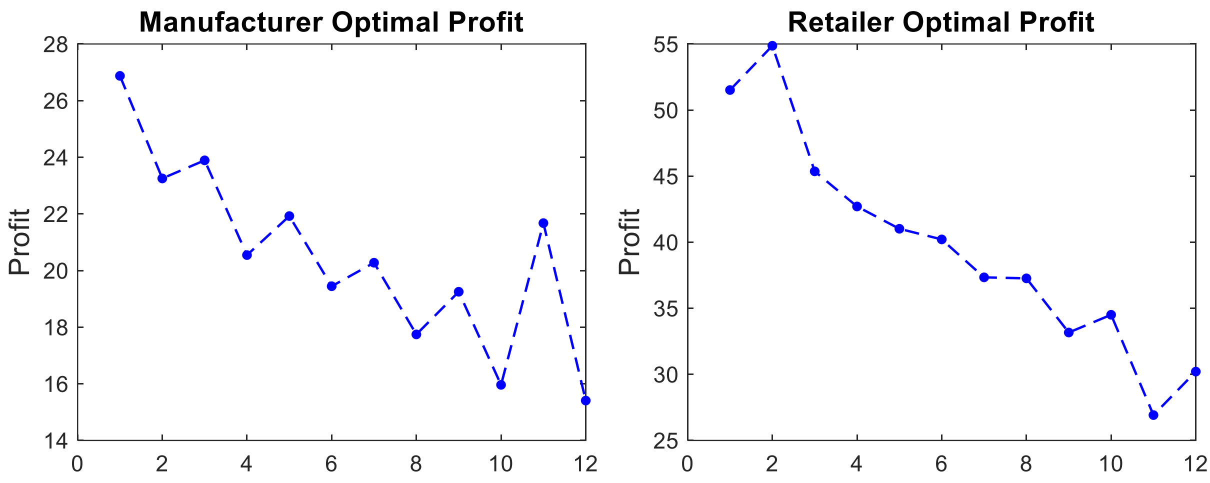

Figure 4: Optimal profits

The total values, denoted as $J^{m} = 246$ and $J^{r} = 475$, are obtained from the analysis. The observed pattern reflects the scaling factor structure. The players strategically make decisions as a volatile set to maximize their profits. It is worth mentioning that this strategy capitalizes on the fact that any price changes are forgotten after a span of two periods (Figure 5).

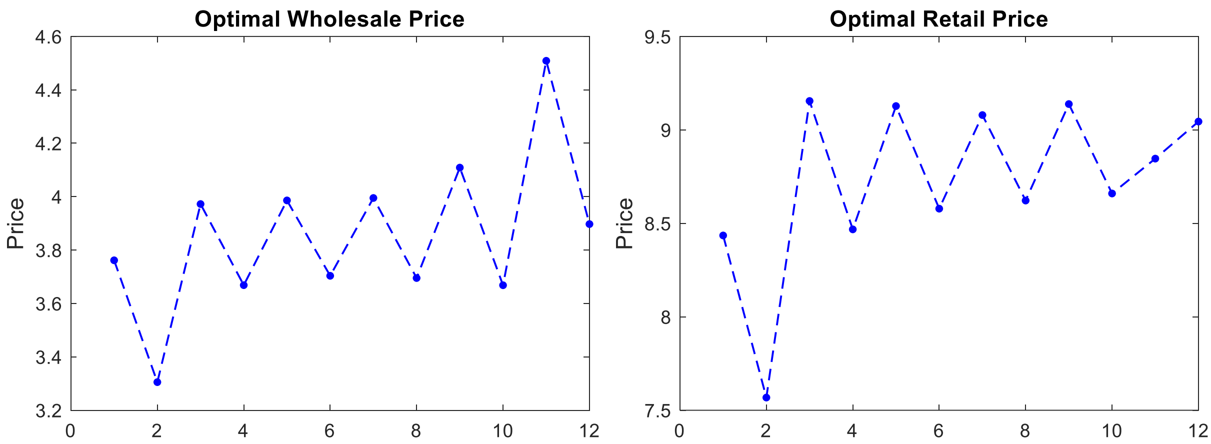

Figure 5: Optimal prices

The retail price decisions result in the scaling factor that is illustrated in Figure 6.

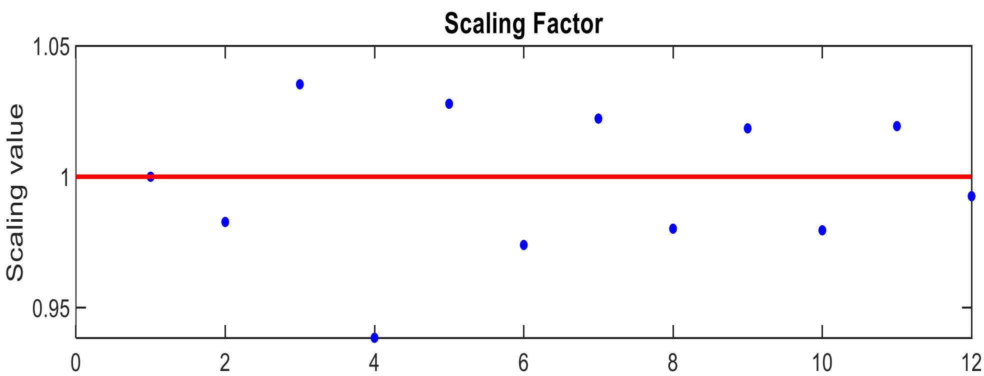

Figure 6: Scaling factor for a short memory

The red line separates the region where the retail prices are boosting /shrinking the market. The volume ordered at each period is mirrored in Figure 7.

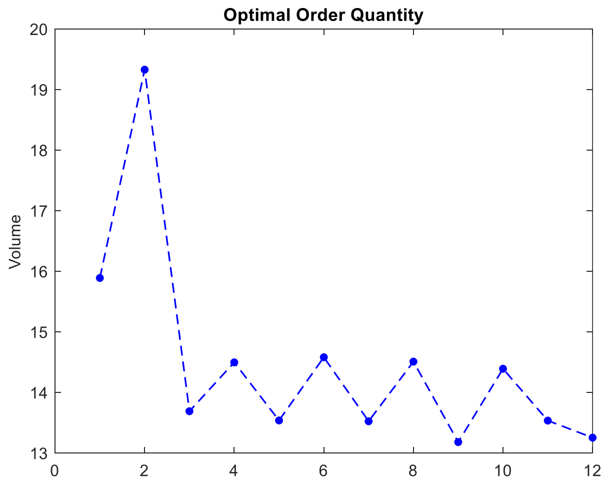

Figure 7: Optimal order quantity

3.3. Case 3, Single Contract under Capacity Constraints

The pollution capacity constraint, determined by the regulator and denoted as q^c (associated with the emissions amount), represents the maximum allowance of production, serving an upper bound for the maximum pollution that might be generated by the manufacturer. It is crucial to adhere to this constraint and ensure that it is not violated. By utilizing Eq. (22), Figure 8 illustrates the profits and volume associated with three distinct capacity constraints (a, b, c).

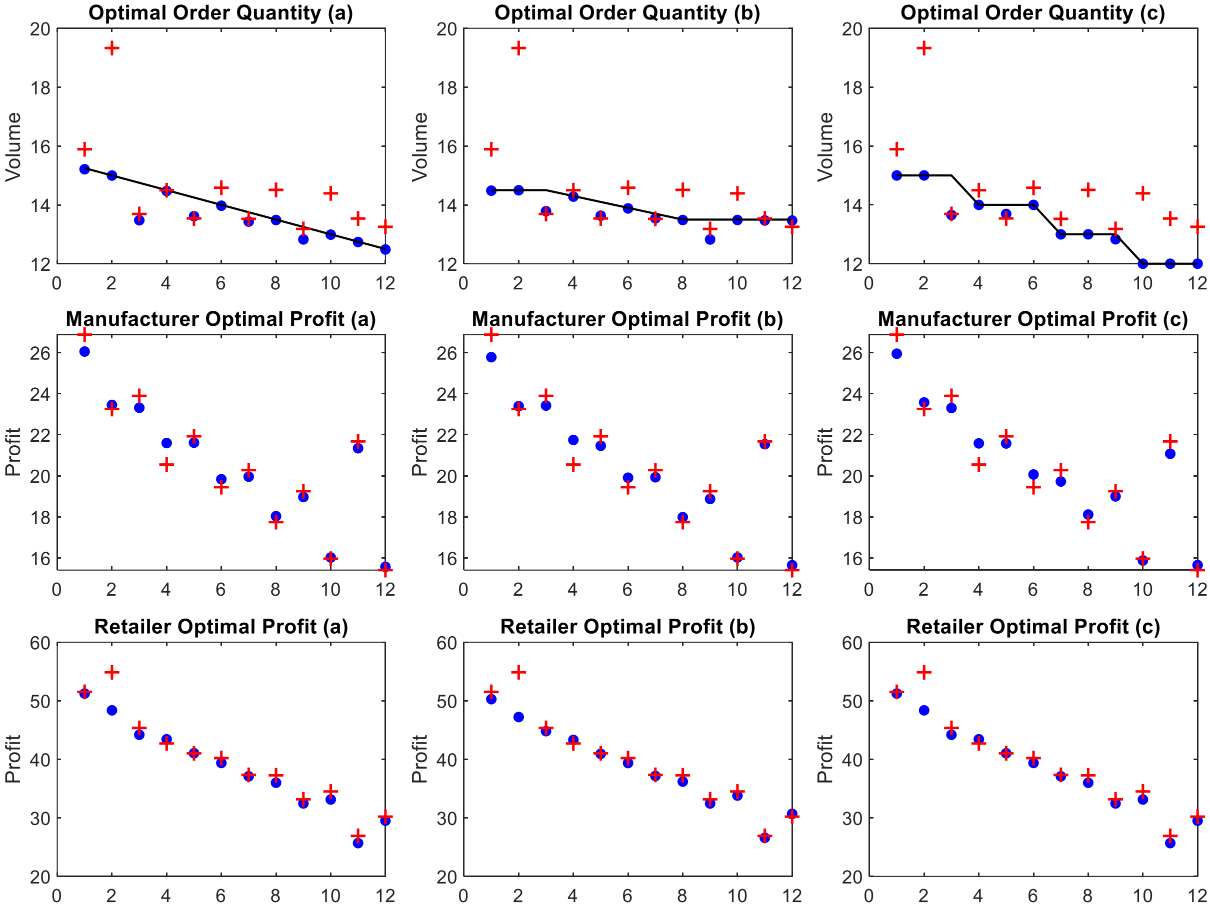

Figure 8: Optimal quantity and profits under capacity constraint policy (red curve)

In the order quantity plot, red crosses represent the unconstrained results, the capacity constraint is depicted by the black line, and the blue circles indicate the optimal solution under capacity constraint. The subscripts a, b, and c correspond to different cases with different capacity constraints. An intriguing finding emerges when comparing the volumes in the constrained and unconstrained models: there are periods when the unconstrained solution operates below the capacity limit, with no requirement to order reduction, but the constrained algorithm intentionally chooses a lower volume, such as period 3 in the plot (a). The total values in each model are

$$J_a^m = 246, \qquad J_b^m = 246, \qquad J_c^m = 245$$

$$J_a^r = 462, \qquad J_b^r = 463, \qquad J_c^r = 457,$$

where the unconstrained problem's results (base model, example 2, section 3.2) are $J^m = 246$ and $J^r = 475$. The findings indicate that in total, these cases reduce the emissions by $5.9, 5.2,$ and 7.9% in a, b, and c cases respectively. Hence, regarding the priorities which can be either pollution reduction or economic growth, case c is the greenest, while case b brings the highest economic return. Figure 9 pictures the price decisions.

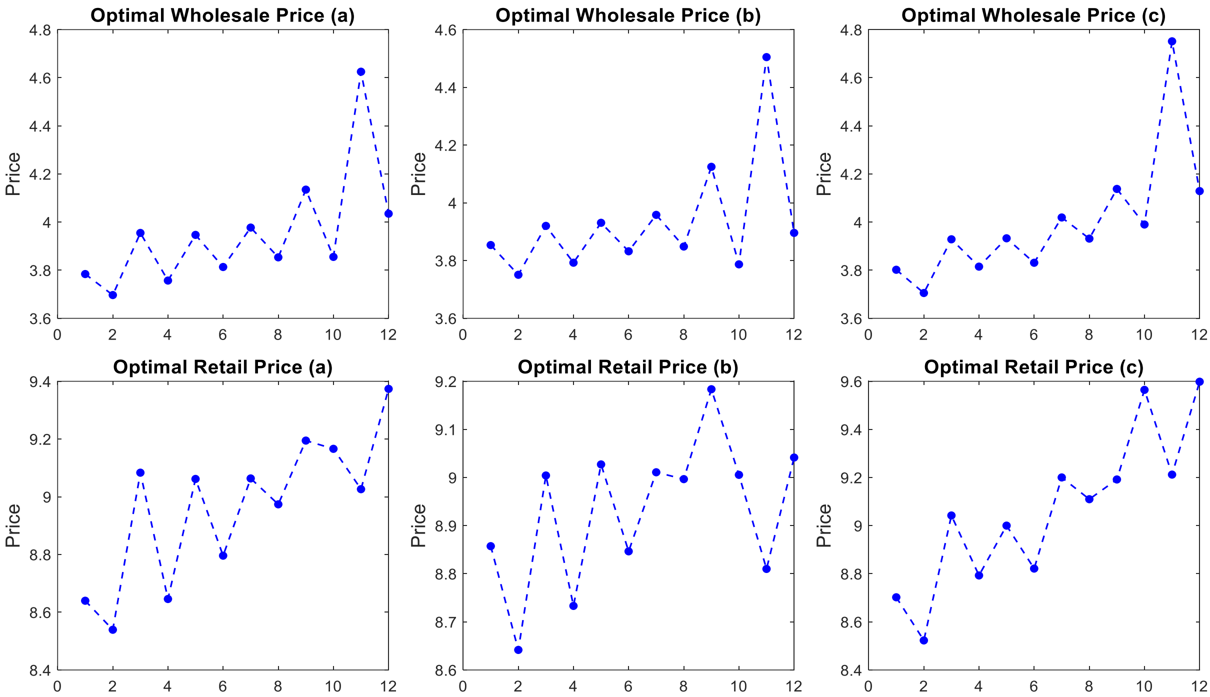

Figure 9: Optimal prices under capacity constraint policy

3.4. Case 4, Pollutions Corrective Tax

According to the discussion in section 2.4, we present an illustrative example where either the manufacturer or the retailer collects a tax following Eq. (12) or Eq. (14). In this example, a player x applies $B_k^x(q_k) = -a_k q_k^2$ in their problem where the damage-intensity factor, a_k, is chosen to be 0.04. Consequently, the tax derived from Eq. (13) or Eq. (15) takes the form of $\tau_k^* = a_k\, q_k^*$, representing the amount paid per unit in period k to mitigate the pollution associated with that unit. Figure 10 illustrates corresponding profits and quantities in both cases where either the manufacturer or the retailer integrates the tax inside their objective functions.

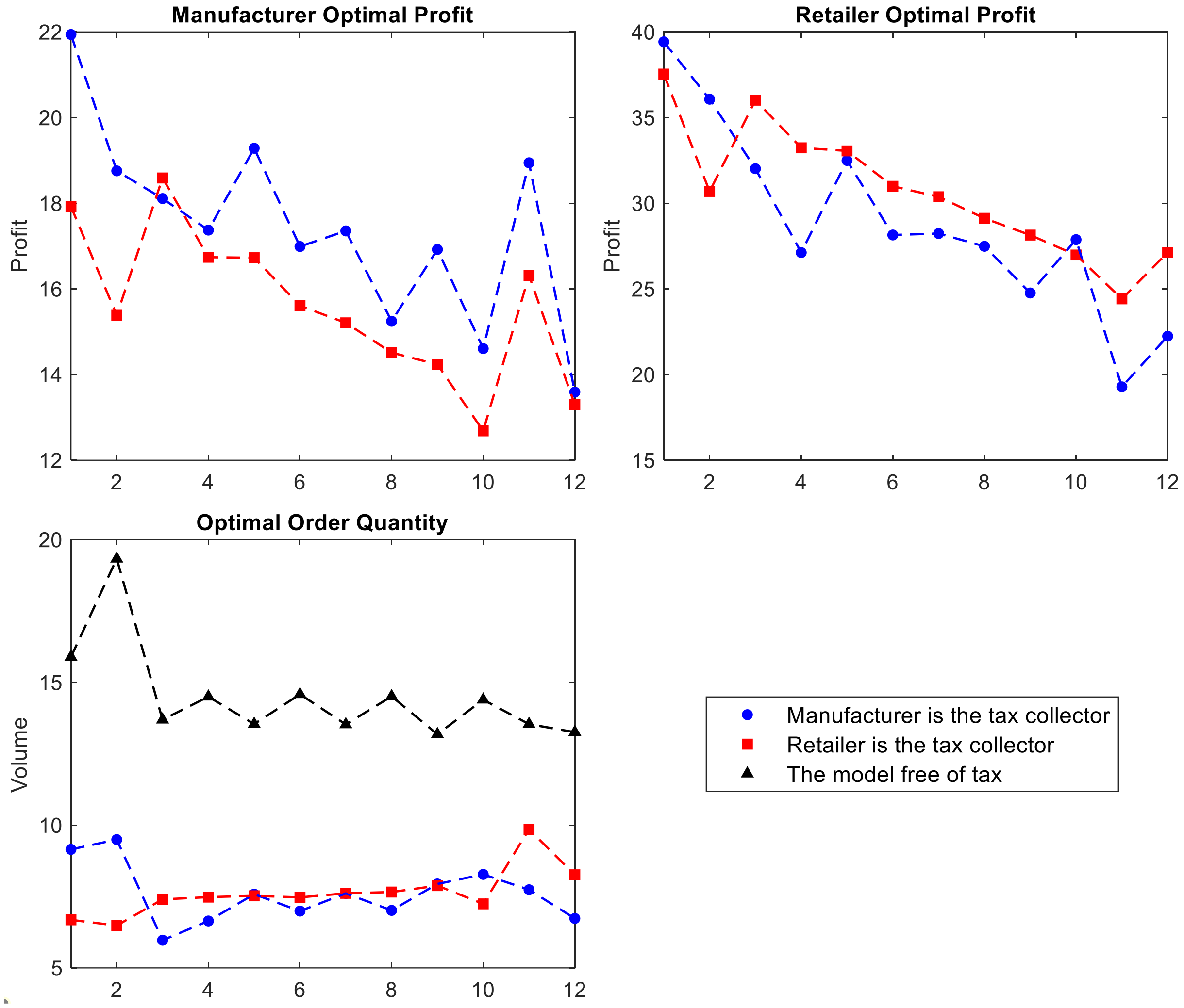

Figure 10: Optimal results

As depicted in Figure 10, the calculated values for this example are

	J^m	J^r
The manufacturer is the tax collector	209	345
The retailer is the tax collector	187	368

Notably, this example showcases the effectiveness of the proposed method in reducing pollution. In comparison to the base model (example 2, section 3.2), the application of tax policy results in a significant 48% and 47% reduction in pollution overall, when the manufacturer and retailer are tax collectors respectively.

The analysis of two tax models within the supply channel, where the responsibility for collecting the pollution tax lies with either the manufacturer or the retailer, reveals that each player achieves higher profits when individually responsible for managing the pollution tax.

The results demonstrate that when the manufacturer takes charge of tax collection, the channel's earnings amount to 554 units of currency, while if the retailer assumes this responsibility, the channel's earnings increase to 555 units, where the model free of tax makes 739 units.

4. Concluding Remarks

We introduce a comprehensive single-contract framework aimed at optimizing a multi-period Stackelberg game with a dynamic, price history-dependent, and DR demand. Our results demonstrate the superiority of the single-contract model over the periodic-contract model, although, the single-contract is not sub-game perfect. This can be attributed to the freedom and awareness embedded within the single contract model. Unlike the periodic contracts model, where players make decisions in each period, the single-contract model identifies an optimal decision at least as good as a periodic-contract framework. The single contract allows for better utilization of the strategic potential in the market.

We illustrate the effectiveness of the single-contract model using two different types of price-history dependency in our examples and observe how this effect is reflected in the output. The algorithm leverages the price history effect to achieve optimal order quantities and maximize values.

Furthermore, we extend the model to address environmental constraints, specifically the pollution capacity constraint and tax. These policies have been widely implemented in many countries. Both systems impose limitations on the channel that may require a reduction in the quantity.

An intriguing finding from the model incorporating a capacity constraint is that there are cases, where the algorithm subject to constraints leads to a lower order quantity compared to the unconstrained solution, even though the specified cap permits a higher volume. In other words, there are periods when the unconstrained solution operates below the capacity limit, with no requirement to order reduction, but the constrained algorithm intentionally chooses a lower volume. This behavior highlights a strategic decision-making capability inherent in a single-contract approach that may not be evident in a periodic approach. Furthermore, it underscores the interconnectedness of decisions across different periods, where changes in one period can impact decisions in preceding and subsequent periods.

With the emissions tax policy, the channel faces a cost to mitigate the pollution it has generated, as dictated by the imposed damage function. It is important to note that the constraint framework cannot be effectively implemented in a periodic contract form, highlighting the advantages of the single-contract model in handling environmental constraints.

In conclusion, our proposed single-contract framework outperforms the periodic-contract model in terms of optimization and value maximization but also provides a means to address environmental constraints through policies such as (pollution) capacity constraints and corrective tax inclusion. Our dynamic distributional robust settings are closer to real-world situations and adapted to fully utilizing strategic potentials in the markets with memory. By incorporating these factors, our model offers valuable insights and strategies for decision-making in complex dynamic supply channel scenarios. Future research could explore other environmental policies and different types where the players are sharing exposure to the market risk.

Appendix 1: Notation List

$\beta = \{\beta_1, \dots, \beta_n\}$	Discount factor over individual periods[5]
$c^m = \{c_1^m, \dots, c_n^m\}$	Manufacturer cost
$s = \{s_1, \dots, s_n\}$	Salvage price/cost
$w = \{w_1, \dots, w_n\}$	Wholesale price (Decision variable)
$r = \{r_1, \dots, r_n\}$	Retail price (Decision variable)
$q = \{q_1, \dots, q_n\}$	Order quantity (Decision variable)
$k = \{1, \dots, n\}$	Time or period
$D = \{D_1, \dots, D_n\}$	Demand. $D_k = \mu_k(r) + \sigma_k(r)\, z_k,$ is Demand in period k
$\mu = \{\mu_1, \dots, \mu_n\}$	Mean of demand
$\sigma = \{\sigma_1, \dots, \sigma_n\}$	The standard deviation of demand
$\pi^m = \{\pi_1^m, \dots, \pi_n^m\}$	Manufacturer profit (present value)
$\pi^r = \{\pi_1^r, \dots, \pi_n^r\}$	Retailer profit (present value)
$\tau_k = \{\tau_1, \dots, \tau_n\}$	Emission tax

Appendix 2: Cauchy- Schwartz Inequality

Cauchy-Schwartz inequality reads $|\mathrm{E}(xy)|^2 \leq \mathrm{E}(x^2) \cdot \mathrm{E}(y^2)$. If we choose $x = |q - D| = |(q - \mu) - \sigma\epsilon|$ and $y = 1$ and utilizing that $E(\varepsilon) = 0$ and $E(\varepsilon^2) = 1$, we obtain

[5] The discount factors related to the start (t=0) are $\alpha_k = \beta_1 \cdot \beta_2 \cdot \dots \cdot \beta_k$. Individual periods may be of different length.

$$|E(|q - D|)|^2 \leq E(|q - D|^2) = E[\ (q - \mu)^2 - 2(q - \mu)\sigma\epsilon + \sigma^2\epsilon^2\ ] = (q - \mu)^2 + \sigma^2$$

and thereby $E(|q - D|) \leq \sqrt{\sigma^2 + (q - \mu)^2}$. Applying the equality $(D - q)^+ = \frac{1}{2}\{|D - q| + (D - q)\}$, we obtain directly

$$E(D - q)^+ \leq \frac{1}{2}\left\{\sqrt{\sigma^2 + (q - \mu)^2} - q + \mu\right\}.$$

The equality holds for the deterministic case and certain two valued distributions.

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