# Optimization－based Evacuation on Cruise Vessels Using Two－ way Communication 

BY Andres Velez and Stein W．Wallace

## DISCUSSION PAPER

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# Optimization-based Evacuation on Cruise Vessels Using Two-way Communication 

Andres Velez<br>Department of Business and Management Science<br>NHH Norwegian School of Economics, Bergen, Norway andres.correa@nhh.no ORCID ID 0000-0002-1214-8737

Stein W. Wallace<br>Department of Business and Management Science<br>NHH Norwegian School of Economics, Bergen, Norway stein.wallace@nhh.no ORCID ID 0000-0001-5909-7909

We study the evacuation of large cruise vessels under hazardous conditions. Based on a new sensor mesh technology, allowing wire-less two-way communication also when electricity is lost, we propose an optimization-based procedure that provides real-time guidance to passengers. As a first step, we explore a lifeboat assignment procedure to illustrate some of the aspects linked to pedestrian evacuations on vessels and open the discussion for future work using operations research in a context where human behavioral is part of the problem.

Keywords: Onboard safety, Guidance system, Wireless communication, Lifeboat Assignment, Network flows

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Andres Velez<br>Department of Business and Management Science, NHH Norwegian School of Economics, Bergen, Norway. andres.correa@nhh.no.<br>Stein W. Wallace<br>Department of Business and Management Science, NHH Norwegian School of Economics, Bergen, Norway. stein.wallace@nhh.no.


#### Abstract

We study the evacuation of large cruise vessels under hazardous conditions. Based on a new sensor mesh technology, allowing wire-less two-way communication also when electricity is lost, we propose an optimizationbased procedure that provides real-time guidance to passengers. As a first step, we explore a lifeboat assignment procedure to illustrate some of the aspects linked to pedestrian evacuations on vessels and open the discussion for future work using operations research in a context where human behavior is part of the problem.


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## 1. Introduction

Before the outbreak of the COVID-19 pandemic, the cruise industry was seeing a steady rise in the number of passengers, reaching a total of 29 million worldwide in 2019 (CLIA 2022). It has generated over 160 million passenger movements in world cruise ports since its start in the 1970s (Notteboom et al. 2022). Despite the challenges faced in 2020, it is predicted that 74 percent of the fleet will be back in operation in the coming years. In late 2022, it became clear that the decline in cruise ship operations was only temporary. Cruise companies around the world have restarted their operations and are gradually returning to their pre-pandemic levels of activity. Additionally, several cruise operators are expecting a further increase in hosted activities. As a result, the position of this industry as a significant contributor to the global economy (Pallis et al. 2022) has not changed much.

However, the large number of passengers traveling on ships presents important safety challenges for both travelers and crew members, according to the Maritime Injury Guide (MIG 2021). There have been 448 significant accidents involving cruise ships reported since 2005, and fire is one of the main safety concerns related to cruising. As ships grow larger, it becomes harder for travelers to familiarize themselves with the facilities. But even if they are, their chosen evacuation route may
be inaccessible due to the hazardous situation. Having a quick look at the Overview of the Global Cruise Industry (CLIA 2021) we find that the average cruise length was 6.6 days in 2021; however, this length varies greatly among different markets, being as low as 2-3 days in Asia. This shows that the time passengers spend on board is most likely not enough for them to be acquainted with the layout of all the decks and the resulting path options. In addition, many corridors look exactly the same. To address this, it is natural to think that a guidance system should be implemented onboard as a security measure in case of a disaster. The guidance system should be flexible and able to provide accurate information to all passengers on where they should go, in the short period of time available during a disaster.

A necessary premise for a guidance system to work is to have two-way communication onboard (retrieve information from the real world and be able to feedback valuable insights). Wired communication has major drawbacks, since it can be disrupted by fire and is highly dependent on the ship's power supply (implying that in the event of a fire disaster that damages cables or if there is an interruption in the electrical power on board, data communication within the vessel will be lost). In addition, wired communication is costly to install and maintain. Therefore, wireless communication is essential for bringing analytical results into practice, as it provides a more reliable way for people-tracking and feedback. Until a couple of years ago wireless communication for steel-confined environments was not a realistic option; the technology was not mature enough and failed to overcome the physical phenomena that hinder data transmission through metallic surfaces. However, in recent years, the cutting-edge development of sensor mesh technologies by companies (such as ScanReach from Norway) overcame these difficulties; opening up the possibility of using operation research tools for active evacuation guidance (Luo 2019). To the best of our knowledge, studies on the topic within operations research are scarce, presumably due to this lack of appropriate technology for implementing whatever ideas were developed.

Under the premise of access to this technology, the present paper outlines an optimizationbased system to guide passengers to their destinations in real-time, studies the first of a series of necessary questions, namely the lifeboat assignment problem, and finally makes an open invitation to the operation research community to a challenging topic in which there are plenty of additional questions to be addressed concerning, for example, mathematical modeling, data analytics and machine learning, incorporation of human behavior, and diverse sources of uncertainty.

The evacuation planning approaches by Southworth (1991) and Tuydes (2005) state that disaster management programs must have three phases: The determination of the zones to be evacuated, the definition of a destination for the evacuees, and the allocation of evacuation routes. In the cruise vessel evacuation context, the first phase will be addressed directly by the installation of the sensor mesh network across the decks, providing data about the passengers' location at every point
in time. However, the following two phases present an opportunity for mathematical modeling to help find better options for evacuees and increase their chances to leave the ship safely. We will also consider these two stages separately; there are more reasons for this besides following the protocols. In particular, we seek a flexibility that the current evacuation guidelines, often pasted on the walls, cannot provide. The choice of destinations can be seen as an Assignment procedure in which each passenger is assigned to a specific lifeboat, and procuring a seat for everyone onboard. Subsequently, the evacuation routes are communicated to the passengers, telling them how to get to their designated seats.

There are different kinds of emergencies that can happen onboard, for instance, in 2019, the Viking Sky cruise ship suffered an engine shutdown and had to make an emergency stop at Hustadvika. This region of the sea between Kristiansund and Molde in Norway is well known for its difficult navigability, and that day the winds and waves left the boat adrift without the possibility of launching lifeboats, forcing the evacuation to be performed by helicopter. There could also be other situations where the passengers are not supposed to follow the evacuation paths given to them by the fixed guidelines which are standard today. It can be because the boat is heeling and the lifeboats are stuck in their places. Or they are heading to a lifeboat at full capacity and will be forced to look for another spot, making their path longer and endangering their lives. Therefore, it is important that every passenger has a designated place to go, and this place should be assessed based on the particularities of the ongoing emergency, and possibly the passenger's current location.

For the evacuation system, we propose and analyze an assignment procedure in which each evacuee will be given a destination based on proximity and age distribution. In addition, for later work, we propose a dynamic guidance system, selecting paths for passengers based on the assignment of destinations and periodically reevaluating the network state, looking for unexpected events that may force rerouting the evacuees (for instance, emergency development or non-compliance of the passengers).

## 2. Technical Support and Ship Geometry

### 2.1. Technical Support

As already stated, the feasibility of wireless transmission of data is a necessary technical premise for guiding evacuations on ships. ScanReach has developed a sensor mesh technology that enables wireless data transfer in steel-confined environments, this is a technology that provides instant personnel control during emergencies on ships (ScanReach AS 2023). To achieve this, a special wristband carrying an intelligent chip is worn by each passenger, this chip can detect various
passenger conditions, including body temperature, subtle movements, and falls. The wristband chip's movement detection feature can distinguish between a regular fall and a fall downstairs based on the number of meters the chip has fallen. And the movement detection function can also identify passengers who are trapped in a room or are unable to move by themselves.

In addition to wristbands, several sensors must be installed in each room and corridor to locate and track individually each passenger in real-time. The sensors communicate with each other to collect and transfer the data to a central processing unit. They can be plugged into normal power sockets, and in the event of power outages, there is a battery backup that can last for up to 36 hours. This technology has so far found application on small vessels allowing for the precise and immediate involvement of rescue teams, but it is possible to scale it for larger vessels.

One major concern about the applicability of this technology is whether or not it is legal to track passengers during their trips using wristbands. However, after the health crisis cruise companies have started to implement contact tracing systems on their ships. Royal Caribbean International has, in recent years, introduced a wearable called Tracelet. This device is used to keep a register of the places the passengers were and the people they were in contact with, such information will be used in the event of an onboard health concern. But this is not the only use of the wearable, it can be also used as a key to enter the cabin, or as a contactless payment method. Moreover, the use of this tracking device is not optional for the Royal Caribbean International guest, since any person who refuses to wear it will not be allowed to sail. The idea behind our research is to integrate the ScanReach sensor mesh technology with what is already in use.

### 2.2. Ship Geometry and evacuation simulation

The geometry data used in this study were obtained from different works conducted under the EU framework 7 project SAFEGUARD (Safeguard 2012), which carried out a comprehensive analysis of ship evacuation procedures. As part of this project, a semi-unannounced full-scale assembly was conducted on a vessel operated by Royal Caribbean Cruise Lines International, a ship that operates multiple vacation trips in the Caribbean and Baltic Sea regions. The vessel consists of thirteen decks, including seven decks for passenger cabins and additional decks dedicated to entertainment and recreational facilities, such as restaurants, bars, swimming pools, a casino, a theater, a cinema, a spa, a business center, a gym, a climbing wall, a mini-golf course, a card room, and shops (Galea et al. 2013).

To ensure the accuracy and reliability of evacuation models, the SAFEGUARD project answers the invitation from The International Maritime Organization (IMO) for sharing full-scale data to be used for validation and calibration. The methodologies utilized for the data collection for the
data sets were clearly depicted in the work by Deere et al. (2012), and experiments related to the response time of the passengers to the evacuation alarm were run by Brown et al. (2021).

The validation data sets, including the ship's geometries, can be accessed at the Fire Safety Engineering Group (FSEG) associated with Greenwich University through the following link: https://fseg.gre.ac.uk/validation/ship_evacuation. By utilizing the geometry data derived from the Royal Caribbean cruise vessel described in the aforementioned references, this study ensures the incorporation of validated and reliable information. The obtained geometry data will be appropriately transformed into a suitable data structure to be utilized in the optimization models conducted in this research.

## 3. Modeling the Assignment Procedure

### 3.1. General Definitions

Literature related to the dynamic guidance of passengers on cruise vessels is scarce, nevertheless, Cisek and Kapalka (2014) presented a two-layer approach for an active dynamic evacuation signage system. This early approach was not considering the jamming effect when several passengers are using the same corridor at the same time and also ignored the individual differences of each evacuee. It can be naive to consider a crowd as a group of people with homogeneous characteristics, as there are people with different mobility restrictions and physical capacities. If we consider the development of an emergency, a corridor might be safe to cross in the near future but inaccessible after some minutes; thus, suggesting this corridor to people with fast walking speed might be a desirable option to clear the decks quickly, but suggesting this to a different group of people would guide them to a dead end. In addition, some passengers on cruise vessels are wheelchair users or have all kinds of disabilities, for those cases, the suggestion of routes including staircases or narrow corridors could not be ideal (if the nature of the disaster allows the use of elevators).

Having this in mind, the multi-commodity flow problem is a natural candidate for modeling the Assignment procedure. The multi-commodity flow problem is a formulation with several applications in which we wish to send from sources to sinks, either different physical goods or a single type of good with multiple pairs of origins and destinations that need to send the good to each other. These goods share the same network, but are reigned by their own particular features. Hence, we cannot treat one commodity at a time if we want to find an overall optimal flow (Ahuja et al. 1993).

The basic formulation for the multi-commodity flow problem defines particular cost functions and flow vectors for each commodity, restricting the total flow of all commodities on an arc by some "bundle" constraints tied to a common maximum arc capacity. This formulation also allows us to restrict individually the flow of a specific commodity along an arc, making it easy to model
exclusive arcs for a group of commodities, and the ability or inability of a commodity to reach the upcoming node.

We defined three commodities - groups of people - in the model, namely, Young adults, Elders and children, and Wheelchair users. These are merely labels, thus there might be in the samples elders with good physical conditions, or on the contrary, young people with mobility restrictions (for instance, obesity, fractures, among others). Correct labeling of the passengers is important for a more accurate assignment, but this process lies beyond the scope of this paper. Young adults describe persons with full mobility (the ability to traverse all possible connections between two nodes) and the highest walking speed. Elders and children describe persons with reduced walking speed but without any restriction to use the staircases. Finally, Wheelchair users have their own category due to the mobility restrictions they face and the large area the wheelchairs occupy in the corridors.

In the classical formulation of the multi-commodity problem, we know the sources and the sinks. In our model we need to determine the sinks (destinations) for the flow. We are achieve this the classical way, depicted in Figure 1. We connect all lifeboat nodes to a single imaginary node (supersink) with their respective directed fake arcs. The capacity of a fake arc equals the capacity of the corresponding lifeboat. This means the passengers can be assigned freely to fake arcs (and therefore to the lifeboats) as long as the lifeboat capacity is not exceeded, allowing the objective function to effectively drive the assignment of passengers based on walking distance, and network congestion.

### 3.2. Equivalent length

Usually, the objective of evacuation planning is to find individual suggestions informing about the shortest available route or find a way to allocate evacuation routes in order to minimize the clearing time of the decks. These objectives in traffic assignment research come from the early work by Wardrop (1952) and his definition of the user equilibrium (UE) and system optimality (SO). In both setups, it is necessary to assess time, either taking it into account directly or finding a suitable representation of its effects in static networks.

Originally used in evacuation planning for buildings, the equivalent length is a concept seeking a representation of how difficult it is to traverse a corridor for individuals. This is achieved by adding penalties to various factors that contribute to slowing down the walking speed, and finally multiplying these penalties by the real length of the corridor. Terms such as congestion, harmful gas accumulations, obstacles, the inclination of the ship, or danger coefficients can be considered in this approach (Xuan 2012). In other words, a corridor would become "longer" when it is more difficult to walk through it, and any route using this corridor would be considered more expensive in the optimization model.


Figure 1 Super-sink network transformation: All lifeboats are connected to a single node through arcs with null length and capacity equal to the number of seats of each individual lifeboat.

In the model that will be presented in this paper, we will make use of this definition to represent the different walking speeds of the groups of people and the difficulty of traveling congested corridors and staircases. In practice, this means that the parameter involving the cost of traversing a given arc is higher for Elders and children and Wheelchair users, and going either upstairs or downstairs is more physically demanding. The equivalent length also provides us a way to take into account congestion in our model as a decision variable, by penalizing the corridors with high crowd density.

### 3.3. Corridor's capacity relaxation

In the context of traffic flow applications, imposing hard constraints on arc capacities for modeling purposes is often deemed insufficient to accurately capture the underlying dynamics of congestion. The reason is that these constraints fail to account for the interactions between individual flow units, such as vehicles or evacuees. Moreover, hard constraints are not a favorable choice for evacuation scenarios in which our main objective is to ensure the safety and well-being of individuals. Such constraints often result in infeasible solutions for certain evacuees, potentially leading to the abandonment of people in the face of life-threatening events. Reaching the capacity of a corridor or highway will not prevent new units from entering the connection, but will have an effect on the travel speed. Therefore, in many cases, there is no such thing as a forbidden connection, but rather the undesirable effect of congestion.

Another potentially negative effect of using hard constraints is that people of the same age group, starting and ending in the same place, maybe split as the arc capacity is reached. This is less likely to happen if capacity constraints are soft. In a system affected by human behavior, any suggested action that can cause confusion will increase the chance that people do not do as suggested, and hence that the overall evacuation time increases.

A penalty approach replaces the hard bundle constraints with (linear) penalties for exceeding the arcs' capacities. In practice, this allows evacuees to surpass the bound and traverse the arc but they are subject to a lower walking speed (or an increase of the equivalent length of the corridors). This kind of behavior is described in the field of pedestrians and evacuation dynamics when exploring the hydrodynamic relation between the density and flow of pedestrian streams.

The flow of a pedestrian stream gives the number of pedestrians crossing a specific point within a facility over a given period of time. In other words, it represents the rate at which pedestrians are moving through that location. However, defining density is more difficult, an initial approach would count for the number of pedestrians within a selected area, but it is also possible to provide more detailed definitions. Several studies since 1968 have tried to describe the relationship between flow and density. In the diagrams for the different studies, shown in Figure 2, a common behavior can be observed; the flow of pedestrians increases when the density of the corridor also increases, and this happens until the maximum flow is reached (defining the capacity of the corridor); from this point, increasing density within a corridor impacts negatively the flow of pedestrians. As it can be observed as well, the velocity is always decreasing with the increase of pedestrian density within the corridor across all the models (Schadschneider et al. 2009).

We used this definition of the capacity of a corridor to determine the threshold from where the penalty will start to be paid. Thus, we are interested in avoiding the assignment of people to a corridor when it will have a negative impact on the outflow, rather than a negative impact on the walking speed which was proven monotonically decreasing. This also means the penalty cannot be seen as an approximation of the walking speed but as a measure of how much the model formulation is willing to compromise the outflow. During our computational experiment, the density at the corridor's capacity will be approximated from the data reported by Helbing et al. (2007), this study is the most recent among all the studies plotted in Figure 2.

### 3.4. Model formulation

We define $G$ as the undirected network graph consisting of all the vertices and arcs, representing the structure of a cruise ship layout. The set $\mathcal{V}=\{1, \ldots, V, V+1\}$ represents the set of vertices in the ship layout, and the vertices can be different facilities, for example, a cabin or an intersection. Node $V+1$ is a super-sink. The set $\mathcal{S}=\{1, \ldots, S\}$ represents the sink nodes, typically lifeboats to


Figure 2 Fundamental diagrams for pedestrian movement in planar facilities for a number of studies. The figure is taken from Schadschneider et al. (2009). The parameter $\rho$ shows the average density accounting for the relation between the number of pedestrians and the available area. The flow $J$ of a pedestrian stream represents the number of pedestrians crossing a fixed location per unit of time. Finally, the average speed of the pedestrian stream is represented by $v$.

Table 1 Sets in the proposed model

| Symbol | Description |
| :---: | :---: |
| $G$ | $G=(\mathcal{V}, \mathcal{A})$, a graph consisting of the vertices and arcs, representing the ship's layout. |
| $\mathcal{V}$ | Collection of vertices, representing a logical space (e.g. a cabin, a hall, a doorway, and an intersection of corridors) and the super-sink node. |
| $\mathcal{S}$ | Collection of sink nodes (e.g. lifeboats), $\mathcal{S} \subseteq \mathcal{V}$. |
| $\mathcal{A}$ | Collection of arcs between nodes (e.g. a corridor and a stairway), which can also be expressed as $(i, j)$, where $(i, j) \in \mathcal{V}$. |
| $\mathcal{K}$ | Collection of the group of evacuees labeled according to their age and mobility restrictions. |
| Fake | Collection of arcs that connect all the nodes $i \in \mathcal{S}$ to the super-sink node. |
| Elevators | Collection of arcs that represent the elevators on the vessel. |
| Out ${ }_{i}$ | Collections of arcs that go out from node $i \in \mathcal{V}$. |
| $I n_{i}$ | Collections of arcs that go in to node $i \in \mathcal{V}$. |

which the demand is allocated. However, in this formulation, these sink nodes are transformed into transshipment nodes and they are connected to the single super-sink by a special set of arcs. All demand is assigned to the super-sink. $\mathcal{A}$ is the set of arcs linking the vertices. They can be either corridors, stairways, or elevators; arcs are represented by the combination of tail and head vertices so that $\mathcal{A}=\{(i, j)$ with $i, j \in \mathcal{V}\}$. The set Fake represents all arcs connecting the sink nodes with the super-sink, and the set Elevators gathers all the arcs that represent elevators on board. In
addition, for every $\operatorname{arc}(i, j) \in A$ there is an $\operatorname{arc}(j, i) \in A$, a necessary network transformation to allow bi-directional flow in the mathematical formulation.

The mathematical formulation for the collection of evacuees on board, as it was stated in the previous section, will consider up to three different commodities $k \in \mathcal{K}=\{1,2,3\}$. For $k=1$ we designate the collection of young adults and teenagers, who have full mobility at the highest walking speed. For $k=2$ we designate the collection of elder passengers, children, and persons with full mobility but a relatively low walking speed, in comparison to $k=1$. And lastly, $k=3$ represent the wheelchair users who have mobility restriction through stairways and narrow corridors.

We also defined two sets for each node $i \in \mathcal{V}$, namely $O u t_{i}$ and $I n_{i}$, defined as follow: Out ${ }_{i}$ represents the set of all arcs going out from node $i$, and consequently, $I n_{i}$ represents the set of all arcs going into node $i$.

A penalty approach for the multi-commodity flow problem is formulated as follows:

$$
\min \sum_{i j \in A} \sum_{k \in K} C_{i j}^{k} \cdot x_{i j}^{k}+\psi \sum_{i j \in A} f_{i j}+\gamma \sum_{i j \in \text { Fake }} \sum_{k \in K} s_{i j}^{k}
$$

Subject to:

$$
\begin{align*}
& \sum_{j \in O u t_{i}} x_{i j}^{k}-\sum_{j \in I n_{i}} x_{j i}^{k}=B_{i}^{k} \quad \text { for all } k \in \mathcal{K} \text { and } i \in \mathcal{V} .  \tag{1}\\
& f_{i j} \geq \sum_{k=1}^{K}\left(x_{i j}^{k} \cdot \rho^{k}\right)-\left(U_{i j} \cdot \phi(i, j)\right) \quad \text { for all } i, j \in\{i j \mid i j \notin \text { Fake } \cup \text { Elevators }\} .  \tag{2}\\
& x_{i j}^{k} \leq \mu_{i j}^{k} \quad \text { for all } i, j \in\left\{i, j \mid \mu_{i, j}^{k} \geq 0\right\} \text { and } k \in \mathcal{K} .  \tag{3}\\
& \sum_{k}^{K} x_{i j}^{k} \leq \text { BoatMax } \quad \text { for all } i, j \in \text { Fake. }  \tag{4}\\
& s_{i j}^{k} \geq x_{i, j}^{k}-D^{k} \quad \text { for all } i, j \in \text { Fake and } k \in \mathcal{K} .  \tag{5}\\
& f_{i j} \geq 0 \quad \text { for all } i, j \in \mathcal{A} \backslash\{\text { Fake }\} .  \tag{6}\\
& s_{i j}^{k} \geq 0 \quad \text { for all } i j \in \text { Fake and } k \in \mathcal{K} . \tag{7}
\end{align*}
$$

Constraints (1) are node balance constraints. Constraints (2) penalize the assignment of flow to the corridors above their individual capacities. Constraints (3) ensure that flow is not assigned to those corridors that cannot be traversed or are forbidden for a given group of passengers, such as elevators. Constraints (4) state the maximum capacity of a lifeboat as a hard constraint. Constraints (5) penalize the excess of evacuees of vulnerable groups within one lifeboat. Constraints (6) and (7) define the domain of the variables.

All the fake $\operatorname{arcs}(i, V+1)$ with $i \in \mathcal{S}$ are capacitated, making sure the flow of evacuees is less than or equal to the lifeboat physical capacity. Additionally, the soft constraint (5) for the sake
of the implementation presented in this paper, will make sure that not more than 80 percent of passengers of the lifeboats are labeled as Elders and children, and no more than 10 percent of the passengers are wheelchair users; if these thresholds are surpassed, a penalty must be paid. A lifeboat that is completely full of young adults is permitted.

Table 2 Variables and Parameters

| Symbol | Description |
| :--- | :--- |
| Variables | Flow of evacuees from group k traversing the arc $(i, j) \in \mathcal{A}$. |
| $x_{i j}^{k}$ | Total excess of flow in the arc $(i, j) \in \mathcal{A}$. |
| $f_{i j}$ | Total excess of evacuees from group $k \in \mathcal{K}$ in the $\operatorname{arc}(i, j) \in$ Fake. |
| $s_{i j}^{k}$ | Maximum capacity of one lifeboat for people in group $k \in \mathcal{K}$. |
| Parameters |  |
| $D^{k}$ | External flow vector on node $i \in \mathcal{V}$ for the group $k \in \mathcal{K}$. |
| $B_{i}^{k}$ | Equivalent length of the arc $(i, j) \in \mathcal{A}$ for each age group $k \in \mathcal{K}$. |
| $C_{i j}^{k}$ | Area of the $(i, j) \in \mathcal{A}$, except for arcs $(i, j) \in F a k e \cup E l e v a t o r s$. |
| $\phi(i, j)$ | Inflection bound for the flow on the arc $(i, j) \in \mathcal{A}$, except for arcs $(i, j) \in$ |
| $U_{i j}$ | Fake $\cup$ Elevators $($ Measured as density $)$. |
| $\mu_{i j}^{k}$ | Individual capacity constraint on the arc $(i, j) \in \mathcal{A}$ for each age group |
| $\rho^{k}$ | $k \in \mathcal{K}$. |
| $\psi$ | Relationship of occupied area by an evacuee of each group $k \in \mathcal{K}$. |
| $\gamma$ | Penalty for the excess of flow. |

We would like to highlight the parameter $D^{k}$ in connection with the previous paragraph. $D^{k}$ represents a soft constraint on the maximum number of people from group $k$ that is allowed to board a lifeboat, determined by a desired distribution of evacuees on the lifeboats. This might seem odd, but let us consider that a secondary disaster might happen after the lifeboats were launched into the ocean. In this case, it is desirable to have young adults able to help in the new situation.

Let us also define the external flow as the vector $B_{i}^{k}$, representing inflow and outflow for all nodes $i \in \mathcal{V}$. By convention origin-nodes are set as positive values, whereas, on sink nodes the values are negative. Note for the formulation with a super-sink only node $V+1$ has a negative value. All the other nodes are called transshipment nodes since inflow is equal to outflow. Parameter $C_{i j}^{k}$ represents the cost of traversing a corridor and it is defined using the concept of equivalent length.

The parameter $\phi(i, j)$ is the total area of the corridor or room represented by an arc. It is an important measurement in order to assess the congestion in the network, along with the upper bound $U_{i j}$ and space occupied for every kind of passenger $\rho^{k}$. Note that the term $\rho^{k}$ was added to constraint (2), as wheelchair users occupy a larger area in the corridors. Finally, the parameter $\mu_{i j}^{k}$ is the individual capacity on every arc for people from each age group, this parameter is useful for
the definition of arcs that can be traversed by people from a given group but not from another; either because of physical restrictions or evacuation policies defining dedicated paths not applicable to all evacuees.

For this multi-commodity flow formulation, three variables are considered, one main variable and two auxiliary variables. The variables $x_{i j}^{k}$ answer the formulation's main question: the number of persons traversing a corridor from each age group. The variables $f_{i j}$ and $s_{i j}^{k}$ and their respective parameters $\psi$ and $\gamma$ serve for the relaxation of formerly hard constraints that will be explained shortly.

## 4. Flow decomposition

After solving the multi-commodity flow problem, the next step involves breaking down the flow into paths to determine individual assignments for evacuees. The transformation of flows into paths can be done in various ways, with paths and cycles representing nonnegative flows traversing arcs. Therefore, while every nonnegative flow can be represented as paths and cycles, they may not necessarily have a unique representation (Ahuja et al. 1993). This process, known as the Flow Decomposition Problem, can be modeled using either edge-based or path-based formulations and is NP-hard for certain formulations (Williams et al. 2021).

We have chosen a path-based formulation of the flow decomposition problem, mainly for the reasons that will be outlined in the next paragraph; but in addition, this type of formulation allows the use of non-exact methods or column generation procedures for finding a solution, and this could be particularly relevant for those versions of the problem that are relatively hard to solve given time restrictions.

When a path formulation is defined for a network problem, it is done under the assumption that there is a pool of paths to choose from. Such a pool is supposed to be finite and contain all the possible ordered node combinations able to represent a solution. This characteristic often makes the number of variables quite large. But in the context of evacuations, most of these paths are not promising suggestions for the evacuees; an acyclic path suggesting to go up and down stairs several times before leading the person to a safe place will hardly be considered optimal, nor followed by a rational person. Thus, the selection of a set of promising paths sufficiently large to provide flexibility on the route assignments is the initial step for decomposing successfully the flow of evacuees into paths.

While network flow theory guarantees that any solution to the multi-commodity flow problem can be decomposed, the choice of decomposition procedure is crucial in real-life scenarios. It is tempting to use the reduced network, including only arcs with positive flow, to scale down the problem size during the decomposition phase. However, this approach can make it difficult to assess
the solution's quality in certain parameter configurations. To address this, we propose calculating the overall top 100 shortest paths using the original network as an initial filter against counterintuitive path recommendations. For example, if a parameter configuration leads to assigning flows to arcs that cannot be found in any of the best optional paths for the entire network, the resulting solution is unlikely to be effective or realistic from the passenger's perspective.

In our work, we aim to reduce non-compliance by using a limited search space that prioritizes solutions considered intuitive by passengers. With this pool selection, it is possible that the different formulations for the Flow decomposition are not able to encounter feasible solutions, for certain cases and/or parameter values. This situation can be an indication either that the pool size is too small and didn't provide enough flexibility, or that the selected penalty for congestion is too high and would suggest the passengers use corridors that are not present within their overall best optional paths.

Until now, researchers have mainly focused on finding the decomposition with the minimum number of paths, a problem that has been proven to be NP-hard in the strong sense (Pieńkosz and Kołtyś 2015, Vatinlen et al. 2008). Nevertheless, other objectives can be pursued when formulating this problem, for instance: decomposing an arc flow into a path flow with a minimum length of the longest path (the min-max criterion often used in other network problems), or even minimizing the concept of unfairness borrowed from Roughgarden and Tardos (2002), defined as the ratio between the time it takes to travel on the suggested path and the time it would have taken to travel on the shortest possible path. In this paper, the difference between those lengths will be considered for measuring the unfairness instead of the ratio. The selection of objective function will have an effect on the arrival time distribution, or the average walked distance, and by extension the willingness of people to comply with the assignment.

An algorithm for finding the k-shortest routes from any point to a given destination was developed by Yen (1970). We argue that a pool formed by a sufficient number of ranked shortest paths is the most promising for solving the flow decomposition problem. The pool size must be large enough to provide flexibility, but not so large that it includes non-viable solutions, thereby only increasing the problem size and consequently, increasing the computing time. During our numerical tests, we found that the top 100 shortest paths (from each node to each lifeboat station) were enough to find feasible solutions for the considered cases.

In the Ph.D. thesis by Ohst (2016) we can find formal definitions for path-based formulations of the flow decomposition problem. Also, this work shows the necessary constraint to ensure the sum of the flow assigned to certain paths must equal the flow traversing the corridors they all have in common. Thus, it is necessary to construct a two-dimensional incidence matrix $I_{a, i}$ having all the arcs in the network on the rows and all the considered paths on the columns, from now
on denoted $\mathcal{P}$. A number 1 in this matrix indicates that path $i$ crosses corridor $a$, therefore the added flow of all the paths traversing corridor $a \in \mathcal{A}$ corresponds to the flow calculated for the multi-commodity flow problem described before. Notice that since we have separated flows for each group of passengers, it is necessary to solve independently the optimization models for each group. Consequently, the forthcoming formulations do not explicitly reference this particular set.

The flow decomposition problem minimizing the unfairness in the system is formulated as follows:

$$
\min \sum_{i=0}^{|P|}\left(l_{i}-l_{i}^{*}\right) \cdot x_{i}
$$

Subject to:

$$
\begin{align*}
& \sum_{i=0}^{|P|} x_{i} \cdot I_{a, i}=F_{a} \quad \text { for all } a \in(0,|A|) .  \tag{8}\\
& x_{i} \geq 0 \quad \text { for all } i \in(0,|P|) . \tag{9}
\end{align*}
$$

Constraint (8) is the previously described arc balance constraint with $F_{a}$ representing the flow traversing arc $a$, found by solving the multi-commodity flow problem, the term $l_{i}-l_{i}^{*}$ represents unfairness of the assignment, where $l_{i}$ is the length of the current path, and $l_{i}^{*}$ represent the length of the shortest path for a passenger located at the origin node of path $i$. Lastly, $x_{i}$ is the decision variable accounting for the flow assigned to each individual path.

For the other two formulations that will be developed, it is necessary to define extra decision variables. The variable $\zeta$ is used to minimize either the number of selected paths or the longest path used in the solution. Additionally, variable $y_{i}$ is a binary variable acting as a connection between the objective function and the flow decomposition constraint. With these definitions, the min-max formulation is as follows:

$$
\min \zeta
$$

Subject to:

$$
\begin{align*}
& L_{i} * y_{i} \leq \zeta \quad \text { for all } i \in(0,|P|) .  \tag{10}\\
& x_{i} \leq M \cdot y_{i} \quad \text { for all } i \in(0,|P|) .  \tag{11}\\
& \sum_{i=0}^{|P|} x_{i} \cdot I_{a, i}=F_{a} \quad \text { for all } a \in(0,|A|) . \tag{12}
\end{align*}
$$

$$
\begin{align*}
& x_{i} \geq 0 \quad \text { for all } i \in(0,|P|) .  \tag{13}\\
& y_{i} \in\{0,1\} \quad \text { for all } i \in(0,|P|) .  \tag{14}\\
& \zeta \geq 0 . \tag{15}
\end{align*}
$$

Constraint (10) ensure that $\zeta$ takes as value the length of the longest selected path in the solution. Constraint (11) is the classic big M constraint linking the variables $x_{i}$ and $y_{i}$. Constraint (12) is the flow decomposition constraint. Constraints (13)-(15) are the domain of the variables.

For the last formulation, if we want to find the minimum number of paths necessary for decomposing the flow just constraint (10) in the previous model should be modified. Thus, the flow decomposition problem with minimum number of paths is formulated as follows:

$$
\min \zeta
$$

Subject to:

$$
\begin{align*}
& \sum_{i=0}^{|P|} y_{i} \leq \zeta \quad \text { for all } i \in(0,|P|) .  \tag{16}\\
& x_{i} \leq M \cdot y_{i} \quad \text { for all } i \in(0,|P|) .  \tag{17}\\
& \sum_{i=0}^{|P|} x_{i} \cdot I_{a, i}=F_{a} \quad \text { for all } a \in(0,|A|) .  \tag{18}\\
& x_{i} \geq 0 \quad \text { for all } i \in(0,|P|) .  \tag{19}\\
& y_{i} \in\{0,1\} \quad \text { for all } i \in(0,|P|) .  \tag{20}\\
& \zeta \geq 0 . \tag{21}
\end{align*}
$$

In this formulation, constraint (16) sums up the number of selected paths, therefore, minimizing $\zeta$ minimizes the number of variables $y_{i}$ that can have a value of 1 , this constraint is what makes this formulation an NP-hard problem.

## 5. Case study

As mentioned before, we have access to the layout of a cruise vessel belonging to Royal Caribbean International, with a capacity for 2500 passengers and 842 crew members. Starting from this layout, a network was constructed following two simple rules: every intersection must be a node, and a node should be placed every 10 meters. These rules were considered due to the importance of precisely locating the possible turns, and a distance in which they do not interfere with each other (but there are several sensors as communication backup, in case of failure of one sensor). Following this, a network with 557 nodes and 1464 arcs (recall that for each connection between two points
two arcs are needed, one in each direction) is generated. This vessel has 18 lifeboats with a capacity of 150 passengers each, 9 lifeboats on each side of the ship with a total of 7 possible entries. To reduce the number of decision variables, we decided to use the entries as the nodes before the super-sink, instead of using the lifeboats directly. We argue that having 18 possible destinations will provide no additional information to the model, but it will almost triple the number of paths (and decision variables) considered in the flow decomposition stage.

The initial location of the passengers is necessary to initialize the optimization problem, for the purpose of the numerical test only two groups of people are considered, however, it can be easily expanded to the three groups described in the paper or consider a completely different definition for the groups. Also, three different cases were considered for this initial location. In the first case, all the passengers were assumed to be in their cabins when the alarm went off; in other words, the 2500 passengers with 50 percent being young adults and 50 percent elders and children (this distribution is close to the average number of passengers by age group reported by CLIA (2021)), were placed evenly across the decks where the cabins are located. This is the classic case used during simulation studies because this situation replicates an emergency in the middle of the night. In the second case, the alarm goes off during daytime and most of the passengers are assumed to be enjoying the facilities on the top decks (for instance, the pool, the training center, the sports court, or the bars) and a few of them are located in their cabins. Finally, in the third case, the alarm goes off around dinner time, thus the passengers are assumed to be in the restaurants or ready to go to the theater. It is important to mention that for this ship the largest restaurants and the theater are located in the vicinity of the lifeboats, and in the study by Brown et al. (2021) these locations were used as assembly stations.

In addition, the values for the parameters that penalize surpassing the capacity levels must be determined for the study case. The parameter $\psi$ is important in determining the shortest routes. On top of that, as was mentioned before, this penalty should be seen as how permissive the model is to having congestion within a corridor, rather than an exact way to calculate the extra time spent by people when traversing a congested connection. Thus, different values for $\psi$ were analyzed when solving the model, but $\gamma$ remained constant. All the cases were solved using the Python API of Gurobi.

## 6. Results

For each of the three cases presented in the previous section, the multi-commodity flow formulation was solved for different values of $\psi$, starting from $\psi=1$. Once the value of the parameter was increased, it was clear that the model was almost insensitive to variation of $\psi$ between 1 and 10 (this can be seen in the tables reported in the Appendix). It can easily be explained by realizing
that relatively small values of $\psi$ will not have a significant impact on the total route length. This is because the penalty should be greater than the cost of taking any alternative route, in order to find more attractive a detour rather than the original path. Thus, we generally observe that the solutions do not change with respect to the non-penalized model because the formulation is willing to pay the penalty.

Therefore, it is expected that as $\psi$ goes up, the number of positive variables $f_{i j}$ goes down since the system is actively avoiding having any kind of congestion in its corridors. The other extreme case is achieved when the value of $\psi$ is large, and increasing it further will not change the solutions anymore. This happens because there are some arcs in which it does not matter how much you penalize the congestion, they are necessary to reach the destination. For instance, all the staircases will pay for the congestion because it is the only way to go up and down the vessel. It is unavoidable, no matter how high you set $\psi$. This is the situation we observed with values above 50 , for Cases 1 and 2. Nevertheless, Case 3 presents a different behavior that will be explained later in the section. In conclusion, we found that values for $\psi$ between 20 and 30 would provide the most reasonable solutions for all three cases; however, this might change when considering different ships with their unique features. In order to obtain fast solutions, good values for the penalty should be found in advance for each ship, instead of having a tuning process in the middle of a disaster.

Figure 3 presents histograms for the three different formulations considered for the flow decomposition problem. The first thing to notice is that the longest path for the group of elders and children is shorter than the one for the other group, this result is expected given the benefits within the model for this age group. It is also clear that there are significant differences between the results found within each group, indicating that the different formulations lead to different approaches to getting the evacuees to the lifeboats. For example, we can see on the bottom figure how all the formulations seemed to suggest shorter evacuation routes than what the fewer paths decomposition does (orange), but in the end, no major differences in the extreme cases can be observed.

This is not the case for the elders and children group, although something similar can be observed with the min-max decomposition. After initially suggesting longer routes to many evacuees it managed to assign the last person to a shorter path than the other formulations. This is the classic behavior of any kind of min-max optimization, since we are minimizing the longest path it is not that important what is happening with the people who do not belong to the extreme cases.

It might easily happen that the suggestions for passengers relatively close to the lifeboats will be considered sub-optimal by them, and undermine the trust in the system, ultimately increasing the non-compliance. In this regard, it is noticeable how the fair approaches seem to give the best options to as many people as possible, but leave a small number of people with longer paths (and potentially more dangerous paths) on the tails of the distribution. Analysis as the previous ones can


Figure 3 Histograms for different Flow Decomposition Formulations (solved for Case 2 with $\psi=40$ ). Length traveled by Elders and Children (top). Length Traveled by Young Adults (bottom).
be done for all three different cases, and for different $\psi$; therefore, the selection of a decomposition approach must take into account that different situations on the vessel will lead to differences in the performance of these formulations.

The fewer paths decomposition was expected to provide a smaller or simpler pool of paths going to the lifeboats. Independently of its proven costly computation time, a simpler way to guide people will always be a desirable feature. However, we found no evidence of a significant improvement in this regard; as was mentioned before the problem is more difficult to solve, and after computing time to 5 minutes the optimality gap was 1.3 percent and the best lower bounds found were between 260 and 278 used paths depending of the case. On average, the optimal solutions found using the other formulations used no more than 5 extra paths in comparison with the fewer paths decomposition, thus this formulation is not leading to significantly simpler solutions than the others.

Finally, we would like to describe briefly the results for Case 3 . For this cruise vessel, the largest restaurants and the theatre are located on the same deck as the lifeboats. This means that finding the solution for the shortest path is trivial from the passengers' point of view (since they are able
to locate the closest lifeboat with ease) and they are likely to disregard complex paths in this setup. In particular, for Case 3 , when $\psi$ is increased above 30 , the model finds itself in the situation described in Section 4 regarding the feasibility of the flow decomposition formulation (see Table 7 and Table 8 in the Appendix). In those cases with large crowds in few spots, when the penalty for congestion is too high, it is very easy for the Multi-commodity problem to assign flow to corridors that are not used for the top quickest possible routes. Perhaps, this outcome suggests that locating restaurants and theaters on the same deck as the lifeboats could be a favorable design decision. Nevertheless, this emphasizes the importance of the selection of appropriate penalties based on the passengers' initial positions during an onboard emergency.

## 7. Conclusions and outlook

Modeling evacuations (or any procedure highly dependent on human behavior) is a difficult task. Despite all the details taken into account while formulating this model, it is evidently simple and more considerations are needed for it to capture reality in a better way. To begin with, one of the main issues in the formulation presented in this paper is the use of a static network; the overall goal is to clear all decks of passengers as quickly as possible, not to minimize the distance walked by the evacuees. Thus, time is the key factor during an evacuation; not only because the variables we want to minimize should be measured in time units, but because the state of the network changes over time.

In order to provide a good example of this, let us go back to the multi-commodity flow problem; our decision variables are accounting for the number of passengers traversing a corridor, but we cannot answer the question of when they walked through the corridor. The passengers engaged in the evacuation start from different locations, therefore it is obvious that they will arrive at a certain corridor at different times. Moreover, as was shown in the paper by Brown et al. (2021), passengers do not start the evacuation immediately after the alarm goes off but the starting time follows a log-normal distribution (with different parameters depending on the boat and trip type). This fact alone indicates that the model can likely overestimate the congestion within a corridor; it might seem that a large number of passengers are in it but in reality, they were evenly distributed in time thus the corridor was never jammed. The opposite can be true as well, punctual traffic jams could be generated but they cannot be seen by a static formulation. This means, that quicker (and safer) paths can end up being neglected when the effects of time on parameters and variables are not being modeled.

Additionally, even though making an analysis of the distance walked by evacuees can provide interesting insights regarding the performance of different flow decomposition approaches, it is still more adequate to use time-dependent variables. The next step would be the expansion of the model
into a dynamic network formulation; this will bring the flexibility to model more precisely some interactions and events happening during an evacuation, but also comes with its own particular challenges. Solving mixed integer optimization models to optimality is a highly time-demanding process, even for very simple models. Increasing the network size would exponentially increase the time required to solve a problem, and during an emergency, we do not have much time. Fortunately, network problems have nice properties when it comes to computation times, and more recent non-exact methods can also be considered.

In an ideal world, all the different parameters in a model are well defined, either because they are constant in time or because we have complete certainty on how they will develop over the study horizon. However, often this is not the case, for instance, we do not know for sure how a fire is going to spread on board. To be sure about this it would be necessary to have a complete understanding of fire behavior, both current ship status and layout distribution, awareness of any other possible source that can cause explosions, and an endless list of factors that may have influence. And even if we know the development of the disaster for sure, the evacuees' behavior is stochastic, no matter if they were provided with the best possible solution or not. We cannot guarantee that all people are going to follow the provided indications. There can be many good reasons to not do so, such as helping others, looking for a family member, or having important belongings inside the cabin. These are important issues that remain open to exploration by modelers in the operation research community, and possibly in collaboration with experts in human behavior.

Once the evacuees were given a destination for their evacuation, it is necessary then to select paths through the corridors that take them safely to their destinations. In the best scenario, people will use the routes found from the Assignment Problem; however, it would be naive to expect the state of the network and the evacuation conditions to remain unaltered during the evacuation. Here the installation of the mesh sensor technology and two-way communication is crucial since the system can perform frequent updates of the location of passengers and the availability of the corridors. Under unexpected events, this updated information allows us to react and calculate new suggestions for the evacuees, take corrective measures, and provide them with safer routes through wristband devices or dynamic signaling on walls, all connected to the sensor mesh network.

Once we consider active guidance, it is imperative to recognize that a good solution not only takes the passengers quickly to their destination, and assesses future risks, but also avoids confusing instructions that could undermine the trust in the system, this way encouraging non-compliance, since the choice of evacuation route by the evacuees is influenced by the level of confidence they have in the information available to them ( Lu et al. 2014). Dealing with situations such as counterflows, U-turns, and people's preferences, among other factors; can be quite challenging to handle during a dynamic guidance stage.

Proposing evacuation plans that deviate significantly from human instinct, such as directing individuals towards the origin of a disaster or instructing them to make a U-turn when there is no visible threat ahead, often face significant resistance. Such instructions can create the impression for the passengers that the guidance system lacks understanding of the situation. However, there are instances when these seemingly counterintuitive instructions become necessary or the most viable option due to unexpected events. These unforeseen events can disrupt the original evacuation plan for all passengers on board.

For instance, if a fire suddenly blocks the suggested evacuation route, it becomes imperative for evacuees to backtrack and, in some cases, find an alternative lifeboat as their new best option. In such scenarios, it is crucial to ensure that the process of rerouting passengers is conducted in a manner that minimizes psychological distress for those passengers who were not directly affected by the fire but still need to be redirected due to the unforeseen circumstances. While passengers directly exposed to the fire will certainly comprehend the need for a change in plans, those compelled to switch positions may struggle to grasp the rationality behind this adjustment. As for this one, there are other situations that may arise in which a multidisciplinary understanding is necessary to design a good solution approach, bringing a dynamic guidance system closer to the users, and increasing the chances of a successful implementation. To ensure a truly successful implementation of the evacuation system, the integration of dynamic signaling is crucial.

## Compliance with Ethical Standards

Conflict of interest The authors have no competing interests to declare that are relevant to the content of this article.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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## Appendix. A

|  | Function | $\leq 25$ (m) | $\leq 40$ (m) | $\leq 55$ (m) | $\leq 70$ (m) | $\leq 75$ (m) | $\leq 90$ (m) | $\leq 105$ (m) | $\leq 120$ (m) | $\leq 135$ (m) | $\leq 150$ (m) | $\leq 165$ (m) | $\leq 180$ (m) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ส่ | Fair | 0.0 | 18.56 | 41.46 | $\overline{64.02}$ | 82.83 | 89.94 | 94.19 | 98.69 | 100.0 | 100.0 | 100.0 | 100.0 |
| $\stackrel{7}{0}-1$ | Fewer | 0.49 | 17.58 | 41.46 | 62.96 | 81.28 | 90.43 | 95.75 | 99.67 | 100.0 | 100.0 | 100.0 | 100.0 |
| $\stackrel{\sim}{\sim}$ | MinMax | 0.0 | 18.56 | 40.8 | 63.94 | 82.26 | 89.94 | 94.77 | 99.67 | 100.0 | 100.0 | 100.0 | 100.0 |
| Tె | Fair | 0.0 | 18.56 | 41.46 | 63.37 | 81.19 | 88.23 | 93.3 | 98.69 | 100.0 | 100.0 | 100.0 | 100.0 |
| $\underset{\sim}{0} 10$ | Fewer | 0.98 | 17.09 | 39.98 | 62.06 | 81.19 | 89.53 | 94.77 | 99.67 | 100.0 | 100.0 | 100.0 | 100.0 |
| ค | MinMax | 0.0 | 18.56 | 40.47 | 62.88 | 80.7 | 88.55 | 93.79 | 99.67 | 100.0 | 100.0 | 100.0 | 100.0 |
| Tี่ | Fair | 0.0 | 18.56 | 41.46 | 63.37 | 80.87 | 87.9 | 93.3 | 98.69 | 100.0 | 100.0 | 100.0 | 100.0 |
| $\stackrel{7}{0}$ | Fewer | 0.49 | 18.23 | 40.8 | 61.57 | 79.89 | 88.88 | 94.77 | 99.18 | 100.0 | 100.0 | 100.0 | 100.0 |
| $\bigcirc$ | MinMax | 0.0 | 18.23 | 39.49 | 61.41 | 79.89 | 89.21 | 95.26 | 99.67 | 100.0 | 100.0 | 100.0 | 100.0 |
| नె | Fair | 0.49 | 16.43 | 39.49 | 58.87 | 76.7 | 84.96 | 92.48 | 96.65 | 98.45 | 98.69 | 100.0 | 100.0 |
| वृㅇ | Fewer | 0.49 | 16.27 | 37.69 | 57.65 | 76.12 | 85.28 | 93.46 | 98.36 | 99.67 | 100.0 | 100.0 | 100.0 |
| 0 | MinMax | 0.49 | 17.01 | 36.96 | 54.62 | 73.34 | 85.61 | 95.42 | 99.67 | 100.0 | 100.0 | 100.0 | 100.0 |
| ศี่ | Fair | 0.98 | 16.19 | 39.41 | 59.44 | 77.68 | 85.69 | 90.52 | 95.26 | 98.04 | 99.02 | 100.0 | 100.0 |
| ${ }^{6}$ | Fewer | 0.98 | 15.94 | 38.02 | 57.15 | 75.14 | 84.79 | 92.48 | 98.86 | 99.67 | 99.67 | 100.0 | 100.0 |
| ${ }^{\circ}$ | MinMax | 0.0 | 15.94 | 34.59 | 54.21 | 73.34 | 86.26 | 95.09 | 99.67 | 100.0 | 100.0 | 100.0 | 100.0 |
|  | Fair | 0.98 | 16.52 | 38.27 | 57.73 | 75.55 | 84.71 | 91.17 | 95.91 | 97.71 | 98.69 | 100.0 | 100.0 |
| $\stackrel{1}{9}$ 아 | Fewer | 0.0 | 16.76 | 37.53 | 56.66 | 74.65 | 83.48 | 91.17 | 98.04 | 99.84 | 99.84 | 100.0 | 100.0 |
| $\stackrel{\square}{\sim}$ | MinMax | 0.98 | 15.62 | 35.24 | 52.41 | 72.2 | 86.26 | 95.09 | 99.67 | 100.0 | 100.0 | 100.0 | 100.0 |
| శె | Fair | 0.98 | 16.52 | 38.27 | 57.73 | 75.55 | 84.71 | 91.17 | 95.91 | 97.71 | 98.69 | 100.0 | 100.0 |
| ${ }_{2} 8$ | Fewer | 0.0 | 16.76 | 37.53 | 56.66 | 74.65 | 83.48 | 91.17 | 98.04 | 99.84 | 99.84 | 100.0 | 100.0 |
| $\stackrel{1}{2}$ | MinMax | 0.98 | 15.62 | 35.24 | 52.41 | 72.2 | 86.26 | 95.09 | 99.67 | 100.0 | 100.0 | 100.0 | 100.0 |

Table 3 Case 1 for Elder people and Kids. Percentage of people assigned to paths up to a given length, for different penalties.

|  | Function | $\leq 25$ (m) | $\leq 40$ (m) | $\leq 55$ (m) | $\leq 70$ (m) | $\leq 75$ (m) | $\leq 90$ (m) | $\leq 105$ (m) | $\leq 120$ (m) | $\leq 135$ (m) | $\leq 150$ (m) | $\leq 165$ (m) | $\leq 180$ (m) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ๘ֹ่ | Fair | 0.0 | 8.5 | 19.13 | 31.56 | 54.37 | 69.42 | 80.54 | 87.74 | 93.13 | 97.06 | 100.0 | 100.0 |
| $\stackrel{7}{0}$ | Fewer | 0.0 | 7.2 | 18.64 | 31.73 | 56.5 | 69.91 | 80.87 | 87.57 | 93.13 | 97.06 | 100.0 | 100.0 |
| 0 | MinMax | 0.34 | 6.07 | 17.54 | 30.19 | 55.14 | 69.31 | 81.11 | 87.77 | 92.92 | 96.96 | 100.0 | 100.0 |
| F | Fair | 0.0 | 10.22 | 19.71 | 30.83 | 53.15 | 67.54 | 79.72 | 86.26 | 93.13 | 97.06 | 100.0 | 100.0 |
| $\underset{\sim}{0} 5$ | Fewer | 0.0 | 9.57 | 19.87 | 32.95 | 52.25 | 69.42 | 80.05 | 86.43 | 92.64 | 96.57 | 100.0 | 100.0 |
| $\bigcirc$ | MinMax | 0.98 | 9.24 | 19.71 | 30.83 | 54.7 | 70.4 | 80.21 | 86.43 | 92.31 | 96.73 | 100.0 | 100.0 |
| శె | Fair | 0.0 | 10.07 | 19.23 | 31.51 | 52.29 | 68.9 | 80.36 | 84.45 | 91.98 | 95.09 | 100.0 | 100.0 |
| $\underset{\sim}{2}$ | Fewer | 0.0 | 8.59 | 20.36 | 31.97 | 52.25 | 68.93 | 80.7 | 86.92 | 91.5 | 96.08 | 99.35 | 100.0 |
| $0^{\circ}$ | MinMax | 0.98 | 9.57 | 20.69 | 32.46 | 51.92 | 66.64 | 79.07 | 85.94 | 91.5 | 96.24 | 100.0 | 100.0 |
| - | Fair | 0.0 | 8.42 | 20.52 | 31.64 | 47.18 | 65.17 | 75.96 | 83.97 | 91.17 | 96.4 | 99.84 | 100.0 |
| $\stackrel{\sim}{0}$ 아 | Fewer | 0.0 | 8.75 | 19.87 | 32.3 | 48.98 | 62.22 | 76.61 | 85.28 | 91.33 | 95.58 | 99.35 | 100.0 |
| $\bigcirc$ | MinMax | 0.0 | 9.24 | 19.05 | 31.48 | 48.65 | 63.86 | 74.49 | 84.46 | 92.8 | 96.57 | 100.0 | 100.0 |
| T | Fair | 0.0 | 8.91 | 18.4 | 29.68 | 47.51 | 60.83 | 73.43 | 80.21 | 87.74 | 94.11 | 98.04 | 98.69 |
| $\underset{\sim}{1}$ | Fewer | 0.49 | 10.23 | 20.38 | 29.87 | 47.87 | 61.7 | 73.98 | 81.34 | 86.74 | 92.96 | 97.55 | 98.85 |
| ค | MinMax | 0.0 | 8.91 | 19.05 | 29.68 | 44.24 | 58.3 | 69.99 | 81.28 | 89.86 | 94.93 | 100.0 | 100.0 |
| - | Fair | 0.0 | 9.89 | 19.54 | 29.03 | 45.79 | 58.79 | 71.05 | 80.21 | 87.65 | 94.52 | 97.38 | 97.38 |
| $\stackrel{\square}{0}$ | Fewer | 0.49 | 8.91 | 18.07 | 28.86 | 46.2 | 61.9 | 72.04 | 81.36 | 84.63 | 91.01 | 95.91 | 96.89 |
| - | MinMax | 0.0 | 9.73 | 18.07 | 28.21 | 45.87 | 57.65 | 69.26 | 77.6 | 82.99 | 92.64 | 100.0 | 100.0 |
|  | Fair | 0.0 | 9.4 | 20.2 | 30.66 | 44.81 | 56.83 | 70.24 | 78.5 | 84.96 | 90.52 | 95.75 | 96.73 |
| $\cdots$ | Fewer | 0.0 | 9.89 | 19.54 | 28.37 | 44.73 | 56.5 | 67.21 | 78.41 | 84.96 | 93.46 | 97.22 | 98.53 |
| 0 | MinMax | 0.0 | 9.89 | 19.05 | 29.19 | 43.74 | 54.54 | 66.07 | 75.8 | 84.96 | 94.11 | 100.0 | 100.0 |

Table 4 Case 1 for Young People. Percentage of people assigned to paths up to a given length, for different penalties.

|  | Function | $\leq 25$（m） | $\leq 40$（m） | $\leq 55$（m） | $\leq 70$（m） | $\leq 75$（m） | $\leq 90$（m） | $\leq 105$（m） | $\leq 120$（m） | $\leq 135$（m） | $\leq 150$（m） | $\leq 165$（m） | $\leq 180$（m） |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| बं่ | Fair | 0.08 | 3.52 | 14.5 | 24.9 | 46.52 | 69.07 | 74.6 | 83.65 | 96.73 | 99.33 | 100.0 | 100.0 |
| 唇 - | Fewer | 0.08 | 3.19 | 13.41 | 23.47 | 45.26 | 70.08 | 77.03 | 85.41 | 96.31 | 98.83 | 99.16 | 100.0 |
| $\bigcirc$ | MinMax | 0.17 | 3.35 | 14.5 | 24.73 | 44.01 | 67.31 | 73.93 | 85.16 | 97.82 | 100.0 | 100.0 | 100.0 |
| テं่ | Fair | 0.0 | 3.52 | 14.08 | 23.89 | 45.68 | 70.08 | 75.27 | 83.49 | 95.56 | 99.83 | 100.0 | 100.0 |
| \％ | Fewer | 0.17 | 3.44 | 12.24 | 22.05 | 45.6 | 70.24 | 77.54 | 86.17 | 96.14 | 99.16 | 99.16 | 100.0 |
| － | MinMax | 0.17 | 3.35 | 12.99 | 22.72 | 44.17 | 68.82 | 75.94 | 86.17 | 96.98 | 100.0 | 100.0 | 100.0 |
| －் | Fair | 0.08 | 3.35 | 13.33 | 22.97 | 44.01 | 68.99 | 76.03 | 85.25 | 96.23 | 99.83 | 100.0 | 100.0 |
| 중앙 | Fewer | 0.0 | 3.52 | 13.58 | 23.22 | 44.09 | 69.07 | 76.36 | 85.83 | 96.81 | 99.08 | 99.16 | 100.0 |
| ค | MinMax | 0.17 | 3.34 | 13.53 | 22.97 | 43.53 | 67.92 | 75.36 | 85.55 | 96.99 | 100.0 | 100.0 | 100.0 |
| F］ | Fair | 0.08 | 3.44 | 13.91 | 23.55 | 45.26 | 68.32 | 75.27 | 83.82 | 95.39 | 99.83 | 100.0 | 100.0 |
| ส్व̃ ค | Fewer | 0.08 | 3.52 | 13.83 | 23.3 | 45.1 | 69.57 | 77.12 | 85.92 | 94.3 | 98.32 | 100.0 | 100.0 |
| ${ }_{\sim}^{0}$ | MinMax | 0.17 | 3.27 | 14.08 | 23.47 | 43.42 | 67.56 | 75.1 | 85.83 | 96.14 | 100.0 | 100.0 | 100.0 |
| F | Fair | 0.0 | 2.35 | 12.24 | 21.71 | 43.76 | 67.06 | 73.34 | 77.79 | 88.6 | 98.32 | 99.83 | 99.83 |
| ส | Fewer | 0.0 | 2.51 | 12.41 | 18.86 | 39.9 | 59.93 | 71.42 | 82.15 | 94.97 | 98.66 | 99.66 | 99.83 |
| 0 | MinMax | 0.17 | 1.51 | 11.48 | 19.03 | 35.79 | 59.68 | 69.99 | 85.0 | 96.14 | 99.83 | 100.0 | 100.0 |
| － | Fair | 0.0 | 2.35 | 12.24 | 21.71 | 43.17 | 67.73 | 74.35 | 77.37 | 88.35 | 97.99 | 99.5 | 99.83 |
| 중가 | Fewer | 0.17 | 2.51 | 14.5 | 21.79 | 42.75 | 65.8 | 76.11 | 79.88 | 87.34 | 95.14 | 95.39 | 99.33 |
| － | MinMax | 0.17 | 1.51 | 12.91 | 20.45 | 36.71 | 58.42 | 68.73 | 82.82 | 96.14 | 99.83 | 100.0 | 100.0 |
| － | Fair | 0.0 | 2.35 | 13.33 | 22.63 | 43.67 | 65.13 | 71.25 | 76.61 | 88.43 | 95.98 | 97.65 | 100.0 |
| 중웅 | Fewer | 0.17 | 2.51 | 12.49 | 19.78 | 43.67 | 65.13 | 74.94 | 82.31 | 89.44 | 92.46 | 94.72 | 98.91 |
| － | MinMax | 0.17 | 1.51 | 12.91 | 20.45 | 35.46 | 56.83 | 67.14 | 82.48 | 95.89 | 99.58 | 100.0 | 100.0 |

Table 5 Case 2 for Elder people and Kids．Percentage of people assigned to paths up to a given length，for different penalties．

|  | Function | $\leq 25$（m） | $\leq 40$（m） | $\leq 55$（m） | $\leq 70$（m） | $\leq 75$（m） | $\leq 90$（m） | $\leq 105$（m） | $\leq 120$（m） | $\leq 135(\mathrm{~m})$ | $\leq 150$（m） | $\leq 165$（m） | $\leq 180$（m） |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ส่ं | Fair | 0.0 | 1.69 | 8.68 | 14.42 | 30.78 | 47.81 | 55.65 | 67.71 | 77.32 | 84.57 | 91.23 | 99.41 |
| 局 - | Fewer | 0.08 | 1.85 | 8.77 | 14.59 | 30.78 | 47.81 | 55.65 | 67.71 | 77.32 | 84.57 | 91.23 | 99.41 |
| － | MinMax | 0.0 | 1.52 | 8.68 | 14.42 | 30.78 | 47.81 | 55.65 | 67.71 | 77.32 | 84.57 | 91.23 | 99.41 |
| F］ | Fair | 0.08 | 1.35 | 7.67 | 13.24 | 26.22 | 42.66 | 54.3 | 66.27 | 76.05 | 84.06 | 86.17 | 98.9 |
| $\underset{\sim}{10}$ | Fewer | 0.08 | 1.69 | 8.68 | 14.0 | 27.07 | 41.23 | 51.77 | 64.17 | 76.9 | 83.73 | 88.45 | 99.41 |
| $\bigcirc$ | MinMax | 0.0 | 1.52 | 7.5 | 13.07 | 27.32 | 41.65 | 52.28 | 65.35 | 76.22 | 84.06 | 88.45 | 99.41 |
| － | Fair | 0.0 | 1.43 | 5.82 | 11.64 | 25.8 | 39.63 | 51.52 | 66.27 | 75.55 | 83.81 | 86.59 | 98.23 |
| 줄 | Fewer | 0.0 | 1.1 | 4.89 | 9.95 | 24.96 | 41.32 | 52.95 | 64.0 | 76.56 | 84.06 | 88.7 | 98.48 |
| $\bigcirc$ | MinMax | 0.0 | 1.52 | 5.82 | 11.47 | 27.32 | 39.38 | 49.49 | 65.77 | 76.81 | 83.56 | 89.29 | 98.82 |
| － | Fair | 0.0 | 1.52 | 6.24 | 10.46 | 24.54 | 40.22 | 51.77 | 63.41 | 72.93 | 81.53 | 86.68 | 95.87 |
| \％ | Fewer | 0.0 | 1.26 | 6.07 | 10.12 | 24.2 | 40.39 | 51.85 | 64.0 | 72.93 | 81.62 | 86.76 | 96.12 |
| － | MinMax | 0.0 | 0.42 | 5.56 | 11.2 | 23.0 | 39.09 | 53.41 | 66.55 | 73.63 | 80.88 | 86.35 | 95.62 |
| － | Fair | 0.0 | 1.68 | 6.66 | 11.04 | 24.6 | 36.48 | 48.1 | 61.16 | 68.41 | 76.33 | 80.2 | 91.49 |
| 梟 | Fewer | 0.0 | 0.84 | 7.76 | 13.24 | 24.03 | 36.68 | 48.57 | 57.76 | 65.94 | 75.55 | 80.61 | 92.75 |
| － | MinMax | 0.0 | 1.01 | 7.34 | 12.56 | 24.79 | 40.56 | 46.96 | 58.85 | 67.71 | 73.52 | 77.66 | 93.0 |
| － | Fair | 0.0 | 1.52 | 5.48 | 9.61 | 24.28 | 39.71 | 50.42 | 56.58 | 66.02 | 75.21 | 76.56 | 86.0 |
| 중아 | Fewer | 0.0 | 1.01 | 7.67 | 12.98 | 23.27 | 39.04 | 45.95 | 53.46 | 64.0 | 74.79 | 78.67 | 88.7 |
| ค | MinMax | 0.0 | 0.93 | 6.32 | 11.13 | 24.7 | 41.57 | 50.67 | 57.84 | 63.49 | 70.24 | 76.98 | 87.52 |
| － | Fair | 0.0 | 0.84 | 3.29 | 6.5 | 20.0 | 35.19 | 47.68 | 50.89 | 57.13 | 68.52 | 75.78 | 79.41 |
| \％ | Fewer | 0.0 | 1.01 | 5.56 | 9.19 | 18.13 | 30.19 | 41.48 | 47.47 | 55.48 | 65.43 | 68.72 | 81.45 |
| ค | MinMax | 0.0 | 0.0 | 4.21 | 6.15 | 13.06 | 27.3 | 42.29 | 49.54 | 59.22 | 63.77 | 68.83 | 77.76 |


|  | Function | $\leq 25$ (m) | $\leq 40$ (m) | $\leq 55$ (m) | $\leq 70$ (m) | $\leq 75$ (m) | $\leq 90$ (m) | $\leq 105$ (m) | $\leq 120$ (m) | $\leq 135$ (m) | $\leq 150$ (m) | $\leq 165$ (m) | $\leq 180$ (m) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| बं่ | Fair | 1.6 | 12.0 | 81.6 | 83.2 | 89.6 | 94.4 | 96.0 | 96.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| $\stackrel{7}{0}$ | Fewer | 1.6 | 12.0 | 81.6 | 83.2 | 89.6 | 94.4 | 96.0 | 96.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| $\bigcirc$ | MinMax | 1.6 | 12.0 | 81.6 | 83.2 | 89.6 | 94.4 | 96.0 | 96.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| - | Fair | 1.6 | 11.2 | 76.8 | 79.2 | 86.4 | 94.4 | 95.2 | 95.2 | 98.4 | 99.2 | 99.2 | 100.0 |
| \% | Fewer | 1.6 | 11.2 | 76.8 | 79.2 | 85.6 | 92.8 | 94.4 | 95.2 | 100.0 | 100.0 | 100.0 | 100.0 |
| - | MinMax | 1.6 | 11.2 | 75.6 | 78.0 | 86.4 | 94.0 | 95.6 | 95.6 | 100.0 | 100.0 | 100.0 | 100.0 |
| - | Fair | 1.6 | 11.2 | 77.2 | 79.6 | 85.76 | 93.2 | 94.0 | 94.0 | 99.2 | 100.0 | 100.0 | 100.0 |
| \% | Fewer | 1.6 | 11.2 | 78.4 | 80.8 | 84.8 | 92.0 | 92.8 | 94.4 | 100.0 | 100.0 | 100.0 | 100.0 |
| - | MinMax | 1.6 | 11.2 | 78.4 | 80.16 | 86.4 | 91.2 | 92.8 | 92.8 | 100.0 | 100.0 | 100.0 | 100.0 |
| F | Fair | 1.6 | 11.2 | 76.64 | 78.4 | 85.84 | 93.76 | 94.56 | 94.56 | 99.2 | 100.0 | 100.0 | 100.0 |
| ส్రై | Fewer | 1.6 | 11.2 | 76.0 | 78.4 | 85.2 | 94.4 | 95.2 | 95.2 | 100.0 | 100.0 | 100.0 | 100.0 |
| ${ }_{\sim}^{0}$ | MinMax | 1.6 | 11.2 | 77.84 | 80.24 | 85.2 | 91.76 | 93.36 | 93.36 | 100.0 | 100.0 | 100.0 | 100.0 |
| F | Fair | 1.6 | 11.2 | 77.2 | 79.6 | 85.76 | 93.2 | 94.0 | 94.0 | 99.2 | 100.0 | 100.0 | 100.0 |
| ส | Fewer | 1.6 | 11.2 | 78.4 | 80.8 | 84.8 | 92.0 | 92.8 | 94.4 | 100.0 | 100.0 | 100.0 | 100.0 |
| 0 | MinMax | 1.6 | 11.2 | 78.4 | 80.16 | 86.4 | 91.2 | 92.8 | 92.8 | 100.0 | 100.0 | 100.0 | 100.0 |
| - | Fair | 1.6 | 11.2 | 77.2 | 79.6 | 85.76 | 93.2 | 94.0 | 94.0 | 99.2 | 100.0 | 100.0 | 100.0 |
| ${ }_{\square}^{\text {g }}$ 아 | Fewer | 1.6 | 11.2 | 78.4 | 80.8 | 84.8 | 92.0 | 92.8 | 94.4 | 100.0 | 100.0 | 100.0 | 100.0 |
| - | MinMax | 1.6 | 11.2 | 78.4 | 80.16 | 86.4 | 91.2 | 92.8 | 92.8 | 100.0 | 100.0 | 100.0 | 100.0 |
| - | Fair | 1.6 | 11.2 | 78.4 | 80.8 | 86.4 | 92.8 | 92.8 | 92.8 | 99.2 | 99.68 | 99.68 | 99.68 |
| 중웅 | Fewer | 1.6 | 11.2 | 76.0 | 78.4 | 84.4 | 93.6 | 94.88 | 95.68 | 99.68 | 99.68 | 100.0 | 100.0 |
| 0 | MinMax | 1.6 | 11.2 | 78.4 | 80.16 | 86.4 | 91.2 | 92.48 | 92.48 | 99.68 | 99.68 | 100.0 | 100.0 |

Table $7 \quad$ Case 3 for Elder People and Kids. Percentage of people assigned to paths up to a given length, for different penalties.

|  | Function | $\leq 25$ (m) | $\leq 40$ (m) | $\leq 55$ (m) | $\leq 70$ (m) | $\leq 75$ (m) | $\leq 90$ (m) | $\leq 105$ (m) | $\leq 120$ (m) | $\leq 135$ (m) | $\leq 150$ (m) | $\leq 165$ (m) | $\leq 180$ (m) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | Fair | 0.8 | 7.6 | 38.4 | 40.8 | 75.2 | 81.2 | 88.0 | 89.6 | 95.2 | 96.0 | 96.0 | 100.0 |
| $\stackrel{\text { ¢ }}{\sim}$ | Fewer | 1.6 | 4.8 | 38.4 | 40.8 | 78.0 | 84.0 | 88.0 | 89.6 | 95.2 | 96.0 | 96.0 | 100.0 |
| $\bigcirc$ | MinMax | 1.6 | 4.8 | 38.4 | 40.8 | 78.0 | 84.0 | 88.0 | 89.6 | 95.2 | 96.0 | 96.0 | 100.0 |
| นี่ | Fair | 1.6 | 11.2 | 44.8 | 47.2 | 78.0 | 84.0 | 88.0 | 89.6 | 95.2 | 96.0 | 96.0 | 100.0 |
| ¢ | Fewer | 1.6 | 11.2 | 44.8 | 47.2 | 78.0 | 84.0 | 88.0 | 89.6 | 95.2 | 96.0 | 96.0 | 100.0 |
| -1 | MinMax | 1.6 | 11.2 | 44.8 | 47.2 | 78.0 | 84.0 | 88.0 | 89.6 | 95.2 | 96.0 | 96.0 | 100.0 |
| - | Fair | 1.6 | 11.2 | 31.68 | 43.84 | 71.44 | 79.68 | 88.0 | 89.6 | 95.2 | 96.0 | 96.0 | 100.0 |
| $\underset{\sim}{1}$ | Fewer | 1.6 | 11.2 | 31.68 | 44.64 | 72.24 | 78.24 | 88.0 | 89.6 | 94.4 | 96.0 | 96.0 | 100.0 |
| - | MinMax | 1.6 | 11.2 | 31.68 | 44.64 | 71.44 | 83.04 | 88.0 | 89.6 | 95.2 | 96.0 | 96.0 | 100.0 |
| 家 | Fair | 1.6 | 11.2 | 31.2 | 43.04 | 70.64 | 78.88 | 88.0 | 89.6 | 95.2 | 96.0 | 96.0 | 100.0 |
| ${ }_{\text {¢ }}^{\text {¢ }}$ ¢ | Fewer | 1.6 | 11.2 | 31.2 | 44.64 | 71.44 | 77.44 | 88.0 | 89.6 | 94.4 | 96.0 | 96.0 | 100.0 |
| - | MinMax | 1.6 | 11.2 | 31.2 | 43.84 | 70.64 | 82.24 | 88.0 | 89.6 | 95.2 | 96.0 | 96.0 | 100.0 |
| - | Fair | 1.6 | 11.2 | 31.2 | 43.52 | 70.32 | 78.24 | 88.0 | 89.6 | 94.32 | 94.32 | 94.32 | 98.08 |
| ${ }_{\sim}^{6}$ | Fewer | 1.6 | 11.2 | 31.2 | 43.84 | 69.92 | 81.44 | 88.0 | 88.8 | 94.32 | 94.32 | 94.32 | 98.96 |
| $\bigcirc$ | MinMax | 1.6 | 11.2 | 31.2 | 43.52 | 69.52 | 81.92 | 88.0 | 88.8 | 92.48 | 94.08 | 94.08 | 99.2 |
|  | Fair | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\stackrel{1}{\sim}$ | Fewer | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ค | MinMax | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | Fair | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | 0 | 0 | 0 |
| 梟 | Fewer | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | MinMax | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table $8 \quad$ Case 3 for Young People. Percentage of people assigned to paths up to a given length, for different penalties.

## NORGES HANDELSHØYSKOLE

Norwegian School of Economics

Helleveien 30
NO－5045 Bergen
Norway
T＋4755959000
E nhh．postmottak＠nhh．no
W www．nhh．no

