Essays on Dynamic Games: Impacts of Different Contracts and Policy Constraints in a Distributional Robust Approach

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Contents

List of Figures

List of Tables

Abstract

This research encompasses three articles that specifically tackle decentralized supply channels and propose comprehensive solution algorithms for multi-periodic bilevel equilibrium problems. The supply channel consists of two members, an upstream member (manufacturer) and a downstream member (retailer), who assume the roles of leader and follower, respectively, in a Stackelberg game. The primary objective of the channel is to effectively manage dynamic demand, which is dependent on price history, within a multi-period time frame. Due to the price history effect on the uncertain demand, the problem turns out to be highly nested. The first article presents a channel facing dynamic and price-dependent demand, where the demand information is incomplete, and the only information provided is the mean and the standard deviation of the demand. To address this challenge, a distributional-robust (DR) approach is proposed, which provides a lower bound on the channel's expected profit for the problem with known distribution. The retailer bears the uncertainty of the demand, while the manufacturer perceives it through the retailer's order quantity. In the second article, we extend this framework by considering a single contract that covers all periods, enabling simultaneous optimization of decisions for the entire periods. The leader's expected payoff of this type of contract, logically, is not lower than the subgame perfect result. For the follower on the other hand, we did not observe any counterexample to demonstrate that he may be worse off by using a single contract. The algorithm optimally addresses concerns related to environmental corrective actions. It incorporates pollution capacity constraints and tax, where the algorithm is constrained to produce below a predetermined cap in the first policy, and in the second policy the tax, as a decision variable, is obtained for each period. The third article, as an extension of the first two, introduces a buyback price into the channel to share risks between the players. The proposed algorithm addresses a problem within a cap-and-trade system. In addition to proposing equilibrium-finding algorithms for various problems and introducing new methods to enhance value, the provided numerical illustration for multi-period scenarios offers valuable insights for decision-makers.

Keywords: Multi-Periodic Stackelberg Game, Distributional-Robust Approach, Price-History Dependent Demand, Single Contract, Buyback Contract, Capacity Constraint, Pollution tax, Cap-and-Trade system.

Introduction

Recent technological advancements have accelerated the pace of product development, resulting in the rapid obsolescence of old products when new ones are introduced to the market. These short-life products, which cannot be stored for future sales, contribute to increased demand uncertainty (Khan & Sarkar 2021). In such cases, the order volume is determined with the objective of minimizing the costs associated with overordering or underordering.

The demand is typically characterized by a distribution, although it is not always feasible or cost-effective to ascertain the exact distribution. In 1957, Scarf et al. introduced a novel ordering rule for situations where only the mean and variance of the demand are known, without any information about the demand distribution. His maxmin distributional-robust (DR) approach was later simplified and extended by Moon and Gallego. The profit obtained through the DR approach is a tight lower bound for models with known distribution (Moon & Gallego 1994).

The scope of this essay encompasses a multi-period Stackelberg Game, where the manufacturer leads the market, and the retailer follows. Both the wholesale and retail prices are decision variables for the manufacturer and the retailer, respectively, in addition to the order quantity which is determined based on these prices after optimization. The demand is dynamic and influenced by price history, meaning that not only the current price affects demand, but also past prices can shape demand pattern. This interdependency results in a nested structure in an additive multiplicative form of demand and consequently the channel (Azad Gholami et al. 2019). Uncertainty in demand can lead to overordering or underordering. Overordering of perishable products results in unsold items that cannot be carried over to subsequent periods. On the other hand, underordering may harm the channel's reputation due to shortages and dissatisfied customers. This uncertainty can be further amplified by historical price data, where each demand is a function of current and previous prices. This feature allows for strategic pricing. In this essay, all parameters and variables can be time dependent. The three papers included in this essay address these challenges as follows.

The first paper introduces the DR approach and extends it to address the multi-period problem where demand is influenced by all previous prices. The wholesale price is determined by the leader (manufacturer), while the retail price and order quantity are set by the follower (retailer) in a DR structure. The solution procedure employs a bilevel optimization algorithm, starting from the last period and moving backward to the first. This approach allows for obtaining optimized solutions for each period, effectively simulating periodic contracts sequentially. The proposed algorithm showcases dynamic problem settings and examines the impact of parameters on the channel's value. This paper was presented at the 19th International Symposium on Dynamic Games and Applications.

The second paper focuses on environmental corrective actions within the supply channel. Given that human-generated pollution significantly contributes to climate change, international treaties such as the Kyoto Protocol in Japan (1997) and the Paris Agreement in New York (2017) have emphasized the urgency of addressing this issue (Bai et al. 2022). Governments and organizations have committed to adopting regulatory policies to reduce pollution by at least 50% by 2050 (Liu et al. 2015). Additionally, growing consumer awareness about the environmental impact of products necessitates the revision of supply channel models. This paper explores two widely used pollution reduction policies: capacity constraints and tax. Capacity constraints involve imposing a mandatory cap (command-and-control policy) on the total pollution level in each period. On the other hand, the tax policy represents the cost incurred by the supply channel for each unit of pollution or production (Kannan et al. 2022). To address these issues, a single-contract framework is introduced, enabling the players to observe the consequences of their decisions and make simultaneous modifications. Comparing algorithms with long and short memories demonstrates the algorithm's effectiveness in addressing models with dynamic demands and various forms of price history dependency. A single-contract framework introduced in this paper is compared with the subgame perfect case. The single contract allows for simultaneous decision-making wherein all periods' decisions and their consequences can be tracked and changed if necessary. Consequently, this may lead to different strategic decisions which may increase the manufacturer's total payoff. This paper was presented at the 13th ISDG Workshop.

Building on the first two papers, the third paper analyzes a multi-period Stackelberg game with a DR approach, dynamic and price-history dependent demand under a single contract with a cap-and-trade system. Since massive carbon emissions have caused serious global environmental damage, governments have promoted the development of low-carbon policies to maintain sustainability and reduce the effects of this problem. The cap-and-trade policy is among the widely adopted systems that manufacturers implement to curb carbon dioxide emissions while optimizing their profit. This alters the supply channels' optimal solution. With a cap-and-trade policy, the manufacturer is allocated a production capacity allowance. It means they have a tradable quota. This quota can be sold if the optimal order quantity is lower than the production allowance and has to be bought if the channel's optimal order quantity is higher than the determined quota. According to the prices of the selling or buying allowance, the manufacturer may gain or lose profit.

It is the downstream member who faces the uncertain and dynamic demand. To share this risk, the upstream member may agree on a buyback price for the unsold commodity. Therefore, a buyback contract is considered in the manufacturer's function. Hence, in addition to the wholesale price, a non-negative buyback price is declared by the manufacturer for each period. The buyback agreement allows the retailer to return the unsold items to the manufacturer (or the manufacturer pays a buyback price to the retailer and the retailer salvages/discards the unsold items). This paper was presented at the 20th EUROpt Workshop.

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Chapter 1

A Subgame Perfect Approach to a Multi-Period Stackelberg Game with Dynamic, Price-Dependent, Distributional-Robust Demand

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Abstract
This paper investigates a multi-periodic channel optimization facing uncertain, price-dependent, and dynamic demand. The picture of the market uncertainty is incomplete, and only the price and time-dependent mean and standard deviation are known and may depend on the price history. The actual demand distribution itself is unknown as is typically the case in real-world problems. An algorithm finding the optimized decentralized channel equilibrium is developed when the downstream member optimizes her expected profit stream by a distributional-robust approach, and the upstream member (leader) considers it as the follower's reaction function. The algorithm allows for strategic decisions whereby the current demand is scaled by the previous price setting.

JEL classification: C61, C62, C63, C72, C73, D81.

Keywords Multi-Periodic Stackelberg Game, Subgame Perfect Distributional-Robust Approach, Supply Chain Management, Dynamic Price History-Dependent Demand.

1.1 Introduction

A supply channel is often accompanied by a time-varying and uncertain demand. The upstream member (manufacturer) and the downstream member (retailer) are exposed to this market uncertainty in different manners. The retailer directly faces the uncertainty, while the manufacturer senses it through the order quantity made by the retailer. The uncertainty in demand often leads the chain to a lost sale or unsold quantity which can potentially be salvaged (Khan & Sarkar 2021). Therefore, anticipating the trend of the future market and how to satisfy the stochastic demand creates a challenge for the supply channel players.

The simplest case occurs when the demand is structured from a distribution of price-independent quantities. However, in reality, demand varies as time goes by. As a result, strategies like offering cheaper commodities during specific periods to stimulate future market demand can be implemented. On the other hand, customers, being aware of the price trends, may adjust their purchasing plans based on historical product prices over time. In our study, market demand can be adjusted to increase (decrease) with the influence of price history (path). Strategic pricing, hence, occurs where the demand contains current and historical prices. Therefore, the current price remains important, however, the prior prices can affect the demand critically. Leveraging this characteristic strategically, we can increase future demand by lowering prices in the present. This is a strategy for market penetration. The main challenge in a multi-period discrete-time model with dynamic price-dependent demand lies in the interdependence among all price values. This nestedness in the demand model comes into play through the notion of scaling functions that capture the effects of prior prices.

To make it computationally convenient, one might consider a deterministic demand as a function of price, however, this deviates from real-world dynamics. In our paper, demand is modeled by a random variable in which the mean and variance of the demand, scaled by previous prices, are time and price-dependent. For instance, to capture a greater share of the market, one may offer products at lower prices (even below their cost) to achieve a higher potential market in the future. Although this approach may result in negative initial revenue, it can be a viable strategy if the demand increases sufficiently in the future.

A channel is normally not fully equipped with comprehensive demand distribution information, either the information is unavailable or prohibitively costly to obtain. In such cases, a distributional-robust (DR) approach becomes necessary. Regardless of the demand distribution, a DR approach allows for the identification of the supply channel solution while establishing a lower bound for the retailer gain under any distribution but with the same mean and standard deviation. In our study, the DR approach is extended to address a multi-periodic setting, where each period's gain may depend explicitly on the price history.

The present study focuses on the game between upstream and downstream parties within a decentralized channel. The manufacturer is the leader, and the retailer follows him. The demand of this channel is dynamic and price-dependent for a perishable commodity where the demand distribution is not fully known. The problem is addressed in a multi-periodic revenue management framework. In each period (k) the leader initiates by determining the wholesale price (w_k) and the follower immediately follows up by deciding the order quantity (q_k) and retail price (r_k). Both parties aim to optimize their holistic profits over all periods. Due to the perishable nature, the commodity cannot be stored for later use. Thus, any unsold item must be salvaged (discarded) at a reduced price (cost) s_k.

The retailer encounters the market risk through stochastic demand (D_k) and may endure not meeting the market by missing an opportunity to sell $(D_k - q_k)^+$ more, or salvaging/discarding $(q_k - D_k)^+$ leftover inventory. In each period the uncertainty is unveiled after the decisions on w_k,r_k,and q_k are made. The primary contribution of this paper lies in addressing such a multi-periodic supply channel where the means and standard deviations are the only information about the demand encompassing the effect of antecedent price setting. This novel approach incorporates the effects on future market demand by previous decisions (price-setting). This is implemented by setting the means and standard deviations as functions of all previous retail prices, such that previous prices scale the demand, but only the current retail price determines the current coefficient of variation (CV). The potential dependence on previous prices authorizes strategic pricing, e.g., lowering prices to enhance demand by attracting more customers. Over time, prices can be adjusted if the customer base increases sufficiently.

The mathematical model presented in section 1.3.1 accommodates a stochastic price history-dependent demand. Regarding the DR approach, supply channel objective functions are appropriately modified. Later in section 1.3.2, the DR model is extended to encompass multiple periods. The model structure adheres to a general form and may undergo alterations from one period to another. This design enables the model's underlying functions to systematically adapt and evolve, reflecting the dynamics of the channel.

The price-history dependency on the demand is elaborated in section 1.3.3. It plays a crucial role in the nestedness of the model. The presence of this 'price memory' is integral, as, without it, the problem devolves into a repeated game, i.e., each period forms its own decuple single-period game. Hence, no strategic pricing can happen when the periods operate independently from each other.

In section 1.4, we delve into the resolution and analysis of economic decision-making problems. This section focuses on examining the Stackelberg game between a manufacturer and a retailer, where the manufacturer leads the channel. The algorithm can solve a time-dependent parameter set and a fixed set. We implement the model to address two specific scenarios: one involving a problem with known distribution and another under the DR model. By applying the developed algorithm, we aim to provide insights and solutions for the examples.

1.2 Literature Review

In 1957, Scarf et al. proposed a method to address an inventory problem characterized by limited demand information where the only available knowledge of demand was the mean and standard deviation and the demand distribution was uncharted (Scarf et al. 1957). Building upon Scarf's work, Gallego and Moon revised and extended Scarf's method to tackle a newsvendor problem with three specific conditions: the problem incorporated the possibility of a second purchasing opportunity arising after demand was revealed, a multi-item case, and a random yield case (Gallego & Moon 1993). In another study, Gallego discussed a maxmin distributional-robust approach to acquire order and inventory levels minimizing the cost of holding/shortage in a newsvendor problem (Gallego 1992). Gallego collaborated with Moon to analyze both continuous and periodic inventory models, incorporating backorders and lost sales. They employed a price-independent demand and a maxmin DR approach to optimize the order volume and retail price (Moon & Gallego 1994).

Godfrey and Powell optimized the newsvendor DR problem involving repeated inventory management using the concave adaptive value estimation (CAVE) algorithm (Godfrey & Powell 2001). Similarly, Mostard et al. studied the DR newsvendor problem considering the possibility of reselling returned items before the end of the season if not damaged and salvaging any remaining unsold items. They also accounted for the potential harm caused by shortages, incorporating a shortage cost from the retailer's perspective (Mostard et al. 2005). Additionally, Pal et al. inset a DR newsvendor problem focusing on inventory management with a non-linear holding cost, aiming to reduce the inventory level (Pal et al. 2015). Sarkar et al. explored the DR Stackelberg newsvendor problem under a make-to-order and consignment policy, where both parties shared a portion of the holding cost (Sarkar et al. 2018). Khan and Sarkar presented a DR newsvendor problem, incorporating back-ordering and stochastic and price-dependent demand (Khan & Sarkar 2021). Their proposed approach involved the retailer paying an additional price per product to transfer risk associated with unsold items to the manufacturer. Finally, Govindarajan et al. addressed a DR multi-location newsvendor problem to optimize the inventory level minimizing cost (Govindarajan et al. 2021).

Our paper contributes to this research area by optimizing the multi-period supply chain Stackelberg game in which demand is time and price history-dependent, although the distribution of demand is unknown. The only available information is the mean (μ) and the standard deviation (σ) of the demand as functions of time and prices when the price history impacts the future demand, i.e., price history dependent demand. Practically speaking, figuring out the stochastic drivers in a time-dependent demand distribution may not be available or economically viable. Hence, the distributional-robust model is a maxmin-formulation to generate a weak lower bound on optimal expected value. Section 1.4 works out an illustration comparing distributional-robust results with the alternative fully informed cases exemplified by uniform distributions. As far as we are aware, the DR model has solved a maximum 2-period independent problem with linear demand. The contribution of this paper, hence, covers a multi-period DR game where the demand is dynamic and price-path dependent. The solution scheme allows demand engineering to occur through the optimal pricing strategy when the demand distribution is unknown.

Table 1.1: A literature review on distributional-robust problem

Authors	**Perishable**	**Periods**	**Demand**[a]	**PHD**[b]
Scarf et al., 1957	✓	1	S	×
Gallego, 1992	×	1	S	×
Gallego & Moon, 1993	✓	1	S	×
Moon & Gallego, 1994	×	1	S	×
Godfrey & Powell, 2001	✓	2	S	×
Mostard, et al., 2005	✓	1	S	×
Pal, et al., 2015	✓	1	S	×
Sarkar, et al., 2018	✓	1	S	×
Khan & Sarkar, 2021	✓	1	S	×
Govindarajan, 2021	✓	1	S	×
This paper	✓	**Any**	**TPD**	✓

[a] **S** represents static demand, and **TPD** stands for the demand that is both time and price-dependent.

[b] The abbreviation **PHD** corresponds to Price history dependent.

1.3 Model Framework

This work considers the normal flow of retail; The product is produced (supplied) by a manufacturer (wholesaler) and sold by a retailer to the customers. Both parties are risk-neutral and want to maximize their expected discounted total profits. The main part of the solution effort is the computation of equilibrium prices leading to order quantities that maximize expected profits. It leads to a subgame-perfect optimization that can be decomposed into a sequence of connected decisions. A trivial subclass of our approach covers the multi-period supply channel games with stochastic demands that are only dependent on the current price and time. The non-negative property of demand excludes all distributions with compact support not limited from below, e.g. the normal distribution. This is particularly important when the volatility is dependent on decision variables (e.g. the price). The discrete-time structure in our model represents a timespan that is divided into intervals called periods. Since the commodity is perishable, it is either sold in the current period or salvaged/discarded (the unsold items).

The DR supply channel problem has received considerable attention, according to the existing literature, although it has mostly been limited to a maximum of two periods and has primarily focused on the newsvendor structure. In this paper, the proposed novel DR algorithm efficiently determines the optimal equilibrium prices and quantities in the subgame-perfect framework. Our approach addresses the problem within a multi-periodic framework incorporating explicit time-dependent model parameters (non-autonomous). To provide a comprehensive understanding, we begin by explaining the single-period problem in section 1.3.1, followed by the extension to a multi-period context in sections 1.3.2 and 1.3.3 To facilitate comprehension through this section, we introduce the following notation list, where n denotes the number of periods.

Notation

$\beta = \{\beta_1, \ldots, \beta_n\}$	Discount factor over individual periods[1]
$c^m = \{c^m_1, \ldots, c^m_n\}$	Manufacturer cost
$s = \{s_1, \ldots, s_n\}$	Salvage price/discarding cost
$w = \{w_1, \ldots, w_n\}$	Wholesale price (decisions)
$r = \{r_1, \ldots, r_n\}$	Retail price (decisions)
$q = \{q_1, \ldots, q_n\}$	Order quantity (decisions)
$k \in \{1, \ldots, n\}$	Time or period
$D = \{D_1, \ldots, D_n\}$	Demand
$\mu = \{\mu_1, \ldots, \mu_n\}$	Mean of demand
$\sigma = \{\sigma_1, \ldots, \sigma_n\}$	Standard deviation of demand
$\varepsilon = \{\varepsilon_1, \ldots, \varepsilon_n\}$	Stochastic and independent drivers with mean 0 and variance 1
$\pi^m = \{\pi^m_1, \ldots, \pi^m_n\}$	Manufacturer profit (running value)
$\pi^r = \{\pi^r_1, \ldots, \pi^r_n\}$	Retailer profit (running value)
JR^x	The total expected value of player x, in the DR model
JD^x	The total expected value of player x, in the model with known distribution

[1]The discount factors related to the start ($k = 1$) are $\alpha_k = \beta_1 \cdot \beta_2 \cdot \ldots \cdot \beta_k$ and $0 < \beta$. Individual periods may be of different lengths.

1.3.1 Single-Period Distributional-Robust Game

In this supply channel under the Stackelberg game, the channel leader, the manufacturer, acts first and offers the price w that maximizes his profit $E[\pi^m(w,q)]$. Then the follower, the retailer, decides on the optimal volume q and optimal retail price r that maximizes his expected profit $E[\pi^r(r,w,q)]$. It addresses a single-order opportunity, and the market cannot be replenished; Consequently, the unmet demand is considered backlogged and is not involved in the algorithm. The unsold items, on the other hand, can be salvaged/discarded at a reduced price/cost s. We have dropped the time index since this section explains a single-period problem. The general form of demand is

$$D = \mu(r) + \sigma(r)\varepsilon \geq 0, \tag{1.1}$$

where μ and σ are deterministic given functions of retail price r, and ε is a given stochastic variable with a mean and standard deviation of 0 and 1 respectively. Noticing the stochastic demand, the retailer orders q and sells $\min(D,q)$ at price r to maximize his profit

$$\pi^r(r,w,q) = r\min(D,q) + s(q-D)^+ - wq. \tag{1.2}$$

The leftover inventory $(q-D)^+$ is salvaged at $s(>0)$ or discarded at $s(<0)$. To optimize the problem, the expected value is illustrated as [2]

$$\begin{aligned} E[\pi^r(r,w,q)] &= (r-s)E[\min(D,q)] - (w-s)q \\ &= (r-s)E(D-[D-q]^+) - (w-s)q \\ &= (r-s)\mu - (w-s)q - (r-s)E[D-q]^+. \end{aligned} \tag{1.3}$$

If the demand is accompanied by a known distribution, the value of $E[D-q]^+$ can be simply calculated (Gholami et al. 2021). Otherwise, when the demand distribution is unknown but the mean and standard deviation are provided, a distributional-robust approach offers an optimal way to solve the problem. In general, the following hold

I. $(D-q)^+ = \frac{1}{2}\{|D-q| + (D-q)\}$

II. $E(D-q)^+ = \frac{1}{2}\{E[|D-q|] + E(D) - E(q)\}$

III. $E[|D-q|] \leq \sqrt{E[(D-q)^2]} = \sqrt{(q-\mu)^2+\sigma^2}$ (Cauchy-Schwartz inequality)

A simple consequence of these relations is

[2] $\min(D,q) = D - (D-q)^+$ and $(q-D)^+ = (q-D) + (D-q)^+$

$$E[D-q]^+ \leq \frac{\sqrt{\sigma^2+(q-\mu)^2}+\mu-q}{2}. \tag{1.4}$$

This inequality gives a tight lower bound on expected retailer profit for any distribution with the same μ and σ. Hence,

$$E[\pi^r] \geq (r-s)\mu-(w-s)q-(r-s)\left(\frac{\sqrt{\sigma^2+(q-\mu)^2}+\mu-q}{2}\right) \equiv \Pi^r. \tag{1.5}$$

From now on we use the term 'profit' for this bound. The DR approach is defined by replacing $E[\pi^r]$ with Π^r. It has been demonstrated that equality holds in Eq. (1.5) for some special distributions (Gallegol & Moon, 2016) and trivially for a deterministic demand.

The μ and σ approach zero when prices turn to large values[3]. The manufacturer optimizes his problem to find the optimal price w, manipulating the retailer to order q in the Stackelberg game, such that this pair (w, q) maximizes his profit

$$\pi^m(w,q) = (w-c^m)q = E[\pi^m(w,q)]. \tag{1.6}$$

To have consistent notation, $E[\pi^m] = \pi^m = \Pi^m$.

1.3.2 Multi-Period Distributional-Robust Game

In a multi-periodic channel, players endeavor to maximize their total discounted expected profit streams

$$\begin{gathered} J_k^x = \alpha_k \Pi_k^x + \alpha_{k+1}\Pi_{k+1}^x + \alpha_{k+2}\Pi_{k+2}^x + \ldots + \alpha_n \Pi_n^x \quad \text{for } x \in \{m, r\}, \\ \text{where } \alpha_k = \beta_1 \cdot \beta_2 \ldots \beta_k, \end{gathered} \tag{1.7}$$

and n represents the number of periods that may be of different duration, and β_k is the discount rate for period k. J_k^m and J_k^r are the present values of the streams for the manufacturer and the retailer, respectively, from period k and onward. The players optimize their J_k^x at each period (i.e., subgame perfect).

1.3.3 A Multi-period Distributional-robust Game with Price history-Dependent Demand

Demand is usually sensitive to price, and this may evolve over time. The current price and time are normally not the only factors impacting the current demand. Previous price

[3]Real demand is non-negative with compact support on a finite interval.

settings may scale the market by, e.g., boosting or reducing the upcoming demands. This impact is likely to be time-dependent. In this work, we assume that the price history affects the size of the demand, while the present price modifies the coefficient of variation ($CV_k = CV_k(r_k)$), i.e.,

$$\begin{aligned} D_k(\vec{\mathbf{r}}_k) &= \Phi_k(\vec{\mathbf{r}}_{k-1}) d_k(r_k), \quad k \in \{1, \dots, n\} \\ d_k(r_k) &= \hat{\mu}_k(r_k) + \hat{\sigma}_k(r_k)\varepsilon_k \\ \Phi_1 &\equiv 1, \end{aligned} \tag{1.8}$$

where $\vec{\mathbf{r}}_k = \{r_1, \dots, r_k\}$. The hat sign represents the scaled variable (without the price history effect). The Φ_k is a scaling function, addressing a cumulative relation of previous market price settings. As an example, if the relevance between periods' memories is multiplicative, the cumulative scaling at each period forms as

$$\Phi_k(\vec{\mathbf{r}}_{k-1}) = g_k(r_{k-1})\Phi_{k-1}(\vec{\mathbf{r}}_{k-2}) = \prod_{i=2}^{k} g_i(r_{i-1}), \tag{1.9}$$

where g_i carries the effect of the previous price r_{i-1}. Strategic pricing to boost future demand may occur optimally in some model specifications. The case $\Phi_k \equiv 1$ for all k implies that the demand in each period only depends on the current price, i.e., $D_k = d_k(r_k)$, and no strategic pricing can occur. Hence, Eq. (1.7) can be summarized as

$$J_k^x = \sum_{i=k}^{n} \alpha_i \Phi_i \hat{\Pi}_i^x, \quad x \in \{m, r\}, \tag{1.10}$$

where $\hat{\Pi}_k^r, \hat{\Pi}_k^m$ only depend on decision variables in period k, i.e.,

$$[\hat{\mu}_k(r_k), \hat{\sigma}_k(r_k), \hat{q}_k] = \frac{[\mu_k(\vec{\mathbf{r}}_k), \sigma_k(\vec{\mathbf{r}}_k), q_k]}{\Phi_k(\vec{\mathbf{r}}_{k-1})}, \tag{1.11}$$

The term $\alpha_k \Phi_k$ is known at the beginning of period k. Viewing the problem from an arbitrary period (k) and onward, Eq. (1.10) implies the scaled value, j_k^x.

$$\begin{aligned} j_k^x &= \hat{\Pi}_k^x + \beta_{k+1} g_{k+1} j_{k+1}^x, \quad \text{for } x \in \{m, r\} \\ \text{where } j_k^x &= \frac{J_k^x}{\alpha_k \cdot \Phi_k}, \text{ and } \hat{\Pi}_k^x = \frac{\Pi_k^x}{\Phi_k}. \end{aligned} \tag{1.12}$$

By starting at the last period (n), the sequence of leader-follower games defined by the payoffs $j_n^m, j_n^r, \dots, j_1^m, j_1^r$ is solved. Each of these games has objectives to be maximized in the form $j_k^x = \hat{\Pi}_k^x + \beta_{k+1} g_{k+1} j_{k+1}^x = \hat{\Pi}_k^x(r_k, w_k, \hat{q}_k) + \beta_{k+1} g_{k+1}(r_k) A_{k+1}^x$, where $A_{k+1}^x = [j_{k+1}^x]^*$, which is zero at the end ($A_{n+1}^x = [j_{n+1}^x]^* = 0$), and is a known constant at each period in the backward induction process where it is calculated from a higher period.

When the scaled games are solved, and the decisions r^*, w^*, and q^* are known, the Φ^* are determined and then quantities and profits are rescaled to their proper values. In this optimal control problem, Φ represents the state variable, and prices play the control role. We summarize the findings so far in the following propositions.

Proposition 1.3.1. *In a single-period newsvendor (fixed prices) problem, a fully equipped demand creates more expected profit for the retailer compared to the DR model, i.e.*

$$\pi R^*(q_R^*) \leq \pi D(q_R^*) \leq \pi D^*(q_D^*), \tag{1.13}$$

where πR and πD represent the expected profit of the distributional-robust model and the model with a known distribution, respectively, and the star addresses the optimal value. It implies that the DR optimal profit is a lower bound for the problem with known distribution. Furthermore, the DR model's policy is not optimal for the model with a known distribution, i.e., $\pi D(q_R^*) \leq \pi D^*(q_D^*)$.

In a multi-periodic price history-dependent supply channel problem, the relation

$$JR^{r*}(r_R^*, w_R^*, q_R^*) \leq JD^r(r_R^*, w_R^*, q_R^*) \leq JD^{r*}(r_D^*, w_D^*, q_D^*) \tag{1.14}$$

holds. The JR^{r*}, JD^r, and JD^{r*} address the retailer's expected optimal value of the DR model, the model with known distribution before optimization, and the model with known distribution after optimization, respectively. JR and JD follow Eq. (1.10)*. The indexes R and D, used for w, r, and q, represent the distributional robust model and the model with known distribution results, respectively. Hence, $JD^r(r_R^*, w_R^*, q_R^*)$ determines the result of the model with known distribution for the DR model policy.*

Proposition 1.3.2. *The distributionally robust profit for the retailer at each period k is*

$$\begin{aligned} \hat{\Pi}_k^r(r_k, w_k, \hat{q}_k) = {} & (r_k - s_k)\hat{\mu}_k(r_k) - (w_k - s_k)\hat{q}_k \\ & - \frac{(r_k - s_k)}{2}\left(\sqrt{\hat{\sigma}_k^2(r_k) + (\hat{q}_k - \hat{\mu}_k(r_k))^2} + \hat{\mu}_k(r_k) - \hat{q}_k\right), \end{aligned} \tag{1.15}$$

for any given pair (r_k, w_k), the retailer's scaled return $\hat{\Pi}_k^r$ is maximized by choosing the order quantity as

$$\begin{aligned} & \hat{q}_k = \hat{q}_k(r_k, w_k) = \hat{\mu}_k(r_k) + \hat{\sigma}_k(r_k)\Lambda_k(r_k, w_k), \\ & \Lambda_k = \frac{\eta_k - 1/2}{\sqrt{\eta_k(1 - \eta_k)}} \quad \text{and} \quad \eta_k = \frac{r_k - w_k}{r_k - s_k}. \end{aligned} \tag{1.16}$$

Proof. See Appendix A. □

Proposition 1.3.3. *Let (r^*, w^*, q^*) be the equilibrium solution resulting in the payoffs $[J^m, J^r]^*$. Then the subsequence $\{(r_i^*, w_i^*, q_i^*), [J_i^m, J_i^r]^*, i = k : n\}$ is the solution of the subgame starting at period k.*

Scaling the order quantities by Φ decouples the sequence of subgames that reveals (r^*, w^*) by utilizing $j_k^x = \hat{\Pi}_k^x + \beta_{k+1} g_{k+1} j_{k+1}^x = \hat{\Pi}_k^x(r_k, w_k, \hat{q}_k) + \beta_{k+1} g_{k+1}(r_k) A_{k+1}^x$, where $A_{k+1}^x = [j_{k+1}^x]^*$, which is zero at the end, i.e., $A_{n+1}^x = [j_{n+1}^x]^* = 0$.

1.4 Numerical Implementation

The simplest case occurs when the demand depends only on the current price. This family of problems decouples into a series of independent single-period problems. Albeit most markets depict some dependency on the price history affecting customers' behavior. In this section, we offer examples with a price history-dependent demand to show how to implement the proposed algorithm. Our numerical illustration is given by applying the algorithm to optimize problems with the scaled mean and standard deviation of demand given by (see Eq.(1.8)).

$$\hat{\mu}_k(r_k) = \frac{1000\left(1 + \frac{1}{1+k}\right)}{r_k^2} \quad \text{and} \quad \hat{\sigma}_k(r_k) = \frac{\hat{\mu}_k(r_k)}{2\sqrt{3}}. \tag{1.17}$$

To assess the DR results, we assume that the uniform distribution (UD) is the true distribution and solve the model with UD incorporating the DR policy and call it UR model results (similar to $JD^r(w_R^*, r_R^*, q_R^*)$ in Eq. (1.14)).

From Eq. (1.9), the scaling factor has the general structure delineated below

$$\begin{aligned} \Phi_k(\vec{r}_{k-1}) &= \prod_{i=2}^{k} g_i(r_{i-1}), \\ \text{where } g_i(r_{i-1}) &= e^{\gamma_i (K_i - r_{i-1})}. \end{aligned} \tag{1.18}$$

The time-dependent parameter K_k is a kind of current time preference price, and γ_k represents the strength of a current deviation to the future demand. The scale factor g_i acts similarly to a discount factor, though the retailer can manipulate it by setting the price to modify future demand.

To optimize the manufacturer-retailer problem, an n-value parameter set has been applied for each time-dependent parameter c^m, s, β, K, γ, where n represents the number of periods (15 in this illustration).

$$\begin{aligned}
c^m &= [2, 2, 2, 2.2, 2.2, 2.2, 2.5, 2.5, 2.5, 2.5, 2.8, 2.8, 2.8, 3, 3] \\
s &= [1, 1, 1, 1, 1, 1.2, 1.2, 1.2, 1.2, 1.2, 1.3, 1.3, 1.3, 1.3, 1.3] \\
\beta &= [1, 0.96, 0.96, 0.96, 0.97, 0.97, 0.97, 0.97, 0.98, 0.98, 0.98, 0.98, 0.98, 0.98, 0.98] \\
K &= [5.6, 5.6, 5.4, 5.4, 5.4, 5.3, 5.3, 5.3, 5.3, 5.1, 5.1, 5.1, 5.1, 5.1, 5.1] \\
\gamma &= [0.05, 0.05, 0.05, 0.05, 0.04, 0.04, 0.04, 0.04, 0.03, 0.03, 0.03, 0.03, 0.03, 0.03, 0.03]
\end{aligned}$$

The decision variables (r^*, w^*, q^*) in the equilibrium state are depicted in Figure 1.1. Figure 1.1(a) provides a visual presentation of the retailer's profit in each period. The blue line represents the result of the model with the uniform distribution (UD), the red line indicates the distributional-robust model (DR), and the green line depicts the implementation of the UD model with the DR policy (UR). Furthermore, Figure 1.1(b) showcases the manufacturer's profits in both the DR and UD cases. The figures offer a comparative view of the expected profitability achieved under each model.

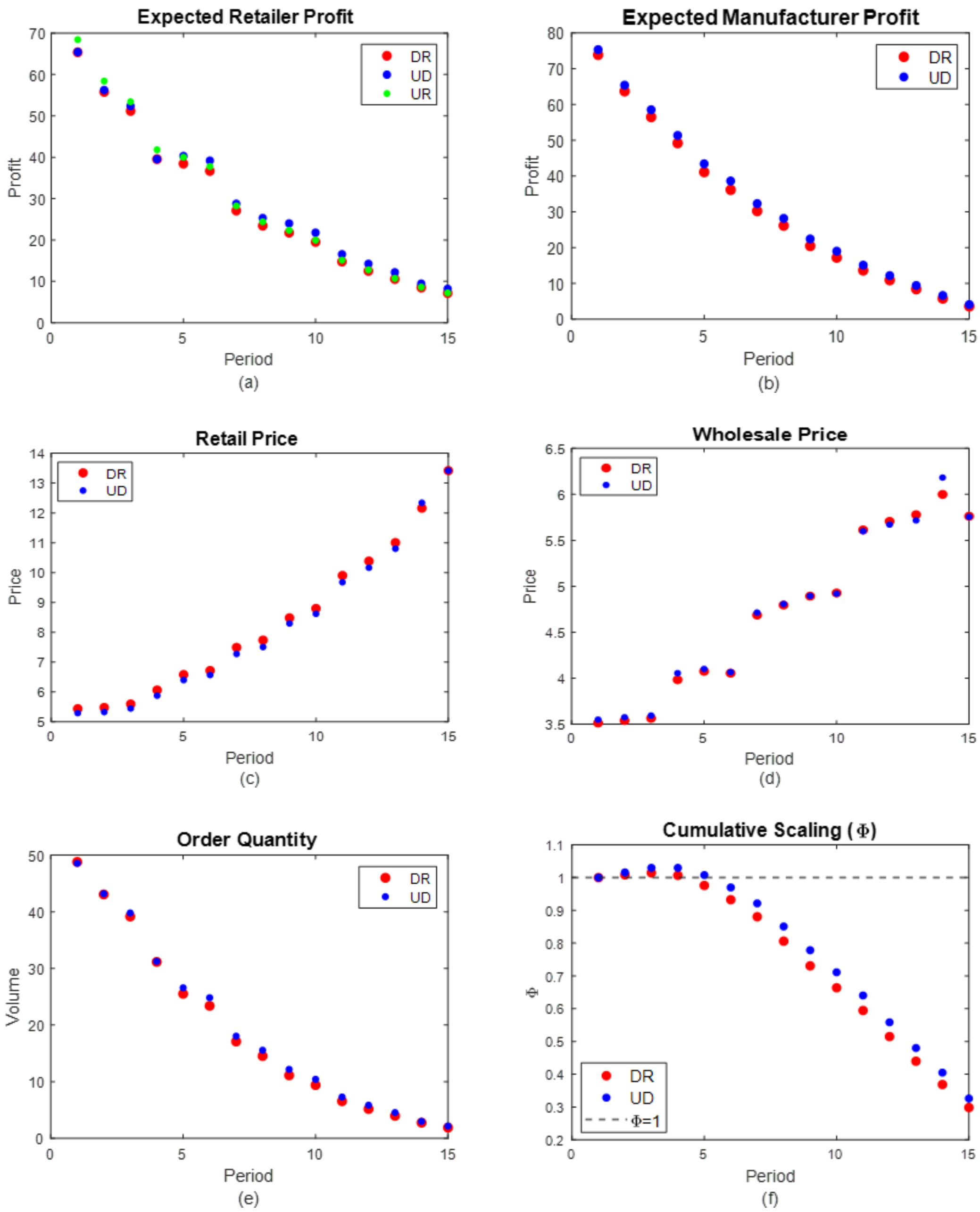

Figure 1.1: Optimal results

The total expected profit of the retailer satisfies $JR^{r*}(=432) \leq JD^{r}(=449) \leq JD^{r*}(=454)$, as stated in Eq. (1.14). The manufacturer, although not directly affected by the incomplete information, experiences the market's volatility as a result of the retailer's order volume decision. The manufacturer achieves $JR^{m*}(=456.1) \leq JD^{m*}(=482.1)$.

The difference between $JD^{x*}(r_D^*, w_D^*, q_D^*)$ and $JD^x(r_R^*, w_R^*, q_R^*)$ quantifies the profit deviation of DR from UD model. This discrepancy highlights the impact of incomplete information. The loss incurred due to the incomplete information is referred to as the Expected Value of Additional Information (EVAI), and in this example, it equals

$$EVAI^r = 454 - 449 = 5$$
$$EVAI^m = 482.1 - 456.1 = 26.$$

The implementation of the DR policy results in a 0.94% deviation for the retailer and a 5.2% deviation for the manufacturer from their actual values [4]. The retailer is willing to invest up to 5 units of currency to obtain complete information. However, despite the presence of these deviations, the DR policy proves to be a highly effective heuristic.

Examining plot (c), in the DR model, the retailer prices increase by 147%, whereas in the UD model, they increase by 153.9% over time. The wholesale price decision improves by 64% in DR and 62.1% in UD models (plot (d)). The price decisions led to a 96.1% quantity decline in the DR model and 95.5% in the UD model (plot (e)) over time. Looking at plot (f), the retail prices exceed the market price preference from period 4 leading to a downward trend in cumulative scaling.

Plot (d) displays that the wholesale price follows the same pattern as the manufacturer's cost vector. However, apart from the final step, an increase in cost typically results in a higher increase in the wholesale price. For instance, a 10% increase in the cost in the first jump (period 3 to 4) raises the wholesale price by 11.8% and 12.8% in DR and UD models respectively. The reason may stem from the fact that the increase in cost implies both higher cost and salvage loss $(w - s)$ for the retailer, leading to a lower order quantity. Hence, the manufacturer sets a price to also partly compensate for this quantity reduction.

1.4.1 Time Independent Model Parameters

We have set the parameter vectors as, $c^m = 2$, $\beta = 0.96$, $s = 1$, $K = 5.6$, and $\gamma = 0.05$ for any period. Then, it is only the scaling factor that changes the results.

[4] If the problem with complete information (known distribution) is considered actual.

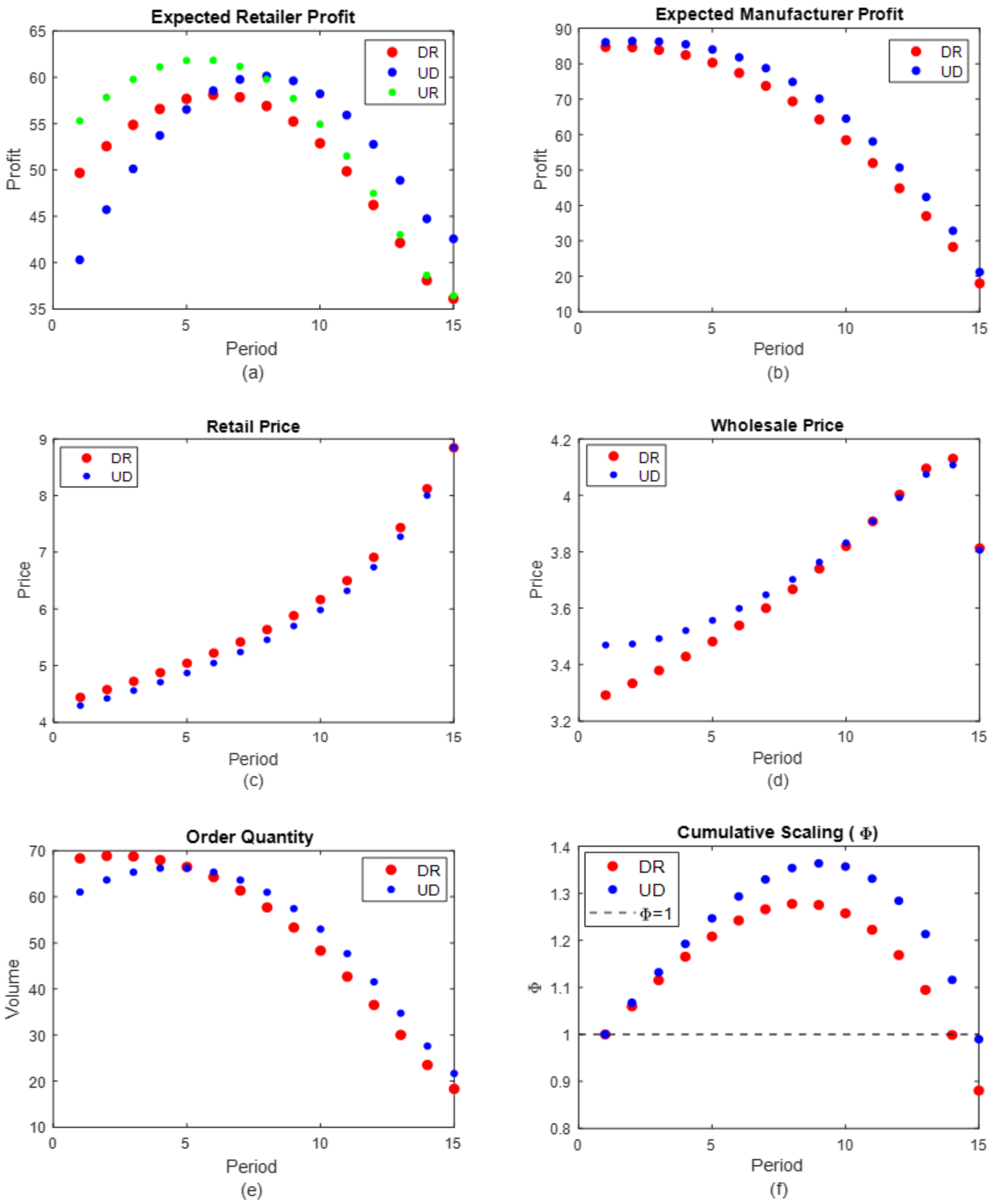

Figure 1.2: Results with time-independent model parameters

In this experiment, the retailer obtains 769.8 from the DR model and 787.6 from the UD model (plot (a)). Meanwhile, the manufacturer gains 939.5 from implementing the DR model and 1000.4 from the UD model (plot (b)). The wholesale price increases by 25.5% until period 14, but by 15.8% overall (plot (d)), and the retail price increases by 99.1% (plot (c)) in the DR model. These price changes have led to a 73.2% decline in ordering (plot (e)), while the market has been in a state of prosperity for a long time (wherever higher than 1 in plot (f)).

1.4.2 Impact of Key Model Parameters on Total Profit

In this section, we evaluate the effect of varying salvage value (s), manufacturer cost (c^m), and current time preference price (K) on the players' values within the proposed DR model. These parameters can exhibit increases, decreases, or remain unchanged. Table 2 presents 27 different scenarios, where scenario 27 represents the reference problem (baseline) previously solved in Section 1.4.

Table 1.2: The effect of key parameters on outputs

Scenario	c^m(%)	S(%)	K(%)	JM	JR	ΔJM(%)	ΔJR(%)
1	10	10	10	432.9	400	-5.1	-7.4
2	10	10	-10	319.5	315.4	-29.9	-27
3	10	10	0	369.95	353.4	-18.9	-18.2
4	10	-10	10	419.4	387.2	-8.1	-10.4
5	10	-10	-10	309.8	305.7	-32.1	-29.2
6	10	-10	0	358.7	342.5	-21.3	-20.7
7	10	0	10	426	393.4	-6.6	-8.9
8	10	0	-10	314.6	310.5	-31	-28.1
9	10	0	0	364.2	347.8	-20.2	-19.5
10	-10	10	10	703.8	643.8	54.3	49
11	-10	10	-10	505.4	492.4	10.8	14
12	-10	10	0	593.4	560.2	30.1	29.7
13	-10	-10	10	677.1	617.6	48.5	43
14	-10	-10	-10	468.7	473.1	6.7	9.5
15	-10	-10	0	571.1	537.6	25.21	24.5
16	-10	0	10	690	630.2	51.3	45.9
17	-10	0	-10	495.8	482.5	8.7	11.7
18	-10	0	0	581.9	548.6	27.6	27
19	0	10	10	546.8	501.6	19.9	16
20	0	10	-10	397.9	390	-12.77	-9.8
21	0	10	0	464.1	440	1.75	1.8
22	0	-10	10	528	483.6	15.8	12
23	0	-10	-10	384.9	376.3	-15.6	-13
24	0	-10	0	448.5	424.4	-1.7	-1.7
25	0	0	10	537.2	492.2	17.8	14
26	0	0	-10	391.5	382.9	-14.2	-11.4
27(Baseline)	0	0	0	456.1	432	0	0

The parameters in Table 2 are varied by ±10%, while a value of 0% indicates no change. The last two columns of the table present the deviation percentage in value compared to scenario 27. The results for total expected profits are displayed in Figure 1.3, where the horizontal red and black lines indicate scenario 27 total expected profits for the manufacturer and the retailer respectively.

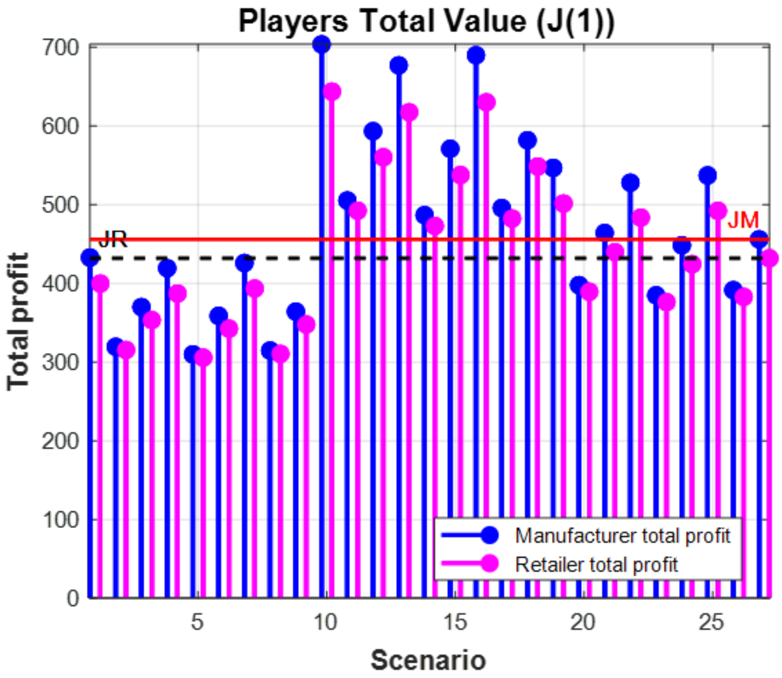

Figure 1.3: Total expected profits in each scenario

Both players are represented in this plot, with blue stems standing for the manufacturer and magenta for the retailer in each scenario. In this example, scenarios 1-9, 20,23-24, and 26 result in a decline in channel value, while the remaining scenarios display a beneficial effect. Scenario 14 demonstrates that a cost reduction can compensate for the decrease in salvage value and preference price. However, an increase in salvage value and preference price is unable to offset the impact of a cost increase, as reflected in scenario 1, indicating a higher sensitivity to cost. Figure 1.4 pictures the order quantities generated by different scenarios. The highest and lowest order quantities, like the total profit, are observed in scenarios 10 and 5 respectively. Howbeit the trend of ordering almost follows the same trend.

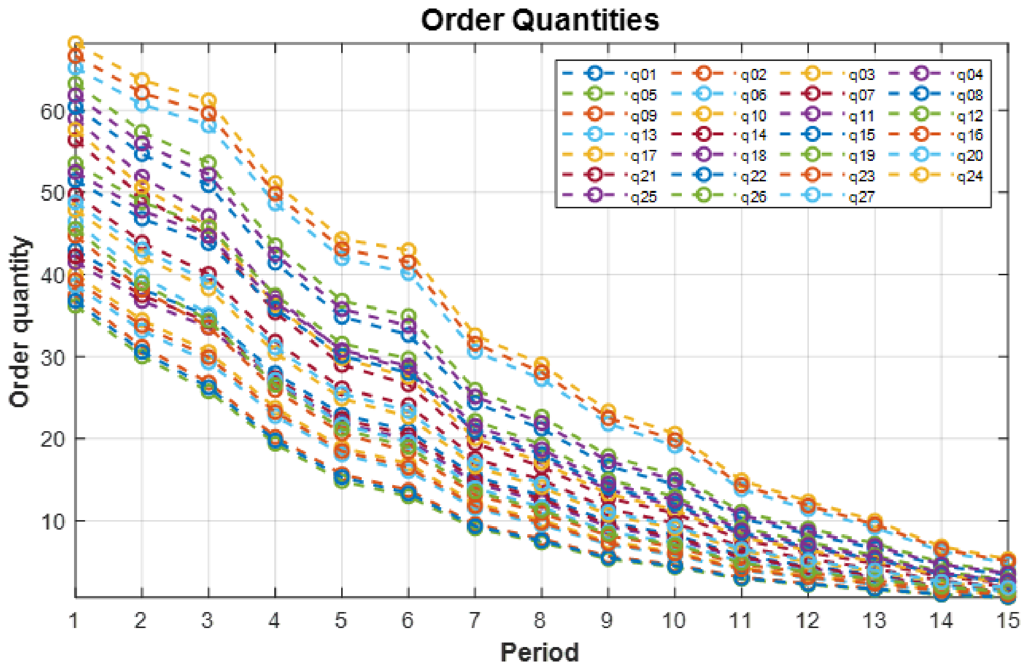

Figure 1.4: Order quantities in each scenario

1.5 Concluding Remarks

We have introduced a comprehensive framework for solving multi-periodic manufacturer-retailer games where the dynamic and stochastic demand is influenced by price history but lacks knowledge about the distribution of the stochastic drivers. Our proposed algorithm effectively addresses this challenge through a price history-dependent distributional-robust approach, providing valuable insights for decision-making. All parameters defining the (Stackelberg) game are allowed to vary with time. The algorithm solves the distributional-robust (DR) model in a subgame-perfect manner through backward induction and provides a weak lower bound on the retailer's expected value.

In each period, the players initiate a new contract considering a price history-dependent demand with a subgame perfect structure. However, this periodic contract structure does not accommodate order/production capacity constraints. This limitation arises due to the specific approach employed in solving the problem, wherein a scaled quantity is computed within the algorithm and then rescaled to its actual amount. Furthermore, in the calculation phase, the players decide for each period and proceed to the next without having the opportunity to modify their policy if needed. Addressing this limitation and addressing a more flexible decision-making process will be of key interest in future research.

Through an illustrative example, we evaluated the model's response to a $\pm 10\%$ change in parameters, manufacturer cost (c^m), salvage value (s), and current time preference price (K). The results reveal that the manufacturer cost alone strongly affects the overall outcome, with a 10% increase (decrease) leading to a 19.2% (28.3%) decrease (increase) in the channel's total profit. Changes in preference price result in a 16.8% (12.1%) increase (decrease) in the channel's value while the salvage value has a relatively minor influence, causing a 2.6% (0.9%) increase (decrease) in the channel's value. Additionally, the sensitivity to cost is also evident in the wholesale price, where an increase in cost leads to a proportionately higher increase in the wholesale price. Although the manufacturer does not directly face market stochasticity, he realizes its influence through the retailer's order volume. The market is improved by $\Phi > 1$ and whenever $r > K$ the cumulative scaling factor begins to decrease and in $\Phi < 1$ shrinks the market, as depicted in the example.

Future research endeavors could explore the incorporation of optimal buyback and quantity discount schemes to enhance the decision-making process. An interesting improvement would be to enable the players to consider single contracts for the entire time horizon while embedding realistic constraints that are not easily incorporated in the multi-periodic contract scheme with the subgame-perfect approach.

Appendix A

From the retailer's expected profit with the DR framework,

$$\Pi^r = (r-s)\mu - (w-s)q - \frac{(r-s)}{2}\left(\sqrt{\sigma^2 + (q-\mu)^2} + \mu - q\right), \tag{1.19}$$

then

$$\frac{\partial \Pi^r}{\partial q} = -(w-s) - \frac{r-s}{2}\left(\frac{q-\mu}{\sqrt{\sigma^2+(q-\mu)^2}} - 1\right) = 0, \tag{1.20}$$

which yields

$$q = \mu + \sigma\Lambda, \quad \text{where } \Lambda = \frac{\eta - \frac{1}{2}}{\sqrt{\eta(1-\eta)}} \text{ and } \eta = \frac{r-w}{r-s}. \tag{1.21}$$

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Chapter 2

Distributional-Robust Single Contract Game with Price History-Dependent Demand and Corrective Actions

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Abstract

The paper investigates a multi-period supply channel facing uncertain and price-history-dependent demands and environmental regulations. The knowledge about the demands is limited to its mean and standard deviation in each period, i.e., there is incomplete information on the actual distribution. A distributional robust approach is conducted to address incompleteness. The chain is incorporating environmental policies such as pollution constraints and (optimal) corrective taxes. A single contract covers all periods. Numerical examples highlight the benefits of a single contract.

JEL classification: C61, C62, C63, C72, C73, D81, Q52.

Keywords Dynamic Games, Single Contract, Distributional-Robust Demand, Price History-Dependent Demand, Environmental Corrective Actions, Sustainability.

2.1 Introduction

With rapid global economic development, environmental challenges have been deteriorating constantly (Yang et al. 2014). The major source of this problem has been regarded as greenhouse gas emitted by production, services, and consumption (Song & Leng 2012). This problem has attracted more countries' attention since the 1980s (Ma et al. 2022). Nowadays, sustainability is a key subject for environmentalists, economists, industrialists, consumers, academia, and governments (Manupati et al. 2019, Yang et al. 2014). With the aim of environmental protection and reduction of pollution, many governments have agreed to contribute to the goal of emissions reduction by at least 50% by 2050 as reported by the International Energy Agency (Liu et al. 2015, Song & Leng 2012). Furthermore, because of consumer awareness development, many governments and companies have implemented pollution reduction policies and displayed their attempts to reduce their footprint by pasting a tag on their products, like Tesco and Boots. The actions that increase consumer awareness of environmental concerns, encourage them to opt for a product with a lower environmental footprint if its price is affordable. However, actions with lower pollution may result in a higher cost for the channel players (Yang et al. 2014).

Among all environmental pollution reduction policies, two types of policies, emissions capacity constraint, and emissions tax regulation have been analyzed by many countries or regions, such as the European Union (EU), Canada, China, and the IMO (Bai et al. 2022). Emissions capacity constraints involve setting a maximum limit, or carbon cap, on the level of emissions allowed within the supply channel. In contrast, emissions tax regulation imposes a cost on each unit of production or pollution, requiring the channel to pay a tax fee for each unit of pollution generated through production or consumption. According to Luo, et al., emissions tax is considered one of the most effective market-based mechanisms and enjoys widespread acceptance worldwide. More than 20 countries, including Canada, Australia, the United Kingdom, and the United States, have already implemented emissions tax policies (Luo et al. 2022). For instance, the Dutch government has planned to impose a CO_2 emissions tax on industrial companies starting in 2021, initially set at 30 euros per ton of CO_2 emitted. This amount would increase to 125-150 euros by 2030, ensuring the sustainability of industrial firms (Blomberg[1]).

As economic globalization has deepened, the world economy has evolved into a complex and interconnected system. However, a significant challenge remains in establishing crucial pollution reduction targets within such a complex economic framework (Jiang 2022) and encouraging industries to collaborate. This underscores the importance of developing an algorithm capable of identifying optimal solutions while considering envi-

[1] https://www.bloomberg.com/news/articles/2019-06-28/dutch-government-plans-co2-emissions-levy-for-industrial-firms

ronmental pollution reduction constraints. Supply channels have increasingly prioritized ecological sustainability in alignment with the United Nations' Sustainable Development goals (Kannan et al. 2022). Therefore, it is vital to examine the impact of pollution reduction activities on economic players (Yang et al. 2014).

Kannan et al. analyzed the barriers to implementing pollution reduction policies in India. They employed the Best Worst method to determine the relative importance of the barriers, focusing on regulatory policies. They establish interrelationships among the barriers of pollution reduction policies, using the Decision-Making Trial and Evaluation Laboratory. Regarding their categories, the economic category was found to carry the highest weight, followed by the organizational and environmental categories. Their finding highlights several key observations, including the lack of initial funding, hidden costs, uncertain carbon market price, lack of research and development, lack of support from the authorities, lack of alternative energy sources, unaccountability of production waste, fear to shift to a new system, lack of in-house reverse logistics, irrational current taxes, unaccountability of supply chain actors and lack of social demand (Kannan et al. 2022).

Wu et al. review the progress made in carbon neutrality efforts. They note that 120 countries worldwide have proposed carbon-neutral goals, such as China, accompanied by national development, different cities and industries have actively included carbon neutrality in their development plans (Wu et al. 2022). In a study by Xu et al., the performance of emissions tax policy in a supply chain composed of one supplier and two financially asymmetric manufacturers was investigated under Cournot competition. The researchers argue that emissions tax plays a crucial role in restricting carbon emissions and improving environmental performance for climate change mitigation (Xu et al. 2022).

Luo et al. developed the Stackelberg game to evaluate the impact of the emissions tax on (re)manufacturing decisions within a closed-looped supply chain. They examined both scenarios with no investment in pollution reduction technology, as well as with investment in centralized and decentralized closed-loop supply chains. Their finding suggests that emissions tax encourages manufacturers to invest in pollution reduction technology or engage in remanufacturing. Moreover, when the tax is low, the pollution level in the centralized closed-loop supply chain exceeds that of the decentralized model (Luo et al. 2022). Choi and Cai discuss the environmental challenges arising from shorter lead times in the production process, which can result in inadequate control of chemical and material processing operations. To address this issue, they propose the imposition of an environmental tax on suppliers to incentivize investment in green technologies (Choi & Cai 2020). Zhang et al. explore a single-period Stackelberg model considering three regulatory approaches, tax, subsidy, and tax-subsidy policies. Their study aims to assess the effects of each method on channel profit and environmental pollution (Zhang et al. 2020).

Hong et al. explore the integration of tax regulations and green product design strategies, where the decision variables are the degree of product greenness and retail price being made by the manufacturer and the retailer respectively (Hong et al. 2019). Manupati et al. investigate different production-distribution and inventory problems in a multi-echelon supply chain, considering three pollution reduction policies viz tax, strict capacity capping, and a cap-and-trade system. They also incorporate lead-time considerations using a non-linear mixed integer programming model (Manupati et al. 2019). In a study by Song et al., a stochastic production capacity problem is expanded to incorporate cap-and-trade and pollution tax regulations. The researchers found that firms increase their capacity when capacity investment is low enough, leading to higher unit profit (Song et al. 2017). The manufacturer in Chen et al.'s model employs two different techniques, standard and green technology using cap-and-trade and capacity constraints to reduce pollution. They demonstrate that emissions trade yields higher profits compared to capacity constraints (Chen et al. 2016). Choi incorporates pollution tax policy into a fashion apparel problem and examines the implementation of a quick response system through reduced lead time, faster delivery mode, and local sourcing instead of offshore sourcing (Choi 2013). Zhang and Xu study a newsvendor multi-item production plan with stochastic, but constant demand[2], considering both cap-and-trade and pollution tax regulations (Zhang & Xu 2013).

In our model, a Stackelberg game composed of a manufacturer and a retailer is considered, wherein the manufacturer leads the channel in a multi-period setting. The retailer faces a demand that is time and price history-dependent, i.e., the current and prior prices determine the demand in each period. Moreover, the incomplete information on demand distribution leads to opting for a distributional-robust (DR) approach. The proposed algorithm operates under a single contract, wherein the players consider their decisions and the resulting consequences across all periods simultaneously. The primary objective is to ensure the attainment of the highest possible value. In the following, we study two environmental pollution reduction policies: pollution tax and capacity constraint and propose optimal algorithms satisfying the environmental policies. The decision variables are the prices. However, in the pollution tax model, the tax fee is also a decision made by the regulator. He addresses the chain problem by endogenizing the pollution externalities. The contributions of this paper are hence

- to solve multi-period Stackelberg distributional-robust game,
- with a dynamic and price history-dependent demand,
- under a single contract, compared to periodic contracts, which makes significantly

[2]A stochastic constant demand can be formed as $D = \mu + \sigma\varepsilon$, where μ and σ are constant values representing the mean and standard deviation of the demand and ε is a distribution with mean zero and variance 1.

different results by utilizing more information and freedom to decide,

- propose an algorithm with environmental constraints, viz capacity constraints and pollution tax,
- acquiring the corrective tax that the regulator puts on the leader to fully compensate for the pollution produced.

2.2 Model Framework

In our model, demand is dynamic and price history-dependent for a perishable commodity produced by the manufacturer and sold by the retailer. The manufacturer leads and the retailer follows him, while both aim at maximizing their expected values by making pricing policies, leading to order quantity decisions under a single contract. With a single contract, the players can improve their expected value by regulating their decisions when they can observe the connected decisions' reactions. The players have a certain number of periods and decide on all their variables simultaneously. Unlike periodic contracts, a single contract is not subgame perfect, but if the players cling to the contract, both may obtain higher values. This can be exploited in DR settings as well as in more unrealistic situations with complete demand information.

The channel produces an externality in the form of pollution such that the order quantity may be obliged to follow environmental protection policies to reduce the environmental footprint. We employ two policies: capacity constraints and environmental taxes. In the capacity constraint system, the pollution produced in period k by manufacturing the ordered quantity q_k is $e_k q_k$ which cannot exceed a certain cap M_k, i.e., $0 \leq e_k q_k \leq M_k$. The tax can be split into two subcases: Any given tax on a unit of production or order quantity (environmental or not) and a corrective tax that exactly endogenizes the cost of eliminating pollution flow in the chain. The latter tends to depend on quantity or production volume. It depends on the damage function or the flow of externality costs. We consider a corrective tax where either the manufacturer or the retailer incorporates this decision in their formulation and pays the tax. All taxes and pollution or production caps are allowed to be dynamic.

The next subsection deals with the price history-dependent demand structure. The DR model under a single contract is introduced in subsection 2.2.2. It is formed for the DR model, but the algorithm is applicable in the case with complete information by replacing the profits and quantity functions corresponding to the actual demand distribution. In the following, subsections 2.2.3 and 2.2.4 organize the single contract model for capacity constraints and corrective taxes. Each proposed model is provided with a non-trivial example in section 2.3.

2.2.1 Demand Structure

The demand in period $k \in \{1, \ldots, n\}$ as a dynamic function of prices is modeled as

$$D_k(\vec{\mathbf{r}}_k) = \mu_k(\vec{\mathbf{r}}_k) + \sigma_k(\vec{\mathbf{r}}_k)\varepsilon_k, \quad \vec{\mathbf{r}}_k = (r_1, \ldots, r_k) \tag{2.1}$$

where μ_k and σ_k are deterministic functions of time and retail prices and represent the mean and standard deviation of demand at period k. The epsilons are uncorrelated random variables independent of prices with mean and standard deviation equal to 0 and 1 respectively. Incomplete information in the present setting means that the distributions for the ε_k are unknown. This is typically the situation in most real-world cases, either the complete information is not accessible, or it is too costly to obtain it. Hence, it is worthwhile to implement an approach that does not rely on the specificities of the ε_k-distributions, i.e., a distributional-robust (DR) approach is the way forward. It implies replacing the retailer's expected value/profit with a tight lower bound, i.e., at least one set of distributions results in an expected value equal to this bound and no other distribution implies a lower expected value. That is, no fully informed situation will have a lower expected profit for the same set of means and variances.

2.2.2 Model Formulation

The retailer orders q_k from the manufacturer with the wholesale price w_k considering the demand D_k and sells the amount of $\min(D_k, q_k)$ to the customers at the price r_k in period k. The time scope is divided into n discrete intervals referred to as periods. If q_k exceeds the demand, $(q_k - D_k)^+$ can be salvaged (discarded) at a price (cost) of s_k. The retailer's profit function in period k is[3]

$$\pi_k^r(\vec{\mathbf{r}}_k, w_k, q_k) = r_k \min(D_k, q_k) + s_k(q_k - D_k)^+ - w_k q_k + B_k^r(q_k), \tag{2.2}$$

where the first term represents the revenue of sold items, the second indicates the revenue (or cost) of the leftovers, the third term is the cost of purchase, and the last term addresses any other gains or costs by acquiring q_k. The function B_k^r may be non-linear. An example of this non-linear part can be a damage function. The retailer's objective function is then the expected values of profits,

$$\begin{aligned} E[\pi_k^r] &= r_k \left(\mu_k - E(D_k - q_k)^+\right) + s_k \left(q_k - \mu_k + E(D_k - q_k)^+\right) - w_k q_k + B_k^r(q_k) \\ &= (r_k - s_k)\mu_k - (w_k - s_k)q_k - (r_k - s_k)E[D_k - q_k]^+ + B_k^r(q_k). \end{aligned} \tag{2.3}$$

The demand distribution, if known, provides a solution for the term $E[D_k - q_k]^+$.

[3]See list of notations in Appendix A.

However, Cauchy- Schwartz inequality (Fakhrabadi & Sandal 2023)[4] assists with

$$E[D_k - q_k]^+ \leq \frac{1}{2}\left(\sqrt{\sigma_k^2 + (q_k - \mu_k)^2} - q_k + \mu_k\right). \tag{2.4}$$

By replacing $E[D_k - q_k]^+$ in Eq. (2.3) by the right-hand side of Eq. (2.4), the expected DR approach for the retailer is obtained as a lower bound for the model with known distribution as follows

$$\begin{aligned}\Pi_k^r(\vec{\mathbf{r}}_k, w_k) &\equiv E[\pi_k^r]_{DR} \\ &= (r_k - s_k)\mu_k - (w_k - s_k)q_k - \\ &\frac{r_k - s_k}{2}\left(\sqrt{\sigma_k^2 + (q_k - \mu_k)^2} - q_k + \mu_k\right) + B_k^r(q_k) \quad \leq E[\pi_k^r]_D,\end{aligned} \tag{2.5}$$

where $E[\pi_k^r]_D$ represents the profit of the same model with known distribution. From now on we use the term 'profit' for this bound (Π_k^r).

The manufacturer's expected profit is calculated as

$$\Pi_k^m(q_k, w_k) = E[\pi_k^m] \ = (w_k - c_k^m)q_k + B_k^m(q_k), \tag{2.6}$$

where the first term represents the manufacturer revenue, and the second term addresses any other linear or non-linear gains or losses associated with q_k. In the Stackelberg game, the manufacturer declares his price first, conditioned on the retailer's optimal reaction (r_k, q_k). In the multi-period problem, both the retailer and manufacturer aim to optimize their values given by

$$J_1^x = \alpha_1\Pi_1^x + \alpha_2\Pi_2^x + \alpha_3\Pi_3^x + \ldots + \alpha_n\Pi_n^x \quad \text{for } x \in \{m, r\}, \tag{2.7}$$

where n is the number of periods and m, r indicates the manufacturer (m) and the retailer (r). The parameter α represents

$$\alpha_k = \beta_1 \cdot \beta_2 \cdot \ldots \cdot \beta_k, \tag{2.8}$$

where β_k is discounting factor for period k, and $\alpha_1 = \beta_1 = 1$. In a single contract, $w^* = [w_1^*, \ldots, w_n^*]$ is revealed and then $r^* = [r_1^*, \ldots, r_n^*]$ and $q^* = [q_1^*, \ldots, q_n^*]$ are declared. The players can observe the consequences of their decisions simultaneously and change their decisions, if necessary, before finalizing the optimization process and signing a contract. The price history-dependent demand may allow for strategic decisions by manipulating future demand to improve optimal return. The manufacturer knows exactly his profit when the single contract is written. The retailer has all the risk by knowing a lower bound on his expected total return. The key findings are summarized in the following

[4]See Appendix B.

propositions.

Proposition 2.2.1 (Optimal Order Quantity)**.** *The optimal order quantity q_k for any pair (w_k, r_k) is given by*

$$(w_k - s_k) - \frac{1}{2}(r_k - s_k)\left[\frac{q_k - \mu_k}{\sqrt{\sigma_k^2 + (q_k - \mu_k)^2}} - 1\right] + \frac{\partial B_k^r(q_k)}{\partial q_k} = 0. \tag{2.9}$$

The special case $B_k^r(q_k) \equiv 0$ yields

$$q_k(\vec{\mathbf{r}}_k, w_k) = \mu_k(\vec{\mathbf{r}}_k) + \frac{\sigma_k(\vec{\mathbf{r}}_k)}{2}\frac{2\eta_k - 1}{\sqrt{\eta_k(1-\eta_k)}} \text{ and } \eta_k = \frac{r_k - w_k}{r_k - s_k}. \tag{2.10}$$

Proposition 2.2.2 (Single Contract Key Feature)**.** *The manufacturer gains at least a payoff equal to the subgame perfect total payoff.*

Proof. A single-contract model benefits from taking into consideration all decisions simultaneously. The subgame perfectness restricts the choice of decisions. Hence, the decision space for a single contract covers the subgame perfect choices.

A single contract may create significantly different results by utilizing this freedom to allow for strategic pricing when all periods are considered simultaneously, i.e., optimizing without a fixed term structure. This leads to the manufacturer's benefits $J_{\mathrm{SC}}^m \geq J_{\mathrm{PC}}^m$, where SC and PC represent the single and periodic contracts respectively. □

2.2.3 The Model Formulation Under Capacity Constraints

If the regulator's strategy to reduce pollution generated by the manufacturer is defined as a capacity constraint, $e_k q_k \leq M_k$ must hold, where e_k represents the pollution from producing one unit of product, and M_k is the maximum pollution permitted in period k. Hence, the manufacturer has to constrain his optimization by $q_k \leq q_k^c = \frac{M_k}{e_k}$. Hence utilizing Eqs.(2.5) and (2.6) as the players' profits and Eq.(2.7) as their payoffs, the game is

$$\max_{w \in \mathcal{W}} J_1^m \quad \text{s.t.} \quad (r, q) = \arg \max_{(r,q) \in \mathcal{R}} J_1^r, \tag{2.11}$$

where $\mathcal{W}$ and $\mathcal{R}$ represent the feasible spaces for the wholesale and retail prices and order quantities compatible with all constraining conditions, e.g., $q_k \leq q_k^c$.

2.2.4 The Model Formulation Under the Pollution Tax Policy

If the government decides to implement a corrective tax policy, the channel is required to pay tax for each unit of pollution or spend a cost to clean the pollution it has caused. Since our channel consists of a manufacturer and a retailer, we address the problem of each player when facing the pollution tax.

2.2.4.1 Manufacturer as Tax Collector

The regulator, cognizant of the problem formulations faced by the players, imposes a Pigouvian tax (Corrective tax). Notice that a damage function can be internalized by the manufacturer, by issuing the quantity-dependent tax $\tau_k = \tau_k(q_k)$

$$\Pi_k^m(w_k, q_k) = (w_k - c_k^m - \tau_k)q_k = (w_k - c_k^m)q_k + B_k^m(q_k), \tag{2.12}$$

and the retailer's problem stays unchanged. Here $B_k^m(q_k)$ are the damage functions implied by the tax issued. The Corrective tax is

$$\tau_k(q_k) = -\frac{B_k^m(q_k)}{q_k}. \tag{2.13}$$

Hence, this tax is issued as a non-fixed tax that depends on the actual production. It automatically generates a cost that exactly pays for the damage and the manufacturer considers it when he makes his decisions. Therefore, the manufacturer internalizes the pollution damage (B_k^m) and his optimization gives the optimal quantity (q_k^*) and thereby the tax $\tau_k^*(q_k^*)$ that is imposed on the manufacturer to mitigate their environmental footprint or the cost they would incur to remove the pollution.

2.2.4.2 Retailer as Tax Collector

According to the retailer profit function in Eq. (2.5), any given periodic tax (τ_k) can be accommodated by setting $B_k^r(q_k) = -\tau_k q_k$, resulting in

$$\begin{aligned}\Pi_k^r(\vec{\mathbf{r}}_k, w_k, q_k) = (r_k - s_k)\mu_k - (w_k + \tau_k - s_k)q_k - \\ \frac{(r_k - s_k)}{2}\left(\sqrt{\sigma_k^2 + (q_k - \mu_k)^2} - q_k + \mu_k\right).\end{aligned} \tag{2.14}$$

This is equal to the case without B_k^r, but with w_k replaced by $w_k + \tau_k$. The best order quantity q_k for any given set of parameters (r_k, s_k, τ_k, w_k), is then given by Eq. (2.10) where w_k is replaced by $w_k + \tau_k$. If the tax is only on sold items, it is equivalent to $B_k^r = 0$ and r_k replaced with $r_k - \tau_k$ in Eq. (2.10).

A Pigouvian tax will endogenize a damage cost function $B_k^r(q_k)$, and Eq. (2.10) determines the best order quantities for any given set of (r_k, s_k, w_k). The damage function is revealed by the quantity-dependent tax $\tau_k(q_k)$ issued by the regulator as

$$B_k^r(q_k) = -q_k \tau_k(q_k). \tag{2.15}$$

In this case, there are no shortcuts to determine the best order quantities. The full version of Eq. (2.9) must be applied.

Both sections 2.2.3 and 2.2.4 are solved under a single contract. The periodic backward induction algorithm, which is commonly utilized to solve such problems, cannot solve the price history-dependent problems under the ordering/production constraints.

2.3 Numerical Implementation

We begin the numerical illustration by comparing an unconstraint single contract with a periodic contract. Later, in section 2.3.2 we move on to the unconstraint single contract model with a short memory and its extension to the models with capacity constraint and tax policies. As mentioned before, the demand may be affected by previous periods' price decisions. Indeed, an increase in the price today may bring about a decrease (increase) in the customer base tomorrow. Hence, price settings in one period may change the future customer base, and therefore change the future demand and modify the supply channel's values. Although, the effect of each period's price might fade out over time. This effect can be labeled as memory and denoted by $\Phi_k(\vec{\mathbf{r}}_{k-1})$, where $\Phi_1 = 1$ and $\vec{\mathbf{r}} = (r_1, \ldots, r_k)$ and $k \in \{2, \ldots, n\}$.

In the rest of this paper, we deal with a demand scaled by the price history such that the coefficient of variation only depends on the current price, i.e.,

$$\begin{aligned} D_k(\vec{\mathbf{r}}_k) &= \Phi_k(\vec{\mathbf{r}}_{k-1}) d_k(r_k) \\ \text{where} \qquad d_k(r_k) &= \tilde{\mu}_k(r_k) + \tilde{\sigma}_k(r_k)\varepsilon_k. \end{aligned} \tag{2.16}$$

So, from Eq. (2.1)

$$\begin{aligned} \mu_k(\vec{\mathbf{r}}_k) &= \Phi_k(\vec{\mathbf{r}}_{k-1})\tilde{\mu}_k(r_k) \\ \sigma_k(\vec{\mathbf{r}}_k) &= \Phi_k(\vec{\mathbf{r}}_{k-1})\tilde{\sigma}_k(r_k). \end{aligned} \tag{2.17}$$

For the numerical examples, we apply the following scaled demand terms

$$\tilde{\mu}_k(r_k) = \frac{100(10 + \frac{1}{(1+k)})}{r_k^2}, \quad \tilde{\sigma}_k(r_k) = \frac{\tilde{\mu}_k(r_k)}{2\sqrt{3}} \tag{2.18}$$

The parameters are set to constant over time by $c_k^m = 2$, $s_k = 1$, $\beta_k = 0.96$ for all $k \in \{1, \ldots, 12\}$ in all examples. We utilize two different kinds of scaling factors representing long-term (section 2.3.1) and short-term (the rest of the examples) memory. We have limited the scaling factor to perform in the range $[0.7, 2]$, meaning $\Phi_k = \max(\min(2, \Phi_k), 0.7)$ at each arbitrary period k.

2.3.1 Case 1, Single Contract vs. Periodic Contract

We have structured this paper on a single contract in section 2.2.2, but it is worth comparing the same problem set with a periodic contract where the algorithm commences from the last period and steps back to the first. We exemplify these cases by the scaling factor

$$\Phi_k(\vec{\mathbf{r}}_{k-1}) = \prod_{i=2}^{k} g_i(r_{i-1}) \quad \text{and } g_k = e^{\gamma_k(K_k - r_{k-1})}. \tag{2.19}$$

K_k is the market price preference and γ_k represents the strength of a current deviation on the future demand (marginal log scale). The parameters are set to $K_k = 6, \gamma_k = 0.04$. For a DR periodic problem, the players' total value,

$$j_k^x = \pi_k^x + \beta_{k+1} \cdot g_{k+1} \cdot j_{k+1}^x \quad \text{for } x \in \{m, r\} \tag{2.20}$$

is optimized in each period k, i.e., the players optimize their current situation in the game, ensuring a subgame-perfect solution by starting at the end (Fakhrabadi & Sandal 2023, Gholami et al. 2021). In Eq. (2.20), J^m and J^r address the manufacturer and retailer values respectively. Utilizing Eq. (2.7) for the single contract and Eq. (2.20) for the periodic contract, Figure 2.1. illustrates players' profits.

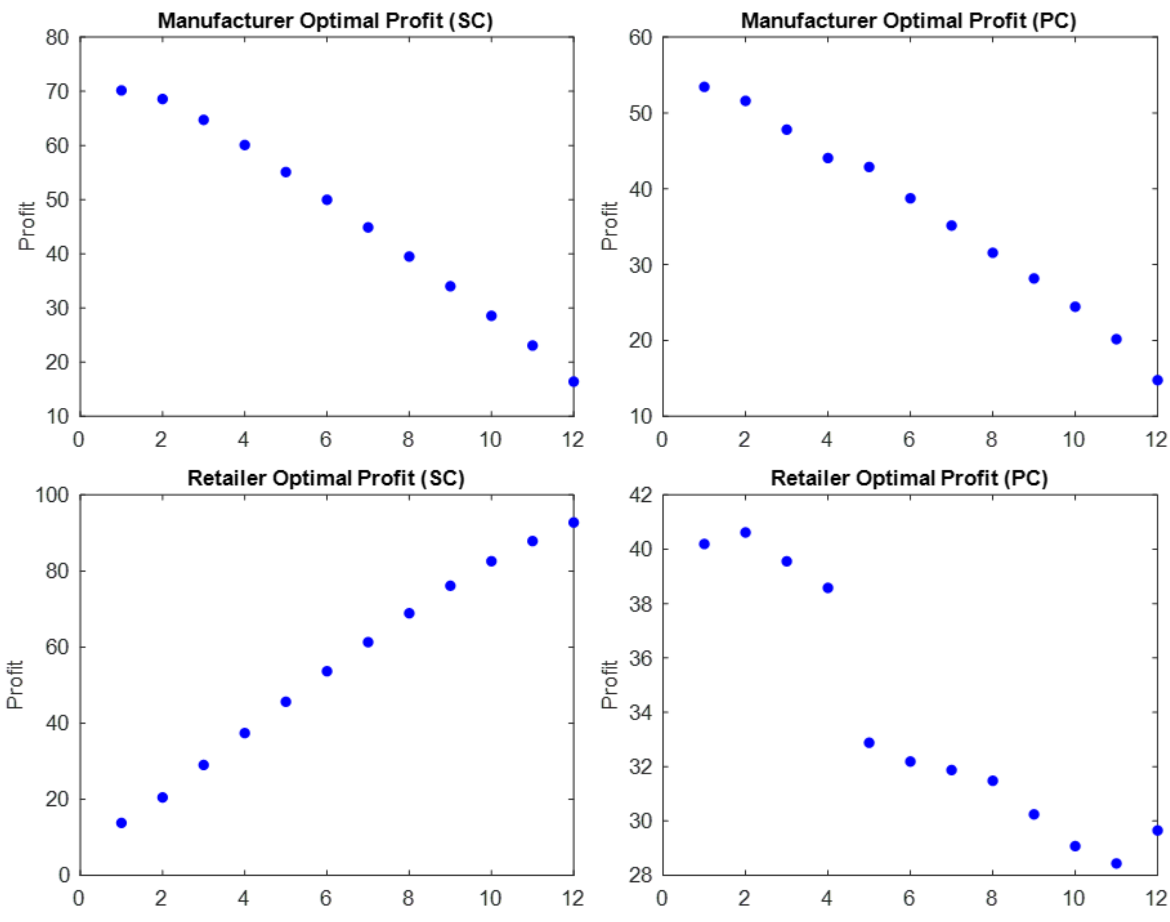

Figure 2.1: Optimal profits, single contract (SC) vs. periodic contract (PC)

SC and PC are the models with single and periodic contracts respectively. The players' total value was revealed as

$$J^m_{\text{SC}} = 555 > J^m_{\text{PC}} = 433,$$
$$J^r_{\text{SC}} = 670 > J^r_{\text{PC}} = 408$$

This is equivalent to a relative increase of 28% and 64% in the total returns of the manufacturer and the retailer by utilizing a single contract instead of periodic ones. It is observed that this single contract is beneficial for both the manufacturer and retailer which is compatible with the statement in section 2.2.4.

Embedding the optimal SC wholesale prices (w^*) into the periodic contract algorithm and solving the problem for retail price yields $J^m = 471$ and $J^r = 491$. This outcome highlights a significant finding: the periodic framework fails to recognize the superior values identified by the SC, even when the optimal wholesale prices w^* are provided.

The optimal prices are plotted in Figure 2.2. The lower prices obtained by the single contract (SC) are accompanied by higher quantities leading to a larger market (Figure 2.3) and higher returns. The leader collects more profit in the beginning and the follower in the end in the SC case.

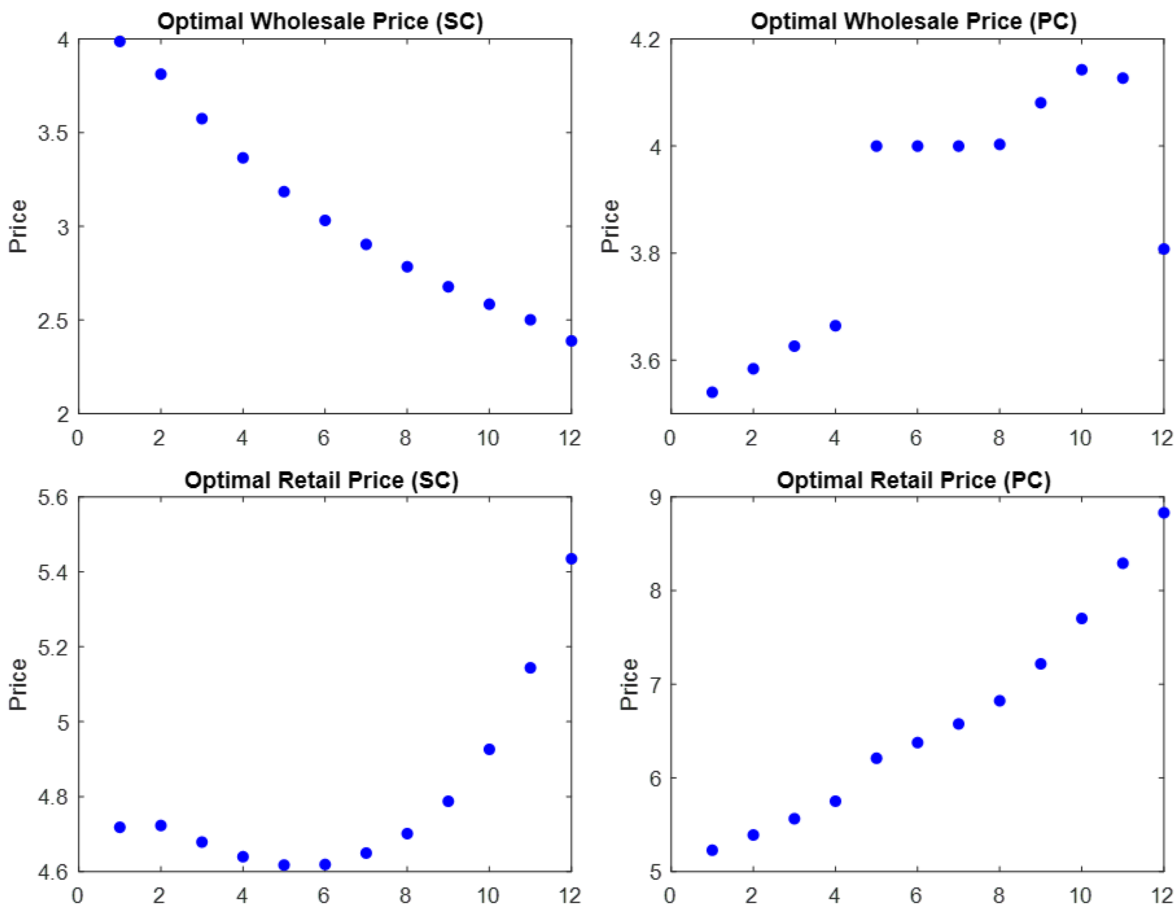

Figure 2.2: Optimal prices, single contract (SC) vs. periodic contract (PC)

Figure 2.3 mirrors the effectiveness of each contract form in stimulating market growth. The results reveal that the single contract model consistently bolsters the market, while the periodic contract deviates from this trend from the third period and begins to contract the market at $k = 6$. The red line represents the threshold that separates the market stimulation and market contraction phases based on the price history effects.

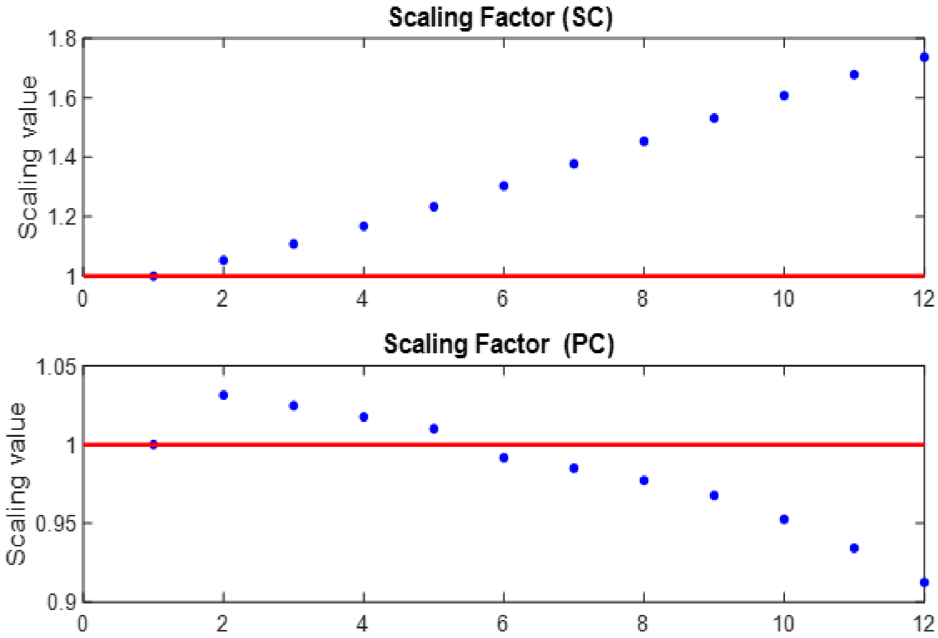

Figure 2.3: Scaling factor, single contract (SC) vs. periodic contract (PC)

2.3.2 Case 2,Unconstrained Single Contract and Short Memory

In this example, we only implement a single contract with scaling factors

$$\Phi_k = e^{\gamma_k(r_{k-2}-r_{k-1})}. \tag{2.21}$$

The explicit memory effect of each price only lasts for two periods. Therefore,

$$\Phi = \{1, e^{\gamma_2(R-r_1)}, e^{\gamma_3(r_1-r_2)}, e^{\gamma_4(r_2-r_3)}, \ldots\}. \tag{2.22}$$

Φ is the effective scaling factor and R is a given reference price (model parameter) for the second period. The parameters are set to $\gamma_k = 0.04$ and $R = 10$ for $k \in \{1, \ldots, 12\}$. Figure 2.4 illustrates the optimal profits for the manufacturer and retailer.

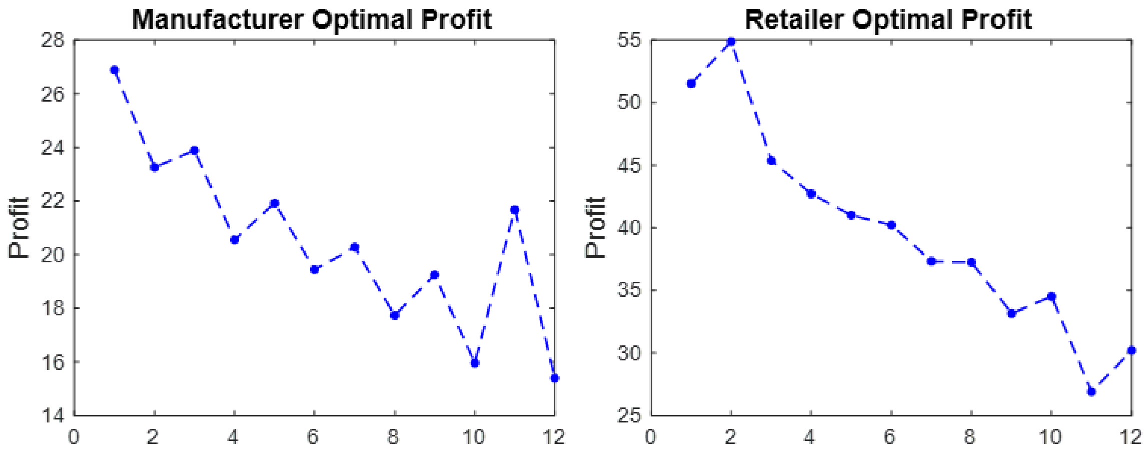

Figure 2.4: Optimal profits

The total values, denoted as $J^m = 246$ and $J^r = 475$, are obtained from the analysis. The observed pattern reflects the scaling factor structure. The players strategically make decisions as a volatile set to maximize their profits. It is worth mentioning that this strategy capitalizes on the fact that any price changes are forgotten after a span of two periods (Figure 2.5).

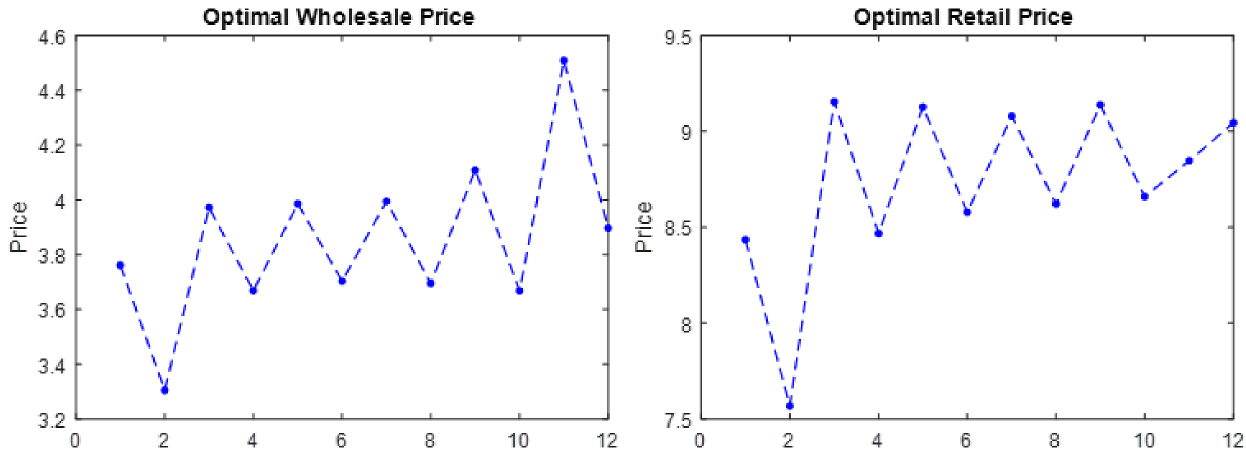

Figure 2.5: Optimal prices

The retail price decisions result in the scaling factor that is illustrated in Figure 2.6.

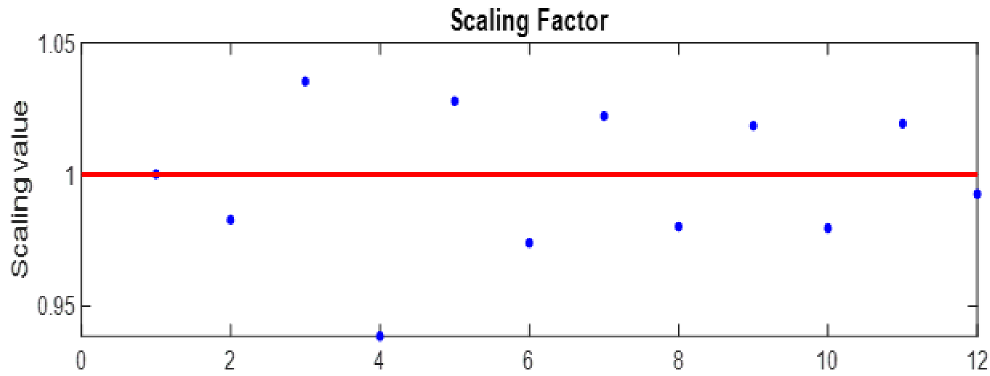

Figure 2.6: Scaling factor for a short memory

The red line separates the region where the retail prices are boosting /shrinking the market. The volume ordered at each period is mirrored in Figure 2.7.

Figure 2.7: Optimal order quantity

2.3.3 Case 3, Single Contract and Capacity Constraints

The pollution capacity constraint, determined by the regulator and denoted as q^c (associated with the emissions amount), represents the maximum allowance of production, serving an upper bound for the maximum pollution that might be generated by the manufacturer. It is crucial to adhere to this constraint and ensure that it is not violated. By utilizing Eq. (2.22), Figure 2.8 illustrates the profits and volume associated with three distinct capacity constraints (a, b, c).

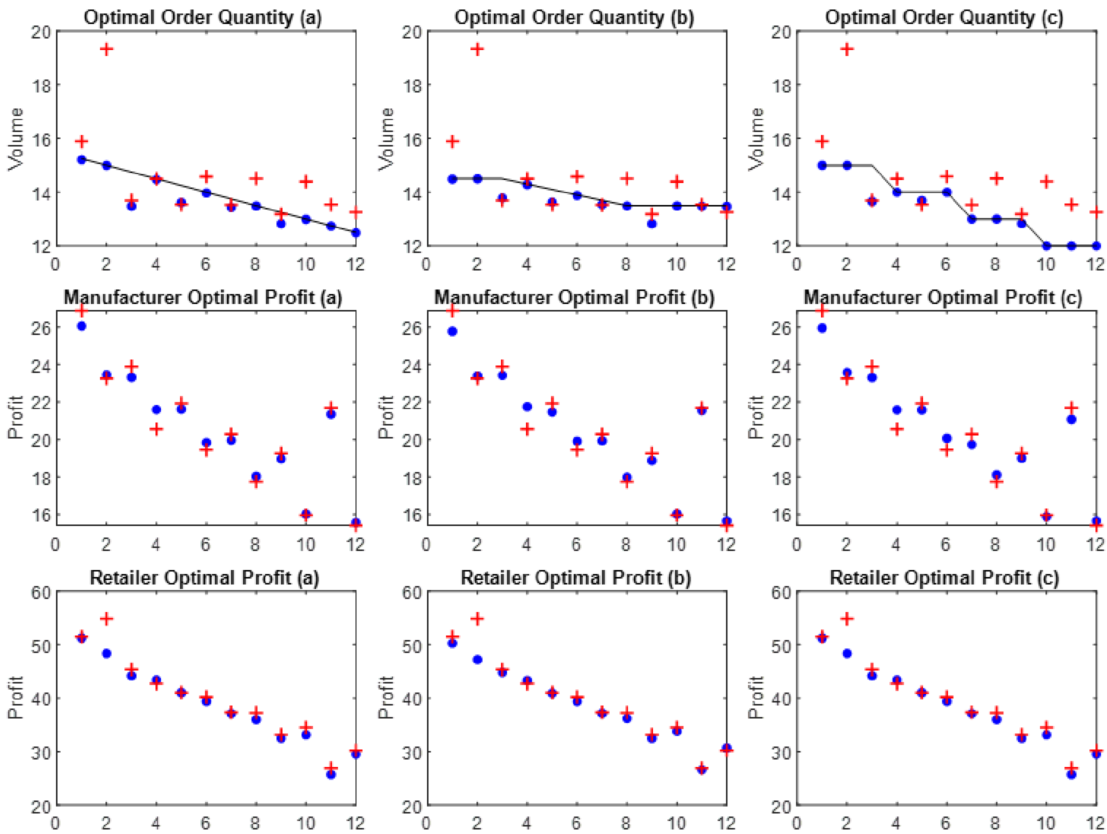

Figure 2.8: Optimal quantity and profits under capacity constraint policy (red curve)

In the order quantity plot, red crosses represent the unconstrained results, the capacity constraint is depicted by the black line, and the blue circles indicate the optimal solution under capacity constraint. The subscripts a, b, and c correspond to different cases with different capacity constraints. An intriguing finding emerges when comparing the volumes in the constrained and unconstrained models: there are periods when the unconstrained solution operates below the capacity limit, with no requirement to order reduction, but the constrained algorithm intentionally chooses a lower volume, such as period 3 in the plot (a). The total values in each model are

$$J_a^m = 246, \qquad J_b^m = 246, \qquad J_c^m = 245,$$
$$J_a^r = 462, \qquad J_b^r = 463, \qquad J_c^r = 457,$$

where the unconstrained problem's results (base model, example 2, section 2.3.2) are $J^m = 246$ and $J^r = 475$. The findings indicate that in total, these cases reduce the emissions by 5.9, 5.2, and 7.9% in a, b, and c cases respectively. Hence, regarding the priorities which can be either pollution reduction or economic growth, case c is the greenest, while case b brings the highest economic return. Figure 2.9 pictures the price decisions.

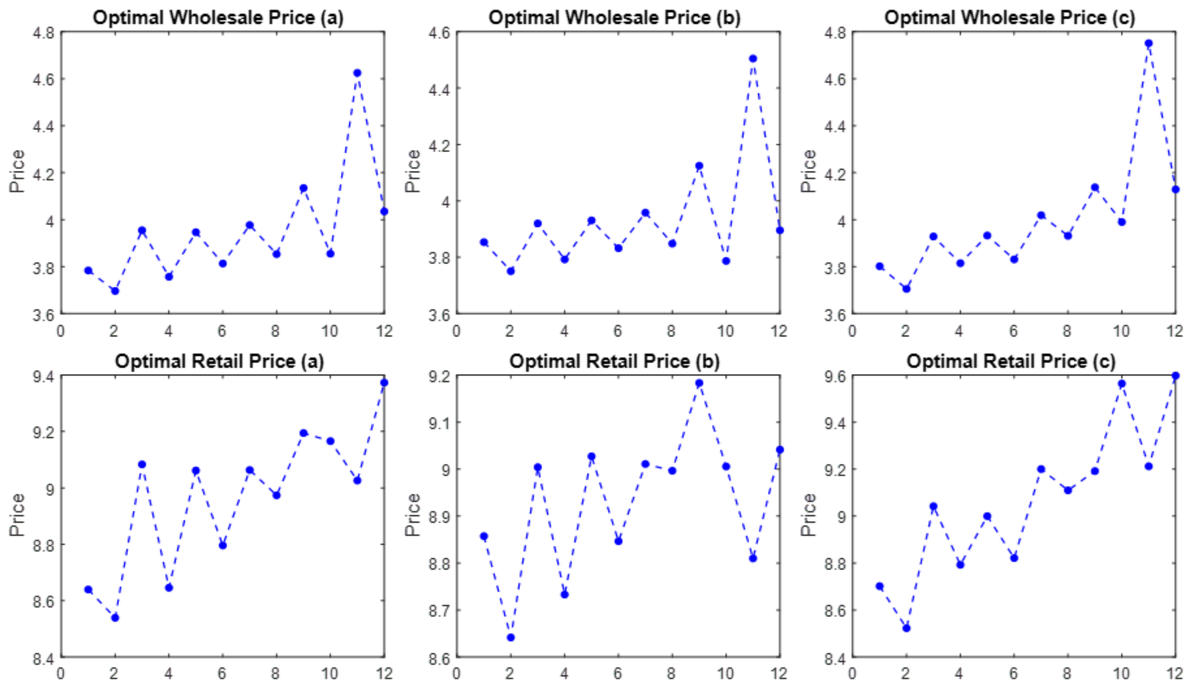

Figure 2.9: Optimal prices under capacity constraint policy

2.3.4 Case 4, Corrective Tax

According to the discussion in section 2.2.4, we present an illustrative example where either the manufacturer or the retailer collects a tax following Eq. (2.12) or Eq. (2.14). In this example, a player x applies $B_k^x(q_k) = -a_k q_k^2$ in their problem where the damage-intensity factor, a_k, is chosen to be 0.04. Consequently, the tax derived from Eq. (2.13) or Eq. (2.15) takes the form of $\tau_k^* = a_k q_k^*$, representing the amount paid per unit in period k to mitigate the pollution associated with that unit. Figure 2.10 illustrates corresponding profits and quantities in both cases where either the manufacturer or the retailer integrates the tax inside their objective functions.

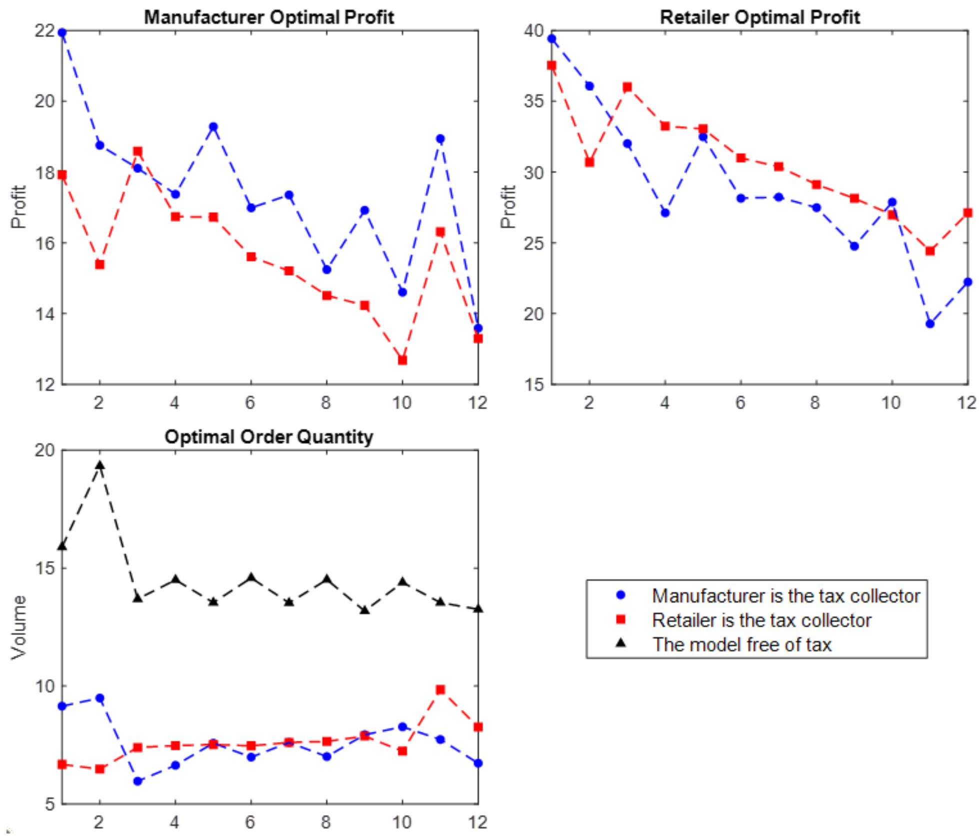

Figure 2.10: Optimal results

As depicted in Figure 2.10, the calculated values for this example are

	J^m	J^r
The manufacturer is the tax collector	209	345
The retailer is the tax collector	187	368

Notably, this example showcases the effectiveness of the proposed method in reducing pollution. In comparison to the base model (example 2, section 2.3.2), the application of tax policy results in a significant 48% and 47% reduction in pollution overall, when the manufacturer and retailer are tax collectors respectively.

The analysis of two tax models within the supply channel, where the responsibility for collecting the pollution tax lies with either the manufacturer or the retailer, reveals that each player achieves higher profits when individually responsible for managing the pollution tax.

The results demonstrate that when the manufacturer takes charge of tax collection, the channel's earnings amount to 554 units of currency, while if the retailer assumes this responsibility, the channel's earnings increase to 555 units, where the model free of tax makes 739 units.

2.4 Concluding Remarks

We introduce a comprehensive single-contract framework aimed at optimizing a multi-period Stackelberg game with a dynamic, price history-dependent, and DR demand. Our results demonstrate the superiority of the single-contract model over the periodic-contract model, although, the single-contract is not sub-game perfect. This can be attributed to the freedom and awareness embedded within the single contract model. Unlike the periodic contracts model, where players make decisions in each period, the single-contract model identifies an optimal decision at least as good as a periodic-contract framework. The single contract allows for better utilization of the strategic potential in the market.

We illustrate the effectiveness of the single-contract model using two different types of price history dependency in our examples and observe how this effect is reflected in the output. The algorithm leverages the price history effect to achieve optimal order quantities and maximize values.

Furthermore, we extend the model to address environmental constraints, specifically the pollution capacity constraint and tax. These policies have been widely implemented in many countries. Both systems impose limitations on the channel that may require a reduction in the quantity.

An intriguing finding from the model incorporating a capacity constraint is that there are cases, where the algorithm subject to constraints leads to a lower order quantity compared to the unconstrained solution, even though the specified cap permits a higher volume. In other words, there are periods when the unconstrained solution operates below the capacity limit, with no requirement to order reduction, but the constrained algorithm intentionally chooses a lower volume. This behavior highlights a strategic decision-making capability inherent in a single-contract approach that may not be evident in a periodic approach. Furthermore, it underscores the interconnectedness of decisions across different periods, where changes in one period can impact decisions in preceding and subsequent periods.

With the emissions tax policy, the channel faces a cost to mitigate the pollution it has generated, as dictated by the imposed damage function. It is important to note that the constraint framework cannot be effectively implemented in a periodic contract form, highlighting the advantages of the single-contract model in handling environmental constraints.

In conclusion, our proposed single-contract framework outperforms the periodic-contract model in terms of optimization and value maximization but also provides a means to address environmental constraints through policies such as (pollution) capacity constraints and corrective tax inclusion. Our dynamic distributional robust settings are closer to real-world situations and adapted to fully utilizing strategic potentials in the markets with memory. By incorporating these factors, our model offers valuable insights and strategies for decision-making in complex dynamic supply channel scenarios. Future research could explore other environmental policies and different types where the players are sharing exposure to the market risk.

Appendix A: Notation List

$\beta = \{\beta_1, \ldots, \beta_n\}$	Discount factor over individual periods[5]
$c^m = \{c_1^m, \ldots, c_n^m\}$	Manufacturer cost
$s = \{s_1, \ldots, s_n\}$	Salvage price/discarding cost
$w = \{w_1, \ldots, w_n\}$	Wholesale price (decision variable)
$r = \{r_1, \ldots, r_n\}$	Retail price (decision variable)
$q = \{q_1, \ldots, q_n\}$	Order quantity (decision variable)
$k \in \{1, \ldots, n\}$	Time or period
$D = \{D_1, \ldots, D_n\}$	Demand. $D_k = \mu_k(r) + \sigma_k(r)\varepsilon_k$ is demand in period k
$\mu = \{\mu_1, \ldots, \mu_n\}$	Mean of demand
$\sigma = \{\sigma_1, \ldots, \sigma_n\}$	Standard deviation of demand
$\varepsilon = \{\varepsilon_1, \ldots, \varepsilon_n\}$	Stochastic and independent drivers with mean 0 and variance 1
$\pi^m = \{\pi_1^m, \ldots, \pi_n^m\}$	Manufacturer profit (present value)
$\pi^r = \{\pi_1^r, \ldots, \pi_n^r\}$	Retailer profit (present value)
$\tau_k = \{\tau_1, \ldots, \tau_n\}$	Emission tax

Appendix B: Cauchy- Schwartz Inequality

The Cauchy-Schwartz inequality reads $|E(xy)|^2 \leq E(x^2) \cdot E(y^2)$. If we choose $x = |q - D| = |(q - \mu) - \sigma\varepsilon|$ and $y = 1$, and utilize that $E(\varepsilon) = 0$ and $E(\varepsilon^2) = 1$, we obtain

$$|E(|q - D|)|^2 \leq E(|q - D|^2) = E[(q - \mu)^2 - 2(q - \mu)\sigma\varepsilon + \sigma^2\varepsilon^2] = (q - \mu)^2 + \sigma^2 \quad (2.23)$$

and thereby $E(|q - D|) \leq \sqrt{\sigma^2 + (q - \mu)^2}$. Applying the equality

$$(D - q)^+ = \frac{1}{2}\{|D - q| + (D - q)\}, \quad (2.24)$$

[5] The discount factors related to the start ($t = 0$) are $\alpha_k = \beta_1 \cdot \beta_2 \cdot \ldots \cdot \beta_k$. Individual periods may be of different length.

we obtain directly

$$E(D-q)^{+} \leq \frac{1}{2}\left(\sqrt{\sigma^{2}+(q-\mu)^{2}}-q+\mu\right) \tag{2.25}$$

The equality holds for the deterministic case and certain two valued distributions.

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Chapter 3

A Distributional Robust Analysis of Buyback and Cap-and-Trade Policies

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Abstract
This study delves into a dynamic Stackelberg game comprised of a manufacturer and a retailer, operating in an environment with fluctuating demand and price-dependent consumer behavior. The multi-period optimization challenges the manufacturer to strategically set wholesale and buyback prices, while the retailer determines the retail price and order quantities within a single contract. In this dynamic framework, the players operate under the constraints of a cap-and-trade policy, with limited knowledge of demand distributions, characterized only by mean and standard deviation parameters. To address this inherent uncertainty, we employ a distributionally robust approach. Additionally, we explore the enduring effects of historical decisions on present-day demand, reflecting a memory-like market behavior. Through numerical examples, we illuminate the influence of buyback contracts and cap-and-trade policies on decision-making processes within this setting.

JEL classification: C61, C62, C63, C72, C73, D81, Q52.

Keywords : Cap-and-Trade Policy, Multi-Period Stackelberg Game, Price History-Dependent Demand, Distributional-Robust Demand, Single Contract, Buyback Contract, Sustainability.

3.1 Introduction

3.1.1 Cap-and-Trad Policy

Rapid industrialization and urbanization have accelerated environmental pollution and incurred purification costs in many countries. Indeed, economic growth has come at the expense of increased environmental pollution. China, one of the largest polluters, has performed an assessment that reveals the damaging effects on various sectors such as agriculture, global temperature, and life expectancy (Du et al. 2016). The primary cause of global warming has been carbon dioxide and this prompts governments to consider the urgent need to reduce this pollution. In recent years, authorities have concentrated on measuring pollution levels and investigating potential mechanisms for carbon emissions reduction and addressing associated risks (Wang et al. 2021, Taleizadeh et al. 2021, Cao et al. 2017, Xu et al. 2017, Du et al. 2015,0).

While efforts to combat global climate change have been initiated worldwide, environmental policies and sustainability initiatives have become a competitive advantage for manufacturers. Furthermore, customers are increasingly aware of the importance of low-carbon products and are willing to pay more for products that have a minimal environmental impact (Wang et al. 2021, Tong et al. 2019). This growing awareness has led supply chain stakeholders to incorporate sustainable development and low-carbon environmental policies into their operations and decision-making processes to align with market changes (Wang et al. 2021). Companies such as HP, Dell, and Acer are actively working to reduce e-waste, energy consumption, and carbon emissions, while others like Ford and Volkswagen are exploring the production of vehicles powered by alternative energy sources. For example, Siemens adopted cleaner technologies in 2016, leading to a reduction of 521 million tons of carbon emissions, which accounted for over 60% of Germany's annual (Tong et al. 2019). Retailers such as Walmart and Tesco have also engaged in green activities and implemented carbon footprint labeling for their products (Mondal & Giri 2022*b*).

The first international agreement addressing greenhouse gas emissions and the reduction of carbon emissions footprint was the UNFCCC[1]. It aimed to establish official obligations and support measures to reduce emissions impact on the environment (Du et al. 2015,0). In the UK, fiscal policies such as the Climate Change Agreement (CCA), Climate Change Tax (CCT), and Carbon Price Support (CPS) have been implemented to control greenhouse gas emissions (Xu et al. 2018). Also, the European Union's Emissions Trading System (EU ETS) lowered the emissions cap by 15% in 2015 since its inception in 2005 (Mondal & Giri 2022*b*).

[1]United Nations Framework Convention on Climate Change

The Kyoto Protocol was introduced to guide decision-makers in regulating companies' activities related to carbon pollution. The cap-and-trade ($C\&T$) policy is the main framework mechanism of this protocol and is considered one of the most effective policies (Mondal & Giri 2022*a*, Du et al. 2016). Although, among all carbon reduction policies, the common carbon policies have usually been introduced as $C\&T$ and carbon tax (Feng et al. 2021), the carbon tax does not limit the emissions by an emissions cap. Under the $C\&T$ regulation, manufacturers are allocated a maximum allowance of free emission credits. If this allowance capacity is insufficient to achieve optimal results, manufacturers can purchase emission credits or implement greener production methods to reduce their carbon emissions. They can also sell the surplus quotas to generate profit (Taleizadeh et al. 2021, Li et al. 2021). Accordingly, companies that actively reduce their emissions are rewarded economically and environmentally (Mondal & Giri 2022*b*). According to a report by the European Commission in 2013, the EU Emissions Trading Scheme covered 31 countries and limited nearly 50% of carbon emissions. In this system, the government establishes the necessary policies for emission trading quotas, while companies are responsible for regulating their allocated quotas (Cao et al. 2017, Xu et al. 2017, Du et al. 2015). For example, in 2013, Foxconn invested less than 50 million RMB in energy-saving retrofits but made a profit of 10 million RMB (60 million RMB in revenue) by selling surplus carbon credits (Tong et al. 2019).

It is argued that the $C\&T$ policy offers more profit potential compared to other environmental policies and it has been widely adopted in recent years in countries such as Norway, Netherlands, Sweden, Denmark, and China (Chen et al. 2020). A well-designed $C\&T$ system can improve the efficiency of emissions reduction goals when the regulator sets appropriate emissions cap and trading price (Du et al. 2015, Chen et al. 2020).

To evaluate the effectiveness of the $C\&T$ policy, Mondal and Giri examined a supply channel with green activities and price-dependent deterministic demand. They studied four models: centralized, decentralized, bargaining revenue sharing, and retailer-led revenue sharing under the $C\&T$ policy. Their findings indicate that a higher carbon emissions allowance price motivates manufacturers to improve their green operations, leading to a reduction in carbon dioxide emissions (Mondal & Giri 2022*a*). Feng et al. investigated cooperation in a supply chain using a joint replenishment game, where two or more dependent or independent firms cooperate horizontally under the $C\&T$ system to identify the optimal joint ordering strategy. They found that the retailer with the highest altruistic parameter value benefits from the surplus of carbon emissions allowance (Feng et al. 2021).

Zhao et al. propounded a remanufacturing problem under the $C\&T$ policy and proposed three production decision models: single-product remanufacturing with fixed carbon emissions, extended single-product remanufacturing with variable carbon emissions, and multi-product remanufacturing models (Zhao et al. 2021). Wang et al. combined the $C\&T$ policy with the customers' low-carbon preferences with a differential game model, considering three different scenarios: non-cooperation (coop) scenario where the manufacturer is the leader of the two-echelon supply chain, the supplier's emissions reduction efforts are supported by the manufacturer and a two-way coop contract when both channel members support each other's emissions reduction efforts (Wang et al. 2021).

Taleizadeh et al. examined a supply chain problem involving a retailer and a manufacturer, under $C\&T$, where they could either compete or cooperate in pricing and production decisions. Their model suggests that cooperation yields greater benefits, considering environmental concerns (Taleizadeh et al. 2021). Li et al. investigated two types of subsidy policies based on fixed green technology investment cost (FC subsidy) and the amount of emissions reduction (ER subsidy) under the $C\&T$ mechanism, using Stackelberg game models. The results indicated that government subsidy policies alone cannot guarantee green technology investment and total carbon emissions reduction (Li et al. 2021).

Chen et al. compared the effects of carbon emissions tax policy with a $C\&T$ system using a static optimal model. They found that the $C\&T$ system is more efficient for emission reduction than the carbon tax. However, the impact of the $C\&T$ system on a manufacturer's profit is uncertain and dependent on the carbon cap. Therefore, selecting an appropriate emissions cap and carbon trading price is crucial for ensuring the efficiency of this policy (Chen et al. 2020). Even though former research by Wittneben implies the opposite argument stating that a carbon emissions tax might be a faster and economically more beneficial approach to reducing greenhouse gas emissions. They believe that a carbon emissions tax can generate more income for the government to invest in green projects, while income from $C\&T$ policy is more uncertain. Additionally, implementing a new tax is less complex than the process required for implementing a $C\&T$ system (Wittneben 2009).

Wang and Han proposed a dual mechanism of $C\&T$ and subsidies/penalties for a (re)manufacturing problem with stochastic return and random yield rates, considering four different distribution functions (Wang & Han 2020). Similarly, Mondal and Giri studied competition and cooperation among retailers and a manufacturer under government invention and the $C\&T$ policy. They developed a centralized policy and three manufacturer-led decentralized policies viz. Collusion, Cournot (Nash), and Stackelberg (Mondal & Giri 2022*b*). Aghaie et al. concentrated on the application of the $C\&T$ policy in groundwater extraction management with four different monitoring scenarios. They simulated this model using agent-based modeling addressing interactions between social, institutional, economic, and groundwater systems (Aghaie et al. 2020).

Kushwaha, et al. proposed a mixed-integer linear programming model for a remanufacturing system. They determined the optimal combination of channels for collecting used products from different regions in a finite multi-period setting under $C\&T$ regulation (Kushwaha et al. 2020). Tong, et al. employed the evolutionary game and the $C\&T$ policy considering customer preference for low-carbon products, to examine the behavior of a powerful retailer in a retailer-led supply chain. They used system dynamics to simulate and analyze dynamic and transient behaviors. Their results indicate that the emissions cap, market price of carbon credits, and consumers' preferences for low-carbon products are key factors affecting retailer and manufacturer behaviors (Tong et al. 2019).

Li et al. applied a Stackelberg game between the government and the manufacturer. They indicated that the manufacturer is more incentivized to upgrade its purification technology in a high-carbon preference market compared to a low-carbon preference market (Li et al. 2018). Turki et al. investigated a (re)manufacturing plan considering the differences between new and remanufactured items, random machine failures, the $C\&T$ policy, and distinct random customer demands for both types of products. Their results revealed that a lower carbon cap and/or a high price of carbon trading, impel the producer to collect and remanufacture used items and limit carbon emissions (Turki et al. 2018). Xia et al. incorporated reciprocal preferences and consumers' low-carbon awareness (CLA) into a dynamic supply chain where the manufacturer plays a Stackelberg game with a retailer. Their results demonstrate that the optimal wholesale price increases with CLA, while the optimal emissions level decreases with CLA (Xia et al. 2018). Xu et al. studied the decision-making and coordination of a centralized and decentralized supply chain under $C\&T$ regulation and the Stackelberg game. They investigated pricing and carbon emissions abatement decisions, considering the preferences for low-carbon products (Xu et al. 2018). Cao et al. investigated the impacts of the $C\&T$ policy and low-carbon subsidy policy on the production and level of carbon emissions reduction of a manufacturer under the Stackelberg game. Their findings indicated that the level of carbon emissions reduction is positively related to the carbon trading price. They also discussed that a low-carbon subsidy policy is more beneficial for society when the environmental damage coefficient is below a certain threshold; otherwise, the $C\&T$ policy is preferred (Cao et al. 2017).

Ji et al. studied three decision models: one without $C\&T$ regulation, one based on grandfathering mechanism and $C\&T$ regulation, and one with $C\&T$ regulation based on benchmarking mechanism. They concluded that the benchmark model, compared to grandfathering, can more effectively incentivize manufacturers to produce low-carbon products and motivate retailers to promote low-carbon products (Ji et al. 2017).

Xu et al. addressed the coordination problem of a make-to-order (MTO) supply chain, which includes a manufacturer and a retailer, with wholesale price and cost-sharing contracts under $C\&T$ policy. Their findings indicated that the manufacturer and retailer optimal profits decrease (increase) by buying (selling) prices of emissions allowance (Xu et al. 2017). Du, et al. assessed the trade-off between reducing and incrementing carbon emissions while considering economic considerations in a single period. They studied the factors that could impact the optimal production strategy and profit where customers prefer low-carbon products. Their analysis assumed equal buy and sell prices in an oligopoly market (Du et al. 2016).

Du, et al. conducted studies in 2013 and 2015 to investigate the impact of $C\&T$ emissions regulation on a single-period supply chain problem. In their models, the channel follows a Stackelberg game between an emission permit supplier and an emission-dependent manufacturer. The supplier and manufacturer made decisions regarding permit pricing and production quantity, respectively (Du et al. 2013,0). Du, et al. considered a supplier and a manufacturer in a Stackelberg game framework, where the emission cap is allocated to the manufacturer by the government. Their findings revealed that optimal production and the manufacturer's profit had a positive relationship with the emissions cap increment, while the supplier's profit had a negative relation to the emission cap increment (Du et al. 2013). Du, et al., On the other hand, illustrated that the supplier began the first step considering the high permit price inspired the manufacturer to reduce the production quantity to satisfy the imposed emissions cap which resulted in the supplier's profit deduction (Du et al. 2015).

This paper investigates the impact of $C\&T$ on players' decisions and profits. The channel consists of a manufacturer and a retailer in a Stackelberg game, and the manufacturer is the leader. In our market, the demand for a perishable commodity is stochastic and dynamic and a function of historical retail prices. The demand function has a distribution that may change over time. However, it is often improbable to have complete information about the distribution either because comprehensive information is not available, or it is too costly to obtain. A distributional-robust (DR) approach assists in coping with this kind of incomplete information. The expected profit for the retailer is replaced by a lower (weak) bound relative to the obtainable value with complete knowledge of the distribution. In our proposed framework, future demands are influenced by historical price choices. This effect operates as a kind of market memory, and it adds a property to dynamic demand models reflecting a fundamental aspect of many real-world markets. Consequently, there are opportunities for strategic pricing aimed at shaping demand in subsequent periods.

The players sign a single contract covering all associated decisions for all periods. Compared to multi-periodic contracts, a single contract optimization requires monitoring all decision variables and their effect on each period simultaneously. The difference between the value of single and periodic contracts provokes the players to select a single contract over periodic ones (Fakhrabadi & Sandal 2023a).

If the retailer faces over-ordering, the leftovers might be salvaged or discarded. It means the retailer carries the demand stochasticity and the manufacturer only feels it through the quantity ordered. Even though, after supplying this order, the manufacturer does not observe any risk. Hence, to split the risk of overordering, the manufacturer offers a non-negative buyback value at each period for unsold items. This transfers part of the risk to the manufacturer. Hence, the manufacturer decides the wholesale and buyback prices, and the retailer decides the retail price and the order quantities. In short, the contributions of this paper include:

- Addressing the multi-period DR Supply Chain with a single contract under $C\&T$ regulation.
- Determining wholesale and buyback prices by the manufacturer, and retail prices and order quantities by the retailer for all periods.
- Employing price-dependent and dynamic demands where current demand depends on the price history as well as the current price.
- Obtaining optimal buyback values and risk sharing in the presence of strategic pricing opportunities.

3.1.2 Buyback Contracts

Buyback contract is prevalent in many commodities such as fashion apparel, books, and CDs. The mechanism operates such that the channel members deal in a single contract wherein the manufacturer provides all wholesale and buy-back prices. Contingent upon this information, the retailer decides on all retailer/market prices and order quantities. This may encourage the retailer to order more while sharing the demand uncertainty with the manufacturer (Qin et al. 2021, Xue et al. 2019). Otherwise, with no buyback contract, only the retailer is directly facing the uncertainty of demand, while the manufacturer only senses it through the order quantity (Azad Gholami et al. 2019). The buyback contract shares the risk of demand stochasticity between the upstream (manufacturer) and downstream (retailer) of the channel and improves the efficiency of the channel (Qin et al. 2021).

For a perishable good, at the end of each period, the unsold items are to be salvaged at a lower price, bought back by the manufacturer, or sent to the destruction center at manufacturer cost (at buyback price). When the manufacturer offers a buyback price, the retailer is incentivized to order more, and this may increase the manufacturer's profit. Inversely, without a buyback contract, the retailer may order less. Finding optimal buyback prices for a multi-periodic problem can be a challenge due to the nestedness caused by the price history-dependent demand (Azad Gholami et al. 2019).

Hou et al. studied coordination between one manufacturer and two suppliers in the presence of demand uncertainty and supply risks. They study a firm with two sources of the same product, a main and a backup, where the former is cheaper but is accompanied by disruption risks. They argued that the buyer benefits from a backup supplier through a buyback policy to deal with the risks (Hou et al. 2010). Wu examined the effect of the buyback contract (as a parameter) on retail price, order quantity, and wholesale price in a vertical integration case (chain optimizing) and a Stackelberg game. Their single-period formulation revealed that buyback contracts can yield a higher profit in both approaches (Wu 2013). Wei and Tang analyzed the buyback contract as a risk-sharing tool in a single-period Stackelberg game and compared it with the chain maximizing output and found that supply chain profit enhanced while using the buy-back strategy (Wei & Tang 2013). One manufacturer and two competing retailers in the Xu et al. study illustrated the value of buyback contracts. They created three scenarios as a buyback contract is offered to neither one, one, or both retailers with a price-dependent static demand. They indicated that offering a buyback contract to both retailers benefits all channel members even in high-level competition (Xue et al. 2019). In another attempt to optimize the supply channels with buyback contracts, Azad Gholami et al. considered a multi-periodic channel with delayed information. Their Stackelberg game compromised a manufacturer and a retailer in a multi-periodic setting. They found that too generous a buyback price can decrease the expected profit for the retailer and create a sub-optimal profit for the manufacturer as well (Azad Gholami et al. 2019).

Qin et al. built a supply chain with buyback contracts and fairness concerns under stochastic demand and employed the Bayesian theorem. Their findings indicated that both the retailer's first order quantity and total order quantity decreased with the wholesale price and increased with the buyback price (Qin et al. 2021). Momeni et al. Investigated a buyback coordination mechanism to encourage the channel to participate in operations regeneration to reuse the expired products in other productions. Their results illustrated that the optimal solution could happen only if the revenue of a reused product in addition to the saving on its disposing cost, was greater than its reproducing cost (Momeni et al. 2022). Gong et al. analyzed inventory management where the demand arrives continuously with a drifted Brownian motion and buyback contract. They found that the supplier usually does not benefit from a low buyback price because the optimal policy is conservative when the buyback price is low. It leads to a lower chance of the products to be expired and hence the rate of profit is not affected by the buyback price (Gong et al. 2022). We embed the buyback contract as a decision variable into the manufacturer optimization problem to share the demand risks and increase fairness. The manufacturer's decision variables then are wholesale and buyback prices and those of the retailer are the retail price and order volume.

3.1.3 Demand Structure

Our demand is structured as a dynamic function in a multi-periodic setting. It can be of different forms in each period. The time horizon consists of n discrete intervals (referred to as periods). Considering an arbitrary period k when $k \in \{1, \ldots, n\}$, the general form of demand is given as

$$\begin{aligned} D_k(\vec{\mathbf{r}}_k) &= \mu_k(\vec{\mathbf{r}}_k) + \sigma_k(\vec{\mathbf{r}}_k)\varepsilon_k \\ \text{where } \vec{\mathbf{r}}_k &= (r_i, \ldots, r_n), \quad \forall i \in \{1, \ldots, n\}. \end{aligned} \tag{3.1}$$

The mean μ_k and standard deviation σ_k are known functions of retail price history. The stochastic part of the demand ε_k is normalized to have a mean and standard deviation of 0 and 1 and they are independent of each other (between periods). This problem can be solved when the distribution of demand is known (Fakhrabadi & Sandal 2023*b*). This paper investigates situations with incomplete demand information because it is either impossible to obtain all the information or it is too costly. The distributional-robust (DR) approach for a multi-periodic price history-dependent problem is introduced in a seminal paper by Fakhrabadi and Sandal, 2023 (Fakhrabadi & Sandal 2023*b*). We provide more information regarding DR approach formulation in section 3.2.

3.2 Model Formulation

Notation

$w = \{w_1, \ldots, w_n\}$	Wholesale price (decision variable)
$b = \{b_1, \ldots, b_n\}$	Buyback price (decision variable)
$r = \{r_1, \ldots, r_n\}$	Retail price (decision variable)
$q = \{q_1, \ldots, q_n\}$	Order quantity (decision variable)
$c^m = \{c^m_1, \ldots, c^m_n\}$	Manufacturer cost
$c^r = \{c^r_1, \ldots, c^r_n\}$	Retailer cost
$\beta = \{\beta_1, \ldots, \beta_n\}$	Discount factor over individual periods
$D = \{D_1, \ldots, D_n\}$	Demand $D_k = \mu_k(r) + \sigma_k(r)\varepsilon_k$
$\mu = \{\mu_1, \ldots, \mu_n\}$	Mean of demand
$\sigma = \{\sigma_1, \ldots, \sigma_n\}$	Standard deviation of demand
$\varepsilon = \{\varepsilon_1, \ldots, \varepsilon_n\}$	Stochastic and independent drivers of the demand
$s = \{s_1, \ldots, s_n\}$	Salvage price/discarding cost
$k \in \{1, \ldots, n\}$	Time or period
$q^c = \{q^c_1, \ldots, q^c_n\}$	Maximum allowance for production
$u = \{u_1, \ldots, u_n\}$	The unit cost of buying allowances for producing extra

$v = \{v_1, \ldots, v_n\}$	The unit price selling unused allowances
$\pi^m = \{\pi_1^m, \ldots, \pi_n^m\}$	Manufacturer's profit (present value)
$\pi^r = \{\pi_1^r, \ldots, \pi_n^r\}$	Retailer's profit (present value)

We have adopted the short notation in this paper: For any vectors A and B, we define $AB = BA = \{A_i B_i\}_{i=1}^n$.

To address our proposed model, the algorithm is built for a perishable product in a multi-period Stackelberg game. In this game, the upstream (manufacturer) is the leader and the downstream (retailer) follows him. The manufacturer, first, declares the wholesale and buyback prices, and then the retailer decides on the retail prices and order quantities. The unsold items cannot be restored at the end of each period and sold at the next period. Therefore, for the retailer, any unsold item is discarded at cost s, salvaged at price s, bought back by the manufacturer at price b, or the manufacturer pays cost b to the retailer to discard the unsold items at price s. All variables and parameters remain constant within each period but may vary between periods. The players agree on a single contract where they can observe their decisions and the consequences across all periods simultaneously and improve their decisions. The nucleus's objective is to ensure the attainment of the highest possible value, and a single contract creates a higher value for the channel compared to a periodic contract (Fakhrabadi & Sandal 2023*a*).

The $C\&T$ policy structures this channel where the manufacturer is constrained with a maximum allowance of pollution generating, but he is permitted to buy the extra allowance required or sell the surplus allowance he has not consumed. This trade can be categorized either as an income (when selling surplus allowance) or as an additional cost (when buying extra allowance) which may increase or decrease the channel's profit. The prices of buying and selling the allowance can be unequal.

For simplicity in exposition, we drop the time index k whenever an equation is held by just adding subscript k to all quantities involved. Since the channel consists of a manufacturer and a retailer, the bilevel optimization algorithm maximizes the manufacturer's value subject to the retailer's value maximization. The algorithm allows only non-negative values and variables; however, the profit may be negative for a period. The parameters and functions can vary at each period. The manufacturer's operation is constrained to a maximum production allowance, q^c, where he can trade it. The manufacturer's profit function is

$$\pi^m = (w - c^m)q - b(q - D)^+ - u(q - q^c)^+ + v(q^c - q)^+, \tag{3.2}$$

where u is the purchase price and v the selling price of the production allowance. The manufacturer purchases production allowance when $(q^* - q^c)^+$ is non-zero and sells when $(q^c - q^*)^+$ is non-zero (q^* denotes the optimal order quantity). In the first case, u is a unit cost and in the second case, v is a unit income. The manufacturer then expects to make a profit of

$$E[\pi^m] = (w - c^m - b)q + b\mu - bE(D - q)^+ - u(q - q^c)^+ + v(q^c - q)^+. \tag{3.3}$$

When the distribution of the demand is known, Eq. (3.3) can be simplified by $E(D - q)^+ = \int_\Omega (x - q) f(x)\, dx$, where $f(x)$ is the probability density function of demand D with compact support on Ω.

With a buyback contract, the demand stochasticity permeates the manufacturer's profit in addition to the retailer's profit. The manufacturer decides on wholesale price and buyback values. Even though a high buyback price may encourage the retailer to order more, a too-generous buyback price is detrimental to the manufacturer's expected profit.

The inner level optimization occurs with the retailer's profit function as,

$$\pi^r = r \min(D, q) + (b + s)(q - D)^+ - wq - c^r q. \tag{3.4}$$

The terms in Eq. (3.4) depict the revenue, unsold items income, the purchase cost of the order, and the retailer's other costs for units ordered, respectively. The retailer's expected profit is

$$E[\pi^r] = (r - s - b)\mu - (w + c^r - b - s)q - (r - s - b)E(D - q)^+, \tag{3.5}$$

where $r > b + s$ due to economic feasibility. The key conclusions are summarized in the following propositions.

Proposition 3.2.1. *The bi-level optimization in general is (from Eqs.* (3.3) *and* (3.5)*)*

$$\begin{gathered} \max_{(w,b)\in W} JD^m \quad s.t. \quad (r,q) = \arg\max_{(r,q)\in R} JD^r, \\ where \quad JD^x = \alpha_1 E[\pi_1^x] + \alpha_2 E[\pi_2^x] + \ldots + \alpha_n E[\pi_n^x] \quad for \quad x \in \{m, r\}, \\ and \quad \alpha_k = \beta_1 \cdot \beta_2 \cdot \ldots \cdot \beta_k. \end{gathered} \tag{3.6}$$

β_k represents the discounting factor for the period k, and m and r correspond to the manufacturer and retailer, respectively. W and R are constraints on the manufacturer and retailer.

The distributionally robust (DR) bi-level optimization is

$$\begin{gathered} \max_{(w,b)\in W} J^m \quad s.t. \quad (r,q) = \arg\max_{(r,q)\in R} J^r, \\ where \quad J^x = \alpha_1 \Pi_1^x + \alpha_2 \Pi_2^x + \ldots + \alpha_n \Pi_n^x \quad for \quad x \in \{m, r\}, \end{gathered} \tag{3.7}$$

and Π^m and Π^r, where $r > b + s$, are players' expected profits' tight lower bounds for the *case with full information;*

$$E\left(\pi^m(q,w,b)\right) \geq (w-c^m-b)q + b\mu - \frac{b}{2}\left(\sqrt{\sigma^2+(q-\mu)^2}-q+\mu\right) - \\ u(q-q^c)^+ + v(q^c-q)^+ \equiv \Pi^m \tag{3.8}$$

$$E\left(\pi^r(q,w,b,\vec{\mathbf{r}})\right) \geq (r-s-b)\mu - (w+c^r-b-s)q - \\ \frac{(r-b-s)}{2}\left(\sqrt{\sigma^2+(q-\mu)^2}-q+\mu\right) \equiv \Pi^r. \tag{3.9}$$

Hence, $J^x \leq JD^x$, i.e., both DR players payoffs are a tight lower bound for the case with full information.

Proof. See Appendix A. □

Proposition 3.2.2. *The following holds in a DR framework:*

For any feasible decision set $(w_k, \vec{\mathbf{r}}_k, b_k)$ at period k, the optimal order quantity is

$$q_k(w_k,\vec{\mathbf{r}}_k,b_k) = \mu_k(\vec{\mathbf{r}}_k) + \sigma_k(\vec{\mathbf{r}}_k)\Lambda_k(w_k,r_k,b_k), \\ \Lambda_k = \frac{2\eta_k-1}{2\sqrt{\eta_k(1-\eta_k)}}, \quad \eta_k = \frac{r_k-w_k-c_k^r}{r_k-s_k-b_k}. \tag{3.10}$$

Proof. See Appendix B. □

Proposition 3.2.3. *The optimal order quantity is increasing in buyback price.*

Proof. Following from Eq. (3.10),

$$\frac{\partial q}{\partial b} = \frac{\sigma\eta}{4(r-s-b)(\eta(1-\eta))^{\frac{3}{2}}}. \tag{3.11}$$

□

3.3 Numerical Implementation

In this section, we offer illustrative instances of the solution algorithm expounded in Section 3.2 From Eq. (3.1), $D_k(\vec{\mathbf{r}}_k) = \mu_k(\vec{\mathbf{r}}_k) + \sigma_k(\vec{\mathbf{r}}_k)\varepsilon_k$ we exemplify a price history-dependent demand where the retail price of period k influences periods $k, k+1$, and $k+2$, i.e., $D_k = D_k(r_{k-2}, r_{k-1}, r_k)$. In our numerical illustrations, we choose the following form of the demand.

$$D_k(r_{k-2},r_{k-1},r_k) = \Phi_k(r_{k-2},r_{k-1})\tilde{\mu}_k(r_k) + \Phi_k(r_{k-2},r_{k-1})\tilde{\sigma}_k(r_k)\varepsilon_k, \\ \Phi_1 \equiv 1, \quad \Phi_2 = e^{\gamma_2(R-r_1)}, \quad \Phi_k = e^{\gamma_k(r_{k-2}-r_{k-1})} \quad \text{for} \quad k \in \{3,\dots,n\}, \\ \tilde{\mu}_k(r_k) = 100-2r_k, \quad \text{and} \quad \tilde{\sigma}_k(r_k) = 0.2\tilde{\mu}_k(r_k). \tag{3.12}$$

The parameters γ and R represent the strength of a current deviation to the future demand and reference retail price respectively. This choice aims to streamline complexity while enabling a comprehensive exploration of the independent role also the interplay between buyback, $C\&T$, and the effective price history.

A multi-period model sans the price history effect examines a recurring game scenario; To this extent, all periods adhere to the same optimal policy, leaving no opportunity for strategic pricing maneuvers. In contrast, the model incorporating the influence of the historical prices not only steers the channel towards outcomes that mirror reality but also exhibits the potential to enhance the channels' value. This elevation is facilitated by its ability to stimulate future demand through the strategic reduction of current prices.

The parameters set of $c_k^m = 10, c_k^r = 2, \beta_k = 0.97, R = 40, \gamma_k = 0.02, s_k = 0, k \in \{1, \ldots, n\}$ and $n = 12$ is used in upcoming cases. More information about the parameters and other functions, employed in examples, are provided in the next sections.

3.3.1 The Effect of Buyback Contracts

We initiate our numerical exploration by introducing unconstrained models that encompass both scenarios with and without a buyback contract (the model with buyback is named the base model later in this paper). In this context, we operate under the assumption of an absence of environmental constraints while the participating entities remain engaged in a buyback contract (and non-buyback) that accounts for historical price influences. As outlined in section 3.2, our approach encompasses dependent bilevel optimization, including manufacturer optimization at the outer level and retailer optimization at the inner level. For this example, the player's expected profits with a buyback contract from the expressions in Eqs. (3.8) and (3.9) are

$$\begin{aligned}
&\Pi_k^m(w_k, b_k, \vec{\mathbf{r}}_k) = (w_k - c_k^m) q_k(w_k, b_k, \vec{\mathbf{r}}_k) - \\
&\frac{b_k}{2} \left(\sqrt{\sigma_k^2(\vec{\mathbf{r}}_k) + (q_k(w_k, b_k, \vec{\mathbf{r}}_k) - \mu_k(\vec{\mathbf{r}}_k))^2} + q_k(w_k, b_k, \vec{\mathbf{r}}_k) - \mu_k(\vec{\mathbf{r}}_k) \right).
\end{aligned} \tag{3.13}$$

$$\begin{aligned}
\Pi_k^r(w_k, b_k, \vec{\mathbf{r}}_k) &= (r_k - s_k - b_k)\mu_k(\vec{\mathbf{r}}_k) - (w_k + c_k^r - s_k - b_k) q_k(w_k, b_k, \vec{\mathbf{r}}_k) - \\
&\frac{(r_k - s_k - b_k)}{2} \left(\sqrt{\sigma_k^2(\vec{\mathbf{r}}_k) + (q_k(w_k, b_k, \vec{\mathbf{r}}_k) - \mu_k(\vec{\mathbf{r}}_k))^2} - \right. \\
&\left. q_k(w_k, b_k, \vec{\mathbf{r}}_k) + \mu_k(\vec{\mathbf{r}}_k) \right).
\end{aligned} \tag{3.14}$$

The non-buyback results are derived from,

$$\Pi_k^m(w_k, \vec{\mathbf{r}}_k) = (w_k - c_k^m) q_k(w_k, \vec{\mathbf{r}}_k) \tag{3.15}$$

$$
\begin{aligned}
&\Pi_k^r(w_k, \vec{\mathbf{r}}_k) = (r_k - s_k)\mu_k(\vec{\mathbf{r}}_k) - (w_k + c_k^r - s_k)q_k(w_k, \vec{r}_k) - \\
&\frac{(r_k - s_k)}{2}\left(\sqrt{\sigma_k^2(\vec{r}_k) + (q_k(w_k, \vec{\mathbf{r}}_k) - \mu_k(\vec{\mathbf{r}}_k))^2} - q_k(w_k, \vec{\mathbf{r}}_k) + \mu_k(\vec{\mathbf{r}}_k)\right).
\end{aligned} \tag{3.16}
$$

Using Eqs. (3.13) and (3.14) for the model with buyback contract and Eqs. (3.15) and (3.16) for the non-buyback model, and parameter set $\{c_k^m, c_k^r, \beta_k, R, \gamma_k, s_k, n\}$, the players' profits are illustrated in Figure 3.1.

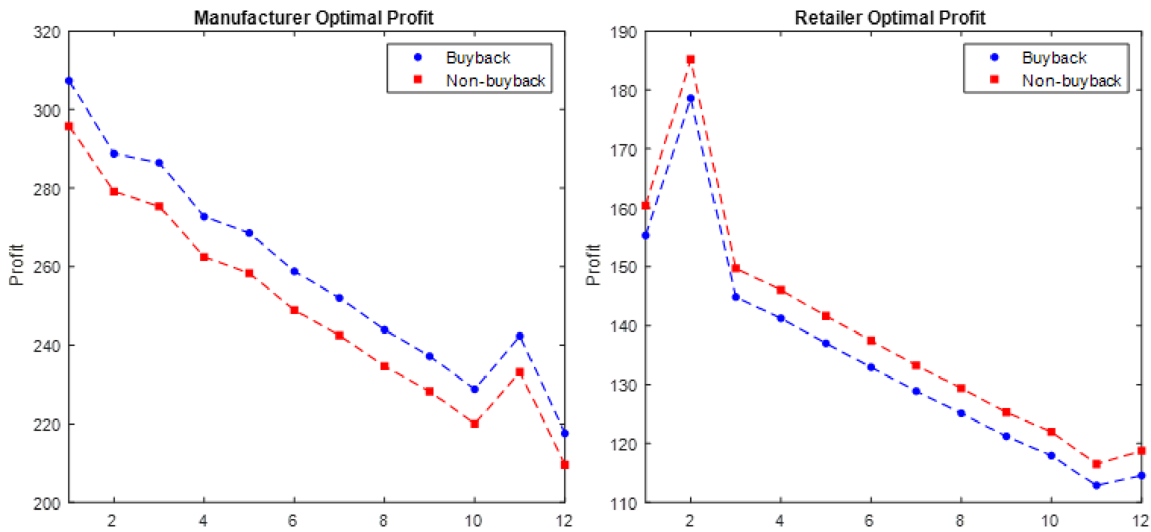

Figure 3.1: Optimal profits, the models with and without buyback contracts

In this example, while the manufacturer obtains a higher value through a buyback contract, the retailer pays the cost of carrying lower risk;

$$
\begin{aligned}
J^m_{Buyback} &= 3105, \quad J^r_{Buyback} = 1611, \\
J^m_{Non-buyback} &= 2988, \quad J^r_{Non-buyback} = 1666.
\end{aligned}
$$

The manufacturer observes a 4% gain, while the retailer experiences a 3.3% loss. This outcome becomes evident upon examining the decisions illustrated in Figure 3.2, denoted as (r^*, w^*, b^*). The manufacturer strategically introduces a non-zero buyback price, which is paired with a higher wholesale price. This approach compensates for the additional incurred risk due to the buyback arrangement promoting the retailer to respond by raising the retail price and the order quantity (Figure 3.3). This dynamic reveals that the buyback pricing in this scenario stimulates the retailer to ramp up their order volume.

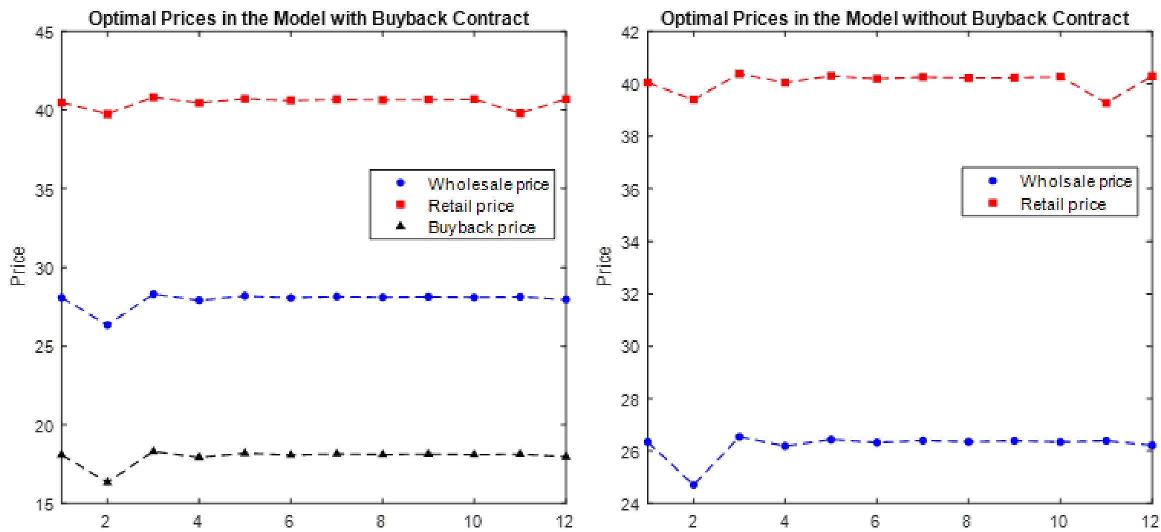

Figure 3.2: Optimal prices, the models with and without buyback contracts

In a model with the buyback contract, the wholesale prices operate within the range of $[26.3, 28.2]$ while the corresponding buyback prices fall within the span of $[16.3, 18.2]$. Initially, this buyback price- equivalent to $64 - 65\%$ of the wholesale price- might appear overly generous or surprising. However, when considering the progression of wholesale price increments compared to the model lacking the buyback contract, it becomes evident that the cost associated with the buyback is offset by an average wholesale price increase of $\simeq 7\%$.

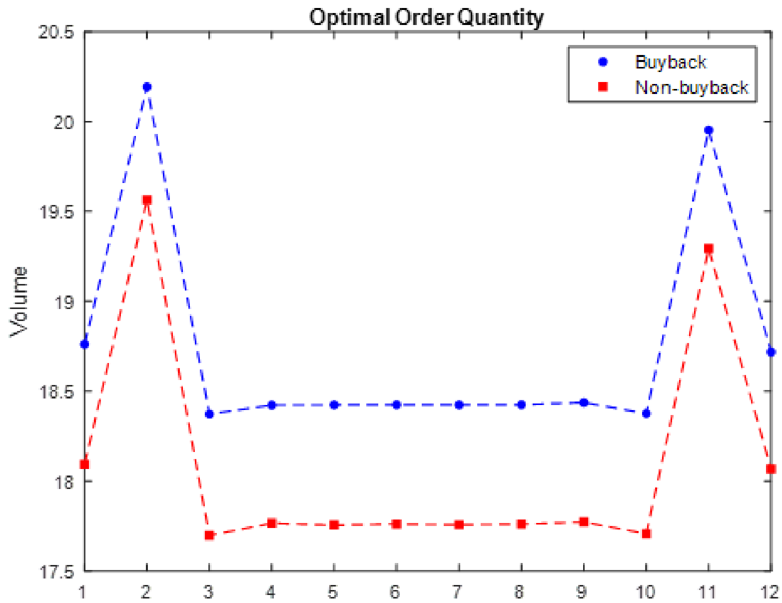

Figure 3.3: Optimal order quantity, the models with and without buyback contracts

3.3.2 Buyback and Cap-and-Trade Policy versus only Buyback

Within the framework of the $C\&T$ policy, the cost of procuring a production allowance commonly surpasses its sales price. We have assumed that any excess production allowance cannot be rolled over or utilized in subsequent periods. Hence, when the manufacturer encounters an excess allowance situation and determines that the optimal solution falls below the allowed capacity, the prudent course of action is to sell the surplus. Failing to do so would result in the forfeiture of potential revenue.

To illustrate this example, Eqs. (3.8) and (3.9) are considered. The parameter configuration $\{c_k^m, c_k^r, s_k, \beta_k, R, \gamma_k, n\}$ is the same as in the previous section (the base model, only with buyback). The selling and buying prices used for this case are $u_k = 1.5$, $v_k = 1$. The insights drown from this example are embodied in Figure 3.4, which elucidates the profit trajectories of two distinct models: the model subject to the constraints of the $C\&T$ policy and buyback contract and the model only with buyback contract.

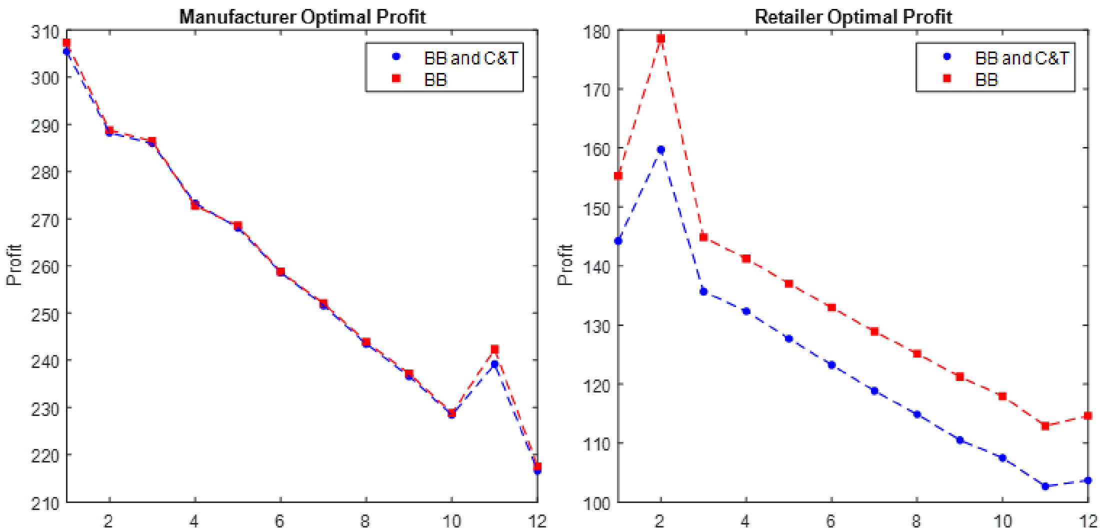

Figure 3.4: Optimal profits, buyback with $C\&T$ policy model (CT) vs. buyback only (BB)

Within this scenario, the application of the $C\&T$ policy results in a reduction of the players' values leading to $J_{CT}^m = 3096$, $J_{CT}^r = 1481$. In contrast, without the influence of the $C\&T$ policy, the values are different, with $J_{no-CT}^m = 3105$, $J_{no-CT}^r = 1611$.

Interestingly, despite the overall diminishment in value, the manufacturer secured higher profits during period 4 under the $C\&T$ policy. However, the shift in strategy translates to a marginal 0.29% decrease in the manufacturer's overall value, while the retailer experiences a more substantial decline of 8.1%. These values are rooted in the price dynamics and order quantity delineated in Figures 3.5, 3.6,

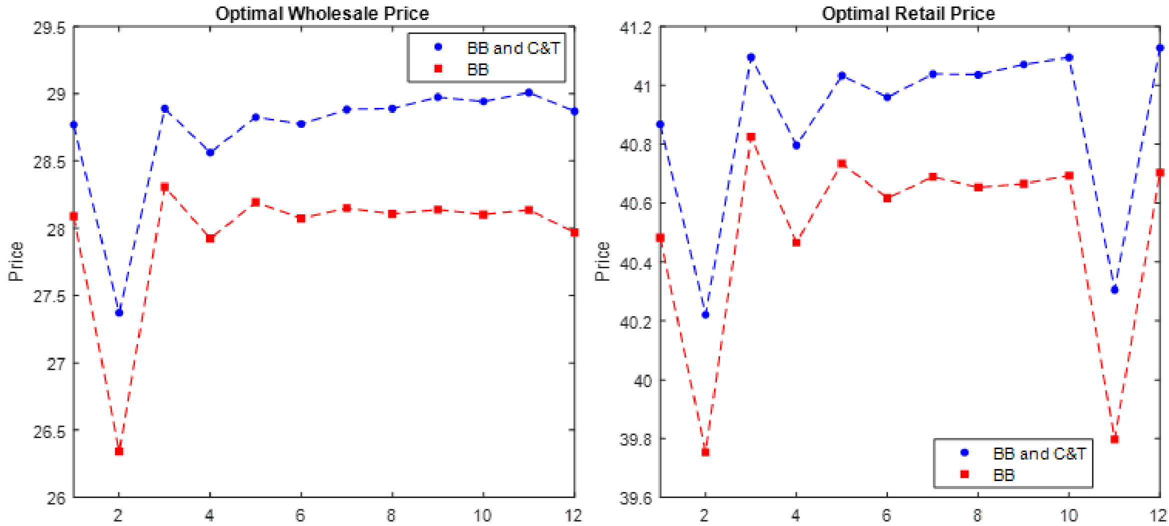

Figure 3.5: Optimal prices with and without $C\&T$ constraint

where the imposition of the capacity constraint is met with heightened prices for both players. The manufacturer price spectrum which initially ranged from $[26.3, 28.3]$ in the model only with a buyback contract, undergoes a shift to $[27.4, 29]$ in the presence of the $C\&T$ policy's constraints in addition to the buyback contract. Similarly, the retailer price span, initially $[39.8, 41]$ in the model only with buyback, adjusted to $[40.2, 41.1]$ under the influence of the $C\&T$ policy plus buyback contract.

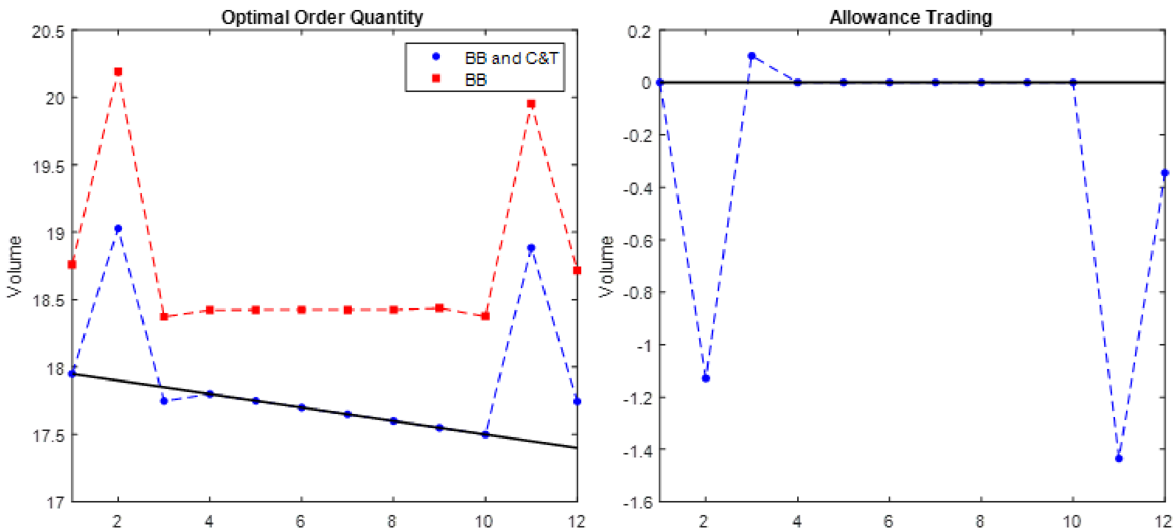

Figure 3.6: Optimal volumes with and without $C\&T$ constrained

Observing the combined insights offered by both plots in Figure 3.6, the optimal channel behavior becomes evident. This optimal configuration emerges when the manufacturer chooses to sell the surplus proportion of their production allowance during periods 1, 3, and 5-7 while opting to buy during the remaining periods. The dynamic is depicted in the right plot of Figure 3.6, where positive values correspond to the selling volume and negative numbers denote the buying volume.

3.3.3 Price Sensitivity of the Cap-and-Trade Policy

The pricing structure within the $C\&T$ policy wields a substantial influence over the strategic choices undertaken by players in the channel. One significant implication emerges when the selling price surpasses $w - c^m - b$. In this scenario, if production is not mandated, the manufacturer may opt to sell production allowance more than engaging in production activities. Conversely, the impact of a low marginal purchase price lies in its potential to stimulate heightened production levels within the channel provided this aligns with optimality. Employing the base parameters set $\{c_k^m, c_k^r, s_k, \beta_k, R, \gamma_k, n\}$, profits are illustrated in Figure 3.7.

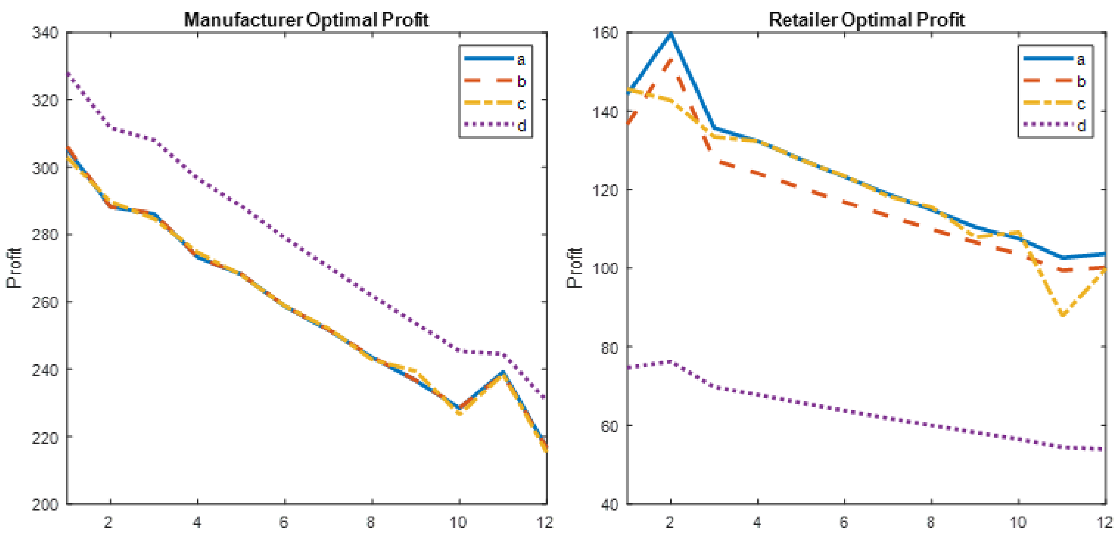

Figure 3.7: Optimal profits

The scenarios mentioned yield values

a:	v = 1, u = 1.5	3095.7	1481
b:	v = 2, u = 2	3096.4	1412
c:	v = 1, u = 8	3094	1444
d:	v = 10, u = 12	3318	763
Base model:	v = 0, u = 0	3105	1611

Within this context, our base model serves as the reference point, characterized by $J^m = 3105$ and $J^r = 1611$.

Scenario 'd' emerges as advantageous for the manufacturer, although it conversely diminishes the retailer's value to the lowest point (compared to the other scenarios). In contrast, scenario 'a' presents the manufacturer with the lowest value while elevating the retailer's position within other scenarios. To provide a visual understanding, we refer to Figures 3.8, 3.9, where the optimal order quantities and prices are depicted.

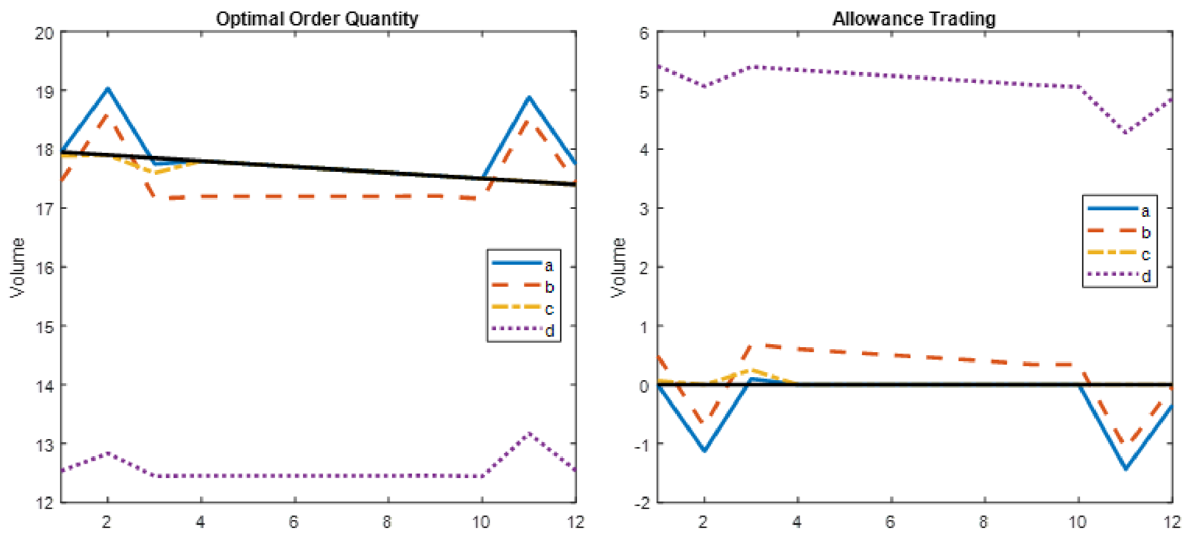

Figure 3.8: Optimal Volumes

The maximum allowance is depicted by a black line in the optimal order quantity figure (left). Following the pattern of ordering, the right figure showcases the trading type.

Notably, in scenario 'd' the allowance trading prices act as an incentive for the manufacturer to adopt a higher pricing strategy throughout each period. This strategic move effectively curtails the retailer's order volume, thereby aligning with the intent to limit capacity allowance consumption. Consequently, the new optimal decisions (r^*, w^*, q^*) along with the selling profit of production allowance make a higher profit for the manufacturer (in all scenarios and baseline model). Elevating the trading prices inevitably leads to a corresponding increase in the manufacturer's payoff while concurrently diminishing the retailer's payoff.

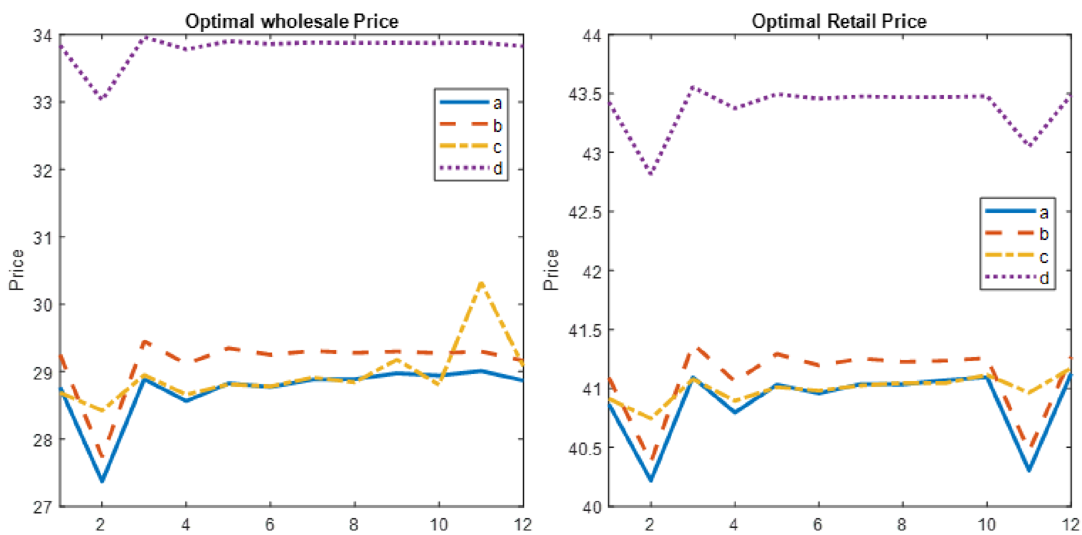

Figure 3.9: Optimal prices

3.4 Concluding Remarks

Our study delves into a multi-period Stackelberg game imbued with distributional-robust price-history dependent demand, unraveling intricate dynamics within the context of our proposed model. This innovative framework encapsulates a unified contract strategy (single contract) that effectively addresses all periods' decisions simultaneously. Additionally, the introduction of a buyback contract represents a strategic risk-sharing mechanism, where the manufacturer also undertakes the uncertainty inherent in demand fluctuations. To further enhance its environmental impact, our model embraces a Cap-and-Trade ($C\&T$) policy, serving to regulate pollution.

The exploration unfolds through numerical examples that illuminate the profound impact of a buyback contract on channel results. Moreover, we delve into the intricate interplay of trading prices within the $C\&T$ policy on channel behavior.

For instance, the model elucidates that the viability of a generous buyback price can hinge on the probability of leftover inventory—referring to the scenario where the retailer orders less than the demand mean. In this context, a seemingly high buyback price, which comes initially along with an elevated wholesale price and order volume, can ultimately generate heightened profits for the manufacturer.

Importantly, the interplay between the players through a buyback contract doesn't universally induce increased order volume from the retailer. because a lower risk for the retailer is fulfilled by a higher wholesale price. Thus, optimizing the buyback price is a crucial strategic consideration.

The model also illuminates that the production capacity constraints enforced by the $C\&T$ policy do not uniformly impose restrictions. Instead, their impact shifts based on the prevailing prices of production allowances. When the selling price remains below the manufacturer's profit from production, the allowance trade-off fails to yield higher profits for the players compared to the baseline model. However, as the selling price of the production allowance aligns with and exceeds the manufacturer's profit from production, the manufacturer reaps amplified profits from production allowance trading, while the retailer consistently faces a disadvantageous position.

Appendix A

To compute the expected value of the retailer and manufacturer profits, from Eqs. (3.3) and (3.5), the value of $E(D-q)^+$ is required. Referring to the paper of Fakhrabadi and Sandal (Fakhrabadi & Sandal 2023*b*),

$$(D-q)^+ = \frac{1}{2}\left\{|D-q|+(D-q)\right\}, \tag{3.17}$$

$$E(D-q)^{+}=\frac{1}{2}\left\{E[|D-q|]+E(D-q)\right\}. \tag{3.18}$$

From Cauchy-Schwartz inequality

$$E\left[|D-q|\right]\leq\sqrt{E\left[(D-q)^{2}\right]}=\sqrt{\sigma^{2}+(q-\mu)^{2}}. \tag{3.19}$$

Therefore

$$E(D-q)^{+}\leq\frac{\left(\sqrt{\sigma^{2}+(q-\mu)^{2}}-q+\mu\right)}{2}. \tag{3.20}$$

The inequality in Eq. (3.20) introduces a tight lower bound on expected retailer profit for any distribution with the same μ and σ. Hence Eqs. (3.3) and (3.5) are recast as

$$E[\pi^{m}]\geq(w-c^{m}-b)q+b\mu-\frac{b}{2}\left(\sqrt{\sigma^{2}+(q-\mu)^{2}}-q+\mu\right)-u(q-q^{c})^{+}+v(q^{c}-q)^{+}\equiv\Pi^{m}, \tag{3.21}$$

$$E[\pi^{r}]\geq(r-b-s)\mu-(w+c^{r}-b-s)q-\frac{(r-b-s)}{2}\left(\sqrt{\sigma^{2}+(q-\mu)^{2}}-q+\mu\right)\equiv\Pi^{r}. \tag{3.22}$$

There is at least one distribution (namely the worst distribution) that Eqs. (3.21) and (3.22) hold with equality.

Appendix B

Notice that for the economic feasibility ($r > b+s$), Π_k^r is strictly concave in q_k. For any feasible set of $(w_k, \vec{\mathbf{r}}_k, b_k)$, the unique nonnegative solution of $\frac{\partial \Pi_k^r}{\partial q_k}=0$ is the global maximum given by

$$q_k=\mu_k(\vec{\mathbf{r}}_k)+\sigma_k(\vec{\mathbf{r}}_k)\Lambda_k,\quad \Lambda_k=\frac{2\eta_k-1}{2\sqrt{\eta_k(1-\eta_k)}},\quad \eta_k=\frac{r_k-w_k-c_k^r}{r_k-s_k-b_k}. \tag{3.23}$$

Since $\max\limits_{q}\left(\alpha_1\Pi_1^r+\alpha_2\Pi_2^r+\cdots+\alpha_n\Pi_n^r\right)\leq\max\limits_{q_1}\left(\alpha_1\Pi_1^r\right)+\cdots+\max\limits_{q_n}\left(\alpha_n\Pi_n^r\right)$ holds with equality by Eq. (3.23), the result is guaranteed to yield the maximum.

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