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# Price-dependent profit sharing as an escape from the Bertrand paradox

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## **JEL classification:**

**Keywords:** Profit-sharing, vertical restraints, investments, competition.

**Abstract:** In this paper we show how an upstream firm can prevent destructive competition among downstream firms producing relatively close substitutes by implementing a price-dependent profit-sharing rule. The rule also ensures that the downstream firms undertake investments which benefit the industry in aggregate. The model is consistent with observations from the market for content commodities distributed by mobile networks.

# 1 Introduction

The Bertrand paradox may provide a plausible explanation why the majority of the content commodities on the Internet are offered for free (marginal costs). The rival is just “one click away”, and competing content providers have strong incentives to undercut each other as long as there are positive profit margins. In contrast, we observe that prices for mobile phone content commodities like ring tones, football goal alerts and jokes are well above marginal costs (the sales value of such services in Norway in 2006 was twice as high as the total value of Internet ads). One potential explanation why the Bertrand paradox is not observed for such goods, is the price-dependent profit-sharing rule used by some upstream mobile access providers. The rule implies that the upstream firms charge a share of the end-user price per unit of content instead of for instance a unit wholesale price from the content providers. The crucial feature of this rule is that the share accruing to a given content provider is increasing in the end-user price. The table below shows the profit-sharing rule used by the dominant Norwegian mobile operator Telenor; if a content provider sells his good for 1 NOK he receives 45 % of the revenue, while he receives 80 % if he sells the good for 70 NOK.<sup>1</sup>

End-user price (NOK)	1.0	1.5	3	5	10	20	70
Share to the content provider	45%	54.%	62%	66%	68%	70%	80%

In the formal model we consider an upstream firm selling an input (access) to downstream firms producing differentiated services. The upstream firm determines the access conditions, while the downstream firms decide end-user prices and investments in for instance marketing. We show that by using a price-dependent profit-sharing rule, the upstream firm induces the retailers to behave as if demand has become less price elastic. A price-dependent profit-sharing rule is sufficient to

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<sup>1</sup>In addition to this revenue-sharing rule, the content providers are charged a fixed fee (but no unit wholesale price). Strand (2004) emphasizes that the revenue-sharing scheme creates incentives to promote new services. The Norwegian business model is now widely taken up in Europe and Asia (Strand, 2004).

achieve the vertical integration outcome also in presence of investment spillovers. A fixed fee determines the allocation of the total industry profit.

The upstream firm could alternatively use a combination of resale price maintenance (RPM), a fixed fee and a wholesale price below marginal costs to achieve the vertical integration outcome (see Mathewson and Winter, 1984). The Bertrand paradox is then avoided by indirectly limiting the retailers' strategy choices. The novelty in the above proposal is that the sharing-scheme reduces the undercutting incentives among retailers directly by reducing the perceived price elasticity, and is less likely to raise anti-trust concerns compared to RPM.<sup>2</sup>

## 2 The Model

We consider an upstream firm selling an input to  $n$  downstream firms. The demand curve faced by downstream firm  $i = 1, \dots, n$  is given by  $q_i = q_i(p)$ , where  $p$  is the vector of prices charged by the  $n$  downstream firms. We assume that the demand functions are well behaved and downward sloping in own price ( $\partial q_i / \partial p_i < 0$ ). The consumers perceive the goods sold by the downstream firms as imperfect substitutes ( $\partial q_i / \partial p_j > 0$ ).

Marginal costs both at the upstream and downstream levels are set equal to zero; however, this does not matter for the qualitative results. Hence, we can write total operating profit in the industry as

$$\Pi(p) = \sum_{i=1}^n p_i q_i(p). \quad (1)$$

Below, we consider a two-stage game where the upstream firm at stage 1 determines the conditions for access to the upstream good, and where the downstream firms subsequently compete in prices. In Section 3 we extend the model by allowing the downstream firms to make market-expanding investments.

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<sup>2</sup>Moreover, with RPM the retail prices are not decided by the players with hands on market experience. This may obviously be detrimental to the total channel outcome.

The upstream firm uses a two-part tariff, consisting of a fixed fee  $F$  and a profit-sharing rule. We specify the profit-sharing rule such that downstream firm  $i$  keeps a share  $\beta(p_i)$  of its operating profit, while the upstream firm gets the share  $(1 - \beta(p_i))$ . We later show that  $\beta'(p_i) > 0$ .<sup>3</sup>

### Stage 2

The operating profit of downstream firm  $i$  equals  $\pi_i(p) = \beta(p_i)p_iq_i$ , and at the last stage each downstream firm solves  $p_i^* = \arg \max \pi_i$ . This yields the FOCs

$$\left[ q_i^* + p_i^* \frac{\partial q_i}{\partial p_i} \right] + \frac{\beta'(p_i^*)}{\beta(p_i^*)} p_i^* q_i^* = 0. \quad (2)$$

The second term in (2) would vanish if  $\beta$  were constant ( $\beta' = 0$ ), in which case we would get the standard result that a profit maximizing price  $\hat{p}_i$  satisfies  $[\hat{q}_i + \hat{p}_i \frac{\partial q_i}{\partial p_i}] = 0$ . With  $\beta' > 0$  the second term on the left-hand side of equation (2) is positive, implying that the marginal profit at any given price is higher than if  $\beta' = 0$ . This induces each of the downstream firms to behave less aggressively, and we can state:

**Proposition 1:** *The profit-maximizing prices will be higher for  $\beta'(p_i) > 0$  compared to  $\beta'(p_i) = 0$ .*

By defining  $\varepsilon_{ii} \equiv \frac{p_i}{q_i} \frac{\partial q_i}{\partial p_i}$  as the price elasticity of demand for good  $i$ , we can rewrite (2) as

$$\varepsilon_{ii}^* = - \left( 1 + \frac{\beta'(p_i^*)}{\beta(p_i^*)} p_i^* \right) \quad (3)$$

Equation (3) characterizes the profit-maximizing equilibrium price for firm  $i$ . It is well known that revenue - and thus profit for a firm facing zero marginal costs - other things equal is maximized by choosing a price for which the elasticity is equal to minus one. However, since  $\frac{\beta'(p_i^*)}{\beta(p_i^*)} p_i^* > 0$ , we see from (3) that the profit sharing rule induces the downstream service provider to behave as if the demand has become less price elastic. This confirms that the profit-maximizing prices will be higher if  $\beta' > 0$  than if  $\beta' = 0$ :

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<sup>3</sup>In contrast to the present paper, the literature on revenue-sharing as a vertical restraint conventionally assumes that the revenue share is a constant; i.e.  $\beta' = 0$  (see e.g. Lal, 1990).

**Proposition 2:** *A profit-sharing rule  $\beta'(p_i) > 0$  reduces the perceived elasticity of demand for the downstream firms, making them behave less aggressively.*

In the sequel we assume an isoelastic sharing rule so that  $\beta_i(p_i) = \theta p_i^\lambda$ , where  $\lambda$  is the elasticity parameter determined by the upstream firm at stage 1 (for the moment we treat  $\theta$  as a positive scalar). With this specification we can reformulate (2) and (3) as

$$(1 + \lambda) q_i^* + p_i^* \frac{\partial q_i}{\partial p_i} = 0 \quad (4)$$

$$\varepsilon_{ii}^* = -(1 + \lambda) \quad (5)$$

### Stage 1

The upstream firm will use  $\lambda$  to induce the downstream firms to set the prices that maximize total industry profit. The fixed fee  $F$  is then used as a profit distribution parameter. Thus, we first derive the hypothetical equilibrium under vertical integration (VI). Solving  $p_i = \arg \max \Pi(p)$  yields the FOCs

$$\left[ q_i + p_i \frac{\partial q_i}{\partial p_i} \right] + \sum_{j \neq i} p_j \frac{\partial q_j}{\partial p_i} = 0 \quad (i = 1, \dots, n). \quad (6)$$

The term in the square bracket of (6) measures the marginal profit on good  $i$  and is analogous to the term in the square bracket of (2). The second term of (6) internalizes the horizontal pecuniary externality when products are imperfect substitutes. Let  $\omega_{ji}^p = - \frac{\partial q_j}{\partial p_i} / \frac{\partial q_i}{\partial p_i}$  measure the increased demand for good  $j$  per unit reduction in the demand for good  $i$  when  $p_i$  increases. The higher these ratios, the higher  $p_i$  should be set in order to maximize aggregate industry profit. The challenge for the upstream firm in a vertically separated market structure is to set conditions inducing the downstream firms to internalize this effect at stage 2.

Inserting for  $\omega_{ji}^p$  into (6) we can now characterize industry optimum as

$$q_i + \left[ p_i - \sum_{j \neq i} p_j \omega_{ji}^p \right] \frac{\partial q_i}{\partial p_i} = 0. \quad (i = 1, \dots, n).$$

By imposing symmetry this expression can be reformulated as (with subscript VI for vertical integration)

$$q_{VI} + p_{VI} \left[ 1 - (n - 1) \omega_{ji}^p \right] \frac{\partial q_i}{\partial p_i} = 0. \quad (7)$$

The optimal value of  $\lambda$  ensures that aggregate profit is the same in the vertically separated market structure as in the hypothetical equilibrium with vertical integration. This value can be found by using equations (4) and (7) and setting  $q_i^*/p_i^* = q_{VI}/p_{VI}$ .<sup>4</sup> We then have

$$\lambda = \lambda^* \equiv -1 + \frac{1}{1 - (n-1)\omega_{ji}^p}. \quad (8)$$

Inserting for (8) into (5) we further find

$$\varepsilon_{ii}^* = -\frac{1}{1 - (n-1)\omega_{ji}^p}.$$

If a price reduction of good  $i$  does not affect demand for good  $j$ , we have  $\frac{\partial q_j}{\partial p_i} = \omega_{ji}^p = 0$ . The downstream firms thus choose prices such that  $\varepsilon_{ii}^* = -1$ , which is optimal also from the industry's point of view ( $\lambda^* = 0$ ). However, if the goods are imperfect substitutes (such that  $\frac{\partial q_j}{\partial p_i} > 0$ ), each downstream firm fully internalizes the effect its price has on the profit of the other firms when  $\lambda = \lambda^* > 0$ . Hence, the downstream firms will not engage in destructive price competition even if they produce close substitutes, and the Bertrand paradox is avoided.<sup>5</sup>

**Proposition 3:** *The profit-sharing rule  $\beta_i(p_i) = \theta p_i^\lambda$  with  $\lambda = \lambda^*$  solves the Bertrand paradox, and induces the downstream firms to maximize aggregate industry profit.*

### 3 Market-expanding investments with spillovers

We now extend the model to allow each downstream firm to undertake market-expanding (or quality-enhancing) investments with potential spillovers. At the outset, it is not clear how one firm's investments affect sales and profits of the other

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<sup>4</sup>Setting  $q_i^*/p_i^* = q_{VI}/p_{VI}$  uniquely determines the prices, since  $q_i/p_i$  is monotonically decreasing in  $p_i$  when  $\partial q_i/\partial p_i < 0$ .

<sup>5</sup>As long as the horizontal pecuniary externality is the only problem to solve, we see from (8) that the scalar  $\theta$  has no impact on the outcome (but in absence of the fixed fee it could be used as an instrument to allocate aggregate industry profit).

firms. The investing firm's product will typically become relatively more attractive than those of the rivals. Thereby the latter could be harmed. However, there might also be technological or marketing spillovers from an investment such that one firm's investment may be to the benefit of all the downstream firms. A given firm's marketing of ring tones, for instance, is also likely to benefit other firms selling ring tone services. We thus open up for both positive and negative spillovers from investments.

We assume that the downstream profit function of firm  $i$  net of any fixed fee is given by

$$\pi_i = \beta(p_i)p_iq_i(p, x) - \phi(x_i), \quad (9)$$

where the new variable  $x$  denotes the vector of market-expanding investments undertaken by the  $n$  downstream firms, and  $\phi(x_i)$  is the investment cost function. The more a firm invests, the higher is the demand it faces;  $\partial q_i/\partial x_i > 0$ . We assume that  $\phi'(x_i) > 0$ , and that it is sufficiently convex to satisfy all second-order conditions for a profit maximum.

Total industry profit is now given by

$$\Pi(p, x) = \sum_{i=1}^n [p_iq_i(p, x) - \phi(x_i)]. \quad (10)$$

The upstream firm determines the input conditions at stage 1, with  $\theta$  and  $\lambda$  as strategic variables, and at stage 2 the downstream firms decide non-cooperatively on end-user prices and investment levels.

At stage 2 the first-order condition  $\partial\pi_i/\partial p_i = 0$  is given by equation (4). Simultaneously solving  $\partial\pi_i/\partial x_i = 0$  we further find

$$\theta (p_i^*)^{\lambda+1} \frac{\partial q_i}{\partial x_i} = \phi'(x_i^*). \quad (11)$$

where  $\theta p_i^{\lambda+1}$  is the profit margin per unit sold.

To find the optimal profit-sharing rule at stage 1, we again use vertical integration as a benchmark. Maximizing (10) with respect to  $x_i$  we find the FOCs

$$p_i \frac{\partial q_i}{\partial x_i} + \sum_{j \neq i} p_j \frac{\partial q_j}{\partial x_i} = \phi'(x_i) \quad (i = 1, \dots, n). \quad (12)$$



If there were no investment spillovers the term  $\partial q_j / \partial x_i$  would in general be negative, and more so the closer horizontal substitutes the goods. This effect, which will not be taken into account by independent downstream firms, tends to generate overinvestments in a decentralized market structure. However, if one firm's investment increases demand also for its rivals, we have  $\partial q_j / \partial x_i > 0$ . This is more likely to be the case the poorer horizontal substitutes the goods are and the stronger the investment spillovers.

Analogous to our procedure above, we define  $\omega_{ji}^x = \frac{\partial q_j}{\partial x_i} / \frac{\partial q_i}{\partial x_i}$ . The variable  $\omega_{ji}^x$  measures the increase in demand for good  $j$  per unit change in the demand for good  $i$  resulting from a higher investment by downstream firm  $i$ . With perfect spillovers an investment by firm  $i$  benefits all firms equally ( $\partial q_i / \partial x_i = \partial q_j / \partial x_i > 0$ ), and we then have  $\omega_{ji}^x = 1$ . Otherwise we have  $\omega_{ji}^x < 1$  (and  $\omega_{ji}^x$  is negative if  $\partial q_j / \partial x_i < 0 \forall i$ ).

Imposing symmetry, we can now reformulate (12) as

$$p_{VI} [1 + (n-1)\omega_{ji}^x] \frac{\partial q_i}{\partial x_i} = \phi'(x_i). \quad (13)$$

The first-order condition  $\partial \Pi / \partial p_i = 0$  is given by equation (7), and thus  $\lambda^*$  in equation (8) still applies. Clearly, aggregate profit is maximized also in the decentralized market structure if it yields the same prices and investment levels as under vertical integration. We can therefore use equations (11) and (13) to find that the upstream firm at stage 1 should set

$$\theta = \theta^* = \frac{1 + (n-1)\omega_{ji}^x}{p_{VI}^{\lambda^*}}. \quad (14)$$

Abstracting from the distribution of the fixed fee  $F$ , the downstream firms' participation constraint requires that  $\theta > 0$  (c.f. equation (9)). The range of permissible values for  $\theta^*$  is thus in the interval  $(0, n/p_{VI}^{\lambda^*}]$ . In the extreme case where an investment by one downstream firm increases its demand by as much as the other firms loose in sales ( $\partial q_i / \partial x_i = -(n-1)\partial q_j / \partial x_i$ ), the investment is a waste of resources from the industry's point of view. Then the upstream firm should set  $\theta^*$  close to zero. In the other extreme case, where we have perfect technological spillovers ( $\omega_{ji}^x = 1$ ), we see that  $\theta^* = n/p_{VI}^{\lambda^*}$ . More generally, the upstream firm should specify a profit-sharing rule which gives each downstream firm a higher profit margin, as captured

by  $\theta^*$ , the more beneficial its investments are for its rivals.

We can state:

**Proposition 4:** *The profit-sharing rule  $\beta_i(p_i) = \theta p_i^\lambda$  with  $\lambda = \lambda^*$  and  $\theta = \theta^*$  yields the downstream firms pricing and investment incentives which maximize industry profit.*

## 4 Concluding Remarks

A major problem in many network industries is that firms may end up with destructive competition because they produce relatively close substitutes. This may prevent the firms from undertaking investments which could benefit the industry in aggregate. Such an outcome can be avoided by implementing a profit-shifting rule which reduces the downstream firms' perceived elasticity of demand. Optimal investment levels are ensured by giving the downstream firms an appropriate profit margin that depends on how one firm's investments affect its rivals.

Another merit of our approach is that it is easy to implement when marginal costs are low, since profit sharing then approaches revenue sharing. A general limitation of revenue sharing is the costs of monitoring the retailer's revenue (Cachon and Lariviere, 2005, and Dana and Spier, 2001). In the case at hand, this problem is rarely significant, since the upstream mobile provider collects the revenue from the end users (but it is the content providers who decide end user prices).

## 5 Literature

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