On the Modelling of the Public Sector in a Walrasian General Equilibrium

By

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CONTENTS

OUTLINE OF THE THESIS

This thesis is a discussion of how the public sector can be modelled within the theoretical framework of a Walrasian general equilibrium system. In the elementary textbook model of general equilibrium, there are usually only two economic agents, the producer and the consumer. The public sector is either not present at all, or it is perhaps seen as just another firm or household. The question posed here is whether it can be argued that the public sector is an economic agent different enough from the two others to warrant its own theoretical formulation. Although by no means definitive, the analysis in Chapter 2 of this thesis suggests that the answer is yes.

At this point one could argue that there are many models and theories of public economic behaviour, and that the whole field of public economics (not to mention macroeconomics) is indeed concerned with just that -- public economics. This is true. But still the literature may be said to leave the impression that the public sector is in some sense an exogenous phenomenon in the economy and not an independent agent that can be given a rationale along with the consumer and the producer. Furthermore, many existing models of the public sector are of a rather partial nature, analysing tax policy, public production efficiency, privatization and provision of public goods etc. as separate questions. There seems to be less widespread agreement on how the public sector should be modelled if the level of abstraction is that of Walrasian general equilibrium.

Following a short philosophical essay in Chapter 1, Chapter 2 is an attempt to rationalize the formulation of the public sector as a third economic agent after the consumer and the private producer. There are two arguments, both of which are quite simple. First, it is argued that the distinguishing characteristic of the public sector is *authority*, i.e. the ability to enforce economic transactions by law. This is not a great surprise in itself, of course, but authority is here seen as the only reason for considering the possibility of defining the public sector as a distinct economic agent. Secondly, it is argued that placing production activities in the public sector is rational if the use of authority results in lower total production costs than would be the case if the goods or services in question were supplied by private firms. Chapter 2 of the thesis is an elaboration of these two arguments.

One of the implications of the discussion in Chapter 2 is that in a Walrasian general equilibrium model the provision of public goods and the level of public consumption should be regarded as exogenously given results of the political decision process. (The level of public production may still be determined by first-order conditions from economics, of course.) This in turn implies that economists may have more to say about the financing of public expenditures than about the composition and level of public consumption. Chapter 3 thus takes up some aspects of optimum taxation formulae in a general equilibrium context. Special attention is devoted to the common assumption in optimum taxation theory that producer prices are constant, with the conclusion that this assumption is not strictly necessary. (What is important, is to assume constant returns to scale in all firms.)

Another issue in Chapter 3 is how optimal tax rates change when the level of public production rises or falls. In most articles on optimal taxation, public expenditures are simply assumed to be a fixed, exogenous number. In a Walrasian general equilibrium model, however, it seems more natural to define such expenditures as the total costs of providing public goods. Normally, the consumers' preferences are defined over both private and public goods, and there may be an impact on the demand for private goods from an increase in the supply of public goods. This leads to the following chain of effects: If the exogenous level of public production is increased, total public production costs rise,[1)] and the increase in the public revenue requirement leads to a higher general level of taxation. But optimal, relative tax rates may be altered as well, because producer prices and/or demand elasticities are likely to change. In Chapter 3 such effects are analysed briefly in order to identify some of the variables which determine whether each optimal tax rate rises or falls.

A question neglected so far is why the Walrasian general equilibrium is interesting. There are several reasons, in spite of the observation that very few real economies, if any, are perfectly competitive. First, most models of real economies are general equilibrium models in the sense that they consist of a system of equations with some exogenous and endogenous variables and, hopefully, a solution. Among these, the Walrasian scheme is quite general and flexible. Secondly, the traditional optimum taxation formulae are directly applicable only to a perfectly competitive economy with constant returns to scale in all firms. Therefore, if one wants to analyse simple, but general optimal taxation, the Walrasian model seems to be the choice alternative. A third reason may be that the analysis of

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1) It is assumed that the public sector minimizes costs at given market prices in all its production activities.

change is often at least as interesting as the characterization of a static equilibrium. Then the Walrasian model is useful to the extent that it captures the effects of changes in exogenous variables in a better way than it describes each static equilibrium.

Comparative statics with taxation is more commonly called tax incidence in the literature. The purpose of Chapter 4 is to look into the incidence of taxation in general equilibrium. In the first part of the Chapter, it is argued that firm, analytical conclusions about tax incidence are extremely hard to come by in a general equilibrium model, unless one is willing to make rather strong assumptions. The reason is simply that the model is "too" general; it has very few *a priori* predictions about the direction of change in equilibrium prices. Instead, comparative statics results depend on the actual magnitudes of demand, supply and substitution elasticities, factor intensities, initial endowments and so on. General equilibrium tax incidence is therefore illustrated by three different numerical simulation models in the second part of Chapter 4.

In the first simulation model, tax rates are exogenously given, and not necessarily optimal. The most interesting result here is perhaps that a Laffer curve is easily generated, showing that total tax revenues first rise and then fall as the tax rate in one (factor) market is increased. The second simulation model has the same parameters and structure in utility and production functions as the first, but now taxation is optimal. In the absence of a Laffer curve the most interesting result from this simulation is that producer prices remain constant when public production increases, whereas relative, optimal tax rates change because demand elasticities do. It turns out, however, that the constancy of producer prices is merely a numerical coincidence: The third simulation model has optimal taxation, but a slightly different utility function, and there producers' factor prices change with the public production level. This model thus shows that producer prices are not necessarily constant in comparative statics, even if taxation is optimal.

Numerical examples only demonstrate what is possible and not what is probable. To say something about the likely effects of optimal taxation, we need a model with empirically plausible estimates of the parameters in utility and production functions. In Chapter 5, which is the final chapter of the thesis, the model GEMPS (General Equilibrium Model with a Public Sector) is presented. GEMPS is very far from describing the true condition of the Norwegian economy, but it has the same parameters, exogenous variables and production structure as a model that does, viz., MISMOD (see references in Chapter 5). So GEMPS may be seen as a large, comparative statics "exercise" where the exogenous tax system of MISMOD (and Norway in 1984) is replaced by optimal taxation. If we believe that a Walrasian general equilibrium model is a good description of the economy, GEMPS tells us what the Norwegian economy would have been in 1984 had taxation been optimal. Focussing, more prudently, on the change from one system to another, the most striking result is the effect on production efficiency: marginal costs in GEMPS are about 50% of the marginal costs in MISMOD, very much due to the lack of intermediate goods taxation.

To summarize, the line of thought throughout the thesis is the following: The state can be defined as a distinct economic agent on the same level of abstraction as the model consumer and producer. As a consequence of this definition, the composition and level of public production and consumption are regarded as exogenously given, and the analysis concentrates on how public expenditures can be financed in an optimal way. This analysis should end up in an empirical model which takes the theoretical results into account, since the ultimate scientific aim is to increase our insight into the real economy.

Chapter 1.
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FOREWORD ON INDIVIDUALISM, LANGUAGE AND AGREEMENT

"Andre derimod er videnskabeligt anlagt. Deres evne til at "tro" er svigtende, hvilket igen har affødt en anden tendens, nemlig trangen til at "<u>vide</u>". Sådanne individer kan ikke suggereres til at finde hvile i andres opfattelser, i andres påstande, ligegyldigt af hvor stort et flertal disse opfattelser og påstande i forvejen så end er akcepterede. ...... Disse væsener er således i realiteten med hensyn til tænkning frit stillet og har til basis for deres opfattelse og tænkning de mere eller mindre intellektuelle faciter, som de i deres begær efter viden tilsidst søger. Sådanne væsener kender vi under begrebet "<u>videnskabsmænd</u>" eller forskere. Og deres resultater udgør det, vi kalder "videnskaben"."

Martinus
("Bisættelse")

FOREWORD ON INDIVIDUALISM, LANGUAGE AND AGREEMENT

As the first word in its title indicates, this essay is an introduction to the models and analyses in the dissertation. As an approach to later essays, I shall be concerned with some quasi-philosophical thoughts regarding the research process in a social science such as economics.

One of the main ideas underlying my work is that a society can be viewed as a collection of individuals. I shall call this idea <u>individualism</u>. An assumption of individualism is made explicitly or implicitly in most of the dissertation. It is also the point of departure for a characterization of science itself considered as a social process. Thus, I start with a few reflections on individualism and then speculate on how scientific theories can be regarded as the result of individuals' agreement on scientific concepts and rules.

Individualism

I assume that a social phenomenon can be modelled as a process generated by individuals. This does not mean that there is nothing more in the world than individuals. But social processes are nevertheless seen as an outcome of individuals' choices, interaction and manipulation of physical things.

Individualism could no doubt be opposed in several ways. One possibility is to assume that society is a kind of organism of its own, and that the actions of every individual (if the word is meaningful) are completely determined by the social group to which he belongs. Another possibility is the assumption that human behaviour is governed by conditions which are exogenous to social processes, e.g. rules for moral correctness. I do not deny that these assumptions are possible, nor that they could be used for some theory. But I do not make them - I believe that individualism is a better point of departure for a theory which sets out to describe or explain social processes.

Individualism means that a social organization is conceived as an aggregate of its constituent, individual members. It asserts that social phenomena can be seen as caused by individual choices and actions, and in this way focusses on the individual as an autonomous agent in the social organization. A more fundamental assumption is therefore that every individual is autonomous and has <u>freedom of choice</u>, defined here as the ability to choose among the options available to him.

Freedom of choice does not mean that the individual is necessarily able to define his options or that he is in fact physically or psychologically free. The point is that the individual is always <u>able</u> to choose another action than the one he actually chooses, and that in this sense he is responsible for his choice. In other words, I assume that in a theory about social phenomena the responsibility for an individual's actions and thoughts may be placed with the individual himself. I reject the viewpoint that <u>since</u> his behaviour may be influenced by a social organization, he is not to be made responsible for his choices.

In a more philosophical or perhaps psychological perspective, the assumptions which I call individualism and freedom of choice rest on the identification of each individual with a self which is the basis of individual worth. Thus, each individual has a responsible self, capable of choosing. It is recognized that the self is influenced by and probably determined by the socialization of the individual. But this does not mean that he is not responsible for his own actions, since the self may criticize itself by inquiring into all those past experiences which established it.

As a digression I wish to emphasize that these assertions are intended as assumptions for a theory and not as a complete description of how people ordinarily behave or think. First, it seems unreasonable to require children to be personally responsible for everything they do. In most of their social roles, children do not conform very well with my concept of an individual. Second, rather few adults seem to be willing to take the full responsibility for all their actions. Much energy is instead devoted to blaming others or the circumstances. Third, it is an important research project in psychology to find out how the self is established and how it is influenced by social interaction.

I regard social science in general and economics in particular as social phenomena, and apply the ideas of individualism, freedom of choice and individual worth to characterize them as social processes generated by scientists. (The proliferation of science among non-scientists is ignored.) The most important part of such a characterization is the concept of a scientific language as a subset of language in general.

Language

Although no precise definition will be attempted here, language may be seen rather trivially as a collection of concepts conveying information, together with a set of rules for how the concepts may be used in a meaningful way. In addition, however, language is a social phenomenon and thus results from individuals' actions and interaction. There are two important aspects of language in this context. First, I believe that every individual has his own version of the language and that each individual version is unique. Second, I believe that concepts cannot reside elsewhere than in individual minds. Therefore, a new concept cannot be discovered, it must be invented by someone. Logical implications can be discovered, but not the names of the results. Although one discovers by mathematical methods that there "should be something there" in quantum mechanics, one cannot possibly discover that this something has the name "quark".[1)]

One reason why individual versions of language are unique, is that the individual is unique: the meaning of each concept he knows depends on the whole set of thoughts and connotations it invokes in his mind, and this set in turn is determined by his particular history and experiences. It would perhaps be possible to define concepts objectively if there existed a type of definition which did not use concepts that had to be defined themselves. I do not believe that such definitions exist, and I doubt that they can be found. Metaphorically speaking, language is a self-supporting chassis.

[1)] There may be a distinction between a concept and its name, but it will not be discussed here.

The individual uniqueness of language should not be exaggerated. For a surprising lot of words, the degree of commonness in meaning is apparently very high across the members of the society in question, and seems to depend - among other things - on the concept's level of abstraction. To illustrate, most of us know very precisely what a table is, because we can see and touch it in order to test whether the thing we have in front of us is what we were once taught to call a table. It is more difficult to be sure that we have a correct interpretation of abstract concepts like nothingness and infinity. The physical symbol ∞ does not demonstrate the properties of infinity in the same way as a table displays the features of a table. So the individual aspect of language is probably less important in everyday life than in science, since science requires greater precision and scientific concepts are often more abstract.

Agreement

I believe that a major force of social coherence is agreement. Indeed, with an individualistic view of the world one hardly avoids seeing some kind of agreement on language, social conduct etc. as a prerequisite for the very existence of a society. In this context, agreement is a relation between two individuals A and B such that if A communicates some concept to B, and B acts in a way which A perceives as consistent with his own understanding of the concept, then they agree as far as A is concerned. If the same applies to B, then they agree. Agreement (thus "defined") does not guarantee that A and B really act according to the same concept. They only behave consistently from each other's point of view.

Agreement is important because it is closely related to the extent to which theories are accepted as fruitful, interesting, explanatory or good. I do not believe that much else than this can be said in general about how science evolves. The next section is confined to a few subjective remarks.

Some remarks on social science

In my opinion there is no great difference between a scientific and an ordinary manner of thinking. The difference is mainly the willingness in science to comply systematically with the rules of logic. Superficially, scientific knowledge may be regarded as a set of scientific concepts and a set of scientific rules for how the concepts should be used. (The rules must include logic.) In this respect scientific knowledge is a subset of language - a scientifically structured subset, so to speak. But then scientific knowledge also has an individual aspect. In particular, there is no absolute objectivity, and no assertion is necessarily true.[2)] Truth is just what follows from logical manipulation and analysis, and even this is not quite without qualifications, since there has to be agreement on the use of logic.

Given the use of logic in scientific reasoning, there is not much interest in objections to the implications of a set of assumptions. Assumptions, on the other hand, are very interesting. Somewhat drastically one could even identify a theory with the explicit and implicit assumptions on which it rests, because deductions cannot be disputed as long as one agrees to the use of logic. Then it is intriguing to observe that the judgement of a set

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[2)] I do not discuss whether science is objective or not here, but allude to the distinction between assumptions and their implications.

of assumptions is not based on criteria within the theory itself; the criteria must lie on the outside. Although there are such criteria in the philosophy of science,[3] I think that in practice the evaluation of theories is markedly individual, and in the last resort, it must be.

I assume that this does not matter to any individual who decides what to think about some theory. To be more specific, I find it meaningless to doubt one's own thinking; provided that it is logical. To deny the validity of one's own thoughts seems to defy the existence of one's self.[4] What I do doubt, on the other hand, is that two individuals' thoughts are the same. We can be less certain of social or aggregate phenomena than we can be of individual observations. Therefore, when it comes to judgement, the opinion of each individual is as valid as the opinion of any or all other individuals, because all individuals are of equal worth.

It would be mistaken to conclude that there is no point in discussing assumptions. Quite to the contrary, I consider it an important task to justify and criticize assumptions in order to find more general or fundamental axioms from which they can be deduced.[5] But I believe that *de facto* and at a given point of time the main indicator of better and poorer theories is the greater or lesser agreement among scientists as to the usefulness of their assumptions.

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[3] An example is the falsifiability criterion by Popper (1934/1980), which roughly says that it should be logically possible to determine whether the predictions of a theory are false.

[4] This idea partly resembles the notion of being advocated by Descartes (1637/1966, p. 60): "Je pense, donc je suis."

[5] This does not mean that logical analysis of the implications of a set of assumptions is a task of minor importance.

Popper's falsifiability criterion, which was briefly mentioned in footnote 3), is rather obvious. If it is not fulfilled, then it is impossible to know whether a theory or its predictions are false and the theory may be just a tautology with no substance. Falsification can be regarded as an alternative to verification, which may be interpreted as a confirmation that the predictions of a theory are true. Of course we have to define "confirmation" precisely in such an interpretation, but in most cases verification will be hampered by the well-known problem of induction. In short, the induction problem is that one cannot infer from a finite set of observations to a possibly infinite universe. Induction is illegitimate in a truly infinite universe, whereas if the universe is finite, induction produces hypotheses for further testing. To illustrate, let us suppose that 99 black ravens have been observed, whereupon the assertion is made that all ravens are black. If we know that there are 100 ravens, we then have to test the assertion by finding the last one. If the number of ravens is infinite (or possibly so), the assertion cannot even be tested for a firm conclusion.

The problem of induction seems to be widely ignored among economists. Inferences from a finite (and in some cases small) set of observations are often presented as indisputable facts called empirical "evidence", and much would be gained by a more critical attitude towards econometric results. On the other hand, I believe that social science would have a dismal future if we were to avoid induction altogether in every scientific activity. Inductions are made all the time by scientists who observe some phenomenon in a limited social group and infer from this group to society as a whole, and induction is therefore a major source of perspectives and hypotheses for the development of new theories. Induction generates ideas. This is an important reason why language and concepts matter: induction often boils down to inventing a name (a concept) for a social phenomenon and bringing it to scientific attention in a new and original way. Whether or not the new concept will be regarded as fruitful and interesting depends on its appeal to the imagination of individual scientists.

A final word on the analysis in the thesis

In the above perspective, chapter 2 and to some extent chapter 3 of the thesis are discussions of assumptions underlying certain economic models. Chapter 2 presents a few, common formulations of the public sector in economics and tries to provide a rationale for modelling the state as an autonomous economic agent in general equilibrium models. Chapter 3 is concerned with the interpretation of the traditional formulae for optimal taxation in public economics. Hopefully, these chapters shed some light on some of the numerous difficult questions arising when one tries to understand the complex relationship between authoritative, collective action and decentralized, individual choice.

What is left to the reader, is to agree or disagree.

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Chapter 2.
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TOWARDS A WALRASIAN MODEL OF THE STATE

"La Nation ne fait pas corps en France; elle réside entière dans la personne du roi. L'Etat, c'est moi."

Louis XIV

TOWARDS A WALRASIAN MODEL OF THE STATE

PART 1: OVERVIEW

Introduction

The purpose of this essay is to introduce a fundamental concept of the state. The discussion is neither very detailed nor a thorough survey of the literature, but concentrates on the general framework for analysis of public economic activity. Hopefully, the essay still serves to point out some of the many interesting problems in the theory of public action. A few of these problems will be taken up in other parts of the dissertation.

In the first part four strands of economic literature will be sketched with the emphasis on their implicit view of the state: the theory of the social welfare function, the theory of public choice, game theory and the theory of clubs, and optimum taxation theory. No attempt is made at a thorough review of the literature. The point is rather to provide a background for a theory of authority, and part 1 should not be read as a critique of existing models.

One of the conclusions in the essay is that economists are not obliged to take the economic state's existence as given, and that relatively little can be said in general about what productive activitites the state should undertake. The state is seen as the execution of authority, which is a social construct. In this respect the state can be compared with the microeconomic firm, which is a way to organize the transformation of resources into consumption goods. Analogously the state is a way to organize the solving of certain economic problems that cannot or would not be solved equally well by private agents. The theory of authority may be broken down into three parts: an outline of the kind of problems the state can solve; recommendations of how these problems should be handled; and a description of institutions needed to handle them.

Part 1 of this chapter contains a discussion of the definition of the state where the fields of economic literature mentioned above represent four important aspects of public behaviour. The second part is an attempt at synthesis of these and other characteristics of the state, and leads to a formal (but fairly abstract) definition of the state as the application of a set of legal rights and duties which regulate the enforcement of social and economic measures. Part 2 also presents some thoughts on how the set of rights and duties may be established as well as an analysis of some of the reasons why public economic activity may be warranted.

<u>The state as a social welfare function</u>

The rationale of a social welfare function was given by Bergson (1938) and Samuelson (1947). Bergson's main argument is that the fundamental economic problem of how to allocate scarce resources to alternative ends can be solved by maximization of a social welfare function. Bergson defines an optimal allocation as an economic situation where the total differential of a properly formulated welfare function is zero, whereupon he deduces the standard marginal conditions for efficient production and consumption.

The first basic welfare theorem says that these efficiency conditions are fulfilled in a perfectly competitive economy (Debreu, 1959, ch. 6), so it seems that the government needs no social welfare function if only competition is perfect. But one of the main implications of Samuelson's analysis (1947, ch. VIII) is that this is not so. There may be several Pareto efficient allocations in an economy, corresponding to the different sets of general equilibrium prices at which trade and production take place. Therefore, a social welfare function is needed to pick one of the allocations in a set of Pareto optima.

Mathematically, a Bergson-Samuelson individualistic social welfare function may be written as $W = W[u^1(x^1), \ldots, u^I(x^I)]$, where u^i, $i = 1, \ldots, I$ are the utility levels of I individuals given the consumption bundles x^i. If W is defined so as to be increasing in all its arguments, then maximization of W will obviously lead to a Pareto optimal situation where it is impossible to increase u^i by reallocation of resources without decreasing u^j, for all i and some j. It may be worth noting that the converse does not generally hold: Even if the allocation x^i, $i = 1, \ldots, I$, is Pareto optimal, W is not necessarily maximal. (If it <u>necessarily</u> were, then this would mean that the social welfare function left society indifferent to all Pareto optima.)

The social welfare function is primarily needed for the selection of one Pareto optimum from a set of efficient allocations, since this set itself can be established by enumeration of all perfectly competitive equilibria. These Pareto optima differ in the interpersonal distribution of utility levels, and the maximization of W therefore inevitably involves some kind of interpersonal comparison. (Samuelson (1947, p. 244): "...without assumptions concerning interpersonal comparisons of utility, it is impossible to decide which of these [Pareto optimal] points is best.") A rather obvious way to make such interpersonal comparisons is to assume that all utility functions $u^i(\cdot)$ are cardinally measurable and comparable. Although analytically helpful, this does not seem to be a very good solution in practice, since cardinal utility functions are as yet rather difficult to observe. Only an approximate measure of ordinal utility can be obtained through observation of individual demand and a test of whether the individual conforms with the generalized axiom of revealed preference (Varian, 1984, pp. 141-143). Every cardinal representation of approximate utility measures would imply an implicit interpersonal comparison when used in a social welfare function. In the absence of observed cardinal utility functions the comparison could never become explicit, and this fact could perhaps be critized on philosophical grounds.

A social welfare function can be interpreted as a theory of the state because it points out that there may exist economic problems which are not automatically solved by individual, decentralized actions. In a perfectly competitive economy it is not enough that each consumer's utility is maximized; we must also determine the interpersonal distribution of utility levels. But viewed as a description of the state's behaviour a social welfare function still has its shortcomings in that it leaves several interesting questions unanswered: Are there important public economic activities in the real world that cannot be described by a social welfare function and the implications of maximizing it? What are the political objectives for which a social welfare function is an adequate expression? What are the inherent costs of the procedure of selecting a preferred Pareto optimum? And most important of all: How is W itself determined? Some of these questions have found their answer in the literature on public choice.

The state as a public choice

According to Mueller (1979, p. 1), "Public choice can be defined as the economic study of nonmarket decisionmaking, or simply the application of economics to political science. The subject matter of public choice is the same as that of political science: the theory of the state, voting rules, voter behavior, party politics, the bureaucracy, and so on. The methodology of public choice is that of economics, however. The basic behavioral postulate of public choice, as for economics, is that man is an egoistic, rational utility maximizer." Mueller thus regards public choice and economics as two separate scientific activities with certain methodical elements in common. As an example of this relationship between public choice and economics a brief presentation of the median voter model may be illustrative.

The median voter model is based on an idea from Hotelling (1929) and describes the electoral support of two political parties in a representative democracy. It is assumed that all individuals vote for one of the parties, and the problem is to predict the winner of the election. A party's winning chances obviously depend on its politics. Let us assume that the issue at stake, the political action α, can take on any real value from 0 to 1, i.e. $\alpha \in [0,1]$. Suppose further that the electorate's attitudes towards α are continuously distributed according to some symmetric density function like the one, e.g., in Figure 2.1, and that a voter's satisfaction with an alternative α is $u(|\alpha^* - \alpha|)$, where u is an increasing, real-valued function and α^* is the preferred alternative. A party offering α_1 would receive the votes from individuals favouring any policy in the interval $[0, \frac{1}{2}(\alpha_1 + \alpha_2)]$; the rest of the votes would go to the party offering α_2. The latter would win the election since α_2 lies

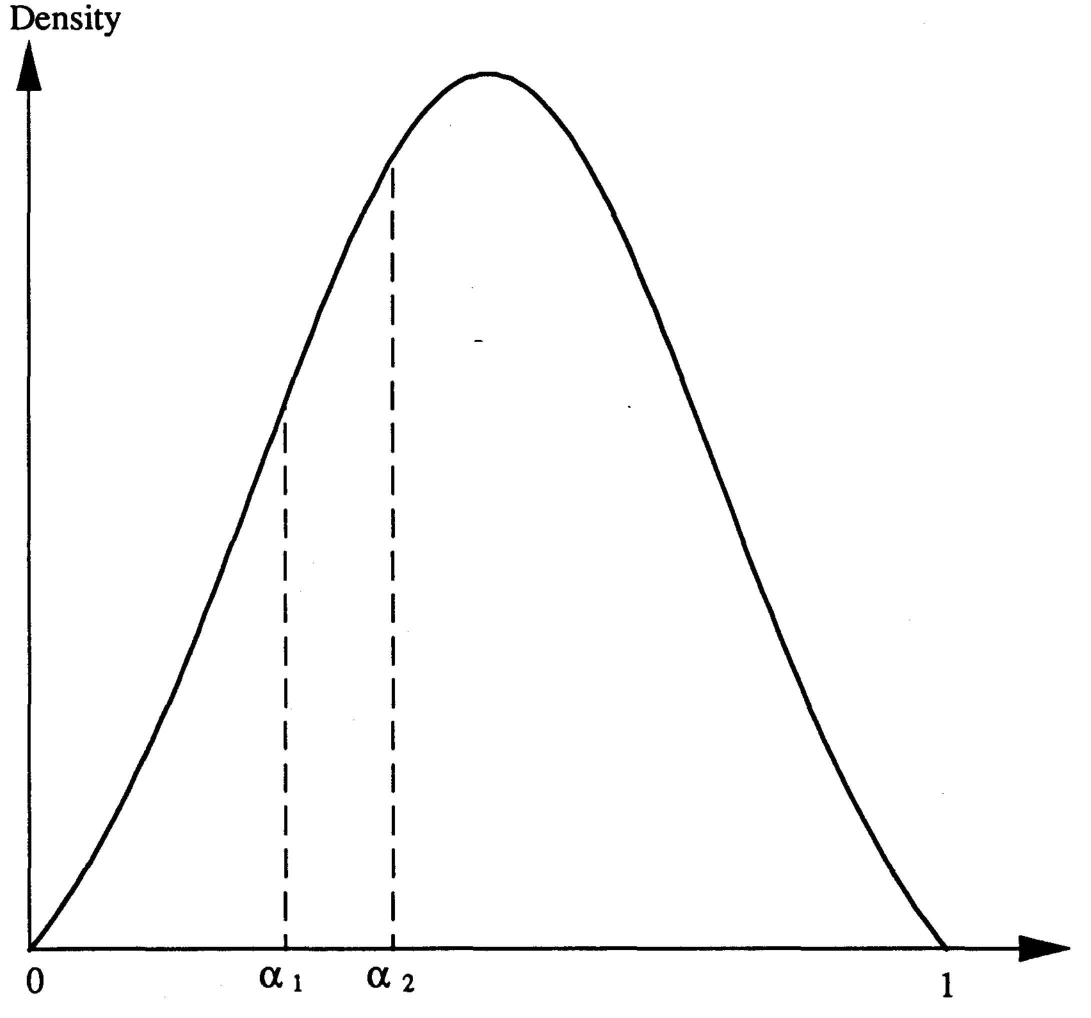

Figure 2.1. A distribution of political attitudes.

closer to the median policy than α_1 does. With repeated elections the long run winner will be precisely the party offering the median policy. Therefore, all we need to finally determine the model's outcome is an assumption that any political party aims at maximizing its electoral support.

It seems somewhat farfetched to classify a theory as "economic" as soon as it contains some agents with maximizing behaviour, so if the median voter model deals with economics, it must be because the policy action can be interpreted as an economic variable. For instance, it could be defined as the general degree of taxation in the economy, and the median voter model could then be used to predict the likely future level of taxes. Although the real political world is often more complicated than the median voter model (see Mueller, 1979), the model still represents a theory of the state: The state is identified with that political party which offers the policy favoured by the median voter, and the state's behaviour is the median policy. In economic terms the level of taxes is such that the median voter's utility is maximized, given a certain democratic constitution.

The median voter model is a theory of the consistency between preferences and governmental action rather than an explanation of the existence of the state itself. The political parties are taken as given, and the model does not tell us why there is a state in the first place. An interesting step towards remedy of this weakness has been taken by Brennan and Buchanan (1977).

In the Brennan-Buchanan model the political issue towards which the median voter must determine his attitudes is the tax constitution or the laws that regulate the levy of taxes. This is an issue because of the state's behaviour: "The characteristic assumption of our Leviathan-type model is that, in each post-constitutional budgetary period, the government will attempt to maximize total revenue collections (and hence total spending), within the constitutionally-appointed regime." (p. 260). In addition to this behavioural assumption Brennan and Buchanan make the supposition that the government's purpose - from the voters' point of view - is to provide public goods, but at the same time individuals' and the state's interests are in conflict over the level of taxation. Brennan and Buchanan, apparently on the individuals' side, consequently deduce the advice that individuals vote for tax constitutions that put bounds on the state's ability to tax.

Brennan and Buchanan do not elaborate on their assumptions about the state, but it is possible within the framework of their model to imagine the state as a set of consumers who receive their income from public tax revenues. It is not clear whether these consumers are also voters. Another question is why the voters accept the state at all; it is not self-evident that a state is the best social device for producing public goods. Finally, the assumption that tax revenues are maximized may be criticized for being too simple. It may be true in some countries, but undoubtedly there are other objectives politicians could pursue. A slightly different approach is to assume that the state undertakes productive activities with increasing returns to scale.

The state as a producing coalition or club

It is well known that for a perfectly competitive equilibrium to exist, the aggregate production set should be convex (Debreu, 1959, section 5.7); there should be constant or decreasing returns to scale in production. If, on the other hand, there are relatively large producers with increasing returns, the production set may be non-convex, and the equilibrium may be upset. Boiteux (1956), Baumol and Bradford (1970) and others before them have argued that this situation calls for public action.

Game theory is well suited for analysis of firms with increasing returns to scale (Littlechild, 1970; Faulhaber, 1975; and Sharkey, 1982). In such analyses a theory of the state may be said to emerge from an application of the theory of multi-player cooperative games.

Suppose that the consumers in the economy unanimously agree that any production activity should be efficient, given a set of factor prices. This means that total production costs are to be minimized at every level of demand. Consider a firm producing one good, let y be the production level and c(y) the minimum production cost (given factor prices). The cost game of the firm is a cooperative game where each unit of the product is defined as a player whose objective is to avoid having cost shares allocated to him.[1)] If there are increasing returns to scale a decrease in production will induce a less than proportionate decrease in costs, and the cost function will be subhomogeneous:

[1)]The introduction of a cost game is an analytical trick with the same effect as assuming that the consumers themselves are players, each demanding one unit of the good. A consumer demanding more than one unit must be modelled as a coalition in the cooperative game.

$$c(\delta y) \geqq \delta c(y),\ 0 < \delta < 1.$$

Let y^j be the production of firm j, y total demand (at some prevailing price), and define $\delta^j > 0$ such that $\delta^j y = y^j$ and $\Sigma_j \delta^j = 1$. Then

$$c(\delta^j y) \geqq \delta^j c(y) \quad \forall j$$

$$\Rightarrow$$

$$\Sigma_j\ c(y^j) \geqq \Sigma_j\ \delta^j c(y) = c(y).$$

Thus, when the cost function is subhomogeneous, total production costs are lower when one large firm produces alone than when there are several smaller firms. The cost function supports a natural monopoly. Identifying each unit of the good with a consumer, the basic assumption now is that a coalition of consumers is needed to set up the production of y. These consumers try to minimize the total amount of pooled resources which are transformed into y. The inequalities above then mean that any single coalition member can obtain a better position (i.e. a lower imputed cost share) in the grand coalition including all individuals than in any subcoalition, since total costs can always be distributed according to the δ's. The first inequality implies that an individual will lose if he breaks out of a coalition. The second inequality implies that all members of a coalition will gain from an increase in the coalition's size. Thus, increasing returns to scale imply that the only stable coalition is the one with all consumers included.

The authors mentioned earlier; Littlechild, Faulhaber and Sharkey; do not explicitly argue that their theories are theories of the state. The question they try to answer is: What should the price of a publicly produced good be if no subset of consumers is to subsidize any other subset of consumers? The line of argument seems to be that if there is a public firm with increasing returns to scale, then it can be described as a coalition of all consumers in the economy pooling their resources to achieve an efficient production of their total demand. A theory of the state emerges implicitly by the way the problem is posed; the state is the coalition of all consumers.

In club theory the state has been defined explicitly as a club with all consumers as members (Sandler and Tschirhart, 1980). The essential point in club theory is that a club is formed to benefit from economies of scale in production, and membership size is determined by the condition that marginal crowding costs equal the marginal reduction in average production costs. The theory of clubs roughly corresponds to the situation discussed above if we interpret the cost function c(•) as production costs plus some kind of crowding costs (which may be zero). The state may be defined as a club where every citizen happens to be a member.

This theory of the state is of course open to criticism. Two rather trivial objections are that the model does not tell us <u>how</u> coalitions are formed, since the rationale of a coalition is merely the outcome of forming it; and that empirically, coalitions of consumers rarely supply their own demand for a good in full, but only small parts of it. (The latter point does not apply to Sharkey's article (1982), where the matter of interest is precisely this problem.) A more serious weakness is that the definition of the state as the coalition of all consumers is rather arbitrary: If there were I = 1 000 000 individuals, and the technology was such that the core coalition contained I - 1 members, would it not be natural to call it a state despite the exclusion of one person? More generally, how large must a coalition be before it is defined as a state?

This is an important question, since any firm size (and, in an imperfect market economy, any firm's size) may result from the game theoretic analysis. The model does not explain the difference - if there is any - between a public and a private firm.

A possible difference between a public and a private firm, both with increasing returns to scale, is that the latter often sets monopolistic prices, whereas the former, belonging to the public sector, may cover its costs through taxation. This difference is analysed in the theory of optimal taxation.

The state as public consumption and a set of tax rates

The state appears as an economic agent in almost any model in macroeconomics and very often in public economics. In most cases it is represented by a set of exogenous tax rates and an exogenous level of public expenditures. To illustrate one such formulation of the state it is instructive to sketch the very simplest model of taxation in a perfectly competitive economy as presented by Sandmo (1976).

Assume that there are constant returns to scale so that producer j's profit-maximizing production plan y^j yields zero profit if producer prices p are given: $py^j = 0$, all j. Let there be only one consumer, whose maximum utility is v(q) if consumer prices are q. q is normalized by setting one consumer price, e.g. q_0, equal to 1. The state's tax revenues in terms of the numéraire are given by T = tx, where t is a vector of tax rates and x is the consumer's net demand (i.e. consumption less endowments) at q. The tax rates drive wedges between consumer and producer prices, q = p + t.

Our problem is: If T is given exogenously, and lump-sum taxation is infeasible, what is the optimal structure of consumer prices? It turns out that the solution to this problem also yields formulae for optimal tax rates:

$$\underset{q}{\text{maximize}}\ v(q) \text{ subject to } tx = T \text{ and } q_0 = 1,$$

yields first-order conditions which can be expressed as

$$(1) \qquad \Sigma_m\ t_m \partial x_m/\partial q_n = \frac{\lambda - \mu}{\mu} x_n \ ,$$

where λ is the consumer's marginal utility of income and μ is the shadow price of the restriction $tx = T$. This formula will be interpreted in closer detail in Chapter 3. To characterize the optimum taxation economy, however, it is useful to state Walras' law:

$$T = tx = (q - p)x = -px = p(y - x),$$

where the facts that $qx = 0$ and $py^j = 0$ are used together with the definition $y \equiv \Sigma_j\ y^j$. Walras' law says that if private supply equals private demand in <u>all</u> markets, then public tax revenues are always zero, no matter what the tax rates are. What the state collects in taxation, it redistributes as subsidies. If the assumption of private general equilibrium is made, the state has no other purpose than to distort prices. This is simply inefficient, so the optimal state is no state.

Let us now assume that the state buys goods in private markets for public consumption, g, so that the general equilibrium is $x + g = y$. This raises a few questions which are not explicitly dealt with by Sandmo (1976).

Which set of prices does the public sector face - do public firms pay taxes to the public sector? How can g be determined so as to be consistent with equilibrium? If tax revenues depend on the pattern and level of public expenditure, how - if at all - does this affect the optimal tax rates?

One simple way around these questions is to assume that the tax rate of the numéraire is equal to zero and that net tax revenues are expended on this good alone. The optimum taxation formula of course continues to hold in the version written above, and we have a model of the state: The state "is" the vector [t, T], where $\underline{t}$ is optimal and T is exogenous.

As in much of public choice theory, the state's existence is taken as given in models of optimum taxation. The theory does not attempt to explain or model the state as such. Instead, its main view seems to be that if there is an empirical state that wants to confront allegedly competitive consumers and producers with optimal taxes, then it may find out about these taxes in the model's formulae (like (1)). The theory does not question the state's existence, but rather is a calculation of the best set of general equilibrium prices given that a certain amount of real resources is to be removed from the private sector.

<u>The state as an autonomous economic agent</u>

Two recurring questions in the previous paragraphs have been: What is the justification of the state's existence, and what is the difference between private and public institutions? The theory which comes closest to answering these questions, seems to be the Brennan-Buchanan model of tax constitutions for Leviathan (Brennan and Buchanan, 1977). However, in their model, the state seems to be something of an inescapable evil, and the consumers must act so as to hamper its behaviour. This is a rather pessimistic view of the world, and it is also implausible that no more reasonable justification for the state should exist, in view of the empirical observation that the public sector is the most important economic agent in the greater part of the industrialized countries.

It is proposed here that the sole distinguishing feature of the state is authority. Authority is the essential characteristic that separates the state from a private economic agent (even though consumers can be said to exercise authority over their initial endowments). Thus, a description of the state must be a description of authority and the process of executing actions which are made possible by application of authority.

In an abstract sense, authority may be defined as a set of legal rights to enforce social measures. In an economy, one of the most important legal rights is the right to tax, but of course there are others, like direct price regulation, the right to expropriate, confiscate inheritance, print money, etc. The list may be made arbitrarily long. But considering the right to tax, we realize that authority very easily makes the state an extremely powerful economic agent. Taxation - or application of the right to tax - is to force others to pay for something they may not desire or even get. Therefore, it is almost trivial to find behavioural assumptions (e.g. maximization of tax revenues) which make the state an adversary of consumers and producers.

Authority is analogous to technology. Just like the producer's application of technology, formalized by a given production set, the state applies authority, formalized by a set of given legal rights and duties. Like the producer, the state is impersonal and has to be run by consumers. It may also be observed that the state does not have more or different information about the economy than private agents have, nor does it command a technology which is necessarily inaccessible to the private sector.

A step towards explanation of the state's existence is to justify the application of authority. Hence, the key to analysis of the state is to look for economic problems that may be solved by applying authority, and, preferably, better so than by decentralized private actions. Whether or not authoritative action is warranted of course depends on the economic problem considered, but nevertheless a few general remarks can be made:

It will be postulated that there is a preference for decentralized decision-making in the economy. Then authority is not appropriate when private agents can and do achieve optimal solutions to economic problems by decentralized action. This is what happens when every possible transaction which is beneficial to both sides of a dyadic relation is carried out, so that there is no conflict of interests impeding the decentralized solution; and when the costs of collective decision-making are not prohibitive. As an example, consider a small park, a public good to the people living in the neighbourhood. Decision costs could hardly prevent these few consumers from coming together, agreeing to hire an economist to reveal true preferences for the park and then build it if the project turned out to be profitable. In other words it is not a principle or necessity that public goods should always be provided by the state (as the theory of clubs clearly demonstrates).

However, when the interests of two or more private agents are in direct conflict and the agents are unable to solve this conflict on their own, authority may have a task to fulfil. It is precisely when universally beneficial decisions are <u>not</u> possible that authority is needed. Indeed, this is the rationale of the concept of authority, for, if no economic decision could be made to the detriment of anyone, then it is hard to imagine what the significance of authority would be.

Authority may also be warranted when collective action is prohibited by high decision or transaction costs. For instance, the customers of a monopolist may be too numerous to meet and compensate the producer for the profit he loses if the product price instead of marginal revenue is set equal to marginal cost. More precisely, even if there is a positive difference between the efficiency loss under monopoly and monopoly profits, the costs of coordinating consumers' actions towards the monopolist may exceed this difference. Authority may then be applied to the effect that price equals marginal cost by regulation.

It is also interesting to ask how the authority set (the set of legal rights and duties of the state) is or can be established. If a legislative source is defined very broadly as all legal rights and duties a state could possibly have, then the authority set in a particular economy can be defined as a subset of the legislative source. In a representative democracy this subset is the outcome of some process of aggregation of individual preferences, a process which will in general be extremely complicated. For instance, if each individual's preferences are represented by a utility function $u^i(\cdot)$ with consumption bundles as arguments, then such a process ideally determines an authority set which will guarantee that individual consumption levels are socially optimal. In some cases it is impossible to construct a social decision process that will yield such a result (Arrow, 1963). This problem will be further discussed in part 2.

Examples

The literature of course contains many examples of how authority may or should be applied to solve economic problems. Three such examples will be presented here.

1. Optimum taxation.

Maximization of economic welfare in a perfectly competitive economy usually involves a choice of one Pareto efficient allocation among several Pareto optima which differ in the interpersonal distribution of utility levels. Although elementary theories of social welfare functions are not always explicit on the point, this choice is often imagined as being made by lump-sum taxation of consumers' incomes. A social welfare function thus offers a place for authority: authority institutes lump-sum taxation. More precisely, the authority set has as one of its elements the legal right to tax, which is applied to the private economy in order to maximize economic welfare.

Optimum taxation theory may be interpreted as a second best version of this kind of authoritative action. It is second best because lump-sum taxation is assumed to be infeasible and the alternative - commodity taxation - entails an efficiency loss in the private sector. Optimum taxation theory has more to say about the way authority should be applied, however, by presenting formulae for welfare-maximizing tax rates. In addition the theory goes some steps towards an explanation of the existence of the state, since we must believe that there is a good reason for the tax revenue requirement (T). The state is "explained" by the purpose of taxation, and the authority set contains the right to tax and the duty to tax optimally.

2. Natural monopoly.

It was shown in the discussion of firms with increasing returns to scale that if the production is set up by a coalition of consumers, the only stable coalition is the one including all individuals. In technical terms this is because economies of scale imply a non-empty core in the cost game where consumers coalesce to minimize the total cost of supplying their demand for a good. When costs are subhomogeneous, application of authority consequently cannot outdo what the consumers achieve on their own (assuming that decision costs are zero).

There is, however, an important difference between subhomogeneity and subadditivity of the cost function. Subhomogeneity is defined as decreasing average costs: $c(\delta y) \geqq \delta c(y)$ for all $\delta \in]0,1[$. Subadditivity is a weaker property: $\Sigma_j c(y^j) \geqq c(\Sigma_j y^j)$, saying that total costs do not fall when a given production is split up among several producers. With subhomogeneity a non-empty core always exists; with subadditivity this is not necessarily so. As long as the cost function is subadditive, total production costs are minimized in the grand coalition. But the gains from forming this coalition may still not be sufficiently large that some smaller coalition could not do better by going it alone, so a grand <u>core</u> may not exist. This situation is illustrated in Figure 2.2, which is taken from Baumol (1977). The total cost of producing y (at given factor prices) is c(y), and marginal costs are infinite at y'. It is easy to see that

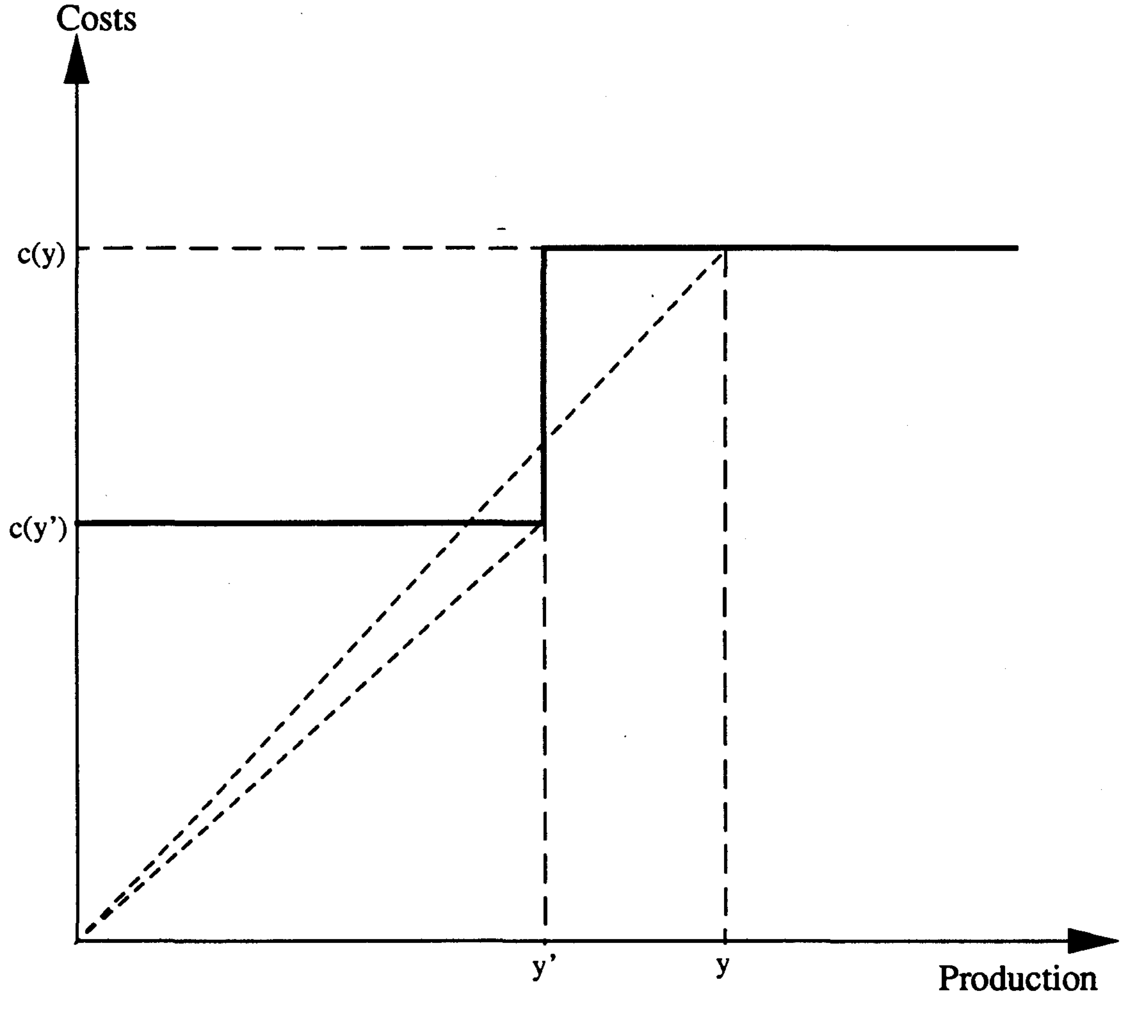

Figure 2.2. A subadditive cost function for production level y.

average costs are lower at y' than at y; that is, a subcoalition producing y' would achieve a lower imputed unit cost than the grand coalition which produces y. Assuming for simplicity that each consumer demands one unit of the good, we realize that the grand coalition will tend to break down. If this happens, and there are two coalitions; one producing y' with costs c(y') and the other producing y - y' with costs c(y - y') = c(y'); the total cost of providing y will not be minimized since c(y') + c(y - y') = 2c(y') > c(y). Both y and (y', y - y') are Pareto optimal production schemes, since at both points it is possible to improve the situation of some consumers only at the expense of others.

Authority may be applied here to choose one Pareto optimum, just like maximization of a social welfare function would do. If it is a general social value that production should require a minimum of resources, then authority may be needed to establish and sustain a grand coalition which produces y. The authority set A contains the right to control the production of y and the duty to minimize total production costs.

In concluding it is worth noting that <u>natural monopoly</u> is defined as a firm which can produce the total market demand with a subadditive cost function (Baumol, 1977). In view of the discussion above this definition implies a choice of Pareto optimum in those cases where the subadditivity is not "strong enough" for a grand core to exist. If no core exists, the competitive conditions of a natural monopoly may be rather complicated, as is evidenced by Panzar and Willig (1977). In particular, no sustainable product prices may exist if the firm must recover its costs. The argument raised in this paragraph is that authoritative action may be warranted in such cases.

3. External effects.

The role of authority in dealing with external effects is described in the classical article by Coase (1960). Coase makes two main points. First, he observes that the identification of those liable for the economic consequences of external effects is not always an indisputable matter. When a firm pollutes the air, it is not obvious that it should pay for this; it may be instead that neighbours who are harmed should buy units of clean air from the firm. The conflict of interests arises because nobody owns the air, and the external effects can therefore be interpreted as the result of lack of markets. Coase's second point is that one reason why a market is missing is that transactions costs are prohibitive: "In order to carry out a market transaction it is necessary to discover who it is that one wishes to deal with, to inform people that one wishes to deal and on what terms, to conduct negotiations leading up to a bargain, to draw up the contract, to undertake the inspection needed to make sure that the terms of the contract are being observed, and so on. These operations are often extremely costly, sufficiently costly at any rate to prevent many transactions that would be carried out in a world in which the pricing system worked without cost." (Coase, 1960, p. 15). In response, alternative structures are formed: "It is clear that an alternative form of economic organisation which could achieve the same result at less cost than would be incurred by using the market would enable the value of production to be raised. As I explained many years ago, [Coase (1937)] the firm represents such an alternative to organising production through market transactions." (Coase, 1960, p. 16). In today's terms, this would perhaps correspond to the method of internalizing external effects. However, it is not the only possibility: "An alternative solution is direct Government regulation. ... Such authoritarian methods save a lot of trouble (for those doing the organising). Furthermore, the government has at its disposal the police and the other law enforcement agencies to make sure that its regulations are carried out. It is clear that the government has powers which might enable it to get some things done at a lower cost than could a private organisation (or at any rate one without special governmental powers)." (p. 16 - 17).

PART 2: A THEORY OF THE STATE

Part 2 of this essay is intended as a synthesis of part 1 and restates in a more precise and detailed manner the arguments raised there. The aim is to construct a theory of the state which can be used as a description of some aspects of real-world governments. The perspective will be to formulate the theory so as to provide a <u>framework for analysis</u> of whether the state or the private sector ought to take care of certain economic activities.

<u>The distinguishing characteristic of the state</u>

We seek a definition of the state that will work in general equilibrium models, where the state is viewed as an autonomous economic agent like the consumer and the producer. If the state is not simply a consumer or a producer, it must be because the state has some characteristic which distinguishes it from private agents. As observed in part 1, this distinguishing characteristic of the state is authority, and, moreover, nothing but authority. To define the state we therefore need a definition of authority. It will be useful to separate between the legal basis or source of authority and the authoritative regulations to which an economy is subjected, since we shall then be able to take account of legal rights and duties which could be but are not in use.

Let the legislation set L be a set containing all known rights and duties that might conceivably be part of a legal basis for authority at a certain point of time. Some rights which belong to L are: to form dictatorship (enlightened or not), to be king or queen, to rule by democratic representation, to tax, to expropriate real goods from consumers and producers, and to regulate prices and production. Some duties in L are: to maximize collective or individual welfare, to maximize the welfare of the least favoured individual, to use resources and endowments efficiently, to avoid environmental damage, to secure absolute and relative justice, and to guard the liberty of individuals and groups. The legislation set is analogous to technology, and the distinction between L and authority is of the same kind as that between technology and the production technique used in a particular firm.

An authority set A can now be defined as a subset of the legislative source L. Among all conceivable legal rights and duties, those which are in fact applied in a given economy constitute the authority set, and will be called <u>rules</u>. This leads to a formal definition of the state:

> The state is the application of authority, a set of legal rules which regulate the enforcement of social and economic measures.

It should be emphasized that this concept of a state is quite analogous to the microeconomic model of a producer. The parallel between the state's authority set and the private production set has been indicated both here and in part 1. The analogy also extends to the definition of the state, which points out that the significance of authority lies in its application to the economy. It is only physically that the state consists of consumers who maximize their individual utility. The state is authoritative action, regulated by the authority set and executed by consumers; just as the firm is productive action, regulated by the production set and executed by consumers.

Formation of an authority set

An important question is how the authority set is established in a given economy. One possibility, which, in spite of its frequent empirical occurrence, will not be discussed here, is to seize power by violent means. Another possibility is that the formation of an authority set is due to a peaceful collective decision process. Unfortunately, Arrow's possibility theorem (Arrow, 1963) implies that in general, there is no acceptable social decision process which yields a rational social ordering of given alternatives, depending on what the individual orderings of these alternatives are. Arrow describes a social decision process by the following 7 assumptions:

(1) The social ordering is defined over a given choice set of mutually exclusive alternatives.

(2) All elements in the choice set are ordered.

(3) The social ordering is transitive.

(4) If individual orderings change, and the change is only that a certain alternative α rises in every individual ordering; then the new social ordering ranks α before all those alternatives to which α was preferred in the original social ordering.

(5) If individual orderings change, and the change is such that every individual preserves his ordering of those alternatives which are relevant to the social choice, then the social choice is the same.

(6) There is no pair of alternatives the social ordering of which is independent of all individual orderings of those alternatives.

(7) There is no individual whose ordering of all alternatives is the social ordering irrespective of other individuals' orderings.

Assumptions (2) and (3) define rationality in Arrow's model, assumption (4) is called positive association of social and individual values, (5) independence of irrelevant alternatives, (6) citizens' sovereignty, and (7) non-dictatorship. Arrow's possibility theorem states that if we are to construct a social decision process in keeping with (1) - (7), we must have some information about individual preferences. In other words, (1) - (7) are inconsistent with the following assumption of unrestricted domain:

> Any individual ordering of the alternatives is admissible in the social decision process.

There is no restriction on what individuals are allowed to think about the alternatives. Unless we postulate something about the structure of individual preferences, there is then no way to reach social decisions in accordance with requirements (1) - (7). Arrow also argues (1963, p. 61) that it is not enough to assume that each individual's preferences are defined over his consumption set and exhibit local non-satiation. More severe restrictions are needed.

The relationship between Arrow's possibility theorem and a Bergson - Samuelson individualistic social welfare function has been the subject of much discussion and some confusion. An attempt at clarification is made in Appendix 1; suffice it here to say that Arrow's theorem does not imply that no Bergson-Samuelson individualistic social welfare function can exist.

The assumption of unrestricted domain is attractive because it permits the design of a social decision process to be independent of specific sets of individual orderings. Thus, we should look for other ways around Arrow's possibility theorem. For instance, restrictions on the set of alternatives may prove useful in certain cases. Arrow shows (1963, p. 48) that when there are only two alternatives, the majority-of-voters rule is a social decision process consistent with (1) - (7). This is interesting to the extent that the decision process may be formulated as a sequence of either-or choices where the voting individuals do not have preferences over authority <u>sets</u> and there are no more than two mutually exclusive alternatives involved in each choice. Roberts (1980) has noted that information about individuals' cardinal utility functions can be used to avoid Arrow's theorem, which presupposes that individual preference orderings cannot be compared. Other assumptions, such as independence of irrelevant alternatives or transitivity of the social ordering, have also been criticized.

<u>Social decisions with restricted rationality</u>

The core of Arrow's possibility theorem is that when individuals' preferences are sufficiently disparate, there is no way to aggregate them into a well-behaved social preordering. This indicates that we should investigate the relationship between individuals' preferences and the set of choice alternatives over which they are defined. One approach to such an analysis is to bring together Rawls' concept of original position (Rawls, 1971) and Elster's concept of commitment (Elster, 1979).

Rawls argues that individuals are in a socalled original position behind a veil of ignorance when they discuss and determine the basic structure of their society. Behind the veil, individuals do not know what their position in the economy will be after determination of the basic structure; nor do they know the likelihood of entering a particular position. Therefore, their strategy (in Rawls' theory) is to maximize their welfare, given that the worst positional result obtains. The main social objective becomes that of maximizing the welfare of the least favoured individual in the economy (i.e. in front of the veil), since anyone behind the veil may become that individual. This objective hinges on the notion of a veil of ignorance, and we must ask why the veil is there in the first place. Rawls emphasizes that the original position is imaginative: "It is clear, then, that the original position is a purely hypothetical situation. Nothing resembling it need ever take place, although we can by deliberately following the constraints it expresses simulate the reflections of the parties." (Rawls, 1971, p. 120). A possible interpretation of this seems to be that individuals <u>deliberately</u> place themselves behind a hypothetical veil of ignorance.

To place oneself in the original position behind a veil of ignorance is the same as a redefinition of the choice set. In front of the veil, the choice set contains allocations in the real economy. Behind the veil, the choice set contains different constitutions, e.g. different social welfare functions. By accepting the notion of an original position, individuals select basic rules for the allocation process instead of choosing economic allocations. Thereby restrictions on the range of the economic result pertaining to any individual are induced, although their significance will only be known in front of the veil. By entering the original position, the individual binds himself to a choice among those allocations that are compatible with the basic rules.

Elster (1979) argues that it may be individually rational to bind oneself: Ulysses, wanting to hear the Sirens and knowing the perilous effects of their song, had his men bind him to the mast of his ship. In this way, he took steps ex ante to overcome temporary irrationality and secure overall rationality. Elster defines this kind of commitment by five criteria:

> To bind oneself is to carry out a certain decision in order to increase the probability that one will carry out another decision at a later time;
>
> if the first decision changes the set of options available at the later time, then this does not count as binding oneself if the new feasible set contains the old one;
>
> the effect of the first decision must be to set up some causal process in the external world, since the incentives to carry out the second decision must be enhanced;
>
> the resistance against the first decision must be smaller than the resistance that would have opposed the second decision had not the first decision intervened; and

binding oneself is an act of commission, not of omission.

In his discussion of commitment, Elster questions the meaning of individual rationality. Ulysses not only orders his men to bind him, he also orders them to ignore his orders to be released when he hears the Sirens. The first Ulysses claims priority over the second, who will undoubtedly claim priority over the first, and it seems that Ulysses has more than one self. (In fact, Elster argues that for practical purposes, 3 is a reasonable number of selves.) This philosophical complication is not important here, however, and it will be assumed that the members of an economy may pretend to be in the original position when they determine the authority set. Their attitude is rational by definition, so to speak, because the alternative - being unable to establish any authority set and consequently having no constitution - is worse.

A formal theory of authority

This section presents an abstract model of the formation of the legal rules establishing the state's authority and describes the basic structure of the authority set introduced above. The authority set is a subset of all possible constitutional rules and is chosen by an assembly of individuals who participate in a voting process. The rules chosen influence the public production possibilities as well as the selection of goods to be produced by the state.

Let L be the finite and non-empty set of all known legal rights and duties which can belong to a legal basis for public authority at a certain point of time. L_s is the collection of all subsets of the legislation set L. The electorate is also a finite and non-empty set, denoted by E; and contains the individuals who are to choose a number of rules (which constitute one of the elements in L_S). Each individual $i \in E$ has a reflexive and transitive preference relation $\lesssim_i$ defined on L_S. P is a profile of all individuals' preference relations: $P = \{\lesssim_i\}$, $\forall\ i \in E$. Let E_S be the collection of all subsets of E, and define a set $D \subset E_S$ such that $\emptyset \notin D$. Then the pair (E,D) is a voting game whenever D is the set of winning coalitions (Nakamura, 1979). Voting under majority rule is an example of a voting game, where a coalition S is a winning coalition if and only if $|S| > |E|/2$. The social choice is made according to the following concept of domination (Le Breton and Salles, 1986, Definition 5):

> If there exists a coalition $S \in D$ such that for $\alpha \in L_S$,
>
> (i) $\alpha <_i \alpha_1$ for all $i \in S$ and $\alpha \in L_S$, in which case we write $\alpha <^0 \alpha_1$;
>
> (ii) $\alpha \lesssim_i \alpha'$ for all $i \in S$ and all $\alpha' \in L_S$ for which there is a finite sequence $\{\alpha'_k\}$ in L_S such that $\alpha_1 <^0 \alpha'_1 <^0 \alpha'_2 <^0 \ldots <^0 \alpha'$;
>
> then α_1 dominates α, and we write $\alpha < \alpha_1$.

The relation $<^0$ may be called zero-order domination. Intuitively, the definition of domination contains two requirements: First, α_1 must dominate α to the order of 0. Second, there must exist no other alternative α' to which α is preferred by some individual in the winning coalition S and which eventually dominates α_1, even if this happens through an arbitrarily long, finite sequence of zero-order dominations. If such an alternative α' did exist, then the decision process would exhibit the well-known paradox of voting. Thus the definition of domination is designed so as to prevent voting cycles, no matter how many alternatives are involved in a cycle. In the special case where any sequence has only one term (i.e. when $\alpha_1' = \alpha'$, which we may call first-order domination and write as $\alpha <^1 \alpha_1$), Rubinstein (1980) has motivated the definition as follows: "True, I prefer b to a, but if b is adopted, a situation arises where the majority prefers c. Since c is worse than a from my point of view, I will not take any chances and will not vote for b in place of a."

Now we are in a position to define a solution concept for voting games due to Rubinstein (1980) and Le Breton and Salles (1986, Definition 6):

> The <u>stability set</u> is the set of alternatives which are not dominated by any other alternatives. The stability set is $C(E,D,P) \equiv \{\alpha \in L_s \mid \nexists\, \alpha_1 \in L_s \text{ such that } \alpha < \alpha_1\}$.

According to Le Breton and Salles (1986, Theorem 7), the stability set of a proper voting game is always non-empty. A voting game is proper if the intersection of any two winning coalitions is non-empty. (Voting under majority rule is a proper voting game since two winning coalitions must include more than half the voters each and therefore must have at least one member in common.) Further details on stability sets are given in Appendix 2. The formal definition of the set of rules establishing the state is:

> The authority set A of the economy belongs to the stability set which results from a proper voting game over all subsets of the legislation set: $A \in C(E,D,P)$, where E, L_s and consequently A are finite.

Obviously, A may not be unique in this definition, since C(E,D,P) may contain more than one element. The uniqueness of A will not be questioned here. It seems reasonable to assume that C(E,D,P) contains one element if the voting game is such that every individual must adhere to only one out of several alternatives to which he is indifferent and if the rule selecting winning coalitions is such that two opposing coalitions cannot win at the same time.

Redefine I to be the finite set of individuals in the whole economy (I contains the electorate E), and let G be a finite set of goods. G contains a set of private goods, M. A technological structure is defined for M by a convex production set $\bar{Y}$. M is the set of ordinary private goods which are inputs and/or outputs in the production processes of private firms. These private goods are traded in a perfectly competitive part of the economy, where each agent takes the vector of market prices q as a given parameter. Furthermore G contains K, a set of goods which are not produced by private, profit-maximizing firms. The members of K may be pure public goods, "impure" public goods (Sandler and Tschirhart, 1980, p. 1487), or even private goods other than those in M; but every $k \in K$ will be called a public good. (What is of interest here is how commodities are produced, not how they are consumed.) Some of the goods in K, K_1, are produced by coalitions (clubs) S^1, S^2,.... of consumers. This happens when decision costs are not prohibitive for a club which could produce some $k \in K$. Let the decision costs for good k be the values of a function $d: I_S \times \{k\} \to \mathbb{R}_+$, where I_S is the collection of all subsets of I. We would normally expect decision costs to increase with coalition size, so that $d(S,k) \geqq d(S^1,k)$ whenever $|S| \geqq |S^1|$. Let the total costs of producing k be $c^k(z_k^S)$, where z_k^S is the amount of k demanded by some coalition S when the coalition members must cover the costs c^k themselves. Finally, for each $i \in I$, u_0^i is the utility level if $i \notin S$ and u_S^i the utility level if $i \in S$. Then a coalition procuring a public good k is characterized by a membership condition and an aggregate budget constraint:

$$\forall\ i \in S,\ u_S^i \geqq u_0^i$$
$$\sum_{i \in S} q(x^i - w^i) + c^k(z_k^S) + d(S,k) = 0$$

where x^i and w^i are individual i's consumption and initial endowments, respectively, of the goods in M. Denote by S_k the set of all coalitions which satisfy these conditions, given k. Then K_1 can be defined as the set $K_1 = \{k \in K \subset G \mid S_k \neq \emptyset\}$.

Another subset of K is K_2 - public goods which are produced by the state. Define <u>administration costs</u> by a function $a: \{k\} \rightsquigarrow \mathbb{R}_+$. Administration costs result from the public activities needed to <u>set up</u> the production of k. Assume first that individual utility levels are observable. Then a public good will be produced by the state if it would have been produced by a club had decision costs been equal to administration costs: $k \in K_2$ if there is an $S \in \{I_S \backslash S_k\}$ which would belong to S_k if $d(S,k)$ were equal to $a(k)$. It is possible in such a case to finance the production of k, e.g. by charging $[c^k(z_k^S) + a(k)]/|S|$ from each member of S, and the state's budget will balance. This situation seems too simple, however, mainly because individual utility levels are assumed to be observable. With perfect information the state's administration costs are likely to be approximately the same as a private club's decision costs, and the difference between public and private production would probably be negligible.

In practice individual utility levels are not observable (or at least they are not observed). A basic assumption will therefore be that the state can only observe each individual's consumption of private and public goods. Let $[x^i, z^i]$ be the vector of consumption levels of private (x) and public (z) goods for all individuals $i \in I$. The state measures social welfare by a real-valued function $U = U([x^i, z^i])$. The optimal level of public production results from a maximization of U, although it is not known whether this moves the economy towards or away from a Pareto efficient allocation. Whether or not a good $k \in K$ will be produced by the state is determined by the requirement that $U_k \geqq U_{-k}$, where U_k is social welfare when the state produces k and U_{-k} is social welfare when it does not. It is important to note that all x^i, $i \in I$, as well as all z_n^i, $i \in I$, $n \in K_1$, may change when the state increases its provision of k. The reasons are that public production must be financed with some kind of (tax) revenues, and that some consumers may choose to leave their club if it happens to produce k and they can get it cheaper from the state.

Let the <u>tax revenue function</u> be $T = T([x^i, z^i])$, where x^i and z^i depend on market prices, q; initial endowments, w^i; the production possibilities for private goods, Y; and the cost functions for public goods, $c^k(\cdot)$. Prices of publicly produced goods count as taxes. Suppose that $k \notin K_2$ initially, and let z_n be the public production of good n. Corresponding to U_k is the public budget restriction

$$T([x^i, z^i]_k) = \sum_{n \in K_2} [c^n(z_n) + a(n)] + c^k(z_k) + a(k),$$

whereas U_{-k} corresponds to

$$T([x^i, z^i]_{-k}) = \sum_{n \in K_2} [c^n(z_n) + a(n)].$$

The condition for k to become a member of K_2 is that $U_k \geqq U_{-k}$ under the restriction that the budget balances in both situations.

The final step is to relate administration costs, social welfare, and tax revenues to the authority set A. Formally, this may be done by making A one of the arguments in the functions a(•), U(•), and T(•). The point, however, is simply that the rules in the authority set must determine criteria for evaluating social welfare (i.e. U), establish the state's ability to raise revenues (i.e. T), and institute the public sector's administrative framework (i.e. a).

A brief restatement and some remarks

An essential assumption in the model of the state is that the individuals selecting legal rules have enough information about the voting process they participate in to avoid voting cycles. This information enables them to establish an authority set even if their preferences are diverse. The objective of the state being to increase social welfare, the authority set should make it possible to raise tax revenue and produce public goods efficiently. The state may produce a good if a social decision process involving a large number of consumers and high decision costs would otherwise be required to set up a private firm or form a private club with an optimal production level of the good. With observable individual utility functions social welfare can be increased if the state takes over the production of goods for which its administration costs are lower than private decision costs. If individual utility functions are unobservable, the state's political and social objectives may instead be expressed by a welfare function defined over individual consumption levels. Whether a good is produced by the public or the private sector is then determined by maximization of this social welfare function.

A few remarks on the formulation in the previous section are perhaps necessary. First and foremost, the theory is extremely simplified. It does not consider imperfect competition or asymmetric information at all: competition is either perfect or production takes place in clubs, and the only imperfection in the information system is that individual utility levels are unobservable. However, imperfect competition and asymmetric information would not lead to substantial alterations in the essentials of the theory, although it would undoubtedly become much more complex. The main point - that authority is the distinguishing feature of the state and that public behaviour must be based on legal authority - would probably remain the same.

It could be argued that administration costs should depend on the set of consumers who receive a good produced by the state. This can easily be incorporated in the theory above by redefining the domain of $a(\cdot)$ to be $I_s \times \{k\}$. Another possible objection could be that the separation between production costs $c^k(z_k)$ on the one hand and decision costs $d(S,k)$ or administration costs $a(k)$ on the other is artificial. This contention is to some extent irrelevant, since if $d(\cdot)$ or $a(\cdot)$ were included in $c^k(\cdot)$, there would be no interesting difference between clubs and the state. It might well be that there is no interesting <u>empirical</u> difference between clubs and the state, but the possible result that the state produces nothing ($K_2 = \emptyset$) is not inconsistent with the theory as such.

A public good k may be produced both by clubs and by the state, i.e. the intersection of K_1 and K_2 may be non-empty. This poses no problem when individual utility levels are observable, since public production then leads to a Pareto improvement. With unobservable utility levels the case is less obvious. As already noted the state does not know whether maximization of U yields a Pareto efficient allocation, and thus cannot determine whether leaving the production of k to some private club alone would lead to a Pareto improvement. It is therefore difficult to say something in general about privatization in the model. (This is to some extent reflected in recent articles on privatization, which for the most part deal with partial theoretical aspects or empirical examples. See Kay and Thompson (1986), Yarrow (1986), and Bös (1987).) Even in the full information case where utility can be observed, the state's efficiency at production depends on the specific rules in the authority set. Without knowledge of individual utility its position is worse since it does not know what the "true" social welfare is. Therefore it must be expected that public production levels are not necessarily optimal, and in a normative analysis the true welfare loss due to inoptimality of public production would have to be weighed against the cost saving d(S,K) - a(k).[2)]

The problem of existence of general equilibrium in the model has not and will not be treated in the general case. Instead, it will be discussed briefly with special versions of the social welfare function U and the tax revenue function T in the next chapter. There U is defined as the utility function of a "representative" consumer, as is often done in optimum taxation theory. Furthermore it is convenient to assume that there are no

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[2)] Of course, private clubs may find it equally difficult to observe the members' welfare, but they differ from the state in that membership is voluntary.

clubs, i.e. every public good that is produced, is produced by the state. Finally the values of T may be the revenues from sale of public goods, lump sum transfers from consumers, or ordinary, distortive commodity taxation.

Conclusion

The state has been defined as the application of authority, a set of legal rights and duties which regulate the enforcement of social and economic measures. The authority set may be established by a collective decision process where the individuals deliberately place themselves in a Rawlsian original position in order to choose authority rules from a broader set of exogenously given choice alternatives. In the Rawlsian original position, the individuals do not think strategically about their subjective economic position. Hence it is easier to reach a social decision than it would be if no restrictions were placed on the decision process. Economic authority typically concerns the social welfare level in the economy, and institutes e.g. the rights to tax and the right to control prices and production. Such rights are used if an economy with rational public economic activity is better than an entirely private economy, according to normative criteria defined in the authority set.

APPENDIX 1: SOCIAL WELFARE FUNCTIONS

The Arrow social welfare function

Define:

X a set of choice alternatives.

$\precsim$ the set of all logically possible orderings of X. Any member $\precsim_i$ of $\precsim$ is complete: $\forall\ \alpha,\alpha_1 \in X$, $\alpha \precsim_i \alpha_1$ or $\alpha_1 \precsim_i \alpha$; and transitive: $\forall\ \alpha,\alpha_1 \in X$, $\alpha \precsim_i \alpha_1$ and $\alpha_1 \precsim_i \alpha_2 \Rightarrow \alpha \precsim_i \alpha_2$. If it is not the case that $\alpha \precsim_i \alpha_1$, then we write $\alpha_1 <_i \alpha$. Hereafter the subscript i denotes individual i, and there are I individuals.

P a profile of I individual orderings of X.

Π the I-dimensional set of all logically possible profiles.

$\precsim_s \in \precsim$ a social ordering of X.

$\mathcal{F}: \Pi \rightsquigarrow \precsim_s$ a social decision process specifying $\precsim_s$ as a function of P; this is the Arrow social welfare function. If there is a pair α,α_1 in X such that $\alpha \neq \alpha_1$ and $\alpha \precsim_s \alpha_1$ for any profile P (where $\precsim_s$ corresponds to P), then $\mathcal{F}$ is said to be <u>imposed</u>. If there is an i such that for all α,α_1 in X, $\alpha <_i \alpha_1 \Rightarrow \alpha <_s \alpha_1$ regardless of the orderings of all other individuals than i, then $\mathcal{F}$ is said to be <u>dictatorial</u>.

$C(X)$ the set of alternatives $\alpha \in X$ such that $\alpha_1 \precsim_s \alpha$ for all $\alpha_1 \in X$; this is the social choice.

Arrow (1963) makes the following assumptions:

(1) The domain of $\mathcal{F}$ is Π (unrestricted domain).

(2) Let $<_s$ correspond to P; $<_s'$ to P'; $\alpha,\alpha_0,\alpha_1,\alpha_2 \in X$; and let

$\alpha_0 \neq \alpha \neq \alpha_1$. If for all i,

$\alpha_0 \precsim_i \alpha_1 \Leftrightarrow \alpha_0 \precsim_i' \alpha_1$,

$\forall\ \alpha_1,\ \alpha \precsim_i \alpha_1 \Rightarrow \alpha \precsim_i' \alpha_1$, and

$\forall\ \alpha_1,\ \alpha \prec_i \alpha_1 \Rightarrow \alpha \prec_i' \alpha_1$, then

$\alpha \prec_s \alpha_2 \Rightarrow \alpha \prec_s' \alpha_2$ (positive association of social and individual values).

(3) Let C(X) correspond to P and C'(X) to P'. If for all i and all α, α_1 in X, $\alpha \precsim_i \alpha_1 \Leftrightarrow \alpha \precsim_i' \alpha_1$, then C(X) = C'(X) (independence of irrelevant alternatives).

Arrow's possibility theorem is that a social decision process satisfying conditions (1), (2) and (3) is either imposed or dictatorial.

The Bergson-Samuelson social welfare function

Samuelson (1947) defines the Bergson-Samuelson social welfare function as a real-valued function $W = W(x^1, x^2, \ldots, x^I)$, where the x^i's are individual consumption bundles. If it is required that "individuals should count" (Samuelson, 1947, pp. 223 and 229), W takes the special form $W = V[u^1(x^1), u^2(x^2), \ldots, u^I(x^I)]$, where the u^i's are individual utility functions. Thus, what makes the Bergson-Samuelson welfare function individualistic, is that it is composed of individual utility functions and some function aggregating individual utility levels. The functions u^i express the economy's unique profile P of individual orderings of different consumption bundles. The set X of choice alternatives consists of all attainable sets of I individual consumption bundles. Therefore, the precise connection between Arrow's $\mathcal{F}$ and Samuelson's W is:

$$C(X) = \{\alpha \in X \mid V[u^1(x^1), u^2(x^2), \ldots, u^I(x^I)] \text{ is maximal}\},$$

where C is the social choice corresponding to P. Maximization of the Bergson-Samuelson individualistic social welfare function leads to the Arrow social choice corresponding to the economy's unique set of individual preferences (provided, of course, that fundamental ethical values are the same).

Samuelson has argued that Arrow's possibility theorem is irrelevant to welfare economics: "... the Arrow result is much more a contribution to the infant discipline of mathemathical politics than to the traditional mathematical theory of welfare economics. I export Arrow from economics to politics because I do not believe that he has proved the impossibility of the traditional Bergson welfare function of economics, even though many of his less expert readers seem inevitably drawn into thinking so." (Samuelson, 1967, p. 42). It is of course true that Arrow has not proved the non-existence of W, his possibility theorem concerns $\mathcal{F}$, which has individual orderings as its domain and not the set of choice alternatives. But this does not necessarily imply that Arrow's result is irrelevant to welfare economics. It is rather the other way around, that Arrow's possibility theorem represents a limitation of the generality of a Bergson-Samuelson individualistic social welfare function, since the specification of the latter must be ad hoc relative to the economy's preference profile P <u>as long as this is not known</u>. Put differently, the point is that there is no way to specify how V should be altered when the functions u^1, u^2, ..., u^I change, without restrictions on the kind of changes which are allowed. (This question has also been discussed by Johansen (1969).)

APPENDIX 2: VOTING GAMES

The definition of domination which is used in the essay reflects an assumption that voting individuals are able to see through all possible voting cycles and modify their own voting accordingly. This assumption may seem strong, but if it is weakened, the stability set may be empty. Since they are quite recent, some results on the non-emptiness of stability sets are summarized below.

Define the <u>Nakamura number</u> as an integer h(E,D) such that

$$h(E,D) = \infty \text{ if } \bigcap_{S \in D} S \neq \emptyset,$$

$$h(E,D) = \min \{|D'|: D' \subset D, \bigcap_{S \in D'} S = \emptyset\} \text{ if } \bigcap_{S \in D} S = \emptyset.$$

Intuitively, the Nakamura number is the size of the smallest set of winning coalitions with no veto players - a veto player being defined as an individual who appears in every winning coalition. The voting game is proper if and only if $h(E,D) \geqq 3$.

The main theorem in this context is that the stability set of a proper voting game is non-empty (Le Breton and Salles, 1986, Theorem 7). The proof is so short and simple that it will be replicated here:

> Assume that $<$ has a cycle, i.e. there is an index $k \leqq |L_s|$ and a sequence $\alpha_1, \alpha_2, \ldots, \alpha_k$ in L_s such that $\alpha_1 <_{C^1} \alpha_2 <_{C^2} \ldots <_{C^{k-1}} \alpha_k <_{C^k} \alpha_1$, where $<_C$ signifies that one alternative is preferred to another due to the voting of coalition C. According to the definition of domination, this must imply that $\alpha_1 \lesssim_i \alpha_k$ for all $i \in C^1$, and $\alpha_k <_i \alpha_1$ for all $i \in C^k$. Since the voting

game is proper, $C^1 \cap C^k \neq \emptyset$, so there is an individual $i \in E$ to whom $\alpha_1 \precsim_i \alpha_k$ and $\alpha_k \prec_i \alpha_1$. This is a contradiction, so it cannot be the case that $\prec$ has a cycle. Hence $\prec$ has maximal elements on L_s.

Weaker theorems are:

Le Breton and Salles, Theorem 5.

(i) If $|L_s| \leqq 2h(E,D) - 3$, then the stability set with first-order domination is non-empty.

(ii) If $|L_s| \geqq 2h(E,D) - 1$, then there is a triplet (E,D,P) such that the stability set with first-order domination is empty.

Le Breton and Salles, Theorem 1.

(i) If $|L_s| \leqq h(E,D) - 1$, then the stability set with zero-order domination is non-empty.

(ii) If $|L_s| > h(E,D) - 1$, then there is a triplet (E,D,P) such that the stability set with zero-order domination is empty.

The stability set in (i) and (ii) coincides with the core of the voting game.

Le Breton and Salles, Theorem 4.

If every individual in a proper voting game has a preference ordering (and not a preordering) on L_s - i.e. indifference as between alternatives is precluded - then the stability set with first-order domination is non-empty.

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GENERAL EQUILIBRIUM ASPECTS OF OPTIMUM TAXATION FORMULAE

"La propriété c'est le vol."

Pierre Joseph Proudhon

GENERAL EQUILIBRIUM ASPECTS OF OPTIMUM TAXATION FORMULAE

Introduction

This chapter is an attempt to point out and clarify some of the general equilibrium aspects of optimum taxation formulae. In Section 1 of the essay the assumption of constant producer prices is examined. This assumption has been discussed by several authors: Dixit (1970) argues that the formulae obtained with constant producer prices are the same as those obtained if we assume that there are constant returns to scale in production. Atkinson and Stiglitz (1976, p. 102) write: "For ease of exposition, it is ... assumed that producer prices are fixed for all commodities and labour ..., although the results in no way depend on this assumption." Sandmo (1982, p. 93) says that "The assumption of constant producer prices is obviously a strong one. However, it could fairly easily be relaxed to an assumption of constant returns to scale, the important point being zero profit for distribution to the consumers." Sandmo's point is reformulated in Section 1, in order to discuss the sense in which it is correct.

Section 2 examines the traditional interpretation of the formula for optimal taxation and contains a brief consideration of its practical usefulness. The focus is on the relationship between substitution effects and income effects of taxation.

The third main section deals with the significance of public production in optimum taxation theory. One of the main results from this theory is the inverse elasticity rule (Ramsey, 1927): Taxes should be relatively high in markets where the elasticity of demand is relatively low (in absolute value). Modern versions of the inverse elasticity rule are most often calculated under the assumption that the composition and magnitude of public expenditure are constant. In other words, the possible influence of public production and consumption on the demand for private goods and hence on tax revenues is only implicit in the theory. Diamond and Mirrlees (1971, p. 271) touch upon this question, but seem to regard it as one of little interest: "... the presence of alternative bundles of public consumption does not alter the rules for the optimal tax structure." An intuitive comment to this assertion could be that if additional tax revenue is needed to finance an <u>increase</u> in public production, and this increase results in greater demand for some private goods, then these goods should be taxed more heavily than others to the extent that the demand increase counteracts the own price effect. The purpose of Section 3 is to shed some light on this intuition.

Institutional set-up

Goods.

There is a finite set M of private goods and a finite set K of publicly produced goods in the economy. K may contain both pure public goods in the sense of Samuelson (1954) and ordinary private goods. However, any $k \in K$ will be called a public good, and $K \cap M = \emptyset$.

Prices and taxes.

Corresponding to M there are two $|M|$-dimensional price vectors: consumer prices q and producer prices p. Both are strictly positive: $q > 0$ and $p > 0$. No public goods have prices. Tax rates are defined by $t \equiv q - p$. (The assumption that $q > 0$ and $p > 0$ may thus restrict the set of permissible tax rates. Of course, more sophisticated tax systems are conceivable. In principle the one outlined here can model any kind of taxation of market transactions in a static context.) Good 1 is chosen as the numéraire, so that $q_1 = p_1 = 1$ and $t_1 = 0$.

The state.

The state produces the set K of public goods, the production levels of which are given by the vector $z \in \mathbb{R}_+^{|K|}$, where $\mathbb{R}_+^{|K|}$ is the non-negative orthant of $|K|$-dimensional Euclidean space. z is produced from inputs bought in private markets at producer prices which are taken as given; and public purchases are financed with the tax revenues. The public input requirement vector is $g(p,z)$, where $g(p,z) \in \mathbb{R}_-^{|M|}$ denotes the level of all $|M|$ inputs.[1] The components of g are non-positive real numbers by convention, so that $\partial g_m/\partial z_k < 0$ when m is used in the production of k. (Otherwise, $\partial g_m/\partial z_k$ is zero.) g as a function characterizes the public production technology, which is given exogenously. There is free disposal of z.

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[1] g is written somewhat imprecisely with p and z as arguments to signify the possible importance of these variables in the public decision process determining g. Technologically z implies an upper bound on $g(\cdot)$. Later, g will be regarded as a mathematically well-defined function of (p,z).

The private producer.

For simplicity it is assumed that there is only one producer. All inputs to and outputs from private production belong to M; and their levels are given by the vector $y \in \mathbb{R}^{|M|}$. The producer maximizes profits py subject to the restriction that $y \in Y$, the production possibilities set. Y and, from the producer's point of view, p are given exogenously. Y is a convex cone, so that $py = 0$ whenever y is optimal for p.

The consumer.

It will be assumed that there is only one consumer, whose final consumption is given by the vector $x \in \mathbb{R}_{+}^{|M|}$. Initial endowments are given by an $|M|$-vector $w \geq 0$. There are no initial endowments of public goods. The consumer has a differentiable, strictly quasiconcave and increasing utility function u, defined over all consumption bundles (x,z). x is chosen so as to maximize u subject to a budget constraint $q(x - w) = b$, where q is taken as given and b is a positive, negative or zero lump-sum transfer. The ensuing indirect utility function is v(q,z,w,b) and the demand function x(q,z,w,b). It is assumed that these functions are differentiable to any required order.

General equilibrium.

The analysis must apply to an economy where demand equals supply in every private market;

$$x - w = y + g$$

Premultiplying both sides by q, we obtain

$$q(x - w) = (p + t)(y + g)$$

and by the consumer's budget restriction (assuming that $b = 0$) and the fact that profits are zero, we have

$$t(x - w) + pg = 0$$

Thus, it is necessary that tax revenues equal public expenditure if general equilibrium is to be maintained. This is simply Walras' law in a three-agent economy.

1: Constant producer prices.

To elucidate the assumption of constant producer prices in optimum taxation theory, a simplification of the institutional framework will be introduced in keeping with the Samuelson-Diamond-Mirrlees approach (Samuelson, 1951, and Diamond and Mirrlees, 1971). Assume that $z = 0$ and that public consumption is fixed at $g^0 \in \mathbb{R}_-^{|M|}$. (Note that some of the components of g^0 are privately produced commodities.) Public expenditure becomes $-pg^0$, a constant (see below), and the economy can be partly characterized by the following system, where the maximum utility function v is taken as the measure of social welfare and market demand may be positive, zero, or negative:

Welfare:	$v(q,0,w,b)$
Market demand:	$x(q,0,w,b) - w$
Private production:	y such that $y \in Y$ and py is maximized
Public budget constraint:	$(q - p)(x - w) - b + pg^0 = 0$

The problem is to find an optimal set of consumer prices. The vector q appears thrice in the system above; in the consumer's utility function, in the final demand function and in the public budget constraint, and a reasonable formulation of the optimization problem would be to

$$\text{maximize} \quad v(q,0,w,b) \qquad \text{with respect to } q,$$
$$\text{subject to } (q - p)[x(q,0,w,b) - w] - b + pg^0 = 0.$$

For this problem to be well-defined something must be said about the vector of producer prices p. The most important thing to note is that p does not affect consumer welfare directly; it is not among the arguments of v. The choice of producer prices only has a bearing on the existence of general equilibrium in the economy (and in this respect p indirectly influences welfare). p is less relevant for the optimal <u>structure</u> of consumer prices, and the more interesting question is not whether p is constant, but whether it exists. The existence of an optimal set of producer prices will be taken up below; for the moment, however, let us make the somewhat strong assumption that for each demand vector $x(q,0,w,b)$ and given public consumption g^0 there is a p which sustains general equilibrium with a profit-maximizing producer. Let us also assume that lump-sum transfers b are infeasible. Then the complete optimization problem may be stated as follows:

Let the production possibilities set Y, initial endowments $w \geq 0$ and public consumption $g^0 \leq 0$ be given exogenously. Then, among all permissible pairs (q,p), find (q^*,p^*) with the following properties:

(i) $v(q^*,0,w,0)$ is maximal subject to the restriction that
$(q^* - p^*)[x(q^*,0,w,0) - w] + p^*g^0 = 0$,

(ii) $x(q^*,0,w,0) - w = y^* + g^0$, where y^* maximizes profits p^*y subject to the restriction that $y \in Y$.

(i) is a constrained maximization problem where it is analytically convenient to take p^* as a given constant; whereas (ii) is an existence problem. If there is an interior solution to the complete problem, the maximum in (i) can be characterized by ordinary first-order conditions, and the optimal consumer price vector is indicated by means of gradients (see Section 2). However, the existence problem (ii) is more complicated, and was analysed in detail by Diamond and Mirrlees (1971, pp. 273 - 276). Indeed, Diamond and Mirrlees' "Optimal Taxation Theorem" is precisely a demonstration of the existence of p^*. Later, Mantel (1975) has established a more general existence theorem for overall market equilibrium in economies with a public sector. Mantel's results imply that by viewing the problems (i) and (ii) as one, we may replace the assumption that there is an equilibrium-preserving p for <u>each</u> demand vector x by a demonstration that there is such a p for one specific demand vector, viz., $x(q^*,0,w,0)$.

Not all conceivable tax systems are necessarily consistent with general equilibrium. Mantel (1975) and Gale and Mas-Colell (1975, 1979) emphasize two particular requirements which seem difficult to avoid: No agent should face negative market prices, and the consumer's (set-valued) budget function should be lower hemicontinuous with convex values. Negative market prices pose a problem if e.g. consumer preferences are monotonic, since demand could be infinite if the consumer were paid to consume. With limited resources and finite production possibilities it could be that no equilibrium existed in this case. Unless the consumer is or may become satiated in some goods, it is therefore required of the tax system that no subsidy exceeds 100%.

The set-valued budget function is defined as

$$B(p,s) \equiv \{x \in X | \; q(p,s)x - \beta(p,s) \leqq 0\},$$

where s is a parameterization of the tax system, X is a convex consumption set, $q(p,s)$ is the consumer price vector resulting from producer prices p and taxation s, and $\beta(p,s)$ is the consumer's income. If the tax system is $s = (t,b)$ as in the model above, s is fixed for all p and the consumer's income is $\beta(p,s) = (p + t)\bar{w} + b$, then the budget function is both continuous in p and convex-valued. (For $b < 0$ and $|b|$ sufficiently large, $B(p,s)$ may of course be empty.) Regressive taxation, on the other hand, introduces non-convexities in the budget set, as illustrated in Figure 3.1. The consumer owns w_1 of good 1 and nothing of good 2. If he sells more than $w_1 - x_1$ he is not taxed. But if his supply is less than $w_1 - x_1$ he faces the tax vector $t = (t_1,0)$, where $t_1 < 0$. His budget set is therefore non-convex. We are not guaranteed that a general equilibrium price vector exists, since it might be p if only the consumption function is well-behaved at these prices.

To take p^* as a given constant in (i) would be unacceptable if producer prices were arguments in the indirect utility function v. This happens when there are decreasing returns to scale and the positive profits from private production are distributed to the consumer. Then, both an increase in the consumer price and a decrease in the producer price of some commodity will generally decrease consumer welfare. The former entails the ordinary price distortion effects, and the latter has an income effect

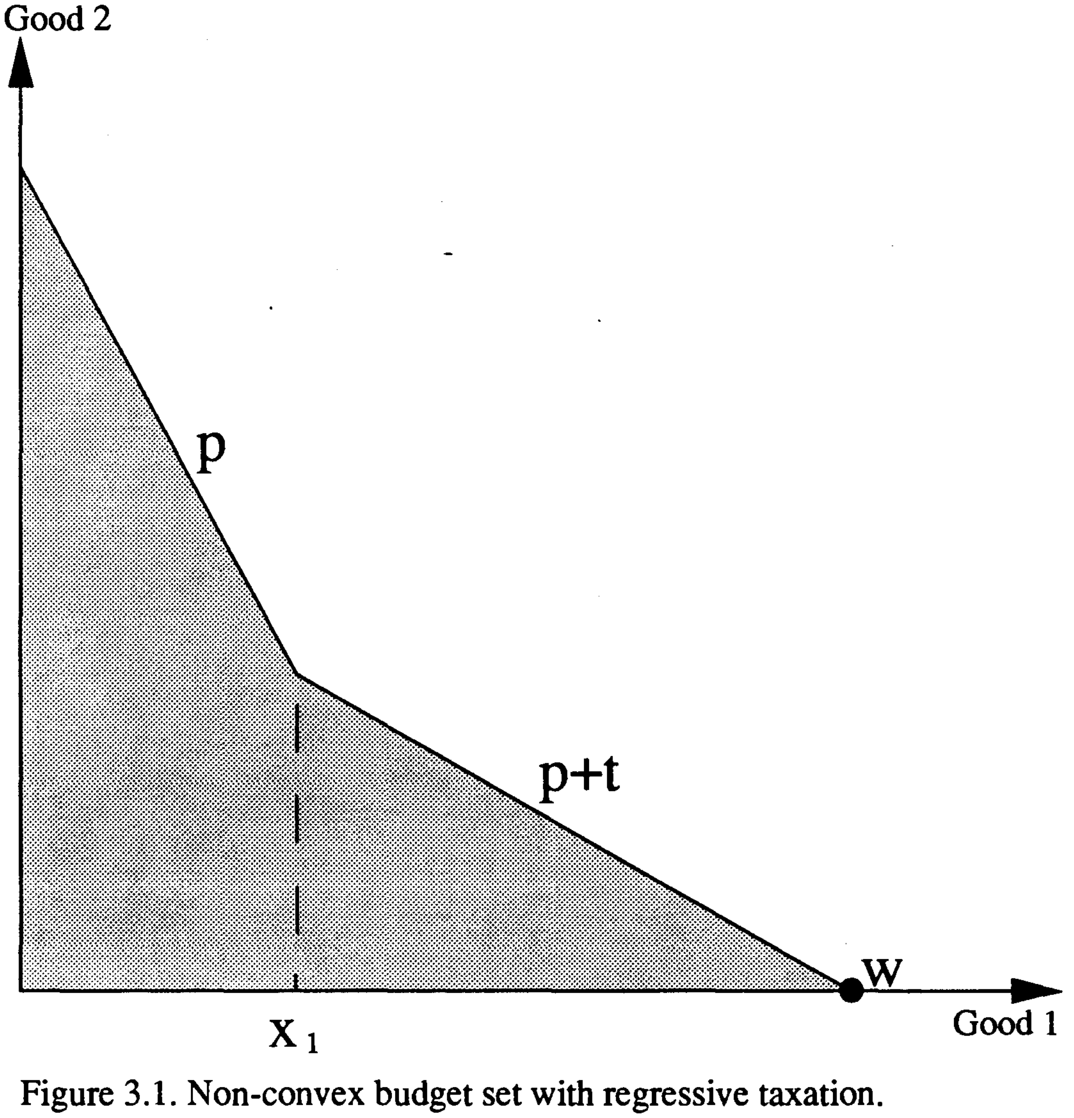

Figure 3.1. Non-convex budget set with regressive taxation.

insofar as the producer's profits decrease. A similar question has been discussed by Mirrlees (1972) and Hagen (1985), as well as by Dixit (1970) and Atkinson and Stiglitz (1976). Except for Mirrlees these authors take a primal approach, showing that the first-order changes in producer prices which are relevant for optimal taxation vanish when the transformation function (the surface of Y) is linearly homogeneous. The interpretation offered above may perhaps be viewed as a dual alternative.

2: Income effects

Let us now assume that there is positive public production: $z \geq 0$. To begin with, z, public inputs, $g(p,z)$, and lump-sum transfers, b, are determined by the state and are exogenous to the analysis. To solve the maximization problem (i), form the Lagrangian

$$\Lambda = v(q,z,w,b) + \mu\{(q - p)[x(q,z,w,b) - w] - b + pg(p,z)\}$$

where the Lagrange multiplier μ may be interpreted as a measure of the marginal cost of public funds.[2)] Given the existence of an appropriate set of producer prices, the first-order conditions with respect to consumer prices can be written as

$$\frac{\partial \Lambda}{\partial q_m} = \frac{\partial v}{\partial q_m} + \mu\{(x_m - w_m) + \Sigma_n t_n \frac{\partial x_n}{\partial q_m}\} = 0, \qquad m,n \in M$$

Since $\partial v/\partial q_m = -\lambda(x_m - w_m)$, where λ is the consumer's marginal utility of

---

[2)] This formulation implicitly presupposes that μ exists, which is not trivial since the expression in braces is a function of prices and public production in general equilibrium. It is not certain that such a function is concave or quasiconcave.

lump-sum income, the condition can be rewritten for each $m \in M$ as

$$(1) \qquad \Sigma_n \; t_n \frac{\partial x_n}{\partial q_m} = \frac{\lambda - \mu}{\mu} (x_m - w_m)$$

With non-negative initial endowments, the Slutsky equation is:

$$\frac{\partial x_n}{\partial q_m} = \frac{\partial \tilde{x}_n}{\partial q_m} - (x_m - w_m)\frac{\partial x_n}{\partial b}$$

where $\sim$ denotes compensated demand (Berg, 1987). Using this and the symmetry of the Slutsky matrix the formula for optimal taxation becomes

$$(2) \qquad \frac{\Sigma_n \; t_n \partial \tilde{x}_m / \partial q_n}{x_m - w_m} = \frac{\lambda - \mu}{\mu} + \Sigma_n t_n \frac{\partial x_n}{\partial b}, \qquad m,n \in M$$

which is the well-known result from Samuelson (1951). If public production is financed in its entirety with lump-sum taxation, i.e. $b = pg$, all tax rates may be set equal to zero. In that case it is easy to see that $\lambda = \mu$, the marginal cost of public funds equals the marginal utility of consumer income. (Λ is defined to make μ a positive number, although it is a cost item.)

Expression (2) is often interpreted as follows: "The percentage reduction, along the compensated demand curve, of the consumption of all commodities be the same, relative to what they would have been had the consumer prices been equal to the producer prices." (Stiglitz and Dasgupta, 1971, p. 156, cf. Sandmo, 1976, p. 42.) Diamond and Mirrlees (1971, p. 262) have offered more details: Assume that producer prices are constant in the rather strong sense that $\partial \tilde{x}_m / \partial q_n = \partial \tilde{x}_m / \partial t_n$. Assume also that $\partial \tilde{x}_m / \partial q_n$ is constant for all n. Then

$$\Sigma_n \frac{\partial\tilde{x}_m}{\partial q_n} t_n = \Sigma_n \frac{\partial\tilde{x}_m}{\partial q_n} \int_0^{t_n} d\tau_n = \Sigma_n \int_0^{t_n} \frac{\partial\tilde{x}_m}{\partial q_n} d\tau_n = \Sigma_n \int_0^{t_n} \frac{\partial\tilde{x}_m}{\partial t_n} d\tau_n = d\tilde{x}_m, \quad m,n \in M$$

This indicates two reasons why the traditional interpretation of equation (2) is valid only when tax rates are relatively small. The first is that the assumption that a tax increase is fully carried over into an increase in the consumer price may be rather strong if m is an input to private production and the tax increase is large. The second is that $\partial\tilde{x}_m/\partial q_n$ cannot be a constant for <u>all</u> n since $\Sigma_n q_n\partial\tilde{x}_m/\partial q_n = 0$ irrespective of q by the zero-degree homogeneity of the compensated demand function (note that the public production vector z is a constant in the utility function, and that public goods do not have prices). Thus, when tax rates are no longer small, formula (2) is only an approximate characterization of optimal taxation. In itself it is also a characterization in quantity terms and not in terms of tax rates since its right-hand side depends on the tax vector t. Although it is often said to show that the efficiency aspect of optimal taxation has to do with substitution effects, formula (2) does not seem to be very useful as a prescription for practical determination of tax rates.

To characterize tax rates it should be observed that the reason for taxation is the public tax revenue requirement, and that income effects may be relevant when this requirement is to be met. If a given revenue must be raised, it may be that tax rates should be relatively high in markets where their imposition has a moderate effect on the tax base. These are markets where the <u>un</u>compensated demand elasticities are low in absolute value. Since the prices and the tax rate of the numéraire are given exogenously, let $\bar{x}_q$ be the Jacobian matrix of first derivatives of the demand function with respect to consumer prices with the first row and column deleted, and let $\bar{x} - \bar{w}$ be the vector of net demand for goods 2, ..., $|M|$. Then for the optimal tax vector $\bar{t} \equiv [t_2, \ldots, t_{|M|}]$ the $|M|-1$ first-order conditions of type (1) may be written as

$$\bar{t}\bar{x}_q = \frac{\lambda - \mu}{\mu} (\bar{x} - \bar{w})$$

or, provided that $(\bar{x}_q)^{-1}$ exists:

$$\bar{t} = \frac{\lambda - \mu}{\mu} [\bar{x}(q^*, z, w, b) - \bar{w}] [\bar{x}_q(q^*, z, w, b)]^{-1} \tag{3}$$

Formula (3) is probably the simplest characterization of optimal tax rates that can be obtained in a general model where taxes are not necessarily small and no special assumptions are made regarding the form of the consumer demand functions.[3)] It shows that income effects matter for the absolute level of tax rates, because $\bar{x}_q$ contains first derivatives of the uncompensated demand functions. Atkinson and Stiglitz (1980, p. 373) note that "... the income effect would arise with any form of taxation, ...", which is quite true, of course; but the income effect does arise, and this has implications for the level of tax rates.

The familiar inverse elasticity rule results from an assumption that all cross-price effects are zero;[4)] i.e. that the matrix $\bar{x}_q$ is diagonal. Then

---

3) Note that without the normalization of both consumer and producer prices no formula like (3) could be obtained, since the ordinary Jacobian of the consumer's demand is singular.

4) This may seem unrealistic, but in practice it is often found that the absolute values of cross-price effects are quite small in comparison to own-price effects.

its inverse is very easily calculated (assuming that none of the own-price effects are zero), and (3) may be restated as

$$
\begin{bmatrix}
t_2^*/q_2^* & 0 & \cdots & 0 \\
0 & t_3^*/q_3^* & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
0 & 0 & \cdots & t_{|M|}^*/q_{|M|}^*
\end{bmatrix}
$$

(4) $$=$$

$$
\frac{\lambda - \mu}{\mu}
\begin{bmatrix}
\frac{x_2 - w_2}{q_2^*} & 0 & \cdots & 0 \\
0 & \frac{x_3 - w_3}{q_3^*} & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
0 & 0 & \cdots & \frac{x_{|M|} - w_{|M|}}{q_{|M|}^*}
\end{bmatrix}
\begin{bmatrix}
\frac{1}{\partial x_2/\partial q_2^*} & 0 & \cdots & 0 \\
0 & \frac{1}{\partial x_3/\partial q_3^*} & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
0 & 0 & \cdots & \frac{1}{\partial x_{|M|}/\partial q_{|M|}^*}
\end{bmatrix}
$$

3: Public production

The importance of complementarity between public and private goods has been emphasized by Atkinson and Stern (1974) and King (1986), who have focussed on optimality rules for public goods rather than the formulae for optimal taxation. Now the question is whether such a complementarity also has a bearing on the optimal adjustment of tax rates. A private good m and a public good k may be called complements if $\partial x_m/\partial z_k > 0$, and substitutes if $\partial x_m/\partial z_k < 0$. Note that this definition differs from the usual one which involves prices. Here, complementarity is interpreted as a more technical or physical property.

In the optimum taxation formula (1), public production does not appear explicitly, which is probably why Diamond and Mirrlees (1971) do not elaborate on this point (see the Introduction). Whether z is zero or not does not alter the formula. On the other hand, z is an exogenous variable affecting final consumption, the responsiveness of demand to price changes, the equilibrium prices, as well as the Lagrange multipliers. The role of public production must therefore be illustrated by differentiating equation (1):

$$\Sigma_n \left[\frac{\partial x_n}{\partial q_m} dt_n \quad + \quad t_n \, \Sigma_h \, \frac{\partial^2 x_n}{\partial q_m \partial q_h} dq_h \quad + \quad t_n \, \Sigma_k \, \frac{\partial^2 x_n}{\partial q_m \partial z_k} dz_k\right]$$

$$(5) \qquad =$$

$$d(\frac{\lambda}{\mu})(x_m - w_m) \quad + \quad (\frac{\lambda}{\mu} - 1)\{\Sigma_h \frac{\partial x_m}{\partial q_h} dq_h \quad + \quad \Sigma_k \frac{\partial x_m}{\partial z_k} dz_k\}$$

There is not much to be said in general from this expression about the direction of change in each tax rate when the public production level increases. The second term on the left-hand side represents the effect on the price responsiveness of demand from changes in general equilibrium consumer prices. The third term shows the same kind of impact from the production levels of the public goods. On the right-hand side, the first term reflects the change in the (inverse of the) marginal cost of public funds in terms of private income, and will be discussed below. The first term in the braces is the direct change in demand caused by the consumer price differentials. Finally, the second term in braces is the effect of complementarity between public and private goods: *Ceteris paribus*, the taxation of complements to a public good normally tends to increase with the production level of that good; and the taxation of substitutes tends to decrease. "Normally" means that the conclusion holds if the own-price derivative $\partial x_m/\partial q_m$ is negative, cross-price effects do not outweigh the own-price effect, and the marginal cost of public funds exceeds the private marginal utility of income, $\frac{\lambda}{\mu} < 1$. Condition (5) could be interpreted as an earmarking condition for public funds, saying e.g. that it may be optimal to finance an increase in road construction by increasing the taxation of petrol more than the taxation of other goods. Still, two qualifications must be kept in mind: The first is that this is not earmarking in the usual sense which would only mean that restrictions were added to the state's optimization problem; the second is that the result only applies *ceteris paribus*; direct and indirect price effects may well neutralize the impact from complementarity between public and private goods.

If the public revenue requirement is T, then the Lagrange multiplier in the state's optimization problem is given by $\mu = -\partial v/\partial T$. The consumer's marginal utility of income is of course $\lambda = \partial v/\partial b$. As noted above, if lump-sum taxation is feasible, public expenditures can be financed by reducing b, and the marginal welfare loss of taxation will be $\mu = \lambda$. Assuming that lump-sum taxation is infeasible, however, we find that

$$\mu > \lambda \iff -\partial v/\partial T > \partial v/\partial b$$

This equivalence will be taken as a definition of the efficiency loss from distortive taxation: A given tax system entails an aggregate, marginal efficiency loss if and only if the decrease in social welfare caused by a marginal increase in the tax revenue requirement is greater in absolute value than it would have been under lump-sum taxation. Hence, if there is a marginal loss of efficiency in the aggregate, then μ will exceed λ, and if μ exceeds λ, then there is a marginal, aggregate efficiency loss.

To discuss the sign of $(\lambda-\mu)$ further, write the first-order condition (2) as follows:

$$(\frac{\lambda}{\mu} - 1 + \Sigma_n t_n \frac{\partial x_n}{\partial b})(x_m - w_m) \quad = \quad \Sigma_n t_n \frac{\partial \tilde{x}_m}{\partial q_n}$$

If we multiply by t_m and sum over all $m \in M$, we have:

$$(\frac{\lambda}{\mu} - 1 + \Sigma_n t_n \frac{\partial x_n}{\partial b}) \ \Sigma_m t_m (x_m - w_m) \quad = \quad \Sigma_m \Sigma_n t_n \frac{\partial \tilde{x}_m}{\partial q_n} \ t_m \quad \leqq \quad 0$$

by the negative semi-definiteness of the Slutsky matrix. Since the tax revenues are positive, this yields another necessary condition for optimal taxation:

$$(6) \qquad \frac{\lambda}{\mu} \ \leqq \ 1 - \Sigma_n t_n \frac{\partial x_n}{\partial b}$$

Atkinson and Stern (1974) use this inequality to observe that depending on tax rates and the normality or inferiority of different goods, λ may be greater or less than μ. But then it must be remembered that the tax rates in (6) are supposed to be optimal. Thus if λ is greater than μ, then an increase in the wedges between producer and consumer prices is better, on the margin, than an increase in lump-sum taxation. Without denying the possibility of such a result, it seems rather implausible. If, on the other hand, there is a marginal efficiency loss from distortive taxation in the aggregate, then $\lambda < \mu$; and (6) is consistent with a tax system where normal consumption goods have positive tax rates and normal goods which are supplied to factor markets by the consumer have negative tax rates.

Optimal public production

In order to complete the model and illustrate the role of the inefficiency indicator μ/λ, the main result from the literature on optimal production of public goods will be reviewed. To produce efficiently, the state must determine the optimal input requirement in public production. An *ad hoc* argument in this connection is that if taxation reduces welfare and public goods increase it, the state should minimize production costs:

$$\text{minimize } -pg \text{ with respect to } g, \text{ subject to } z(g) = \bar{z},$$

where $z(g)$ is public production as a function of public inputs, and $\bar{z}$ is any non-negative output vector. A more precise discussion of the public efficiency problem has been carried out by Diamond and Mirrlees (1971, pp. 19-23). Their argument is very simple: If a welfare-increasing change in consumer prices exists in the general equilibrium with optimal taxation, and aggregate demand functions are continuous, then the optimum must be on the frontier of the production set. Since social welfare may be increased by changing the consumer prices, the only reason it is not done must be that the resulting demand would lie outside the production set. Diamond and Mirrlees conclude that aggregate production efficiency is desirable. An assumption of public cost minimization at given producer prices is consistent with their conclusion, at least as long as there are no other imperfections than optimal taxes in the economy. The minimization of costs yields a public input requirement vector $g(p,z)$.

As several authors have observed, the fact that public production is financed by distortionary taxation may modify the elementary optimality rule for public goods (Atkinson and Stern, 1974; and King, 1986; cf.

Samuelson, 1954). Suppose that the state wishes to choose z optimally, given the input requirement $g(p,z)$ and exogenous lump-sum transfers b. The first-order conditions are:

$$(7) \qquad \frac{\partial \Lambda}{\partial z_k} = \frac{\partial v}{\partial z_k} + \mu(\Sigma_n t_n^* \frac{\partial x_n}{\partial z_k} + \Sigma_n p_n^* \frac{\partial g_n}{\partial z_k}) = 0, \qquad k \in K,\ n \in M,$$

where t_n^* and p_n^* are determined for all n in the optimization problem in Section 1. Following King (1986) we can define the marginal willingness to pay for public good k as

$$- \frac{\partial b}{\partial z_k}\bigg|_{v=\bar{v}} ,$$

where $\bar{v}$ is a given maximum utility level. Differentiating with respect to z_k in $v(q,z,w,b) = \bar{v}$, we find:

$$\frac{\partial v}{\partial z_k} = - \frac{\partial v}{\partial b} \frac{\partial b}{\partial z_k}\bigg|_{v=\bar{v}} = \lambda \, (-\frac{\partial b}{\partial z_k}\bigg|_{v=\bar{v}})$$

Substituting for $\partial v/\partial z_k$, the first-order condition (7) can be rewritten as

$$- \frac{\partial b}{\partial z_k}\bigg|_{v=\bar{v}} = - \frac{\mu}{\lambda} \, (\Sigma_n t_n^* \frac{\partial x_n}{\partial z_k} + \Sigma_n p_n^* \frac{\partial g_n}{\partial z_k})$$

or in matrix notation:

$$(8) \qquad - b_z^{\bar{v}} = - \frac{\mu}{\lambda} \, (t^* x_z + p^* g_z)$$

where $-b_z^{\bar{v}}$ is the consumer's marginal willingness to pay for the $|K|$ public goods at utility level $\bar{v}$, x_z is the $|M| \cdot |K|$-matrix of first derivatives of x w.r.t. z, and g_z is the $|M|\cdot|K|$-matrix of first derivatives of g w.r.t. z. Formula (8) is analogous to equation [3] in Atkinson and Stern (1974, p. 122) and to equation [21] in King (1986, p. 279). p^*g_z is the vector of marginal costs in the production of each public good, and formula (8) states that the marginal willingness to pay for public goods should be proportional to their marginal cost, adjusted by the change in tax revenues following an infinitesimal variation in the provision of each public good.

In real terms, p^*g_z is the marginal opportunity cost of public inputs, and t^*x_z is the marginal resource cost (positive or negative) due to the wedges between consumer and producer prices. However, there is also the general equilibriun aspect that the public budget must balance, and in that sense p^*g_z may be called an expenditure effect and t^*x_z a revenue effect of the increase in public production.

An important point is that the marginal willingness to pay for public goods may exceed the marginal production costs measured as the net change in the public budget equation brought about by the increase in z. This happens when distortive taxation causes a marginal efficiency loss in the aggregate, in which case $\mu/\lambda > 1$. Then the marginal willingness to pay must be sufficient to outweigh the decrease in social welfare resulting from the increase in taxation needed to finance the marginal unit of public production.

The complete second best equilibrium

It is clear that the first-order condition (8) cannot determine both the optimal production level for public goods and the inefficiency indicator $\frac{\mu}{\lambda}$ at the same time. Mathematically, μ and λ are determined along with all the other endogenous variables in the private and public optimization problems: consumer and producer prices, consumption and production of private and public goods. To describe this system mathematically, it should be noted that the optimum taxation problem concerns the optimal selection of two sets of general equilibrium prices, viz., p and q. A first-order condition which differs slightly from equation (1) then seems more convenient: Since $t_1 = 0$ and $\bar{t} = q - p$, (1) may be rewritten as

$$t_1 \frac{\partial x_1}{\partial q_m} + \sum_{n=2} t_n \frac{\partial x_n}{\partial q_m} = \left(\frac{\lambda}{\mu} - 1\right)(x_m - w_m)$$

$$\Rightarrow$$

$$\sum_{n=1} q_n \frac{\partial x_n}{\partial q_m} - \sum_{n=1} p_n \frac{\partial x_n}{\partial q_m} + (x_m - w_m) = \frac{\lambda}{\mu}(x_m - w_m)$$

By the consumer's budget restriction,

$$\sum_{n=1} q_n \frac{\partial x_n}{\partial q_m} + (x_m - w_m) = 0$$

so that

$$(x_m - w_m) + \frac{\mu}{\lambda} \sum_{n=1} p_n \frac{\partial x_n}{\partial q_m} = 0$$

Thus, given the exogenous variables w, $\bar{z}$ and b, as well as the technology implicit in the private production possibilities set Y and the public production function $z(\cdot)$, the endogenous variables q, p, x, y, g, λ and μ are determined by the following conditions and simultaneous equations:

$u(x,\bar{z})$ is maximal subject to $q(x - w) = b$, for any $\bar{z}$ and q; thus determining x and λ.

py is maximal subject to $y \in Y$, for any p.

$-pg$ is minimal for any p and every $\bar{z}$ such that $z(g) = \bar{z}$; thus determining g.

$p_1 = q_1 = 1$; thus defining the numéraire and its tax rate.

$x_n - w_n = y_n + g_n$, for $n = 2,\ldots,|M|$; (market equilibrium).

$(q - p)(x - w) + pg = 0$; (the public budget equation).

$\mu = -\lambda(x_2 - w_2)/[\sum_{n=1} p_n \partial x_n/\partial q_2]$; (the marginal cost of public funds).

$(x_m - w_m) + \frac{\mu}{\lambda} \sum_{n=1} p_n \frac{\partial x_n}{\partial q_m} = 0$, $m = 3,\ldots,|M|$; (optimal taxation).

If the public production level is to be optimal, and K only contains pure public goods, then $\bar{z}^*$ may be determined by the first-order condition (8).

Conclusion

This chapter discusses some features of a three-agent general equilibrium model. General equilibria with consumers and producers being well known, the focus is on the behaviour of the public sector, or the state. The state's purpose is to provide public goods to the consumers, financing its production by imposing taxes on the private sector. Therefore it must solve three optimization problems: how to tax optimally, how to produce efficiently, and how to supply an optimal amount of public goods. Three points are made in the note. The first is that the existence of an optimal set of producer prices is a more interesting question than whether these prices are constant or not. The second is that income effects are relevant for the determination of the absolute level of tax rates. The third is that complementarity between public and private goods influences the change in optimal tax rates and the optimal level of production of pure public goods.

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GENERAL EQUILIBRIUM TAX INCIDENCE: ANALYTICAL FORMULAE AND THREE NUMERICAL SIMULATIONS

It is no joke to have to analyze the incidence of anything, a tax change or a factor-supply change, in a full general equilibrium model."

Paul A. Samuelson

GENERAL EQUILIBRIUM TAX INCIDENCE:
ANALYTICAL FORMULAE AND THREE NUMERICAL SIMULATIONS

PART 1: ANALYSIS

Introduction

In comparison with the literature on optimal taxation, general equilibrium incidence analysis seems a relatively small branch of public economics. One reason why the number of contributions in this field appears to be moderate, is perhaps that the original Harberger model (Harberger, 1962) was not only a pioneering work but offered simple and intuitively appealing answers to most of the questions which seem natural to pose in this context. It was not until 1976 that one of the main weaknesses of Harberger's model - the assumption of zero initial taxes - was amended (Vandendorpe and Friedlaender, 1976). In addition, minor modifications were introduced by Mieszkowski (1967) and McLure (1970, 1972): In the Harberger model, it is assumed that the demand pattern (i.e. marginal and average propensities to consume) is the same for all private consumers and the government. Mieszkowski (1967) relaxed this assumption and studied the implications of differing preferences as between workers (those who receive wages) and capitalists (those who receive returns to capital). McLure (1970) emphasized the change in absolute as opposed to relative prices in the Harberger model by adding a monetary equation where total factor payments equal money supply times its velocity. In his second article (1972), McLure focussed on the incidence of public expenditure rather than taxes; public expenditure being defined as the government's purchases of factors and final goods. Both authors use a model with only two factors and two final goods. This assumption is certainly crucial to the analysis, and it may also be essential for the results.

The purpose of the present essay is to discuss a traditional tax incidence model in view of the reflections on the role of the state in preceding chapters. Some attention will also be devoted to the close connection between incidence analysis and the so-called Laffer curve. The emphasis will not be so much on the kind of economic variables which determine price changes (such as factor intensities, elasticities of substitution etc.) as on the kind of economic changes following increases and decreases in taxation or public production. The chapter has two main parts. In part 1 a general analytical model is presented. It has many goods, exogenously given initial taxes, and significant differences between private and public production. Part 2 presents the results of simulations with three different numerical models, the first of which illustrates the analytical formulae in part 1, where the tax rates are exogenous. In the other two numerical models, tax rates are optimal and thus endogenous.

A general equilibrium model with exogenous taxation

Some aspects of the model which will be used to analyse the incidence of public economic activity have already been discussed at some length in previous chapters. The modifications of the model in Chapter 3 which will be introduced mainly lie in the formulation of private production, since explicit factor demand functions will be needed, and in the tax system.

Private production and consumption.

Assume that M private commodities are produced in the economy, and that there are N production factors. There are no initial endowments of the private commodities. Each of them is produced by one private firm under constant returns to scale. The minimum cost function of producer m can then be written as $y_m c^m(p)$, where p is the N-vector of factor prices taken as given by the producer and y_m is the production level.[1] Since there is no well-defined supply function, y_m is determined from the demand side. Given c^m and p, the firm's conditional demand for input n is $y_m \partial c^m/\partial p_n$. The input coefficients for producer and commodity m are defined as

$$a_n^m(p) \equiv \frac{\partial c^m(p)}{\partial p_n}, \qquad n = 1,\ldots,N, \quad m = 1,\ldots,M$$

Hence the private production technology is completely characterized by the matrix of input coefficient functions

$$a(\ ) \equiv \begin{bmatrix} a_1^1(\ ) & \cdots & a_N^1(\ ) \\ \vdots & & \vdots \\ a_1^M(\ ) & \cdots & a_N^M(\ ) \end{bmatrix}$$

M equations relating producers' commodity prices P to unit costs close the production side of the private sector:

$$P_m = p \cdot [a^m(p)]', \qquad m = 1,\ldots,M$$

where a^m is the N-vector of input coefficients in firm m.

---

[1] The cost functions are assumed to be independent of public production. Note also that the activity level of a firm, its production level and the market supply are the same because one and only one firm produces each good.

As regards the consumer, y denotes his demand for commodities and x his demand for factors which can also be used as inputs. x and y are functions of an N-vector of consumer's factor prices, q, an M-vector of consumer's commodity prices, Q, the K-vector of public goods, z, lump-sum transfers from the state, b, and the N-vector of initial endowments, w. The budget restriction equating income and expenditure is

$$\sum_n q_n(w_n - x_n) + b = \sum_m Q_m y_m$$

The public sector.

The public production level, z, is a function of an N-vector g of inputs which the state buys in private factor markets at given producer prices. In order to simplify it is assumed that there is only one public good. In accordance with the conclusions in Chapter 3, the state is assumed to minimize public production costs, given p and z, and the resulting factor demand function $g = g(p,z)$ will be homogeneous of degree zero in p. Factor tax rates are given by an N-vector t, and commodity taxes are given by the M-vector s. The tax rates drive wedges between consumer and producer prices: $q = p + t$, $Q = P + s$. Note that t_n will be negative if factor n is taxed. Both public production and tax rates are exogenous, but the level of lump-sum transfers to the consumer, b, will be regarded as an endogenous variable and is thus determined in general equilibrium.

General equilibrium.

With the definitions above, general equilibrium is characterized by the following system:

(1) $w - x(q,Q,b,z,w) = y(q,Q,b,z,w)\cdot a(p) + g(p,z)$

(2) $P = p\cdot[a(p)]'$

(3) $[q, Q] \equiv [p, P] + [t, s]$

(4) $q\cdot[w - x(q,Q,b,z,w)] + b = Q\cdot y(q,Q,b,z,w)$

(5) $s\cdot y(q,Q,b,z,w) - t\cdot[w - x(q,Q,b,z,w)] = p\cdot g(p,z) + b$

Equation (1) describes the equilibrium in factor markets, and (2) is the clearing condition for commodity markets. (3) formalizes the tax system, (4) is the consumer's budget restriction, and (5) is the government budget equation.[2] The exogenous variables of the system are t, s, z and w, and the endogenous variables are p, P and b. Although a simple counting rule does not apply here, it may be noted that the N+M+1 unknowns are matched by N equations in (1), M equations in (2) and one equation in (5). (1), (2), and (5) can only be solved for relative prices, and one equation in the system is redundant if (4) holds (which it does, since the consumer maximizes his utility). The price system may therefore be normalized, e.g. by setting $P_1 \equiv 1$. Given s_1, this also implies a normalization of consumer prices.

Differential tax incidence formulae

Upon solution of equations (1)-(5) the lump-sum transfers and each market price will in general be functions of all the exogenous variables. If prices and lump-sum transfers given by these functions are substituted

[2] Vectors and matrices will be enclosed in brackets, whereas arguments of functions are put within parentheses or braces. Prices and tax rates are always row vectors, and their differentials are column vectors. g, w, x, y and z will be regarded interchangeably as row or column vectors, depending on the context in which they appear, but their differentials are always column vectors.

back into (1), (2) and (5), the latter become identities which are assumed to be differentiable with respect to tax rates and the public production level. Thus, in general equilibrium the differential form of an equation from (2) is

$$dP_m = \sum_n a_n^m \, dp_n + \sum_n p_n \sum_k \frac{\partial a_n^m}{\partial p_k} dp_k \qquad (dP_1 = 0)$$

Since the cross price effects are symmetric (i.e. $\partial a_n^m/\partial p_k = \partial a_k^m/\partial p_n$) and the input coefficient functions are homogeneous of degree zero, the double sum is zero. Hence the price differentials become

(6) $\quad dP = a(p)\cdot dp \qquad (dP_1 = 0)$

(7) $\quad dq = dp + dt$

(8) $\quad dQ = a(p)\cdot dp + ds \qquad (dQ_1 = ds)$

Assuming that initial endowments are fixed, we can write the differential of equation (1) as

$$dx + \sum_m y_m \cdot da^m + a' \cdot dy + dg = 0 \ .$$

Let subscripts (except m) indicate first partial derivatives, i.e. x_Q is the NxM-matrix of first partial derivatives of x with respect to Q, y_b is the column vector of first partial derivatives of y with respect to b etc. Then

$$dx = x_q \cdot [dp + dt] + x_Q \cdot [a \cdot dp + ds] + x_b \cdot db + x_z \cdot dz$$

$$da^m = a_p^m \cdot dp$$

$$dy = y_q \cdot [dp + dt] + y_Q \cdot [a \cdot dp + ds] + y_b \cdot db + y_z \cdot dz$$

$$dg = g_p \cdot dp + g_z \cdot dz$$

so that

$$
(9)\qquad
\begin{aligned}
&[x_q + x_Q a + \sum_m y_m a_p^m + a'y_q + a'y_Q a + g_p]dp + \\
&[x_b + a'y_b]db + \\
&[x_q + a'y_q]dt + [x_Q + a'y_Q]ds + [x_z + a'y_z + g_z]dz \\
&= 0
\end{aligned}
$$

To close the system, we must express db by the differential of the state's budget equation, (5):

$$
y\cdot ds + s\cdot dy - [w - x]\cdot \underline{d}t + t\cdot dx = g\cdot dp + p\cdot dg + db
$$

$\Leftrightarrow$

$$
(10)\qquad
\begin{aligned}
&[tx_q + tx_Q a + sy_q + sy_Q a - g - pg_p]dp + \\
&[sy_b + tx_b - 1]db + \\
&[tx_q - [w - x] + sy_q]dt + [tx_Q + sy_Q + y]ds + [sy_z + tx_z - pg_z]dz \\
&= 0
\end{aligned}
$$

In (10) the factors in front of dp and dt are row vectors with N elements, those in front of db and dz are scalars, and the one in front of ds is a row vector with M elements. $pg_p = 0$ by the symmetry of cross price effects and the zero-degree homogeneity of the public factor demand functions. (9) and (10) may now be compiled in one matrix expression:

$$
(11)\qquad
\begin{aligned}
&\begin{bmatrix} [x_q + a'y_q + x_Q a + a'y_Q a + \sum_m y_m a_p^m + g_p] & [x_b + a'y_b] \\ [tx_q + sy_q + tx_Q a + sy_Q a - g] & [tx_b + sy_b - 1] \end{bmatrix} \begin{bmatrix} dp \\ db \end{bmatrix} \\
&= \\
&- \begin{bmatrix} [x_q + a'y_q] & [x_Q + a'y_Q] & [x_z + a'y_z + g_z] \\ [tx_q - [w - x] + sy_q] & [tx_Q + sy_Q + y] & [tx_z + sy_z - pg_z] \end{bmatrix} \begin{bmatrix} dt \\ ds \\ dz \end{bmatrix}
\end{aligned}
$$

Equation (11) is the fundamental differential tax incidence formula, where the endogenous variables dp and db are such that the changes they cause in factor markets and the public budget equal the direct effects of small variations in tax rates and public production. In the next paragraph these direct and indirect effects are interpreted in closer detail.

Interpretation

As already observed, the right-hand side of equation (11) may be viewed as the direct effects of changes in factor taxes t, commodity taxes s and the public production level $\bar{z}$. When t increases, consumer's factor prices q also increase, since $q \equiv p + t$. Therefore, the consumer's factor demand changes by the rate x_q and his commodity demand by the rate y_q, and the latter effect causes private firms' factor demand to change by $a'y_q$. Thus, the first term on the right-hand side of (11) is $[x_q + a'y_q]dt$. The reasoning behind the two other terms in the first row, $[x_Q + a'y_Q]ds$ and $[x_z + a'y_z + g_z]dz$, is quite similar. E.g., when z increases, the factor demand changes are $x_z dz$ (the consumer), $a'y_z dz$ (private firms) and $g_z dz$ (the state). In the second row on the right-hand side of (11) are the direct effect on the government budget of a small variation in t, s and z. Since an increase in a (negative) factor tax rate increases the consumer price of the factor, the direct revenue impact is $tx_q dt + [x - w]dt$ from the factor markets and $sy_Q dt$ from the commodity markets. Likewise, the direct revenue effects of an increase in commodity taxes are $tx_Q ds$ from the factor markets and $sy_Q ds + yds$ from the commodity markets. Finally, an increase in public production entails a net change in tax revenues which is the sum of commodity market effects $sy_z dz$, factor market effects $tx_z dz$, and public expenditure effects $pg_z dz$.

The main point in the differential tax incidence equation (11) is that the changes in factor prices and lump-sum transfers must be such that the direct effects of dt, ds and dz mentioned above are outweighed. Thus, the left-hand side of (11) may be interpreted along much the same lines as its right-hand side. The first term in the upper row represents the factor market reactions to price changes. An increase in producers' factor prices causes factor adjustment effects $\sum_m y_m a_p^m$ dp in private firms and $g_p$dp in the public sector. The terms $x_Q a$ and $a'y_Q a$ are a little more complicated: When p_n rises, unit costs and the commodity price increase by $a_n^k dp_n$ in firm k ($k \neq 1$). Then consumer price Q_k also increases, inducing a change in the demand for factor 1, say, which is $(\partial x_1/\partial Q_k)a_n^k dp_n$ on the part of the consumer and $\bar{a}_1^m(\partial y_m/\partial Q_k)a_n^k dp_n$ in firm m. Both are summed over k = 2,...,M since unit costs increase in all firms except firm 1, and the producers' factor adjustments are added together to yield the total increase or decrease in demand for the input. The two terms $x_Q a$dp and $a'y_Q a$dp reflect the fact that equation (2) (commodity market equilibrium) has been substituted into equation (1). It is the assumption of constant returns to scale which makes this substitution possible, since commodity prices are related to unit costs in a simple way when production functions are linearly homogeneous.

The second term in the upper row on the left-hand side, $[x_b + a'y_b]$db, is an income effect in the factor markets. When lump-sum transfers increase, the consumer alters his factor demand by $x_b$db and his commodity demand by $y_b$db. The latter effect causes producers' factor demand to change by $a'y_b$db.

The second row on the left-hand side shows the impact of dp and db on the public budget. tx_qdp and sy_qdp are the revenue effects from factor and commodity markets when factor prices vary, and $tx_Q adp$ and $sy_Q adp$ are the corresponding effects from commodity price variation caused by the changes in factor prices. gdp is the change in public expenditures apart from lump-sum transfers. Finally, $[tx_b + sy_b - 1]db$ is the net income effect on tax revenues from the endogenous increase or decrease in the lump-sum transfers.

The role of the state

The presence of public production alters the traditional tax incidence formulae. To illustrate this, a few simplifying assumptions will be introduced. First, let the consumer's factor demand be identically zero ($x \equiv 0$) and assume that his demand for private goods is insensitive to factor prices ($y_q = 0$). Then equation (11) can be written as

$$\begin{bmatrix} [a'y_Q a + \sum_m y_m a_p^m + g_p] & [a'y_b] \\ [sy_Q a - g] & [sy_b - 1] \end{bmatrix} \begin{bmatrix} dp \\ db \end{bmatrix} = - \begin{bmatrix} [0] & [a'y_Q] & [a'y_z + g_z] \\ [-w] & [sy_Q + y] & [sy_z - pg_z] \end{bmatrix} \begin{bmatrix} dt \\ ds \\ dz \end{bmatrix}$$

Furthermore, assume that there are only two factors and two private goods ($N = M = 2$); that all tax rates except t_1 are zero ($s_1 = s_2 = t_2 = 0$); and that only t_1 is changed in order to finance an increase dz in the public production level ($ds_1 = ds_2 = dt_2 = 0$). Thus the matrix equation above becomes:

$$\begin{aligned} & e_{11}dp_1 + e_{12}dp_2 + e_{13}db = e_1 dz \\ (12) \qquad & e_{21}dp_1 + e_{22}dp_2 + e_{23}db = e_2 dz \\ & g_1\, dp_1 + g_2\, dp_2 + \quad db = e_3 dz - w_1 dt_1 \end{aligned}$$

where

$$\begin{aligned}
e_{hi} &\equiv \sum_m \left[a_h^m \frac{\partial y_m}{\partial Q_2} a_h^2 + y_m \frac{\partial a_h^m}{\partial p_i} \right] + \frac{\partial g_h}{\partial p_i}\,, && h,i = 1,2 \\
e_{h3} &\equiv \sum_m a_h^m \frac{\partial y_m}{\partial b}\,, && h = 1,2 \\
e_h &\equiv -\sum_m a_h^m \frac{\partial y_m}{\partial z} + \frac{\partial g_h}{\partial z}\,, && h = 1,2 \\
e_3 &\equiv -\sum_n p_n \frac{\partial g_n}{\partial z}
\end{aligned}$$

(Note that $dP_1 = 0$.) Define E as the matrix of left-hand side coefficients in (12):

$$E \equiv \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ g_1 & g_2 & 1 \end{bmatrix}$$

Provided that $|E| \neq 0$, the solution to (12) is:

$$\begin{aligned}
dp_1 &= \frac{1}{|E|}\left[e_1(e_{22}-e_{23}g_2) + e_{12}(e_{23}e_3-e_2) + e_{13}(e_2g_2-e_{22}e_3) \right]dz \\
&\quad - \frac{1}{|E|}(e_{12}e_{23}-e_{13}e_{22})w_1\ dt_1
\end{aligned}$$

$$\begin{aligned}
dp_2 &= \frac{1}{|E|}\left[e_2(e_{11}-e_{13}g_1) + e_{21}(e_{13}e_3-e_1) + e_{23}(e_1g_1-e_{11}e_3) \right]dz \\
&\quad + \frac{1}{|E|}(e_{11}e_{23}-e_{13}e_{21})w_1\ dt_1
\end{aligned}$$

$$\begin{aligned}
db &= \frac{1}{|E|}\left[e_1(e_{21}g_2-e_{22}g_1) + e_2(e_{12}g_1-e_{11}g_2) + e_3(e_{11}e_{22}-e_{12}e_2) \right]dz \\
&\quad - \frac{1}{|E|}(e_{11}e_{22}-e_{12}e_{21})w_1\ dt_1
\end{aligned}$$

where

$$|E| = e_{11}(e_{22} - e_{23}g_2) + e_{12}(e_{23}g_1 - e_{21}) + e_{13}(e_{21}g_2 - e_{22}g_1)$$

It is fairly clear that the signs of dp_1, dp_2 and db are ambiguous. Indeed, in the original Harberger model, which is even simpler than the one above, the signs of the factor price differentials depend on the relative magnitudes of factor intensities, elasticities of substitution and demand elasticities (Atkinson and Stiglitz, 1980, pp. 165 - 170). One could perhaps generalize the stability argument from partial equilibrium analysis; assuming that excess demand falls in each market when the market price increases and that cross price effects are not large enough to outweigh such direct effects (see e.g. Dixit and Norman, 1980, p. 131). But we could just as well assume directly that $|E| > 0$ etc., the more so since referring to stability appears to be reasoning outside a model where there is no price adjustment mechanism.

The solutions for dp_1, dp_2 and db are simple only if $dz = g_1 = g_2 = 0$, in which case $|E| = e_{11}e_{22} - e_{12}e_{21}$ and $db = -w_1dt_1$. Then, under "normal" circumstances, it seems reasonable to conjecture that $dp_1/dt_1 < 0$ and $dp_2/dt_1 > 0$ since $dt_1 > 0$ means that the taxation of factor market 1 becomes less severe. If g_1 or g_2 are positive, but $dz = 0$, db may be smaller or greater than $-w_1dt_1$, depending on whether public expenditures increase or decrease with t_1. If $dz \neq 0$, the solution is considerably more complex, with several counteracting effects. The price variation caused by changes in tax rates or public production will therefore be further discussed in the second part of this chapter.

The Laffer curve

The Laffer curve depicts total tax revenues as a function of some measure of the degree of taxation in the economy, often defined as the average or marginal tax rate on labour income. This functional relationship was mentioned by Adam Smith (1776) and described explicitly by Jules Dupuit (1844, p. 370):

"Si on augmente graduellement un impôt depuis 0 jusqu'au chiffre qui équivaut à une prohibition, son produit commence par être nul, puis croît insensiblement, atteint un maximum, décroît ensuite successivement, puis devient nul. Il suit de là que quand l'état a besoin de trouver une somme donnée au moyen d'un impôt, il y a toujours deux taxes qui satisfont à la condition, l'une au-dessus, l'autre au-dessous de celle qui donne le maximum de produit."

In more recent years the Laffer curve has been analysed by several authors, notably Canto, Joines and Laffer (1981), Fullerton (1982), Bender (1984) and Malcolmson (1986). The emphasis has been on the curve's slope, the existence of revenue-maximizing levels of taxation, and whether there are two or more values of each tax rate which will yield the same revenue. It is difficult to draw unambiguous conclusions to these questions in a general equilibrium model. However, they are closely related to the incidence analysis. Let T be the total revenues from taxation of market transactions. Then from equation (5) we have that

$$T(t,s,z,w) \equiv s \cdot y^*(t,s,z,w) - t \cdot [w - x^*(t,s,z,w)] \ ,$$

where x^* and y^* are the consumer's factor demand and commodity demand considered as functions of the exogenous variables in general equilibrium. The graph of T can be defined as the Laffer curve in general equilibrium. (It is a matter of opinion whether the lump-sum transfers, b, should be included in this definition. In principle, b may be negative, since it is the difference between two independent numbers; tax revenues and public expenditures.) Assuming that w is constant, the differential of T can be found in equation (10):

$$\begin{aligned} dT \equiv & [tx_q - [w - x] + sy_q]dt + [tx_Q + sy_Q + y]ds + [sy_z + tx_z]dz \\ & + [tx_q + tx_Q a + sy_q + sy_Q a]dp + [sy_b + tx_b]db \end{aligned}$$

where [dp db] is the solution of the fundamental incidence formula (11). In general, the sign of dT is ambiguous. On the other hand, if we make the same simplifying assumptions as above, i.e. $x \equiv 0$, $y_q = 0$, $N = M = 2$, $s_1 = s_2 = t_2 = 0$, and $ds_1 = ds_2 = dt_2 = 0$, the differential is trivial: $dT = -w_1 dt_1$. The significance of public production comes to the fore if e.g. $s_1 \neq 0$. Retaining the other simplifying assumptions (and $dP_1 = 0$), the differential of T is:

$$(13) \qquad dT = -w_1 dt_1 + s_1 \left[\frac{\partial y_1}{\partial Q_2} a_1^2 \, dp_1 + \frac{\partial y_1}{\partial Q_2} a_2^2 \, dp_2 + \frac{\partial y_1}{\partial b} \, db + \frac{\partial y_1}{\partial z} \, dz\right]$$

where dp_1, dp_2 and db solve a modified version of (12):

$$\begin{aligned}
&e_{11}dp_1 + e_{12}dp_2 + e_{13}db = e_1 dz \\
&e_{21}dp_1 + e_{22}dp_2 + e_{23}db = e_2 dz \\
&(s_1\frac{\partial y_1}{\partial Q_2}a_1^2 - g_1)dp_1 + (s_1\frac{\partial y_1}{\partial Q_2}a_2^2 - g_2)dp_2 + (s_1\frac{\partial y_1}{\partial b} - 1)db = \\
&\qquad\qquad - (e_3 + s_1\frac{\partial y_1}{\partial z})dz + w_1 dt_1
\end{aligned}$$

Now there is a logical possibility that the state's revenues first increase and then decrease, since each of the five terms in (13) may be either negative or positive. Suppose, e.g., that $dt_1 < 0$. Then we will normally expect dp_1 to be positive and dp_2 to be negative, so the terms $s_1(\partial y_1/\partial Q_2)a_1^2 dp_1$ and $s_1(\partial y_1/\partial Q_2)a_2^2 dp_2$ work in opposite directions. In addition, $(\partial y_1/\partial b)db$ is positive only if y_1 is normal and b increases, or y_1 is inferior and b decreases; whereas $\partial y_1/\partial z$ may be positive or negative according to whether y_1 and z are complements or substitutes in the sense of Chapter 3. Thus, T may increase as well as decrease. In the first numerical simulation in part 2 it turns out that a bell-shaped Laffer curve is easily generated when a factor tax rate is increased, but not when commodity taxes rise.

PART 2: THREE NUMERICAL SIMULATIONS

Parameterization of the model in part 1

There are several ways to simplify the model in part 1 so as to make the interpretation of the incidence formulae easier. One approach, which has been used by Harberger (1962), Shoven and Whalley (1972) and Atkinson and Stiglitz (1980), is to introduce a simpler economic structure. Harberger's main assumptions may serve as an example (the other analyses are not identical with the Harberger model, but similar to it):

- There are two factors ($N = 2$), two commodities ($M = 2$), no public goods ($K = 0$) and no public consumption ($\bar{g} = 0$).
- Factor supply is independent of factor prices ($\dot{x} \equiv 0$).
- Initially all tax rates are zero, and there is no efficiency loss from taxation. Then one of the factor tax rates is increased: $[t,s] = [0,0,0,0]$, $[dt,ds]' = [dt_1,0,0,0] \leq 0$.
- The state spends tax revenues in the same way as consumers would do if faced with the same prices. Public expenditure is taken into account by considering consumer demand as compensated. (As Atkinson and Stiglitz (1980, p. 173) put it, "the proceeds are returned to the consumers as a lump-sum subsidy".)

In the present chapter a different approach will be chosen, by using a computer to simulate small changes in general equilibrium prices. The analytical validity of the simulation approach of course depends on how flexible the parameterization of the model is. The main restriction of this kind in the present model is that elasticities of substitution are assumed to be constant in all productive activities. In addition the results of course depend on the actual numbers chosen for the parameters and exogenous variables.

Simple CES production functions.

A simple, single-level CES form will be assumed for private and public production functions. The minimum unit cost function of firm m can thus be written as

$$c^m(p) \equiv \{\Sigma_n \ (p_n)^{1-\sigma_m}\}^{1/(1-\sigma_m)}, \quad m = 1,\ldots,M$$

where σ_m is the elasticity of substitution between any pair of inputs. If f_1 and f_2 denote the optimal input levels of two production factors, then σ_m is defined as the negative of the relative rate of change in f_1/f_2 as the marginal rate of substitution changes:

$$\sigma_m \equiv - \frac{\partial(f_1/f_2)}{\partial(p_1/p_2)} \frac{p_1/p_2}{f_1/f_2}$$

The input coefficients are the derivatives of $c^m(p)$:

$$a_n^m(p) = \{c^m(p)/p_n\}^{\sigma_m}, \quad m = 1,\ldots,M, \ n = 1,\ldots,N$$

For simplicity it is assumed that there is only one public good (K = 1). On the other hand the public production function z(g) expressing z as a function of the vector of public inputs can be formulated so as to allow for decreasing, constant or increasing returns to scale. This may be done by introducing the elasticity of scale through a monotone transformation of the simple CES functional form. Cost minimization in the public sector will then yield conditional factor demand functions:

$$g_k(p,z) = \left[\Sigma_n \left(\frac{p_n}{p_k}\right)^{1-\gamma}\right]^{\gamma/(1-\gamma)} \cdot z^{1/\varepsilon}, \quad k = 1,\ldots,N$$

where γ is the constant elasticity of substitution between any pair of public inputs and ε is the scale elasticity.

A single-level CES function is strongly separable and the elasticity of substitution is the same for any pair of private goods, so substitutes and complements cannot occur simultaneously in the production functions. It is convenient to assume as simple a production structure as possible since the main focus is on the consumer's behaviour. As regards the consumer's utility function, however, both substitutes and complements should be allowed. It is also interesting to examine different ways in which the consumer's demand for private goods may be made to depend on the level of public production.

The utility function.

As already indicated, a CES function must have (at least) two levels before it is sufficiently flexible to allow for both substitutes and complements. Furthermore, if we are to define complementarity and substitution between public and private goods as in Chapter 3, the CES utility function must not be strongly separable in all its arguments. One of the simplest functional forms satisfying these requirements seems to be

$$u(x,y,z) = \left[\sum_n x_n^{\rho} + \sum_{m\neq h} y_m^{\rho} + y_h^{\rho} z^{\rho}\right]^{1/\rho}$$

where $\rho \equiv (\alpha - 1)/\alpha$ and α is the elasticity of substitution at the top level, i.e. between any pair in the set $\{x_1,\ldots,x_N,y_1,\ldots,\{y_h z\},\ldots,y_M\}$. Since it is increasing in all its arguments, the utility function will be strictly quasi-concave if $\alpha > 0$. The Lagrange function for maximization of u subject to the budget constraint (4) is

$$\mathcal{L} = \left[\sum_n x_n^{\rho} + \sum_{m \neq h} y_m^{\rho} + y_h^{\rho}\, z^{\rho}\right]^{1/\rho} - \lambda\left[\sum_n q_n x_n + \sum_m Q_m y_m - (qw + b)\right]$$

and the first-order conditions are

$$\left[\sum_n x_n^{\rho} + \sum_{m \neq h} y_m^{\rho} + y_h^{\rho}\, z^{\rho}\right]^{\frac{1}{\rho} - 1} x_i^{\rho-1} = \lambda q_i \;,\; i = 1,\dots,N$$

$$\left[\sum_n x_n^{\rho} + \sum_{m \neq h} y_m^{\rho} + y_h^{\rho}\, z^{\rho}\right]^{\frac{1}{\rho} - 1} y_j^{\rho-1} = \lambda Q_j \;,\; i = 1,\dots,M,\; j \neq h$$

$$\left[\sum_n x_n^{\rho} + \sum_{m \neq h} y_m^{\rho} + y_h^{\rho}\, z^{\rho}\right]^{\frac{1}{\rho} - 1} y_h^{\rho-1}\, z^{\rho} = \lambda Q_h \;.$$

x_i can be expressed as a function of y_h by eliminating λ, the marginal utility of income:

$$x_i = Q_h^{\alpha}\, q_i^{-\alpha}\, y_h\, z^{1-\alpha}$$

With the corresponding function for y_j, the budget restriction can be rewritten as

$$\left\{\left[\sum_n q_n^{1-\alpha} + \sum_{m \neq h} Q_m^{1-\alpha}\right] Q_h^{\alpha}\, z^{1-\alpha} + Q_h\right\} y_h = qw + b$$

so that

$$y_h = \frac{qw + b}{Q_h^{\alpha}\left[\sum_n q_n^{1-\alpha} + \sum_{m \neq h} Q_m^{1-\alpha}\right] z^{1-\alpha} + Q_h}$$

$$x_i = \frac{qw + b}{q_i^{\alpha}\left[\sum_n q_n^{1-\alpha} + \sum_{m \neq h} Q_m^{1-\alpha} + (\frac{Q_h}{z})^{1-\alpha}\right]}, \quad i = 1,\dots,N$$

$$y_j = \frac{qw + b}{Q_j^{\alpha}\left[\sum_n q_n^{1-\alpha} + \sum_{m \neq h} Q_m^{1-\alpha} + (\frac{Q_h}{z})^{1-\alpha}\right]}, \quad j = 1,\dots,h-1,h+1,\dots,M$$

Let the consumer's budget shares be defined as $\varphi_n \equiv q_n x_n/(qw + b)$ and $\theta_m \equiv Q_m y_m/(qw + b)$. Then the most important partial derivatives are:

$$\frac{\partial x_i}{\partial q_k} = (\alpha - 1)\ \varphi_i\ (\frac{q_i}{q_k})^{\alpha}\ \frac{x_i}{q_i} + \varphi_i\ \frac{w_k}{q_i}, \qquad i = 1,\dots,N,\ i \neq k$$

$$\frac{\partial x_i}{\partial q_i} = \left[\alpha(\varphi_i - 1) - \varphi_i\right]\frac{x_i}{q_i} + \varphi_i\ \frac{w_i}{q_i}, \qquad i = 1,\dots,N$$

$$\frac{\partial x_i}{\partial Q_j} = (\alpha - 1)\ \varphi_i\ (\frac{q_i}{Q_j})^{\alpha}\ \frac{x_i}{q_i}, \qquad \begin{array}{l} i = 1,\dots,N \\ j = 1,\dots,M,\ j \neq h \end{array}$$

$$\frac{\partial x_i}{\partial Q_h} = (\alpha - 1)\ \varphi_i\ (\frac{q_i}{Q_h})^{\alpha}\ \frac{x_i}{q_i}\ z^{\alpha-1}, \qquad i = 1,\dots,N$$

$$\frac{\partial x_i}{\partial b} = \frac{1}{q_i^{\alpha}\left[\sum_n q_n^{1-\alpha} + \sum_{m \neq h} Q_m^{1-\alpha} + (\frac{Q_h}{z})^{1-\alpha}\right]}, \quad i = 1,\dots,N$$

$$\frac{\partial x_i}{\partial z} = (1 - \alpha)\ \varphi_i\ (\frac{q_i}{Q_h})^{\alpha-1}\ x_i\ z^{\alpha-2}, \qquad i = 1,\dots,N$$

$$\frac{\partial y_h}{\partial q_k} = (\alpha - 1)\ \theta_h\ (\frac{Q_h}{q_k})^{\alpha}\ \frac{y_h}{Q_h}\ z^{1-\alpha} + \theta_h\ \frac{w_k}{Q_h}, \quad k = 1,\dots,N$$

$$\frac{\partial y_h}{\partial Q_j} = (\alpha - 1)\ \theta_h\ (\frac{Q_h}{Q_j})^{\alpha}\ \frac{y_h}{Q_h}\ z^{1-\alpha}, \qquad j = 1,\dots,M,\ j \neq h$$

$$\frac{\partial y_h}{\partial Q_h} = \left[\alpha(\theta_h - 1) - \theta_h\right]\frac{y_h}{Q_h}$$

$$\frac{\partial y_h}{\partial b} = \frac{1}{Q_h^{\alpha}\left[\sum_n q_n^{1-\alpha} + \sum_{m \neq h} Q_m^{1-\alpha}\right] z^{1-\alpha} + Q_h}$$

$$\frac{\partial y_h}{\partial z} = (\alpha - 1)\ (1 - \theta_h)\ \frac{y_h}{z}$$

From these it is easily seen that the own price derivative of y_h is unambiguously negative, as is that of x_i if the wealth effect $\varphi_i w_i/q_i$ is not too large. (Note that this is not the same as the income effect in a Slutsky equation. All goods are normal here.) The sign of the cross price derivatives is the same as the sign of $(\alpha - 1)$. Equalling $b/(qw + b)$, elasticities with respect to lump-sum income depend on prices, but are identical for all private goods. Finally, $\partial x_i/\partial z$ and $\partial y_h/\partial z$ have opposite signs which are determined by the sign of $(\alpha - 1)$, so if x_i and z are complements in the sense of chapter 3, then y_h and z are substitutes, and vice versa.

In principle, it is now possible to find the parameterized versions of the analytical results from part 1. However, the resulting expressions would be completely intractable unless employed in a numerical simulation. For this purpose the numerical modelling tool COMPAK (Lensberg and Rasmussen, 1986) will be used.

A numerical model of a 5-good economy with exogenous taxes

The point of departure for the numerical simulation is a simple model with two production factors, two private commodities and one public good. To determine an equilibrium, some taxation rules must also be introduced. In the basic equilibrium of the first model presented here, the rule is to let all tax rates be equal. In the COMPAK algorithm the model must be formulated as a complementarity problem. This is done by introducing a slack variable in each equation along with a complementary slackness condition involving the slack variable and its dual variable corresponding to the equation. Choosing commodity 1 as a numéraire, the model thus becomes

$$
\begin{array}{ll}
w_1 - x_1 - \sum_m a_1^m y_m - g_1 - \omega_1 = 0 & \omega_1 \cdot p_1 = 0 \\
w_2 - x_2 - \sum_m a_2^m y_m - g_2 - \omega_2 = 0 & \omega_2 \cdot p_2 = 0 \\
P_2 - c^2 - \psi_2 = 0 & \psi_2 \cdot P_2 = 0 \\
\sum_m s_m y_m - \sum_n t_n (w_n - x_n) - \sum_n p_n g_n - b_1 + b_2 = 0 & b_1 \cdot b_2 = 0
\end{array}
$$

where ω_1, ω_2 and ψ_2 are the respective slack variables for factor market 1, factor market 2, and the profits per unit in firm 2. Either b_1 or b_2 may be the slack variable for the public budget, depending on the sign of the lump-sum transfers to the consumer. If $b > 0$, then $b_2 = 0$, and b_1 will be equal to b. If $b < 0$, then $b_1 = 0$, and b_2 will be equal to $|b|$. The normalization rule is $P_1 = Q_1 = 1$, and the taxation rule is $t_1 = t_2 = s_2$. (All tax rates are non-negative in the numerical model, the computer program for which is reproduced in Appendix A.) The utility function is given by:

$$u(x,y,z) = \left[x_1{}^\rho + x_2{}^\rho + y_1{}^\rho + y_2{}^\rho z^\rho \right]^{1/\rho}$$

Exogenous variables and parameters are listed as a reference case below, along with market prices, tax revenues and lump-sum transfers, supply and demand, the consumer's budget shares, supply and demand elasticities, the value shares of the inputs and factor intensities in the initial general equilibrium. From now on the COMPAK variable names will be used in order to facilitate the interpretation of the diagrams with the simulation results. The COMPAK variable names are defined in the following list of initial equilibrium values for exogenous and endogenous variables:

Initial equilibrium and reference case

Parameters	
Consumer's substitution parameter (α):	ALF = 1.1
Private firms' substitution elasticity (σ_m):	SIG1 = 0.5
	SIG2 = 0.5
Public substitution elasticity (γ):	GAM = 0.5
Public scale elasticity (ε):	EPS = 1.0
Exogenous variables	
Initial factor endowments (w_n)	W1 = 10.000
	W2 = 10.000
Public production level (z):	Z = 1.000
Tax rate on commodity 1 (s_1):	S1 = 0.000
Tax rate on commodity 2 (s_2):	S2 = 0.104
Factor tax rates (t_n):	T1 = 0.104
	T2 = 0.104
Producer price of commodity 1 (P_1):	PY1 = 1.000
Main endogenous variables	
Producers' factor prices (p_n):	PX1 = 0.250
	PX2 = 0.250
Producer price of commodity 2 (P_2):	PY2 = 1.000
Consumer's factor prices (q_n):	QX1 = 0.146
	QX2 = 0.146
Consumer price of commodity 1 (Q_1):	QY1 = 1.000
Consumer price of commodity 2 (Q_2):	QY2 = 1.104
Lump-sum transfers (b):	TRAN = 0.000

<u>Other endogenous variables</u>

Consumer's factor demand (x_n):	X1 = 5.492
	X2 = 5.492
Consumer's demand for commodity 1 (y_1):	Y1 = 0.661
Consumer's demand for commodity 2 (y_2):	Y2 = 0.593
Public factor demand (g_n):	G1 = 2.000
	G2 = 2.000
$(-\partial x_1/\partial q_1)(q_1/(w_1-x_1))$:	ESX1X1 = 0.698
$(-\partial x_2/\partial q_2)(q_2/(w_2-x_2))$:	ESX2X2 = 0.698
$(-\partial x_1/\partial q_2)(q_2/(w_1-x_1))$:	ESX1X2 = -0.643
$(-\partial x_2/\partial q_1)(q_1/(w_2-x_2))$:	ESX2X1 = -0.643
$(-\partial x_1/\partial Q_1)(Q_1/(w_1-x_1))$:	ESX1Y1 = -0.028
$(-\partial x_2/\partial Q_1)(Q_1/(w_2-x_2))$:	ESX2Y1 = -0.028
$(-\partial x_1/\partial Q_2)(Q_2/(w_1-x_1))$:	ESX1Y2 = -0.027
$(-\partial x_2/\partial Q_2)(Q_2/(w_2-x_2))$:	ESX2Y2 = -0.027
$(\partial y_1/\partial q_1)(q_1/y_1)$:	EDY1X1 = 0.527
$(\partial y_1/\partial q_2)(q_2/y_1)$:	EDY1X2 = 0.527
$(\partial y_2/\partial q_1)(q_1/y_2)$:	EDY2X1 = 0.527
$(\partial y_2/\partial q_2)(q_2/y_2)$:	EDY2X2 = 0.527
$(\partial y_1/\partial Q_1)(Q_1/y_1)$:	EDY1Y1 = -1.077
$(\partial y_1/\partial Q_2)(Q_2/y_1)$:	EDY1Y2 = 0.022
$(\partial y_2/\partial Q_1)(Q_2/y_1)$:	EDY2Y1 = 0.023
$(\partial y_2/\partial Q_2)(Q_2/y_2)$:	EDY2Y2 = -1.078

Since ALF = 1.1 > 1, the cross price derivatives will be positive, and the factor (ALF - 1) will be rather small. The demand for Y2 increases with Z, as does the supply of both production factors, since X1 and X2 decrease with Z. The substitution elasticities SIG1, SIG2 and GAM are moderate, so that the economy will probably be quite well-behaved. Numerically there is no difference between the two production factors, and their prices are equal in the initial equilibrium. Since the tax rate S2 makes QY2 somewhat higher than QY1, the demand for Y2 is slightly lower than the demand for Y1. The absolute values of the own price and cross price factor supply elasticities are perhaps larger than one would have expected to see in an empirical model, but still not too unrealistic. On the other hand, there are negligible cross price elasticities of factor supply and commodity demand with respect to commodity prices. The level of public production being equal to 1, all goods enter symmetrically in the utility function. The exogenous tax rates S2, T1 and T2 have been adjusted so that lump-sum transfers are approximately zero.

Harberger incidence

Several exercises in comparative statics are now possible. We start with a series of 1% increments of T1, in the spirit of the Harberger model.[3] T1 is increased until the consumer price QX1 is so low that it is no longer possible to find an equilibrium. This happens at T1 = 0.454. The level of public production, Z, is held constant, and excess tax revenues are returned to the consumer as a lump-sum transfer. The main results of these tax changes are shown in Figures 4.1-4.8. Note that the origins of the diagrams are not in the point [0,0].

Figure 4.1 demonstrates what is usually meant by tax incidence; the effect on the market prices of increasing the tax rate in the market (all tax rates are defined as non-negative numbers in the diagrams). The general equilibrium result here is as expected. The producer price of factor 1, PX1, increases, and the consumer price, QX1, decreases. It is also clear that the slope of the PX1 curve is greater than the negative of the slope of the QX1 curve. In this sense the greater part of the tax incidence is on the producers.

---

[3] In the Harberger model, a factor tax rate is increased in one of the firms. Here it is increased in one of the markets. The point, however, is that tax rates are exogenous and that the revenues are returned to the consumer as lump-sum transfers.

Paths of PX1 and QX1 as T1 increases

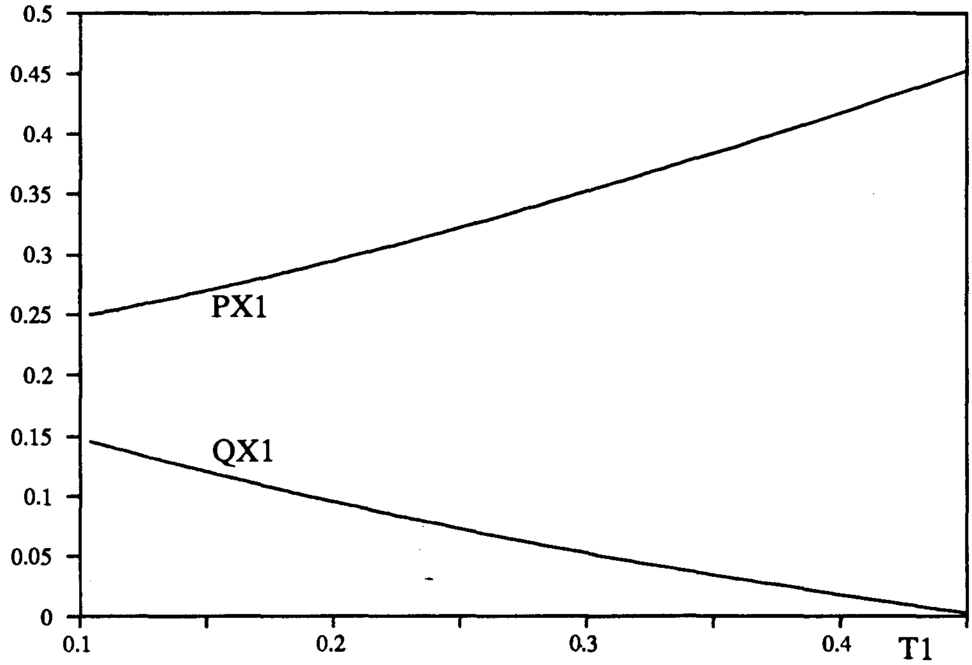

Figure 4.1

Paths of PX2 and QX2 as T1 increases

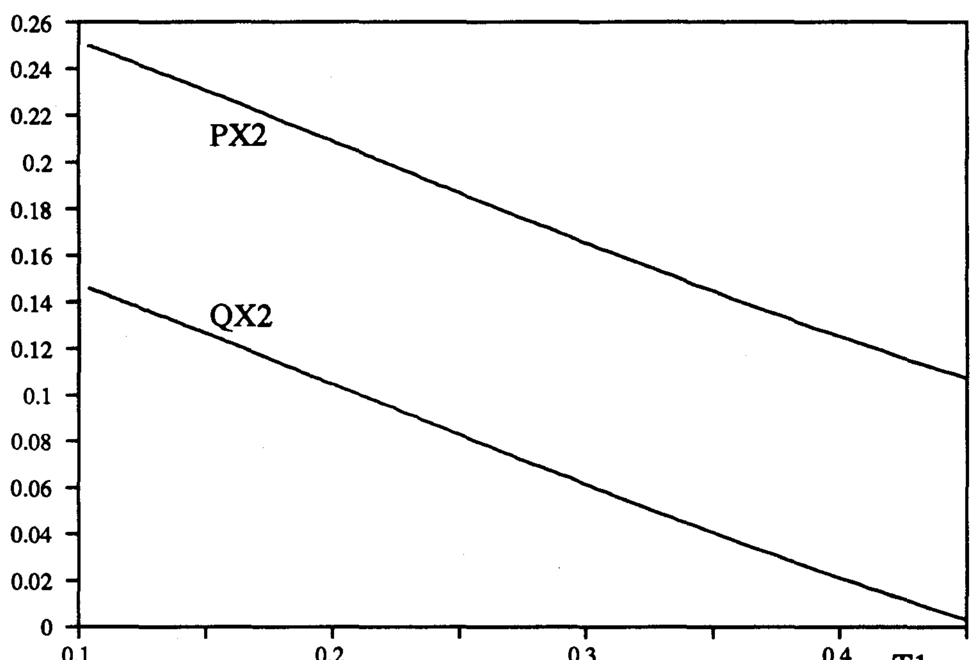

Figure 4.2

Figure 4.2 illustrates the price changes in factor market 2 when the tax rate is increased in factor market 1. There are several counteracting effects behind the downward-sloping curves for the producer price PX2 and the consumer price QX2. E.g., when PX1 increases, there is a substitution effect which increases the producers' demand for factor 2 and thus works towards an increase in PX2. On the other hand, there is a scale effect working in the opposite direction. As figure 4.2 shows, the net effect of these and other adjustments is a decrease in PX2 and QX2. The distance between the two curves is of course constant since T2 is not changed.

In Figure 4.3 a rather surprising result is displayed: When T1 increases, total tax revenues TREV first rise, then fall, exactly as described by Dupuit (1844). The result is surprising for two reasons. First, many authors have argued that total tax revenues fall only if tax rates and/or supply elasticities are very high. Second, it is often seen that such effects are somewhat greater in partial equilibrium than in a general equilibrium model because of the dampening repercussions of the latter. In Figure 4.3, which depicts a general equilibrium result, maximum tax revenues are 1.217 (or 21.7% above the reference case) at the tax rate T1 = 0.256 (or 78.5% of the producer price, which is PX1 = 0.326). At the maximum the own price supply elasticity of factor 1 is ESX1X1 = 1.555, and the two cross price elasticities are ESX1X2 = -1.158 and ESX2X1 = -0.657.

It seems that the magnitude of the cross price elasticities in factor supply is an important cause of both the Dupuit-Laffer effects and the declining prices in factor market 2. As T1 increases and QX1 falls, the consumer withdraws his supply of factor 1 and increases his supply of factor 2. He also reduces his commodity demand (the cross elasticity is 0.435 at the maximum). The rising supply of factor 2 induces a downward

Dupuit-Laffer curve as T1 increases

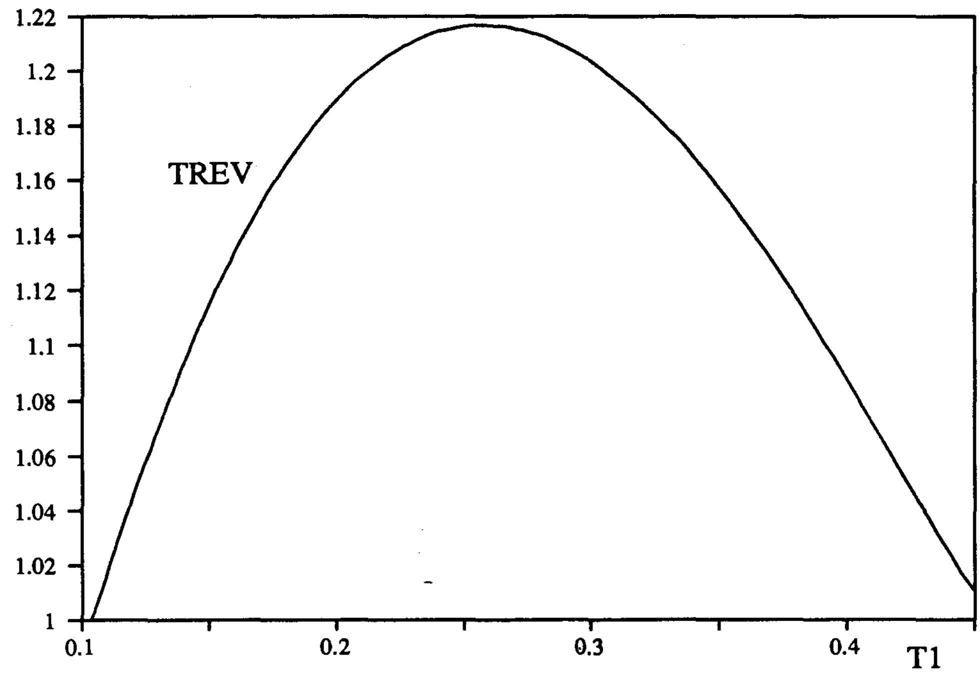

Figure 4.3

Path of ESX1X1 as T1 increases

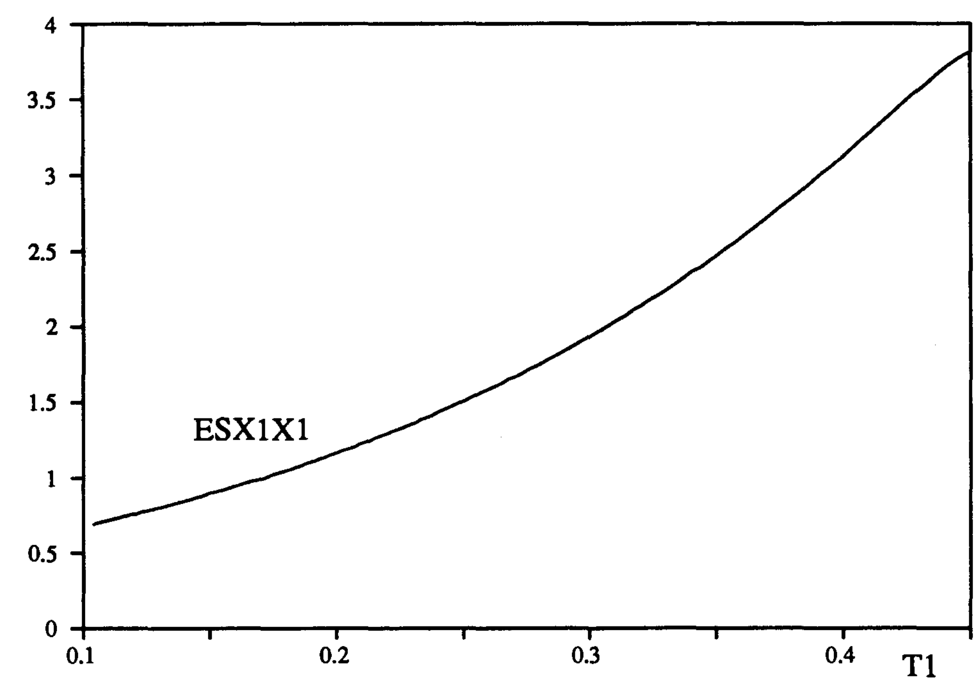

Figure 4.4

pressure on PX2 and QX2, and the greater tax revenues from this market do not outweigh the decline in revenues from factor market 1 and commodity market 2.

The variations in different elasticities as T1 rises are shown in Figures 4.4-4.8. We see that the own price supply elasticity of factor 1, ESX1X1, increases, and its cross price supply elasticity, ESX1X2, decreases. The cross price supply elasticity of factor 2, ESX2X1, first rises slightly, then it falls. This factor's own price supply elasticity rises at first, but seems to attain a maximum just before the equilibrium breaks down. The cross price supply elasticity of factor 2 with respect to QY1, ESX2Y1, falls initially, but clearly reaches a minimum and starts to increase before the breakdown of equilibrium. Note, however, that the interval of variation is very small: [-0.0409, -0.0276].

If T2 were changed instead of T1, exactly the same results would occur, since numerically there is no difference between the production factors in the model. Therefore only the effects of increasing the commodity tax rate S2 remain to be seen. Some of them are illustrated in Figures 4.9-4.10. First and foremost, Figure 4.9 shows that the incidence of taxation is entirely on the consumer, since the producer price PY2 is not affected by S2 at all. Thus, even though increases in QY2 could have repercussion effects in factor markets through cross elasticities and the consumer's real income, this does not appear to have any impact on the firm's total costs. (The reason may be that such an impact is numerically negligible, since the cross elasticities are small: -0.03. But as will become clear later, there is a remarkable "separability" between the commodity side and the factor side of this particular model.) Figure 4.10 demonstrates the Dupuit-Laffer curve for an increase in S2: tax revenues do not decline at

Path of ESX1X2 as T1 increases

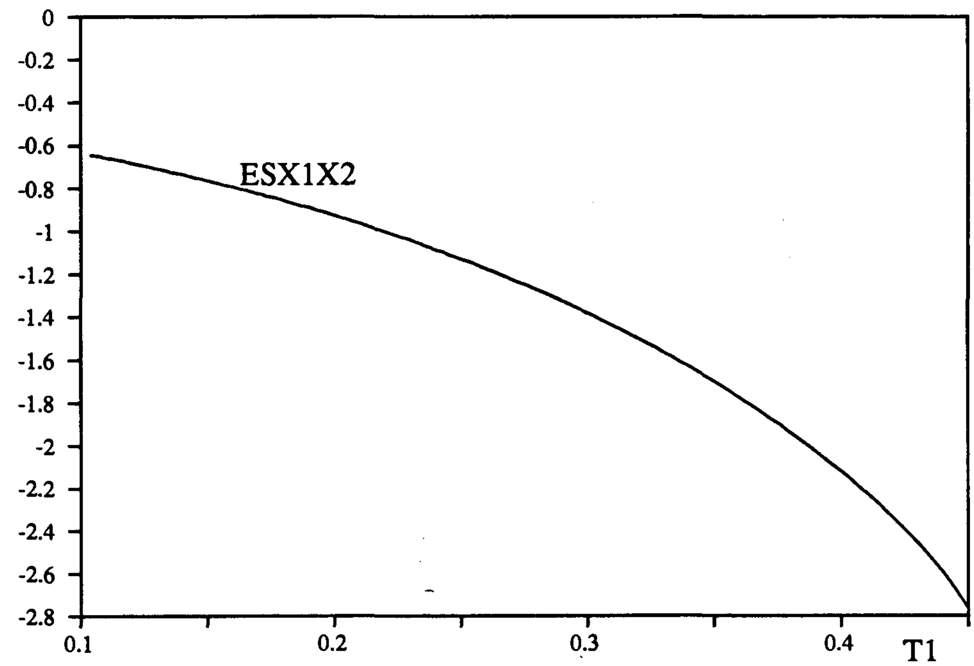

Figure 4.5

Path of ESX2X1 as T1 increases

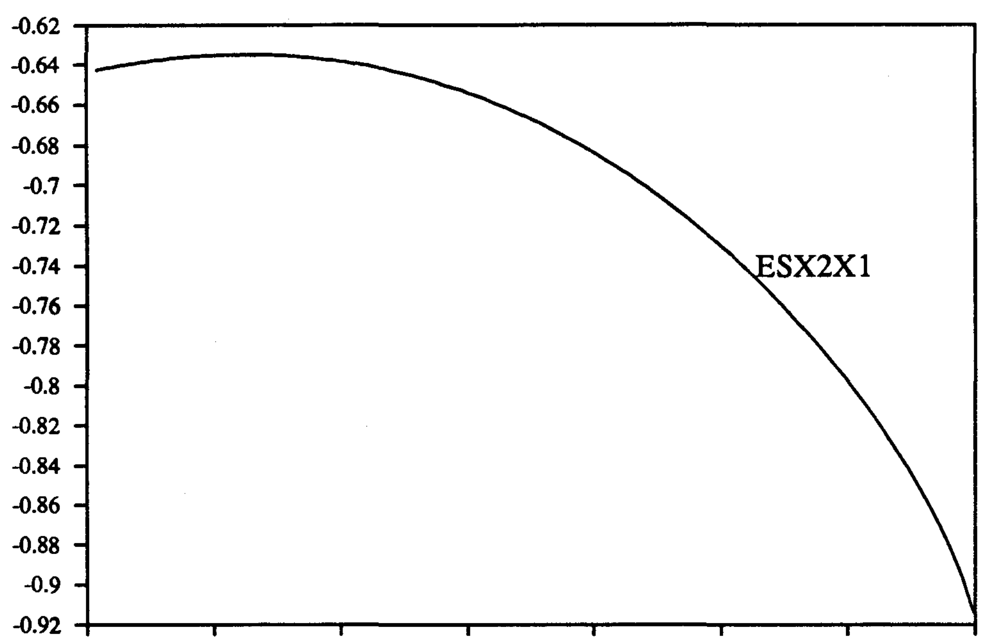

Figure 4.6

Path of ESX2X2 as T1 increases

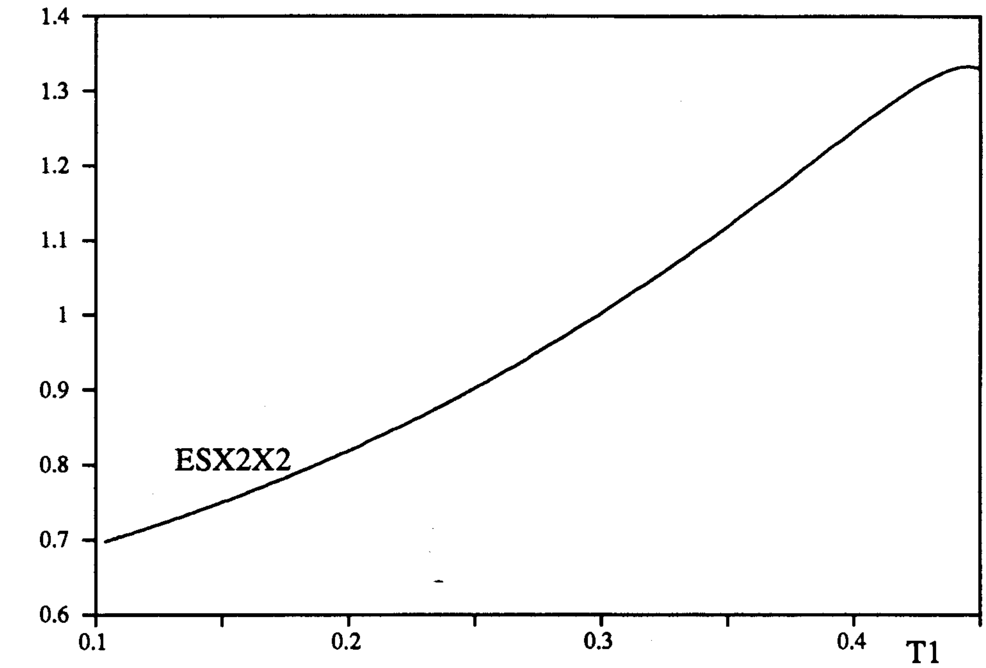

Figure 4.7

Path of ESX2Y1 as T1 increases

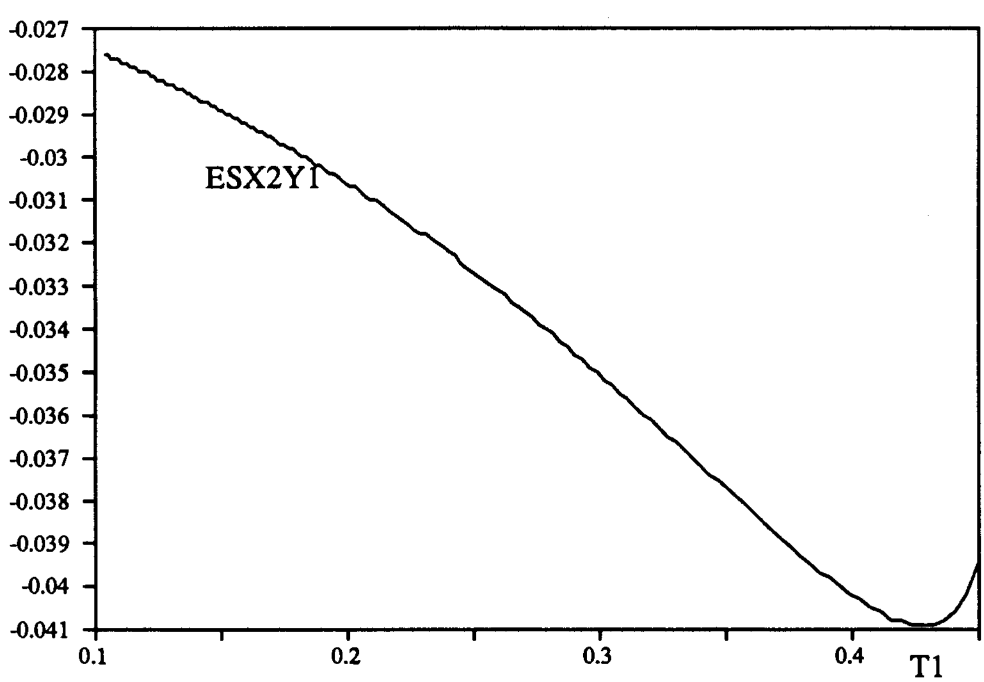

Figure 4.8

Paths of PY2 and QY2 as S2 increases

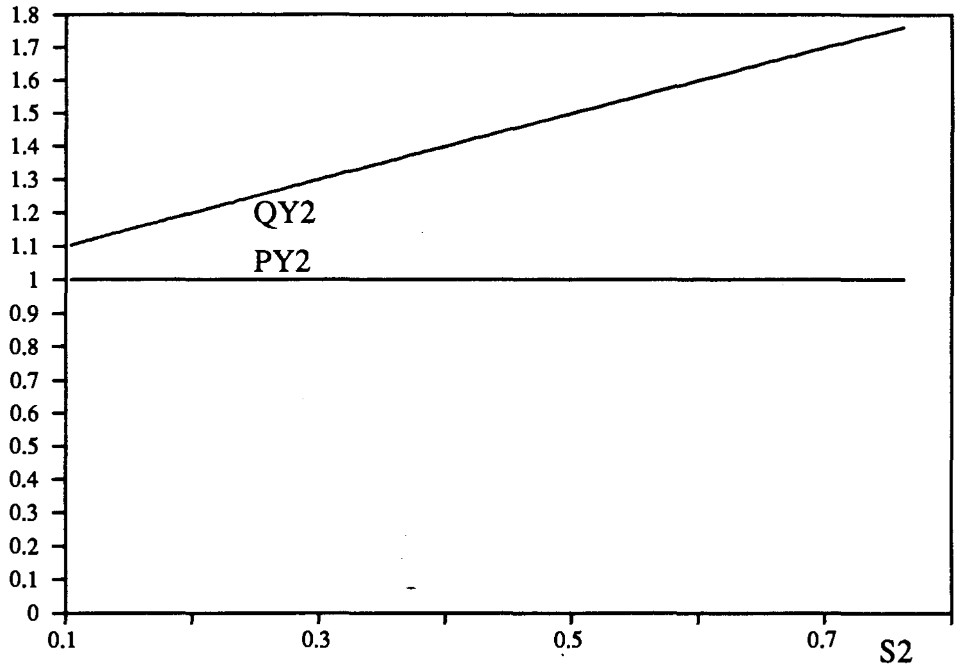

Figure 4.9

Dupuit-Laffer curve as S2 increases

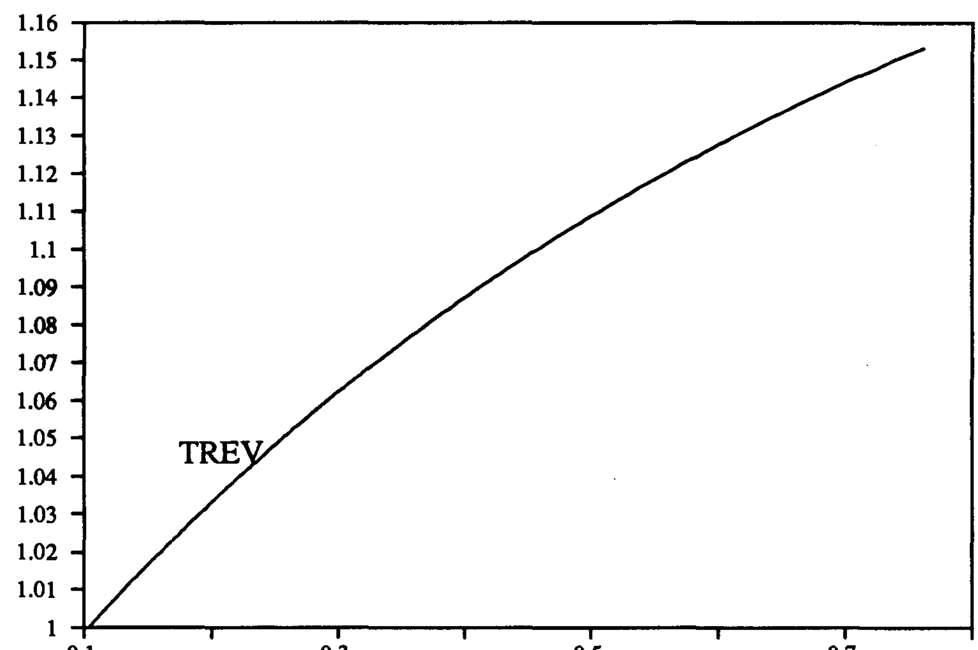

Figure 4.10

any point. (The simulation was stopped after incrementing S2 by 1% 200 times, without any breakdown of equilibrium.) It also turns out that the own price and cross price supply elasticities develop in a straighforward manner, and that all factor prices remain constant at their initial levels.

A modified model with endogenous tax rates

The purpose of this section is to compare the results from the Harberger incidence analysis with those obtained when the taxation level increases and tax rates are endogenous. The numerical model in the previous section can of course not be used to compute optimal tax rates unless completed with a set of equations representing the new tax rules. These equations will be the first-order conditions for optimal taxes which were discussed in Chapter 3. Thus, equation (3) in part 1 of this chapter is replaced by

$$(3') \qquad P_1 = Q_1 = 1$$

$$(3'') \qquad x_k - w_k + \frac{\mu}{\lambda}\left[\sum_n p_n \frac{\partial x_n}{\partial q_k} + \sum_m P_m \frac{\partial y_m}{\partial q_k}\right] = 0, \qquad k = 1,\dots,N$$

$$(3''') \qquad y_j + \frac{\mu}{\lambda}\left[\sum_n p_n \frac{\partial x_n}{\partial Q_j} + \sum_m P_m \frac{\partial y_m}{\partial Q_j}\right] = 0, \qquad j = 2,\dots,M$$

Here λ is the consumer's marginal utility of income, determined by utility maximization:

$$v \equiv \max u(x,\ z) \text{ subject to } q(w - x) + b = Qy \Rightarrow \lambda = \partial v/\partial b$$

As indicated in Chapter 3, μ is the marginal cost of public funds, i.e. if T is the public revenue requirement and the consumer's indirect utility function v is taken as a measure of social welfare, then $\mu = -\partial v/\partial T$. μ is determined along with p, P, q and Q by equations (1), (2), (3')-(3'''), (4), and (5). In equation (5), the tax vectors are defined by

$$[t, s] \equiv [q, Q] - [p, P]$$

Finally, lump-sum transfers (or taxes) are exogenously given. Normally, b will equal zero, since if lump-sum taxation were feasible, there would be no need for other optimal taxes. The computer program for this model is reproduced in Appendix B.

If all the remaining exogenous variables are held at the same levels as in the previous section (including the public production level), the main endogenous variables in the equilibrium with optimal taxation are:

<u>Equilibrium with optimal tax rates</u>

Producers' factor prices (p_n):	PX1 = 0.250
	PX2 = 0.250
Producer price of commodity 2 (P_2):	PY2 = 1.000
Consumer's factor prices (q_n):	QX1 = 0.139
	QX2 = 0.139
Consumer price of commodity 2 (Q_2):	QY2 = 1.000
Consumer's factor demand (x_n):	X1 = 5.492
	X2 = 5.492
Consumer's commodity demand (y_m):	Y1 = 0.627
	Y2 = 0.627

There are only two important differences between this equilibrium and the reference case. First, the optimal tax rates are T1 = 0.1109, T2 = 0.1109 and S2 = 0. Second, the demand for Y1 equals the demand for Y2, and their prices are now the same. This symmetry between the two commodities is reflected in the consumer's demand and supply elasticities, which also differ very little from the reference case. It is not obvious why the optimal commodity tax rate S2 is equal to zero, but it may be related to the fact that as long as Z = 1, the commodities enter symmetrically into the consumer's utility function, and the tax rate S1 is constrained to equal zero. In the same way we may note that factor tax rates are equal and that the two factors also enter symmetrically in the utility function.

In the numerical simulations which follow, the public production level is repeatedly increased by 1% until the consumer's factor prices are so low that it is no longer possible to find an equilibrium. It is assumed that the returns to scale in public production are constant: EPS = 1. Some of the main results are shown in Figures 4.11-4.16. In Figure 4.11 we see that the factor tax rates are always equal, and they appear to increase almost linearly until Z approaches the level where the general equilibrium breaks down (the end point is at Z = 1.891). In Figure 4.12 the paths of the optimal factor prices are displayed to demonstrate the incidence of optimal factor taxes. It turns out that the incidence is entirely on the consumer; the producers' factor prices remain constant when T1 and T2 increases. As will become clear later, this is due to the specification of this particular numerical example.

Figure 4.13 shows that under optimal taxation and constant returns to scale, tax revenues increase in exact proportion to the public production level. In general equilibrium, tax revenues are always equal to public expenditures, which in this case are the total costs of public production. Since the public sector minimizes costs (by assumption), the tax revenue function will be an ordinary minimum cost function, <u>provided that the producers' factor prices are constant</u>. Figure 4.12 shows that they are, Path of optimal T1 (T2) as Z increases

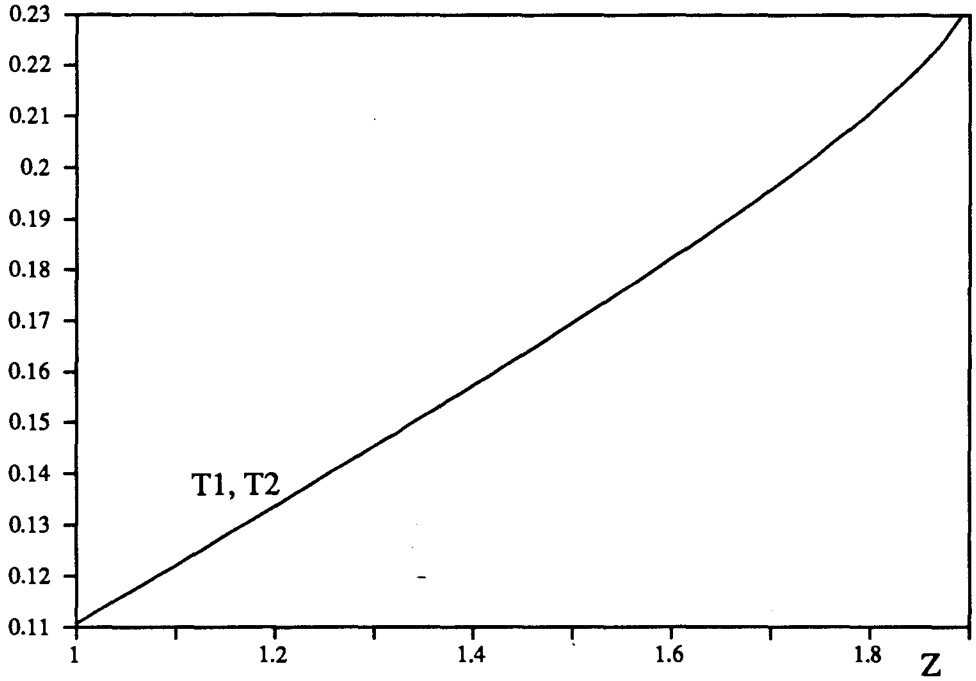

Figure 4.11

Optimal factor prices as Z increases

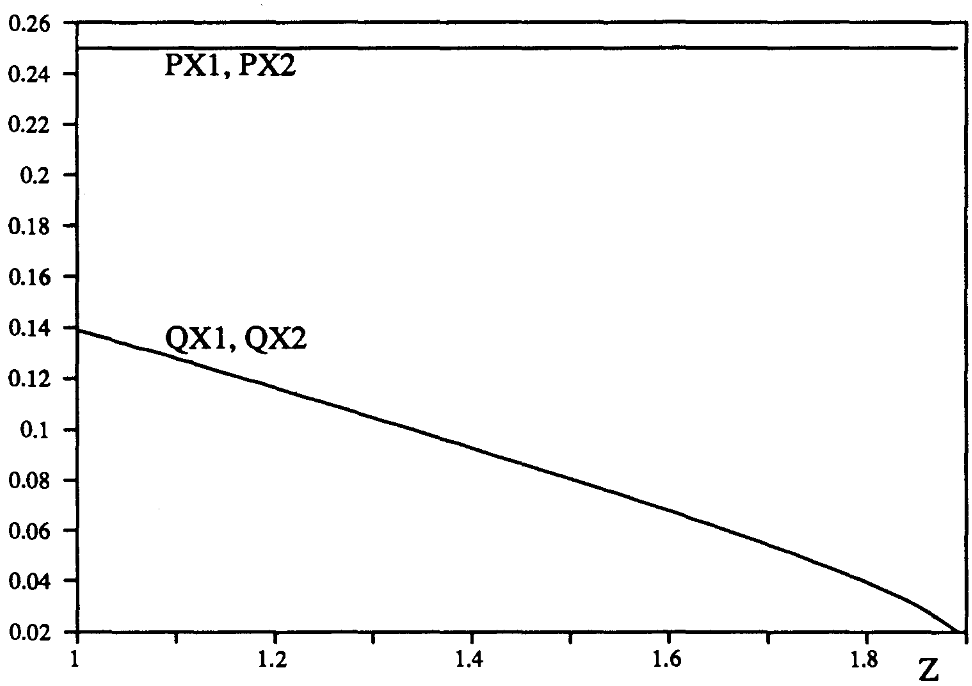

Figure 4.12

Optimal tax revenues as Z increases

EPS = 1 (Constant returns to scale)

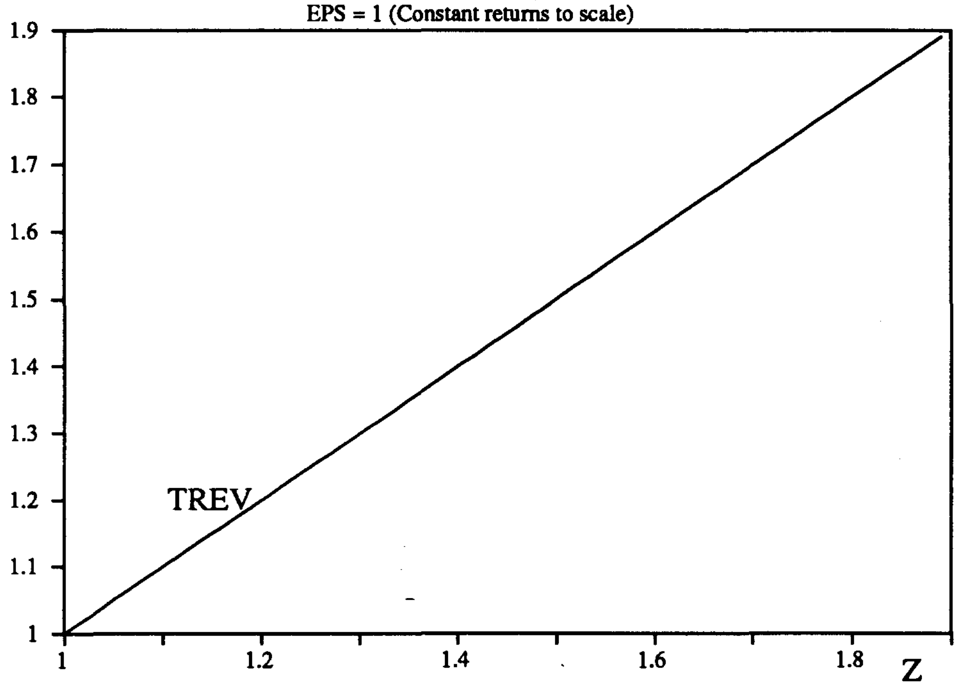

Figure 4.13

Total tax revenues plotted against optimal T1

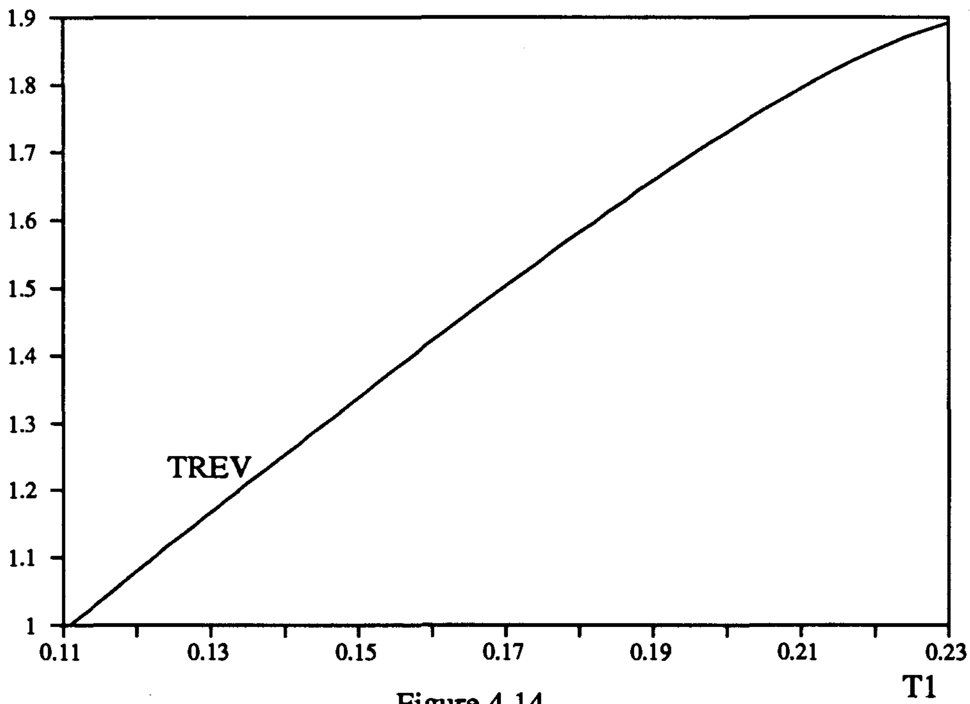

Figure 4.14

and we can therefore use the public factor demand functions to express the public cost function and the tax revenues as

$$T = p_1 g_1 + p_2 g_2 = p_1 \left[1+\left(\frac{p_2}{p_1}\right)^{1-\gamma}\right]^{\frac{\gamma}{1-\gamma}} z^{1/\varepsilon} + p_2 \left[1+\left(\frac{p_1}{p_2}\right)^{1-\gamma}\right]^{\frac{\gamma}{1-\gamma}} z^{1/\varepsilon}$$

Since $p_1 = p_2 = 0.25$ and $\gamma = 0.5$, we have that

$$\begin{aligned} T &= 0.25\left[(1 + 1)^{\gamma/(1-\gamma)} + (1 + 1)^{\gamma/(1-\gamma)}\right] z^{1/\varepsilon} \\ &= z^{1/\varepsilon} \end{aligned}$$

Thus, as long as producers' factor prices remain constant at their initial level, the curvature of the tax revenue function is determined by the public scale elasticity.

There is no Dupuit-Laffer curve in this model, since the tax rates are endogenous. However, for the sake of comparison with Figure 4.3, total tax revenues are plotted against T1 in Figure 4.14. We see that revenues rise almost linearly with T1, but with a slight, concave curvature towards the breakdown of the equilibrium.

Figure 4.15 depicts the development of the two Lagrange multipliers, LAM (λ) and MU (μ). No clear conclusion was drawn in the discussion in Chapter 3 of how the consumer's marginal utility of income and the marginal cost of public funds develop when the public production level rises. Here we see that μ exceeds λ everywhere, and it becomes very large as Z approaches its end point 1.891. The consequences of this for NU (ν), the coefficient $(\lambda - \mu)/\mu$, are shown in Figure 4.16. NU is the percentage marginal loss from not using lump-sum taxes instead of distortive tax rates and appears

Paths of LAM and MU as Z increases

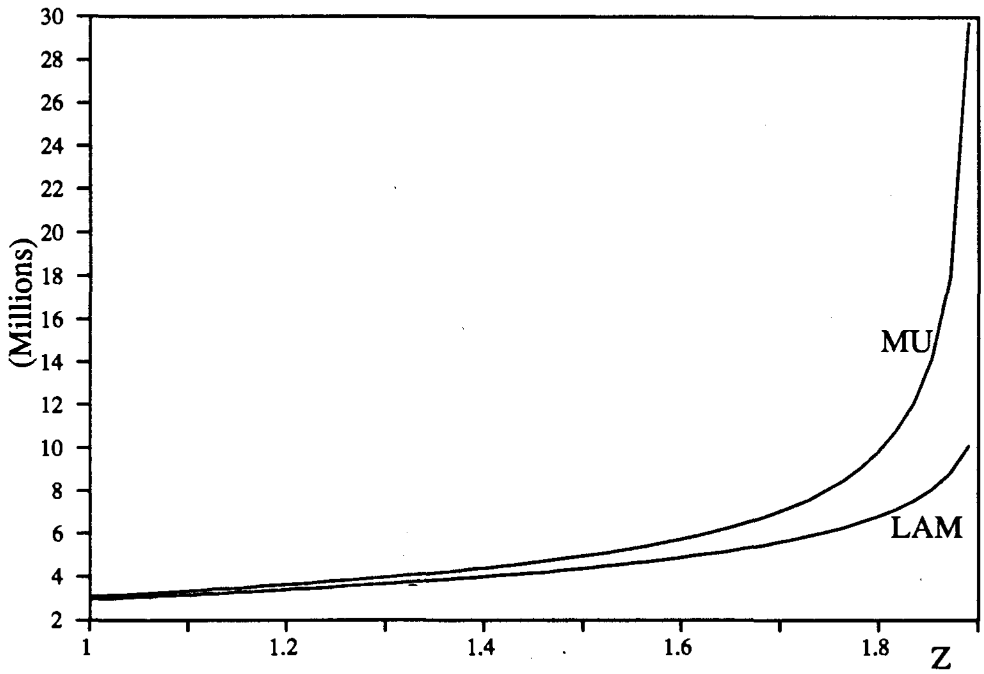

Figure 4.15

Path of NU = (LAM-MU)/MU as Z increases

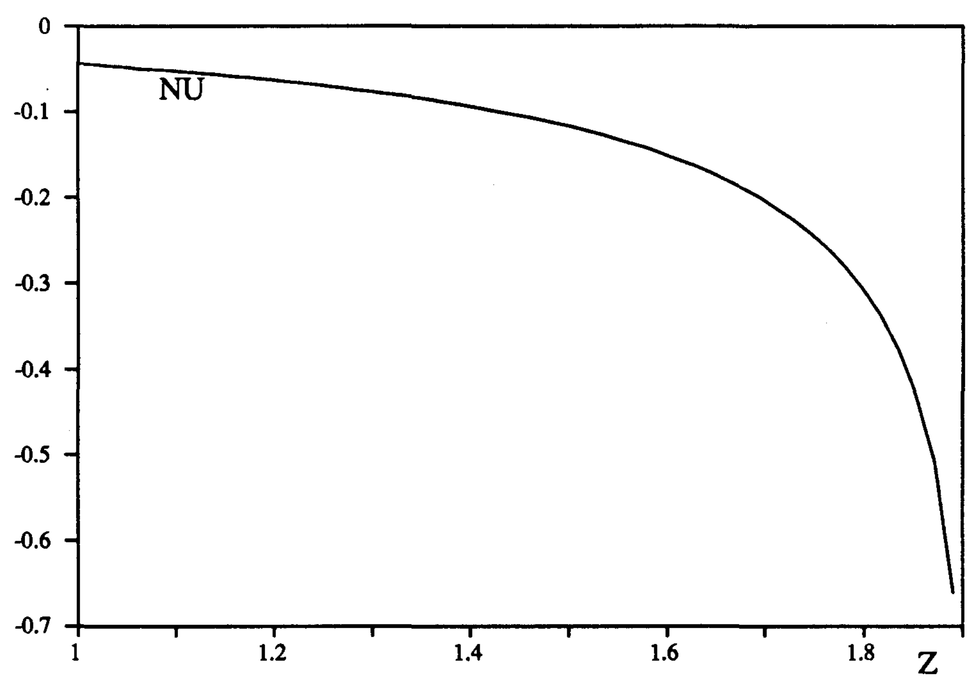

Figure 4.16

in the optimal tax formula (1) in Chapter 3. NU is negative, and its absolute value grows progressively with Z. This indicates that the real cost of optimal taxation increases with the overall tax level.

Other results of the simulation (not shown in the diagrams) are that all four commodity prices, PY1, PY2, QY1 and QY2, remain constant at their initial level (which is 1) when Z is increased and taxation is optimal. In other words, the optimal S2 is still 0, although Y2 and Z are complements in the sense of Chapter 3: The demand for Y1 falls as taxation increases, and so does the demand for Y2; but less rapidly.

Simulation 3: Optimal taxes and non-constant producer prices

The third numerical simulation model is included here just to show that the constancy of optimal producer prices in the previous section is probably a coincidence of numbers and not a theoretical property. In addition the example will serve to point out that producer prices are not necessarily constant in a comparative statics sense when the tax rates are optimal. The model is taken from Berg (1989), but will be sketched briefly below.

There are 6 goods in the model: 2 production factors denoted x_1 and x_3; 3 privately produced commoditites, denoted x_0, x_2, and x_4; and one public good, denoted by z. The consumer prices of the private commodities are Qn, n = 0,...,4, and the producer prices are Pn, n = 0,1,...,4. Commodity 0 is chosen as a numéraire, so that Q0 = P0 = 1. The three private production functions are of the single-level CES type, with substitution elasticities 0.5, 0.6 and 0.4 for good 0, 2 and 4, respectively. The public production function is also single-level CES, with the elasticity of substitution equal to 0.2. Returns to scale are constant in all production activities. The one representative consumer has a two-level CES utility function which is independent of the public production level:

$$u(x) \equiv \left\{ x_0^{\frac{\rho-1}{\rho}} + \left[x_1^{\frac{\eta-1}{\eta}} + x_2^{\frac{\eta-1}{\eta}} \right]^{\frac{\rho-1}{\rho}\frac{\eta}{\eta-1}} + \left[x_3^{\frac{\varphi-1}{\varphi}} + x_4^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\rho-1}{\rho}\frac{\varphi}{\varphi-1}} \right\}^{\frac{\rho}{\rho-1}}$$

where $\rho = 1.2$, $\eta = 0.7$ and $\varphi = 0.3$. The consumer's initial endowments of the production factors are $w_1 = w_2 = 5$, and the public production level is equal to 1. Then the optimal taxation equilibrium is:

P1 = 0.9392	Q1 = 0.8392
P2 = 1.1312	Q2 = 1.1655
P3 = 1.0627	Q3 = 0.8870
P4 = 1.0585	Q4 = 1.1775

The marginal cost of government funds in terms of private income is 1.0044 in this initial equilibrium. Now the public production level is repeatedly incremented by 1% from 1.0000 to 3.0177, and the corresponding paths for optimal producer and consumer prices in the model are computed. The most important results are displayed in Figures 4.17-4.22. Figure 4.17 shows that the producer price of factor 1 rises from 0.9392 to 0.9554 when z is increased. The incidence is mainly on the consumer price, however, as is evident form Figure 4.18. In the same way, the producer price of factor 2 falls with increasing public production (Figure 4.19), but less so than the consumer price of that factor (Figure 4.20). Figures 4.21 and 4.22 depict the equilibrium paths of the commodity prices. The producers' output prices turn out to be constant, and the incidence is entirely on the consumer.

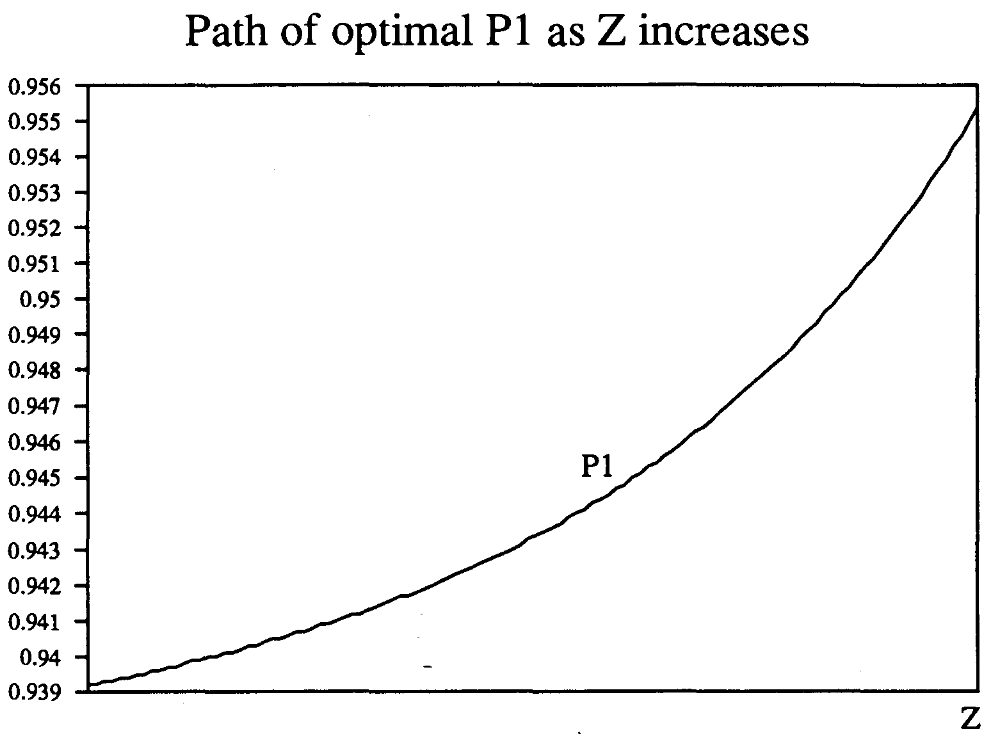

Figure 4.17

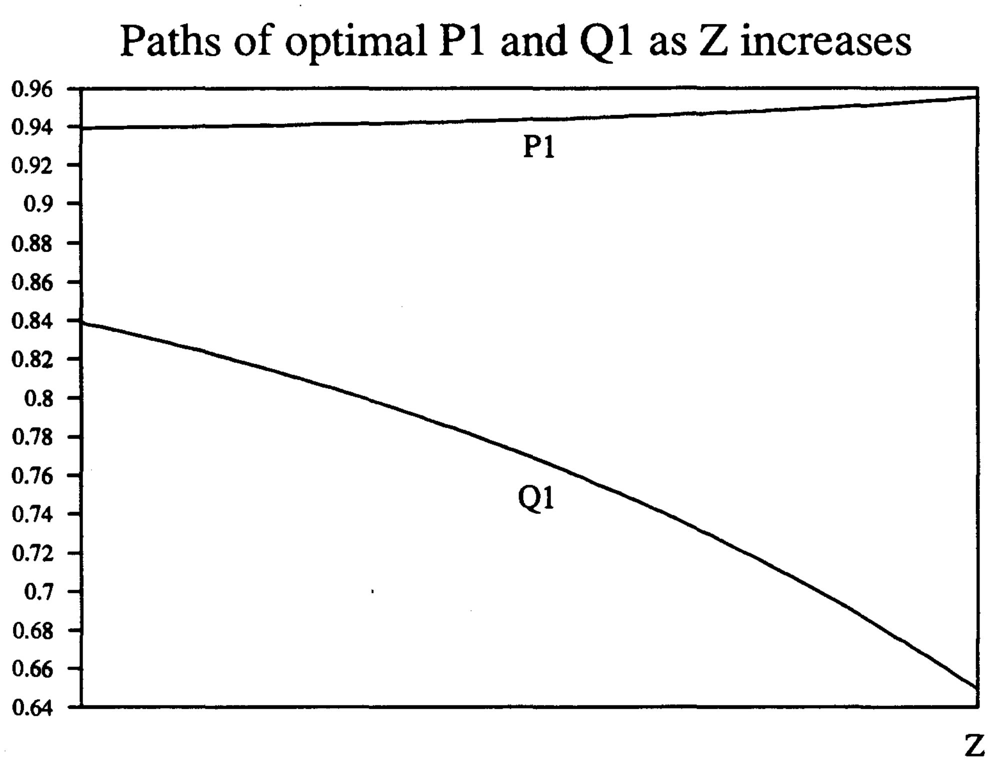

Figure 4.18

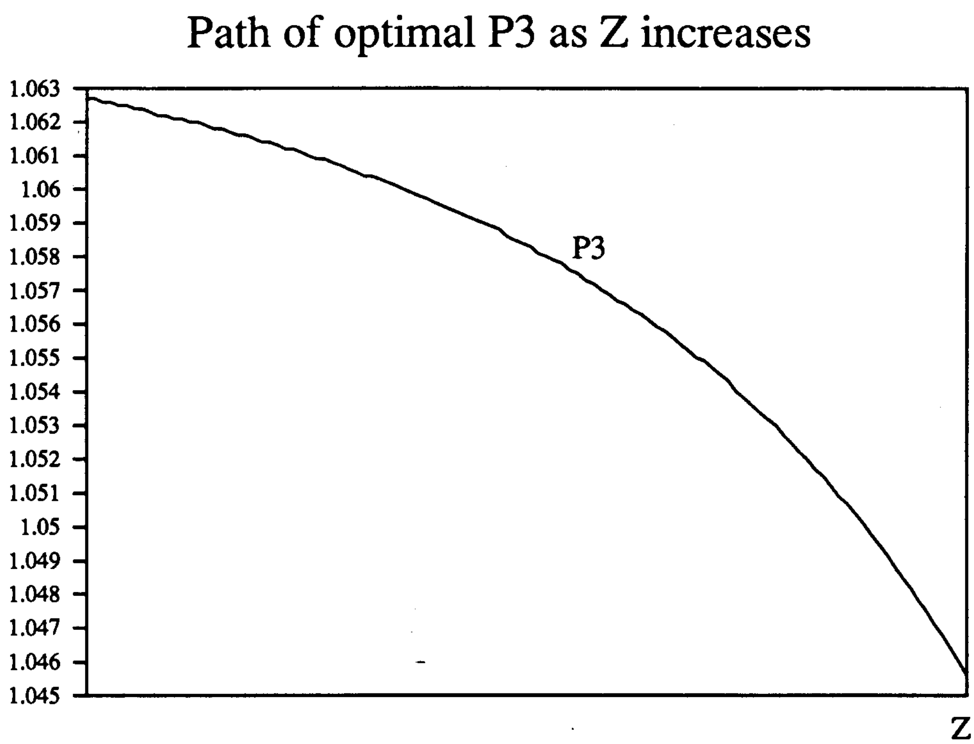

Figure 4.19

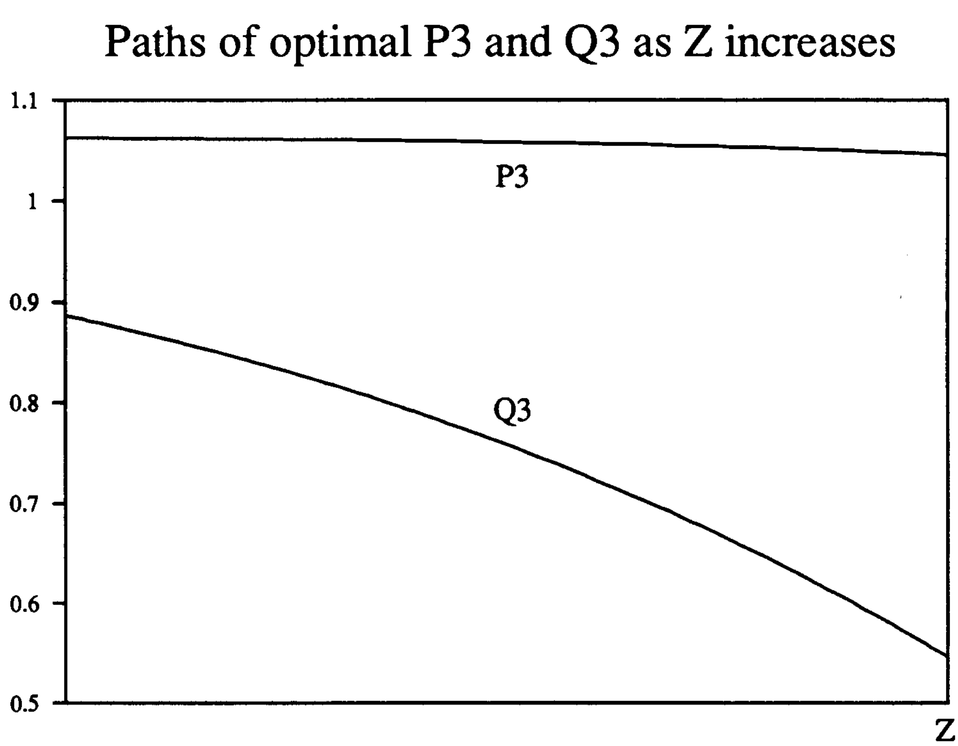

Figure 4.20

Paths of optimal P2 and Q2 as Z increases

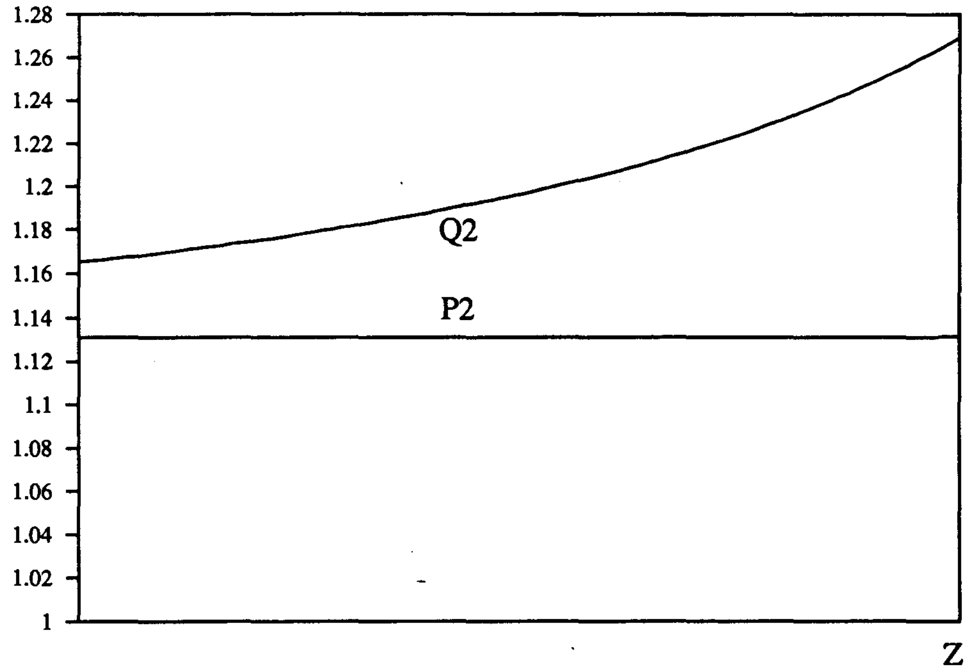

Figure 4.21

Paths of optimal P4 and Q4 as Z increases

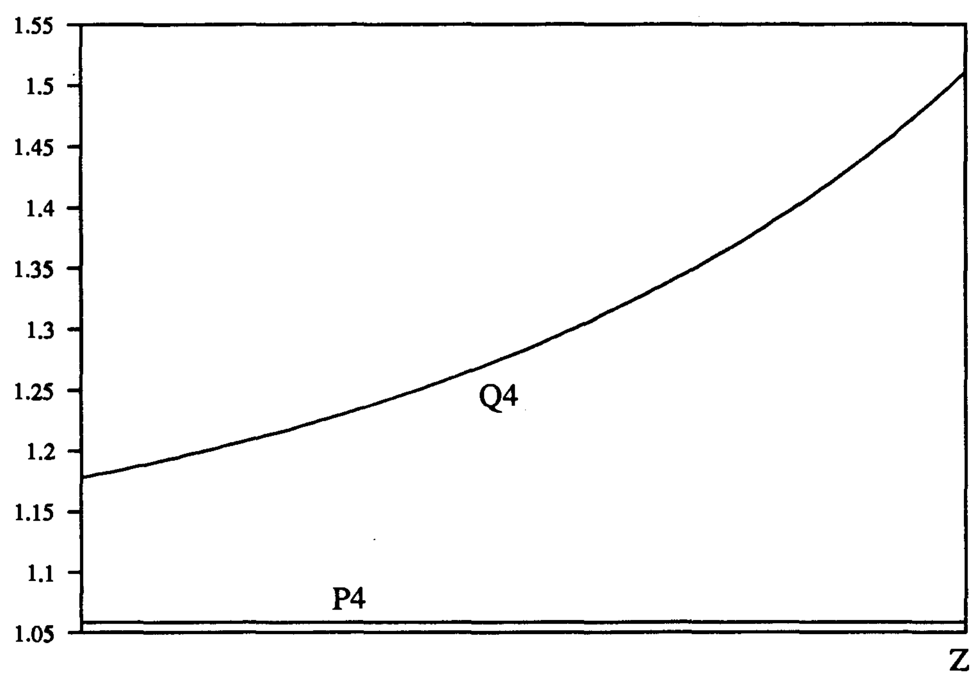

Figure 4.22

Concluding remarks

Any numerical simulation result depends critically on the values chosen for the exogenous variables of the model, as well as on the functional forms being used. The present models are no exception, and the diagrams in this chapter can only be taken as demonstrations of what is possible in general equilibrium. There are several ways to assess whether the results are plausible. One is to investigate the sensitivity with respect to the functional specification and the values of exogenous variables and parameters. This is a demanding task, and will not be attempted here. Another possibility is to formulate a model with empirically plausible estimates of the parameters. Such a model will be discussed in the next chapter.

Appendix A: Computer program for the model with exogenous tax rates

```
C This is a COMPAK V2.1 input file.
C Programmer: Morten Berg, Centre for Applied Research.

C W1 and W2 are initial endowments; of which the consumer
C demands X1 and X2, producers demand F1 and F2, and the
C state demands G1 and G2. Private firms and the state face
C factor prices PX1 and PX2, the consumer faces QX1 and QX2.
C The consumer's commodity demand is Y1 and Y2 at prices QY1
C and QY2. Producers' commodity prices are PY1 and PY2. T1,
C T2, S1 and S2 are the factor and commodity tax rates, and
C Z is the level of public production. TRAN is a lump-sum
C transfer from the state to the consumer, and LUMP is a
C lump-sum tax.

C Commodity 1 is the numéraire, so that PY1=QY1=1 and S1=0.

MODEL
      FCTR1,XSW1,QX1,2
      FCTR2,XSW2,QX2,2
      DFCT2,PY2,XSC2,1
      BUDG,TRAN,LUMP,1

PARAMETERS
C ALF is the elasticity of substitution in the utility
C function, SIG1, SIG2 and GAM are the elasticities of
C substitution in firm 1, firm 2, and public production.
C EPS is the public scale elasticity.

      ALF=1.1
      SIG1=0.5
      SIG2=0.5
      GAM=0.5
      EPS=1.0

C The level of initial endowments is an origin for the model:

      W1=10
      W2=10

C Choice of numéraire involves one producer price and
C its slack variable:

      PY1=1.0
      XSC1=0

C The level of public production is 1 at the outset,
C making Z a dummy parameter:

      Z=1

C The tax system is exogenously given by the level of
C each tax rate; or half endogenously by some tax formula:
```

```
      T1=0.1041
      T2=0.1041
      S1=0
      S2=0.1041

DEFINITIONS
****************
C The consumer
****************
C Consumer's commodity prices:

      QY1=PY1
      QY2=PY2+S2

C Value of initial endowments and lump-sum transfers:

      GRIN=(QX1*W1)+(QX2*W2)+TRAN-LUMP

C Consumer's demand:

      A=1-ALF
      PRC=(QY2^ALF)*((QX1^A)+(QX2^A)+(QY1^A))

      Y2=GRIN/((PRC*(Z^A))+QY2)
      Y1=((QY2/QY1)^ALF)*Y2*(Z^A)
      X2=((QY2/QX2)^ALF)*Y2*(Z^A)
      X1=((QY2/QX1)^ALF)*Y2*(Z^A)

C Consumer's budget shares:

      PHI1=(QX1*X1)/GRIN
      PHI2=(QX2*X2)/GRIN
      ZET1=(QX1*W1)/GRIN
      ZET2=(QX2*W2)/GRIN
      THE1=(QY1*Y1)/GRIN
      THE2=(QY2*Y2)/GRIN

C Consumer's demand derivatives:

      DX1X1=(((ALF*(PHI1-1))-PHI1)*(X1/QX1))+(PHI1*(W1/QX1))
      DX1X2=((-A)*PHI1*((QX1/QX2)^ALF)*(X1/QX1))+(PHI1*(W2/QX1))
      DX1Y1=(-A)*PHI1*((QX1/QY1)^ALF)*(X1/QX1)
      DX1Y2=(-A)*PHI1*((QX1/QY2)^ALF)*(X1/QX1)*(Z^(-A))
      DX2X1=((-A)*PHI2*((QX2/QX1)^ALF)*(X2/QX2))+(PHI2*(W1/QX2))
      DX2X2=(((ALF*(PHI2-1))-PHI2)*(X2/QX2))+(PHI2*(W2/QX2))
      DX2Y1=(-A)*PHI2*((QX2/QY1)^ALF)*(X2/QX2)
      DX2Y2=(-A)*PHI2*((QX2/QY2)^ALF)*(X2/QX2)*(Z^(-A))
      DY1X1=(ZET1-(A*THE1*((QX1/QY1)^A)))*(Y1/QX1)
      DY1X2=(ZET2-(A*THE1*((QX2/QY1)^A)))*(Y1/QX2)
      DY1Y1=((ALF*(THE1-1))-THE1)*(Y1/QY1)
      DY1Y2=(-A)*THE1*((QY1/QY2)^ALF)*(Y1/QY1)*(Z^(-A))
      DY2X1=(ZET1-(A*THE2*((QX1/QY2)^A)*(Z^A)))*(Y2/QX1)
      DY2X2=(ZET2-(A*THE2*((QX2/QY2)^A)*(Z^A)))*(Y2/QX2)
      DY2Y1=(-A)*THE2*((QY2/QY1)^ALF)*(Y2/QY2)*(Z^A)
      DY2Y2=((ALF*(THE2-1))-THE2)*(Y2/QY2)

C Consumer's factor SUPPLY elasticities:
```

```
      ESX1X1=DX1X1*(QX1/(X1-W1))
      ESX1X2=DX1X2*(QX2/(X1-W1))
      ESX1Y1=DX1Y1*(QY1/(X1-W1))
      ESX1Y2=DX1Y2*(QY2/(X1-W1))
      ESX2X1=DX2X1*(QX1/(X2-W2))
      ESX2X2=DX2X2*(QX2/(X2-W2))
      ESX2Y1=DX2Y1*(QY1/(X2-W2))
      ESX2Y2=DX2Y2*(QY2/(X2-W2))

C Consumer's commodity demand elasticities:

      EDY1X1=DY1X1*QX1/Y1
      EDY1X2=DY1X2*QX2/Y1
      EDY1Y1=DY1Y1*QY1/Y1
      EDY1Y2=DY1Y2*QY2/Y1
      EDY2X1=DY2X1*QX1/Y2
      EDY2X2=DY2X2*QX2/Y2
      EDY2Y1=DY2Y1*QY1/Y2
      EDY2Y2=DY2Y2*QY2/Y2

*****************
C The producers
*****************
C Producers' factor prices:

      PX1=QX1+T1
      PX2=QX2+T2

C The production functions are single-level CES, with
C constant returns to scale. Unit cost functions:

      C1=((PX1^(1-SIG1))+(PX2^(1-SIG1)))^(1/(1-SIG1))
      C2=((PX1^(1-SIG2))+(PX2^(1-SIG2)))^(1/(1-SIG2))

C Input coefficient functions:

      A11=(C1/PX1)^SIG1
      A12=(C1/PX2)^SIG1
      A21=(C2/PX1)^SIG2
      A22=(C2/PX2)^SIG2

C Factor demand:

      F11=A11*Y1
      F12=A12*Y1
      F21=A21*Y2
      F22=A22*Y2

C Factors' value shares:

      KSIF11=(PX1*A11)/C1
      KSIF12=(PX2*A12)/C1
      KSIF21=(PX1*A21)/C2
      KSIF22=(PX2*A22)/C2

C Factor intensities:
```

```
      IOTA1=F11/F12
      IOTA2=F21/F22

*************
C The state
*************
C The public production function is single-level CES.
C Contingent factor demand functions:

      G=1-GAM

      G1=((1+((PX2/PX1)^G))^(GAM/G))*(Z^(1/EPS))
      G2=((1+((PX1/PX2)^G))^(GAM/G))*(Z^(1/EPS))

C Tax revenues and public expenditure:

      TREV=(S1*Y1)+(S2*Y2)+(T1*(W1-X1))+(T2*(W2-X2))
      PEXP=(PX1*G1)+(PX2*G2)

***********************
C General equilibrium
***********************

      FCTR1=W1-X1-F11-F21-G1-XSW1
      FCTR2=W2-X2-F12-F22-G2-XSW2
      DFCT1=C1-PY1+XSC1
      DFCT2=C2-PY2+XSC2
      BUDG=TREV+LUMP-PEXP-TRAN

INITIAL GUESS
      QX1=0.1459
      QX2=0.1459
      PY2=1.
      TRAN=0.

RUN DATA
      TMAX=147
      DT=1
      PRINT=1
      LOGFIL=0
      BDR=0

DYNAMICS
      T1=T1*1.01

OUTPUT
      W1,W1
      ALF,SIG1,SIG2,GAM,EPS
      T1,T2,S1,S2,LUMP
      PX1,QX1,PX2,QX2
      PY1,QY1,PY2,QY2
      X1,X2,Y1,Y2
      PHI1,PHI2,ZET1,ZET2,THE1,THE2
      ESX1X1,ESX1X2,ESX1Y1,ESX1Y2
      ESX2X1,ESX2X2,ESX2Y1,ESX2Y2
      EDY1X1,EDY1X2,EDY1Y1,EDY1Y2
      EDY2X1,EDY2X2,EDY2Y1,EDY2Y2
      C1,C2,F11,F12,F21,F22
```

```
     KSIF11,KSIF12,KSIF21,KSIF22,IOTA1,IOTA2
     Z,G1,G2,TREV,PEXP,TRAN

END
```

Appendix B: Computer program for the model with optimal tax rates

```
C This is a COMPAK V2.1 input file.
C Programmer: Morten Berg, Centre for Applied Research.

C W1 and W2 are initial endowments; of which the consumer
C demands X1 and X2, producers demand F1 and F2, and the
C state demands G1 and G2. Private firms and the state face
C factor prices PX1 and PX2, the consumer faces QX1 and QX2.
C The consumer's commodity demand is Y1 and Y2 at prices QY1
C and QY2. Producers' commodity prices are PY1 and PY2. T1,
C T2, S1 and S2 are the factor and commodity tax rates, and
C Z is the level of public production. TRAN is a lump-sum
C transfer from the state to the consumer.

C Commodity 1 is the numéraire, so that PY1=QY1=1 and S1=0.

MODEL
     EQU1,PX1,REST1,1
     EQU2,PX2,REST2,1
     EQU3,PY2,REST3,1
     EQU4,QX1,REST4,1
     EQU5,QX2,REST5,1
     EQU6,QY2,REST6,1

PARAMETERS
C ALF is the elasticity of substitution in the utility
C function, SIG1, SIG2 and GAM are the elasticities of
C substitution in firm 1, firm 2, and public production.
C EPS is the public scale elasticity.

     ALF=1.1
     SIG1=0.5
     SIG2=0.5
     GAM=0.5
     EPS=1

C The level of initial endowments is an origin for the model:

     W1=10
     W2=10

C Choice of numéraire involves the producer and consumer price
C of one good:

     PY1=1
     QY1=1

C The level of public production is 1 at the outset,
```

```
C making Z a dummy parameter:

      Z=1

DEFINITIONS
****************
C The consumer
****************
C Value of initial endowments and lump-sum transfers:

      GRIN=(QX1*W1)+(QX2*W2)

C Consumer's demand:

      A=1-ALF
      PRC=(QY2^ALF)*((QX1^A)+(QX2^A)+(QY1^A))

      Y2=GRIN/((PRC*(Z^A))+QY2)
      Y1=((QY2/QY1)^ALF)*Y2*(Z^A)
      X2=((QY2/QX2)^ALF)*Y2*(Z^A)
      X1=((QY2/QX1)^ALF)*Y2*(Z^A)

C Marginal utility of income (times 10^-6):

      RHO=(-A)/ALF
      L=((X1^RHO)+(X2^RHO)+(Y1^RHO)+((Y2*Z)^RHO))^(1./(-A))
      LAM=(L*(X1^(1./(-ALF))))/QX1
      LM6=LAM/(10^6)

C Consumer's budget shares:

      PHI1=(QX1*X1)/GRIN
      PHI2=(QX2*X2)/GRIN
      ZET1=(QX1*W1)/GRIN
      ZET2=(QX2*W2)/GRIN
      THE1=(QY1*Y1)/GRIN
      THE2=(QY2*Y2)/GRIN

C Consumer's demand derivatives:

      DX1X1=(((ALF*(PHI1-1))-PHI1)*(X1/QX1))+(PHI1*(W1/QX1))
      DX1X2=((-A)*PHI1*((QX1/QX2)^ALF)*(X1/QX1))+(PHI1*(W2/QX1))
      DX1Y1=(-A)*PHI1*((QX1/QY1)^ALF)*(X1/QX1)
      DX1Y2=(-A)*PHI1*((QX1/QY2)^ALF)*(X1/QX1)*(Z^(-A))
      DX2X1=((-A)*PHI2*((QX2/QX1)^ALF)*(X2/QX2))+(PHI2*(W1/QX2))
      DX2X2=(((ALF*(PHI2-1))-PHI2)*(X2/QX2))+(PHI2*(W2/QX2))
      DX2Y1=(-A)*PHI2*((QX2/QY1)^ALF)*(X2/QX2)
      DX2Y2=(-A)*PHI2*((QX2/QY2)^ALF)*(X2/QX2)*(Z^(-A))
      DY1X1=(ZET1-(A*THE1*((QX1/QY1)^A)))*(Y1/QX1)
      DY1X2=(ZET2-(A*THE1*((QX2/QY1)^A)))*(Y1/QX2)
      DY1Y1=((ALF*(THE1-1))-THE1)*(Y1/QY1)
      DY1Y2=(-A)*THE1*((QY1/QY2)^ALF)*(Y1/QY1)*(Z^(-A))
      DY2X1=(ZET1-(A*THE2*((QX1/QY2)^A)*(Z^A)))*(Y2/QX1)
      DY2X2=(ZET2-(A*THE2*((QX2/QY2)^A)*(Z^A)))*(Y2/QX2)
      DY2Y1=(-A)*THE2*((QY2/QY1)^ALF)*(Y2/QY2)*(Z^A)
      DY2Y2=((ALF*(THE2-1))-THE2)*(Y2/QY2)
```

```
****************
C The producers
****************
C The production functions are single-level CES, with
C constant returns to scale. Unit cost functions:

      C1=((PX1^(1-SIG1))+(PX2^(1-SIG1)))^(1/(1-SIG1))
      C2=((PX1^(1-SIG2))+(PX2^(1-SIG2)))^(1/(1-SIG2))

C Input coefficient functions:

      A11=(C1/PX1)^SIG1
      A12=(C1/PX2)^SIG1
      A21=(C2/PX1)^SIG2
      A22=(C2/PX2)^SIG2

C Factor demand:

      F11=A11*Y1
      F12=A12*Y1
      F21=A21*Y2
      F22=A22*Y2

*************
C The state
*************
C The public production function is single-level CES.
C Conditional factor demand functions:

      G=1-GAM

      G1=((1+((PX2/PX1)^G))^(GAM/G))*(Z^(1/EPS))
      G2=((1+((PX1/PX2)^G))^(GAM/G))*(Z^(1/EPS))

C Optimal tax rates:

      T1=PX1-QX1
      T2=PX2-QX2
      S1=QY1-PY1
      S2=QY2-PY2

C Tax revenues and public expenditure:

      TREV=(S1*Y1)+(S2*Y2)+(T1*(W1-X1))+(T2*(W2-X2))
      PEXP=(PX1*G1)+(PX2*G2)

C Optimal tax formula coefficients and marginal cost of
C public funds (times 10^-6):

      SUMX1=(PX1*DX1X1)+(PX2*DX2X1)+(PY1*DY1X1)+(PY2*DY2X1)
      SUMX2=(PX1*DX1X2)+(PX2*DX2X2)+(PY1*DY1X2)+(PY2*DY2X2)
      SUMY2=(PX1*DX1Y2)+(PX2*DX2Y2)+(PY1*DY1Y2)+(PY2*DY2Y2)
      MU=(LAM*(W1-X1))/SUMX1
      MU6=MU/(10^6)
      NU=(LAM-MU)/MU
```

```
********************************************
C General equilibrium with optimal taxation
********************************************

      EQU1=X1-W1+F11+F21+G1+REST1
      EQU2=X2-W2+F12+F22+G2+REST2
      EQU3=C2-PY2+REST3
      EQU4=PEXP-TREV+TRAN
      EQU5=REST5-X2-((MU/LAM)*SUMX2)+W2
      EQU6=REST6-Y2-((MU/LAM)*SUMY2)

      DFCT1=C1-PY1+XSC1

INITIAL GUESS
      PX1=0.25
      PX2=0.25
      PY2=1
      QX1=0.1391
      QX2=0.1391
      QY2=1

RUN DATA
      TMAX=63
      DT=1
      PRINT=1
      LOGFIL=0
      BDR=0

DYNAMICS
      Z=Z*1.01

OUTPUT
C     W1,W2
C     ALF,SIG1,SIG2,GAM,EPS
     T1,T2,S2,TREV
     LM6,MU6,NU,PEXP
     PX1,QX1,PX2,QX2
     PY1,QY1,PY2,QY2
     X1,X2,Y1,Y2
     F11,F12,F21,F22,G1,G2
C     DX1X1,DX1X2,DX1Y1,DX1Y2
C     DX2X1,DX2X2,DX2Y1,DX2Y2
C     DY1X1,DY1X2,DY1Y1,DY1Y2
C     DY2X1,DY2X2,DY2Y1,DY2Y2

END
```

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Chapter 5
April 1990

GENERAL EQUILIBRIUM WITH OPTIMAL TAXATION: A COMPLEMENTARITY FORMAT AND A NORWEGIAN MODEL

"The world is all that is the case."

Ludwig Wittgenstein

GENERAL EQUILIBRIUM WITH OPTIMAL TAXATION: A COMPLEMENTARITY FORMAT AND A NORWEGIAN MODEL

Introduction

In the last sections of the previous chapter, the incidence of optimal tax rates was illustrated in a simplified general equilibrium model. In the present chapter, an empirical optimum taxation model will be developed and described in detail. It is called GEMPS, General Equilibrium Model with a Public Sector. No references have been found in the literature to models with exactly the same tax structure as GEMPS, even though much of the work in this field is especially aimed at the analysis of tax systems. (Surveys are given by Shoven and Whalley (1984), Bergman (1985), Whalley (1987) and Pereira and Shoven (1988).) In existing empirical models tax rates are often given exogenously, or perhaps by an extremely simplified tax rule (e.g. uniform income or commodity taxes), and not by necessary conditions for maximum social welfare. One reason for this is of course that these models are built for practical purposes and thus attempt to incorporate real-world tax systems which are rarely optimal.

There are a few articles on the computation of optimal taxes. Atkinson and Stiglitz (1972) discuss a model with labour as the only production factor and constant returns to scale in production, where all producer prices are fixed. They are then able to analyse optimal taxes which only depend on the properties of a representative consumer's utility function. They also present two numerical illustrations based on empirical estimates of the parameters in a direct addilog utility function as well as in the linear expenditure system (LES). Similarly, in a many-consumer model, Deaton (1977) uses utility functions associated with the linear expenditure system. He assumes that the supply of labour is exogenously given, that there are constant returns to scale in production, and that all producer prices are fixed. Finally he presents illustrative calculations of optimal tax rates in the United Kingdom, based on empirical estimates of the LES parameters.

Apart from the practical considerations mentioned above, there is also a more technical reason for the lack of numerical optimum taxation models. To deduce the ordinary first-order conditions for optimal tax rates one has to choose a numéraire and determine one of the tax rates exogenously. Then the resulting formulae are not directly applicable if prices are normalized to the simplex $\{p \in \mathbb{R}^n_+ \mid \Sigma_i p_i = 1\}$, where n is the number of goods having a market price. To solve numerical models, one often uses some kind of fixed point algorithm where the price simplex is searched for a set of equilibrium prices. Thus, the optimum taxation formulae are not easily implemented in models with fixed point solution algorithms. This problem is discussed by Harris and MacKinnon (1979) and Heady and Mitra (1980), who present algorithms for the computation of general equilibrium with optimal tax rates. (Their algorithms are not implemented in GEMPS.) Harris and MacKinnon illustrate their results with numerical examples very much like those of Atkinson and Stiglitz (1972) and Deaton (1977). One of their main conclusions (Harris and MacKinnon, 1979, pp. 211-212) is that "optimal tax rates can be extremely sensitive to the specification of the model used to derive them".

Several numerical models with optimal taxation are described by Heady and Mitra (1980, 1982, 1986). In their 1980 article, they present an algorithm to solve a general equilibrium model with optimal taxation, illustrating their results by two numerical examples. Only the first example is based on empirical data, establishing the parameters in a Stone-Geary utility function. There are eight consumer goods, and labour is the only input. Thus, with constant returns to scale in private production, relative producer prices are fixed.

The computable general equilibrium model developed by Heady and Mitra (1982) is quite similar to GEMPS from a theoretical viewpoint, although empirically it is much simpler. Again, the linear expenditure system is used, but private firms use both labour, capital and intermediate goods as inputs, so that producer prices are not necessarily constant when public revenue requirements change. Two numerical examples are presented. The first is based on a mixture of data for Brazil and the United States, whereas the second "draws on illustrative figures from Indian data" (Heady and Mitra, 1982, p. 18). In the first example there are three consumption goods, capital and labour; in the second there are two consumption goods and labour in each of the two sectors Urban and Rural.

There are many articles investigating the marginal cost of public funds within the framework of a numerical general equilibrium model, e.g. Stuart (1984), Ballard, Shoven and Whalley (1985), Hansson and Stuart (1985), Clarete and Whalley (1987) and Ballard (1988). The approach in these works is typically to model the real-world tax system of a particular benchmark year, and change one or more of the exogenous tax rates. The (marginal) excess burden of each tax rate may then be computed as an equivalent variation, or there may be more direct estimates of the reductions in different consumers' utility levels. In principle it would be possible to find optimal tax rates in this way, by a trial and failure process leading to equalization of the marginal excess burdens of all tax rates.

A recent paper by Vennemo (1990) may perhaps be seen as an extension of these works. With a primal approach (i.e. with quantities and not prices as the control variables) Vennemo analyses optimal tariffs in a general equilibrium model where the parameters are estimated from Norwegian data. The supply of labour and capital is given exogenously, and there are 31 production sectors. Several numerical experiments are carried out, notably a comparison of different tariff structures. Lump-sum taxes are endogenous in the model.

GEMPS differs from the models above in that optimal consumer and producer prices are computed endogenously as part of the solution of a complex general equilibrium equation system. Thus, there will be only one number for the marginal cost of public funds, equal for all tax rates; and it is not found by a numerical experiment, but is given by the value of one of the first-order conditions for optimal tax rates in general equilibrium. The first section of this chapter describes the complete set of equations constituting the general equilibrium with optimal taxation. Then in the next section the main structure of GEMPS is sketched, i.e. its sectoral division, the social welfare function, the export demand functions, and the production functions for intermediate and final goods. After this the basic general equilibrium with optimal tax rates is computed. Finally, three comparative, static analyses are carried out, the results of which are presented in diagrammatic form.

The non-linear complementarity problem of optimal taxation

The basic theoretical structure of the empirical model presented here is: There is one consumer, who maximizes a direct utility function u. His consumption vector is $[x,z] \in \mathbb{R}^{N+K}$, where x is his demand for private goods and z is his consumption of publicly produced goods. The consumer's budget equation is $q[x - w] = 0$, where q is the vector of consumer prices taken as given by the consumer, and w is the vector of initial endowments. q is endogenous in the model, whereas w is exogenous.

There are J private firms, with at least one firm producing each traded commodity. Private production exhibits constant returns to scale, and the production levels are therefore determined from the demand side provided that profits are non-negative. Each producer maximizes profits, taking the vector of producer prices, p, as given. Since the production functions are homothetic, private minimum cost functions are separable in prices and the production level. Indeed, if π_j denotes profits per unit produced, total profits in firm j may be written as

$$\pi_j y_j \equiv \left[\sum_n p_n a_n^j(p)\right] y_j \ , \qquad\qquad j = 1,\ldots,J$$

where $a_n^j(p)$ is the input-output coefficient of good n in firm j and y_j is the firm's activity level. If good n is an input, the input coefficient is negative: $a_n^j < 0$. If n is an output, on the other hand, the output coefficient is positive: $a_n^j > 0$. Both output and input coefficients may in general depend on prices, but in the empirical model below, only the latter do. In order to simplify notation in the first part of this chapter it is assumed that there is only one producer of each commodity, i.e. firm j produces commodity j. (In the numerical model some of the commodities are produced by two or more firms at the same time.) Commodity markets are numbered from 1 to J and factor markets from J+1 to N.

The vector of public production levels, $z \in \mathbb{R}^K$, is exogenously given, for instance by some political decision process. Interpreting the state as a producer, it faces producer prices in private markets, and it minimizes its production costs taking these prices as given. The public factor demand vector g is thus determined by p and z, and all elements of g are non-positive: $g(p,z) \in \mathbb{R}^N_-$. The tax system is defined by $q_n \equiv p_n + t_n$, n = 1, ..., N. Each t_n may be negative, positive, or zero. If good n is an input, $t_n < 0$ means that it is taxed; if good n is an output, $t_n < 0$ means that it is subsidized. The public budget equation is:

$$\sum_n t_n(x_n - w_n) + \sum_n p_n g_n = 0$$

The non-linear complementarity problem (NLCP) has five sets of equations. Even though the system is simultaneous and non-linear, it is convenient to think of each endogenous variable as assigned to a particular equation. First, one of the goods, e.g. commodity 1, is chosen as a numéraire; and the other commodity prices are assigned to the second set of equations:

(1) $\quad p_1 = 1$

(2) $\quad -\pi_j + \psi_j = 0 \qquad\qquad \psi_j \cdot p_j = 0, \quad j = 2,\dots,J$

If $\pi_j < 0$, i.e. if there is a deficit in firm j, then the first part of (2) implies that $\psi_j < 0$. This is impossible since ψ_j is constrained to be a non-negative variable. If $\pi_j > 0$, the second part of (2) ensures that $p_j = 0$. Thus, a firm with a positive commodity price earns zero profits.

In the third set of equations, excess demand is assumed to be zero in all commodity markets. The activity levels are assigned to this set:

$$(3) \qquad x_n - w_n - \sum_j a_n^j y_j - g_n + \omega_n = 0 \qquad\qquad \omega_n \cdot y_n = 0, \quad n = 1,\ldots,J$$

The interpretation of the variable $\omega_n > 0$ in condition (3) is: If there is excess supply of commodity n, the activity level of firm n should be zero. If, on the other hand, $y_n > 0$, then $\omega_n = 0$ and demand equals supply.

In the fourth set of equations excess demand is assumed to be zero in all factor markets, and the assigned variables are the factor prices:

$$(4) \qquad x_n - w_n - \sum_j a_n^j y_j - g_n + \omega_n = 0 \qquad\qquad \omega_n \cdot p_n = 0, \quad n = J+1,\ldots,N$$

The interpretation of the variable ω_n now is: If there is excess supply of factor n, its market price should be zero. If $p_n > 0$, then $\omega_n = 0$ and (4) implies market equilibrium.

To close the system, producer prices must be related to consumer prices. This will be done by introducing the ordinary formulae for optimal tax rates (Diamond and Mirrlees, 1971). It should be observed that two assumptions in the model implicitly restrict the optimality of the tax rates. First, it is an <u>assumption</u> that the state minimizes production costs and that it faces producer prices in factor markets; and there is no claim that this is optimal. Second, a (public or private) producer faces producer prices in <u>all</u> markets. Thus if both the consumer and some firm demand commodity n, the consumer pays $p_n + t_n$ per unit whereas the firm pays p_n. From a computational viewpoint this restrictive assumption could easily be relaxed so that all buyers pay the same price irrespective of whether they are consumers or producers. However, Diamond and Mirrlees' formulae for optimum taxation do not seem to be immediately applicable to the more general case where tax rates vary from one individual to another.

Diamond and Mirrlees may be said to impose a social structure on the economy, where producers are in one "tax sector" and consumers are in another. But there are many other logically possible social structures. One could for instance place all sellers in one sector and all buyers in another, or consumers could be grouped according to a progressive income taxation scheme. There is of course one truly optimal form of taxation, viz., lump-sum taxes which do not distort prices. If it is assumed that lump-sum taxation is impossible, then a social structure must be chosen explicitly or implicitly in order to construct an optimum taxation model. Economists often believe, e.g., that initial endowments cannot be (and that market transactions can be) taxed directly. In comparison to lump-sum taxation, the optimality of any tax system is in general restricted by the socio-economic structure implied by the model.

Let u^* be the consumer's indirect utility function, and choose good 1 as a numéraire so that $q_1 = p_1 = 1$ and $t_1 = 0$. The optimum taxation problem is to find consumer prices q_m, $m = 2,\ldots,N$, which maximize u^* subject to the public budget restriction and given the existence of producer prices that sustain the equilibrium. The Lagrangian for this problem is:

$$\mathcal{L} = u^* + \mu\left[\sum_n t_n(x_n - w_n) - T\right]$$

where T is the public revenue requirement. From the first-order conditions

$$\sum_n (q_n - p_n)\frac{\partial x_n}{\partial q_m} = \frac{\lambda - \mu}{\mu}(x_m - w_m) \qquad m = 2,\ldots,N$$

where λ is the consumer's marginal utility of income and μ is the marginal cost of public funds; and using the fact that

$$\sum_{n=1}^{N} q_n \frac{\partial x_n}{\partial q_m} + (x_m - w_m) = 0$$

from the budget restriction, we have:

$$(^*) \qquad x_m - w_m + \frac{\mu}{\lambda} \sum_{n=1}^{N} p_n \frac{\partial x_n}{\partial q_m} = 0 \qquad\qquad m = 2,\ldots,N$$

Here the magnitude of the coefficient μ/λ depends on the public revenue requirement. In the NLCP, optimal consumer prices may thus be assigned to the following N equations, where μ/λ is found by using $(^*)$ with $m = 2$:

$$(5) \left\{ \begin{array}{lll} q_1 = p_1 & & \\ \sum_n p_n g_n - \sum_n (q_n - p_n)(x_n - w_n) + \zeta_2 = 0 & & q_2 \cdot \zeta_2 = 0 \\ w_m - x_m - \frac{\mu}{\lambda} \sum_{n=1}^{N} p_n \frac{\partial x_n}{\partial q_m} + \zeta_m = 0 & & q_m \cdot \zeta_m = 0 \end{array} \right.$$

$$m = 3,\ldots,N$$

The optimal tax rates will be given implicitly by $t_m = q_m - p_m$ for each m, and the dual variables ζ_m will be zero in an interior solution. The system of equations (1)-(5) is the nonlinear complementarity problem of general equilibrium with optimal taxation.

GEMPS: A numerical model with optimal taxation

The empirical model developed in this chapter is called GEMPS (General Equilibrium Model with a Public Sector). GEMPS is a modification of another empirical model of the Norwegian economy: MISMOD (Mathiesen, 1986). MISMOD is a static, one-period Walrasian model with 3 geographic regions. It has 12 different production activities which produce 12 commodities and foreign currency under constant returns to scale. There are three types of labour, and each firm's capital stock is given exogenously. (Capital stocks result from depreciation and investment in some previous period, and do not depend on investment demand in the period analysed.) MISMOD has four different households; households with children under 10, households where individuals are between 16 and 45, households where individuals are between 45 and 70, and households where individuals are more than 70 years old. The households are the consumers of the model, and they demand 5 consumption goods in addition to bonds and leisure. A household's income is the sum of wages and a share of the total capital income in the economy. The five consumption goods are aggregates of the 12 produced commodities and imports. In MISMOD the public sector is modelled as a household consuming services which are produced by a public firm. The public firm maximizes profits, and the public household's income is simply the net revenue from taxation of private producers and consumers. In addition to domestic demand, each producer faces an export demand curve.

Just a few of the parameters in MISMOD are econometric estimates. Instead, the Norwegian economy in 1984 is used as a benchmark, and it is assumed that the Norwegian economy in 1984 can be described by a Walras model. Given estimates of the elasticities of substitution in production and consumption, other coefficients are then defined so that all sellers' prices are equal to 1. The method of benchmarking can be criticized from a methodological point of view. First and foremost, there is no test of the hypothesis that a Walras model is a valid description of the economy. Second, the entire system of equations should be estimated on time-series data instead of being based on one single year, since aspects of the real economy which are not captured by an ordinary general equilibrium model may appear as random fluctuations. Thus, in using MISMOD, one implicitly assumes that 1984 was in some sense a normal year.

GEMPS has the same production structure as MISMOD, but only one region. The sectors aggregating the 12 produced commodities into 5 consumption goods in MISMOD are interpreted as firms in GEMPS, so that GEMPS has 5 main products. Furthermore, GEMPS has only one kind of labour input, only one private household, and no public household (i.e. no public utility function). In what follows, the formal equation system of GEMPS is described. The motivation for the model is discussed along with some of its weaknesses after the formal presentation.

The utility function.

Since it may be impossible to find explicit demand functions from a nested CES utility function with given consumption levels for public goods, the consumer's utility function is assumed to be separable in the consumption of private and public goods. Then the maximum attainable utility, but not the demand for private goods, will depend on the public production level. Let the consumer's utility level U be a nested CES function of consumption X, leisure L, wealth B (interpreted as bonds), and public goods Z:

$$U = \left[\alpha_1 X^{\frac{\rho-1}{\rho}} + \alpha_2 L^{\frac{\rho-1}{\rho}} + \alpha_3 B^{\frac{\rho-1}{\rho}} + \alpha_4 Z^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

where the α's are constants and ρ is the consumer's top-level substitution parameter. X is a CES function of five goods; X_1 (Food and beverages), X_2 (Clothes), X_3 (Housing), X_4 (Transport) and X_5 (Services):

$$X = \left[\sum_{n=1}^{5} \alpha_{1n} X_n^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

where η is the substitution parameter at the second level. The consumer's preference structure can be illustrated as follows:

Utility
(CES)

Consumption (CES) | Leisure | Bonds | Public goods

Food | Clothes | Housing | Transport | Services

The utility maximization problem is solved in two steps. First, utility is maximized with respect to consumption, leisure and bonds, given the provision of public goods, Z, initial endowments of leisure and bonds, L_0 and B_0, a consumer price index QX and market prices QL and QB. This yields the Lagrangian function

$$\begin{aligned}\mathcal{L} = & \left[\alpha_1 X^{\frac{\rho-1}{\rho}} + \alpha_2 L^{\frac{\rho-1}{\rho}} + \alpha_3 B^{\frac{\rho-1}{\rho}} + \alpha_4 Z^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \\ & - \lambda\left[QX \cdot X + QL(L - L_0) + QB(B - B_0)\right]\end{aligned}$$

and demand functions

$$X = \frac{QL \cdot L_0 + QB \cdot B_0}{(QX/\alpha_1)^{\rho}(\alpha_1^{\rho}\, QX^{1-\rho} + \alpha_2^{\rho}\, QL^{1-\rho} + \alpha_3^{\rho}\, QB^{1-\rho})}$$

$$L = \frac{QL \cdot L_0 + QB \cdot B_0}{(QL/\alpha_2)^{\rho}(\alpha_1^{\rho}\, QX^{1-\rho} + \alpha_2^{\rho}\, QL^{1-\rho} + \alpha_3^{\rho}\, QB^{1-\rho})}$$

$$B = \frac{QL \cdot L_0 + QB \cdot B_0}{(QB/\alpha_3)^{\rho}(\alpha_1^{\rho}\, QX^{1-\rho} + \alpha_2^{\rho}\, QL^{1-\rho} + \alpha_3^{\rho}\, QB^{1-\rho})}$$

Next, consumption expenditure is minimized, given X and the consumer's market prices of the five products, Q_1, ..., Q_5:

$$\text{minimize} \sum_{n=1}^{5} Q_n X_n \qquad \text{s.t.} \quad \left[\sum_{n=1}^{5} \alpha_{1n} X_n^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} = X$$

yields the conditional demand functions

$$X_n = \alpha_{1n}^{\eta} Q_n^{-\eta} \left[\sum_{m=1}^{5} \alpha_{1m}^{\eta} Q_m^{1-\eta}\right]^{\frac{\eta}{1-\eta}} X, \qquad n = 1,\ldots,5$$

The solution is completed by defining the price index QX as the minimum unit cost of X:

$$QX \equiv \left[\sum_{n=1}^{5} \alpha_{1n}^{\eta} Q_n^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

The expressions for the partial derivatives of the consumer's demand functions resemble those in the previous chapter and will not be reported here. They are given in the computer program in Berg (1990).

The production functions.

There are two kinds of producers in GEMPS. First, the five main goods are nested Cobb-Douglas functions of 12 intermediate products as well as imports (the role of which will be discussed below):

$$Y_n = AA_n \prod_m YY_{nm}^{\beta_{nm}} \qquad n = 1,\ldots,5$$

$$YY_{nm} \equiv AB_{nm} \cdot YD_{nm}^{\beta_{nm}^1} \cdot VD_{nm}^{1-\beta_{nm}^1} \qquad m \in \{01,02,\ldots,12\}$$

where AA_n, AB_{nm} and the β's are constants, Y_n is producer n's supply; YD_{nm} is his demand for intermediate product m, and VD_{nm} denotes his currency demand. Let the market price of intermediate good m be PY_m, define the price index PYY_{nm} as the minimum unit cost of YY_{nm}, and let PV be the price of foreign currency. Then the conditional demand functions for inputs in sector n are:

$$YD_{nm} = (PV/(1-\beta^1_{nm}))^{1-\beta^1_{nm}} (PY_m/\beta^1_{nm})^{\beta^1_{nm}-1} (YY_{nm}/AB_{nm})$$
$$VD_n \equiv \sum_m VD_{nm} = \sum_m (PY_m/\beta^1_{nm})^{\beta^1_{nm}} (PV/(1-\beta^1_{nm}))^{-\beta^1_{nm}} (YY_{nm}/AB_{nm})$$

where the assumption of profit maximization (and hence cost minimization) implies that

$$YY_{nm} = (\beta_{nm}/PYY_{nm}) \prod_k (PYY_{nk}/\beta_{nk})^{\beta_{nk}} (Y_n/AA_n)$$

and

$$PYY_{nm} = (PY_m/\beta^1_{nm})^{\beta^1_{nm}} (PV/(1-\beta^1_{nm}))^{1-\beta^1_{nm}} (1/AB_{nm})$$

The list of main products and producers is:

1 Food and beverages
2 Clothes
3 Housing
4 Transport
5 Services

Second, the activity levels in the 12 intermediate production sectors, IA_{00}, ..., IA_{12}, are either CES or Cobb-Douglas functions of capital, labour and other products. The top-level elasticity of substitution in activity 00 (which corresponds to "Shipping" in MISMOD) is for instance 0.5, whereas activity 01 (Electricity) is formulated as a Cobb-Douglas function with the top-level elasticity of substitution equal to 1. Taking sector 00 as an example, the production function is given by

$$IA_{00} = \left[\delta_{001} II_{00}^{\frac{\sigma_{00}-1}{\sigma_{00}}} + \delta_{002} KI_{00}^{\frac{\sigma_{00}-1}{\sigma_{00}}} + \delta_{003} NI_{00}^{\frac{\sigma_{00}-1}{\sigma_{00}}}\right]^{\frac{\sigma_{00}}{\sigma_{00}-1}}$$

where σ_{00} is the top-level elasticity of substitution in sector 00, the δ's are constants, KI_{00} is the input of capital, NI_{00} the labour input, and II_{00} the input of other intermediate products as well as imported goods (foreign currency). Let the prices of these three inputs be PK, PN and PII_{00}, respectively, and write σ without subscripts for notational simplicity. Then minimization of total costs in sector 00 yields the following minimum unit cost and conditional input demand functions:

$$
\begin{aligned}
CI_{00} &= \left[\delta_{001}^{\sigma} PII_{00}{}^{1-\sigma} + \delta_{002}^{\sigma} PK^{1-\sigma} + \delta_{003}^{\sigma} PN^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\
II_{00} &= (CI_{00} \cdot \delta_{001}/PII_{00})^{\sigma}\ YI_{00} \\
KI_{00} &= (CI_{00} \cdot \delta_{002}/PK)^{\sigma}\ YI_{00} \\
NI_{00} &= (CI_{00} \cdot \delta_{003}/PN)^{\sigma}\ YI_{00}
\end{aligned}
$$

On the next level II_{00} is a Leontief function of three intermediate goods and currency, and the price PII_{00} is interpreted as the minimum unit cost in the activity producing II_{00}:

$$
\begin{aligned}
I_{00m} &= \delta_{00m}^{i}\ II_{00}, \quad m = 03,\ 07,\ 11 \\
PII_{00} &\equiv \sum_{m} \delta_{00m}^{i} \cdot PI_{00m}, \quad m = 03,\ 07,\ 11
\end{aligned}
$$

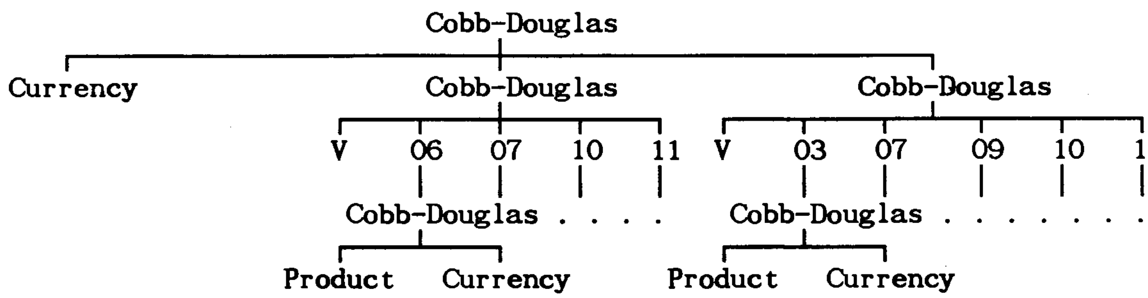
where I_{00m} is a Cobb-Douglas function of currency and product m and PI_{00m} is the minimum unit cost of I_{00m}. In other sectors than Shipping, there may be a Cobb-Douglas function instead of a Leontief function. All CES and Cobb-Douglas functions in the model are linearly homogeneous. Each intermediate sector produces one or more of the 12 products, with fixed output coefficients. Their cost structures are sketched in the Appendix. The intermediate products and producers are:

V Currency	00 Shipping
01 Electricity production	01 Electricity
02 Electricity distribution	
03 Petroleum	03 Petroleum
04 Primary goods	04 Primary industries
05 Food	05 Food
06 Textiles	06 Textiles
07 Ships and rigs	07 Shipbuilding
08 Chemicals	08 Chemicals
09 Metals	09 Metals
10 Manufactured goods	10 Manufacture
11 Private services	11 Private services
12 Public services	12 Public services

The formulation of imports in the 17 production sectors in GEMPS may seem odd, since the good "Currency" appears in numerous aggregates in the production functions. First, all kinds of imported goods are regarded as currency because the model only incorporates the trade balance effects and not the actual commodity flows from (and to) the world market. Second, when currency appears in some aggregate on a low level in the nested cost structure, it should be interpreted as the foreign counterpart of the domestic good in the same aggregate. This formulation has been chosen so as to facilitate the empirical estimation of coefficients and substitution parameters in the production functions.

Saving, investment and depreciation.

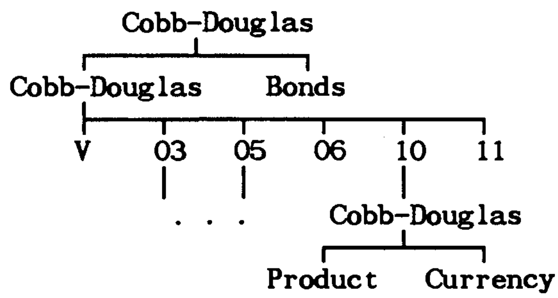
The formulation of saving and investment decisions is extremely simple in GEMPS. The consumer, foreigners and the state demand bonds, which are supplied by a producer. In MISMOD, this production sector uses currency, oil investment goods and other investment goods. The latter two are outputs from other sectors which in turn demand the twelve intermediate products in the model as inputs. In GEMPS, all these activities are formulated as a single firm, with the following structure:



GEMPS is a static, one-period model, so the production of investment goods does not increase capital stocks in the economy. The consumer, foreigners and the authorities presumably have some idea about future consumption, since they demand bonds, but such expectations are not explicit in GEMPS. Capital is not an argument in the consumer's utility function, and the supply of capital to each sector is completely inelastic. Depreciation is exogenously given, and is deducted from capital income. It results in an immediate demand for bonds which may be interpreted as investment for replacement. The state owns all the capital in sectors 01 Electricity and 12 Public services, and 29.5% of the capital stock in sector 03 Petroleum. Foreigners own 28% of the capital in sector 03 Petroleum, and the consumer owns all other capital stocks.

Exports and foreigners' demand.

There are 13 foreigners in GEMPS. Twelve of the export demand functions are each directed towards one intermediate good, and are simple CES or Cobb-Douglas functions of this good and its foreign counterpart. Domestic and foreign goods are thus not perfect substitutes. It is assumed that all foreign currency world market prices are constant and equal to 1, so that the foreign demand for an intermediate good is determined by its <u>domestic</u> (i.e. Norwegian) price and the domestic price of foreign currency. The 13th function corresponds to foreigners' demand in Norway in MISMOD, and has the following structure:



The state.

The most important part of the state's economic activity, viz., public production, has already been described as intermediate sector 12. It has constant returns to scale and minimizes costs. Some of its production is sold to private firms at unit cost, some is distributed to the consumer. Any deficit is financed by optimal taxation. The (ad valorem) tax rate on domestic capital income is an exogenous constant TC. Thus, the state keeps a fraction TC of returns to capital which are not paid to foreigners and distributes a fraction 1 - TC to the consumer as lump-sum transfers. As in the private sector, public saving decisions result in public demand for bonds. Both the demand for bonds and the demand for public services are exogenously given.

Discussion of the model

The motivation behind the choice of MISMOD as a point of departure for the design of GEMPS is that MISMOD seems to be the only genuine Walrasian, numerical model of the Norwegian economy. Its purpose is to describe and analyse the Norwegian system of taxes and subsidies as of 1984, with special emphasis on changes in industry structure as a consequence of tax changes. Although MISMOD is not necessarily well suited for analyses of optimal taxation, GEMPS has exactly the same production structure and the same parameters. In part, the reason is that this makes it possible to interpret the transition from MISMOD to GEMPS as a comparative statics exercise. All parameters and the main economic framework being the same, the results from the two models are comparable.[1)] The 1984 benchmark of MISMOD is described in general terms by Mathiesen (1986), but there is no published documentation of the exact values of all the model's exogenous parameters. All exogenous variables in GEMPS are given in Berg (1990).

There are a few differences between MISMOD and GEMPS. The three types of labour in MISMOD are represented by one variable in GEMPS, there is only one geographic region instead of three, and the two commodities "Electricity production" and "Electricity distribution" are regarded as one intermediate commodity in GEMPS. The aggregation of labour has simply been done by regarding the three types of labour in MISMOD as one good. The geographic aggregation is then insignificant because MISMOD's three regional production functions for each good have the same parameters as soon as labour is aggregated into one factor. The aggregation of electricity has been done by equalizing prices and summing up demand and supply, respectively, across all sectors.

Another difference is that GEMPS has only one household, whereas MISMOD has four private utility functions. The consumer's utility function in GEMPS, however, has the same substitution parameters as the MISMOD

[1)] However, the main focus in this chapter is not on such a comparison. Another reason for the present approach is that the construction of a new computable general equilibrium model is an extremely demanding task.

households (viz., 1.2 at the top level and 0.48 at the second level), and the endowments of leisure and bonds are the sum of the households' endowments in MISMOD. A more important difference is that the MISMOD households have positive or negative endowments of consumption goods in order to induce empirically plausible demand elasticities. For theoretical reasons there are no such endowments in GEMPS. (The elasticities would not be the same anyway, since equilibrium prices are quite different in the two models. As will be shown below, the consumer's demand and supply elasticities in GEMPS are not unreasonable.)

Although the levels of public consumption and saving are the same, a further difference between GEMPS and MISMOD is that these levels are exogenously given in GEMPS, and do not vary with market prices as in MISMOD. Finally, the tax system in GEMPS is of course completely different from that of MISMOD.

The main weaknesses of GEMPS lie in formulations that are inherited from MISMOD and implicitly restrict the optimality of the tax system. In particular, the taxation of the consumer's income is subject to three rather restrictive assumptions. First, there is no progressive taxation of labour income. The labour income tax is simply a wedge between the wage rate firms pay and the wage rate the consumer receives, and the tax base is total labour supply. Secondly, the taxation of capital income is exogenous and not necessarily optimal. The reasons are that capital supply is fixed, and that capital is not an argument in the consumer's utility function. Consequently, the formulae for optimal taxation do not apply (at least not directly) to capital. The tax rate on capital income in GEMPS, TC, is an exogenous parameter in the interval [0,1]. In the equilibrium below, TC = 0.5.

The third restriction is the formulation of saving, which is extremely simplified. Saving is to some extent regarded as a residual in GEMPS and MISMOD, even though the consumer's wealth is formally represented by a variable in the utility function. This formulation is intended as a substitute for a more realistic (and more complex) model with several time periods. In the present version of GEMPS, wealth is viewed as bonds, and has a market price. There is also a bond producer facing two kinds of demand: bond demand from the consumer, foreigners and the state; and investment demand due to depreciation, which is given exogenously. A possible interpretation of this firm is that it provides investment goods which will be available in a hypothetical future period, issuing bonds now as claims on future returns on that investment. The good "Bonds" is chosen as a numéraire.

The balance of payments may perhaps seem a little obscure, since it is not explicit in the model. If the value of exports, i.e. the net supply of currency, exceeds the value of imports, i.e. the demand for currency, then the bond producer's currency demand will cover the difference in general equilibrium. At the end of the market period, the consumer, the state and foreigners hold bonds, some of which are financial claims on foreigners. The model does not keep track of the ownership of such claims.

As already mentioned, there are 12 export demand functions in GEMPS, but the actual flows of physical goods to foreigners are not recorded. Exports are generated because foreigners have initial endowments of currency. The currency endowments are not the same in GEMPS as in MISMOD, however. In the transition from MISMOD to GEMPS, it turns out that the average price level of intermediate goods falls sharply and export demand increases accordingly. In fact, it seems to be impossible to find any equilibrium solution of GEMPS unless the currency endowments are substantially lower than in MISMOD. The initial endowments of currency in MISMOD are chosen arbitrarily to generate the actual Norwegian export levels of 1984. In GEMPS, the endowments are also arbitrary, and generate approximately the same physical volumes as in MISMOD. The export value will differ between the two models, however, since the equilibrium prices do.

The basic equilibrium

GEMPS is fully documented in the computer program in Berg (1990). Computer programs being difficult to read, some of the exogenous variables will be listed here as well:

Public consumption	83.9714
Public saving	49.6614
Consumer's labour endowment	353.6990
Consumer's bond endowment	53.9804
Capital stocks:	
00 Shipping	4.6543
01 Electricity	10.2750
03 Petroleum	62.1820
04 Primary industries	11.5289
05 Food	2.0635
06 Textiles	1.4679
07 Shipbuilding	0.9654
08 Chemicals	2.8474
09 Metals	5.6765
10 Manufacture	5.8351
11 Private services	37.6629
12 Public services	3.4111

Let us now turn to the endogenous variables in the basic equilibrium of GEMPS. The producer prices of currency and the 12 intermediate goods are:

Currency	PV	= 2.1355
Electricity production	PY_{01}	= 0.2454
Electricity distribution	PY_{02}	= 0.2454
Petroleum	PY_{03}	= 0.6846
Primary goods	PY_{04}	= 0.4218
Food	PY_{05}	= 0.3766
Textiles	PY_{06}	= 0.5275
Ships and rigs	PY_{07}	= 0.6793
Chemicals	PY_{08}	= 0.6151
Metals	PY_{09}	= 0.8423
Manufactured goods	PY_{10}	= 0.4935
Private services	PY_{11}	= 0.3945
Public services	PY_{12}	= 0.4573

The equilibrium prices of capital in GEMPS are:

Shipping	PK_{00}	= 7.4611
Electricity	PK_{01}	= 0.4298
Petroleum	PK_{03}	= 0.9407
Primary industries	PK_{04}	= 0.3178
Food	PK_{05}	= 0.1679
Textiles	PK_{06}	= 0.6633
Shipbuilding	PK_{07}	= 0.7733
Chemicals	PK_{08}	= 0.7455
Metals	PK_{09}	= 0.8838
Manufacture	PK_{10}	= 0.7816
Private services	PK_{11}	= 0.6798
Public services	PK_{12}	= 0.4516

If capital were a homogeneous good, these prices would indicate where the marginal return on investment is the largest. The most profitable sector is Shipping, mainly because the price of its product, foreign currency, is relatively high. The least profitable sectors seem to be Food production, Primary industries and Public services. Note that depreciation allowances have not been deducted from the prices listed here.

Perhaps the most important part of the equilibrium is the solution for producer and consumer prices of labour and the five main goods, and the resulting optimal tax rates:

Labour	$PL = 0.8532$	$QL = 0.8311$	
Food	$P_1 = 0.3312$	$Q_1 = 0.5094$	
Clothes	$P_2 = 0.8468$	$Q_2 = 1.3024$	
Housing	$P_3 = 0.3676$	$Q_3 = 0.5654$	
Transport	$P_4 = 0.6915$	$Q_4 = 1.0636$	
Services	$P_5 = 0.4172$	$Q_5 = 0.6417$	
Labour	$TL = -0.0221$	$TL/PL = -2.59\%$	$TL/QL = -2.66\%$
Food	$T_1 = 0.1782$	$T_1/P_1 = 53.81\%$	$T_1/Q_1 = 34.99\%$
Clothes	$T_2 = 0.4557$	$T_2/P_2 = 53.81\%$	$T_2/Q_2 = 34.99\%$
Housing	$T_3 = 0.1978$	$T_3/P_3 = 53.81\%$	$T_3/Q_3 = 34.99\%$
Transport	$T_4 = 0.3721$	$T_4/P_4 = 53.81\%$	$T_4/Q_4 = 34.99\%$
Services	$T_5 = 0.2245$	$T_5/P_5 = 53.81\%$	$T_5/Q_5 = 34.99\%$

Two results are worth noting in the table above. The first is that the tax rate on labour is quite small in absolute value, perhaps contrary to what is commonly assumed or estimated in the literature. The second is that the percentage tax rates on the 5 main commodities are exactly the same. The uncompensated demand and supply elasticities are also quite similar:

	QL	Q_1	Q_2	Q_3	Q_4	Q_5	QB
EL	0.5465	-0.0172	-0.0134	-0.0207	-0.0546	-0.0171	-0.2991
EX_1	0.8832	-0.5708	-0.0707	-0.1094	-0.2877	-0.0902	0.1734
EX_2	0.8832	-0.0908	-0.5507	-0.1094	-0.2877	-0.0902	0.1734
EX_3	0.8832	-0.0908	-0.0707	-0.5894	-0.2877	-0.0902	0.1734
EX_4	0.8832	-0.0908	-0.0707	-0.1094	-0.7677	-0.0902	0.1734
EX_5	0.8832	-0.0908	-0.0707	-0.1094	-0.2877	-0.5702	0.1734
EB	40.5519	0.4581	0.3568	0.5520	1.4520	0.4551	-47.1347

We see that the labour supply elasticity is 0.5465, and that the own-price demand elasticities for consumption goods are between -0.55 and -0.77. The elasticities of the demand for bonds are difficult to interpret from an empirical viewpoint, since the model's price of bonds does not have an obvious counterpart in the Norwegian economy. The relatively large numbers for the own-price elasticity and the cross elasticity with respect to the price of labour partly reflect the fact that the consumer's net market demand for bonds is only 1.2019.

The compensated elasticities of <u>consumption</u> (not market demand) are:

	QL	Q_1	Q_2	Q_3	Q_4	Q_5	QB
CL	-0.6045	0.0599	0.0466	0.0721	0.1897	0.0595	0.1766
CX_1	0.5955	-0.5209	-0.0318	-0.0493	-0.1296	-0.0406	0.1766
CX_2	0.5955	-0.0409	-0.5118	-0.0493	-0.1296	-0.0406	0.1766
CX_3	0.5955	-0.0409	-0.0318	-0.5293	-0.1296	-0.0406	0.1766
CX_4	0.5955	-0.0409	-0.0318	-0.0493	-0.6096	-0.0406	0.1766
CX_5	0.5955	-0.0409	-0.0318	-0.0493	-0.1296	-0.5206	0.1766
CB	0.5955	0.0599	0.0466	0.0721	0.1897	0.0595	-1.0234

The two relevant compensated <u>market</u> elasticities are:

	QL	Q_1	Q_2	Q_3	Q_4	Q_5	QB
CL'	1.0428	-0.1033	-0.0804	-0.1244	-0.3273	-0.1026	-0.3047
CB'	27.3435	2.7489	2.1406	3.3120	8.7119	2.7307	-46.9875

The consumer's net market demand (supply) is:

Labour	(129.7909)
Food	36.7222
Clothes	11.1853
Housing	39.8678
Transport	55.7472
Services	28.9629
Bonds	1.2019

The demand for labour is distributed as follows among the intermediate sectors:

00 Shipping	10.5253
01 Electricity	1.4482
03 Petroleum	4.6399
04 Primary industries	3.5299
05 Food	1.9484
06 Textiles	3.9920
07 Shipbuilding	5.3369
08 Chemicals	2.6705
09 Metals	3.3639
10 Manufacture	18.8221
11 Private services	47.4605
12 Public services	26.0533

The final endogenous variables in the basic equilibrium of GEMPS are the marginal cost of public funds, μ, the consumer's marginal utility of income, λ, and his maximum utility level, U. They are μ = 0.002931; λ = 0.001766; and U = 49763.0654. This means that the marginal cost of public funds in terms of private income, μ/λ, is equal to 1.6598.

Comparative statics

1. Increasing public consumption under optimal taxation.

The empirical validity of any numerical model, viewed as a description of the real economy, can be questioned. GEMPS is no exception, and it would be unwarranted to assert that the Norwegian economy would have looked like GEMPS if only tax rates had been optimal. This does not mean, however, that a model such as GEMPS is empirically worthless. Even though it may not describe each static equilibrium in a fully satisfactory way, it may nevertheless be useful in the analysis of change; i.e. in comparative statics.

In this section, three comparative analyses are considered. First, the level of public consumption is increased while being financed by optimal taxation until it is no longer possible to compute an equilibrium. Next, the transition from the basic equilibrium reported in the previous section to a situation with only lump-sum taxation is analysed. This is done by increasing exogenous lump-sum taxes gradually, covering the public budget deficit with decreasing, optimal tax rates. In the third simulation the level of public consumption is increased once again, but this time there are only lump-sum taxes.

Large amounts of data may be output from these comparative analyses. The focus here being on the general equilibrium aspects of optimal taxation, however, only a few of the solution paths of different variables have been selected and are displayed in the following diagrams.

Figure 5.1 shows the equilibrium paths of the consumer price QL and the producer price PL in the labour market. Both prices increase with the public consumption level Z; the producer price almost linearly, and the consumer price progressively. In this sense the greater part of the incidence is on the consumer. The paths stop at Z = 181.78 because no equilibrium could be found when incrementing Z by 2% beyond this point.

The solution paths of the prices of commodity 1 are drawn in Figure 5.2. The consumer price Q1 rises progressively, but contrary to the case of labour, the producer price P1 now falls slightly. The solution paths of the other commodity prices are quite similar, except for commodity 5, the prices of which are shown in Figure 5.3. Here the producer price grows with the level of public production, although the increase is not very great.

The most interesting result of the present simulation, the optimal tax rates, are depicted in Figure 5.4. The tax rate on labour is negative by definition: $TL \equiv QL - PL$. We see that all tax rates rise with the public consumption level, no paths crossing each other. It also seems that the rise in the overall taxation level is evenly distributed among the five commodity tax rates, whereas the increase in the absolute value of the labour tax rate is more modest.

The development of percentage tax rates is shown in Figure 5.5. It turns out that the five main commodities continue having equal percentage tax rates when the public consumption level increases. Since the percentage tax rate on labour differs from the commodity rates, the equality of the latter is likely to be due to the specific functional forms which are used

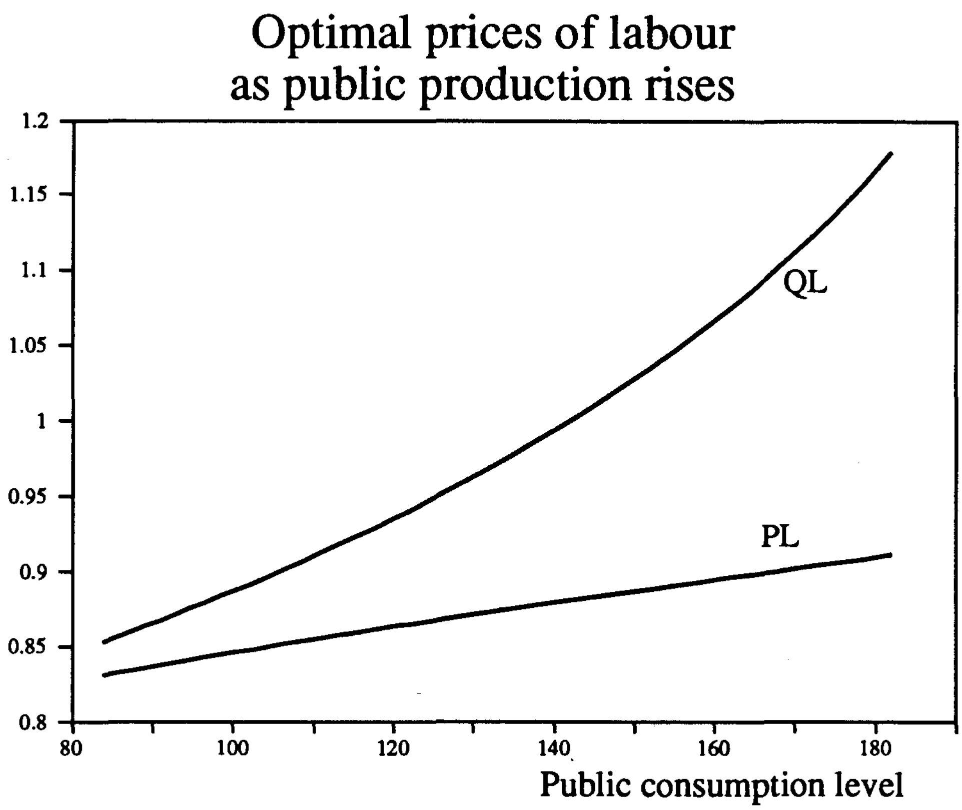


Figure 5.1

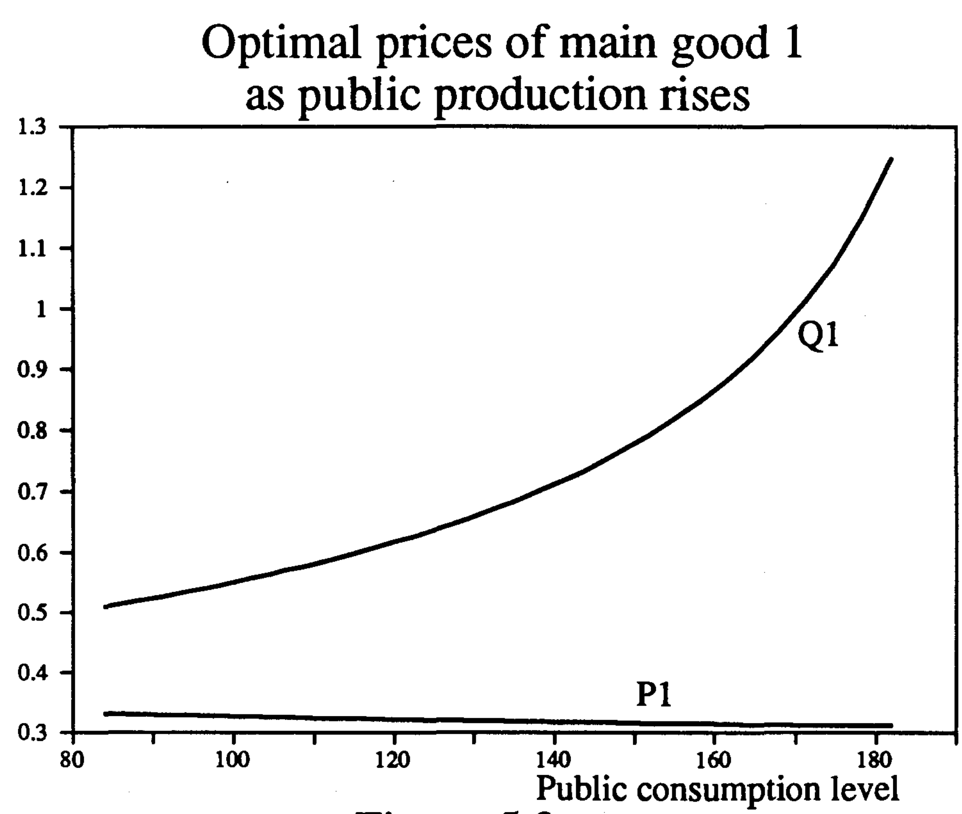


Figure 5.2

Optimal prices of main good 5 as public production rises

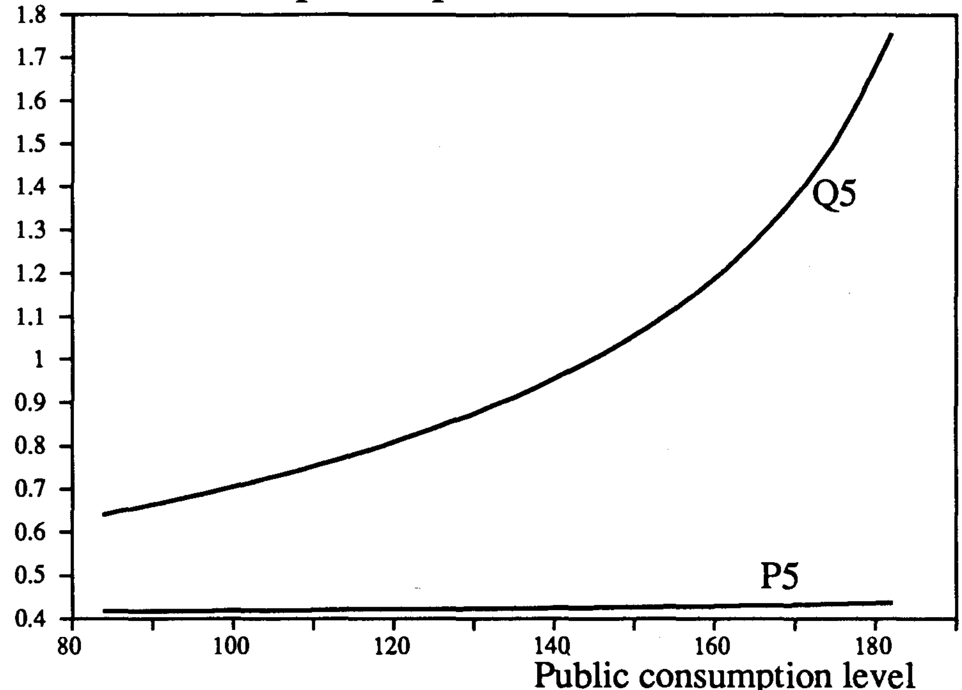


Figure 5.3

Optimal tax rates as public production rises

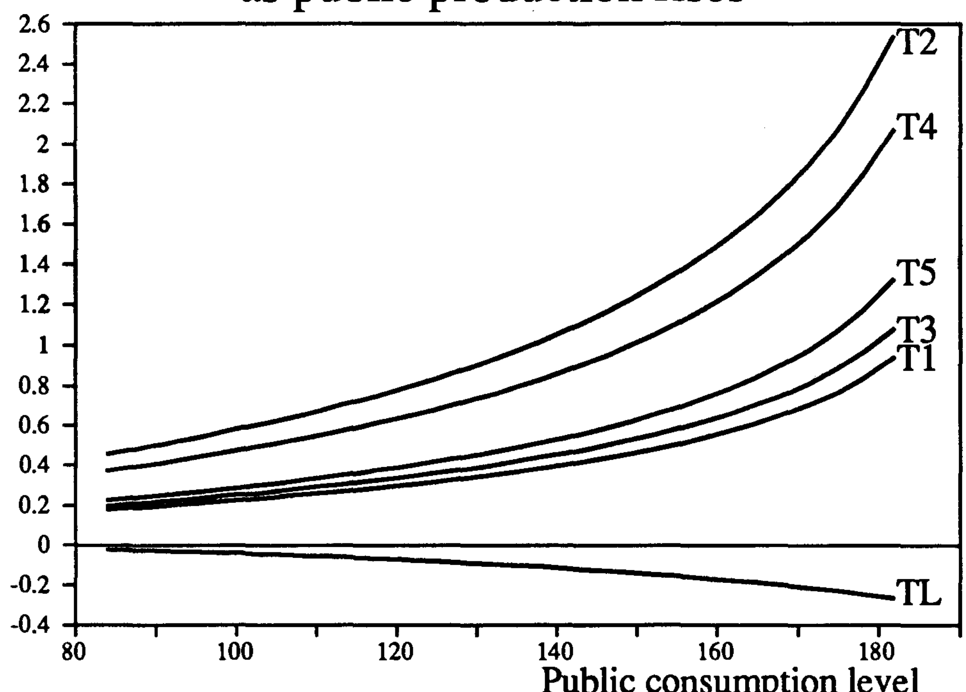


Figure 5.4

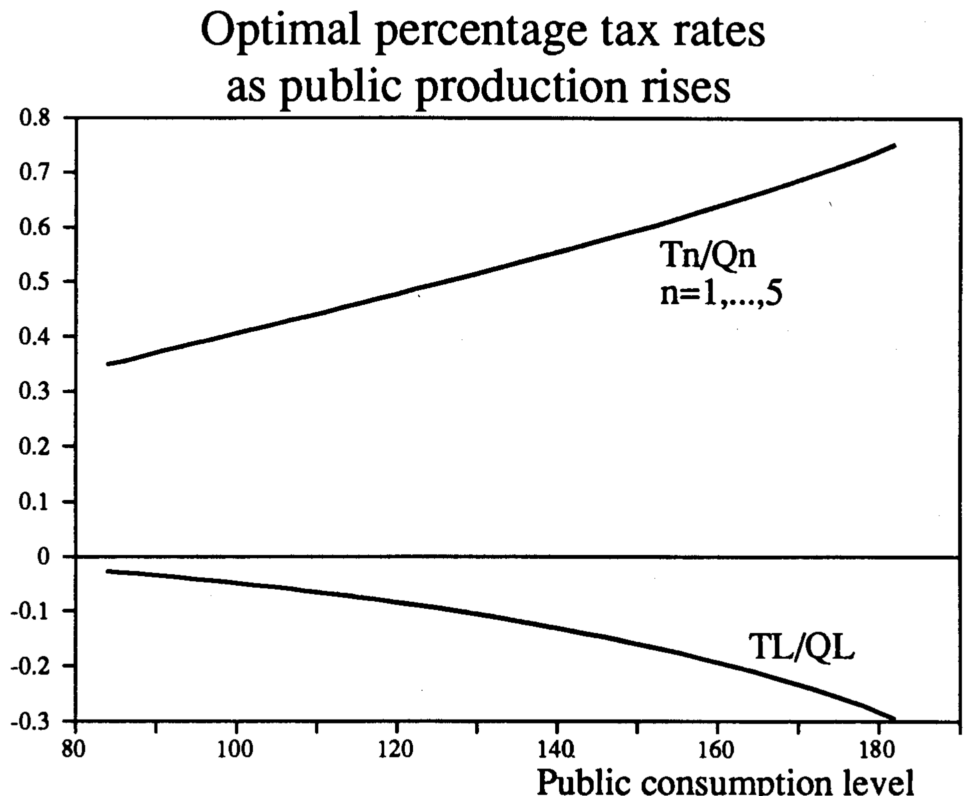


Figure 5.5

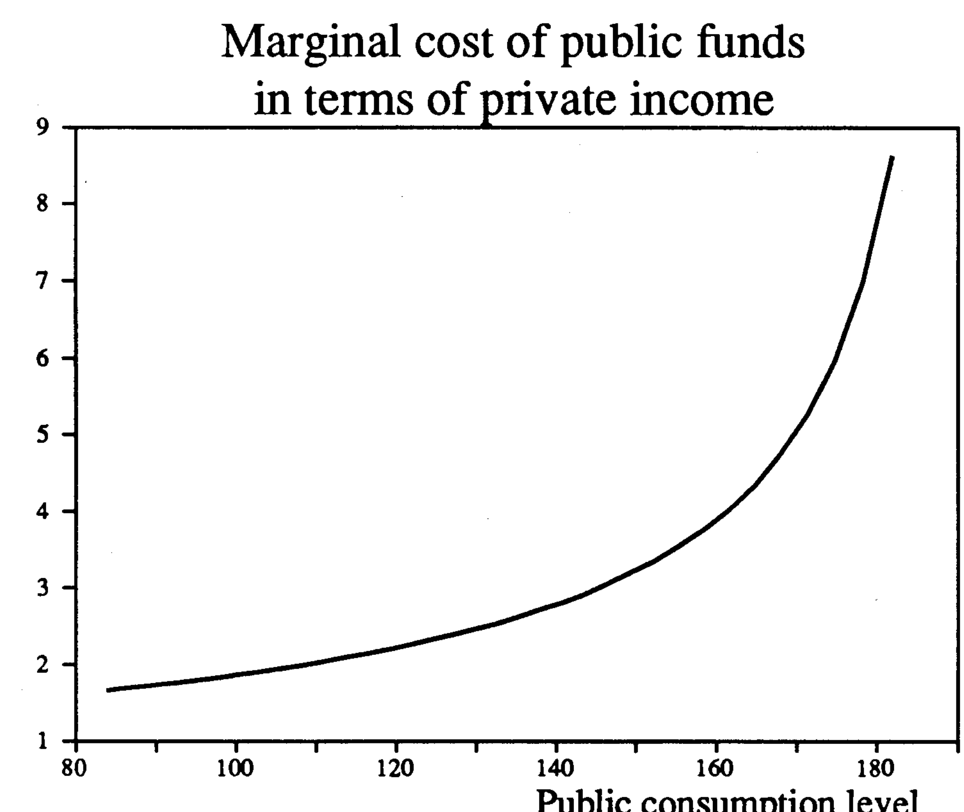


Figure 5.6

in the model. T_n/Q_n rises from 34.99% to 75.06% in the simulation, and TL/QL varies between -2.66% and -29.31%.

Figure 5.6 demonstrates that the normalized marginal cost of public funds, $\frac{\mu}{\lambda}$, increases progressively with Z, from 1.6598 to 8.6033. This reflects the growing marginal welfare loss due to distortive (although optimal) taxation. The marginal cost of public funds in terms of private income becomes very high as the level of public consumption approaches the point where the equilibrium breaks down. In this context the beneficial side of government production should not be neglected, however. Although the marginal cost of public funds increases, so does public consumption, thus yielding a higher utility level on the part of the consumer. The effect is shown in Figure 5.7, where the consumer's (maximum) utility level is seen to rise with Z, from 49763.07 to 55362.38; before falling towards 53857.74 at the end point. The first-order conditions for optimal public production which were discussed briefly in Chapter 3 imply that the maximum point is reached when the marginal utility of Z is equal to the marginal cost of public funds (μ) times the marginal increase in the public deficit when Z increases. Therefore the progressive increase in μ/λ depicted in Figure 5.6 may well bring about an interior maximum such as the one in Figure 5.7.

2. Transition from optimal tax rates to lump-sum taxation.

The next comparative analysis is to investigate the development of taxes and prices if the optimal tax rates are gradually replaced by lump-sum taxation. This is interesting because lump-sum taxation is often regarded as a yardstick against which the welfare effects of various tax regimes are compared. In GEMPS this comparative statics exercise has been carried out by incrementing lump-sum taxes 100 times, from zero to the level where all optimal market tax rates are zero, viz., 39.1083. The level of public consumption is the same as in the basic equilibrium. Some of the results are reported in Figures 5.8-5.12.

In Figure 5.8 we see that the distance between the two prices of labour stays almost constant until lump-sum taxes have grown to 70% of their final level, then it starts to diminish. In addition it turns out that both the producer price and the consumer price of labour are lower under lump-sum taxation than under optimal market taxation. In the commodity markets, things are different, as Figure 5.9 demonstrates. The consumer price of commodity 1 declines sharply, and the producer price increases only slightly. The picture is the same in the other commodity markets, except for market 5, where the producer price falls (although by small amounts; from 0.4172 to 0.4105). Judging from the diagrams, the incidence of optimal taxes is mainly on the consumer, at least in the five commodity markets.

In Figure 5.10 the paths of the optimal market tax rates are drawn. In accordance with the two previous diagrams, the tax rates on the five main commodities decrease smoothly towards zero; whereas the absolute value of the labour tax rate first increases before it starts to fall. Figure 5.11 shows the consequences as far as the normalized marginal cost of public funds is concerned: $\frac{\mu}{\lambda}$ decreases monotonically from 1.6598 to 1. Of course, with only lump-sum taxation, the marginal welfare cost of raising public revenue is equal to the consumer's marginal utility of (lump-sum) income.

Figure 5.12 displays the path of the consumer's maximum utility level in the transition from optimal market taxes to lump-sum taxation. The result here is unexpected, but not inexplicable. Since there is a welfare loss

Social welfare with optimal taxes
as public production rises

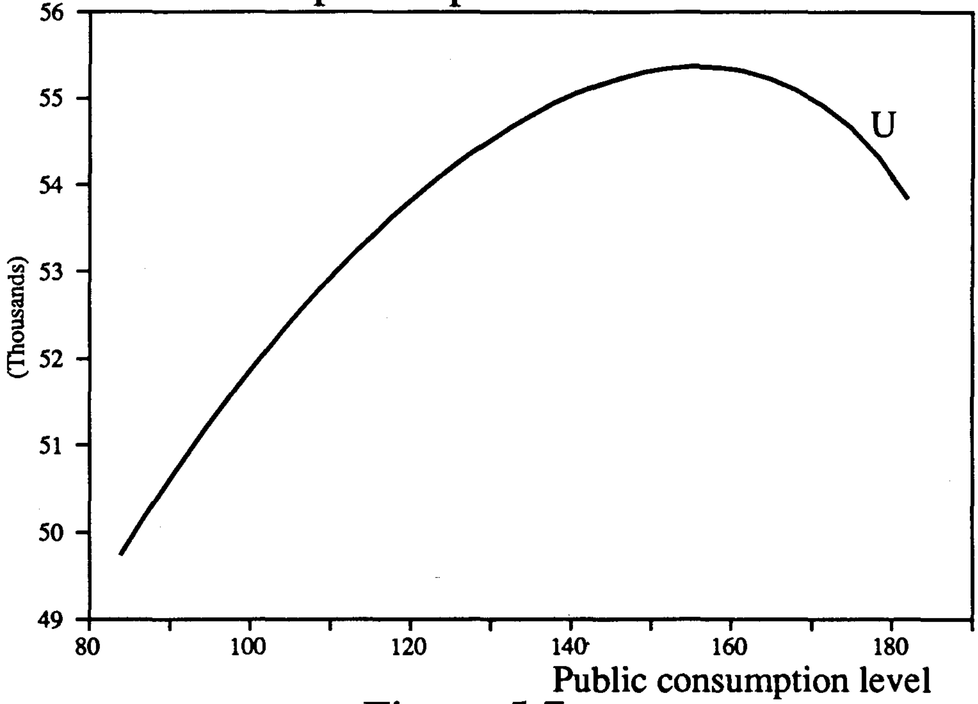


Figure 5.7

Transition from optimal tax rates
to lump-sum taxation

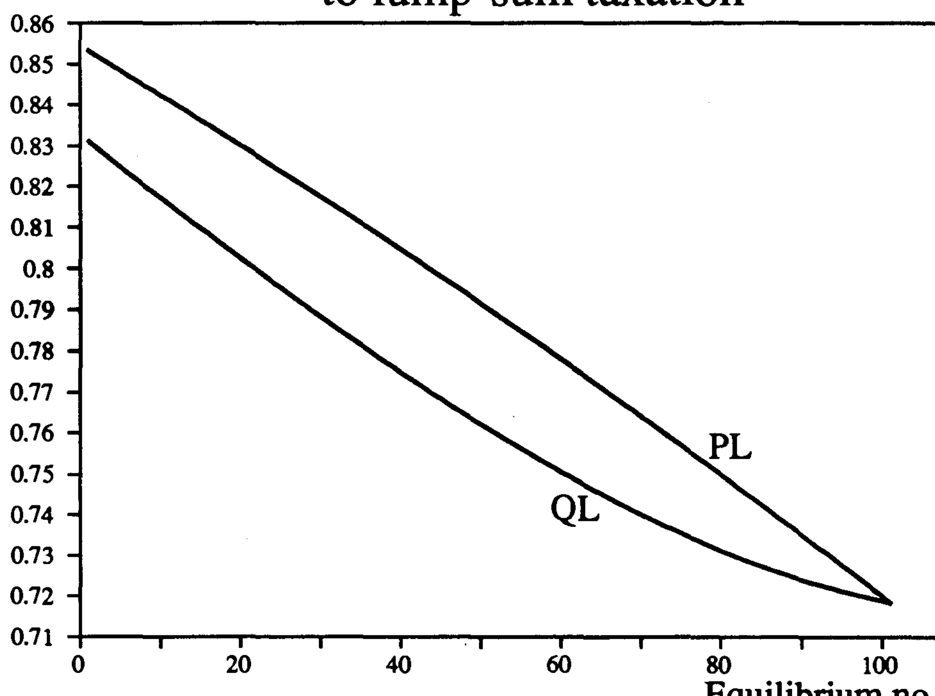


Figure 5.8

Transition from optimal tax rates
to lump-sum taxation

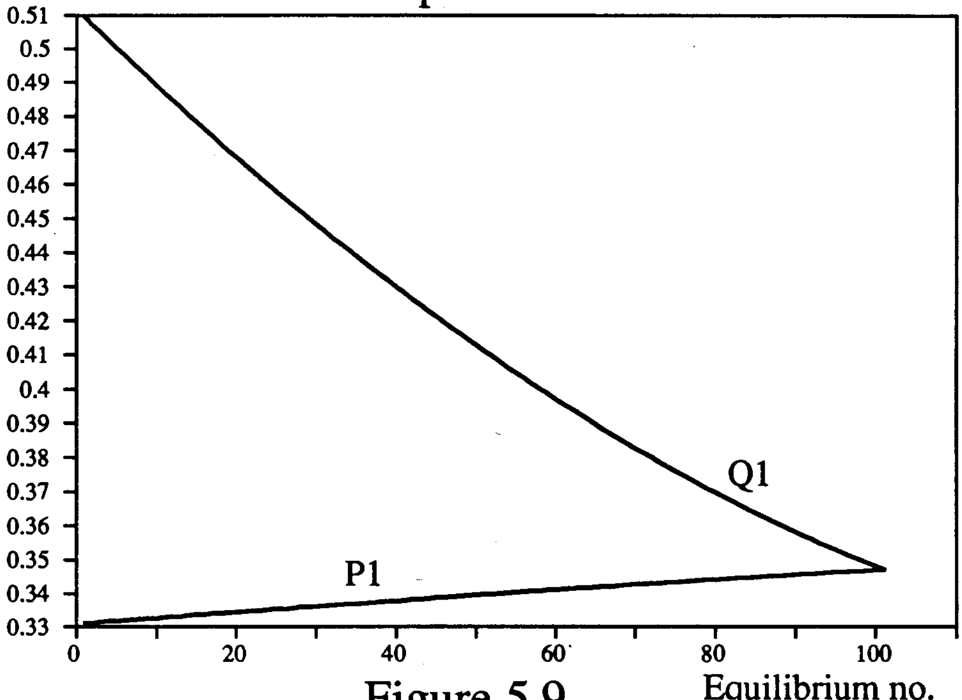


Figure 5.9

Transition from optimal tax rates
to lump-sum taxation

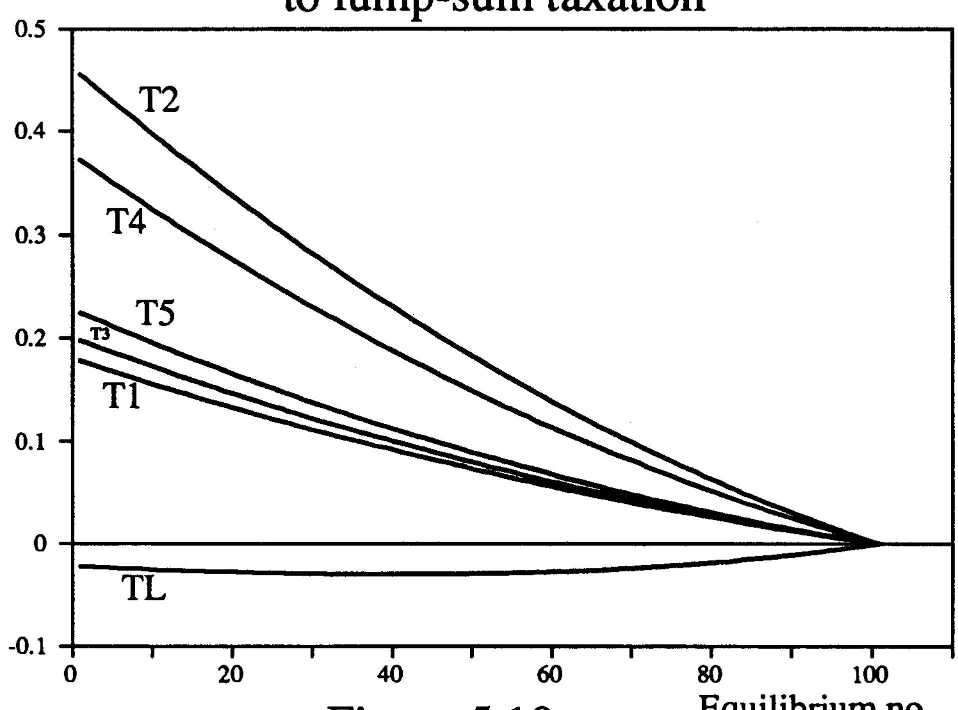


Figure 5.10

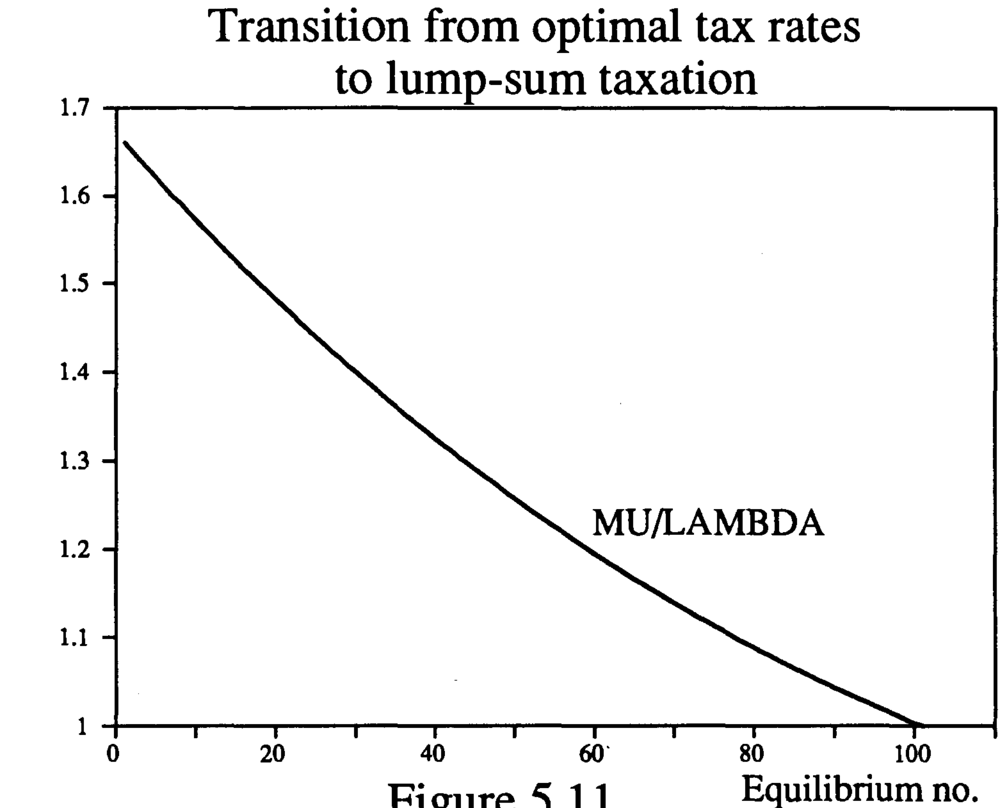


Figure 5.11

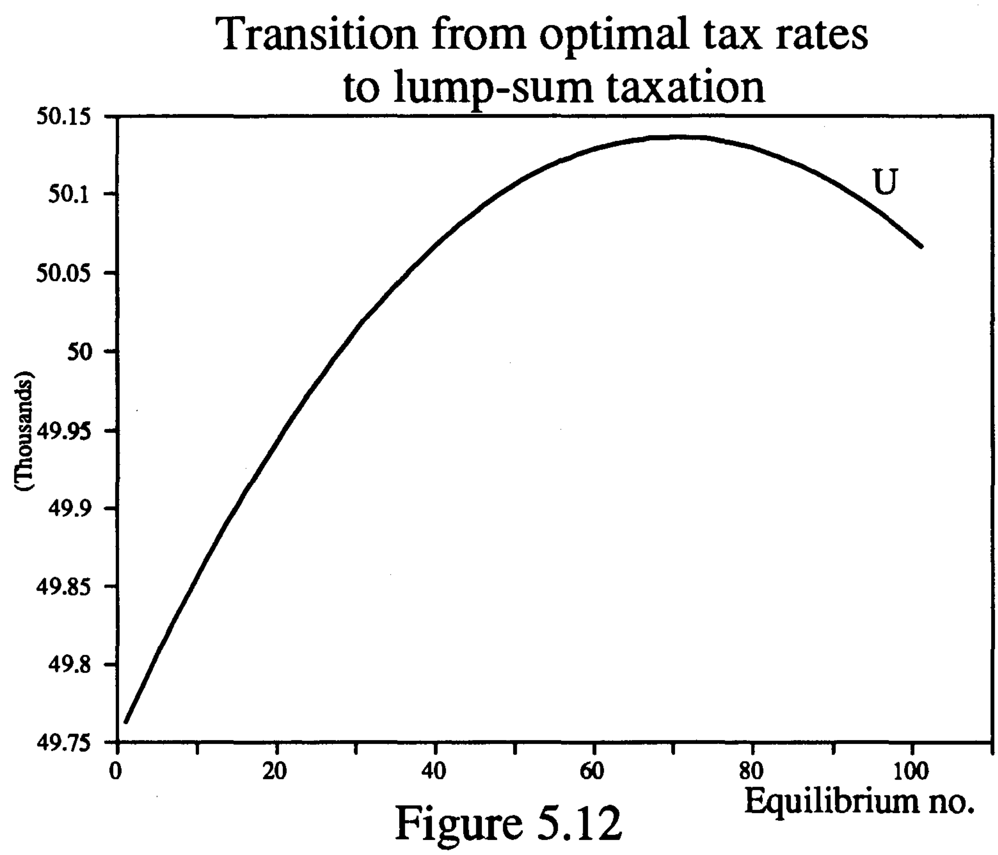


Figure 5.12

from distortive taxation even if tax rates are optimal, we would expect that the representative consumer's utility level would increase when lump-sum taxation is introduced. So it does, but only up to a point where market tax rates are still non-zero. In fact, in this point, we have that

TL = -0.0225	TL/QL = -3.06%
T1 = 0.0357	T1/Q1 = 9.42%
T2 = 0.0882	T2/Q2 = 9.42%
T3 = 0.0385	T3/Q3 = 9.42%
T4 = 0.0721	T4/Q4 = 9.42%
T5 = 0.0429	T5/Q5 = 9.42%

A possible explanation is that optimal taxation has a terms-of-trade effect which may outweigh the welfare loss due to price distortion. The mechanism is: Levying domestic market taxes indirectly reduces imports which in the case of GEMPS are used as inputs in intermediate sectors. A reduction of imports means that the demand for foreign currency falls. If domestic and foreign export goods were perfect substitutes, the supply of foreign currency would be perfectly elastic, and the reduced currency demand would have had no effect on the exchange rate. But in GEMPS foreign and domestic export goods are not perfect substitutes, and there is an upward-sloping currency supply curve. Taxation of imports will induce a downward pressure on the exchange rate, and the terms of trade improve. It turns out that the price of foreign currency increases throughout the simulation, from 2.1355 to 2.1798.

3. Increasing public production under lump-sum taxation.

The third comparative statics analysis is carried out to shed more light on the differences between lump-sum and optimal taxation. Now the level of public consumption is increased in the same way as in simulation 1, but government expenditures are financed with lump-sum taxes. Some of the main results are given in Figures 5.13-5.15. Figure 5.13 shows that all main

Equilibrium prices with lump-sum taxes as public production rises

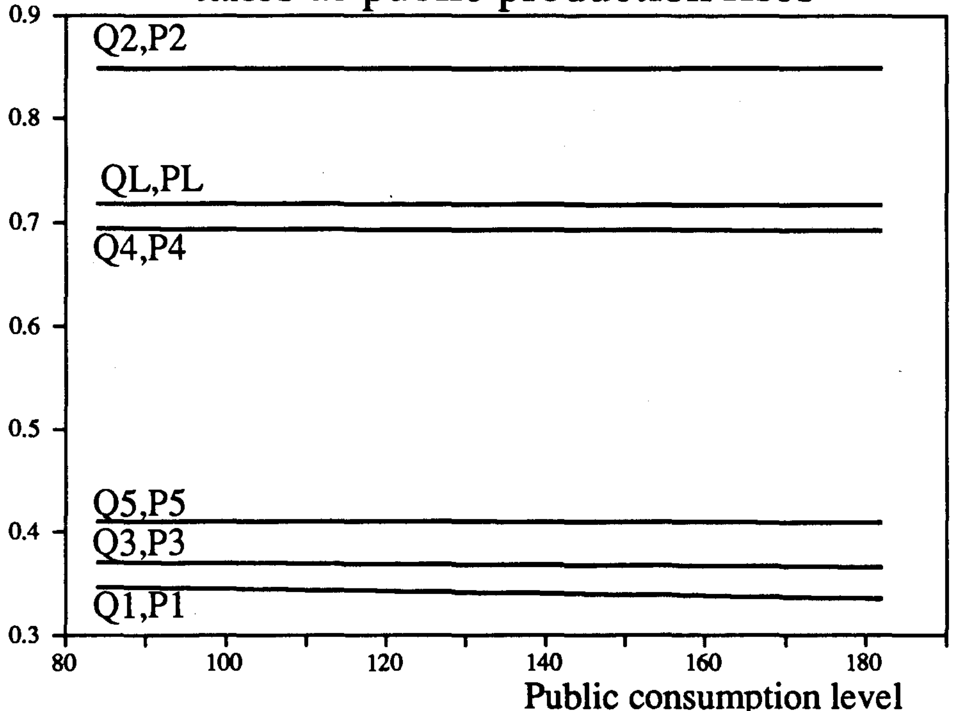


Figure 5.13

Social welfare with lump-sum taxes as public production rises

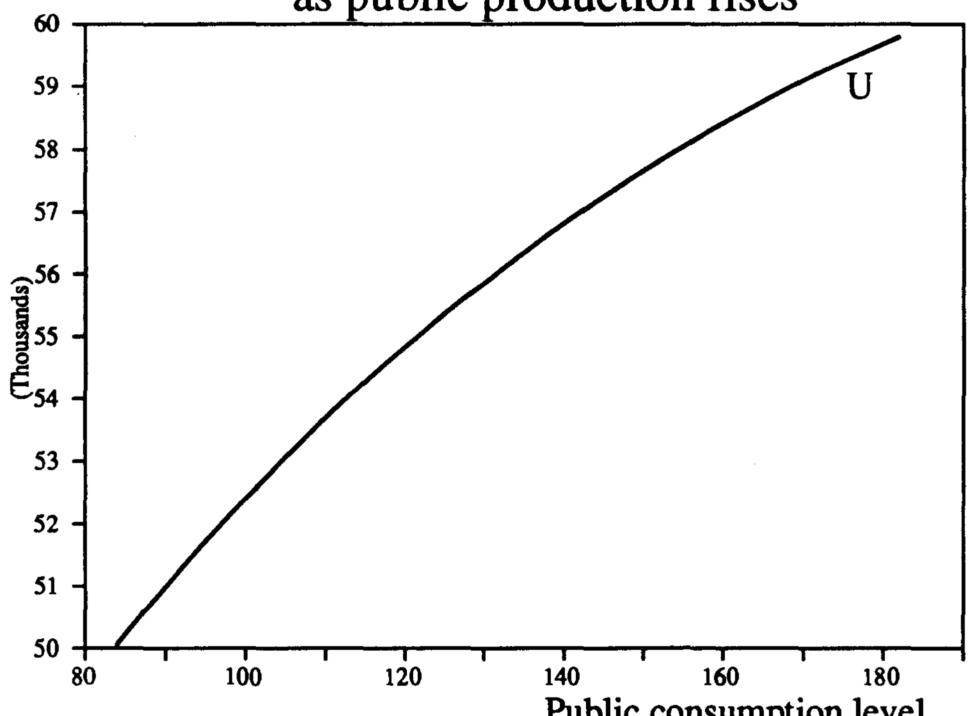


Figure 5.14

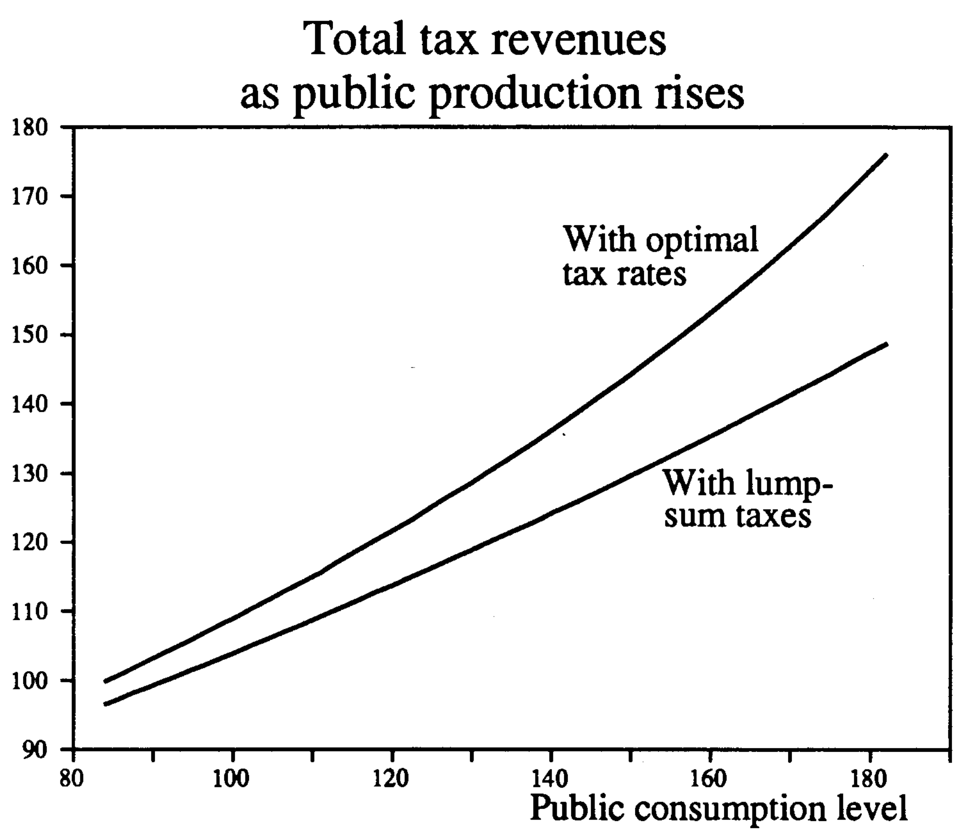

Total tax revenues
as public production rises
180
170
160
150
140
130
120
110
100
90
80
100
120
140
160
180
Public consumption level
With optimal
tax rates
With lump-
sum taxes

Figure 5.15

consumer and producer prices are nearly constant when Z rises; and in Figure 5.14 the consumer's utility level increases monotonically. Finally, we see in Figure 5.15 that the tax revenues required to finance Z are greater and rise more rapidly under optimal market taxation than under lump-sum taxation.

Concluding remarks

This chapter describes a computable general equilibrium model, GEMPS, that can be used to analyse the likely behaviour of optimal market tax rates in Norway when the consumption of public goods increases. The model has one representative consumer who supplies labour and demands 5 main commodities in addition to bonds, 5 main producers and 12 intermediate sectors as well as a bond producer. Exports follow from export demand functions in which foreign and domestic goods are imperfect substitutes. The level of public consumption and saving is given exogenously, and government revenues may be raised by either optimal market taxes or lump-sum taxation.

From a numerical viewpoint, the simulations carried out seem to indicate that the general equilibrium with optimal taxes is quite well-behaved. For instance, comparative statics solution paths for optimal tax rates appear to be continuous. The incidence of optimal taxation is mainly on the consumer in GEMPS, in the sense that consumer prices change more than producer prices when taxes increase. But taxation is evenly distributed among the five main commodities and labour; indeed, percentage commodity tax rates are always the same.

The model obviously has its weaknesses. For one thing, the formulation of saving is not very sophisticated, and no conclusions about the optimal taxation of interest and capital income have been drawn. Secondly, the foreign sector plays a more important role than one would perhaps expect in practice. In particular, the welfare effects of the tax system seem to depend on the specification of the export demand functions, since some degree of market taxation may improve the terms of trade.

Coming as no surprise, the latter point may lead to a conclusion. The foreign sector is a difficult part of the formulation of most computable general equilibrium models. One reason is that there is no easy way to describe the imperfect competition in export markets without making it possible for a small country to influence world market prices. At the same time, the theory of optimal taxation is developed within the framework of a closed economy - there is only one government or authority with the ability to tax in the model. Thus, more work could be done to develop theories of optimum taxation in open economies, aiming at implementation in computable general equilibrium models.

Appendix

There are several cost structures in the 12 intermediate production sectors. The different structures are illustrated below, with partial elasticities of substitution given in parentheses and the numbers in the Cobb-Douglas or Leontief aggregates indicating the products that each sector demands as inputs.

00 Shipping:

```
                      CES (0.5)
        ┌─────────────────┼─────────────────┐
    Leontief           Capital            Labour
        ├──────┬──────┬──────┐
        V     03     07     11
               |      |      |
             ....  Cobb-Douglas
                    ┌─────┴─────┐
                 Product     Currency
```

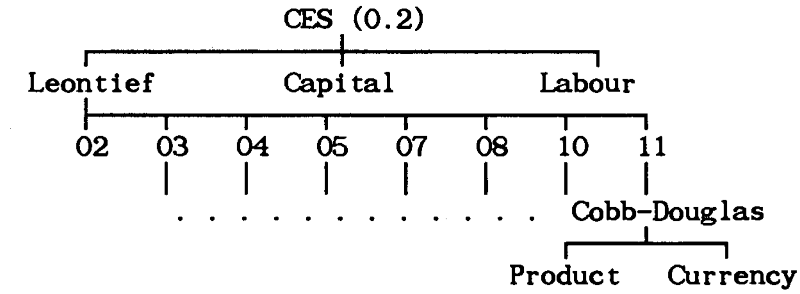
01 Electricity:

```
                    Cobb-Douglas
        ┌─────────────────┼─────────────────┐
    Leontief           Capital            Labour
        ├──────┬──────┐
       01     10     11
               |      |
          ..  Cobb-Douglas
               ┌──────┴──────┐
            Product      Currency
```

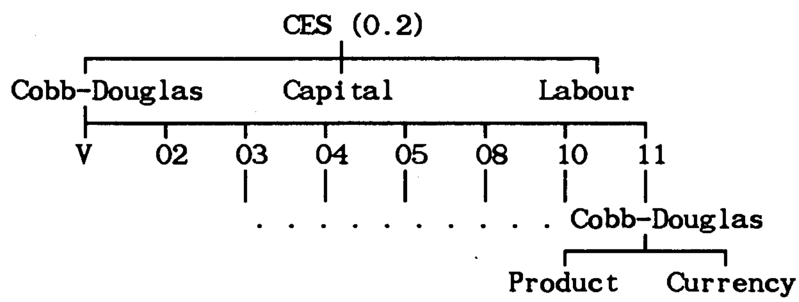
03 Petroleum:

```
                    Cobb-Douglas
        ┌─────────────────┼─────────────────┐
  Cobb-Douglas         Capital            Labour
        ├──────┬──────┬──────┬──────┐
        V     03     07     10     11
               |      |      |      |
             . . . . .  Cobb-Douglas
                         ┌──────┴──────┐
                      Product      Currency
```

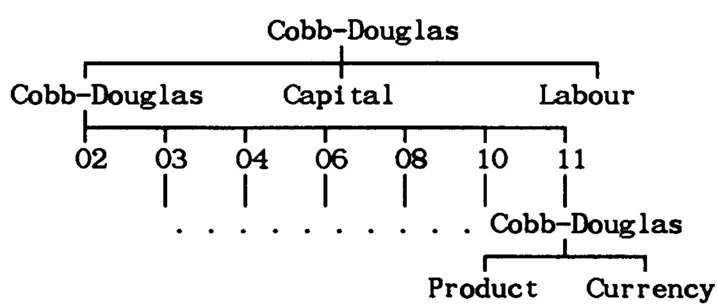
04 Primary industries:



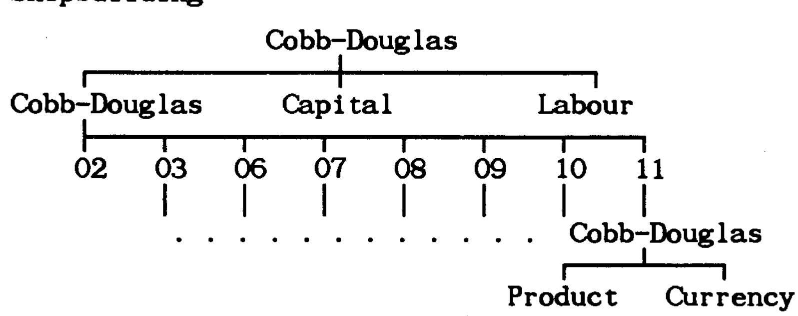
05 Food:



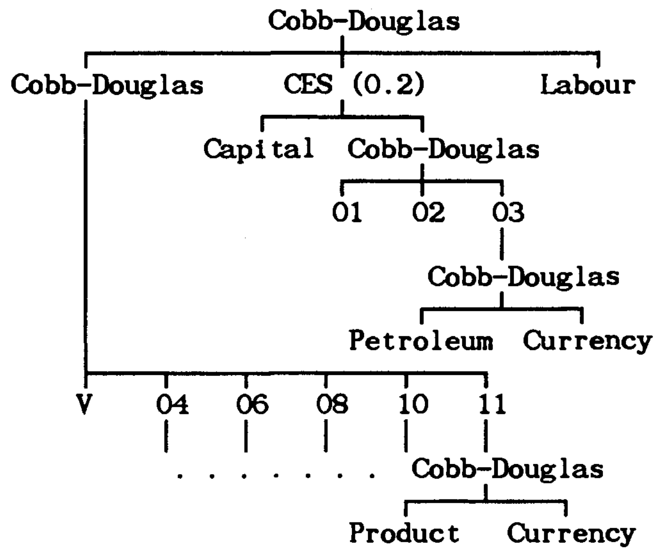
06 Textiles:



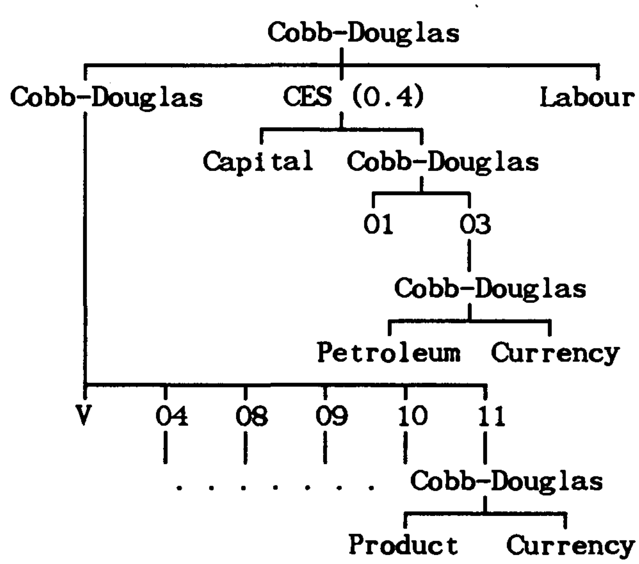
07 Shipbuilding:



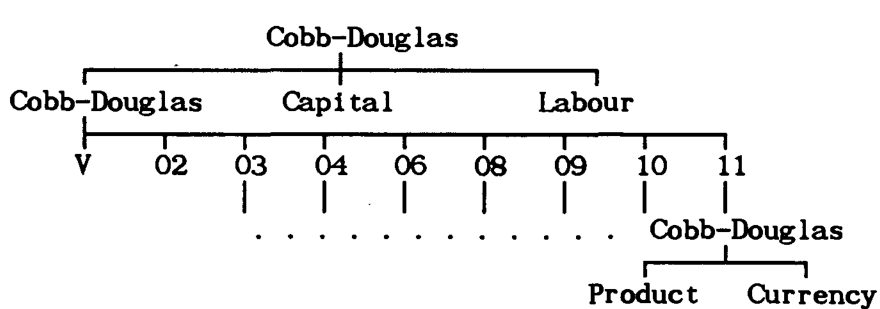
08 Chemicals:



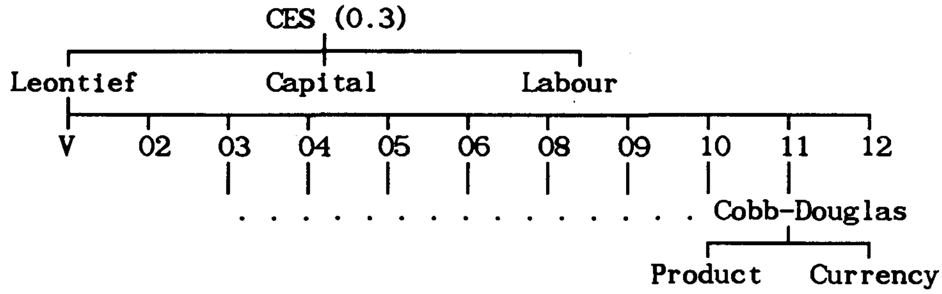
09 Metals:



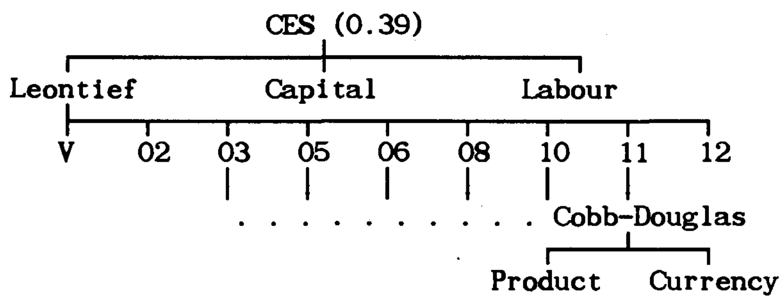
10 Manufacture:



11 Private services:



12 Public services:



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CONCLUSION

As a whole, the five main chapters in this thesis may be seen as an argument about how the public sector can be modelled within a Walrasian, microeconomic framework. The theoretical idea underlying much of the argument is that if the public sector (or "The State", which is the term used here) merits its own formulation in economics, the reason must be that it has or may be given a real economic significance apart from that of the traditional private agents, the consumer and the producer. This comes out clearly if one thinks in terms of alternatives. If the state could perform no economic task of importance or significance, then there would or should be no economic state. If the state could perform some economic tasks, but all of them were better left to the private sector, then there would or should be no economic state. (The qualification "economic" is introduced because there might still be political, social or even historical reasons for the state's existence, of course.)

The thesis is also a discussion of how elements of economic theory within the Walrasian framework can be added or modified on a general level, and not so much a presentation of theorems on logical relationships between such elements. Thus, the chapters following the first reflections on the role and modelling of the public sector try to shed some light on a few of the consequences of the steps taken towards a Walrasian concept of the state. Special interest is devoted to the formulation of the state in so-called computable general equilibrium models. One reason is that models must be related to empirical data in order to improve insight into complex economic phenomena. Another reason is that the types and extent of the economic tasks that the state is best suited to perform, cannot be easily determined from theoretical considerations alone. What the public sector should do depends on the specific consumer preferences and the production technology available in the economy. It seems difficult to obtain general theorems on privatization.

The argument in the thesis ends with a computable general equilibrium model with optimal taxation. Apart from demonstrating the existence of such an equilibrium, the contribution of this model is first and foremost to indicate that the optimum taxation equilibrium is well-behaved as long as utility and production functions are. The comparative statics solution paths of endogenous variables bear every sign of being continuous if judged by their geometric appearance. In addition, however, the model is in fact an empirical description of the possible effects on the Norwegian economy of introducing and changing optimal tax rates. To be sure, the model is simplified and not ideal for optimum tax analysis. But it is important to note that any empirical model must be enormously simplified as a description of real-world phenomena. Otherwise it would simply not be tractable in practical decision-making. The third contribution of the model is that it illustrates theoretical results some of which are most often only discussed in an intuitive manner. The actual magnitudes of the changes in the marginal cost of public funds when government consumption increases are for instance not easy to express analytically. Likewise, the consumer and producer prices as dependent on exogenous variables and parameters in general equilibrium are complicated functions.

Obviously, not all the work that could have been done, has been done in this thesis. The discussion of the economic role of the state in Chapter 2 could no doubt be extended into theorems on privatization; and it is also possible to say more about internal efficiency in the public sector. Both administration and direct production costs should in general be brought to a minimum, and even the computation of optimal tax rates is not a trivial question in practice. As already noted in the discussion of the numerical model GEMPS in Chapter 5, the model specification could be altered in many ways. To analyse the optimal taxation of savings one would e.g. prefer a model with at least two time periods. To investigate the effects on the income distribution of changing tax rates we would have to include several households in a more general social welfare function. One could question the relevance of the Walrasian framework itself, arguing that tax analysis in oligopolistic or monopolistic markets is more interesting. And there are numerous other questions and problems that have not been mentioned. They will have to be taken up in future work.

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