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Economic Evaluation of Alternative Harvesting Strategies for Fish Stocks

By

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Abstract

The main objective of the thesis is to compare different strategies for management of fluctuating fish stocks with respect to economic performance. The economically optimal strategy is found and this is compared with strategies based on simple rules. Main emphasis is put on the two alternatives constant annual catch and constant fishing effort. The comparisons are performed under a variety of different assumptions concerning economic and biological relationships. Both an aggregated biomass model and a disaggregated year-class model are used in the study. A case-study of the Arcto-Norwegian cod stock is performed.

The main conclusion is that although the theory suggests that constant effort is economically superior to constant catch in the simple model, numerical simulations indicate that the difference between these two strategies is surprisingly small. The economically optimal strategy, however, yields a discounted net revenue that is 20 to 40 per cent higher than for the two alternatives. Constant catch is easier to implement than constant effort, but reservation has to made for the possible higher risk implied by this strategy.

Preface

The present text is a slightly revised version of a dissertation submitted for the degree of dr. oecon. at the Norwegian School of Economics and Business Administration.

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1. INTRODUCTION

1.1 Objectives

Fishing is a production process which, broadly speaking, is characterized by two input factors, the fish stock biomass and the fishing effort, and one output, the catch. The term fishing effort refers to some aggregate measure of factors of production, e.g. the total number of standardized vessels per unit time.

Most fish stocks are characterized by biomass fluctuations. The main source of these fluctuations is usually variation in recruitment to the stock. As the causes of the fluctuations are complex and hard to trace, they will in most of this work be treated as exogenous. If, for some reason, one would want the catch to be stable over time, this would normally call for compensating variations in fishing effort[1]. If, one the other hand, one would want fishing effort to be stable over time, this will result in fluctuating catch. It is normally difficult to stabilize effort and catch simultaneously.

Do variations in effort and catch cause problems? There are several reasons why stock fluctuations may cause problems for the fishing industry. The fishing industry may be broadly divided into two sectors, the harvesting sector and the processing sector, and the effects of variations in the fish stock will be different for these two sectors. Variations in effort in the harvesting sector may be a problem as a part of the fleet will have to be idle for shorter or longer periods, resulting in uneven use of capital and labour. Variations in the catch cause problems for the harvesting sector because they imply variations in gross revenue and possibly also in net revenue. In real life the fluctuations are stochastic and uncertainty about future income makes long-run planning and investment decisions difficult.

Variations in the harvest also cause problems for the processing sector because they imply uneven use of resources and make planning more difficult. Such variability also makes marketing more difficult because a stable and secure supply to the markets can not be guaranteed. Many fish products can not be stored for a long time without deteriorating, and this effect can therefore not be compensated for by inventory buffers. There will be a

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[1] This is not the case if catch is independent of the stock size; in that special case stabilization of effort means stabilization of catch.

trade-off between the harvesting and the processing sector when it comes to stabilizing effort. Stabilizing effort in the harvesting sector will destabilize effort in the processing sector and vice versa. This is because a more stable effort in the harvesting sector will in most cases enhance the variability in catches due to the fluctuations in the fish stock, which in turn implies more unstable effort in the processing sector.

Formulating a model.

In practice the management of a fishery is based on multiple objectives. A tentative set of objectives may be:

1. Maximization of the social benefits from the fishery (consumers' and producers' surplus).
2. Conservation of the fish population (sustainability).
3. Securing stable employment both in the harvesting and processing sector.
4. Securing stable supply to the markets.

A mathematical optimization model usually consists of one objective function and a set of constraints. A practical way around the mathematical complexities accruing from the multi-objective problem is to convert all the objective functions except one into constraints.

The main objective of this thesis is to compare different management strategies primarily with respect to profitability but, as will be seen, also with respect to certain other aspects. Considerable emphasis will be put on the effects of stabilizing effort or catch. The Norwegian cod fishery in the Barents Sea will often be used as an example. The objective function will be to maximize the social benefits accruing from the fishing industry. As about 90 % of fish products in Norway are exported, the analysis can be simplified by excluding the consumers' surplus from the social benefit function. However, there are still problems in defining the social benefit function correctly. Switching from high levels of effort to low levels and vice versa may imply both direct and indirect costs. As these costs are difficult to quantify, the approach taken here is to place constraints on yearly variation in effort or catch. Another problem is that the opportunity cost of labour and other factors may not be the same as the private cost. As these switching costs, the opportunity cost of labour, etc., may be hard to measure, there will always be imperfect knowledge about the parameters in the social benefit function. In addition to these are the different kinds of uncertainty pertaining to stock assessment. Further, it may be difficult to formulate the social benefit function mathematically in such a way that we are guaranteed to find the global optimum.

Due to the theoretical nature of optimization models and the uncertainty about the parameter estimates used in such models, complex pulse fishing strategies resulting from the models may be judged unacceptable by the fishermen involved and will hardly be recommended by fisheries managers. Instead history shows that fisheries managers are inclined to rely on different biological reference points in formulating harvesting strategies. Such strategies are in fact based on more or less optimally chosen rules of thumb. Examples of such strategies are F_{max}, the fishing mortality supposed to maximize yield per recruit or sustainable yield in aggregate models (also called F_{MSY}), $F_{0.1}$, the fishing mortality corresponding to a point on the yield-per-recruit function with a slope that is 10% of the slope through the origin. Constant escapement policies[2], constant effort and constant catch policies are other examples of reference point based strategies. The constant F policies referred to may or may not imply constant effort depending on the production function. Elimination of the undesirable effects of fluctuations in stock biomass which translate into fluctuation in catch or effort, has high priority in the present work. Therefore, inspired by the reference point based strategies mentioned above, the main emphasis will be put on the constant effort and constant catch policies. To my knowledge no such comprehensive study comparing these two strategies has been performed earlier. The literature is spotty when it comes to explicit comparison of alternative harvesting strategies (Hilborn and Walters, 1992: p.470), and most of the papers that exist are mainly concerned with biological aspects, not economic, e.g. Ricker (1958), Ludwig (1981) and Murawski and Idoine (1989).

The present work will deal with controlling the catch through catch quotas, i.e. Total Allowable Catches (TAC). It will be assumed that also effort can be controlled indirectly through such schemes. This is based on the assumption that any quota will be taken by the minimum effort required in order to maximize total profits.

Constant catch.

The idea of implementing constant catch is not new to managers or scientists. In fact, the

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[2] Escapement is defined as the part of the fish stock that survives from one period to the next after fishing has taken place.

concept Maximum Sustainable Yield (MSY) has long been the predominant reference point in fisheries management. In a deterministic world this will imply both constant effort and constant harvest. The world is not deterministic, however, and the use of MSY as a target often results in a yield that is neither maximum nor sustainable. Many fish stocks have been characterized by overexploitation in relation to MSY, and the first step towards implementing constant catch has often been to reduce the exploitation rate and let the stock accumulate to a secure level. On the other hand, with constant effort also the variability of yield will increase with the exploitation rate and therefore there is in most cases a trade-off between the yield and the variance in yield. This relationship has been pointed out by several authors, see Section 1.3.

Constant catch has had a reputation of increasing risk and instability to the fish population and of being one of the least profitable strategies. It is quite likely that this will be true if the catch is kept at the MSY level, but as the present work will show this is not true under many other circumstances. One of the main purposes of the present work is to point out the circumstances under which constant catch can be a viable alternative that can compete with the other strategies with respect to profitability. In other words, the main task of the work presented here is to compare these strategies with the optimal strategy and with each other under a wide range of assumptions.

One advantage of the constant catch policy is that it requires a minimum of information in order to be implemented. Yearly stock assessments are all that is required to see whether the stock is large enough to support the constant catch or whether the total allowable catch has to be adjusted.

Both the constant catch and the constant effort strategy mentioned above imply a reduction in the variability of both catch and effort compared to a pulse fishing strategy. However, they also represent a reduction in profitability compared to the profit maximizing strategy. It is by no means clear that the reduction in profitability will be the same for both strategies. It is therefore a main concern of this study to compare these two strategies with respect to their profitability and to see how this compares to the optimal (pulse fishing) strategy. Other aspects that may be of interest when the strategies are compared include the stability of a possible equilibrium, variation in stock biomass and variation in net revenue. In order to draw any general robust conclusions, the comparisons have to be performed under a wide range of different assumptions concerning recruitment and density dependence in the stock relationships, such as density dependent natural mortality, individual growth and sexual maturity. Also a diversity of economic relationships have to be analysed.

Risk.

Risk deserves special attention, as it has to do with the possibility of implementing these strategies in practice. Risk is defined here as the probability that a critical or detrimental event will occur, in this case that the fish stock will be reduced to a critically low level. This aspect may exclude short-lived and highly fluctuating fish stocks, among which are many of the pelagic species like herring and capelin, as candidates for constant catch harvesting. For these stocks it may be impossible to guarantee that the stock will be big enough in any period to support a constant catch large enough to be of economic interest, and constant escapement is therefore usually considered to be the most appropriate management strategy. In order to implement a constant catch strategy it is necessary to have a fish stock with a long life-span such that the surviving part of the population can act as a buffer against undesirable events which may occur, e.g. low recruitment in several subsequent periods. The stock to be considered here, the Arcto-Norwegian cod stock, is a good example and representative of similar stocks. This is a very important stock economically. Furthermore, it is long-lived; for practical modelling purposes it consists of at least 12 year classes, though individual fish can be up to 50 years old (Tande & Tande, 1986).

To analyse the question of risk, stochastic methods are called for. These will be applied under different assumptions. Risk imposes a new constraint, namely that the stock can not be reduced below a certain level. This constraint may be incompatible with other constraints present. In that case the model may have no solution or alternatively the resulting constant catch may be so low that it is of no economic interest.

Risk aversion among fishermen or managers can be included in the objective function by letting this be concave instead of linear in net revenue. The consequences of having a concave objective function will be analysed in several places in this study. Also the consequences of some degree of monopoly-power on the output side and monopsony on the input side can and will be analysed by an appropriate formulation of the objective function, that is, by letting the price per unit of catch and effort depend on their respective quantities.

In addition to the constraints imposed by constant effort and constant catch, there are other constraints on the maximization which have to be taken into account. These are either of a biological or an economic nature. The most fundamental constraint on the biological side is the dynamic constraint, i.e., how does the fish stock reproduce and how does this reproduction depend upon the stock itself? Assumptions about natural growth, natural mortality, recruitment, cannibalism and so on are embedded in the dynamic constraint, and various assumptions about these factors will be examined to see how they affect the choice of harvesting strategy.

Summary of objectives.

To sum up, there is often a difference between how fisheries are managed in practice and what scientists claim to be the optimal management strategy from a purely economic viewpoint. Practical management is typically based on simple but sensible reference points. It is the purpose of this work to compare some of the reference points suggested for practical management with each other and with the optimal strategies based on optimization models under a variety of assumptions. Sometimes such reference point strategies are well suited for incorporating objectives that are not easily included in a mathematical model.

1.2 Traditional fisheries economics

The purpose of this section is to make reference to some models commonly applied in fisheries economics. We want to see under what circumstances such models lead to sustained fishing and under what circumstances pulse fishing patterns may occur.

A discrete-time model.

The model used here for the purposes of illustration is described as follows. Let the objective function be

$$J = \sum_{t=1}^{T} [ph(x_t,E_t) - cE_t] \tag{1.2.1}$$

where T is the length of the time horizon, x and E are stock biomass and effort respectively. c is the unit cost of effort, p is the unit price of harvest and h denotes the harvest function. Eq. (1.2.1) is to be maximized subject to the dynamic constraint

(1.2.2) $$x_{t+1} = x_t + G(x_t) - h(x_t, E_t).$$

where G is a discrete time surplus growth function. In addition there is a constraint on the stock biomass in period, $x_{T+1} \geq \bar{x}$ and an initial condition, $x_o = \bar{x}_o$. Let the production (harvest) function be given by

(1.2.3) $$h(x,E) = qEx,$$

and let G be the logistic growth function,

(1.2.4) $$G(x) = rx(1 - \frac{x}{K})$$

where r is the intrinsic growth rate (the maximum rate of growth obtained when $x \approx 0$) and K is the carrying capacity of the environment. If the solution of this model is restricted to a sustainable policy, i.e. $h = G(x)$, then the optimality conditions are given by:

(1.2.5) $$x^* = \frac{1}{2}(K + \frac{c}{pq}),$$

(1.2.6) $$E^* = \frac{1}{2}\frac{r}{q}(1 - \frac{c}{pqK})$$ and

(1.2.7) $$h^* = \frac{r}{4}(K - \frac{c^2}{p^2q^2K}).$$

Due to the sustainability requirement, such a policy will never be more profitable than the optimal pulse fishing pattern. The reason why an increase in net return can be achieved is that abandoning the sustainability requirement makes it possible to exploit the economies of scale implied by the production function. This is obtained by letting the stock build up over a certain period and then harvesting. The gain from this approach can be illustrated by a numerical example.

Let $T = 10$, $q = 1$, $r = 0.5$, $K = 10$, $p = 1$ and $c = 4$. Then $x^* = 7$, $E^* = 0.15$ and $h^* = 1.05$. Net return in the static model is 0.45 each year or 4.5 over a 10-year period.

By applying pulse fishing this result can be improved by more than 20%. The following optimal pattern emerges:

$E_3 = 0.671$, $h_3 = 5.926$, $\pi_3 = 3.244$,

$E_8 = 0.534$, $h_8 = 4.298$, $\pi_8 = 2.165$.

In all other years we have $E = h = 0$. The net revenue over 10 years then becomes 5.4. The problem with this model is that it does not take into account the direct and indirect costs of switching from zero effort to high effort and vice versa. If this cost is included, a much more even fishing pattern will occur. Alternatively there can be a constraint on the relative change of effort or harvest from one period to the next.

Discounting of the future will change the optimal fishing pattern but it will not change the line of reasoning made above. With a positive discount rate, δ, the optimal equilibrium stock level is given by:

$$(1.2.8) \qquad x^* = \frac{K}{4}\left\{\left(\frac{c}{pqK} + 1 - \frac{\delta}{r}\right) + \left[\left(\frac{c}{pqK} + 1 - \frac{\delta}{r}\right)^2 + \frac{8c\delta}{pqKr}\right]^{1/2}\right\},$$

(Clark, 1990: p.45). It is easily seen that Eq. (1.2.8) is equal to Eq. (1.2.5) when $\delta = 0$. However, since the benefits obtained by pulse fishing are somewhat delayed, they will be a bit reduced by discounting of the future.

Optimal control theory.

In the previous section it was seen that optimal pulse fishing patterns may occur in discrete time models when the sustainability requirement is abandoned due to economies of scale. Applying optimal control theory, it can be shown that pulse fishing may also occur in continuous time models when there are nonlinearities in the objective function. The traditional linear fisheries model

$$(1.2.9) \qquad \text{maximize} \int_o^\infty e^{-\delta t}(pqx - c)E\,dt$$

subject to

$$(1.2.10) \qquad \frac{dx}{dt} = F(x) - qEx$$

and

(1.2.11) $$0 \leq E(t) \leq E_{max}$$

leads to the optimal equilibrium condition

(1.2.12) $$F'(x^*) - \frac{cF(x^*)}{qx^*(pqx^* - c)} = \delta$$

where δ is the discount rate. This is derived in a straightforward manner using Pontryagin's maximum principle (see e.g. Clark (1990)). The optimal approach to equilibrium is a so-called bang-bang approach. That means maximum harvest when $x < x^*$ and zero harvest when $x > x^*$. In equilibrium both effort and harvest are constant. Also any deviation from this equilibrium due to unexpected distortions requires a bang-bang return.

In the case of nonlinearities in the objective function the optimum conditions may be different. Of particular interest is the case of nonincreasing marginal cost. In general, given the objective functional

(1.2.13) $$\text{maximize } J\{h\} = \int_{t_o}^{t_1} g(x,t,h)dt$$

subject to the state equation

(1.2.14) $$\frac{dx}{dt} = f_o(x,t) + f_1(x,t)h$$

an optimal control exists if the integrand $g(x,t,h)$ is concave in h. Let the functional for socially optimal management be given by

(1.2.15) $$g(x,t,h) = e^{-\delta t}[U(h) - C(x,h)],$$

with the cost function being separable

(1.2.16) $$C(x,h) = c(x)\phi(h)$$

where $\phi(h)$ is an increasing function of the harvest rate, $\phi'(h) > 0$. The social utility function, $U(h)$, is supposed to be increasing and concave in h. If the marginal cost is increasing, i.e. $\phi''(h) > 0$, then $g(x,t,h)$ is concave. If, on the other hand, $\phi''(h) < 0$, then the equilibrium solution x^* and h^* will no longer be optimal. In the latter case a so-called "chattering" control may occur. Assuming a maximum harvest rate,

(1.2.17) $$0 \leq h(t) \leq h_{max},$$

this means that $h(t) = 0$ for a certain period and then $h(t) = h_{max}$ for a certain period. For the proof of the optimality of this strategy see Clark (1990: p.149ff).

The Beverton-Holt model.

The Beverton-Holt model (Beverton & Holt, 1957) is a year-class model, and is described in more detail in Chapter three. Dynamic optimization is not as straightforward here as it is with the simpler aggregate biomass model. Clark (1990) states that: "Even with the simplifying assumption ... that recruitment is independent of stock size, an analytic solution for the general problem seems completely unattainable. We therefore make an additional assumption that *the costs of fishing are negligible;..*". Clark concludes that unless there is perfect selectivity in the fishery, the optimal pattern of fishing consists of a sequence of impulse controls at specific times. Perfect selectivity is rarely met in real fisheries. This result can easily be modified to allow for variable recruitment since the optimal age of capture is in fact independent of the recruitment level. Clark (1990: p.299f) has proved analytically that optimal pulse fishing produces a greater average yield than any sustained yield harvest using a simple non-selective multi-cohort model. Numerical optimization models indicate that optimal pulse fishing patterns occur also when costs are positive. Hannesson (1975) and Horwood (1987) prove this point applying non-linear programming. Hannesson uses constant recruitment and Horwood includes a stock-recruitment relationship. This question is also addressed in Chapter three using a fluctuating recruitment pattern under a variety of different assumptions.

The reason why pulse fishing may be optimal in the Beverton-Holt model is quite different from the other models referred to above. The main reason is that the selectivity of fishing gear is not optimally adjusted. Pulse fishing has also been observed in practice, especially by the large fishing fleets of Japan and the Soviet Union prior to the advent of the 200-mile extended economic zones in the late 1970s. Pulse fishing in practice, however, requires that there are alternative stocks to go to. In general we may conclude that the motivation for pulse fishing is more based on economic and technical factors than on biological factors.

Fluctuating or stochastic recruitment.

It has been shown above that a varying fishing pattern may well be optimal although there is constant recruitment or a deterministic stock-recruitment relationship. There is reason to believe that an optimal varying fishing pattern may be even more pronounced when the recruitment is characterized by exogenous fluctuations or stochasticity. This may be the case even without such factors as cost-nonlinearities or less than perfect selectivity.

Real fluctuations are in general stochastic although it may be possible to find traces of certain patterns using empirical analysis, see Chapter four. In this section reference will be made to a completely stochastic model analysed by Reed (1974, 1979). In the model recruitment in period k+1 is a function

$$R_{k+1} = Z_k F(R_k - H_k) \tag{1.2.18}$$

where Z_k are independent, identically distributed random variables and H_k is harvest. Define escapement as

$$S_k \equiv R_k - H_k, \quad 0 \leq S_k \leq R_k. \tag{1.2.19}$$

Using stochastic dynamic programming it can be shown that for all k < T (where T is the terminal period) the optimal harvest strategy is given by:

$$H^* = \begin{cases} R - S^* & \text{if } R > S^* \\ 0 & \text{otherwise} \end{cases}. \tag{1.2.20}$$

S* is given by

$$F'(S^*) \cdot \frac{p - \mathcal{E}_z\{ZC(ZF(S^*))\}}{p - C(S^*)} = \frac{1}{\delta}, \tag{1.2.21}$$

see Clark (1990: p.346). $\mathcal{E}$ denotes expectation, δ is the discount rate, p is a constant price and $C(\cdot)$ is the harvesting cost function. The optimal strategy in period T is to harvest all of the resource that is profitably captured. Once again it is seen that the optimal strategy implies both varying catch and varying effort.

The main conclusion from this section is that an optimum solution implying a sustained yield fishing, although commonly referred to in the literature, is based on quite special assumptions and may be a rare occurrence in practice. Under a variety of perhaps more realistic assumptions a varying fishing pattern will turn out to be optimal. This, however, is based on the assumption that switching the level of effort from year to year is free. Since fisheries management often are based on reference points such as constant effort or (less common) constant catch, it may be of interest to see how these alternatives compare to each other and to the optimal management strategy.

1.3 Review of the literature.

Most of the literature on harvest strategies has been concerned with finding the optimal strategy under different assumptions, e.g. Reed (1974 & 1979), Clark (1990) and Parma (1990). These works have mainly been of a theoretical nature and have had little impact on actual fisheries management. An attempt will be made here to give a review of some of the literature dealing with comparisons of harvesting strategies or with how to find the best harvesting strategy. Particular attention is devoted to articles with emphasis on stabilizing undesirable fluctuations in catch and effort or on the management implications of fluctuations in recruitment and stock biomass in general. (The review is given in a more or less chronological order.)

Ricker (1958) was one of the first to make comparisons of alternative harvesting strategies, namely constant effort versus constant escapement. His conclusions are:

1. The average catch obtained by constant escapement is greater than the catch obtained by constant fishing effort.
2. The variability of the catch obtained by constant escapement is greater than the one obtained by constant effort.

In Gatto and Rinaldi (1976) the mean value of the catch and its variability due to environmental fluctuations are analysed for a very general stock-recruitment model. Particular attention is devoted to comparison of the two fishing strategies, constant effort and constant escapement, in terms of mean catch, variance in catches and maximum deviation of catch. The authors demonstrate analytically that constant escapement policies will always give a higher mean catch, though such policies would give higher catch variance and more extreme catches under certain conditions of environmental variability. In other words, Gatto and Rinaldi prove analytically what Ricker (1958) showed by numerical methods.

Walters (1975) develops a method for incorporating the effects of environmental variability and judgmental uncertainty about future production parameters into the design of optimal harvesting strategies. For the case of the Skeena River sockeye the method suggests that optimal strategies are insensitive to judgmental uncertainty about the Ricker stock production parameters, but are very sensitive to objectives related to the mean and variance of catches. Strategies for reducing the variance of harvests imply higher and more predictable stocks, and trade-offs between the mean and variance of catches can be quantified along a Pareto frontier for negotiations. Walters also claims that the method outlined should be applicable to many fisheries, not only the Skeena River sockeye.

Doubleday (1976) examines the effect of random fluctuations in biological production on the success of fisheries management schemes using an aggregate biomass (Schaefer) model. Control of stock biomass, catch and effort are considered. After comparing the results from the different schemes the author concludes that the average yield taken is inversely related to yearly fluctuations in yield. Control of stock biomass maximizes the average yield but at the same time induces large fluctuations in catch. Control of catch requires a large reduction in average catch in order to obtain stability. The effects of controlling effort lie between those of controlling biomass and controlling catch. Doubleday also examines the recovery potential of an exploited stock to deviations from equilibrium and finds that a good compromise between yield, stability and conservation is to fix fishing effort at a level corresponding to an equilibrium biomass at two thirds of the virgin stock biomass. This conclusion may, however, be dependent on the particular model used by Doubleday.

Lett and Doubleday (1976) state that cycles and fluctuations in recruitment are the most important factors in determining the relative merits of different management strategies, with reference to the Southern Gulf of St. Lawrence Cod. Management by constant catch quotas was the poorest strategy considered, resulting in large fluctuations in stock size and effort, and leading to a collapse of the stock at lower levels of exploitation than could be maintained by other strategies. Controlling the fishing mortality rate (constant effort quotas) opposed the natural cyclic tendency resulting in stability of catch and stock biomass and high levels of yield.

Beddington and May (1977) find that as effort and yield are increased, animal populations that are being harvested for sustained yield will take longer to recover from environmentally imposed disturbances. One consequence is that the coefficient of variation of the yield increases as the point of maximum sustainable yield (MSY) is approached. When overexploitation has resulted in a population smaller than that of MSY, high effort produces a low average yield with a high variance. These effects are expected to be more pronounced for a harvesting strategy based on constant quotas than for one based on constant effort. The results are derived in an MSY context but are expected to apply also when the aim is to maximize discounted net economic revenue.

May et al. (1978) conclude on the basis of eight different stock-recruitment relationships that an increase in the level of exploitation (effort) will increase the absolute fluctuations in yield, especially when the rate of exploitation exceeds the MSY level.

Beddington (1978) considers the equilibrium conditions associated with different harvesting regimes using a mathematical model of the type applied for assessing quotas for baleen whales. That is, a time-delayed non-linear difference equation is used for updating. Different equilibria will have different degrees of stability and the relative stability of different strategies may then be used as an index to estimate the likely risks of driving a stock beyond MSY in a randomly varying environment. By comparing quota and effort management, the author shows that effort management is considerably more stable.

Beddington and Grenfell (1979) consider the problem of the safety of whale management strategies in the face of environmental variability. They find that even a rather small amount of variability in the natural mortality rate has a significant impact on $\bar{t}$ compared to a model with constant natural mortality where $\bar{t}$ is the mean time to move to protection status from some specified population level. They also question the use of $\bar{t}$ as a measure of the safety of a regime. They show that the frequency distribution of $\bar{t}$ is far from normal and changes markedly with slight changes in parameters. Therefore the mean value of $\bar{t}$ is a very poor measure of the risk. Using the probability of the population moving to the target level in t years, i.e. using the probability as a management criterion, is investigated as an improved alternative. Beddington and Grenfell conclude that there is still a need to compromise between yield and safety, particularly since the results are based upon the assumption of perfect information about stock performance and dynamics.

Horwood et al. (1979) use a time logistic model with stochastic variations in recruitment to calculate the time taken for a whale population to fall into the protected category (90% of the MSY level) for different management strategies including constant quota. They conclude that even with a constant quota strategy the practical effects of stochastic variability in recruitment in the sense that the stock is endangered would on average not be noticed over a period of several hundred years.

Doubleday (1980) observes that year-to-year fluctuations in recruitment have substantial economic consequences for the fishing industry. While fishery managers cannot eliminate or even accurately predict variations in fish stocks, means are available to reduce the unfavourable economic impact of such fluctuations. These options include catch insurance, control of fishing gear, limitation of fishing mortality and influencing the composition of fishing fleets. Doubleday claims that such measures, in some instances, can reduce year-to-year variation by 50% or more.

Horwood and Shepherd (1981) analyse the sensitivity of age-structured populations to perturbations in recruitment. They find that the sensitivity to such noise increases monotonically as exploitation increases and they also conclude that in general there is a trade-off between yield and stability of yield. These conclusions are reconfirmed in Horwood (1982) with application to the North Sea herring.

In Ludwig (1981) five different harvesting strategies are compared according to their effect on the expected discounted yield, the coefficient of variation of the yield and the expected time to reach 10% of carrying capacity. The five harvesting strategies are i) constant effort (at the level that maximizes the expected discounted yield), ii) the optimal strategy for maximizing expected discounted yield (pulse fishing), iii) the strategy of the International Whaling Commission (IWC) which reduces harvest quotas as the estimated stock drops below the MSY level, iv) a modified version of ii) with the switching point adjusted to increase the expected time to reach 10% of the carrying capacity and v) a version of ii) where the rate of change of effort is adjusted according to the current population and effort levels. Ludwig considers these strategies using three different models for population dynamics, namely the Beverton-Holt model, the logistic model and the Pella-Tomlinson model. Also two levels of population fluctuation are applied.

Differences in the performance of the strategies with respect to discounted yield are slight for small levels of fluctuation or where the return time for displacements from equilibrium is short. Substantial differences between the strategies appear, however, for high levels of random fluctuation in the population. Ludwig also shows how stochastic control theory can be used to improve harvesting strategies. The constant effort strategy has the poorest performance when judged by its expected discounted yield.

Mendelssohn (1982) compares the effects of changes in the discount rate with the effects of changes in risk aversion for a simple stochastic model of fish population dynamics. The numerical results suggest that optimal harvesting strategies and population dynamics are insensitive to changes in the discount factor but sensitive to changes in the degree of risk aversion. The higher the degree of risk aversion, the smoother the optimal harvesting policy.

In Deriso (1985) results of harvest strategies that optimize a risk averse management objective are presented. Harvest policies that maximize a logarithmic utility function are shown to differ substantially from the fixed escapement policies. Deriso finds that the logarithmic utility function will be appropriate when the fishery constitutes a major portion of the fishermen's income or price and costs vary with volume. He concludes that constant harvest rate strategies may have a number of advantages under these circumstances.

Hilborn (1987) presents an analysis of the types of uncertainties faced by resource managers and classifies the different types of uncertainties according to the frequency of occurrence. Managers develop ways for dealing with frequently occurring uncertainties. Infrequently occurring uncertainties are dealt with by an adaptive learning approach whereas uncertainties that occur rarely, called surprise, are very difficult to deal with.

Getz et al. (1987) evaluate the performance of three harvest policies in managing fisheries with different dynamics and conclude that policies that have very different implications for short-term yields achieve close to maximum expected sustainable yield. For example, the policy that maximized yield over a 10-year period resulted in only a 1% higher biomass than either the best constant effort or best constant escapement policy. The latter minimizes variation in annual stock size since each year the stock is harvested down to a certain level and the fishery is closed whenever the stock falls below this level. The constant effort policy has more variation in annual stock size but much less variation in annual catch. It is relatively easy for the manager to ensure long-term maximization of the yield; the problem is to find a compromise between short-term stability of the fishery and long-term viability. Getz et al. also claim that the most important source of uncertainty in estimating long-term productivity is density dependence in the stock-recruitment relationship, not environmental factors causing year-to-year fluctuations.

Sissenwine et al. (1988) look at some fisheries management implications of recruitment variability. One of their conclusions is that a constant catch strategy generally will result in a stock collapse if a sequence of poor recruitment occurs. This problem is most likely to occur if there is non-stationarity and autocorrelation in the recruitment series.

Murawski and Idoine (1989) use Monte Carlo yield models with stochastic recruitment functions to examine the effects of constant catch policies on fishing mortality, catch and stock size. Three additional variables were used as indices of fishery variability and risk: i) the change in catch between years, ii) the number of successive years the target constant catch was attained and iii) the relative number of years the target constant catch was not attained. Two stochastic recruitment models were used; one in which recruitment was independent of spawning stock biomass and one with a stock-recruitment relationship included. For the stock-recruitment model the ability to sustain moderate yields was critically dependent on rehabilitation strategies for years in which the total stock size was too low to support the constant catch target. Fluctuations in annual yields can be reduced by accumulating harvestable stock and setting constant catch targets that result in moderate to low fishing mortality. The desirability of such strategies is determined by the relative value of averting the risk of undesirable stock and catch fluctuations versus the costs of forgone yields due to natural mortality and density dependent processes.

In addition to comparing different constant catch policies, Murawski and Idoine also compare these with what they call the most common alternative, namely constant fishing effort. Two constant fishing mortalities are considered, $F_{0.1}$ and F_{max}. $F_{0.1}$ is the fishing mortality corresponding to a point on the yield-per-recruit function with a slope that is 10% of the slope through the origin, and F_{max} is the fishing mortality that maximizes yield per recruit. Not surprisingly they find that such policies lead to more variation in catch, less variation in stock biomass and lower average catch. The best constant catch policy compares with a $F_{0.1}$ policy when it comes to average fishing mortality and average stock size. Murawski and Idoine also mention the benefits of constant catch policies coming from a simplified system of stock assessment and management advice compared to constant fishing mortality. Conservative policies of constant catch should require only routine assessment updates to ensure stock and fishery performance within acceptable bounds whereas constant fishing mortality policies require annual projections of stock and catch to establish catch quotas.

Hannesson (1989) poses the question of whether it may sometimes be desirable to have some variability in fish stock abundance and catches. This question is analysed in the context of purely stochastic variations in the stock with overlapping generations, and also in the context of more sophisticated population dynamics (year-class model). The conclusion is that variability of fish stocks is not necessarily a bad thing since a larger stock may mean a lower unit cost of fishing; risk aversion and irreversible investments are not taken into account. A lower natural mortality rate increases the desirability of stable catch quotas compared to stable effort or fishing mortality.

In Horwood et al. (1990) the objective function is to minimize a weighted sum of the variances in effort and catch and then find a feed-back control law to determine annual quotas. A target level for catch and effort is agreed upon and the objective is to minimize variations around this level. One of the conclusions is that at low target levels of fishing mortality it is optimal to put high emphasis on stabilizing deviations in catch relative to deviations in effort, but completely constant catch does not turn out to be optimal. As the target level of fishing mortality increases, the optimal policy tends towards a constant effort policy; and for the majority of target fishing mortalities the optimal regime seems to lie closer to constant effort than to constant catch. The costs of variability also increase with increasing target fishing mortality.

Pelletier and Laurec (1990) point out that classical management strategies try to maximize different criteria such as production in weight or in value, or stabilization of fishing effort or yield, and that these objectives are in principle incompatible. They try to determine intermediate TAC rules that constitute a compromise between several criteria with reference to the North Sea cod. Pelletier and Laurec conclude that compound strategies can be found that outweigh classical rules like F_{max} and $F_{0.1}$ when several objectives are taken into consideration, and that gains can be obtained with respect to stability of yield and fishing effort without necessarily implying losses with regard to other criteria.

In MacLennan et al. (1991) the effect of fishing mortality on the variation in yield is examined with particular attention to the changes in stock size from one year to the next, and it is shown that the variability of the stock increases with increased fishing mortality. The authors also use a model in which fishing mortality is a function of age to show that the optimum gear selectivity for the most stable yield depends on the level of exploitation. When the exploitation rate is high, it is better to have a selectivity function which increases steadily with age rather than a knife-edged selectivity pattern.

In conclusion, the most striking common agreement in the papers reviewed here seems to be that there will always be a trade-off between the size of the catch and the variability in catch. Of the three strategies constant catch, constant effort and constant escapement, constant catch is ranked as the poorest and constant escapement is ranked as the best when it comes to catch in biomass. When it comes to stability of the catch, the ranking is reversed. Furthermore, the most dominant common conclusion seems to be that the higher the exploitation rate, the higher the variation in catch and effort. It is also a common conclusion that there is a trade-off between variability in catch and effort and that different strategies cause different degrees of variation in the stock biomass. Complete simultaneous stability of both catch and effort is unachievable in practice except at the level zero or with the special production function mentioned in Footnote 1. Common for many of these papers, however, is that they explicitly or implicitly assume that constant catch is synonymous to MSY. It has already been pointed out by several authors that trying to apply MSY as a constant catch will perform poorly and may even be detrimental. Other authors have found that a constant catch corresponding to a fishing mortality of $F_{0.1}$ may perform well without inducing too high risk and may be interesting to compare with a fixed mortality at the level $F_{0.1}$. Economists noticed long ago that MSY is not the appropriate reference point except perhaps when the cost of effort and the discount rate are both zero. Many, if not most, of the papers mentioned above do not explicitly take economic variables or relationships into account. Maximization of sustainable yield (in weight) seems to be the most dominant objective, which means that the value of the yield or the costs of effort are not considered. Furthermore, most of the papers ignore the effect that a positive discount rate may have and do not consider risk aversion.

The work presented here attempts to compare different harvesting strategies using a diversity of models and a wide range of assumptions concerning biological and economic relationships. The strategies considered are chosen in a manner that puts the main emphasis on compensating for the undesirable effects of large fluctuations in stock biomass. The comparisons are mainly concerned with profitability but also consider other aspects such as risk, stability and the possibility of implementation of the strategies. The study may be said to differ from earlier work by being more comprehensive. It includes a single-cohort as well as a multi-cohort model and it includes both deterministic and stochastic analysis. Furthermore, it considers non-malleable fixed capital and multi-species modelling, areas which have not received much attention in earlier works. The study consists of a theoretical/analytical part as well as a numerical/quasi-empirical part. Altogether this makes the conclusions fairly robust and applicable to many different situations.

1.4 Methods and organization

A wide range of methods are used in this thesis, but most of the analysis is based on bio-economic modelling. This means that a biological model explaining the population dynamics is combined with an economic model which contains the objective function and some economic constraints and relationships. The two models are linked by the production function in the economic model which produces the fishing mortality in the biological model affecting the population dynamics. Mathematical tools used include difference and differential equations. These are used for the updating of the fish stock; difference equations in the case of aggregated biomass models and differential equations for the year-class models. Non-linear programming is used to find the optimal harvesting strategy subject to various conditions in the case of a year-class model. In Chapter two the use of dynamic programming to find the optimal harvesting strategy is outlined for the aggregate model. Monte Carlo simulations are used both in Chapters two and three to evaluate performances in the stochastic versions of the models. In Chapter four spectral analysis is applied in order to analyse time series of recruitment data. Software tools used include simulation programmes written in Fortran 77 and numerical optimization programmes using GAMS-MINOS for the non-linear programming.

The outline of the thesis is as follows. Chapter two contains an aggregate biomass model, that is, the stock is not divided into different cohorts or year-classes. In this chapter the impact on strategy choice of a downward sloping demand function for fish products, increasing cost per unit effort, risk-aversion, alternative production functions and investment in non-malleable capital are analysed on the economic side of the model. Density dependent natural mortality and a spawning stock-recruitment relationship are analysed on the biological side of the model, and the question of multi-species modelling is briefly considered. In addition, different aspects of stochasticity are included and stability properties of the dynamic model are investigated.

In Chapter three a multi-cohort model is introduced. Many of the problems dealt with in Chapter two are revisited and some new problems are addressed, such as the importance of an age-structure in the population, density dependent individual growth, density dependent sexual maturity and selective fishing according to the age of the fish. In this chapter recruitment of new cohorts to the stock is supposed to be either fluctuating according to a deterministic pattern or stochastic.

Chapter four is an empirical analysis of recruitment. As it is hard to find any significant relationship between recruitment and the spawning stock, effort is devoted to finding significant cyclical fluctuations in recruitment and also relating these to variations in temperature.

In Chapter five the results are summarized and some management implications are discussed.

2 ANALYSIS OF AN AGGREGATE BIOMASS MODEL

2.1 An outline of the aggregate model.

This section starts with a simple biomass model with some basic properties which depend upon the assumptions about the production function only. This model will be useful as a reference when more sophisticated elaborations are introduced later. The main results in this section can also be found in Hannesson and Steinshamn (1991).

The purpose is to investigate the results of fixing effort versus fixing catch when the biomass varies in some way. Whether the variations in biomass are stochastic or deterministic is, in fact, of minor importance here. Starting with an aggregated biomass model here proves helpful both as a useful introduction to and as a necessary supplement to the disaggregated model in Chapter three. That is because many of the results that are unequivocal in the aggregated model become ambiguous in the disaggregated model. This is also the reason why this chapter is devoted to the aggregated model when, in fact, such a model is only a special case of the disaggregated model in Chapter three, i.e. the special case of one instead of n year-classes.

This simple model consists of three variables and a production function combining them. The variables are: u (catch), E (effort) and B (biomass). All units are arbitrary. u is a function of B and E:

$$u = f(B,E), \tag{2.1.1}$$

and the inverse of this function with respect to E is supposed to exist and is defined as:

$$E = g(B,u). \tag{2.1.2}$$

g(B,u) is the minimum effort required to take a given catch u when the stock size is B. It is assumed that both f and g are continuous functions which are at least twice differentiable in both their arguments. Define $\mathcal{E}$ as the expectation operator although $\mathcal{E}$ can also be thought of as taking some average of future periods in a deterministic, intertemporal model; i.e. a model in which B varies but the future values of B are known with certainty. Then it can be shown that the following proposition is true:

Proposition

If $\partial^2 f/\partial B^2 \geq 0$ and $\partial^2 g/\partial B^2 \geq 0$, and one of the $\geq$ are strict inequalities, then

$$\{\mathcal{E}_B g(B;\bar{u}) \,|\, (\bar{u} = \mathcal{E}_B f(B;E_c))\} > E_c. \qquad (2.1.3)$$

Proof

If $\partial^2 g/\partial B^2 > 0$ then $\mathcal{E}_B g(B;\bar{u}) > g(\mathcal{E}B;\bar{u})$. If $\partial^2 f/\partial B^2 = 0$ then $\bar{u} = \mathcal{E}_B f(B;E_c) = f(\mathcal{E}B;E_c)$. $g(\mathcal{E}B;\bar{u}) = g(\mathcal{E}B;f(\mathcal{E}B;E_c)) = E_c$ since g and f are inverse functions. Thus we have $\mathcal{E}_B g(B;\bar{u}) > E_c$.

If, on the other hand, $\partial^2 g/\partial B^2 = 0$ then $\mathcal{E}_B g(B;\bar{u}) = g(\mathcal{E}B;\bar{u})$. However, if $\partial^2 f/\partial B^2 > 0$ then $\bar{u} = \mathcal{E}_B f(B;E_c) > f(\mathcal{E}B;E_c)$ and $g(\mathcal{E}B;\bar{u}) > g(\mathcal{E}B;f(\mathcal{E}B;E_c)) = E_c \Rightarrow \mathcal{E}_B g(B;\bar{u}) > E_c$.

If both $\partial^2 f/\partial B^2$ and $\partial^2 g/\partial B^2$ are strictly greater than zero, we have by the same reasoning that $\mathcal{E}_B g(B;\bar{u}) > E_c$. QED.

Put in words, this proposition says that the expected effort required to take a given constant catch, $\bar{u}$, is greater than the constant effort, E_c, which gives rise to $\bar{u}$ as an expected catch.

This proposition could, as an alternative, equivalently be formulated as:

$$\{\mathcal{E}_B f(B;\bar{E}) \,|\, (\bar{E} = \mathcal{E}_B g(B;u_c))\} > u_c \qquad (2.1.4)$$

given the same assumptions about the second derivatives. This states that the expected catch of a given constant level of effort is greater than the constant catch requiring the same expected level of effort. As mentioned earlier, the expectation here can also be thought of as averages over time in a deterministic model.

The assumptions that the second derivatives with respect to B should be greater than or equal to zero are sufficient but not necessary conditions. E.g. the assumption that

$\partial^2 f/\partial B^2 > 0$ is quite unlikely other than perhaps within a small range of its domain. It may be more likely that $\partial^2 f/\partial B^2 < 0$, but it is still easy to find examples that the proposition above is valid.

Since g is the inverse of f, we also have the relationship:

$$g_{BB} = \frac{-f_{BB}f_E^2 + 2f_{EB}f_Ef_B - f_{EE}f_B^2}{f_E^3} \tag{2.1.5}$$

where subscripts are used to denote partial derivatives. This clearly shows that $g_{BB} > 0$ if $f_{BB} \leq 0$ since it is assumed that f_E, f_B and f_{EB} are all greater than zero and $f_{EE} \leq 0$.

What then are the limitations and the usefulness of the model and the proposition presented here? The main reason why this proposition is useful is that it offers a hypothesis which one may want to test within the framework of more detailed, complex and sophisticated models. The model above also suggests something about the economic performance of the alternative strategies of fixing either effort or catch in a world where prices and other parameters are constant. Since the variables are measured in arbitrary units, one can easily imagine that units are chosen such that per unit price of catch and per unit cost of effort both equal one. The proposition above then states that a fixed effort strategy is expected to be more profitable than a fixed catch strategy, and this is the hypothesis which the rest of this study is devoted to testing under different assumptions.

The following points may have an impact on the choice of harvesting strategy and will therefore be included in the analysis in due course.

On the economic side:
- Non-linearities in revenues and costs, i.e. a downward sloping demand curve or costs which are convex in effort.
- Risk-aversion, i.e. a utility function which is concave in revenue.
- The implications of changing the discount rate.
- The implications of changing the production function.
- The implications of investment in fixed capital.

On the biological side:

- Using a disaggregated model, i.e. introducing several cohorts.
- Introducing population dynamics, i.e. a spawning-stock/recruitment relationship which may include a density dependent sexual maturity pattern.
- Density dependent natural mortality.
- Density dependent individual growth.
- Multi-species modelling.

2.2 The Economic Model

In the economic part of the model the consequences of some of the natural extensions of the model can be analysed by simple analytic means. The intention is therefore not to go into any great detail with this model here but rather to list some of the extensions which may be of interest and analyse their implications. The magnitude of these implications is, however, an empirical question.

Consider the downward sloping demand function first. If the total revenue is not simply given by pu where p is the per unit price of fish but rather by p(u)u and $p'(u) < 0$, the partial derivative $TR'(u) = p'(u)u + p(u)$ and the second partial derivative $TR''(u) = p''(u)u + 2p'(u)$. Assuming that TR is a continuous and twice differentiable function of u, the condition that the second derivative is everywhere negative is sufficient to make the revenue of a constant catch, u_c, higher than the revenue of a variable catch with the same expected value. This condition can be written

$$-\frac{p''(u)}{p'(u)}u < 2. \tag{2.2.1}$$

The lefthand side of this equality is a relative measure of concavity. This condition is certainly fulfilled if $p''(u) \leq 0$. Usually it will be assumed that TR is a concave function whenever p is a function of u. Whether this condition is fulfilled or not therefore depends on whether the fishermen have market power or whether they are price takers. If they have market power, then it is reasonable to assume that the total revenue function is concave. This will be a question for empirical investigation in each case.

Thus the existence of market power, i.e a downward sloping demand function, will offset the result of the proposition. If the degree of concavity in the total revenue function is large enough, then the result concerning constant effort versus constant catch may be reversed.

Another way concavity of the revenue function can be induced is by evaluating not the revenue as such but rather the utility of the revenue. If this utility function is concave, then the fishermen are risk-averse and therefore prefer a stable income to an unstable income with the same expectation. However, it is not satisfactory to use total revenue as an argument in the utility function; rather the net revenue should be used, and this will vary when both effort and catch are fixed. As will be pointed out in a later section, net revenue is likely to vary more when effort is fixed than when catch is fixed and it is therefore reasonable to assume that a concave utility function with net revenue as its argument will favour constant catch at the expense of constant effort.

Another non-linearity which may affect the choice of strategy occurs when costs are not simply given by cE where c is the per unit price (cost) of effort but rather by c(E)E, i.e. the per unit cost of effort is a function of the level of effort applied. $c'(E) > 0$ since the price paid per unit input tends to increase the higher the demand is, in other words, c(E) is the supply function for E. In the case of constant effort this will have no effect whereas in the case where E is variable the net revenue is given by $\pi = p\bar{u} - c(E)E$, $\pi'(E) = -c'(E)E - c(E)$, and $\pi''(E) = -c''(E)E - 2c'(E)$. π is concave in E if $\pi''(E) < 0$, i.e. if

$$- \frac{c''(E)E}{c'(E)} < 2. \tag{2.2.2}$$

Since $c'(E) > 0$ this is certainly fulfilled when $c''(E) > 0$. It is also fulfilled, however, if $c''(E) < 0$ but $|c''(E)| < 2c'(E)/E$. In this case a fixed effort will be preferred to a variable effort yielding the same catch on average.

If the objective is not merely to maximize the expected net return but rather the present value of the future net income stream, then the discount rate and changes in the discount rate will be of importance. These effects are not so easily predicted. First of all they depend upon the expectations regarding the near future. If a relatively abundant stock of fish is expected in the near future then the net revenue expectations are above normal for this period. Since variations in net revenue are assumed to be greater with fixed effort than with fixed catch, and a high discount rate puts more weight on the near future at the expense of the more distant future, a high discount rate will favour fixed effort to fixed catch in this case. It is assumed here that the expected net revenue in the long run is the same with both strategies. On the other hand, if less than normal abundance in the stock is expected for the near future, net revenue expectations for this period are below normal and the effect of a high discount rate is reversed. Another way to view this matter is to say that a constant catch strategy is more resistant to a high discount rate since variations in net revenue are smaller. However, also in this case an exact assessment of the importance of the question can only be obtained by considering a real world fishery.

Another question which may be of importance in the economic model is what technology is prevailing in the fishery. In other words, is it possible to specify the production function any further and what implications will this have concerning the choice of quota strategy?

The traditional way to classify fisheries according to their technologies is to distinguish between interactive and non-interactive fisheries. These are also known by other names; e.g. search fishery for the interactive fishery and schooling fishery for the non-interactive (see e.g. Neher(1990)). There is a continuum between interactive and non-interactive fisheries, but in its extreme the interactive fishery is characterized by the fact that instantaneous fishing mortality increases proportionally with effort and is independent of the size of the fish stock whereas total fishing mortality, i.e. fishing mortality over some time interval, increases less than proportionally with effort. At the other extreme, in the case of a completely non-interactive fishery, the instantaneous fishing mortality is inversely related to stock size; i.e. fishing mortality approaches infinity as the stock size approaches zero, whereas the total mortality is proportional to effort and independent of stock size. In other words, the term interactivity refers to the situation in which adding a unit of effort will decrease the catch per unit of effort over time as the stock is depleted.

These concepts and their implications in a multi-cohort fishery will be elaborated further in Chapter three. It suffices here to state that within an aggregate biomass model the question of how to choose between fixed effort quotas and fixed catch quotas is only interesting to the extent that the fishery is interactive. With a completely non-interactive fishery the catch is proportional to effort and fixing one is equivalent to fixing the other. This is also the case in a multi-cohort fishery, but since interactivity can be measured along a continuum, it may be interesting to study empirically the effect of a change in the degree of interactivity. In general one may conclude that the smaller the degree of interactivity, the more equal the two strategies, and in the extreme case of non-interactivity they are the same.

The last topic that will be dealt with to some extent in the economic model is the role of investment in fixed capital. The reason why this may affect the choice of strategy is that capital used in fisheries tends to be nonmalleable. If this were not the case, and capital in the form of fishing vessels could be bought and sold at any time, this question would be of no importance. Intuitively it seems clear that with a fixed level of effort, the problem connected with irreversible investments will not arise since the catch quotas are based on this level of effort and therefore there is always sufficient capacity to take the quotas. In other words, the problem of excess capacity or lack of capacity is absent. On the other hand, if there are constant catch quotas from year to year and a fluctuating fish stock, then it may well be possible that the level of effort is too small to catch the quota in a year when the stock is small, such that one has to invest in new vessels, whereas some years later the stock may be more abundant and one is faced with excess capacity incurring "unnecessary" costs.

This is considered further in sections 2.6.3 and 3.4, where the purpose is to investigate the extent to which the degree of irreversibility affects the choice between strategies. It will be shown that there are several ways to model this irreversibility, e.g. depending upon whether the age structure of the fishing fleet is taken into account or not. It is quite clear that the level of irreversibility may be of importance and that a high degree of irreversibility probably will favour constant effort. However, the extent to which this will have a significant impact on the choice of strategy for a particular fishery, is an empirical matter.

When it comes to the expected importance of these different extensions of the economic model, it seems to be in order here to quote from Mendelssohn (1982). He compares the effects of changes in the discount factor with the effects of changes in risk-aversion for a stochastic model of fish population dynamics applying dynamic programming. His main conclusion is that: "..policy often is insensitive to changes in the discount factor over a range of values likely to be found in practice." Further, "..when the utility functions are viewed as total revenue curves with marginal price sensitive to supply, the results suggest that for optimally managed fisheries highly supply-sensitive prices will lead to stabilized harvests, whether the fishery is economically or biologically stochastic." (Mendelssohn, 1982: p.1256). These conclusions are derived from numerical calculations and are in fact interesting results. However, it remains to be seen whether the results also apply in a model with several cohorts. Another question which will be addressed in the following, is whether a stabilization of effort may be a substitute for stabilizing harvest when it comes to analysing the effects of risk-aversion, since both tend to stabilize net income, although not to the same degree. As Mendelssohn does not consider the possibility of stabilizing effort as an alternative to stabilizing harvest, it may well be that the introduction of risk-aversion does not have such a great impact when these two policies are to be compared.

2.2.1 Revenue variations and choice of strategy

The purpose of this section is to investigate the variations in net revenues resulting from different management strategies with main emphasis on the two relatively extreme strategies which are prominent in this work, viz. the constant effort and constant catch strategy. As pointed out in Chapter one there are several reasons why we would want to reduce the variability of income. Fluctuations in net revenue may have an impact on the performance of the two strategies when this is measured by the discounted total net revenue since the discounted net revenue can be compared with a weighted average where income in the near future is more heavily weighted than income further into the future. Therefore great fluctuations in the near future may have a great impact on the net present value.

Looking at these two strategies it is seen that annual net revenue from a constant effort strategy can be written

$$\pi_t(E_c) = pu_t(B_t;E_c) - cE_c \tag{2.2.3}$$

where p is price per unit catch and c is price per unit effort. It is assumed that instantaneous fishing mortality, F, is proportional to the level of effort, i.e. $F = qE$, where q is the catchability coefficient. Here q is chosen so that E and F are the same units, in other words $q = 1$ and c has to be chosen accordingly. cE_c is constant and will be defined as C. The variations in net revenue with this strategy come from variations in catch which again come from variations in the fish population, B. The expected net revenue from this strategy can be written

(2.2.4) $$\mathcal{E}\pi(E_c) = p\mathcal{E}u(B;E_c) - C,$$

with $\mathcal{E}$ denoting expectations. The variance is given by

(2.2.5) $$\text{var } \pi(E_c) = p^2 \text{var } u(B;E_c).$$

Annual net revenue from the alternative strategy, constant catch, is given by

(2.2.6) $$\pi_t(u_c) = pu_c - cE_t(B_t;u_c).$$

Variations in net revenue come from the cost side due to variations in the effort required to take the given catch when the fish population B varies. The gross revenue, pu_c, is constant and will be denoted R. The expected net revenue with this strategy is given by

(2.2.7) $$\mathcal{E}\pi(u_c) = R - c\mathcal{E}E(B;u_c),$$

and the variation is given by

(2.2.8) $$\text{var } \pi(u_c) = c^2 \text{var } E(B;u_c).$$

If variations in net revenue are greater with fixed effort than with fixed catch, this can be stated as

(2.2.9) $$p^2 \text{var } u(B;E_c) > c^2 \text{var } E(B;u_c).$$

Obviously the probability that the inequality above is valid is higher the lower the c/p ratio.

First of all, it must be stated that the expected net revenue from a constant effort strategy is higher than the expected net revenue from the constant catch strategy under relatively weak assumptions when the strategies are obliged either to yield the same average catch or to use the same average level of effort. This is the case, for example, when the effort required to take a given catch is a convex function of the fish stock, in other words, the catch-isoclines in the effort/stock diagram are convex curves. In this case the average catch coming from a fixed level of effort is higher than the highest constant catch which requires the same average level of effort. For this to be true it is sufficient that the catch is a linearly increasing function of the stock for a given level of effort and that the catch is a concave function of effort for a given stock level as is the case with the Beverton-Holt model. For a proof see Section 2.1 or Hannesson and Steinshamn (1990).

However, the whole story is not told by the expected net revenues. If, for example, gross revenue is concave in catch instead of linear (e.g. due to a downward sloping demand curve), this may improve the case for the constant catch strategy. On the other hand, if costs increase more than proportionally when effort increases, i.e. cost as a function of effort is convex, this may improve the case for the constant effort strategy. Also discounting of the future may make it more difficult to decide which strategy is better.

Another point which can make it difficult to decide between the two strategies is that their performance need not be measured by their net revenues but by the utility yielded by these net revenues. A concave utility function indicating risk-aversion among fishermen, means that a stable income will be preferred to a more variable income. In other words, the strategy with less variability in net revenue measured by its variance will be preferred, and this can even outweigh a higher expected net revenue. It has already been mentioned that the c/p ratio is of importance when variability in income is being discussed. Since units of catch and units of effort can be chosen freely, however, this ratio itself is of little help.

It is then more helpful to compare the two variances var $u(\cdot;E_c)$ and var $E(\cdot;u_c)$. The following formula is valid assuming that yield, u, is linear in stock size, B:

$$\text{(2.2.10)} \qquad \text{var } u(\cdot;E_c) = \left[\frac{\partial u}{\partial B}\right]^2 \text{var (B)}$$

since $\partial u/\partial B$ is a constant. On the other hand, assuming that u is concave in effort, E, the approximate formula:

$$\text{(2.2.11)} \qquad \text{var } E(\cdot;u_c) \simeq \left[\frac{\partial E}{\partial B}\right]^2 \text{var (B)},$$

which often is useful in applications, is unfortunately of little help here since $(\mathcal{E}E)^2$ is small compared to var (B) (Hammersley and Handscomb, 1967: p. 14). Therefore application of this formula here would underestimate var $E(\cdot;u_c)$.

The first conclusion which may be drawn from this is that some experimentation with numerical examples is necessary in order to answer the question posed in the beginning. The second conclusion is that the result of this numerical experimentation is likely to show that the $\text{var}\pi(u_c)$ is almost always less than the $\text{var}\pi(E_c)$ when choosing reasonable values of the economic parameters and applying fishing mortality production functions with the same characteristics as the Beverton-Holt model. In other words, the catch is a linearly increasing function of the fish stock and an increasing and concave function of effort. The numerical examples also indicate that $\text{var}\pi(u_c)$ is only greater than $\text{var}\pi(E_c)$ when the c/p ratio is so low that the fishery yields a negative net return. In section 2.4 an analytic proof for this is provided under certain assumptions. The numerical results occur as a by-product of the simulations in section 2.5.

2.3 Population fluctuations and choice of strategy.

If the main objective is to stabilize income in the harvesting sector, results from earlier sections indicate that constant catch may be the most efficient way to achieve this. Intuition suggests, however, that the problem with this policy is that under certain circumstances it may endanger the population. In other words, such a policy may reduce the population to a critically low level and create undesirably large fluctuations in the population. The purpose of this section is to investigate under which assumptions these claims are most likely to hold true.

It seems rather obvious that the degree of fluctuation in the total population will depend upon the harvesting strategy chosen. Furthermore, it seems likely that the constant catch strategy will imply greater fluctuations than the constant effort strategy, but, as will be shown, this is not always the case.

If the population consists of non-overlapping generations with little or no significant relationship between the parent stock and subsequent recruitment such that the population varies independently of any prior harvesting pattern, constant catch will imply greater fluctuations. With constant catch the same quantity will be harvested whether the population is large or small, and the absolute variation in population will be the same with harvesting as without (the variation relative to the total biomass will even be larger since the population is smaller when harvesting takes place). With constant effort, on the other hand, the variation in population size will be reduced since the harvest is proportional to biomass level, i.e. the harvest is large when the population is large and small when the population is small.

This argument does not necessarily carry over to a fish stock with overlapping generations as the following simple model will show. Suppose that the dynamics are given by the equation:

$$B_{t+1} = s(B_t - u) + w_t \tag{2.3.1}$$

with constant catch u, or

$$B_{t+1} = s(1 - E)B_t + w_t \tag{2.3.2}$$

with constant effort E. Here B denotes stock biomass, s is the survival parameter and w is recruitment. Implicitly (2.3.1) and (2.3.2) assume the production function $u = EB$. Suppose, further, that recruitment is given by the sequence

$$\{w_t\} = \{w_1, w_2, w_1, w_2, \ldots\} \tag{2.3.3}$$

with only two values that repeat themselves such that the population will be a corresponding, oscillatory sequence

$$\{B_t\} = \{B_1, B_2, B_1, B_2, \ldots\}. \tag{2.3.4}$$

This sequence represents a special kind of stable equilibrium under certain conditions, namely that $|s| < 1$. The conditions for stability are the topic of Section 2.3. With constant catch

$$B_i = \frac{s(w_i - u) + w_j - s^2 u}{1 - s^2} \qquad i,j = 1,2 \qquad i \neq j, \tag{2.3.5}$$

and average biomass is given by

$$\bar{B}_u = \frac{(1+s)(w_1 + w_2 - 2su)}{2(1 - s^2)}. \tag{2.3.6}$$

The variance in biomass is given by

$$\begin{aligned} \operatorname{var}(B_u) &= \frac{(1-s)^2 (w_1 - w_2)^2}{4(1 - s^2)^2} \\ &= \frac{(1-s)^2}{(1-s^2)^2} \operatorname{var}(w). \end{aligned} \tag{2.3.7}$$

It is seen that the variance in biomass given by Eq. (2.3.7) is independent of the constant harvest u and is therefore equal to the variance in biomass without any harvesting. With constant effort, E:

$$(2.3.8) \qquad B_i = \frac{s(1-E)w_i + w_j}{1-s^2(1-E)^2} \qquad i,j = 1,2 \qquad i \neq j$$

and average biomass is given by:

$$(2.3.9) \qquad \bar{B}_E = \frac{(w_1+w_2)(1+s(1-E))}{2[1-s^2(1-E)^2]}.$$

The variance in biomass is then given by

$$(2.3.10) \qquad \mathrm{var}(B_E) = \frac{[1-s(1-E)]^2(w_1-w_2)^2}{4[1-s^2(1-E)^2]^2} \mathrm{var}\ (w)$$

$$= \frac{[1-s(1-E)]^2}{[1-s^2(1-E)^2]^2} \mathrm{var}\ (w).$$

It is readily seen that $\mathrm{var}(B_u) = \mathrm{var}(B_E)$ if $E = 0$. Further, it can be seen that $\mathrm{var}(B_u) < \mathrm{var}(B_E)$ whenever $0 < E < 1$. The conclusion so far is that with constant catch the variance in biomass is the same as without any harvesting, whereas with constant effort the variance will never be less than this. This result runs counter to the intuition derived from a model without overlapping generations. The next question is: does this argument carry over to longer recruitment cycles, and not only the one which is used here for simplicity as an introduction. The answer is that it does not carry over in general but rather that it depends on the situation. If a longer repeated sequence of recruitment is used, constant effort may imply less variation in biomass than constant catch. This is certainly the case if the repeated sequence is long and marked by a high degree of auto-correlation. This can be shown numerically by, for example, using a trigonometric function to create the repeated recruitment sequence. The reason why this is so, is that with a long sequence and with high auto-correlation there will be many consecutive periods with low recruitment, and the constant catch strategy may reinforce the effect this has on the total population more than the constant effort strategy will, thus forcing the biomass down to a critically low level. The constant effort strategy compensates for consecutive periods with low recruitment by lowering harvest, since total harvest here is proportional to the biomass level.

Note that no matter how long the repeated recruitment sequence is, $\mathrm{var}(B_u)$ will be independent of u. When the recruitment sequence is short, $E > 0$ will imply an increase in $\mathrm{var}(B_E)$. For example when the length of the sequence is three, $\mathrm{var}(B_u) < \mathrm{var}(B_E)$ for all $0 < E < 1$. This is seen by the fact that

$$\mathrm{var}(B_u) = \frac{1-s}{1-s^3}\,\mathrm{var}\,(w) \tag{2.3.11}$$

whereas

$$\mathrm{var}(B_E) = \frac{1-s(1-E)}{1-s^3(1-E)^3}\,\mathrm{var}\,(w). \tag{2.3.12}$$

Further, if we define

$$g(s) = \frac{1-s}{1-s^3},$$

then $g'(s) < 0$ for $0 \leq 1$, and since $s(1-E) < s$ for $0 < E < 1$ the claim above follows. When the recruitment sequence is longer,

$$\{w_t\} = \{w_1,\ldots, w_n, w_1,\ldots, w_n,\ldots\},$$

$E > 0$ may well imply decreased $\mathrm{var}(B_E)$ compared to when $E = 0$. However, for n greater than three, analytical expressions for the relationship between var(B) and var(w) are not readily found, and the relationship between them will certainly no longer be given by a constant. For example with $n = 4$ and $s = 0.8$, numerical calculations show that with constant or zero harvest, $\mathrm{var}(B_u)$ is approximately 50% of var(w) no matter what var(w) is. A constant effort is equivalent to a decrease in s. When $E = 0.5$, the calculations indicate that $\mathrm{var}(B_E)$ is about 70-80% of var(w).

For larger values of n, numerical calculations show that $\mathrm{var}(B_u)$ may be much larger than $\mathrm{var}(B_E)$, and the question of which strategy causes largest variation in total biomass depends heavily upon the presence of auto-correlation in recruitment. The following example illustrates this. Assume that the repeated sequence of recruitment has a length of ten; i.e. it takes ten periods before the recruitment pattern repeats itself. Suppose further that the recruitment sequence is the random sequence

$$\{w_t\} = \{2,4,8,5,4,5,8,2,1,6\}$$

and s = 0.8. In this case $\mathrm{var}(B_u)$ for any u is 5.018, whereas $\mathrm{var}(B_E)$ when E = 0.5 is 5.028. On the other hand, if the recruitment sequence is substituted by the cyclical sequence

$$\{w_t\} = \{5,4,3,2,1,1,2,3,4,5\},$$

then $\mathrm{var}(B_u) = 5.748$ whereas $\mathrm{var}(B_E) = 3.88$ (E = 0.5). With a cyclical recruitment pattern $\mathrm{var}(B_u)$ has increased, $\mathrm{var}(B_E)$ has decreased and the ordering of the two strategies with respect to variation in biomass has been reversed.

The conclusion of this section therefore is that the higher the degree of auto-correlation and the longer the repeated recruitment sequence, the larger the variation in biomass resulting from a constant catch strategy will be compared to the variation resulting from constant effort. As mentioned earlier, the problem with the constant catch strategy is that at some point the total biomass may reach a critically low level so that the escapement is too small to secure future recruitment. This, however, is a problem that has to be taken into account when the optimal constant catch is decided upon, and is therefore separate from the analysis above and will not affect the conclusion in this section.

Appendix to section 2.3.

The specific kind of repeated equilibrium referred to in this section can be derived as follows:

$$\mathbf{B} = \sigma \mathbf{S}\mathbf{w}$$

where B is the n-vector of repeated biomass values, w is the n-vector of repeated recruitment, σ is the scalar $1/(1-s^n)$ and S is the (n x n)-matrix

$$\begin{bmatrix} s^{n-1} & s^{n-2} & \cdots & s^{1} & s^{0} \\ s^{0} & s^{n-1} & \cdots & s^{2} & s^{1} \\ s^{1} & s^{0} & \cdots & s^{3} & s^{2} \\ s^{2} & s^{1} & \cdots & s^{4} & s^{3} \\ \vdots & \vdots & \vdots\vdots & \vdots & \vdots \\ s^{n-2} & s^{n-3} & \cdots & s^{0} & s^{n-1} \end{bmatrix}.$$

In the case of constant effort, the equilibrium is found by replacing all instances of s by s(1-E). In the case of constant catch, each element in the vector **w**, w_i, has to be replaced by $(w_i\text{-}su)$.

2.4 Delayed recruitment and stability.

Fluctuations in the population are the very core of the analysis throughout this work. If there are no fluctuations and recruitment is given by a spawning stock/recruitment relationship, it can be shown, using a delayed recruitment model, that given certain assumptions about the parameters, an optimal equilibrium solution can be found as a constant escapement policy. See e.g. Clark (1976 and 1990). Such an equilibrium solution will also result in both constant effort and constant harvest as soon as the equilibrium is reached. However, the assumptions about the parameters are not always fulfilled, and in that case a stable equilibrium will not be attained. An overlapping generations model is likely to be more stable than a model with non-overlapping generations because in this model the surviving part of the spawning stock works as a buffer against fluctuations in recruitment and thus exerts a stabilizing effect.

Stability analysis is defined as finding the range of initial values under which a stable equilibrium will result. A model is said to be more stable than another if this range is greater than that of the other model.

With non-overlapping generations it has been established that constant effort exerts a stabilizing effect on the population (Goh, 1977). This, however, is not necessarily the case in the delayed recruitment model with overlapping generations. The delayed recruitment model may be used as a vastly simplified but not unreasonable description of the Atlantic cod fishery, and the dynamics of the model are given by:

$$B(t+1) = s[B(t) - u(t)] + R(B(t-\sigma) - u(t-\sigma)) \tag{2.4.1}$$

where $B(t)$ is the biomass, $u(t)$ is the harvest and then $B(t) - u(t)$ is the escapement, all at time t. Further, s is a parameter showing the part of the stock that survives after natural mortality has taken place and $R(\cdot)$ is the recruitment function with a delay of σ periods. The reason why a difference equation is judged appropriate here instead of a differential equation is that, in the case of the cod, the spawning takes place during a relatively short, limited season. According to Goh and Agnew (1978) a constant effort policy may have a destabilizing effect for some values of the parameters because such a policy may reduce the efficiency of the surviving part of the breeding stock as a buffer, and this impact on the population can be stronger than the stabilizing effect through the recruitment function.

Goh and Agnew's conclusion is that an increase in the survival parameter, s, only has a stabilizing effect when the time period between spawning and recruitment, σ, is small. When this period is longer, an increase in s exerts a destabilizing effect except for values of s close to one.

Thus it may be possible, even when the stock/recruitment relationship obeys some mathematical function $R(\cdot)$, that the population shows fluctuating behaviour.

In this section a simple delayed recruitment model will be used to decide under which policy a stable equilibrium is most likely to occur, constant effort or constant catch. The production function assumed here, is of the form:

(2.4.2) $$u(t) = E(t)B(t).$$

With constant catch the dynamics are given by:

(2.4.3) $$B(t+1) = s[B(t) - u] + R(B(t-\sigma) - u)$$

where u is a constant, and with constant effort:

(2.4.4) $$B(t+1) = s[(1-E)B(t)] + R((1-E)B(t-\sigma))$$

where E is a constant. analysing the stability properties becomes progressively more complicated as the delay in the model, σ, increases. For the sake of completeness, a review of the case with no delay will first be given, i.e. the case in which $\sigma = 0$ and we have an arbitrary non-linear recruitment function $R(\cdot)$. The approach taken here is to linearize R about some equilibrium level $\hat{B}$, and assuming this equilibrium is chosen optimally, $R((1-E)\hat{B} = R(\hat{B} - u)$, i.e. the optimal equilibrium biomass will be the same with both strategies since the model is deterministic and there are no exogenous fluctuations.

Using this approach in the case of constant effort, we have:

(2.4.5) $$B(t+1) - [s(1-E) + R'((1-E)\hat{B})]B(t) = R((1-E)\hat{B}) - R'((1-E)\hat{B})\hat{B}$$

and the characteristic equation is given by

(2.4.6) $$b - a = 0$$

where

(2.4.7) $$a = [s(1-E) + R'((1-E)\hat{B})].$$

Since the root is $b = a$, in order to have local dynamic stability, we must have $|b| < 1$ which implies

(2.4.8) $$-1 - s(1-E) < R'((1-E)\hat{B}) < 1 - s(1-E).$$

In the case of constant catch:

(2.4.9) $$B(t+1) - [s+R'(\hat{B}-u)]B(t) = R(\hat{B}-u) - R'(\hat{B}-u)\hat{B} - su.$$

The characteristic equation of the complementary solution is again given by (2.4.6) where, in this case:

(2.4.10) $$a = s + R'(\hat{B} - u).$$

The condition for local stability is therefore:

(2.4.11) $$-1 - s < R'(\hat{B} - u) < 1 - s.$$

As mentioned above, with an optimally chosen equilibrium, $R'(\hat{B} - u) = R'((1-E)\hat{B})$. In the case of constant catch the region of stability, illustrated in Fig. 2.4.1, will not change unless s changes, and the length of the interval in which R' must lie, will always be two since $0 \leq s \leq 1$. With constant effort the interval will also be of length two, but in this case the position of the interval will depend on E, and only coincide with the interval for constant catch if $E = 0$. Thus the region of stability with constant catch is not different from the region of stability without harvesting, but in the case without harvesting $R'(\hat{B})$ will be involved instead of $R'(\hat{B} - u)$. In order to check the likelihood of R' falling within the region of stability, a particular recruitment function has to be investigated.

If $\sigma = 1$, i.e. there is one period delay between spawning and recruitment, the general characteristic equation can be written:

$$b^2 + a_1 b + a_2 = 0 \quad (2.4.12)$$

and according to the Schur-Cohn criterion (see e.g. LaSalle, 1986: pp.27) the roots lie inside the unit circle if and only if

$$|a_2| < 1 \quad \text{and} \quad |a_1| < 1 + a_2.$$

In the case of constant effort $a_1 = -s(1-E)$ and $a_2 = R'((1-E)\hat{B})$ and since both s and E lie in the interval [0,1], these two conditions can be taken together and imply:

$$-1 < R'((1-E)\hat{B}) < 1 - s(1-E). \quad (2.4.13)$$

With constant catch $a_1 = -s$ and $a_2 = -R'(\hat{B} - u)$, and again the two conditions can be combined to imply:

$$-1 < R'(\hat{B} - u) < 1 - s. \quad (2.4.14)$$

It is seen that in this case the region of local stability is smaller than with constant effort provided $E > 0$ since $s(1-E) < s$. This is illustrated in Figs. 2.4.2a and b.

The conclusion so far is that constant effort in a model with a one period delay may exert a stabilizing effect compared to constant catch, and that the region of stability with constant catch is the same as without harvesting. However, in a model without delayed recruitment this is not necessarily the case.

A conventional assumption with regard to the cod stock is that fish are recruited to the fishery at the age of three, and thus recruitment in period t+1 depends on the spawning stock in period t-2; i.e. there is a three-period delay, $\sigma = 2$. This case will be considered now as it is of particular interest. The population dynamics can be described by the recurrence equations

(2.4.15) $$B(t+1) = s(1-E)B(t) + R((1-E)B(t-2))$$

in the case of constant effort, and

(2.4.16) $$B(t+1) = s(B(t) - u) + R(B(t-2) - u)$$

with constant catch. In both cases the characteristic equation can be written in general form as

(2.4.17) $$b^3 + a_1 b^2 + a_2 = 0$$

after linearizing the recruitment function R at the equilibrium population level $\hat{B}$. Again, according to the Schur-Cohn criterion the roots lie inside the unit circle if and only if

$$(1)\ |a_1 + a_2| < 1 \quad \text{and} \quad (2)\ |-a_1 a_2| < 1 - a_2^2.$$

The coefficients a_1 and a_2 are defined in the same manner as when $\sigma = 1$, and the regions of stability defined above are most easily analysed using graphical illustrations. a_1 is defined as s or s(1-E) and is assumed to lie in the interval [0,1]. Condition (2) above implies that a_2 is in the interval [-1,1]. The conditions above give the regions of stability as illustrated in Fig. 2.4.3.

Fig. 2.4.3a illustrates the region with constant catch which is the same as without harvesting. This coincides with the region for constant effort (Fig. 2.4.3b) only if E = 0. As in the case where $\sigma = 1$, the region of local stability in the R',s - space is increased when constant effort is applied since a higher E allows s to be higher and still maintain the same value for $a_1 = s(1-E)$; in other words, multiplying s by (1-E) implies a scaling effect along the horizontal s-axis. This can be shown to be valid also for higher values of the delay-parameter, σ, but higher values of σ will not be considered here.

As indicated, the main conclusion here is that a constant level of effort exerts a stabilizing effect compared to constant catch. It must, however, be emphasized that this conclusion is based on local stability analysis at an equilibrium and is therefore only valid in a small neighborhood around this equilibrium.

The next step will be to consider a specific recruitment function, and the Beverton-Holt (B-H) recruitment function is chosen here since this is one which is considered reasonably realistic for the cod stock. The B-H recruitment function is of the form

(2.4.18) $$R(t+1) = \frac{\alpha B(t-\sigma)}{1+\beta B(t-\sigma)}$$

where α and β are parameters. Dropping the time variable for a moment, it is seen that

(2.4.19) $$R'(B) = \frac{\alpha}{(1+\beta B)^2}$$

and in particular $R'(0) = \alpha$. This is the reason why α is also called the intrinsic or maximum growth rate. In order to have $R'(B) > 0$ we must have $\alpha > 0$. If the restriction $\beta > 0$ is imposed, then it will be guaranteed that $R''(B) < 0$. In order to make the 'recruitment curve' cross the 45^0 line, we must have $\alpha > 1$, otherwise $R'(0) < 1$ and declining as B increases, and the stock would never be able to reproduce itself. This, however, does not mean that the stock necessarily becomes extinct since some of the spawning stock can survive into the next period. Also notice that

$$\lim_{B\to\infty} R = \frac{\alpha}{\beta}.$$

For completeness a short description of the stability properties of the model in the case of no harvesting will be given first. With $\sigma = 0$:

(2.4.20) $$B(t+1) = sB(t) + \frac{\alpha B(t)}{1+\beta B(t)}.$$

This expression can be solved for the equilibrium stock $\hat{B}$:

(2.4.21) $$\hat{B} = \frac{\alpha - M}{\beta M}$$

where $M \equiv 1\text{-}s$ is the natural mortality. Linearization of the recruitment function at the equilibrium point gives:

(2.4.22) $$R(B) = R(\hat{B}) + R'(\hat{B})(B - \hat{B})$$

where $R(\hat{B}) = (\alpha - M)/\beta$ and $R'(\hat{B}) = M^2/\alpha$. It is seen that the equilibrium population $\hat{B}$ and $R(\hat{B})$ are positive if and only if $\alpha > M$ when $\beta > 0$. This, however, is already implicit since it is assumed that $M < 1$ and $\alpha > 1$. The characteristic equation in this case is

$$b + a = 0 \tag{2.4.23}$$

where

$$a = -R'(\hat{B}) - s. \tag{2.4.24}$$

The criterion for stability $|b| < 1$ yields $M < \alpha$ as a lower limit on α and no upper limit. In other words, the intrinsic growth rate must be greater than the natural mortality, which is fulfilled by the assumption.

If $\sigma = 1$, the Schur-Cohn criterion for stability gives $M < \alpha$ which is the same as above, and $-\alpha < M^2$ which is always fulfilled since $\alpha > 0$. The region of stability is unchanged compared to the $\sigma = 0$ case, since $R'(B) > 0$ always. When $\sigma = 2$ the condition for stability is $M < \alpha$ and

$$\alpha^2 - \alpha(M^2 - M^3) - M^4 > 0 \tag{2.4.25}$$

which does not lend itself very easily to interpretation.

However, we shall proceed and look at the more interesting case where constant effort is compared with constant catch using the B-H recruitment function. First, the simplest possible case is investigated, namely that in which the recruitment lag, σ, is zero. With constant effort the dynamics are described by equation (2.4.4), and the equilibrium stock can be found to be:

$$\hat{B} = \frac{\alpha(1-E) - [1-s(1-E)]}{\beta(1-E)[1-s(1-E)]}. \tag{2.4.26}$$

Furthermore,

(2.4.27)
$$R'(\hat{B}(1-E)) = \frac{[1-s(1-E)]^2}{\alpha(1-E)^2}.$$

These expressions look similar to the case without harvesting except that the parameters α, β and s are all multiplied by (1-E). In this case a necessary and sufficient condition for stability is:

(2.4.28)
$$\frac{1-s(1-E)}{(1-E)^2} < \alpha.$$

This constitutes a lower limit for α whereas there is no upper limit.

In the case of constant catch the dynamics are given by (2.4.3) with $\sigma = 0$. The equilibrium stock is no longer unique, but is given by:

(2.4.29)
$$\hat{B}_1, \hat{B}_2 = \frac{(2M-1)\beta u + (\alpha-M) \pm \sqrt{\beta^2 u^2 - 2\beta(\alpha+M)u + (\alpha-M)^2}}{2\beta M}.$$

$\hat{B}_1$ and $\hat{B}_2$ are only real if:

(i)
$$u \geq \frac{(\sqrt{\alpha} + \sqrt{M})^2}{\beta}$$

or:

(ii)
$$u \leq \frac{(\sqrt{\alpha} - \sqrt{M})^2}{\beta}.$$

The first of these conditions, (i), can be ruled out since it can be shown that the constant yield, u, in this case always will be greater than the equilibrium stock, $\hat{B}$, which does not make sense.[1]

---

[1] If $u = (\sqrt{\alpha} + \sqrt{M})^2/\beta$ we have $\hat{B}_1 = \hat{B}_2 = u - (\sqrt{\alpha} + \sqrt{M})/(\beta\sqrt{M})$ which is clearly less than u. If $u > (M + \alpha)^2/\beta$, say $u = [(M+\alpha)^2 + x]/\beta$, then the greatest of the two roots, $\hat{B}_1$, will be given by: $\hat{B}_1 = u - Z$ where

$$Z = \frac{x + 2(\sqrt{\alpha M} + M) - (4x\sqrt{\alpha M} + x^2)^{1/2}}{2\beta M} > 0 \qquad \text{for all } x > 0.$$

Let us therefore proceed by looking at case (ii) and consider first the case with strict equality. In this case there is only one equilibrium stock given by:

$$\hat{B}_1 = \hat{B}_2 = u + \frac{(\sqrt{\alpha} - \sqrt{M})}{\beta\sqrt{M}}. \tag{2.4.30}$$

Clearly $\hat{B} > u$ if and only if $\alpha > M$. Furthermore, $R'(\hat{B} - u) = M$. In this case the criterion for stability is given by

$$-1\text{-}s < R'(\hat{B} - u) < 1\text{-}s. \tag{2.4.31}$$

The leftmost inequality is always fulfilled since R' is always positive, but the rightmost has to be replaced by an equality. In other words, the local stability analysis shows a non-oscillatory pattern that is neither divergent nor convergent.

If (ii) above is a strict inequality, there will be two distinct equilibrium stocks, $\hat{B}_1$ and $\hat{B}_2$, whose stability properties have to be investigated separately. Assume that:

$$u = \frac{(\sqrt{\alpha} - \sqrt{M})^2 - x}{\beta} \tag{2.4.32}$$

with $0 < x < (\sqrt{\alpha} - \sqrt{M})^2$. Then,

$$\hat{B}_1 - u = \frac{x + 2\sqrt{\alpha M} - 2M + [x^2 + 4x\sqrt{\alpha M}]^{1/2}}{2\beta M} \tag{2.4.33a}$$

and

$$\hat{B}_2 - u = \frac{x + 2\sqrt{\alpha M} - 2M - [x^2 + 4x\sqrt{\alpha M}]^{1/2}}{2\beta M}. \tag{2.4.33b}$$

$\hat{B}_2 - u > 0$ only if $x < (\sqrt{\alpha} - \sqrt{M})^2$ supposing $x > 0$ and $\alpha > M$. However, if $x = (\sqrt{\alpha} - \sqrt{M})^2$, then $u = 0$; thus both $\hat{B}_1 - u$ and $\hat{B}_2 - u$ is greater than zero for all reasonable choices of x, since $\hat{B}_1 > \hat{B}_2$.

The next step then is to find $R'(\hat{B} - u)$ for the two equilibrium stocks in order to investigate their local stability properties:

$$R'(\hat{B}_1 - u) = \frac{2\alpha M^2}{x(x + 4\sqrt{\alpha M}) + 2M\alpha + \sqrt{x}\,(x + 2\sqrt{\alpha M})(x + 4\sqrt{\alpha M})^{1/2}}. \tag{2.4.34}$$

The condition for local stability of the reduced, linearized first-order difference equation is:

$$-1 < (1-M) + R'(\hat{B} - u) < 1, \tag{2.4.35}$$

and the left inequality is always fulfilled since both $(1-M)$ and $R'(\hat{B} - u)$ are non-negative. Also the right inequality is satisfied since this can be rewritten:

$$x(x + 4\sqrt{\alpha M}) + \sqrt{x}(x + 2\sqrt{\alpha M})(x + 4\sqrt{\alpha M})^{1/2} > 0 \tag{2.4.36}$$

which is clearly satisfied with x, α and M non-negative. On the other hand:

$$R'(\hat{B}_2 - u) = \frac{2\alpha M^2}{x(x + 4\sqrt{\alpha M}) + 2M\alpha - \sqrt{x}\,(x + 2\sqrt{\alpha M})(x + 4\sqrt{\alpha M})^{1/2}}. \tag{2.4.37}$$

and again the left inequality above is always satisfied. However, the right inequality is never satisfied since this condition after some algebra reduces to

$$4\alpha M < 0 \tag{2.4.38}$$

which clearly is contradictory given the assumptions above.

The conclusion therefore is that of the two equilibrium stocks $\hat{B}_1$ represents a stable equilibrium and $\hat{B}_2$ represents an unstable equilibrium, or in other words, it is always possible to choose a constant catch with this recruitment function such that there is a unique, stable equilibrium population.

The essence of the analysis above is that in the case of constant catch the region of stability in the α, s - space is given by

$$u < \frac{(\sqrt{\alpha} - \sqrt{M})^2}{\beta} \tag{2.4.39}$$

which can be rewritten:

$$\alpha > \beta u + 2\sqrt{\beta u M} + M, \tag{2.4.40}$$

and there is no upper limit on α. This has to be compared with the region of stability in the case of constant effort, (2.4.28), rewritten here as:

$$\alpha > \frac{E + M(1-E)}{(1 - E)^2} \tag{2.4.41}$$

To compare these two regions notice that by setting

$$\alpha = \beta u_c + 2\sqrt{\beta u_c M} + M$$

in the case of a constant catch u_c (defining the limit of the region of stability) the equilibrium stock, which is unique, can be written:

$$\hat{B} = u_c + \sqrt{\frac{u_c}{\beta M}} \tag{2.4.42}$$

assuming $M > 0$. Thus the effort in this case, defined as $E = u_c/\hat{B}$, can be written:

$$E = \frac{u_c \sqrt{\beta M}}{u_c\sqrt{\beta M} + \sqrt{u_c}}. \tag{2.4.43}$$

Equation (2.4.43) above defines the corresponding E to any value of u_c and M (or s), and by letting this represent the constant catch E_c, this expression can be substituted into (2.4.41) to define the constant-effort region of stability corresponding to the constant-catch region. This yields:

$$\alpha > \beta M u_c + \sqrt{\beta M u_c} + M\sqrt{\beta M u_c} + M \qquad (2.4.44)$$

and by comparing this with (2.4.40) it is easily seen that the lower limit on α is smaller in the constant effort case and thus the region of stability is greater.

The conclusion that the region of stability is greater in the case of constant effort comes perhaps as no surprise, and it can also be shown that all the equilibria in the case of constant catch that had to be ruled out as unstable (with equilibrium stock $\bar{B}_2$) in fact can be approached as a stable equilibrium using constant effort.

Another thing to note is that although the conclusion above is not very surprising, it was nontrivial to reach this conclusion formally, even when there was no delay in the model ($\sigma = 0$) with a B-H recruitment function. Further, using only a one period time-delay it is fairly easy to show that the regions of stability actually coincide with the regions of stability without time delay, applying the same procedures as above. This suggests that instead of trying to repeat this exercise with a greater delay in the model ($\sigma > 1$), it is perhaps more fruitful to rely on the conclusions from the model with a general recruitment function R(B), particularly since all the conclusions so far point in the same direction. The intuitive explanation of why the region of stability is larger with a constant effort policy is straightforward: the catch is reduced as soon as the stock is below its equilibrium level and increased as soon as it is above the equilibrium, instead of being independent of the stock as is the case with constant catch.

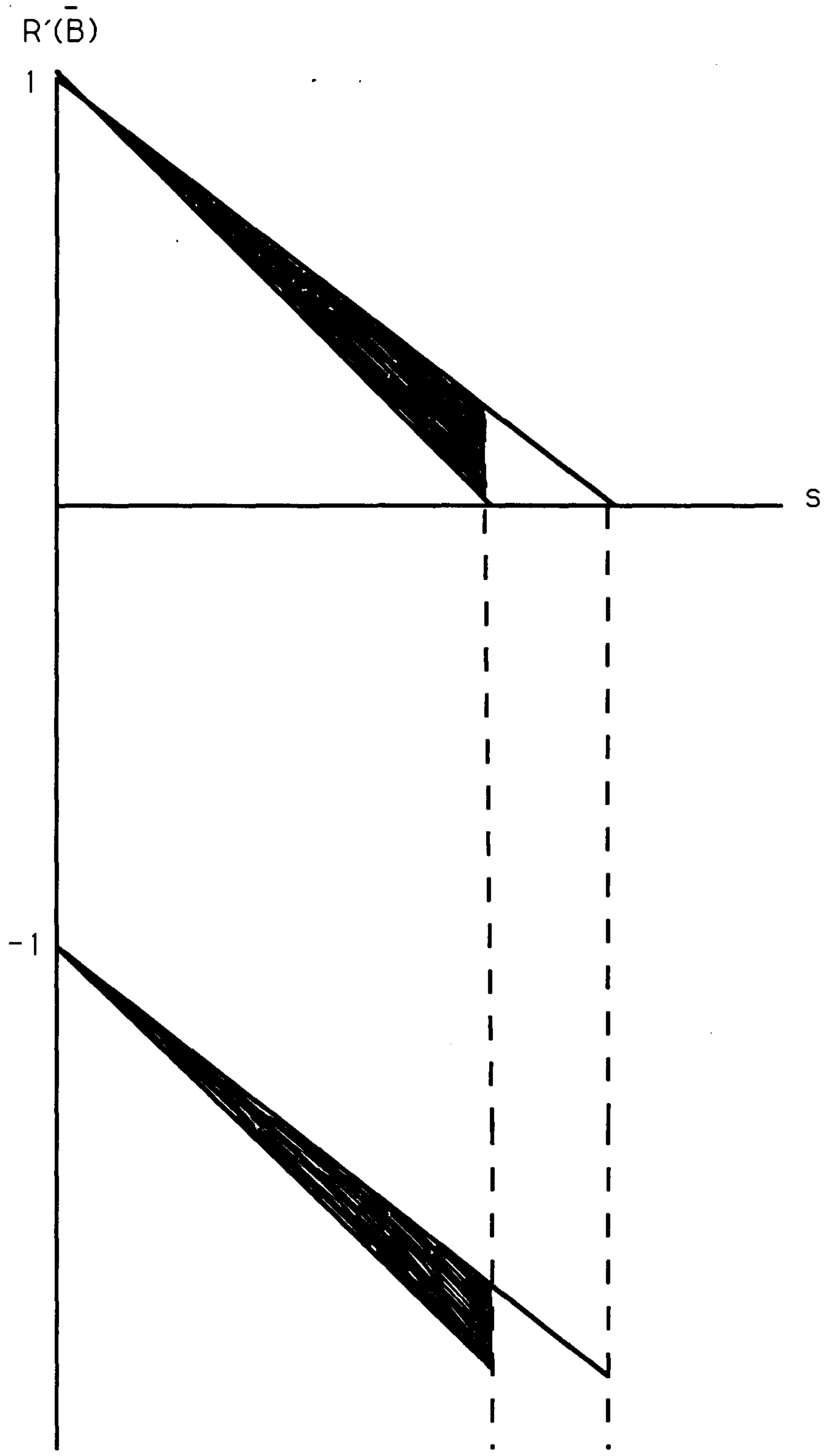
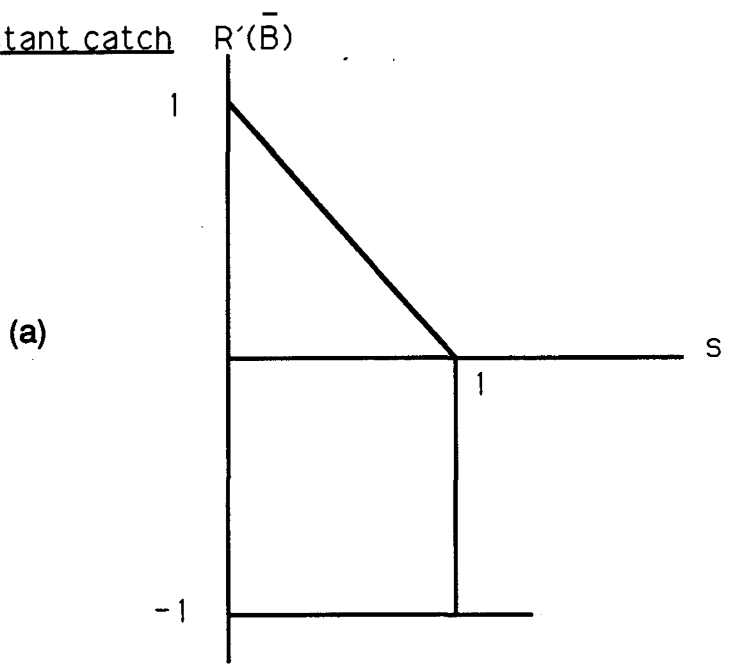


Fig. 2.4.1

When $E = 0.2$ the region of stability is increased by the upper shaded area and decreased by the lower shaded area.

Constant catch



Constant effort

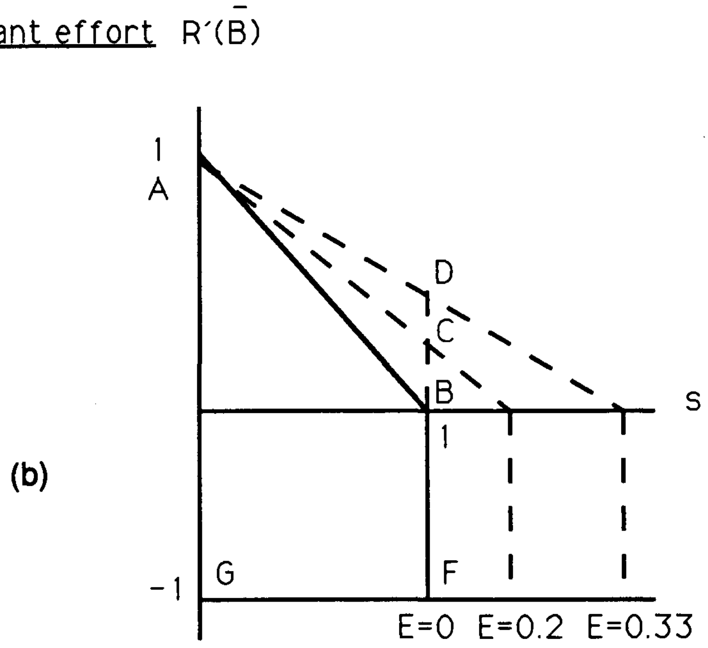
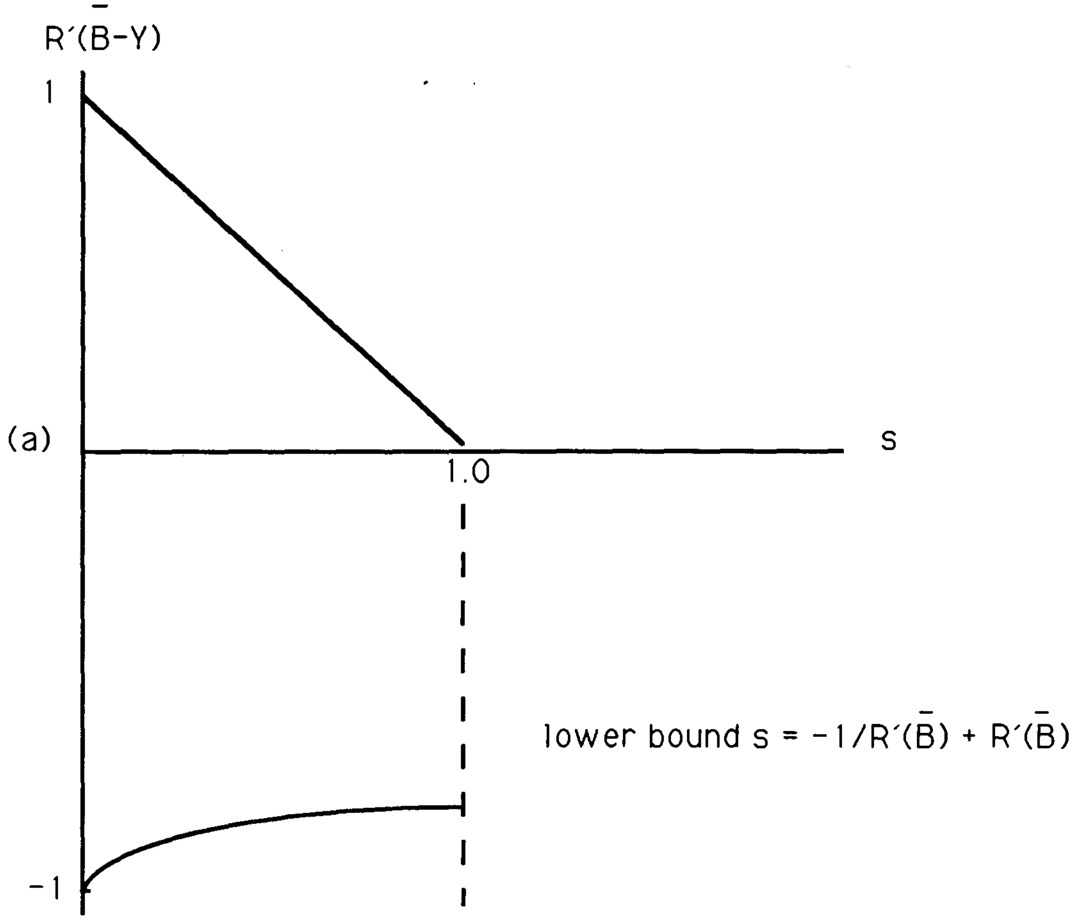


Fig. 2.4.2

σ = 1. Fig. 2b shows how the region of local stability increases when the level of constant effort, E, increases. When E = 0.2 the region of stability is increased by the triangle ABC and when E = 0.33 by the triangle ABD compared to the original region ABFG.

Constant catch



Constant effort

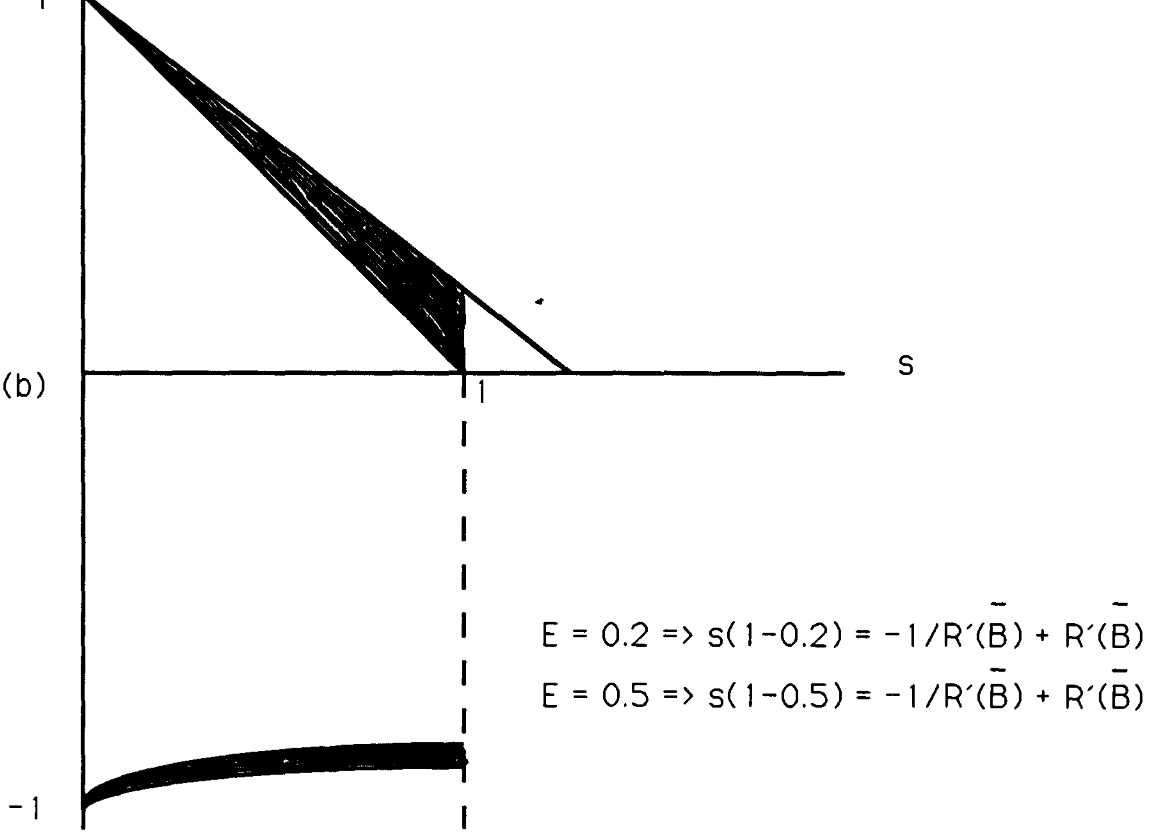


Fig. 2.4.3

2.5 Some results from a two-period model.

After reviewing the stability properties of the model, it is now time to point out some of the different effects of choosing constant catch versus constant effort using a model with a time horizon of only two periods. Results from the two-period model may illustrate some interesting aspects of the problem considered here, and may indicate the need to develop a more realistic and complex model in order to pursue further the questions that arise. These results particularly emphasize the effects of overlapping generations.

Assume first that the production function in the fishery is

$$u_t = qE_tB_t \tag{2.5.1}$$

where u is harvest, E is effort and B is biomass in period t, and q is the catchability coefficient. Since effort is here measured in arbitrary units, it is possible to define $q \equiv 1$ and adjust the units of effort accordingly. Denote the price per unit harvest p, the cost per unit effort c. Then the net revenue function can be written either

$$\pi(u_t, B_t) = (p - c/B_t)u_t \tag{2.5.2a}$$

or

$$\pi(E_t, B_t) = (pB_t - c)E_t \tag{2.5.2b}$$

depending on whether u or E is considered to be the control variable. Assume also a survival parameter s, and a recruitment w, such that the dynamics in the model can be written either

$$B_{t+1} = s(B_t - u_t) + w_t \tag{2.5.3a}$$

or

$$B_{t+1} = s(1 - E_t)\,B_t + w_t. \tag{2.5.3b}$$

First the importance of the survival parameter will be considered, that is whether there are overlapping generations or not. If s=0, then

(2.5.4) $$B_{t+1} = w_t$$

and there is no dependence between B_{t+1} and B_t. An optimal policy will be to apply maximal effort (or maximal harvest) if $B_t > c/p$ and zero effort (or harvest) if $B_t < c/p$.[1]

It is easily seen that any constant effort policy will yield a better result in terms of net revenue than the constant catch policy with the same average harvest. Denote the biomass in the two periods as B_1 and B_2 with an average $\bar{B}$. The average harvest with a constant effort E, is

(2.5.5) $$\bar{u} = E\bar{B}.$$

In order to take $\bar{u}$ as a constant harvest, an average effort level $\bar{E}$ is needed, which is given by

(2.5.6) $$\bar{E} = \frac{E\bar{B}}{2}\left(\frac{1}{B_1} + \frac{1}{B_2}\right)$$

which is always greater than E unless $B_1 = B_2 = 0$. This also holds true for most of the other production functions commonly applied in fisheries analysis, e.g.

(2.5.7a) $$u = E^{\alpha}B^{\beta}$$

or

(2.5.7b) $$u = B(1 - e^{-E}).$$

---

[1] An optimal constant policy will be either zero or maximal effort depending on whether $\Sigma(B_t - c/p)$ is greater than or less than zero.

This point is illustrated in Fig. 2.5.1 for a concave production function. The argument may be extended to a similar model with an arbitrary number of periods.

If, on the other hand, $s > 0$, then the biomass in the next period is no longer independent of the harvesting strategy applied in the present period. B_2 will not be the same with constant harvest as with constant effort. In this case the optimal constant policy may have an interior solution that can be found analytically or numerically. For example, in the two-period case with B_1 given, the optimal constant effort is

$$E^* = \frac{p[B_1(1+s) + w] - 2c}{2psB_1} \tag{2.5.8}$$

Since E is restricted by $E \in [0,1]$, the c/p ratio will determine whether E* is the optimal solution. E* from Eq. (2.5.8) is an interior solution as long as

$$\frac{1}{2}[B_1(1+s) + w] - sB_1 < \frac{c}{p} < \frac{1}{2}[B_1(1+s) + w]; \tag{2.5.9}$$

otherwise $E^* = 0$ or $E^* = 1$. The optimal constant harvest, u*, in a two-period model is given by

$$u^* = B_1 + \frac{w}{s} - \frac{1}{s}\left\{\frac{c(sB_1+w)}{2p - c/B_1}\right\}^{1/2} \tag{2.5.10}$$

The constraints on u* are:

i) $u^* \geq 0$,

ii) $u^* \leq B_1$,

iii) $u^* \leq B_2$.

i) is fulfilled if

$$\frac{c}{p} \leq \frac{2(sB_1 + w)}{1 + s + \frac{w}{B_1}}. \tag{2.5.11}$$

ii) is fulfilled if

(2.5.12)
$$\frac{c}{p} \geq \frac{2B_1(sB_1 + w)}{B_1(1+s)^2 + (sB_1+w)}.$$

iii) is fulfilled if

(2.5.13)
$$\frac{c}{p} \geq \frac{2w^2B_1}{w^2 + B_1(sB_1+w)}.$$

It is easy to check that iii) is fulfilled whenever ii) is fulfilled as long as $B_1 \geq w$. With a time horizon longer than two, it will be necessary to resort to numerical solution methods to find E* or u*. The maximum net revenue from harvesting with the optimal constant effort is given by

(2.5.14)
$$J(E^*) = \frac{[pB_1(1+s) + pw - 2c]^2}{4psB_1}.$$

Let us leave alone the optimal constant catch for the moment and apply instead the average harvest from the constant effort policy as a constant catch. This will be given by

(2.5.15)
$$\bar{u} = \frac{[pB_1(1+s) + pw]^2 - 4c^2}{8p^2sB_1}$$

The necessary effort in each period to take this harvest is $\bar{u}/B_i$ (i=1,2). The net value of harvesting with constant catch is given by

(2.5.16)
$$J(\bar{u}) = (p - \frac{c}{B_1})\,\bar{u} + (p - \frac{c}{s(B_1-\bar{u}) + w})\,\bar{u}.$$

$J(\bar{u})$ will be higher than J(E*) if the average of $\bar{u}/B_i$ (i=1,2) is smaller than E*. Given that

$B_1(1-s) < w < B_1$, an equilibrium effort, $0 < E^{\dagger} < 1$, can be found such that $B_1 = B_2$. This equilibrium effort is given by

$$E^{\dagger} = \frac{w - (1-s)\, B_1}{sB_1} \tag{2.5.17}$$

and applying this results in constant harvest as well as constant effort.[2] Further, it can be shown that with any E lower than $E^{\dagger}$, $J(\bar{u}) < J(E)$, and vice versa, where $\bar{u}$ is the average harvest generated from any effort level applied, though not necessarily the optimal one (see appendix to this section). The interesting question now is to see how E* in particular relates to $E^{\dagger}$. It can easily be shown that $E^* > E^{\dagger}$ if and only if

$$\frac{c}{p} < \frac{B_1(3-s) - w}{2} \tag{2.5.18}$$

where c/p is the cost-price ratio. In this case there will always exist a constant catch policy that can outperform the optimal constant effort policy, since the average catch produced by the optimal effort is such a catch. Furthermore, this average catch may not be the optimal constant catch; there might exist another constant catch that can perform even better. Note that the right side of (2.5.18) will always be less than the right side of (2.5.9) as long as $w > B(1-s)$ which is necessary for $E^{\dagger} > 0$. The right side of (2.5.18) is obviously increasing in B and decreasing in w and s.

Let us now summarize the five different cases that may appear in the two-period model:

(i) $$\frac{c}{p} < \frac{1}{2}[B_1(1+s) + w] - sB \Rightarrow E^* = 1$$

(ii) $$\frac{1}{2}[B_1(1+s) + w] - sB < \frac{c}{p} < \frac{B_1(3-s) - w}{2} \Rightarrow E^{\dagger} < E^* < 1$$

(iii) $$\frac{c}{p} = \frac{B_1(3-s) - w}{2} \Rightarrow E^* = E^{\dagger}$$

---

[2] $E^{\dagger}$ is increasing in w, decreasing in B_1 and increasing in s provided that $B_1 > w$ which is necessary for $E^{\dagger} < 1$.

(iv) $$\frac{B_1(3-s) - w}{2} < \frac{c}{p} < \frac{1}{2}[B_1(1+s) + w] \Rightarrow 0 < E^* < E^\dagger$$

(v) $$\frac{c}{p} > \frac{1}{2}[B_1(1+s) + w] \Rightarrow E^* = 0.$$

Case (i) is not very interesting since the escapement, B - u, in this case will be zero. Although recruitment is not directly linked to the escapement, there has to be at least some minimum escapement in order to secure future recruitment, a question that will be addressed further in a moment.

In case (ii) constant catch is better than constant effort in the two-period case.

Case (iii). Obviously in this case where $E^* = E^\dagger$, the two strategies will perform equally well, and the biomass will remain at its equilibrium level. Case (iii), however, is a knife-edge case.

Case (iv). In this case constant effort is a better choice than constant catch.

Case (v). In the case that $E^* = 0$, the net revenue in each period is zero. However, in the long run the biomass level will build up without harvesting and ultimately the system will move from case (v) to case (iv). This assumes that the biomass is not already at or above its natural carrying capacity, in which case it is a low-valued species that is not worth harvesting at all, and hence not worth any attention in this study either.

In a two-period model an important difference between the two strategies considered here lies in the escapement after period two is over. One reason why constant catch performs better in case ii) (and constant effort in case iv)), is that it leaves a lower escapement, (B-u), at the end of the time horizon. In some cases the optimal constant policy may be the one that results in B - u = 0, and some constraint therefore has to be placed on the minimum allowable escapement, e.g. $B - u \geq \xi_{min}$.

$E^\dagger$ plays a crucial role also with respect to escapement. Defining the escapement at the end of period two with constant effort as

(2.5.19) $$\xi(E) = [sB_1(1-E) + w](1-E)$$

and with constant catch as

(2.5.20) $$\xi(u) = s(B_1 - u) + w - u$$

it is easily shown that $\xi(\bar{u}(E)) < \xi(E)$ whenever $E > E^\dagger$ where

$$\bar{u}(E) = \frac{1}{2}[B_1 + sB_1(1-E) + w]E$$

is the average catch for any E. In other words, the price that is paid for a better economic performance is smaller escapement at the end of the two periods since whenever $E > E^\dagger$, $J(E) < J(\bar{u}(E))$ and $\xi(E) > \xi(\bar{u}(E))$ and vice versa. It may be considered worthwhile to pay this price if it is hard to prove any relationship between escapement and future recruitment, and a minimum escapement is the only necessary requirement. Depending on the cost-price ratio, the optimal constant harvest may be the one that drives the escapement down to its minimum level at the end of the time horizon. Only in this case of a corner solution or when $E^* = 1$, will it be possible to increase the economic performance of any of the strategies by decreasing ξ_{min}.

Even when the constant catch is calculated as the average harvest produced by some constant effort level, there may be a significant difference in the net revenues yielded by the two strategies. If an optimal constant catch is applied, this difference becomes even more pronounced. Another point worth noting is that the average biomass level is higher with constant catch than with constant effort whenever $E > E^\dagger$ and vice versa.

Now, it may be interesting to compare these constant policies with the optimal unconstrained policies. Regarding the harvest, u, as the control variable, the optimal policy is easily found using Bellman's principle for dynamic optimization. Denoting the harvest in the two periods u_1 and u_2 respectively, the optimal solution requires

$$u_2 = \begin{cases} u_{2max} & \text{if } p - c/B_2 > 0; \\ 0 & \text{if } p - c/B_2 < 0. \end{cases}$$

u_{max} is given by $B_2 - \xi_{min}$. Applying the optimal policy in period two gives

(2.5.21) $$J_2^*(B_2) = \max_{u_2} \pi(B_2, u_2).$$

π is given by Eq. (2.5.2a). In period 1

(2.5.22) $$J_1^*(B_1) = \max_{u_1} \{\pi(B_1, u_1) + J_2^*(B_2)\}.$$

Whenever $u_1 < B + \frac{w}{s} - \frac{c}{ps}$, $u_2 = u_{max}$. Further, when this is satisfied

(2.5.23) $$\frac{\partial J_1^*(B_1)}{\partial u_1} = \frac{2c\xi s^2}{[s(B-u_1) + w]^3} > 0$$

such that the only two values of u_1 in the interval $[0, B + \frac{w}{s} - \frac{c}{ps}]$ that need to be taken into account are the end values. If $u_1 > B + \frac{w}{s} - \frac{c}{ps}$, then $u_2 = 0$ and

(2.5.24) $$J_1^*(B_1) = \pi_1(B_1, u_1)$$

implying that

$$u_1 = \begin{cases} u_{1max} & \text{if } p - c/B_1 > 0; \\ 0 & \text{if } p - c/B_1 < 0. \end{cases}$$

In order to find the optimal policy, the value of $J_1(B_1)$ has to be calculated for these three values of u_1:

$u_1 = 0,$

$u_1 = B + \frac{w}{s} - \frac{c}{ps}$ and

$u_1 = u_{1\max} \equiv B_1 - \xi_{\min}.$

Consider some numerical examples to illustrate the consequences of the optimal constant policies and the unconstrained optimal policy. Table 2.5.1 gives biomass, harvest and net revenue in each period and also average values. The values of the parameters are chosen arbitrarily as $B_1 = 10$, $s = 0.8$, $w = 4$, $p = 1$ and $c = 7$.

Table 2.5.1

Optimal constant effort policy: $E^* = 0.5$

	$B_1 = 10$	$u_1 = 5$	$\pi_1 = 1.5$	
	$B_2 = 8$	$u_2 = 4$	$\pi_2 = 0.5$	
average	9	4.5	1	$\xi = B_2 - u_2 = 4$

Constant harvest policy: $\bar{u} = 4.5$

	$B_1 = 10$	$E_1 = 0.45$	$\pi_1 = 1.35$	
	$B_2 = 8.4$	$E_2 = 0.54$	$\pi_2 = 0.75$	
average	9.2	0.49	1.05	$\xi = B_2 - u_2 = 3.9$

Optimal constant harvest policy: $u^* = 4.952$

	$B_1 = 10$	$E_1 = 0.495$	$\pi_1 = 1.49$	
	$B_2 = 8.04$	$E_2 = 0.616$	$\pi_2 = 0.64$	
average	9.02	0.556	1.06	$\xi = B_2 - u_2 = 3.09$

Optimal unconstrained policy: $\xi = 3.1$

	$B_1 = 10$	$u_1 = 0$	$\pi_1 = 0$	
	$B_2 = 12$	$u_2 = 8.1$	$\pi_2 = 3.375$	
average	11	4.05	1.69	$\xi = B_2 - u_2 = 3.1$

The constant catch policy with the same average as the optimal constant effort gives 5% higher average net revenue than constant effort, and the optimal constant catch gives 6%

higher net revenue. The unconstrained (not constant) optimal policy on the other hand gives 69% higher net revenue than the optimal constant effort. The constant catch policies clearly have the lowest variance in net revenues, and this may weigh heavily when the best policy has to be chosen. The role of the variance in net revenues will be pursued further in a following section.

Is there anything to be learned from this two-period model that can be of practical use and is there any reason to apply this further? In order to answer this question it is necessary to compare the two-period model with a model with a longer time horizon. At the other extreme, the time horizon may be infinite. The result from a model with an infinite time horizon is that optimal constant effort and optimal constant catch coincide with the equilibrium solution, assuming that recruitment varies stochastically. The optimal long-run equilibrium biomass level is given by:

$$B^{\infty} = \sqrt{\frac{cw}{p(1-s)}} \tag{2.5.25}$$

and the corresponding optimal long-run catch and effort are

$$u^{\infty} = \frac{w - \sqrt{(1-s)cw/p}}{s} \tag{2.5.26}$$

and

$$E^{\infty} = \frac{1}{s}\{\sqrt{\frac{(1-s)pw}{c}} - (1-s)\}. \tag{2.5.27}$$

respectively (see next section for the derivation of these). In the equations above w can be both a constant recruitment or the expectation of a stochastic recruitment sequence. It is relatively easy to check that $E^{\infty} < E^{*}(B^{\infty})$ as long as the effort level is subject to $0 \leq E \leq 1$. The optimal effort level for a finite time horizon will only approach E^{∞} as the time horizon approaches infinity.

The two-period model clearly differs from this result, and a short horizon model may therefore be considered as a substitute for a high discount rate, i.e. it emphasizes the effects of a myopic decision pattern. A two-period model is chosen since this may give some interesting analytical results as opposed to models with a longer but finite horizon, given the dynamics assumed here. An objective of this section is to check by numerical simulations whether these results also hold true in a stochastic model with a long time horizon.

Discounting of the net revenue in the second period may influence some of the conclusions above. A very high discount rate may reverse some of the results whereas a moderate or low discount rate (less than 10 per cent) probably will have no effect on the qualitative conclusions.

So far the economic parameters p and c have not played any important role. The main results were based on physical units of effort and catch, not their economic values. The economic parameters were mainly used to find the optimal effort, E*. If, however, p and c cease to be constant parameters and become variables depending on u and E respectively, their role will become more important.

Consider a downward sloping demand function, i.e. $p = p(u)$. In order to investigate the partial effect of a downward sloping demand curve, it is only necessary to look at the gross revenue. It is easy to find examples of downward sloping demand functions that will make the constant catch strategy a relatively better choice compared to the situation with a fixed price[3]. A linear demand function is one such example:

$$p(u) = a - bu. \tag{2.5.28}$$

With a linear demand function the following will always be satisfied, except when $u_1 = u_2$:

$$p(u_1)u_1 + p(u_2)u_2 < 2p\left(\frac{u_1+u_2}{2}\right)\frac{u_1+u_2}{2}. \tag{2.5.29}$$

If $-p''(u)u/p'(u) < 2$, (2.5.29) will always be fulfilled. However, if $p''(u) > 0$, (2.5.29) may not be satisfied. E.g. if $p(u) = au^{-1}$, the increased price when u is low exactly offsets the decreased price when u is high, and (2.5.29) becomes an equality and thus equivalent to a fixed price. As may be expected from this, if $p(u) = au^{\beta}$, (2.5.29) will only be satisfied if $-1 < \beta < 0$. Thus if $\beta < -1$, a downward sloping demand function will favour the constant

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[3] Remember that we are here only interested in maximizing the producer's surplus and ignoring the consumer's surplus.

effort strategy. It is tempting therefore to suggest the conclusion that an inelastic demand will favour constant effort whereas an elastic demand will favour constant catch. This conclusion is a bit premature since, as was seen above, a linear demand function (which can be both elastic and inelastic) will unequivocally favour constant catch. In the case of a demand function with constant demand elasticity, $1/\beta$, of the type above, the conclusion holds true. The conclusion therefore is that the following types of demand functions at least will favour constant catch: 1) linear demand functions, 2) concave demand functions and 3) constant elasticity demand functions which are elastic.

The whole story, however, is not told by this partial analysis of the gross revenue. The downward sloping demand function may force the optimal constant values of harvest and effort down in order to take advantage of the higher price resulting from a lower output. It is known that when the optimal effort is forced below the equilibrium value $E^\dagger$, such that $B_2 > B_1$, constant effort will be a better alternative than constant catch, and this argument is based on the physical characteristics of the system. This means that a demand function that in a partial perspective favours constant catch may force the optimal constant harvest or effort below the equilibrium level, and the effect of this may outweigh the partial effect of the demand function on the gross revenue.

Continuing the numerical example from the previous section may illustrate this. For example, the demand function $p(u) = 4 - 0.5\,u$ results in an optimal constant effort level $E^* = 0.33$, implying $J(E^*) = 10.72$, whereas the optimal constant catch $u^* = 3.2$ implies $J(u^*) = 10.75$. Clearly the catch rate is reduced and the difference between the strategies is smaller compared to a fixed price $p = 1$. If the demand curve is shifted down and given by $p(u) = 3 - 0.5\,u$, then $E^* = 0.22$ and $J(E^*) = 5.320$ whereas $u^* = 2.25$ and $J(u^*) = 5.318$. The difference between the two strategies is still small, but it is now reversed, making constant effort the better choice, as a result of the linear demand function which by itself was supposed to favour constant catch. This is due to the fact that the catch rate is forced below the equilibrium level given by $E^\dagger = 0.25$ and $u^\dagger = 2.5$.

Next, take a look at the situation where cost per unit effort is a function of the effort level applied, i.e. a situation where there is some degree of monopsony. This may be relevant here because many of the inputs to the harvesting sector are rather specialized and the fishing industry is therefore the sole buyer of these inputs. For this purpose it is only necessary to look at the cost side, and the reasoning will in many respects be similar to the downward sloping demand function case. The function c(E) is increasing, and if it is linear, i.e. $c(E) = a + bE$, the following will always hold true:

$$c(E_1)E_1 + c(E_2)E_2 > 2c\left(\frac{E_1+E_2}{2}\right)\frac{E_1+E_2}{2} \qquad (2.5.30)$$

except when $E_1 = E_2$. In other words, a linear cost function will favour the constant effort strategy.[4] The inequality in (2.5.30) will also be fulfilled with a convex cost function, i.e. $c''(E) > 0$, and in fact any cost function satisfying

$$\frac{-c''(E)E}{c'(E)} < 2. \qquad (2.5.31)$$

In the case of a function with constant cost elasticity, $c(E) = aE^{\beta}$, (2.5.31) reduces to $-(\beta - 1) < 2$ implying $\beta > -1$, which is satisfied since c(E) is supposed to be increasing. The only case in which (2.5.31) is not fulfilled, is if c(E) is highly concave, which a priori does not seem very likely. A volume dependent cost function will, however, reduce the optimal constant effort. In this case this will reinforce the most likely effect of such a cost function, namely that it favours constant effort.

For these reasons it seems a priori quite likely that an increasing marginal cost of effort will favour constant effort, whereas the effect of a downward sloping demand function is a bit more ambiguous. The analysis makes it clear, however, that in neither case will it be possible to obtain an unambiguous result, even within the framework of this simple two-period model. It is therefore necessary to undertake an empirical study to make the analysis complete. This will be the subject of the next section.

So far it has been assumed that there is no risk-aversion. In other words, the utility function has been assumed to be linear in net revenue. If this is not the case, i.e. if the utility function is concave, this will favour the strategy with less variation in net revenue. The numerical example above indicates that this will be the constant catch policy, but this is not necessarily the case, and the effect of a concave utility function will therefore not be totally unambiguous either. With a constant catch strategy, the gross revenue will be

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[4] The use of the term cost function is, strictly speaking, not correct here since a cost function is supposed to be a function of output, not the input in production. To use the term in the way it is used above is, however, established as common in fisheries economics and will therefore be retained.

constant, but costs will vary. With constant effort it is the gross revenue that varies when costs are constant, thus making the total effect ambiguous. However, under the relatively mild assumption that the fishery yields a non-negative expected net revenue, the following proposition will hold true in the two-period context:

Proposition

Assuming a time horizon of two periods, a non-negative expected revenue, a production function given by (2.5.1) and dynamics given by (2.5.3a) or (2.5.3b), a constant effort policy will never have less variance in net revenues than the constant harvest policy yielding the same average catch.

Proof

Denote the constant levels of u and E by u_c and E_c. Since with constant effort only gross revenues vary and with constant catch only costs vary, the proposition above is equivalent to saying (cf. Eq.(2.2.9), p.30)

$$c^2 \text{ var } E(B; u_c) < p^2 \text{ var } u(B; E_c)$$

or

$$c\sigma_E < p\sigma_u \tag{2.5.32}$$

where σ is the standard deviation. Since it is assumed that

$$c\bar{E} < p\bar{u}$$

(where the bar indicates expected or in this case average values), it is sufficient to show that

$$\sigma_E/\bar{E} < \sigma_u/\bar{u}. \tag{2.5.33}$$

Using the definition of a standard deviation, we have

$$\frac{\sigma_E}{E} = \left| \frac{B_1 - [s(B_1-u_c) + w]}{B_1 + [s(B_1-u_c) + w]} \right| = \left| \frac{B_1 - B_2(u_c)}{B_1 + B_2(u_c)} \right|$$

and

$$\frac{\sigma_u}{\bar{u}} = \left| \frac{B_1 - [s(1-E_c)B_1 + w]}{B_1 + [s(1-E_c)B_1 + w]} \right| = \left| \frac{B_1 - B_2(E_c)}{B_1 + B_2(E_c)} \right|$$

Condition (2.5.33) can be rewritten

(2.5.33') $$\left| \frac{B_1 - B_2(u_c)}{B_1 + B_2(u_c)} \right| < \left| \frac{B_1 - B_2(E_c)}{B_1 + B_2(E_c)} \right|$$

where $u_c = \frac{E_c}{2}[B_1 + sB_1(1-E_c) + w]$ by definition in order to yield the same average catch. The function $g(B_2) = |(B_1 - B_2)/(B_1 + B_2)|$ is monotonically decreasing for $B_2 < B_1$ and monotonically increasing for $B_2 > B_1$;

(2.5.34) $$|B_1 - B_2(u_c)| < |B_1 - B_2(E_c)|$$

is thus necessary and sufficient for (2.5.33) (and (2.5.32)) to be fulfilled. In (2.5.34) both sides will have the same sign when the absolute values are not taken into account. Assuming that both sides are positive, (2.5.34) (without absolute values) reduces to

(2.5.35) $$E_c > \frac{w - (1-s)B_1}{sB_1} \equiv E^{\dagger}.$$

However, letting $E < E^{\dagger}$ contradicts the assumption that both sides are positive since $E^{\dagger}$ is the equilibrium level of effort, (and vice versa assuming both sides are negative). (2.5.34), (2.5.33) and (2.5.32) are therefore satisfied for all values of E. Q.E.D.

It is also interesting to note the effect that risk-aversion will have in combination with a downward sloping demand function or increasing cost function under the different policies. In the case of constant catch the demand function will have no effect, whereas an increasing cost function will have an amplifying effect on the variance in net revenue. With constant effort, on the other hand, the cost function will have no effect but a downward sloping demand curve will reduce the variance in net revenue. Thus both a downward sloping demand function and an increasing cost function will counteract the difference in variance between the two strategies pointed out above, making unambiguous a priori results even harder to obtain.

Appendix (section 2.5)

The purpose of this appendix is to show that

$$J(\bar{u}) < J(E) \quad \text{when} \quad E < E^{\dagger}$$

and

$$J(\bar{u}) > J(E) \quad \text{when} \quad E > E^{\dagger}.$$

E is here any constant effort level and $E^{\dagger}$ is defined such that $B_2(E^{\dagger}) = B_1$. $\bar{u}$ is the average catch caused by E, that is,

$$\bar{u}(E) = \frac{E}{2}[B_1 + B_2(E)].$$

$J(\bar{u})$ and J(E) are both increasing and concave functions in E. The only two values of $E \in [0,1]$ for which $J(\bar{u}) = J(E)$ are $E = 0$ and $E = E^{\dagger}$. The proposition made above will therefore be proved if it can be confirmed that

$$\left.\frac{\partial J(\bar{u})}{\partial E}\right|_{E=0} < \left.\frac{\partial J(E)}{\partial E}\right|_{E=0}$$

and

$$\left.\frac{\partial J(\bar{u})}{\partial E}\right|_{E=E^{\dagger}} > \left.\frac{\partial J(E)}{\partial E}\right|_{E=E^{\dagger}}.$$

Since the revenue parts of $J(\bar{u})$ and J(E) are equal by definition, it is sufficient here to look at the cost parts. The costs with constant catch are given by

$$C(\bar{u}) = \frac{c\bar{u}}{B_1} + \frac{c\bar{u}}{B_2(\bar{u})}$$

and the costs with constant effort are simply

$$C(E) = 2cE.$$

Since c is a constant it can be removed from these expressions without influencing the line of reasoning. It is now sufficient to show that

$$\left.\frac{\partial C(\bar{u})}{\partial E}\right|_{E=0} > \left.\frac{\partial C(E)}{\partial E}\right|_{E=0}$$

and

$$\left.\frac{\partial C(\bar{u})}{\partial E}\right|_{E=E^{\dagger}} < \left.\frac{\partial C(E)}{\partial E}\right|_{E=E^{\dagger}}.$$

We have

$$\frac{\partial C(\bar{u})}{\partial E} = \frac{1}{B_1}\cdot\frac{\partial \bar{u}}{\partial E} + \frac{\partial \bar{u}/\partial E \cdot B_2 - \partial B_2/\partial E \cdot \bar{u}}{(B_2)^2}$$

and

$$\frac{\partial C(E)}{\partial E} = 2.$$

Further,

$$\frac{\partial \bar{u}}{\partial E} = \frac{B_1}{2} + \frac{sB_1}{2} - sB_1E + \frac{w}{2}$$

and

$$\frac{\partial B_2(\bar{u})}{\partial E} = -\frac{sB_1}{2} - \frac{s^2B_1}{2} + s^2B_1E - \frac{sw}{2}.$$

Substituting this into the expressions above and using the facts that $\bar{u} = 0$ and $B_2 = sB_1 + w$ when $E = 0$ and $B_1 = B_2$ when $E = E^{\dagger}$, the expressions

$$\left.\frac{\partial C(\bar{u})}{\partial E}\right|_{E=0} > \left.\frac{\partial C(E)}{\partial E}\right|_{E=0}$$

and

$$\left.\frac{\partial C(\bar{u})}{\partial E}\right|_{E=E^{\dagger}} < \left.\frac{\partial C(E)}{\partial E}\right|_{E=E^{\dagger}}$$

both reduce to the same; namely

$$[sB_1 + w - B_1]^2 > 0,$$

which is obviously fulfilled.

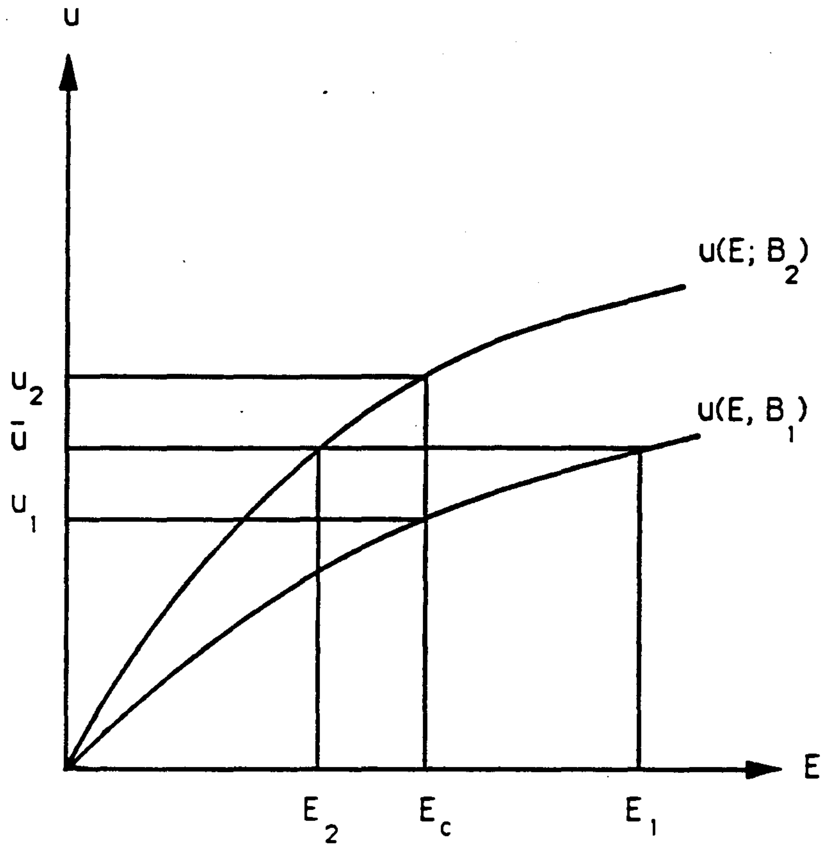

Fig. 2.5.1

The constant effort E_c gives $\bar{u}$ on average. $\bar{u}$ applied as constant catch on the other hand requires E_2 and E_1 in the two periods which has a higher average than E_c

2.6 Stochastic analysis.

The objective of this section is to explore the effects of stochasticity in the analysis. The difference between the constant effort and the constant catch strategy will be explored within this framework and reference to the deterministic counterpart of the model will be made in order to find the cost of uncertainty. The basic model will be extended in several directions, e.g. biological and economic complexities will be included in the model, one at the time. The basic model is in other words a special case of a more general model, in which most of the biological and economic parameters are treated as constants. Experience shows that analytical results from such models are rare except for some special cases, and therefore numerical results will be provided in most of this section.

2.6.1 The model

The model is an aggregate biomass model where the dynamics are given by

$$B_{t+1} = s\xi_t + w_t. \tag{2.6.1}$$

B_t is biomass, ξ_t is escapement and w_t is recruitment, all at time t, and s is the survival parameter. In the basic model the following production function is assumed:

$$u_t = E_t B_t \tag{2.6.2}$$

where E_t is the effort level at time t. The objective function is given by

$$V^* = \max_{u_t} \sum_{t=0}^{T} \alpha^t \pi(u_t, B_t) \tag{2.6.3}$$

where π is the net revenue function and α is the discount factor $(1+r)^{-1}$, r is the discount rate. The net revenue function can be written

$$\pi(u_t, B_t) = pu_t - C(u_t, B_t) \tag{2.6.4}$$

where p is the price per unit harvest and C() is the cost function. The cost function is defined as

(2.6.5) $$C(u_t,B_t) = cE(u_t,B_t)$$

where c is the cost per unit effort, and effort is here written as a function of u and B. Given equation (2.6.2) above, this function is simply $E_t = u_t/B_t$. The net revenue function can therefore be rewritten

(2.6.6a) $$\pi(u_t,B_t) = (p - c/B_t)u_t$$

or

(2.6.6b) $$\pi(E_t,B_t) = (pB_t - c)E_t.$$

$\partial\pi/\partial B > 0$ whereas $\partial\pi/\partial u$ and $\partial\pi/\partial E$ are only greater than zero if $(p - c/B) > 0$, that is when the biomass level is higher than the level that results in an open access fishery.

To start with the simplest possible model as a reference, assume constant recruitment, $w_t = w\forall t$, and no discounting, $r = 0$ (i.e. $\alpha = 1$). The objective is first to derive the optimal equilibrium by maximizing

(2.6.7) $$V = \sum_{t=0}^{\infty} \pi(u_t,B_t).$$

It is obvious that with a constant recruitment the optimal equilibrium will imply both constant effort and constant catch. In other words, the three optimal solutions, unconstrained optimum, optimal constant effort and optimal constant catch will coincide. However, with zero discounting it is clear from (2.6.7) that the objective function will not converge, and it will therefore be replaced by the long-term average

(2.6.8) $$V = \frac{1}{T}\sum_{t=0}^{T} \pi(u_t,B_t),$$

see e.g. Clark (1976: p.386). This has to be maximized subject to the dynamic constraint, equation (2.6.1), and subject to $B_t - u_t \geq \xi_{min}$ for all t where ξ_{min} is the minimum escapement level required to secure future recruitment. Assuming that an equilibrium solution exists, this problem can be solved as a non-linear programming problem using the method of Lagrange multipliers. The Lagrange expression is given by

(2.6.9) $$L = (p-c/B)u + \lambda_1[s(B-u)+w-B] + \lambda_2[B-u-\xi_{min}]$$

where B and u now represent equilibrium levels. The Kuhn-Tucker conditions are

(2.6.10) $$\frac{\partial L}{\partial B} = \frac{c}{B^2} u + \lambda_1 s - \lambda_1 + \lambda_2 = 0$$

(2.6.11) $$\frac{\partial L}{\partial u} = p - \frac{c}{B} - \lambda_1 s - \lambda_2 = 0$$

(2.6.12) $$\frac{\partial L}{\partial \lambda_1} = s(B-u) + w - B = 0$$

(2.6.13) $$\frac{\partial L}{\partial \lambda_2} = B - u - \xi_{min} \geq 0, \qquad \frac{\partial L}{\partial \lambda_2} \lambda_2 = 0.$$

From the dynamic constraint the equilibrium catch is given by

(2.6.14) $$u = \frac{w - (1-s)B}{s}.$$

Substituting this into (2.6.13) and rearranging it is seen that

(2.6.15) $$B > w + \xi_{min} s$$

with $\lambda_2 = 0$, or

(2.6.16) $$B = w + \xi_{min} s$$

with $\lambda_2 > 0$. This is the minimum escapement constraint stated only with the biological parameters w, s and ξ_{min}. Assuming $\lambda_2 = 0$, i.e. assuming an interior solution, the optimal catch and biomass level can be found from the equations (2.6.10), (2.6.11) and (2.6.12), and they are

$$B^* = \sqrt{\frac{cw}{p(1-s)}} \tag{2.6.17}$$

and

$$u^* = \frac{w - \sqrt{(1-s)cw/p}}{s}. \tag{2.6.18}$$

Substituting B* and u* into (2.6.13), it is seen that this interior solution is only in effect if

$$\frac{c}{p} > \frac{(w+\xi_{min}s)^2(1-s)}{w} \tag{2.6.19}$$

otherwise $B - u = \xi_{min}$, implying

$$B^* = s\xi_{min} + w \tag{2.6.20}$$

and

$$u^* = w - (1-s)\xi_{min} \tag{2.6.21}$$

In other words, the escapement level will only be larger than the required minimum if the cost/price ratio is smaller than some value depending on the biological parameters w, s and ξ_{min}.[1] In particular, if $\xi_{min} = 0$, condition (2.6.19) can be rewritten

$$\frac{c}{p} > w(1-s). \tag{2.6.22}$$

The optimal equilibrium effort level will be given by $E^* = u^*/B^*$ which implies that

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[1] This value will be increasing with ξ_{min}, increasing with w given that $w > \xi_{min}s$ and decreasing in s given that $s > (2\xi_{min}-w)/3\xi_{min}$.

(2.6.23) $$E^* = \frac{1}{s}\left\{\sqrt{\frac{p(1-s)w}{c}} - (1-s)\right\}$$

from (2.6.17) and (2.6.18) when the minimum escapement is not binding, and

(2.6.24) $$E^* = 1 - \frac{\xi_{min}}{s\xi_{min}+w}$$

when the minimum escapement constraint is binding.[2] However, for the purposes of this study the optimal equilibrium with constant recruitment is not of interest in itself, and is only meant to serve as a reference.

Comparing constant effort with constant catch will only be of interest if the fish stock biomass varies for some reason, and in this section the assumption will be that this variation is due to stochasticity in recruitment. The dynamics are still given by equation (2.6.1), but now the recruitment w_t is a stochastic variable with expectation μ. However, if recruitment is a stochastic variable and there is no stock - recruitment relationship at all, the optimal policy will be given by the optimal solution from the deterministic equivalent of the model. This is rather obvious since in this case recruitment will be an additive stochastic variable in the dynamic equation. The objective is to maximize the expected net present value of future income, and the dynamics can be written

(2.6.25) $$\mathcal{E}B_{t+1} = s(\mathcal{E}B_t - u_t) + \mu.$$

$\mathcal{E}$ is the expectation operator. The minimum escapement constraint may play a role here. If the constraint on minimum escapement is

(2.6.26) $$\mathcal{E}(B_t - u_t) \geq \xi_{min},$$

the optimal solution will be equivalent to the constant recruitment case with the constant recruitment w replaced by the expected recruitment μ. Adjustment in the initial and final

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[2] Optimality conditions (23) and (24) can also be found by maximizing the Lagrange expression

$$L = (pB-c)E + \lambda_1[s(1-E)B + w - B] + \lambda_2[(1-E)B-\xi_{min}]$$

with respect to E and B.

periods is not regarded here. In other words, it is assumed that the initial biomass is at the optimal equilibrium level B*. The optimal approach to this level, e.g. from a severely depleted stock level, is usually a so-called bang-bang approach when we are dealing with linear models. The above constraint on minimum escapement is simply that expected escapement in any period shall not be less than ξ_{min}. If this is replaced by the stronger condition that actual escapement in each period shall not be less than ξ_{min}, a more cautious policy is required. In this case the worst possible recruitment pattern must be taken into account, although the probability that such a recruitment pattern will in fact occur may be infinitely small. The expected recruitment, μ, has to be replaced by the minimum recruitment, w_{min}. A policy based on w_{min} will yield a much lower income than a policy based on μ, since this policy will probably overshoot the minimum escapement goal. A constant policy based on w_{min} will also yield a lower income than the same policy allowing the catch and effort to vary from period to period. In this case knowledge about the stock level in each period may be used to ensure that optimal effort is applied at the same time as the escapement level is not jeopardized.

Another way to put this is to say that a constant policy with respect to the control variable will only be optimal if knowledge about the current state of the resource is not available beyond period 0. In all other cases such knowledge may be used to improve the performance of the system. The combination of a constant policy and a constraint on actual escapement in each period will not be applied here because it has to be based on a recruitment pattern which with all probability will not occur, and thus is too restrictive.

Although the optimal solution is the same in the stochastic model as it is in its deterministic equivalent, the expected performances of the constant catch and the constant effort strategies may be different in the stochastic case. If expected recruitment is μ, the expected biomass in each period will be $\bar{B}$. Suppose that the actual B_t lies in the range $[B^L, B^u]$ which is symmetric around $\bar{B}$. Then the expected net revenue with constant catch will be

$$(2.6.27) \qquad \mathcal{E}\pi(CC) = \int_{B^L}^{B^u} (p - c/x)uf(x)\,dx$$

and with constant effort

(2.6.28) $$\mathcal{E}\pi(CE) = \int_{B^L}^{B^u} (px - c)Ef(x)\,dx.$$

It can easily be shown (see appendix to this section) that $\mathcal{E}\pi(CE) > \mathcal{E}\pi(CC)$ when f(x) is a constant and given that $B^u > B^L$.

Another way the model above will show a difference in performance between the two strategies considered here is in the approach to the long term equilibrium given by Eqs. (2.6.17), (2.6.18) and (2.6.23). If there were no constraints on the adjustment to the equilibrium, a bang-bang approach would be optimal in this kind of model, meaning that zero harvest is applied if the biomass is below the level given by (2.6.17) and maximum harvest is applied if it is above this level, until B* is reached. However, here the problem is to find either the constant catch or the constant effort that is the best substitute for the bang-bang solution. If the time horizon tends to infinity, the optimal constant effort will approach E* given by (2.6.23) and the optimal constant catch will approach u* given by (2.6.18) for any c/p ratio satisfying (2.6.19) and any initial population satisfying the minimum escapement condition. E* and u* may, however, be zero if the c/p ratio is high enough.

With a relatively long but finite time horizon, T=100, numerical calculations confirm that constant effort yields better profitability than constant catch for all relevant values of c/p. In other words, with a long time horizon and stochastic recruitment, constant effort is the better alternative just as it is with only two periods if the total biomass is stochastic. This also indicates that a constant effort policy is the best way to approach the long term equilibrium. This conclusion depends upon the assumption that all economic and biological parameters remain constant.

What has been said so far is only meant as an introduction to this section. In order to make the analysis more interesting consider two possibilities:

1) including a spawning stock/recruitment relationship which is influenced by stochasticity and

2) including the aspect of irreversible investment in fixed capital.

The following presents an investigation of both of these cases.

2.6.2 Stock - recruitment relationship.

First, take a look at the spawning stock/recruitment relationship. With a deterministic discrete time model the conditions for optimal harvest policies are well known from e.g. Clark (1990). Assuming w_t in Eq. (2.6.1) is a function

(2.6.29) $$w_t = f(B_t - u_t)$$

the dynamic recurrence equation can be written as a function

(2.6.30) $$B_{t+1} = F(\xi_t) = s\xi_t + f(\xi_t)$$

where

$$\xi_t \equiv B_t - u_t.$$

ξ_t is the escapement in period t. Given an objective function

(2.6.31) $$J = \sum_{t=0}^{\infty} (1+r)^{-t}\pi(B_t,u_t)$$

where

(2.6.32) $$\pi(B_t,u_t) = \int_{\xi_t}^{B_t} [p - C(x)]dx,$$

the optimal escapement, ξ^*, is given by

(2.6.33) $$F'(\xi^*)\frac{p - C(F(\xi^*))}{p - C(\xi^*)} = 1 + r.$$

r is the annual rate of discount. In this case the optimal solution implies both constant effort, constant catch and constant escapement. With a specific recruitment function, however, this condition soon becomes quite messy[3], and therefore lends itself naturally to numerical solution methods.

Unfortunately the real world is not as simple as the deterministic model suggests. If annual recruitment is influenced by stochasticity, the optimum conditions will change. Assume that the recurrence equation is given by

(2.6.34) $$B_{t+1} = F(B_t, u_t, \varepsilon_t) = s\xi_t + f(\xi_t)\varepsilon_t$$

where ε is a random variable. Assume further that revenue is linear in catch and that there is a constant cost function; i.e. the objective function is given by

(2.6.35) $$J = \mathcal{E}[\sum_{t=1}^{T} (1+r)^{1-t} u_t].$$

Here $\mathcal{E}$ denotes expectation. According to Parma (1990) and Reed (1974 and 1979), if the recruitment function is continuously differentiable and strictly concave, the optimal policy is to harvest in any year t:

(2.6.36) $$u_t^* = \begin{cases} B_t - \xi_t^* & \text{if } B_t \geq \xi_t^* \\ 0 & \text{if } B_t < \xi_t^* \end{cases}$$

where $\xi_t^* = g(\xi_{t-1})$. If there is a lag in the recruitment function of k periods, then the escapement levels $\xi_{t-2} \dots \xi_{t-k}$ have to be included in this function as well. The stochastic model with a lag has been analysed by Parma (1990), the stochastic model without a lag by Reed (1974 and 1979) and the deterministic model with a lag by Clark (1976). They all conclude that the optimal policy has to be based on a target escapement level, ξ_t^*. In the stochastic models $\xi_t^* \geq \xi^d$ where ξ^d is the optimal solution in the deterministic

---

[3] The optimum condition when r = 0 in the case of a Beverton-Holt recruitment function, $f(\xi) = \alpha\xi/(1+\beta\xi)$, is given by
$ps(s-1)(1+\beta\xi^*)^3 + p\alpha(s-1)(1+\beta\xi^*)^2 + (p\alpha s + c\alpha\beta)(1+\beta\xi^*) + p\alpha^2 = 0$
which gives three roots that have to be investigated for ξ^*.

counterpart of the model. Except in deterministic models, an optimal policy implies both variable catch and variable effort.

In the deterministic version of this model the two strategies are merely two different ways of stating the same objective function. Any differences between the two strategies have to stem from the dynamics of the system when the equilibrium is affected by exogenous perturbations. These differences are more likely to manifest themselves in the time profile and the variance of the variables than in their average values.

Assuming that recruitment is given by the Beverton-Holt recruitment function, the dynamics in the absence of harvest are:

$$B_{t+1} = f(B_t) = sB_t + \frac{\alpha B_t}{1+\beta B_t}. \tag{2.6.37}$$

The production function is $u_t = E_t B_t$, hence the escapement will be either $B_t(1-E_t)$ or (B_t-u_t) depending on whether E or u is considered to be the control variable.

Although the deterministic optimum solution is the same in both cases, there exist interesting differences in the dynamics. For one thing, with constant effort, $0 \le E \le 1$, there will always be a single, stable equilibrium stock. On the other hand, with constant catch, u, there will be zero, one or two equilibrium stocks depending on the size of u. If $u < (\sqrt{\alpha} - \sqrt{M})^2/\beta$, there will be two equilibrium stocks of which one is stable and the other is unstable (see Eq. (2.4.29)). $M \equiv 1 - s$ is the natural mortality. Of the two equilibria, the one with the higher biomass will be the stable one, and this will also be the economically more profitable.

It is seen from Fig. 2.6.1 that the system will approach the equilibrium stock $\overline{B}$ faster following the $f(B_t(1-E^*))$ path than following the $f(B_t-u^*)$ path because the vertical distance between the curve given by $f(B_t(1-E^*))$ and the 45^o line is always greater than the vertical distance between $f(B_t-u^*)$ and the 45^o line. E^* and u^* are assumed to be the optimal E and u respectively. There will always exist an $0 \le E^* \le 1$ that maximizes

$\pi(E) = (pB(E) - c)E$ subject to the dynamic constraint since $\pi(E)$ is a continuous function on this interval. Further, $E^* < E_o < 1$, where

$$E_o = \frac{2(c/p)\beta s+\alpha+s-(c/p)\beta-\{4(c/p)\beta s+[(c/p)\beta-\alpha-s]^2\}^{1/2}}{2(c/p)\beta s} \tag{2.6.38}$$

is defined such that $\pi(E_o) = 0$. It is easy to check that $E_o \leq 1$ as long as the parameter values c, p, β and s are non-negative. The existence of an interior solution, $E^* \in (0,1)$ maximizing $\pi(E)$, follows then simply from continuity and the facts that $\pi(0) = 0$, $\pi(E_o) = 0$, $\pi'(0) > 0$ and $\pi'(E_o) < 0$. For a certain E the equilibrium stock is given by

$$\bar{B}(E) = \frac{\alpha(1-E) - [1-s(1-E)]}{\beta(1-E)[1-s(1-E)]}. \tag{2.6.39}$$

In the same manner it is possible to show that there exists a u^* maximizing $\pi(u) = (p - (c/B))u$. For given parameter values this u^* coincides with the equilibrium catch implied by E^*.

In a stochastic model of the kind suggested above, the optimal catch, effort and equilibrium stock will be the same as in the deterministic equivalent of the model. What happens in the stochastic model is that the recruitment is continuously affected by random perturbations and hence the biomass level is steadily kept away from its equilibrium level. Since the adjustment to equilibrium is faster with constant effort than with constant catch, the movement away from equilibrium will be faster with constant catch (the recurrence curve in the case of constant effort lies closer to the 45^o line). If the recruitment steadily is higher than required by equilibrium, the biomass will increase faster with constant catch and also decrease faster if recruitment is less than required. In the latter case this will increase the danger of extinction; a danger that is not present in the case of constant effort.

The higher the biomass level, the higher the profitability will be with either strategy. A plausible guess therefore would be that if the recruitment, by some accident, is for several periods higher than that required by the equilibrium, then the total biomass level will be higher with constant catch and hence this will be the more profitable strategy. If the recruitment is below this level for several periods, it will be the other way around. However, this guess turns out to be incorrect. In fact it is possible to provide examples where the system is simulated over for example 20 periods, and the biomass is consistently higher with one of the strategies but at the same time the net revenue is higher with the other strategy. The reason is that except in two cases[4], the strategies require different biomass levels in order to yield the same net revenue for given parameter values. The biomass level B_E with constant effort, and the biomass level B_u with constant catch that gives the same net revenue can be written as functions of each other; B_E as a function of B_u is given by:

$$B_E = \bar{B} + \frac{c}{p} - \frac{c}{p}\frac{\bar{B}}{B_u}. \tag{2.6.40}$$

Equation (2.6.40) is derived by setting $pu - cu/B_u = pEB_E - cE$, substituting $u = E\bar{B}$ and rearranging. The parameters in this function will be the equilibrium stock $\bar{B}$, which is assumed to be the same with both strategies and the cost-price ratio, c/p. This function will be increasing and concave in B_u, see Fig. 2.6.2.

The question here is what happens to net revenue and biomass when there is a deviation from the optimal equilibrium. Due to the nonlinear recurrence equation, the dynamics may soon become quite complex. A natural way to approach this question therefore, is to look at it step by step. Here the first three periods, 0, 1 and 2, are analysed and then an attempt to generalize is made from there.

First the optimal constant effort, E^*, and the optimal constant catch, u^*, have to be decided from the deterministic equivalent of the model given the parameters α, β, s, c and p. This gives the equilibrium stock $\bar{B}(E^*)$ which will be used as initial stock. Since $u^* = E^*\bar{B}$, the two strategies will leave the same escapement behind in this period:

$$\bar{B}(1-E^*) = (\bar{B} - u^*), \tag{2.6.41}$$

and hence the biomass level in period 1 will be the same under both strategies regardless of which exogenous shocks the system is exposed to. However, unlike period 0 the

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[4] Namely, $B_u = c/p$ and $B_u = \bar{B}$.

profitability may be different for the two strategies, and in fact will be unless B_1 equals $\bar{B}$ or c/p. B = c/p is the lowest stock level that will give a non-negative profit, and it is also the stock level that will be the outcome of an open access fishery. If the actual biomass, B_1, is less than $\bar{B}$, $\pi_1(CC) > \pi_1(CE)$ and vice versa. Assume therefore that B_1 is uniformly distributed on the interval $[c/p, 2\bar{B}\text{-}c/p]$. Then it can be shown that expected net revenue with constant effort, $\mathcal{E}\pi_1(CE)$, is higher than expected net revenue with constant catch, $\mathcal{E}\pi_1(CC)$, see appendix.

Since the result referred to above is based on a uniform probability distribution, and this is a rather special (but not unrealistic) assumption, it may be of interest to comment on the implications of using other probability distributions, e.g. a normal distribution or a log-normal (skewed) distribution. A log-normal distribution is positively skewed and will therefore increase the expected profitability of the constant catch strategy, but at the same time increase the danger of extinction. If the uniform distribution is substituted by a normal distribution, a natural hypothesis is that this will make the strategies yield more similar net revenues since the stock level is now more likely to cluster around $\bar{B}$, and at $\bar{B}$ we have $\pi(CC) = \pi(CE)$. The main conclusion, however, from period 1 is that with a rectangular probability distribution, the expected net income with constant effort is higher than with constant catch.

The next step then is to look at period 2. In this period we have

$$\mathcal{E}B_2(\xi_{CC}) > \mathcal{E}B_2(\xi_{CE}) \tag{2.6.42}$$

if $B_1 > \bar{B}$ and vice versa. Since B_1 is the same with both strategies, see Eq. (2.6.41), it is now possible to calculate the expected net revenue, $\mathcal{E}\pi$, for both strategies with different values of B_1 varying from c/p to $2\bar{B}$ - c/p. The expected net revenue in period 2 with the constant effort strategy is

$$\mathcal{E}\pi_2(CE) = E^* \int_{B_2^L(CE)}^{B_2^u(CE)} (px - c)dx \tag{2.6.43}$$

where

$$B_2^L(CE) = s\xi_1(CE) + f(\xi_1(CE))\cdot 0.5 \tag{2.6.44}$$

and

$$B_2^u(CE) = s\xi_1(CE) + f(\xi_1(CE))\cdot 1.5 \tag{2.6.45}$$

which can be rewritten

$$\mathcal{E}\pi_2(CE) = E^*f(\xi_{CE})\{ps\xi_{CE} + f(\xi_{CE}) - c\}. \tag{2.6.46}$$

Accordingly, expected net revenue with constant catch is

$$\mathcal{E}\pi_2(CC) = u^*\int_{B_2^L(CC)}^{B_2^u(CC)} (p - c/x)dx \tag{2.6.47}$$

$$= u^*\{pf(\xi_{CC}) - c\ \ln[B_2^u(CC)/B_2^L(CC)]\}$$

with $B_2^u(CC)$ and $B_2^L(CC)$ calculated in an equivalent manner. The condition

$$\mathcal{E}\pi(CE) > \mathcal{E}\pi(CC)$$

can be rewritten

$$f(\xi_{CE})\{ps\xi_{CE} + f(\xi_{CE}) - c\} > \bar{B}\{pf(\xi_{CC}) - c\ \ln[B^u(CC)/B^L(CC)]\}. \tag{2.6.48}$$

Both sides of this inequality are functions of B_1 and the parameters. As this expression is tricky to manipulate analytically, numerical methods have been resorted to. Since the expected biomass level in period 1 is $\bar{B}$, the expected biomass level in period 2 will be $\bar{B}$, hence the argument above for period 2 will be valid also for later periods. However, when there is an actual deviation from this equilibrium in one period, the whole system is affected. Instead of trying to trace these effects period by period for all possible cases, Monte Carlo simulations are resorted to here. When complicating factors such as non-linearities are added to the system, a priori expectations based on analytical methods become harder to find and the numerical simulations become even more important. The analysis above, however, suggests that under certain assumptions a constant effort strategy is more profitable than a constant catch strategy. The Monte Carlo simulations will answer whether this hypothesis holds true under other assumptions too.

In the following the difference between constant effort and constant catch in such a model will be discussed under varying assumptions using Monte Carlo simulations.

These are the basic parameter values in the model: $p = 1$, $c = 6$, $s = .8$, $\alpha = 1$, $\beta = .1$ where α and β are the parameters in the Beverton-Holt recruitment function (see footnote 3). First the optimal catch, effort and equilibrium stock are calculated for the deterministic equivalent in each case. This equilibrium stock is then used as initial stock in the stochastic model, and the optimal catch and effort from the deterministic model are used as fixed catch and effort in the stochastic model. By comparing the economic result, i.e. present value of net revenue from the stochastic model with the deterministic model, the cost of uncertainty is found.

Simulation 1.

The first simulation is based on the basic model with a zero discount rate. The optimal deterministic solution is found by maximizing

$$\pi = pEB - cE \tag{2.6.49}$$

subject to Eq. (2.6.39). This gives $E^* = 0.148$, $B^* = 19.7$, $u^* = 2.91$ and $\pi^* = 2.023$. In the stochastic case the simulation is performed over 50 periods with recruitment in each period multiplied by a random, uniformly distributed variable ε. In the Monte Carlo simulations 100 different patterns of the random variable are drawn. Two different levels of stochasticity are investigated. With relatively low stochasticity, i.e. $\varepsilon \in [0.75, 1.25]$, there is no significant difference between the constant effort (CE) and the constant catch (CC) strategy. In this case the 95% confidence interval for J with the two strategies is:

CC: $[98.92, 100.52]$ $\bar{J} = 99.72$,

CE: $[100.23, 101.76]$ $\bar{J} = 100.99$.

$\bar{J}$ is the average J for all random patterns. However, with increased stochasticity, i.e. $\varepsilon \in [0.5, 1.5]$, there is not only a difference between the two strategies, but in the case of CC, 23 out of the 100 recruitment patterns that are drawn result in extinction of the stock.

These 23 patterns are removed from the simulations of both strategies in order to make them comparable. The resulting confidence intervals are:

CC: [97.07, 100.12] $J = 98.59$,

CE: [101.59, 104.24] $J = 102.91$.

J with CC has decreased and J with CE has increased, in other words increased stochasticity decreases the profitability of CC but increases the profitability of CE. Further, since the intervals are no longer overlapping, there is a significant difference between the strategies. In comparing with the net revenue in the deterministic case, which over 50 periods is $J = 101.16$, only the result obtained with the low level of stochasticity can be taken into account since the 23 simulations that were removed will be favourable for the stochastic case compared to the deterministic (only the "worst" random patterns leading to extinction were removed). In any case, the cost of uncertainty seems to be relatively small with the low level of stochasticity.

Simulation 2.

So far the discount rate has been zero. The next question therefore comes naturally: What is the effect of a positive discount rate? The answer is twofold. With a discount rate of $r = 0.1$, the deterministic optimum is $E^* = 0.19$, $B^* = 16.06$ and $u^* = 3.05$. The first part of the answer is that due to the high catch rate, the CC strategy leads to extinction with 59 out of the 100 recruitment patterns with the high stochasticity level. Even with the moderate level of stochasticity, 30% of the recruitment patterns lead to extinction and are therefore discarded. This excludes CC as a practical alternative when both the size of the discount rate and the level of stochasticity are considerable, unless there exist other factors that may modify this conclusion. The result based on the remaining 70 observations in the case of moderate stochasticity is reported anyway and the following confidence intervals are found:

CC: [20.71, 21.13] $J = 20.92$,

CE: [21.01, 21.41] $J = 21.21$.

Again the intervals are overlapping. In other words, the reason why the CC strategy has to be ruled out is not lack of profitability but the high risk of extinction. The reason why a higher discount rate increases this risk, is the higher optimal catch rate implied by a high discount rate.

Simulation 3.

A factor which may work in the opposite direction is a downward sloping demand function. According to the theory from a previous section, a linear demand function for example is supposed to work unequivocally in favour of CC. This is tested in the stochastic case by using the following demand function:

(2.6.50) $$p(u) = 2.5 - 0.5u.$$

From the deterministic model, the optimum is given by $E^* = 0.082$, $B^* = 26.72$, $u^* = 2.20$ and $J^* = 28.20$. It is noticed that the optimal catch rate is drastically reduced due to the downward sloping demand function even though a positive discount rate, $r = 0.1$, is retained. With the values for E, B and u above as input, the Monte Carlo simulations give the following intervals for J:

CC: [27.98, 28.15] $J = 28.07$,

CE: [27.77, 27.94] $J = 27.85$.

These results are interesting. Even though the high level of stochasticity is applied and a positive discount rate is used, the downward sloping demand function makes the optimal harvest so low that no instances of extinction are observed in the case of CC. Further, the result from the Monte Carlo simulations is that the intervals are not overlapping, thus making CC the preferred strategy. In other words, a downward sloping demand function may outweigh both the effect of high stochasticity and a high discount rate, not only to the extent that the two strategies can compete but even to the extent that CC is better than CE. This result is based on a model that takes only the producers' surplus into account and ignores the consumers' surplus. Taking the consumers' surplus into account; i.e. letting the objective be to maximize the sum of the producers' and consumers' surplus, will not change the result as long as this function also is concave in catch. This will certainly be the case when the demand function is linear.

Simulation 4.

A linear marginal cost function, in the sense that cost per unit effort is a function of the effort level applied, is supposed to work in favour of constant effort. However, it will reduce the optimal effort level and catch rate, thus reducing the risk of extinction in the case of CC. The following marginal cost function is suggested:

(2.6.51) $$C(E) = 4.5 + 10E,$$

and the optimum from the deterministic model is given by $E^* = 0.171$, $B^* = 17.59$, $u^* = 3.02$ and $J^* = 21.27$. Only the low level of stochasticity is used in the Monte Carlo simulations. The result is that 11 out of the 100 simulations have to be discarded because they cause extinction in the case of CC. However, the 95% confidence intervals based on the 89 remaining observations are overlapping for the two strategies:

CC: [20.75, 21.26] $\bar{J} = 21.01$,

CE: [21.17, 21.56] $\bar{J} = 21.36$.

The conclusion is that an increasing marginal cost function does not significantly increase the difference between the strategies with respect to net present value of income. On the other hand, an increasing marginal cost function does imply a lower effort level and catch rate which reduce the risk of extinction in the case of CC. Accordingly, it is not possible to conclude that an increasing cost function works in favour of constant effort in this case but it could happen if, for example, the marginal cost function is strongly convex.

Simulation 5.

The next question is: What is the effect of making the survival parameter (or natural mortality) density dependent? In this case it is hard to say anything a priori about the likely difference between the two strategies. It is, however, obvious that if the natural mortality is an increasing function of the stock size, the optimal catch rate will be higher in order to increase the survival of the remaining stock. As has been seen earlier, a high catch rate combined with a high level of stochasticity may be detrimental if a CC strategy is applied. Only the moderate level of stochasticity is used here to compare the two strategies. The following linear density dependent survival function is applied:

$$s = 1 - 0.01B. \tag{2.6.52}$$

This gives as an optimal equilibrium in the deterministic case: $E^* = 0.235$, $B^* = 15.32$, $u^* = 3.60$ and $J^* = 23.89$. Even with the low level of stochasticity, a CC strategy leads to complete extinction in 31 out of 100 simulations. Thus the CC strategy has to be ruled out in this case, not because it performs worse in the presence of density dependent natural mortality, but because the higher optimal catch rate implied endangers the stock.

CC: [23.61, 24.01] $\bar{J} = 23.81$,

CE: [23.94, 24.31] $\bar{J} = 24.13$.

Simulation 6.

The next step is to take a closer look at the production function. So far it has been assumed that catch is proportional to effort for a given stock and vice versa. This is at best a simplification. Numerical experiments with other functional forms indicate that this simplification does not have any important consequences for the conclusions so far.

For one thing, in the profit function in Eq. (2.6.32), the profit is given as an integral from ξ_t to B_t. In the simulations presented here, the profit in each period is calculated according to the expression in Eq. (2.6.49). This simplification does not seem to make a big difference. Another way in which the form of the production function may affect the choice of strategy is by the degree of interactivity between the fish stock and the fishing effort. The larger the "role" of the fish stock in the production function, the greater the difference between the two strategies may be. If the catch, on the other hand, is proportional to the effort level and independent of the stock, there will be no difference between the two strategies.

As an alternative to the production function used so far, u = EB, the following function is suggested:

$$u = B(1 - e^{-E}). \tag{2.6.53}$$

In this case u is still linear in B but concave in E. Further, regardless of the units chosen for E, it will not be possible to harvest more than the entire stock, thus there need not be any limitations on the range of E. This function is derived from an instantaneous Cobb-Douglas function; i.e. dB/dt = -EB and du/dt = EB.

With the same parameter values as earlier for p, c, α and β, the optimum in the deterministic case is given by: $E^* = 0.19$, $B^* = 17.46$, $u^* = 3.02$ and $J^* = 20.51$ over 50 periods with discount rate r = 0.1. Except for the production function this simulation is equivalent to simulation 2 above. If the values here are used as inputs in the MC-simulations and the high rate of stochasticity is applied, the stock becomes extinct in 49 out of 100 observations with CC. Using the low level of stochasticity, 14 observations have to be discarded due to extinction. Based on the remaining 84 observations the following comparison of the two strategies can be made:

CC: [20.19, 20.59] J = 20.39,

CE: [20.47, 20.86] J = 20.67.

Also in this case there is no significant difference between the strategies since the intervals are overlapping. The chief difference the introduction of this production function seems to have made is to reduce the risk of extinction compared to simulation 2 and the reason is that the optimal catch is lower whereas the optimal target biomass level is higher.

Some conclusions from the simulations may be appropriate at this point. For one thing, with a low level of stochasticity, there seems to be no significant difference between the economic performances of the two strategies at a 5% significance level. If the degree of stochasticity is increased, this will affect the choice of strategy in two ways. Firstly, in many cases there will now be a significant difference between the strategies measured by the net present value of income. Secondly, and perhaps more importantly, this will severely increase the probability of depleting the total population when constant catch quotas are applied.

Further, in most of the simulations presented here, comparisons of the stochastic model with its deterministic equivalent indicate that the cost of uncertainty is relatively small. These comparisons have to be based only on simulations where 100% of the observations are retained; i.e. where there are no cases of extinction. In the simulations reported here the introduction of uncertainty does not reduce the net present value by more than at most 1.5%. The points made above indicate that the higher the level of stochasticity, the better the case for constant effort. In many cases a high level of stochasticity combined with constant catch may be detrimental.

So far all the comparisons between strategies have been based on one measure: the discounted net revenue. This is an aggregate measure that only takes the variable net income into account. However, even though the net present value (NPV) of income is quite similar for the two strategies in many cases, there may still exist interesting differences, especially in the time profile of net income and population biomass level.

2.6.3 Non-malleable capital and irreversible investments.

Another factor that may be of importance in the stochastic analysis is the role of investment in fixed or non-malleable capital. There are two types of fixed capital; namely perfectly irreversible investment decisions on the one hand, and so-called quasi malleable capital on the other hand. In this study the latter will be concentrated on since perfectly irreversible investment implies a depreciation rate equal to zero which is not very realistic. Furthermore, it is important to distinguish between fixed (capital) cost and variable (harvesting) costs. Analyses of optimal strategies in the presence of irreversible investments are made by Charles(1983a and 1983b) and Clark et al. (1979). The problem is also considered in another part of this study within a deterministic setting and a multicohort model. Some simplifying assumptions are made here, e.g. that the size of the stock is perfectly known at the beginning of each season and that the necessary non-negative investments can be made immediately on the basis of this knowledge.

In the general model the net revenue function is as follows:

$$\pi_t = pu_t(E_t,B_t) - VC(E_t) - FC(E_t^{max}) - IC(I_t) \tag{2.6.54}$$

where VC denotes the variable cost function, FC the fixed cost function and IC the investment cost function. The new terms introduced here are E_t^{max} which is the maximum capacity and I_t which is the gross investment in period t.

There are two kinds of dynamics in the model; the dynamics of the resource stock and the dynamics of the capital stock. Another way to put this is to say that the two types of capital, the resource and the conventional capital, are both included as state variables. The investment in both stocks, the catch rate and capital investment respectively, are control variables. In each period

$$0 \le E_t \le E_t^{max}$$

where

$$E_t^{max} = (1-\delta)E_{t-1}^{max} + I_t \tag{2.6.55}$$

and the gross investment, I_t, is defined as

$$I_t = \max(0, E_t(u_t,B_t) - (1-\delta)E_{t-1}^{max}). \tag{2.6.56}$$

It is not possible to have negative investments in any period; i.e. there is no second-hand market for capital (fishing vessels).

Assuming a particular model of the same kind that has been used earlier in this section, i.e. dynamics given by (2.6.1) and the production function given by (2.6.2), it is possible to derive some analytical results based on a deterministic version of the model. In particular it is possible to find the optimal equilibrium catch, effort and biomass level. In the optimal deterministic equilibrium there will never exist excess capacity and hence fixed costs will play no role. The net revenue function can be rewritten

$$\pi_t = pEB_t - [c_v + c_i(\delta+r)]E, \tag{2.6.57}$$

and optimal biomass, catch and effort will be given by Eqs. (2.6.17), (2.6.18) and (2.6.23) with c substituted by $[c_v+c_i(\delta+r)]$ where c_v is the variable cost per unit of effort, c_i is the cost per unit of capital and r is the rate of interest. The user cost of capital is thus defined as $c_i(\delta+r)$. These values will be calculated and used as inputs in the stochastic model. The total net revenue from this model will also serve as a reference for the stochastic model. In the stochastic model the updating of fish-stock biomass can be given by

$$B_{t+1} = s\xi_t + w\varepsilon_t \tag{2.6.58}$$

where ξ is escapement and ε is a uniformly distributed random variable with expectation 1. That is, recruitment is a completely random, uniformly distributed variable with expectation w. However, it has already been seen that under such circumstances the constant effort strategy can be proved to be more profitable than constant catch even in the absence of irreversible investments. Introducing irreversible investments and fixed costs will not alter the situation under the constant effort strategy and will only serve to decrease the profitability of constant catch. The only interesting question that can be posed here is how sensitive the profitability of the constant catch strategy is to changes in parameter values like c_i, δ and c_f which is the per unit cost of excess capacity. c_f does only affect the part of the fleet that is idle at any point in time, $(E_t^{max} - E_t)$, and is therefore not equivalent to the FC term in Eq. (2.6.54). The results are reported in Table 2.6.1. In the initial model the economic parameter values are $p = 1$, $c_v = 6$, $c_f = 3$ and $c_i = 12$. The depreciation rate plus the interest rate, $(\delta+r)$, is assumed to be 0.1 and the survival parameter $s = 0.8$. The optimal biomass, catch and effort from the deterministic equivalent of the model are reported as B*, u* and E* respectively. J denotes the accumulated net revenue over a time period of 50 years. When stochasticity is introduced, 100 different recruitment patterns are drawn with expected recruitment 4 and standard deviation 2. The

95% confidence interval for the average accumulated net revenue with these recruitment patterns can be calculated as $\bar{J} \pm 2s_{\bar{J}}/\sqrt{n}$ where $n = 100$ and $s_{\bar{J}}$ is the standard deviation for J. These intervals and the average J are reported in the rightmost column in Table 2.6.1.

c_f is supposed to be significantly less than c_v since it is less costly to have idle fishing vessels than it is to actually use these fishing vessels. In the initial model c_f is one half of c_v, but in order to check how sensitive the model is to changes in c_f, the values $c_f = 0$ and $c_f = c_v$ are applied. The deterministic optimum and the profitability of CE are independent of c_f.

If investment c_i and δ are changed in such a way that $c_i(\delta+r)$ remains unchanged, this will not have any effect on the deterministic model or on the profitability of the CE strategy, but the profitability of the CC strategy may be affected by the mix of c_i and δ. In the simulations reported in the table c_i and $(\delta+r)$ are reduced by 50%, one at a time. The profitability of CC increases faster when c_i is reduced than when $(\delta+r)$ is reduced. That is, the constant catch strategy is more sensitive to a change in the cost of investment than to a change in the depreciation rate. It is not intuitively clear why this is so. Part of the explanation is that increased c_i will either reduce the profitability due to higher costs when investment takes place or will have no effect at all when no investment takes place. An increased depreciation rate will reduce profitability when investment takes place simply because a higher δ implies higher investment. On the other hand, when the level of fixed capital is already too high, an increased δ will reduce fixed costs because it implies some degree of disinvestment and thus will increase the profitability.

It is also of interest to compare these results with the result of using the alternative production function given by Eq. (2.6.53). The effort requirement function, which is the inverse of the production function, becomes

$$E = \ln\left(\frac{B}{B - u}\right). \qquad (2.6.59)$$

The escapement in the case of constant effort is now written:

(2.6.60) $$\xi_t = B_t e^{-E}.$$

With this production function the optimal equilibrium effort, E*, in the deterministic model is given by

(2.6.61) $$E^* = \ln\left(\frac{-2s^2c}{\sqrt{p^2w^2M^2+4scpwM} - 2sc - pwM}\right).$$

$M \equiv (1-s)$ and $c = [c_v + c_i(\delta+r)]$. This value can be substituted into the production function in order to find u* and into

(2.6.62) $$B^* = \frac{w}{1 - se^{-E^*}}$$

in order to find the optimal equilibrium stock. The parameter values are changed such that $c_v = 5$ and $c_i = 10$. It is interesting to note that the introduction of this production function does not make any large difference compared to the other results reported in the table.

It is common for all the results that the regions (95% intervals for average net revenue) are not overlapping. Further, the regions with constant catch do not in any case include the net revenue resulting from the deterministic model whereas the regions with constant effort do not only include this value, but the average J is higher in all cases. In other words, there is a positive cost of stochasticity with constant catch, and this cost increases with fixed costs but it is positive even when $c_f = 0$. With constant effort the cost of uncertainty is not significantly different from zero.

It may be even more interesting to combine irreversible investments with the presence of a spawning stock / recruitment relationship. A Beverton-Holt recruitment function is applied for this purpose. In this case it is not easy to find an analytical expression for the deterministic optimum. The optimal equilibrium can, however, be calculated using numerical methods. With the same economic parameter values as in the initial model and $\alpha = 1$ and $\beta = 0.1$ in the B-H recruitment function, the result is: E* = 0.139, B* = 20.55

and u* = 2.85. Net revenue in each period is 1.85. The problem now is that in 12 out of 100 simulations, the CC strategy is not feasible because the stock ultimately becomes smaller than the allowable catch. In other words, the stock will become extinct if this strategy continues. Higher costs will reduce the optimal catch rate and increase the probability that CC is a feasible strategy. The result reported here is calculated using the following values: $c_v = 10$, $c_f = 10$, $c_i = 20$ and in this case the stock does not become extinct. Now E* = 0.107, B* = 23.84, u* = 2.55 and J* = 63.21. The confidence intervals are

CC [57.45, 60.70] $\bar{J} = 59.07$,
CE [61.47, 63.94] $\bar{J} = 62.71$.

The only noticeable difference from the results reported earlier is that the average J with the CE strategy is less than J* from the deterministic optimum.

Another interesting aspect of including non-malleable capital is that it is no longer obvious that the argument about risk aversion being in favour of constant catch holds true. This argument is based on the assumption that net revenue with constant catch will have less variance than net revenue with constant effort, but when there are fixed costs pertaining to investments and the biomass is varying stochastically, this assumption may be false. The argument above is only valid if it is plausible that a low variance of net revenues outweighs a lower value of the average net revenue. Calculations of the average variance of net revenue based on 100 different stochastic recruitment patterns with the initial model mentioned above, show that $\overline{\text{var}}(\pi(CC)) = .086$ whereas $\overline{\text{var}}(\pi(CE)) = .056$. It is important to note that the increased variance of the CC strategy is entirely caused by the investment costs and not by the fixed costs of having excess capacity. The fixed costs of excess capacity actually work in the opposite direction; that is to reduce the variance. This can be explained as follows. In a period when the biomass level is lower than the equilibrium, there is a need for new investments in fishing effort in order to take the constant catch which will make the already low profit in this period even lower, thus increasing the variance of net revenues. In such a period there will be no excess capacity costs. On the other hand, when the biomass level is higher than the equilibrium level, there will be no need for new investments but the excess capacity costs will reduce the net revenue which already is higher than average, and therefore the variance of net revenues will be reduced. It is clear from this that the higher c_i and the lower c_f, the higher the variance with the CC strategy will be. It seems reasonable to suggest that c_i is high relative to c_f, or in other words, that the buying cost per unit effort is higher than the per unit fixed cost of idle effort.

The conclusion from this part of the analysis is that when there are irreversible investments in fixed capital this will, not unexpectedly, significantly reduce the profitability of the constant catch strategy. Even an assumption of strong risk aversion will not be able to reverse this conclusion. The analysis above assumes somewhat unrealistically that new investments can be made almost instantaneously. This of course is not true, but it enables us to illustrate the main effects of having non-malleable capital.

Table 2.6.1

	Deterministic	Stochastic	
Initial model	u*=2	CC [37.43, 38.98]	J = 38.21
	E*=.167	CE [39.41, 40.77]	J = 40.09
	B*=12		
	J = 40		
$c_f = 6$	"	CC [37.07, 38.68]	J = 37.88
$c_f = 0$	"	CC [37.78, 39.29]	J = 38.54
$c_i = 24$	u*=1.76	CC [28.44, 29.88]	J = 29.16
	E*=.136	CE [30.45, 31.64]	J = 31.04
	B*=12.96		
	J=30.96		
$c_i = 6$	u*=2.13	CC [41.49, 43.27]	J = 42.38
	E*=.185	CE [44.64, 46.10]	J = 45.37
	B*=11.49		
	J=45.27		
$(\delta+r)=.05$	"	CC [37.64, 39.81]	J = 38.72
$u=B(1-e^{-E})$	u*=2	CC [41.17, 42.96]	J = 42.06
	E*=.182	CE [44.71, 46.08]	J = 45.40
	B*=12		
	J=45.30		

Appendix (section 2.6)

In this appendix we want to compare

(A1) $$\mathcal{E}\pi(CC) = \int_{B^L}^{B^u} (p - c/x)uf(x)\,dx$$

and

(A2) $$\mathcal{E}\pi(CE) = \int_{B^L}^{B^u} (px - c)Ef(x)\,dx$$

where $u = E(B^u + B^L)/2$. If $f(x)$ is a constant, i.e. $f(x) \equiv k$, then

(A3) $$\mathcal{E}\pi(CE) > \mathcal{E}\pi(CC).$$

Proof:
Taking the integrals, it is seen that this is equivalent to saying

(A4) $$\frac{2(B^u - B^L)}{B^u + B^L} < \ln(B^u/B^L).$$

Assume that B^L is fixed and let the left side of this inequality be a function $g(B^u)$ and the right side a function $h(B^u)$. Then it is seen that $g(B^L) = h(B^L) = 0$, and the derivatives are

(A5) $$\frac{dg(B^u)}{dB^u} = \frac{4B^L}{(B^u + B^L)^2}$$

and

(A6) $$\frac{dh(B^u)}{dB^u} = \frac{1}{B^u}.$$

The condition in Eq. (A3) is fulfilled if

(A7) $$\frac{dh(B^{u})}{dB^{u}} > \frac{dg(B^{u})}{dB^{u}},$$

and by inserting the expressions for the derivatives above and simplifying it can be seen that this condition reduces to

(A8) $$(B^{u})^{2} - 2B^{u}B^{L} + (B^{L})^{2} > 0$$

which is always satisfied.

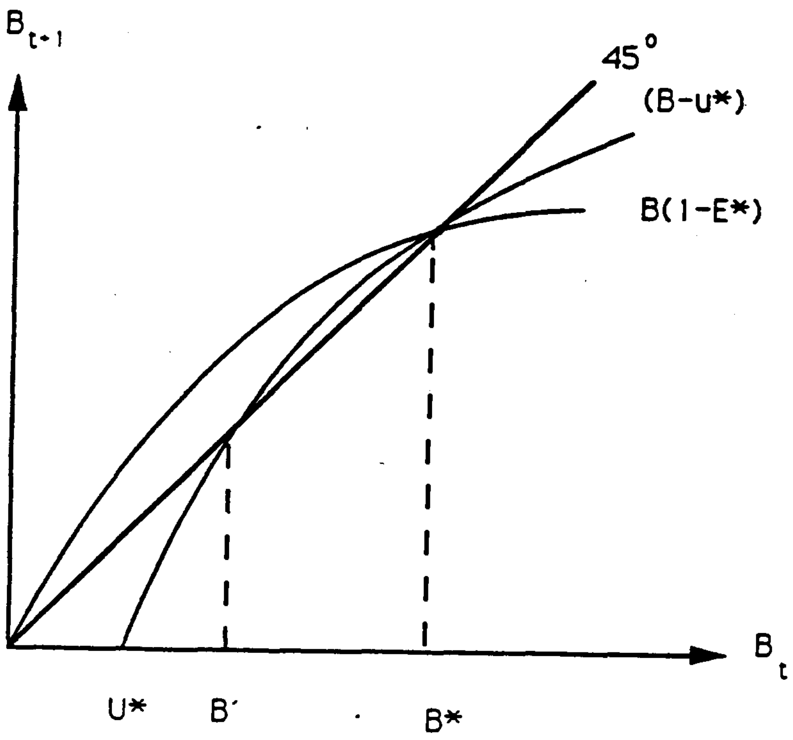


Fig. 2.6.1

B* is assumed to be the optimal equilibrium stock with both strategies. B' is an unstable equilibrium with constant catch.

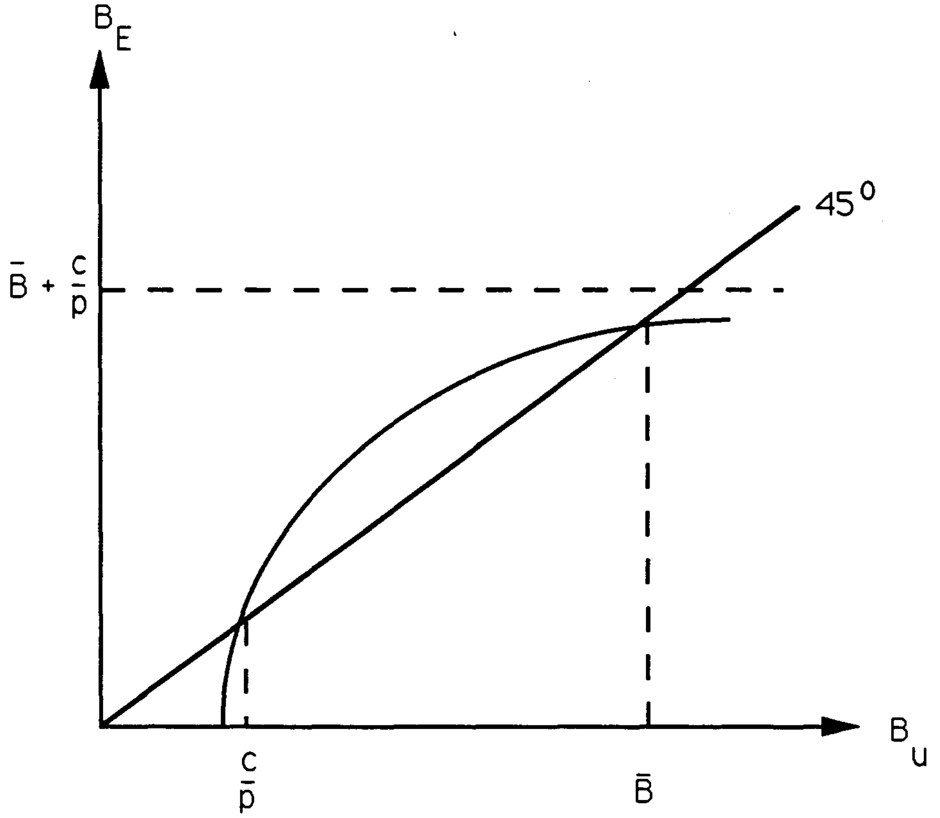


FIG. 2.6.2

2.7 Multi-species modelling

In recent years the importance of including multi-species models in bioeconomic analysis has been widely recognized and accepted. The reason why multi-species modelling has not received more attention earlier, is that such models quickly become quite complicated and good estimates of the parameter values needed may be hard to obtain. This is one of the reasons why such models are not given more attention than they are here either. This section is, however, meant at least to indicate the kind of model that is relevant and to provide an indication of the kind of results we would expect from the model by making some qualified guesses of the parameters.

The main advantage of multi-species models compared to single-species models is that they include a larger part of the ecosystem (by definition) whereas single-species models take the effect of the interaction between species as exogenously given. In this study so far the interaction with other species has been incorporated in the natural mortality parameter and in the stochasticity of recruitment, but this of course also includes environmental effects such as climatic changes, etc. The only interaction that has been included is the interaction a fish stock has with itself which has been included by making several of the parameters density dependent variables. In other words, the main advantage of multi-species models is increased realism and the main disadvantage is increased complexity and lack of data.

The objective of this section is to construct a model that has at least some qualitative characteristics in common with the ecosystem in the Barents Sea. The main fish stock is the Arcto-Norwegian cod stock and the intention here is to include this stock's main predator, the sea mammals, and the main prey species, capelin. Sea mammals include seals and whales, and capelin is here an umbrella term for capelin, herring and some other species with similar characteristics; i.e. pelagic plankton-eaters. Assume that only cod is harvested. This is not quite realistic, but extensive harvesting of sea mammals in this area has not taken place for some time. With respect to capelin, it can be assumed that this stock is harvested according to a constant fishing mortality regime (constant catch can be ruled out for such a short lived species with highly fluctuating recruitment). This section will not be concerned with calculating the simultaneous optimal harvesting rate for both cod and capelin. Since the purpose is to compare different harvesting strategies for the cod, it is sufficient to maximize net revenue in this fishery taking into account the stock dynamics for the other species.

For sea mammals it is assumed that the growth rate is low but stable and that there is a relatively strong stock-dependent recruitment and also that these animals are long-lived. As previously mentioned, the capelin is a short lived species, i.e. it has a high natural mortality rate. It is assumed that the recruitment is hardly stock dependent at all. Also with respect to cod there is assumed to be no spawning stock / recruitment relationship.

The model is described as follows. Denote cod: x, sea-mammals: y and capelin: z. Then the dynamics in the model are given by these equations:

$$x_{t+1} = s_x \xi_t + w_t - \alpha \xi_t y_t + \beta \xi_t z_t, \tag{2.7.1}$$

$$y_{t+1} = s_y y_t + \frac{a y_t}{1+b y_t} + \gamma \xi_t y_t + \delta y_t z_t, \tag{2.7.2}$$

$$z_{t+1} = s_z z_t + v_t - \varepsilon \xi_t z_t - \zeta y_t z_t, \tag{2.7.3}$$

where ξ is the escapement of the x population after harvesting. α, β, γ, δ, ε and ζ are parameters describing the predator/prey relationships and they are assumed positive. a and b are parameters in the Beverton-Holt recruitment function used for the y population. s_x, s_y and s_z are survival parameters that account for the natural mortality not embedded in the predator/prey relationships. If v and w, which denote recruitment of z and x respectively, are assumed to be constants, it is easy to provide examples of locally stable equilibria of the system with or without harvesting of x.

The stability properties of the equilibria may be of importance, and a sufficient condition for local asymptotic stability around an equilibrium x_o, y_o and z_o is that the eigenvalues of the matrix A in the linearized system $(\chi_{t+1} - \chi_o) = A(\chi_t - \chi_o)$ where χ_t is the vector $[x_t, y_t, z_t]'$ are all inside the unit circle; i.e. have modules less than one, see e.g Sydsaeter (1990: p. 307).

Assume the following parameter values:

$\alpha = 0.031$
$\beta = 0.055$
$\gamma = 0.008$
$\delta = 0.008$,
$\varepsilon = 0.0125$,
$\zeta = 0.014$.

$s_x = 0.5$
$s_y = 0.72$
$s_z = 0.75$

$v = 5$
$w = 4$

$a = 0.06$,
$b = 0.1$,

The predator/prey parameters are derived from Flaaten (1988), and adjusted to take account of the fact that Flaaten's model is based on differential equations while the model here is based on difference equations. The small value of a is meant to reflect the relatively slow intrinsic growth rate of sea mammals. Without any harvesting this will result in one stable equilibrium with all stocks positive:
$x = 21.48$,
$y = 4.96$ and
$z = 8.50$.

Introducing the harvesting function $u = xE$ and the net revenue function $\pi = pu - cE$ with $p = 1$ and $c = 6$, we obtain the following optimal equilibrium:
$x = 19.93$, $y = 0$ and $z = 11.816$. This gives $E^* = 0.305$, $u^* = 6.089$ and $\pi = 4.246$. It is obvious why the optimal level of the predator, y, should be zero. Since sea mammals are not harvested, they have no value and in addition they are detrimental to the cod stock both directly by eating cod and indirectly by competing with the cod for the capelin stock. If it is considered unacceptable to harvest the stock of sea mammals in the area down to a critically low level, it is necessary to put a minimum constraint on the size of this stock. Assume for example, that this constraint is approximately 50% of the initial equilibrium stock; i.e. $y \geq 2.5$. This will result in an equilibrium: $x = 22.42$, $y = 2.5$ and $z = 9.44$, with $E^* = 0.128$, $u^* = 2.86$ and $\pi = 2.1$. This equilibrium will be locally stable both with E^* applied as constant effort and u^* applied as constant catch.

Assume now a positive discount rate and look at the problem of maximizing discounted net present value over a finite time horizon and with catch and effort allowed to vary freely. Then the objective function is given by

$$J = \sum_{t=0}^{T} (1+r)^{-t} \pi(E_t, x_t) \tag{2.7.4}$$

subject to the dynamic constraints. This optimum is calculated using a numerical method (non-linear programming with GAMS-MINOS) and $x_o = 22.4$, $y_o = 2.5$ and $z_o = 9.4$ as input. Further, there is a constraint on the minimum escapement for x which is set at 2, 10% of the virgin stock. Even with constant recruitment, a pulse fishing pattern proves to be optimal in this case, and this pattern is shown in Fig. 2.7.1. Fig. 2.7.2 shows the development of the fish stocks over time using this pattern. The maximum value of the objective function in this case is $J^* = 49.88$ when the discount rate is $r = 0.1$.

Let us now compare this result with the result of a constant catch and constant effort policy. The optimal constant catch is $u^* = 5.2$ which results in $J^* = 31.03$ and the optimal constant effort is 0.35 which results in $J^* = 38.36$.

It is seen from Fig. 2.7.2, however, that without any constraints, the y stock is driven almost to extinction during this period. This is because the equilibrium obtained on the basis of parameter values from Flaaten (1988) results in a quite low y stock. Fishing will reduce the availability of food for the y stock and thus slowly drive it into extinction. It is therefore interesting to note that the constraint $y_t \geq 2.5$ for all t is sufficient to make a constant policy optimal, and this will result in the same optimum as without discounting with the possible exception for some adjustment in the initial and the final periods. That is, with this constraint pulse fishing is no longer optimal.

Letting recruitment vary according to a deterministic sinusoidal function results in the following. Assume that

$$w_t = 4 + 2\sin(0.7854t) \tag{2.7.5}$$

and

$$v_t = 5 + 2.5\sin(0.7854t). \tag{2.7.6}$$

The optimal solution is then illustrated in Fig. 2.7.3 and the maximum discounted net revenue is $J^* = 30.3$. The optimal constant effort in this case is $E^* = 0.115$ resulting in $J^* = 22.9$. The time paths of the x, y and z stock are illustrated in Fig. 2.7.4. The minimum constraint on y is only binding in one period, namely period 43. The optimal constant catch is $u^* = 2.5$ resulting in $J^* = 20.2$. Comparing these two strategies, it is seen that CE reduces the discounted profit by 24% and CC reduces discounted profit by 33%. The CC strategy is about 12% less profitable than CE. The development of the fish stocks is remarkably similar in the two cases, as illustrated in Table 2.7.1. Also in the case of constant catch, the minimum constraint on y is only binding in period 43.

Table 2.7.1

Constant Effort

	x	y	z
Average	22.74	2.82	9.20
Variance	39.75	0.05	8.98

Constant Effort

	x	y	z
Average	22.65	2.85	9.24
Variance	42.26	0.07	10.01

The x stock is slightly smaller and the y and z stock are slightly larger with constant catch whereas the variance of the biomass is a bit larger for all stocks when constant catch is applied. The average catch with constant effort, $\bar{u} = 2.61$, is a bit higher than the optimal constant catch, $u^* = 2.54$. With respect to the variance in net revenue, this will be 0.53 with constant effort and only 0.05 with constant catch; in other words the variance of net revenue with CC is one tenth of the variance with CE.

2.7.1 Stochastic Recruitment

Suppose now that recruitment for the x-population is a random, uniformly distributed variable in the range [1, 7]. Then 100 different recruitment patterns are drawn and u^* is applied as constant catch (CC) and E^* as constant effort (CE). Net revenue is calculated over a period of 100 years and the 95% confidence intervals for net revenue are

CC [207.94, 208.33] $\bar{J} = 208.14$
CE [209.19, 210.25] $\bar{J} = 209.72$.

Although these intervals are not overlapping so that there is a significant difference, the CE strategy is on average less than 1% more profitable than CC.

Assume now that stochastic recruitment is introduced also for the z stock with recruitment uniformly distributed in the range [1.25, 8.75]. In the example presented here, recruitment levels for the x and z stocks are highly correlated because the same underlying stochastic variable is used, but this is not unrealistic as the external, environmental conditions affecting recruitment for one stock also affect the recruitment of the other. Now the difference between the two strategies becomes greater, and this is reflected in the respective confidence intervals and average values for net revenue:

CC	[193.78, 196.56]	$J = 195.77$
CE	[211.68, 214.47]	$J = 213.08$.

In this case the CE strategy becomes almost 9% more profitable on average. In other words, increased stochasticity and variations in the biomass make constant effort more profitable.

The conclusion from this section seems to be that inclusion of multi-species modelling does not change the fact that constant effort turns out to be the more profitable strategy whereas constant catch leads to less variance in the profits (but to higher variance in the fish stocks). This conclusion holds both when recruitment is constant and pulse fishing is the optimal strategy, and when there are deterministic or stochastic fluctuations in the recruitment. The greater the variation in the biomass level, due to, for example, fluctuations in recruitment, the more profitable CE is compared to CC. It is also interesting to note that the more stochastic recruitment of the prey species is, the better CE will be compared to CC. The development of the x, y and z stocks over time with constant effort and constant catch is illustrated in Figs. 2.7.5 and 2.7.6 respectively.

Appendix (section 2.7)

The existence of equilibria in the system

$$x_{t+1} = (1-M_x)x_t + w - \alpha x_t y_t + \beta x_t z_t$$

$$y_{t+1} = (1-M_y)y_t + \frac{ay_t}{1+by_t} + \gamma x_t y_t + \delta y_t z_t$$

$$z_{t+1} = (1-M_z)z_t + v - \varepsilon x_t z_t - \zeta y_t z_t$$

can be analysed as follows. First define xy, yz and xz as linear functions in x, y and z in equilibrium:

$$xy = l_{11}x + l_{12}y + l_{13}z + l_{14}$$

$$zy = l_{21}x + l_{22}y + l_{23}z + l_{24}$$

$$xz = l_{31}x + l_{32}y + l_{33}z + l_{34}.$$

The l_{ij} parameters are defined as:

$$l_{11} = (\varepsilon M_x \delta b - \beta\zeta\gamma)/\Omega$$

$$l_{12} = (\beta\zeta b M_y)/\Omega$$

$$l_{13} = (\beta M_z \delta b - \beta\zeta\delta)/\Omega$$

$$l_{14} = [\beta\zeta(M_y-a) - \delta b(\varepsilon w+\beta v)]/\Omega$$

$$l_{21} = (\varepsilon\alpha\gamma - \gamma\varepsilon b M_x)/\Omega$$

$$l_{22} = (-\varepsilon\alpha b M_y)/\Omega$$

$$l_{23} = (\varepsilon\alpha\delta - \gamma\beta b M_z)/\Omega$$

$$l_{24} = [-\varepsilon\alpha(M_y - a) - \gamma b(\varepsilon w + \beta v)]/\Omega$$

$$l_{31} = (\gamma\zeta b M_x - \alpha\zeta\gamma)/\Omega$$

$$l_{32} = (\alpha\zeta b M_y)/\Omega$$

$$l_{33} = (\alpha\delta b M_z - \alpha\zeta\delta)/\Omega$$

$$l_{34} = [\alpha\zeta(M_y - a) - b(\alpha\delta v + \gamma\zeta w)]/\Omega$$

with Ω defined as

$$\Omega = b(\beta\gamma\zeta - \alpha\delta\varepsilon).$$

The system can now be rewritten

$$z = A(x - l_{12})(y - l_{11}) - K_1$$

$$y = B(x - l_{33})(z - l_{31}) - K_2$$

$$x = C(y - l_{23})(z - l_{22}) - K_3$$

with $A = l_{13}^{-1}$, $B = l_{32}^{-1}$ and $C = l_{21}^{-1}$. Further

$$K_1 = l_{13}^{-1}(l_{12}l_{11} + l_{14})$$

$$K_2 = l_{32}^{-1}(l_{33}l_{31} + l_{34})$$

$$K_3 = l_{21}^{-1}(l_{22}l_{23} + l_{24}).$$

From this it is possible to derive x and y as rational functions of each other by eliminating z from the system. Then x and y can be rewritten

$$x = \frac{m_2 y^2 + m_1 y + m_o}{n_2 y^2 + n_1 y + n_o}$$

and

$$y = \frac{i_2 x^2 + i_1 x + i_o}{j_2 x^2 + j_1 x + j_o}.$$

Here we have

$$m_2 = -CAl_{12}$$

$$m_1 = C[Al_{12}(l_{11}+l_{23}) - K_1 - l_{22}]$$

$$m_0 = -CAl_{12}l_{23}l_{11} + Cl_{23}(K_1+l_{22})$$

$$n_2 = -CA$$

$$n_1 = CA(l_{11}+l_{23})$$

$$n_0 = 1 - CAl_{11}l_{23}$$

$$i_2 = -BAl_{11}$$

$$i_1 = BAl_{11}(l_{12}+l_{33}) - B(K_1 + l_{31})$$

$$i_0 = -BAl_{12}l_{11}l_{33} + K_1Bl_{33} + Bl_{31}l_{33} - K_2$$

$$j_2 = -BA$$

$$j_1 = BA(l_{12}+l_{33})$$

$$j_0 = 1 - BAl_{12}l_{33}.$$

From the two equations above it is possible to eliminate x and find y from the 5th degree polynomial:

$$k_5y^5 + k_4y^4 + k_3y^3 + k_2y^2 + k_1y + k_o = 0,$$

and thus the system has at most 5 equilibrium solutions. The parameters here are defined:

$$k_5 = j_2m_2^2 + j_1m_2n_2 + j_on_2^2$$

$$k_4 = 2j_2m_1m_2 + j_1m_1n_2 + j_1m_2n_1 + 2j_on_2n_1 - i_2m_2^2 - i_1m_2n_2 - i_on_2^2$$

$$k_3 = 2j_2m_2m_o + j_2m_1^2 + j_1m_2n_o + j_1m_1n_1 + j_1m_on_2 + 2j_on_2n_o + j_on_1^2$$
$$-2i_2m_1m_2 - i_1m_2n_1 - i_1m_1n_2 - 2i_on_2n_1$$

$$k_2 = 2j_2m_1m_o + j_1m_1n_o + j_1m_on_1 + 2j_on_1n_o - 2i_2m_2m_o - i_2m_1^2 - i_1m_2n_o$$
$$-i_1m_1n_1 - i_1m_on_2 - 2i_on_2n_o - i_on_1^2$$

$$k_1 = j_2m_o^2 + j_1m_on_o + j_on_o^2 - 2i_2m_1m_o - i_1m_1n_o - i_1m_on_1 - 2i_on_1n_o$$

$$k_o = -i_2m_o^2 - i_1m_on_o - i_on_o^2.$$

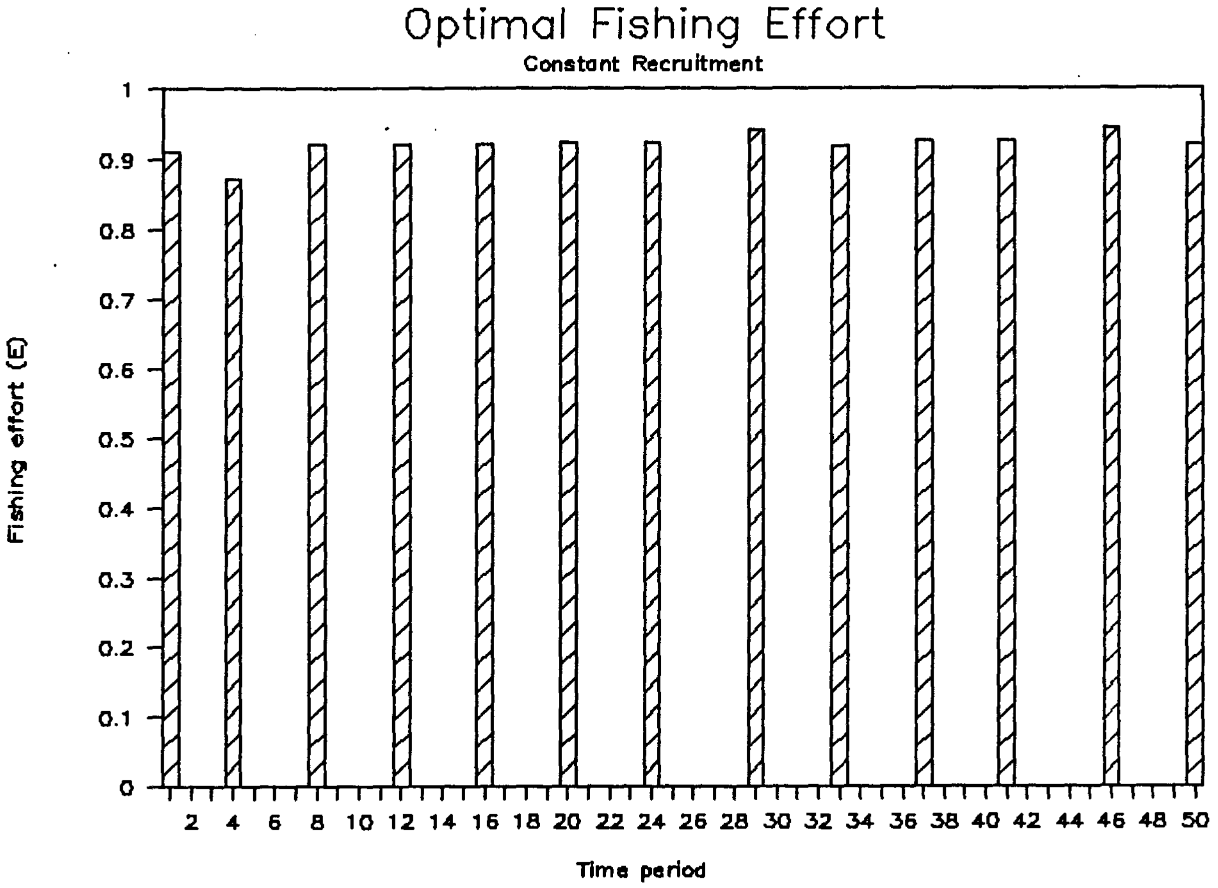


Fig. 2.7.1

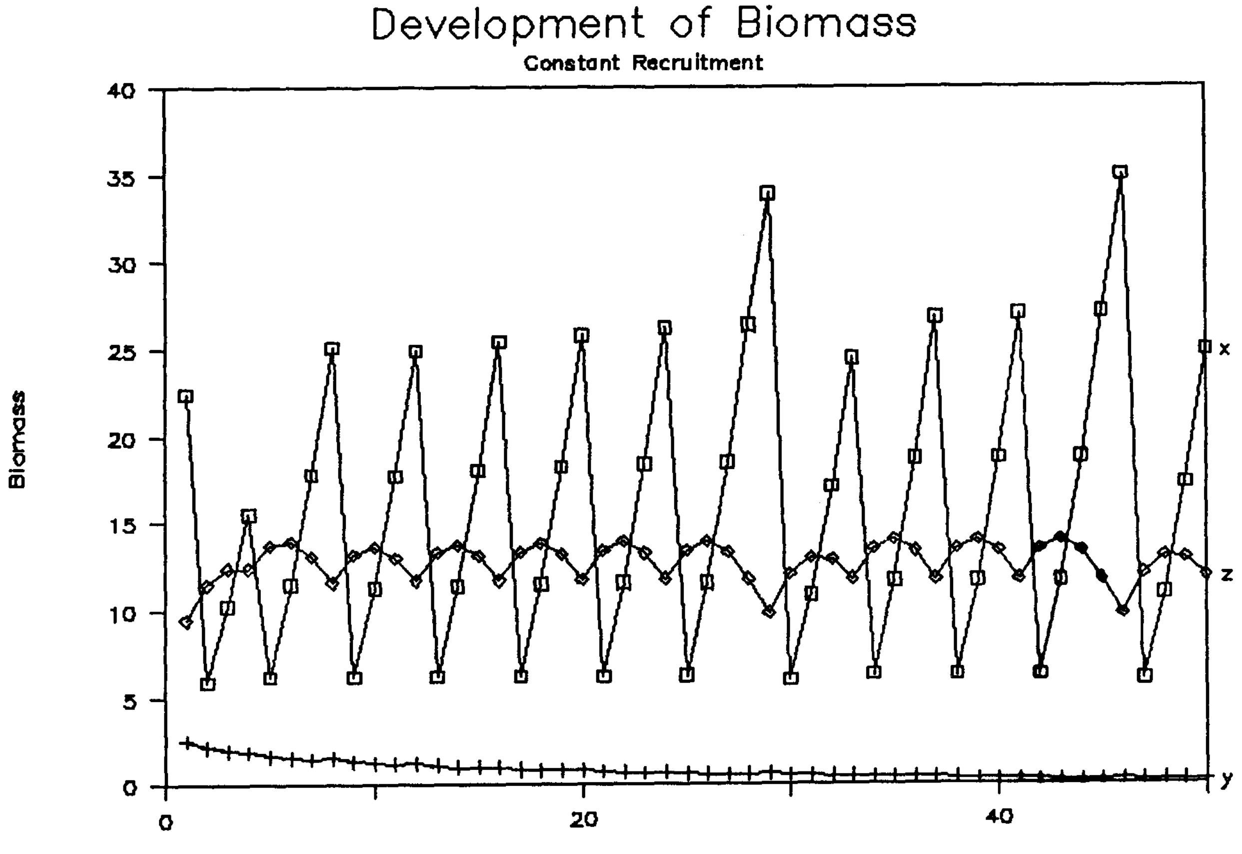


Fig. 2.7.2

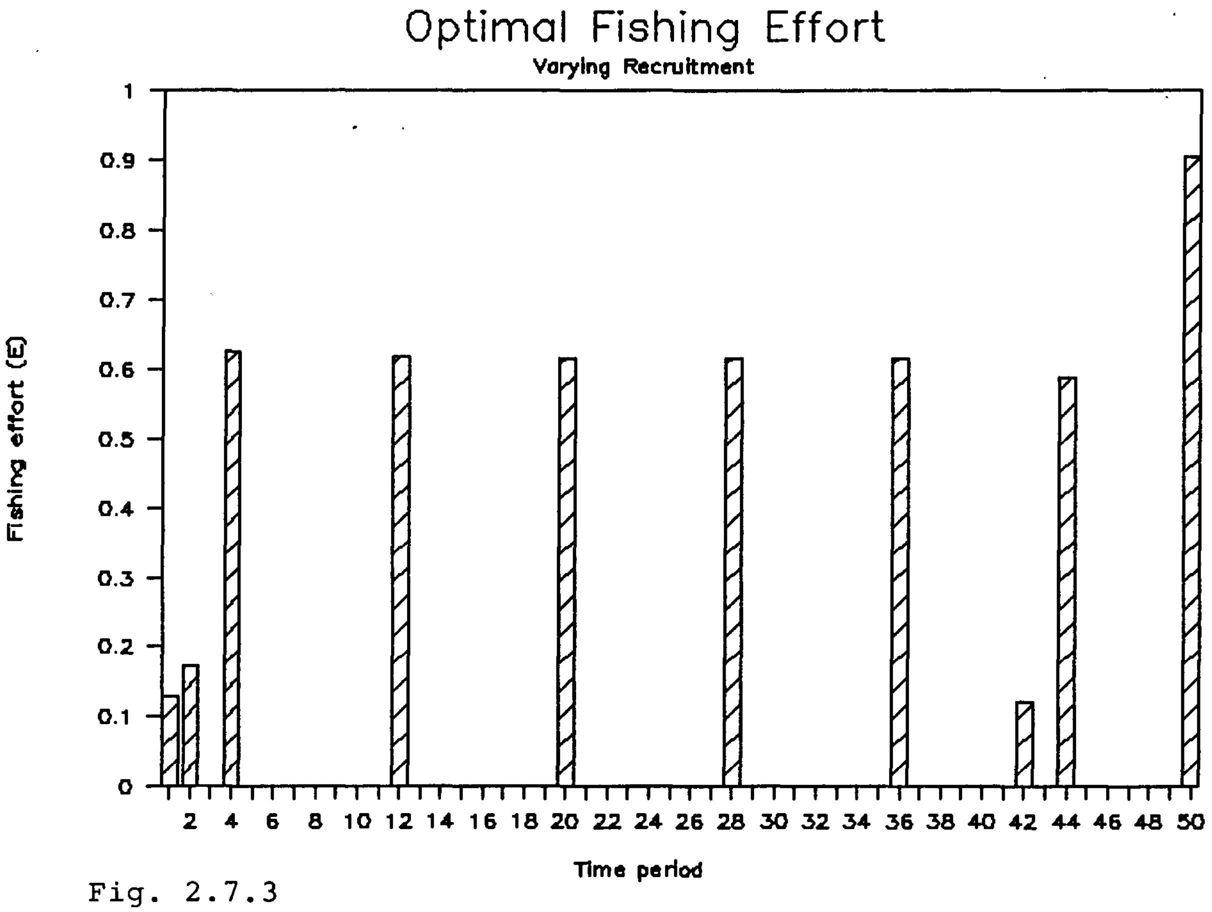


Fig. 2.7.3

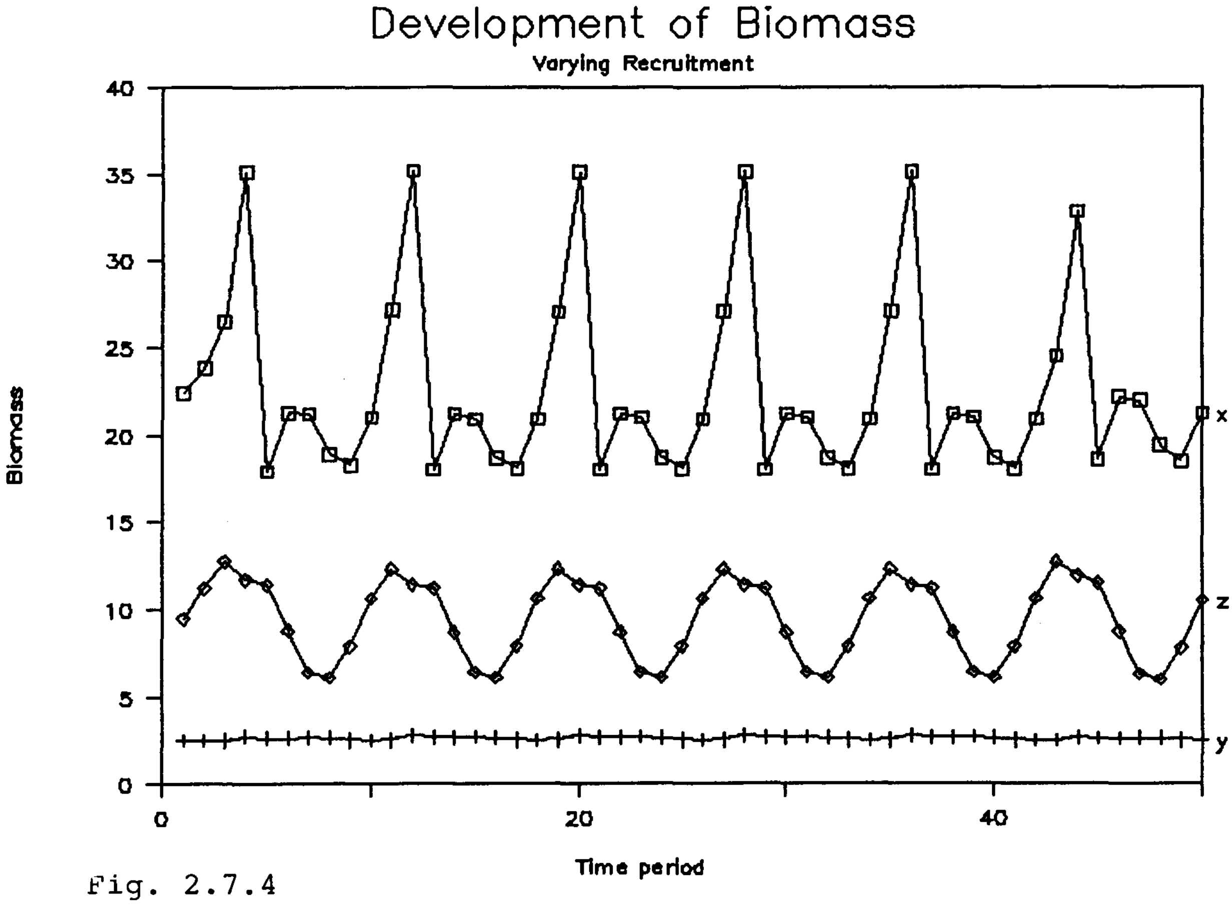


Fig. 2.7.4

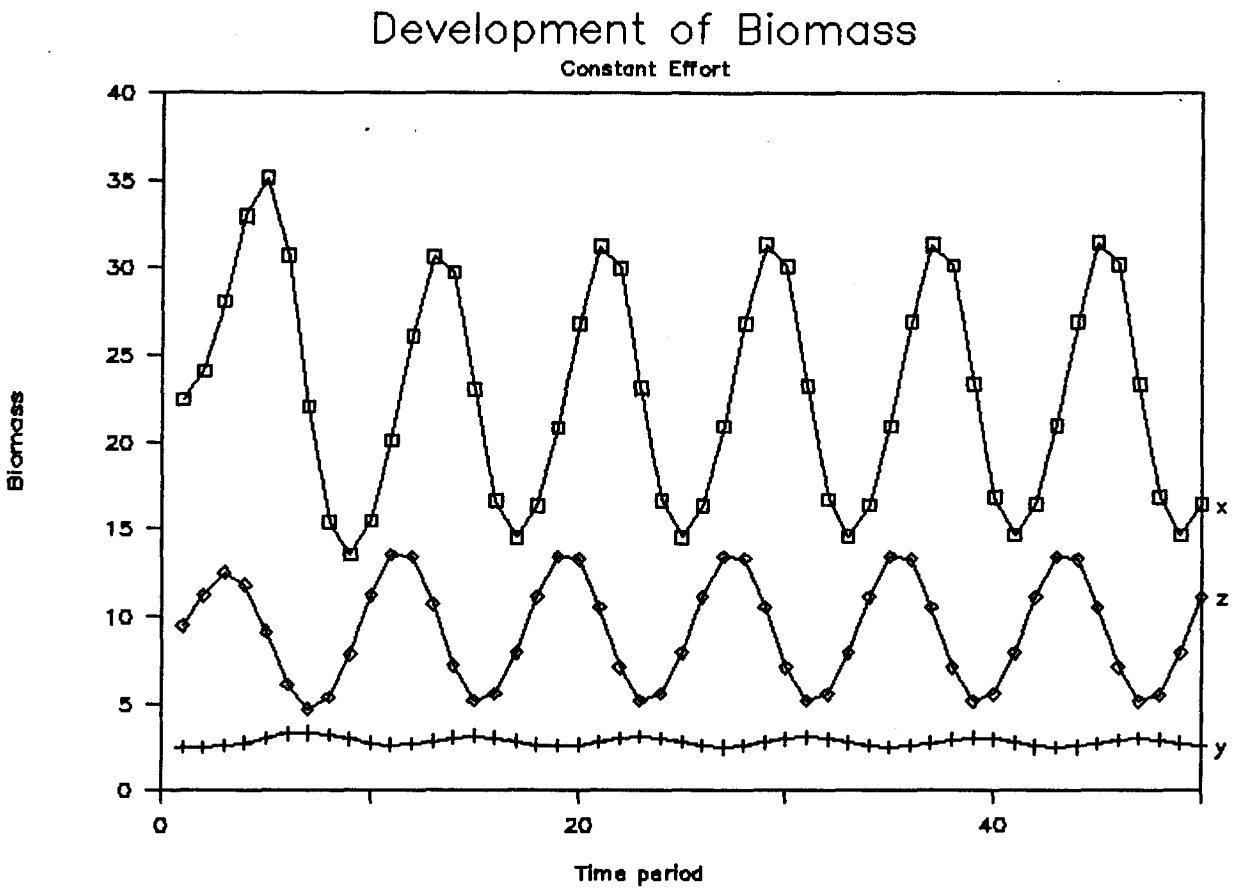


Fig. 2.7.5

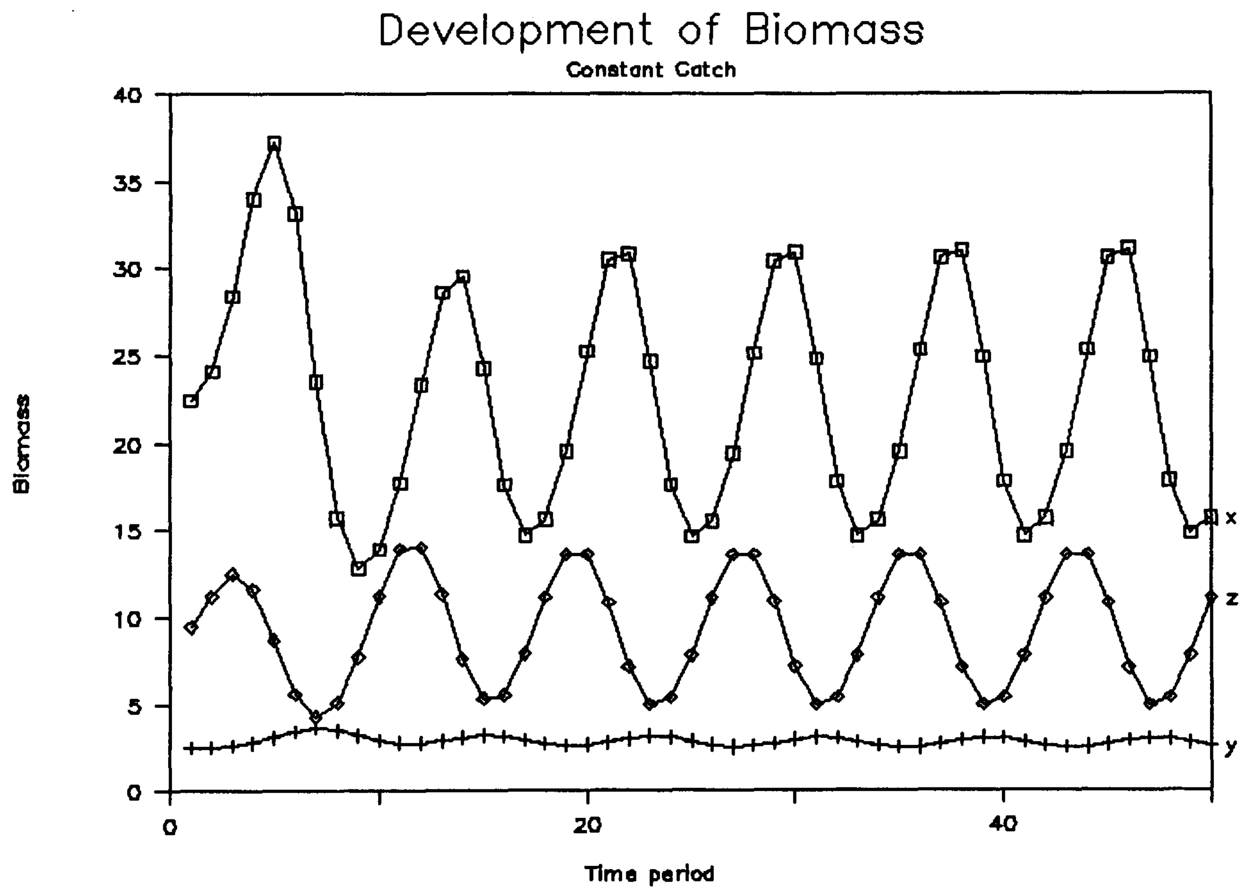


Fig. 2.7.6

3. ANALYSIS OF A YEAR-CLASS MODEL

3.1 The Beverton - Holt Model

The Beverton-Holt model was originally formulated by the biologists R.H.J. Beverton and S.J. Holt in 1957 and has since been a quite popular model within fisheries management although it has certain obvious limitations. We shall here see some of the reasons why it has been so popular and also comment on some of the limitations of the model. This section gives a short description of the general Beverton-Holt model whereas the model described in section 3.3 in many respects differs from the original model formulated by Beverton and Holt.

The main characteristic of the model is that it is an age-structured model operating on several cohorts or year-classes. Fishing within this model will generally affect the age-composition of the population so that younger cohorts become more predominant. The stock number of an individual cohort of age i at time t is a function of the original number of fish recruited to that cohort at time $t-i+\sigma$, $R_{t-i+\sigma}$, where σ is the age of recruitment, and the mortality this cohort has since suffered. Time is measured in years and it is assumed that recruitment occurs on the 1st of January each year as a simplification. The total mortality is composed of a vector of fishing mortalities, **F**, and natural mortalities, **M**, which are specific for this cohort. Thus we can write:

$$N_{i,t} = N_{i,t}(R_{t-i+\sigma}, \mathbf{F}, \mathbf{M}). \tag{3.1.1}$$

Note that the subscript i denoting cohort relates to the age of the cohort and not to the time of spawning or recruitment which may be an alternative.

The total biomass of the population at time t is the sum of the biomass of the individual cohorts, i.e. stock numbers times average weights:

$$B_t = \sum_i N_{i,t}\overline{w}_{i,t} \tag{3.1.2}$$

where $\overline{w}_{i,t}$ is the average weight of fish of age i at time t. Applying average weights adds a simplification to the model as we are ignoring the fact that weight is a continuous function of time. This approximation yields correct results only for given values of fishing mortality.

The catch in numbers from year-class i at time t, or more precisely during some time interval at t, is a function of the stock number of the year-class and the fishing mortality during that interval. Therefore the catch may be written:

$$C_{i,t} = C_{i,t}(N_{i,t}, F_{i,t}). \tag{3.1.3}$$

$F_{i,t}$ is the fishing mortality affecting year-class i at time t. This is considered constant during the given interval. The total yield in biomass at time t is the sum of the individual yields from each year-class:

$$Y_t = \sum_i C_{i,t}\overline{w}_{i,t}. \tag{3.1.4}$$

These are the general formulations of the population and yield functions. Special formulations of these functions may be derived analytically, in the spirit of Beverton and Holt.

Assume that the instantaneous rate of change, i.e. the time derivative, of the stock is given by the differential equation:

$$\frac{dN_{i,t}}{dt} = -(F_{i,t} + M_{i,t})N_{i,t}. \tag{3.1.5}$$

By integrating equation (3.1.5) over one time period from t-1 to t, we derive an expression for $N_{i,t}$ which is the number of fish of age i at the beginning of period t supposing t is an integer:

$$N_{i,t} = N_{i-1,t-1}e^{-Z_{i-1,t-1}}. \tag{3.1.6}$$

$Z_{i,t}$ is the total instantaneous mortality suffered by fish of age i in period t during which it is kept constant:

(3.1.7) $$Z_{i,t} = F_{i,t} + M_{i,t}.$$

Fish of age i at time t belong to the same cohort as fish of age i-1 at time t-1, thus the analysis is here restricted to one cohort only recruited at time $t-i+\sigma$. By repeatedly substituting for $N_{i-1,t-1}$ in equation (3.1.6) we finally arrive at an expression for $N_{i,t}$:

(3.1.8) $$N_{i,t} = R_{t-i+\sigma}\, e^{-\sum_{j=\sigma}^{i-1} Z_{j,t-i+j}}.$$

Recruitment is defined as the stock number of age σ at time t:

$$R_t \equiv N_{\sigma,t}.$$

The instantaneous rate of change in catch from year-class i at time t is given by the instantaneous fishing mortality multiplied by the stock number:

(3.1.9) $$\frac{dC_{i,t}}{dt} = F_{i,t} N_{i,t}.$$

Integrating this over one period for which the age, i, and mortalities, $F_{i,t}$ and $M_{i,t}$, are constant using

(3.1.10) $$N_{i,t} = N_{i,\bar{t}}\, e^{-Z_{i,\bar{t}}(t-\bar{t})}$$

and substituting from (3.1.8) gives us the catch from year-class i from t to t+1:

(3.1.11) $$C_{i,t} = \frac{F_{i,t}}{Z_{i,t}} R_{t-i+\sigma}(1 - e^{-Z_{i,t}})\, e^{-\sum_{j=\sigma}^{i-1} Z_{j,t-i+j}}.$$

For a given initial population and recruitment n periods hence and a given average weight function, we are now able to deduce the future yield, population and its age structure.

Thus the dynamic path of the yield from fishing and the age-structured population depends on the variables R, w, M and F. We will return to each of these variables in later sections. Note also that the fishing mortality, F, provides the link to the economic model.

The Beverton-Holt model has both advantages and disadvantages compared to other models. As stated above the model is a mere analytic deduction, and as long as R, w, M and F are regarded as fixed parameters, the relationships deduced from the model are quite trivial. According to the British economist F.Y. Edgeworth, many of the most severe mistakes in economics arise from treating parameters as fixed when they are in fact variable, and the argument may probably also be transferred to this biological model. If we therefore modify the assumption of R, w, M and F as fixed parameters and treat them as variables, the dynamic relationships in the Beverton-Holt model may soon become complex. There are many good reasons why this should be done and some of them are mentioned below.

The natural mortality, M, consists of several different components which may be affected both by the own stock and by other stocks as well as by environmental factors. The same applies to the growth and weight functions and to recruitment. In our version of the Beverton-Holt model we will treat these variables not as continuous variables of time but as constant averages within each time period and variable between time periods. This is already indicated above, and will require much better data than would be necessary if we treated the parameters as constant throughout. Unfortunately this approach also requires better data than that available at present. However, it is to be hoped that such data will be available in the future and then the model will be ready.

In conclusion, the model is able to take into account many of the complexities met with in reality although, of course, it still represents a huge simplification of the ecosystem in the Barents Sea. Another argument in favour of the Beverton-Holt model is that it is well suited for long-lived species such as the Arcto-Norwegian cod as it continuously calculates the age structure of the population. Thus the consequences of changing gear selectivity as well as different behaviour among year-classes may be accounted for.

3.2 A note on the concept fishing effort and short description of the fleet.

The fishing industry may be divided into two broad categories; the harvesting sector and the processing sector. The harvesting sector, or the fishing fleet, consists of several kinds of vessels and they perform the harvesting of the fish stock, including searching for fish, actual fishing, transport and in some cases a certain degree of processing. After the fish is landed it is sold to the processing sector which transforms the raw material into manufactured products. Thus the harvesting sector supplies the raw material and the processing sector demands it. Since the quantity supplied is generally constrained by catch quotas or the catch capacity and the fishermen's sales organizations have great power in determining the prices, the supply and demand conditions for raw material differ quite a lot from traditional factor markets. The demand function confronted on the output market for manufactured products, on the other hand, conforms quite well to neoclassical theory. The processing sector, however, will not be paid much attention in this study, instead attention will be concentrated on the harvesting sector.

3.2.1 The harvesting sector

The harvesting sector consists of vessels that differ in many respects. The main characteristics of the vessels are probably the size, age, crew number, technological sophistication and type of special gear. These characteristics are not independent of each other and the overall catch capacity of a vessel may be considered as a function of all these characteristics. The size of a vessel may be measured by gross register tons or by carrying capacity e.g. in hectoliters, and the crew number is related to the size. The level of technological sophistication depends on the speed (transport efficiency), searching equipment, efficiency of special gear etc., and is closely connected to the age of the vessel. All these characteristics determine the catch capacity, i.e. the amount of fish which can be caught within a given time period, and the fixed costs are an increasing function of the catch capacity.

As the catch capacity of a vessel i, E_i, is a function of the vector of characteristics pertaining to this specific vessel, κ_i, this may be written:

$$E_i = h(\kappa_i). \qquad (3.2.1)$$

The total catch capacity of the fishing fleet is then merely the sum of the individual capacities:

(3.2.2) $$E = \sum_i E_i$$

The total effort which may be devoted to fishing from the Norwegian harvesting sector is thus limited by the total catch capacity. In addition, the fishing effort exerted from foreign countries has to be taken into account, and together these two constitute the total effort causing fishing mortality. The concept of fishing effort in this study will also include a somewhat broader concept, namely everything that causes costs for the harvesting sector, e.g. when the effects of fixed costs and non-malleable capital are taken into account and analysed. The narrower concept, everything that causes fishing mortality, is mainly used by biologists, whereas economists also use the broader concept. With the simple production function used in much of this study, these two concepts will be equivalent, that is when effort, E, is proportional to average fishing mortality.

The catch capacity for each vessel as a function of its characteristics may be practically impossible to estimate, and therefore some simplifications will have to be applied. For the purposes of this study only an aggregate measure of fishing effort is used. In reality the Norwegian fleet consists of six groups: The oceanic trawlers comprise three groups and the coastal fisheries consist of three groups. These can shortly be described as follows:

1. Saltfish trawlers 200 GRT and above. The average size of vessels in this group was about 450 GRT and the number of vessels was 9 in 1984. These vessels mainly operate around Spitsbergen, and the fish is salted on board and then delivered to stock fish producers in Western Norway.

2. Fresh fish trawlers 200 GRT and above. The average size was about 300 GRT and the number of vessels 45 in 1984. These vessels operate mainly off the Norwegian coast and deliver fresh or round frozen fish to the fillet industry in Northern Norway. The industry firms usually have vested interests in the vessels.

3. Factory trawlers 200 GRT and above. The average size of such vessels was about 1000 GRT and the number of vessels 9 in 1984. The vessels operate mainly in

the Barents Sea and the fish is filleted and frozen on board and the products usually sold abroad.

4. Fisheries with gillnet, hand-line and Danish seine on and off the coast of Northern Norway 13 - 20.9 meters over all length (o.a.l.). These vessels are registered in the three northernmost counties in Norway, and the total number of vessels was 264 in 1984.

5. Fisheries with long line on and off the coast of Nordland 13 - 20.9 m o.a.l. The total number of vessels in 1984 was 93.

6. Other coastal fisheries; vessels 8 - 12.9 m o.a.l. The total number of vessels was about 2600.

The reason why these vessel groups are described here, although only an aggregate measure of fishing effort is applied, is that this description can serve as background information when different selectivity patterns are compared later in this chapter. Each of these vessel groups operates with particular gear selectivity patterns in particular areas at particular times. Thus the cod fisheries may be specified according to vessel (gear) type, time and fishing area, i.e. by whom, where and when the fish are caught. Different classifications of fishery may overlap.

The trawlers, which dominate the harvesting sector measured by catch capacity and often may be treated as one group, operate throughout the year in the open sea. In the trawl fishery selectivity is to some extent controllable by regulating mesh sizes. The gillnet, hand-line and Danish seine vessels operate mainly from February until the end of June. The long line vessels operate from February until the end of May. The other coastal fisheries operate all year.

The main seasonal fisheries are the spawning cod (skrei) fishery in Lofoten from February to April and the spring cod fishery in Finnmark from April to the end of June. Traditionally, the spawning cod fishery in Lofoten has been most important economically. In the spring cod fishery in Finnmark the fish is about 3 - 6 years old.

The structure of the fleet is continually changing due to investment, disinvestment, deterioration and technological development. After WW II the tendency has been towards increased relative importance of the efficient trawlers, but the gillnet, hand-line, long line and Danish seine have also become more efficient and these types of gear are by no means obsolete. The introduction of auto-line, for example, has rationalized the long liners considerably.

The construction of an oceanic trawler fleet has not been uncontroversial. The building of freshfish trawlers during the period from 1965-75 was met with great resistance, especially from the owners of smaller vessels. The building of the saltfish and factory trawlers did not arouse so much controversy, mainly because they were originally intended to operate in distant waters such as Newfoundland. However, this plan failed in the mid-seventies when the fleet moved to Norwegian waters, mainly the Barents Sea. It is worth noting that the decrease in catch from trawlers in later years is due to binding catch quotas.

The owner structure of the fleet is dominated by small firms and the owners are often part of the crew, especially for smaller vessels. In some instances firms in the processing sector have vested interests in the fleet, mainly in the trawler fleet.

In addition to the Norwegian fisheries mentioned here in which cod is the main species, mortality is also induced in the cod stock when cod appear as bycatch in other fisheries. Most important of these instances is the bycatch of cod fry by shrimp trawlers.

3.3 A multi-cohort analysis

This section gives a description of a generalized bio-economic model consisting of a biological and an economic submodel. The biological part represents the population dynamics, calculating the number of fish, total biomass, spawning stock biomass, catch in numbers and yield in biomass for each year given weight-relationships, maturity-relationships and natural and fishing mortality. In other words, the population dynamics is the growth and decrease in the population of fish from year to year.

The economic part of the model calculates the value of the yield or the gross revenue from fishing and fixed and variable costs as functions of fishing effort and the utilization of the fishing effort respectively. Thus the net income for each year and the discounted net revenue for all future years for a given time horizon may be calculated. The link between these two models is given by the fishing mortality production function which translates total fishing effort into instantaneous fishing mortality.

The model is formulated in annual discrete time. This is an appropriate approach in the case of the Arcto-Norwegian cod since spawning and recruitment is supposed to take place once a year and most data have a base period of one year. The discrete time model also provides a convenient framework for modelling of management policies such as annual catch quotas. The simulations will be based on quasi-empirical specifications of the different relationships in the model. These applications are not supposed to be numerically exact representations of the real world but rather reasonable approximations to actual relationships. In any case, a main interest of this section concerns the effects of including or excluding various biological and economic relationships.

3.3.1 Objective function and control variables.

The objective of this study is to analyse optimal long-term management of the Arcto-Norwegian cod stock in the Barents Sea. To achieve this end, the performance of different management strategies will be measured and compared. The traditional economic approach to dynamic optimality problems is to maximize discounted future benefits subject to the relevant constraints. We will not depart from this tradition here, but instead of using an optimization model, we will compare the performance of different strategies using a simulation model in this section and then return to optimization in a later section. A natural procedure is to maximize the discounted sum of the producers' net revenues or profits, assuming that the final product is sold on the international market and treating the international price as an exogenous variable.

The discounted net revenue function or net present value (NPV) can be written mathematically:

$$\text{NPV} = \sum_{t=0}^{T} (1+r)^{-t}\pi_t dt \tag{3.3.1}$$

where

$$\pi_t = R_t - C_t.$$

R_t and C_t are the revenues and costs, π_t the net revenue in period t and r is the rate of discount. In the analysis the qualitative effect of changes in the discount rate on the ranking of management strategies will be studied among other things.

An efficient strategy will be the strategy that maximizes the discounted net revenue function. The main decision variables for the fishermen are choice of the level of catching capacity by investment and disinvestment and the use of this capacity, i.e. fishing effort.

The fisheries which are described by the model are essentially decentralized fisheries consisting of many small agents. They attempt to maximize their own net revenue functions subject to biological, technological, economic and legal conditions. It is therefore an objective for the government to manipulate the legal and economic environment in such a way that the social welfare function will be maximized. As already suggested, the social welfare function can be defined as the total net surplus function for the producers. Maximization of this function will by definition guarantee efficient use of the resources in the industry.

The government has several instruments for controlling the legal and economic environment such as issuing quotas and licensing-schemes, imposing taxes and paying subsidies, regulating mesh sizes or directly controlling the fishing effort.

In this study the primary focus is on the issue of quotas and thereby indirect control of fishing effort. A well-defined combination of the control variables will constitute a management strategy.

3.3.2 A general outline of the biological model

When working with biological models it is important to distinguish between aggregated models which explain the growth of a population only as a function of the total biomass and disaggregated models which divide the total population into cohorts. The choice of model depends on the purpose of the analysis and the species in question. In the case of the Arcto-Norwegian cod, a species consisting of several year-classes where each year-class is exposed to fishing for several years, a disaggregated cohort model of the Beverton-Holt type seems appropriate.

The biological model represents the population dynamics, i.e. changes in biomass and cohort sizes over time. This involves the calculation of the number of fish in each cohort for each period. Given an initial recruitment, the number of individuals in each cohort decreases over time due to fishing and natural mortality. The decrease in each cohort size due to fishing and natural mortality for each year will be calculated. In this way not only the total catch but also the age structure of the catch as well as the age-structure in the population can be found. Moreover, knowing individual weights and the sexual maturity ogive, the total biomass of the population, the spawning stock biomass and the yield in biomass may be calculated.

In this section the following relationships will be treated: updating of the stock and calculating the catch, stock-dependent weight relationships, stock-dependent maturity ogives, recruitment and natural mortality. The examples here concentrate on a single cohort. The total stock, total yield, etc. may be found by summing over all cohorts. The following gives a description of the special version of the Beverton-Holt model used here.

3.3.2.1 Updating

The updating of a single cohort, recruited at time 0, from one period to the next is in the tradition of Beverton and Holt (1957) given by the equation:

$$N_t = R \exp[-\sum_{i=1}^{t} (M_{t-i} + F_{t-i})], \tag{3.3.2}$$

where $R = N_0$ is the number of fish in this cohort recruited to the stock at time 0. N_t is the number of fish in the cohort at the beginning of period t. M_t is the average natural mortality and F_t the fishing mortality affecting the cohort during period [t-1,t].

The instantaneous catch rate is given by

$$\dot{C} = F_t N_t, \tag{3.3.3}$$

and the total catch during one period is given by integrating over one period:

$$C_t = \int_t^{t+1} F_t N_t dt = \frac{F_t}{Z_t} N_t (1 - e^{-Z_t}). \tag{3.3.4}$$

$Z_t = M_t + F_t$ is the total mortality affecting the cohort during [t,t+1]. In the case of several vessel-groups

$$F_t = \sum_{j=1}^{m} F_{jt} \tag{3.3.5}$$

where F_{jt} is the fishing mortality exerted by vessel-group j in period t and m is the total number of vessel-groups.

3.3.2.2 The stock-dependent weight relationship

To calculate the total biomass of the stock, the yield from the stock and the spawning stock biomass, it is necessary to know the individual weights. Here we will rely on average individual weights for each cohort. The total biomass of a single cohort is then simply the total number of fish in that cohort multiplied by the average individual weights. This is only correct as an approximation as growth is a continuous process while time is here being treated as a discrete variable. A more appropriate, but more complex, approach would be to apply a mixed model with time being both discrete and continuous. On the other hand, as fishing to a large extent is a seasonal activity, treating weight as a constant parameter is a convenient simplification.

Factors affecting individual weights in a cohort may be many and complex. The weight and the gain in weight for an individual from one period to another is strongly dependent on the abundance of food available. The food available to each fish is again dependent on the total amount of food in the feeding area and the total number of fish and animals of different species competing for this food. The cod changes its diet throughout its life-span and thus one indicator of the number of competitors for the same food may be the size of the stock of cod at about the same age. The stock size of other species preferring a similar diet is also important but this will be ignored since our attention in this chapter is restricted to a single-species model. As the cod is a cannibal, the availability of food for one cohort increases with the size of the stock in younger cohorts.

The weight of a fish of age t is given by

$$w_t = w_\sigma + \sum_{i=\sigma}^{t-1} Dw_i \qquad Dw \in [Dw_{min}, Dw_{max}], \tag{3.3.6}$$

where w_σ is the weight at the age of recruitment, σ, and Dw_i is the weight-gain during the i'th year of its life. If the food availability is extremely small, $Dw_i \rightarrow Dw_{min}$, which may be zero.

One possibility is to write Dw as an increasing function of the stock-size of cohorts significantly younger than the one under consideration and a decreasing function of the stock-size of cohorts of the same age and older:

$$Dw_i = Dw(N_{<i}, N_{\geq i}), \tag{3.3.7}$$

$$N_{\geq i} \equiv \sum_{j=1}^{J} N_j, \qquad \frac{\partial Dw_i}{\partial N_{\geq i}} < 0,$$

$$N_{<i} \equiv \sum_{j=\sigma}^{i-1} N_j, \qquad \frac{\partial Dw_i}{\partial N_{<i}} > 0,$$

where J is the age of the oldest cohort. With relevant data on stock size these functions may be estimated by statistical methods, one function for each year-class.

However, a simpler weight relationship is employed here; namely that the gain in weight for each cohort in a particular year only depends on the total biomass of cod, B, in this year:

$$(3.3.8) \qquad Dw_i = Dw(B), \qquad \frac{dDw_i}{dB} < 0.$$

The following relationship is applied in this model:

$$(3.3.9) \qquad Dw_i = g(B)\cdot a\cdot i(1 - \frac{i}{b})$$

where

$$(3.3.10) \qquad g(B) = 1 + (A-B)/(A+B).$$

All parameter values are given in Table 3.3.3. B is the total stock biomass as before and A denotes a reference level for total biomass, e.g. the average stock biomass over a long period. Thus if A = B growth will be normal. If B > A growth will be reduced and if $B \rightarrow \infty$ growth will be zero. A function similar to (3.3.10) is used by Arnason (1987). Eqs. (3.3.9) and (3.3.10) are illustrated in Figs. 3.3.1 and 3.3.2 respectively.

3.3.2.3 Stock-dependent sexual maturity

The maturity ogive, i.e. the relative number of sexually mature fish in each year-class, may also depend on the total stock size. This should not be surprising since sexual maturity is likely to be closely related to individual weight. The faster the growth, the earlier the fish mature and, as was shown above, in this model the growth is dependent on the stock size. Therefore, the way sexual maturity is modelled here will not be too different from the way individual growth is modelled.

If $s_{i,t}$ is used to denote the relative number of mature fish of age i at time t, then we may postulate:

(3.3.11) $$s_{i,t} = s_{i-1,t-1} + \varphi_{i,t} \qquad s_{i,t} \in [0,1],$$

where $\varphi_{i,t}$ is the relative number of first time spawners of age i at time t. Obviously $s_{i,t}$ may be rewritten:

(3.3.12) $$s_{i,t} = \sum_{j=\sigma}^{i} \varphi_{j,t-i+j}$$

where σ is the age of recruitment. $\varphi_{i,t}$ may depend both on the stock size of cohort i at time t and on the total biomass at time t as follows:

(3.3.13) $$\varphi_{i,t} = \varphi(N_{i,t}, B_t), \qquad \frac{\partial \varphi}{\partial N_{i,t}} < 0, \qquad \frac{\partial \varphi}{\partial B_t} < 0.$$

The derivatives in (3.3.13) are supposed to be negative due to increased competition for food when stock increases. The reason why average weight is not chosen as an argument instead of N and B is that then it would not be possible to single out the effect of stock dependent sexual maturity independent of stock dependent growth. In other words it would not be possible to separate the effect of an increased total biomass and the effect of a larger spawning stock and subsequent recruitment. To simplify, at this stage φ will be explained solely by N, and the following relationship is suggested:

(3.3.14) $$\varphi_i = a_i + b_i/N_i \quad \text{for all t},$$

where a_i and b_i are parameters to be estimated. One expects both parameters to be nonnegative. For Arcto-Norwegian cod and $i < 6$, a_i and b_i are probably both close to zero.

A more realistic way to model sexual maturity may be to let maturity depend functionally on the stock size of fish younger and older than the cohort in focus as suggested in connection with the weight function. Another way is to let sexual maturity depend directly on individual weight and consequently indirectly on stock size. In this study a numerical specification of equation (3.3.14) will be applied.

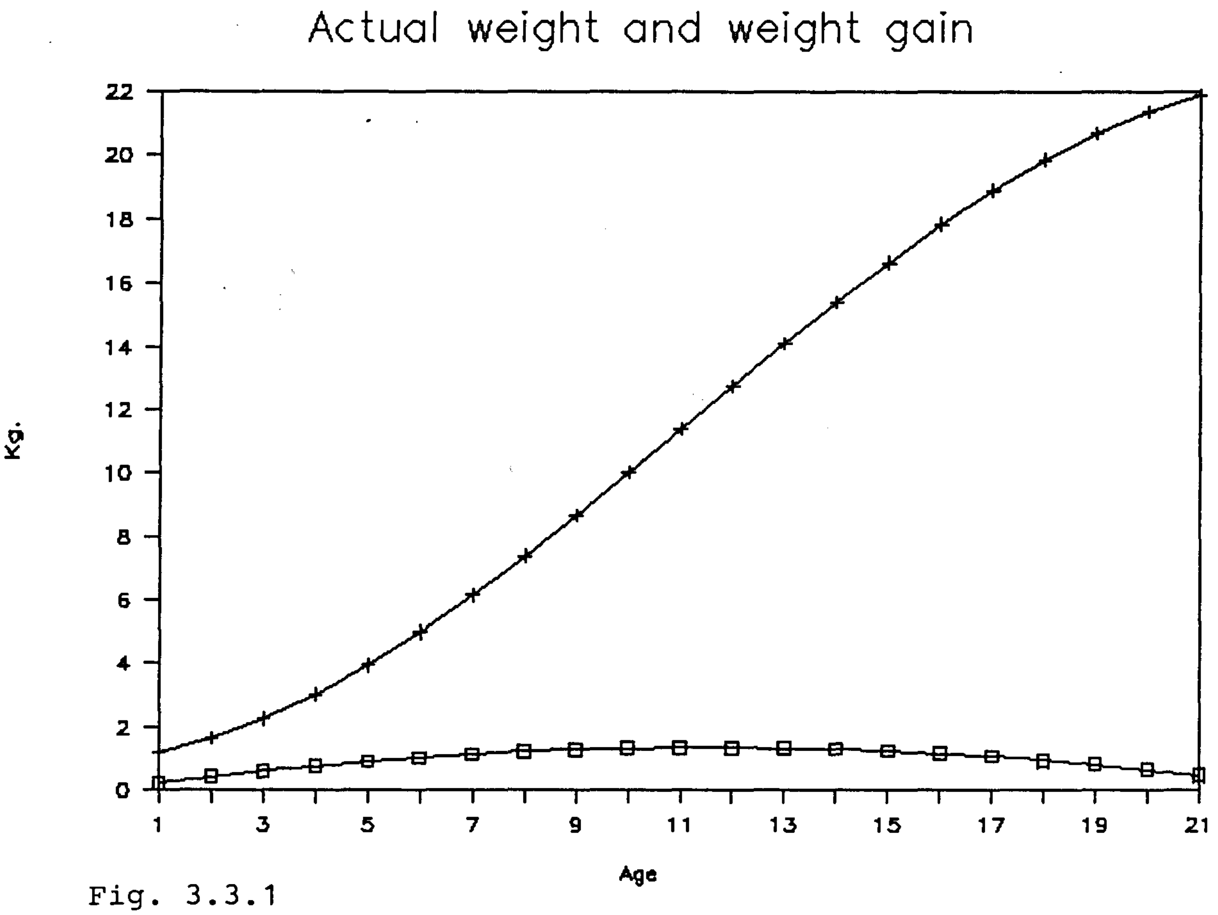


Fig. 3.3.1

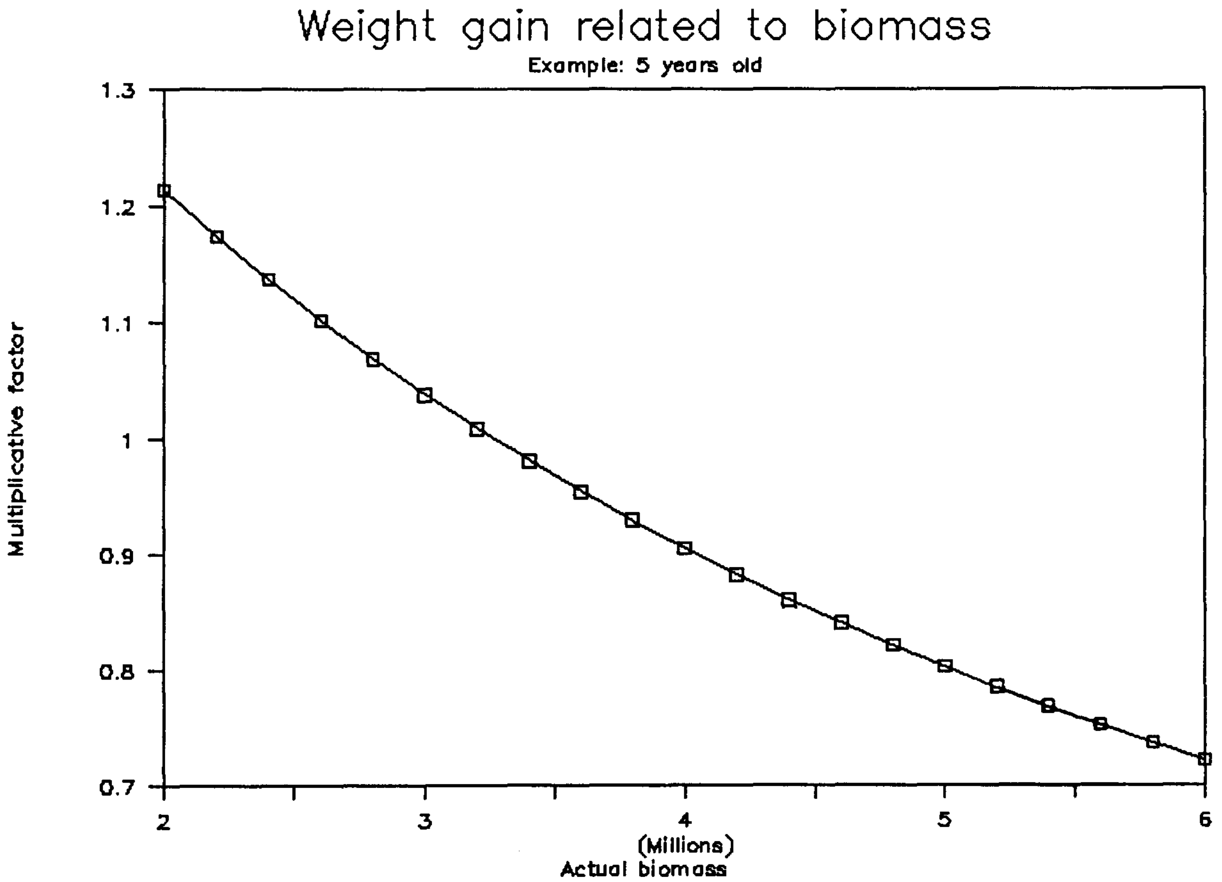


Fig. 3.3.2

Calculating the sexual maturity ogive and the size of the spawning stock is not only important because recruitment in subsequent years may depend on the size of the spawning stock, but also because the migratory pattern of the population depends on sexual maturity. Mature fish migrate to the coast of Norway north of 62^{o} northern latitude to spawn, mainly in Lofoten, while immature fish at the age of 3-6 years migrate to the coast of Finnmark to prey on capelin. Each of these migrations provide a basis for different seasonal coastal fisheries and thus the maturity ogive may be important in determining the total stock available in each fishery.

3.3.2.4 Recruitment

Intuitively, there should be a relationship between the size of the spawning stock and the subsequent recruitment. This is the traditional way to specify recruitment functions in fishery models, and the two functions most often used are the Beverton-Holt recruitment function,

$$R_{t+\sigma} = \frac{aS_t}{1+bS_t}, \tag{3.3.15}$$

and the Ricker recruitment function,

$$R_{t+\sigma} = aS_t e^{-bS_t}. \tag{3.3.16}$$

R_t is recruitment at time t and S_t is the spawning stock at time t. a and b are parameters, and σ is the age at which the fish is recruited to the stock. The Ricker recruitment function is usually applied to short-lived species, where the fishing affects only one or two cohorts at the same time, whereas the Beverton-Holt recruitment function usually is applied to long-lived species such as the cod where the fishing affects several cohorts at the same time. However, in the case of the Arcto-Norwegian cod the intuition that recruitment is a function of the spawning stock is not supported by empirical studies (see Chapter 4). One important reason is that recruitment typically represents a very small fraction of individuals spawned while the rest perish, quite a few due to predation by other cod. Just a marginal difference in the predation pattern, e.g. such that 0.2% survive instead of 0.1, may therefore outweigh a decline in the spawning stock to one half. Thus, in the case of the Arcto-Norwegian cod, only when the spawning stock is at an extremely low level does there appear to be a critical relationship between spawning and recruitment.

Fig. 4.2 (Chapter 4, p.208) indicates long or medium term cyclicality in recruitment of the cod over time. Since the factors that may possibly have an influence on recruitment are numerous, ranging from environmental factors such as weather, temperatures, ice cover and currents, to ecological factors such as predation, these fluctuations will be treated as exogenous. An empirical time-series analysis of recruitment will be discussed at some length in the next chapter. At this stage the following recruitment function will be suggested:

$$R_t = a + b \sin(\lambda t). \tag{3.3.17}$$

b and λ are parameters representing amplitude and frequency in radians respectively. The minimum recruitment is given by a-b. In this study equation (3.3.17) will be applied both in a deterministic and a stochastic version.

3.3.2.5 **Natural mortality**

Individual gain in weight and yearly recruitment give rise to increases in population size. Now attention will be directed to an important factor reducing the population, namely natural mortality.

Natural mortality is made up of several parts, the most important being mortality due to predation and senescent and spawning stress mortality. As might be expected, young immature fish are mostly exposed to mortality due to predation while older mature fish are mostly exposed to senescent and spawning stress mortality. The youngest and the oldest fish have the highest natural mortality; the youngest because of severe predation and the oldest because senescent mortality increases with age. This is the reason why a U-shaped natural mortality curve during the life-span of a cohort is chosen here. A much more sophisticated way to model natural mortality would be to estimate the total amount of different predators and prey species in a multi-species model. However, the following relationship is used in this chapter:

$$M_i = i^a + bi + c \qquad (3.3.18)$$

where M_i is the natural mortality for the year-class at age i. Parameter values are again given in Table 3.3.3. This curve may shift or change its slope over time due to changes in ecological conditions. This may occur in several ways, e.g. simply by weighting M_i with a weighting function similar to Eq. (3.3.10) but with opposite sign, or by letting one or more of the parameters a, b and c depend on the stock size or total biomass. In the model presented here, the former method is applied.

3.3.3 **The habitat: The Barents Sea**

In this study interest is limited to the biota of the Barents Sea and a short description of the habitat will therefore be appropriate. The Barents Sea, named after the Dutch explorer Willem Barents (1549-97), is characterized by extreme environmental and climatic conditions as it is located close to the Arctic. It borders on the Norwegian and Russian coasts in the south, Novaya Semlya in the southeast, Franz Josef Land in the northeast and Spitsbergen in the northwest. The border between the Barents Sea and the Norwegian Sea in the west goes from North Cape through Bear Island to Spitsbergen. A complex current system can be observed in this area, although it is dominated by two main currents. In the southern part the currents are towards the north and east while in the northern part the current directions are towards south and west. This means that the main currents run counterclockwise and these are illustrated in Figure 3.3.3. The Barents Sea covers 1.3 million square kilometres and the depth is on average 300 m. The greatest depth is about 500 m, but some banks are not more than 100 m.

Since the Barents Sea is very shallow, temperature variations in the air relatively soon result in temperature changes in the water, and as many species live near their existence limit, this results in fluctuations in stock size and recruitment. Thus fluctuations in recruitment of cod here can be up to seven times greater than the fluctuations in the area around Iceland.

Important characteristics of a marine area are salinity and climate. At the entrance of the Barents Sea the mean salinity in the autumn from 1966-77 was 35.13^{o}/oo, and the mean temperature in the same period was 6.2^{o}C. (Midttun & Loeng, 1986). The temperature and salinity decreases towards the east and north. The formation of ice usually starts in

October or late September and the melting of ice starts in May or June, but the variation in the extension of the ice cover from year to year is considerable. It is only the northern and eastern parts which are covered by ice during winter, thus the Barents Sea is mainly ice free throughout the year. Also the temperature and the salinity show great variations both in the short and in the long run, and some of these variations seem to be of a cyclical nature (ibid.).

The main effect of climatic variations is in determining the timing of the blooming period of phytoplankton and subsequent growth of zoo plankton. In fact, cold years are characterized by an early spring bloom of phytoplankton and a relatively late development of zoo plankton whereas warm years are characterized by a late phytoplankton bloom and an earlier development of zoo plankton (Rey, Skjoldal & Slagstad, 1987: p. 29). The cod larvae feed mainly on zoo plankton, and the survival of the cod larvae during the first half year, seems to be crucial for the subsequent recruitment to the exploitable cod stock. However, the relationship between temperatures and recruitment will be investigated in more detail later on.

3.3.4 **The Arcto-Norwegian Cod (Gadus morhua L.)**

The Arcto-Norwegian cod is a boreal species, migrating between the feeding grounds in the Barents Sea and the spawning grounds in the Norwegian coastal waters. It spawns on average for the first time at an age of 7-8 years and thereafter once a year until death. Spawning takes place mainly in the Lofoten area from February to April, but also to a certain extent from 62^{o} northern latitude in the south to the coast of Finnmark in the north. The eggs drift with the Gulf stream northeastwards along the coast close to the surface for about 2-3 weeks. After hatching they drift as larvae or, later on, fry into the Barents Sea and toward Spitsbergen where the juvenile fish grow up. The pelagic stage lasts for 4-5 months at the end of which the fry seek the bottom. At an average age of about 3-4 years the cod becomes vulnerable to mesh sizes of 135 mm which is the present minimum size for Norwegian trawlers.

The fry mainly feed on zoo plankton whereas the juvenile cod feed on smaller fish, crustaceans and worms. Fish older than three years prey upon other fish (also cod) and shellfish. The young cod, of about 4-6 years, migrate to the eastern coast of Finnmark every winter chasing the spawning capelin. Cod are also preyed upon by other animals such as whales and seals and are exposed to human exploitation in addition to cannibalism.

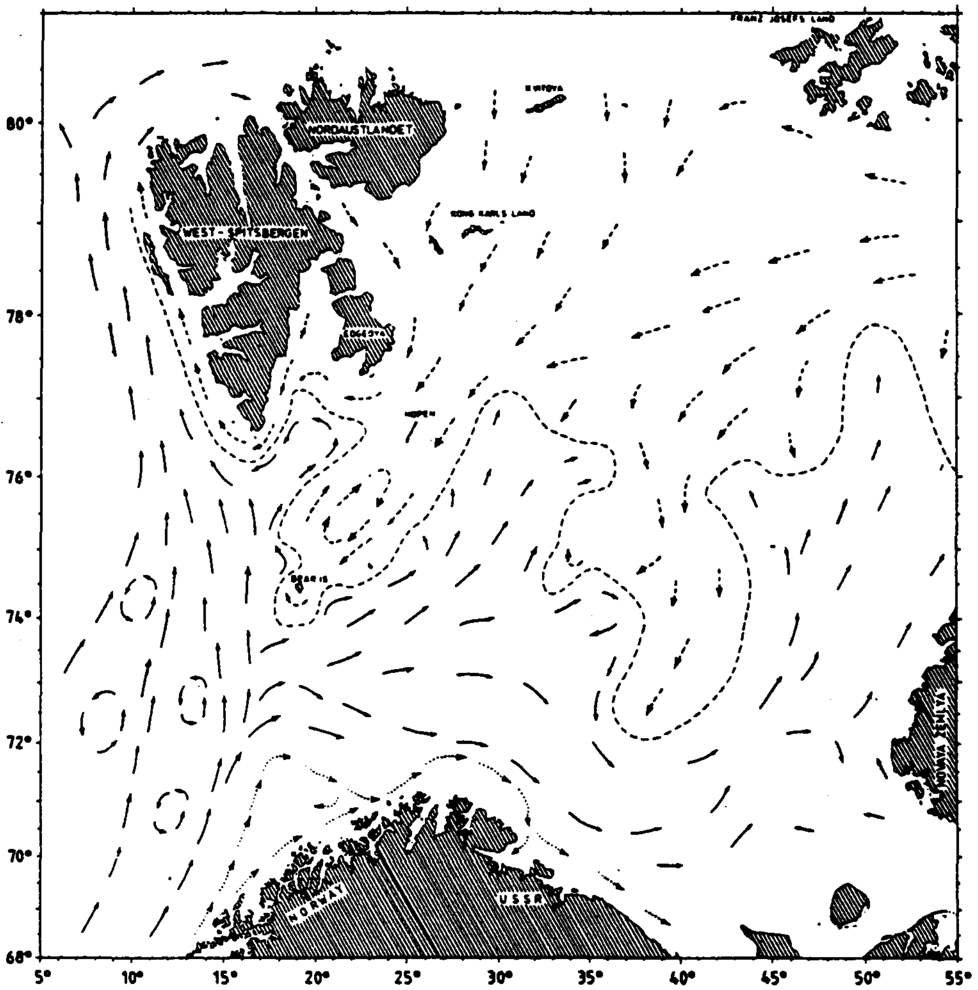


Fig. 3.3.3. Surface currents in the Barents Sea. Arctic currents (--->), Atlantic currents (——>) and Coastal currents (····>).

SOURCE: Midtun and Loeng 1986.

The adolescent cod prefers temperatures from 2 to 10°C and a depth of about 5-600 meters. The first-time spawner is on average 5-7 kg and 70-100 cm. At the age of 15 only 5% have survived with an average weight of 15 kg, but specimens reaching 150 cm length and weighing 60 kg have been caught.

Cod fishery is by far the most valuable and important fishery in the Barents Sea. In fact, the Atlantic cod, which includes the Arcto-Norwegian cod, is among the five or six economically most important fish species in the world (Tande and Tande, 1986: p. 244). In addition to the year-round trawl-fishery in the Barents Sea, there are two seasonal fisheries in Norwegian coastal waters. First, there is a fishery on the spawning stock which mainly takes place in Lofoten from February to April, though also along the coast of Møre. The second important seasonal fishery takes place at the coast of Finnmark from April to June, the socalled spring-cod fishery. This fishery is directed at immature cod of about 4 - 6 years which migrate to the coast of Finnmark every year following the capelin to its spawning grounds.

3.3.5 An outline of the economic model

The economic model is based upon the objective of maximizing the discounted value of the future net income flow from fishing subject to the biological and technological constraints pertaining to the fishery.

The annual gross revenue, TR_t, is calculated by multiplying an (nx1) vector of products, $\mathbf{x}_t$, by the relevant (nx1) price-vector, $\mathbf{p}_t$:

$$TR_t = x_t'p_t \qquad (3.3.19)$$

where ' is used to denote the transpose. The net revenue, π_t, each year is obtained by subtracting total costs consisting of fixed costs and variable costs from gross revenue. Fixed costs depend on the aggregate fishing capacity and will be addressed in a later section whereas variable costs depend on the use of this capacity. In addition to these costs there are capital costs related to the degree of gross investment which will also be addressed in a later section. Net revenue in year t may be written:

(3.3.20) $$\pi_t = x_t'p_t - FC(E_{max,t}) - VC(E_t) - I_t.$$

FC, VC and I are fixed, variable and gross investment costs respectively. E_{max} is maximum capacity and E is actually applied fishing effort. In this section fixed costs and investment costs are set at zero.

Discounted net revenue in this discrete-time model has already been formulated in equation (3.3.1). Different values of r will be considered later and in this model there is assumed to be a single, homogeneous output from the fishing industry.

3.3.6 The fishing mortality production function

The link between the biological and the economic submodels is given by a fishing mortality production function which translates fishing effort into fishing mortality. Before this can be done, precise definitions of fishing effort have to be provided. As indicated this concept may be defined in several ways. Clark (1985: pp.38-40) has proposed a set of definitions and the following draws upon these:

E_{max} = nominal fishing effort or capacity, e.g. number of standardized vessels that can be put in operation in a particular year but which may also incur costs when they are idle.

E = actual fishing effort, e.g. volume of water screened by E_{max} during a certain time interval. This may be written

(3.3.21) $$E = aE_{max}$$

where a is the total volume of seawater screened by E_{max} during one time period and thus denotes the actual use of this capacity. E and E_{max} are aggregate measures and the actual fishing effort is directly related to fishing mortality.

The instantaneous rate of catch from one single cohort is given by

(3.3.22) $$\dot{C}_j = k_j\rho_j E$$

where k_j is a selectivity parameter and ρ_j is a density parameter for cohort j within the area the fishing is performed. ρ_j may as a special case be written N_j/V where N_j is the stock number of this cohort and V is the total volume of water in the fishing area. Thus we may write

(3.3.23) $$\dot{C}_j = k_j \frac{N_j}{V} E$$

as the instantaneous catch rate of cohort j. The instantaneous fishing mortality may here be found by dividing the instantaneous catch rate by the total stock number:

(3.3.24) $$F_j = \frac{\dot{C}_j}{N_j}.$$

Using the special definition of ρ_j above, it is seen from equation (3.3.23) that F_j is proportional to nominal fishing effort

(3.3.25) $$F_j = q_j E$$

given the implicit assumptions in this outline, e.g. that the density of fish is proportional to the abundance of fish. q is the catchability coefficient, defined as

$$q_j \equiv \frac{k_j}{V}.$$

This definition of fishing mortality is equivalent to the definition in the Beverton-Holt model, as may be seen by comparing equation (3.3.24) with equation (3.3.3). It is important to note that whereas equation (3.3.24) is a mere definition of fishing mortality, equation (3.3.25) is a hypothesis concerning the relationship between effort and fishing mortality.

Given that this hypothesis is true, the fishing mortality function may be estimated by linear regression:

$$Z = \hat{M} + \hat{q}E \tag{3.3.26}$$

where Z = M + F is the sum of natural mortality and fishing mortality. $\hat{M}$ is an estimate of M and $\hat{q}$ is an estimate of q. However this will undoubtedly prove difficult to estimate empirically, especially since relevant data on mortality and effort will be hard to obtain. The data are mainly on a yearly basis and a year is too long a period to expect the natural mortality and the catchability coefficient to be constant.

Theoretically, however, it should be possible to calculate the catchability coefficient and the fishing mortality if the exact values for the parameters k, a and V are known and a proper definition of fishing effort is applied.

However, the assumption of a constant density or concentration profile and thus a constant catchability coefficient may not be very realistic, in fact it is probably quite unrealistic. Therefore, instead of assuming that density, ρ, is proportional to the fish abundance within a given area, i.e. $\rho = N/V$, it will here be assumed that there is variable concentration profile; i.e. ρ varies over an area for a given N. (For a more thorough discussion of concentration profiles, see Clark (1985: pp. 42-50)). This means that the overall abundance of fish within an area is a function of the concentration profile as follows:

$$N(\rho) = \int_0^{\bar{\rho}} f(\rho)d\rho, \tag{3.3.27}$$

where $f(\rho)$ is the total amount of fish within a small concentration area $d\rho$. It follows that $N'(\rho) = f(\rho) > 0$. It may be even more interesting to study the inverse function $\rho(N)$ as indicated above, and this is a non-decreasing function which may be concave, convex or linear.

For demersal species like the cod, interactive fisheries with uniform density or linear concentration profiles are often assumed. In the case of pelagic, schooling species, constant concentration profiles are often assumed. Both these assumptions are quite extreme, and the less extreme assumption of a concave density profile will be adopted here. A simple specification of a concave concentration profile may be:

(3.3.28) $$\rho(N) = hN^{\alpha} \qquad 0 \leq \alpha \leq 1.$$

Substituting this into equation (3.3.22) and omitting the subscript j, the instantaneous catch rate may then be formulated as a Cobb-Douglas production function:

(3.3.29) $$\dot{C} = \varepsilon N^{\alpha} E,$$

where $\varepsilon = hk$. The catchability coefficient, q, defined according to the hypothesis in equation (3.3.25), can now be stated as a function:

(3.3.30) $$q(N) = \varepsilon N^{\alpha - 1}.$$

Three different cases with respect to the value of α will be considered: i) $\alpha = 1$, ii) $\alpha = 0$, and iii) $0 < \alpha < 1$.

i) $\alpha = 1$: $\dot{C} = \varepsilon NE$.

This is the case corresponding to equation (3.3.25) with q constant. In this case the fishery may be described as an interactive fishery (see Beverton and Holt (1957) or Hannesson (1976: pp. 21-22)). This means each unit of fishing effort is competing with all the other units, and the instantaneous rate of fishing mortality is proportional to effort and independent of stock size:

(3.3.31) $$F = \varepsilon E.$$

The total fishing mortality or catch is increasing less than proportionally with an increase in effort. This may be seen from equation (3.3.4) which is an expression for catch in numbers in this case.

ii) $\alpha = 0$: $\dot{C} = \varepsilon E$.

This case represents the opposite of an interactive fishery, namely a non-interactive fishery. Each unit of fishing effort is not competing with the other units. The instantaneous fishing mortality is no longer independent of the stock size; in fact, as the stock size in numbers, N, approaches zero, the fishing mortality, F, will approach infinity for a given level of effort:

(3.3.32) $$F = \varepsilon E/N.$$

The total fishing mortality or catch will increase proportionally with effort. This can be seen by first solving the differential equation which expresses the immediate change in stock size:

$$\dot{N} = -\varepsilon E - MN.$$

Solving, we find

(3.3.33) $$N(t) = [N_o + \varepsilon E/M]e^{-Mt} + \varepsilon E/M.$$

Using this expression for N(t), the catch in numbers during a certain time period can be calculated as:

(3.3.34) $$C = \int_{t_1}^{t_2} FN\,dt = \varepsilon E(t_2 - t_1).$$

From (3.3.34) it is seen that the catch or total mortality is independent of stock size.

<u>iii) $0 < \alpha < 1$: $\dot{C} = \varepsilon N^{\alpha}E$.</u>

This third case is perhaps a bit trickier than the two preceding ones, but probably also the most realistic. Since

(3.3.35) $$F = \varepsilon N^{\alpha-1}E,$$

the stock in numbers may be calculated from a non-linear differential equation as follows:

$$\dot{N} = -\varepsilon N^{\alpha}E - MN.$$

Solving this differential equation we find:

$$(3.3.36) \qquad N(t) = \{[N_0^{1-\alpha} + \varepsilon E/M]e^{-M(1-\alpha)t} - \varepsilon E/M\}^{1/(1-\alpha)}.$$

It is easily seen that (3.3.36) is equal to (3.3.33) when $\alpha = 0$. The catch in numbers during the period $[t_1,t_2]$ may be calculated by using the expression in (3.3.36) as follows:

$$(3.3.37) \qquad C = \int_{t_1}^{t_2} FN\,dt = \int_{t_1}^{t_2} \varepsilon N^{\alpha}E\,dt$$

$$= \varepsilon E\int_{t_1}^{t_2} \{[N_0^{1-\alpha} + \varepsilon E/M]e^{-M(1-\alpha)t} - \varepsilon E/M\}^{\alpha/(1-\alpha)}dt.$$

This integral is most easily solved by choosing particular values for α. When e.g. $\alpha = 1/3$ the catch during period $[t_1,t_2]$ may be written:

$$(3.3.38) \qquad C = (-\varepsilon E/b)\Big|_{t_1}^{t_2} [2x^{1/2} - 2\sqrt{c}\tan^{-1}\sqrt{x/c}],$$

where $x = (ae^{-bt} - c)$,

$a = N_0^{1-\alpha} + c,$

$b = M(1-\alpha)$

and $c = \varepsilon E/M.$

This may be rewritten:

$$(3.3.39) \qquad C = 2\varepsilon E/b\left\{N(t_1)^{\alpha} - N(t_2)^{\alpha} - \sqrt{c}\left[\tan^{-1}\sqrt{\frac{N(t_1)^{1-\alpha}}{c}} - \tan^{-1}\sqrt{\frac{N(t_2)^{1-\alpha}}{c}}\right]\right\}.$$

For the case of $\alpha = 0.6$, one obtains:

$$C = (-\varepsilon E/b)\Big|_{t_1}^{t_2}\left[\frac{2}{3}x^{3/2} - 2cx^{1/2} + 2c^{3/2}\tan^{-1}\sqrt{x/c}\right]. \tag{3.3.40}$$

which may be rewritten:

$$C = 2\varepsilon E/b\Big\{1/3[N(t_1)^{\alpha} - N(t_2)^{\alpha}] - c[N(t_1)^{\alpha/3} - N(t_2)^{\alpha/3}] \tag{3.3.41}$$

$$+ c^{3/2}\left[\tan^{-1}\sqrt{\frac{N(t_1)^{1-\alpha}}{c}} - \tan^{-1}\sqrt{\frac{N(t_2)^{1-\alpha}}{c}}\right]\Big\}.$$

In the interest of simplicity, and for the purposes of this study, α will be restricted to these two values.

This section was meant to illustrate the way fishing effort is mapped into fishing mortality in the model. In this approach a Cobb-Douglas production function is chosen where the instantaneous catch rate is homogeneous of degree $\alpha + 1$, thus indicating certain economies of scale if $\alpha > 0$. This production function could have been substituted by a slightly more general one:

$$C = kN^{\alpha}E^{\beta}, \tag{3.3.42}$$

being homogeneous of degree $\alpha + \beta$, or an even more general one not necessarily of the Cobb-Douglas type:

$$C = C(N,E).$$

For further discussion of fishing mortality production functions, see e.g. Arnason (1984). The production function in equation (3.3.29) was chosen for the sake of simplicity in this version of the model.

3.3.7 Results based on stylized examples

In this section some results obtained on the basis of the bioeconomic model described above will be presented. The main question is: what are the economic and biological effects of a constant catch quota compared to those of a constant effort quota strategy? This question is answered with the help of computer simulations under different assumptions. The assumptions adopted here give rise to 11 different simulations or runs which are listed in the Table 3.3.1:

Table 3.3.1

Simulation no.:	1	2	3	4	5	6	7	8	9	10	11
Cyclicality	+	+	+		+	+	+	+	+	+	+
Dens.dep.ind.growth		+									
Dens.dep.nat.mort.			+								
Dens.dep.sex.mat.				+							
Value of α	1	1	1	1	1/3	0.6	1	1	1	1	1
Sloping demand							+				
Convex var.cost								+			
Changed selectivity									+		
Stochasticity										+	
Risk aversion											+

Each simulation is performed with two different discount rates, namely 0 and 0.1. The results are based on a rather stylized empirical version of the model. Thus the numerical results do not pretend to represent reality very closely. The focus of this study is on qualitative results, not quantitative.

The initial parameter values are chosen as follows: The recruitment age is set at 3, and the number of cohorts is chosen to be 20, thus the oldest fish in the model are 22 years old. The initial stock is an equilibrium stock found by performing simulation 1 over a period of 42 years which equals 5 recruitment cycles. The initial values of the maturity ogive and weight are based on ICES estimates (Anon. 1987).

The effects of the two strategies, constant effort quotas and constant catch quotas, are then analysed. In the case of a constant effort strategy, a number of standardized fishing units (SFU) or E in the production function (3.3.29) are specified. Knowing the values of k and

α, the catch from each cohort may then be found by solving equations (3.3.4), (3.3.34) or (3.3.37). This can be done for each of the cohorts as their size and natural mortality are known. It is assumed that the parameter k reflects the selectivity pattern in the respective fisheries. In this way the stock size and all the other variables concerning population dynamics may be calculated along with the economic results. All k's are assumed to be unity except in simulation 9.

In the case of a constant catch quota strategy, catch quotas replace fishing effort as the main control variable. Knowing the maximum quota which can be caught in each period, the minimum fishing effort necessary to catch this quota may then be calculated by an iterative method.

The model above is able to simulate any catch or effort quota strategy; these quotas need not be constant from year to year. Some advantages resulting from stabilizing one or the other have been pointed out earlier, but in fact there is little reason to believe that either constant catch quotas or constant effort quotas constitute an economically optimal management strategy. Comparison with the unconstrained optimal policy will be made in a later section.

Here optimization is carried out in the sense that the optimal constant catch and the optimal constant effort over a long period (50 years) are found, and the present values of the net revenues (NPV) are compared. This can be done since NPV is a unimodal function of effort, and the results are reported in Table 3.3.2. The initial stock is the equilibrium stock mentioned above. The differences in the E-values when α is varied are due to the fact that effort is measured in arbitrary units and these units are changed when the production function is changed. For the same reason the cost per unit effort has to change. The resulting average yield is then used as constant catch quota input when the two strategies are compared. An alternative would be to compare the result from the constant effort strategy with the result from the optimal constant catch, since NPV is also a unimodal function of constant catch. This, however, would not be a fair comparison as the constant catch that is optimal over a 50-year period will not be viable indefinitely.

Table 3.3.2
Results of Simulations

			Const.effort		Const.catch		
No.	Quota	r	effort	NPV	effort	NPV	Comp
1	516884	0.1	0.17	37.655	0.17	37.700	0
1	497737	0.0	0.13	183.869	0.13	184.105	0
2	777649	0.1	0.28	52.533	0.28	49.879	CE
2	751110	0.0	0.24	255.555	0.25	252.965	CE
3	582007	0.1	0.23	38.338	0.23	38.335	0
3	558440	0.0	0.20	179.220	0.20	179.605	0
4	459636	0.1	0.11	37.486	0.12	37.535	0
4	528532	0.0	0.09	219.266	0.09	219.224	0
5	595081	0.1	205	30.877	206	30.804	0
5	554940	0.0	168	151.470	168	151.217	0
6	610043	0.1	15	41.841	15	41.838	0
6	587779	0.0	12	203.890	12	203.768	0
7	488568	0.1	0.12	40.723	0.12	40.745	0
7	462616	0.0	0.10	190.409	0.10	190.540	0
8	488568	0.1	0.12	43.772	0.12	43.741	0
8	477001	0.0	0.11	203.961	0.11	203.978	0
9	551511	0.1	0.26	30.316	0.27	29.639	CE
9	514160	0.0	0.21	152.080	0.21	149.688	CE
10	504559	0.1	0.17	37.192	0.17	37.127	0
10	486909	0.0	0.13	178.454	0.13	178.693	0
11	516884	0.1	0.17	164.125	0.17	164.196	0
11	497737	0.0	0.13	755.743	0.13	755.940	0

The results from each of the simulations described above are reported below. All simulations are performed for a period of 50 years and in all, except 5 and 6, $\alpha = 1$, $p = 10$ and $VC = 10^7$. These results are reported in the Table 3.3.2. NPV is calculated with $r = 0.1$ and $r = 0$. The first column in Table 3.3.2 gives the number of the simulation. The second column gives the average quota applying the optimal constant effort based on a system in equilibrium. This is also taken as the quota in the constant catch case, and the economic result of this strategy is calculated using the same initial population. The third column gives the discount rate. The following columns give effort and net present values (in millions) for a period of 50 years with the respective strategies. In the case of constant effort, the effort reported is the optimal constant effort, and in the case of constant catch, the effort reported is the average effort necessary to take the given catch quota. The last column gives a comparison of the two strategies; 0 indicating that there is less than one per cent difference between the respective NPVs, CE indicating that constant effort is the better strategy and CC indicating that constant catch is the better strategy. Let us now consider each simulation in turn.

Simulation 1: Cyclical fluctuations in recruitment.

In all simulations except no. 4 and 10, recruitment is calculated according to equation (3.3.17) with parameters $a = 620{,}000$, $b = 520{,}000$ and $\lambda = 0.748$ giving a recruitment cycle of 8.4 years. Average recruitment in the period 1946-1987 was about 620,000 individuals according to ICES (Anon. 1987). The minimum recruitment in this period was 112,000 in 1966 and in this example it is 100,000. The maximum recruitment in the period was about 1,800,000 in 1970 which is quite a bit higher than the 1,140,000 (=a+b) in this example. However, 1970 was a rather outstanding year with respect to recruitment. As is seen from Table 3.3.2 the constant catch strategy seems to be the more profitable but only marginally so and the difference reported is not regarded as significant.

Simulation 2: Density dependent individual growth.

When density dependent individual growth is applied, the weight function used is given by the equations (3.3.9) and (3.3.10) with $a = 0.234$ and $b = 23.35$. The reference biomass used in the weighting function, g(B), is 3,882,064. Now the constant effort yields better results both with $r = 0.1$ and $r = 0$. The reason may be that with constant effort the harvest rate is relatively high when the biomass level is high compared to constant catch, and this counteracts the effect of reduced individual growth.

Simulation 3: Density dependent natural mortality.

Natural mortality is calculated according to equation (3.3.18) with $a = -.75$, $b = .016$ and $c = -.14$. The natural mortality has been made stock-dependent by multiplying it by a function similar to Eq. (3.3.10), namely 1+(B-A)/(B+A), where A is the reference biomass, 3,882,064, and B is the actual biomass.

According to theory, density dependent natural mortality should have an effect similar to density dependent natural growth due to reduced biomass variance. However, the simulations here indicate that the effect is smaller, and there does not seem to be a significant difference between the two strategies in this case. It is also interesting to note that a stronger density dependent relationship obtained by using the weighting function 1+(B-A)/(B+0.25A) increases the profitability of both strategies whereas the relative difference between the two remains remarkably constant.

Simulation 4: Density dependent sexual maturity and recruitment.

In this simulation sexual maturity is made density dependent and the relative share of first-time spawners is calculated using equation (3.3.14) with $a_i = b_i = 0$ for $i \leq 6$ and $a_i = 11.5$ and $b_i = 392{,}881$ for $i > 6$. In addition recruitment is made dependent on the spawning stock using a modified Beverton-Holt recruitment function of the form:

$$R = (a + S)/(b + cS), \qquad (3.3.43)$$

where R is recruitment and S is the spawning stock biomass three years earlier. a = -900,000, b = 0.1635 and c = 9.3E-7. Using this recruitment function will ensure that a stable natural equilibrium will be reached in the absence of fishing. Cyclicality in recruitment is disregarded in this simulation. If anything, theory suggests that a stock recruitment relationship will favour constant effort since variations in spawning stock biomass are reduced with this strategy and recruitment is a concave function in spawning stock. Table 3.3.2, however, shows that there is not much difference between the two strategies.

Simulation 5: $\alpha = 1/3$.

In this simulation the assumption that $\alpha = 1$ is departed from. When α is changed, the units for measuring effort and variable costs have to be changed accordingly. The initial values of E are chosen as the optimal E in Table 3.3.2 at different values of α using the respective variable costs. Variable costs are chosen from the rule of thumb that total variable costs on average shall be about 50% of total revenue. According to theory, the smaller α is, the more the two strategies will coincide, since the fishery then becomes more non-interactive with catch being directly proportional to effort. Therefore, one would expect the difference between the strategies to be even smaller than in the previous simulations. This is hard to confirm due to the arbitrary units in which effort and costs are measured, but in any case the difference between the strategies seems to be negligible.

Simulation 6: $\alpha = 0.6$.

The comments made above apply also here.

Simulation 7: Including price/quantity relationship.

An elastic demand function is assumed:

(3.3.44) $$p = Ay^{-0.5}.$$

When the yield is 500,000, the price is set equal to ten, thus A = 7071. As demonstrated in Chapter 2, one would expect the constant catch strategy to be favoured by a convex demand function of the type above, but even in this case the difference between the strategies is very small, and is not reported as significant.

Simulation 8: Increasing cost per unit effort.

Introducing convex variable costs should favour constant effort as demonstrated in Chapter 2 under certain conditions. Here a variable cost function

(3.3.45) $$c(E) = AE^{1.5}$$

is applied where A = 172,132,590, thus $c(0.15) = 10^7$. It is seen that the difference between the strategies is negligible also in this case.

Simulation 9: Changed selectivity pattern.

So far it has been assumed that the fishing mortality affects each year class equally. This may not be true. Young fish may escape the fishing gear because of their small size and older fish may migrate from the fishing area. This last point is especially true for the Barents Sea trawl fishery. Fishing mortality should therefore be multiplied by a selectivity parameter, k, particular for each cohort, j, thus constituting a selectivity pattern. Here the following selectivity pattern will be suggested:

$k_3 = 0.05$,

$k_4 = 0.15$,

$k_5 = 0.3$,

$k_6 = 0.6$,

$k_7 = 1.0$,

$k_8 = 0.75$,

$k_j = 0.5$ when $j \geq 9$.

This selectivity is supposed to represent the selectivity pattern of the trawl fishery to some extent. With a selectivity pattern like this the growth potential in the population is not exploited fully since younger cohorts are harvested at a relatively heavier rate than older cohorts. This has greater effect when a constant catch strategy is applied because the catch in this case is not reduced when the population level is low. In the constant effort case this effect is not so heavy since exploitation of younger cohorts is decreased when the stock is at a low level. This is also reflected in the results in Table 3.3.2 where it is seen that constant effort yields a significantly better profitability.

Simulation 10: Including stochastic effects.

In reality recruitment is not a deterministic process which may be described by some simple trigonometric function as suggested above. Recruitment involves huge random fluctuations from year to year, and therefore this analysis will not be complete without a simulation including stochastic effects. This is achieved simply by multiplying recruitment each year by a random number, $0.5 \leq z \leq 1.5$, with a uniform probability distribution. One such simulation is reported in Table 3.3.2 where the same stochastic recruitment is used with both strategies, and the result is not very different from simulation 1. Stochastic effects on recruitment increase the variations in biomass and this will favour constant effort, but no significant difference is reported in Table 3.3.2. Stochasticity, however, is a complex matter, and the question of uncertainty will therefore be analysed further in a later section.

Simulation 11: Risk aversion.

Risk aversion will under relatively mild assumptions favour constant catch since this strategy usually implies less variation in net revenue. This is tested in our model by replacing net revenue with its logarithm in the objective function. However, even in this case there seems to be only a small difference between the strategies.

Discussion

It is time to summarize the results from this section. In Chapter 2 it was demonstrated that in most cases we will expect higher profitability from constant effort than from constant catch. In this section, however, it is seen that using stylized numerical examples with a multi-cohort model the difference between the two strategies is in most cases small, meaning that the opportunity cost of stability is small, and for this reason perhaps a constant catch quota system would not be less preferable than a quota system based on constant effort. In the two cases in which there is a significant difference between the strategies, however, constant effort is the better of the two.

Table 3.3.3.
List of Parameters

Equation no. or expression	Name	Parameter			
		a	b	c	other
15	Cyclical recruitm.	620,000	520,000		$\lambda = 0.748$
9	Dens.dep. growth	0.234	23.35		
10					B = 3,882,064
16 1+(B-A)/(B+A)	Nat.mort.	-0.75	0.016	-0.14	 B = 3,882,064
(a+S)/(b+cS)	Stock.dep. recruitm.	-9E5	0.1635	9.3E-7	
14	Sex.mat.	0	0	for $i \leq 5$	
		11.5	392,881	for $i \geq 6$	
Constant price		$p = 10$			
Cost per unit effort		$\alpha = 1$	→	$VC = 10^7$	
		$\alpha = 0.6$	→	$VC = 150{,}000$	
		$\alpha = 1/3$	→	$VC = 15{,}000$	
Demand function		$p = Ay^{-0.5}$	→	$A = 7071$	
Convex var.cost		$VC = AE^{1.5}$	→	$A = 172{,}132{,}590$	

3.4 The role of investment in fixed capital

An important aspect in discussing optimal harvesting strategies, which has not yet been considered, is the role of investment in fixed capital. Investment will now be included as one of the control variables. The contribution made in this section is to compare the two strategies, constant effort and constant catch and to investigate the importance of irreversible investment when deciding between the two strategies. Optimal investment in this section means that the optimal constant effort or catch is calculated and then investment is made to fulfil the objective of these strategies.

The main reason why investment is important is that the level of investment, and especially disinvestment, can not be chosen freely due to the rather special technology used in the fisheries. In other words, the implicit assumption made so far that the cost of investment and disinvestment can be embedded in the general cost function, is abandoned. When a fishing vessel is built, there is little, if any, alternative use for it, thus investment in fixed capital in fisheries can be considered as a highly irreversible investment. However, capital in fisheries deteriorates as all other capital, so that investment can be treated as quasi-reversible, i.e. the only way disinvestment can occur is through depreciation.

In some cases there exists a foreign market for at least some of the vessels withdrawn from the fisheries or the vessels may be transferred to other fisheries. It is, however, difficult to give a general description of these markets and for this reason the second-hand market will be disregarded here. The mobility of capital between fisheries seems, in fact, to be rather limited.

The aspects of irreversibility or quasi-reversibility mentioned above imply some constraints on the optimization compared to the case where investment and disinvestment can take place freely.

In addition to the irreversibility it is also possible that there is an upper limit on the level of investment that can take place within a certain time period, at least at a fixed unit price. This aspect will also be included in the model, and it can be modelled either by letting the unit price of investment increase sharply when the investment is higher than a certain level or by excluding the possibility of investing above a certain level. The last approach will be applied here.

In the case of constant effort, the role of investment in fixed capital does not make any great difference. The objective of this strategy is to keep the capacity or effort at a given level which has been calculated as the optimal constant effort, and investment occurs only in order to replace the depreciation. In equilibrium the age structure of the fleet will remain the same from year to year or follow a pattern that repeats itself, so it is quite conceivable that investment can be treated as a fixed rate each year.

When the total catch, on the other hand, is kept at a fixed level each year, the necessary effort will have to vary, and this variation can be restricted upwards by the maximum allowable investment. In other periods excess effort may exist causing excess costs. In this case the assumption of a fixed rate of depreciation is no longer realistic. The reason is that a certain ratio of each fishing vessel does not deteriorate each year, rather the fishing vessel is in operation for a certain period of time and is then scrapped. The age structure of the capital stock no longer needs to remain the same from year to year, and this calls for an age-structured measure of effort. However, this aspect will have to wait until later in this section.

In previous sections the relationship between costs and effort has been included in the model in a rather simple way. In order to analyse the role of investment, this relationship has to be made slightly more sophisticated. It is no longer sufficient to look only at variable costs as a function of actual effort. First of all it is necessary to make a distinction between actual effort or the utilization of the fleet on one hand, and the maximum effort or fleet capacity on the other hand. The fleet capacity is the capital stock and investment adds to this capital stock. If a constant catch strategy is applied, the goal of the decision-makers is to keep the capital stock on a high enough level to take the catch with a varying fish stock. Due to the irreversibility of investment, the capital stock may in some periods be higher than necessary to reach this goal and thus induce higher costs.

A priori, it seems reasonable that constant catch will be less effective compared to constant effort when irreversible investment is taken into account due to the reduced flexibility of capital. The question posed here is to what extent this is true according to our model. To investigate this question three different costs have to be included; variable costs, fixed costs and investment costs. Variable costs are the costs which mainly have been considered in earlier sections, and these have been assumed to be linear in actual effort. Fixed costs relate to the maximum effort or catch capacity and consist of maintenance costs and other costs independent of the actual catch. Investment costs are the costs incurred by gross investment.

A completely irreversible investment scheme, meaning that no depreciation of capital takes place, is clearly not realistic and effort will therefore not be spent on this case. Rather, the more realistic and interesting case of quasi-reversible investment will be analysed.

First some key variables and relationships have to be introduced: E denotes actual effort as earlier whereas the maximum capacity or capital stock is denoted by K, i.e. $E_t \leq K_t$ for all t where t denotes time period. I denotes gross investment. c denotes variable cost per unit actual effort, C denotes fixed cost per unit K, and i denotes investment cost per unit I. p is price, Y is yield and δ is the depreciation rate.

In the case of a constant effort denoted by $\bar{E}$, $K_t = \bar{E}$ for all t. In this case net investment equals zero and gross investment equals $\delta\bar{E}$ each period. Net profit in one period may be written:

$$\pi_t(\bar{E}) = pY_t(\bar{E}) - c\bar{E} - C\bar{E} - i\delta\bar{E} \tag{3.4.1}$$

which can be rewritten

$$\pi_t(\bar{E}) = pY_t(\bar{E}) - \psi\bar{E} \tag{3.4.2}$$

where $\psi \equiv c + C + i\delta$. All cost components can be included in the term ψ, and the situation has not changed from the one analysed in previous sections.

Turning to the constant catch case with the constant catch denoted by $\bar{Y}$, net profit from one period can be written:

$$\pi_t(\bar{Y}) = p\bar{Y} - cE_t(B_t;\bar{Y}) - CK_t - iI_t. \tag{3.4.3}$$

The capital dynamics are given by:

(3.4.4) $$K_t = (1-\delta)K_{t-1} + I_t, \qquad I_t \geq 0.$$

Irreversibility is imposed by the last inequality. For simplicity there is not supposed to be any lag between the actual investment and the time the new capital comes into use. This is perhaps somewhat unrealistic but is not assumed to have any great impact on the results of the model.

The effort necessary to take the constant yield decides the level of gross investment which has to take place. Since uncertainty has not been included in the model yet, this is a known value, E_t. E_t is the capital stock which is just large enough to take the fixed catch with a given fish stock. Thus the level of investment can be formulated:

(3.4.5) $$I_t = \max[0, E_t - (1-\delta)K_{t-1}]$$

and the capital stock dynamics are:

(3.4.6) $$K_t = \max[(1-\delta)K_{t-1}, E_t].$$

The two strategies, constant effort or constant catch, can now be compared applying the extension of the model suggested here and appropriate data for costs and depreciation rate. The following data has been chosen for illustrative purposes:
$c = 10^7$,
$C = .25\text{x}10^7$
$i = 2.5\text{x}10^7$ and
$\delta = 0.1$.
Thus $\psi = 15\text{x}10^6$. In the following the optimal constant catch and the optimal constant effort are calculated and the economic results from applying the respective strategies are compared. The results when the depreciation rate, δ, is .1 are shown in Table 3.4.1. The constant catch strategy yields a better result both when the discount rate is zero and .1. This may be due to the fact that there is no surplus capacity at the beginning of any period and therefore no costs are induced by surplus capacity. This may indicate that .1 is a relatively high rate of depreciation and gross investment has to be made in every period. The average effort in the constant catch case is the same as the optimal constant effort, but

the optimal constant catch is a bit higher than the average catch yielded by the constant effort. Thus the average cost is the same with both strategies, but the average income is a bit higher with constant catch. The relative difference is also a bit higher when r is increased from zero to .1.

Table 3.4.1

Net present values. $\delta = 0.1$

		r=0.0	r=0.1
Const.	opt. Y	481,000	515,000
catch	av. E	0.11	0.14
	NPV	158.9 mill.	34.1 mill.
		r=0.0	r=0.1
Const.	av. Y	475,144	510,052
effort	opt. E	0.11	0.14
	NPV	158.2 mill.	33.7 mill.

The results reported in Table 3.4.1 may seem a bit counter-intuitive, and therefore it can be of interest to compare this with the case when the depreciation rate is lower, i.e. when the degree of irreversibility is higher. The results of this are shown in Table 3.4.2 where δ is lowered to 0.03. It is seen from the table that the result is now reversed so that constant effort is a marginally better strategy, but the difference between the strategies is still very small. This means that a higher degree of irreversibility (a lower capital flexibility) not surprisingly favours constant effort. The net present values in each case have increased due to lower replacement costs. Also here the relative difference between the two strategies is higher when the discount rate is 0.1.

From this comparison it may be concluded that the more inflexible or irreversible investment in fixed capital, the better is constant effort compared to constant catch. However, the difference in profitability between the two strategies is not very great and is not very sensitive to a relatively large change in the depreciation rate. It must be mentioned that these calculations assume, somewhat unrealistically, the ability to predict future development of the recruitment and stock size.

Table 3.4.2

Net present values. $\delta = 0.03$

		r=0.0	r=0.1
Const.	opt. Y	489,000	526,000
catch	av. E	0.12	0.16
	NPV	168.0 mill.	36.1 mill.
		r=0.0	r=0.1
Const.	av. Y	489,018	517,816
effort	opt. E	0.12	0.15
	NPV	168.3 mill.	36.5 mill.

It may also be interesting to test how sensitive the economic results are to an increase in the unit cost of fixed capital, C. The values of the unit cost parameters are changed in such a way that the cost parameter ψ with constant effort remains the same, but now
$c = .25x10^7$
$C = 10^7$ and
$i = 2.5x10^7$.
That is the unit cost of fixed capital has taken the value of the unit cost of actual effort in Table 3.4.1 (and 3.4.2), and vice versa. This is supposed to make the situation less favourable for the constant catch strategy since the cost of having idle capital has increased, but the question is how sensitive the economic results are to this change. The depreciation rate of 0.03 is retained from Table 3.4.2. Thus the result with a constant effort strategy is the same as in Table 3.4.2 whereas the result with a constant catch strategy, reported in Table 3.4.3, has to be compared with the result in Table 3.4.2.

The effect of the change in the cost parameters is that the profitability of the constant catch strategy is marginally lower. The interesting result, however, is that the economic performance of the constant catch strategy is to a very small extent sensitive to a rather comprehensive change in the cost structure. The results so far indicate that the constant catch strategy may still be able to compete economically even when irreversible investment is an important characteristic of the fishery.

Table 3.4.3

Net present values. $\delta = 0.03$

		r=0.0	r=0.1
Const.	opt. Y	488,000	526,000
catch	av. E	0.12	0.16
	NPV	167.7 mill.	36.0 mill.

Another factor which can increase the desirability of constant effort compared to constant catch, is the fact that the decision-makers may not always be able to invest as much as they want, e.g. because the cost of investment increases sharply when the level of investment is high or because investment is impossible above a certain level. The first instance will unequivocally increase the cost of a constant catch strategy because this is the strategy which sometimes makes high levels of investment necessary. In the second instance such high cost will not be induced, but income may be lost if the necessary level of actual effort is too high to be achieved by the existing capital and further investment is impossible. The case outlined here is investigated in Table 3.4.4.

Table 3.4.4

Net present values. $\delta = 0.1$

		r=0.0	r=0.1
Const.	opt. Y	481,000	533,000
catch	av. E	0.11	0.14
	NPV	158.8 mill.	34.1 mill.

The situation now is comparable with the situation in Table 3.4.1, i.e. a relatively high degree of depreciation, $\delta = 0.1$, and the same unit cost parameters. The difference from Table 3.4.1 is that gross investment each year can not exceed 0.015 units. The restricted ability to invest imposes a new constraint upon the system, and it therefore seems reasonable to think that this lack of opportunity to invest would reduce the income possibilities and thus the profitability of the constant catch strategy. However, this is not the case, and the reason is that the obligation to take a given constant catch also constitutes a constraint since the constant catch is not optimal compared to a situation where both catch and effort can vary freely. To meet this constraint a certain capital stock is necessary and this requires investment costs. When investment is restricted, some income is lost but at the same time investment and effort costs are reduced, and the constant catch now represents only an upper limit on the allowable catch. In this case the constant catch strategy consists of calculating an optimal target constant catch, a target which is not fulfilled in all periods due to lack of effort.

The restricted investment ability has no impact on the constant effort strategy, on the other hand, since the only investment under this strategy is replacement due to depreciation and this is not restricted by the 0.015 units. Thus the economic result of this strategy is the same as in Table 3.4.1.

As is seen from Table 3.4.4, the result from the constant catch strategy is hardly changed at all when $r = 0$. On the other hand, when $r = 0.1$, the target quota is set higher, the restriction on investment is binding in several periods (33 out of 50) and the overall profitability has increased slightly (due to rounding this is not seen in the tables). It is worth noting that the actual average yield is 510,000 whereas the target yield (which is reported in the table) is 533,000. Another way to state this is to say that the profitability of constant catch has increased since one constraint (the obligation to take a constant catch) is overruled by another (restricted investment opportunities). Another interesting thing to note is that the average yield is lower and average effort is about the same as when effort is constant (cf. Table 3.4.1) but nevertheless profitability is slightly higher since both effort and catch is allowed to vary a bit.

In the last part of this section the assumption of a constant depreciation rate will be departed from. The alternative to using a fixed depreciation rate is to keep track of the age structure of the fishing fleet and then to scrap the part of the fleet which is older than a certain age. With a constant effort strategy this means that the age structure of the fleet is a pattern which will repeat itself after a certain number of periods, viz. the same number of periods as the "scrapping-age" of the vessels. A special case is when there is the same number of boats in each year-class; then a constant number of vessels will be replaced each year, thus this is equivalent to a constant depreciation rate. In the interest of simplicity this assumption will be made in the case of constant effort and therefore it will not be necessary to repeat the simulations in the constant effort case. A 10% depreciation rate, for example, is equivalent to a fleet consisting of ten year-classes. With a constant catch strategy on the other hand, the fleet structure will have to change in order to meet the effort requirements of the strategy, i.e. investment has to be made in each period so that the total capacity is large enough to take the constant catch given the fish-stock.

In the initial period the age structure of the capital stock is the same with both strategies. With a constant catch strategy the necessary effort is calculated and then divided by the number of year-classes so that initially each year-class has the same number of units of effort. Subsequent changes in the age structure of the fleet is due to the applied investment scheme. In this model it is assumed that investment, the number of vessels in the youngest year-class, is non-negative, and is made in order to make the total capacity large enough to take the constant quota in each period. When the vessels are older than a certain age they are scrapped and are no longer of any value. If, for example, ten year-classes are assumed, vessels are scrapped after ten years and replaced by new vessels if necessary. It is assumed that the quality of effort is independent of age. The investor can buy any amount of new capital but is only allowed to scrap the part of the fleet which has reached the "scrapping-age", and this is where irreversible investment comes into play. A part of the fleet may therefore be idle in some periods without being scrapped and this overcapacity will incur fixed but not variable costs. The total capacity is always large enough to secure that the quota is caught and additional costs making the constant catch strategy more expensive than the constant effort strategy may come from the fixed costs in connection with idle vessels.

Table 3.4.5

Net present values. Age-structured capital. 10 year classes.

		r=0.0	r=0.1
Const.	opt. Y	482,000	517,000
catch	NPV	158.1 mill.	33.5 mill.

Table 3.4.6

Net present values. Age-structured capital. 33 year classes.

		r=0.0	r=0.1
Const.	opt. Y	491,000	526,000
catch	NPV	167.7 mill.	35.8 mill.

In Tables 3.4.5 and 3.4.6 the results of the constant catch strategy in the presence of an age-structured capital stock are reported. The values of the cost parameters are the same as in Table 3.4.1 (and 3.4.2). In Table 3.4.5 the number of year-classes of vessels is ten and the results here can therefore be compared with Table 3.4.1 which has a depreciation rate $\delta = 0.1$. In Table 3.4.6 the number of year-classes has been increased to 33 and can be compared with Table 3.4.2 with a fixed depreciation rate $\delta = 0.03$.

Comparisons of the respective tables show that in the case of an age-structured capital stock, which after all is the most realistic representation of depreciation in fishing vessels, the economic performance of the constant catch strategy has been made worse relative to constant effort.

Table 3.4.5 compared to Table 3.4.1 shows that the optimal constant catch quota has increased a bit both when $r = 0$ and when $r = 0.1$ and at the same time the net present value has decreased. In fact, with a fixed depreciation rate the NPV is marginally higher with a constant catch strategy whereas in the presence of an age-structured capital stock it is marginally lower.

Comparing Table 3.4.6 with Table 3.4.2 it is seen that the optimal constant quota has only increased when $r = 0$ whereas the NPV has decreased both when $r = 0$ and when $r = 0.1$. The relative difference between the economic performance of the constant effort versus constant catch strategy is also higher in the case of 33 year-classes instead of ten. An increased number of year-classes is equivalent to a lower depreciation rate or, in other words, to a more irreversible capital-stock. 33 year-classes is also supposed to be more realistic than ten since fishing vessels can become quite old before they have to be replaced. The conclusion therefore must be the same as earlier: increased irreversibility of the capital stock favours constant effort compared to constant catch.

3.5 Calculation of optimal catches and effort.

As pointed out in Section 3.3, neither constant annual catch quotas nor variable catch quotas based on constant annual effort are likely to be optimal in an unconstrained setting. The justification for putting so much emphasis on the constant effort/constant catch strategies is that these strategies may be a far more realistic representation of the real world possibilities than the strategy resulting from, for example, an unconstrained optimization. In other words, some sort of stabilization of either catch or effort (or other variables) may be a reasonable constraint on the optimization possibilities, and the most realistic model is perhaps one which allows a certain degree of variation in the main variables but which is not totally unrestricted.

The restrictions on the relative change in effort or catch do imply greater stability with regard to these variables. In the case of constant effort the net revenue will vary more while the stock becomes more stable. The closer the technology in the fishery is to a non-interactive fishery, the more constant effort will coincide with constant catch. In the case of constant catch net revenue becomes more stable while the stock will vary more. The variations in net revenue with the two strategies are illustrated in Figs. 3.5.1 - 3.5.4 for different values of α (degrees of interactivity). In this section the alternative cost of increased stability will be analysed.

Optimization in a multicohort model is not a trivial task, and an analytic solution to the dynamic optimization problem seems almost impossible to attain when effort costs are positive or density-dependent biological relationships are included in the model (Clark, 1990). For this reason a simulation model was developed and used to analyse and compare the constant policies mentioned above. The model was used to analyse the effects of such relationships as density-dependent individual growth, natural mortality and recruitment and price/quantity relationships. Many of these relationships can not be included in an optimization model without introducing severe difficulties of non-concavity.

It may be argued that constant catch and effort strategies are not very realistic or likely to be implemented. The counter-argument, however, is that pulse-fishing, a common result in optimization models, is perhaps even less realistic.

The purpose of this section is to calculate the optimal fishing effort applying non-linear programming. The possibility of putting constraints on the relative change of effort and catch from year to year will be retained since this may yield a more realistic result than unconstrained optimization. This may then be called a semi-constant or semi-stable policy and will be compared to the result of unconstrained optimization. In this way it is possible to calculate the opportunity cost of stability.

A similar model has been developed by Horwood (1987) but his study was limited to a constant catchability coefficient. In fact almost all studies of this kind have considered either a completely interactive or a completely non-interactive fishery (Horwood 1987, Hannesson 1975). In the work presented here the Cobb-Douglas production function from section 3.3.6 is applied, and consequently catchability is a function of stock size, i.e. the fishery can also be somewhere between interactive and non-interactive.

The objective function in the programming model should be familiar:

$$J = \sum_{t=0}^{T} (1 + r)^{-t}\{pY(t) - cE(t)\} \tag{3.5.1}$$

where

$$Y(t) = \sum_{i=1}^{n} C_{it}, \tag{3.5.2}$$

and n is the number of cohorts exposed to fishing. The catch from cohort i at time t, C_{it}, is calculated according to the equations in section 3.3.6. Thus the objective function is concave in effort assuming linear (or convex) costs in effort. The programming model is evaluated subject to possible constraints primarily on the relative change of effort from year to year and on the total biomass in period T+1.

This represents a discrete time, non-linear, dynamic optimization problem but due to its discrete nature it may also be interpreted as a static, T-dimensional optimization problem and solved as such, treating effort in each time period, E_t, as an independent variable.

As indicated, costs are supposed to be linear in effort. However, the introduction of a convex cost function would not violate the concavity of the objective function. Nor would the introduction of a selectivity pattern. These possibilities will be considered in turn.

The updating of each cohort from year to year is also given by the equations in section 3.3.6. Since effort is assumed to be constant within a time period, the updating equation can be rewritten:

$$(3.5.3) \qquad N_{i,t} = R_{t-i+1}\exp[-\sum_{j=1}^{i-1}(F_{t-j} + M)],$$

and the catch equation:

$$(3.5.4) \qquad C_{i,t} = \frac{F_t}{F_t+M} R_{t-i+1}\exp[-\sum_{j=1}^{i-1}(F_{t-j}+M)](1-\exp(-F_t-M)) \quad \text{if } i \le t.$$

If $i > t$, $N_{i,t}$ has to be calculated from the given initial stock. The recruitment to the stock, R_t, is given by the cyclical recruitment function in section 3.3.2.4. In Horwood (1987) a spawning stock/recruitment relationship is applied, and this will also be investigated here as an alternative to cyclical fluctuations.

Because of the monotonic relationship between effort and catch in this model, the optimal path of catches will be the path resulting from the optimal path of effort. As an alternative to putting constraints on the relative change of fishing effort one could put constraints on the relative change in catches from one period to the next. This would require a more general model than the one presented above, since both the objective function and the constraints would be non-linear.

A non-linear optimization model of this kind requires much more computer capacity than the previous simulation model, and therefore can not be as large as the model presented earlier, i.e. the number of periods and the number of cohorts have to be decreased. The model will, however, be made as large as possible and hopefully large enough to draw some qualitative conclusions.

A non-interactive fishery.

The total catch during one time period is directly proportional to the level of effort. Using a proportionality factor of one, the catch in numbers from each cohort is equal to fishing effort, F, and the discounted net present value may be written:

$$J = \sum_{t=0}^{T} (1+r)^{-t}\{p\sum_{i} F_t w_i - cF_t\}. \tag{3.5.5}$$

The maximization rule becomes then:

$$\begin{aligned} &F_t = F_{max} \text{ whenever } p\sum_{i} w_i > c \quad \text{and} \\ &F_t = 0 \text{ whenever } p\sum_{i} w_i < c. \end{aligned} \tag{3.5.6}$$

As this case is not very interesting, the non-interactive fishery will not be analysed any further.

An interactive fishery.

This kind of fishery has been analysed by different authors in similar settings earlier, but it will still be considered here, both for the sake of thoroughness and because factors such as selectivity pattern, fluctuations in recruitment, convex costs and so on should be studied.

The main biological characteristics of the optimization model are as follows. Cyclical fluctuations in recruitment are applied, and the length of the recruitment cycle is eight years. As it has been necessary to limit the total number of periods, the time horizon has been set equal to two full recruitment cycles, i.e. 16 years. The number of cohorts has been decreased and set equal to 12. The age of the youngest cohort is set at four, so the cohorts range from ages four to 15. The last cohort is in fact a socalled 15+ group, i.e. it comprises whatever remains of cohorts aged 15 years or older. The individual weight at each age is the same as earlier, and natural mortality is set equal to 0.2 for all cohorts.

Some economic characteristics should also be mentioned. In most optimizations a zero discount rate is applied. The number of fish in each cohort is measured in millions, the price is set equal to 1, and the unit cost of effort is set at 1000. These initial conditions on the biological and economic parameter values will be relaxed when necessary.

Eight different optimizations have been performed, and these are characterized as:

(1) Unconstrained optimization
(2) Increased discount rate
(3) Different selectivity pattern
(4) Recruitment as a function of the spawning stock
(5) Convex cost function
(6) Constraints on the relative change of effort from one period to the next
(7) Constraints on the relative change of catch from one period to the next
(8) Cannibalism

Table 3.5.1 **Optimal pattern of fishing effort. An interactive fishery. $\alpha = 1$.**

	Optimization run number							
	1	2	3	4	5	6	7	8
Period								
1	.51	.67			.15	.10	.17	1.66
2					.13	.08	.11	
3			2.01		.12	.09	.09	
4					.12	.12	.09	
5					.13	.15	.10	
6					.15	.18	.13	2.205
7		2.16			.17	.23	.18	
8	2.34				.17	.25	.25	
9					.15	.19	.21	
10			2.34		.14	.14	.15	
11					.13	.11	.13	2.29
12					.14	.13	.15	
13					.17	.17	.18	
14					.19	.21	.26	
15		1.39		1.18	.20	.26	.42	
16	2.47		.78		.21	.32		1.03
NPV	14306	5774	13426	12358	11641	11003	10707	15830

The discount rate is zero in all runs except the second. The resulting optimal path of effort from each optimization is given in Table 3.5.1. The initial stock in each optimization is calculated by an iterative process giving the equilibrium stock resulting from applying the optimal constant effort in each case. One constraint applied in all the optimizations is that the total biomass at the end of the last period should at least be as large as the total biomass of the initial stock. This may be written:

$$\sum_i N_{i,T}\exp(-F_T-M)w_{i+1} \geq \text{initial biomass},$$

where each $N_{i,T}$ is a function of the recruitment in year T-i+1 and the mortality it has suffered since. The function on the left-hand side of the inequality sign is convex in F, and since the inequality is of the "≥" type, the feasible region for the non-linear program is concave. This makes the program harder to solve with respect to a global maximum. The problem can however still be solved by using separable programming or reverse convex programming techniques, see e.g. Avriel (1976).

The result of the unconstrained optimization is, not surprisingly, a high degree of pulse fishing. Actually, the fishing effort is only active in the first, the eighth and the final period, but the optimal level of effort is quite high in these periods; in fact the average effort per year is 0.24 which is higher than the optimal constant effort, 0.21. The main reason why pulse fishing is optimal is that there are economies of scale in the production function with respect to the size of the stock and the level of effort. The net present value (NPV) in this case is 14306 measured in arbitrary units.

Introducing a positive discount rate does not alter this pattern to any large extent. The only important effect is that the fishing effort should be applied a bit earlier which is an obvious result. A high degree of pulse fishing is still optimal. The NPV in this case is only 5774 due to the discounting.

The introduction of more selectivity in the fishery does not make any great difference. In optimization (3) a knife-edge selectivity pattern is applied, only catching fish older than 8 years. This leads to a delay in the optimal fishing, but there are still heavy pulses. The reason for the delay is that a more selective fishery will more effectively exploit the growth potential of the fish, and as a result the value of the objective function is higher in this case than when the fishing effort affects all cohorts to the same extent, viz. 13426. Run (1), however, gives a higher NPV indicating that the forgone income from not fishing cohorts younger than 8 years is higher than the gain coming from more effective exploitation of the growth potential.

It is seen that neither a higher discount rate nor a sharper selectivity pattern will alter the result that heavy pulse fishing is optimal. However, combining these two conditions will indeed do so. When a positive discount rate and a sharp selectivity are combined, the optimal path of effort becomes much more even though there are still pulses. The reason must be that these two factors are drawing in opposite directions; a higher discount rate calls for earlier application of fishing effort while more selectivity calls for a delay. The combined effects imply a more even utilization of the fishing effort.

In optimization (4) a spawning stock/recruitment relationship is applied according to the Beverton-Holt recruitment function. In this case all fishing is delayed to period 13, and the reason why this is so seems quite obvious; it is optimal to build up the spawning stock as much as possible to secure a high recruitment. In this simulation cyclical fluctuations are not considered. NPV in this case is 12358.

In the next simulation convex costs are included according to the function,

(3.5.7) $$c(E) = 10391E^{1.5}.$$

This equation is calculated from the condition that c(.21) = 1000, since .21 is the optimal constant effort when the unit cost of effort is set equal to 1000. It should come as no surprise that the result is a much more even fishing path. It is perhaps more surprising that the level of effort is well below .21 in all periods except the last even though the initial stock is a bit larger than in the unconstrained case. Convex costs reduces the NPV to 11641.

In optimization (6) the relative change in effort from one period to the next is constrained as follows:

(3.5.8) $$0.75 \leq F_{t+1}/F_t \leq 1.25 \qquad t = 1,..,T\text{-}1.$$

(3.5.8) represents 2x(T-1) linear constraints implying that the effort or fishing mortality can not increase or decrease by more than 25% from one period to the next. From Table 3.5.1 it is seen that these constraints are almost always binding. In the previous optimization including convex costs, however, F_{t+1}/F_t ranged from 0.88 to 1.16; thus, if there is a high degree of convexity in the cost function, these constraints will not become binding. The introduction of these constraints decreases the value of the objective function or NPV by 16% compared to optimization (1).

If, instead of putting constraints on the relative change of effort there are constraints on the relative change of annual catch, the optimal fishing path will be as reported in column (7) in Table 3.5.1. In this optimization the constraints were formulated according to the inequalities:

$$0.75 \leq Y_{t+1}/Y_t \leq 1.25 \qquad t = 1,..,T\text{-}1 \tag{3.5.9}$$

applying that annual catch is not allowed to increase or decrease by more than 25%. Also here the constraints are binding most of the time. In this case the value of the objective function is decreased by 18% compared to optimization (1), so stabilizing catch seems to be slightly more expensive in terms of opportunity cost than stabilizing effort.

In optimization (8) stock dependence in some of the parameters is considered. It is formulated in a way that is meant to represent some possible effects of cannibalism. In lack of other food the fish may become very cannibalistic, and this will primarily affect the youngest cohorts, the fry not yet recruited to the fishery, and in turn this will influence the recruitment to the stock. Since the main predators are the mature and the old fish, this will be modelled as a negative relationship between recruitment and the spawning stock in the previous year:

$$R_t = \left(\frac{\bar{S} - S_t}{\bar{S}} + 1\right)620 \tag{3.5.10}$$

where $\bar{S}$ is the reference spawning stock biomass given from the initial population and S_t is the actual spawning stock. 620 is the average recruitment from the cyclical recruitment function, so in this optimization run no fluctuations are considered. The result is again a pulse fishing pattern with quite heavy fishing in periods 1, 6, 11 and 16, and no fishing in between. The rather odd result here is that applying an optimal effort pattern in the presence of cannibalism will yield a higher net revenue from fishing than the maximum net revenue when cannibalism is not present. The reason is that it pays off to catch a great part of the mature stock not only directly but also indirectly in the form of higher recruitment to younger cohorts. This is at best only true under very special conditions and a stringent quantitative study of the consequences of cannibalism will be necessary to investigate this further. Recent evidence from the Barents Sea, however, suggests that cannibalism is an important factor influencing the size of annual recruitment. Compared to the optimization runs (1) - (4), it is seen that in the presence of cannibalism in this quasi-empirical setting the fishing pulses become both stronger and more frequent because it is no longer optimal to have a large spawning stock. However, with greater availability of other food, the degree of cannibalism would probably be reduced, but this can only be analysed in a multi-species model.

<u>Generalization of the production function.</u>

Leaving the assumption of linearity between fishing effort and fishing mortality, i.e., no longer assuming a constant catchability coefficient, the non-linear programming model presented here can be reformulated according to Eqs. (3.3.36) and (3.3.37) in section 3.3.6 giving the updating of the fish stock and the calculation of the catch. Since fishing effort is assumed to be fixed within a time period, the updating equation can now be written:

(3.5.11)

$$N_{i,t} = \{R_{t-i+1}e^{-M(1-\alpha)(i-1)} - \sum_{j=1}^{i-1} \frac{F_{t-j}}{M}(1 - e^{-M(1-\alpha)})e^{-M(1-\alpha)(j-1)}\}^{1/(1-\alpha)}$$

where $N_{i,t}$ is the number of fish of age i at time t and R_t is recruitment at time t. When $i > t$, $N_{i,t}$ has to be calculated from the given initial stock. This is then appropriately substituted into Eq. (3.3.37) and solved for $\alpha = 1/3$ and $\alpha = 0.6$. Thus the catch from each cohort is calculated. As is seen, this involves a high degree of non-linearity, and also the production function is discontinuous, as it is not defined when effort is equal to zero. Fortunately,

(3.5.12) $$\lim_{E \to 0} C_{i,t} = 0,$$

where $C_{i,t}$ is catch from cohort i at time t, so this problem can be overcome by setting the lower limit on E equal to 1E-6. The reason for this discontinuity is that the degree of interaction between stock size and effort in producing the catch becomes lower as α goes from one to zero. When α is zero, there is no interaction at all (non-interactive fishery).

This move from an interactive fishery towards a non-interactive fishery as α becomes smaller also increases the danger of extinction of some of the cohorts. Thus, when F is "too high" in one or more periods the expression in (3.5.11) is not defined either, nor is the catch equation. This is equivalent to $C_{i,t} = N_{i,t}$, i.e. the rest of the cohort is taken as a whole.

The objective function is the same as earlier, (3.5.1), but it has now become far more complex than in the case of an interactive fishery. The high degree of non-linearity will strain the limit of the available computer capacity if the model is large. For this reason the total number of cohorts has been reduced to 8, but this will not distort the main characteristics of the solution.

The constraint that the total stock biomass must be at least as large at the end of the final period as the initial biomass is maintained. Because of the higher complexity of this model, it will not be possible to perform all the runs that were performed in the case of the interactive fishery, but some of them will be repeated with this production function.

The results when $\alpha = 0.6$ are reported in Table 3.5.2. Four optimization runs are considered in this case:
(1) Unconstrained optimization
(2) Increased discount rate
(3) Constraints on the relative change of effort from one period to the next
(4) Convex cost function

The first optimization run is unconstrained with the exception of the restriction that biomass in period T+1 shall be at least as large as the initial biomass and this is reported in the first column of the table. The discount rate in this run is set equal to zero and the number of periods is 16. It is seen from the table that a high degree of pulse fishing is still optimal. The reason for this is that there is still a certain degree of interactivity in the fishery between stock size and effort, i.e. economies of scale, so it pays off to let the stock build up for a time and then capture a large part of it by exerting a relatively large effort. This depends on the assumption that variable costs are linear in effort. Compared to the optimal constant effort which is 1.9 units of effort each period, it is seen that the introduction of pulse fishing increases the value of the objective function from 8536 to 11450 or by 34%.

In the second run the discount rate has been increased from 0 to 10 per cent. This does not alter the optimal pattern of fishing effort, but the effort is shifted in time so it becomes higher in early periods and the NPV is much lower due to discounting. This is in accordance with intuition and the results from the completely interactive fishery.

In the third run a constraint on the change of effort from year to year equivalent to (3.5.8) has been introduced. The value of the objective function is reduced by 20 per cent compared to optimization run number one, which is greater than the 16 per cent reduction in the case of a totally interactive fishery. This indicates that stabilizing effort becomes more expensive as the technology moves from interactive to non-interactive since stabilizing effort then increasingly coincides with stabilizing catch.

Next a convex variable cost function is introduced. Also in the case of this production function the optimal pattern of fishing effort becomes much more even, in fact the constraint that the relative change of effort from period to period should not exceed 25% would only be binding in one single period. The average change of effort with this cost function is only 14%. Another interesting point to note is that the constraint on the level of total biomass in period T+1 is no longer binding, a depletion of the stock is now prevented by the cost function.

Table 3.5.3 gives the results from the same runs when $\alpha = 1/3$. In this case the optimal constant effort is 6 units when cost per unit is 90. Applying this effort for 16 periods yields a net revenue of 4671 with zero discount rate. The first column of Table 3.5.3 shows that the net revenue has increased to 7511, or nearly 61%, when effort is free to vary, so the cost of having a completely constant effort pattern is not insignificant within this simple setting.

When the discount rate is increased from 0 to 10 %, the optimal fishing pattern is also in this case higher in the near future. There is not much difference between this pattern and the pattern arising when $\alpha = 0.6$.

When the restriction on the relative change of effort from period to period given by equation (3.5.8) is imposed on the system, this becomes binding all the time as was the case earlier. Because of this constraint the value of the objective function is decreased by 29 per cent. From this it is possible to draw the conclusion that the lower α, the higher the opportunity cost of stabilizing effort. In other words, the higher the degree of interactivity in the fishery, the lower this opportunity cost. Since the lower the degree of interactivity, the more stabilizing effort coincides with stabilizing catch, this may indicate that the opportunity cost of stabilizing the catch is higher than stabilizing the effort.

In run number 4 a convex variable cost function is introduced. This results in a much more even fishing pattern; the maximum relative change in either direction is 20%, and the average relative change in either direction is 12%. In this case the constraint in (3.5.8) would not be binding in any period, and the opportunity cost of stabilization is lower since the cost function itself implies a certain stabilization. Also here the constraint on the biomass in period T+1 is no longer binding. There is little reason to believe that a convex cost function should be less realistic than a linear one, and if this is the case, stabilization comes as a consequence of the technology in the fishery.

It may be interesting to compare these results with the optimal policy in the case of a non-interactive fishery, i.e. when $\alpha = 0$. Since catch then is proportional to effort, it is sufficient to maximize catch assuming that costs are linear in effort and assuming that the fishery yields a non-negative net return. In the optimization presented here we are assuming that the catch from each cohort is proportional to the size of that cohort in numbers, such that the relative rate of harvest is the same for all cohorts in a given year. A lower bound on the size of the spawning stock biomass is imposed in all years including year T+1, otherwise the stock would be harvested down to zero at some time. It is supposed here that all fish of eight years or older are sexually mature and thus comprise the spawning stock. The lower bound on the spawning stock is 1500 which is approximately one half of the initial spawning stock. Only one optimization is performed with $\alpha = 0$, and the result is illustrated in Fig. 3.5.5. Note that the pulse fishing strategy is obviously optimal also in this case. This result, together with the similarity of the results for $\alpha = 1$, $\alpha = .6$ and $\alpha = 1/3$ suggests that it is optimal to let the stock grow to a certain level and then take as much as possible under the given constraints. These results fit quite well with the results of Hannesson (1975) and Horwood (1987).

In Table 3.5.4 the main parameters used in these runs are reported. The initial stock is calculated as the equilibrium stock resulting from using the optimal constant effort over a period of at least two recruitment cycles, i.e. 16 years. Since this is more than the largest number of cohorts, viz. 12, it is guaranteed that an equilibrium is reached in this case. The linear cost function is chosen rather arbitrarily by the rule that total costs shall amount to about 50% of total revenue if a constant effort is applied. Estimates of individual average weights were collected from ICES-data.

Table 3.5.2 **Optimal pattern of fishing effort.**
A generalized production function. $\alpha = .6$

	Optimization run number			
	1	2	3	4
Period				
1	7.29	7.78	2.09	1.37
2			1.56	1.27
3			1.17	1.16
4			1.12	1.14
5			1.40	1.30
6			1.75	1.60
7		1.44	2.19	1.90
8	8.22	6.94	2.74	2.16
9	7.27	8.01	2.10	1.50
10			1.57	1.34
11			1.18	1.18
12			1.12	1.14
13			1.40	1.29
14			1.75	1.59
15			2.19	1.89
16	8.21	8.21	2.74	2.15
NPV	11450	5368	9152	9643

Table 3.5.3 **Optimal pattern of fishing effort.**
A generalized production function. $\alpha = 1/3$

	Optimization run number			
	1	2	3	4
Period				
1	25.78	25.90	8.09	4.05
2		1.34	6.06	3.84
3			4.55	3.51
4			3.41	3.22
5			4.26	3.21
6			5.33	3.90
7		2.14	6.66	4.67
8	23.42	21.46	8.33	5.30
9	27.01	28.24	8.09	4.36
10			6.07	4.07
11			4.55	3.68
12			3.41	3.35
13			4.27	3.28
14			5.33	3.84
15			6.66	4.61
16	23.44	23.43	8.33	5.23
NPV	7511	3819	5316	7111

Table 3.5.4 Table of main parameters

Coh.	Init.stock $\alpha = 1$	Init.stock $\alpha = 0.6$	Init.stock $\alpha = 1/3$	weight
1	620	620	620	1.00
2	167	165	174	1.55
3	44	31	29	2.35
4	74	60	69	3.45
5	120	125	166	4.70
6	127	151	219	6.17
7	97	117	185	7.70
8	56	54	95	9.25
9	23			10.85
10	6			12.50
11	2			13.90
12	3			15.00
Var. costs	1000	180	90	
opt.const. effort	0.21	1.9	6.0	

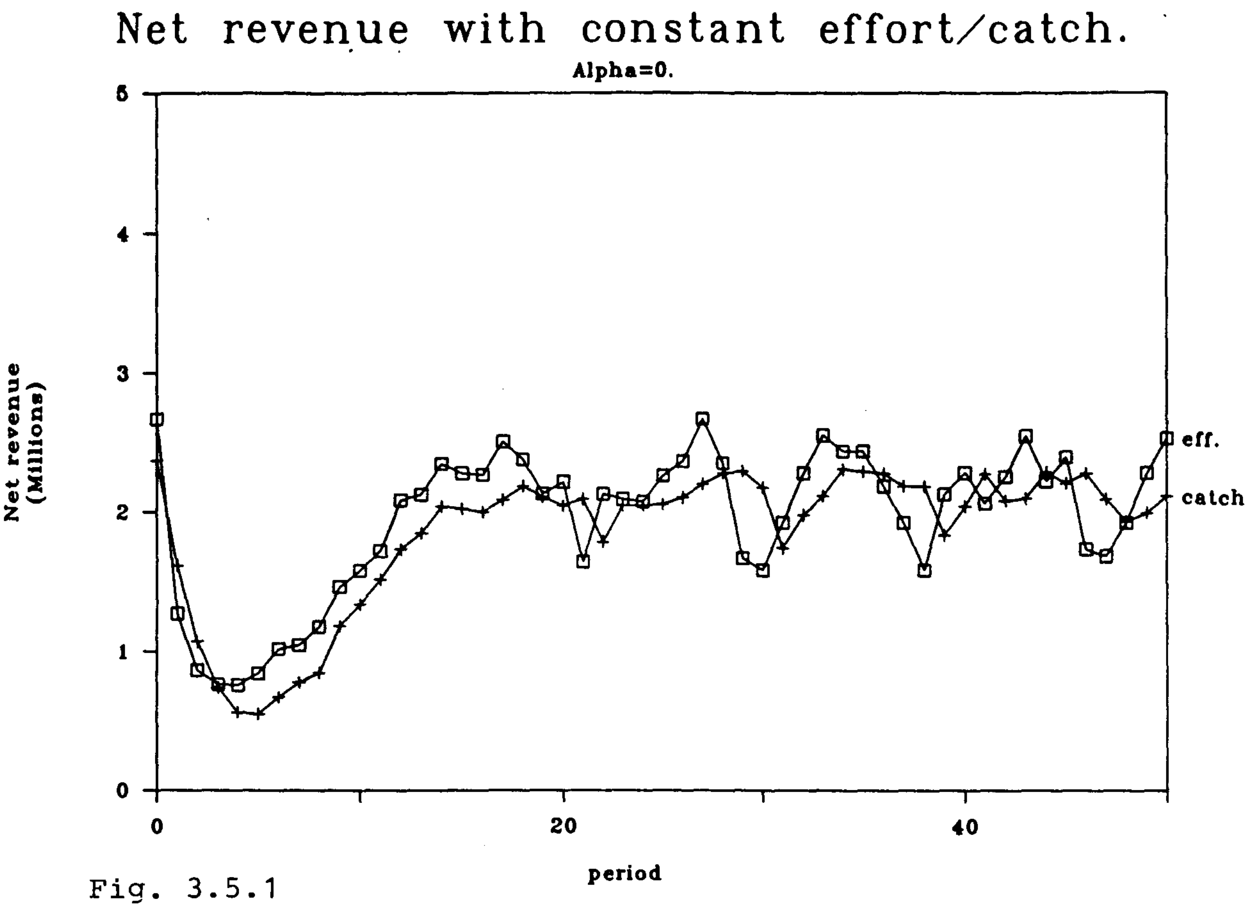


Fig. 3.5.1

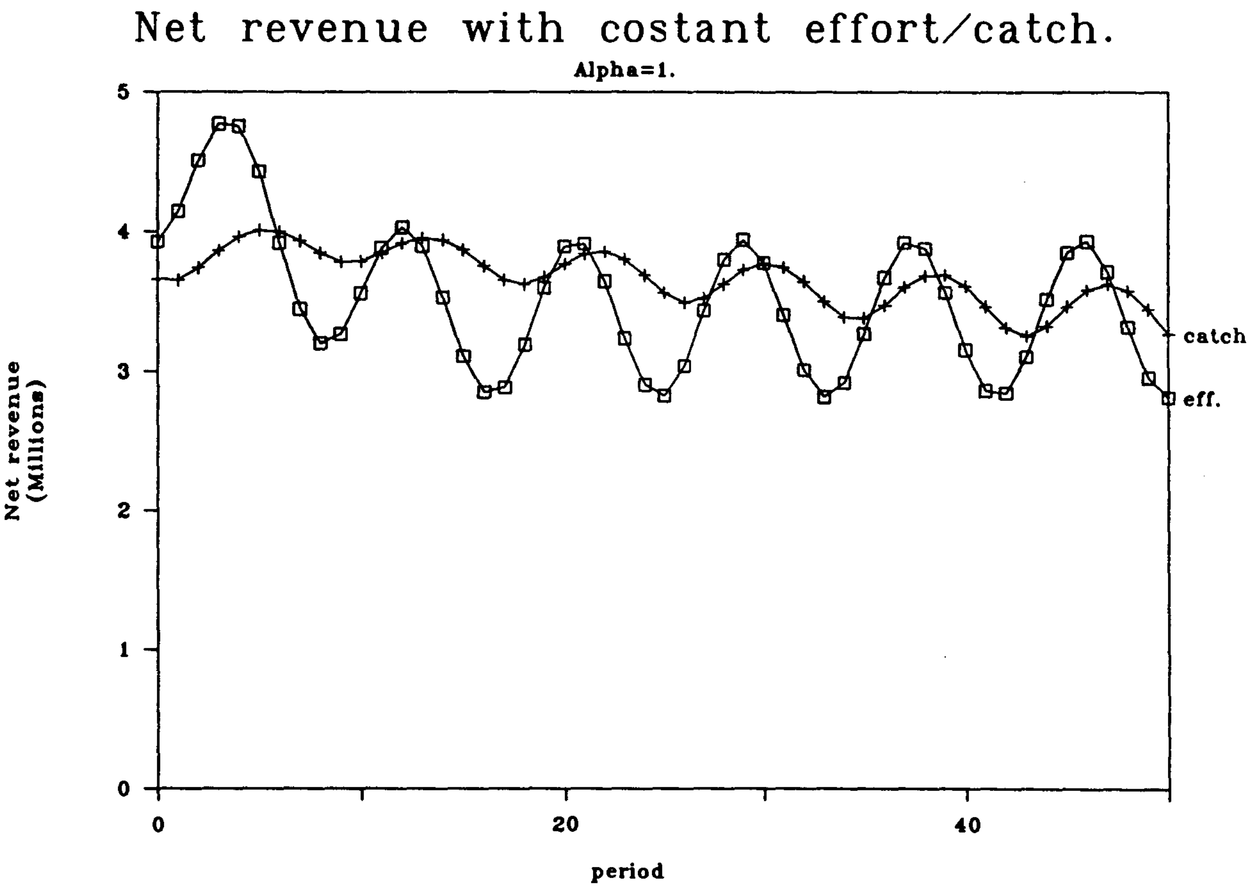


Fig. 3.5.2

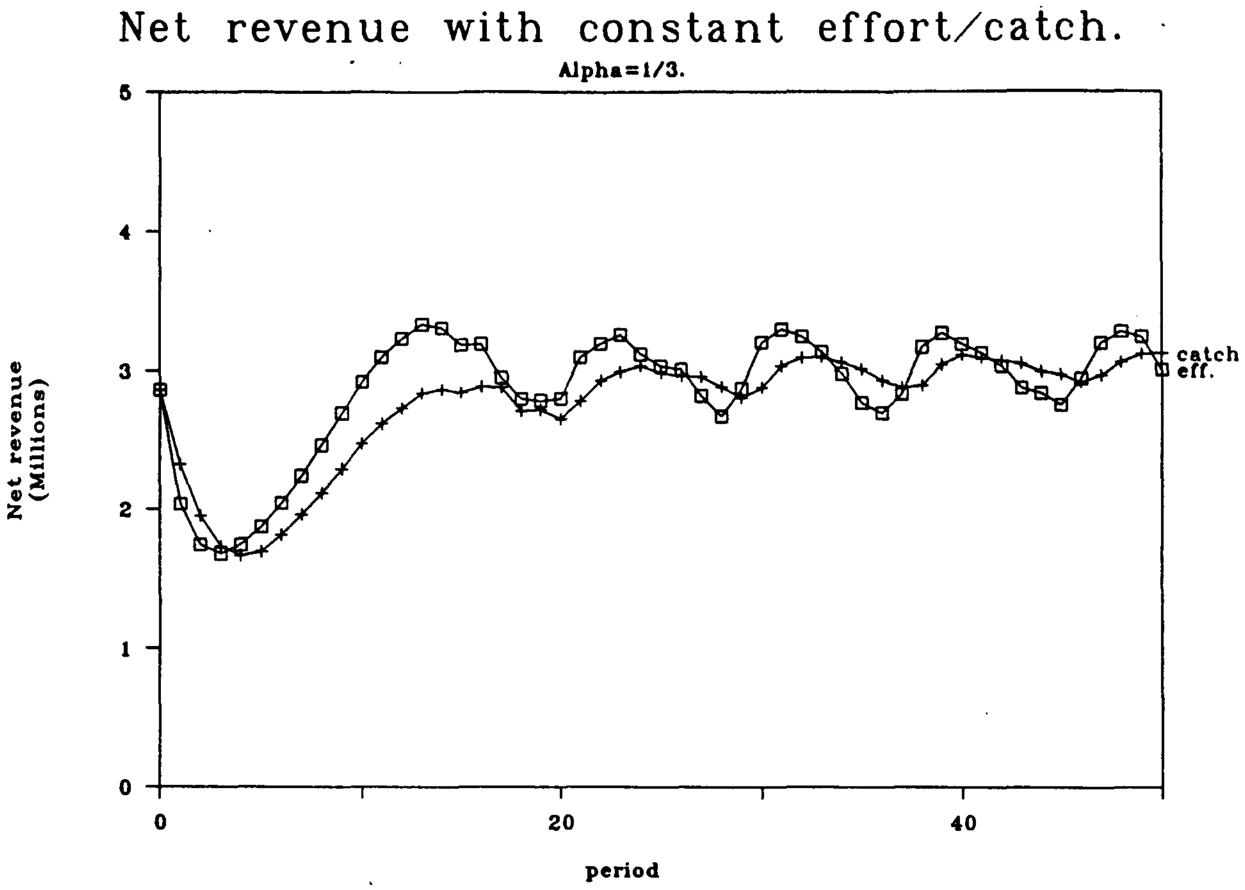


Fig. 3.5.3

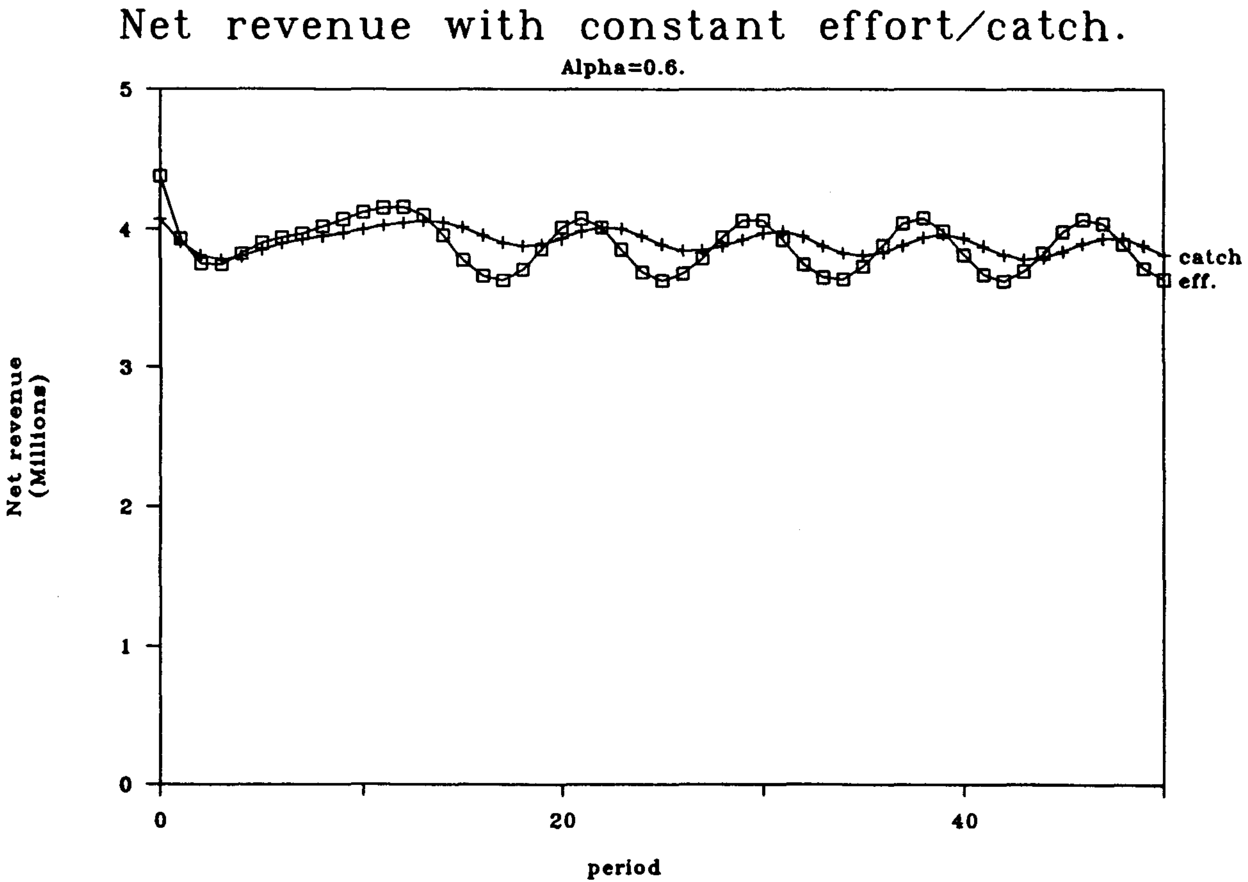


Fig. 3.5.4

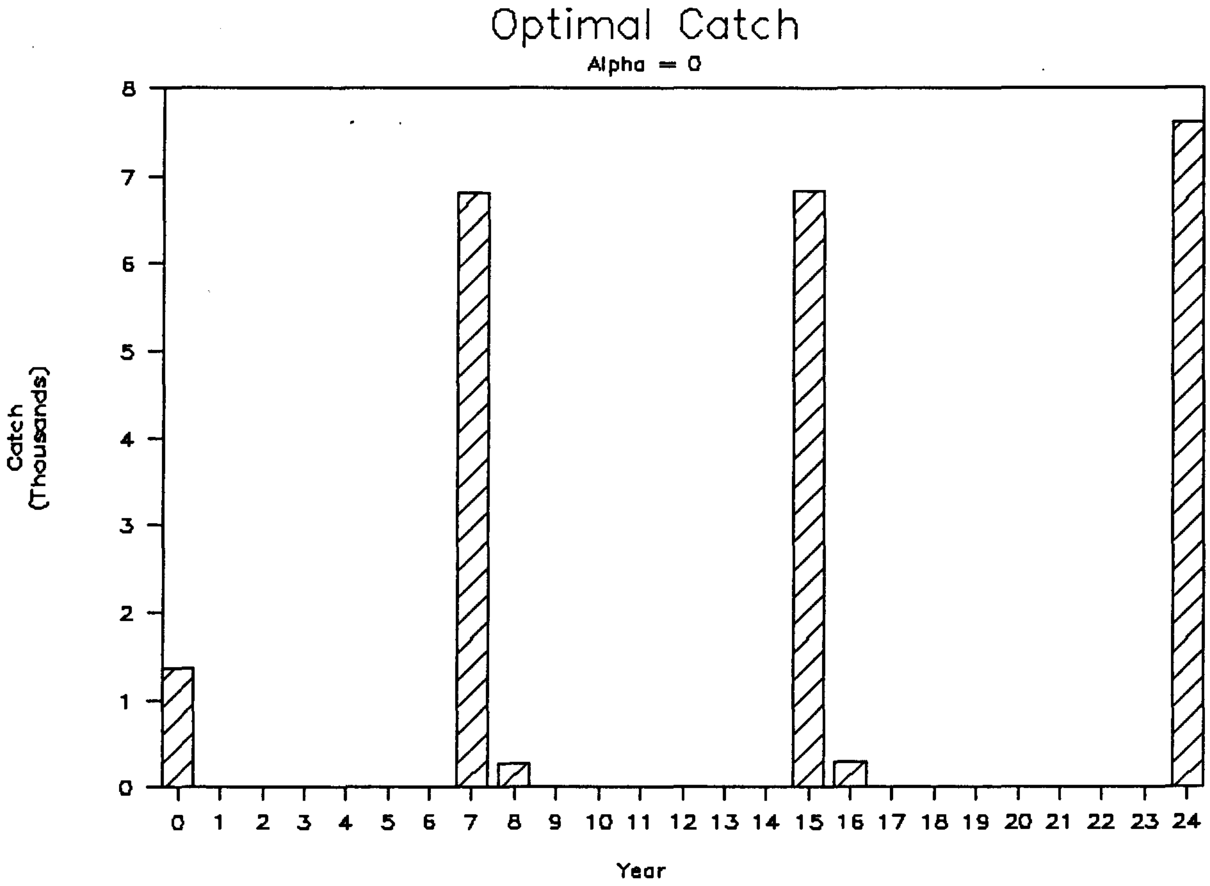

Optimal Catch
Alpha = 0
Catch (Thousands)
0
1
2
3
4
5
6
7
8
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24
Year

Fig. 3.5.5

3.6 Uncertainty

Common for the approaches which have been applied in much of this study is that a deterministic world has been assumed. This is obviously a violence against reality. In the sections to come uncertainty will play an important role, but the approach towards the topic of uncertainty will be gradual. Therefore a quite simple question will be asked here: how will the performances of the two main strategies mentioned above be affected when a sudden and unexpected pulse goes through the system?

Uncertainty can apply to many parts of the model. On the economic side neither prices nor costs can be predicted with full confidence. On the biological side both the development of individual weights and natural mortality may be subject to unexpected changes and this will in turn influence the total number of fish or biomass. Even more important for the stock development is the recruitment to the youngest cohort of the stock. Looking at historical data it is obvious that the recruitment is subject to a large degree of uncertainty if not the largest among the parameters considered here. So, regarding the question mentioned above, the unexpected pulses will in this study be limited to the recruitment of fish.

Two kinds of pulses will be considered: a sudden increase in recruitment in a single year and a sudden disappearance of recruitment. These pulses will be sent through the system for a number of years at least as large as the number of cohorts.

The effects of the pulses will then be measured by comparing the two strategies with and without such pulses in a situation where the system is initially in equilibrium. Except for the pulses a cyclical fluctuating recruitment pattern is assumed and the time period chosen is 24 years, which is equal to three full recruitment cycles, whereas the number of cohorts is 20.

As the system is supposed to be in equilibrium in the absence of pulses, different initial stocks have to be applied for the two strategies. For the constant effort case the optimal fixed effort of $E = 0.18$ and the initial equilibrium population have been calculated by an iterative process (assuming price=10, cost per unit effort is 10^7 and the discount rate is 10%). In the constant catch case the situation is a bit different as the optimal constant catch for a limited time period will probably not be viable in the very long run or indefinitely. Therefore a lower constant catch of 500,000 has been chosen as this is viable indefinitely and the corresponding equilibrium population can be calculated and used as the initial population.

The values of the net discounted revenues will not be of particular interest since each strategy is compared with itself with and without pulses and it is the effect on the relative change in average yield and average effort requirement which is interesting. How this will influence net revenues depends on the magnitude of costs relative to gross revenues and therefore depends on how cost per unit effort is chosen, a topic on which some general remarks will soon be made.

The results of the simulations are shown in Table 3.6.1. Except for the initial population the same biological parameters have been used for both strategies. A positive pulse means that in the second year a recruitment of 1,900,000 individuals are suddenly introduced, which is approximately equal to the historical maximum. It is more than one million individuals more than the average of the corresponding cyclical recruitment function. A negative pulse means that the recruitment is zero in the second year.

Table 3.6.1
Effects of sudden pulses in recruitment

		Without pulses	With pos.pulse		With neg.pulse	
Constant effort	Av. Y	518,070	549,833	6.1%	483,682	-6.6%
	E_{fixed}	0.18	0.18		0.18	
Constant catch	Y_{fixed}	500,000	500,000		500,000	
	Av. E	0.13	0.11	-15.4%	0.17	30.8%

Looking first at the reaction of the constant effort strategy to these pulses it is seen that the average yield increases by 6% with a positive pulse while it decreases by 7% with a negative pulse. In other words, the relative magnitude of the reaction is about the same both with positive and negative pulses. With the constant catch strategy the decreased effort requirement given a positive pulse is about 15% whereas the increased effort requirement with a negative pulse is about 31%. This illustrates both the higher relative magnitude in the case of average effort with a constant catch and the lack of symmetry with this strategy; i.e. the effect of a negative pulse is much larger than the effect of a positive pulse.

The effect on the net revenues or the net present value, however, is not told by these figures. In the case of constant effort the increased yield due to a positive pulse directly affects gross revenue whereas costs are constant. With a constant catch strategy, on the other hand, the reduced effort requirement due to a positive pulse affects the costs whereas gross revenue remains constant. Since the costs are hopefully less than gross revenue (otherwise there would be no reason to continue fishing) the effect on the net revenue from a relative change in effort is less than the effect of the same relative change in catch. Since the models presented so far have been based on arbitrary units of effort, it is not possible to say anything exact about this question here, but some general notes can be made. First, the effect of a change both in effort and catch is larger, the larger costs are relative to gross revenue. The reason is obvious; in this case net revenue is small and even a small effect will be large in relative terms. Second, the difference between the effect on net revenue from a change in yield and the effect on net revenue from a change in effort is bigger in relative terms when costs are small compared to gross revenue, i.e. if the cost-price relationship (c/p) is small. This is also an obvious result, because when c/p is small, a change in effort (costs) will only have a minor influence on net revenue while a change in yield (gross revenue) will have about the same effect on net revenue (the difference between gross revenue and net revenue is small when costs are relatively small).

The conclusion which may be drawn from the results in Table 3.6.1 and the general remarks made here must be that even though the relative effect on effort with a constant catch is larger than the relative effect on catch with constant effort, the relative effect on net revenues will be smaller with constant catch than with constant effort unless the c/p ratio is very high.

Another point to note is that the constant catch strategy is very sensitive to negative pulses since the additional effort required to keep a constant catch with a negative pulse (reduced stock) is much higher than the saved effort due to a positive pulse (increased stock) of the same magnitude. This is due to the "production function" which is assumed in the

Beverton-Holt model and which implies that effort requirement to keep a constant catch is convex in the stock size, and it may have detrimental effects on profitability if several more or less successive negative pulses should occur. For a discussion of cyclicality in recruitment see Chapter 4.

3.6.1 Uncertainty and Monte Carlo Simulations

Large fluctuations are an important characteristic pertaining to the recruitment of very many fish stocks and not least to the Arcto-Norwegian cod stock. A high level of uncertainty adheres to the recruitment process and in order to make the analysis more realistic this uncertainty has to be taken into account.

In this part of the study (which still deals with numerical simulations) the deterministic sinusoidal recruitment function which has been useful earlier to introduce fluctuations in the recruitment, will be abandoned and the recruitment each year will be drawn at random from a hopefully representative probability distribution. In other words, uncertainty will be included by using socalled Monte-Carlo simulations. Monte-Carlo simulation is an other word for stochastic simulation including sampling stochastic variates from a probability distribution, i.e. experimenting with the model over time. Monte Carlo simulation is a statistical sampling experiment with the model. This has also been called simulation in a narrow sense as opposed to simulation in a wide sense which is performing sampling experiments on the model of the system as has been done earlier in this study. The way the word simulation has been used so far is therefore in the wide sense whereas the way it is about to be used now is in the narrow sense (Rubinstein, 1981: p.11).

In the Monte-Carlo simulations performed here different recruitment patterns for the Arcto-Norwegian cod have been drawn from a lognormal probability distribution. This distribution seems to be a relatively good choice since, at least, it reproduces an important qualitative characteristic of the historical recruitment data, viz. a positively skewed distribution as can be seen from Fig. 3.6.1. In addition, it is perhaps also the natural distribution to reflect the large number of multiplicative effects included in the recruitment process (Charles, 1983: p. 2081). The parameters in the lognormal probability distribution, σ and μ, have been calculated from historical recruitment data as $\sigma = 0.788$ and $\mu = 13.1$.

In this study 189 independent recruitment patterns have been drawn from the lognormal distribution meant to represent future recruitment for a period of 51 years, the period for which the net present values (NPV) are calculated. The optimal constant catch and the optimal constant effort are then calculated as the respective constant catch and constant effort which maximize the NPV for this period according to the Beverton-Holt model using 20 cohorts and the same economic and biological parameter values as earlier. Data has been gathered on the NPVs and on the fixed and average catch and effort from each of the strategies respectively. The same 189 recruitment patterns have been used to calculate both optimal constant effort and optimal constant catch.

This can be stated more formally: The optimal constant catch quota is calculated for each of the 189 recruitment patterns as

$$(3.6.1) \qquad NPV(Q^{*}) = \max_{Q}\{ \sum_{t=0}^{T} \alpha^{t}[pQ - cE_{t}(Q,N_{t})]\}$$

defining optimal constant catch in biomass, Q^{*}. α is the discount factor $(1+r)^{-1}$ where r is the discount rate, and N_t is the vector giving the number of fish in each cohort at time t. p is price per unit biomass and c is cost per unit effort as earlier, and E_t is the minimum effort which is necessary to take the quota at time t from a given fish population. The catch and the updating of the individual cohorts are calculated according to the Beverton-Holt model (described in section 3.3) including a U-shaped natural mortality as a function of age, and a constant vector of individual weights. Thus Q consists of the sum of the catches from each individual cohort measured in biomass.

The optimal constant effort is calculated in a similar manner:

$$(3.6.2) \qquad NPV(E^{*}) = \max_{E}\{ \sum_{t=0}^{T} \alpha^{t}[pQ_{t}(E,N_{t}) - cE]\}$$

defining the optimal constant effort E^{*}.

For each Q^{*} there exists an average $\bar{E}$ which is the average effort over the T+1 periods necessary to take the catch Q^{*}, and accordingly for each E^{*} there exists an average $\bar{Q}$ which shows the average catch for each of the 189 recruitment patterns. Thus we have 189 observations on the NPVs from each strategy and Q^{*}, $\bar{Q}$, E^{*} and $\bar{E}$. Some of the statistical properties of these observations are listed in Table 3.6.2. The discount rate used is 0.1.

Table 3.6.2
Statistical data gathered from Monte Carlo simulations

	NPV^{Q^*}	Q^*	$\bar{E}$	NPV^{E^*}	E^*	$\bar{Q}$
Average	38.761 mill	520,587	.178	40.732 mill	.178	549,037
$s/\sqrt{n}$	445,031	4588	.003	535,195	.0007	4683
Max.	55.040 mill	649,000	.257	74.239 mill	.21	738,518
Min.	25.517 mill	372,000	.109	27.217 mill	.16	381,513

The main question now is which of the two strategies is more profitable measured by the NPVs. From Table 3.6.2 it is seen that the NPV from the constant effort strategy is higher, but in order to analyse this question properly it is necessary to establish confidence intervals around the respective means of the NPVs from the two strategies and see whether these actually are significantly different.

The standard error of the NPV is $\sigma_{NPV}/\sqrt{n}$ where n = 189. The true value of σ_{NPV} is not known, so this has to be estimated from the formula

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} [NPV_i(\cdot *) - \overline{NPV}(\cdot *)]^2 \tag{3.6.3}$$

giving an estimate, s, for σ_{NPV} and $s/\sqrt{n}$ for $\sigma/\sqrt{n}$. This estimate is recorded in the second row in Table 3.6.2. By the Central Limit Theorem it is expected that the distribution of NPV(·*) is normal and therefore it can be said with 95% confidence that the true average of NPV(·*) must lie within two standard deviations from its mean. Thus, from the data in Table 3.6.2 it is possible to establish confidence intervals for

$NPV(Q^*)$: [37.871 m, 39.651 m]
and
$NPV(E^*)$: [39.661 m, 41.802 m].

Since these intervals do not overlap the conclusion may be drawn from the simulations that under these assumptions there actually exists a difference in profitability between the two strategies, and that this difference is in favour of constant effort, but it must be mentioned that this difference is not large.

Some other interesting differences between the two strategies can also be pointed out by looking at Table 3.6.2. For example, the average catch applying the optimal constant effort for each of the recruitment patterns is significantly higher than the optimal constant catch on average for all the 189 recruitment patterns. On the other hand, the average effort with constant catch and the optimal constant effort have in fact almost exactly the same mean (exactly the same mean up to three decimal points). But with the constant effort strategy the deviation of the effort around the mean is much smaller than with the constant catch strategy, which is seen both from the $s/\sqrt{n}$ estimate and from the maximum and minimum values. This may be due to the effort requirement function implied by the Beverton-Holt model, which is convex in stock size and means that the effort requirement increases sharply when the stock size is low.

The standard deviations for the optimal constant catch and for the average catch with constant effort, on the other hand, are not much different though the difference between the maximum and minimum values is a bit higher with constant effort than with constant catch.

The net present value is more stable around its mean with the constant catch strategy measured both by its standard deviation and by the difference between the maximum and minimum values.

These confidence intervals have been calculated applying socalled crude Monte Carlo (Hammersley and Handscomb, 1967: pp. 51f.), i.e. no variance-reducing techniques have been applied. One very powerful variance-reducing technique is correlated sampling and this seems to be relevant for the problem considered here (Rubinstein, 1981: pp. 124f.). The reason is that the difference between the outcomes of the two strategies is the main interest in this analysis and the same sequence of random number vectors (recruitment patterns) is being used with both strategies. The calculated difference between the strategies is often small compared to the separate results while the variance of the difference, which is the sum of the variances in the two sets of runs, may often be large if the results are independent. Since the same random number vectors are being used, however, the results will not be independent but rather highly positively correlated and the variance of the difference will not be given by the sum of the variances but by:

$$\sigma^2_\Delta = \sigma^2_{E^*} + \sigma^2_{Q^*} - 2\mathrm{cov}(NPV(E^*),NPV(Q^*)) \tag{3.6.4}$$

If the runs had been independent, cov(NPV(E^*),NPV(Q^*)) would be zero and nothing would be gained by this method. In the case considered here there is good reason to believe that there is positive correlation between the NPVs from the two sets of runs since high recruitment will increase the profitability of both strategies and the same sequence of random recruitment patterns is being used. This is also indicated by a high correlation coefficient: $R^2 = 0.91$.

The estimated average difference between the economic results from the two strategies is:

$$\Delta\widehat{NPV} = (1/n)\sum_{i=1}^{n} (NPV_i(E^*) - NPV_i(Q^*)) = 1.970 \text{ mill.} \tag{3.6.5}$$

The estimated variance is:

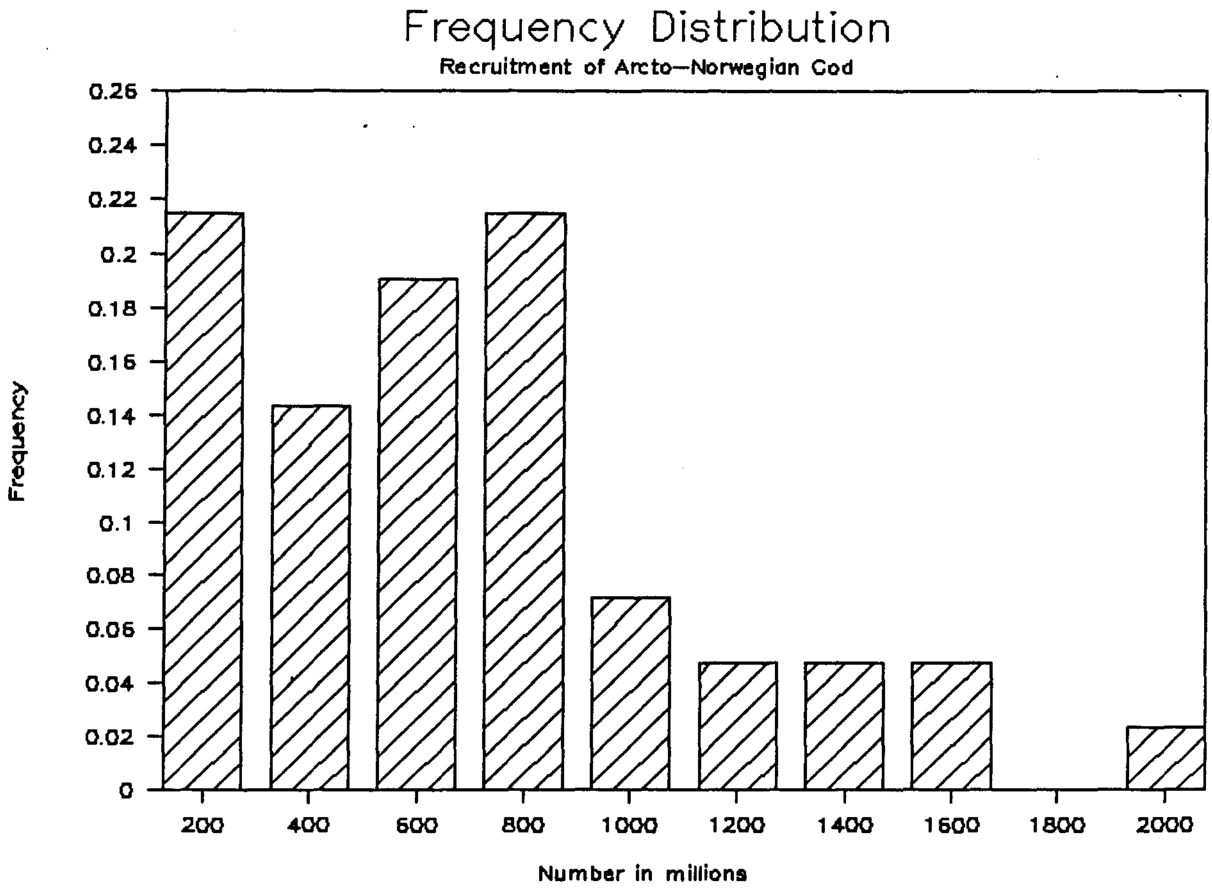
$$s^2_\Delta = s^2_{E^*} + s^2_{Q^*} - 2\mathrm{cov}(NPV(E^*),NPV(Q^*)) = 6.1E12 \tag{3.6.6}$$

and thus the estimated $s/\sqrt{n} = 179{,}806$. This means that a 95% confidence interval for the difference between the strategies is given by

1.970 mill. ± 2x179,806 or [1.611 mill., 2.330 mill.].

Since the values of the economic parameters used are chosen arbitrarily, the relative numbers may be of more interest here. The conclusion therefore is that the constant effort strategy under these assumptions is from 4.2% to 6.0% more profitable in the long run with a 10% discount rate than the constant catch strategy.

An important criticism which may be raised against the analysis performed here is that it is based upon an ex post evaluation. Each time a random recruitment pattern is drawn, it is assumed that the yearly recruitment is known 50 periods hence, and then the optimal catch or optimal effort is calculated. This can, by definition, not be true since recruitment is stochastic and in the real world it is not possible to calculate the optimal values as has been done here and therefore the optimal strategies can not be implemented. The only values that are known in the real world are estimates of the parameters of the probability distribution based on historical data, and even these will change with time. Therefore perhaps the same constant catch and constant effort should be used for all the different recruitment patterns and these should be based upon the known values of the probability distribution.



Frequency Distribution
Recruitment of Arcto—Norwegian Cod
Frequency
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Number in millions

Fig. 3.6.1

4. EMPIRICAL ANALYSIS OF RECRUITMENT

4.1 Factors influencing recruitment of Arcto-Norwegian cod

As was seen in Sections 2.6 and 3.6, the risk implied by a constant catch quota strategy increases with the existence of a strong stock recruitment relationship or serial correlation in recruitment. Also the variation in population size coming from the different management strategies may depend upon the existence of serial correlation. This was shown in Section 2.3. This is the reason why some effort is devoted to the question of stock recruitment and serial correlation of recruitment. Also one of the factors that may explain possible serial correlation, viz. the temperature, is analysed.

The relationship between recruitment and different explanatory variables will be investigated. It has been mentioned earlier that a statistically significant relationship between recruitment and the size of the spawning stock has not been found. Therefore some alternative hypotheses must be applied instead of or at least in addition to this. Among the most obvious factors which may affect recruitment are environmental conditions such as currents, ice coverage and temperatures as well as ecological conditions such as the size of the cod stock and other species. This last point is due to the fact that a large adult population not only produces a large number of eggs but also eats a great number of eggs and fry, which decreases the survival rate of adolescent cod and subsequent recruitment to the cod stock. All these factors have to be taken into consideration although not all of them will occur in the final formulation of the recruitment function.

Data for all conceivable environmental factors does not exist. It is therefore necessary to choose an indicator of the environmental conditions for which good data exists such as temperature or ice coverage. As was to be expected, these two indicators show a high level of correlation with each other and one of them has to be chosen.

In this section average temperature is chosen as the main indicator of climatic regime. The relationship between long-term fluctuations in recruitment and variations in the climatic regime in the reproduction area will be investigated. Recruitment is defined as the number of three-year-old fish every year. This problem is complex, on one hand negative temperature anomalies on the spawning grounds can delay and displace spawning, but on the other hand such negative anomalies may cause fewer predators of fry to be present. Negative temperature anomalies may also cause less food to be available for the fry and juvenile fish. If any such correlation is to be found, it is most likely to be found at the extreme latitudes such as the Barents Sea where the fish live near their environmental (e.g. temperature) limits.

There are many reasons to believe that a positive relationship between these two variables may exist and several hypotheses can be stated. Two such hypotheses will be proposed:

1. Recruitment will increase when there is a positive shift in climatic regime, i.e. when the average temperature rises. This is because the size of the feeding area for the fish stock is increased to the north and east. This will happen irrespective of the initial climatic regime, only the relative shift matters. It is supposed that the fish stock has adapted to the initial area where it has been feeding for some time and food has become scarce. Outside this area food is more plentiful but it has been too cold for the fish to feed, especially to the north and east.

2. An alternative hypothesis is to say that there exists a more direct relationship between recruitment and temperature. If this is true, recruitment can be perceived as an immediate or lagged function of temperature which implies high recruitment when temperature is high and vice versa. This may be the result of an increased fertility or survival among fish when the temperature is high.

The first approach to this analysis will be based on rather crude data from Sætersdal and Loeng (1987). These data are illustrated in Fig. 4.1. Considering Fig. 4.1, it is tempting to suggest that moving from a cold climatic regime to a relatively warmer regime is in most cases a necessary condition for high recruitment since these two occurrences coincide in most cases. However, this pattern might just be accidental and the probability that this pattern may occur by accident will therefore be calculated. There are 79 yearly observations of temperature from 1900 to 1983 with the exception of five years during World War II, and in the entire period there are nine transitions from a cold to a relatively warmer climatic regime. The variable is given as an index from one to four. With the exception of the year 1964 every regime lasts for more than one year. The recruitment data is an index given on a scale from one to three: low, medium and high; and of the 82 observations from 1902 to 1983 there are nine observations of high recruitment, 19 observations of medium recruitment and the remaining 54 observations are low recruitment (not shown in the figure). Thus there are a total of 77 joint observations of the recruitment and temperature index for the periods 1902 - 1939 and 1945 - 1983. Seven of the observations with high recruitment coincide with a transition to a warmer climatic regime. The remaining two cases of regime transition coincide with medium recruitment, thus there is no occurrence of transition to a relatively warmer climate combined with low recruitment. If the letter z is used to symbolize coincidence between high recruitment and transition to a warmer regime and k is the number of regime transitions, m is the number of high recruitment and n is the total number of joint observations, then the probability formula for a hypergeometric distribution can be written:

$$(z) = \begin{bmatrix} m \\ z \end{bmatrix} \begin{bmatrix} n - m \\ k - z \end{bmatrix} \Big/ \begin{bmatrix} n \\ k \end{bmatrix} \tag{4.1}$$

and the cumulative probability of z being greater than or equal to some number $j < m$ is given by:

$$P(z \geq j) = P(z = j) + P(z = j+1) + \ldots + P(z = m-1) + P(z = m) \tag{4.2}$$

when $z \leq m \leq k$. The probability of $z \geq 7$ when $k = m = 9$ and $n = 77$ has been calculated to 5.122×10^{-7} or about one to two million. This is a rather promising result as far as the simple probability model is concerned, but this model can be criticized for several reasons. The probability is calculated on the basis of a pattern found in the figure and it may be argued that perhaps any figure can show some sort of pattern with an equivalent low probability of accident. However, this pattern corresponds quite well with the theory stated in the hypothesis above, thus it can at least be regarded as a good starting point for further investigation.

Time series analysis of recruitment

It is reasonable to believe, and is indeed a widely held view, that stock recruitment is strongly influenced by environmental factors such as sea temperature. This view is supported by the analysis above. Forecasts of environmental data are, however, within the current state of the art of a very short-term nature, and therefore predictions of future recruitment based on a direct relationship to environment will also be so. On the other hand, if it is possible to detect significant long-term cycles in the recruitment variable and perhaps also a significant relationship between these fluctuations and variations in temperature, this may provide the base for long-term recruitment forecasts. Improved recruitment predictions, e.g. reduced variance in the probability distribution, may also improve some aspects of fishery management. For the present purpose it does not matter whether high recruitment or stock abundance occur in the year 2000 or 2005 as long as the frequencies (periods) and amplitudes of future stock fluctuations and how these are influenced by different management regimes can be predicted with some degree of certainty. A univariate analysis of recruitment showing more or less regular cycles could of course do, but the best thing would be to provide some insight into the causality behind these fluctuations in recruitment.

The data consists of time-series data for recruitment of cod and temperature for 1946 - 1987. The stock size in number of three-year-old Arcto-Norwegian cod (Anon. 1988) estimated by virtual population analysis (VPA) has been chosen as the dependent variable, i.e. the number of three-year-olds in year t+3 is used as an indicator of recruitment in year t. The independent variable is the mean temperatures at the Kola section 70°30' - 72°30'N in the 0-200 m. layer on the 15th day of the month (Bochkov, 1982).

Visual inspection of a plot of the raw data indicates no clear relationship. However, after log-transformations have been taken of the VPA numbers and temperatures, visual inspection indicates a slight positive relationship. This suggests some sort of log-linear formulation of the regression equation, e.g.

$$R_t = a_o T_t^{a_1} \tag{4.3}$$

where R_t is recruitment and T_t is temperature at time (year) t. Taking logarithms and performing OLS-regression gave the following estimators:

		st.d.	t	
$\widehat{(\log_e a_o)}$ =	2.15	1.06	2.03	$R^2 = 0.27$
$\hat{a}_1$ =	2.99	0.78	3.83	DW = 1.11

This result is not very good; R^2 is low and the Durbin-Watson statistic indicates autocorrelation. However, alternative specifications of the equation did not improve the result. By taking first order differences of the data or following the Cochrane-Orcutt iterative procedure the Durbin-Watson statistic may be improved but then the explanatory power measured by R^2 almost disappears, i.e. DW = 2.07 but $R^2 = 0.07$. By using temperature averages for some selected months (e.g. April - June) the explanatory power of the regression model was even poorer. This may indicate that recruitment is in some way sensitive to environmental influences throughout the year and not only in the spawning season.

The high degree of autocorrelation measured by the DW statistic may suggest inclusion of lagged values of the variables. Lagged values of the temperatures, however, did not show any sign of relationship upon testing different kinds of distributed lag hypotheses. The models used for this purpose were the Almon polynomial distributed lag and the Koyck geometric distributed lag. Both these models showed better fit the fewer lags included. Inclusion of lagged values of the dependent variable, recruitment, is not suggested by the theory.

Since the specification of the log-linear model above was the best to be found from the original data and it lacks explanatory power, this suggests inclusion of more explanatory variables. According to theory, the size of the spawning stock is the most obvious factor behind recruitment although this has seldom been successfully confirmed by empirical studies.

The traditional way to model recruitment is to express it as a function of the spawning stock at an earlier time, e.g. $R_t = R_t(S_{t-\sigma})$, where S_t is recruitment at time t and σ is the period between spawning and recruitment to the fish stock. Several attempts at relating recruitment in numbers to the size of the spawning stock one or more periods earlier were made. The Beverton-Holt recruitment function, Eq. (3.3.15), as well as several other functional forms were tried. These attempts, however, invariably failed to yield a meaningful recruitment function with proper signs of the parameters.

In light of this and what has been said earlier, a recruitment function including both the spawning stock and temperatures will be tried: $R_t = R_t(S_{t-\sigma}, T_t)$. Based on trial and error the following non-linear formulation of the function was found:

$$R_t = a_o S_{t-3}^{a_1} e^{a_2 T_t}. \tag{4.4}$$

OLS-regression performed after log-transformation gave the following results:

		st.d.	t	
$\widehat{(\log_e a_o)}$ =	2.65	0.99	2.68	DW = 1.08
$\hat{a}_1$ =	0.14	0.21	0.68	$R^2 = 0.29$
$\hat{a}_2$ =	0.77	0.23	3.38	

The period between spawning and recruitment is set at three years; thus the data covers 39 observations in the period from 1949 - 87. The explanatory power has only increased by 2% and the autocorrelation is still high. Taking first differences of the data increased DW to 1.77 but decreased R^2 to 0.13. In addition the estimated coefficient for the spawning stock is not significantly different from zero.

One of the most important results from this part of the analysis was that the explanatory power measured by R^2 and the fit between estimated and actual numbers seemed to be much greater for the period after 1965 than for the period from 1945 to 1965. Equation (4.4) estimated for the period from 1966-87 gave $R^2 = 0.37$ but DW = 0.95. After taking first order differences the result was $R^2 = 0.27$ and DW = 1.89. The coefficient for the spawning stock, $\hat{a}_1$, was still not significantly different from zero. A Chow-test did not show any significant differences between coefficients obtained from the two periods. It is therefore not allowed to split the sample into two parts, which perhaps could solve part of the problem of lacking explanatory power. The reason why the explanatory power seems to be better in the latest part of the data series is not obvious, but improved sampling methods for biological data may be one explanation.

There is no indication of a significant direct linear, non-linear or lagged relationship in the data, but there is another possibility left, viz. to move from the time domain to the frequency domain. First, an estimation of the sample spectral density for the individual time series will be undertaken to detect the degree of autocorrelation and the frequencies of any cycles present in the data. The existence of more or less regular cycles will by itself be an interesting result as far as recruitment is concerned. Second, a cross-spectral analysis of the two series will be performed which is the frequency domain analogue to correlation analysis, and this bivariate analysis may say something about the causal relationship between the series. More about spectral analysis can be found in e.g. Fishman (1968), Harvey (1981 B) and Shumway (1988).

In the frequency domain interest is centred on the contributions made by various periodic components in the series. Since regular cycles are not to be expected in the series, interest is concentrated on any tendency towards cyclical movements centred around a particular frequency. Visual inspection of the recruitment data in Fig. 4.2, indicates that the existence of such a tendency is quite likely.

Univariate Analysis

First the periodogram will be estimated, or rather the sample spectral density which is proportional to the periodogram. This is based on the idea of correlating a known periodic series such as a sine or cosine-function to the observed series. If the observations of one of the samples are denoted x_t, t=1,..,T, this can be transformed into two random variables of varying but known frequencies, assuming that this series is stationary. The random variables that can be formed are the sine transform

$$X_S(\lambda_j) = (2/T)^{1/2} \sum_{t=1}^{T} x_t \sin(\lambda_j t) \qquad j=1,..,n-1 \tag{4.5}$$

and the cosine transform

$$X_C(\lambda_j) = (2/T)^{1/2} \sum_{t=1}^{T} x_t \cos(\lambda_j t). \qquad j=1,..,n-1. \tag{4.6}$$

These can combined into the periodogram defined as

$$P_x(\lambda_j) = [X_S^2(\lambda_j) + X_C^2(\lambda_j)] \qquad j=1,..,n-1, \tag{4.7}$$

which gives a measure of the sample variance as a function of the frequency λ_j. The frequencies have been defined as

(4.8) $$\lambda_j = 2\pi j/T, \; j = 0,1,..,n$$

where

$$n = \begin{cases} T/2 & \text{if T is even} \\ (T-1)/2 & \text{if T is odd.} \end{cases}$$

It is now convenient to use the discrete Fourier transform:

(4.9) $$X(\lambda_j) = (2/T)^{1/2} \sum_{t=1}^{T} x_t e^{-i\lambda_j t}.$$

This can also be written in terms of the sine and cosine-transforms, applying the conventional trigonometric identity

$$e^{-i\lambda_j t} = \cos \lambda_j t - i \sin \lambda_j t$$

and thus

(4.10) $$X(\lambda_j) = X_C(\lambda_j) - iX_S(\lambda_j).$$

The periodogram may then be written:

(4.11) $$P_x(\lambda_j) = |X(\lambda_j)|^2.$$

The spectral density (or power spectrum) is a continuous function

(4.12) $$f_x(\lambda) = (2\pi)^{-1} \sum_{\tau=-\infty}^{\infty} R_x(\tau)e^{-i\lambda\tau}$$

where $R_x(\tau)$ is the autocovariance function

(4.13) $$R_x(\tau) = E[(x_{t+\tau}-\mu)(x_t-\mu)],$$

and μ is the mean of the x_t series. If it is assumed that the series is real instead of complex, the spectral density can be defined by

$$f(\lambda) = (2\pi)^{-1} [R(0) + 2 \sum_{\tau=1}^{\infty} R(\tau)\cos \lambda\tau], \tag{4.14}$$

and the sample spectral density can be estimated by

$$I(\lambda) = (2\pi)^{-1} [C(0) + 2 \sum_{\tau=1}^{T-1} C(\tau)\cos \lambda\tau], \tag{4.15}$$

where the theoretical autocovariances R() are replaced by the sample autocovariances C(). In order to develop an (average) spectral estimator (of the ordinates in the neighbourhood) at some frequency λ, some smoothing (in the frequency domain) or weighting (in the time domain) is necessary. In the frequency domain smoothing can be performed by seeing the spectrum through a spectral window as a weighted average of adjacent frequencies. The spectral window however is the Fourier transform of some weighting function or lag window in the time domain. Thus smoothing and weighting are two equivalent techniques that can be used. A window is characterized by its width which is simply the length of its base, and this can be measured in two ways: by the range and by the bandwith. For a white noise process a simple average of adjacent frequencies is a sufficient spectral window, and a possible estimator of $f(\lambda)$ is

$$\hat{f}(\lambda) = m^{-1} \sum_{i=-m'}^{m'} I(\lambda_{j-i}) \tag{4.16}$$

where $m = 2m'+1$, the number of spectral points used in averaging, defines the range, and the bandwith is defined as the width of the window in radians, $2\pi m/T$. For a white noise spectrum any weighted average will be an unbiased estimator. Furthermore the variance will be smaller the larger the range, and this latter property also holds when the series is not generated by a white noise process. However if the process is not white noise, there is no guarantee that the estimator will always be unbiased. In this case a trade-off between bias and variance emerges, influencing the choice of range. Choosing a too large range will result in poor resolution, implying leakage of the power spectrum from one frequency to another and may introduce the possibility of a large bias. Applying a triangular window instead of a rectangular will give more weight to the ordinates closest to the frequency of interest.

The alternative approach of weighting the autocovariances in the time domain has been chosen. The estimator of f(λ) is then given by

$$\hat{f}(\lambda) = (2\pi)^{-1}\,[k(0)c(0) + 2\sum_{\tau=1}^{T-1} k(\tau)C(\tau)\cos\lambda\tau] \qquad (4.17)$$

where k(0),..,k(T-1) are the weights. A number of weighting procedures have been suggested in the literature, and one of the most widely used, the Tukey-Hanning weighting scheme, has been chosen here:

$$k(\tau) = k_{M,\tau} = \begin{cases} 1/2[1 + \cos(\pi\tau/M)] & \tau < M \\ 0 & \tau \geq M. \end{cases} \qquad (4.18)$$

The result of the estimation when M = 6 is shown in Fig. 4.3 for recruitment and Fig. 4.4 for temperature. The Tukey-Hanning weighting scheme has a bandwith of $2\pi/M = 1.0472$ and the corresponding range is $1.0472\,(T/2\pi) = 7$, (Fishman, 1968: p.94). From Fig. 4.3 it is seen that the estimated spectrum for recruitment has a maximum at λ_5=0.748 corresponding to a recruitment period of 8.4 years. The sample spectral density for temperature in Fig. 4.4 shows high power for the low frequencies but it is steadily declining until $\lambda \approx \pi/2$ and thereafter has a local maximum at $\lambda_{15} = 2.24$ corresponding to a period of 2.8 years. The plot of the sample spectral density describes an analysis of variance of the time series as a function of the variable λ since the definition of the power spectrum is essentially that it is the variance of the time series in a frequency interval dλ centred at λ, and the area under the power spectrum over the range $[-\pi,\pi]$ is equal to the variance, σ^2. The main conclusion so far is that there seems to exist a relatively strong tendency towards an irregular recruitment cycle of about 8 years.

The rationale behind these cycles can be explained in different ways. One explanation includes only the cod itself and is related to the degree of cannibalism. Since recruitment is mainly determined by the survival of the eggs and not so much by the original quantity of eggs, predation and cannibalism are a major determining factor. A large adult stock will, by cannibalism, depress the stock of prefishery juveniles, thus recruitment decreases. This new adult stock resulting from low recruitment will be smaller than the long-term average and will therefore exercise a lower predation pressure on the new juvenile stock which in turn results in high recruitment, high adult stock and so on, and the cycle will repeat itself (Laevastu and Favorite, 1988: p. 112f.).

This is at best only part of the explanation, as migration of adults away from the nursery grounds is quite prevalent for Arcto-Norwegian cod. Predation pressure from other species may be just as significant and so may fishing, and these factors will probably not cause regular cycles. However, as the cod is recruited at the age of three years, a cycle of this kind of about eight years is not unlikely. Therefore this will be retained as a possible part of the causality behind the cycles.

Bivariate Analysis

The next step is to extend the analysis to relations between the two original series, recruitment and temperature, using the notion of a frequency-dependent correlation function. For two jointly stationary series, x_t and y_t, the cross covariance function

$$(4.19) \qquad R_{yx}(\tau) = E[(y_{t+\tau} - \mu_y)(x_t - \mu_x)]$$

and the cross-spectrum, $f_{yx}(\lambda)$, are related by

$$(4.20) \qquad f_{yx}(\lambda) = (2\pi)^{-1} \sum_{\tau=-\infty}^{\infty} R_{yx}(\tau)e^{-i\lambda\tau}.$$

The cross-spectrum is generally a complex function which is normally broken down into two real quantities, the co-spectrum

$$(4.21) \qquad c(\lambda) = (2\pi)^{-1} \sum_{\tau=-\infty}^{\infty} R_{yx}(\tau)\cos \lambda\tau$$

and the quadrature-spectrum

(4.22) $$q(\lambda) = (2\pi)^{-1} \sum_{\tau=-\infty}^{\infty} R_{yx}(\tau) \sin \lambda\tau$$

thus leading to the representation

(4.23) $$f_{yx}(\lambda) = c(\lambda) - iq(\lambda).$$

The cross-spectrum plays the role of a frequency-dependent measure of covariance between the two series, and the simplest interpretation of the cross-spectrum is probably in terms of gain, phase and coherence. The coherence,

(4.24) $$\gamma^2(\lambda) = \frac{|f_{yx}(\lambda)|^2}{f_x(\lambda) f_y(\lambda)}$$

is a measure of the fraction of $f_y(\lambda)$ which can be systematically accounted for by movements in x at a certain frequency. This is a number between 0 and 1 for all values of λ, and can be interpreted as the frequency domain analogue of the ordinary squared correlation coefficient. The gain is defined by

(4.25) $$G(\lambda) = |f_{yx}(\lambda)|/f_x(\lambda)$$

and the phase shift by

(4.26) $$\varphi(\lambda) = \tan^{-1}[q(\lambda)/c(\lambda)].$$

To interpret these quantities it is useful first to think of the problem as finding the impulse response function, a_t t=0,±1,.., which minimizes the mean square error (MSE) between the output series, recruitment, and some linear filtered version of the input series, temperature:

(4.27) $$\text{MSE} = E\left[(y_t - \sum_{s=-\infty}^{\infty} a_s x_{t-s})^2\right].$$

For computational simplicity the infinite Fourier-transform of the impulse response function is used,

$$A(\lambda) = \sum_{s=-\infty}^{\infty} a_s e^{-i\lambda s},$$

which can be written as

$$A(\lambda) = f_{yx}(\lambda)/f_x(\lambda), \tag{4.28}$$

see Shumway (1988: p.100). It can now be seen that the gain $G(\lambda) = |A(\lambda)|$. The gain and the phase can be interpreted as the twofold effect of linear filtering. The linear transformation may imply amplification or attenuation of each frequency component due to the gain. If low frequency contributions are amplified and high frequency contributions are attenuated, the resulting sequence will have less variation over short intervals and more variation over long intervals and the degree of autocorrelation will increase (and vice versa). The other effect of a filter, the phase shift, concerns the possibility of a shift in the series with respect to its position in time.

Given the data on recruitment and temperature, the coherence, gain and phase functions may all be estimated. There need be no assumption that the two series are linearly related; the aim of the discussion of linear filters has mainly been to illustrate some theoretical aspects. The motivation for applying spectral analysis was in fact the lack of relationship in the time domain. The Tukey-Hanning weighting scheme has been used also in the estimation of the co-spectrum and the quadrature-spectrum:

$$\hat{c}(\lambda) = (2\pi)^{-1} \sum_{\tau=-M}^{M} k_{M,\tau} C_{yx}(\tau) \cos \lambda\tau \tag{4.29}$$

$$\hat{q}(\lambda) = (2\pi)^{-1} \sum_{\tau=-M}^{M} k_{M,\tau} C_{yx}(\tau) \sin \lambda\tau, \tag{4.30}$$

except that here the weighting scheme is centred on $\tau = 1$ instead of $\tau = 0$. The reason for this is that in cross-spectrum estimation there may be adverse results if the weighting function is centred on the zero-lagged cross-covariance C_{yx} when there is a peak in the cross-covariance function at a nonzero lag (Fishman, 1968: p. 122). Analogous to the univarite case, the components of the sample cross-spectrum are obtained by replacing the theoretical cross-covariances by the sample cross-covariances and here the sample cross-covariance function has a peak at lag $\tau = 1$. The estimated coherence, gain and phase functions are shown in Figs. 4.5, 4.6 and 4.7 respectively.

First a test for coherence has to be performed. Only if the null-hypothesis of zero coherence is rejected will it be meaningful to analyse the gain and phase functions. The test for zero coherence will for simplicity be performed on an individual basis for each frequency. The strength of the linear relation can be assessed by comparing the estimated coherence $\hat{\gamma}^2(\lambda)$ with the constant

$$K_\alpha = \frac{B_\alpha}{1 + B_\alpha} \quad \text{where} \tag{4.31}$$

$$B_\alpha = \frac{2F_{2,2(m-1)}(\alpha)}{2(m - 1)}. \tag{4.32}$$

m is the range of the lag window and α is the type I error probability in testing the zero coherence hypothesis. In this case $m = 7$ and when $\alpha = 5\%$ is chosen, it can be found from the F-statistic table in Koutsoyiannis (1977: p. 663) that $F_{2,12} = 3.89$, thus $B_\alpha = 0.6483$ and $K_\alpha = 0.39$. From Fig. 4.5 it is now clear that only at very low frequencies is there a significant coherence at a 5% significance level; in fact only the three lowest frequencies including $\lambda=0$ have significant coherence at this level corresponding to recruitment cycles with periods greater than 14 years. The recruitment period of 8.4 years which was detected in the univariate analysis has a coherence of about 0.2 meaning that about 1/5 of this can be systematically accounted for by temperature variations. Fortunately it is the long-term changes corresponding to low frequencies that are of interest in this analysis. There is also a local but insignificant maximum at the frequency $\lambda = 1.65$ corresponding to a period of 3.8 years.

The sample gain, Fig. 4.6, indicates that relatively low frequencies ($\lambda < \pi/2$) are amplified and high frequencies are attenuated. This means that medium and long-term changes in temperature (periods greater than 4 years) are associated with even larger changes in recruitment whereas short-term changes in temperature are associated with relatively small changes in recruitment.

The phase angle curve shown in Fig. 4.7 has at least two characteristics. First, the phase angle is positive for all frequencies and second, the slope of the curve is decreasing with increasing frequencies almost over the whole range. The positive phase angle indicates that temperature changes are leading recruitment changes, which is consistent with theory. The decreasing slope of the curve indicates that low-frequency components are delayed longer in time than high-frequency ones.

Reservations have to be made about the small number of observations, but as observations of recruitment are only meaningful on a yearly basis and these have only been recorded since 1946, it will still be some time before a satisfactory number of joint observations on recruitment and temperatures will be available. Until then allowance has to be made for this problem. The spectral analysis indicates that this may be a useful method, and even more useful some time in the future when the sufficient number of observations are available.

The main conclusion from the spectral analysis is that a recruitment cycle with a period of about 8 years is quite prevalent compared to other cycles and part of this (about 20%) can be explained by prior changes in climate. However these cycles are quite irregular and the rest of the variation in recruitment has to be explained by several other factors. This is hardly surprising and is in accordance with the stochastic nature of the problem considered.

RECRUITMENT AND TEMPERATURES FROM 1900- 1983

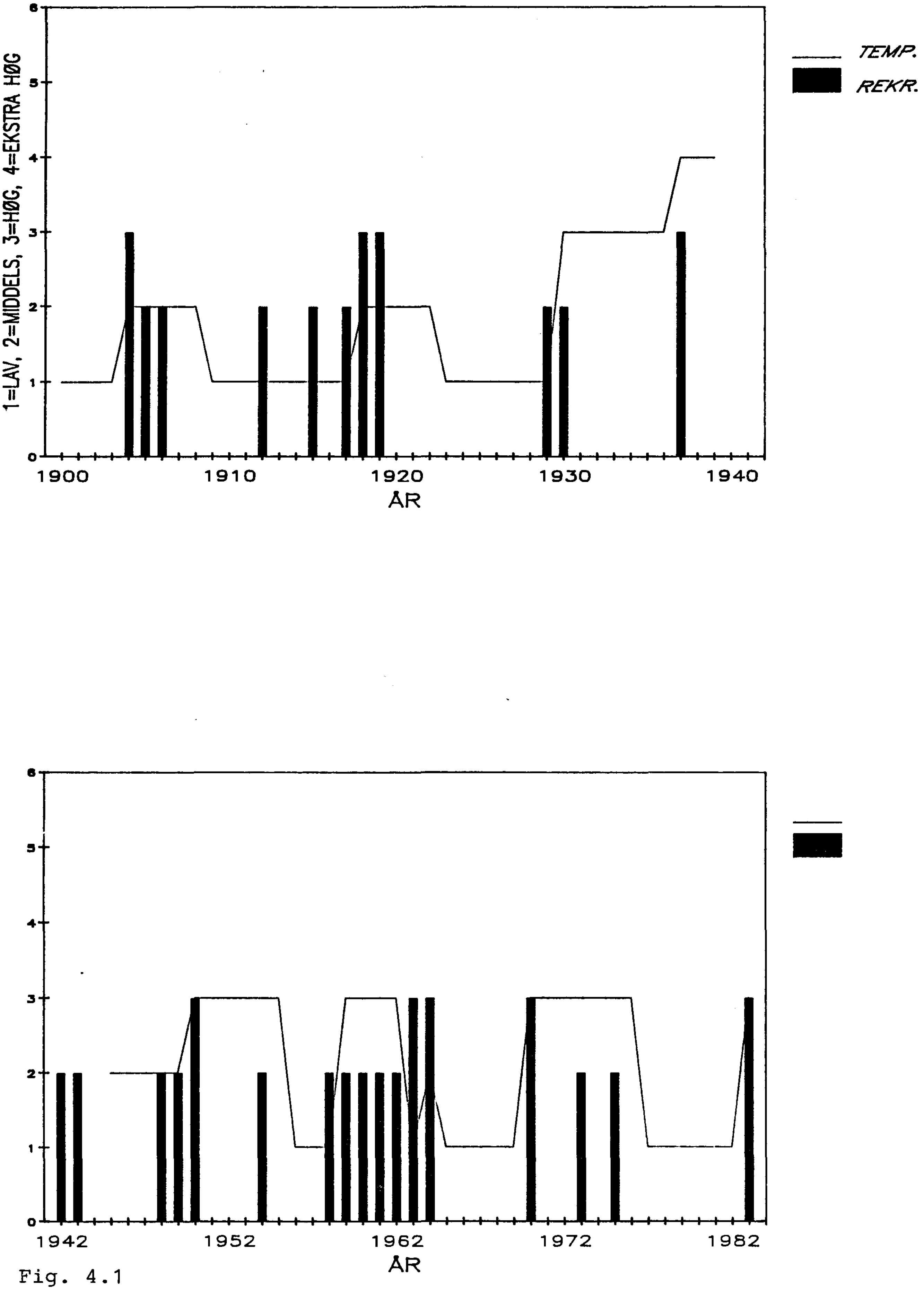


Fig. 4.1

RECRUITMENT

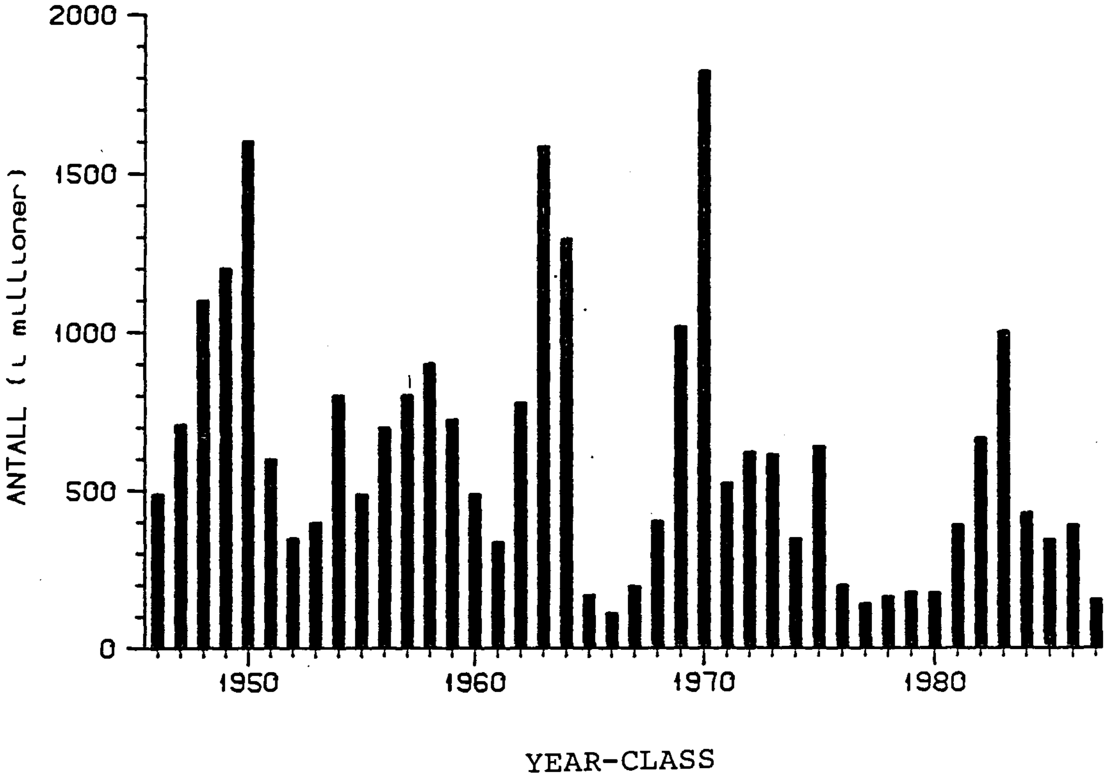


Fig. 4.2

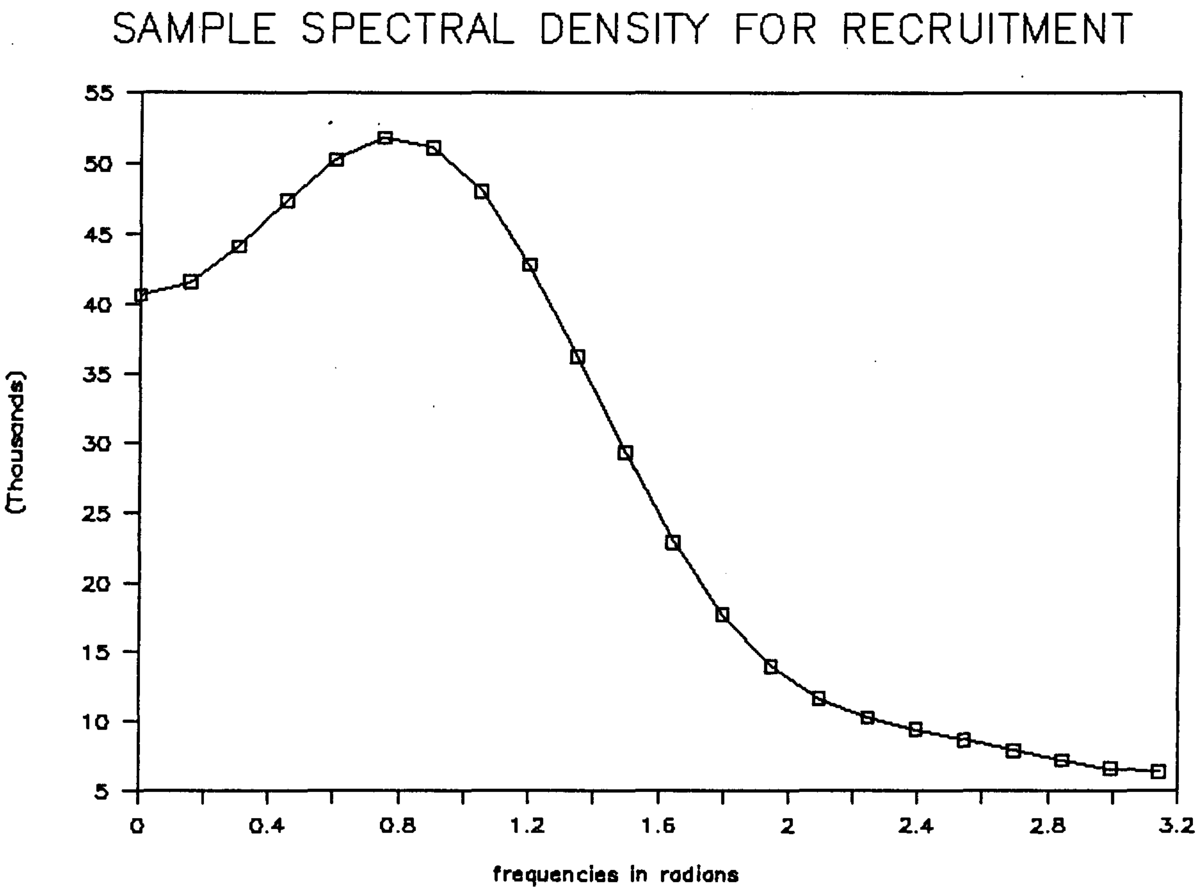

SAMPLE SPECTRAL DENSITY FOR RECRUITMENT
(Thousands)
55
50
45
40
35
30
25
20
15
10
5
0
0.4
0.8
1.2
1.6
2
2.4
2.8
3.2
frequencies in radians

Fig. 4.3

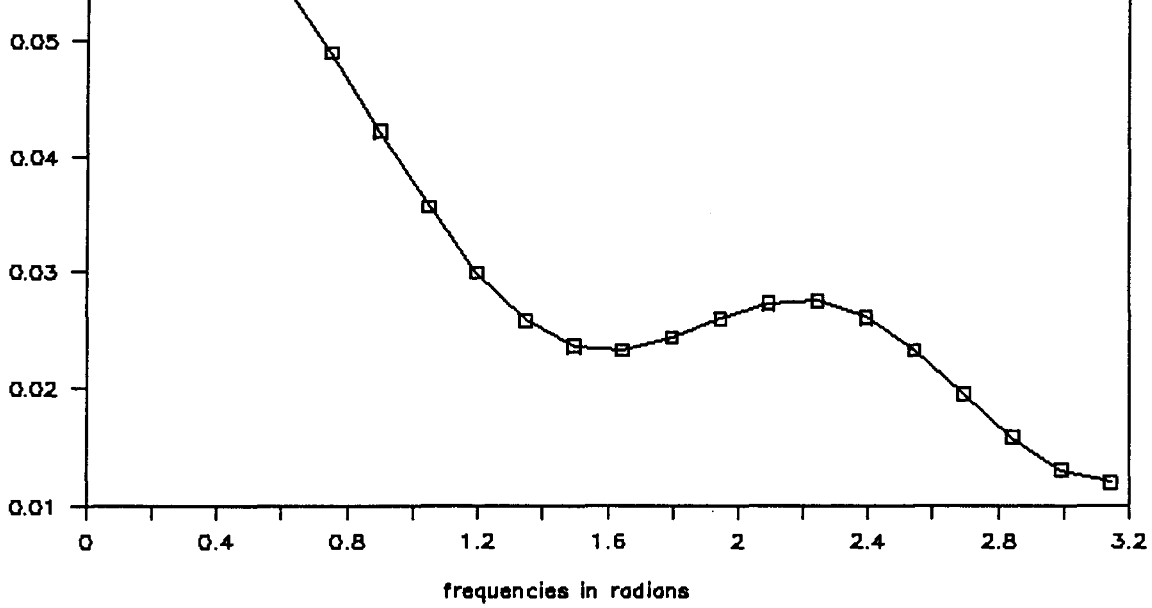

SAMPLE SPECTRAL DENSITY FOR TEMPERATURE
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.4
0.8
1.2
1.6
2
2.4
2.8
3.2
frequencies in radians

Fig. 4.4

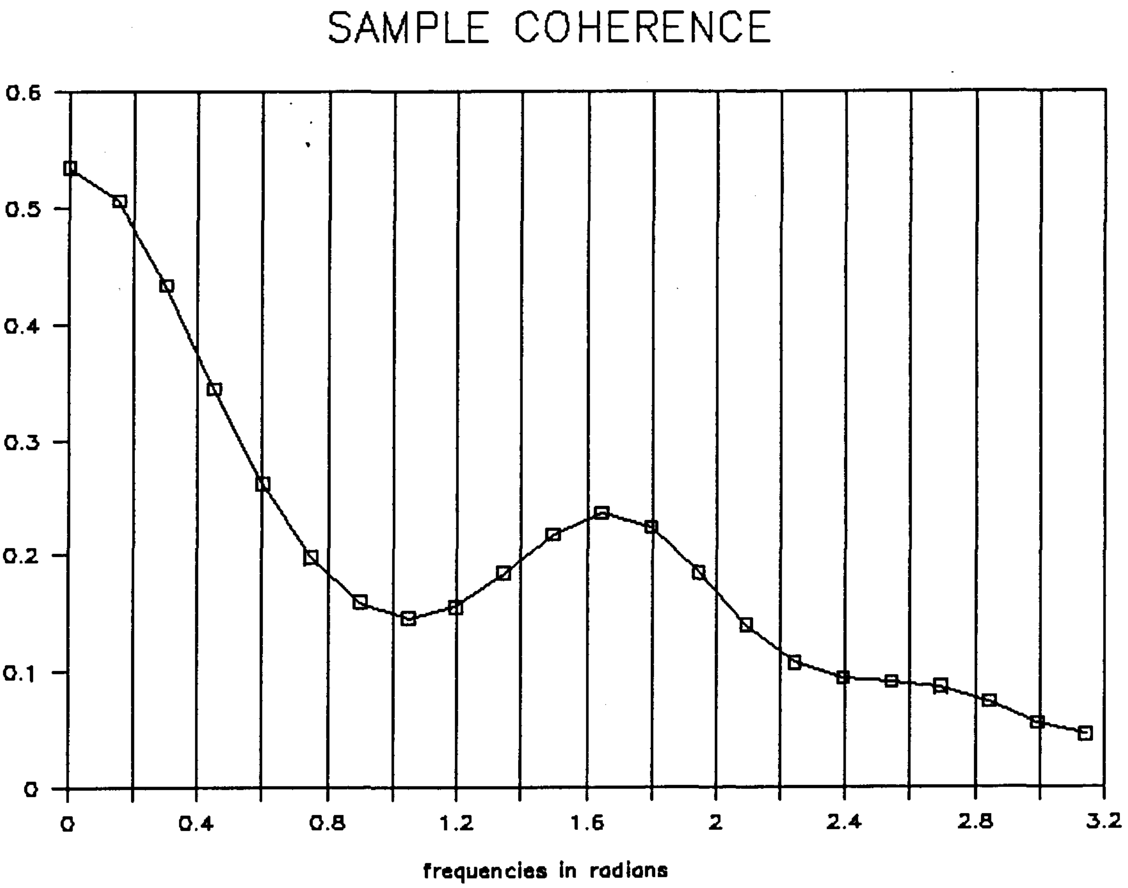

SAMPLE COHERENCE
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.4
0.8
1.2
1.6
2
2.4
2.8
3.2
frequencies in radians

Fig. 4.5

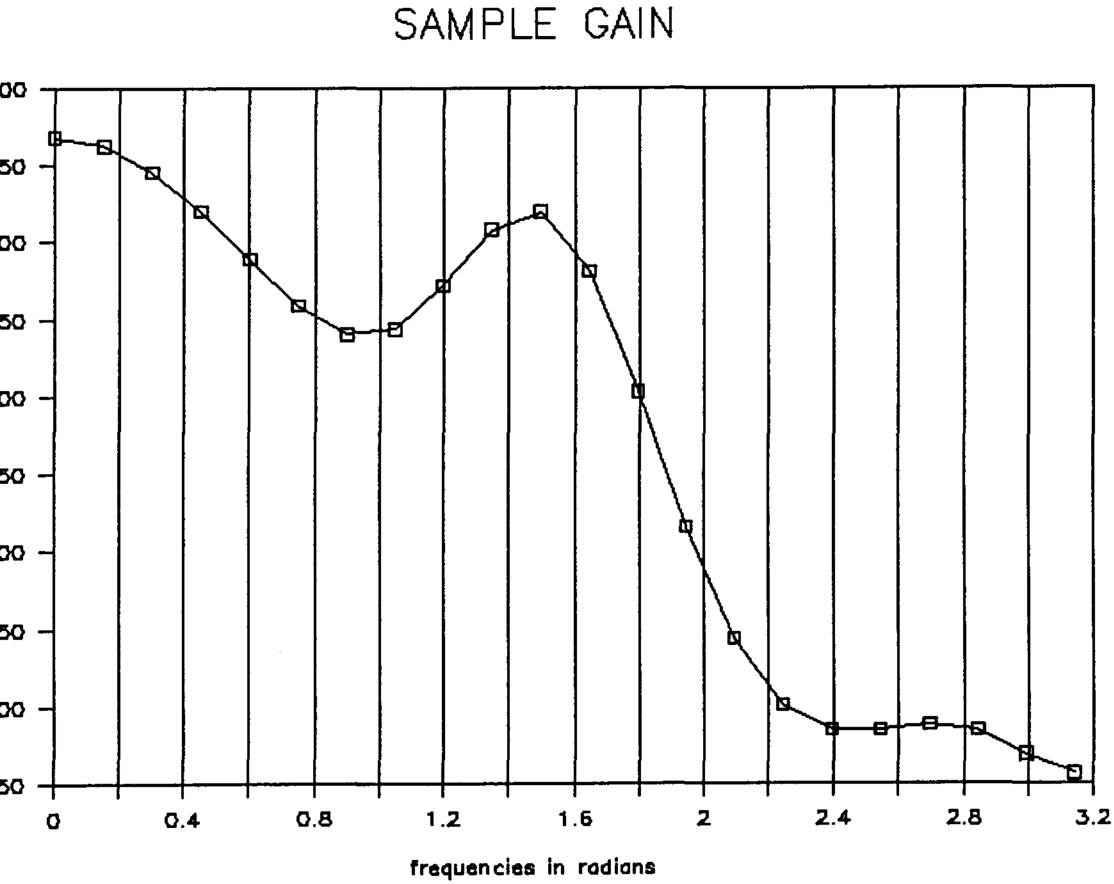

SAMPLE GAIN
600
550
500
450
400
350
300
250
200
150
0
0.4
0.8
1.2
1.6
2
2.4
2.8
3.2
frequencies in radians

Fig. 4.6

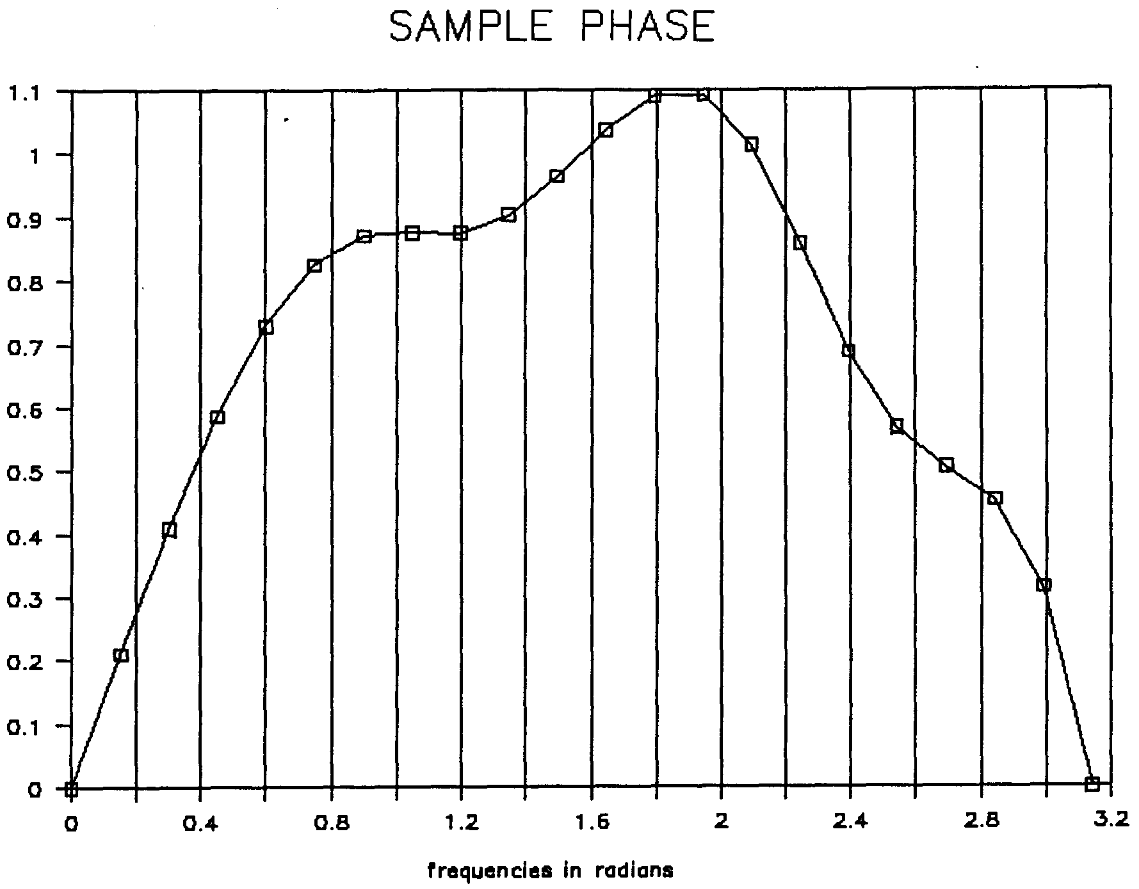

SAMPLE PHASE
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.4
0.8
1.2
1.6
2
2.4
2.8
3.2
frequencies in radians

Fig. 4.7

5 SUMMARY AND CONCLUSIONS

In this chapter the main conclusions from the thesis are briefly summarized. The objective of the presented work was to compare different management strategies for a commercial fishery. As priority was put on the aspect of stabilizing effort, catch and income, and as fisheries management in practice commonly is ruled by socalled reference points, emphasis was put on two quota strategies, viz. constant annual catch quotas (CC) and annual catch quotas based on a constant level of fishing effort (CE). The case study analysed is the case of the Arcto-Norwegian cod fishery in the Barents Sea.

Using a simple aggregated bio-economic model with constant parameters, the theory presented in Chapter 2 suggests that CE is superior to CC with respect to profitability. It is then the purpose to investigate whether this result holds true also when biological and economic parameters are no longer constant. The main exceptions from this result consist of circumstances that render the gross revenue function concave. These are primarily risk-aversion among fishermen, implying a concave utility function in net revenue, and some (but not all) price-quantity relationships. With stock-dependent biological parameters or a disaggregated model, it is hard to predict anything from theory, and numerical simulations have to be resorted to. This is done in the last part of Chapter 2 and in Chapter 3 using cyclical or stochastic recruitment to the stock. These simulations suggest that the difference between the strategies with respect to profitability is surprisingly small.

The cases reported in Chapter 2 with a significant difference between the strategies are:
1) the case of a linear price-quantity relationship where constant catch quotas are superior and
2) the case of fixed costs pertaining to non-malleable capital and irreversible investments.
However, with respect to the risk of extinction implied by either strategy, the constant effort strategy is, not surprisingly, less risky than the constant catch strategy. This result applies in particular to the case with either a strong stock - recruitment relationship, high auto-correlation in recruitment or both. Also factors making the optimal harvesting rate higher imply a higher risk induced by the CC strategy.

In Chapter 3 a more realistic disaggregated model is analysed. Results from this chapter suggest that the differences in profitability between the strategies are even smaller. For example the price - quantity relationship applied does not imply a significant difference between the strategies. The only factors implying a significant difference are a stock dependent individual growth function and a changed selectivity pattern in the fishery. The first aspect is due to the fact that reduced variation in biomass increases the relative profitability of constant effort. The second aspect has to do with the introduction of a selectivity pattern similar to the pattern found in the trawler fleet. Such a selectivity pattern reduces the exploitation of the growth potential in the fish stock. With constant catch the effect of this reduction is more pronounced, and therefore the difference between the strategies considered is larger for the trawler fleet.

Section 3.4 analyses the effects of irreversible investments. The conclusion from this section is that the difference between the strategies is still surprisingly small and that it becomes smaller the higher the depreciation rate. This is also the case when the capital stock is disaggregated into year-classes.

Uncertainty is also revisited in Chapter 3 and the conclusion here is that in addition to increasing the risk implied by constant catch, introducing uncertainty also renders constant catch less profitable relative to constant effort.

In Chapter 4 the objective is to find a possible stock - recruitment relationship, auto-correlation in the recruitment or cyclicality in recruitment related to environmental variables. No trace of a stock recruitment relationship or autocorrelation was found. A slight trace of cyclicality, viz. an irregular eight-year cycle, was found, but the evidence is rather weak due to the limited data set.

Possible extensions

Although rather comprehensive, the work presented here has not exhausted all possibilities. Some suggestions for extended work in this area are therefore given below. Such work may be both theoretical and empirical.

On the theoretical side, perhaps the most obvious extension is to apply more complex dynamical systems. Such an extension would hardly have any impact on the practical results and conclusions drawn here, but it might be interesting to see, for example, if one of the strategies is more prone to result in chaotic dynamic behaviour than the other. It is well known that chaotic behaviour can result from surprisingly simple dynamic systems such as, for example, the discrete logistic growth function. Therefore, although the constant effort strategy has been shown to have a stabilizing effect on the population than the constant catch strategy, using an overlapping generations model, this does not guarantee that this strategy is less prone to result in chaos.

The empirical results presented here are not meant to represent an exact reproduction of the real world. They are all based on quasi-empirical models, and sensitivity analysis has been resorted to to see how valid they are. One suggestion for further work would therefore be to undertake a comprehensive empirical study of economic data from a particular fishery, e.g. the Barents Sea cod fishery. Experience from this work indicates that further elaboration on biological data from this fishery would hardly be successful at present.

Of particular interest in an empirical study would be to estimate a demand function and a production function for the fishery. Some work has, however, already been done in this area. For example Hannesson (1983) estimates the Cobb-Douglas function $y = AE^{\beta}W^{\alpha}$ where E is effort, W is the fish stock and A, β and α are parameters. He concludes that the null hypothesis $\beta = 1$ can not be rejected. There is however reason to believe that α is significantly less than 1, and estimated values range from 0.74 to 0.89. As we remember from Chapter 3, the lower α is, the more similar the economic performances of the strategies will be.

Overall conclusion

An overall conclusion from this thesis is that the possibility of constant annual catch quotas should not be ruled out at the outset for economic reasons if such a strategy is regarded as desirable for whatever other reasons. The detailed model in Chapter three proves that the difference between this strategy and the constant effort strategy in fact is smaller than one would expect from the more general model in Chapter two. If such a strategy is to be implemented, however, it ought to be done with great care because of the possible risk implied. The constant quota should be set at a moderate level and only for a rather short period, in the case of the Arcto-Norwegian Cod, for example, four to five years with the possibility of modification if unexpected events should occur.

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