



**Norges
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**ESSAYS IN
CORPORATE FINANCE**

by

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¹Chapters 1 thru 5 are arranged in order of conception.

I dont have nothing only words to put down on paper. Its so hard. Some times theres mor in the empty paper nor there is when you get the writing down on it. You try to word the big things and they tern ther backs on you. Yet youwl see stanning stoans and ther backs wil talk to you. The living stoan wil all ways have the living wood in it I know that. With the hart of the chyld in it which that hart of the chyld is in that same and very thing what lives inside us and afeart of being beartht.

from the novel *Riddley Walker* by Russell Hoban

INTRODUCTION

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This dissertation contains five independent essays out of which four contribute to the literature on incomplete contracting in financial economics and one that contributes to the literature on initial public offerings (IPOs). The present chapter offers a synthesis of these essays and a review of that part of the literature that directly relates to this dissertation.¹

1 Financial Contracting

1.1 Financial Contracting: An overview

The financial contracting literature can be categorized into: (*i*) the traditional agency cost literature as represented by Jensen and Meckling (1976), Myers (1977), and synthesized by Barnea, Haugen, and Senbet (1978), which deals with conflicts of interest between the various claimants of the firm; (*ii*) the literature that deals with asymmetric information and adverse selection as represented by Ross (1977) and Myers and Majluf (1984); it examines how a firm's choice of financial contract (or financial structure) may convey private information to investors; (*iii*) the literature that considers the role of securities in the allocation of control rights; see Aghion and Bolton (1992), Harris and

¹See Harris and Raviv (1991, 1992), Allen and Winton (1995), and Hart [1995] for extensive reviews on the literature on financial contracting and Ibbotson and Ritter (1995) for a review on the IPO literature; Nærland (1994) provides an review of the IPO literatue in Norwegian.

Raviv (1988), and Zender (1991); and, finally, (*iv*) the literature that assumes incomplete contracting because states are either costly to observe and to verify or because states cannot be verified at all, even at a cost; important papers in this category include Townsend (1979), Gale and Hellwig (1985), and Hart and Moore (1989).

The contracting part of this dissertation contributes to category (*iii*) (chapters 1 and 3) and to category (*iv*) (chapters 2, 3, and 4).

1.2 Incomplete Contracting

A contracting environment is said to be *incomplete* whenever payoffs and/or actions are contingent on states that are either costly to verify or cannot be verified at all, even at a cost. Analogously, a contracting environment is said to be *complete* if states are observable and verifiable at zero cost. A state is said to be *verifiable* if it can be ascertained in a court of law whether or not it has occurred and is said to be *non-verifiable* otherwise.² Contracts written directly on non-verifiable information are not enforceable, even if this information is jointly observable. The questions of interest in this literature relate to the existence as well as the ability of incentive constrained contracts—such as debt and equity—to approximate or possibly replicate contracts written under complete contracting.

Two leading models in the literature on incomplete contracting in finance are Hart-Moore (1989), where cash flows are jointly observable but cannot be verified even at a cost, and the costly state verification (CSV) framework developed by Townsend (1979) and extended by Gale and Hellwig (1985). In the CSV framework, cash flows are costlessly observable to the entrepreneur and observable to outsiders only if verified, which is costly. Both Hart and Moore and Gale and Hellwig examine the ability of the debt contract to induce the firm (or entrepreneur) to make funds available to the investor *ex post*.

1.2.1 Cash Flows Observable but Non-Verifiable

Hart and Moore (1989) introduce a three date model of an entrepreneurial owned project which yields non-verifiable (though jointly observable) cash flows on future dates 1 and

²See Grossman and Hart (1986) for a distinction between observability and verifiability.

2. Since cash flows are non-verifiable, the entrepreneur has both the ability and the incentive to divert cash flows away from the lender. However, in addition to cash flows generated, the firm has assets in place that can be seized (and sold) by the lender on the intermediate date should the borrower fail to pay the intermediate debt payment in full. Since (partial) liquidation of assets yields a reduction in future (date 2) cash flows, liquidation reduces the amount of cash that the borrower will be able to divert in the future, in turn inducing him to pay out as much cash as possible (until the scheduled debt payment is fully satisfied) on the interim date—in other words, the threat of default is sufficient to induce the borrower to make cash available to the creditor on the interim date even though this cash cannot be verified. As a result, as shown by Hart and Moore, the debt contract is renegotiation proof and *strategic defaults* will never occur.³

The noticeable features of the Hart-Moore setup are that the debt contract derived is renegotiation proof (i.e. strategic defaults will not occur) and default costs arise endogeneously (i.e. liquidity defaults may occur). As will be explained below, these features of the Hart-Moore model can be contrasted with those of the debt contract derived by Gale and Hellwig (1985), which may fail to be renegotiation proof and for which default costs are imposed exogenously.

Harris and Raviv (1995) extend Hart and Moore in several directions. For example, whereas Hart and Moore focus on the case in which the creditor makes a take-it-or-leave-it offer to the debtor ('creditor favored debt') on the interim date, Harris and Raviv also examine the case in which the debtor is able to make a take-it-or-leave-it offer to the creditor ('debtor favored debt'). They show that debtor favored debt will in general dominate (i.e. yield less asset sales) creditor favored debt, the reason for which is that creditor favored debt gives too much control in the hands of the creditor and thus too much asset sales on the interim date. They then examine a general contract ('the universal contract') which seeks to provide the optimal balance of interim bargaining power between the creditor and the debtor.

As already explained, Hart and Moore examine the ability of debt to induce the

³A strategic default occurs when the borrower has sufficient cash at hand to avoid default but refuses to make it available to the lender and instead diverts it for his own consumption. A strategic default is different from *liquidity default*, which occurs if the borrower does not have sufficient cash at hand to avoid default. For example, Noe and Wang (1997) refer to strategic and liquidity defaults as "won't pay" and "can't pay" defaults.

borrower (firm) to make funds available to the investor, the enforcement mechanism of which is given as a contingent right on the part of the investor to impose (costly) bankruptcy on the entrepreneur should he fail to pay the promised amount. Using the Hart-Moore setup, Fluck (1995) examines an alternative, and widely used, contract: outside equity. This contract is different from the debt contract both in terms of its control structure and the type of cash flow right that it confers to investors. She shows that outside equity is incentive compatible provided that it is issued with unlimited life, which is a result that is consistent with what one generally observes in practice. Unlike debt, which is equipped with a contingent control right on the part of the creditor to intervene if the scheduled payment is not paid in full, the control right associated with outside equity is non-contingent (or 'tacit') and gives shareholders a right to dismiss the manager whenever dissatisfied with the proposed dividend payout. Fluck then compares debt and equity and shows that outside equity will be used when cash flows are volatile and debt will be used when cash flows are more stable.

1.2.2 Cash Flows Costly to Verify and to Observe

In formulating the *costly state verification* (CSV) framework, Townsend (1979) considers a situation in which a risk averse entrepreneur needs cash on an initial date to implement a project that yields a random date 1 cash flow. Cash flow realizations are observable to the entrepreneur without cost but are observable to the investor only if verified, which is costly. Townsend derives the optimal incentive compatible contract in this environment and shows that it is characterized by a fixed payment in the non-verification region and a state-contingent payment in the verification region. Thus, Townsend derives from first principles a debt-like contract that induces verification (or bankruptcy) if the fixed payment contracted upon ex ante is not paid in full.⁴

Gale and Hellwig (1985) extend the model by Townsend (1979) to the case in which the entrepreneur is risk neutral.⁵ They derive the optimal contract for this case and

⁴To see why the payment to the investor must be fixed in the non-verification region, assume to the contrary that it is state contingent and based on the entrepreneur's report of the true state. Since now the entrepreneur's report will not be subject to a costly verification in the non-verification region, the entrepreneur will have an incentive to report that state which will imply the lowest possible payment to the investor, in turn ensuring that the effective payment in the non-verification region is fixed and not state contingent.

⁵See Winton (1995) for a generalization and extension of Townsend (1979) and Gale and Hellwig

show that it takes the form of the standard debt contract “with bankruptcy.” The optimal contract, as in Townsend, calls for a fixed payment in the non-verified state but, unlike Townsend, calls for a zero payment to the entrepreneur in the verified state. A zero payment to the (risk neutral) entrepreneur in the verified state is optimal in this setting of risk neutrality because it minimizes the contractual debt payment, which in turn minimizes the trigger point for a costly verification and minimizes therefore expected verification (or default) costs. A zero payment to the entrepreneur in the event of verification also correspond to a scenario in which bankruptcy allows creditors to seize the firm’s assets in the event that the firm does not pay its debt in full; that is, the optimal contract correspond to the standard debt contract “with bankruptcy.”

Hart (1995), in his recent book on financial contracting, summarizes the criticisms that have been directed towards the CSV framework. I list and adress these next.

(i) The debt contract derived by Gale and Hellwig is not renegotiation proof.

One problem with the CSV setup is that strategic defaults are possible and that the standard debt contract as derived by Gale and Hellwig therefore fails to be incentive compatible. Although this is perhaps a serious short-coming of the CSV framework, it need not mean its demise. For example, lenders (such as banks or other institutional lenders) usually have reputation at stake that prevent them from making concessions to borrowers. In addition, the many lender involved in the case of publicly traded debt may concessions very difficult or even impossible (see Gertner and Sharfstein [1991] for an analysis of free rider problems in debt renegotiations).

(ii) The CSV framework “does not seem to be able to explain the existence of dividends and (outside) equity.”

This (potential) shortcoming of the CSV framework is adressed directly in Chapter 2, where the CSV setup is used to derive and examine an outside equity contract with dividend payments tied to a noisy information signal regarding the true cash flow (see Section 2.2 for further elaboration).

(iii) The CSV setup is unlikely to yield debt like contracts if extended to multiple periods.

Chapter 4 adresses this critique directly by extending the CSV framework to multiple

(1985).

periods. It shows that the standard debt contract in the multiperiod case will induce the borrower to provide truthful reports of cash flows generated until the firm is either debt free or its debt capacity has been exhausted. In the latter case the creditor steps in to verify and issues the maximum amount of debt permitted by the the firm's debt capacity (see Section 2.1 for further elaboration).

(iv) As shown by Mookherje and Png (1989), random verification yields lower expected verification costs than does the debt contract derived by Gale and Hellwig.

In other words, the standard debt contract as derived by Gale and Hellwig as the optimal contract under CSV is not optimal once stochastic verification schemes are allowed. Taken at face value, this is a serious critique of the CSV setup. However, if one is to study debt contracts and develop insights as to how actual firms are financed, one may argue (as does Winton [1995]) that stochastic verification is difficult to implement and rarely observed in practice and therefore focus on non-stochastic verification schemes. In any case, as already mentioned, Chapter 4 shows that if extended to a multiperiod framework, the standard debt contract will induce truthful cash flow reports until either the firm's debt capacity has been exhausted or until the firm is debt free. In other words, when considering the multiperiod case, it is not at all clear that random verification gives less verification compared to the standard debt contract.

(v) The costs of verifying cash flows (or 'bankruptcy costs') are specified exogenously (rather than occurring endogenously as in the Hart-Moore setup).

Following Cantillo (1997) one can argue that the exogenously specified verification costs of the CSV setup arise from a time consuming process of renegotiating existing contracts and that it is "just as natural to assume that renegotiation is time consuming as it is to require that contracts be renegotiation-proof." The empirical evidence indicates that such debt renegotiations can be costly; for example, Gilson et al. (1990) document that such renegotiations can be quite prolonged (and therefore costly), taking on average 15 to 28 months to complete. Furthermore, evidence presented by Frank and Torous (1994) and Tashjian et al. (1996) show that the recovery rates in various types of financial distress may vary between 51 and 80 percent.

2 Four Essays on Financial Contracting

2.1 Multiperiod Debt Contracts under Costly State Verification

Chapter 4 extends the one-period model of Gale and Hellwig to a multiperiod world and shows that the borrower will be able to use the standard debt contract to construct debt structures that induce truthful cash flow reports on each date as long as the firm's debt capacity permits further debt issuance to cover cash short-falls. Furthermore, the model shows that verification (or intervention) occurs only when the firm's debt capacity has been exhausted and the borrower declares bankruptcy—verification thus occurs only in bankruptcy.

Relatedly, Chang (1990) extends the one period CSV framework of Gale and Hellwig by deriving the optimal contract (from 'first principles') for the two period case. He shows that the optimal contract exhibits a number features associated with debt contracts as they appear in practice, such as coupon payments, call features, and sinking fund requirements. As noted by Gjesdal (1994), however, Chang's results are very sensitive to the structure he assumes for verification costs. In addition, while Chang rules out both additional debt issuance and dividend payments on the interim date, such restrictions are not imposed in the model of Chapter 4.

More closely related to my paper is Webb (1992), who extends the CSV framework to the two periods case and shows that the borrower will be able to make a credible promise to make a state contingent payment to the initial lender on the interim date. As in my model, this contract induces the borrower to make a truthful report of the interim cash flows. However, in terms of extending Gal and Hellwig, the approach taken in Chapter 4 has two advantages relative to Webb's. One is that it is somewhat easier to reconcile with actual debt markets: while Webb shows that truthful reports obtain by making the interim payment to the lender directly contingent on the report made by the borrower—the enforcement mechanism of which is that any cash diverted cannot be used to reduce the amount of debt issued on the interim date (thus preventing maximum equity participation for the final project)—the approach of Chapter 4 uses well known features observed in actual debt markets, such as debt maturity, renegotiation, and callable debt. Two, while Webb's model does not easily extend to the multiperiod case,

the model developed in Chapter 4 specifically examines the general multiperiod case.

2.2 Outside Equity and the Role of Accounting Information

Chapter 2 examines the role of outside equity in the CSV setup. The model developed introduces outside equity into the one period CSV framework by allowing the investor to observe noisy information about realized cash flows. To ensure that enforceable contracts cannot be written directly on the nature of the information available to the investor (thus ruling out trivial solutions) the information observed by the investor is assumed to be non-verifiable. In the limit, however, it becomes perfectly correlated with cash flows (almost surely), in which case cash flows are (in effect) jointly observable while still being costly to verify.⁶ The paper shows that there exists an outside equity contract with dividend payments partly contingent on the non-verifiable information observed by the investor; indeed, as the information observed by the investor becomes perfectly correlated with true cash flows, dividend payments become contingent on it with probability one and first best obtains.

Chapter 2 provides a role for outside equity within a version of the traditional CSV framework. In addition, on a more general level, it provides a theory of outside equity and dividends based on incomplete contracting. To motivate this result, note that dividend payments as observed in practice are generally contingent, albeit loosely, on the firms' earnings reports. Yet from an incomplete contracting perspective the underlying mechanism that allows such payments are not well understood; indeed, the financial contracting literature has essentially ignored the role of accounting information despite its role in determining the amount of cash to distribute to external owners. The model developed in Chapter 2 derives outside equity with dividend payments contingent on jointly observable information regarding realized cash flow rather than on cash flow realizations per se and thus contributes to a seemingly ignored area of financial contracting.

2.3 Outside Equity and Debt under Costly State Verification

Chapter 3 examines outside equity and debt in a model in which cash flows are costly

⁶In comparison, as we recall, in the Hart and Moore (1989) model cash flows are jointly observable but non-verifiable even at a cost.

to verify but in which investors and the entrepreneur jointly observe interim information regarding future cash flows. The paper focuses on the ability of debt and equity to allow interim payments contingent on non-verifiable interim information. The idea behind the model is that to the extent that contracts are issued with different types of control and cash flow rights, their abilities to allow state contingent (interim) payments should differ as well, which turns out to be the case.

Indeed, consistent with what one observes, the model shows that the structure associated with the outside equity contract allows interim payments that are more responsive to interim information compared the structure associated with the debt contract. In addition, the model shows that the debt contract is more robust than the outside equity contract and is thus able to fund projects that cannot be funded with outside equity. Taken together these results imply that equity will be the least costly funding alternative and will thus be issued whenever both debt and equity are feasible, while debt will be used only if the project cannot feasibly be funded with equity.

Chapter 3 is related to Chiesa (1992), who develops an agency model in which an entrepreneur's effort decision is made on the interim date after the (non-verifiable) state of nature has been revealed. She shows that while the standard debt contract is not optimal in this setting, a debt contract with warrants for the lender and cash/equity settlement options for the borrower allows the entrepreneur to commit to first best effort choice and is thus optimal. The efficiency gain generated by the alternative contract in relation to the standard debt contract occurs because the alternative contract allows a more efficient allocation of payments across future states. Relatedly, although there are no effort decisions to be made, the outside equity contract in the model developed in Chapter 3 yields lower expected default costs than does the standard debt contract. As in Chiesa (1992), this occurs because the 'alternative' contract allocates higher payments to higher states more efficiently than does the standard debt contract.

Chapter 3 is also related to Chang (1993), who develops a model in which the firm generates cash on the interim date. Part of this cash will be optimally reinvested in the firm's operations by the firm's manager and the rest will be paid out to the firm's investors. A contracting problem arises in this setting because the optimal payout (or the optimal amount of funds to be invested), although jointly observable, is non-contractible

and can therefore not be included in a written contract. Chang shows that outside equity can be used in this setting along with a compensation contract for the manager in which the manager's pay is contingent on the amount he pays out in dividends on the interim date.

There exists a large literature on the choice between debt and equity, which (generally) assumes *ex ante* asymmetric information (category (ii)).⁷ In contrast to this literature, Chapter 3 follows the lead of Gale and Hellwig (1985), Hart and Moore (1989), and Bolton and Sharfstein (1992) and assumes *ex post* asymmetric and/or non-contractible information. One important strand of the *ex ante* view of financial contracting deals with the information content of a firm's choice of financial contract. Ross (1977) argues that a larger debt proportion serves as a favorable signal to the market about firm quality, which is private information to the firm's insider. Ross develops his argument in a model in which managerial risk aversion implies managerial preference for equity over debt under symmetric information. Managerial equity preference provides the necessary signalling costs to allow 'good' firms to use debt to truthfully reveal greater than average confidence about future prospects. The similarity between Ross and the model developed in Chapter 3 is that while in the former equity preference is imposed via the assumption of risk aversion, in Chapter 3 equity preference arises endogenously from the underlying structure of the available contracts. This similarity between the two strands of the literature suggests that the model developed in Chapter 3 may offer a way of integrating the *ex ante* view of financial contracting to which Ross (1977) belongs and the *ex post* view to which my model belongs.

2.4 A Control Theory of Financial Structure

While the models contained in chapters 2-4 deal with the ability of financial contracts to induce the firm's insider to make funds available to the firm's investors *ex post*, the model developed in **Chapter 1** follows in the tradition of Aghion and Bolton (1992), who examine the optimal allocation of control and cash flow rights between the various claimants of the firm when there may be disagreement between the various claimants as to the appropriate action to choose as a response to jointly observable but non-verifiable

⁷See e.g. the extensive review provided by Harris and Raviv (1991).

interim information.

The story in Chapter 1 is as follows. A founder is to sell (part of) his firm to investors on the initial date. It is assumed that the founder is essential to the firm in its development stages, which implies that he will be retained as the its manager until at least the interim date, but not essential and even harmful to the firm's profitability in later stages. On the interim date the firm's security holders and the founder both observe non-verifiable information. This information reveals the type of operating policy that is consistent with maximizing firm value. In particular, it is assumed that this information will call either for the firm to be liquidated, for an expansion of the firm's existing operations, or stay with the operating policy determined on the initial date. In the first case, because liquidation implies a termination of operations and, in the second case, because an expansion requires management skills that the initial founder do not have, the founder will have to step down if either of these two policies are implemented. Thus, only in the case in which the firm's initial operating policy is kept, will the founder be able to stay with the firm without having a negative impact on the firm's profitability.

Because stepping down implies loss of future control rents, the founder will not do so voluntarily as this implies the loss of future control rents.⁸ It will, however, be optimal for the founder to agree ex ante to give up control in certain interim states; however, because interim states (or information) is nonverifiable and therefore non-contractible, the founder must design the firm's initial financial structure such that the control structure associated with this financial structure transfer control to investors in states for which this is optimal ex ante. The contribution of the paper is that the type of financial structure needed to implement the optimal policy closely resembles the type of financial structures observed in practice: short-term debt, long-term junior debt, and outside equity with voting rights.

The paper extends Chang (1992), who considers the ability of short-term debt to transfer control to investors from a restructuring averse manger who will resist to restructure the firm ex post but will agree ex ante to allow the firm to be restructured

⁸The assumption of private control rents that cannot be assigned to third parties is a common modeling device (for a formal exposition, see Aghion and Bolton [1992]). Examples of such control rents include managerial perquisites, returns to firm-specific investment in human capital, reputation effects from succesfully running the firm etc.

in certain states. Unlike the paper appearing in Chapter 1, however, Chang does not consider debt maturity and priority structure and outside equity and limits his analysis to the ability of short-term debt to transfer control to investors.

The results derived in Chapter 1 on the maturity and priority structure of debt are similar to those derived by Diamond (1992), in which there is *ax ante* asymmetric information and in which the benefit of short-term debt is to allow repricing on the interim date while the cost of short-term debt arises from the possibility that the firm is erroneously liquidated. However, apart from the fact that structure of the model by Diamond and the model presented in Chapter 1 are very different, Diamond limits his analysis to debt arguing that equity control substitutes debt holder control. In contrast, Chapter 1 shows that the control mechanism associated with outside equity complements (rather than substitutes) the control transfer mechanism associated with debt.

The model is also related to Zwiebel (1996), who considers debt as a bonding device for managers who values control; issuing an appropriate amount of debt allows the manager to commit to a policy of undertaking only profitable projects in the future. Although debt introduces a risk of default and thus represents to the manager a strictly positive probability of having to give up control, refraining from doing so triggers a (costly) takeover and thus loss of managerial control with probability one. In other words, outside equity control along with the bankruptcy mechanism attached to debt financing provides an optimal balance of control. In addition, as in Chapter 1, outside equity control and debt holder control complement rather than substitute each other in implementing the optimal policy.

3 Initial Public Offerings

The underpricing of IPOs of common stocks is a well known empirical fact. For example, Nærland (1994) finds excess returns of around 12 % over the first two trading days in a sample of IPOs of common stocks at the Oslo Stock Exchange. From a theoretical perspective such large degree of underpricing is puzzling. Why do issuers (apparently) leave such large sums of money on the table for investors? Adding to the underpricing puzzle are results of overpricing in the IPO markets for Real Investment Trusts (REITs)

(Wang et al. [1992]) and investment grade bonds (Datta et al. [1997]).⁹ Existing IPO theories are able to explain underpricing but are unable to account for overpricing.¹⁰ In contrast, the model developed in **Chapter 5** is able to account for overpricing as well as underpricing.

Rock (1986) explains the underpricing phenomenon documented for IPOs of common stocks by winner's curse. In his model there are two types of firms—'good' and 'bad'—and there are informed investors who are able to distinguish between the two types of firms but their number is insufficient to ensure that the IPO will go through. As a result, the issuer must price the issue in a way that entices uninformed investors to submit bids along with informed investors. In the presence of informed investors, uninformed investors will be allocated a disproportionate large share of issues that are overpriced and thus suffer winner's curse. As a result, to induce uninformed investors to submit bids, the issuer must underprice the issue until the expected return to uninformed investors is zero.

Chemmanur (1993) develops a three date model in which a given fraction of the firm is sold off to investors at the initial date and then the rest at the interim date—the post-issue date.¹¹ The issuer sets the IPO price so as to maximize total proceeds over the two dates. A lower initial price will attract a greater number of investors to produce costly information about the firm. This is beneficial in that it increases the informational efficiency of the firm's post-issue market value but is also costly because information costs must be absorbed by the issuer through a lower IPO price. Underpricing occurs in this case (if it does) so as to compensate investors for their information costs.

Chapter 5 elaborates on the dynamic information production ideas of Sherman (1992) and Chemmanur (1993) by developing a general IPO model in which informed as well as

⁹Other IPO puzzles (or 'anomalies') include hot issue markets (Ibbotson and Jaffe [1975] and Ritter [1984]) and long-run underperformance (Ritter [1991]), though the IPO literature seems more inclined to regard overpricing as an 'ugly fact.'

¹⁰Existing IPO theories include winner's curse (Rock [1986], Beatty and Ritter [1986], and Carter and Manaster [1990]), signalling (Allen and Faulhaber [1989], Grinblatt and Wang [1989], and Welch [1989]), information cascades (Welch [1992]), incomplete markets (Mauer and Senbet [1992]), litigation avoidance (Tinic [1988]; though Hughes and Thakor [1992] show that the litigation avoidance hypothesis of Tinic is not time consistent), and costly information production. Commenting on their findings of positive excess returns of IPOs of REITs, Wang et al. call into question "the completeness of [existing] IPO pricing models."

¹¹A similar argument is found in Sherman (1992)

uninformed investors are allowed to submit bids. In this model, underpricing is shown (as in Rock) to occur either as a response to winner's curse (in the case when there are both informed and uninformed investors participating IPO) or (as in Sherman [1992] and Cemmanur [1993]) so as to compensate informed investors for their information costs. In addition, and more importantly, the model is able to account for IPOs overpricing (on average).

More specifically, in the formal model the issuer chooses the initial price and the number of shares to sell at the initial stage in a way that induces the optimal amount of informed and uninformed bidding. The model shows that if the issue is priced to induce bids from both informed and uninformed investors, then it will be underpriced on average. Just as in Rock (1986)—but in a very different and much more general setting—underpricing occurs because uninformed investors must compete against informed investors for shares and will be allocated a larger than average fraction IPOs that are overpriced and a smaller than average fraction of issues that are underpriced: uninformed investors suffer winner's curse will demand underpricing on average in order to be willing to submit bids. The model shows further that if information costs low and investors observe more precise information than the issuer, then the issuer will optimally market the issue exclusively to informed investors. When this occurs, then the IPO price may (but need not) set sufficiently low to allow the IPO to be overpriced on average (or in expectation).

In conclusion, while existing IPO models are designed to explain underpricing and are therefore unable to account for the fact that some types of IPOs are overpriced on average, the model developed in Chapter 5 is able to account for IPO underpricing as well as overpricing. Importantly, IPO underpricing and overpricing are derived from general principles in a unified framework and thus escapes the “special purpose” critique that has sometimes been directed towards existing IPO models.

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CHAPTER 1

A Control Theory of Financial Structure: Outside Equity Control and the Priority and Maturity Structure of Debt*

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Abstract

Firms' financial structures typically consist of debt claims of different priority and maturity as well as of outside equity with voting rights. The present paper develops a simple control theory of financial structure in which these features arise endogeneously to allocate control rights and cash flow rights optimally among the various claimants of the firm. Short-term debt is the senior claim and is issued to ensure that the firm is liquidated in certain states. While outside equity with voting rights enables investors to seize control in states for which this is optimal ex ante, long-term debt serves as a defensive by which the founder optimally regulates the extent of ex-post shareholder involvement.

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1 Introduction

Firms' financial structures typically consist of debt claims of different priority and maturity as well as outside equity with voting rights. In this paper I present a simple control theory of financial structure in which these features arise endogeneously to optimally allocate control and cash flows rights among the various claimants of the firm. In particular, the paper derives a model in which the combination of contingent debt holder control, non-contingent shareholder control, and the defensive role of long-term debt arise as a way of optimally balancing firm value and managerial control.

In the formal model, an entrepreneur-manager (the 'founder') is to sell (part of) his firm to investors while at the same retaining control as retaining control may put him into a position to collect control rents in the future. As is common, these control rents are assumed to be non-assignable and can therefore not be sold to investors along with future cash flows on the initial date. On the interim date, the founder as well as the firm's security holders jointly observe non-verifiable information about future cash flows. This information will determine whether it is optimal on the interim date to liquidate the firm, expand operations, or continue the operating policy already in place. Because liquidation implies termination of the firm's operations and expansion requires the founder to step down, only the latter will allow the founder to remain with the firm and thus collect control rents. On the interim date therefore, to the extent that the founder is in control, in order to protect his control rents, the founder will stick with the firm's initial operating policy even though this policy does necessarily maximize firm value. However, at the ex ante, while selling the firm to investors, the founder will balance increased control rents against lower firm value and will in general desire a policy that allows control to be transferred to the firm's security holders—who are pure value maximizers—in certain states. It is shown that it will be possible to credibly commit to the optimal policy via an initial financial structure that consists of short-term debt, long-term (junior) debt, and voting (outside) equity.

Liquidating the firm will be optimal on the interim date if expected future earnings are low. Because low future earnings prospects (naturally) adversely affects the firm's ability to raise funds to refinance current obligations, the bankruptcy mechanism associated with short-term debt becomes an efficient mechanism by which control is transferred

to the firm's security holders when the firm's future earnings prospects are sufficiently low to warrant termination of operations.¹ It is shown that a necessary condition for short-term debt to provide this role is that the firm is allowed to refund (senior) short-term debt on the interim date by issuing another senior short-term debt claim, though there must be a covenant in place protecting long-term lenders from too much dilution of their claim.

While the control rights associated with debt are generally contingent on the firm not being able to meet its contractual debt payment in full, outside equity provides investors with the right to interfere regardless of the firm's future earnings prospects.² Thus, if the founder designs a financial structure that consists exclusively of short-term debt and outside equity (or outside equity alone), the firm's security holders will be able to liquidate the firm or oust the founder-manager whenever this is consistent with value maximization. In practice, however, control contests can be quite costly, both because of the direct costs involved and because other security holders are often able to free ride on improvements implemented by perhaps a small group of shareholders. As a result, shareholders will generally allow deviations from value maximization so long as such deviations are not too large. In the current paper, although the direct costs of interfering are insufficient to prevent shareholders from interfering too frequently relative to ex ante optimality, the founder will nonetheless be able to implement the optimal policy by issuing long-term debt and thus create a debt overhang.

As shown by Myers (1977), the presence of a debt overhang may induce shareholders to pass up valuable investment opportunities if too much of the resulting gain in firm value goes to the firm's bondholders. In the present setting, the debt overhang created by long-term debt reduces the incentives of shareholders to exercise their control rights, which enables the founder to use long-term debt to optimally adjust the extent of shareholder involvement.³ More long-term debt provides a larger debt overhang and thus

¹For other models in which debt transfers control in the event of poor operating performance see e.g. Harris and Raviv (1990), Chang (1992), Aghion and Bolton (1992), Diamond (1993), Dewatripont and Tirole (1994), and Hart and Moore (1989, 1994).

²Though Berkovitch and Israel (1996) examine the non-contingent nature of debt holder control, the characterizing feature of debt is its contingent control feature.

³The defensive role of debt financing suggested here is, of course, not new. In Israel (1991) a higher level of debt has the effect of reducing the probability that a takeover will happen while at the same time increasing the value collected by the target's shareholders in the event of a successful bid. In my

more protection against shareholder interference.

Many of the results on debt maturity and priority structure generated in the present paper are similar to results derived by Diamond (1993). However, the structure of the present model is different than his. For example, while in his model the founder (or borrower) is better informed than prospective lenders about the firm's repayment ability, in my model all information is symmetric. Furthermore, Diamond ignores the role of outside equity and focuses instead "on the effects of debt on transfer of control, thus avoiding takeovers as another way of transferring control." In contrast, the role of outside equity is explicitly examined in the present model and it is shown that the role of outside equity in transferring control to investors complements (rather than substitutes) the role of debt. Finally, in his model the amount of long-term debt arises as a residual from the investors' participation constraint after the optimal level of short-term debt has been determined. Long-term debt, therefore, has no specific role in the firm's financial structure other than not being short-term. In my model, the presence of long-term debt creates a debt overhang without which the optimal contract cannot be implemented.

It is well known that in frictionless markets the choice of financial structure is irrelevant (Modigliani and Miller [1958]). In the present paper, the frictions that give rise to financial structure relevance include non-verifiable information and non-assignable benefits of control. The contribution of the paper is to show that these frictions give rise to a financial structure characterized by non-trivial priority and maturity structures of debt as well as outside equity with voting rights.⁴ In other words, the assumed frictions give rise to a financial structure that is sufficiently comprehensive to resemble the types of financial structures observed in practice.

The paper is organized as follows. Section 2 presents the basic model. Section 3 derives the optimal contract between the entrepreneur and investors under complete

model the presence of *long-term* debt, as opposed to debt in general, is purely defensive: it reduces the probability that shareholders will take actions to increase firm value.

⁴Other control theories of financial structure include (but are not limited to) Aghion and Bolton (1992), Chang (1992), Diamond (1993), Hart and Moore (1994), Dewatripont and Tirole (1994), Berglöf (1994), and Zwiebel (1996); however, none of these papers derive the type of comprehensive financial structure derived in the present paper. The present model is closely related to Chang (1992), who shows that short-term debt can be used to implement the optimal contract between a firm's investors and a restructuring averse manager. Chang, however, ignores the role of outside equity and consequently the potential roles of debt maturity and priority.

contracting. Section 4 examines the design of the firm's financial structure to implement the optimal contract under incomplete contracting. Finally, Section 6 concludes the paper.

2 The Model

The model has three dates and contains a firm, whose founder holds an initial ownership stake of $\alpha \in (0, 1)$, which is assumed large enough to leave him with a controlling stake on date 0. We may think of $\alpha < 1$ as having resulted from the founder having sold part of the firm to an outside investor in order to fund the firm's initial development stages prior to date 0.

It is assumed that the founder needs funds on date 0 (for private consumption) and that he raises the required funds by selling part (or the entire) of his ownership stake in the firm. Although in the model the founder's need for funds arises exogenously, the argument will go through (at some cost in complexity) if the securities issued on date 0 are issued in order to implement an investment project rather than simply raising cash to the founder. To further streamline the argument, I assume that the founder sells his entire stake in the firm on date 0, while retaining his position as its manager.⁵ It is assumed that the financial market is competitive, that investors and the founder are both risk neutral, and that the riskless rate of return is zero.

It is further assumed that the founder derives utility both from the amount of cash that he receives initially from selling the firm to investors as well as from future control rents. Let $Q > 0$ denote the value of the founder's control rents. We may think of Q as being the pecuniary equivalent of (possibly) non-pecuniary rents. It is further assumed (as is common) that the assumed control rents cannot be assigned to the firm's security holders and that they can be collected by the founder if and only if he retains control until date 2. In other words, if the founder walks on the interim date, he receives no rents from control.

On date 1, everybody observes an information signal \tilde{x} , whose realization x is non-verifiable and therefore cannot be the basis of an enforceable contract (Grossman and

⁵Note that it is not necessary that the founder holds a zero stake on date 1 for the results to come through; it is sufficient that his date 1 ownership is less than his initial stake α .

Hart [1986]). \tilde{x} is a random variable with cumulative distribution function $F(x)$ and density $f(x)$; $f(x) > 0$ for all $x \in X = [\underline{x}, \bar{x}]$ and $f(x) = 0$ for $x \notin X$. As will be explained below, x provides information regarding the date 2 cash flow. In addition, it will be useful in determining the firm's date 1 value maximizing operating policy.

On date 1, after \tilde{x} has been observed, the set of actions (or operating policies) available to the firm are *liquidation* (L), *continuation* (C), and *expansion* (E), where actions L and E represent changes in the firm's operating policy and C does not. While actions L and E require that the founder is replaced on the interim date, action C allows him to stay and thus collect Q .

On date 0, the founder designs the firm's financial structure in such a way as to maximize the combined value of expected future control rents and the cash received from selling his stake α . On date 1, the founder holds a zero stake in the firm and will therefore, if given the choice, always choose C as both L and E imply that he will be unable to collect the control rent Q .

The date 2 cash flow under *continuation* is given by the random variable $x + \tilde{\omega}$, where $\tilde{\omega}$ has zero mean and range $[\underline{\omega}, \bar{\omega}]$. Its distribution and density functions are denoted $G(\omega)$ and $g(\omega)$, where $g(\omega) > 0$ for all $\omega \in \Omega$ and $g(\omega) = 0$ for $\omega \notin \Omega$. It is assumed that $\text{cov}(\tilde{\omega}, \tilde{x}) = 0$ so that x represents the expected date 2 cash flow and therefore the date 1 value of the firm under continuation.

If the firm is *liquidated*, its assets are sold at the non-random liquidation value $l > 0$, which is distributed to the firm's claimants according to the priority of their claims. Comparing liquidation and continuation, we observe that liquidation will maximize the value of the firm whenever $x \in [\underline{x}, l)$. However, by the presence of $Q > 0$, the founder will at the ex ante stage prefer a liquidation region $x \in X_L \equiv [\underline{x}, x_L)$ with $x_L < l$. In other words, because he is unable to collect control rents if the firm is liquidated, the founder will put less emphasis on firm value and thus choose a liquidation policy that deviates from one that implies strict value maximization.

Expansion is assumed to create a date 2 cash flow of $J(x) + \tilde{\omega}$, where $J'(x) > 1$. It is assumed that there exists an $x^* \geq \underline{x}$ such that $J(x) > x$ for all $x > x^*$ and $J(x) \leq x$ otherwise. In other words, expansion (or the alternative operating policy under a different manager) adds value to the firm if the firm shows sufficient promise on the

interim date. Note, however, that our specification of $J(x)$ allows for the possibility that $J(x) \geq x$ for all $x \in X$. In other words, our specification is sufficiently general to allow for the possibility that there appears on date 1 with probability one a management team that is able to improve the operating performance of the firm relative to that of the initial founder. As already indicated, it is assumed that the founder is unable to undertake the expansion and that he must be replaced for it to be implemented. The idea here is that although the founder may be essential to the firm in its initial development stages, his management abilities may run short once (or if) the firm becomes successful and its operations too complex for the initial founder to be able to pursue value maximization.⁶

Comparing expansion and continuation, we observe that value maximization will call for expansion whenever $x \in (x^*, \bar{x}]$. However, since the founder will be unable to collect control rents if he is replaced on the interim date, it will be optimal to allow expansion only for $x \in X_E \equiv (x_E, \bar{x}]$, where $x_E > x^*$ in which case the probability of expansion under the optimal policy than under value maximization.

To the extent that X_E and X_L are non-empty so that it will be (ex ante) optimal for the founder to give up control on the interim date in certain states, he must make a credible promise to do so. If x can be verified without cost there is no enforcement problem because the parties can simply write an enforceable contract contingent x . If x cannot be verified (except possibly at a high cost), however, then such contracts are not possible. In the next section I derive the optimal policy assuming that x is verifiable and then proceed in Section 4 to show that the optimal policy can be implemented via an initial financial structure that consists of short-term senior debt, long-term junior debt with covenants restricting the amount of new senior date 1 debt that the firm can issue, and equity with voting rights.

3 The Optimal Policy

Assume that x is verifiable. This allows the founder to write an enforceable contract directly on x . Recall from the previous section that pairwise comparisons between liquidation and continuation and expansion and continuation yielded subsets $X_L = [x, x_L)$

⁶A well known example of this is *Apple's* co-founder Steven Jobs, who was ousted in part because of a perception of that the size of the company had outpaced his management skills.

and $X_E = (x_E, \bar{x}]$ of X for which the founder prefers, ex ante, liquidation over continuation and expansion over continuation, respectively. The present section finds the optimal sizes of the subsets x_L and x_E . Although it is quite possible that $x_L > x_E$ so that continuation is never desired, it is assumed that $x_L < x_E$ so that the control transfer policy desired by the founder is characterized by non-empty subsets $Z \equiv (X_L, X_C, X_E)$, where $X_C \equiv [x_L, x_E]$ gives the subset of X over which the founder retains control.

The founder chooses Z to maximize his date 0 expected utility, which is given by the function $E(U) = \alpha V + E(Q)$, where V denotes the date 0 firm value and $E(Q)$ the founder's expected control rent. As is made clear below, both V and $E(Q)$ depend on Z . The founder on date 0 now chooses the optimal Z by solving

$$\max_Z E(U)$$

or, equivalently, by solving

$$\max_{x_L, x_E} \int_{\underline{x}}^{x_L} \alpha l f(x) dx + \int_{x_L}^{x_E} (\alpha x + Q) f(x) dx + \int_{x_E}^{\bar{x}} \alpha J(x) f(x) dx. \quad (\mathcal{P})$$

The first order conditions of \mathcal{P} are:

$$l = x_L + Q/\alpha \quad (1)$$

and

$$J(x_E) = x_E + Q/\alpha, \quad (2)$$

where the second order conditions for maximum are easily shown to be satisfied.

We observe from first order conditions (1) and (2) that the determinants of x_L and x_E are Q , α , l , and $J(x)$. These variables influence x_L and x_E as follows. A larger value of the founder's control rent, Q , decreases x_L and increases x_E . This occurs because an increase in Q makes control more valuable thus inducing the entrepreneur to increase the probability of retaining control. A larger value of the entrepreneur's ownership rate, α , decreases the relative value of control. This compels the entrepreneur to substitute cash for control by increasing x_L and decreasing x_E . An increase in l makes liquidation more valuable relative to continuation. This leads the founder to increase the probability that the firm will be liquidated, increasing x_L . Finally, an increase in $J(x)$ for each x makes expansion more valuable and leads the founder to decrease x_E thus increasing the probability that the expansionary operating policy will be implemented.

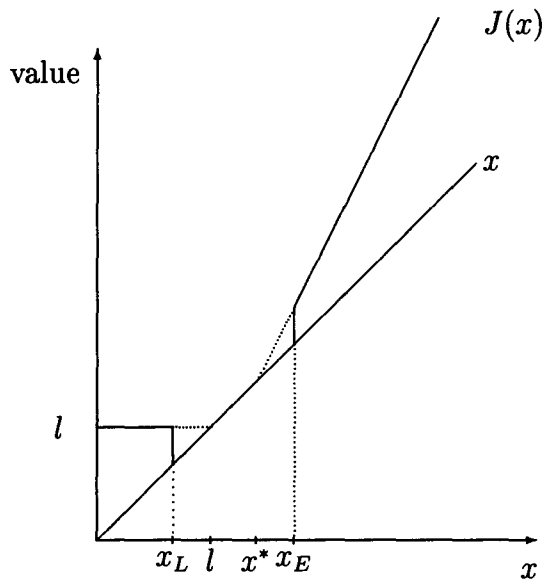


Figure 1: The figure depicts the date 1 value of the firm as a function of x and the optimal policy.

Note that the need to create an optimal control transfer policy ex ante arises from the desire of the project owner to sell his firm (fully or partly) to investors while at the same time retaining control. If he instead refrains from issuing new securities on date 0, he will on his own account implement the optimal policy determined by \mathcal{P} on the interim date. To see this, suppose that the founder retains the equity position α and refrains from issuing new securities on date 0. Faced now with the decision to liquidate or continue on date 1, the founder receives αl if he liquidates and $Q + \alpha E(x + \tilde{\omega}) = Q + \alpha x$ if he continues. He will therefore choose to liquidate whenever $x < l - Q/\alpha$, which is the same rule as provided by the solution to \mathcal{P} . Similarly, the founder will allow expansion (by selling the firm on date 1) if $x < J(x) - Q/\alpha$, which is again the same rule given by the solution to \mathcal{P} .

4 Using Financial Claims to Implement the Optimal Policy

Suppose then that x is cannot be verified and therefore that enforceable contracts written on x do not exist. However, the present section shows the existence of a financial structure that implements the optimal policy derived under the assumption that x is verifiable.

Recall that the optimal contract calls for a control transfer to investors and liquidation whenever $x \in [\underline{x}, x_L)$. Such a control transfer can be induced by issuing a short-term

debt claim with face value face value $D_s = x_L$ due on date 1. Since the firm generates no cash on the interim date, this claim must be refinanced. As will be discussed below, it will be necessary to allow the firm to refinance the initial short-term debt claim by issuing senior debt on the interim date. Let $D_{s1}(i(x))$, denote the face value of debt issued on date 1 to refund the initial short-term debt claim D_s given that the value of the firm is $i(x) = x, J(x)$; that is, the debt payment required to refund the initial short-term claim depends on the particular operating policy chosen on the interim date.⁷

The short-term claim issued on date 1 is issued so as to raise just enough funds to pay off the date 0 short-term claim D_s . This implies that $D_{s1}(i(x))$ will be determined from

$$\int_{\omega}^{D_{s1}-x} (i(x) + \omega)g(\omega)d\omega + D_{s1}(1 - G(D_{s1} - x)) = D_s. \quad (4)$$

Although it may be in the interest of the founder (or the firm's shareholders) to raise more than D_s , it is assumed that there is a covenant in place that restricts the amount of new borrowing not to exceed D_s .

Proposition 1 describes the financial structure needed to implement the optimal policy.⁸

Proposition 1 *Suppose that the amount of senior short-term debt issued on date 1 is restricted not to exceed the amount necessary to raise exactly x_L and suppose that this restriction (covenant) is enforceable. Then the optimal policy can be implemented by a combination of senior short-term debt with face value $D_s = x_L$, (zero-coupon) junior long-term debt with face value $D_l = J(x_E) + \bar{w} - D(J(x_E))$, and voting equity; all issued on date 0.*

The enforceability of the covenant restricting the amount of new senior debt issued not to exceed $D_s = x_L$ requires that the amount of funds raised on date 1 as well as the amount owed to date 0 short term lenders can be verified. I assume this to be the case noting that in practice the amount owed to creditors as well as the amount of funds raised through new debt issues is usually verifiable public information.

⁷We may view $D_{s1}(J(x))$ as arising either from the existing short-term debt claim being refunded before shareholders take action or as the face value of an 'imaginary' debt claim that shareholders must issue in order to pay off existing short-term lenders. In either case, the amount D_s is owed to the short-term creditor on the interim date

⁸All proofs are to be found in the Appendix.

Although long-term debt holders allow the firm to issue senior debt on date 1 thus diluting their claim, the firm is not allowed to raise more than the amount needed to pay off date 0 short-term lenders.⁹ Since x represents the date 1 value of the date 2 cash flow under continuation, the maximum amount of funds that the firm will be able to raise on date 1 under continuation is given by $\min(x, x_L) = \min(x, D_s)$, from which it is apparent that the firm will be able to raise the funds necessary to avoid default for all $x \geq x_L = D_s$ if and only if the new claim issued is senior to the long-term debt claim already in place on date 1.¹⁰

To enable shareholders to replace the founder so as to implement an expansion, the equity claim must be given (majority) voting rights. Ideally, shareholder control would be contingent on whether or not x exceeds x_E . Since x is non-verifiable, however, shareholders must be assigned control rights either for all realizations of x or for none. While assigning control rights for no x ensures that expansion will never occur, assigning control rights for all values of x creates the potential that shareholders ignore the optimal policy and instead implement expansion or liquidation whenever this is consistent with maximizing the value of the firm. However, the use of long-term debt in this setting creates a debt overhang on date 1 that reduces the incentives of shareholders to exercise their control rights. By appropriately setting the level of long-term debt on the initial date, the founder is able to create a debt overhang on date 1 that ensures that shareholders exercise their control rights if and only if $x > x_E$. In other words, long-term debt creates a debt overhang which effectively converts the non-contingent control right of shareholders into a contingent control right, contingent on the event $x > x_E$. By Proposition 1, the level of long-term debt that provides shareholders with such a contingent control right is given by $D_l = J(x_E) + \bar{w} - D_{s1}(J(x_E))$.¹¹ At this level, the amount of

⁹In Hart and Moore (1995) long-term debt is senior to short-term debt in order to prevent the manager from raising funds to invest in unprofitable projects. In my model short-term debt is senior to long-term debt in order to induce efficient liquidation decisions; long-term debt holders prevent the entrepreneur from excessively raising new funds by attaching to the long-term debt claim a covenant restricting the amount of new debt that the firm can issue on date 1 not to exceed the amount needed to pay off the date 0 short-term debt claim coming due.

¹⁰To see this, suppose that $x = x_L$ so that the founder must pledge 100% of the date 2 cash flow in order to avoid default. Since pledging 100 % of the future cash flow can be done only if the claim issued is senior to existing claims, it follows that the optimal liquidation policy can be implemented if and only if debt issued on the interim date is senior to existing claims. This argument follows closely along the lines of Diamond (1991).

¹¹Note that since control transfers are assumed costless in this model, D_l must be set so that the

long-term debt is sufficiently large to ensure that it will be profitable for shareholders to take control in order to to implement *expansion* only if $x > x_E$.

A long-term debt level of $D_l = J(x_E) + \bar{\omega} - D_{s1}(J(x_E))$ gives a total (date 1) debt level $D \equiv D_l + D_{s1}(J(x_E)) = J(x_E) + \bar{\omega}$ due on the interim date. This amount of debt implies that lenders will capture all cash generated by the firm whenever $x \leq x_E$ regardless of the realization of $\tilde{\omega}$. For $x > x_E$, it will be the case that $D < J(x) + \bar{\omega}$, in which case shareholders will receive a positive cash distribution. Since now the costs to shareholders of exercising their control rights are (assumed to be) zero, the founder will be ousted and expansion implemented whenever $x > x_E$.

Using long-term debt to prevent shareholders from taking control in certain states is a straight forward application of Myers' (1977) debt overhang problem (and similar to insights offered by Israel (1991), though he does not distinguish between long-term debt and short-term debt). However, while the presence of a debt overhang in Myers (1977) prevents shareholders from contributing equity capital thereby forcing firms to pass up valuable investment opportunities, the debt overhang in the present model is created with the purpose of preventing over-zealous shareholders from exercising their control rights too frequently.

5 Comparative Statics

The present section relates the composition of the firm's date 0 financial structure to variables Q , α , and l . Noting that empirical work on debt maturity and priority structure is generally performed on book values rather than market values (see e.g. Stohs and Mauer [1996]), the analysis is conducted in terms of the face value of debt rather than market value.

A. The Value of Control. An increase in Q raises the value of control and induces the entrepreneur to substitute away from firm value towards control. As seen directly by first order conditions (1) and (2), a larger value of Q leads to a decrease in x_L and an increase in x_E . Whereas a smaller value of x_L will decrease the amount of short-term

pay-off to shareholders are zero for all $x \in X_L \cup X_C = [x, x_E]$ in which case shareholders weakly prefer not to exercise their control rights. It is possible to include a strictly positive control transfer cost but this complicates the analysis without providing additional insights and is thus avoided.

debt D_s , a larger value of x_E leads to an increase in the long-term debt level, D_l . This leaves the effect of a change in Q on the total amount of debt possibly ambiguous.

Lemma 1 *An increase in the value of control, Q , leads to smaller amount of short-term debt, a larger amount of long-term debt, and a larger amount of total debt.*

The lemma shows that the increase in the amount of long-term debt from a higher value of Q outweighs the accompanying reduction in the amount of short-term debt. This gives the result that the firm's amount of debt will be *increasing* in the value control. In other words, Lemma 1 suggests, contrary to what one might expect, that a high total debt level along with a high proportion of long-term debt indicates large amounts of managerial control rents.

B. The Founder's Initial Equity Position. A larger value of the founder's initial ownership rate, α , decreases the relative value of control. As seen from first order conditions (1) and (2), the effect on x_L and x_E of a change in α has the opposite effect of a change in Q thus leading to the following lemma.

Lemma 2 *An increase in the founder's initial ownership rate, α , leads to a larger amount of short-term debt, a lower amount of long-term debt, and a lower total debt amount.*

Although Lemma 2 is the theoretical counterpart to Lemma 1, because data is available on α while this is not the case for Q , the predictions of Lemma 2 are testable while the predictions of Lemma 1 are not.

C. The Liquidation Value. The next lemma relates the composition of the firm's financial structure to the firm's liquidation value.

Lemma 3 *An increase in the firm's liquidation value, l , increases the amount of short-term debt, decreases the amount of long-term debt, and decreases the total amount of debt.*

A larger liquidation value naturally makes liquidation more profitable relative to continuation. This increases the opportunity cost of control to which the founder responds by increasing the amount of short-term debt and thus increasing the probability that the firm will be liquidated on date 1. While this result can be seen directly from the

expression for D_s , the effect of a change in the firm's liquidation value on the amount of long-term debt is more indirect: a larger value of l leads to an increase in the amount of short-term debt D_s , which in turn increases the amount of date 1 funds needed to refund the initial short-term debt claim. This increases the face value, $D_{s1}(\cdot)$, of short-term debt due on date 2. Since $D_{s1}(\cdot)$ and D_l can be viewed as perfect substitutes in protecting the entrepreneur's control rent on the up-side—as can be seen from (4)—an increase in $D_{s1}(\cdot)$ leads to a one-for-one decrease in D_l . The lemma shows that the decrease in the amount of long-term debt will outweigh the increase in the amount of short-term debt to give an inverse relationship between the firm's liquidation value and its total amount of debt.

The result that a larger liquidation value leads to a lower amount of debt is contrary to results by Harris and Raviv (1990) and Williamson (1988). The empirical evidence on the relationship between liquidation values and the amount of debt is inconclusive (see review article by Harris and Raviv [1991]) thus suggesting that Lemma 3 can be seen as complementing the predictions of Harris and Raviv (1990) and Williamson (1988).

6 Concluding Remarks

This paper develops a simple control theory of financial structure which generates a comprehensive financial structure consisting of short-term senior debt, long-term covenanted debt, and equity with voting rights. As is well known from Modigliani and Miller (1958), the choice of financial structure, and therefore the design of securities, is irrelevant in frictionless markets. Security design is relevant in the present model because information, though costlessly observable to all, is non-verifiable and therefore cannot be contracted upon directly and because the presence of non-assignable entrepreneurial control rents create a non-alignment between the interests of the manager-entrepreneur and the interests of the firm's security holders. The contribution of the paper is to show that the financial structure needed to optimally allocate control rights and cash flow rights in this setting closely resembles the type of comprehensive financial structures observed in practice.

Appendix

Proof of Proposition 1

(i) $x \in [\underline{x}, x_L]$. Then $E(x + \tilde{\omega}) = x < D_s$ so that if choosing continuation the founder will be unable to raise enough cash to satisfy date 0 short-term lenders. Date 0 short term lenders will therefore take control, liquidate the firm and receive $\min(D_s, l) = D_s = x_L$.

(ii) $x \in [x_L, x_E]$. In this case we must show (a) that the firm is able to raise enough cash to satisfy date 0 short-term lenders, (b) that expansion will not be profitable, and (c) that shareholders will refrain from forcing the firm to be liquidated (which is profitable for the firm's security holders for all $x \in (x_L, l)$).

Condition (a) follows since the new claim is senior to existing claims and since $E(x + \tilde{\omega}) = x > D_s = x_L$ for all $x \in [x_L, x_E]$.

Consider then condition (b). The date 1 value of equity under expansion is given by

$$E\{\max[J(x) + \tilde{\omega} - D_l - D_{s1}, 0]\} = 0. \quad (1)$$

Insert the expression for $D_l = J(x_E) + \bar{\omega} - D_{s1}$ into (1) to find that the date 1 equity value is

$$E\{\max[(J(x) - J(x_E)) + (\tilde{\omega} - \bar{\omega}), 0]\},$$

which, since $J(x) \leq J(x_E)$ and $\tilde{\omega} \leq \bar{\omega}$, is zero for all $x \in [x_L, x_E]$

Consider finally condition (c). The face value D_l of long-term debt is determined so that

$$J(x_E) = V_l(J(x_E), D_l(J(x_E))) + D_s, \quad (2)$$

where $V_l(\cdot, \cdot)$ is the date 1 value of a long-term debt claim with face value $D_l(\cdot)$. D_s is similarly the date 1 value of a debt claim issued on date 1 with face value D_{s1} due on date 2.

Let $V_e(l)$ denote the cash received by shareholders if the firm is liquidated on date 1. We want to prove that $V_e(l) = 0$. Suppose to the contrary that $V_e(l) > 0$. If the firm is liquidated, its liquidation proceeds l will be distributed to claimholders according to stated priority rules. By the assumption that $V_e(l) > 0$, this implies that long-term lenders receive D_l and that short-term lenders receive D_s . $V_e(l)$ is therefore the residual

value determined from

$$l = V_e(l) + D_l + D_s. \quad (3)$$

The assumption that $V_e(l) > 0$ implies now that

$$V_e(l) = l - D_l - D_s > J(x_E) - V_l(\cdot; \cdot) - D_s = 0,$$

or that

$$l - D_l > J(x_E) - V_l(\cdot; \cdot);$$

which, by the fact that $V_l(\cdot; \cdot) < D_l$, implies that $l > J(x_E)$, which, by the assumption that X_C is non-empty (i.e. $x_E > x_L$), must be incorrect. This implies that a contradiction has been obtained and therefore that the initial assumption that $V_e(l) > 0$ was wrong. We may therefore conclude that $V_e(l) = 0$. \square

Proof of Lemma 1

The result that D_s is decreasing in Q is seen directly from $D_s = x_L = l - Q/\alpha$.

The result that D_l is increasing in Q can be seen from the expression $D_l = J(x_E) + \bar{w} - D_{s1}(J(x_T))$ and the first order condition $J(x_E) = x_E + Q/\alpha$; a larger value of Q leads to a larger value of x_E (since $J'(x) > 1$), which leads to a greater $J(x_E)$, a lower $D_{s1}(J(x_E))$ and thus higher $D_{s1}(\cdot)$.

To see that $D = D_s + D_l$ is increasing in Q , we note that $D = l - Q/\alpha + J(x_T) + \bar{w} - D_{s1}(J(x_T))$. Taking the total derivative of D with respect to Q yields:

$$\begin{aligned} \frac{dD}{dQ} &= -1/\alpha + J'(x_T)[1 - D'_{s1}(J(x_T))]\frac{dx_T}{dQ} \\ &= -1/\alpha + J'(x_T)[1 - D'_{s1}(J(x_T))]\frac{1}{\alpha[J'(x_T) - 1]} \end{aligned}$$

so that $\frac{dD}{dQ} > 0$ if

$$J'(x_E)[1 - D'_{s1}(J(x_E))] \geq [J'(x_E) - 1] \geq 1,$$

which is the fact since $D'_{s1}(J(x_E)) < 0$. \square

Proof of Lemma 2 The proof of Lemma 2 follows the proof of Lemma 1 with the exception that α takes the place of Q . \square

Proof of Lemma 3

The fact that D_s is increasing in l is seen directly from $D_s = x_L = l - Q$. To see that $D_l = J(x_E) + \bar{\omega} - D_{s1}(J(x_E))$ recall first that $D_{s1}(i(x))$ is determined by

$$\int_{\underline{\omega}}^{D_{s1}-i(x)} (i(x) + \omega)g(\omega)d\omega + D_{s1}(1 - G(D_{s1} - i(x))) = D_s = l - Q/\alpha,$$

from which it can be observed that a larger value of l leads to a larger value of D_{s1} . The larger D_{s1} can in turn be seen from $D_l = J(x_E) + \bar{\omega} - D_{s1}(J(x_E))$ to decrease D_l .

Finally, to see that $D = D_l + D_s$ is decreasing in l , differentiate D with respect to l :

$$\frac{dD}{dl} = 1 - \frac{dD_{s1}}{dl} = 1 - \frac{1}{1 - G(D_{s1} - J(x_E))} < 0.$$

□

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CHAPTER 2

Costly State Verification: Outside Equity and the Role of Accounting Information*

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Abstract

This paper examines outside equity in a costly state verification model in which the investor has access to non-verifiable information about true cash flows. The outside equity contract is shown to be incentive compatible (almost surely) in the limit as the information observed by the investor becomes perfectly correlated with cash flows even though this information cannot be verified, even at a cost. In the limit, dividend payments are contingent (almost surely) on the information observed by the investor, and the outside equity contract achieves first-best.

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1 Introduction

Outside equity with dividend payments tied loosely to firms' earnings reports is a standard funding tool used in practice. Yet the theoretical underpinnings for such an arrangement are not well understood. Indeed, despite the apparent importance of accounting information in determining contractual performance in practice, the contracting literature has essentially ignored its role. The present paper attempts to bridge this gap and does this by analyzing outside equity with dividend payments tied to jointly observable information under costly state verification (CSV).¹ A non-trivial contracting problem arises in this setting because the information available to the investor, though perfectly correlated with true cash flows in the limit, cannot be verified, even at a cost. In addition, the investor is unable to observe the true state unless it is verified, which is costly.

Gale and Hellwig (1985) derive the optimal contract under costly state verification and show that it takes the form of the standard debt contract 'with bankruptcy,' where verification (or bankruptcy) is triggered whenever the fixed payment agreed upon *ex ante* is not paid in full. Since the contractual debt payment is jointly observable and costlessly verifiable, verification under the standard debt contract is triggered by a publicly observable and verifiable event: failure to pay the contractually specified fixed debt payment in full. In contrast, outside equity is issued with an unspecified 'dividend' payment which is contingent on the information observed by the investor (and the entrepreneur). Nonetheless, this information has a role that is analogous to the fixed payment of the standard debt contract: namely that of being a publicly observable variable to which the payment under the contract as well as the verification decision are tied. However, since the information available to the investor is non-verifiable, the control structure associated with outside equity will be different from the one associated with the standard debt contract. Indeed, while the standard debt contract is issued with a fixed payment and a *contingent* right on the part of the lender to call for verification in the event that the entrepreneur does not pay the fixed debt payment in full, the outside equity contract gives the investor a right to call for verification that is *non-contingent* and thus enables him to call for verification regardless of the size of the dividend proposed.

¹The CSV framework is due to Townsend (1979) and Gale and Hellwig (1985); see Allen and Winton (1995) for a recent survey of the CSV literature.

Rather than deriving the outside equity contract from ‘first principles,’ I endow the outside equity contract with characteristics that resemble those associated with outside equity observed in practice: pro-rata cash flow rights and non-contingent control. It is shown that these features are sufficient to induce the entrepreneur to make a enough funds available to the investor ex post to allow the the investor to break even on average, and further that this is obtained under less contracting costs compared to what can be obtained using the standard debt contract so long as the information observed by the investor is sufficiently precise.²

Whereas on a somewhat general level the paper offers a theory of outside equity and dividends under incomplete contracting, on a more specific level it is able to provide a role for outside equity within the traditional CSV framework. As such, the paper addresses an oft cited weakness of the CSV setup: its apparent inability to account for the use of outside equity.³ Of course, to accomodate the use of outside equity it has been necessary to relax the somewhat extreme structure of the CSV setup, which assumes that the information obseved by the investor is uncorrelated with the true state, by giving the investor access to noisy (and non-verifiable) information regarding the true state. Importantly, however, the model retains the defining assumption of the CSV setup that cash flows are costly to verify and indeed coincides with the standard CSV setup in the limit when the information observed by the investor is uncorrelated with the true state.

Mookherjee and Png (1989) show that random verification yields lower verification costs compared to the verification scheme associated with the standard debt contract derived by Gale and Hellwig. As sometimes argued, however, random verification is

²An alternative mechanism to induce payment on outside equity is provided by the signaling role of dividends; see Miller and Rock (1986). One distinction between the signaling argument and the incomplete contracting argument developed in the present paper is that while the signaling argument is based on ex-ante asymmetric information, the incomplete contracting argument developed here is based on ex-post asymmetric information. A second distinction is that though the signaling argument provides an explanation for why dividend surprises are informative, it is silent as to what determines the firm’s dividend policy in the first place. In contrast, the present paper provides a formal theory of outside equity and dividends.

According to the signaling story, dividend payments are larger than what they would be under symmetric information. Interestingly, this feature appears in the current model as well where in certain states the incentive compatible equity contract forces the entrepreneur to make payouts in excess of the corresponding payout associated with symmetric information.

³For example, Hart (1995) notes that “. . . in reality, debt typically coexists with equity as a financial claim on the firm. Yet the CSV model does not seem to be able to explain the existence of dividends and (outside) equity.”

difficult to enforce and rarely observed (see e.g. Winton [1995]). Whether random verification is enforceable or not, the current paper shows that an alternative verification scheme—that represented by an incentive compatible outside equity contract with payments and verification decisions tied to publicly observable information—may yield lower verification costs than either random verification or the verification scheme associated with the standard debt contract, the condition of which is that the information observed by the investor is sufficiently precise.

While the role of debt under incomplete contracting is relatively well understood (see e.g. Hart [1995] and references provided therein), the role of outside equity is not. However, Fluck (1995) examines outside equity in a Hart-Moore (1989) type of model where cash flows are jointly observable but cannot be verified even at a cost. While in her paper outside equity holders are given a ‘tacit’ right to dismiss the manager whenever unsatisfied with the manager’s dividend proposal, in the present paper the outside owner is given a similar tacit (or non-contingent) right to demand costly verification regardless of the state and therefore regardless of the size of the dividend proposal. Moreover, whereas Fluck examines the optimal maturity structure of contracts (and shows that only infinite maturity equity is incentive compatible), the present paper examines the role of accounting information in inducing contractual performance.

Also related is Chang (1993), who develops an agency model in which the manager has an incentive to invest the firm’s interim cash balance in unprofitable projects rather than pay them out as dividends to the firm’s shareholders. Chang shows that the optimal payout, which is based on non-verifiable interim information, can be implemented by funding the firm with a combination of standard debt and outside equity. In contrast to what is done in the present paper (as well as in Fluck), Chang assumes that final cash flows can be verified at zero cost, however.

The paper is organized as follows. Section 2 presents the basic model. Section 3 describes, analyzes, and compares the debt and equity contracts. Section 4 concludes the paper.

2 The Setup

The model has two dates, denoted 0 and 1, and consists of an entrepreneur (insider) and an investor (outsider), who is chosen at random from a competitive financial market. The investor and the entrepreneur are both risk neutral. The entrepreneur is endowed with a profitable project (or firm) but does not have the necessary funds to get it started and must therefore obtain external funds, which he does by contracting with the investor. The project requires a date 0 outlay of I and yields a date 1 random cash flow $x \in [\underline{x}, \infty)$, where $\underline{x} > 0$ and x is an integrable random variable on the general probability space (Ω, \mathcal{F}, P) . Let $E(x)$ denote the mean of x and assume that $E(x) > I$. The NPV of the project is given by $E(x) - I$, which is the value of the project under complete contracting.

On date 0, the investor and the entrepreneur have access to the same amount of information. On date 1, however, while the entrepreneur is able to observe true cash flows costlessly, the investor is able to observe x only after strictly positive verification costs c have been expended.⁴ If verified, x becomes jointly observable and can, in addition, be ascertained in a court of law. It is assumed that the costs of verifying x is picked up by the firm and that the investor incurs zero private costs in having x verified. This assumption implies that if cash flows are verified, there will be a total of $x - c$ to be distributed among the investor and the entrepreneur. To ensure that there are always sufficient resources left in the firm to cover verification costs, it is assumed that $\underline{x} \geq c$.

Although the investor is unable to observe true cash flows unless they are verified, it is assumed that he has access to unbiased, though in general imperfect, information regarding true cash flow. More formally, it is assumed that the information available to the investor is given by the σ -field \mathcal{F}_n of \mathcal{F} and provides the investor with the unbiased estimate $x_n = E(x|\mathcal{F}_n)$ of the true cash flow. So as to rule out trivial outcomes, it is assumed the information contained in \mathcal{F}_n cannot be verified even at a cost. Furthermore, in order to examine the role of information precision, the information available to the investor is arranged as an increasing sequence of σ -fields $\mathcal{F}_0 \subset \mathcal{F}_1 \subset \dots$, with \mathcal{F}_0 being

⁴Note that the results do not depend on verification costs being constant. For example, all the results derived come through with verification costs being linear in x ; such as e.g. $c(x) = a + bx$, where $a \geq 0$ and $b \in (0, 1)$.

the trivial field.⁵

Lemma 1 *The sequence of random variables $\{x_n; n = 0, 1, \dots\}$ converges to x a.s.*

Proof: See Example 35.5 and Thm. 35.6 in Billingsley (1995). □

The random variable x_n represents an unbiased estimate of the true cash flow x . The lemma says that as n is increased, the precision of x_n as an estimate of x increases until, in the limit, x_n becomes a version x and cash flows effectively become jointly observable. Note, though, that since x_n is non-verifiable and cash flows are verifiable only at a cost, the contracting problem between the entrepreneur does not trivially vanish even though cash flows become jointly observable in the limit.⁶ Nonetheless, the paper shows that there exists an *outside equity contract* which will permit dividend payments contingent on x with probability one, in the limit.

The cash flow information contained in \mathcal{F}_n may be viewed as representing information contained in a firm's earnings report and any additional information discernable from this report. Although the information available constitutes a highly stylistic description of the characteristics of earnings information, it nonetheless captures an important aspect of the role of such information; namely, that of providing a publicly observable variable to which dividend payments are linked. Of course, a potential problem arises from the assumption that the information observed is non-verifiable, while the information contained in a firm's earnings reports is (almost by definition) verifiable (though subject to manipulation). Note, however, that dividend payments in practice are usually loosely and rarely contractually linked to firms' earnings reports and are thus partly contingent on non-verifiable information not captured by the firm's earnings report.

⁵Note that this does not mean that the investor has a choice of n , only that $n \in [0, \infty)$.

⁶Indeed, the limiting case is closely related to the environment studied by Hart and Moore (1989), where cash flows are jointly observable but cannot be verified even at a cost. An important distinction between the two setups, though, is that in their model the entrepreneur is able "take the money and run," while this is not possible in the CSV framework. A second distinction is that cash flows are verifiable at a cost in the present paper, while strictly non-verifiable in their setup.

3 The Financial Contracts

3.1 The Standard Debt Contract

Gale and Hellwig (1985) derive the optimal contract under CSV and shows that it takes the form of standard debt contract. This contract yields a constant payment D if cash flows are not verified and $x - c$ if cash flows are verified.⁷ Given an initial investment of I , the payment D is determined by

$$D \int_{V_D^c} dP + \int_{V_D} (x - c) dP = I, \quad (1)$$

where $V_D = \{\omega \in \Omega : x < D\}$ and $V_D^c = \Omega \setminus V_D$. The value of the entrepreneur's inside equity claim is now given by

$$E(W_D) = \int_{V_D} (x - D) dP. \quad (2)$$

Using (1), the expression for $E(W_D)$ can be written

$$E(W_D) = E(x) - I - c \int_{V_D} dP, \quad (3)$$

which shows that the entrepreneur captures the NPV of the project less expected verification costs. It can be observed directly by (3) that the cost of debt, $c \int_{V_D} dP$, is unrelated to the information observed by the investor. This result accords well with the fact, that the optimal contract takes the form of the standard debt contract when the information observed by the investor is uncorrelated with cash flows, as shown by Gale and Hellwig.

3.2 Outside Equity

Let z_n and $\alpha_n \in (0, 1)$ denote, respectively, the 'dividend' payment and the pro-rata cash flow right associated with the outside equity contract. Though α_n applies in principle to actual cash flows, since these are observable to the investors only if verified, it will not

⁷This structure relies on an assumption that the lender does indeed verify if offered a payment that is below the payment contracted upon ex ante. Such an assumption can be rationalized on grounds that the lender has reputational capital at stake (see e.g. Gale and Hellwig [1989]) or that the number of lenders is sufficiently large to make free rider problems in renegotiation sufficiently pervasive that debt renegotiations will fail with probability one (see Gertner and Sharfstein [1991])

be optimal to make the dividend payment z_n directly contingent on actual cash flows in all states.

There will be verification in certain states, though. If verification occurs, then α_n applies to the remaining resources of the firm $x - c$, yielding the payment $z_n = \alpha_n(x - c)$. If there is no verification, on the other hand, so that actual cash flows are unobservable to the investor, then the cash flow right α_n cannot be applied to the actual cash flow but must instead be applied to the jointly observable information signal x_n . In this case, dividend payment in the non-verification region will be $z_n = \alpha_n x_n$.

The sequence of moves on date 1 is as follows. The entrepreneur observes the vector (x, x_n) , while the investor observes x_n . The entrepreneur then proposes a dividend payment $z_n = \alpha_n x_n$, or some other payment $z_n < \alpha_n x_n$. The investor either accepts or rejects the proposed payment. If he accepts it, he receives the proposed payment. If he rejects it, this automatically triggers a verification and he receives $\alpha_n(x - c)$.

On date 0, the entrepreneur seeks to construct an outside equity contract that minimizes the probability of verification (or expected verification costs). Given that the equity payment in the non-verified state is $\alpha_n x_n$, the largest possible non-verification region is the one given by the set of states $V_n^c = \{\omega \in \Omega : x \geq \alpha_n x_n\}$, which requires that the entrepreneur puts $z_n = \alpha_n x_n$ for all values of x for which his ex post budget constraint allows him to do so. The corresponding non-verification set will thus be given by $V_n = \{\omega \in \Omega : x < \alpha_n x_n\}$.⁸

Figure 1 depicts the verification and non-verification sets in (x_n, x) -space. The non-verification set V_n^c is depicted as the union of sets $V_n^{c'}$ and $V_n^c \setminus V_n^{c'}$, where $V_n^{c'}$ represents the set for which the outside equity contract calls for a dividend payment that exceeds the dividend that would be paid out if cash flows were verifiable and observable at zero cost. It is shown below that the probability of verification $P(V_n)$ converges to zero as the precision of the information observed by the investor increases. To see why this should be, note that since x_n constitutes an unbiased estimate of x , ‘observations’

⁸Importantly, the proposed structure requires that the investor will reject any payment offered that is less than $\alpha_n x_n$. Analogous to the assumption made with respect to the standard debt contract, it is assumed that the equity investor is able to commit to call for a verification if offered a payment less than $\alpha_n x_n$. That is, I assume that the investor has either reputation at stake that prevents renegotiation (such as venture capitalists) or that the number of investors is sufficiently large to make renegotiation impossible (such as a publicly held firm).

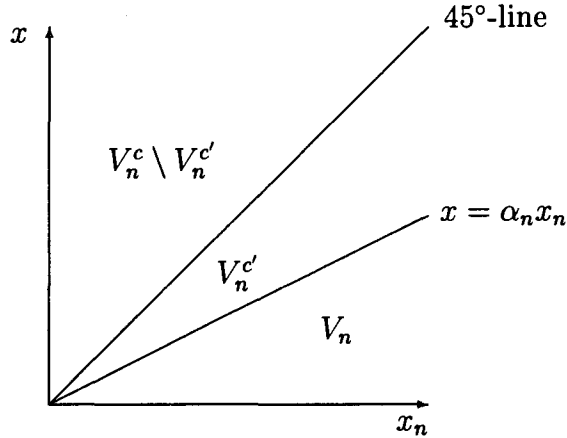


Figure 1: Verification and non-verification regions in (x_n, x) -space.

in (x_n, x) -space will be symmetrically distributed around the 45-degree line. As now the precision of the investor's information increases, so that x_n converges to x , the 'observations' (x_n, x) will cluster closer and closer along the 45-degree until, in the limit, $P(V_n) = 0$.

As will be discussed in more detail below, verification is not necessarily restricted to the set V_n . Indeed, the equity investor holds a non-contingent right to demand verification and potentially has the incentive to demand verification in states that fall outside V_n . In addition, the entrepreneur will have the incentive in certain states to trigger a verification (by offering a dividend that is strictly less than $\alpha_n x_n$) even though his ex post budget constraint would enable him to avoid verification. It turns out, however, that the probability of verification occurring outside the set V_n is strictly decreasing in the precision of x_n and vanishes in the limit as n becomes large.

The incentive compatibility constraints of the investor and the entrepreneur are examined in subsections 3.2.1 and 3.2.2, respectively. Subsection 3.2.3 then examines the general contracting problem associated with outside equity.

3.2.1 Investor's Incentive Compatibility Constraint

As the firm's outside owner, the equity investor holds a non-contingent right to call for verification. This right enable the investor to call for verification regardless of the size of the dividend proposed by the entrepreneur. The potential that the investor will call for verification outside the set V_n arises because upon observing a proposal $\alpha_n x_n$,

the investor will necessarily infer that the actual cash flow x is larger than the size of the dividend that is being proposed. This means that upon observing the dividend offer $\alpha_n x_n$, the investor revise upwards his estimate of the true cash flow. To formalize this, define the σ -fields \mathcal{F}_n' of \mathcal{F} generated by events $A_1, A_2, \dots \in \mathcal{F}_n$ and the event ' $x \geq \max[\underline{x}, \alpha_n x_n]$ ' so that if $A_1, A_2, \dots \in \mathcal{F}_n$, then $A_1 \cap V_n^c, A_2 \cap V_n^c, \dots \in \mathcal{F}_n'$. The random variable $x_n' \equiv E(x|\mathcal{F}_n')$ thus represents the expected value of x given x_n and the event ' $x \geq \max[\underline{x}, \alpha_n x_n]$ '. We note directly by the definition for \mathcal{F}_n' that $x_n' \geq x_n$.

The investor will now reject the entrepreneur's dividend offer $z_n = \alpha_n x_n$ unless

$$\alpha_n(x_n' - c) \leq z_n = \alpha_n x_n, \quad (4)$$

where the leftmost side represents the expected value to the investor of rejecting the entrepreneur's offer, its right hand side represents the value to the investor of accepting the entrepreneur's dividend offer. Condition (4) thus represents the entrepreneur's IC constraint. Cancelling α_n , this constraint can be written

$$x_n' - x_n \leq c. \quad (5)$$

We note by the definition of \mathcal{F}_n' that the left hand side of (5) is non-negative.

Let now M_n denote the set of states for which the investor's IC constraint is not satisfied; that is, let $M_n = \{\omega \in \Omega : x_n' - x_n > c\}$.

Lemma 2 $\lim_n P(M_n) = 0$.

Proof: Note first that the random variable x_n' converges to x almost surely. This follows by the fact that $\mathcal{F}_n \subset \mathcal{F}_n'$ and the fact that x_n converges to x almost surely (Lemma 1). Since now both x_n' and x_n converge to x a.s., their difference, $x_n' - x_n$, converges to zero. We then have that $P(M_n)$ converges to zero by the fact that convergence almost surely implies convergence in probability. \square

Upon observing the dividend offer $z_n = \alpha_n x_n$, the investor infers that x exceeds $\alpha_n x_n$. To the extent that the additional information contained in the entrepreneur's dividend proposal leads the investor to infer that his share of the expected verified cash flow will be larger than the proposed dividend, this will induce him to reject the entrepreneur's dividend offer.

Lemma 2 suggests that the propensity of the investor to call for verification upon observing the entrepreneur's offer will decrease as the investor's information becomes more precise. This occurs because as the precision of the information observed by the investor increases, the additional amount of information that he obtains from observing the event ' $x \geq \alpha_n x_n$ ' goes down. Indeed, in the limit, the dividend proposal is uninformative and the investor has no incentive to reject the entrepreneur's dividend offer.

3.2.2 Entrepreneur's Incentive Compatibility Constraint

The entrepreneur may find it optimal to induce verification even though the firm has sufficient resources at hand to avoid it. This incentive potentially arises when $x \in V_n^c$ (see Figure 1), in which case the scheduled dividend $\alpha_n x_n$ exceeds the dividend $\alpha_n x$ that would be paid out if cash flows were observable and verifiable at no cost.

To examine the incentives of the entrepreneur to trigger a verification outside of the set V_n , suppose that $x \in V_n^c (\subset V_n^c)$, in which case the firm's cash balance is large enough to avoid verification. If now the entrepreneur puts $z_n = \alpha_n x_n$, and thus avoids verification, he receives the remaining cash balance $x - \alpha_n x_n$. However, the entrepreneur may alternatively decide to trigger a verification (by putting $z_n < \alpha_n x_n$), in which case he receives the cash payment $(1 - \alpha_n)(x - c)$. The entrepreneur will refrain from triggering verification only if $(1 - \alpha_n)(x - c) \leq x - \alpha_n x_n$, or

$$x_n - x \leq c(1 - \alpha_n)/\alpha_n, \quad (6)$$

which then constitutes the entrepreneur's incentive compatibility constraint.

Let m'_n denote the set of states for which the entrepreneur's IC condition is not satisfied; that is, let $m'_n = \{\omega \in \Omega : x < x_n - c(1 - \alpha_n)/\alpha_n\}$. As depicted in Figure 2, m'_n intersects V_n . The set $m_n = m'_n \setminus (m'_n \cap V_n)$ constitutes therefore the set of states for which the entrepreneur triggers a verification outside the set V_n .

Lemma 3 $\lim_n P(m_n) = 0$.

Proof: Since $m_n \subset m'_n$, if we can prove that $\lim_n P(m'_n) = 0$, we can conclude that $\lim_n P(m_n) = 0$. To see that $\lim_n P(m'_n) = 0$, write the converse to condition (6) as

$$x - x_n > c(1 - \alpha_n)/\alpha_n. \quad (7)$$

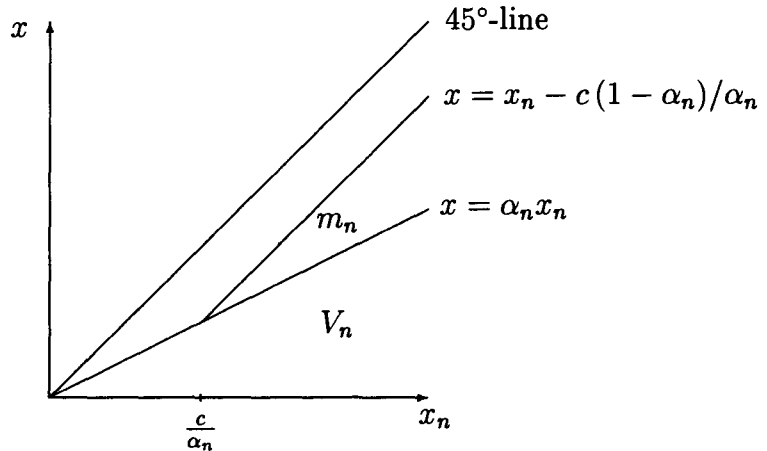


Figure 2: The set m_n for which the investor's IC constraint will not be satisfied.

Now, by the fact that x_n converges to x almost surely (Lemma 1), we know that the left side of (7) converges to zero almost surely. By the fact that convergence almost surely implies convergence in probability, and since the right side of (7) is bounded away from 0 by our restrictions that $\alpha_n < 1$ and $c > 0$, it follows that $\lim_n P(m'_n) = 0$ and therefore that $\lim_n P(m_n) = 0$. \square

In other words, according to the lemma, in the limit as the information observed by the investor becomes perfectly correlated with true cash flows, the entrepreneur has no incentive to induce verification outside the set V_n .

3.2.3 The Cost (and Feasibility) of Outside Equity

Given the structure proposed for the outside equity contract, the expected cash flow which is *accessible* to the investor is given by

$$E_n(x) = \int_{\hat{V}_n^c} x_n dP + \int_{\hat{V}_n} (x - c) dP, \quad (8)$$

where $\hat{V}_n = (V_n \cup M_n \cup m_n) \setminus (M_n \cap m_n)$, $\hat{V}_n^c = \Omega \setminus \hat{V}_n$, and $E_n(x) \leq E(x)$. This expression can be rearranged to

$$E_n(x) = E(x) - \int_{\hat{V}_n^c} (x - x_n) dP - c \int_{\hat{V}_n} dP, \quad (9)$$

where the first term represents expected cash flows and the third term expected verification costs. The second term in (9) can be decomposed into $\int_{\hat{V}_{ne}^c} (x - x_n) dP$ and $\int_{\hat{V}_{nl}^c} (x - x_n) dP$, where $\hat{V}_{ne}^c \equiv \hat{V}_n^c \cap \{x \geq x_n\}$ and $\hat{V}_{nl}^c \equiv \hat{V}_n^c \cap \{x < x_n\}$ so that \hat{V}_{ne}^c

represents the set of states for which the entrepreneur diverts cash from the investor and \hat{V}_{nI}^c represents the set of states for which the equity contract calls for a larger dividend payment than the dividend that would be paid out under complete contracting.⁹

The expected wealth of the entrepreneur can be expressed by

$$E(W_e) = (1 - \alpha_n)E_n(x) + \int_{\hat{V}_n^c} (x - x_n)dP, \quad (10)$$

where the first term constitutes the expected dividend associated with his equity claim and the second term represents a combination of the expected amount of cash that the entrepreneur diverts and the expected amount of excess dividends to be paid out.

As the firm's outside owner, the investor receives $\alpha_n E_n(x)$, where α_n is determined implicitly from

$$I = \alpha_n E_n(x). \quad (11)$$

Note that the α_n implied by (11) need not be unique. Define therefore the set $A \equiv \{\alpha_n : I = \alpha_n E_n(x) \text{ and } 0 < \alpha_n < 1\}$ and further $\alpha_n^* \equiv \inf A$. It can be ascertained directly from \hat{V}_n that $P(\hat{V}_n)$ is decreasing in α_n , and therefore that the entrepreneur will always want to choose $\alpha_n = \alpha_n^*$.

Substituting (11) into (10) yields

$$E(W_e) = E(x) - I - c \int_{\hat{V}_n} dP, \quad (12)$$

which consists of the NPV of the project less verification costs.

As already noted, $E_n(x)$ represents the expected cash flow that is accessible to the outside owner given the structure of the contract and given the optimal strategies of the entrepreneur and the investor. Outside equity is thus feasible if $E_n(x) > I$. Furthermore, to the extent that $E_n(x) = E(x)$, so that $P(\hat{V}_n) = 0$, the outside equity contract will fund any strictly positive NPV project. As indicated by the following proposition, this is indeed the case so long as the information observed by the investor is sufficiently precise.

Proposition 1 $E_n(x)$ converges monotonely to $E(x)$ (a.s.)

⁹As we recall, the equity contract will in certain states call for a payment to the investor that exceed the dividend that would be paid out under complete contracting. The set of states for which this occurs is depicted in Figure 1 as the set V_n^c . This set is cut into by the set m_n (Figure 2), which describes the set of states for which the entrepreneur opts for verification rather than distributing excess dividends.

Proof: Consider

$$E(x) - E_n(x) = \int_{\hat{V}_n^c} (x - x_n) dP + [P(V_n) + P(M_n) + P(m_n) - P(M_n \cap m_n)] c. \quad (13)$$

The proposition requires that each of the terms on the right hand side of (13) converge to zero.

Assume first that $\alpha_n = \alpha \in (0, 1)$. The convergence of $P(m_n)$ and $P(M_n)$ is given by Lemma 2 and Lemma 3, respectively. The convergence of $P(M_n \cap m_n)$ to zero follows accordingly. Consider then $P(V_n)$. To see that $\lim_n P(V_n) = 0$, note that V_n converges to \emptyset by account of $\alpha < 1$ and by the fact that x converges to x_n with probability 1 (Lemma 1). Consider finally the integral $\int_{\hat{V}_n^c} (x - x_n) dP$. To see that this integral converges to zero note first that the (a.s.) convergence of x_n to x (Lemma 1) applies on any subset of Ω (such as V_n^c); the convergence of the integral then follows by standard rules of the integral.

Let now $\alpha_n = \alpha_n^*$. Since $E_n(x)$ is a decreasing function of α_n , to make sure that the convergence is monotone, we need to make sure that α_n^* is decreasing in n (else we may have a range for which $E_n(x)$ is decreasing in n). To see that α_n^* is indeed a decreasing function of n , let $n = n'$ so that α_n^* is determined by

$$\alpha_{n'}^* E_{n'}(x; \alpha_{n'}^*) = I. \quad (14)$$

Increase then n from n' to n'' . Since $E_n(\cdot; \cdot)$ is increasing in n (holding α_n constant), it must be the case that

$$I < \alpha_{n'}^* E_{n''}(x; \alpha_{n'}^*). \quad (15)$$

To see that $\alpha_{n'}^* \leq \alpha_{n''}^*$, and thus that α_n^* is a decreasing function of n , note by the definition of α_n^* that

$$I = \alpha_{n''}^* E_{n''}(x; \alpha_{n''}^*), \quad (16)$$

$$\alpha_{n''}^* E_{n''}(x; \alpha_{n''}^*) \geq \alpha_n E_{n''}(x; \alpha_n) \quad \text{for all } \alpha_n \leq \alpha_{n''}^*, \quad (17)$$

and

$$\alpha_{n''}^* E_{n''}(x; \alpha_{n''}^*) < \alpha_n E_{n''}(x; \alpha_n) \quad \text{for } \alpha_n \in (\alpha_{n''}^*, \bar{\alpha}_n] \quad (18),$$

where $\bar{\alpha}_n$ is such that if $\alpha_{n''}^*$ is a unique element of A then $\bar{\alpha}_n = 1$; otherwise, $\bar{\alpha}_n < 1$.

Now, by (15) and (16) we obtain

$$\alpha_{n''}^* E_{n''}(x; \alpha_{n''}^*) < \alpha_{n'}^* E_{n''}(x; \alpha_{n'}^*),$$

which by (18) implies that $\alpha_n^* < \alpha_{n'}^*$. This leads in turn to the conclusion that α_n^* is a decreasing function of n . \square

Thus, in the limit, as the information observed by the investor becomes perfectly correlated with cash flows, $E_n(x)$ converges to $E(x)$ and the probability of verification, $P(\hat{V}_n)$, converges to zero. This means that first best obtains in the limit and that all positive NPV projects receive funding. Note that this result obtains despite the fact that cash flows are costly to verify and despite the fact that the information observed by the investor cannot be verified, even at a cost. This is in stark contrast to what obtains in the case of debt financing, for which the contracting costs are unrelated to the information observed by the investor.

The fact that the convergence is monotone ensures that the expected verification costs associated with outside equity, $cP(\hat{V}_n)$, are decreasing in n for all $n \geq 0$ and therefore that there exists an $n^* < \infty$ such that $cP(\hat{V}_n) \leq cP(V_D)$ for $n \geq n^*$ and $cP(\hat{V}_n) > cP(V_D)$ for $n < n^*$. That is, there exists a critical level of information precision such that the project will be equity financed if the precision of the information observed by the investor is above this level and otherwise debt financed. Note also that $E_n(x)$ being an increasing function of n implies that it may be the case that outside equity will not be feasible for n sufficiently small. This follows by condition (11), which implies that outside equity will not be feasible unless $E_n(x) > I$. Thus, if $E_0(x) < I$ there exists a critical $n^{**} > 0$ such that outside equity is feasible if only if $n \geq n^{**}$.

Debt preference for sufficiently low n is consistent with Gale and Hellwig (1985) who show that the optimal contract takes the form of the standard debt contract when the information observed by the investor is uncorrelated with cash flows. To compare the standard debt contract and the outside equity contract in this case, observe that outside equity will yield $z_0 = \alpha_0 E(x)$ for $x \geq \alpha_0 E(x)$ and $\alpha_0(x - c)$ otherwise. Thus, outside equity yields a fixed payment in the non-verification region and a fraction α_0 of net assets $x - c$ if cash flows are verified. The probability of verification is therefore given by $P(x < \alpha_0 E(x))$.¹⁰ Consider then the standard debt contract. This contract yields D in the non-verification region but allows the investor 100% of net assets in

¹⁰Note that since the equity contract is not incentive compatible with probability one in this case, $P(x < \alpha_0 E(x))$ under-estimates the actual verification probability; however, the comparison between the two contracts is more easily facilitated by ignoring this.

the verification region—the defining features of the optimal contract. Since now the standard debt contract yields a higher payment than the outside equity contract if cash flows are verified, its fixed payment D will be correspondingly lower. In other words, we have that $D < \alpha_0 E(x)$ and therefore that $P(x < D) < P(x < \alpha_0 E(x))$.

4 Concluding Remarks

Using a CSV setup the present paper develops a model in which the investor-outsider has access to noisy and non-verifiable information regarding true cash flows. The model shows that the outside equity contract permits dividend payments contingent on the information observed by the investor even though the information observed by the investor cannot be verified, even at a cost. In the limit as this information becomes perfectly correlated (almost surely) with true cash flows and the probability that dividend payments are contingent it converges to one. In the limit, therefore, cash flows are never verified, contracting costs associated with outside equity are zero, and first-best (investment levels) obtains.

Moreover, the paper compares the performance of the outside equity contract to that of the standard debt contract. It shows that the entrepreneur will prefer outside equity if the information available to the investor is sufficiently precise and debt otherwise. Consistent with this result, it is well known and perhaps obvious (but not obvious from a contracting point of view) that increased reliance on equity markets relative to bank funding puts greater demands on the quality of firms' accounting reports and disclosure policies. Consider for example the common distinction between the bank based economies of Germany and Japan and the stock based economies of Britain and especially the United States, for which the accounting standards are consider more stringent than those found in the Germany and Japan.

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CHAPTER 3

The Choice Between Debt and Outside Equity in a Costly State Verification Model *

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Abstract

This paper examines and compares the ability of the outside equity and debt contracts to allow payments contingent on non-verifiable interim information in a model in which cash flows are costly to verify. It is shown that high NPV projects will be funded with outside equity, while lower NPV projects will be funded with debt, provided that they are not too risky. Outside equity allows interim payments that are more sensitive to new information than does debt. This ensures that equity is the least costly and thus the preferred funding alternative. Debt, however, emerges as the funding tool of last resort as it is able to fund projects that cannot feasibly be funded with outside equity.

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1 Introduction

Equity payments are generally contingent on non-verifiable, though possibly jointly observable, information. Since non-verifiable information cannot in general be part of an enforceable contract, the fact that payments to shareholders typically are based on non-verifiable information raises the question of what type of enforcement mechanism is in place in practice to induce payments contingent on states that cannot be verified. The present paper explores the structure of an equity-type contract that permits payments contingent on non-verifiable information and compares the performance of this contract to that of debt. The analysis is undertaken in a costly state verification (CSV) model in which cash flows are costly to verify but in which all participants observe noisy interim information that cannot be verified.¹ The paper shows that high NPV projects will be funded with equity, while lower NPV projects will be funded with debt. For the range of risk levels considered, it is shown that while the feasibility constraint of the outside equity contract is unaffected by the level of project risk, projects may be denied debt funding if project risk becomes too high.

Following Aghion and Bolton (1992), a financial contract is described by the types of control and cash flow rights that it confers to the investor.² As such, debt gives the investor fixed a cash flow right and control contingent on the fixed debt payment not being paid in full. Outside equity, on the other hand, carries a proportional cash flow right and non-contingent control.³ The latter creates the potential for the shareholder to extract rents ex-post from the project owner by calling for verification in states for which this is not optimal ex ante. At the same time, since the size of the payment to the equity holder is based on non-verifiable information, the project owner may op-

¹The CSV framework is due to Townsend (1979) and Gale and Hellwig (1985); see Allen and Winton (1995) for a survey of the CSV literature. The present model differs from the standard CSV framework by introducing interim information.

²A control right in the present context provides the investor with a right to call for verification. A (state-) contingent control right allow the investor to call for verification subject to the occurrence of a verifiable event (such as failure to make a required debt payment); if this event does not occur, the investor cannot call for verification. A non-contingent control right, on the other hand, gives the investor a state independent right to call for verification. In the present context such a non-contingent control right is the same thing as an ownership right.

³Common stocks generally carry voting rights. In the present model it is assumed that the investor holds a majority (voting) stake which makes him able to veto any interim payment proposed by the project owner.

opportunistically offer dividend payments below the level implicitly agreed upon ex ante. Despite these concerns, it is shown that the type of proportional cash flow rights and non-contingent control associated with outside equity as observed in practice produce an incentive compatible contract that permits payments contingent on non-verifiable information.

The formal model has three dates. On the initial date, information is symmetric, contracts are signed, and the project is started. On the interim date, investors and the project owner both observe partial information regarding future (date 2) cash flows. Though jointly observable, this information is non-verifiable and thus cannot be used as a basis for an enforceable contract. On the final date, cash flows are realized and become observable to the project owner at no cost while observable to investors only if verified, which is costly. Gale and Hellwig (1985) derive the optimal contract under costly state verification and show that this contract resembles “the standard debt contract with bankruptcy.” The implication of Gale and Hellwig for the present setting is that the project owner will fund interim cash payments by debt, regardless of whether the project is funded initially with debt or equity.

The paper explores two types of funding schemes for the economic environment just described. Under (pure) debt financing the initial investment outlay is funded with a one-period debt claim, which is refunded on date 1 by issuing a second one-period debt claim. To the extent that riskless debt cannot be issued (which will be the case), state-contingent final payments are achieved via the contingent control feature associated with the standard debt contract. This occurs because the initial debt claim will carry a payment that will be repaid in full only in ‘good’ states. In the ‘bad’ states control is transferred to the initial creditor who will fully utilize the firm’s debt capacity by raising as much cash as possible. We will see that this feature of the debt contract makes it possible for the project owner to fully utilize the firm’s debt capacity in all interim states, in turn making debt capable of funding projects that cannot feasibly be funded with equity. However, the same feature that delivers debt as the more robust contract also makes it the more costly one. The reason for this is that the short-term debt contract provides the investor with too much bargaining power on the interim date in the bad state, allocating too much payment to the bad state and too little to the good state. In

short, the payments associated with pure debt financing are not sufficiently responsive to interim information compared to outside equity.

Under outside equity the project owner issues an equity claim on the initial date and then repurchases this claim on the interim date by offering the equity holder a cash payment. The funds needed for this payment is raised by issuing a one period debt claim due on the final date (or, alternatively, by offering the equity holder a debt claim in exchange for the equity claim).⁴ Consistent with what is observed in practice, equity payments are shown to be more sensitive to interim information relative to debt payments. This greater sensitivity to interim information has the effect of producing a more efficient state allocation of expected verification costs and ensures that the use of equity will give a reduction in expected verification costs compared to pure debt financing. However, because there are parameters values for which pure debt financing is feasible while outside equity is not (and the opposite is never the case), debt will be issued in cases in which equity financing is not feasible, which is when the NPV of the project falls below a critical level.

The paper is related to Chiesa (1992), who develops a moral hazard-incomplete contracting model in which cash flows are verifiable at zero costs but in which interim information is non-verifiable and jointly observable. She shows that debt payments can be made contingent on non-verifiable information (and indeed implement first-best) by attaching to the standard debt contract a warrant for the lender and a cash/equity settlement option for the borrower. The present paper shows that an alternative contract—the outside equity contract—allow (incentive compatible) payments that are more sensitive to interim (non-verifiable) information than does debt. As in Chiesa (1992), the resulting contract gives an improvement relative to pure debt financing by allocating higher payments to higher states more efficiently than does debt; however, the control structure associated with this contract is different.

While a number of authors examine the structure debt contracts and their ability to extract payment from the borrower, few examine the outside equity contract (under incomplete contracting) and the ability of equity to extract payment from the project

⁴Although the payment to the equity investor is appropriately interpreted as a payment associated with an equity repurchase in which all of the firm's outside equity is repurchased on the interim date, I will at times refer to this payment as a "dividend" payment.

owner.⁵ Exceptions include Fluck (1995) and Chang (1993). Fluck examines debt and outside equity in the context of the Hart and Moore (1989) model in which cash flows are publicly observable but non-verifiable. Consistent with what is observed in practice, she shows (among other things) that an incentive compatible equity contract must have infinite maturity. Chang (1993) derives the optimal contract between investors and management in a CSV-like model to show that the optimal contract exhibits a payoff function that can be implemented using a combination of debt and outside equity. The present paper analyzes the outside equity contract in an incomplete contracting environment in which cash flows are costly to verify and shows that the structure of the equity contract compared to the debt contract allows interim payments that are more sensitive to new information thus making outside equity the least costly (but least robust) the alternative funding sources available to the project owner.

The paper is organized as follows. Section 2 presents the basic model. Section 3 describes and analyzes the debt and equity contracts. Section 4 analyzes the project owner's optimal contract choice. Section 5 concludes the paper.

2 The Economic Environment

The model has three dates, denoted 0, 1, and 2. An entrepreneur is endowed with a project that yields a random date 2 cash flow x and requires an initial outlay of I . The entrepreneur has zero funds and will need to raise at least I to get the project started. It is assumed that x is uniformly distributed on the interval $[l, h]$, where $0 < l < h$. Let $G(x)$ and $g(x)$ denote the cumulative distribution and density functions for x and let $E(x) = m$ denote its unconditional expectation, where $m = (h + l)/2$ by the properties of the uniform distribution function. It is assumed that the distributional characteristics of x are common knowledge, that all agents are risk neutral, and that the riskless interest rate is zero. Finally, to rule out riskless debt, as we shall see, it is assumed that $I > (m+l)/2$.

Information over the three dates unfolds as follows. On date 0, the size of the initial investment as well as the distributional characteristics of x are common knowledge. On

⁵See Gale and Hellwig (1985) and Hart and Moore (1989) for complementary theories of debt. Bolton and Sharfstein (1996) defend the focus on debt contracts by arguing that firms usually raise capital by issuing debt rather than equity. This argument, however, ignores the fact that equity financing by retaining earnings is the predominant funding source in most countries (see Eckbo and Masulis [1995]).

date 1, the project generates a non-verifiable but jointly observable information signal $S \in \{s_H, s_L\}$, where $S = s_H$ implies $x \in (m, h]$ and $S = s_L$ implies $x \in [l, m]$. The firm generates no cash on date 1 (but may raise cash on this date by issuing a debt claim to be repaid on date 2). On date 2, the entrepreneur observes x without cost while the investor must expend *verification costs*, denoted c , to verify (and observe) the cash flow x .⁶ It is assumed that $c \leq l$ and $c < (h - l)/2$. While the first constraint is imposed to ensure that there are always sufficient resources left in the firm to cover the cost of verifying cash flows, the second is imposed to ensure that the maximum amount of funds that can be raised exceeds the amount that can be raised issuing riskless state contingent debt (see footnote 10).

Gale and Hellwig (1985) derive the optimal contract under costly state verification and show that the optimal contract takes the form of the standard debt contract with bankruptcy. In the present context, the result derived by Gale and Hellwig implies that the optimal contract into date 2, and therefore the optimal contract to fund any interim payments, is the standard debt contract. The standard debt contract is characterized by a fixed payment d_i such that if $x \geq d_i$ then the creditor receives the contractual payment d_i in full, whereas if $x < d_i$, the project owner is unable to satisfy the scheduled debt payment in full, in which case he declares bankruptcy and the creditor verifies and collects the net cash flow $x - c$.⁷

Let $V_i(d_i)$; $i = \{H, L\}$, denote the date 1 value of a debt claim with (date 2) debt payment d_i issued in state i . The expressions for debt values $V_i(d_i)$; $i = \{H, L\}$, are now given by

$$V_H(d_H) = d_H \int_{d_H}^h \frac{1}{h - m} dx + \int_m^{d_H} (x - c) \frac{1}{h - m} dx \quad (1)$$

and

$$V_L(d_L) = d_L \int_{d_L}^m \frac{1}{m - l} dx + \int_l^{d_L} (x - c) \frac{1}{m - l} dx. \quad (2)$$

The firm's debt capacity is $d_i^* = \operatorname{argmax} V_i(d_i)$. The largest amount of cash that can be

⁶Although a more general specification of verification costs is possible in the current setting, this would complicate the analysis without providing further insights.

⁷As is common in the literature, I *assume* that the creditor is able to commit to verify if he does not receive the scheduled debt payment in full. This assumption is usually rationalized on grounds that lenders generally has reputational capital stake that will be lost if accepting payments below those stated in the contract. Note that this assumption does not rule out renegotiation based on jointly observable information, such as the information represented by S .

raised on date 1 in state i is thus $V_i(d_i^*)$.⁸

Lemma 1 $(d_H^*, d_L^*) = (h - c, m - c)$.

Although increasing the date 2 debt payment d_i beyond d_i^* will increase the lender's revenues in the non-bankrupt state, such an increase will also increase the probability that the firm will end up in bankruptcy and thus increase expected default costs. The lemma shows that this balance of higher revenues in the non-bankrupt state against a higher probability of default implies that the debt payment that maximizes the value of debt will provide an optimal default probability that is strictly less than one.⁹

Both (pure) debt and equity financing yield interim payments. To the extent that interim payments are state contingent, the amount of debt issued on the interim date will be state contingent as well. Let p_i^j , $i \in \{H, L\}$, $j \in \{d, e, u\}$, denote the state i date 1 payment associated with contract j , where $j = d$ denotes debt and $j = e$ denotes equity. Finally, $j = u$ will be used for the benchmark case when S is verifiable and enforceable contracts therefore can be written directly on the realization of S . To rule out riskless debt, it is assumed that $p_H^j > m$ and $p_L^j > l$ for all $j \in \{d, e, u\}$.¹⁰

By the fact that S cannot be verified, the payments p_i^j , $i \in \{H, L\}$ associated with a contract $j \in \{d, e\}$ are incentive constrained and therefore determined by the type of control and cash flow rights that contract j confers to the investor. In contrast, the interim payments associated with contract u are derived under the assumption that S is verifiable and are thus not incentive constrained.

The payments p_i^j associated with claim j issued on the initial date give rise to date 2 debt payments $d_i^j = d(p_i^j)$, which are determined from

$$p_i^j = V_i(d_i^j). \quad (3)$$

With payments d_i^j , the expected (date 2) verification costs associated with claim j will now be

$$E^j(c) = \left(\frac{d_H^j - m}{h - l} + \frac{d_L^j - l}{h - l} \right) c. \quad (4)$$

⁸Proofs in the Appendix unless stated otherwise.

⁹Note that the parameter constraint imposed earlier that $c < (h - l)/2$ implies $d_H^* = h - c > m$ and $d_L^* = m - c > l$.

¹⁰It turns out that this will be the case if $I > (m + l)/2 = E(x|s_L)$, which I assume to be the case.

The date 0 value of the project owner's inside equity claim associated with claim j will be $E^j(\max[x - d_i^j, 0])$. His problem on the initial date is to pick the contract $j \in \{d, e\}$ that solves

$$\max_j E^j(\max[x - d_i^j, 0]), \quad (5)$$

subject to the break even constraint

$$\frac{1}{2}p_H^j + \frac{1}{2}p_L^j = I, \quad (6)$$

and subject to feasibility and incentive constraints (both of which will be examined in detail below). Substituting from (3) and (6) into (5) yields for (5) the expression

$$\max_j m - I - E^j(c), \quad (7)$$

which shows that the optimal contract is the one that minimizes expected verification costs. Note that $E^j(c)$ implicitly takes into account the fact that payments $p_i^j, i \in \{H, L\}$ are incentive constrained (and compatible).

3 Verifiable Interim Information

This section examines the benchmark case in which the interim information signal S can be verified at zero cost, in which case enforceable contracts can be written directly on the realization of S .

Let $p_i^u, i = H, L$, denote the optimal date 1 payments associated with the unconstrained contract u . Since these payments are assumed to be contingent directly on S , they are not incentive constrained and are therefore determined as the set of payments that minimizes expected verification costs subject to the zero profit condition $E(p_i^u) = I$. The following lemma presents the solution to this problem.

Lemma 2 *The interim payments (p_H^u, p_L^u) associated with the case in which S can be verified at zero cost are given by*

$$p_H^u = \frac{1}{4}(h - l) + I$$

and

$$p_L^u = -\frac{1}{4}(h - l) + I;$$

furthermore, the resulting debt payments $(d(p_H^u), d(p_L^u))$ allocate expected verification costs equally across states.

The existence of an optimal allocation of debt payments across states is a direct implication of the concavity of $V_i(d_i)$, $i = H, L$. Though the symmetry result of Lemma 2 is a bit special, it is in part a consequence of the special assumptions made with respect to verification costs (being unrelated to the cash flow) and the structure of the interim information signal. Nonetheless, the role of the optimal contract—which is to provide a benchmark to which contracts $j \in \{d, e\}$ can be compared—does not depend on the symmetry result of Lemma 2. Such a comparison between the payments associated with contracts $j \in \{d, e\}$ and the payments associated with contract u will reveal that the latter are ‘more’ state contingent than the payments associated with the incentive constrained contracts $j \in \{d, e\}$ and therefore that $E^j(c) > E^u(c)$ for $j \in \{d, e\}$.¹¹

To compare the contracts in terms of their interim payments, it will be useful to define an arbitrary contract δ , which is issued with payments $(p_H^\Delta, p_L^\Delta) = (p_H^u - \Delta, p_L^u + \Delta)$, where $\Delta \geq 0$, denote interim payments associated with the contract δ that allocates more (less) default probability to the H -state (L -state) compared to the optimal (cost minimizing) payments (p_H^u, p_L^u) of the unconstrained contract. Note that since payments (p_H^Δ, p_L^Δ) are determined from the break-even constraint $E(p_i^\Delta) = I = E(p_i^u)$, any set of payments (p_H^j, p_L^j) associated with actual contracts can be expressed by the vector $(p_H^u - \Delta^j, p_L^u + \Delta^j)$.¹²

Lemma 3 *The expected verification costs associated with payments $(p_H^\Delta, p_L^\Delta) = (p_H^u - \Delta, p_L^u + \Delta)$ are strictly increasing in Δ .*

Δ^j represents a deviation of contract j from the optimal contract. To the extent that $p_L^j \geq p_L^u$ and $p_H^j \leq p_H^u$ (which will be the case), the useful implication of Lemma 3 is that for any two contracts with payments $(p_H^{\Delta_1}, p_L^{\Delta_1})$ and $(p_H^{\Delta_2}, p_L^{\Delta_2})$ with $\Delta_2 > \Delta_1 > 0$ so that $p_H^{\Delta_1} > p_H^{\Delta_2}$ and $p_L^{\Delta_1} < p_L^{\Delta_2}$ it follows that $E^{\Delta_2}(c) > E^{\Delta_1}(c)$. Lemma 3 is thus a formalization of the (perhaps obvious) idea that the further away payments (p_H^j, p_L^j) are from (p_H^u, p_L^u) , the larger are expected verification costs.

¹¹Note that this result extend to the case in which insolvency costs are specified in a more general way than what I have done here, such as letting c be increasing in x .

¹²An implicit assumption here is that actual contracts imply $\Delta > 0$. Lemma 4 in the Appendix shows that this is indeed the case.

4 Non-Verifiable Interim Information

When the interim information signal S is non-verifiable, contracts written directly on S is not enforceable. The present section examines the ability of the outside equity and debt contracts to allow state-contingent payments in the case when S cannot be verified.

4.1 Outside Equity

Outside equity confers a proportional cash flow right and non-contingent control to the investor. Let α denote the proportional cash flow right associated with the outside equity contract. Given α and given that non-contingent control enables the investor to refuse to accept the payment p_j^e offered by the project owner, the shareholder's date 1 state contingent reservation value is given by $U_i = \alpha(E(x|s_i) - c)$, which represents the value of the equity claim in state i if held until date 2 and verification is requested for sure.¹³ ¹⁴ It is assumed that the payment $p_i^e, i = H, L$ is determined by the project owner in a "take-it-or-leave-it" offer on date 1. If this offer is accepted (which it will be in equilibrium) then the project owner issues a debt claim with face value $d_i^e = d(p_i^e)$. This debt payment is due on date 2 and raises on the interim date just enough cash to make the required dividend payment of p_i^e .

In determining the size of the equity payment, the project owner will pick the minimum possible payment that will be accepted by the shareholder. This gives $p_i^e = U_i, i = H, L$ since any $p_i^e > U_i$ is wasteful from the perspective of the project owner and $p_i^e < U_i$ is unacceptable to the shareholder. Hence, α is determined from $I = E(p_i^e)$, which gives $\alpha = I/(m - c)$. The following proposition summarizes the characteristics of outside equity.

Proposition 1 *Suppose that the shareholder contributes I funds in return for a proportional cash flow right α as well as a non-contingent right to demand a verification, then*

¹³One may view 'verification requested for sure' as an assumption. The important point is that Gale and Hellwig (1985) derive the optimal contract as the standard debt contract; the implication of which is that any alternative contract (such as outside equity) will imply a strictly higher verification probability.

¹⁴When dealing with the equity contract we may imagine the costs c as being disclosure costs rather than bankruptcy costs; in particular, costs associated with having to disclose proprietary information. In this case, U_i will represent the date 1 value of outside equity net of disclosure costs. As an alternative, the shareholder may be given the right to dismiss the entrepreneur if he is unsatisfied with the dividend offer. In this case, $E(x|s_i) - c$ may be interpreted as the date 1 value of the firm under new management.

the date 1 payment to the equity holder will be given by

$$p_i^e = \alpha(E(x|s_i) - c), \quad i = H, L$$

where α is determined from $I = E(p_i^e) = \alpha(m - c)$.

While debt is issued with a contractually specified fixed payment, outside equity is not. Nonetheless, the non-contingent control feature associated with the outside equity contract provides the equity investor with a credible threat to impose high costs on the firm should the project owner fail to make an acceptable offer to the outside equity holder. Indeed, by making an acceptable offer, the value of the firm is increased by

$$g_i = c - E(c|d_i^e), \quad (8)$$

where $E(c|d_i^e)$ represents expected verification costs conditional on $S = s_i$.

4.2 Debt

In the case of pure debt financing, the initial claim will carry a fixed payment d_1 , to be paid on date 1. It will be the case, however, that the scheduled payment d_1 is too large to be satisfied in full in the L -state, which implies that control will be transferred to the creditor who in turn will raise $V_L(d_L^*)$ thus fully utilizing the firm's state L debt capacity.¹⁵ Furthermore, since $V_L(d_L^*)$ represents the maximum state L debt value, and thus represents the maximum amount of cash that can be raised on the interim date, it follows that the initial short-term claim will yield state contingent payments $V_L(d_L^*)$ for the L -state and d_1 for the H -state.¹⁶ The debt payment d_1 is now determined from

¹⁵If $d_1 \leq V_L(d_L^*)$, then riskless debt would be possible and the contracting problem between the investor and the project owner would exist. Of course, the fact that the contracting problem disappears if riskless debt can be used is due to the assumption that the lender will verify whenever offered a payment that is less than the payment promised ex ante, which in turn induces the project owner to make the scheduled payment whenever there are funds to do so. See, however, Hart and Moore (1994) for a theory of debt in which cash flows are riskless and non-verifiable and in which the creditor's incentives to intervene are endogenized.

¹⁶We can think of this as the project owner either declaring bankruptcy and the creditor intervening to issue a new debt claim worth $V_L(d_L^*)$ or the project owner offering the payment $V_L(d_L^*)$ to the creditor who will accept the offer since $s = s_L$ is jointly observable. In either case, the project owner is left holding an inside equity claim with positive value. In reality, the creditor may want to extract $V_L(d_L^*)$ and in addition sell the remaining inside equity claim in the market. To avoid this possibility, assume that there is a separation between management skills and funding ability so that although there may be able managers available on the interim date they have no funds and must consequently be given free equity. Alternatively, we may assume that the date 2 cash flow can only be generated under the leadership of the original founder.

the break even constraint

$$\frac{1}{2}d_1 + \frac{1}{2}V_L(d_L^*) = I, \quad (9)$$

for which there exists a solution as long as there exists a d_1 that is less or equal to d_H^* .

5 The Choice of Contract

The project owner has the choice of funding the project with debt or equity and will choose the contract that minimizes expected verification costs, provided, of course, that the contract is feasible. The present section compares the available contracts in terms of expected verification costs and feasibility.

5.1 Expected Verification Costs

Proposition 2 compares the expected verification costs implied by the outside equity contract to the costs implied by short-term and long-term debt.

Proposition 2 *The expected verification costs implied by outside equity are strictly below the expected verification costs associated with debt.*

Proof: According to Lemma 3, the proposition follows if it can be shown that $p_L^e < V_L(d_L^*) < d_1 < p_H^e$. By the date 0 break even constraint, this inequality holds if either $p_L^e < V_L(d_L^*)$ or $d_1 < p_H^e$. To see that $p_L^e < V_L(d_L^*)$, use the expression for p_L^e as given in Proposition 1 and insert $d_L^* = m - c$ into the expression for $V_L(\cdot)$ to find that $p_L^e < V_L(d_L^*)$ implies

$$\alpha\left(\frac{m+l}{2} - c\right) < \frac{m+l}{2} - c + \frac{c^2}{2(m-l)},$$

which is clearly satisfied since $\alpha \leq 1$. □

By Lemma 3, the result that equity is less costly than debt follows because the equity contract allows payments that are more responsive to interim information than what is allowed for by the debt contracts. The result that equity payments are more sensitive to new information than are debt payments squares well with the notion of equity being the riskier claim. Although this is usually attributed to equity being a residual claim while

debt is a fixed claim, here it has been generated in an incomplete contracting framework and shown to result solely from differences in contract structures.¹⁷

5.2 Feasibility

5.2.1 Outside Equity

Outside equity is feasible to the extent that there exists an $\alpha \leq 1$ that solves the investor's break even constraint $I = E(p_i^e)$. This implies that outside equity is feasible if

$$m - I \geq c. \quad (10)$$

Condition (10) results from the fact that the maximum payment that the project owner can credibly offer the equity investor in state i is $E(x|s_i) - c$, the expected value of which is $m - c$, which must exceed the initial investment outlay I .

5.2.2 Debt

Under pure debt financing, the initial debt claim is issued with a date 1 payment d_1 determined by (9). This payment cannot exceed $V_H(d_H^*)$, which represents the maximum cash that can be raised on date 1 in state H . Since the interim debt payment in the L -state is given by $V_L(d_L^*)$, the feasibility condition for short-term debt is given by

$$\frac{1}{2}V_H(d_H^*) + \frac{1}{2}V_L(d_L^*) \geq I, \quad (11)$$

where the left hand side of (11) represents the date 0 value of a one period debt claim issued with a payment $d_1 = V_H(d_H^*)$ and thus pays $V_H(d_H^*)$ in the H -state and $V_L(d_L^*)$ in the L -state. A debt claim issued with a contractual payment $d_1 = V_H(d_H^*)$ fully utilizes the firm's debt capacity in each state. Inserting the expressions for d_i^* from Lemma 1 and using the expressions for $V_H(\cdot)$ and $V_L(\cdot)$ from (1) and (2) turns (11) into

$$m - I \geq c \left(1 - \frac{c}{h - l}\right), \quad (12)$$

which then represents the feasibility condition for short-term debt.

¹⁷I have also considered a long-term debt claim that is renegotiable on the interim date. For the range of parameter values that equity is feasible, the final payments implied by this contract is less variable than the final payments associated with equity financing, in which case Lemma 3 applies to allow the result that equity yields lower default costs on average compared to (long-term) debt. However, for the parameter values for which the outside equity is not feasible, the final payments associated with long-term debt is more variable than the unconstrained contract. This is unlike short-term debt whose final payments are always less variable than the payments implied from the unconstrained contract.

5.3 Debt versus Outside Equity

Comparing the feasibility conditions of outside equity and debt, we observe that debt can feasibly fund projects that will be denied equity funding, while the opposite is never the case. Given the result of Proposition 2 that $E^e(c) < E^d(c)$, the project owner will have a preference for equity over debt and will therefore choose equity whenever equity is feasible. The following proposition summarizes the contract choice of the project owner.

Proposition 3 *The project will be funded with equity if*

(i) $m - I \geq c$, and with debt if

(ii) $m - I \in \left(c \left(1 - \frac{c}{h-l} \right), c \right)$.

Finally, if $m - I \in \left(0, c \left(1 - \frac{c}{h-l} \right) \right)$, the project will receive no funding.

The project owner's choice of funding source is further depicted in Figure 2. Since equity is the least costly alternative (Proposition 2), the project will be funded with equity whenever equity is feasible, which it is provided the project NPV is sufficiently large (i.e. $m - I \geq c$). For a project that is not sufficiently profitable to be funded with outside equity, it will be funded with debt if it is not too risky and sufficiently profitable that $m - I \geq c \left(1 - \frac{c}{h-l} \right)$.

Measuring project risk by $(h - l)$, we observe that while the feasibility condition of outside equity is unaffected by increased risk, the feasibility of debt tightens. In other words, holding project profitability constant, there will be projects that will be denied debt finance if the risk of the project becomes too high, while this is not the case with equity.

The results derived thus reveal preference for equity over debt. This is contrary to the ordering implied by the well known 'pecking order' theory proposed by Myers and Majluf (1984), who argue that a firm in need of external finance should choose debt over equity because the value of equity is more sensitive than the value of debt to new information (and thus asymmetric information). In contrast, the equity dominates debt in the present model precisely because equity payments are more sensitive to new information relative to debt payments. Of course, the specification of asymmetric information differ in the two models. While in the present model there is asymmetric information and non-verifiable information ex post, in Myers and Majluf there is asymmetric information ex

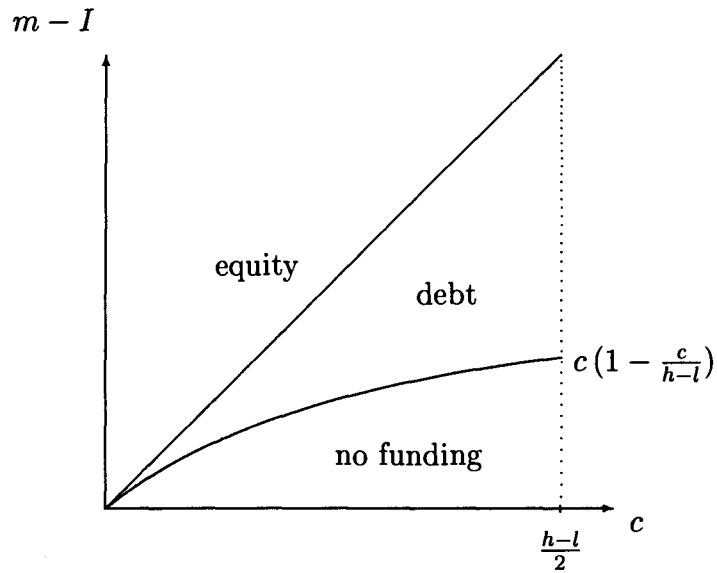


Figure 1: The choice of contract in terms of project NPV and verification costs.

ante while all information ex post is symmetric and verifiable at zero cost.

In the signalling model of Ross (1977) managers are risk averse and prefer equity to debt. Because there is ex ante asymmetric information between the firm's manager and investors, the latter will take a higher debt proportion as a favorable signal of firm quality. In other words, equity preference is utilized by managers of 'good' firms to issue debt so as to credibly convey favorable information about the firm. In contrast to Ross (1977), the present model assumes symmetric information ex ante but non-contractible (and/or asymmetric) information ex post.

Proposition 3 reveals equity preference if debt and equity are both feasible. While Ross imposes equity preference by assuming risk aversion, equity preference in the present model arises endogenously from the underlying structure of the contracts available. The fact that equity preference is needed for debt to be a credible signal of firm quality suggests a possible link between the ex ante view of financial contracting as represented by Ross and the ex post view to which the present model belong. It may be possible to extend the present model to allow for ex ante asymmetric information and then use equity preference as a basis for a signalling model in which firms' use of debt convey positive information to investors.

6 Concluding Remarks

There is a relatively extensive literature dealing with the choice between debt and equity which takes the structure of contracts as given and assumes that it is possible to write enforceable contracts directly on state variables (see Harris and Raviv [1990] for an extensive review). In addition, this literature generally assumes *ex ante* asymmetric information. The current paper—along the lines of Gale and Hellwig [1985], Hart and Moore [1989], Aghion and Bolton [1992] and others—assumes contract incompleteness and examines the choice between debt and outside equity when cash flows are costly to verify and interim information cannot be verified even at a cost. The paper derives an outside equity contract and examines and compares the abilities of outside equity and debt to allow payments contingent on information that cannot be verified.

In comparing the available contracts, it was shown (consistent with what one would expect) that outside equity allows payments that are more sensitive to new (non-verifiable) information than does debt. Although this made outside equity the least costly of funding alternative, the outside equity contract was also shown to be the more fragile contract and debt the more costly but also the more robust contract.

Appendix

Proof of Lemma 1: Solve $\max_{d_H} V_H(d_H)$ and $\max_{d_L} V_L(d_L)$ to find first order conditions $h - d_H - c = 0$ and $m - d_L - c = 0$, which yield the desired expressions. The second order conditions are clearly satisfied. \square

Proof of Lemma 2: We first calculate debt payments $(d(p_H^u), d(p_L^u))$ from

$$p_H^u = d_H^u \int_{d_H^u}^h \frac{1}{h-m} dx + \int_m^{d_H^u} (x-c) \frac{1}{h-m} dx \quad (A.1)$$

and

$$p_L^u = d_L^u \int_{d_L^u}^m \frac{1}{m-l} dx + \int_l^{d_L^u} (x-c) \frac{1}{m-l} dx \quad (A.2)$$

whose solutions are

$$d_H^u = h - c - \sqrt{(h-c)^2 - 2(h-m)p_H^u - m^2 + 2cm} \quad (A.3)$$

$$d_L^u = m - c - \sqrt{(m-c)^2 - 2(m-l)p_L^u - l^2 + 2cl} \quad (A.4)$$

Payments (p_H^u, p_L^u) are now obtained by solving

$$\min_{p_H, p_L} \left(\frac{d_H - m}{h-l} + \frac{d_L - l}{h-l} \right) c \quad (A.5)$$

subject to

$$I = \frac{1}{2}p_H + \frac{1}{2}p_L \quad (A.6)$$

Solving zero profit condition (A.6) for p_L and inserting this expression into the objective function and then differentiating with respect to d_H yields the first order condition

$$2p_H(h-l) = \frac{1}{2}(h-l)^2 + 2I(h-l). \quad (A.7)$$

Equations (A.6) and (A.7) yield expressions for p_H^u and p_L^u reported in Lemma 1.

To see that payments (p_H^u, p_L^u) allocate verification costs equally across states, substitute the expressions for (p_H^u, p_L^u) given in Lemma 1 into (A.3) and (A.4) and insert the resulting expressions for (d_H^u, d_L^u) into (A.7) to find that

$$d_H^u - m = d_L^u - l.$$

□

Proof of Lemma 3: The date 2 debt payments associated with interim payments (p_H^Δ, p_L^Δ) are given by (1) and (2):

$$d_H^\Delta = h - c - \sqrt{(h - c)^2 - 2(h - m)(p_H^u - \Delta) - m^2 + 2mc}$$

and

$$d_L^\Delta = m - c - \sqrt{(m - c)^2 - 2(m - l)(p_L^u + \Delta) - l^2 + 2lc}.$$

The expected verification costs associated with these payments are

$$E^\Delta(c) = \left(\frac{d_H^\Delta - m}{h - l} + \frac{d_L^\Delta - l}{h - l} \right) c.$$

Taking the change in $E^\Delta(c)$ with respect to Δ yields

$$\frac{\partial E^\Delta(c)}{\partial \Delta} = \frac{c}{h - l} \left(\frac{\partial d_H^\Delta}{\partial \Delta} + \frac{\partial d_L^\Delta}{\partial \Delta} \right),$$

where $\partial d_H^\Delta / \partial \Delta < 0$ and $\partial d_L^\Delta / \partial \Delta > 0$. Finally, by the fact that $\partial d_H^\Delta / \partial \Delta$ is strictly increasing and $\partial d_L^\Delta / \partial \Delta$ is strictly decreasing in Δ it follows that $\partial E^\Delta(c) / \partial \Delta > 0$ for $\Delta > 0$. □

Lemma 4 $\Delta^j \geq 0; j \in \{d, e\}$.

Proof: Proposition 2 says that $\Delta^d \geq \Delta^e$, which implies that the proof is complete if we can show that $\Delta^e \geq 0$. By the fact that date 1 payments must satisfy the break even constraint $E(p_i^j) = I, i = H, L$ and $j = e, u$ it is sufficient to show that

$$p_H^e \leq p_H^u, \tag{A.11}$$

or

$$\alpha(E(x|s_H) - c) \leq \frac{1}{4}(h - l) + I, \tag{A.12}$$

which is equivalent to

$$\frac{I}{m - c} \left(\frac{m + h}{2} - c \right) \leq \frac{1}{4}(h - l) + I. \tag{A.13}$$

Using the fact that $m = (h + l)/2$ and rearranging this yields

$$m - c \geq I.$$

In other words, $\Delta^e \geq 0$ whenever the feasibility condition for equity is satisfied (with the inequality being strict whenever $m - c > I$). \square

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CHAPTER 4

Multiperiod Debt Contracts under Costly State Verification*

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Abstract

Gale and Hellwig (1985) derive the optimal contract under costly state verification and show that it takes the form of the standard debt contract 'with bankruptcy.' This paper analyzes a multiperiod version of the one-period model of Gale and Hellwig and shows that the multiperiod standard debt contract—via either renegotiation, short-term debt, or callable debt—induces the borrower to provide truthful reports of realized cash flows on each date so long as the borrower stays solvent and able to cover cash short-falls by issuing additional debt. It is shown that verification occurs if (and only if) the firm's debt capacity has been exhausted and the firm is bankrupt.

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1 Introduction

Gale and Hellwig (1985) derive the optimal contract under costly state verification (CSV) and show that it takes the form of the *standard debt contract* (SDC) “with bankruptcy.”¹ As noted by several researchers, however, the one period result of Gale and Hellwig may not extend too easily to the multiperiod case. For example, Hart (1995; p.125) argues that “...with more periods, one would expect the contract to specify that inspection should take place at several dates—as a (possibly stochastic) function of past events. Moreover, in a multi-period setting there is no reason that inspection should be associated with a termination of the firm’s operations, in the way that bankruptcy is.”² Despite this, the present paper extends the one period model of Gale and Hellwig to a multiperiod setting and shows that inspection (or verification) occurs only when the firm’s debt capacity has been exhausted and the borrower declares bankruptcy—in other words, verification occurs only in bankruptcy. More generally, it shows that the SDC allows the borrower to construct financial structures that induce truthful reports of realized cash flows in each period for as long as the firm’s debt capacity remains intact to allow cash short-falls to be funded by the issuance of additional (junior) debt. Once (and if) the firm’s debt capacity has been exhausted, the mechanism for truthful and costless revelation of cash flows breaks down and, as in the one period case, the borrower defaults and the lender assumes control and verifies.

As defined by Gale and Hellwig, a contract exhibits *maximum equity participation* (MEP) if the borrower contributes all his personal funds towards the project. MEP is optimal in the one period case because it minimizes the required debt payment and thus minimizes expected default costs.³ In the multiperiod context, MEP requires that the borrower applies all cash generated in each period towards debt service rather than towards personal dividend payments. Although such a zero-dividend policy has the effect

¹The optimal contract under costly state verification was first derived by Townsend (1979) and then extended in various ways by Gale and Hellwig (1985); see Allen and Winton (1995) for a recent review of the CSV literature.

²Indeed, in reviewing Hart’s book, Harris (1996) comments: “...most telling, perhaps, is the criticism that the [CSV] model is unlikely to yield debt-like contracts when extended to multiple periods.”

³To be exact, Gale and Hellwig distinguish between unlimited and limited liability and show that if the entrepreneur accepts unlimited liability, then a contract with MEP weakly dominates any contract that does not. At the same time, they argue that if the entrepreneur assumes limited liability then contracts exhibiting MEP will strictly dominate contracts that do not.

of minimizing expected default costs over the life of the project thus maximizing the value of the firm, analogous to the underinvestment problem identified by Myers (1977), the presence of debt in the firm's financial structure potentially induces the borrower to pay himself dividends rather than to use this cash to reduce the firm's level of debt. It turns out, however, as predicted by Myers in a different context, that the borrower will be able to circumvent the debt overhang problem that arises in the multiperiod CSV model by either (i) renegotiating existing debt on each date, (ii) issuing callable debt, or (iii) issuing exclusively on short-term debt.

The renegotiation outcome derived relies on the ability of the borrower to make take-it-or-leave offers on each date along the lender's indifference constraint, as determined by the value of the particular multiperiod SDC in place on the relevant date. Relatedly, Gale (1991) examines a situation in which a supplier and a buyer must agree on the future price and the amount produced of an input when states are random, observable, and non-verifiable. He shows that the efficient outcome (which is optimal risk sharing and first-best production levels) is attainable by renegotiating the initial contract on each date as long as the speed of information arrival is not too fast. In the present paper, renegotiation allows the efficient outcome (which is MEP) so long as the firm's debt capacity permits the borrower to make offers that match the value of the particular multiperiod standard debt contract in place. As in Gale, even though the initial contract is renegotiated on each date, its value is important because it supplies the lender with a reservation value below which the value of future offers cannot fall.

Renegotiation, however, it is not the only avenue through which MEP and truthful reports obtain. Using either a sequence of one-period SDCs or long-term callable debt, it is shown that the borrower will be able to commit to refrain from paying himself interim dividends and instead distribute the cash generated to existing lenders, or, if the firm generates a cash short-fall, use the cash generated to minimize the amount of new debt issued. With short-term debt, as long as the firm is not in bankruptcy, the existing lender receives the full amount owed, with the difference between the payment due and the cash flow generated being funded by issuing of a new one period debt claim. This new claim is priced fairly in the market, in turn inducing the borrower to raise just enough funds to pay off the current lender, thus achieving MEP.

When the firm issues callable debt, though bonds may be left uncalled even though they may trade at a premium relative to their call values and though junior lenders will realize a capital gain when more senior bonds are called, it is possible to construct the call prices in such a way that the borrower effectively makes a credible commitment to pay out *at least* the amount of cash generated on each date. Because the borrower is not allowed to issue senior debt, and thus bears the full cost of increasing the the firm's debt level, debt financed dividend payments will never be optimal. This and the fact that the given call feature ensures that all cash generated on each date is paid out to existing lenders allows truthful revelation of cash flows on each date.

Other papers that examine debt structure under multiperiod CSV are Chang (1990) and Webb (1992), who both consider the two period case. Chang (1990) derives (from 'first principles') the optimal two period contract under costly state verification and shows that the optimal contract has characteristics much like those observed in multiperiod debt contracts used in practice, such as coupon payments, sinking fund provisions, and call features. However, while Chang rules out interim dividend payments and debt issuance, no such restrictions are imposed here. Indeed, allowing additional (junior) debt issuance is critical to the results derived in the present paper.

More closely related to the present paper is Webb (1992), who analyzes a two-period version of the Gale and Hellwig model and shows that the borrower and the initial lender will be able to agree to an interim state contingent payment schedule that induces truthful reports of interim cash flows. Although the present paper and Webb derive the same result—namely the existence of a debt structure within the multiperiod CSV setup that will induce the borrower to report interim cash flows truthfully—the approach taken and the types of debt structures needed for truthful reports are different. For example, while Webb shows that truthful reports obtain by making the interim debt payment directly contingent on the borrower's cash flow report—the enforcement mechanism of which is that any unreported cash must be diverted and not used towards reducing the amount of debt issued on the interim date—the present paper shows that truthful reports may be induced using mechanisms already in use in various debt markets, such as renegotiation, debt maturity, or call features. A second difference lies in the fact that whereas the Webb model may be difficult to extend to the multiperiod case, the present

paper specifically analyzes the general multiperiod case.

Mookherjee and Png (1989) show, for the one period case, that random verification yields lower verification costs compared to what is implied by the standard debt contract; in other words, the contract derived by Gale and Hellwig is not optimal if one allows for stochastic verification. The result derived in the present paper that the borrower in the multiperiod case will report cash flows truthfully until the firm's debt capacity is exhausted, however, indicates that the cost advantage of random verification over the standard debt contract as suggested by Mookherjee and Png is not as clear cut as comparisons based on the one period case seem to reveal.

The rest of the paper is organized as follows. Section 2 describes the basic setup. Section 3 assumes that cash flows are independent over time and examines the ability of renegotiation and short-term debt to induce maximum equity participation on each date. Section 4 assumes that cash flows are positively correlated over time and examines the use of callable debt as well as short-term debt as mechanisms to induce maximum equity participation on each date. Finally, Section 5 concludes the paper.

2 The Setup

The model consists of one borrower and several lenders. The credit market is competitive and open for transactions on each date. The borrower has zero initial funds but is endowed with a project that requires an initial outlay of I and produces random cash flows $\{x_t; t = 1, \dots, \tau\}$, where $\tau \leq \infty$. Although in practice the lack of inside funds may deter a borrower from raising a sufficient amount of external funds to get a project started, it is assumed that the project in question is sufficiently profitable that this does not happen.

Each x_t is defined on \mathcal{R}^+ and $G(x_t|\theta_s)$ and $g(x_t|\theta_s)$ denote, respectively, the distribution and density functions of x_t conditional on date s public information θ_s . The hazard rate of x_t is given by $\rho(x_t|\theta_s) = g(x_t|\theta_s)/(1 - G(x_t|\theta_s))$. As is common, $\rho(\cdot)$ is assumed to be increasing in x_t . The expected date t cash flow conditional on θ_s is denoted $E(x_t|\theta_s)$. It is assumed that everybody is risk neutral. The risk free interest rate is denoted r and is assumed to be strictly positive unless something else is specified.

Cash flows are observed without cost by the entrepreneur but must be verified at a

cost before they are observable to anybody else. Let $c = c(x_t)$ denote the cost of verifying x_t and assume that $c'(x_t) > 0$, $c(x_t) < x_t$, for $x_t > 0$, and $c(0) = 0$. These assumptions imply that the net cash flow $x_t - c(x_t)$ is non-negative for all $x_t \in \mathfrak{R}^+$, which ensures that there are always sufficient resources left in the firm to cover verification costs.

When there are several lenders providing credit to the firm, it is assumed that the process of verifying cash flows is done through a public verification scheme, such as a bankruptcy court, and that each lender incurs zero private costs in the process of verifying cash flows. Since verified cash flows (by definition) can be ascertained in a court of law, stated priority rules become indisputable in this case. This significantly simplifies contract renegotiation in bankruptcy, making it nearly trivial. The assumption that lenders do not incur private costs in verifying cash flows is important because if not, then the borrower would face a trade-off between minimizing expected future verification costs (which would be increasing in the number of lenders) and minimize possible contracting costs arising from a loss in bargaining power vis a vis lenders (which presumably would be increasing in the number of lenders).

It is assumed that the liquidation value of the firm is zero, which implies that firm is always continued after default. Allowing strictly positive liquidation values is possible in the present setting but would make the analysis more complicated without providing additional insights.

It is further assumed that the borrower's credit market transactions in each period are publicly observable but that the amount of dividends extracted by the borrower is not. Although the assumption that dividend distributions are unobservable is unrealistic, it is meant to capture the idea that a firm's insiders are better informed than anybody else about the firm's cash situation and therefore have some degree of discretion with respect to the use of the cash generated in each period. The significance of this assumption is that it rules out contracts written on the amount of cash extracted by the borrower, such as dividend covenants.⁴

Let $v_t(d_{t+1}, \dots, d_\tau; \theta_t)$ denote the date t value of a *multiperiod standard debt contract* (MSDC) with future coupons (d_{t+1}, \dots, d_τ) . This contract yields $d_{t'}$ if $x_{t'} \geq d_{t'}$ and $x_{t'} - c(x_{t'})$ if $x_{t'} < d_{t'}$; in other words, if the period's cash flow exceeds the period's coupon,

⁴See, however, Gjesdal and Antle (1996) for an analysis of optimal dividend covenants in a two-period CSV model in which the borrower values interim dividend payments.

then the lender receives the scheduled payment d_t ; otherwise, control is transferred to the creditor, who verifies and collects the net cash flow $x_t - c(x_t)$.⁵ We note that with one period remaining, the MSDC turns into the one-period contract derived by Gale and Hellwig.

In bankruptcy, the creditor is in control and puts the debt level of the firm to $(d_{t+1}^*(\theta_t), \dots, d_\tau^*(\theta_t))$, where $(d_{t+1}^*(\theta_t), \dots, d_\tau^*(\theta_t)) = \operatorname{argmax}_{(d_{t+1}, \dots, d_\tau)} v_t(d_{t+1}, \dots, d_\tau; \theta_t)$. The particular MSDC characterized by payments $(d_{t+1}^*, \dots, d_\tau^*)$ and the corresponding value $v_t(d_{t+1}^*, \dots, d_\tau^*; \theta_t)$ thus defines the firm's date t debt capacity. The following lemma characterizes the firm's debt capacity.

Lemma 1 (i) *With cash flows i.i.d, payments $(d_{t+1}^*, \dots, d_\tau^*)$ are finite and time invariant: $d_t^*(\theta_t) = \dots = d_\tau^*(\theta_t) = d^* < \infty$.* (ii) *With cash flows positively correlated over time, payments $(d_{t+1}^*, \dots, d_\tau^*)$ are not necessarily finite and are not time invariant.*

Proof: See Appendix A.

With cash flows i.i.d., an increasing hazard rate $\rho(x_t|\theta_s)$ guarantees that the value of the MSDC is single peaked and quasi-concave and therefore that $(d_{t+1}^*(\theta_t), \dots, d_\tau^*(\theta_t)) < \infty$.⁶ Intuitively, as the lender raises interest rates this increases expected revenues as well as expected default costs. At first, the marginal increase in expected revenues exceeds the marginal increase in expected default costs. At some point, however, the marginal increase in expected revenues is overtaken by the marginal increase in expected default costs and a further increase in interest rates will thus depress the value of debt.

With cash flows i.i.d. the multiperiod problem turns into a sequence of one period problems, and, as a result, the payments that maximizes debt value are constant over time. With cash flows positively correlated, the creditor's maximization problem becomes more complex and with the general specification of the model so far, clear statements regarding future debt payments are difficult to make. However, with cash flows

⁵As is common, I assume that the lender willingly steps in to verify whenever the scheduled debt payment is not paid in full. Such an assumption can be rationalized on grounds that the lender has reputational capital at stake or that the number of lenders involved is sufficiently large that the free rider problem associated with debt renegotiations is sufficiently pervasive that any attempts to renegotiate the firm's debt will fail with probability one; see Gertner and Sharfstein (1991) for an analysis of free rider problems in debt renegotiations.

⁶This property is well known from the one period case; see Winton (1995a) and Cantillo (1995).

uniformly distributed and the time structure of information simplified to $\theta_t = x_t$, it can be shown that:

Lemma 2 $d_{t+1}^*(\theta_t) < \dots < d^* < \dots < d_\tau^*(\theta_t)$.

Proof: See Appendix.

The lemma shows that the optimal payments determined by the creditor in a date t bankruptcy are increasing over time. In addition, it shows that earlier payments are below the payments that would obtain if cash flows were i.i.d., while later payments are higher. The intuition for this result is relatively simple. Avoiding default raises the creditor's estimate of future cash flows and hence his estimate of future debt capacity. This will increase the optimal debt payments relative to the case in which cash flows are i.i.d., with the effect being stronger the further into the horizon. At the same time, default destroys the positive information effect just laid out in that it puts the creditor back to square one. This compels the creditor to reduce early payments so as to protect the positive learning effect from avoiding default.

Note that since the debt payments that maximizes the value of the creditor's debt claim when cash flows are i.i.d. implies a future default probability that is strictly less than one, the result of Lemma 2 that earlier payments are below the payments that would obtain under independently distributed cash flows implies that the probability of bankruptcy on date $t + 1$ is strictly less than one. The important implication of this is that the post-bankruptcy value of the borrower's inside equity will be strictly positive.

Let now $v_t^e(d_{t+1}, \dots, d_\tau; \theta_t)$ denote the date t value of the borrower's inside equity claim given debt payments (d_{t+1}, \dots, d_τ) . Note that the fact that $d^*(\theta_t) < \infty$ implies that $v_t^e(d_{t+1}^*, \dots, d_\tau^*; \theta_t) > 0$; that is, the post-bankruptcy value of the borrower's inside equity claim is strictly positive. This result is a direct implication of Lemma 1 and arises because the presence of verification costs has the effect of putting a wedge between realized cash flows and the cash flow that can be appropriated by the lender. Why doesn't the lender sell the firm to an alternative management rather than allowing the original owner to retain his share? Assume for example that the firm is sufficiently more valuable under the original owner compared to what it would be under alternative management. Note, however, that allowing the original owner to be replaced would change none of the results derived in the current model. In either case, bankruptcy

imposes a sufficient penalty on the borrower to induce him to avoid bankruptcy for as long as possible.

On each date the borrower observes the true cash flow x_t , upon which he makes a cash flow 'report' \hat{x}_t of to the lender. This report will always be accompanied by a cash payment, thus ruling out inflated reports ($\hat{x}_t > x_t$).

It is assumed that the borrower is indifferent as to the timing of his consumption, which implies that the borrower is willing to postpone consumption until the firm is debt free (or possibly liquidated, in which case the borrower receives zero) if such delayed gratification weakly increases the value of his inside equity claim. Given the assumption that the borrower is risk neutral, he will on each date choose \hat{x}_t so as to solve

$$\max_{\hat{x}_t} x_t - \hat{x}_t + v_t^e(d_{t+1}(\hat{x}_t), \dots, d_\tau(\hat{x}_t); \theta_t), \quad (\Omega)$$

subject to $\hat{x}_t \leq x_t$.

3 Independent Cash Flows

This section analyzes the case in which cash flows are independently and identically distributed over time. While subsection 3.1 examines renegotiation, subsection 3.2 considers the case in which the firm is funded with a sequence of one period standard debt contracts. With cash flows independently and identically distributed, cash flows are generated by the probability distribution $g(x_t|\theta_0)$, which, for notational convenience, will be denoted $g(x_t)$.

3.1 Renegotiation

This section examines a simple renegotiation game in which the borrower makes take-it-or-leave-it offers to the lender on each date, regardless of whether the firm generates a cash surplus ($x_t > d_t$) or a cash deficit ($x_t < d_t$). Each offer to renegotiate consists of a cash payment \hat{x}_t and rescheduled future debt payments. The creditor either accepts or rejects the borrower's offer. If he accepts it, he receives the cash payment \hat{x}_t along with the rescheduled debt claim $(d'_{t+1}, \dots, d'_\tau)$, while the borrower receives $x_t - \hat{x}_t + v_t^e(d'_{t+1}, \dots, d'_\tau)$. If the creditor rejects the offer, he intervenes, collects the net cash flow

$x_t - c(x_t)$, and puts the debt level of the firm to (d^*, \dots, d^*) , thus receiving the value $v_t(d^*, \dots, d^*)$ while the borrower receives $v_t^e(d^*, \dots, d^*)$.

The date t pre-renegotiated (or cum-coupon) value of the MSDC in effect on date t is given by $d_t + v_t(d_{t+1}, \dots, d_\tau)$. Importantly, it is assumed that the creditor rejects any offer with value $\hat{x}_t + v_t(d'_{t+1}, \dots, d'_\tau)$ less than the cum-coupon debt value $d_t + v_t(d_{t+1}, \dots, d_\tau)$.⁷ This assumption is equivalent to the standard assumption made in the one period CSV setup that the lender rejects any offers less than the payment agreed upon ex ante (see footnote 5): just like accepting a payment less than the scheduled payment d_t would constitute a concession on the part of the lender, so would accepting an offer below the date t cum-coupon debt value $d_t + v_t(d_{t+1}, \dots, d_\tau)$. In any case, the cum-coupon value of the MSDC in effect on a particular date equips the lender with a *reservation value* below which the value of renegotiation offers cannot fall.

Since now any offer for which $\hat{x}_t + v_t(d'_{t+1}, \dots, d'_\tau) > d_t + v_t(d_{t+1}, \dots, d_\tau)$ is wasteful from the borrower's perspective, equilibrium offers will be determined from the lender's date t *indifference constraint*

$$\hat{x}_t + v_t(d'_{t+1}, \dots, d'_\tau) = d_t + v_t(d_{t+1}, \dots, d_\tau). \quad (1)$$

Given $v_t(d^*)$, where d^* denotes the constant coupon associated with a debt claim with maturity date τ , the firm's date t excess debt capacity is given by

$$v_t(d^*) - v_t(d_{t+1}, \dots, d_\tau) \geq 0. \quad (2)$$

In the case of a cash shortfall, the borrower will be able to avoid default so long as the value of firm's excess debt capacity exceeds the size of the cash shortfall:

$$d_t - x_t \leq v_t(d^*) - v_t(d_{t+1}, \dots, d_\tau). \quad (3)$$

Accordingly, the firm is said to be *solvent* on date t if condition (3) is satisfied. On the other hand, if condition (3) is not satisfied, the firm is insolvent and the borrower is unable to come up with an offer to prevent bankruptcy.

⁷We may give the borrower a second-chance by allowing him to switch to the underlying MSDC should the lender reject the borrower's offer, which would enable the borrower to avoid verification despite the lender rejecting the initial offer, provided that the period's cash flow exceeds the period's debt payment. Although this situation will never occur in equilibrium, and can thus be ruled out, it emphasizes the value of the MSDC as providing the basis for the lender's reservation value on each date.

By definition, a solvent firm will always be able to make an offer $(\hat{x}_t, (d'_{t+1}, \dots, d'_\tau))$ that satisfies condition (1). Given the assumption that the lender will reject any offer for which $\hat{x}_t + v_t(d'_{t+1}, \dots, d'_\tau) < d_t + v_t(d_{t+1}, \dots, d_\tau)$, it is clear that the borrower will never declare bankruptcy unless the firm's debt capacity prevents the borrower from making an offer along the lender's indifference constraint. This is so because the value $v_t^e(d^*)$ received by the borrower in bankruptcy is strictly less than the amount $x_t - \hat{x}_t + v_t^e(d'_{t+1}, \dots, d'_\tau)$ the received if bankruptcy is avoided.⁸

To the extent that the solvency condition is satisfied, the question is whether cash flows will be reported truthfully ($\hat{x}_t = x_t$) on each date or if cash flows will be under-reported ($\hat{x}_t < x_t$.) The latter implies that the borrower awards himself a dividend of $x_t - \hat{x}_t > 0$, which includes the possibility that $x_t > d_t > \hat{x}_t$ so that the dividend is in part financed by additional debt issuance. However, according to the following proposition, paying out dividends, even from internal funds, is not optimal.

Proposition 1 *Cash flows will be reported truthfully on each date as long as the firm is solvent.*

The proof is by explanation.⁹ The result of the proposition follows directly by the fact that the value of acceptable offers is determined directly from the cum-coupon debt value $d_t + v_t(d_{t+1}, \dots, d_\tau)$. Since this value is independent of the amount of cash contributed by the borrower, each dollar contributed has the effect of reducing the value of the amount owed by exactly one dollar. The resulting reduction in future debt payments reduces expected verification costs and thus strictly increases the value of the borrower's inside equity claim, which in turn compels the borrower to pay out the entire amount of cash generated on each date. In other words, MEP obtains on each date and cash flows are truthfully revealed.

Even though the MSDC calls for verification whenever the firm incurs a cash shortfall ($x_t < d_t$), it will be renegotiated on each date until (or if) the firm is debt free. Its value $v_t(d_{t+1}, \dots, d_\tau)$ is nonetheless important because it provides a benchmark below which the value of the borrower's offer cannot fall. Similarly, although the MSDC issued by the firm is renegotiated on each date, the fact that this generates a reduction in

⁸This follows by the fact that $x_t \geq \hat{x}_t$ and $v_t^e(d'_{t+1}, \dots, d'_\tau) \geq v_t^e(d^*) > 0$.

⁹Subsection 3.1.2, however, provides a direct proof in the case $\tau = 2$.

future verification costs has no effect on the firm's debt capacity therefore no effect on the firm's ability to implement the project on the initial date. The reason for this is that the lender's reservation value on each date is determined by the cum-coupon value of the MSDC, whose value is unaffected by whether it is renegotiated or not.¹⁰

3.1.1 The Scheduled Repayment Path of Debt

The result that the borrower optimally pays out all cash generated in each period suggests that the scheduled repayment path of debt is, at least to some extent, arbitrary. Indeed, there exists on each date t an infinite number of alternative repayment paths (or MSDCs) that each satisfies the lender's participation constraint. To see this, note first that each (d'_t, \dots, d'_τ) , for $t' = 1, \dots, \tau$ is, in equilibrium, determined from

$$\begin{aligned} & [d'_t + v_t(d'_{t+1}, \dots, d'_\tau)] \int_{d'_t + v_t(d'_{t+1}, \dots, d'_\tau) - v_t(d^*)}^{\infty} g(x_t) dx_t \\ & + \int_0^{d'_t + v_t(d'_{t+1}, \dots, d'_\tau) - v_t(d^*)} (x_t - c(x_t) + v_t(d^*)) g(x_t) dx_t \\ & = (d_{t-1} + v_{t-1}(d_t, \dots, d_\tau) - x_{t-1})(1 + r), \end{aligned} \quad (4)$$

where $d_{t-1} + v_{t-1}(d_t, \dots, d_\tau) - x_{t-1} = I$ for $t = 1$. Suppose that the repayment path (d'_t, \dots, d'_τ) solves (4). To see that this path cannot be unique, note simply that for any (d'_t, \dots, d'_τ) that solves (4), there exists an alternative repayment path (d''_t, \dots, d''_τ) determined by

$$d''_t + v_t(d''_t, \dots, d''_\tau) = d'_t + v_t(d'_t, \dots, d'_\tau),$$

which also solves (4); indeed, any scheduled repayment path (d_t, \dots, d_τ) which satisfies

- (i) $d_t + v_t(d_{t+1}, \dots, d_\tau) = \text{constant}$,
- (ii) $0 \leq d_t$, for $t = 1, \dots, \tau$ and
- (iii) $0 \leq v_t(d_t, \dots, d_\tau) \leq v_t(d^*)$

provides a solution to (4). Given now that each d_t is defined on the real line, the set of scheduled repayment paths that satisfies (i), (ii), and (iii) contains an infinite number of elements.

¹⁰Note that whereas any project for which $E(x) \sum_{t=1}^{\tau} \frac{1}{(1+r)^t} \geq I$ would receive funding under complete contracting, only projects for which $v_0(d^*, \dots, d^*) \geq I$ receives funding under CSV. Since $E(x) \sum_{t=1}^{\tau} \frac{1}{(1+r)^t} > v_0(d^*, \dots, d^*)$, the set of profitable projects that will be denied funding is non-empty. To see that this conclusion is unaffected by allowing the borrower to renegotiate, note simply that renegotiation has no effect on $v_0(d^*, \dots, d^*)$.

3.1.2 An Illustration of the Renegotiation Outcome for $\tau = 2$

Let $\tau = 2$ and suppose that the borrower is endowed with personal funds of $e \in (0, I)$ on the initial date. Let further $\hat{e} \in [0, e]$ denote the borrower's initial equity contribution so that his initial capital need is $I - \hat{e}$. The scheduled coupon payments (d_1, d_2) associated with this amount are determined from the lender's date 0 participation constraint:

$$[d_1 + v_1(d_2)] \int_{d_1 + v_1(d_2) - v_1(d_2^*)}^{\infty} g(x_1) dx_1 + \int_0^{d_1 + v_1(d_2) - v_1(d_2^*)} (x_1 - c(x_1) + v_1(d_2^*)) g(x_1) dx_1 = I - \hat{e}, \quad (5)$$

where $v_1(d_2) = d_2 \int_{d_2}^{\infty} g(x_2) dx_2 + \int_0^{d_2} (x_2 - c(x_2)) g(x_2) dx_2$, and $d_2^* = \operatorname{argmax} v_1(d_2)$.

The borrower upon observing the date 1 cash flow x_1 , puts forth an offer (\hat{x}_1, d'_2) , whose value is determined along the creditor's date 1 indifference constraint

$$\hat{x}_1 + v_1(d'_2) = d_1 + v_1(d_2). \quad (6)$$

Since $v_1(d_2)$ is increasing in d_2 (for $d_2 \leq d_2^*$) and since $d_1 + v_1(d_2)$ is unrelated to \hat{x}_1 , it follows that $d'_2 = d'_2(\hat{x}_1)$ is a strictly decreasing function of \hat{x}_1 . Note that any $\hat{x}_1 \in [0, x_1]$ is possible as long as $d_2(\hat{x}_1) \leq d_2^*$, so that debt financed dividend payments are possible, though, as we shall see, not optimal.

The MEP result of Proposition 1 implies that $(\hat{e}, \hat{x}_1) = (e, x_1)$. To show that this is the case, I will solve the borrower's problem recursively starting with his optimal choice of \hat{x}_1 .

DATE 1: Suppose that the firm is solvent on date 1. The borrower's date 1 wealth as a function of \hat{x}_1 is then given by

$$w_1(\hat{x}_1) = (x_1 - \hat{x}_1) + \int_{d_2(\hat{x}_1)}^{\infty} (x_2 - d'_2(\hat{x}_1)) g(x_2) dx_2, \quad (7)$$

where the first term constitutes the borrower's net cash position and the second term represents the date 1 value of his equity claim as a function of his date 1 equity contribution \hat{x}_1 . Substituting from (6) into (7), gives

$$w_1(\hat{x}_1) = x_1 + E(x_2) - \int_0^{d'_2(\hat{x}_1)} c(x_2) g(x_2) dx_2 - (d_1 + v_1(d_2)), \quad (8)$$

which consists of the cash generated in the first period, the expected cash for the second period, less expected date 2 default costs, and less the date 1 value of the amount owed to the lender.

Since, by (6), d_2' is strictly decreasing in \hat{x}_1 , it follows that $w_1(\hat{x}_1)$ is strictly increasing in \hat{x}_1 and therefore maximized at $\hat{x}_1 = x_1$. In other words, the borrower pays out all the date 1 cash generated and thus reports x_1 truthfully. This result obtains directly from the fact that the value $d_1 + v_1(d_2)$ conferred to the lender is independent of the borrower's date 1 cash contribution \hat{x}_1 , which ensures that the resulting reduction in expected verification costs, $\int_0^{d_2'(\hat{x}_1)} c(x_2)g(x_2)dx_2$, is absorbed in full by the borrower.

DATE 0: With $\hat{x}_1 = x_1$, the borrower's initial wealth can be expressed by

$$w_0(\hat{e}) = (e - \hat{e}) + \int_{d_1+v_1(d_2)-v_1(d_2^*)}^{\infty} w_1(x_1)g(x_1)dx_1 + G(d_1 + v_1(d_2))v_1^e(d_2^*), \quad (9)$$

where $v_1^e(d_2^*)$ represents the post-bankruptcy value of the borrower's inside equity claim, $G(d_1 + v_1(d_2))$ the probability of bankruptcy on date 1, and $(e - \hat{e})$ the borrower's net cash position on date 0. Substituting from (5) and (8), yields

$$\begin{aligned} w_0(\hat{e}) = & e - I + E(x_1) - \int_0^{d_1+v_1(d_2)-v_1(d_2^*)} c(x_1)g(x_1)dx_1 \\ & + (1 - G(\cdot))[E(x_2) - \int_0^{d_2'} c(x_2)g(x_2)dx_2] + G(\cdot)[v_1^e(d_2^*) + v_1(d_2^*)], \end{aligned} \quad (10)$$

which shows that the borrower absorbs 100% of expected default costs. Noting now that $v_1^e(d_2^*) + v_1(d_2^*)$ constitutes the total date 1 post-bankruptcy value of the firm—and therefore the minimum date 1 firm value—we have that $E(x_2) - \int_0^{d_2'} c(x_2)g(x_2)dx_2 \geq v_1^e(d_2^*) + v_1(d_2^*)$. This and the fact that $d_1 + v_1(d_2)$ is strictly decreasing in \hat{e} ensures that the derivative of $w_0(\hat{e})$ with respect to \hat{e} is strictly positive and therefore maximized at $\hat{e} = e$.

We have thus shown that $(\hat{e}, \hat{x}_1) = (e, x_1)$. In other words, MEP obtains on each date and the date 1 cash flow is revealed truthfully.

3.2 Short-Term Debt

The renegotiation outcome relies on the assumption that the borrower makes take-it-or-leave-it offers to the creditor on each date. The present section examines the possibility of replicating the renegotiation outcome by issuing a sequence of one-period standard debt contracts.

Let f_t denote the date t payment associated with a one-period debt contract issued on date $t - 1$. Note that for any $f_t \in [0, d^* + v_t(d^*)]$, there exists at least one MSDC such

that $f_t = d_t + v_t(d_{t+1}, \dots, d_\tau)$. In other words, for any (feasible) one-period standard debt contract with payment f_t , there exists at least one MSDC that matches the value of the short-term contract. This leads to the following result.

Proposition 2 *Suppose that the project is funded with a sequence of one period standard debt contracts with payments $f_t, t = 1, \dots, \tau$ determined by*

$$\begin{aligned} f_t \int_{f_t - v_t(d^*)}^{\infty} g(x_t) dx_t + \int_0^{f_t - v_t(d^*)} (x_t - c(x_t) + v_t(d^*)) g(x_t) dx_t \\ = (f_{t-1} - \hat{x}_{t-1})(1 + r), \end{aligned} \quad (11)$$

where $f_{t-1} - \hat{x}_{t-1} = I$ for $t = 1$, then cash flows will be reported truthfully on each date.

The proof is again by explanation. Note first that if $x_t + v_t(d^*) < f_t$ and the firm is insolvent, the lender intervenes and puts the firm's debt level to $f_{t+1}^* = d^* + v_{t+1}(d^*)$. The ability of the short-term lender to find a MSDC that matches the short-term claim such that $f_{t+1}^* = d^* + v_{t+1}(d^*)$ ensures that the threat of bankruptcy is credible, inducing the borrower to avoid bankruptcy if possible.

Consider then the determination of \hat{x}_t . Recall that with long-term renegotiable debt the lender receives the total amount owed $d_t + v_t(d_{t+1}, \dots, d_\tau)$ in form of a cash payment \hat{x}_t and a new claim $(d'_{t+1}, \dots, d'_\tau)$, where $(\hat{x}_t, (d'_{t+1}, \dots, d'_\tau))$ is determined from

$$\hat{x}_t + v_t(d'_{t+1}, \dots, d'_\tau) = d_t + v_t(d_t, \dots, d_\tau).$$

In contrast, when the project is funded with a sequence of one-period debt contracts, the lender receives the entire amount owed in form of a one-shot cash payment $f_t = d_t + v_t(d_{t+1}, \dots, d_\tau)$. Since the firm in general does not generate enough cash to satisfy the scheduled payment, the borrower must issue a new one period debt claim to make up the balance. MEP (and thus truthful reports) now obtains directly from the fact that the new claim is priced competitively under symmetric information with respect to future (as well as current) cash flows.

4 Positively Correlated Cash Flows

With cash flows are positively correlated, the value of the lender's claim on the firm will be positively related to the size of current cash flow. Since the lender's reservation

value in renegotiation is given by the cum-coupon value of his claim, the fact that the debt value is positively related to the current cash flow implies that the value demanded by the lender in renegotiation will be an increasing function of the borrower's cash flow report (to the extent that this report is informative). This implies in turn that the borrower, when deciding on the size of the cash flow to report, will have to weigh reductions in future default costs against the larger value that he must confer to the creditor in renegotiating the current MSDC—the upshot of which is that renegotiation will not in general yield MEP and truthful reports. Nonetheless, the present section shows that both callable (long-term) debt as well as a sequence of one-period standard debt contracts will induce maximum equity participation and truthful reports on each date.

In order to allow cash flows to be positively correlated over time yet keeping the model tractable, I assume that expectations with respect to future cash flows are conditioned on current cash flow realizations alone rather than on a more comprehensive history of cash flows.¹¹ Thus, we may write $g(x_t|\theta_s) = g(x_t|x_s)$.

4.1 Callable Debt

This section analyzes the use of callable debt. Unlike other cases analyzed, the firm's debt structure will in this case generally contain multiple debt classes ranked by a non-trivial priority structure.

Let $v_t(d^s, u, \hat{x}_t)$ denote the date t (ex-coupon) value of a debt claim issued on date $s \leq t$ with constant coupons d^s , where u denotes its priority status (to be explained in more detail below). Note that to the extent that the borrower's report is informative (or truthful), the debt value is now a function of the borrower's date t cash flow report.¹² Note that since cash flows are positively correlated, $v_t(d^s, u, \hat{x}_t)$ will be positively related to \hat{x}_t (so long as $\hat{x}_t = x_t$).

The borrower is restricted to issue only junior debt, which implies that the priority

¹¹Note that this assumption is necessary only in the case in which the firm is funded with callable debt and is not needed if the firm is funded with a sequence of one-period debt contracts (see Section 4.2), in which case a more general specification of information can be assumed with no added difficulties.

¹²The debt value $v_t(d^s, u, \hat{x}_t)$ is also a function of the total value of senior debt claims (i.e. claims issued before date s but have been not called) but this added notational complexity is ignored with no loss. I will also suppress notation for τ , for the most part.

of an outstanding debt claim (claim 's' say) will either increase over time or remain unchanged. The latter occurs if the borrower refrains from calling bonds that have higher priority than bond s . Notationally, I will write $u = s$ if the priority of bond s does not change from the date s and denote by $u < s$ the case in which bonds of higher priority than bond s have been called, thus increasing the original seniority of bond s .¹³

The firm will in general issue additional bonds and call (not necessarily all) existing bonds in each period. Bonds will be ranked by *absolute priority*, which, here, will imply that more senior lenders are paid 'in full' before less senior lenders receive any payment at all, and where 'in full' will mean receiving the total current cum-coupon value $d^s + v_t(d^s, u, x_t)$. Because absolute priority as defined here makes it impossible for the borrower to transfer wealth to existing bondholders by issuing additional bonds, the borrower will be free to issue as much additional debt as he wants.¹⁴ Indeed, the fact that bonds are ranked by absolute priority allows the borrower at any point to fund personal dividend payments by issuing additional bonds if he so desires.

Cash distributed in bankruptcy is paid out from the firm's verified cash flow and any cash raised by issuing additional debt (or, equivalently, senior lenders receive d^s in cash and maintain their claims unaltered, though I will go with the interpretation that existing lenders will be bought out in bankruptcy). The value of the claim issued in bankruptcy is (analogous to before) given by $v_t(d^{t*}, t, x_t)$, where $d^{t*} = \operatorname{argmax}_{d^t} v_t(d^t, t, x_t)$.¹⁵ Let $S_p = \{0, \dots, s_t - 1\}$ denote the set of lenders who are paid in full in bankruptcy. Lender s_t it thus the lowest priority lender to receive a positive payment in bankruptcy; any

¹³The idea here is that bonds issued in earlier periods have higher priority than bonds issued later. As an example, suppose on some date t that no bonds have been called so that the firm's debt structure will consist of t classes of bonds, with bond 0 being the most senior and bond $t - 1$ the most junior. The priority of bond $s = 0$ is now $u = 0$ while the priority of bond $t - 1$ is ' $t - 1$.' If the borrower decides to call bond 0, then the priority of bond $t - 1$ *increases* to $t - 2$, the priority of bond $t - 2$ increases to $t - 3$, and so forth.

¹⁴Winton (1995b) derives the optimality of absolute priority in a one period CSV model as a way of preventing duplication of verification efforts across lenders. Duplication of verification efforts does not arise in the present setting because of our assumption of a public verification scheme.

Note that the priority rule assumed here is a necessary condition for the results to come through. For example, if senior (or existing) bondholders were to pick up some of the default costs, then the borrower would face restrictions on the amount of junior debt that can be raised, in which case maximum equity participation on each date would not in general occur.

¹⁵Note that d^{t*} represents the vector of coupons that solves the creditor's date t maximization problem. As indicated by Lemma 1, it may be the case that this vector implies verification with probability one on each date, in which case the post-bankruptcy value of the entrepreneur's inside equity claim is zero.

lender holding a claim with lower priority than lender s_l receive zero.¹⁶ The payment to lender s_l is given by $x_t - c(x_t) + v_t(d^{t*}, t, x_t) - \sum_{S_p} (d^s + v_t(d^s, u, x_t))$, which is non-negative.¹⁷

Let $D_t = \sum_{s=0}^{t-1} d^s$ so that D_t denotes the total debt payment due on date t . A bond issued on date s will be issued with a call price of $c_s = v_{s+1}(d^s, s, D_{s+1})$, which represents the date $s + 1$ value of the bond issued on the preceding date should the cash flow x_{s+1} end up being exactly equal to the total debt payment D_{s+1} due. Since this implies that $c_s \leq (<)v_{s+1}(d^s, s, x_{s+1})$ whenever $x_{s+1} \geq (<)D_{s+1}$, the given call price ensures that the bond issued on date s will be trading at a premeium and therefore called on the following date if and only if $x_{s+1} \geq D_{s+1}$. In other words, the call price is specified in such a way that bond s will be called on date $s + 1$ if and only if the firm generates a date $s + 1$ cash surplus.

Note that if bond s is not called on date $s + 1$, then it may on a later date start trading at a premium relative to its call price (and still be left uncalled) or, alternatively, it will be called if its call premium is sufficiently large to exceed the windfall created for remaining less senior bondholders. Let $P_t^s = v_t(d^s, u, x_t) - v_{s+1}(d^s, s, D_{s+1})$, where $t > s + 1$, denote the call premium associated with bond s . Although it is possible that P_t^s is negative, we will be interested in the case for which this call premium is positive. By the definition of the call price c_s and the fact that the bond's market value, $v_t(d^s, u, x_t)$, is an increasing function of its priority (or a decreasing function of u), it follows that $P_t^s \geq 0$ if and only if $x_t \geq D_{s+1}$ for the case $u \leq s$ (and where $t > s + 1$.) If bond s is not called on date $s + 1$, it may (but need not) be left uncalled on later dates as well even if its market value $v_t(d^s, u, x_t)$ rises above its call value c_s . This occurs because by calling bond s the borrower creates a windfall to less senior bondholders whose bonds are not called. Let wf_t^s denote the total value of this windfall. Bond s will now be called if $P_t^s > wf_t^s$; that is, bond s will be called if the value of its call premium exceeds the total

¹⁶In practice, junior lenders whose claims are threatend to be wiped out by adherence to absolute priority are often able to force the borrower (or the bankruptcy court) to make a positive cash payment distribution and in addition avoid that their claims are wiped out; however, in the context of the CSV framework, since verification, by definition, allows both cash flows as well as firm value to be ascertained in a court of law, enforcement of stated priority rules becomes trivial (or costless) leaving attempts to force deviations from absolute priority futile.

¹⁷Note that S_p may be empty, and the set of lenders receiving zero in bankruptcy may be empty as well.

value of the windfall created for remaining bondholders if it is called (see Appendix B for an example).

Let S denote the set of bonds carried over from date $t - 1$ to date t and let S_c denote the set of bonds called on date t . The following proposition describes the type of debt structure and credit market transactions needed to induce MEP and thus truthful reports.

Proposition 3 *Suppose a bond issued on date s is issued with the call price $c_s = v_{s+1}(d^s, s, D_{s+1})$; then, on date $t > s$, if $x_t > D_t$, the borrower will call all outstanding bonds. The resulting net cash distribution,*

$$D_t + \sum_S c_s - \hat{x}_t,$$

is funded by issuing new (callable) debt. If $x_t < D_t$, and the solvency condition is satisfied, then the borrower will call any bond for which $P_t^s > wf_t^s$; the resulting net cash distribution,

$$D_t + \sum_{S_c} c_s - \hat{x}_t,$$

is funded by issuing junior (callable) debt. In either case, the borrower puts $\hat{x}_t = x_t$ on each date thus revealing cash flows truthfully.

Proof: (i) $x_t < D_t$. The total cash payment to bondholders in this case, so long as the firm is solvent, is given by $D_t + \sum_{S_c} c_s$, where the set S_c may be empty. Given the borrower's cash contribution of \hat{x}_t , a total of $D_t + \sum_{S_c} c_s - \hat{x}_t$ in additional debt must be raised. Suppose that the borrower puts $\hat{x}_t = x_t$. To see that this is indeed optimal, note that putting $\hat{x}_t = x_t$ both minimizes the amount of funds that need to be raised and in addition ensures that the borrower and the credit market is symmetrically informed about the current cash flow realization; as a result, additional funds are raised on fair terms. Note finally by absolute priority that existing bondholders will be indifferent as to the size of the new debt issue, which ensures that there will be no wealth transfers to existing bondholders from increasing \hat{x}_t .

(ii) $x_t > D_t$. Note first that all outstanding bonds will be called in this case. To see this, start with the bond issued on date $t - 1$. This bond will be called by the fact that its call price $c_{t-1} = v_t(d^{t-1}, t - 1, D_t)$ is determined in such a way that it will be

called on date t if and only if $x_t > D_t$. Suppose then that the firm has other bonds outstanding as well. To see that these bonds will be called also, observe that for any of these bonds, on the account that $u \leq s$ and $D_{s+1} < D_t$, it will be the case that $c_s = v_t(d^s, s, D_{s+1}) < v_t(d^s, u, x_t)$; in other words, for any of these bonds it will be the case that they are trading above their call prices and will therefore be called. Given \hat{x}_t , the total cash need of the borrower is given by $D_t + \sum_S c_s - \hat{x}_t$. To see that $\hat{x}_t = x_t$, simply observe that this leads symmetric information and minimizes future default costs. \square

Call prices are constructed in such a way that the borrower is able to call all outstanding bonds at a profit whenever the firm generates a cash surplus. The firm's debt will thus be fully refunded in this case. To see that MEP obtains, suppose first that it does, in which case the borrower and lenders become symmetrically informed about current as well as future cash flows. Since now the credit market is competitive, the fact that the firm's debt structure is fully refunded under symmetric information implies that any debt issued is priced fairly in the market and therefore that the borrower absorbs 100 % of expected future verification costs. The borrower, therefore, has no incentive to deviate from MEP.

If the firm generates a cash short-fall, the borrower may call some (but never all) of the firm's outstanding bonds (see Appendix B for an example). Given \hat{x}_t , the borrower will have a date t cash need of $D_t + \sum_{S_c} c_s - \hat{x}_t$, which must be covered by issuing additional debt. The priority rules in place ensure that existing bondholders are unaffected by the amount new debt issued (and will therefore obtain no wealth gain as a result of the borrower increasing his total cash contribution). As a result, the borrower will want to minimize the amount of additional debt issued and thus putting $\hat{x}_t = x_t$.¹⁸

Consistent with what one observes in the market, the model shows that firms may refrain from calling bonds that are trading at a premium relative to their call price.¹⁹ As

¹⁸Again suppose that the borrower puts $\hat{x}_t = x_t$ so that the current cash flow realization will be truthfully revealed. Then any bonds issued will be priced under symmetric information and will thus be priced fairly. Since now, by the stated priority rules, existing bondholders are unaffected by the amount of cash that the borrower pays out, the borrower has no incentive to deviate from $\hat{x}_t = x_t$.

¹⁹Indeed, Longstaff and Tuckman (1994) find that market prices exceed call prices for 35 % of the issues in their sample of 727 issues of callable bonds. Furthermore, the CSV model predicts that more senior issues will trade at larger premia. Consistent with this prediction, Longstaff and Tuckman show that 82 % of the issues selling at a premium "are equal to or senior to other public debt by the same

explained, this occurs whenever the windfall created for remaining junior bondholders exceeds the total value of the premium. An identical rationale for the existence of call premia on callable bonds has been offered by Longstaff and Tuckman (1994). However, whereas such incentives arise exogenously in their paper, in the multiperiod CSV model considered here they arise endogenously from the debt structure created by the borrower seeking to minimize default costs over the life of the firm.

4.2 Short-Term Debt

The use of short-term debt to generate truthful reports when cash flows are positively correlated closely parallels the use of short-term debt when cash flows are independent over time. Indeed, the basic difference between the two cases is that while in the latter the firm's debt capacity is independent of the current cash flow realization, in the first case the firm's debt capacity is positively related to the current cash flow. This difference, however, has no effect on the result that short-term debt induces MEP and thus truthful reports on each date.

Proposition 4 *Suppose that the project is funded with a sequence of one-period debt contracts each with face value f_t ; $t = 1, \dots, \tau$ determined by*

$$\begin{aligned} f_t \int_{f_t - v_t(d^*(x_t); x_t)}^{\infty} g(x_t | x_{t-1}) dx_t + \int_0^{f_t - v_t(d^*(x_t); x_t)} (x_t - c(x_t) + v_t(d^*(x_t); x_t)) g(x_t | x_{t-1}) dx_t \\ = (f_{t-1} - \hat{x}_{t-1})(1 + r) \end{aligned}$$

then $\hat{x}_t = x_t$ on each date.

In renegotiation, when cash flows are positively correlated over time, existing bond holders will demand higher payments (or value) for higher reports. As a result, renegotiation will not in general induce the borrower to provide truthful cash flow reports. With short-term debt, since the payment received by the lender is independent of the period's cash flow (unless the firm is insolvent), this problem does not arise. Instead, the borrower will issue a new one-period claim and MEP arises because this minimizes future debt payments and therefore default costs.

issuer."

5 Concluding Comments

This paper examines debt financing in a multiperiod CSV model and shows that the standard debt contract as derived by Gale and Hellwig (1985) allows the borrower to construct a debt structure which—via either maximum equity participation—induces the borrower to reveal cash flows truthfully on each date. Analogous to the well known under-investment problem identified by Myers (1977), the presence of a debt overhang in the multiperiod CSV framework provides the borrower with a (potential) incentive to under-report the true state. It is shown, however, that it will be possible to avoid such incentives by *(i)* renegotiating existing claims, *(ii)* issuing callable debt, or *(iii)* funding the project with a sequence of one-period standard debt contracts.

Appendix A

Proof of Lemma 1: Specialize to the case for which $\tau = 2$. We need to solve the creditor's maximization problem recursively starting with date 1. Suppose therefore that the lender is control on date 1 and thus chooses d_2 so as to maximize

$$v_1(d_2; \theta_1) = d_2 \int_{d_2}^{\infty} g(x_2|\theta_1)dx_2 + \int_0^{d_2} (x_2 - c(x_2))g(x_2|\theta_1)dx_2 \quad (A.1)$$

It is well known that the solution to this problem implies $d_2^*(\theta_1) < \infty$ (see Winton [1995a] and Cantillo [1995]).

Proceed then to date 0 and suppose that the lender is in control to choose the pair of coupons (d_1, d_2) that maximizes

$$\begin{aligned} v_0(d_1, d_2) &= d_1 \int_{d_1}^{\infty} g(x_1|\theta_0)dx_1 + \int_0^{d_1} (x_1 - c(x_1))g(x_1|\theta_0)dx_1 \\ &+ \int_{d_1}^{\infty} [d_2 \int_{d_2}^{\infty} g(x_2|\theta_1)dx_2 + \int_0^{d_2} (x_2 - c(x_2))g(x_2|\theta_1)dx_2]g(x_1|\theta_0)dx_1 \\ &+ \int_0^{d_1} [d_2^*(\theta_1) \int_{d_2^*(\theta_1)}^{\infty} g(x_2|\theta_1)dx_2 + \int_0^{d_2^*(\theta_1)} (x_2 - c(x_2))g(x_2|\theta_1)dx_2]g(x_1|\theta_0)dx_1 \end{aligned} \quad (A.2)$$

The expression for v_0 reflects the following: if the firm avoids default on date 1, then the debt contract written on date 0 remains in effect until date 2; otherwise, the firm is again in default and the creditor chooses debt level $d_2^*(\theta_1)$ (from (A.1)).

(A.2) allows for the possibility that cash flows are (positively) correlated over time. The next expression specializes (A.2) to the case of i.i.d. cash flows:

$$\begin{aligned} v_0 &= d_1 \int_{d_1}^{\infty} g(x_1)dx_1 + \int_0^{d_1} (x_1 - c(x_1))g(x_1)dx_1 \\ &(1 - G(d_1))[\int_{d_2}^{\infty} g(x_1)dx_1 + \int_0^{d_2} (x_2 - c(x_2))g(x_2)dx_2] \\ &+ G(d_1)[\int_{d_2^*(1)}^{\infty} g(x_2)dx_2 + \int_0^{d_2^*(1)} (x_2 - c(x_2))g(x_2)dx_2], \end{aligned} \quad (A.3)$$

where $d_2^*(1)$ denotes the debt level chosen by the creditor if the firm defaults on date 1.

The first order conditions for maximum of v_0 in (A.3) are given by:

$$\frac{\partial v_0}{\partial d_1} = 1 - G(d_1) - c(d_1)g(d_1)$$

$$\begin{aligned}
& -g(d_1)\left[\int_{d_2}^{\infty} g(x_2)dx_2 + \int_0^{d_2} (x_2 - c(x_2))g(x_2)dx_2\right] \\
& +g(d_1)\left[\int_{d_2^*(1)}^{\infty} g(x_2)dx_2 + \int_0^{d_2^*(1)} (x_2 - c(x_2))g(x_2)dx_2\right] = 0,
\end{aligned} \tag{A.4}$$

whose solution is denoted d_1^* .

$$\frac{\partial v_0}{\partial d_2} = [1 - G(d_2) - c(d_2)g(d_2)][1 - G(d_1)] = 0, \tag{A.5}$$

the solution of which is denoted d_2^* . We note that d_2^* is independent of d_1 and thus coincide with $d_2^*(1)$ from (A.1). This ensures that the last two terms of (A.4) cancels so that

$$\frac{\partial v_0}{\partial d_1} = 1 - G(d_1) - c(d_1)g(d_1) = 0. \tag{A.4'}$$

To see that the second order conditions for maximum are satisfied, rewrite (A.5) and (A.4') to

$$\frac{\partial v_0}{\partial d} = [1 - \rho(d)c(d)][1 - G(d)]$$

and differentiate again with respect to d to obtain

$$\frac{\partial^2 v_0}{\partial d^2} = -[\rho'(d)c(d) + \rho(d)c'(d)][1 - G(d)],$$

which is strictly negative since $\rho'(d), c'(d) > 0$.

We have thus proved that $d_1^ = d_2^* < \infty$. It is obvious that this result extend to the general τ -period case and thus proves part (i) of the lemma.*

Consider then case for which cash flows are positively correlated over time. The first and second order conditions for maximum of $v_0(d_1, d_2)$ with respect to d_1 are given by

$$\frac{\partial v_0}{\partial d_1} = 1 - G(d_1|\theta_0) - c(d_1)g(d_1|\theta_0)$$

$$\begin{aligned}
& -[d_2 \int_{d_2}^{\infty} g(x_2|\theta_1(d_1))dx_2 + \int_0^{d_2} (x_2 - c(x_2))g(x_2|\theta_1(d_1))dx_2]g(d_1|\theta_0) \\
& +[d_2^*(\theta_1(d_1)) \int_{d_2^*(\theta_1(d_1))}^{\infty} g(x_2|\theta_1(d_1))dx_2 + \int_0^{d_2^*(\theta_1(d_1))} (x_2 - c(x_2))g(x_2|\theta_1(d_1))dx_2]g(d_1|\theta_0)
\end{aligned} \tag{A.6}$$

$$= [1 - c(d_1)\rho(d_1|\theta_0) + A\rho(d_1|\theta_0)][1 - G(d_1|\theta_0)] = 0, \tag{A.7}$$

where A is given by the last two terms of (A.6). The second order condition with respect to d_1 becomes:

$$\frac{\partial^2 v_0}{\partial d_1^2} = -[c'(d_1)\rho(d_1|\theta_0) + c(d_1)\rho'(d_1|\theta_0)] + [A\rho'(d_1|\theta_0) + A'(d_1)\rho(d_1|\theta_0)]. \tag{A.8}$$

Consider then the maximization of $v_0(d_1, d_2)$ in (A.2) with respect to d_2 :

$$\frac{\partial v_0}{\partial d_2} = \int_{d_1}^{\infty} [1 - G(d_2|\theta_1) - c(d_2)g(d_2|\theta_1)]g(x_1|\theta_0)dx_1 = 0. \quad (A.9)$$

The second order condition yields:

$$\begin{aligned} \frac{\partial^2 v_0(d_1, d_2)}{\partial d_2^2} = & \\ & - \int_{d_1}^{\infty} [c'(d_2)\rho(d_2|\theta_1) + c(d_2)\rho'(d_2|\theta_1)][1 - G(d_2|\theta_1)]g(x_1|\theta_0)dx_1 \\ & - \int_{d_1}^{\infty} [1 - c(d_2)\rho(d_2|\theta_1)]g(d_2|\theta_1)g(x_1|\theta_0)dx_1. \end{aligned} \quad (A.10)$$

We observe that conditions (A.7) – (A.10) are relatively complex and provide limited insight with respect to debt payments $(d_1^*(\theta_0), d_2^*(\theta_0))$, which ‘proves’ part (ii) of the lemma. \square

Cash Flows Uniformly Distributed

Assume that x_1 is uniformly distributed on the interval $[m_0 - s/2, m_0 + s/2]$ and that x_2 is uniformly distributed on $[x_1 - s/2, x_1 + s/2]$. This means that cash flows (x_1, x_2) are uniformly distributed and that $\theta_1 = x_1$. Assume, in addition, that $c(x) = c$ and that $s/2 > c$, where the first assumption is made for simplicity and the second to ensure the existence of a solution.

Note first that in the case of i.i.d. cash flows it can easily be shown that $d^* \equiv d_1^* = d_2^* = m_0 + s/2 - c$ and further that $d_2^*(x_1) = x_1 + s/2 - c$.

Given the structure imposed, first order conditions (A.7) and (A.9) become, respectively,

$$\begin{aligned} (m_0 + s/2 - d_1 - c) - \frac{1}{s}[d_2(d_1 + s/2 - d_2) + \frac{1}{2}d_2^2 - d_2c] \\ + \frac{1}{s}[\frac{1}{2}(d_1 + s/2 - c)^2] = 0 \end{aligned} \quad (A.7')$$

and

$$\frac{1}{2}(m_0 + s/2)^2 - \frac{1}{2}d_1^2 + (s/2 - d_2 - c)(m_0 + s/2 - d_1) = 0, \quad (A.9')$$

where we have used the fact that $d_2^*(x_1) = x_1 + s/2 - c$. The next lemma summarizes the insights that can be obtained from first order conditions (A.7') and (A.9').

Lemma 2': $d_1^*(\theta_0) < d^* < d_2^*(\theta_0)$.

The lemma says that the optimal post-bankruptcy debt payments are increasing over time and that ‘earlier’ payments are less than what would obtain if cash flows were i.i.d., while ‘later’ payments are higher than what would obtain if cash flows were i.i.d. Note that it can be shown that this result extend beyond the two-period case (to yield Lemma 2 in the text), but not without complexity.

Proof: First order conditions (A.7') and (A.9') may be written

$$d_1 = m_0 + s/2 - c + A, \quad (\text{A.7''})$$

where

$$A \equiv -[d_2(d_1 + s/2 - c) + \frac{1}{2}d_2^2] + \frac{1}{2}[d_1 + s/2 - c]^2,$$

and further

$$d_2 = \frac{1}{2}(m_0 + s/2 + d_1) + s/2 - c. \quad (\text{A.9''})$$

We observe from (A.7'') and (A.9'') that $d_1^*(\theta_0) < d^*$ if $A < 0$ and that $d_2^*(\theta_0) > d^*$ if $d_1^*(\theta_0) > m_0 - s/2$. The latter inequality requires that the lender chooses the debt payment for date 1 in a way that the probability of default on date 1 is strictly positive, I assume to be the case (if this assumption is not made, then the given inequalities remain unchanged except that they need not be strict). In any case, (A.9) implies that $d_2^*(\theta_0) > d^*$.

We then need to show that $A < 0$ so that $d_1^*(\theta_0) < d^*$. To see that this is so, note first that $A < 0$ implies

$$2d_2(d_1 + s/2 - c) > (d_1 + s/2 - c)^2 - d_2^2. \quad (11)$$

To see that inequality (11) is satisfied, note that with $s/2 > c$, this requires that $d_1 + s/2 - c < d_2$. Using the expression for d_2 from (A.9'') we observe that the latter inequality is satisfied provided $d_1 < m_0 + s/2$. Since $m_0 + s/2$ represents the upper bound on the first period cash flow x_1 , we conclude that $A < 0$ and therefore that $d_1^*(\theta_0) < d^*$. \square

Appendix B

An Example of the Borrower's Optimal Call Policy

A bond that trades at a premium relative to its call price earns rents to the bond holder at the expense of the borrower. The borrower may prevent such rents simply by calling the bond as soon as it starts trading above its call price—this is often referred to as the “text book rule.” However, the borrower will refrain from adhering to the text book rule by calling bonds trading a premium creates a windfall for (remaining) junior bond holders that exceeds the value of the premium.

To see the basic argument, consider the following example. Let $t = 3$ and suppose that $x_t < D_t$ for $t = 1, 2, 3$. Suppose also that no bonds were called on dates 1 and 2 and assume that

$$d^0 < x_3 < d^0 + d^1 < D_3 = \sum_{s=0}^3 d^s. \quad (B.1)$$

The date 3 market value of bond 0 is expressed by $v_3(d^0, 0, x_3)$. Given its call price $c_0 = v_1(d^0, 0, d^0)$, this bond trades at a premium of $P_3^0 = v_3(d^0, 0, x_3) - v_1(d^0, 0, d^0)$ by account of $x_3 > d^0$ and the fact that bond 0 is the most senior bond. Unless the borrower calls bond 0, no other bond will trade at a premium since $v_3(d^1, 1, x_3) < c_1 = v_2(d^1, 1, d^0 + d^1)$ by the fact that $x_3 < d^0 + d^1$ for bond 1 and since $v_3(d^2, 2, x_3) < c_2 = v_2(d^2, 2, d^0 + d^1 + d^2)$ for bond 2. This implies that unless the borrower calls the bond issued on date 0, then no bonds will be called.

Suppose, however, that the borrower decides to call bond 0. This increases the priority and therefore market values of bonds 1 and 2 to $v_3(d^1, 0, x_3)$ and $v_3(d^2, 1, x_3)$, creating a total windfall of

$$wf_3^0 = [v_3(d^1, 0, x_3) - v_3(d^1, 1, x_3)] + [v_3(d^2, 1, x_3) - v_3(d^2, 2, x_3)]. \quad (B.2)$$

Bond 0 will now be called only if

$$P_3^0 > wf_3^0. \quad (B.3)$$

If inequality (B.3) is satisfied and the borrower calls bond 0, then the arguments just applied to bond 0 will be applied to bond 1, which will be called if and only if its premium exceed the windfall created if called.

Note finally that bond 2 will never be called. This is because its call price c_2 will be determined on date 2 in a way that it will be called if and only if $x_3 > D_3$; that is, the

borrower on date 2 anticipates the optimal call policy on date 3 as a function and issues a bond that contains a call feature that induces the borrower to call it on date 3 if and only if $x_3 > D_3 = d^0 + d^1 + d^2$.

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CHAPTER 5

A Model of IPO Under- and Overpricing: Endogenous Information Acquisition in the Presence (and Absence) of Uninformed Investors*

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Abstract

This paper develops a model that explains why some types of IPOs (REITs and investment grade bonds) generate negative excess returns while others (common stocks and junk grade bonds) generate positive excess returns. Whereas winner's curse unambiguously forces the issuer to underprice the issue if both informed and uninformed investors submit bids, overpricing (on average) is possible if investors observe information of equal precision. The potential for overpricing arises because the issuer must set the terms of the IPO before the private information observed by investors becomes public. Importantly, the results are derived in a unified framework from general principles.

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1 Introduction

Although the underpricing of initial public offerings (IPOs) of common stocks is a well established empirical fact, the overall IPO evidence reveals statistically significant overpricing as well as underpricing. In particular, while Wang, Chan, and Gau (1992) document negative initial returns for IPOs of Real Estate Investment Trusts (REITs), Datta, Iskander-Datta, and Patell (1997) find positive excess returns for IPOs of speculative grade bonds but negative excess returns for investment grade bonds.¹ Though existing IPO theories are able to explain the well known underpricing phenomenon of common stocks, they are unable to account for the fact that some types of IPOs are overpriced.² In contrast, the present paper develops an IPO model that is able to account for overpricing as well as underpricing. Importantly, the results are derived in a unified framework from general principles and are therefore insulated from the ‘special purpose’-critique that has sometimes been directed towards existing models.

In the formal model, the issuer sets the terms of the issue (the initial price and the number of shares to sell) in a way that attracts the number of informed and uninformed investors that maximizes the combined value of the proceeds received in the IPO and the post-issue market value of the issuer’s remaining claim in the firm. Whereas the participation of uninformed investors increases the probability that the issue will be over-subscribed (and thus succeed), the participation of informed investors has the effect of moving the firm’s post-issue market value towards its fundamental value (which in general will be unknown to the issuer and investors alike). The model implies that if the private costs to investors of evaluating the issue are large, it will be marketed to both informed and uninformed investors. As in Rock (1986), the combination of informed and uninformed investors creates winner’s curse and unamiguously forces the issuer to

¹In addition, Muscarella (1988) finds statistically negative returns in some subsamples of his sample of IPOs of Master Limited Partnerships, while Weiss (1989) and Peavy (1990) document zero excess initial returns for IPOs of closed-end mutual funds.

²Existing IPO theories include winner’s curse (Rock [1986], Beatty and Ritter [1986], Carter and Manaster [1990]), signalling (Allen and Faulhaber [1989], Grinblatt and Wang [1989], and Welch [1989]); information cascades (Welch [1992]), incomplete markets (Mauer and Senbet [1992]), litigation avoidance (Tinic [1988] and Hughes and Thakor [1992]), and costly investor information production (Sherman [1992] and Chemmanur [1993]).

Although in Welch (1992) IPOs may be overpriced ex post, they can never be overpriced on average (or in expectation) as the case is in the present paper.

underprice the issue. If the costs of evaluating the issue are sufficiently low, however, the issuer may find it optimal to solicit bids exclusively from informed investors. With no uninformed investors submitting bids, the issue need not be underpriced and may therefore be overpriced. Empirically, then, the model predicts that IPOs that are on average underpriced (such as those of common stocks and junk grade corporate bonds) will be characterized by a relatively high degree of investor heterogeneity, whereas IPOs that are on average overpriced (such as those of REITs and investment grade corporate bonds) will be characterized by a relatively high degree of investor homogeneity. In addition, it predicts that investors operating in the market for IPOs that are on average overpriced expend less resources in ascertaining value compared to the informed investors operating in the market for underpriced IPOs.

While underpricing is derived as a direct consequence of winner's curse, the potential for overpricing arises because the issuer is forced to determine the terms of the IPO before investors incur the costs that are necessary to become informed and thus before the information generated by investors becomes public. As a result, the initial price must be set in such a way that an investor who decides to acquire information about the issue expects to recover the costs she incurs in acquiring this information. In addition, the fact that the terms of the IPO must be determined before the information observed by investors becomes public implies that an investor who acquires private information about the issue effectively purchases an option to submit a bid if and only if the IPO—as seen from the investor's perspective—is underpriced. The upshot of this, as will be shown, is that IPOs may be overpriced on average.

Indeed, in the special case for which investors are endowed with costless and private information about the firm, it is shown that IPOs will unambiguously be overpriced (on average). This case resembles a situation in which each investor knows the underlying probability distribution from which firms coming to the market are drawn but observes in addition private information about the issue. The fact that the information observed is costless implies that the option embedded in it is costless as well, potentially generating positive rents for investors. However, the existence of positive rents is inconsistent with a competitive market. The issuer will therefore raise the offer price until rents are zero and the issue is overpriced (in expectation).

Whereas overpricing in the case of zero information costs arises directly and unambiguously from the participation constraint of informed investors, IPO overpricing in the general case of positive information costs is not equally immediate. As already argued, if uninformed investors submit bids along with informed investors, then winner's curse leads unambiguously to underpricing. To obtain overpricing in the general case of positive information costs it is therefore necessary that the issuer markets the issue exclusively to informed investors, which is optimal only if the costs to investors of acquiring information are sufficiently low. In addition, it is necessary that the private information observed by individual investors is more precise than the information observed by the issuer. This allows the issuer to optimally increase the initial price as well as the fraction of the firm offered for sale beyond what is strictly necessary to prevent uninformed investors from submitting bids, the result of which is that winner's curse will no longer be a concern and IPOs may (but need not) be overpriced on average.

The formal model elaborates on the information production arguments of Sherman (1992) and Chemmanur (1993), though it is more closely related to the latter. However, while both authors (in different settings) consider exclusively the role of informed investors and show that underpricing may be a consequence of a costly information acquisition process undertaken by investors after the terms of the IPO have been announced, the present paper develops an IPO model in which informed as well as uninformed investors are allowed to submit bids and in which underpricing arises as a direct consequence of winner's curse (but may also for some parameterizations be attributed to the costly information acquisition process undertaken by investors after the terms of the IPO have been announced). In addition, neither of these two papers are able to account for fact that some types of IPOs are overpriced rather than underpriced.

The rest of the paper is organized as follows. Section 2 introduces the basic setup. Section 3 presents the equilibrium conditions for informed and uninformed investors and shows that underpricing and overpricing can be generated directly from these conditions. Section 4 examines overpricing for the general case in which investors information costs are positive and shows that overpricing occurs only if investors' private information costs are sufficiently low and the information observed individual investors is more precise than the information observed by the issuer. Finally, Section 5 concludes the paper.

2 The Setup

The model has three dates, denoted 0, 1, and 2 and contains a firm (the ‘issuer’) whose true value $v = v_i; i = G, B$, where $v_G > v_B$, is not known for sure by anybody until date 2, at which point the firm’s true identity is revealed to everybody. It is commonly known on date 0, however, that the probability of $v = v_G$ is given by $\alpha \in (0, 1)$. The issuer is unsure about her own type but observes a costless information signal $s^F \in \{s_G^F, s_B^F\}$, which is correlated (possibly perfectly) with the true value of the firm. I will be referring to an issuer who observes $s = s_i^F$ as a type i issuer. It is assumed that everybody is risk neutral and that the riskless rate is zero. Table 1 summarizes the sequence of events of the model.

The issuer, regardless of type (as we shall see), undertakes an initial public offering on date 0 by selling a fraction w of the firm at the price P_0 per share. The financial market is competitive and is assumed to contain a large enough number of investors to make it possible to price the issue so that it goes through with probability one, though this will not in general be optimal. Any investor operating in this market has, at cost k , access to a private information signal $s \in \{s_G, s_B\}$, which is correlated with the true value of the firm.

Upon observing the IPO terms (w, P_0) , a total of n investors decide to participate, each submitting either an uninformed bid, an informed bid, or no bid at all. Let n_u denote the number of uninformed investors. Each uninformed investor submits a bid for sure once having decided to participate in the IPO. Let further n_I denote the number of investors who acquire costly private information about the issue before deciding whether or not to submit a bid. An informed investor submits a bid only after obtaining favorable information about the issue; i.e. only after obtaining $s = s_G$.

It is assumed that an investor who obtains an allocation in the IPO is awarded one share at the IPO price P_0 . Let N denote the total number of shares issued by the firm and let n^* denote the number of shares that are put up for sale. The number of shares retained by the issuer is therefore $N - n^* \geq 0$, while the fraction of the firm sold in the IPO is given by $w = n^*/N$. Because the structure of the model requires that n^* is an integer, it is assumed that n^* is given exogenously as an integer greater or equal to one and therefore that N is determined endogenously by the issuer optimally choosing the

Table 1: Sequence of Events

Date 0:	<ul style="list-style-type: none"> • The issuer announces the IPO terms (N, P_0). • n_I investors become informed. • \hat{n}_I investors obtain $s = s_G$ and subsequently submit bids along with n_u uninformed investors. • If $\hat{n}_I + n_u \geq n^*$ the IPO is over-subscribed and goes through; otherwise, the IPO is under-subscribed and fails.
Date 1:	<ul style="list-style-type: none"> • \hat{n}_I is revealed and the firm's post-issue share price is established. • The issuer sells her remaining shares and consumes. • Participating investors close out their positions and consume.
Date 2:	<ul style="list-style-type: none"> • The true value of the firm is revealed.

fraction n^*/N of the firm to sell at the initial stage.

Let \hat{n}_I denote the number of informed investors who obtain $s = s_G$ and consequently end up submitting bids. \hat{n}_I is clearly a binomially distributed random variable with support $[0, n_I]$. With each of the n_u uninformed investors submitting bids for sure, the total number of bidders becomes $n \equiv n_u + \hat{n}_I$. If now $n < n^*$, the issue is under-subscribed and fails to go through. Otherwise, the IPO is over-subscribed and succeeds. Let further $\underline{\hat{n}}_I$ denote the minimum number of informed bids that is required for the issue to come through. Given n_u , we have that $\underline{\hat{n}}_I = \max(n^* - n_u, 0)$, where we note that since the issue will go through with probability one for the case $n_u = n^*$, $n_u > n^*$ will never be optimal and can thus be ruled out.

Although allowing the issue to fail may seem to limit the scope of the analysis to 'best-efforts' issues, as noted by Welch (1992), even 'firm-commitment' offerings are sometimes withdrawn. Moreover, whereas in the present model the issuer sets the terms of the IPO and absorbs 100% of the costs associated with an under-subscribed issue, in a firm-commitment offering the terms of the IPO are generally determined by an investment banker (in consultation with the issuer), who must absorb some of the costs associated with an under-subscribed issue by the fact that she must absorb any unsold shares. As a result, whether the terms of the IPO are set by the issuer, as the case is in the current model, or they are set by an investment banker, as the case is in practice, the IPO terms will reflect the fact that the demand for shares will be a function of the IPO terms as well as of the number of investors who observe favorable information about the issue.³

³Note, however, that although the mathematical structure of the model is inherently linked to the

It is assumed that the issuer as well as the n investors participating in the IPO all consume on date 1 after the post-issue market value of the firm has been established. This assumption ensures that all participants in the IPO make decisions and predictions with respect to the firm's post-issue market value rather than its true value (v_i) and is meant to capture the idea that participants in actual markets attempt to predict future market value and are concerned with 'true' value only to the extent that this is helpful in making predictions regarding the firm's future market value.

Note finally that it may not in all cases be optimal to go public. If it is not desirable to go public, then the issuer retains a 100 % ownership stake in the firm until date 1 at which point she sells (to a private investor) her entire stake at the price $\underline{v} \equiv \alpha v_G + (1 - \alpha)v_B$.

2.1 Information

To become informed, an investor incurs the costs k to observe a private information signal $s = s_i, i = G, B$, whose likelihood probabilities are denoted $P(s_i|v_i); i = G, B$.⁴ The posterior probabilities of s are given by Bayes' rule as follows:

$$P(v_G|s_G) = \frac{P(s_G|v_G)\alpha}{P(s_G|v_G)\alpha + P(s_G|v_B)(1 - \alpha)} \quad (1)$$

and

$$P(v_B|s_B) = \frac{P(s_B|v_B)(1 - \alpha)}{P(s_B|v_B)(1 - \alpha) + P(s_B|v_G)\alpha} \quad (2)$$

The likelihood and posterior probabilities of the issuer's information signal, s^F , are defined analogously.

It is assumed that $P(v_G|s_G), P(v_G|s_G^F) > P(v_G)$ so that signals s and s^F are both strictly informative. No constraints are imposed as to which signal is the more informative, however.

2.2 Post-Issue Firm Value

It is assumed that the total number n_I of informed investors as well as the number \hat{n}_I of investors who obtain $s = s_G$ are revealed in the post-issue market (date 1). This

possibility that the issue is withdrawn, neither the overpricing result derived nor the underpricing result depend on this.

⁴ $P(\cdot)$ and $P(\cdot|\cdot)$ denote unconditional and conditional probabilities throughout.

assumption ensures that the firm's post-issue market value will fully reflect the information acquired by investors but circumvents the question as to how and to what extent this occurs in actual markets.⁵

Let $V(\hat{n}_I)$ denote the (date 1) post-issue value of the firm as a function of \hat{n}_I . To derive the formal expression for $V(\hat{n}_I)$, I start with the probability that \hat{n}_I informed investors observe $s = s_G$ conditional on $v = v_G$:

$$P(\hat{n}_I|v_G) = \binom{n_I}{\hat{n}_I} P(s_G|v_G)^{\hat{n}_I} (1 - P(s_G|v_G))^{n_I - \hat{n}_I}. \quad (3)$$

Using Bayes' Rule, the probability that $v = v_G$ given \hat{n}_I becomes

$$P(v_G|\hat{n}_I) = \frac{P(\hat{n}_I|v_G) \alpha}{P(\hat{n}_I|v_G) \alpha + P(\hat{n}_I|v_B) (1 - \alpha)}. \quad (4)$$

The post-issue value of the firm as a function of \hat{n}_I is then given by

$$V(\hat{n}_I) = P(v_G|\hat{n}_I) v_G + (1 - P(v_G|\hat{n}_I)) v_B \quad (5)$$

which (by assumption) fully reveals the information collected by investors.

Given the total number of shares outstanding of N , the post-issue share price of the firm is given by $V(\hat{n}_I)/N$. The (random) IPO return is therefore

$$r_0(\hat{n}_I) = \frac{V(\hat{n}_I)}{NP_0} - 1, \quad (6)$$

which is increasing in the number of investors who obtain favorable information about the firm. This means that a high initial return represents 'good news' for the issuing firm. In addition, since the number of shares that are put up for sale in the IPO is fixed, it implies that the competition for shares becomes strictly more intense at higher ex-post initial returns, thus giving rise to winner's curse.

The expression for the average (or ex ante expected) post-issue value of the firm is given by

$$\bar{v} = \left(\sum_{\hat{n}_I} P(\hat{n}_I) \right)^{-1} \sum_{\hat{n}_I} P(\hat{n}_I) V(\hat{n}_I), \quad (7)$$

⁵See, however, e.g. Hellwig (1980) and Kyle (1985). Recently, Subrahmanyam and Titman (1997) analyze the linkage between the informational efficiency of the stock market and the benefits of going public in which (part of) investors' private information is revealed via the trading process. They do not, however, analyze IPO returns, which is the objective of the present paper.

where

$$P(\hat{n}_I) = \binom{n_I}{\hat{n}_I} P(s_G)^{\hat{n}_I} (1 - P(s_G))^{n_I - \hat{n}_I} \quad (8)$$

and

$$P(s_G) = P(s_G|v_G)P(v_G) + P(s_G|v_B)P(v_B) \quad (9)$$

Whereas $P(s_G)$ is the unconditional probability that $s = s_G$, $P(\hat{n}_I)$ is the unconditional probability that \hat{n}_I investors obtain $s = s_G$.

Using \bar{v} , the expected initial return, \bar{r}_0 , becomes

$$\bar{r}_0 = \frac{\bar{v}}{NP_0} - 1, \quad (10)$$

which closely resembles the empirical definition of initial return.⁶ Hence, the IPO will be said to be *underpriced* if $\bar{v} > NP_0$ and *overpriced* if $\bar{v} < NP_0$.

The issue will fail if $\hat{n}_I < \hat{n}_I$. Let now the post-issue value of the firm conditional failure be given by

$$\bar{v}_f = P_f v_G + (1 - P_f) v_B, \quad (11)$$

where

$$P_f = \sum_0^{\hat{n}_I - 1} P(v_G | \hat{n}_I). \quad (12)$$

That is, in contrast to the value of the firm conditional on the issue going through, firm value conditional on the issue not going through does not fully impound the information collected by investors. This distinction emphasizes the idea that an important part of the information collected by investors becomes public only via the trades that they do in a public stock market and ensures that there is a cost associated with an under-subscribed issue, in turn providing the issuer with a potential motive to induce uninformed investors to participate in the issue.

2.3 The Issuer's Objective Function

The issuer owns 100% of the firm before taking it public. Selling a fraction $w = n^*/N$, the issuer first receives n^*P_0 at the initial stage and then on date 1 collects additional

⁶This definition, which is critical to the results derived, is in contrast to what is common in the literature of defining the IPO return by relating the IPO value to ex ante value (\underline{v}) rather than to the random post-issue market value, which is what is done here.

proceeds of $(1 - n^*/N)V(\hat{n}_I)$, provided, of course, that the issue is over-subscribed and thus goes through.

Since there are no direct costs of going public, type B firms will always want to copy the IPO strategies of type G firms. This rules out the possibility of separating equilibria in which the terms of the IPO reveals the issuer's information about the firm and thus allows us to ignore the IPO strategy of type B firms and instead concentrate on the pricing decisions of type G firms.

Given the IPO terms of (N, P_0) , the expression for the expected wealth of a type G issuer becomes

$$E(W_G) \equiv P[(1 - n^*/N) \frac{1}{P} \sum_{\hat{n}_I}^{\hat{n}_I} P(\hat{n}_I|s_G^F)V(\hat{n}_I) + n^*P_0] + (1 - P)\bar{v}_f, \quad (13)$$

where

$$P \equiv \sum_{\hat{n}_I}^{\hat{n}_I} P(\hat{n}_I|s_G^F), \quad (14)$$

$$P(\hat{n}_I|s_G^F) = \binom{n_I}{\hat{n}_I} P(s_G|s_G^F)^{\hat{n}_I} (1 - P(s_G|s_G^F))^{n_I - \hat{n}_I}, \quad (15)$$

and

$$P(s_G|s_G^F) = P(s_G|v_G)P(v_G|s_G^F) + (1 - P(s_B|v_B))(1 - P(v_G|s_G^F)). \quad (16)$$

The expression for $E(W_G)$ can be explained as follows. From the perspective of a type G issuer, the IPO succeeds with probability P . If it succeeds, she collects n^*P_0 on the initial date while retaining a claim with an expected value of $(1 - n^*/N) \sum_{\hat{n}_I}^{\hat{n}_I} P(\hat{n}_I|s_G^F)V(\hat{n}_I)$. If the issue is under-subscribed and thus fails, which it does with probability $1 - P$, the issuer is left owning a firm valued at \bar{v}_f .

Formally, then, the issuer chooses on date 0 the IPO terms (N, P_0) so as to maximize $E(W_G)$ subject to informed and uninformed investors breaking even and subject to her own participation constraint: $\max_{(N, P_0)} E(W_G) \geq \underline{v}$. The next section analyzes the break even constraints (or equilibrium conditions) of informed and uninformed investors.

3 Market Equilibrium

3.1 Equilibrium in the Market for Uninformed Investors

The equilibrium condition for uninformed investors is given by

$$\sum_{\hat{n}_I} P(\hat{n}_I)[V(\hat{n}_I) - NP_0] \left(\frac{n^*}{\hat{n}_I + n_u} \right) \frac{1}{N} = 0, \quad (17)$$

which ensures that each of the n_u uninformed investors who decides to submit bids in the IPO breaks even on average. It reflects the fact that uninformed investors are price takers, that they expend zero direct costs in participating in the IPO, and that they face stiffer competition for shares at higher post-issue firm values. For reasons that will become clear, I will be referring to condition (17) as the *underpricing condition*.

Using (10), condition (17) can be expressed in terms of the initial return as follows

$$\sum_{\hat{n}_I} P^*(\hat{n}_I) r_0(\hat{n}_I) = 0, \quad (17')$$

where $P^*(\hat{n}_I) \equiv P(\hat{n}_I) \frac{n^*}{\hat{n}_I + n_u}$ represents the original probability measure adjusted by the *winner's curse factor* $\frac{n^*}{\hat{n}_I + n_u}$. We observe that $\frac{n^*}{\hat{n}_I + n_u}$ is less than or equal to one and that it is decreasing in \hat{n}_I and $V(\hat{n}_I)$, reflecting the fact that uninformed investors are in general rationed and are allocated a disproportionate large (small) fraction of IPOs that are overpriced (underpriced). In any case, condition (17') says that the IPO must be priced so that the expected IPO return from the perspective of an uninformed investor is zero. The following proposition shows that this unambiguously leads to underpricing.

Proposition 1 *If the IPO is priced along the participation constraint for uninformed investors (condition [17]), it will be underpriced.*

Proof: The issue is underpriced if $\bar{v} > NP_0$. Solving (17) for NP_0 gives

$$NP_0 = \left(\sum_{\hat{n}_I} P^*(\hat{n}_I) \right)^{-1} \sum_{\hat{n}_I} P^*(\hat{n}_I) V(\hat{n}_I). \quad (P.1)$$

There is now underpricing if

$$\left(\sum_{\hat{n}_I} P(\hat{n}_I) \right)^{-1} \sum_{\hat{n}_I} P(\hat{n}_I) V(\hat{n}_I) > \left(\sum_{\hat{n}_I} P^*(\hat{n}_I) \right)^{-1} \sum_{\hat{n}_I} P^*(\hat{n}_I) V(\hat{n}_I). \quad (P.2)$$

Both sides of (P.2) is a weighted average of post-issue firm values. To see that the inequality is satisfied, observe simply that the probability measure $P^*(\hat{n}_I)$ places relatively larger weights on low realizations of $V(\hat{n}_I)$ compared to $P(\hat{n}_I)$. \square

IPO underpricing is thus derived as a direct consequence of the fact that uninformed investors are allocated a disproportionate large (small) fraction of IPOs that are overpriced (underpriced). This result is, of course, identical to Rock's (1986) seminal winner's curse insight, but is generated in a very different and more general setting.

Solving condition (17) with respect to NP_0 , we obtain

$$NP_0 = \left(\sum_{\hat{n}_I}^{n_I} P^*(\hat{n}_I) \right)^{-1} \sum_{\hat{n}_I}^{n_I} P^*(\hat{n}_I) V(\hat{n}_I). \quad (18)$$

Denote the right hand side of (18) by $\bar{v}_u = \bar{v}_u(n_u)$ and note that while NP_0 represents the total value of the firm in terms of the initial offering price, \bar{v}_u constitutes the expected post-issue value of the firm from the perspective of an uninformed investor who must take into account the fact that she faces increased competition from informed investors for higher realizations of the firm's post-issue market value.

In order to induce n_u uninformed investors to submit bids, it is necessary that the issuer puts NP_0 equal to $\bar{v}_u(n_u)$. However, as will be discussed in some detail below, if it is not optimal to induce bids from uninformed investors, then the issuer will be free to put $NP_0 > \bar{v}_u(0)$, in which case the IPO need not be underpriced and may therefore be overpriced.

3.2 Equilibrium in the Market for Informed Investors

The equilibrium condition for informed investors is given by

$$P(s_G) \sum_{\hat{n}_I-1}^{n_I-1} P(\hat{n}_I - 1 | s_G) [V(\hat{n}_I) - P_0 N] \frac{n^*}{\hat{n}_I + n_u} \frac{1}{N} = k, \quad (19)$$

where

$$P(\hat{n}_I - 1 | s_G) = \binom{n_I - 1}{\hat{n}_I - 1} P(s_G | s_G)^{\hat{n}_I - 1} (1 - P(s_G | s_G))^{n_I - \hat{n}_I}, \quad (20)$$

$$P(s_G | s_G) = P(s_G | v_G) P(v_G | s_G) + (1 - P(s_B | v_B)) (1 - P(v_G | s_G)). \quad (21)$$

Condition (19) reflects the fact an informed investor submits a bid only after having obtained favorable information about the issue. It further ensures that the n_I investors

who acquire costly information about the issue break even on average. Note that whereas $P(s_G)$ constitutes the unconditional probability that an investor will obtain favorable information about the issue, $P(s_G|s_G)$ represents the probability, as seen from the perspective of an investor who obtains $s = s_G$, that a randomly selected investor from a remaining sample of $n_I - 1$ informed investors obtains $s = s_G$. This gives $P(\hat{n}_I - 1|s_G)$ as the corresponding conditional probability that $\hat{n}_I - 1$ investors, in addition to our ‘representative’ investor, observe favorable information about the firm.

Condition (19) may be written in terms of the IPO return as follows

$$P(s_G) \sum_{\hat{n}_I-1}^{n_I-1} P^*(\hat{n}_I - 1|s_G) r_0(\hat{n}_I) P_0 = k, \quad (19')$$

where $P^*(\hat{n}_I - 1|s_G) \equiv P(\hat{n}_I - 1|s_G) \frac{n^*}{\hat{n}_I + n_u}$. We note by the definition of $P^*(\cdot|s_G)$ that informed investors, like uninformed investors, are exposed to winner’s curse. Despite this, however, as we shall see in Proposition 2, with zero information costs ($k = 0$), condition (19') unambiguously yields overpricing.

Condition (19) can be rearranged to

$$NP_0 = \bar{v}_I \equiv \left(\sum_{\hat{n}_I-1}^{n_I-1} P^*(\hat{n}_I - 1|s_G) \right)^{-1} \left(\sum_{\hat{n}_I-1}^{n_I-1} P^*(\hat{n}_I - 1|s_G) V(\hat{n}_I) - \frac{Nk}{n^* P(s_G)} \right), \quad (22)$$

where we may view $\bar{v}_I = \bar{v}_I(n_I)$ as the value of the firm from the perspective of an investor who is considering acquiring costly information about the firm. By becoming informed, an investor effectively purchases an option (at cost k) to bid for an allocation in the IPO if and only if the private information observed indicates that the issue is underpriced. At the same time the investor must take into consideration the fact that the competition from other informed investors for allocations is more intense for higher degrees of ex-post underpricing.

3.2.1 Zero Information Costs

Assume for the moment that investors’ information costs are zero. Given the opportunity to become privately informed at no cost it would make no sense for an investor to ever submit an uninformed bid. We may therefore in the present subsection ignore the equilibrium condition for uninformed investors (17) and instead focus on the equilibrium condition for informed investors (19).

Recall from Proposition 1 that if the issue is priced along condition (17), it will be underpriced. The next proposition derives an analogous result with respect to overpricing.

Proposition 2 *Suppose that $k = 0$ and $NP_0 = \bar{v}_I(n_I)$, then $\bar{v}_u(0) < \bar{v} < \bar{v}_I(n_I)$ and the issue is overpriced.*

Proof: See Appendix.

In other words, if investors are given access to costless and private information about the firm, the IPO will unambiguously be overpriced. This result is a direct consequence of the fact that the issuer must determine the terms of the IPO before the information observed by investors becomes public, which effectively gives each investor an option to submit a bid if and only if the private information observed indicates that the issue is underpriced. While the value of this option depends on the IPO price in relation to the firm's ex ante expected post-issue share price, the cost of it is equal to the cost of becoming informed, which is zero. Therefore, if the initial price is set less than or equal to the firm's unconditional expected post-issue share price ($NP_0 \leq \bar{v}$), then investors will receive strictly positive rents. The issuer thus increases the initial price until investors' rents are zero and the issue is overpriced.

It is useful here to note that overpricing is a direct result of the fact that the terms of the IPO are set before the information observed privately by investors becomes public and occurs despite the fact that informed investors competing against other informed investors face more intense competition for shares in IPOs that ex post are more underpriced. Put differently, overpricing (on average) occurs despite the fact that informed investors face winner's curse and requires only that private information observed by individual investors is strictly correlated with the true value of the firm.

4 Positive Information Costs

Whereas in the case of zero information costs IPO overpricing may be derived directly from the participation constraint for informed investors, overpricing in the general case of positive information costs requires that the issuer (i) markets the issue exclusively to informed investor, and (ii) prices the issue strictly off the underpricing condition (i.e.

prices the issue so that $NP_0 > \bar{v}_u(0)$). While (i) will be satisfied for sufficiently low k , (ii) requires that the issuer increases the fraction of the firm sold in the IPO relative to the fraction implied by the underpricing condition.⁷ Since it must be optimal to do so, condition (ii) requires that

$$\frac{\partial E(W_G)}{\partial (n^*/N)} \geq 0. \quad (23)$$

Substituting from (19) into the expression for $E(W_G)$ and differentiating with respect to N reveals that $E(W_G)$ is increasing in n^*/N if and only if

$$\frac{1}{\sum_{\hat{n}_I}^{n_I} P(\hat{n}_I|s_G^F)} \sum_{\hat{n}_I}^{n_I} P(\hat{n}_I|s_G^F)V(\hat{n}_I) \leq \frac{1}{\sum_{\hat{n}_I-1}^{n_I-1} P^*(\hat{n}_I-1|s_G)} \sum_{\hat{n}_I-1}^{n_I-1} P^*(\hat{n}_I-1|s_G)V(\hat{n}_I). \quad (24)$$

This result is quite intuitive. Overpricing requires that the issuer floats a larger fraction of the firm in the IPO compared to the fraction that is implied by the underpricing condition $NP_0 = \bar{v}_u(0)$. According to inequality (24), the issuer finds this optimal so long as her valuation of the firm is below that of individual investors who hold favorable information about the firm.⁸

The following lemma expresses condition (24) in terms of the precision of the information observed by individual investors relative to that of the issuer.

Lemma 1 *Condition (24) is satisfied if and only if $P(s_G|s_G) > P(s_G|s_G^F)$; i.e., if and only if the information observed by individual investors is more precise than the information observed by the issuer.*

Proof: See Appendix.

Both the issuer and any investor who bids for shares in the IPO hold favorable information about the issue. As indicated by the lemma, this implies that the party with the more precise information is also the one who values the firm the most.

Although Lemma 1 identifies a necessary condition for overpricing in the general case of positive information costs, we have yet to prove the existence of an equilibrium

⁷To see this, take the change in NP_0 with respect to N along $NP_0 = \bar{v}_I(n_I)$ and observe that this yields $\frac{d}{dN}NP_0 = -P_0 \frac{k}{n^*P(s_G)} < 0$, which in turn implies that $\frac{d}{d(n^*/N)}NP_0 > 0$.

⁸Note that condition (24) is independent of N , which implies that if it is satisfied, then the issuer will want to sell the entire firm at the initial stage. Such a corner result is unlikely to arise in a more complete model in which the fraction retained by the issuer serves as a signal of her private information (Leland and Pyle [1977]) or in which the issuer earns control rents from being the manager-majority shareholder (Zingales [1995]).

in which the issuer markets the issue exclusively to informed investors and in addition prices the IPO sufficiently off the underpricing condition to allow it to be overpriced. This is done next.

Proposition 3 *With $k > 0$, there exists parameter values consistent with the issuer optimally pricing the issue so that it is overpriced.*

Proof: See Appendix.

Propositions 2 and 3 offer a theory of IPO overpricing based on the institutional feature that issuers determine IPO terms before investors reveal their private information. However, to apply to the observed overpricing of IPOs of REITs and investment grade bonds, the theory must be able to account for the fact that these types of securities are different in important respects. In particular, while REITs are relatively risky and generally difficult for investors to value (see Wang et al.), investment grade bonds are safe and relatively easy to value. However, the crucial question in the context of the current model is not so much the degree of difficulties that may be involved in ascertaining value as it is in the amount of resources that investors expend doing so.

More pointedly, the model suggests that the typical informed investor who operates in the market for underpriced IPOs (common stocks and junk grade bonds) incurs higher costs in ascertaining value compared to the typical investor who operates in the market for overpriced IPOs (REITs and investment grade bonds). Although this is clearly the case for investment grade bonds⁹, it may not seem equally obvious in the case of REITs. However, Wang et al. and Datta et al. find that the presence of institutional investors is significantly lower in the IPO market for REITs and investment grade bonds compared to the IPO market for common stocks and junk grade bonds. Hence, to the extent that institutional investors incur higher information costs than non-institutional investors, these findings are consistent with the prediction of the model that investors who operate in the IPO market for REITs and investment grade bonds incur lower information costs than do investors who operate in the IPO market for common stocks and junk grade bonds.

⁹For example, Datta et al. note that “investment grade issues are sold exclusively on bond rating to investors who are interested primarily in safety . . .”

A direct implication of winner's curse and underpricing is that uninformed investors generate zero excess returns over time, despite the fact that the IPOs in question are on average underpriced. Analogously, a direct implication of the overpricing result derived in the present paper is that informed investors generate positive excess returns (equal to their costs of becoming informed), despite the fact that the IPOs in question generate negative excess returns. Unfortunately, these predictions are directly testable only to the extent that the investment performance of individual IPO investors can be tracked over time, which is of course difficult.

There exists, however, an alternative approach to test for winner's curse and underpricing. Using a sample of IPOs from the new issues market in Singapore, Koh and Walter (1989) confirm winner's curse by finding that the returns to uninformed (or 'small') investors weighted by the degree of rationing are not statistically different from the riskless rate. In the context of the current model, data on the degree of rationing that is experienced by informed investors makes it possible to calculate the winner's curse factor ($\frac{n^*}{\hat{n}_I + n_u}$) and thereby, from observed returns, the returns generating process relevant to uninformed investors.¹⁰ Unfortunately, an analogous test of the overpricing result derived in the current paper does not seem to be available. The reason for this is that, unlike what the case is for uninformed investors, the probability distribution that determines the bidding behavior of informed investors does not coincide with the underlying probability distribution that generates actual IPO returns. While the latter is available directly from observed IPO returns, so that information on the degree of rationing experienced by uninformed investors will fill out the missing piece that is needed to confirm condition (17), the first is not.

5 Concluding Remarks

This paper develops a general IPO model in which the issuer chooses the initial price and the fraction of the firm sold so as to attract the number of informed and uninformed investors that maximizes the combined value of the proceeds received in the IPO and the post-issue value of the issuer's remaining claim in the firm. As in Rock (1986), but

¹⁰Using IPO data from the new issues market in Singapore, Koh and Walter (1989) undertakes such a test and find that returns weighted by the degree of rationing experienced by uninformed (or 'small') investors are not statistically different from the riskless rate of return.

in a more general setting, it is shown that whenever informed and uninformed investors submit bids winner's curse will force the issuer to underprice the issue.

Furthermore, consistent with the overall empirical evidence on the pricing of IPOs, the model developed is able to account for the possibility that IPOs are overpriced on average. The potential for overpricing arises because the issuer is forced to determine the final IPO terms before the information observed privately by investors becomes public. It is shown that while IPOs are always overpriced (on average) if the information observed by investors is costless, overpricing arises in the general case of positive information costs if the information observed by individual investors is more precise than that of the issuer, provided investors' information costs are not too high.

Appendix

Proof of Proposition 2: Overpricing requires that $\bar{v}_I(n_I) > \bar{v}$, or

$$\left(\sum_{\hat{n}_I-1}^{n_I-1} P^*(\hat{n}_I - 1|s_G) \right)^{-1} \left(\sum_{\hat{n}_I-1}^{n_I-1} P^*(\hat{n}_I - 1|s_G) V(\hat{n}_I) - \frac{Nk}{n^* P(s_G)} \right) > \left(\sum_{\hat{n}_I}^{n_I} P(\hat{n}_I) \right)^{-1} \sum_{\hat{n}_I}^{n_I} P(\hat{n}_I) V(\hat{n}_I). \quad (\text{A.1})$$

Putting $\star = 0$ and noting that $P(\hat{n}_I - 1|s_G) = P(\hat{n}_I)$ if and only if $P(s_G|s_G) = P(s_G)$, I assume first that $P(s_G|s_G) = P(s_G)$. The idea behind the proof is that if it can be shown that $\bar{v}_I(n_I) = \bar{v}$ for the case $P(s_G|s_G) = P(s_G)$, then it can be concluded that $\bar{v}_I(n_I) > \bar{v}$ whenever $P(s_G|s_G) > P(s_G)$ as this would provide an improvement in $P^*(\hat{n}_I - 1|s_G)$ relative to $P(\hat{n}_I)$ in the sense first order stochastic dominance.

Let $p \equiv P(s_G|s_G) = P(s_G)$ and consider first

$$\frac{P(\hat{n}_I)}{\sum_{\hat{n}_I}^{n_I} P(\hat{n}_I)} = \frac{\binom{n_I}{\hat{n}_I} p^{\hat{n}_I} (1-p)^{n_I-\hat{n}_I}}{\binom{n_I}{\hat{n}_I} p^{\hat{n}_I} (1-p)^{n_I-\hat{n}_I} + \dots + \binom{n_I}{n_I} p^{n_I}} \quad (\text{A.2})$$

$$= \frac{\frac{n_I!}{\hat{n}_I!(n_I-\hat{n}_I)!} p^{\hat{n}_I} (1-p)^{n_I-\hat{n}_I}}{\frac{n_I!}{\hat{n}_I!(n_I-\hat{n}_I)!} p^{\hat{n}_I} (1-p)^{n_I-\hat{n}_I} + \dots + p^{n_I}} \quad (\text{A.3})$$

$$= \frac{p^{\hat{n}_I} (1-p)^{n_I-\hat{n}_I}}{\frac{\hat{n}_I!(n_I-\hat{n}_I)!}{\hat{n}_I!(n_I-\hat{n}_I)!} p^{\hat{n}_I} (1-p)^{n_I-\hat{n}_I} + \dots + \frac{\hat{n}_I!(n_I-\hat{n}_I)!}{n_I!} p^{n_I}} \quad (\text{A.4})$$

Consider then (cancelling n^* on the right hand side)

$$\frac{\frac{n^*}{\hat{n}_I} P(\hat{n}_I - 1|s_G)}{\sum_{\hat{n}_I-1}^{n_I-1} \frac{n^*}{\hat{n}_I} P(\hat{n}_I - 1|s_G)} = \frac{\frac{1}{\hat{n}_I} \binom{n_I-1}{\hat{n}_I-1} p^{\hat{n}_I-1} (1-p)^{n_I-\hat{n}_I}}{\frac{1}{\hat{n}_I} \binom{n_I-1}{\hat{n}_I-1} p^{\hat{n}_I-1} (1-p)^{n_I-\hat{n}_I} + \dots + \frac{1}{n_I} \binom{n_I-1}{n_I-1} p^{n_I-1}} \quad (\text{A.5})$$

$$= \frac{\frac{1}{\hat{n}_I} \frac{(n_I-1)!}{(\hat{n}_I-1)!(n_I-\hat{n}_I)!} p^{\hat{n}_I-1} (1-p)^{n_I-\hat{n}_I}}{\frac{1}{\hat{n}_I} \frac{(n_I-1)!}{(\hat{n}_I-1)!(n_I-\hat{n}_I)!} p^{\hat{n}_I-1} (1-p)^{n_I-\hat{n}_I} + \dots + \frac{1}{n_I} p^{n_I-1}} \quad (\text{A.6})$$

$$= \frac{p^{\hat{n}_I-1} (1-p)^{n_I-\hat{n}_I}}{\frac{\hat{n}_I!(n_I-\hat{n}_I)!}{\hat{n}_I!(n_I-\hat{n}_I)!} p^{\hat{n}_I-1} (1-p)^{n_I-\hat{n}_I} + \dots + \frac{\hat{n}_I!(n_I-\hat{n}_I)!}{n_I!} p^{n_I-1}} \quad (\text{A.7})$$

Cancelling one of the p 's in (A.4), we observe that

$$\frac{P(\hat{n}_I)}{\sum_{\hat{n}_I}^{n_I} P(\hat{n}_I)} = \frac{\frac{1}{\hat{n}_I} P(\hat{n}_I - 1 | s_G)}{\sum_{\hat{n}_I-1}^{n_I-1} \frac{1}{\hat{n}_I} P(\hat{n}_I - 1 | s_G)}, \quad \text{for all } \hat{n}_I \in [\hat{n}_I, n_I],$$

which proves that $\bar{v}_I(n_I) = \bar{v}$ for the case $P(s_G | s_G) = P(s_G)$, leading to the conclusion that $\bar{v}_I(n_I) > \bar{v}$ whenever $P(s_G | s_G) > P(s_G)$.

The proposition also states that $\bar{v}_u(0) < \bar{v}$. To see that this is true, assume to the contrary that $\bar{v}_u(0) \geq \bar{v}$. However, by Proposition 1, this would imply that the IPO is underpriced thus contradicting the overpricing result just proven. \square

Proof of Lemma 1: The proof is analogous to the proof of Proposition 2: I first show that condition (24) holds as an equality for the case $P(s_G | s_G^F) = P(s_G | s_G)$ and then conclude as a consequence that it will hold as a strict inequality (in the desired direction) for the case $P(s_G | s_G) < P(s_G | s_G^F)$.

Let now $p \equiv P(s_G | s_G^F) = P(s_G | s_G)$ and note that

$$\frac{P(\hat{n}_I | s_G^F)}{\sum_{\hat{n}_I}^{n_I} P(\hat{n}_I | s_G^F)} = \frac{\binom{n_I}{\hat{n}_I} p^{\hat{n}_I} (1-p)^{n_I - \hat{n}_I}}{\binom{n_I}{\hat{n}_I} p^{\hat{n}_I} (1-p)^{n_I - \hat{n}_I} + \dots + \binom{n_I}{n_I} p^{n_I}} \quad (\text{A.8})$$

$$= \frac{\frac{n_I!}{\hat{n}_I!(n_I - \hat{n}_I)!} p^{\hat{n}_I} (1-p)^{n_I - \hat{n}_I}}{\frac{n_I!}{\hat{n}_I!(n_I - \hat{n}_I)!} p^{\hat{n}_I} (1-p)^{n_I - \hat{n}_I} + \dots + p^{n_I}} \quad (\text{A.9})$$

$$= \frac{p^{\hat{n}_I} (1-p)^{n_I - \hat{n}_I}}{\frac{\hat{n}_I!(n_I - \hat{n}_I)!}{\hat{n}_I!(n_I - \hat{n}_I)!} p^{\hat{n}_I} (1-p)^{n_I - \hat{n}_I} + \dots + \frac{\hat{n}_I!(n_I - \hat{n}_I)!}{n_I!} p^{n_I}}. \quad (\text{A.10})$$

It has already been shown that

$$\frac{\frac{n^*}{\hat{n}_I} P(\hat{n}_I - 1 | s_G)}{\sum_{\hat{n}_I-1}^{n_I-1} \frac{n^*}{\hat{n}_I} P(\hat{n}_I - 1 | s_G)} = \frac{p^{\hat{n}_I-1} (1-p)^{n_I - \hat{n}_I}}{\frac{\hat{n}_I!(n_I - \hat{n}_I)!}{\hat{n}_I!(n_I - \hat{n}_I)!} p^{\hat{n}_I-1} (1-p)^{n_I - \hat{n}_I} + \dots + \frac{\hat{n}_I!(n_I - \hat{n}_I)!}{n_I!} p^{n_I-1}}. \quad (\text{A.11})$$

Cancelling one of the p 's in (A.10), we observe that

$$\frac{P(\hat{n}_I | s_G^F)}{\sum_{\hat{n}_I}^{n_I} P(\hat{n}_I | s_G^F)} = \frac{\frac{1}{\hat{n}_I} P(\hat{n}_I - 1 | s_G)}{\sum_{\hat{n}_I-1}^{n_I-1} \frac{1}{\hat{n}_I} P(\hat{n}_I - 1 | s_G)}, \quad \text{for all } \hat{n}_I \in [\hat{n}_I, n_I],$$

which proves that condition (24) will hold as an equality for the case $P(s_G|s_G) = P(s_G|s_G^F)$. This allows the conclusion that condition (24) is satisfied as a strict inequality (in the desired direction) whenever $P(s_G|s_G) > P(s_G|s_G^F)$. \square

Proof of Proposition 3: The parameters used are: $v_G = 2,000$, $v_B = 0$, $\alpha = .5$, $P(s_G|v_G) = P(s_B|v_B) = .85$, $P(s_G^F|v_G) = P(s_B^F|v_B) = .8$, $k = 11$, and $n^* = 6$. Consider first

$$\max_{N, P_0} E(W_G)$$

subject to

$$NP_0 = \bar{v}_I \text{ and } NP_0 = \bar{v}_u.$$

The optimal solution to this problem is given by $(n_I, n_u) = (12, 0)$, which yields $\bar{r}_0 = 3.73\%$, $E(W_G) = 1680$, and $n^*/N = 0.52$.¹¹ Condition (24) is satisfied in this case by the parameterization that $P(s_G|v_G) > P(s_G^F|v_G)$, so that it will be optimal to increase the fraction of the firm floated initially relative to $n^*/N = .52$. Increase then n^*/N from .52 to .60 (where .60 is arbitrary beyond the fact that it delivers a negative initial return). This increase in n^*/N happens to increase the optimal number of informed investors to 13. It further raises $E(W_G)$ to 1689 and produces a negative initial return of -11.83%. \square

¹¹This solution was generated via a program written in *Mathematica*.

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