Operational expressions for the marginal cost of indirect taxation when merit arguments matter*

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Abstract

Marginal indirect tax reform analysis evaluates for each commodity (group) the marginal welfare cost (MC) of increasing government revenue with 1 Euro by raising the indirect tax rate on that commodity. In this paper, I propose an adjustment to the MC-expressions to allow for (de)merit good arguments and show how this adjustment can easily be parameterised on the basis of econometric demand analysis.

Keywords: Merit goods; Marginal tax reform

JEL-code: D12, H21

1 Introduction

Marginal indirect tax reform (MITR) analysis is probably one of the most practical applications of public economics. It offers clear-cut guidelines for reform policy and allows itself to an empirical implementation by means of household expenditure data, effective indirect tax rates, estimates for aggregate demand elasticities and a set of welfare weights.[1]

The standard MITR model is very welfaristic in nature in that it assumes the government to endorse the sovereignty of the households in the

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[1]See Ahmad and Stern (1984, 1991) for India and Pakistan, Decoster and Schokkaert (1989) for Belgium, Madden (1995) for Ireland, Kaplanoglou and Newbery (2003) for Greece, Schroyen and Aasness (2002) for Norway.

The standard MITR model is very welfaristic in nature in that it assumes the government to endorse the sovereignty of the households in the economy, fully respecting their decisions regarding the consumption of goods and services. In reality, though, both through statements and through policy measures governments reveal a desire to deviate from consumer preferences for commodities like alcohol and tobacco. Not only try governments to better inform their citizens about the health risks involved, they also try to discourage consumption through excise policy and marketing restrictions. More recently, the World Health Organization has recommended national governments to impose a tax on sugar as an instrument in their battle against obesity. Such arguments are called merit good arguments, and economists have traced out the implications for optimal commodity tax rules. I refer here to an article by Besley (1988) and myself (Schroyen, 2004).

In this paper, I investigate how such merit arguments can be incorporated in MITR analysis. In particular, I show how the central expressions for that analysis–the marginal welfare cost of rasing an extra Euro by means of the indirect tax rate on good i–need to be amended to allow for merit good arguments and how these expressions can be parameterised in terms of aggregate demand elasticities. To model merit goods arguments, I choose the *numéraire* function approach proposed in Schroyen (2004) (where I also explain why Besley's approach is flawed). But since I want to arrive at decision rules that are easily implementable in practise, I will make use of the distance function rather than the *numéraire* function to model the preferences of the government.

2 The model

Households

A representative household has preferences that can be represented by a strongly quasi-concave utility function on n commodities: $u(x_1, ..., x_n)$. Facing a vector of consumer prices $q = (q_1, ..., q_n)$ and having a disposable income m, it solves the problem

$$\max_x u(x) \text{ s.t. } q'x = m. \tag{1}$$

Denoting $\pi \overset{\text{def}}{=} \frac{q}{m}$ as the vector of normalised prices, the solution may be written as $x(\pi)$ yielding a utility level $v(\pi)$. If subscripts with u denote partial derivatives, the first order conditions for (1) may be written as

$$\frac{u_i(x(\pi))}{\sum_j u_j(x(\pi))x_j(\pi)} = \pi_i \ (i = 1, ..., n). \tag{2}$$

Household's preferences may also be represented by the distance function $d(x, \overline{u})$. This function is implicitly defined as

$$u(\frac{x}{d(x, \overline{u})}) = \overline{u} \quad \text{(all } x, \overline{u}); \tag{3}$$

it is the factor by which the commodity bundle x needs to be scaled down to generate a utility level $\overline{u}$.[2] It can be shown that $\frac{\partial d(x(\pi), v(\pi))}{\partial x_i} = \pi_i$ and hence the derivative provides a measure of the household's marginal willingness to pay for commodity i; the cross-partial derivative $\frac{\partial^2 d}{\partial x_i \partial x_j}$ is the so-called Antonelli substitution effect–see Deaton (1979) on these matters.

Government

Suppose now that the government considers commodity x_n as a (de)merit good. Convinced of the (de)merit properties of this commodity, it believes that in order for the consumer to reach utility level $\overline{u}$, all commodities should be scaled down by more (less) than $d(x, \overline{u})$, for instance by the amount

$$D(x, \overline{u}) = d(x, \overline{u}) + \int_0^{x_n} \mu(\chi) \mathrm{d}\chi. \tag{4}$$

In terms of the marginal willingness to pay, we have

$$\frac{\partial D(x, \overline{u})}{\partial x_i} = \frac{\partial d(x, \overline{u})}{\partial x_i} \quad (i \neq n), \text{ and} \tag{5a}$$

$$\frac{\partial D(x, \overline{u})}{\partial x_n} = \frac{\partial d(x, \overline{u})}{\partial x_i} + \mu(x_n) \tag{5b}$$

so that the government believes that the household should be willing to pay $\mu(x_n)$ extra for good n when consuming a bundle (x_{-n}, x_n) yielding utility level $\overline{u}$. It can easily be shown that

$$U(x) = u\left(\frac{x}{1 - \int_0^{x_n} \mu(\chi) d\chi}\right) \quad \text{(all } x). \tag{6}$$

is the utility function to which the government subscribes.[3]

From now on I assume that $\mu(\chi)$ takes the constant value μ so that the denominator becomes $1 - \mu x_n$. If good n is a demerit good, $\mu < 0$. Letting

[2] In contrast, the *numéraire* function specifies the amount of a *numéraire* commodity which, together with the quantities of the other commodities, generates a certain utility level. For empirical purposes, it is desirable to avoid the choice of a commodity (category) as *numéraire*.

[3] Define $U(\cdot)$ as $U(\frac{x}{D(x, \overline{u})}) = \overline{u}$ (all $x, \overline{u}$), then $U(x) = \overline{u}$ if $D(x, \overline{u}) = 1$. From (3) and

$\widetilde{x}_i$ be a shorthand for $\frac{x_i}{1-\mu x_n}$, the marginal utilities for the government are then

$$U_i(x) = \frac{u_i(\widetilde{x})}{1-\mu x_n} + \delta_{in} \sum_j \frac{u_j(\widetilde{x}) x_j \mu}{(1-\mu x_n)^2} \text{ (all } i\text{)}, \tag{7}$$

where $\delta_{in} = 1$ if $i = n$ and 0 otherwise. Normalising these by dividing through by $\sum_k U_k(x) x_k = \sum_k \frac{u_k(\widetilde{x})\widetilde{x}_k}{1-\mu x_n}$ then gives

$$\Pi_i(\widetilde{x}(\mu), \mu) \stackrel{\text{def}}{=} \frac{u_i(\widetilde{x})}{\sum_k u_j(\widetilde{x})\widetilde{x}_j} + \delta_{in}\mu \text{ (all } i\text{)}. \tag{8}$$

Clearly, if $\mu \to 0$, the government's normalised 'demand prices' coincide with those of the household.

I now propose to approximate $\Pi_j(\widetilde{x}(\mu), \mu)$ by a first order Taylor expansion around $\Pi_j(\widetilde{x}(0), 0) = \pi_j$. This gives

$$\Pi_i(\widetilde{x}(\mu), \mu) \simeq \pi_i + \left[\left(\sum_k \frac{\partial \pi_i}{\partial x_k} x_k\right) x_n + \delta_{in}\right] \cdot \mu \text{ (all } i\text{)}. \tag{9}$$

The round bracket term denotes a pure scale effect, i.e. the effect on the normalised demand price for a commodity of an equi-proportional increase in all quantities. I denote these as g_i ($i = 1, ..., n$). It can be shown that $g_i = \frac{\mathrm{d}\pi_i}{\mathrm{d}\log X}$, where X is the Divisia quantity index (see Barten and Bettendorf, 1989, p 1512). We may then write (9) as

$$\Pi_i(\widetilde{x}(\mu), \mu) \simeq \pi_i + g_i x_n \mu + \delta_{in}\mu \text{ (all } i\text{)}. \tag{10}$$

Merit considerations thus affect the marginal willingness to pay in two ways. First, the government's MWP for the merit good (n) will exceed the private one with μ. Second, and less straightforwardly, merit considerations make the government regard the household better off than it is aware off itself, due to all the infra-marginal units consumed of good n. This has a scale effect which, for all normal goods, reduces the MWP.

(4)

$$u\left(\frac{x}{D(x,\overline{u}) - \int_0^{x_n} \mu(\chi)d\chi}\right) = \overline{u} \text{ (all } x, \overline{u}\text{)}$$

so that

$$U\left(\frac{x}{D(x,\overline{u})}\right) = u\left(\frac{x}{D(x,\overline{u}) - \int_0^{x_n} \mu(\chi)\mathrm{d}\chi}\right) \text{ (all } x, \overline{u}\text{)}.$$

Evaluating this at $D(x, \overline{u}) = 1$ finally gives (6).

Notice that $\int_0^{x_n} \mu(\chi)\mathrm{d}\chi = \mu(\widetilde{\chi})x_n$ (some $\widetilde{\chi} \in [0, x_n]$) and therefore that it has the dimension of a budget share (since μ has the dimension of a normalised price).

3 Marginal cost expressions

In MITR analysis, one is interested in the marginal cost in terms of social welfare, W, of raising government revenue, R, with one Euro by changing the tax on commodity i $(i = 1, ..., n)$:

$$MC_i = -\frac{\partial W/\partial t_i}{\partial R/\partial t_i} \quad (i = 1, ..., n) \tag{11}$$

If $MC_i > MC_j$ then welfare can be increased by lowering the indirect tax rate on commodity i and raising the one on commodity j in a budgetary neutral fashion.

Expressions of this kind have been discussed in detail by Ahmad and Stern (1984), who show that a neat parameterisation is obtained by multiplying nominator and denominator by the respective after tax prices q_i. Since the (de)merit good arguments only affect the nominator, I limit myself to this part of the MC-expression.

In this representative household economy, the obvious measure of social welfare is $U(x(\pi))$. The effect of a marginal change in the excise tax rate on commodity i $(i = 1, ..., n)$ on the social welfare is then

$$-\frac{\partial W}{\partial t_i} = -\sum_j U_j \frac{\partial x_j}{\partial \pi_i}\frac{1}{m} = -\left(\frac{\sum_k U_k x_k}{m}\right)\sum_j \frac{U_j}{\sum_k U_k x_k}\frac{\partial x_j}{\partial \pi_i}. \tag{12}$$

The round bracket term shows how the government evaluates the consumer's marginal utility of income. Denoting this as γ, the *rhs* may then be written as $-\gamma \sum_j \Pi_j(\widetilde{x}(\mu), \mu)\frac{\partial x_j}{\partial \pi_i}$ and upon using the approximating expression (10) and the fact that $\sum_j \pi_j \frac{\partial x_j}{\partial \pi_i} = -x_i$ (adding-up), we get

$$-\frac{\partial W}{\partial t_i} \simeq \gamma \left[x_i - \sum_j g_j \frac{\partial x_j}{\partial \pi_i}\mu x_n - \mu \frac{\partial x_n}{\partial \pi_i}\right]. \tag{13}$$

Multiplying through by the consumer price q_i then gives

$$-q_i \frac{\partial W}{\partial t_i} \simeq \gamma \left[q_i x_i - m\mu x_n \left(\sum_j \sigma_j \pi_j x_j \varepsilon_{ji} + \varepsilon_{ni}\right)\right] \tag{14}$$

where $\sigma_j \stackrel{\text{def}}{=} \frac{g_j}{\pi_i}$ is the scale elasticity of good j (remember that $g_i = \frac{\mathrm{d}\pi_i}{\mathrm{d}\log X}$) and $\varepsilon_{ji} \stackrel{\text{def}}{=} \frac{\partial \log x_j}{\partial \log \pi_i}$ is the Marshallian elasticity of the demand for good j w.r.t. the price of good i.

As μ has the dimension of a normalised price, $m\mu$ has the dimension of a price and one may represent it as ηq_n. The reduction on the consumer's welfare–as perceived by the government and measured in Euro–is then

$$-q_i \frac{\partial W}{\partial t_i} \simeq \gamma \left[(q_i x_i) - \eta \, (q_n x_n) \left(\sum_j (q_j x_j) \, \sigma_j \, \varepsilon_{ji} + \varepsilon_{ni} \right) \right] \tag{15}$$

Household welfare goes down to the extent it spends disposable income on commodity i. But the increase in the consumer price q_i has the additional effect of changing the consumption pattern of all goods and to the extent (η) that merit good considerations drive a wedge between the consumer's and the government's MWP, this needs to be accounted for–hence the big round bracket term.

Expression (15) is for a representative household economy, but lets itself easily extend to a heterogenous population. Attaching a social weight λ^h to household h ($h = 1, ..., H$) we get

$$-q_i \frac{\partial W}{\partial t_i} \simeq \sum_h (\lambda^h \gamma^h) \left[(q_i x_i)^h - \eta \, (q_n x_n)^h \left(\sum_j (q_j x_j)^h \, \sigma_j^h \, \varepsilon_{ji}^h + \varepsilon_{ni}^h \right) \right]. \tag{16}$$

The small round bracket terms denote expenditure levels, and are available from household survey data, while the scale and price elasticities can in principle be estimated. Very often though, these estimates are only available from demand analysis at the aggregate level, and should be replaced by them. For a given set of welfare weights ($\lambda^h \gamma^h$) and merit parameter (η), it is then possible to calculate and rank the different MC_is.

4 Retrieving scale elasticities from regular estimation results

Finally, I want to show how estimates for the Marshallian price elasticities (ε_{ji}) and Engel income elasticities (η_i) together with average budget shares (w_i) can be used to construct the corresponding scale elasticities (σ_i).

Let $w = (w_i), E = (\varepsilon_{ij}), \eta = (\eta_i)$ and denote the diagonal matrix with budget shares as $\hat{w}$. The matrix of compensated price elasticities is then given by $E^c = E + \eta w'$. Define now $S \stackrel{\text{def}}{=} \hat{w} E^c$ and $b \stackrel{\text{def}}{=} \hat{w} \eta$. This matrix and vector are the Rotterdam parameterisation of the regular demand system in differential form, i.e.

$$\hat{w} \mathrm{d} \log x = b[-w' \mathrm{d} \log \pi] + S \mathrm{d} \log \pi. \tag{17}$$

If ι denotes the vector of units, then $\iota'b = 1, S = S', S\iota = 0$, and $y\prime Sy < 0$ (all $y \neq \alpha\iota$, α real scalar).

Consider now the bordered matrix $\binom{S \;\; w}{w' \;\; 0}$. This matrix has rank $n{+}1$, and is invertible into $\binom{T \;\; \iota}{\iota' \;\; 0}$. The matrix T has the properties (i) $TS = I - \iota w'$, (ii) $Tw = 0$, (iii) $T = T'$, and (iv) $y'Ty < 0$ (all $y \neq \alpha w$, α real scalar).[4] Pre-multiplying (17) through by T, making use of (i) and rearranging then gives

$$\mathrm{d}\log\pi = T\hat{w}\mathrm{d}\log x - (Tb + \iota)\mathrm{d}\log X, \tag{18}$$

where I used the fact that $w'\mathrm{dlog}\,\pi = -w'\mathrm{dlog}\,x = -\mathrm{dlog}\,X$ (the change in the Divisia quantity index). The vector of scale elasticicities σ is therefore given by $-(Tb + \iota)$ with the property $w'\sigma = -1$.

References

[1] Ahmad E and N Stern (1984) The theory of reform of Indian indirect taxes, *Journal of Public Economics* **25**, 259-98.

[2] Ahmad E and N Stern (1991) The theory and practise of tax reform in developing countries (Cambridge: Cambridge University Press).

[3] Barten A P and L Bettendorf (1989) Price formation of fish: an application of an inverse demand system, *European Economic Review* **33**, 1509-1525.

[4] Besley T (1988) A simple model for merit good arguments, *Journal of Public Economics* **35**, 371-384.

[5] Deaton A (1979) The distance function and consumer behaviour with applications to index numbers and optimal taxation, *Review of Economic Studies* **46**, 391-405.

[6] Decoster A and E Schokkaert (1989) Equity and efficiency of a reform of Belgian indirect taxes, *Recherches Économiques de Louvain* **55**, 155-173.

[7] Kaplanoglou G and D Newbery (2003) Indirect taxation in Greece: evaluation and possible reform, *International Tax and Public Finance* **10,** 511-533.

[8] Madden D (1995) Labour supply, commodity demand and marginal tax reform, *Economic Journal* **105**, 485-497.

[4]The proofs of the invertability and the statements (i)-(iv) are similar to the ones given by Salvas-Bronsard *et al.* (1977, proposition 1).

[9] Salvas-Bronsard L, D Leblanc and C Bronsard (1977) Estimating demand equations: the converse approach, *European Economic Review* **9**, 301-321.

[10] Schroyen F (2004) An alternative way to model merit good arguments, *Journal of Public Economics (forthcoming).*

[11] Schroyen F and J Aasness (2002) Indirect marginal tax reform analysis for Norway, mimeo, Norwegian School of Economics.