

Income tax, consumption value of education, and the choice of educational type

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Abstract

How the tax system might affect the individual's educational level is well studied. But the question of how the tax system affects the individual's choice of educational type is mostly ignored. This is an important issue, since the educational choice of today's young generation determines the skill composition of tomorrow's labor force and hence the future production possibilities of the country. This paper studies the problem in a partial model. A progressive tax system might in fact introduce distortions in the individuals's educational choice and induce him to choose more of the educational type with the higher consumption value. If he also puts more weight on the present than on the future, this effect is strengthened further.

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1 Introduction

The OECD countries as a whole spent 5.8 per cent of their collective GDP on education in 2001, and 12.7 per cent of total public expenditure was devoted to educational institutions¹. Most of these countries offer publicly financed primary and secondary education, and in many countries tertiary education is also provided by the state at no direct cost for the individual. Part of the justification for publicly funded education is the positive effects of education on the productivity of the country². The government encourages the individuals to get higher education, focusing on the amount of human capital in society and to a great extent ignoring its composition. Different types of education yield different rates of private and social return. It therefore ought to be of great interest to the government to learn more about the mechanisms determining the individual's choice of educational direction, and not only the amount of education. At the very least, one should be aware of which kinds of distortions the income tax system imposes on the educational choice of the individuals. Could it in fact be that the tax system induces the individual to choose other kinds of education than it would in the absence of taxes?

The individual's motivation for choosing higher education may be divided into four categories. First, education is an investment that yields higher wages later in life. Individuals invest in education until the expected marginal pecuniary return equals that of other investment alternatives (Nerlove et.al 1993). Second, education is a signal of high abilities of the individual and might correct for information problems in the labor market (Stiglitz 1975). Third, education is insurance against unemployment (Bishop 1994). Fourth, education offers non-pecuniary and non-market types of return, both during the education itself and afterwards (Becker 1964, Lazear 1977). Among these are the joy of learning new things, meeting new people, moving to a new city, enjoying the life as a student, in addition to the increased status in the society that often comes with studying in particular fields. It is important to remember that even if education is treated as homogenous in the literature, it is in fact a heterogenous investment alternative and consumption good. Thus different kinds of education generate different levels of joy or satisfaction during the educational process. Also, different kinds of education require different levels of effort in

¹OECD: Education at a Glance.

²See Lucas (1988).

order to graduate, a factor the student also considers. After its completion, higher education enables the individual to choose among more interesting jobs. Different educational types offer different degrees of flexibility regarding working hours and the regional distribution of jobs. Individuals with strong preferences for where to live or for being able to work part time will value these qualities strongly when choosing type of education. Another feature that differs among the different educational directions is the effort required by the student to complete the education, and thus also the amount of leisure available to the student. Let all these non-market and non-pecuniary types of return to education be summarized as the consumption value of education. Depending on their preferences, individuals put different weight on the consumption value when choosing educational type³.

Fredriksson (1997) shows on Swedish data that the demand for education responds to economic incentives; more students enrolled at the universities in periods with high expected wage returns or with particularly beneficial student loans arrangements. The link between the income tax system and the length of the individual's education is well studied in the literature. Higher education is considered as an investment alternative in which the individual invests until the expected marginal pecuniary return equals that of other investment alternatives. Taxes on financial income increases the relative pecuniary return to education, and taxes on labor income reduces the return to human capital investments (Boskin 1975, Heckman 1976). The nature of the tax schedule also affects the attractiveness of human capital investments. If no direct costs of acquiring education besides foregone labor income are present, a proportional tax on labor income is a neutral tax on the return to human capital investments. But if a positive tax on capital income exists as well, the comprehensive proportional income tax induces the individual to over-invest in human capital (Nielsen and Sørensen 1997). This effect is even stronger if education has a positive consumption value as well (Alstadsæter 2003a). On the other hand, if education requires direct pecuniary investments, a comprehensive proportional income tax discriminates against human capital investments. (Trostel 1993).

Monetary return to the education no doubt is an important factor in the individual's educational choice, but it is a drawback for the explanatory power of the economic models that the other motives behind the educational choice mostly are

³See Alstadsæter (2003b) for a thorough discussion of the concept of the consumption value of education.

ignored. For instance is the Norwegian labor force among the most highly educated in the OECD⁴, but still has a compressed wage structure and moderate wage return to higher education. Where a country as the US at the present has an average wage premium⁵ to an additional year of higher education of 10 %, the corresponding rate in Norway is 5,5 %. This is the average wage return over all kinds of education at the same duration. But different types of education do in fact generate different rates of wage return, even if they have the same duration. As shown by Moen and Semmingsen (1996), some kinds of higher education have negative wage return in Norway compared with having only high school. Still the number of students at universities and regional colleges has more than doubled over the last 20 years⁶. It thus seems like the students are willing to forego future pecuniary return in order to get the non-pecuniary return to the educational type of their choice.

The educational choice of today's young generation determines the skill composition and hence the production possibilities of tomorrow's labor force. Small open economies with high wage levels, as many of the European countries, experience a flagging-out of their industrial production to low-cost countries. A consensus exists in these countries that the future economic growth depends on their abilities to transfer into knowledge-based industries and innovation production. In order to do this, a highly educated labor force with the required skill combination is essential. Little attention has been given the link between the country's income tax system and the individual's choice of educational direction. If it is so that the tax system not only affects how much education the individuals choose to get, but also which kind of education they choose, then the tax system indeed affects future production possibilities.

This paper analyzes how the individual's trade-off between pecuniary and non-pecuniary return in his choice of educational type is affected by the tax system. Depending on the individual's preferences, a progressive tax system might in fact introduce distortions in the individuals's educational choice and induce him to choose more of the educational type with the higher consumption value. If he also puts more weight on the present than on the future, this effect is strengthened further. Section 2 presents the model, the analysis is done in section 3, section 4 presents empirical

⁴OECD: Education at a glance.

⁵Source: Psacharopoulos and Patrinos, 2002.

⁶Hægeland and Møen (2000)

evidence, and section 5 concludes.

2 The model

The representative individual lives for two periods. He already has decided to spend all available time in the first period on acquiring education and getting a bachelor's degree at university level. The remaining decision to make is which subjects to choose for the degree. By modelling the educational choice in this simplified manner, I focus on the choice of educational type and abstract from the decision whether or not to get education in the first place⁷.

Consider the extreme case where the wage return is either low or high, and where the consumption value of the educational type is either low or high. The four different combinations of the educational attributes are:

Type-*A* education: High consumption value and low wage return.

Type-*B* education: Low consumption value and high wage return.

Type-*R* education: High consumption value and high wage return.

Type-*S* education: Low consumption value and low wage return.

No rational individual would choose type-*S* education, since he is much better off by choosing one of the other three alternatives. This educational alternative may hence safely be disregarded in the analysis. In this setting, all individuals would choose type-*R* education, since this offers both high consumption value and high future wages. If this was true in a perfect competitive educational market, all individuals would choose this type of education, and the whole skilled labor force would have identical qualifications. Assume that type-*R* is an education with restricted admission, as the case is for among others for most business schools and medical schools. Then only a selected sample of the individuals may consume this very advantageous education. The following analysis focuses on the educational choice of

⁷A more general and realistic specification would be to allow the individual to choose in the first period how much of his time to spend on getting education, on labour, and on leisure. In this paper I simplify by assuming that the choices of how much and which kind of education to acquire are separable. This is analog to the literature on saving and portfolio choice, where the savings decision is analyzed separately from the portfolio choice.

the individual when perfect competition exists in the educational sector. Both type- A and type- B education have free admission, and the following model analyzes how taxes affect the individual's choice between these two kinds of education. In the following, consider the extreme case where the consumption value of type- A education is positive, while the consumption value of type- B education is negative. So no rational individual chooses type- B education unless he is compensated for the negative consumption value in some way or another. Let the wage return to type- B education compared with getting type- A education be positive. If the individual chooses to get type- A education he puts more weight on the non-pecuniary return to the education and foregoes other consumption since his income is lower than it would have been had he chosen type- B education.

The individual chooses the optimal linear combination of the two types of education, A and B , in the first period. The parameters E_A and E_B denominate the fraction of available time spent on type- A and type- B education, respectively. These fractions are restricted to be between zero and one, and they sum to one⁸:

$$E_A + E_B = 1. \tag{1}$$

and

$$E_A \in \langle 0, 1 \rangle, \quad E_B \in \langle 0, 1 \rangle.$$

By combining the two educational directions, A and B , in different manners, the individual has a continuum of different kinds of bachelor degrees to choose from. Normally one considers the educational choice to be discrete, in which the individual would have to choose either type- A or type- B education, as is the fact in the previously described type R education. In this paper the educational choice is continuous, in which the individual may choose to combine the two kinds of education as he wishes.

The model is competitive where the solution to be described later is the partial equilibrium solution in a competitive model. This puts restrictions on the analysis, and among them the most important one is that no supply effects are considered. Educational institutions mostly have a limited supply of student places within each program, and popular programs introduce admission restrictions. This effect is absent in this model. As long as the individual wishes to acquire more of one type of education, he may do so.

⁸This sum has no particular significance in this model, since the unit of measurement is arbitrary.

The first and second period are not restricted to have the same duration, and so the second period may be much longer than the first period. Most people do spend more of their lifetime working than they do getting education. The individual is also assumed stay in the same job for the whole second period. This is the extreme version of the lock-in effect that to some extent exists in the labour market; the individual has full freedom in his choice of educational type, but he has limited possibility to change this choice after the completion of the education⁹. Since his pre-education qualifications determine for which jobs he qualifies, he has a limited range of jobs to choose from. The time spent working, H , is given in the second period and independent of the educational profile chosen in the first period. Seeing that type- B education leads to a stressful and less enjoyable job that pays better than the alternative, one might also expect that a job requiring type- B qualifications would demand longer hours. That aspect is not considered here. Hence the duration of the second period and the hours worked are independent of the educational profile.

In each period the consumer gets utility from ordinary consumption and education. Education is both a consumption good and an investment alternative. Type A education yields a direct utility gain in the first period because of the advantageous nature of the education. On the other hand, type- B education generates a direct utility loss because it both is a tiring educational process, and because the job it qualifies for has many negative characteristics. At the same time, the educational choice also affects the bundle of goods the individual may consume in the two periods, C_1 and C_2 . Type B education increases the individual's consumption possibilities compared with type- A education. The individual's preferences are represented by the utility function

$$U = U(C_1, C_2, E_A). \quad (2)$$

Utility is increasing in all three consumption goods, C_1 , C_2 , and E_A . First and second period consumption are both assumed to be normal goods, and so is type- A education. It follows from equations (1) and (2) that $U(C_1, C_2, E_A) = U(C_1, C_2, 1 - E_B)$, and hence the marginal utility of type- B education is negative.

A bachelor's degree yield the expected wage return w , where the probability of future unemployment is accounted for. Let type- A education generate zero addi-

⁹This is an analogy to the putty-clay hypothesis in production theory (Johansen, 1972), where there ex-ante is full substitution between labor and capital, while the ex-post production coefficients are given when the capital is installed.

tional wage return in the second period, such that the individual's total expected wage in the second period by investing E_A units in type- A education in the first period is given by $w \cdot E_A \cdot H$. Type B education generates an additional proportional¹⁰ expected wage return e , and the expected second period wage is hence given by $(w + e) \cdot E_B \cdot H$. Different types of education have different probabilities for future unemployment, and this affects the expected wage return to education. E.g., a high probability of unemployment for individuals with type- A qualifications and less probability of unemployment for individuals with type- B qualifications would imply a large difference in the expected marginal wage returns to the two kinds of education, with a low w and a high e .

No tuition fees are paid, but the individual needs to finance his living expenses in the first period. He borrows money in the financial market at a given interest rate r . In the absence of liquidity constraints, he finances all his first period consumption, C_1 , through debt, D . All debt is paid back in the second period¹¹. There exist no non-labor income or intergenerational transfers in the model. His first period budget constraint is hence given by:

$$C_1 = D. \quad (3)$$

The time spent working in the second period, H , is exogenously given and independent of the educational profile. Second period consumption, C_2 , depends crucially on the chosen educational profile. The basic expected w is paid to the individual on all hours he works, independent of his skills. The basic wage is taxed at the rate t_w . In addition, the individual receives a positive wage return e proportional to all units of type- B education he underwent in the first period. This additional wage return to education is also taxed at the basic tax rate t_w , but in addition a surtax of t_e applies. His second period consumption is hence given by:

$$C_2 = [1 - t_w] \cdot w \cdot E_A \cdot H + [1 - t_w] \cdot w \cdot E_B \cdot H + [1 - (t_w + t_e)] \cdot e \cdot E_B \cdot H - [1 + r] \cdot D, \quad (4)$$

¹⁰An alternative is to model the marginal return as a positive and decreasing function of the time spent on type B education, $k(E_B)$, but as the proportional return to education is the simplest way to illustrate our point, that is the method chosen for this paper.

¹¹This two-period model simplifies reality a great deal. It actually means that the individual gets paid in advance, at the beginning of the period, such as to be able to pay back the debt he issued to finance his living expenses in the first period.

If $t_e = 0$, tax on labour income is proportional, and if $t_e > 0$, tax on labour income is progressive. Obviously, if $t_w = t_e = 0$, there is no tax on labour income. Both tax rates are restricted to be larger than or equal to zero, and smaller than one. Regressive income taxation is no option here. r is the exogenously determined net interest rate. A change in the tax rates on labor income hence leaves the tax rate on capital income unaffected¹². Thus the net interest rate and the discount factor are unaffected by the tax on labor income. By combining the equations (3) and (4), we find the individual's life time budget constraint where type-*A* education is a consumption good for which the individual is willing to pay:

$$C_1 + \frac{1}{1+r} \cdot C_2 + \frac{[1-t_w-t_e] \cdot e \cdot H}{1+r} \cdot E_A = \frac{[1-t_w] \cdot [w+e] - t_e \cdot e}{1+r} \cdot H. \quad (5)$$

The right hand side of (5) represents the individual's full income, which is the maximum achievable income had he chosen only type-*B* education. The left hand side is the different kinds of consumption. Type *A* education is now explicitly viewed as a consumption good with a well defined price, namely the present value of the marginal wage premium by choosing the alternative type-*B* education. The price of one additional unit of this type-*A* education is the income he gives up by not choosing type-*B* education. Denote this alternative price of type-*A* education as p_A :

$$p_A \equiv \frac{[1-t_w-t_e] \cdot e \cdot H}{1+r}. \quad (6)$$

The presence of both basic labor income tax, t_w , and the surtax t_e reduces the price of type-*A* education as a consumption good, and the substitution effect of taxes induces the individual to get more type-*A* and less type-*B* education. This effect is even stronger the higher these tax rates are. The individual makes his consumption and investment decisions for the whole of his life span in the first period. The higher his discount rate is, the more weight he puts on the present and less on the future. That is, the higher the net interest rate r is, the more first period consumption matters relative to second period consumption, and the more type-*A* education he chooses to consume. The opposite is the result the higher the wage return to type-*B* education, e , is or the longer the duration of his second period working life, H , is.

¹²This corresponds to the Scandinavian system of dual income taxation, where tax rates on labour and capital income are set separately.

Then the substitution effect induces the individual to choose less type- A education. Even if the income taxes reduce the price of type- A education as a consumption good, they also reduce total net income. This negative income effect would induce the individual to consume less of all goods, including type- A education. The total effect of the taxes on the individual's educational choice is found in the next chapter.

This is a partial model that only investigates the individual's educational decision, and hence the governmental budget constraint is disregarded.

3 The tax analysis.

3.1 The effect of income tax on the educational choice.

In the following, let the prices of first and second period consumption be

$$p_1 \equiv 1, \tag{7}$$

$$p_2 \equiv \frac{1}{1+r}. \tag{8}$$

This allows us to define the price vector $p = (p_1, p_2, p_A)$. Also, let the individual's full income be defined as y :

$$y \equiv \frac{[1 - t_w] \cdot [w + e] - t_e \cdot e}{1 + r} \cdot H. \tag{9}$$

Applying this new notation reduces the individual's life time budget constraint (5) to $p_1 C_1 + p_2 C_2 + p_A E_A = y$. This new notation simplifies the following development of the response function to a tax change in our particular case.

The individual maximizes his utility under the restriction that his lifetime budget constraint must bind. Manipulating the first order conditions and utilizing the first period time constraint, the Marshallian demand functions are found:

$$C_1(p, y), \quad C_2(p, y), \quad \text{and} \quad E_A(p, y).$$

So how does the tax on labor income affect the individual's educational choice? Consider a marginal increase in the tax rates on labour income and investigate how these influence the individual's division of first period time between type- A and type- B education. The effects on the two kinds of education are symmetrical. Since we from (1) have that $E_A + E_B = 1$, it follows that $\Delta E_B = -\Delta E_A$. Hence it is

sufficient to investigate the effect of tax changes on type- A education. The effect of a tax change on the demand for education is then given by

$$\frac{\partial E_A}{\partial t_i} = \frac{\partial E_A}{\partial p_A} \cdot \frac{\partial p_A}{\partial t_i} + \frac{\partial E_A}{\partial y} \cdot \frac{\partial y}{\partial t_i}, \quad i = w, e. \quad (10)$$

As a response function to a tax change, equation (10) is rather unconventional, since the income effect enters twice. A tax increase reduces the price of type- A education as a consumption good. The first element on the right hand side of the equation is this price effect, which consists of the substitution effect and the income effect of a tax increase. But type- A education is also an investment alternative, and the tax reduces the expected return to this investment, measured in expected future wages, and the second element on the right hand side of (10) is this income effect. Thus the tax increase affects the individual's educational choice through two sources; it changes the value of the individual's human capital stock, which in turn determines his income. It also changes the consumption price on education, in which it affects the relative wage return to the two kinds of education. For this reason the second income effect enters the individual's response function.

The first component of the right hand side of (10) reflects how a tax change affects the price of education as a consumption good. This component consists in two factors; the first is the price-effect, which shows how a price change alters the demand for education as a good. The second fraction tells us how much the price of the educational good A is affected by a tax change. The price-effect consists of a substitution effect and an income effect. In this specific case, the Slutsky-equation takes the form¹³

$$\frac{\partial E_A}{\partial p_A} = \frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + \frac{\partial E_A}{\partial y} \cdot (1 - E_A) \quad (11)$$

Total change in the consumption of the educational good A following a price change is given by the substitution effect pluss the income effect. The substitution effect states how much a price change affects the individual's consumption of type- A education when his income is adjusted such that he may achieve the same utility level. The price change affects the real income and the purchasing power of the individual. In turn this affects the achievable consumption bundle of the individual, and this is the income effect.

A tax change also alters the return to education as an investment alternative, namely the second period wage. This is represented by the second component of the

¹³See the Appendix for the deduction of this equation.

right hand side of (10). Increased income induces the individual to consume more of all normal goods, including type- A education, $\frac{\partial E_A}{\partial y} > 0$. But increased taxes reduce total net income, $\frac{\partial y}{\partial t} < 0$. The total of these two effects predicts a negative value on the second component of the right hand side of (10).

Combining all this information, the complete effect of a tax change on the individual's educational decision is given by

$$\frac{\partial E_A}{\partial t_i} = \left\{ \frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + \frac{\partial E_A}{\partial y} \cdot (1 - E_A) \right\} \cdot \frac{\partial p_A}{\partial t_i} + \frac{\partial E_A}{\partial y} \cdot \frac{\partial y}{\partial t_i}, \quad i = w, e. \quad (12)$$

Symmetry implies that if the individual chooses less type- A education, he chooses more type- B education. Also, these changes cancel out, such that the total amount of education is the same. Hence we know that

$$\frac{\partial E_B}{\partial t_i} = -\frac{\partial E_A}{\partial t_i}, \quad i = w, e.$$

This is the general equation; let us now analyze the two cases $i = w$ and $i = e$ separately.

3.2 The effect of increased top marginal income tax, t_e .

The surtax t_e is levied on the additional wage return e that the individual receives by choosing type- B education. The effect this surtax has on the individual's choice of educational type is found from equation (12) by substituting $i = e$. From (9) and (6) we know that

$$\frac{\partial p_A}{\partial t_e} = -\frac{e \cdot H}{1 + r}, \quad \text{and} \quad \frac{\partial y}{\partial t_e} = -\frac{e \cdot H}{1 + r}.$$

Applying the above results reduces equation (12) to:

$$\frac{\partial E_A}{\partial t_e} = -\frac{e \cdot H}{1 + r} \cdot \left\{ \frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + \frac{\partial E_A}{\partial y} (1 + E_B) \right\}. \quad (13)$$

Type A education is a normal good, and the substitution effect of a price increase is negative. With increased income the individual consumes more of all goods, and hence the income effect, is positive. The fraction of the individual's time in the first period spent on type- B education is equal to or smaller than one, and hence $(1 + E_B)$ is larger than one. This tax increase only affects the additional wage return to the education with negative consumption value, and the basic wage is unaffected

by this. The tax reduces the individual's disposable income, but at the same time reduces the price on type- A education as a consumption good. Whether the increased surtax induces the individual to increase or reduce the amount of type- A education he chooses depends entirely on which effect dominates, the substitution effect or the income effect. This is determined by his preferences, and varies among individuals.

$$\text{If } -\frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} \gtrless \frac{\partial E_A}{\partial y} (1 + E_B), \text{ then } \frac{\partial E_A}{\partial t_e} \gtrless 0. \quad (14)$$

If the substitution effect dominates the income effect, the individual's preference structure is of such a kind that he puts great emphasis on the consumption value of type- A education. The tax increase reduces the price on type- A education measured in foregone wage return by not choosing type- B education, and the individual changes his educational profile by choosing more of the education with the tax free consumption return. Then $\frac{\partial E_A}{\partial t_e} > 0$. The more type- B education the individual has in his original educational portfolio, the stronger is the income effect. Increased top marginal tax rate reduces the return to the education with the less advantageous conditions, and hence the individual chooses less type- B education. This follows from the symmetry assumption in equation (1). The individual experiences a net income reduction through two channels; the tax increase and the reduced investment in type- B education. In order for this to be a sustainable solution, the individual hence reduces his consumption of the other consumption goods, represented by first and second period consumption, C_1 and C_2 .

Increased top marginal tax induces the individual to choose less type- A education, and more type- B education if the income effect dominates the substitution effect.

The sign of the effect on the individual's educational portfolio of an increase in the surtax depends entirely on the income and substitution effects. But the amplitude of the effect is partly determined by the fraction $\frac{e \cdot H}{1+r}$. The higher the wage return or the length of the second period are, the higher is the return to type- B education, and the larger is the effect of an tax increase.

The importance of the discount rate. The higher the discount rate, the more does the individual value consumption and income in the present, and the less does he care about the future income when making his educational choice. The present

consumption value of type- A education matters more for the individual than the future expected wages, especially since the price, measured in the present value of future foregone wages, is reduced through this high valuation of the present. A higher discount rate thus dampens the effect of the tax increase. It also alters the relative price between ordinary first period consumption, C_1 , and the education good A . The higher the interest rate, the more expensive is it to borrow in the financial market in order to finance first period ordinary consumption, and this reduces the marginal rate of substitution between type- A education and ordinary first period consumption. In our model the discount rate is the net of tax real interest rate. A high tax rate on capital income would thus reduce the discount rate and increase the relative price on type- A education.

Increased uncertainty of the future has the same effect as an increased discount rate. If the future wage return to higher education is uncertain, the expected wage return to type- B education is reduced, and so is the price of type- A education as a consumption good.

3.3 The effect of increased basic labor income tax, t_w .

The tax rate t_w is levied on all wage income earned by an educated worker. From (9) and (6) it follows that

$$\frac{\partial p_A}{\partial t_w} = -\frac{e \cdot H}{1+r}, \quad \text{and} \quad \frac{\partial y}{\partial t_w} = -\frac{[w+e] \cdot H}{1+r}.$$

Applying the above results in equation (12) with $i = w$ yields

$$\frac{\partial E_A}{\partial t_w} = -\frac{H}{1+r} \cdot \left\{ e \cdot \frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + [e \cdot (1 + E_B) + w] \cdot \frac{\partial E_A}{\partial y} \right\},$$

which is equivalent to

$$\frac{\partial E_A}{\partial t_w} = \frac{\partial E_A}{\partial t_e} - \frac{\partial E_A}{\partial y} \cdot \frac{w \cdot H}{1+r}. \quad (15)$$

As in the previous case, the effect of this increased tax on the composition of the individual's educational portfolio depends on the individual's preference structure. But, since this tax reduces his disposable income from all sources, and not only the wage return to type- B education, the income effect is more dominant in this case. Even if the income effect and substitution effect would cancel out in equation (13),

a tax increase would still induce the individual to consume less type- A education in this case. This is due to the increased importance of the income effect in equation (15) which appears through the additional fraction on the right hand side of equation (15), namely $-\frac{\partial E_A}{\partial y} \cdot \frac{w \cdot H}{1+r}$. This fraction is higher the longer the working period and the wage return to type- B are. The importance of the income effect is somewhat neutralized by a high discount factor when the individual values consumption today more than consumption tomorrow.

If the income effect dominates the substitution effect, then the total effect of an increased basic labor income tax is negative. The reduced income level induces the individual to reduce consumption of all goods, including type- A education, and the educational portfolio changes in the direction of less type- A education and more type- B education. This is also true if the income and substitution effects cancel out.

If the individual has very strong preferences for education as a consumption good, he might choose more type- A education when the tax increases. In that case the substitution effect must be so much larger than the income effect as to compensate for the additional weight put on the income effect through the new fraction on the right hand side of equation (15).

These are general results. Now consider a specific utility function as described below, in order to study more closely the importance and sizes of the substitution and income effects.

3.4 A specific utility function.

Let the utility function be given as the Cobb-Douglas function:

$$U = \alpha \cdot \ln E_A + \theta \cdot \ln C_1 + \gamma \cdot \ln C_2, \quad (16)$$

where both first and second period ordinary consumption and type- A education are normal goods ($\alpha > 0$, $\theta > 0$, and $\gamma > 0$)¹⁴. The individual's lifetime budget constraint is still given by equation (5). The price on type- A education as a consumption good, p_A , and the individual's full income, y , are defined by (6) and (9). In this case the individual's demand function for type- A education is

$$E_A = \frac{y \cdot \alpha}{(\alpha + \theta + \gamma) \cdot p_A}. \quad (17)$$

¹⁴If $\alpha = 0$, the model reduces to the pure human capital model where education is a pure investment alternative and yields no direct consumption value to the individual.

The individual can at most achieve the utility level V , and evaluated at this point, the compensated demand function is identical to (17). Thus the substitution effect is given by

$$\frac{\partial E_A}{\partial p_A} \Big|_{U=V} = -\frac{\alpha \cdot y}{(\alpha + \theta + \gamma) \cdot p_A^2}, \quad (18)$$

while the income effect is given by

$$\frac{\partial E_A}{\partial y} \cdot (1 - E_A) = \frac{\alpha \cdot (\alpha + \theta + \gamma) \cdot p_A - y \cdot \alpha^2}{(\alpha + \theta + \gamma)^2 \cdot p_A^2}. \quad (19)$$

As it turns out, the substitution effect always dominates the income effect. Thus, if type- A education had been an ordinary consumption good, the individual would respond to price increases (decreases) by reducing (increasing) his consumption of this good, $\frac{\partial E_A}{\partial p_A} < 0$. But since it also is an investment alternative, tax changes not only affect the price on type- A education as a consumption good, but also the return to it as an investment alternative. Thus the income effect must be included once more, and this makes the individual's respond to tax changes more uncertain.

The effect on the educational choice of increased surtax. The effect on an increase in the top marginal income tax on labor income on an individual with the previously described preferences is found by combining the equations (13), (18) and (19):

$$\frac{\partial E_A}{\partial t_e} = \frac{\alpha \cdot \{(2 \cdot \alpha + \theta + \gamma) \cdot (1 - t_w) \cdot w - [\theta + \gamma] \cdot [(1 - t_w - t_e) \cdot e]\}}{(\alpha + \theta + \gamma)^2 \cdot (1 - t_w - t_e)^2 \cdot e}. \quad (20)$$

An increase in the surtax induces the individual to choose more type- A education and less type- B education if

$$\frac{e}{w} < \frac{2 \cdot \alpha + \theta + \gamma}{\theta + \gamma} \cdot \frac{1 - t_w}{1 - t_w - t_e}$$

Several factors affect the outcome of an increase in the surtax on the individual's educational choice. The more compressed the wage structure, that is, the lower the expected relative wage return of choosing type- B education over type- A education, $\frac{e}{w}$, the more likely is it that a higher surtax induces him to choose more type- A education. The income effect is less important the lower the relative wage return to type- B education is. Also, the after-tax wage structure is more compressed the higher the surtax is, and this has the same effect as a compressed pre-tax wage structure

on the individual's educational choice. The stronger preferences the individual has for type-*A* education, the higher is α , and the more likely is it that the individual responds to an increased surtax by choosing more type-*A* education.

If the individual chooses more type-*A* and less type-*B* education his disposable income is reduced, and he thus must reduce his ordinary consumption in both periods. The lower the after-tax wage return to investing in type-*B* education, the smaller is the income loss by choosing more type-*A* education, and the less ordinary consumption must he forego in order to increase the consumption of the educational good.

The effect on the educational choice of increased basic labor income tax. The effect on an increase in the basic labor income tax on the educational choice of an individual with the previously described preferences is found by combining the equations (15), (17), (18), and (20):

$$\frac{\partial E_A}{\partial t_w} = \frac{\partial E_A}{\partial t_e} - \frac{\alpha}{\alpha + \theta + \gamma} \cdot \frac{1}{1 - t_w - t_e} \cdot \frac{w}{e}. \quad (21)$$

This is positive if

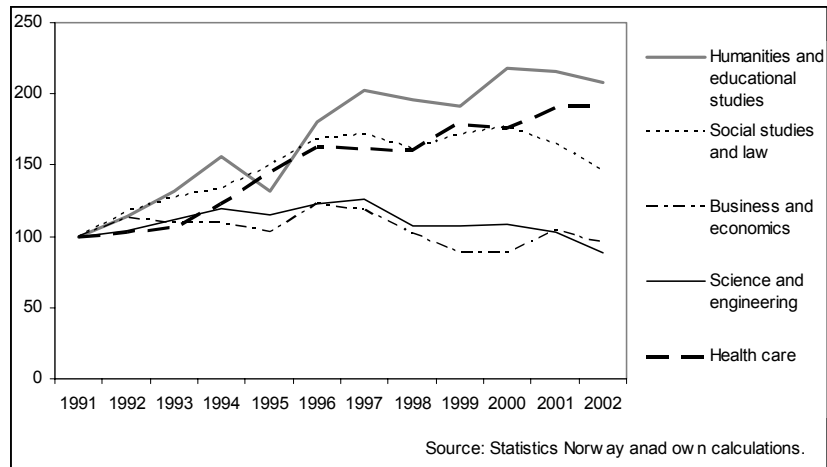
$$\frac{e}{w} < \frac{\alpha \cdot (1 - t_w) + (\alpha + \theta + \gamma) \cdot t_e}{(\theta + \gamma) \cdot (1 - t_w - t_e)}.$$

The higher the surtax, t_e , and the lower the wage return to type-*B* education, e , the lower is the price on type-*A* education as a consumption good. This increases the probability that the substitution effect dominates the income effect when the basic labor income tax is increased. The stronger the individual's preferences for type-*A* education (the higher α), and the weaker his preferences for ordinary consumption (the lower θ and γ) the more likely is it that an increased basic labor income tax induces the individual choose more type-*A* education and less of both type-*B* education and first and second period ordinary consumption. His preferences for type-*A* education must be even stronger than required for the substitution effect to dominate when the surtax is increased.

4 Empirical evidence from Norway.

There has been a 90 percent increase in the number of university graduates in Norway in the period from 1987 to 2002. Even though the number of science graduates is

Figure 1: Annual number of Norwegian university graduates by subject, indexed, 1987=100.



more or less stable over the period, each year a smaller share of the students choose science as their field of study. In 1987 as many as 30.5% of all the graduates were science or engineering majors, while this share had sunk to 17.4% in 2002. Figure 1¹⁵ shows the popularity of different fields of study over time, and all fields except science and business increased their popularity over the period.

The average wage differentials between individuals with low and high education decreased in Norway¹⁶ over the period¹⁷, while it increased in USA, as figure 2 shows. Still, more individuals chose to get higher education over the period. Kahn (1998) finds that the wage setting system in Norway became more centralized in the late 1980's, which lifted the relative wage level of the lower part of the wage distribution, compressing the wage distribution over the period.

¹⁵The figure shows all university graduates, independent of level of the degree. Bear in mind that the figure shows graduated students each year, who made their educational choice 4-6 years prior to graduation, depending on the type of degree completed.

¹⁶It is worth noting though, that a great share of the highly educated individuals work in the public sector, where wages are lower than in the private sector, due to the oligopoly power of the public sector as employer.

¹⁷OECD: Education at a Glance has over some years presented average wage differentials between four groups of the labor force: Below upper secondary education, Upper secondary education (=100), Non-university tertiary education, and University education. Figure 2 shows the index point differences between individuals with below upper secondary education and individuals with university education.

Figure 2: *Income inequality between the labor force participants with the highest and the lowest educational level, men age 25-64.*

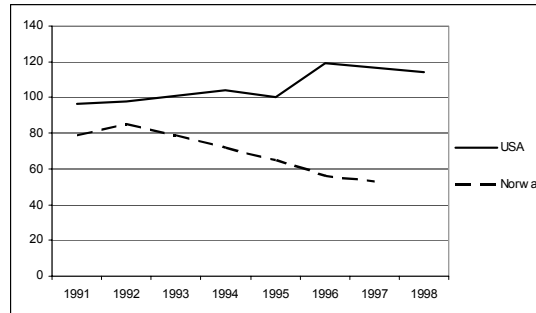
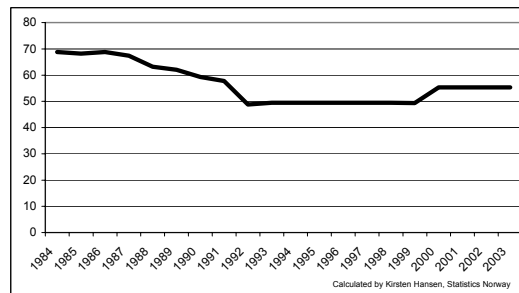


Figure 3: *Top marginal tax rates on wage income in Norway 1984-2003.*



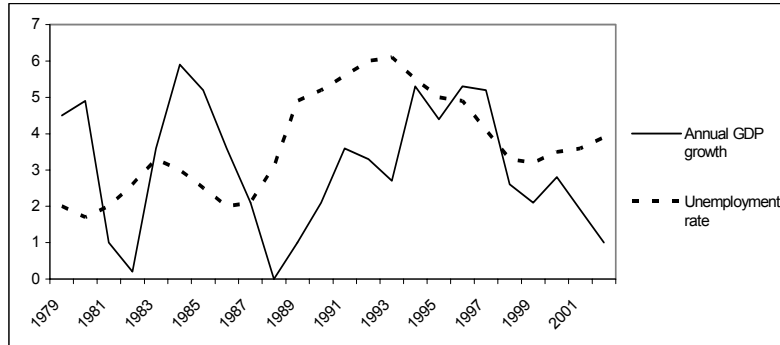
All Scandinavian countries have a tradition for having high taxes and high marginal tax rates on labor in order to finance their large public sectors. Even so, the marginal tax rate on wage income decreased in Norway in the 1980'ies, and were stabilized around 50 % in the 1992 tax reform¹⁸. As figure 3 shows, it later increased to 55,3%. High marginal tax rates makes it more expensive to increase the after-tax income of high-income individuals in centralized wage bargaining, and this serves as a disciplinary factor in the wage negotiations and may lead to a more compressed wage structure¹⁹.

As a country gets richer and the income level of the inhabitants increases, they wish to increase their consumption of all normal goods, including education. It is natural to expect that they put more weight on the consumption value of the

¹⁸Which was a base broadening, rate cutting reform, where the dual income tax with its separation of capital income and labor income was introduced.

¹⁹For an overview on the empirical literature on this issue, see Sørensen (1997).

Figure 4: *Annual GDP growth rates and unemployment rates in Norway, 1979-2002.*



different educational types when making their educational choice, and that they would choose more of the educational types with the higher consumption value. From 1990 to 1997, GDP per capita increased by 30% in Norway, and the country had the highest labor force participation rate (84.4%) in the OECD in 1997, after years of decreasing unemployment. Then one can assume that the individuals choosing field of would care less about the future job possibilities from choosing a particular field of study, and more about how demanding and enjoyable the type of education is. There is a lag here, since individuals choose educational type based on their present expectancy of future wages, and the situation may have changed a great deal until they graduate, such that the actual wage return to their chosen field of study can differ substantially from their expectations. Another important factor here is that Norway has a well developed welfare state with a wide range of benefit programs for unemployed and low income individuals. Hence the importance of the probability of future employment or unemployment in different fields of study is less significant than in countries with a smaller safety net provided by the state.

High marginal tax rates and a compressed wage structure have the same effect; they both reduce the wage differentials in the economy. Thus the price of the educational types with the higher consumption value (type-*A* education in the model) decreases, measured in the foregone wage return by not choosing the educational types generating higher wage return (type-*B* education in the model). The university sector is highly subsidized in Norway and the students face no tuition fees (unless at the private institutions, which are few), and all students are entitled to publicly provided and subsidized student loans. Thus the Norwegian student does not face the actual costs of the higher education, and he in practice faces no credit

Figure 5: *Net of taxes lifetime income for different educational groups relative to the reference group with only high school. Norwegian males and females, 1980 and 1990, 2% and 5% discount rates.*

Type of education	Years after high school	1980 (2%)	1980 (5%)	1990 (2%)	1990 (5%)
Medical school	6	1.43	1.34	1.3	1.19
Law school	6	1.24	1.15	1.33	1.23
MA engineering	5	1.22	1.15	1.23	1.15
MA science	5	1.15	1.07	1.12	1.03
Business school	4	1.1	1.09	1.42	1.34
BA science	3.5	1.1	1.06	1.09	1.03
BA social science	4	1.08	1.01	1.02	0.96
MA humanities	6	1.08	1.01	0.94	0.87
High school	0	1	1	1	1
BA humanities	4	0.96	0.91	0.93	0.89
Teacher's college	4	0.95	0.92	0.89	0.85
Nursing school	3	0.88	0.86	0.87	0.85

Source: Moen and Semmingsen (1996)

constraints. Also, the wage differentials between different kinds of education are small. Figure 5 shows that the net of taxes life time income of the highly skilled relative to the individuals with only high school all decreased over the period, with the exception of law school and business school. Medical school, law school, Business school, and Master of engineering all have restricted admission and very high numbers of applicants each year, and these are the educational types with the higher life time income relative to high school. These types of education are also considered to require a great deal of effort to complete.

One would expect the Norwegian student to weigh the consumption value of education heavily when making his educational choice. To some extent this development seem to have taken place. Even though the wage return to humanities and teacher's college both are smaller than the wage return to finishing the education at high school level, an increasing amount of individuals choose this line of study²⁰, as is seen from figure 1.

The theory model applied in the analysis of the paper describes the extreme case where each type of education has either high or low consumption value and high or low expected wage return. The educational choice is there described as a trade-off

²⁰This raises the question of whether the educational market is able to distribute talent in the optimal manner; do the best skilled individuals choose the more demanding types of education? See Klette and Møen (2002) for a discussion of this issue.

between type-*A* and type-*B* education. But since different individuals have different preferences, some will get the higher consumption value, as well as the higher wage, from type-*B* education, and will thus choose that education independent of the tax system and the compression in the wage system. Apparently some individuals still have higher consumption value from some types of education that generate very low or negative wage return, as seen from the figures 1 and 5. Based on these two figures, one possible organization of the educational types following the theory chapter would be the following:

Type-*R*: Medical school, Law School, and Business School.

Type-*B*: Master of Science and Engineering

Type-*A*: Bachelor of humanities, teacher's college, and nursing school.

The individuals choose education type according to their preferences, which in the theoretical model are assumed to be exogenous. But the preferences for type of education are endogenous, and might change as the individual's understanding of his own abilities and interests changes. For instance would one expect the individual to take account of his success probability in the different educational directions and professions when making his educational choice. To some extent the individual's preferences for education are shaped during primary school, and very much affected by his view of what is interesting or not. In this process, the qualifications and motivation of the teachers are crucial. The decline in the proportion of students choosing natural science can be seen in connection with the rapid decline of qualified mathematics teachers during the last 20 years. In 1997, more than 70 % of the older Norwegian high school teachers had a master's degree, and more than 20 % of all the older teachers had a master of natural sciences²¹. At the same time, only 20 % of the younger high school teachers had a master's degree, and only a couple percent of all the young teachers had a master's of natural sciences. Poor teaching and lower level of mathematics skills among the students increases the comprehension that natural science is a difficult field of study which the students wish to avoid when possible. This could also be a reason for the observed decline in the share of students majoring in science.

²¹Source: Klette and Møen (2002).

5 Conclusion.

Economists have thoroughly discussed how the tax system might affect the individual's educational level. But the question of how the tax system affects the individual's choice of educational type has been mostly ignored. This is an important question, since the educational choice of today's young generation determines the skill composition of tomorrow's labor force and hence the future production possibilities of the country. This paper studies this problem in a simple partial model. Depending on the individual's preferences, a progressive tax system might in fact introduce distortions in the individuals's educational choice and induce him to choose more of the educational type with the higher consumption value. If he also puts more weight on the present than on the future, this effect is strengthened further. Empirical evidence from Norway indicates that individuals value the non-pecuniary return to educational types, and that many are willing to forego future wage return in order to enjoy the educational types with higher consumption value.

The theoretical analysis was done in a partial model. Since so many effects are present side by side with the consumption motive in the educational choice, it is not possible to draw a uniform policy conclusion from this analysis. The main purpose of this paper has been to shed some light on an ignored effect in the literature on taxes, namely the effect on the relative price of different types of education as consumption goods. A natural extension of this model would be to analyze how the presence of uniform and differentiated tuition fees would affect the educational choice of the individual in the presence of taxation when education is considered to be a consumption good.

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7 Mathematical appendix.

Equation (5): By combining the equations (3) and (4), we get:

$$\begin{aligned}
C_1 + \frac{C_2}{1+r} &= D + \frac{[1-t_w] \cdot w \cdot E_A \cdot H + [1-t_w] \cdot w \cdot E_B \cdot H}{1+r} \\
&\quad + \frac{[1-t_w-t_e] \cdot e \cdot E_B \cdot H - (1+r) \cdot D}{1+r} \\
&\Downarrow \\
C_1 + \frac{C_2}{1+r} &= \frac{[1-t_w] \cdot w \cdot E_A + [1-t_w] \cdot w \cdot E_B + [1-t_w-t_e] \cdot e \cdot E_B}{1+r} \cdot H \\
&\Downarrow \quad E_A + E_B = 1 \\
C_1 + \frac{C_2}{1+r} &= \frac{[1-t_w] \cdot w + [1-t_w-t_e] \cdot e \cdot E_B}{1+r} \cdot H \\
&\Downarrow \quad E_B = 1 - E_A \\
C_1 + \frac{C_2}{1+r} &= \frac{[1-t_w] \cdot w + [1-t_w-t_e] \cdot e - [1-t_w-t_e] \cdot e \cdot E_A}{1+r} \cdot H \\
&\Downarrow \\
C_1 + \frac{1}{1+r} \cdot C_2 + \frac{[1-t_w-t_e] \cdot e \cdot H}{1+r} \cdot E_A &= \frac{[1-t_w] \cdot [w+e] - t_e \cdot e}{1+r} \cdot H
\end{aligned}$$

The first order conditions :

$$\begin{aligned}
L &= U(C_1, C_2, E_A) \\
&\quad - \lambda \left(C_1 + \frac{C_2}{1+r} + \frac{[1-t_w-t_e] e H}{1+r} E_A - \frac{[1-t_w] \cdot [w+e] - t_e \cdot e}{1+r} \cdot H \right)
\end{aligned}$$

where λ is the marginal utility of income, which is positive.

$$\frac{\partial L}{\partial C_1} = U_1 - \lambda = 0,$$

$$\frac{\partial L}{\partial C_2} = U_2 - \frac{\lambda}{1+r} = 0$$

$$\frac{\partial L}{\partial E_A} = U_A - \frac{\lambda [1-t_w-t_e] \cdot e \cdot H}{1+r} = U_A - \lambda p_A = 0$$

Finding the Slutsky-equation (11): By substituting for the demand functions in the consumer's utility function, we find the indirect utility function, $V(p, y)$. This

is a scalar, and it expresses the highest achievable utility level at the given budget restriction:

$$V(p, y) \equiv U(C_1(p, y), C_2(p, y), E_A(p, y)) \equiv U(C_1, C_2, E_A)$$

It is also the minimum utility level in the dual problem, namely the consumer's expenditure minimization problem:

$$\min_x p \cdot [C_1, C_2, E_A] \quad \text{given that} \quad U(C_1, C_2, E_A) \geq V(p, y).$$

The Lagrange function of this problem looks as follows:

$$L = p_1 C_1 + p_2 C_2 + p_A E_A - \varphi(U(C_1, C_2, E_A) - V(p, y)),$$

with first order conditions:

$$\frac{\partial L}{\partial C_1} = p_1 - \varphi U_1 = 0 \quad (22)$$

$$\frac{\partial L}{\partial C_2} = p_2 - \varphi U_2 = 0 \quad (23)$$

$$\frac{\partial L}{\partial E_A} = p_A - \varphi U_A = 0 \quad (24)$$

Manipulating equations (22)-(24), we find the demand functions and insert them into the consumer's budget restriction (5). The result is the consumer's expenditure function, $c(p, V)$.

$$c(p, V) \equiv \left\{ \min_x p \cdot [C_1, C_2, E_A], \quad U(C_1, C_2, E_A) \geq V(p, y) \right\}. \quad (25)$$

Since expenditure minimization is equivalent with utility maximization, we get

$$c(\vec{p}, V) = y. \quad (26)$$

The expenditure function is concave in the prices. By Shepard's Lemma, the Hicksian demand functions are found by differentiating the expenditure function:

$$h_i(p, V) = \frac{\partial c(p, V)}{\partial p_i}, \quad i = 1, 2, E_A. \quad (27)$$

The total effect of a price change on the demand of type A education is found by differentiating the Marshallian demand function, $E_A = E_A(p, y)$ when non-labor income μ is held constant:

$$\frac{\partial E_A}{\partial p_A} \Big|_{\mu} = \frac{\partial E_A}{\partial p_A} \Big|_{\bar{y}} + \frac{\partial E_A}{\partial y} \cdot \frac{\partial y}{\partial p_A}. \quad (28)$$

From the expenditure function and the Marshallian demand function it follows that

$$E_A = E_A(p, c(p, V)) = h_E(p, V) \quad (29)$$

The substitution effect of a price change is found by differentiating (29) with regard to the price of type A education:

$$\frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} \equiv \frac{\partial e_A}{\partial y} \frac{\partial c}{\partial p_A} + \frac{\partial E_A}{\partial p_A} \Big|_{\bar{y}} \equiv \frac{\partial h_E}{\partial p_A} \quad (30)$$

From (27) and (29) it follows that

$$\frac{\partial c}{\partial p_A} = E_A,$$

which means that $\frac{\partial E_A}{\partial y} E_A$ is the income effect. If education is a normal good, this effect is positive. By substituting equation (30) and (??) into equation (28), we find the complete Slutsky equation:

$$\frac{\partial E_A}{\partial p_A} \Big|_{\bar{\mu}} = \frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + \frac{\partial E_A}{\partial y} (1 - E_A)$$

Non-labor income is not present in our model, and hence $\mu = 0$ and constant. The above equation then simplifies to equation (11):

$$\frac{\partial E_A}{\partial p_A} = \frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + \frac{\partial E_A}{\partial y} (1 - E_A).$$

Developing equation (13): From (9) and (6) we have that

$$y = \frac{[1 - t_w] \cdot [w + e] - t_e \cdot e}{1 + r} \cdot H$$

and

$$p_A = \frac{[1 - t_w - t_e] \cdot e \cdot H}{1 + r}.$$

Using $i = e$ in equation (12) yields the following:

$$\begin{aligned}
\frac{\partial E_A}{\partial t_e} &= \left(\frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + \frac{\partial E_A}{\partial y} (1 - E_A) \right) \frac{\partial p_A}{\partial t_e} + \frac{\partial E_A}{\partial y} \frac{\partial y}{\partial t_e} \\
&\downarrow \\
&\Downarrow \frac{\partial p_A}{\partial t_e} = -\frac{e \cdot H}{1+r}, \quad \frac{\partial y}{\partial t_e} = -\frac{e \cdot H}{1+r} \\
&\downarrow \\
\frac{\partial E_A}{\partial t_e} &= - \left(\frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + \frac{\partial E_A}{\partial y} (1 - E_A) \right) \cdot \frac{e \cdot H}{1+r} - \frac{\partial E_A}{\partial y} \cdot \frac{e \cdot H}{1+r} \\
&\Downarrow \\
\frac{\partial E_A}{\partial t_e} &= -\frac{e \cdot H}{1+r} \cdot \left(\frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + \frac{\partial E_A}{\partial y} (2 - E_A) \right) \\
&\Updownarrow E_A + E_B = 1 \\
\frac{\partial E_A}{\partial t_e} &= -\frac{e \cdot H}{1+r} \cdot \left(\frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + \frac{\partial E_A}{\partial y} (1 + E_B) \right)
\end{aligned}$$

Developing equation (15): From (9) and (6) we have that

$$\begin{aligned}
y &= \frac{[1 - t_w] \cdot [w + e] - t_e \cdot e}{1+r} \cdot H \\
&\text{and} \\
p_A &= \frac{[1 - t_w - t_e] \cdot e \cdot H}{1+r}
\end{aligned}$$

Using $i = l$ in equation (12) yields the following:

$$\begin{aligned}
\frac{\partial E_A}{\partial t_w} &= \left(\frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + \frac{\partial E_A}{\partial y} (1 - E_A) \right) \frac{\partial p_A}{\partial t_w} + \frac{\partial E_A}{\partial y} \frac{\partial y}{\partial t_w} \\
&\downarrow \\
&\Downarrow \frac{\partial p_A}{\partial t_w} = -\frac{e \cdot H}{1+r}, \quad \text{and} \quad \frac{\partial y}{\partial t_w} = -\frac{[w + e] \cdot H}{1+r} \\
&\downarrow \\
\frac{\partial E_A}{\partial t_w} &= - \left(\frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + \frac{\partial E_A}{\partial y} (1 - E_A) \right) \cdot \frac{e \cdot H}{1+r} - \frac{\partial E_A}{\partial y} \cdot \frac{[w + e] \cdot H}{1+r} \\
&\Downarrow \\
\frac{\partial E_A}{\partial t_w} &= -\frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} \cdot \frac{e \cdot H}{1+r} - \frac{\partial E_A}{\partial y} \cdot \left((1 - E_A) \cdot \frac{e \cdot H}{1+r} + \frac{(w + e) \cdot H}{1+r} \right) \\
E_A + E_B &= 1 \\
\frac{\partial E_A}{\partial t_w} &= -\frac{e \cdot H}{1+r} \cdot \frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} - \frac{\partial E_A}{\partial y} \cdot \frac{w + e \cdot (1 + E_B)}{1+r} \cdot H \\
&\downarrow
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E_A}{\partial t_w} &= -\frac{e \cdot H}{1+r} \cdot \left(\frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + \frac{\partial E_A}{\partial y} \cdot (1 + E_B) \right) - \frac{\partial E_A}{\partial y} \cdot \frac{w \cdot H}{1+r} \\
&\downarrow \\
\frac{\partial E_A}{\partial t_w} &= -\frac{e \cdot H}{1+r} \cdot \left(\frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + \frac{\partial E_A}{\partial y} \cdot (1 + E_B) \right) - \frac{\partial E_A}{\partial y} \cdot \frac{w \cdot H}{1+r} \\
&\downarrow \\
\frac{\partial E_A}{\partial t_w} &= \frac{\partial E_A}{\partial t_e} - \frac{\partial E_A}{\partial y} \cdot \frac{w \cdot H}{1+r}
\end{aligned}$$

Developing the demand function (17): The individual maximizes his utility given that his budget constraint binds, and the Lagrange function is then

$$\mathcal{L} = \alpha \cdot \ln E_A + \theta \cdot \ln C_1 + \gamma \cdot \ln C_2 - \lambda \cdot \left[C_1 + \frac{1}{1+r} \cdot C_2 + p_A \cdot E_A - y \right]$$

with the corresponding first order conditions

$$\frac{\partial \mathcal{L}}{\partial C_1} = \frac{\theta}{C_1} - \lambda = 0 \quad \implies \quad \lambda = \frac{\theta}{C_1} \quad (31)$$

$$\frac{\partial \mathcal{L}}{\partial C_2} = \frac{\gamma}{C_2} - \frac{\lambda}{1+r} = 0 \quad \implies \quad C_2 = C_1 \cdot \frac{\gamma \cdot (1+r)}{\theta} \quad (32)$$

$$\frac{\partial \mathcal{L}}{\partial E_A} = \frac{\alpha}{E_A} - \lambda \cdot p_A = 0 \quad \implies \quad E_A = C_1 \cdot \frac{\alpha}{\theta \cdot p_A} \quad (33)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = - \left[C_1 + \frac{1}{1+r} \cdot C_2 + p_A \cdot E_A - y \right] = 0 \quad (34)$$

The marshallian demand functions are found by combining the first order conditions. From (34) it follows that

$$\begin{aligned}
C_1 &= y - \frac{1}{1+r} \cdot C_2 - p_A \cdot E_A \\
&\Downarrow \text{ (32) and (33)} \\
C_1 &= y - \frac{1}{1+r} \cdot C_1 \cdot \frac{\gamma \cdot (1+r)}{\theta} - p_A \cdot C_1 \cdot \frac{\alpha}{\theta \cdot p_A} \\
&\Downarrow \\
C_1 \cdot \left[1 + \frac{\gamma}{\theta} + \frac{\alpha}{\theta} \right] &= y \\
&\Downarrow \\
C_1 &= y \cdot \frac{\theta}{\alpha + \theta + \gamma}
\end{aligned}$$

Applying this expression in (33) yields

$$\begin{aligned}
E_A &= C_1 \cdot \frac{\alpha}{\theta \cdot p_A} = y \cdot \frac{\theta}{\alpha + \theta + \gamma} \cdot \frac{\alpha}{\theta \cdot p_A} \\
&\Downarrow \\
E_A &= \frac{y \cdot \alpha}{(\alpha + \theta + \gamma) \cdot p_A}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E_A}{\partial p_A} &= \frac{-\alpha \cdot y \cdot (\alpha + \theta + \gamma)}{(\alpha + \theta + \gamma)^2 \cdot p_A^2} \\
&\Downarrow \\
\frac{\partial E_A}{\partial p_A} &= -\frac{\alpha \cdot y}{(\alpha + \theta + \gamma) \cdot p_A^2}
\end{aligned}$$

$$\frac{\partial E_A}{\partial y} = \frac{\alpha}{(\alpha + \theta + \gamma) \cdot p_A}$$

$$\frac{\partial E_A}{\partial y} \cdot (1 - E_A) = \frac{\alpha \cdot (\alpha + \theta + \gamma) \cdot p_A - y \cdot \alpha^2}{(\alpha + \theta + \gamma)^2 \cdot p_A^2}$$

The substitution effect dominates the income effect if

$$-\frac{\alpha \cdot y}{(\alpha + \theta + \gamma) \cdot p_A^2} + \frac{\alpha \cdot (\alpha + \theta + \gamma) \cdot p_A - y \cdot \alpha^2}{(\alpha + \theta + \gamma)^2 \cdot p_A^2} < 0$$

$$(\alpha + \theta + \gamma) \cdot y > (\alpha + \theta + \gamma) \cdot p_A - y \cdot \alpha$$

\Downarrow

$$\frac{2\alpha + \theta + \gamma}{\alpha + \theta + \gamma} > \frac{p_A}{y}$$

\Downarrow

$$\frac{2\alpha + \theta + \gamma}{\alpha + \theta + \gamma} > \frac{\frac{(1-t_w-t_e)e \cdot H}{1+r}}{\frac{(1-t_w)w + (1-t_w-t_e)e}{1+r} \cdot H} = \frac{(1-t_w-t_e)e}{(1-t_w)w + (1-t_w-t_e)e}$$

$\frac{2\alpha + \theta + \gamma}{\alpha + \theta + \gamma} > 1$, and $\frac{(1-t_w-t_e)e}{(1-t_w)w + (1-t_w-t_e)e} < 1$, thus the above inequality holds, and the substitution effect always dominates the income effect in this case.

Developing (20):

$$\frac{\partial E_A}{\partial t_e} = -\frac{e \cdot H}{1+r} \cdot \left\{ \frac{\partial E_A}{\partial p_A} \Big|_{\bar{U}} + \frac{\partial E_A}{\partial y} (2 - E_A) \right\}$$

$$\Downarrow 2 - E_A = 2 - \frac{y \cdot \alpha}{(\alpha + \theta + \gamma) \cdot p_A} = \frac{2(\alpha + \theta + \gamma) \cdot p_A - y \cdot \alpha}{(\alpha + \theta + \gamma) \cdot p_A}$$

$$\frac{\partial E_A}{\partial t_e} = -\frac{e \cdot H}{1+r} \cdot \left\{ -\frac{\alpha \cdot y}{(\alpha + \theta + \gamma) \cdot p_A^2} + \frac{\alpha}{(\alpha + \theta + \gamma) \cdot p_A} \cdot \frac{2(\alpha + \theta + \gamma) \cdot p_A - y \cdot \alpha}{(\alpha + \theta + \gamma) \cdot p_A} \right\}$$

$$\downarrow \frac{\partial E_A}{\partial t_e} = \frac{\alpha \cdot e \cdot H}{(1+r) \cdot (\alpha + \theta + \gamma)^2 \cdot p_A^2} \cdot \{y \cdot (\alpha + \theta + \gamma) - 2(\alpha + \theta + \gamma) \cdot p_A + y \cdot \alpha\}$$

$$\downarrow \frac{\partial E_A}{\partial t_e} = \frac{\alpha \cdot e \cdot H}{(1+r) \cdot (\alpha + \theta + \gamma)^2 \cdot p_A^2} \cdot \{y \cdot (2 \cdot \alpha + \theta + \gamma) - 2(\alpha + \theta + \gamma) \cdot p_A\}$$

Inserting the values for p_A and y yield

$$\frac{\partial E_A}{\partial t_e} = \frac{\alpha \cdot e \cdot H}{(1+r) \cdot (\alpha + \theta + \gamma)^2 \cdot p_A^2} \cdot \left\{ \frac{(1-t_w)w + (1-t_w-t_e)e}{1+r} \cdot H \cdot (2 \cdot \alpha + \theta + \gamma) - 2(\alpha + \theta + \gamma) \cdot \frac{(1-t_w-t_e) \cdot e \cdot H}{1+r} \right\}$$

$$\downarrow \frac{\partial E_A}{\partial t_e} = \frac{\alpha \cdot e \cdot H^2 \cdot \left\{ \begin{array}{l} [(1-t_w)w + (1-t_w-t_e)e] \cdot (2 \cdot \alpha + \theta + \gamma) \\ -2(\alpha + \theta + \gamma) \cdot [(1-t_w-t_e)e] \end{array} \right\}}{(1+r)^2 \cdot (\alpha + \theta + \gamma)^2 \cdot p_A^2}$$

$$\downarrow \frac{\partial E_A}{\partial t_e} = \frac{\alpha \cdot \{(2 \cdot \alpha + \theta + \gamma) \cdot (1-t_w) \cdot w - [\theta + \gamma] \cdot [(1-t_w-t_e) \cdot e]\}}{(\alpha + \theta + \gamma)^2 \cdot (1-t_w-t_e)^2 \cdot e}$$

That is, an increased top income tax induces the individual to choose more type-*A* education and less type-*B* education if

$$(2 \cdot \alpha + \theta + \gamma) \cdot (1-t_w) \cdot w - [\theta + \gamma] \cdot [(1-t_w-t_e) \cdot e] > 0$$

↓

$$\frac{e}{w} < \frac{2 \cdot \alpha + \theta + \gamma}{\theta + \gamma} \cdot \frac{1-t_w}{1-t_w-t_e}$$

Developing (21):

$$\begin{aligned}
\frac{\partial E_A}{\partial t_w} &= \frac{\partial E_A}{\partial t_e} - \frac{\partial E_A}{\partial y} \cdot \frac{w \cdot H}{1+r} = \frac{\partial E_A}{\partial t_e} - \frac{\alpha}{\alpha + \theta + \gamma} \cdot \frac{1}{1 - t_w - t_e} \cdot \frac{w}{e} \\
&\downarrow \\
&= \frac{\alpha \cdot e \cdot H \cdot \{y \cdot (2 \cdot \alpha + \theta + \gamma) - 2(\alpha + \theta + \gamma) \cdot p_A\}}{(1+r) \cdot (\alpha + \theta + \gamma)^2 \cdot p_A^2} \\
&\quad - \frac{\alpha}{(\alpha + \theta + \gamma) \cdot p_A} \cdot \frac{w \cdot H}{1+r} \\
&\downarrow \\
&= \frac{\alpha \cdot H \cdot \left\{ \begin{array}{l} e \cdot y \cdot (2 \cdot \alpha + \theta + \gamma) - e \cdot 2(\alpha + \theta + \gamma) \cdot p_A \\ - (\alpha + \theta + \gamma) \cdot p_A \cdot w \end{array} \right\}}{(1+r) \cdot (\alpha + \theta + \gamma)^2 \cdot p_A^2} \\
&= \frac{\alpha \cdot H \cdot \left\{ \begin{array}{l} e \cdot y \cdot (2 \cdot \alpha + \theta + \gamma) \\ - [2 \cdot e + w] \cdot (\alpha + \theta + \gamma) \cdot p_A \end{array} \right\}}{(1+r) \cdot (\alpha + \theta + \gamma)^2 \cdot p_A^2} \\
&\downarrow \\
&= \frac{\alpha \cdot \left\{ \begin{array}{l} (2 \cdot \alpha + \theta + \gamma) \cdot \{(1 - t_w)w + (1 - t_w - t_e)e\} \\ - (\alpha + \theta + \gamma) \cdot (2e + w) \cdot (1 - t_w - t_e) \end{array} \right\}}{(\alpha + \theta + \gamma)^2 \cdot (1 - t_w - t_e)^2 \cdot e}
\end{aligned}$$

$$\frac{\partial E_A}{\partial t_w} > 0 \quad \text{if}$$

$$\begin{aligned}
&\downarrow \\
(2 \cdot \alpha + \theta + \gamma) \cdot \left\{ \begin{array}{l} (1 - t_w)w \\ + (1 - t_w - t_e)e \end{array} \right\} &> (\alpha + \theta + \gamma) \cdot (2e + w) \cdot (1 - t_w - t_e) \\
&\downarrow \\
\left\{ \begin{array}{l} (2 \cdot \alpha + \theta + \gamma) \cdot (1 - t_w) \cdot w \\ + e \cdot (2 \cdot \alpha + \theta + \gamma) \cdot (1 - t_w - t_e) \end{array} \right\} &> \left\{ \begin{array}{l} 2e \cdot (\alpha + \theta + \gamma) \cdot (1 - t_w - t_e) \\ + w \cdot (\alpha + \theta + \gamma) \cdot (1 - t_w - t_e) \end{array} \right\} \\
&\downarrow \\
\left\{ \begin{array}{l} (2 \cdot \alpha + \theta + \gamma) \cdot (1 - t_w) \\ - (\alpha + \theta + \gamma) \cdot (1 - t_w - t_e) \end{array} \right\} \cdot w &> \left\{ \begin{array}{l} 2 \cdot (\alpha + \theta + \gamma) \cdot (1 - t_w - t_e) \\ - (2 \cdot \alpha + \theta + \gamma) \cdot (1 - t_w - t_e) \end{array} \right\} \cdot e \\
&\downarrow
\end{aligned}$$

$$\begin{aligned}
& \frac{(2 \cdot \alpha + \theta + \gamma) \cdot (1 - t_w) - (\alpha + \theta + \gamma) \cdot (1 - t_w - t_e)}{2 \cdot (\alpha + \theta + \gamma) \cdot (1 - t_w - t_e) - (2 \cdot \alpha + \theta + \gamma) \cdot (1 - t_w - t_e)} > \frac{e}{w} \\
& \qquad \qquad \qquad \downarrow \\
& \frac{(2 \cdot \alpha + \theta + \gamma) \cdot (1 - t_w) - (\alpha + \theta + \gamma) \cdot (1 - t_w - t_e)}{(\theta + \gamma) \cdot (1 - t_w - t_e)} > \frac{e}{w} \\
& \qquad \qquad \qquad \downarrow \\
& \frac{\alpha \cdot (1 - t_w) + (\alpha + \theta + \gamma) \cdot t_e}{(\theta + \gamma) \cdot (1 - t_w - t_e)} > \frac{e}{w}
\end{aligned}$$