Policy measures and storage in a hydropower system

Jostein Skaar

Department of Economics, Norwegian School of Economics and Business Administration, N-5045 Bergen, Norway, jostein.skaar@nhh.no

JEL classification: D43, L13, L94
Keywords: hydropower, storage, reservoirs, oligopoly

Discussion Paper 07/2004

Abstract: In this paper we discuss how three different public policy measures affect water storage controlled by hydropower producing firms. In particular we discuss measures to promote competition, increase transmission capacity and rationing. The analysis is conducted within the framework of an oligopoly model where 2 hydro producing firms engage in dynamic Bertrand competition. We extend this model to be able to analyse how the three policy measures affect storage by hydropower producing firms and focus especially on the probability of hydropower replacing thermal production.

We find that competition, represented by the Bertrand-Nash solution leads to lower storage compared to the monopoly solution. Furthermore, we find that increased transmission capacity and rationing both lead to more fierce competition in situations when water is plentiful and thus to a reduction in storage. These results imply that increased competition, transmission capacity and rationing all contribute to an increased probability of hydropower replacing thermal production.

1 Introduction

Deregulation of electricity markets around the world has been followed by a large number of analyses of competitive strategies in such markets[1]. Most of these studies focus on static analysis of competitive behavior in markets dominated by thermal power. In several electricity markets hydropower contributes a significant share of total production. This includes the Nordic market, New Zealand, South American markets like Chile, Colombia and Argentina, Switzerland and also to some extent markets in the US[2].

The problem facing a hydropower producer is to decide how to allocate a scarce renewable resource between different periods in time. As noted by Garcia et al. (2001), in markets where hydropower plays a significant role, analysis of dynamic pricing behavior is important in order to understand how firms act strategically in such markets.

Garcia et al. refer to an earlier version of a paper by Bushnell (2003) and another by Scott and Read (1996) as some of the few known exceptions focusing on dynamic strategic behavior. In addition we note that Crampes and Moreaux (2001), Johnsen (2001) and Mathiesen et al. (2004) also conduct an analysis of dynamic strategic behavior in electricity markets. These five papers analyze dynamic strategic behavior in a finite-horizon setting where quantity is the strategic variable. Also, only a couple of these papers (Johnsen (2001) and Mathiesen et al. (2004)) address the question of strategic behavior when there is uncertainty with regard to inflow.

Garcia et al (2001) analyze dynamic strategic behavior of hydropower producers in an infinite-horizon setting where two firms engage in dynamic Bertrand competition and where inflow is uncertain. They show that a tightening of the price cap on electricity in a market with significant hydropower production would reduce the alternative value of production in the current period and thus increase the competition between producers when water is plentiful. Furthermore, as an extension to their basic model they show that the price cap also affects the probability of hydropower replacing thermal production of electricity. If the price cap is sufficiently low, prices in situations with full reservoirs become so low that thermal production is eliminated and replaced by hydropower. If there is no inflow in the following peirod this might effect the reliability of the system.

Following the conclusion reached by Garcia et al (2001) it seems natural to ask whether there are any public measures that can be imposed to

[1]This include for example Green and Newbery (1992), Green (1996), Newbery (1998), Borenstein and Bushnell (1999) and Borenstein, Bushnell and Wolak (2002).

[2]See Bushnell (2003).

reduce the potential problem of hydropower replacing thermal production in situations where inflow is plentiful. Of cource, a high price cap would give producers incentives to store water for periods with little inflow. However, a high price cap might be politically undesirable as this would imply a high price for electricity. Thus it is relevant to look at the properties of other public measures. Motivated by the observed energy shortage during the winter 2002/2003 in Scandinavia and the following discussion, we look at three such public measures in this paper.

First, we look at measures to promote competition. Authorities can promote competition for instance by taking actions to prevent collusion or preventing mergers from taking place. We analyse the effect of promoting competition on the probability of hydropower replacing thermal production in the simplest possible way, by comparing the Bertrand-Nash outcome described by Garcia et al (2001) to the monopoly outcome. Even though Garcia et al (2001) describe the collusive outcome which can be identical to the monopoly solution, they make no explicit comparisons of the two outcomes. Our description of the monopoly outcome is also more suitable for comparisons with the Bertrand-Nash solution. We find that competition represented by the Bertrand-Nash outcome implies a higher probability of hydropower replacing thermal production than the monopoly solution.

Another public measure that has been proposed in order to reduce the problem of energy shortage in low inflow situations is increased transmission capacity. The idea is that increased transmission capacity would make it possible to increase production through imports in situations with low inflow and thus reduce the problem of energy shortage. There are several ways this could be modelled. Here, we model transmission between two geographic areas with one hydropower company in each area. In this setting, we find that an increase in transmission capacity leads to a more fierce competition between the two hydro producing companies and thus increases the probability of hydropower replacing thermal production. This effect is similar to a reduction in the price cap as described by Garcia et al (2001).

Finally, we consider the effect on storage by rationing imposed by authorities. Rationing may be thought of as a measure to secure supply of electricity in situations with little or no inflow. We model rationing as an action by authorities to reduce demand in situations when the energy resource is belived to be scarce. Rationing will only affect profits directly in the periods where such rationing is imposed and also affect producers' storage levels. These effects are different from a reduction in the price cap analysed by Garcia et al. We find however, that the effect of rationing is similar to a reduction in the price cap. Increased rationing leads to a more fierce competition when water is plentiful and thus increases the probability of hydropower replacing thermal production of electricity.

In section 2 we restate the basic model developed by Garcia et al (2001) and compare the Bertrand-Nash solution to the monopoly solution. In section 3 we extend the model to include a situation with limited transmission capacity between two different geographic areas. In section 4 we change the model to include an element of rationing. In all three sections we discuss storage when thermal production is included. In section 5 we provide some concluding remarks.

2 Market power and storage

In this section we repeat the basic features of a model developed by Garcia et al (2001). We then develop a monopoly solution and compare this to the basic Bertrand-Nash solution described by Garcia et al. (2001) in a situation with price taking thermal producers present in the market. By doing so we are able to analyse in a simple way how market power affects storage in a hydropower system.

2.1 The Bertrand-Nash solution

The general framework of the Garcia et al (2001) infinite-horizon model is a situation with two hydropower producers, where each producer controls one storage facility. The two reservoirs are of equal size. At the beginning of each period both producers observe how much water that is available for production in the two storage facilities. Thus, there is complete information with regard to the history of the game. At each stage of the game, however, producer $i = 1, 2$ does not observe the other producer's action before the move is made. After observing storage levels the two producers set their prices simultaneously. If they set the same price, it is assumed for simplicity that one of the two producers serves the entire market. This produces asymmetry with regard to storage levels between the producers. However, as Garcia et al (2001) show, the results are unaffected by this assumption[3].

Without additional restrictions there would be a vast number of states (storage levels) visited by the two producers. Garcia et al (2001) solve this problem through a number of assumptions. First it is assumed that demand is equal to one unit in every period and perfectly inelastic. As noted by Garcia et al (2001) this assumption is consistent with the operation reality

[3] Se Garcia et al (2001) section 2.2.

of many real-time wholesale electricity markets. In particular they argue with reference to Train and Shelting (2002) that the widespread use of fixed rate contracts in retail markets makes the responsiveness of market demand extremely limited.[4]

Second, it is assumed that the storage capacity is also equal to one unit. This implies that one producer is not able to store more water than just to cover demand in each period. Garcia et al (2001) also make the assumption that the maximum output capacity is equal to one unit in each period.

Thirdly and finally it is assumed that water inflow w at the end of each period follows a simple binomial process, where $w = 1$ with probability q and $w = 0$ with probability $(1 - q)$. It either rains one unit or it does not. The initial assumption by Garcia et al. (2001) is that both players are identical with respect to the probability of inflow.

The three main assumptions above with regard to demand, storage capacity and water inflow imply that each producer either has one unit available for production or the reservoir is empty in which case production is zero. Thus, there are just two states of storage levels visited by each producer. When we combine the storage levels experienced by the two producers we get four different states, first when both producers have full reservoir, second and third when either producer 1 or 2 have full reservoir and fourth when neither of the two producers have water available for production. If we assume as Garcia et al (2001) that both producers have equal marginal costs normalized to zero, the two states where either producer 1 or 2 have a full reservoir would be identical.

The static game solution to a situation where both producers have empty reservoirs is simply that no production takes place. If one of the two producers has a full reservoir, then this producer is in fact a monopolist. The producer charges the maximum allowed price c^* for the one unit available for production. We interpret this price as the consumers' reservation price[5]. If both producers have full reservoirs, then because the firms are symmetric with respect to marginal costs we have the so-called "Bertrand paradox" where both firms charge a price of zero equal to the marginal cost.

In a dynamic game, actions taken today may affect payoff in future periods. It is assumed that current payoff is unaffected by storage levels in previous periods. It means that the action chosen by producer $i = 1, 2$ based on the current reservoir level would have been taken irrespective of storage

[4] This is also noted by Borenstein et al. (2003) with regard to the electricity market in California.

[5] Garcia et al (2001) interpret this price as either a reservation price or the price cap set by the regulator.

levels experienced in previous periods. The Markov updating of the game makes it possible to express the payoff for producer i as a value function representing the payoff to producer i for the remainder of the game once a certain state of reservoir level has been reached. The value function consists of the current period payoff and the effect that the current action has on the probability of reaching a certain state in the next. The value function for producer i in the state (x, y) is given by V_{xy}, where $x \in \{0, 1\}$ is the reservoir level of producer i and $y \in \{0, 1\}$ is the rival's reservoir level. Future payoff is discounted by the factor $\beta \in (0, 1)$. Because both producers have marginal cost equal to zero and equal probability of inflow the value functions will be symmetric.

Garcia et al. (2001) define value functions for each of the four possible states that can be experienced by producer i. The most interesting state is where both producers have a full reservoir. In this state both producers are able to cover demand and have to decide whether to undercut the rival's price or store the water for future periods. Garcia et al. (2001) derive the following equilibrium price[6]:

$$p^*_{11} = \beta(1 - q)c^*. \tag{1}$$

The intuition here is that the equilibrium price would have to be equal to the alternative value associated with production in the next period. In the next period producer i would receive the reservation price c^* if there is no inflow at the end of the current period. This occurs with probability $(1 - q)$. Also, producer i would have to discount this expected payoff by the factor β. We observe that the equilibrium price is increasing in the reservation price c^* and in the discount factor β. Finally, a higher probability of inflow will reduce the equilibrium price.

When adding thermal production, Garcia et al. (2001) assume that demand in each period is equal to 2 units. Furthermore, it is assumed that the fringe thermal producers have a deterministic capacity in total of one unit and constant marginal cost equal to c^T. They then focus on the state where both hydro producing players have one unit of water available for production, $(1, 1)$ and find that hydropower replaces thermal production when the equilibrium price $p^*_{11} < c^T$.

If the equilibrium price is higher than the marginal cost in thermal production, one unit of water will be saved for production in the next period. Now, if both units of water are produced in the state $(1, 1)$ and there is

[6] See Garcia et al. (2001), equation 2.8.

no inflow before the next period there will be to little capacity available to cover the demand of two units.

2.2 The monopoly solution

According to Garcia et al. (2001) a collusive agreement between the two Bertrand players is sustainable for any price $\widetilde{p}_{11}$, such that $c^* \geq \widetilde{p}_{11} \geq \overset{*}{p}_{11}$, if and only if $\beta \geq \frac{1}{1+q}$. Even though the collusive solution described by Garcia et al. (2001) is identical to the monopoly solution when $c^* = \widetilde{p}_{11}$, it is not straight forward to compare this solution to the Bertrand-Nash case when thermal production is introduced. When thermal production is introduced, this affects the extent to which a player can be punished for deviation from the collusive equilibrium. In order to avoid this problem we describe the monopoly solution.

We let the function V_x represent the monopolist's value for the state (x), where $x \in \{0, 1, 2\}$ denotes the monopolist's storage level. This value function represents the monopolist's value for all remaining periods once the state (x) is reached. We consider first the state where the storage facility is empty. In the current period there is no production. In the next period it either rains one unit with probability q or it does not with probability $(1 - q)$. We want the monopoly case to compare to the two firm Bertrand-Nash outcome described in subsection 2.1 above. Thus we imagine the monopoly case to be the case where the two hydropower producers have merged into one company. Then, if it rains the monopolist would experience an inflow of two units ($w = 2$) and receive the value V_2 for the remaining periods. Otherwise the producer faces the value function V_0. Future payoff is discounted with the factor $\beta \in (0, 1)$. We can now state the monopolist's value function for the state $(x = 0)$ as follows:

$$V_0 = \beta[(1 - q)V_0 + qV_2]. \tag{2}$$

The next state $(x = 1)$ may occur as a result of a situation where the monopolist at the beginning of the previous period had 2 units available for production. In the previous period one unit was produced and there was no water inflow at the end of this period. With only one unit available for production the monopolist could choose between production in the current period at price c^* or save the water for future production. The monopolist would always choose to produce in the current period as long as future profit is discounted with a value less than 1 or the probability of inflow is higher than zero, $1 > \beta(1 - q)$. In the following period the monopolist will either have zero or two units available for production depending on whether it rains or not.

$$V_1 = c^* + \beta[(1-q)V_0 + qV_2]. \tag{3}$$

The last possible state for the monopolist ($x = 2$), is a situation where the producer has more than enough water available to cover demand in the current period. This state is a result of inflow at the end of the previous period. Now, the monopolist would always produce one unit at price c^* during the current period and save one unit for possible production in the future. The choice is between producing now and receiving c^* and producing in two periods from now if there is no inflow in between. The discounted value of the latter option, $\beta^2(1-q)^2c^*$ is always less than the certain payoff today (c^*) as long at either $\beta < 1$ or $q > 0$. Then, the monopolist's value function V_2 can be expressed in the following way[7]:

$$V_2 = c^* + \beta[(1-q)V_1 + qV_2]. \tag{4}$$

The monopolist sets the price equal to the reservation price in every period where the monopolist has at least one unit of water available for production.

The monopoly outcome indicates a higher price in such situations compared to the case where producers compete in setting the price. We should also note that the actual production of electricity is the same in all states of total storage level $\{0, 1, 2\}$, regardless of the model of competition. This is due to the fact that demand is equal to one unit in each period and that the reservation price c^* is equal across periods. However, if we introduce thermal production also in the case of a monopoly producer this will change.

2.3 Introducing thermal production

We increase demand in each period to 2 units and introduce thermal production from a competitive fringe with a production capacity of one unit. The competitive fringe will always bid their marginal cost equal to c^T. Because demand has doubled this will not change the monopolist's value functions for states where the storage level is either 0 or 1. In the state where the reservoir is full however, the monopolist now must choose between an undercutting strategy receiving,

[7] When $c^* = \widetilde{p}_{11}$, the monopoly solution is identical to the collusive solution described in Garcia et al. (2001). We have that $\frac{V_2}{2} = \widetilde{V}_{1,1}$, where $\widetilde{V}_{1,1}$ is defined as one players collusive value (see Garcia et al. equation (3.2)).

$$2c^T + \beta[(1-q)V_0 + qV_2] \tag{5}$$

and charging a higher price than the fringe receiving

$$c^* + \beta[(1-q)V_1 + qV_2]. \tag{6}$$

The undercutting strategy is preferred only when c^T is higher than the level making the monopolist indifferent between the two strategies:

$$c^T > \frac{c^* + \beta(1-q)[V_1 - V_0]}{2}. \tag{7}$$

If we rearrange this equation and use the fact that $V_1 - V_0 = c^*$, we get the following condition for the monopolist to prefer the undercutting strategy:

$$c^T > \frac{c^*(1+\beta(1-q))}{2}. \tag{8}$$

We see that the monopolist would prefer the undercutting strategy as long as he receives a price for his 2 units sold that is higher than the alternative value for this volume. The alternative is to receive c^* for one unit in the current period and the same price for one additional unit if it does not rain in the next period.

As shown by Garcia et al. (2001) hydropower will replace thermal production in the state $(1,1)$ in the Bertrand-Nash case if $c^T > \beta(1-q)c^*$. We can now state our proposition 1:

Proposition 1 *Assuming that* $0 < \beta < 1$, $0 < q < 1$ *and that the reservation price* $c^* > 0$. *Then,*
(i) if $c^T > \frac{c^*(1+\beta(1-q))}{2}$, *hydropower would replace thermal production in both cases.*
(ii) if $c^T < \beta(1-q)c^*$, *hydropower would not replace thermal production in either cases.*
(iii) for intermediate values of c^T, $\frac{c^*(1+\beta(1-q))}{2} > c^T > \beta(1-q)c^*$, *thermal production would only be replaced by hydropower in the case of Bertrand-Nash competition.*

The interesting case is for intermediate values of c^T where thermal production would only be replaced in the case of Bertrand-Nash competition. This means that as long as there is no inflow, then the reservoirs in the Bertrand competition case would be empty in the next period. In the monopoly case however, there will be enough water left in the reservoirs to serve demand. Thus, even though competition may lead to lower prices on electricity in periods with more than enough water, the downside is that less water may be available for periods with little or no inflow.

3 Introducing transmission capacity between two geographic areas

In this section we introduce transmission capacity between two geographic areas where one hydropower producer is located in each area. The objective is to analyse how a change in the transmission capacity affect the possibility of hydropower replacing thermal production of electricity. We continue to assume that both producers experience inflow with the same probability. Furthermore, we assume that demand is equal to $\frac{1}{2}$ in both regions and that the maximum allowed price c^* is the same. Also, there is just one transmission line with capacity $k \in (0, \frac{1}{2})$ between two regions A and B. Electricity flows to the region with demand surplus (high price) until the transmission capacity is binding. The transmission line is operated by a grid operator behaving as a competitive arbitrage agent.

3.1 Bertrand-Nash solution

The restricted transmission capacity implies that we now have a set of new states of reservoir levels. The four original states where $(0,0)$, $(1,0)$, $(0,1)$ and $(1,1)$. Now, if both firms have full reservoirs at the beginning of the current period producer i could either produce $\frac{1}{2}+k$ or $\frac{1}{2}-k$ depending on whether the producer undercuts the rivals price or not. The rival would in both cases serve the residual demand equal to $\frac{1}{2}-k$ or $\frac{1}{2}+k$. In the following period both producers would have either $\frac{1}{2}-k$ or $\frac{1}{2}+k$ left in the reservoir if there is no inflow or one unit if inflow occur. Accordingly, we have the two additional states; $(\frac{1}{2}+k, \frac{1}{2}-k)$ and $(\frac{1}{2}-k, \frac{1}{2}+k)$.

In the state $(\frac{1}{2}+k, \frac{1}{2}-k)$ total production capacity is equal to total demand in both regions. In addition, the existing transmission capacity does not constrain producer i from producing all the available water in its storage facility. Thus, the dominant strategy for producer i is to set the reservation price c^* and sell $\frac{1}{2}+k$ units in the current period. The rival producer would also set the price c^* and sell the remaining $\frac{1}{2}-k$ in his reservoir. In the following period both reservoirs would either be empty or full depending on whether they experience inflow or not[8].

[8]This implies that the states $(1,0)$ and $(0,1)$ would not be visited by the producers

$$V_{(\frac{1}{2}+k),(\frac{1}{2}-k)} = c^*(\frac{1}{2}+k) + \beta[(1-q)V_{0,0} + qV_{1,1}]. \tag{9}$$

In the state $(\frac{1}{2}-k, \frac{1}{2}+k)$ producer i's dominant strategy is to sell $\frac{1}{2}-k$ at price c^* and the rival producer sell $\frac{1}{2}+k$ at the same price.

$$V_{(\frac{1}{2}-k),(\frac{1}{2}+k)} = c^*(\frac{1}{2}-k) + \beta[(1-q)V_{0,0} + qV_{1,1}]. \tag{10}$$

Thus, the difference in value function between the two states $V_{(\frac{1}{2}+k),(\frac{1}{2}-k)} - V_{(\frac{1}{2}-k),(\frac{1}{2}+k)} = c^*2k$. The difference between the two value functions reflects that when producer i charges a lower price than the rival in a state where both producers have full reservoirs, he then forsakes the alternative which is to sell $2k$ at price c^* in the following period if there is no inflow.

In the most interesting state $(1,1)$ where water is plentiful, producer i faces two alternatives in competition with the other producer. First producer i may choose to undercut the price set by the other producer in which case he earns $p_{1,1}(\frac{1}{2}+k)$ in the current period. This would leave $(\frac{1}{2}-k)$ for production in future periods. The other producer will only produce $(\frac{1}{2}-k)$ in the current period and have $(\frac{1}{2}+k)$ left for production in the next period. Thus, the value function for producer i in the state $(1,1)$ can be expressed as follows:

$$p_{1,1}(\frac{1}{2}+k) + \beta[(1-q)V_{(\frac{1}{2}-k),(\frac{1}{2}+k)} + qV_{1,1}]. \tag{11}$$

The second alternative is to charge a price higher than the rival producer. This will result in full import to region A where producer i is located. However, since the transmission line is constrained, there is still positive residual demand facing producer i. Producer i would then charge the reservation price on the residual demand in region A in the current period and leave $(\frac{1}{2}+k)$ for production in the next period. The producer located in region B will only have $(\frac{1}{2}-k)$ left for production in the next period. The value function corresponding to this strategy is:

$$c^*(\frac{1}{2}-k) + \beta[(1-q)V_{(\frac{1}{2}+k),(\frac{1}{2}-k)} + qV_{1,1}]. \tag{12}$$

The two alternatives above, (11) and (12), indicate an equilibrium price $p_{1,1} = \widehat{p}_{1,1}$ where both producers are indifferent between the undercutting strategy and charging a higher price than the rival firm.

when both producers are symmetric with respect to inflow. The four new states are $(0,0)$, $(1,1)$, $(\frac{1}{2}+k, \frac{1}{2}-k)$ and $(\frac{1}{2}-k, \frac{1}{2}+k)$.

$$\widehat{p}_{1,1} = \frac{c^*(\frac{1}{2}-k) + \beta(1-q)[V_{(\frac{1}{2}+k),(\frac{1}{2}-k)} - V_{(\frac{1}{2}-k),(\frac{1}{2}+k)}]}{(\frac{1}{2}+k)}. \tag{13}$$

We observe that the equilibrium price depends on the difference in value between the states $(\frac{1}{2}+k, \frac{1}{2}-k)$ and $(\frac{1}{2}-k, \frac{1}{2}+k)$ which is equal to c^*2k.

We can now restate the equilibrium price in the state $(1,1)$ as follows:

$$\widehat{p}_{1,1} = \frac{c^*(\frac{1}{2}-k) + \beta(1-q)c^*2k}{(\frac{1}{2}+k)}. \tag{14}$$

We note that if $k = \frac{1}{2}$, then the equilibrium price would reduce to the equilibrium price in the case where all demand and production are located at the same node.

Proposition 2 *Assuming that* $0 < \beta < 1$, $0 < q < 1$ *and* $k \in (0, \frac{1}{2})$*, then the equilibrium price* $\widehat{p}_{1,1}$ *is decreasing in* k*.*

Proof. We have that $\frac{\partial \widehat{p}_{1,1}}{\partial k} = -4\frac{c^*(1-\beta+\beta q)}{(1+2k)^2}$. Furthermore, by assumption we have that $0 < \beta < 1$, $0 < q < 1$ and $k \in (0, \frac{1}{2})$. This implies that $(1-\beta+\beta q) > 0$ and that k is positive. Accordingly, we have that $\frac{\partial \widehat{p}_{1,1}}{\partial k} < 0$. ■

The intuition behind the result in proposition 2 is simple. As the transmission capacity increases, competition between our two producers becomes increasingly fierce in the current period because the payoff associated with the strategy of inducing a transmission constraint is reduced. In the extreme case where $k = \frac{1}{2}$, there will be no additional payoff from such a strategy at all.

Now, if we look more closely at the condition stated in equation (14) we can decompose the alternative value in two different components. First we have the value of selling $(\frac{1}{2}-k)$ units of water at the reservation price c^* in the current period. This alternative value is equal to $\frac{c^*(\frac{1}{2}-k)}{(\frac{1}{2}+k)}$. This leaves us with $1-\frac{1}{2}+k$ units for production in the following period. The second part of equation (14), $\frac{\beta(1-q)c^*2k}{(\frac{1}{2}+k)}$, represents the alternative value of selling k units in the other market in the next period and reducing import with the same amount, in total $2k$ units of water. This leaves us with $1-\frac{1}{2}-k$ units that are not represented as part of the alternative value. The reason why this part of the reservoir is not represented in the alternative value is that this amount of water would have to be sold in the second period regardless of whether an undercutting strategy is followed or not. Thus, the payoff received for this part of the inflow is irrelevant when determining the equilibrium price. However, if we compare with other alternatives implying a production of 1 unit we should also account for the last $(\frac{1}{2} - k)$ units that are not accounted for in equation (14).

3.2 Adding thermal production

Again, recognizing the importance of the combination of hydro and thermal based electricity production in real world electricity markets, we add thermal production by a fringe producer in each market. We consider the case where both thermal producers offer their capacity of $\frac{1}{2}$ unit each at the same marginal cost c^T. Demand in each market region is increased to 1 unit in every period.

We look at the state where both hydropower producing firms have 1 unit of water available for production. Now, they can choose to undercut the price set by the two thermal producers and sell 1 unit of power each in the current period at a price slightly below c^T. The alternative is to charge a higher price where we know that $(\frac{1}{2} + k)$ of the units available have an alternative value equal to $\frac{c^*(\frac{1}{2}-k)+\beta(1-q)c^*2k}{(\frac{1}{2}+k)}$, while the remaining $(\frac{1}{2} - k)$ units in the reservoir are sold in the second period at an alternative value equal to $\beta(1-q)c^*$. The undercutting strategy is preferred if

$$c^T > v = (\frac{1}{2} + k)\frac{c^*(\frac{1}{2} - k) + \beta(1-q)c^*2k}{(\frac{1}{2} + k)} + (\frac{1}{2} - k)\beta(1-q)c^*. \qquad (15)$$

Proposition 3 *Assuming that* $0 < \beta < 1$, $0 < q < 1$ *and* $k \in (0, \frac{1}{2})$, *then the alternative value* v *is decreasing in* k.

Proof. We have that $\frac{\partial v}{\partial k} = c^*(\beta - \beta q - 1)$. Furthermore, by assumption we have that $0 < \beta < 1$ and $0 < q < 1$. This implies that $\beta - \beta q - 1 < 0$. Accordingly, we have that $\frac{\partial v}{\partial k} < 0$. ■

It follows from proposition 3 that the probability of observing a situation where hydropower replace thermal production increase when transmission capacity is increased. The intuition behind this result is most easily seen by looking at the two extreme cases where $k = 0$ and $k = \frac{1}{2}$. With $k = \frac{1}{2}$, we have that undercutting is preferred if $c^T > \beta(1-q)c^*$. This is exactly the same as the condition derived in subsection 2.2 where there is only one integrated market. At the other extreme when $k = 0$ we have separate markets. In this situation undercutting becomes a strategy when $c^T > c^* \frac{1}{2} + \frac{1}{2}\beta(1-q)c^*$. This is the same condition as derived for the monopoly case[9] described in subsection 2.3.

4 Rationing

The last public measure we consider in this paper is rationing. If the authorities introduce rationing, demand is set to R, where $0 \leq R \leq 1$. An increase in the level of rationing would in the same way as a reduction in the reservation price c^* lead to a reduction in profits for the two hydropower producing firms. However, in contrast to a change in the reservation price analysed by Garcia et al. (2001), rationing will also affect the producers' reservoir levels. Also, rationing will only affect profits directly in periods where such rationing is imposed while a change in the reservation price will affect profits in all periods.

The level of rationing is set subsequent to the observation of the total reservoir level $s = x + y$, where x denotes the reservoir level of producer 1 and y denotes the reservoir level of producer 2.

Authorities have to follow a set of specific rules when they decide on the level of rationing. We assume no rationing to take place when authorities observe a total reservoir level $s > 1$. When they observe a total reservoir level of $s = 1$ the situation is considered critical and rationing is introduced. This is done by disconnecting some consumers from the network. If the total reservoir level is observed to be below 1 unit, $s < 1$, the authorities will impose rationing where demand is reduced to the available amount of water.

4.1 Bertrand-Nash solution

In the same way as described in the two previous sections, we let $V_{x,y}$ denote the value function of producer $i = 1, 2$ related to the state (x, y), where $x \in \{0, (1-R), 1\}$ represents the reservoir level of producer i. The rival's reservoir level is described by the state variable $y \in \{0, (1-R), 1\}$. Because both producers are symmetric with respect to costs and inflow, they will have identical value functions. Accordingly, it is sufficient to look at the value functions of only one of the two producers.

[9] The monopolist's value functions and alternative value are not affected by the introduction of a possible transmission constraint. Since inflow and demand are evenly distributed between the two regions, the monopolist is able to charge the reservation price in every period.

First, we look at the state where neither of the two producers have water in their reservoirs, $(0,0)$. With no water in the reservoirs nothing is produced in the current period. Both producers will have filled up their reservoirs by the beginning of the next period with probability q. If so, they will receive the value $V_{1,1}$ for the remaining periods of the game. If no inflow occurs with probability $(1-q)$, then none of the two producers will have water in their reservoir at the beginning of the next period. The value of this state is represented by $V_{0,0}$.

$$V_{0,0} = \beta[(1-q)V_{0,0} + qV_{1,1}]. \tag{16}$$

The next state we consider, is where producer i has no water available while the reservoir of the rival firm is full, $(0,1)$. In this state the rival firm will be a monopolist in the current period. If no rationing is introduced, then the rival firm will produce all the available water in the current period and receive the reservation price c^*. However, as described above, the authorities observe that $s = 1$ and impose rationing where demand is reduced to the level R . Now, the rival firm will produce R in the current period and receive c^* for this production. Firm i produce nothing and receive no income in the current period. In the next period the rival firm will have $1-R$ left in the reservoir if there is no inflow. If inflow occurs both firms will have 1 unit of water available for production.

$$V_{0,1} = \beta[(1-q)V_{0,(1-R)} + qV_{1,1}]. \tag{17}$$

We observe that $V_{0,0} - V_{0,1} = \beta(1-q)[V_{0,0} - V_{0,1-R}]$. In the state $(0, 1-R)$ the rival firm will be the only producer with water available for production. Because the total reservoir level is less than 1 unit, rationing will be imposed. The level of demand is set to the observed reservoir level $s = 1-R$. The rival firm will in this state produce the remaining water in the reservoir at the reservation price c^*.

$$V_{0,1-R} = \beta[(1-q)V_{0,0} + qV_{1,1}]. \tag{18}$$

If we compare the value functions defined above we find that $V_{0,1-R} = V_{0,0} = V_{0,1}$. As long at firm i's reservoir is empty the value function is left unaffected by the rival's reservoir level.

The opposite situation is where only firm i has water in the reservoir. In that case the authorities will impose rationing restricting demand to R. Producer i will then produce R and receive the income Rc^* in the current period. If there is no inflow until the next period only firm i will have water in its reservoir. If inflow occurs, both firms will have 1 unit in their reservoirs.

$$V_{1,0} = Rc^* + \beta[(1-q)V_{(1-R),0} + qV_{1,1}]. \tag{19}$$

In the state $(1-R, 0)$ only producer i has water available for production. Because $s < 1$ the authorities impose rationing and set total demand equal to $s = 1 - R$. Producer i will produce the remaining water in the current period, receiving the reservation price. In the next period both producers will either have empty or full reservoirs.

$$V_{(1-R),0} = (1-R)c^* + \beta[(1-q)V_{0,0} + qV_{1,1}]. \tag{20}$$

By substitution from (16),(17), (18) and (19) we can rewrite the value function for the state $(1-R, 0)$ as follows:

$$V_{1-R,0} = (1-R)c^* + V_{0,0} = (1-R)c^* + V_{0,1} = (1-R)c^* + V_{0,1-R}. \tag{21}$$

Furthermore, by the use of equations (20) and (21) we can now express the value function $V_{1,0}$ in the following way:

$$V_{1,0} = Rc^* + \beta(1-q)(1-R)c^* + \beta[(1-q)V_{0,0} + qV_{1,1}]. \tag{22}$$

If the authorities impose rationing, $0 < R < 1$, we can now express the difference between the value functions associated with the two states $(1, 0)$ and $(0, 1)$ as follows:

$$V_{1,0} - V_{0,1} = Rc^* + \beta(1-q)(1-R)c^*. \tag{23}$$

In state $(1, 0)$ only producer i has water available. When rationing is imposed producer i receives the reservation price c^* for the level of rationing imposed. If there is no inflow with probability $1-q$, producer i will receive the reservation price c^* for the remaining water in the reservoir $1-R$. This however, is future payoff that has to be discounted by the factor β.

The last state to consider is where both producers have 1 unit of water available, the state $(1, 1)$. In this state we assume that both producers will set the same price and serve the entire market with probability $\frac{1}{2}$. We look at producer i's optimal response to the price $p_{1,1}$ set by the rival firm. One strategy for producer i would now be to slightly undercut the rival and set the price $p_{1,1} - \varepsilon$. Knowing that no rationing is imposed when the authorities observe that $s > 1$ and assuming that $\varepsilon \to 0$, we can write the value of an undercutting strategy as follows:

$$p_{1,1} + \beta[(1-q)V_{0,1} + qV_{1,1}]. \tag{24}$$

In the current period firm i captures the entire market of 1 unit. In the next period either both firms will have full reservoirs or only the rival firm will have water available.

The alternative strategy for firm i is to let the rival firm take the whole market in the current period. This will produce the following value for firm i:

$$\beta[(1-q)V_{1,0} + qV_{1,1}]. \tag{25}$$

No rationing is imposed in the current period and the rival firm produces the entire water stock of 1 unit. If there is no inflow producer i will be the only producer in the next period. If on the other hand inflow occurs, then both firms will have full reservoirs at the beginning of the next period.

The discussion above indicates an equilibrium price $p_{1,1} = \overline{p}_{1,1}$ where both producers will be indifferent between the two strategies.

$$\overline{p}_{1,1} + \beta[(1-q)V_{01} + qV_{11}] = \beta[(1-q)V_{10} + qV_{11}]. \tag{26}$$

Knowing that $V_{10} - V_{01} = Rc^* + \beta(1-q)(1-R)c^*$, we can rewrite the equilibrium condition as follows:

$$\overline{p}_{1,1} = \beta(1-q)[Rc^* + \beta(1-q)(1-R)c^*] \tag{27}$$

We can now state the following proposition:

Proposition 4 *Assuming that* $0 < \beta < 1$*,* $0 < q < 1$*,* $0 < R < 1$ *and that the reservation price* $c^* > 0$*, then we have that an increased (decreased) level of rationing given by a reduction (increase) in* R *will lead to a reduction (increase) in the equilibrium price* $\overline{p}_{1,1}$*.*

Proof. We have that $\frac{\partial \overline{p}_{1,1}}{\partial R} = (1-q)\beta(c - \beta(1-q)c)$. Furthermore, by assumption we have that $0 < \beta < 1$ and $0 < q < 1$. This implies that $(1-q)\beta(c - \beta(1-q)c) > 0$. Accordingly, we have that $\frac{\partial \overline{p}_{1,1}}{\partial R} > 0$. ■

The intuition here is that the equilibrium price must be equal to the alternative value of producing in a future period. If the producers face rationing in the future this will reduce the value of production in the future. Accordingly, the producers would want to produce more in the current period.

4.2 Thermal production and rationing

We extend the model to include thermal production from a price taking producer (fringe) with a fixed capacity of 1 unit and marginal cost equal to c^T. Demand is increased to 2 units in every period.

In the situation where both hydropower producers have full reservoirs, there is an exess capacity equal to 1 unit. We know that the hydropower producing firm i would want to postpone production if the price received today is lower than the alternative value represented by $\beta(1-q)[Rc^* + \beta(1-q)(1-R)c^*]$. Thus, whether the hydropower producing firms choose to store their water for future production depends among other factors on the level of rationing imposed in situations where the water resource is considered to be scarce.

With demand equal to 2 units, the authorities impose rationing if they observe that the total level of available water resources is less than or equal to 2 units. If just one of the two hydro producing firms has a full water reservoir, then total available capacity is equal to 2 units and rationing is imposed. Demand is set to $1 + R$. If the observed available capacity is observed to be below 2 units, then demand is reduced to match the available capacity.

We look at the state where both hydro producing firms have full reservoirs. With demand equal to 2 units and thermal production present in the market the hydropower producing firms can choose to undercut the price set by the thermal producer and deplete their reservoir in the current period. They will choose to do so if $\widetilde{p}^*_{1,1} < c^T$. If the equilibrium price $\widetilde{p}^*_{1,1}$, decided by the alternative value, is higher than marginal cost in thermal production then the hydro producing firms will have a higher payoff if they store some of their water for future periods.

From proposition 4 we have that the equilibrium price is reduced when rationing is increased (reduction in R). This means that the probability of hydropower replacing thermal production in the state $(1, 1)$ increases as the level of rationing is increased. Thus, by imposing rationing in order to secure water for future periods the authorities might achieve the opposite result. The intuition here is that rationing reduce the alternative value faced by hydropower producing firms. Accordingly, they find it relatively more profitable to produce in the current period when water resources are plentiful.

5 Concluding remarks

The starting point of this paper is an oligopoly model developed by Garcia et al (2001) where 2 hydropower producing firms engage in dynamic Bertrand competition. The basic features of their model was restated in section 2 of this paper.

On the basis of the model developed by Garcia et al (2001) we defined the monopoly outcome in subsection 2.3. We then compared the monopoly outcome to the Bertrand-Nash outcome as defined in the original model. We found that hydropower is less likely to replace thermal production in monopoly case. This result indicates that competition would not necessarily lead to the result that the water resources are allocated to the periods where they are most needed.

Furthermore, we extended the Garcia et al (2001) model to include transmission capacity between two price areas and rationing imposed by the authorities. Both of these extensions we also analyzed in the presence of thermal production.

With regard to transmission capacity described in section 3, we found that the Bertrand-Nash equilibrium price is reduced when transmission capacity is increased. This happens because an increase in the transmission capacity makes competition more fierce. When we included thermal production, we found that an increase in transmission capacity would increase the probability of hydropower replacing thermal production. This indicates that increased transmission capacity may not be a good measure if the aim is to secure enough storage for periods with low inflow.

The same result holds with regard to the extensions made in section 4 where we looked at rationing as a possible measure for securing enough water in periods with low inflow. We found that an increase in the level of rationing would reduce the Bertrand-Nash equilibrium price and thus increase the probability of hydropower replacing thermal production.

References

[1] Borenstein S., J. Bushnell and F. A. Wolak (2002): "Measuring market inefficiencies in California's restructured wholesal electricity market", *American Economic Review*, **92**(5), 1376-1405.

[2] Crampes, C. and M. Moreaux (2001): "Water Resource and Power Generation", *International Journal of Industrial Organization*, **19**(6), 975-997.

[3] Bushnell, J (2003): "A mixed complementary model of hydro - thermal electricity competition in the Western U.S., *Operations Research*, **51**(1), 80-93.

[4] Borenstein S. and J. Bushnell (1999): "An Empirical Analysis of the Potential for Market Power in California's Electricity Industry", *Journal of Industrial Economics*, **47**(3), 285-323.

[5] Garcia, A., J. D. Reitzes, E. Stacchetti (2001). "Strategic Pricing when Electricity is Storable.", *Journal of Regulatory Economics* **20**(3), 223-247

[6] Green R.J. and D.M. Newbery (1992): "Competition in the British Electricity Spot Market", Journal of Political Economy, **100**(5), 929-953.

[7] Green R. J. (1996): "Increasing competition in the British electricity spot market", The Journal of Industrial Economics, XLIV(2), 205-216.

[8] Newbery D.M. (1998): "Competition, contracts and entry in the electricity spot market", Rand Journal of Economics, **29**(4), 726-749.

[9] Scott, T. J. and E. G. Read (1996). "Modelling Hydro Reservoir Operation in a Deregulated Electricity Market.", *International Transactions on Operations Research* **3**(3/4), 243-253.

[10] Train, K. and A. Selting (2000). "The Effect of Price on Residential Consumer Choice in Competitive Retail Energy Markets: Evidence from Specific Markets to Date." *Edison Electric Institute Research.*