Social origins of a work ethic: Norms, mobility and urban unemployment*

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Abstract

Neighbourhood effects and worker mobility have been proposed as explanations for the pattern of employment in cities. This paper presents a theoretical framework within which the joint impact of these two factors can be analyzed. The evolution of unemployment patterns is modelled as a stochastic process, where workers sometimes make employment decisions influenced by local norms, and sometimes decisions of where to live based on neighbourhood characteristics. A long run outcome of full employment and complete segregation is found to be robust to a wide range of process specifications. More non-segregated long run outcomes are possible if mobility decisions are based on neighbourhood employment rates than if they are based on other neighbourhood characteristics.

Keywords: Norms; Mobility; Unemployment; Urban studies

* Comments from Bertil Tungodden, Oddvar Kaarbøe and Gaute Torsvik are greatly appreciated. For any remaining errors, the author is responsible.

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Introduction

In many cities, there is a concentration of social problems to certain inner-city areas. The disproportionate presence of poverty, crime and unemployment in central urban areas has been extensively documented by a number of empirical studies (e.g. Glaeser, Kahn and Rappaport (2000), Glaeser, Sacerdote and Scheinkman (1996), Fieldhouse (1998), Immergluck (1998), Reingold (1999) and Raphael (1998)). A number of explanantions for this spatial pattern of social problems have been suggested, many of which view unemployment as a key problem. The explanatory factors used are commonly some variation on notions of opportunity, influence and/or mobility. The traditional spatial mismatch hypothesis of Kain (1968) argues that suburban job growth has increased the distance an average inner city worker must commute to work, and thereby increased their costs of employment. In a much cited contribution, Wilson (1987) suggests that unemployment is part of a greater tangle of social problems, but deems salient the exodus of good role models from poorer neighbourhoods as an explanation of these problems. There have also been suggestions of an inflow of poor people to inner city areas, due to lower housing costs (see e.g. Glaeser, 1999) or access to public transportation for those too poor to own a car (Glaeser, Kahn and Rappaport, 2000). O'Regan and Quigley (1998) find human capital and exposure to the employed the most important factors for employment, from which we can surmise that low skill levels in inner cities lead to unemployment, which leads to more unemployment as others follow suit. Conversely, Bertrand, Luttmer and Mullainathan (1999) find that the probability of being on welfare increases with your exposure to social networks in which welfare use is more common.

The notion that there are neighbourhood or peer group effects in the spatial pattern of unemployment, has lately received much attention. Though empirical studies documenting neighbourhood effects face some methodological challenges, there nevertheless seems to be a consensus that such effects are real and important (see e.g. Glaeser, 1999). If indeed there are neigbourhood effects in employment, one implication is that the spatial distribution of employment might exhibit multiple equilibria. Or, in the words of Glaeser and Scheinkman (2000), there might exists a social multiplier, where small changes in the fundamental causes of individual employment might have a large impact on the aggregate level of employment. That small changes in employment policy might have a large impact on aggregate employment, is obviously something of which policy makers should take note. However, to correctly heed these neighbourhood effects, we need some theoretical framework within which to study their implications.

In this paper, I propose a theoretical framework by means of which we can study the joint impact of neighbourhood influence and worker mobility on the level and spatial distribution of urban unemployment. The purpose of the paper is to provide a foundation for a systematic treatment of the issues involved, rather than to draw precise policy implications. The notion of influence used is consistent with that of Lindbeck, Nyberg and Weibull (1999), where workers are assumed to be influenced by a social norm against being unemployed, and where the strength of the norm depends on the number of employed workers. In contrast to Lindbeck et al, however, I assign workers locations on a social grid, and assume that each worker is influenced only by his closest neighbours. The norms are thus local, rather than global, in scope. The existence of a social grid also permits the study of worker mobility, which in the model takes the form of pairs of workers exchanging locations, as in the neighbourhood segregation model of Schelling (1971). A variety of ways in which workers might decide to move is explored, some of which are consistent with the idea of Wilson (1987) that good role models leave depressed neighbourhoods, some of which are not.

The basic approach of the paper is to model the locations and employment status of workers as a stochastic process, where workers are repeatedly drawn at random to make either decisions of whether or not to be employed, or of whether or not to move to another location. The limit sets of the process are taken to represent the patterns of employment and worker locations we can expect to see in the long run, when the process has run for a sufficiently large number of periods. The objective of the paper is to see how different assumptions about the manner in which workers make employment and mobility decisions can lead to different long-run outcomes. Though essentially a model of interdependent preferences, many of the elements of the model developed below were inspired by models in the field of evolutionary game theory, specifically those of Kandori, Mailath and Rob (1993), Ellison (1993) and Young

(1993, 1998). In particular, the notion of local interaction is similar to that introduced by Ellison. Since the time horizon within which it makes sense to study employment and mobility decisions is restricted, however, we focus only on long run outcomes and do not introduce error terms into decisions to select between long run outcomes, the way the aforementioned models do. The basic model also has similarities to that of Bala and Goyal (2001), but has a different object of study.

In the following section I present an initial version of the model in which workers are immobile and segregated according to their level of education, and have their employment decisions influenced by their neighbours. In section three, this simple introductory version is used to illustrate that reducing the sample of neighbours observed by a worker when making employment decisions, in effect works as a means of selection between long run outcomes. In particular, if sample sizes are below a certain level, the state of full employment is the only possible long run outcome. Section four specifies the general model in which workers make both employment decisions and decisions of where to live, and section five suggests a range of different ways in which decisions of whether or not to move can be made. In section six, I show that a state of full employment and complete segregation according to education is a long run outcome for almost all of the motives for mobility specified. Moreover, if workers move to locations that are strictly better on some characteristic, very different long run outcomes are possible, including states of full employment, states of full unemployment among those with a low level of education, states of full segregation according to education and states of full integration. In addition, more non-segregated long run outcomes are possible when mobility decisions are based on neighbourhood employment rates rather than neighbourhood composition in terms of education.

A model of neighbourhood effects in a segregated city

Consider a finite population of N workers, who inhabit equally many locations of a circular city. The workers are heterogeneous in some characteristic $e \in \{L, H\}$, which we take to be education, though it might also be productivity or some other characteristic. Denote by N_1 the number of workers with a high level of education, $e = H$. And let $N_2 = N - N_1$ be the number of workers with a low level of education, $e = L$. Initially, we will assume that workers are completely segregated in terms of education, with the high education workers occupying positions 1 through N_1, and the low education workers inhabiting positions $N_1 + 1$ through *N*. In this formulation, workers thus do not have a choice of where to live.

Workers do, however, have a choice between working full time (*E*) and being unemployed (*U*). We will assume time is discrete, and in each period each worker has a probability $d \in \langle 0,1 \rangle$ of being called upon to revise his current employment status. When revising, a worker perceives the rewards from working as the utility $u(.)$ of consuming his net wages $w(.)$. We assume that wages are increasing in levels of education, $w(H) > w(L)$, and for a worker with education e_i we write the payoffs $p_i(E)$ from being employed as

$$p_i(E) = u[w(e_i)] \tag{1}$$

The rewards from being unemployed are the utility of consuming unemployment benefits *T*. There is also a social cost to being unemployed, which depends on the composition of the neighbourhood of a worker in terms of employment. We assume each worker has *k* neighbours to each side of him on the circle, 2*k* neighbours in all. A revising worker at location i observes a sample $s \in [1, 2k]$ of his neighbours, and assumes the proportion $\overline{q}_i$ of employed workers in this sample is representative for his neighbourhood. The social cost to being unemployed is an increasing function $v(\overline{q}_i)$ of this proportion. The payoffs $p_i(U)$ from choosing unemployment can then be written as[1]

$$p_i(U) = u(T) - v(\overline{q}_i) \tag{2}$$

[1] In equation (2) we assume additive separability. This means that we view the utility from benefits and the social costs as distinct elements which do not influence each other.

The social cost $v(\overline{q}_i)$ might have several interpretations. Lindbeck, Nyberg and Weibull (1999), who employ a similar payoff structure, suggest that the cost $v(\overline{q}_i)$ might reflect some social norm in favour of working, a norm whose strength depends on the number of agents adhering to it. Alternatively, if we view $v(\overline{q}_i)$ as a relative social cost, capturing the difference in socially derived payoffs when unemployed as compared to when employed, $v(\overline{q}_i)$ might represent some advantage in acting similarly to one's neighbours. Being the deviant can expose you to the resentment or distrust of others, but there are also more tangible rewards from acting in a manner similar to others. Being employed while having a network of employed neighbours might for instance provide you with more opportunities for finding a better paying job or with better ways of doing your current job. And if you are unemployed in a neighbourhood of unemployment, your chances of discovering better ways of exploiting the system of benefits might increase.

However social costs are construed, payoffs translate into actions in the following way. A revising worker at location *i* chooses employment if $p_i(E) > p_i(U)$, and unemployment if the opposite inequality holds. If $p_i(E) = p_i(U)$, the worker is indifferent and tosses a coin to select his employment status.

For given forms of the functions $u(.)$, $w(.)$ and $v(.)$ and a given value of the parameter *T*, we can derive the minimum proportion q_e^* of employed workers needed to induce a worker with education *e* to choose employment. In other words, there is some q_H^* such that a high education worker chooses employment if $\overline{q}_i > q_H^*$, and unemployment if $\overline{q}_i < q_H^*$. Similarly, there is some q_L^* such that a low education worker chooses employment if $\overline{q}_i > q_L^*$ and unemployment if $\overline{q}_i < q_L^*$. To add some further structure to the model, assume that $q_H^* < 0$, which means that a high education worker always chooses employment no matter how much or how little employment there is in his neighbourhood. This restriction eases analysis, by decreasing the number of states we have to consider. For low education workers, on the other hand, $q_L^* \in \langle 0,1 \rangle$, which implies that their choice of employment status does differ according to the employment situation of their neighbourhood.[2] Utilities and social costs fitting these restrictions are illustrated by the below figure.

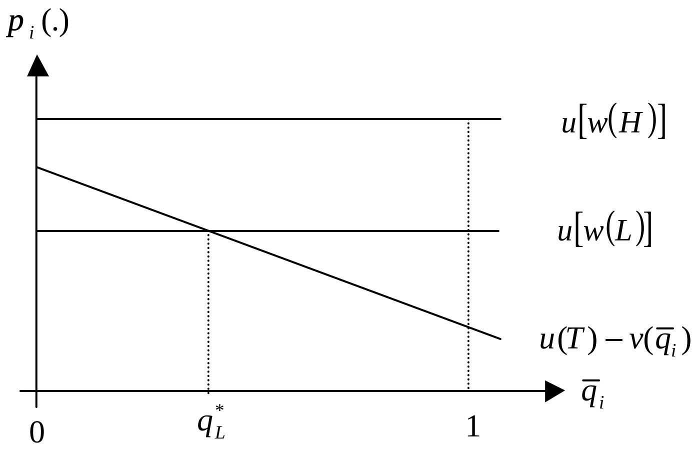


Figure 1: Restrictions on payoffs

Given the assumption that agents are immobile and segregated, we can represent the state of play in period *t* by a vector $\boldsymbol{m}^{\boldsymbol{t}}$, whose *i*th element $m_i^t \in \{E,U\}$ is the employment status of the agent at position *i* on the circle at time *t*. The state space $\overline{\mathbf{O}}$ consists of all state vectors $\boldsymbol{m}$ such that each element in $\boldsymbol{m}$ is either E or U.

$$\overline{\mathbf{O}} = \{\boldsymbol{m} : m_i \in \{E,U\}, \forall i \in [1,N]\} \tag{3}$$

For ease of exposition, let us name a few states. Denote by $\boldsymbol{m}^{\boldsymbol{EE}}$ the state in which everyone is employed, i.e.

$$\boldsymbol{m}^{\boldsymbol{EE}} \equiv \{\boldsymbol{m} : m_i = E, \forall i \in [1,N]\} \tag{4}$$

Similarly, let $\boldsymbol{m}^{\boldsymbol{EU}}$ represent the state in which only the high-education agents are employed, while the low-education agents are unemployed

[2] The set of which q_L^* is an element does not contain its boundaries, which means that low education workers are not indifferent if everyone in their neighbourhood is employed or unemployed. Including the boundaries would not alter the gist of the results that follow, but would make them significantly less tidy, as the limit sets in the boundary cases could be cycles containing a large number of states.

$$\boldsymbol{m}^{EU} \equiv \{\boldsymbol{m} : m_i = E, \forall i \in [1, N_1] \,\&\, m_i = U, \forall i \in [N_1 + 1, N]\} \quad (5)$$

Given the manner in which agents revise their employment status, the evolution of the state vector $\boldsymbol{m}^t$ constitutes a Markov chain on the state space $\overline{\mathbf{O}}$. For any given neighbourhood sample size *s*, let $\overline{\mathbf{P}}(s)$ be the transition matrix implied by the process of revision, where element *jk* of $\overline{\mathbf{P}}(s)$ is the probability of going from state *j* to state *k* from one period to the next. For any given *s*, we can then represent the process by a transition matrix $\overline{\mathbf{P}}(s)$ on a state space $\overline{\mathbf{O}}$, which we can sum up as $(\overline{\mathbf{O}}, \overline{\mathbf{P}}(s))$.

Long run behaviour in a segregated city

Our object of study is the evolution of play as agents repeatedly reconsider their employment status. The long run outcomes of this process, i.e. where we end up after the process of revisions has run for a large number of periods, is represented by the limit sets of the process. A limit set is a set of states which once reached, the process never leaves.[3] Even more strictly, an absorbing state is a limit set consisting of only a single state. In other words, once we have reached an absorbing state, we remain in that state in all later periods. A limit set that contains several states, is often referred to as a limit cycle.

For the above process, the following proposition captures the long run behaviour of agents:

PROPOSITION 1

Consider the process $(\overline{\mathbf{O}}, \overline{\mathbf{P}}(s))$. *For* N_1 *and* N_2 *sufficiently large:*

i) $\boldsymbol{m}^{EE}$ *is an absorbing state for all* $q_L^* \in \langle 0,1 \rangle$ *and all* $s \in [1, 2k]$.

ii) $\boldsymbol{m}^{EU}$ *is an absorbing state if and only if* $q_L^* \in \langle 0.5, 1 \rangle$ *and* $s > \frac{k}{q_L^*}$.

iii) There are no other limit sets for any $q_L^* \in \langle 0,1 \rangle$ *and* $s \in [1, 2k]$.

[3] Markov chains and limit sets are defined more rigorously in an appendix.

A formal proof of the proposition is presented in an appendix, as are the proofs of later propositions.

The first part of the proposition tells us that the state of full employment, $\boldsymbol{m}^{EE}$, is an absorbing state for all relevant values of q_L^* and *s*. The state of full employment is thus robust to variations in these parameters. The intuitive reason $\boldsymbol{m}^{EE}$ is an absorbing state in all these cases, is that a revising worker in this state draws a sample of only employed workers, and thus chooses to remain employed. Once we are in the state of full employment $\boldsymbol{m}^{EE}$, no worker ever alters his employment status, which means that we stay in $\boldsymbol{m}^{EE}$.

In contrast, the state of full unemployment in the low education group $\boldsymbol{m}^{EU}$ is only an absorbing state for a restricted range of values of q_L^* and *s*. Specifically, $q_L^* \in \langle 0.5,1 \rangle$ means that for a low-education worker the required number of unemployed neighbours that would make him choose unemployment is lower than the required number of employed neighbours that would make him choose employment. Moreover, the sample of workers cannot be too small, $q_L^* \in \langle 0.5,1 \rangle$ and $s > \frac{k}{q_L^*}$ imply that $s > k$, so workers must sample more than half their neighbourhood for unemployment to be a stable long-term outcome.

The reason $\boldsymbol{m}^{EU}$ is not an absorbing state when low education workers are more easily persuaded to choose employment than unemployment, is as follows. Imagine that a low education worker chooses employment if exactly half or more of his neighbours are employed, $q_L^* = 0.5$. Assume that there are at least k employed high education workers. In any given period there is a chance an unemployed low education worker living next to a high education worker is called upon to revise his employment status. If he samples his entire neighbourhood, $s = 2k$, he perceives a neighbourhood employment rate of 50% and thus chooses employment. If he samples less than his entire neighbourhood, $s < 2k$, there is still a chance that half or more than half his sample are employed, upon which he chooses employment. The same argument applies if in the next period the next unemployed worker on the circle revises his employment status, so there is a chance he chooses employment as well. And thus we can continue around the circle until all low education workers have chosen employment, and we have reached the state of full employment $\boldsymbol{m}^{EE}$. The state of full employment among those with a low level of education thus unravels as the workers at the edges of the unemployed segment switch to employment.

A similar argument tells us why neighbourhood samples must be of a certain size for $\boldsymbol{m}^{EU}$ to be an absorbing state. Imagine the smallest possible sample size, $s = 1$, and consider once more the unemployed low education worker living next to a high education worker. The sample drawn by this worker might consist of a high education employed worker, which would make him choose employment for any relevant value of q_L^* . The same is true for the next unemployed worker on the circle, and so on until we reach the state of full employment $\boldsymbol{m}^{EE}$. As the second part of proposition 1 tells us, the minimum sample size needed to prevent such an unraveling of the unemployed segment decreases as it gets harder to make low education workers choose employment.

The unemployed segment does not unravel from its edges in the above manner, if a low education worker chooses employment only if more than half his neighbours do $q_L^* > 0.5$, and if sample sizes are sufficiently large $s > \frac{k}{q_L^*}$. In that case, $\boldsymbol{m}^{EU}$ is an absorbing state. Note that this hinges on the size of the low education group being sufficiently large for the unemployed to sustain each other's choices. As the last part of the proposition establishes, $\boldsymbol{m}^{EU}$ and $\boldsymbol{m}^{EE}$ are in fact the only possible limit sets of the process. The reason is that if there are two low education workers living next to each other who differ in their employment status, then they have the same number of employed neighbours. If called upon to revise, at least one of them might therefore want to alter his status. Repeated revisions of this sort can bring us to $\boldsymbol{m}^{EE}$ or $\boldsymbol{m}^{EU}$.

For certain values of the parameters of the model, we thus have two absorbing states, whereas for other values we have only one. In particular, a notable implication of proposition 1 is that by reducing the sample size of the agents in the model, we can reduce the number of absorbing states. Reductions of sample size can thus be viewed as a means of selection between absorbing states in the present model. As small sample sizes might be taken to represent imitative behaviour of agents, and larger sample sizes more rational best reply deliberations, a reasonable interpretation of this result is that less rationality entails a more unique prediction of long run outcomes.

A model of neighbourhood effects and mobility

The above assumption of full segregation and immobile agents is rather extreme, yet serves as a useful introductory case. We now abandon this assumption, allowing any initial configuration of residences for high and low education agents, and affording agents the opportunity to switch locations. Workers thus sometimes revise their employment status, and sometimes their place of residence. The choice of employment status takes place much as in the above model, whereas for the choice of residence a range of different rules that might govern mobility are proposed.

As in the preceding model, there are N agents occupying as many locations on a circle, N_1 of whom have a high level of education (*H*) and the remainder a low level of education (*L*). The idea that workers sometimes revise their employment status and sometimes their place of residence can be modelled in a variety of ways, yet we choose the following simple variant. In each period there is a random draw, which with probability p puts us in a situation mode (*S*) and with probability $(1-p)$ puts us in a residence mode (*R*). The size of p might then reflect the frequency with which choices of employment are made relative to choices of mobility.

In a period in which we are in a situation mode, each worker has a probability $d \in \langle 0,1 \rangle$ of being selected to revise his employment status. The choice between employment (*E*) and unemployment (*U*) is then made the same way as in the preceding model, with one modification. Having made the above point about sample sizes, we now abandon this element and let $s = 2k$. A worker revising his employment status now observes the proportion of employment in his entire neighbourhood (i.e. across all $2k$ neighbours), and if we denote by q_i the proportion employed in the neighbourhood of the worker currently occupying location *i*, the payoffs to this worker from unemployment become

$$p_i(U) = u(T) - v(q_i) \tag{6}$$

A revising worker compares these payoffs with the payoffs from employment given by equation (1), and makes the choice which maximizes his payoffs, tossing a coin if indifferent. The restriction on payoffs imposed earlier remain in place, so a high education worker always chooses employment, whereas a low education worker is influenced by the level of employment in his neighbourhood. In the situation mode, no worker changes his place of residence.

In a period where the random draw puts us in a residence mode, two workers are drawn at random to consider switching locations with each other. The basic idea is that a move is made if both find the residence of the other more desirable than their own, or if one of the two finds the residence of the other more desirable and has the means to compensate the other for making the switch. In this respect, the model resembles the residential segregation model of Schelling (1971). In the present model, there is a variety of ways in which workers can assess the desirability of locations. In the next section, we discuss a range of these. The different ways of assessing locations are captured by rules of mobility, stating that two workers exchange locations if they and their neighbourhoods have certain characteristics. If the two do not have the required characterstics, the workers remain in their current locations. In the residence mode, no worker revises his employment status.

With mobile agents, we can represent the state of play at time t by a matrix $\mathbf{M}^t$, whose ith column $\boldsymbol{m}_i^t \in \{E, U\} \times \{H, L\}$ captures the employment status and the level of education of the agent at location i on the circle at time t. The state space $\mathbf{O}$ consists of all state matrices $\mathbf{M}$ such that each column $\boldsymbol{m}_i$ of $\mathbf{M}$ has E or U in its first row, and H or L in its second row.

$$\mathbf{O} = \{\mathbf{M} : \boldsymbol{m}_i \in \{E, U\} \times \{H, L\}, \forall i \in [1, N]\} \tag{7}$$

For expositional convenience, we name a few sets of states. Denote by $\mathbf{M}^{\mathbf{EE}}$ the set of states in which all workers are employed.

$$\mathbf{M}^{\mathbf{EE}} \equiv \{\mathbf{M} : \boldsymbol{m}_i \in \{E\} \times \{H, L\} \forall i \in [1, N]\} \tag{8}$$

Moreover, let $\mathbf{M}^{\mathbf{EESEG}}$ be the set of states in which every worker is employed, and workers are completely segregated according to their level of education. Note that $\mathbf{M}^{\mathbf{EESEG}}$ is a subset of $\mathbf{M}^{\mathbf{EE}}$. If A is a set of locations on the circle, and **?** the set of all such sets A that contain N_1 adjacent locations on the circle only, then we can define $\mathbf{M}^{\mathbf{EESEG}}$ as follows.

$$\mathbf{M}^{\mathbf{EESEG}} \equiv \{\mathbf{M} : \boldsymbol{m}_i = (E, H) \forall i \in A \,\&\, M_i = (E, L) \forall i \notin A | A \in \mathbf{?}\} \tag{9}$$

Similarly, let $\mathbf{M}^{\mathbf{EU}}$ denote the set of states in which all workers with a high level of education are employed, and all workers with a low level of education are unemployed.

$$\mathbf{M}^{\mathbf{EU}} \equiv \{\mathbf{M} : \boldsymbol{m}_i \in \{(E, H), (U, L)\} \forall i \in [1, N]\} \tag{10}$$

The set of states in which all high education workers are employed, all low education workers unemployed, and workers are completely segregated according to levels of education, we call $\mathbf{M}^{\mathbf{EUSEG}}$. It follows that $\mathbf{M}^{\mathbf{EUSEG}}$ is a subset of $\mathbf{M}^{\mathbf{EU}}$.

$$\mathbf{M}^{\mathbf{EUSEG}} \equiv \{\mathbf{M} : \boldsymbol{m}_i = (E, H) \forall i \in A \,\&\, M_i = (U, L) \forall i \notin A | A \in \mathbf{?}\} \tag{11}$$

In contrast, let $\mathbf{M}^{\mathbf{INT}}$ be the set of states of perfect integration, where every other worker has a high level of education and is employed, and the locations in-between are occupied by low education workers who are unemployed.

$$\mathbf{M}^{\mathbf{INT}} \equiv \left\{ \begin{array}{l} \mathbf{M} : (\boldsymbol{m}_i = \{E, H\}, \forall i \text{ odd} \,\&\, \boldsymbol{m}_i = \{U, L\}, \forall i \text{ even}) \\ \text{or } (\boldsymbol{m}_i = \{E, H\}, \forall i \text{ even} \,\&\, \boldsymbol{m}_i = \{U, L\}, \forall i \text{ odd}) | N \text{ even} \end{array} \right\} \tag{12}$$

This set exists only when there are equally many agents with each type of education, which implies that the total number of agents must be even. Nevertheless, when these states do exist, they are a candidate to consider when calculating limit sets. Note that $\mathbf{M}^{\mathbf{INT}}$ is a subset of $\mathbf{M}^{\mathbf{EU}}$.

Rules of mobility

To know the properties of the process that governs the evolution of the state of play matrix, we must specify how decisions to move are made. To this end, we define a number of rules of mobility, each of which captures a different motive for moving. In the context of the above model, a worker revising his place of residence basically has two characteristics by which to evaluate how attractive a neighbourhood is to live in. One is the level of employment in the neighbourhood (or conversely the level of unemployment), the other is the proportion of high education workers in the neighbourhood (or conversely the proportion of low education workers). For any state $\mathbf{M}^{\mathbf{t}} \in \mathbf{O}$ and any neighbourhood size k, let q_i^t denote the proportion of employed workers in the neighbourhood of the worker residing at i, and let h_i^t denote the proportion of high education workers in that neighbourhood. A revising worker can use one of these characteristics, or a combination of both, to calculate whether another location is better than his own.

Even if a worker desires to move to another location, the worker currently occupying that location might be unwilling to switch. In this case, the worker desiring to switch might compensate the other party, if he has the means to do so. Whether a switch is made thus depends on characteristics of the revising workers. One assumption is that employed workers have the means to compensate unemployed workers, and high education workers have the means to induce a switch with low education workers. The below rules of mobility capture variations of these ideas, depending on the neighbourhood characteristics by which workers evaluate the attractiveness of locations.

Let us start with the case where workers evaluate locations solely by the employment rate of their neighbourhoods. Here we make the basic assumption that workers are upwardly mobile, and thus desire to move to a location with an employment rate no lower than their current location. We also make the assumption that when one worker wants to move but not another, an employed worker can compensate an unemployed worker in order to induce a switch. For the mobility rules defined below, it would not make that much of a difference if we added the possibility that high education workers can compensate low education workers. Let us define three mobility rules based on upward mobility in terms of employment, starting with the one that requires the least in terms of an improvement in employment.

Mobility rule r^1 states that workers want to move to locations that have at least as many employed neighbours as their current location, where the employment rate of a neighbourhood is gauged by its level before a move is made. In other words, r^1 supposes a limited amount of rationality in workers, since a location that is as good as your current one before a move is made, might actually prove worse after the move is made.

DEFINITION 1:

Suppose that at time t we are in state $\mathbf{M^t} \in \mathbf{O}$, *and that two agents at locations* $a, b \in [1, N]$ *are drawn to revise their locations.*

Then by rule r^1 $\boldsymbol{m_a^{t+1}} = \boldsymbol{m_b^t}$ *and* $\boldsymbol{m_b^{t+1}} = \boldsymbol{m_a^t}$

If i) $q_b^t \geq q_a^t$, *ii)* $\boldsymbol{m_a^t} \in \{E\} \times \{H, L\}$ *and iii)* $\boldsymbol{m_b^t} = (U, L)$.

The definition of rule r^1 thus says that if two workers are drawn to revise their locations, they switch if one is employed and the other unemployed, and the latter is currently in a location with at least as many employed neighbours as the former. Note that this and the following definitions describe only the columns in which $\mathbf{M^{t+1}}$ differs from $\mathbf{M^t}$, i.e. the locations that are affected by workers revising their locations, for all locations i unaffected by such revisions $\boldsymbol{m_i^{t+1}} = \boldsymbol{m_i^t}$, as implicitly specified by the general description of the residence mode.

The next rule r^2 holds agents to be slightly more rational, comparing the rate of employment in their current location to what would be the rate of employment in a prospective location after they had moved there. As in the preceding rule, workers desire to move to locations with at least as many employed neighbours, and two workers switch locations if one is employed and desires to move and the other is unemployed.

DEFINITION 2:

Suppose that at time t we are in state $\mathbf{M^t} \in \mathbf{O}$, *and that two agents at locations* $a, b \in [1, N]$ *are drawn to revise their locations.*

Then by rule r^2 $\boldsymbol{m_a^{t+1}} = \boldsymbol{m_b^t}$ *and* $\boldsymbol{m_b^{t+1}} = \boldsymbol{m_a^t}$

If thereby i) $q_b^{t+1} \geq q_a^t$, *ii)* $\boldsymbol{m_a^t} \in \{E\} \times \{H, L\}$ *and iii)* $\boldsymbol{m_b^t} = (U, L)$.

A third rule r^3 states that workers want to move to locations where there are strictly more employed neighbours. Whether workers gauge employment by its level before or after a move is made, does not matter that much here, but we assume that they use the after-move level. As the below definition explains, an employed and an unemployed worker switch locations if the employed worker so desires.

DEFINITION 3:

Suppose that at time t we are in state $\mathbf{M^t} \in \mathbf{O}$, *and that two agents at locations* $a, b \in [1, N]$ *are drawn to revise their locations.*

Then by rule r^3 $\boldsymbol{m_a^{t+1}} = \boldsymbol{m_b^t}$ *and* $\boldsymbol{m_b^{t+1}} = \boldsymbol{m_a^t}$

If thereby i) $q_b^{t+1} > q_a^t$, *ii)* $\boldsymbol{m_a^t} \in \{E\} \times \{H, L\}$ *and iii)* $\boldsymbol{m_b^t} = (U, L)$.

Of course, workers need not be upwardly mobile. Frank (1985) suggests that it can be better to be a large fish in a small pond than vice versa. Let us include a rule reflecting this idea, where employed workers crave the status of being employed in a neighbourhood where few others are. Rule r^4 states that an employed and an unemployed worker switch locations if the employed worker gets strictly less employed neighbours this way. This rule is then in a sense the opposite of the preceding rule r^3.

DEFINITION 4:

Suppose that at time t we are in state $\mathbf{M^t} \in \mathbf{O}$ *, and that two agents at locations* $a,b \in [1,N]$ *are drawn to revise their locations.*

Then by rule r^4 $\boldsymbol{m}_{\boldsymbol{a}}^{\boldsymbol{t+1}} = \boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{t}}$ *and* $\boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{t+1}} = \boldsymbol{m}_{\boldsymbol{a}}^{\boldsymbol{t}}$

If thereby i) $q_b^{t+1} < q_a^t$*, ii)* $\boldsymbol{m}_{\boldsymbol{a}}^{\boldsymbol{t}} \in \{E\}\times\{H,L\}$ *and iii)* $\boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{t}} = (U,L)$.

Having introduced a few rules based solely on neighbourhood levels of employment, let us now turn to rules that rely only on proportions of high and low education workers. In what follows, we will adopt the basic idea of Schelling (1971) that agents gravitate towards neighbourhoods that hold a greater number of agents of their own type, where the type of an agent is his level of education. High education workers thus prefer to live in neighbourhoods richer in high education workers, and low education workers prefer neighbourhoods poorer in high education workers. For the mobility rules to come, we will assume that a high education and a low education worker switch positions when the former so prefers, in order to keep definitions minimalistic. However, we could equivalently have assumed that a switch is made when both find it beneficial. Moreover, adding the possibility that low education employed workers compensate low education unemployed workers for making a switch the former finds desirable, would not significantly affect the results.

The first rule based on neighbourhood composition in terms of education, r^5, states that workers desire to move to locations where the number of neighbours currently sharing their level of education is at least as high as in their current locations. In the rationality awarded agents, this rule thus resembles r^1, since agents assess locations by their neighbourhood composition before a move is made.

DEFINITION 5:

Suppose that at time t we are in state $\mathbf{M^t} \in \mathbf{O}$ *, and that two agents at locations* $a,b \in [1,N]$ *are drawn to revise their locations.*

Then by rule r^5 $\boldsymbol{m}_{\boldsymbol{a}}^{\boldsymbol{t+1}} = \boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{t}}$ *and* $\boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{t+1}} = \boldsymbol{m}_{\boldsymbol{a}}^{\boldsymbol{t}}$

If i) $h_b^t \geq h_a^t$*, ii)* $\boldsymbol{m}_{\boldsymbol{a}}^{\boldsymbol{t}} \in \{E,U\}\times\{H\}$ *and iii)* $\boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{t}} \in \{E,U\}\times\{L\}$.

The next rule r^6 presupposes a higher degree of rationality, in stating that workers prefer to leave their current location if a prospective location holds more neighbours sharing their level of education, after the move is made.

DEFINITION 6:

Suppose that at time t we are in state $\mathbf{M^t} \in \mathbf{O}$ *, and that two agents at locations* $a, b \in [1, N]$ *are drawn to revise their locations.*

Then by rule r^6 $\boldsymbol{m_a^{t+1}} = \boldsymbol{m_b^t}$ *and* $\boldsymbol{m_b^{t+1}} = \boldsymbol{m_a^t}$

If thereby i) $h_b^{t+1} \geq h_a^t$*, ii)* $\boldsymbol{m_a^t} \in \{E, U\} \times \{H\}$ *and iii)* $\boldsymbol{m_b^t} \in \{E, U\} \times \{L\}$.

A third rule based on neighbourhood composition in terms of education, r^7, states that workers move only to locations with strictly more of their own type. The definition assumes neighbourhood compositions are compared after a move is made, but comparisons being made before the move would not affect the results to come.

DEFINITION 7:

Suppose that at time t we are in state $\mathbf{M^t} \in \mathbf{O}$ *, and that two agents at locations* $a, b \in [1, N]$ *are drawn to revise their locations.*

Then by rule r^7 $\boldsymbol{m_a^{t+1}} = \boldsymbol{m_b^t}$ *and* $\boldsymbol{m_b^{t+1}} = \boldsymbol{m_a^t}$

If thereby i) $h_b^{t+1} > h_a^t$*, ii)* $\boldsymbol{m_a^t} \in \{E, U\} \times \{H\}$ *and iii)* $\boldsymbol{m_b^t} \in \{E, U\} \times \{L\}$.

Once more, one might entertain the possibility that a worker would rather be a big fish in a small pond, than blend in with their neighbours. In the present context, this would mean that workers prefer locations poorer in neighbours sharing their level of education. Rule r^8 captures a variant of this idea, where locations with strictly fewer neighbours of their own type are preferred by workers.

DEFINITION 8:

Suppose that at time t we are in state $\mathbf{M^t} \in \mathbf{O}$, *and that two agents at locations* $a, b \in [1, N]$ *are drawn to revise their locations.*

Then by rule r^8 $\boldsymbol{m_a^{t+1}} = \boldsymbol{m_b^t}$ *and* $\boldsymbol{m_b^{t+1}} = \boldsymbol{m_a^t}$

If thereby i) $h_b^{t+1} < h_a^t$, *ii)* $\boldsymbol{m_a^t} \in \{E, U\} \times \{H\}$ *and iii)* $\boldsymbol{m_b^t} \in \{E, U\} \times \{L\}$.

Thw two characteristics of a neighbourhood, the rate of employment and the proportion of high education workers, can also be combined in a variety of ways, to gauge how attractive locations are. Let us explore a few simple rules that combine the two. The first two of these rules are lexicographic orderings according to the two characteristics; workers prefer a location better to another according to a first characteristic, but if two locations are equally good according to the first characteristic, then workers prefer the location that is better according to the second characteristic. In this manner, rule r^9 states that an employed and an unemployed worker switch locations if the former worker gets a strictly higher number of employed neighbours that way; if he gets as many employed neighbours, a switch is made if he is a high education worker who gets strictly more high education neighbours if he moves.

DEFINITION 9:

Suppose that at time t we are in state $\mathbf{M^t} \in \mathbf{O}$, *and that two agents at locations* $a, b \in [1, N]$ *are drawn to revise their locations.*

Then by rule r^9 $\boldsymbol{m_a^{t+1}} = \boldsymbol{m_b^t}$ *and* $\boldsymbol{m_b^{t+1}} = \boldsymbol{m_a^t}$

If thereby i) $q_b^{t+1} > q_a^t$, *ii)* $\boldsymbol{m_a^t} \in \{E\} \times \{H, L\}$ *and iii)* $\boldsymbol{m_b^t} = (U, L)$

or i) $q_b^{t+1} = q_a^t$ *and* $h_b^{t+1} > h_a^t$, *ii)* $\boldsymbol{m_a^t} \in \{E, U\} \times \{H\}$ *and iii)* $\boldsymbol{m_b^t} \in \{E, U\} \times \{L\}$.

Rule r^{10} is just the reverse, a high and low education worker switch locations if the former gets strictly more neighbours of his own type; if he gets at least as many, a switch is made if the low education worker is unemployed and the high education worker gets at least as many employed neighbours.

DEFINITION 10:

Suppose that at time t we are in state $\mathbf{M^t} \in \mathbf{O}$, *and that two agents at locations* $a, b \in [1, N]$ *are drawn to revise their locations.*

Then by rule r^{10} $\boldsymbol{m}_{\boldsymbol{a}}^{\boldsymbol{t+1}} = \boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{t}}$ *and* $\boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{t+1}} = \boldsymbol{m}_{\boldsymbol{a}}^{\boldsymbol{t}}$

If thereby i) $h_b^{t+1} > h_a^t$, *ii)* $\boldsymbol{m}_{\boldsymbol{a}}^{\boldsymbol{t}} \in \{E, U\} \times \{H\}$ *and iii)* $\boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{t}} \in \{E, U\} \times \{L\}$

or i) $h_b^{t+1} = h_a^t$ *and* $q_b^{t+1} > q_a^t$, *ii)* $\boldsymbol{m}_{\boldsymbol{a}}^{\boldsymbol{t}} \in \{E\} \times \{H, L\}$ *and iii)* $\boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{t}} = (U, L)$.

The final rule r^{11} does not rank characteristics, but states that a move is made whenever an employed workers can get strictly more employed neighbours by switching places with an unemployed worker, and whenever a high education worker can get strictly more neighbours with a high level of education by switching places with a low education worker.

DEFINITION 11:

Suppose that at time t we are in state $\mathbf{M^t} \in \mathbf{O}$, *and that two agents at locations* $a, b \in [1, N]$ *are drawn to revise their locations.*

Then by rule r^{11} $\boldsymbol{m}_{\boldsymbol{a}}^{\boldsymbol{t+1}} = \boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{t}}$ *and* $\boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{t+1}} = \boldsymbol{m}_{\boldsymbol{a}}^{\boldsymbol{t}}$

If thereby i) $q_b^{t+1} > q_a^t$, *ii)* $\boldsymbol{m}_{\boldsymbol{a}}^{\boldsymbol{t}} \in \{E\} \times \{H, L\}$ *and iii)* $\boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{t}} = (U, L)$

or i) $h_b^{t+1} > h_a^t$, *ii)* $\boldsymbol{m}_{\boldsymbol{a}}^{\boldsymbol{t}} \in \{E, U\} \times \{H\}$ *and iii)* $\boldsymbol{m}_{\boldsymbol{b}}^{\boldsymbol{t}} \in \{E, U\} \times \{L\}$.

Denote by $\mathbf{T}$ the set containing all eleven rules of mobility.

$$\mathbf{T} = \{r^j : j \in \{1, \ldots, 11\}\} \tag{13}$$

Given the way in which agents revise their employment status and place of residence, for any of the mobility rules $r^j \in \mathbf{T}$, the evolution of the state matrix $\mathbf{M^t}$ constitutes a Markov chain on the state space $\mathbf{O}$. If we denote by $\mathbf{P}(r^j)$ the transition matrix of the process when rule r^j is in place, we can sum up the process as $(\mathbf{O}, \mathbf{P}(r^j))$.

Long run behaviour when agents are mobile

For each of the mobility rules defined above, the limit sets can be computed. For each rule, there can be several limit sets, depending on the value of the parameter q_L^*. Rules that are stricter in their requirements for mobility, typically have more limit sets. Limit sets for a specific few of the above rules are explored below, but let us start by examining some regularities across mobility rules.

PROPOSITION 2

Consider the process $(\mathbf{O},\mathbf{P}(r^j))$. *For* N_1 *and* N_2 *sufficiently large:*

i) A state $\mathbf{M}$ *is contained in a limit set for all rules* $r^j \in \mathbf{T}-\{r^8\}$, *if and only if* $\mathbf{M}\in \mathbf{M}^{\mathbf{EESEG}}$.

ii) No state $\mathbf{M}\in \mathbf{O}$ *is contained in a limit set for all rules* $r^j \in \mathbf{T}$.

The first part of the proposition captures the fact that all states in the set $\mathbf{M}^{\mathbf{EESEG}}$, i.e. states of full employment and total spatial segregation according to education, are absorbing states or contained in a limit cycle for every mobility rule defined above except r^8. States of this kind are thus remarkably robust to variations in motives of mobility, in fact more so than the states of any other set. However, no set of states is contained in a limit set for all the previously defined mobility rules, as the second part of the proposition posits. Even for states in $\mathbf{M}^{\mathbf{EESEG}}$, there are thus bounds to robustness.

The intuitive reasons why states of full employment and full segregation remain in place almost whatever motive workers have for moving, are as follows. If everyone is employed, no worker has any unemployed neighbours, and thus no worker chooses to be unemployed. The local employment norm is everywhere too strong for unemployment to be an attractive option. No unemployed workers also means that there is no available location for an employed worker to move to, so no moves are made on the basis of neighbourhood employment rates.

With full segregation, the high education workers already occupy the locations with the most high education neighbours, and they therefore cannot gain more neighbours of their own type by switching locations with low education workers. They could get as many neighbours of their own type by moving, but since larger segments of high education workers are at least as attractive as smaller ones, there is always a chance that we return to a state of full segregation. The states of full employment and full segregation thus cannot be forever abandoned if moves are made on the basis of employment, or if workers prefer to live with their own kind. If, on the other hand, workers prefer to live with the other kind, they will move away from concentrations of their own kind and not return, in which case a fully segregated state can be forever abandoned.

In more technical terms, the reason states in $\mathbf{M}^{\mathbf{EESEG}}$ are robust to all rules of mobility but one, can be explained in the following way. First, notice that when all players are employed, employment is the optimal choice for any worker drawn to revise his employment status. If we are in a state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$, no worker thus ever changes his employment status, which means that in all later periods, we remain within the set of states where everyone is employed $\mathbf{M}^{\mathbf{EE}}$. Second, for any of the above mobility rules, save rule r^8, either no location switches are possible by which we go from a state in $\mathbf{M}^{\mathbf{EESEG}}$ to a state unsegregated according to education, or if such switches are possible there exists some series of switches which brings us back to a segregated state. For the four mobility rules based solely on neighbourhood employment rates, r^1, r^2, r^3 and r^4, this is a fairly trivial matter, since according to these rules one agent must be unemployed for a location switch to occur. As there is no unemployment in a state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$, there is thus no possibility of a switch happening, and each state in the set is thus an absorbing state.

For the first two rules based on education type, r^5 and r^6, location switches are possible in any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$. Consider the following figure, where there are twelve locations, and six workers of each type forming contiguous segments.

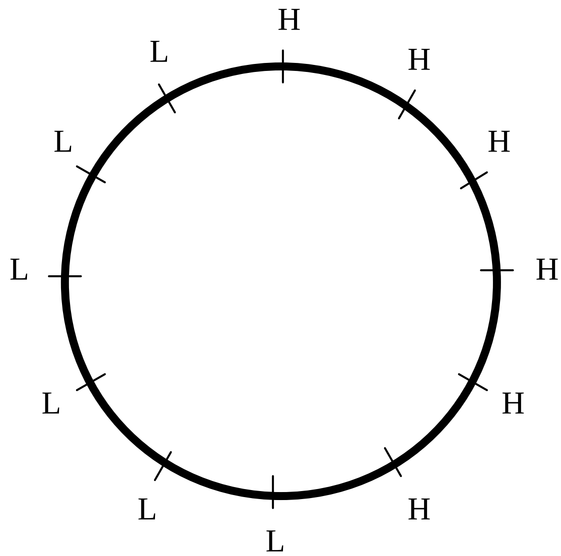


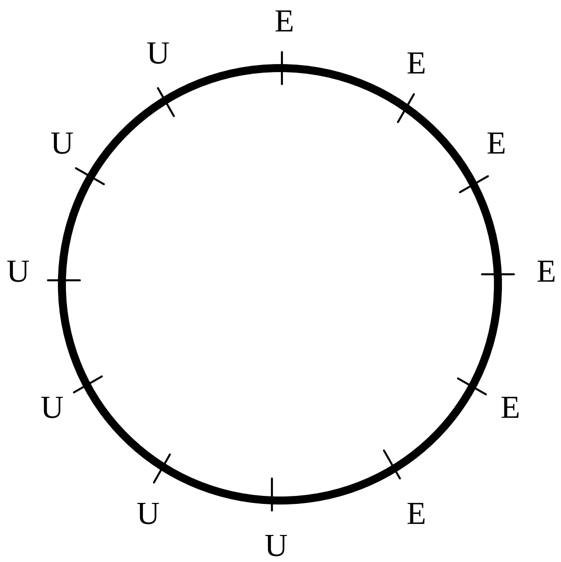
Figure 2. Illustration of a state of segregation.

From the figure, we see that a high education agent at the edge of the high education segment has as many high education neighbours as his closest low education neighbour. Under rule r^5, this implies that two workers of this kind would exchange locations if called upon to consider this option. This particular rule thus allows us to leave a state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$ for one without complete segregation. One can show, however, that from any state that is not segregated, one can reach any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$. Loosely, the reason why this happens is that any stray high education worker finds more high education neighbours in a contiguous high education segment than elsewhere. This means that the states in the set $\mathbf{M}^{\mathbf{EESEG}}$ must be part of a limit set under rule r^5. Under rule r^6, adjacent high and low education workers in the above figure would not exchange locations, as they would get fewer neighbours of their own type after such an exchange. However, a high education worker at one edge of the high education segment could exchange locations with a low education worker at the other edge of that segment. Thus, location switches can rotate the high education segment around the circle, which implies that under rule r^6, the states $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$ form a limit cycle.

As figure 2 reveals, there are no locations to which a high education worker can move and get strictly more high education neighbours. In a state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$, no moves are thus possible under rule r^7, which makes each state in the set an absorbing state.

Rules r^9, r^{10} and r^{11} just combine strict requirements of employment and high education neighbours in various ways, and thus do not allow any location switches, making any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$ an absorbing state under any of these rules. In sum, then, for rules r^1 through r^7 and r^9 through r^{11}, any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$ is contained in a limit set.

The reason why no other state is contained in a limit set for all these ten rules, can be explained in two steps. First, we can show that for rule r^1, only states of full employment $\mathbf{M} \in \mathbf{M}^{\mathbf{EE}}$ are contained in limit sets. Consider the following figure in which the Ls and Hs of figure 2 have been replaced by Us and Es, respectively.



Figur 3. Illustration of a state containing employment and unemployment.

For similar reasons as in the above discussion of rule r^5, through location switches under rule r^1, we can reach a state in which there is total segregation according to employment status. Note that an employed worker at the top of the circle would now want to change locations with the unemployed worker to his left. If this happens, the next employed worker to the right on the circle would also want to switch positions with the unemployed worker. Thus we can continue until the unemployed worker has only employed neighbours, and chooses employment if called upon to revise his employment status. Every unemployed worker can be brought into the employed fold in this manner, and made to choose employment. Once everyone is employed, noone wants to switch back to unemployment. For the rule r^1, a state is contained in a limit set only if $\mathbf{M} \in \mathbf{M}^{\mathbf{EE}}$.

Second, under rule r^6, only states segregated according to education are contained in limit sets. Through a slightly more complicated argument than in the above case of rule r^5, one can show that under r^6 any state in which there is incomplete segregation according to education can be transformed into one of complete segregation through a series of location switches. Again the main reason is that high education workers prefer to move to locations where the concentration of high education neighbours is greater, which it is in contiguous segments. Once a state of segregation is reached, r^6 does not permit segregation to be abandoned. In sum, then, since under r^1 only states of employment are contained in limit sets, and under r^6 only states of segregation according to education are contained in limit sets, no state $\mathbf{M} \notin \mathbf{M}^{\mathbf{EESEG}}$ can be contained in a limit set for all rules r^1 through r^7 and r^9 through r^{11}.

Finally, no state $\mathbf{M} \in \mathbf{O}$ is contained in a limit set across all mobility rules $r^j \in \mathbf{T}$, due to the fact that no state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$ is contained in a limit set under rule r^8. From figure 2, it is obvious that any high education agent would want to exchange locations with any low education agent under rule r^8, since the former agent would thereby reduce his number of high education neighbours. However, any high education agent thus separated from a high education segment would not want to rejoin that segment, since his number of high education neighbours would then rise. In a sense, high education workers want to avoid congregations of their own kind. Thus any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$ can be left for a state outside that set, but since the final switch that would lead us back to a state in that set from any other set cannot be made, the states $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$ cannot be part of any limit set.

As noted earlier, though states in $\mathbf{M}^{\mathbf{EESEG}}$ are particularly robust to variations in mobility rules, specific mobilty rules permit a variety of limit sets, sometimes including states that are not in $\mathbf{M}^{\mathbf{EESEG}}$. The mobility rules requiring prospective neighbourhoods to be weakly better are what drives the restriction of limit sets seen in proposition 2. A greater range of limit sets exists under the rules that require a prospective neighbourhood to be strictly better for a worker to want to move there. A closer examination of long run outcomes under these rules is therefore warranted. A full characterization of limit sets is difficult for the rules in question, yet the following propositions adequately capture the variety in possible long run outcomes.

PROPOSITION 3

Consider the process $\left(\mathbf{O}, \mathbf{P}(r^j)\right)$.

For $j = 3$, *and* N_1 *and* N_2 *sufficiently large:*

i) Any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EE}}$ *is an absorbing state.*

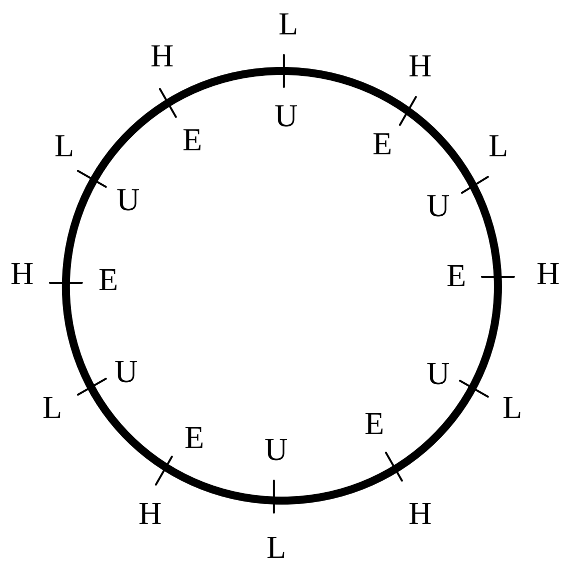
ii) Any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EUSEG}}$ *is an absorbing state if and only if* $q_L^* \in \langle 0.5, 1 \rangle$.

iii) Any state $\mathbf{M} \in \mathbf{M}^{\mathbf{INT}}$ *is an absorbing state if* $q_L^* \in \langle 0.5, 1 \rangle$ *and k is even.*

This proposition addresses rule r^3, by which employed workers move to locations that are strictly better in terms of employed neighbours. As the first part of the proposition indicates, any state of full employment, regardless of the spatial location of high and low education agents, is an absorbing state. The reasons for this are that when everyone is employed, no worker ever chooses unemployment, and since there are no unemployed workers to switch locations with, no two workers ever exchange locations.

According to the second part of the proposition, any state in which there is total segregation according to education and every low education worker is unemployed, is an absorbing state provided low education workers are more easily persuaded to choose unemployment than employment, $q_L^* > 0.5$. As figure 3 tells us, no employed worker in such a state would get more employed neighbours by switching places with an unemployed worker. And the proportion of employed neighbours for any low education worker in such a state is one half or less, which implies that if $q_L^* > 0.5$, all low education workers stay unemployed. If, on the other hand, $q_L^* \leq 0.5$ the low education workers at the edges of the unemployed segment could switch to employment, and a succession of such switches would make the unemployed segment crumble from its edges.

The third part of the proposition claims that a state in which all low education workers are unemployed, and there is full integration in the sense that employed high education workers and unemployed low education workers occupy alternate locations on the circle, is an absorbing state provided $q_L^* > 0.5$ and the number of neighbours to each side k is an even number. To appreciate why this is, consider the following figure.



Figur 4. Illustration of integrated state

Imagine that $k = 2$, so each player has four neighbours, two to each side. Exactly half the neighbours of every worker are then employed. Thus if $q_L^* > 0.5$, low education workers remain unemployed. Every employed worker has two employed neighbours, and would get two or less by switching locations with an unemployed worker, so no location switches will occur. This line of reasoning extends to any case in which k is even. If, on the other hand, k were odd, less than half the neighbours of an employed person would be employed, whereas half or more than half his neighbours would be employed if he switched locations with an unemployed worker. For k odd, then, a state of total integration would crumble.

Very similar results to those of proposition 3 can be derived when mobility decisions are motivated by neighbourhood levels of education, or by a combination of employment rates and levels of education.

PROPOSITION 4

Consider the process $(\mathbf{O},\mathbf{P}(r^j))$.

For $j \in \{7,9,10,11\}$*, and* N_1 *and* N_2 *sufficiently large:*

i) Any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$ *is an absorbing state.*

ii) Any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EUSEG}}$ *is an absorbing state if and only if* $q_L^* \in \langle 0.5,1 \rangle$.

iii) Any state $\mathbf{M} \in \mathbf{M}^{\mathbf{INT}}$ *is an absorbing state if* $q_L^* \in \langle 0.5,1 \rangle$ *and k is even.*

The second and third parts of proposition 4 mirror those of the preceding proposition. When we allow for the fact that here mobility is (also) based on levels of education, the reasons why segregated and integrated states of full unemployment among those with a low level of education are absorbing states, are very similar to those recounted in the above justification of proposition 3. Let us instead focus on where mobility based on education levels produces a different result from mobility based on education, as captured by the first part of the two propositions. When mobility happens due to differences in levels of education, or such differences provide an added reason to move, all states of full employment need not be contained in limit sets. Intuitively, this can be explained by imagining a state of full employment in which all high education workers but one form a contiguous segment. The high education worker isolated from his peers would then gain high education neighbours by switching locations with a low education worker at the edge of the contiguous segment of high education workers. Once such a move is made, we reach a state of full segregation, in which no high education worker can get more high education neighbours through further moves. In other words, while segregated states of full employment are absorbing states, not all non-segregated states of full employment are absorbing states.

In conjunction, propositions 3 and 4 reveal that if moves are made to locations that are strictly better on some characteristic, a wide range of long run outcomes can be observed. Both states of full employment and of full unemployment among those with a low level of education can be absorbing states, as can both fully segregated and fully integrated states. Moving processes of this kind thus permit a wide variety of worlds to exist in the long run. However, variety is greater where decisions to move are based solely on employment levels, than where these decisions feature a comparison of neighbourhood education levels. In a sense, then, mobility based on education generates a bias towards more segregated long run outcomes.

Concluding remarks

The results obtained in this paper show that complete segregation and full employment is a long run outcome robust to variations in sample size, payoffs to workers and mobility motives. Though sample size was studied only in the initial model with fixed locations, where reduced samples were heralded as a means of selection among limit sets, a similar point could be made if employment decisions in the models including mobility were based on limited samples. However, then we would also have to tackle the question of whether only employment decisions should reflect limited samples, or if samples ought also to be assumed limited in mobility decisions. If so, more moves would be permitted under each of the above mobility rules, which on the one hand could mean that segregated states would be easier to reach, while on the other hand segregated states could also be easier to leave. Though limited samples might in this context reduce the number of limit sets, the number of states included in each limit set might rise, which makes the selection effect more dubious.

As noted initially, the purpose of the above framework is to study the joint impact of employment and mobility decisions in an urban context. A few of the assumptions on which the analysis is based are of course highly stylized, in particular the idea that workers inhabit a circular space, and that their payoffs from employment and unemployment are restricted they way they are. A more general model would let workers inhabit a more general social grid, and one way of analyzing such a model would be by means of the concept of contagion thresholds introduced by Morris (2000). Assuming greater variation in the characteristics that determine the payoffs to individuals would make the analysis richer, but also more complex as long run outcomes would vary according to how the population is distributed across these characteristics. Finally, as matters of education or productivity are influenced by the choices, opportunities and social situation of workers, making these characteristics endogenous would also constitute an improvement to the framework proposed here.

Appendix: Proof of propositions 1 through 4

The two processes $(\overline{\mathbf{O}},\overline{\mathbf{P}}(s))$ and $\left(\mathbf{O},\mathbf{P}(r^j)\right)$ are discrete time Markov processes on finite state spaces, since the probability of transiting between two states from the current period to the next, depends on the properties of no state other than the current. A state $\boldsymbol{m'}$ (or $\mathbf{M'}$) of such a process is accessible from another state $\boldsymbol{m}$ (or $\mathbf{M}$), if there is a positive probability of reaching $\boldsymbol{m'}$ (or $\mathbf{M'}$) from $\boldsymbol{m}$ (or $\mathbf{M}$) in a finite number of periods. Two states communicate if each is accessible from the other. A limit set is defined as a set of states such that all states in the set communicate, and no state outside the set is accessible from any state in the set. A limit set is thus a set of states which once reached, the process never leaves. An absorbing state is a limit set consisting of a single state, whereas we call a limit set consisting of several states a limit cycle.

For the process $(\overline{\mathbf{O}},\overline{\mathbf{P}}(s))$, an absorbing state is a state in which no worker would alter his employment status, for any sample he could draw of his neighbours. For the process $\left(\mathbf{O},\mathbf{P}(r^j)\right)$, an absorbing state is a state in which no worker would alter his employment status, and no two workers would switch locations by rule r^j. In the below proofs of the propositions, we typically establish some absorbing states (or limit sets), and then proceed to rule out further limit sets by showing that an absorbing state (or a limit set) is accessible from the remaining states.

PROOF OF PROPOSITION 1:

i) In state $\boldsymbol{m}^{EE}$, for any revising worker, the proportion of employed neighbours observed is one, $\overline{q}_i = 1 > q_L^*$. No worker ever changes his employment status, which means that no other state is accessible from $\boldsymbol{m}^{EE}$. The state $\boldsymbol{m}^{EE}$ is thus an absorbing state.

ii) If at time t we are in state $\boldsymbol{m}^{\boldsymbol{EU}}$, then

$$q_{N_1+1}^t \geq q_i^t \text{ for all } i \in [N_1 + 2, N] \tag{A1}$$

The sequence of revisions in which agents at positions $N_1 + 1$ through N successively revise their employment status has positive probability. If with positive probability $\overline{q}_{N_1+1}^t \geq q_L^*$, then with positive probability we get $m_{N_1+1}^{t+1} = E$, which implies that with positive probability $\overline{q}_{N_1+2}^{t+1} = \overline{q}_{N_1+1}^t$ and $m_{N_1+2}^{t+2} = E$, which by repeated application implies that with positive probability $m_{N_1+i}^{t+i} = E$ for all $i \in [1, N - N_1]$. With positive probability we thus reach $\boldsymbol{m}^{\boldsymbol{EE}}$ in a finite number of periods, which implies that $\boldsymbol{m}^{\boldsymbol{EE}}$ is accessible from $\boldsymbol{m}^{\boldsymbol{EU}}$, and since $\boldsymbol{m}^{\boldsymbol{EE}}$ is an absorbing state, $\boldsymbol{m}^{\boldsymbol{EU}}$ can therefore not be contained in any limit set.

If on the other hand the probability that $\overline{q}_{N_1+1}^t \geq q_L^*$ is zero, then by virtue of (A1), $\overline{q}_i^t < q_L^*$ for any revising player at location $i \in [N_1 + 2, N]$. No sequence of revisions thus exists, for which $m_{N_1+i}^t = E$ for any $t > t$ and $i \in [1, N - N_1]$. In this case, no other state is accessible from $\boldsymbol{m}^{\boldsymbol{EU}}$, it is consequently an absorbing state.

The state $\boldsymbol{m}^{\boldsymbol{EU}}$ is thus an absorbing state if and only if $\overline{q}_{N_1+1}^t < q_L^*$ for all possible samples the player at position $N_1 + 1$ could draw. For any sample size $s \in [1, 2k]$, $\overline{q}_{N_1+1}^t \geq 0.5$ with positive probability, since it is always possible that the sample the player at the edge of the employed segment draws contains all employed neighbours or only employed neighbours. For $\boldsymbol{m}^{\boldsymbol{EU}}$ to be an absorbing state, we must therefore have $q_L^* > 0.5$. Moreover, since at most $\overline{q}_{N_1+1}^t = 1$ if $s < k$ and $\overline{q}_{N_1+1}^t = \frac{k}{s}$ at most if $s \geq k$, we must have $\frac{k}{s} < q_L^*$ for $\overline{q}_{N_1+1}^t \geq q_L^*$ to have zero probability. Thus, for $\boldsymbol{m}^{\boldsymbol{EU}}$ to be an absorbing state, we must have $s > \frac{k}{q_L^*}$.

iii) Imagine that at time t we are at some state $\boldsymbol{m}' \notin \{\boldsymbol{m}^{EE}, \boldsymbol{m}^{EU}\}$. Starting at location $N_1 + 1$ and moving clockwise, find the first two locations for which $m'_i \neq m'_{i+1}$ where $i \in [N_1 + 1, N - 1]$. Two adjacent agents have $2k - 2$ neighbours in common, they have each other as neighbours, and their final neighbour they do not have in common. For two adjacent agents with different employment status, the employed agent then has at least as many unemployed neighbours as the unemployed agent, and the unemployed agent has at least as many employed neighbours as the employed agent. Thus, if $m'_i = E$ and $m'_{i+1} = U$, then $q^t_{i+1} \geq q^t_i$, and vice versa. For at least one of the two agents there must then exist some sample which would make him alter his strategy upon revision. If the player at location i alters his strategy, then by implication all players from $i - 1$ counter-clockwise to $N_1 + 1$ might successively alter their strategies. If the player at location $i + 1$ alters his strategy, we proceed clockwise to the next pair of adjacent agents with different employment status. By repeated applications of this procedure, we eventually end up in a state where all low-education agents have the same employment status, i.e. in $\boldsymbol{m}^{EE}$ or $\boldsymbol{m}^{EU}$. No state $\boldsymbol{m}' \notin \{\boldsymbol{m}^{EE}, \boldsymbol{m}^{EU}\}$ can thus be contained in a limit set.?

PROOF OF PROPOSITION 2:

i) First we prove that any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$ is contained in a limit set under all mobility rules $r^j \in \mathbf{T} - \{r^8\}$. Note that in any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EE}}$, $q_i = 1 > q^*_L$ for any location $i \in [1, N]$, so no state outside $\mathbf{M}^{\mathbf{EE}}$ is accessible from a state in $\mathbf{M}^{\mathbf{EE}}$. In words, no worker ever switches to unemployment since all his neighbours are employed. Since $\mathbf{M}^{\mathbf{EESEG}} \subset \mathbf{M}^{\mathbf{EE}}$, no state outside $\mathbf{M}^{\mathbf{EE}}$ is thus accessible from a state in $\mathbf{M}^{\mathbf{EESEG}}$.

For the mobility rules based on employment, r^1, r^2, r^3 and r^4, location switches occur only between unemployed and employed workers, and since there are no unemployed workers in any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$, these states must be absorbing states.

For rules r^5 and r^6, location switches are possible. However, we can prove that from any state in $\mathbf{M}^{\mathbf{EE}}$ that is not in $\mathbf{M}^{\mathbf{EESEG}}$, i.e. that is not segregated, we can transit to a state in $\mathbf{M}^{\mathbf{EESEG}}$. Hence, states in $\mathbf{M}^{\mathbf{EESEG}}$ must be contained in some limit cycle. Consider any state $\mathbf{M'} \in \mathbf{M}^{\mathbf{EE}} - \mathbf{M}^{\mathbf{EESEG}}$, and note that it has at least two segments of adjacent high education agents, and two segments of low education agents, otherwise it would be segregated. By implication, there are at least four pairs of high and low education workers residing at adjacent locations. By virtue of an argument similar to that used in the proof of proposition 1iii), the low education worker of such a pair must have at least as many high education neighbours as the high education worker of that pair. This due to the fact that they have $2k-2$ neighbours in common, they have each other as neighbours, and only one neighbour that they do not share.

From the pairs of adjacent high and low education workers, take the pair with the highest number of high education neighbours (if there are several such pairs, pick any one of them). Let us say their proportion of high education neighbours is $\hat{h}$. If the number of workers with each type of education, N_1 and N_2, are large, there now exists some other pair of adjacent high and low education workers that have $\hat{h}$ or less high education workers, and that do not have the former pair in their neighbourhood. Both under rule r^5 and r^6, the high education worker of the latter pair and the low education worker of the former would switch locations.

We thus reach a new state $\mathbf{M''}$, where the high education worker that just moved has $\hat{h}$ high education neighbours, and any low education worker living next to him also has $\hat{h}$ high education neighbours. Furthermore, for any worker that does not have this high education worker as a neighbour, the proportion of high education workers is equal to or less than what he had in state $\mathbf{M'}$. Among these, there thus exists some pair of high and bw education adjacent agents, where the high education worker would switch locations with the low education worker adjacent to the high education worker that just moved.

Thus we reach a new state $\mathbf{M'''}$, from which we can repeat the argument a finite number of times until a low education segment is eradicated. Then we start all over again by finding the pair of high and low education workers with the highest number of high education neighbours, and gradually eradicate the segment associated with this low education worker as well. A finite number of repetitions of this procedure eradicates all low education segments but one, and we have reached a state in $\mathbf{M}^{\mathbf{EESEG}}$.

For rules r^7, r^9, r^{10} and r^{11}, in a state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$ no location switch is ever made on the basis of employment, since under rule r^7 it is not permitted, and under rules r^9, r^{10} and r^{11} there are no unemployed workers with whom an employed worker can switch positions. Any switches would have to be made on the basis of education. However, if at time *t* we are in a state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$, then

$$h_i^t \leq h_j^t \text{ if } m_i^t = (\cdot, L) \text{ and } m_j^t = (\cdot, H) \qquad \text{(A2)}$$

By implication, no high education worker can get strictly more neighbours with a high level of education by switching locations with a low education worker, and no switches are thus ever made. Under rules r^7, r^9, r^{10} and r^{11}, any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$ is thus an absorbing state.

Next we prove that only states $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$ are contained in limit sets for all rules $r^j \in \mathbf{T} - \{r^8\}$. This is done in two steps, first by showing that only states in $\mathbf{M}^{\mathbf{EE}}$ are contained in limit sets under rule r^1, and second by showing that of the states in $\mathbf{M}^{\mathbf{EE}}$, only those in $\mathbf{M}^{\mathbf{EESEG}}$ are contained in limit sets under rule r^6.

Under mobility rule r^1, any state $\mathbf{M}' \in \mathbf{O}$ that is not segregated according to employment, can be transformed into one that is thus segregated by a series of location switches. Start with any employed agent, and number his position on the circle 1. Then move clockwise to the first location occupied by an unemployed agent, say location *a*. Then proceed clockwise to the first subsequent location occupied by an employed agent, say at location b. The employed agent at location b has at least as few employed neighbours as the unemployed worker at location $b-1$, and would thus want to exchange locations with him. Having moved to location $b-1$, the employed worker has at least as few employed neighbours as any unemployed worker immediately preceding him on the circle, and would want to move once more. Thus we can continue until the employed worker reaches location a. By repeating this process for each employed worker, a contiguous segment of employed workers forms from location 1 onwards, a segment that eventually holds all employed workers, which means that we are in a state of full segregation according to employment.

From a state of full segregation according to employment, we can proceed to eradicate unemployment through further location switches. Imagine that the employed segment stretches from location 1 to location c. The unemployed worker at location $c+1$ has at least as many employed neighbours as the employed worker at location c, and the two might therefore exchange locations. The unemployed worker now at location c has at least as many employed neighbours as the employed worker at location $c-1$, and the two might exchange locations. Thus we can keep moving the unemployed worker into the employed segment. If the number of high education workers is sufficiently high, the unemployed worker in question eventually has only employed workers in his neighbourhood. If selected to revise his employment status, he would then choose employment. In a similar manner we can move every single unemployed worker at locations $c+2$ through N into the employed segment one at a time, and make them choose employment, which means that we eventually reach some state of full employment $\mathbf{M} \in \mathbf{M}^{\mathbf{EE}}$.

Under rule r^1, from any state that is not segregated according to employment we can move to one that is segregated, and from any segregated state we can move to one of full employment. A state in $\mathbf{M}^{\mathbf{EE}}$ is thus accessible from any state outside that set. But as argued above, no state outside $\mathbf{M}^{\mathbf{EE}}$ is accessible from a state in $\mathbf{M}^{\mathbf{EE}}$, which implies that no state outside $\mathbf{M}^{\mathbf{EE}}$ is contained in a limit set under rule r^1.

For rule r^6, we have already proved that from any state in $\mathbf{M}^{\mathbf{EE}}$, we can transit to a state in $\mathbf{M}^{\mathbf{EESEG}}$. We now add a proof of the fact that no state outside $\mathbf{M}^{\mathbf{EESEG}}$ is accessible from any state in $\mathbf{M}^{\mathbf{EESEG}}$ under rule r^6. Consider the high education workers in a state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$. If not on the boundary of the high education segment, a high education worker has at least $k+1$ high education neighbours, and switching locations with any low education worker would leave him with at most k high education neighbours. If on the boundary of the high education segment, a high education worker has exactly k high education workers, and would get as many high education neighbours if he exchanged locations with the low education worker at the opposite boundary of his segment, otherwise he would get strictly fewer. The only location switches permitted by rule r^6 in a state $\mathbf{M} \in \mathbf{M}^{\mathbf{EESEG}}$ are thus between high and low education workers at opposite boundaries of the high education segment. This implies that from any state in $\mathbf{M}^{\mathbf{EESEG}}$ we can move only to other states in $\mathbf{M}^{\mathbf{EESEG}}$, which implies that the states in $\mathbf{M}^{\mathbf{EESEG}}$ form a limit cycle. Moreover, since a state in $\mathbf{M}^{\mathbf{EESEG}}$ is accessible from any state in $\mathbf{M}^{\mathbf{EE}}$, no state in $\mathbf{M}^{\mathbf{EE}}$ is contained in a limit set under rule r^6.

Summing up, only states in $\mathbf{M}^{\mathbf{EE}}$ are contained in limit sets under rule r^1, and of the states in $\mathbf{M}^{\mathbf{EE}}$ only those in $\mathbf{M}^{\mathbf{EESEG}}$ are contained in limit sets under rule r^6, which implies that no state not in $\mathbf{M}^{\mathbf{EESEG}}$ can be contained in a limit set for all rules $r^j \in \mathbf{T} - \{r^8\}$.

ii) Here we need only prove that no state in $\mathbf{M}^{\mathbf{EESEG}}$ is contained in a limit set for rule r^8. Note that any high education worker in a state $\mathbf{M'} \in \mathbf{M}^{\mathbf{EESEG}}$ has k or more high education neighbours, where the workers at the boundary of the high education segment have exactly k and those not at the boundary more than k. By switching locations with any low education worker, they would get k or less high education neighbours. Thus any high education worker not at the edge of the high education segment would switch locations with any low education worker. Moreover, a high education worker on the boundary would want to exchange locations with the low education neighbour next to him on the circle, since he would then get $k-1$ high education neighbours. (The only location switch between high and low education workers that is not permitted under rule r^8 is between a low and high education worker at opposite edges of the high education segment.) Thus from a state $\mathbf{M'} \in \mathbf{M}^{\mathbf{EESEG}}$, we can transit to a state $\mathbf{M''} \notin \mathbf{M}^{\mathbf{EESEG}}$.

However, any switch that caused such a transition cannot be undone, since the high education worker who moved now has at most k high education neighbours if he moved from the interior of the high education segment or at most $k-1$ high education neighbours if he moved from the boundary. To make the high education segment complete again, he would have to move to a location where he would get at least $k+1$ high education neighbours in the former case, and k high education neighbours in the latter. Under rule r^8, such a move would not be made. And since location switches happen sequentially, one at a time, such a move is needed as the final switch in a series through which an unsegregated state is supplanted by a segregated state. Under rule r^8, then, from a state in $\mathbf{M}^{\mathbf{EESEG}}$ we can transit to a state in $\mathbf{M}^{\mathbf{EE}} - \mathbf{M}^{\mathbf{EESEG}}$, but no state in $\mathbf{M}^{\mathbf{EESEG}}$ is accessible from a state in $\mathbf{M}^{\mathbf{EE}} - \mathbf{M}^{\mathbf{EESEG}}$, which means that no state is contained in a limit set for all rules $r^j \in \mathbf{T}$.?

PROOF OF PROPOSITION 3:

i) In any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EE}}$, $q_i = 1 > q_L^*$ for a worker at any location $i \in [1, N]$. Moreover, no worker is unemployed, so no two workers ever exchange locations by rule r^3. Any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EE}}$ is therefore an absorbing state.

ii) If at time t we are in any state $\mathbf{M}^{\mathbf{t}} \in \mathbf{M}^{\mathbf{EUSEG}}$, and we let $N_1 + 1$ be the location of the unemployed worker who has an employed worker before him and an unemployed worker after him on the circle, then (A1) holds. By implication, since $q^t_{N_1+1} = 0.5$, then no worker would ever alter his employment status if $q_L^* \in \langle 0.5, 1\rangle$. If on the other hand, $q_L^* \in \langle 0, 0.5]$, then upon revision the player at position $N_1 + 1$ could choose employment, $\boldsymbol{m}^{t+1}_{N_1+1} = (E, L)$, which implies $q^{t+1}_{N_1+2} = 0.5$, which could mean $\boldsymbol{m}^{t+2}_{N_1+2} = (E, L)$, and so on until $\boldsymbol{m}^{t+i}_{N_1+i} = (E, L)$ for all $i \in [1, N - N_1]$. We have thus reached some state $\mathbf{M}^{\mathbf{t+i}} \in \mathbf{M}^{\mathbf{EESEG}} \subset \mathbf{M}^{\mathbf{EE}}$, and no state $\mathbf{M}^{\mathbf{t}} \in \mathbf{M}^{\mathbf{EUSEG}}$ is therefore contained in a limit set.

In any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EUSEG}}$, any employed worker has at least k employed neighbours. By switching locations with an unemployed worker, the most employed neighbours he could get is k. No two workers would therefore exchange locations under rule r^3. In conlusion, any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EUSEG}}$ is absorbing if and only if $q_L^* \in \langle 0.5,1 \rangle$.

iii) In any state $\mathbf{M} \in \mathbf{M}^{\mathbf{INT}}$, if k is even then $q_i = 0.5$ for all locations $i \in [1, N]$. By implication, no revising worker changes his employment status if $q_L^* \in \langle 0.5,1 \rangle$. Any employed worker in a state $\mathbf{M} \in \mathbf{M}^{\mathbf{INT}}$ has k employed neighbours, and would get k or less by switching locations with an unemployed worker, so under rule r^3 no location switches occur in such a state. Any state $\mathbf{M} \in \mathbf{M}^{\mathbf{INT}}$ is therefore an absorbing state for k even and $q_L^* \in \langle 0.5,1 \rangle$.?

PROOF OF PROPOSITION 4:

i) See the proof of proposition 2.

ii) From the proof of proposition 3 we know that in any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EUSEG}}$, no worker changes his employment status if $q_L^* \in \langle 0.5,1 \rangle$, whereas all unemployed workers could change sequentially to employment if $q_L^* \in \langle 0, 0.5]$. Any high education worker has at least k high education neighbours in a state $\mathbf{M} \in \mathbf{M}^{\mathbf{EUSEG}}$, and would get k or less by switching locations with a low education worker. From the proof of proposition 3ii) we know that no employed worker can get more employed neighbours by switching locations with an unemployed worker. Under rules r^7, r^9, r^{10} and r^{11}, no location switches thus occur in a state $\mathbf{M} \in \mathbf{M}^{\mathbf{EUSEG}}$. In conclusion, any state $\mathbf{M} \in \mathbf{M}^{\mathbf{EUSEG}}$ is absorbing if and only if $q_L^* \in \langle 0.5,1 \rangle$.

iii) The proof of proposition 3iii) tells us that in a state $\mathbf{M} \in \mathbf{M}^{\mathbf{INT}}$, no worker changes his employment status if k is even and $q_L^* \in \langle 0.5,1 \rangle$, and no employed worker could get more employed neighbours by switching locations with an unemployed worker. If

k is even, in any state $\mathbf{M} \in \mathbf{M}^{\mathbf{INT}}$ a high education worker has k high education neighbours, and would get k or less if by switching with a low education worker. Under rules r^7, r^9, r^{10} and r^{11}, then, any state $\mathbf{M} \in \mathbf{M}^{\mathbf{INT}}$ is absorbing if k is even and $q_L^* \in \langle 0.5,1 \rangle$.?

References

Bala, Venkatesh and Sanjeev Goyal (2001), "Conformism and diversity under social learning", *Economic theory*, 17, 101-120

Bertrand, Marianne, Erzo F.P. Luttmer, Sendhil Mullainathan (1999), ”Network effects and welfare cultures”, mimeo, NBER

Binmore, Kenneth G., Larry Samuelson and Richard Vaughan (1995), ”Musical chairs: Modelling noisy evolution”, *Games and economic behavior*, 11, 1-35

Ellison, Glenn (1993), «Learning, local interaction, and coordination», *Econometrica*, 61, 1047-71

Ellison (2000), ”Basins of attraction, long-run stochastic stability, and the speed of step-by-step evolution”, *Review of Economic Studies*, 67, 17-45

Fieldhouse, Edward A. (1999), ”Ethnic minority unemployment and spatial mismatch: The case of London”, *Urban Studies*, vol. 36, no. 9, 1569-1596

Frank, Robert (1985), *Choosing the right pond: Human behavior and the quest for status*, Oxford University Press, New York

Glaeser, Edward L., B. Sacerdote and J. Scheinkman (1996), ”Crime and social interactions”, *Quarterly Journal of Economics*, 111:2, 508-548

Glaeser, Edward L. (1999), ”The future of urban research: Non-market interactions”, Mimeo, Harvard University/NBER

Glaeser, Edward L., Matthew E. Kahn and Jordan Rappaport (2000), ”Why do the poor live in cities”, Discussion Paper No 1891, Harvard Institute of Economic Research

Glaeser, Edward L., José A. Scheinkman (2000), ”Non-market interactions”, Working Paper 8053, NBER

Immergluck, Daniel (1998) , ”Job proximity and the urban employment problem: Do suitable nearby jobs improve neighbourhood employment rates?”, *Urban Studies,* vol. 35, no. 1, 7-23

Kain, J. F. (1968), ”Housing segregation, negro employment, and metropolitan decentralization”, *Quarterly Journal of Economics*, 82, 175-197

Kandori, Michihiro, George J. Mailath and Rafael Rob (1993), «Learning, mutation, and long run equilibria in games», *Econometrica*, vol. 61, no. 1, 29-56

Lindbeck, Assar, Sten Nyberg and Jörgen W. Weibull (1999), ”Social norms and economic incentives in the welfare state”, *Quarterly Journal of Economics*, vol. CXIV, issue 1, 1-35

Morris, Stephen (2000), "Contagion", *Review of economic studies*, 67, 57-78

O’Regan, Katherine M. and John M. Quigley (1998), ”Where youth live: Economic effects of urban space on employment prospects”, *Urban Studies*, vol. 35, no. 7, 1187-1205

Raphael, Steven (1998), ”The spatial mismatch hypethesis and black youth joblessness – Evidence from the San Francisco Bay Area”, Journal of Urban Economics, vol. 43, iss. 1, 79-111

Reingold, David A. (1999), ”Social networks and the employment problem of the urban poor”, *Urban Studies*, vol. 36. no. 11, 1907-1932

Schelling, Thomas C. (1971), “Dynamic models of segregation”, *Journal of mathematical sociology*, 1, 143-186

Wilson, William Julius (1987), *The truly disadvantaged: The inner city, the underclass and public policy*, University of Chicago Press, Chicago

Young, H. Peyton (1993), ”The evolution of conventions”, *Econometrica*, vol. 61, no. 2, 57-84

Young, H. Peyton (1996), ”The economics of convention”, *Journal of economic perspectives*, vol. 10, no. 2, 105-122

Young, H. Peyton (1998), *Individual strategy and social structure – An evolutionary theory of institutions*, Princeton University Press, Princeton, New Jersey