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Industrial Clusters:
equilibrium, welfare, and policy.*

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Abstract: This paper studies the size and number of industrial clusters that will arise in a multi-country world in which one sector has a propensity to cluster because of increasing returns to scale. It compares the equilibrium with the world welfare maximum, showing that the equilibrium will generally have clusters that are too small, while there are possibly too many countries with a cluster. Allowing national governments to subsidize will move the equilibrium to the world welfare maximum, so there is no 'race to the bottom'. If subsidy rates were capped then there would be a proliferation of too many and too small clusters.

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1: Introduction:

There is a presumption among practising economists and policy makers that national subsidies to encourage clusters, while possibly in the national interest, are detrimental to world economic efficiency. To the extent that such policy simply causes a cluster to locate in one country rather than another, policy is zero sum game. It becomes negative sum if there is a 'race to the bottom', distorting policy instruments away from cooperatively set levels.

The purpose of this paper is to investigate whether this presumption is well founded. To do this we analyse a world of many countries and two production sectors. One sector is subject to increasing returns to scale at the national level, arising because of a spatially concentrated technological externality between producers. As a consequence, this sector clusters in a subset of available countries, even if all countries have identical underlying characteristics. The first set of issues we investigate are to do with the number and size of these clusters. How do the number and size of clusters at the equilibrium compare the world welfare maximising configuration of the industry? The second set of issues are to do with policy. What are the incentives for the use of active policy to attract a cluster, and what is the equilibrium of policy competition between countries? From the international point of view, should national industrial policy be encouraged or constrained?

The market failure in the model derives from an externality which creates increasing returns to scale to national production. The world welfare maximising number and size of clusters is determined by a cost-benefit calculation comparing output from each cluster with output foregone in the other sector of the economy. In contrast, the equilibrium number is determined by private agents deciding in which sector to work. We assume that entry by coalitions of agents is possible, so that there are no coordination failures encountered in establishing the industry in a country. However, increasing returns are not fully internalised, as individual agents are free to enter and exit the industry, making their calculation on the basis of private not social returns.

We show that equilibrium produces too little of the increasing returns good, and does so in clusters that are too small: depending on demand conditions, there may be too many or too few of these clusters. Turning to policy, we show that there is an incentive for governments to use policy to attract or enlarge an industrial cluster. However, in a central case, the Nash policy equilibrium

between governments supports the world welfare maximum. There is no 'race to the bottom', and if some international authority were to cap subsidy rates this would lead to a proliferation of too many too small clusters. It is better to have relatively high subsidies, this increasing the size of clusters, reducing the price of the good, and thereby removing the incentive for further countries to try to acquire a cluster.

The analysis of this paper connects with two quite old strands of literature. The basic model we use is one of increasing returns (in one of two sectors) and price taking behaviour. This is similar to the modelling of increasing returns undertaken in trade theory in the late 1970s and early 1980s (for example, Panagariya 1981 and Markusen and Melvin 1981). However, these papers look at a single small open economy or at trade between two countries, asking questions quite different from those addressed in this paper. Our policy analysis connects to results on city size in the urban economics literature. Henderson (1974), Vickrey (1977), and others show that entry of cities, each controlled by a single large agent, leads to an efficient outcome. While different in many aspects (eg infinitely many potential city sites, spatially mobile labour, cities fully specialized each in a different activity), these models share with ours the property that the Nash equilibrium of an entry game in which decision takers maximise the total income of the city/ country supports world efficiency.[1]

Throughout the paper we maintain the assumption that countries are small, and we look at policy equilibrium between these countries This is in contrast to the few policy papers that use 'new economic geography' to analyse industrial policy. Baldwin and Krugman (2000) and Kind et al (2000) have a richer modelling of the micro-foundations of clustering than we have in this paper, but focus on two countries, looking at policy to tax or to sustain an existing agglomeration. We address different questions, looking at the number of clusters that operate and the policy game between countries that are seeking to develop clusters of, eg, high technology activity. We regard the present paper as setting out the simplest possible benchmark case, to which richer economic geography modelling can later be added.

2. Optimum and equilibrium:

The model:

There are *K* countries, and country specific variables are subscripted *i*. All countries have the same underlying characteristics and are endowed with the same quantity, *L*, of a single factor of production. There are two sectors, *y* and *x*, both producing output that is freely traded; *y* will be used as numeraire, the price of *x* being denoted *p*. Utility in country *i* is

$$u_i = v(p) + m_i \tag{1}$$

where m_i is income, preferences are assumed quasi-linear, and $v(p)$ is the indirect utility function for good *x*. Country *i* demand for good *x* is therefore $-v'(p)$.

Production of *y* takes place according to the production function $Y(L - n)$, where *L* is labour endowment, *n* is employment in the *x*-sector, and *Y* is increasing and strictly concave. In the *x*-sector there are increasing returns, arising as workers create positive externalities for other workers. This is modelled by assuming that if *n* workers are active in the sector, then the average product of each is $a(n)$, increasing and concave in *n*. The income of country *i* is therefore

$$m_i = Y(L - n_i) + pn_i a(n_i). \tag{2}$$

In general, not all countries will be active in the *x*-sector, and we denote the number of *x*-active countries by *k*. If each of these countries employs *n* workers in *x*-production and the remaining *K* - *k* countries specialise in *y*, then product market clearing is

$$-Kv'(p) = kna(n). \tag{3}$$

World welfare maximisation:

We look first at the optimal arrangement of production in the world economy. This amounts to choosing the number of *x*-active countries, *k*, and the employment levels in each, *n*, to maximise

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world welfare. Welfare is simply the sum of all countries' utilities,

$$W \equiv Kv(p) + (K - k)Y(L) + k\big[Y(L - n) + pna(n)\big]. \tag{4}$$

The first term is consumer surplus on x consumption, the second the income of countries with no x production, and the third the income of the k countries with x-sector employment at level n. Price is determined by equation (3), although small changes in price have no effect on (4) as they are simply transfer payments.

Choosing n and k to maximise W gives first order conditions:

$$Y'(L - n^o) = p^o\big[a(n^o) + n^o a'(n^o)\big] \tag{5}$$

$$Y(L) = p^o n^o a(n^o) + Y(L - n^o) \tag{6}$$

where the superscript ^{o} refers to the fact that variables are taking their optimum values.[2] The first condition, (5), equates the value marginal products of labour in each sector. The second, (6), says that the number of x-active countries should be set so that income is the same in all countries, implying that no income is gained by having another country start x-production. Adding more x-active countries would reduce the price of x, meaning that the value of extra x-output produced is less than the y-output foregone. We assume an interior solution with some countries active and others inactive, i.e., that the solution of (3), (5) and (6) for n^o, p^o and k^o has $k^o \in (0, K)$. Notice that, eliminating p^o from (5) and (6), the welfare maximising value of n^o can be implicitly defined in terms of technology alone.

Although the world welfare maximum equalises income across countries it does not equalise the marginal product of labour, which is higher in x-active countries. This means that – despite free trade in goods – there would be further gains from international labour mobility to equate marginal products across countries.

Equilibrium:

Either single workers or coalitions of workers are free to enter and exit the *x*-sector, although no transfers are possible between workers in the *x*-sector and those in the *y*-sector. The entry process is best thought of as involving two stages. At the first, a coalition of any size may enter; this overcomes coordination failure problems associated with setting up increasing returns to scale activities. At the second stage individual workers may enter or leave the *x*-sector. The coalition is therefore unable to force workers to stay in the *x*-sector if they can do better by working in *y*. These assumptions ensure two things. Mobility of individual workers at the second stage ensures that in *x*-active countries each worker's average value product in the *x* sector equals the marginal product in the *y*-sector, i.e.

$$Y'(L - n^e) = p^e a(n^e),\qquad\qquad(7)$$

where the superscript ^{e} indicates equilibrium. Entry by coalitions at the first stage means that there exists no value of *n* (coalition of workers) at which there is unexploited profit from entry, $p^e a(n) > Y'(L - n)$.[3]

    This is illustrated in figure 1. The horizontal axis (of length *L*) gives employment in each sector, the curve $Y'(L - n)$ is the marginal product schedule in the *y*-sector, and the curves $pa(n)$ the value average product schedules in the *x*-sector, drawn for different values of *p*. The point marked E is the equilibrium. If fewer countries had an *x*-sector, then the price would be high and the upper schedule, $p'a(n)$, would apply. However, it would then be profitable for a coalition of workers to enter, this creating a new *x*-industry and driving the price down. Equilibrium is at the tangency point, where no further surplus can be extracted by *x*-sector workers: at any price lower than this workers would quit the industry and work in the *y*-sector. Formally then, equilibrium is characterised by (7) and (8),

$$- Y''(L - n^e) = p^e a'(n^e).\qquad\qquad(8)$$

Using (7) in (8) to eliminate p^e, gives an equation which is the first order condition for the

minimization of average costs of *x*-sector production (where average costs are the wage divided by output per worker, $Y'(L - n)/a(n)$); the equilibrium value of *n* depends just on technologies.

It is apparent from figure 1 that, in countries with *x*-production, the value of *x*-sector output exceeds the value of *y*-sector output it displaces, since rectangle $p^e a(n^e) n^e$ is larger than shaded area $Y(L) - Y(L - n^e) = \int_{L-n^e}^{L} Y'(L - z)dz$. This means that, at equilibrium, *x*-active countries have higher income and utility than do countries that specialize in *y*. It is worth asking two questions about this; what is it that makes this consistent with equilibrium, and what determines the magnitude of the income loss? On the first, an additional country entering the *x*-industry would see its workers becoming better off since (even following a marginal reduction in *p*), $pa(n) > F'(L)$, the wage when there is no *x*-industry. However, once the *x*-industry exists and employs *n* workers, the wage rises to $F'(L$-$n)$, and workers then quit the *x*-sector to work in *y*, since $pa(n) < F'(L$-$n)$. Thus, it is inability to force workers to stay in the *x*-sector that constrains the number of *x*-active countries and supports the equilibrium.

The size of the income loss depends on the slope of the $F'(L$-$n)$ schedule. If this were horizontal, then workers could be employed in the *y*-sector without encountering diminishing returns, and there would be no income loss. In our model $F'(L$-$n)$ is diminishing, most naturally because of the presence of a sector-specific factor in *y*-production. It is worth stepping outside the model, and asking what else would generate diminishing returns to *y*-sector employment. One possibility is that expansion of the *y*-sector encounters deteriorating terms of trade, as would occur if each country produced a distinct variety of *y*-output.[4] Then $F'(L$-$n)$ would be replaced by the value marginal product schedule, and downwards slope would come from price change even if the marginal physical product were constant. In addition, if there were non-tradeables then countries with no *x*-production would have a larger non-tradeable sector, this being associated with lower non-tradeable prices and diminishing marginal utility in non-tradeable consumption.

Comparison of equilibrium and optimum:

Figure 1 illustrates the equilibrium, and figure 2 is the analogous figure illustrating the optimum. At the optimum *x*-sector employment in each *x*-active country is at n^o, the point where the full value marginal product in *x*-production (including the external effect) equals the marginal product in the

y-sector (equation (5)). The number of *x*-active countries has adjusted until the price is p^o, at which the curve $p^o a(n)$ is positioned so that the two shaded areas in the figure are of equal size. At this point n^o workers in the *x*-industry produce the same value of output (the rectangle $n^o p^o a(n^o)$) as they would employed in the *y*-industry, $\left(\int_{L-n^o}^{L} Y'(L-z)dz \right)$, ensuring that equation (6) holds. Notice that for this to be true it must be the case that there is a range of values of *n* at which $p^o[a(n)+na'(n)] > Y'(L-n)$, creating the lens shape area containing point *C*.

Comparing figures 1 and 2, it can be proved that if $a(n)$ is concave and $Y'(L-n)$ convex, then $p^o < p^e$, so that the curve $p^o a(n)$ lies strictly below $Y'(L-n)$, as illustrated (see appendix 1). This price inequality means that the optimum produces more *x*-output in total than does the equilibrium. What about the employment level in each *x*-active country? Since the optimum internalises the externality, we would expect that the optimum level of employment in each cluster would be greater than the equilibrium, so $n^o > n^e$. This is generally, but not necessarily, the case; the effect can be reversed as the lower price at the optimum reduces the value of the externality. Appendix 2 explores this further and shows that sufficient conditions for $n^o > n^e$ are that $Y'(L-n)$ is linear, or *n* is small.

What about the number of *x*-active countries, k^o compared to k^e? In the neighbourhood of the equilibrium it is certainly the case that a marginal increase in the number of *x*-active countries raises world welfare, since *x*-active countries have higher income and utility than do countries that specialize in *y*. However, the full comparison of k^o to k^e can go either way; total output is lower at the equilibrium, but so too is output per cluster (when $n^e < n^o$). The outcome depends on, amongst other things, the price elasticity of demand. If the elasticity is low enough then the equilibrium will have more clusters than the optimum, $k^e > k^o$; if the elasticity is very high then overall *x*-production is low and it may be the case that $k^e < k^o$. The case of an iso-elastic $a(n)$ function and linear $Y'(L-n)$ is worked out explicitly in appendix 3, which shows that the equilibrium has clusters smaller than the optimum ($n^e < n^o$), and more clusters than the optimum, ($k^e > k^o$), if the demand elasticity is less than some critical value which is greater than unity. Thus, a possible case is illustrated in figure 3. The four curves illustrated correspond to conditions (5) - (8), and incorporate the market clearing price, solved through (3). A world welfare contour is illustrated and it is clear that, in the neighbourhood of equilibrium, a small increase in either *n* or *k* raises welfare. Comparison of the equilibrium with the optimum indicates that the equilibrium has too many and too small clusters, k^e

$> k^o$, $n^e < n^o$.

Figure 4 gives a further way of illustrating outcomes. The bold curve illustrated is the production possibility frontier of a single country. The price associated with the welfare maximum is illustrated by the slope of the budget line, p^o. Some countries specialise in y, and others produce at point O (where the MRT equals the price p^o, i.e. equation (5) holds). Indifference curves are illustrated, and at the optimum all countries have utility level u^o. Comparison of levels of demand and supply determines k, the number of x-active countries. In the equilibrium countries either specialise in y or produce at point E (which is on the production possibility frontier but at the point given by equation (7)). The price p^e is higher, and we see that countries with x industry are on a higher indifference curve (u^k) than are those specializing in y (u^{K-k}).

3. International policy competition:

We now turn to policy, looking briefly at the choice of action by a single country, and then turning to the policy equilibrium that will arise if countries engage in simultaneous policy competition. The policy instrument we consider is an ad valorem subsidy on output in the manufacturing sector, with subsidy factor denoted s, paid for out of a lump sum tax on the endowment.

To see the effect of unilateral action, suppose that at the equilibrium one country that does not have x-production introduces a subsidy, $s > 1$. Potential producers in the x-sector in this country are now faced with return $sp^e a(n)$ rather than $p^e a(n)$, so entry becomes strictly profitable. If no other country uses the subsidy the price remains p^e and some other country is forced out.[5] The country using the subsidy experiences a strict welfare gain, and the exiter a welfare loss.

This suggests modelling the policy game between countries. We look for the Nash equilibrium of a two stage game. At the first stage governments choose a subsidy rate, and we will see that generally some countries choose to use this subsidy, so have $s > 1$, while others will not, leaving $s = 1$. At the second stage the market equilibrium is established.

To find the policy equilibrium, consider first a single country with x-production. If a subsidy s is in place, then employment in the sector is determined by equilibrium condition (7) which becomes

$$Y'(L - n) \ = \ spa(n) \tag{9}$$

where p is the consumer price, (equation (3) is unchanged). The country's optimum policy is to set s^* to satisfy

$$s^* \ = \ 1 \ + \ n^s a'(n^s)/a(n^s), \tag{10}$$

ensuring that n^s satisfies

$$p[a(n^s) \ + \ n^s a'(n^s)] \ = \ Y'(L - n^s). \tag{11}$$

This is the first order condition for maximisation of the income of an x-active country, (m_i, equation (2)). Thus, for a given value of p, x-active countries set s to induce the size of the country's x-industry employment to go to the level that equates marginal value products in the two sectors.

Suppose now that countries have x-industry if and only if government uses the subsidy. How many countries will employ the subsidy (with $s = s^*$), and how many not (with $s = 1$)? Countries are indifferent between having the industry or not when p has adjusted to level p^s at which,

$$Y(L) \ = \ p^s n^s a(n^s) \ + \ Y(L - n^s), \tag{12}$$

Notice that equations (11) and (12) are the first order conditions for a global welfare maximum (equations (5) and (6)) and, with (3), they determine values of n, k, p. This means that $n^s = n^o$, $k^s = k^o$, and $p^s = p^o$.

Finally, we have to check that x-industry is active in countries that have the subsidy, and only those countries. This comes from inspection of figure 2. In countries with the subsidy the x-sector has average value product schedule $s^* p^o a(n) = p^o[a(n)+na'(n)]$, and since this lies above $Y'(L - n)$ over some range, entry is profitable. Conversely, in countries where the subsidy is not in use the x-sector faces $p^o a(n)$ which lies below $Y'(L - n)$ everywhere, so entry is not profitable.

We summarise this in the following proposition.

Proposition: A Nash policy equilibrium decentralises the world welfare maximum. It has k^s countries setting $s = s^*$, and the remaining $K - k$ setting $s = 1$, with n^s, k^s, p^s determined by (3), (11) and (12).

The fact that the Nash equilibrium in industrial subsidies decentralises the global welfare maximum may seem surprising, and a few remarks are in order. First, a single policy instrument secures the optimal scale of the *x*-industry in each active country, *and* the optimal number of such countries. Attaining two targets with one instrument is not as surprising as it first seems, because the instrument is set separately by each country and takes different values for countries that are active and that are inactive in *x*-production. Expressed differently, countries choose two instruments -- a value of *s* and a probability of using it ($= k^s/K$) and the equilibrium outlined above is the mixed strategy equilibrium of this game in two policy instruments. Second, the countries are assumed to be small, so no country influences the terms of trade; this removes the prisoners' dilemma aspect of trade and industrial policy that might otherwise be expected.

International policy regulation:

It is often suggested that the presence of an internationally mobile *x*-industry, as modelled in this paper, is conducive to a 'race to the bottom', with countries over-using subsidies in an attempt to attract a cluster. A proposed solution to the problem is international regulation to cap the levels of subsidy that can be given. This turns out to be a bad idea in this model, and it is quite instructive to consider why.

The effect of a cap on the subsidy can be found by comparative statics on equations (9), giving the effect of a given level of subsidy on employment; (12), the indifference condition for countries to use the subsidy or not; and (3), supply and demand. It is straightforward to show that $dn/ds > 0$, and $dp/ds \leq 0$ if $s \leq s^*$, with $dp/ds = 0$ if $s = s^*$. In the neighbourhood of $s = s^*$ it must also be the case that $dk/ds < 0$, although globally the sign of this relationship depends on demand elasticities. These relationships are illustrated on figure 5, on which the lines $n(sub)$ and $k(sub)$ show

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the dependence of n and k on the subsidy rate, and come from solving equations (9) and (12) with (3). The optimal subsidy, s^*, gives point O, and we see that constraining countries to use a lower subsidy (i.e. moving along the horizontal axis to points $s < s^*$) reduces x-industry employment in each active country, this raising p and inducing more countries to offer the subsidy. Thus, international regulation to cap industrial subsidy rates would have the effect of reducing welfare as industrial centres shrink in size but also proliferate in number. Conversely, de-regulation (at $s < s^*$) would cause an increase in subsidy rates, causing countries to drop out of the industry. Pareto improvements would follow because as remaining centres expand prices fall.

For completeness, we also point out that a regime change occurs at a low value of the subsidy. If the subsidy is low enough it is not profitable for x-activity to become established even in countries using the subsidy, so the number of x-active countries becomes less than the number of countries offering the subsidy. The dashed lines $n(entry)$ and $k(entry)$ are solutions to the subsidy inclusive equilibrium conditions,

$$Y'(L - n) \ = \ s\,p\,a(n), \qquad -Y''(L \ - \ n) \ = \ s\,p\,a'(n). \qquad\qquad (9), (8^*)$$

Since government cannot force entry simply by offering a subsidy, the outcome is given by the lower of the $k(sub)$ and $k(entry)$ curves and higher of the $n(sub)$ and $n(entry)$ curves. Thus, points E at $s = 1$ are simply the equilibrium of section 2.

Robustness:

In this model changes in one country affect others only via their effect on the price of x. The fundamental reason why the global optimum and the policy equilibrium are identical is that each individual country is a price taker and there is no global benefit from a price change. In this section we make two minor extensions to the model that relax these conditions.

The first change is simply to assume that a country perceives that an increase in its subsidy rate (and hence x-sector employment) will depress the world price of good x. The second is to suppose that there is a shadow premium on government revenue in each country. This is an additional distortion and it has the implication that, when subsidies are in place, a change in the price

of x is no longer just a transfer, but also has real income effects.

Denoting the shadow premium on government funds used to support the x-industry by λ, the welfare of a single country with x-industry is $v(p) + m(p,n)$, where $m(p,n)$ is income net of the additional cost of subsidy,

$$m(p,n) = Y(L - n) + pna(n) - \lambda n \left[Y'(L - n) - pa(n) \right]. \tag{13}$$

The final term is the premium on public funds times the cost of covering any gap between wages and average value product in the x-sector (the term is square brackets is equal to $(s - 1)pa(n)$). For simplicity, let us control n directly rather than indirectly through s. The first order condition for national welfare maximisation is now

$$\frac{\partial m(n,p)}{\partial n} + \left[v'(p) + \frac{\partial m(n,p)}{\partial p} \right] \left(\frac{dp}{dn} \right)^c = 0. \tag{14}$$

where $(dp/dn)^c \leq 0$ is the conjectured change in world price from an increase in one country's employment. Countries are indifferent between having industry or not when

$$Y(L) = m(n,p), \tag{15}$$

so the policy equilibrium is defined by (14) and (15), with (3).

Should there be a cap on the subsidy rate? World welfare is

$$W \equiv Kv(p) + (K - k)Y(L) + km(n,p) \tag{16}$$

and totally differentiating this with respect to n and k gives,

$$\frac{dW}{dn} = k\frac{\partial m(n,p)}{\partial n} + \left[Kv'(p) + k\frac{\partial m(n,p)}{\partial p}\right]\frac{dp}{dn}$$

$$\frac{dW}{dk} = -Y(L) + m(n,p) + \left[Kv'(p) + k\frac{\partial m(n,p)}{\partial p}\right]\frac{dp}{dk}$$

(17)

We evaluate this at the policy equilibrium, where (14) and (15) hold. We also note that, using (3) with derivatives of (13),

$$v'(p) + \partial m/\partial p = v'(p) + (1+\lambda)na(n) > 0,$$

$$Kv'(p) + k\partial m/\partial p = \lambda kna(n) \geq 0,$$

(18)

where the first inequality holds because x-active countries export x. Equations (17) therefore become,

$$\frac{dW}{dn} = -k\left[v'(p) + (1+\lambda)na(n)\right]\left(\frac{dp}{dn}\right)^c + \lambda kna(n)\frac{dp}{dn}$$

$$\frac{dW}{dk} = \lambda kna(n)\frac{dp}{dk}$$

(19)

These expressions indicate first, that if $\lambda = 0$ but $(dp/dn)^c < 0$, then there is a world welfare gain from an expansion of x-sector employment, $dW/dn > 0$. Thus, at the policy equilibrium, subsidies are too low (from the point of view of world welfare) as countries perceive that an increase in the subsidy rate will worsen their terms of trade. Alternatively, if there is a premium on public funds, $\lambda > 0$ (and setting $(dp/dn)^c = 0$), then there is world welfare gain from capping the subsidy, so reducing n and k, raising p and hence raising welfare. The reason is that there is a strategic complementarity in the subsidy game -- as one country raises its subsidy so the best response subsidy rates of other countries increases -- combined now with a real cost of using the subsidy, due to the premium on public funds.

These extensions illustrate that adding extra distortions, through manipulation of the terms

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of trade or revenue constraints, can either weaken or strengthen the case for international policy regulation. However, they also illustrate the sense in which our basic model , and the consequent efficiency of the policy equilibrium, captures a useful benchmark case.

4. Conclusions:

The paper examines the welfare economics of the simplest possible model in which the size and number of clusters of industrial activity is endogenously determined. In the absence of policy the equilibrium output of the sector is less than the level that maximises world welfare, and the division of this output between countries also differs from the world optimum. At equilibrium clusters are typically too small, and (unless demand is quite price elastic) there will be too many of them. Real income is higher in countries that have a cluster of activity than in countries that do not, and this creates an incentive for subsidisation (or other active industrial policy) to attract a cluster. Although intuition might suggest that the subsidy is over-used as governments compete to have one of these clusters in their country, this turns out to not be the case. Competition for clusters will increase the supply and reduce the price of the output of the sector. Since clusters arise because of increasing returns this is welfare increasing, and the paper shows that in an important central case the policy equilibrium coincides with the world welfare maximum.

Appendix 1:

We want to prove that $p^o < p^e$. Suppose that p is the same at both, then in figure A1 the equilibrium value of n is at E (equations (7) and (8)) and the optimum value of n is at O, (equation (5)), greater by amount Δn. We now show that, if Y' is convex and a is concave, then the increase in $pna(n)$ between points E and O is greater than the decrease in $Y(L - n)$, so that the equality required by equation (6) cannot be satisfied. To satisfy it requires that p is lower at O than at E.

Changes are illustrated by areas in figure A1. Changes in both $pna(n)$ and $Y(L-n)$ share area EGMQ, so we need to prove that area HFGJ is greater than area OEG. Area HFGJ is,

$$HFGJ = pa'(\tilde{n})\Delta n[n + \Delta n] \tag{A1}$$

where $\tilde{n}$ is obtained by a mean value theorem. If Y' is convex, then area OEG satisfies,

$$OEG \leq p\Delta n\left[a'(\tilde{n})\Delta n + a'(n + \Delta n)[n + \Delta n]\right]/2 \tag{A2}$$

The right hand side of this is the area of the triangle shape bounding OEG. In square brackets, the firms term is the height FG, and the second is the vertical distance between the two curves, distance OF. Subtracting,

$$HFGJ - OEG \geq p\Delta n\left(a'(\tilde{n})[n + \Delta n/2] - a'(n + \Delta n)[n + \Delta n]/2\right) \geq 0 \tag{A3}$$

Since a is concave and $n + \Delta n \geq \tilde{n}$, this expression is positive, and strictly positive if the convexity of Y' or concavity of a is strict anywhere in the interval $n, n + \Delta n$.

Appendix 2:

The equilibrium selects n in a manner that minimises a function, $E(n)$, defined as

$$E(n) = Y'(L - n)/a(n)$$

while the optimum minimises

$$O(n) = \left(Y(L) - Y(L - n)\right)/na(n)$$

Combining these, we have

$$O(n) = E(n)\left(Y(L) - Y(L - n)\right)/nY'(L - n)$$

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Differentiating gives, with some rearrangement

$$O'(n) = E'(n)\frac{Y(L) - Y(L-n)}{nY'(L-n)} + \frac{E(n)}{n}\left[1 - \frac{Y(L) - Y(L-n)}{nY'(L-n)}\left(1 - \frac{nY''(L-n)}{Y'(L-n)}\right)\right]$$

If both $O(n)$ and $E(n)$ have a unique minimum, then the minimum of $O(n)$ will be at a higher value of n than the minimum of $E(n)$ if $0'(n) < 0$ at $E'(n) = 0$. Defining $A(n)$ as,

$$A(n) = \left[1 - \frac{Y(L) - Y(L-n)}{nY'(L-n)}\left(1 - \frac{nY''(L-n)}{Y'(L-n)}\right)\right]$$

We want to prove that $A(n) < 0$ at the equilibrium point. Using a third order Taylor's expansion for $Y(L) - Y(L - n)$ (where $\tilde{n}$ makes the expansion exact), gives

$$A(n) = \frac{nY''(L-n)}{2Y'(L-n)}\left[1 + \frac{nY''(L-n)}{2Y'(L-n)}\right] - \frac{n^2Y'''(L-\tilde{n})}{6Y'(L-n)}\left[1 - \frac{nY''(L-n)}{2Y'(L-n)}\right]$$

This is negative if $Y(L - n)$ takes the quadratic form given in (A5) below, or if n is small.

Appendix 3: Examples

The figures of the text are produced using the following functional forms:

$$K = 1, \quad v'(p) = -0.3p^{-2}, \quad a(n) = 0.2 + n^{0.2}, \quad Y(L-n) = (2-n)^{-0.25}. \quad \text{(A4)}$$

Closed form solutions can be derived for the following example. Let y production be given by,

$$Y = \alpha + \beta(L-n) - \gamma(L-n)^2/2, \quad Y' = \beta - \gamma(L-n), \quad Y'' = -\gamma. \quad \text{(A5)}$$

so the marginal product schedule is linear. x- production is,

$$a(n) = n^{\theta}, \quad \theta \in (0, 1).$$

Eliminating p by taking the ratio of equations (5) and (6),

$$\big(Y(L) - Y(L - n)\big)/Y'(L - n) = na(n)/\big(a(n) + na'(n)\big),$$

from which the welfare maximising value n^o can be derived as

$$n^o = 2\theta(\beta - \gamma L)/\gamma(1 - \theta). \tag{A6}$$

Eliminating p by taking the ratio of equations (7) and (8),

$$- Y'(L - n)/Y''(L - n) = a(n)/a'(n),$$

from which the equilibrium value n^e can be derived as

$$n^e = \theta(\beta - \gamma L)/\gamma(1 - \theta). \tag{A7}$$

Thus the number of employees in each cluster at the optimum is ½ the number at the equilibrium.

Now let world demand for x-output be $Kp^{-\eta}$, so that equation (3) is,

$$p = \big(n^{\theta+1}k/K\big)^{-1/\eta}.$$

Equation (5) then becomes

$$\beta - \gamma(L - n^o) = (1 + \theta)\big(k^o/K\big)^{-1/\eta}\big(n^o\big)^{\theta - (\theta+1)/\eta} \tag{A8}$$

If $\eta = 1$, this gives,

$$k^o = \frac{\gamma K}{2\theta}\left(\frac{1 - \theta}{\beta - \gamma L}\right)^2.$$

Similarly, equation (7) then becomes

$$\beta - \gamma(L - n^e) = \big(k^e/K\big)^{-1/\eta}\big(n^e\big)^{\theta - (\theta+1)/\eta} \tag{A9}$$

If $\eta = 1$, this gives,

$$k^e = \frac{\gamma K}{\theta}\left(\frac{1 - \theta}{\beta - \gamma L}\right)^2.$$

Thus, the equilibrium number of x-active countries is twice the optimum number, $k^e = 2k^o$. The equilibrium total quantity of output is $2(\frac{1}{2})^{\theta+1} = 2^{-\theta}$ times that at the optimum and the price is 2^θ times higher. Under what conditions is $k^e > k^o$? Taking the ratio of (A8) to (A9) with $k^e = k^o$ gives

$$\frac{\beta - \gamma(L - n^o)}{\beta - \gamma(L - n^e)} = (1 + \theta)\left(\frac{n^o}{n^e}\right)^{\theta - (\theta+1)/\eta}$$

which, using (A6) and (A7) can be shown to hold when

$$\eta = (1 + \theta)/\theta. \tag{A10}$$

Thus, there are too many clusters, $k^e > k^o$, providing $\eta < (1 + \theta)/\theta$, (including $\eta = 1$). Only if demand is more elastic than this will we have $k^e < k^o$; in this case the higher equilibrium price chokes off quantity sufficiently that a smaller number of countries are x-active.

Endnotes:

1. For a good modern statement of this see Becker and Henderson (2000). The fact that at the optimum city land rents should be transferred to the increasing returns activity has given rise to the label 'Henry George' theorems for these results.

2. The second order condition is clear from figure 2, discussed below. On this figure the curves $Y'(L - n)$ and $p[a(n) + na'(n)]$ intersect twice. The second order condition is that the maximum is the upper intersection.

3. There cannot be a situation in which $p^e a(n^e) < Y'(L - n^e)$ as the coalition knows that workers would leave at the second stage.

4. An 'Armington' structure of y-sector demand.

5. There is a continuum of countries, so this discussion is more accurately phrased as a measure of countries introducing the subsidy, and a measure exiting.

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Figure 1: Equilibrium

Figure 2: Optimum

Eqn. (5):
$Y'(L-n) = p[a(n) + na'(n)]$

Equil

World welfare contour

Opt

Eqn. (6):
$Y(L) = pna(n) + Y(L-n)$

Eqn. (8):
$Y``(L-n) = pa'(n)$

Eqn. (7):
$Y'(L-n) = pa(n)$
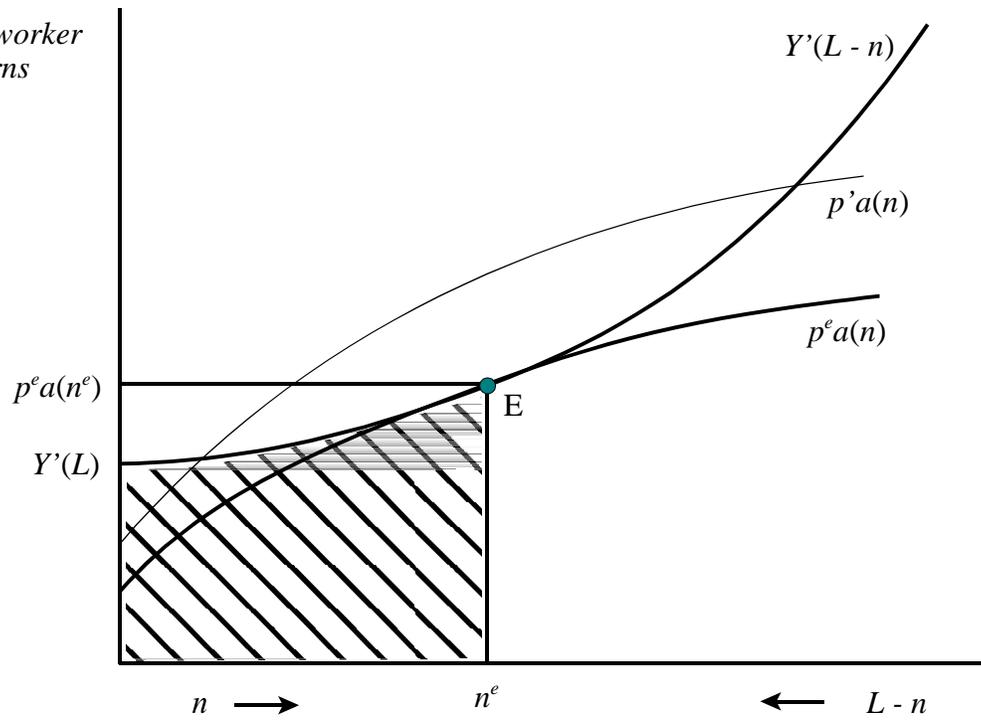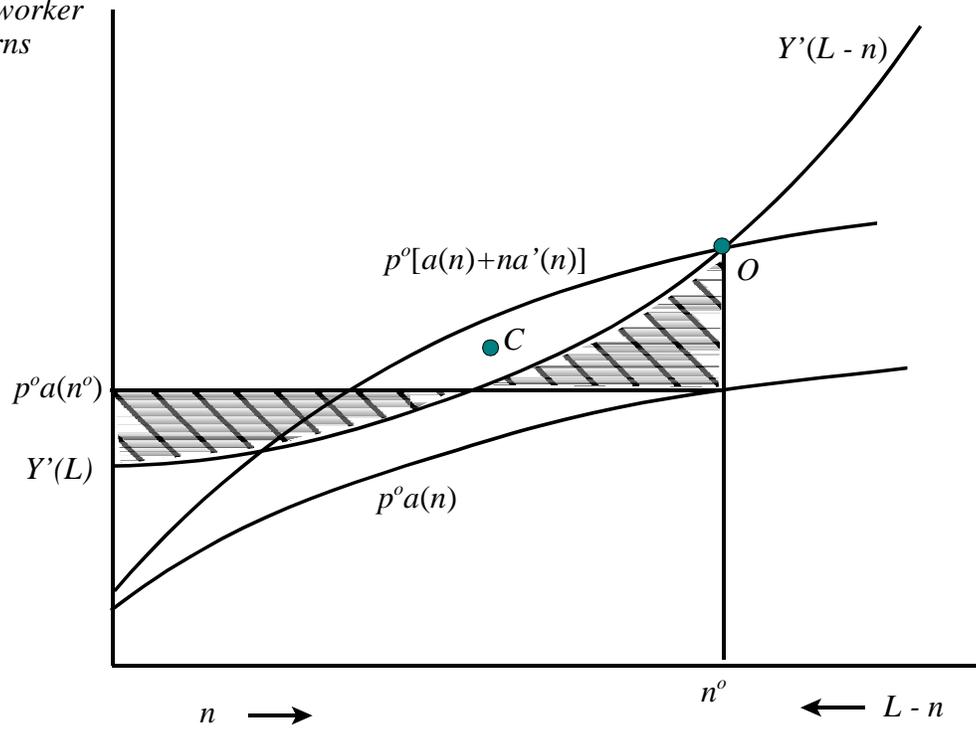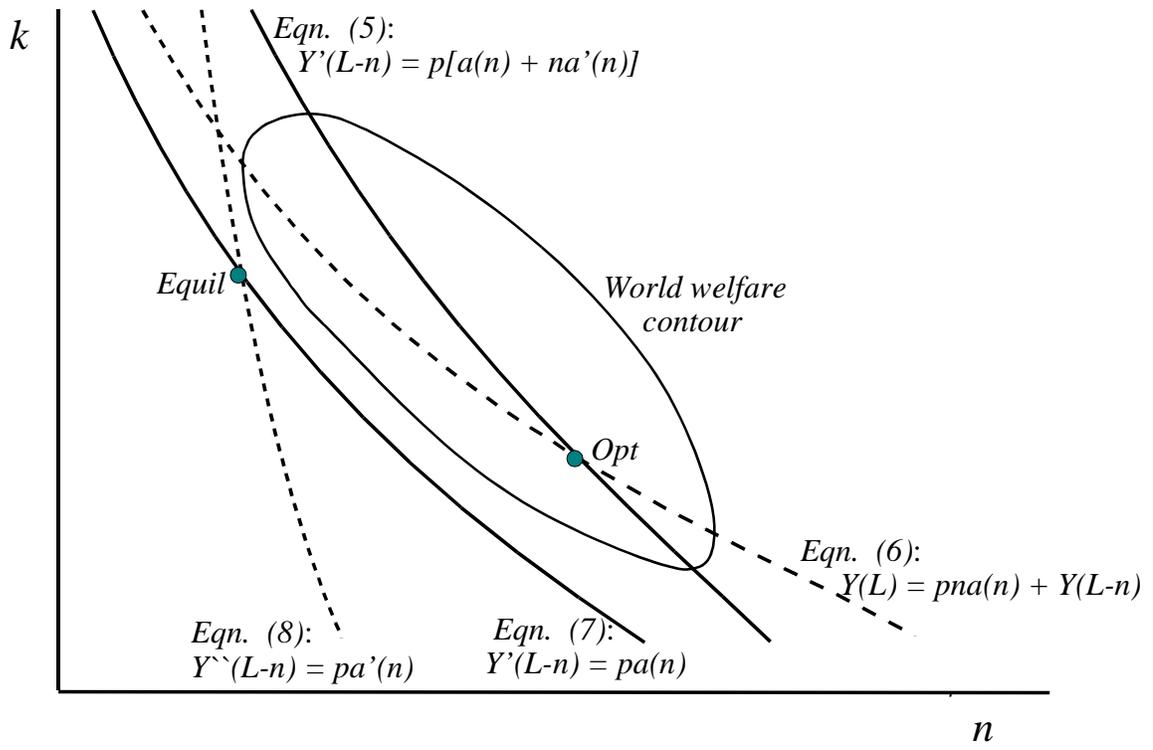
Figure 3: *Equilibrium and Optimum*

u^k

u^o

u^{K-k}

p^e

E

O

p^e

p^o
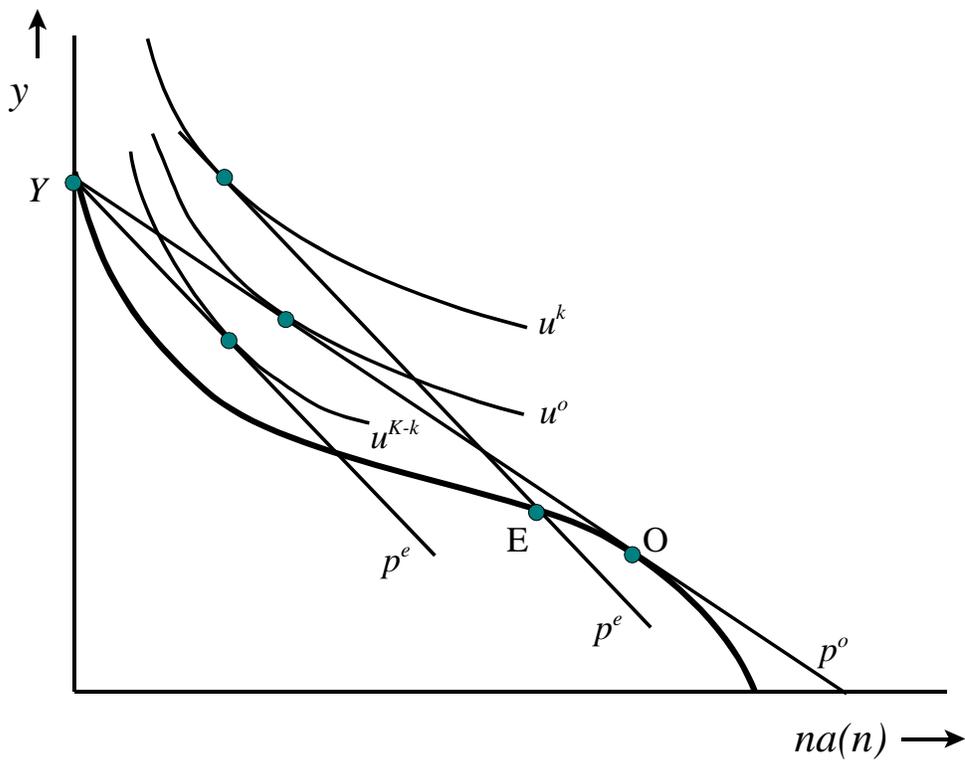
Figure 4: *Equilibrium and Optimum*
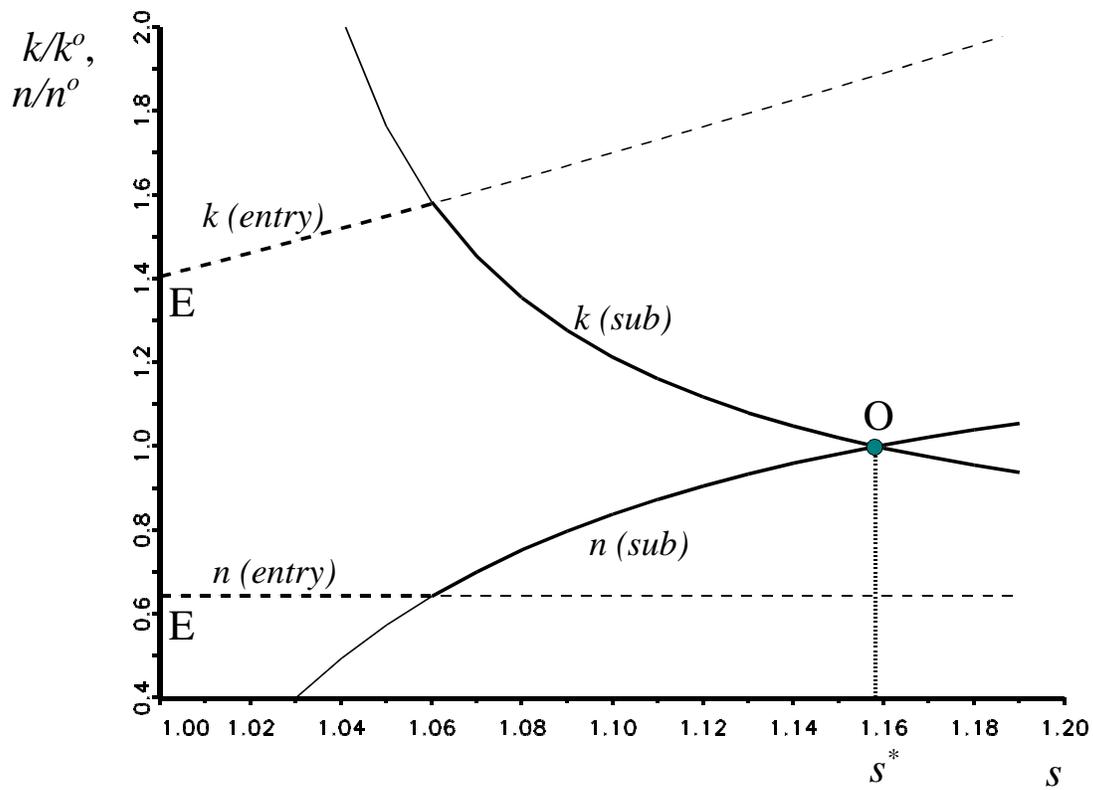
Figure 5: Policy regulation

Figure A1