RESOLVING DISTRIBUTIONAL CONFLICTS BETWEEN GENERATIONS

GEIR B. ASHEIM AND BERTIL TUNGODDEN

Abstract. We describe a new approach to the problem of resolving distributional conflicts between an infinite and countable number of generations. We impose conditions on the social preferences that capture the following idea: If indifference or preference holds between truncated paths for infinitely many truncating times, then indifference or preference holds also between the untruncated infinite paths. In this framework we show (1) how such conditions illustrate the problem of combining Strong Pareto and impartiality in an intergenerational setting, and (2) how equity conditions well-known from the finite setting can be used to characterize different versions of leximin and utilitarianism.
Journal of Economic Literature Classification Numbers: D63, Q01.
Keywords: Intergenerational justice, Leximin, Utilitarianism.

1. Introduction

The Suppes-Sen grading principle captures both a concern for equal treatment of generations and the demand for efficiency. And it turns out that this is all that is needed in order to justify sustainable solutions within reasonable technological frameworks.[1]

However, there are two problems with this approach. First, it has been argued that the Suppes-Sen grading principle cannot capture impartiality among an infinite and countable number of generations in a satisfactory manner (Liedekerke and Lauwers [7]). Second, even if we should accept this justification for sustainability, there exists the further problem about how

Acknowledgements: We thank David Miller for comments. Asheim gratefully acknowledge financial support from the Hewlett Foundation.

Addresses: Geir B. Asheim, Dept. of Econ., University of Oslo, P.O. Box 1095 Blindern, 0317 Oslo, Norway, and SIEPR, 579 Serra Mall, Stanford University, CA 94305, USA. *Email*: g.b.asheim@econ.uio.no

Bertil Tungodden, Dept. of Econ., Norwegian School of Economics and Business Administration, Hellev. 30, 5045 Bergen, Norway. *Email*: bertil.tungodden@nhh.no

Date: 7 May 2002.

[1]See Asheim, Buchholz, and Tungodden [2].

to resolve distributional conflicts between generations that go beyond the sustainability question. In the following, we consider both these problems.

In Section 3, we look at the problem of intergenerational impartiality. According to Liedekerke and Lauwers [7, p. 163], formal impartiality is ensured by imposing the axiom of Strong Anonymity (entailing indifference to any permutation of utilities of an *infinite* number of generations). As is well-known, this demand cannot be combined with the Strong Pareto axiom. Liedekerke and Lauwers suggest to establish a framework where an acceptable trade-off between the demands of impartiality and Strong Pareto can be made. This implies a rejection of the Suppes-Sen grading principle, which is characterized by Weak Anonymity (entailing indifference to any permutation of utilities of only a *finite* number of generations) and Strong Pareto.

One might think that it should be possible to find some intermediate position, where impartiality is extended beyond Weak Anonymity (by entailing indifference to *some* – but not all – permutations of an infinite number of generations) within a framework satisfying Strong Pareto, and hence, the Suppes-Sen grading principle. As we illustrate in the present paper, this is not an easy task. We attain a seemingly reasonable extension of Weak Anonymity by considering conditions that specify that one infinite utility path should be deemed indifferent to another infinite utility path if the head of the former utility path is indifferent to the head of the latter at infinitely many truncating times. However, as reported in Section 3, it is not possible to establish an equivalence relation without coming in conflict with Strong Pareto if impartiality is extended in this manner.

In Section 4, we look at how to resolve distributional conflicts between generations that go beyond the sustainability question. In this respect, we introduce similar conditions on (strict) preference that turn out to bring the infinite intergenerational setting into line with the framework for distributive justice in the finite setting. These conditions capture the idea that one infinite utility path should be considered strictly better than another infinite utility path if the head of the former is considered strictly better than the latter at infinitely many truncating times. Within this framework, we show how equity conditions well-known from the finite setting can be applied to the debate on infinite intergenerational justice. Moreover, we provide characterizations of some of the prominent positions in the literature.

The formal framework and the justification for sustainability based on the Suppes-Sen grading principle is introduced in Section 2, and concluding remarks are provided in Section 5.

2. The framework

There is an infinite number of generations $t = 1, 2, \dots$. The utility level of generation t is given by u_t, which should be interpreted as the utility level of a representative member of this generation. Initially (in Section 3 and the first part of Section 4), we assume that the utilities need not be more than ordinally measurable and level comparable, whereas later (in the second part of Section 4) we consider utilities that are also cardinally measurable and unit comparable.

A binary relation R over paths ${}_1\mathbf{u} = (u_1, u_2, \dots)$ starting in period 1 expresses social preferences over different intergenerational utility paths. Any such binary relation R is throughout assumed to be reflexive and transitive on the infinite Cartesian product $\mathbb{R}^\infty$ of the set of real numbers $\mathbb{R}$, where $\infty = |\mathbb{N}|$ and $\mathbb{N}$ is the set of natural numbers. The social preferences R may be complete or incomplete, with I denoting the symmetric part, i.e. indifference, and P denoting the asymmetric part, i.e. (strict) preference. For any path ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and any time T, ${}_1\mathbf{u}_T = (u_1, u_2, \dots, u_T)$ denotes the truncation of ${}_1\mathbf{u}$ at T, and ${}_1\tilde{\mathbf{u}}_T$ is a permutation of ${}_1\mathbf{u}_T$ having the property that ${}_1\tilde{\mathbf{u}}_T$ is non-decreasing. Refer to ${}_1\mathbf{u}_T$ as the T-head and ${}_{T+1}\mathbf{u}$ as the T-tail of ${}_1\mathbf{u}$.

In order to define sets of feasible paths we assume that the initial endowment of generation $t \geq 1$ is given by a $n-$dimensional ($n < \infty$) vector of capital stocks k_t. A generation t acts by choosing a utility level u_t and a vector of capital stocks k_{t+1} which is bequeathed to the next generation $t+1$. For every t, the function F_t gives the maximum utility attainable for generation t if k_t is inherited and k_{t+1} is bequeathed; i.e., $u_t \leq F_t(k_t, k_{t+1})$ has to hold for any feasible *utility-bequest pair* (u_t, k_{t+1}) of generation t. Furthermore, it is assumed that the utility level of each generation cannot fall below a certain lower bound $\underline{u}$. This lower bound serves two purposes. First, $\underline{u}$ can be interpreted as the subsistence level of any generation. Second, since there are technological limitations on the accumulation of stocks in the course of one period, $F_t(k_t, k_{t+1}) < \underline{u}$ can be used to capture that the bequest k_{t+1} is infeasible given the inheritance k_t. Hence, generation t's utility-bequest pair

(u_t, k_{t+1}) is said to be *feasible* at t given k_t if $\underline{u} < u_t \leq F_t(k_t, k_{t+1})$. The sequence ${}_1\mathbf{F} = (F_1, F_2, ...)$ characterizes the technology of the economy under consideration. Given the technology ${}_1\mathbf{F}$, a utility path ${}_t\mathbf{u} = (u_t, u_{t+1}, \dots)$ is feasible at t given k_t if there exists a path ${}_{t+1}\mathbf{k} = (k_{t+1}, k_{t+2}, \dots)$ such that, for all $s \geq t$, generation s's utility bequest pair (u_s, k_{s+1}) is feasible at s given k_s.

A utility path ${}_1\mathbf{v}$ weakly Pareto-dominates another utility path ${}_1\mathbf{u}$ if every generation is weakly better of in ${}_1\mathbf{v}$ than in ${}_1\mathbf{u}$ and some generation is strictly better off. A utility path ${}_1\mathbf{v}$ is said to be efficient if there is no other feasible utility path that weakly Pareto-dominates this path. A feasible utility path ${}_1\mathbf{v}$ is said to be *R-maximal*, if there exists no feasible path ${}_1\mathbf{u}$ such that ${}_1\mathbf{u}\, P\, {}_1\mathbf{v}$. A feasible utility path ${}_1\mathbf{v}$ is said to be *R-optimal*, if ${}_1\mathbf{v}\, R\, {}_1\mathbf{u}$ for any feasible path ${}_1\mathbf{u}$. Any R-optimal path is R-maximal, while the converse need not hold if R is incomplete.

Within this framework, the justification for sustainability in Asheim, Buchholz, and Tungodden [2] rests on one technological assumption and two conditions on the social preferences. First, the following domain restriction is imposed on the technological framework.

Assumption 1 (Immediate Productivity of ${}_1\mathbf{F}$)**.** If ${}_t\mathbf{u} = (u_t, u_{t+1}, \dots)$ is feasible at t given k_t with $u_t > u_{t+1}$, then $(u_{t+1}, u_t, u_{t+2}, \dots)$ is feasible and inefficient at t given k_t.

This assumption means that if a generation has higher utility than the next, then its excess utility can be transferred at negative cost to its successor. It thus generalizes positive net capital productivity to a setting where utilities need not be more than ordinally measurable and level comparable. Second, the following two conditions are imposed on the social preferences.

Condition SP (Strong Pareto)**.** For any ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in \mathbb{R}^\infty$, if $v_t \geq u_t$ for all t and $v_s > u_s$ for some s, then ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$.

Condition WA (Weak Anonymity)**.** For any ${}_1\mathbf{u}$, ${}_1\mathbf{v} \in \mathbb{R}^\infty$, if for some finite permutation π, $v_{\pi(t)} = u_t$ for all t, then ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$.[2]

Conditions SP and WA generate the *Suppes-Sen grading principle* R^{S}. The binary relation R^{S} deems two paths to be indifferent if one is obtained from

[2]A permutation, i.e., a bijective mapping of $\{1, 2, \dots\}$ onto itself, is finite whenever there is a T such that $\pi(t) = t$ for any $t > T$.

the other through a finite permutation, and one utility path to be preferred to another if a finite permutation of the former weakly Pareto-dominates the other. The Suppes-Sen grading principle R^{S} is a subrelation[3] to the social preferences R if and only if R satisfies SP and WA.

Define sustainability in the following standard way.

Definition 1 (Sustainability)**.** Generation t with inheritance k_t is said to *behave in a sustainable manner* if it chooses a feasible utility-bequest pair (u_t, k_{t+1}) so that the constant utility path $(u_t, u_t, \dots)$ is feasible at $t+1$ given k_{t+1}. The utility *path* ${}_1\mathbf{u} = (u_1, u_2, \dots)$ is called *sustainable* given k_1 if there exists ${}_2\mathbf{k} = (k_2, k_3, ...)$ such that every generation behaves in a sustainable manner along $({}_1\mathbf{k}, {}_1\mathbf{u}) = (k_1, (u_1, k_2), (u_2, k_3), ...)$.

We can now state the justification for sustainability.

Proposition 1 (Asheim, Buchholz and Tungodden [2])**.** *If the social preferences R satisfy Strong Pareto and Weak Anonymity, and the technology satisfies immediate productivity, then only sustainable utility paths are R-maximal.*

Proof. See Asheim, Buchholz, and Tungodden [2]. □

3. Intergenerational Impartiality

It has been argued that in order to capture the whole idea of impartiality in an intergenerational setting, we have to deem any two utility paths as equally good if they can be derived from each other by a permutation of the utilities. The problem is that by making a permutation of the utilities of an infinite number of generations, we may end up in a direct conflict with Strong Pareto. By way of illustration, ${}_1\mathbf{v} = (1, 0, 1, 0, 1, 0, \dots)$ can be attained from ${}_1\mathbf{u} = (0, 0, 1, 0, 1, 0, \dots)$ by a permutation where generation 2 gets the utility of generation 1, generation t the utility of generation $t+2$ when t is an odd number, and generation $t+2$ the utility of generation t when t is an even number. On the basis of this fact, most economists have chosen to adopt a weaker version of impartiality, which only endorses indifference when one of the utility paths can be derived from the other by

[3] R' is said to be a *subrelation* to R'' if (i) ${}_1\mathbf{v}\, I'\, {}_1\mathbf{u}$ implies ${}_1\mathbf{v}\, I''\, {}_1\mathbf{u}$ and (ii) ${}_1\mathbf{v}\, P'\, {}_1\mathbf{u}$ implies ${}_1\mathbf{v}\, P''\, {}_1\mathbf{u}$, with I' and I'' and P' and P'' denoting the symmetric and asymmetric parts of R' and R'', respectively.

a *finite* number of permutations. It is this weak version of impartiality that is captured by Weak Anonymity.

This move, however, does not seem to be entirely satisfactory, because as pointed out by Liedekerke and Lauwers [7, p. 165] (in the present terminology and notation): "Weak [Anonymity] only affects T-head of the utility stream and not its T-tail for large enough T. But the T-tail of an infinite utility stream is infinitely larger than its T-head, therefore the [W]eak [Anonymity] condition only guarantees impartiality for a (negligibly) small part of the utility stream". Liedekerke and Lauwers accordingly suggest that we should adopt a framework that incorporates a trade-off between Weak Anonymity and Strong Pareto.

Weak Anonymity can certainly be criticized along the lines suggested by Liedekerke and Lauwers. Here we will illustrate the problems associated with extending indifference between infinite paths without coming in conflict with Strong Pareto, through an analysis that will set the stage for the characterizations we present in Section 4.

Consider the following condition.

Condition WIC (Weak Indifference Continuity)**.** If ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and ${}_1\mathbf{v} = (v_1, v_2, \dots)$ are two utility paths, and there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, I\, {}_1\mathbf{u}$, then ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$.

WIC states that an infinite utility path should be considered indifferent to another infinite utility path if the head of the former is considered indifferent to the latter *at every point in time* beyond a certain initial phase.

It is straightforward to see that WIC and WA generate the following equivalence (i.e., reflexive, symmetric, and transitive) relation.

Definition 2. For any two utility paths ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, the relation ${}_1\mathbf{u}\, R^{\mathrm{M}}_{\mathrm{W}}\, {}_1\mathbf{v}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, ${}_1\tilde{\mathbf{u}}_T = {}_1\tilde{\mathbf{v}}_T$.

Proposition 2. *The relation $R^{\mathrm{M}}_{\mathrm{W}}$ satisfies Weak Anonymity and Weak Indifference Continuity, and $R^{\mathrm{M}}_{\mathrm{W}}$ is a subrelation to any R satisfying Weak Anonymity and Weak Indifference Continuity.*

Proof. Trivial. □

However, if our aim is to extend impartiality to cases involving an infinite number of generations, nothing is gained by combining Weak Anonymity with Weak Indifference Continuity.

Proposition 3. *For any* ${}_1\mathbf{v}$ *and* ${}_1\mathbf{u}$*,* ${}_1\mathbf{v}\, I^{\mathrm{M}}_{\mathrm{W}}\, {}_1\mathbf{u}$ *if and only if for some* $T \geq 1$*,* ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$ *and* ${}_{T+1}\mathbf{v} = {}_{T+1}\mathbf{u}$*.*

Proof. The if-part of the proposition is trivial, and thus we will only prove the only-if part. Suppose ${}_1\mathbf{v}\, I^{\mathrm{M}}_{\mathrm{W}}\, {}_1\mathbf{u}$. In that case, there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$ and ${}_1\tilde{\mathbf{v}}_{T+1} = {}_1\tilde{\mathbf{u}}_{T+1}$. Hence, $v_{T+1} = u_{T+1}$, and thus ${}_{\hat{T}+1}\mathbf{v} = {}_{\hat{T}+1}\mathbf{u}$. The result follows. $\square$

Proposition 3 shows that if Weak Anonymity and Weak Indifference Continuity deem two paths indifferent when one is derived from the other through an infinite permutation, then the permutation has no consequence for the levels of utilities that are assigned to generations beyond some finite point in time. Hence, these cases are already covered by Weak Anonymity.

A stronger operationalization of impartiality can be attained by combining Weak Anonymity with the following condition.

Condition SIC (Strong Indifference Continuity)**.** If ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and ${}_1\mathbf{v} = (v_1, v_2, \dots)$ are two utility paths, and for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, I\, {}_1\mathbf{u}$, then ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$.

SIC means that an infinite utility path should be considered indifferent to another infinite utility path if, at *any* point in time, there is a *future* point in time at which the head of the former is considered indifferent to the latter.

If we combine Strong Indifference Continuity and Weak Anonymity, then we are able to express impartiality in a large number of cases covering permutations of an infinite number of generations. As an illustration, compare ${}_1\mathbf{v} = (1, 0, 1, 0, 1, 0, \dots)$ and ${}_1\mathbf{w} = (0, 1, 0, 1, 0, 1, \dots)$, where ${}_1\mathbf{v}$ can be attained from ${}_1\mathbf{w}$ by a permutation of the utilities of generation 1 and 2, 3 and 4, 5 and 6, and so on. In this case, as argued by Liedekerke and Lauwers [7, p. 162], equal treatment of generations should imply indifference between ${}_1\mathbf{v}$ and ${}_1\mathbf{w}$. This cannot be captured by Weak Anonymity alone. While if we combine WA and SIC, then it appears that we attain the desired conclusion. However, any equivalence relation that satisfies WA and SIC comes in conflict with Strong Pareto.

Proposition 4. *If* $R^{\mathrm{M}}_{\mathrm{S}}$ *is the transitive closure of some* R *satisfying Weak Anonymity and Strong Indifference Continuity, then* $R^{\mathrm{M}}_{\mathrm{S}}$ *does not satisfy Strong Pareto.*

Proof. Consider the following paths: ${}_1\mathbf{u} = (0,0,1,0,1,0,\dots)$, ${}_1\mathbf{v} = (1,0,1,0,1,0,\dots)$, and ${}_1\mathbf{w} = (0,1,0,1,0,1,\dots)$. Since R satisfies WA and SIC, it follows that ${}_1\mathbf{v}\, I\, {}_1\mathbf{w}$ and ${}_1\mathbf{w}\, I\, {}_1\mathbf{u}$. Since $R^{\mathrm{M}}_{\mathrm{S}}$ is the transitive closure of R, we have that ${}_1\mathbf{v}\, I^{\mathrm{M}}_{\mathrm{S}}\, {}_1\mathbf{u}$. This shows that $R^{\mathrm{M}}_{\mathrm{S}}$ does not satisfy SP. □

Props. 2–4 illustrate the problems associated with extending indifference between infinite paths. On the one hand, if we add WIC to Weak Anonymity we obtain the same equivalence relation that is implied by Weak Anonymity alone. On the other hand, any equivalence relation that satisfies SIC and Weak Anonymity comes in conflict with Strong Pareto.

Let I and P be the symmetric and asymmetric parts of a binary relation R. Write $\mathcal{I}_T := \{({}_1\mathbf{v},\, {}_1\mathbf{u}) |\, ({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, I\, {}_1\mathbf{u}\}$ and $\mathcal{I}_\infty := \{({}_1\mathbf{v},\, {}_1\mathbf{u}) |\, {}_1\mathbf{v}\, I\, {}_1\mathbf{u}\}$. Then WIC means that lim inf of the sequence $\mathcal{I}_T$ is included in $\mathcal{I}_\infty$,

$$\bigcup_{\hat{T}=1}^{\infty} \bigcap_{T=\hat{T}}^{\infty} \mathcal{I}_T \subseteq \mathcal{I}_\infty\,,$$

while SIC means that lim sup of the sequence $\mathcal{I}_T$ is included in $\mathcal{I}_\infty$,

$$\bigcap_{\hat{T}=1}^{\infty} \bigcup_{T=\hat{T}}^{\infty} \mathcal{I}_T \subseteq \mathcal{I}_\infty\,.$$

Hence, WIC and SIC correspond to different kinds of continuity at infinity for the symmetric part I. The analogous kinds of continuity at infinity can be defined for the asymmetric part P. We show in the next section that such preference continuity can be used to characterize different versions of leximin and utilitarianism.

4. Characterizations

As stated in Section 2 we can, following Asheim, Buchholz, and Tungodden [2], justify sustainability through the conditions of Strong Pareto and Weak Anonymity. The relation generated by SP and WA – the Suppes-Sen grading principle, R^S – is, however, incomplete. As reported in Section 3 it seems difficult to extend indifference beyond WA without coming conflict with SP. In the present section, we pose another problem: how to resolve distributional conflicts between generations when comparing paths that are R^S-maximal. This problem amounts to extending (strict) preference beyond what is implied by the Suppes-Sen grading principle.

From a technical viewpoint, WIC and SIC establish a nice link to the standard finite setting of distributive justice. Within the framework of WIC and SIC, the comparison of any two infinite utility paths amounts to comparing an infinite number of utility paths each containing a *finite* number of generations. Similarly, we may apply well-known equity conditions and arguments from the traditional literature on distributive justice more generally if we introduce conditions similar to WIC and SIC for the asymmetric part of the social preferences. In this respect, there are two options.

Condition WPC (Weak Preference Continuity)**.** If ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and ${}_1\mathbf{v} = (v_1, v_2, \dots)$ are two utility paths, and there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, P\, {}_1\mathbf{u}$, then ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$.

Condition SPC (Strong Preference Continuity)**.** If ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and ${}_1\mathbf{v} = (v_1, v_2, \dots)$ are two utility paths, and there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, R\, {}_1\mathbf{u}$, and for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, P\, {}_1\mathbf{u}$, then ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$.

These conditions can alternatively be formulated as follows. Write

$$\mathcal{R} := \bigcup_{\hat{T}=1}^{\infty} \bigcap_{T=\hat{T}}^{\infty} \left\{ ({}_1\mathbf{v},\, {}_1\mathbf{u}) \middle|\, ({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, R\, {}_1\mathbf{u} \right\}.$$

If R is complete for comparisons between paths having the same tail, then $\mathcal{R}$ denotes the set pairs $({}_1\mathbf{v},\, {}_1\mathbf{u})$ satisfying that beyond some $\hat{T}$ there exists no T such that ${}_1\mathbf{u}$ is preferred to $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})$. Write $\mathcal{P}_T := \{({}_1\mathbf{v},\, {}_1\mathbf{u}) \in \mathcal{R} |\, ({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, P\, {}_1\mathbf{u}\}$ and $\mathcal{P}_\infty := \{({}_1\mathbf{v},\, {}_1\mathbf{u}) |\, {}_1\mathbf{v}\, P\, {}_1\mathbf{u}\}$. Then WPC means that lim inf of the sequence $\mathcal{P}_T$ is included in $\mathcal{P}_\infty$,

$$\bigcup_{\hat{T}=1}^{\infty} \bigcap_{T=\hat{T}}^{\infty} \mathcal{P}_T \subseteq \mathcal{P}_\infty\,,$$

while SPC means that lim sup of the sequence $\mathcal{P}_T$ is included in $\mathcal{P}_\infty$,

$$\bigcap_{\hat{T}=1}^{\infty} \bigcup_{T=\hat{T}}^{\infty} \mathcal{P}_T \subseteq \mathcal{P}_\infty\,.$$

In the following we illustrate how this framework can be used to characterize the intergenerational versions of the Rawlsian leximin principle and the utilitarian principle.

4.1. **Leximin.** We start with the Rawlsian leximin principle. It has been stated as follows in the infinite case (see, e.g., Asheim [1, p. 2]), where "S" indicates that $R^{\mathrm{L}}_{\mathrm{S}}$ will be shown to correspond to the *Strong* Preference Continuity:

Definition 3 (S-Leximin)**.** For any two utility paths ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, the S-Leximin relation ${}_1\mathbf{v}\, R^{\mathrm{L}}_{\mathrm{S}}\, {}_1\mathbf{u}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, either ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$ or there is a $s \in \{1, \ldots, T\}$ with $\tilde{v}_t = \tilde{u}_t$ for all $1 \leq t < s$ and $\tilde{v}_s > \tilde{u}_s$.

Alternatively, we can give a weaker formulation of leximin, $R^{\mathrm{L}}_{\mathrm{W}}$, that will be shown to correspond to the *Weak* Preference Continuity.

Definition 4 (W-Leximin)**.** For any two utility paths ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, ${}_1\mathbf{v}\; I^{\mathrm{L}}_{\mathrm{W}}\, {}_1\mathbf{u}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$. Moreover, ${}_1\mathbf{v}\, P^{\mathrm{L}}_{\mathrm{W}}\, {}_1\mathbf{u}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, there is a $s \in \{1, ..., T\}$ with $\tilde{v}_t = \tilde{u}_t$ for all $1 \leq t < s$ and $\tilde{v}_s > \tilde{u}_s$.

Let us start out by characterizing $R^{\mathrm{L}}_{\mathrm{S}}$. It is well-known that the leximin principle covering finite cases can be characterized by the Suppes-Sen grading principle and the equity condition suggested by Hammond [4, 5].

Condition HE (Hammond Equity)**.** If ${}_1\mathbf{u} = (u_1, u_2, \ldots)$ and ${}_1\mathbf{v} = (v_1, v_2, \ldots)$ are two utility paths, and there exists j, k such that $u_j > v_j > v_k > u_k$ and $u_i = v_i$ for all $i \neq j, k$, then ${}_1\mathbf{v}\, R\, {}_1\mathbf{u}$.

However, in general it is not straightforward to translate this result into the infinite case.[4] However, by applying SPC, we obtain the following characterization.

Proposition 5. *$R^{\mathrm{L}}_{\mathrm{S}}$ is a subrelation to R if and only if R satisfies Strong Pareto, Weak Anonymity, Hammond Equity, and Strong Preference Continuity.*

Proof. (If) Assume that R satisfies SP, WA, HE, and SPC. According to the definition of a subrelation (cf. footnote 3), we have to show that, for any ${}_1\mathbf{u}$, ${}_1\mathbf{v}$, ${}_1\mathbf{v}\, I^{\mathrm{L}}_{\mathrm{S}}\, {}_1\mathbf{u}$ implies ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$ and ${}_1\mathbf{v}\, P^{\mathrm{L}}_{\mathrm{S}}\, {}_1\mathbf{u}$ implies ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$. This naturally divides the if part of the proof into two subparts.

[4] Lauwers [6] characterizes the maximin relation by a version of Hammond Equity within a framework where Strong Pareto is relaxed.

(1) Consider any ${}_1\mathbf{u}$, ${}_1\mathbf{v}$ such that ${}_1\mathbf{v}\, I^{\mathrm{L}}_{\mathrm{S}}\, {}_1\mathbf{u}$. By definition of $R^{\mathrm{L}}_{\mathrm{S}}$, there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, ${}_1\tilde{\mathbf{v}}_T = {}_1\tilde{\mathbf{u}}_T$. By WA and Prop. 3, it follows that ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$.

(2) Consider any ${}_1\mathbf{u}$, ${}_1\mathbf{v}$ such that ${}_1\mathbf{v}\, P^{\mathrm{L}}_{\mathrm{S}}\, {}_1\mathbf{u}$. By definition of $R^{\mathrm{L}}_{\mathrm{S}}$, for any $\hat{T} \geq 1$, there exist $T \geq \hat{T}$ and $s \in \{1, ..., T\}$ such that $\tilde{v}_t = \tilde{u}_t$ for all $1 \leq t < s$ and $\tilde{v}_s > \tilde{u}_s$. I.e., for any such $T \geq \hat{T}$, the sth worst off in ${}_1\tilde{\mathbf{u}}_T$ is worse off than the sth worst off in ${}_1\tilde{\mathbf{v}}_T$. For given $T \geq \hat{T}$, we can now construct a utility path ${}_1\hat{\mathbf{u}}$ by means of a sequence of steps involving conflicts between two generations. The first step involves a conflict between the sth worst off and the $(s+1)$st worst off, the second step a conflict between the sth worst off and the $(s+2)$nd worst off, and so on until the conflict between the sth worst off and the best off is included in the sequence. In each step, let the the sth worst off generation in ${}_1\tilde{\mathbf{u}}_T$ gain less than $1/(T-s+1)$ of the difference between the utility level this generation attains in ${}_1\tilde{\mathbf{u}}_T$ and ${}_1\tilde{\mathbf{v}}_T$, and let the better off generation in ${}_1\tilde{\mathbf{u}}_T$ attain the same as the minimum of what this generation gets in ${}_1\tilde{\mathbf{u}}_T$ and ${}_1\tilde{\mathbf{v}}_T$. Let ${}_1\hat{\mathbf{u}}_T$ be the path derived from ${}_1\tilde{\mathbf{u}}_T$ by this sequence. By HE (and SP in cases for which the better off generation does not have higher utility in ${}_1\tilde{\mathbf{u}}_T$), each such step is at least a weak improvement according to R. Hence, by transitivity we have that

$$({}_1\hat{\mathbf{u}}_T,\, {}_{T+1}\mathbf{u})\, R\, ({}_1\tilde{\mathbf{u}}_T,\, {}_{T+1}\mathbf{u})\,.$$

Since R satisfies SP, it follows that

$$({}_1\tilde{\mathbf{v}}_T,\, {}_{T+1}\mathbf{u})\, P\, ({}_1\hat{\mathbf{u}}_T,\, {}_{T+1}\mathbf{u})\,,$$

while WA implies that

$$({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, I\, ({}_1\tilde{\mathbf{v}}_T,\, {}_{T+1}\mathbf{u}) \;\text{ and }\; ({}_1\tilde{\mathbf{u}}_T,\, {}_{T+1}\mathbf{u})\, I\, {}_1\mathbf{u}\,.$$

Hence, by transitivity, $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, P\, {}_1\mathbf{u}$. Since, for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that this holds and R satisfies SPC, it now follows that ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$.

(Only if) Assume that $R^{\mathrm{L}}_{\mathrm{S}}$ is a subrelation to R. Then it is trivial to establish that R satisfies SP, WA, and HE. To show that R satisfies SPC, assume that for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, P\, {}_1\mathbf{u}$. Since $R^{\mathrm{L}}_{\mathrm{S}}$ is a subrelation to R and $R^{\mathrm{L}}_{\mathrm{S}}$ is complete for comparisons between paths having the same tail, we have that for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, P^{\mathrm{L}}_{\mathrm{S}}\, {}_1\mathbf{u}$. By definition of $R^{\mathrm{L}}_{\mathrm{S}}$, this entails that ${}_1\mathbf{v}\, P^{\mathrm{L}}_{\mathrm{S}}\, {}_1\mathbf{u}$, which in turn implies ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$ since $R^{\mathrm{L}}_{\mathrm{S}}$ is a subrelation to R. Hence, ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$

holds if for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that $({}_1\mathbf{v}_T, {}_{T+1}\mathbf{u}) \, P \, {}_1\mathbf{u}$. Thus, we have established that R satisfies SPC. □

This result deals with an immediate objection to the Rawlsian leximin position — that the leximin principle is implausible because it assigns *absolute* priority to the interests of the *worst off* generation in cases where it is in conflict with the interest of an *infinite number of future generations*. Intuitively, this seems a reasonable counterargument. However, the proposition shows that this objection does not approach the problem head-on, at least not if we accept the framework of a reflexive and transitive binary relation as the basis of normative evaluation of intergenerational justice. In this framework, Prop. 5 tells us that our view on intergenerational justice in general can be determined by considering a particular set of two-generation conflicts. If we agree on assigning absolute priority to the worse off in such a conflict, then we have to assign absolute priority to the worse off in general. Hence, our result provides a defense for the leximin principle in the infinite setting since it seems less difficult to accept the two-generation claim.

An analogous result can be established for $R^{\mathrm{L}}_{\mathrm{W}}$.

Proposition 6. *$R^{\mathrm{L}}_{\mathrm{W}}$ is a subrelation to R if and only if R satisfies Strong Pareto, Weak Anonymity, Hammond Equity, and Weak Preference Continuity.*

Proof. The result follows by trivial modification of the parts of the proof of Prop. 5 that involve SPC. □

We must impose an assumption on the technological framework in order to ensure that there exists a maximal path according to $R^{\mathrm{L}}_{\mathrm{S}}$ (and thus $R^{\mathrm{L}}_{\mathrm{W}}$, since $R^{\mathrm{L}}_{\mathrm{W}}$ is a subrelation to $R^{\mathrm{L}}_{\mathrm{S}}$). In this respect, the following domain restriction is of particular interest.

Assumption 2 (Eventual Productivity of ${}_1\mathbf{F}$)**.** For any t and k_t, there exists a feasible and efficient path with constant utility.

Eventual productivity is fulfilled for technologies usually considered in the context of sustainability (see Asheim, Buchholz, and Tungodden [2]). If we take this restriction into account, we can provide a complete justification for an egalitarian approach to intergenerational justice.

Proposition 7. *If the technology satisfies Eventual Productivity, and R satisfies Strong Pareto, Weak Anonymity, Hammond Equity, and Weak Preference Continuity, then the feasible and efficient path with constant utility is the unique R-optimal path.*

Proof. Eventual productivity guarantees the existence of an efficient utility path ${}_1\mathbf{v}$ with constant utility. Since any alternative feasible path ${}_1\mathbf{u}$ provides at least one generation with lower utility than this constant level, it follows that ${}_1\mathbf{v}\, P^{\mathrm{L}}_{\mathrm{W}}\, {}_1\mathbf{u}$. Hence, ${}_1\mathbf{v}\, P^{\mathrm{L}}_{\mathrm{W}}\, {}_1\mathbf{u}$ for any other feasible path. It follows from Prop. 6 that $R^{\mathrm{L}}_{\mathrm{W}}$ is a subrelation to R. Therefore, ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$ for any other feasible path. Hence, ${}_1\mathbf{v}$ is R-optimal, and no other feasible path is R-optimal. □

It follows that the feasible and efficient path with constant utility is preferred to any other feasible path according to any binary relation to which $R^{\mathrm{L}}_{\mathrm{W}}$ is a subrelation; in particular, this holds for $R^{\mathrm{L}}_{\mathrm{S}}$. Hence, the egalitarian path is the unique optimal path also under the stronger version of leximin.

4.2. **Utilitarianism.** The utilitarian overtaking criterion, introduced by von Weizsäcker [8], represents an important alternative approach to intergenerational justice.[5] As with leximin, there are two versions to consider.

Definition 5 (Catching Up)**.** For any two utility paths ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, the Catching Up relation ${}_1\mathbf{v}\, R^{\mathrm{U}}_{\mathrm{S}}\, {}_1\mathbf{u}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, $\sum_{t=1}^{T} v_t \geq \sum_{t=1}^{T} u_t$.

Definition 6 (Overtaking)**.** For any two utility paths ${}_1\mathbf{u}$ and ${}_1\mathbf{v}$, ${}_1\mathbf{v}\, I^{\mathrm{U}}_{\mathrm{W}}\, {}_1\mathbf{u}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, $\sum_{t=1}^{T} v_t = \sum_{t=1}^{T} u_t$. Moreover, ${}_1\mathbf{v}\, P^{\mathrm{U}}_{\mathrm{W}}\, {}_1\mathbf{u}$ holds if there exists some $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, $\sum_{t=1}^{T} v_t > \sum_{t=1}^{T} u_t$.

As an illustration, compare ${}_1\mathbf{v} = (2, 0, 2, 0, \dots)$ and ${}_1\mathbf{u} = (1, 1, 1, 1, \dots)$. Here ${}_1\mathbf{v}\, P^{\mathrm{U}}_{\mathrm{S}}\, {}_1\mathbf{u}$ since ${}_1\mathbf{u}$ never catches up with ${}_1\mathbf{v}$, while the utility paths are incomparable according to $R^{\mathrm{U}}_{\mathrm{W}}$ since ${}_1\mathbf{v}$ never overtakes ${}_1\mathbf{u}$. It is of interest to note that von Weizsäcker [8, p. 85] defines preference by overtaking (i.e., ${}_1\mathbf{v}$ is preferred to ${}_1\mathbf{u}$ if ${}_1\mathbf{v}\, P^{\mathrm{U}}_{\mathrm{W}}\, {}_1\mathbf{u}$), and optimality by catching up (i.e., ${}_1\mathbf{v}$ is optimal if ${}_1\mathbf{v}\, R^{\mathrm{U}}_{\mathrm{S}}\, {}_1\mathbf{u}$ for any feasible path ${}_1\mathbf{u}$).

[5] To analyze "overtaking" and "catching up" we must consider utilities that are also cardinally measurable and unit comparable.

Also in the case of these utilitarian criteria, we may appeal to a set of two-generation conflicts in order to provide characterization.[6]

Condition 2U (2-Generation Utilitarianism)**.** If ${}_1\mathbf{u} = (u_1, u_2, \dots)$ and ${}_1\mathbf{v} = (v_1, v_2, \dots)$ are two utility paths, and there exists j, k such that $0 < u_j - v_j \leq v_k - u_k$ and $u_i = v_i$ for all $i \neq j, k$, then ${}_1\mathbf{v}\, R\, {}_1\mathbf{u}$.

By introducing 2-Generation Utilitarianism we overcome an immediate objection to the catching up and overtaking criteria, namely that these criteria allow a large number of smaller gains for many generations to outweigh a greater loss for a single generation. The following results show that this is only a consequence of wanting to follow in two-generation conflicts the interests of the generation that experiences the greater loss.

Proposition 8. *$R^{\mathrm{U}}_{\mathrm{S}}$ is a subrelation to R if and only if R satisfies Strong Pareto, 2-Generation Utilitarianism, and Strong Preference Continuity.*

Proof. (If) Assume that R satisfies SP, 2U, and SPC. According to the definition of a subrelation (cf. footnote 3), we have to show that, for any ${}_1\mathbf{u}$, ${}_1\mathbf{v}$, ${}_1\mathbf{v}\, I^{\mathrm{U}}_{\mathrm{S}}\, {}_1\mathbf{u}$ implies ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$ and ${}_1\mathbf{v}\, P^{\mathrm{U}}_{\mathrm{S}}\, {}_1\mathbf{u}$ implies ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$. This naturally divides the if part of the proof into two subparts.

(1) Consider any ${}_1\mathbf{u}$, ${}_1\mathbf{v}$ such that ${}_1\mathbf{v}\, I^{\mathrm{U}}_{\mathrm{S}}\, {}_1\mathbf{u}$. By definition of $R^{\mathrm{U}}_{\mathrm{S}}$, there exists a $\hat{T} \geq 1$ such that for all $T \geq \hat{T}$, $\sum_{t=1}^{T} v_t = \sum_{t=1}^{T} u_t$ and $\sum_{t=1}^{T+1} v_t = \sum_{t=1}^{T+1} u_t$. Hence, $v_{T+1} = u_{T+1}$, and thus ${}_{\hat{T}+1}\mathbf{v} = {}_{\hat{T}+1}\mathbf{u}$. Since $\sum_{t=1}^{\hat{T}} v_t = \sum_{t=1}^{\hat{T}} u_t$ we can establish, for each of ${}_1\mathbf{v}_{\hat{T}}$ and ${}_1\mathbf{u}_{\hat{T}}$, a sequence of steps involving conflicts between two generations. In each step, let the gain for the worse off generation be equal the loss for the better off generation. This procedure leads to two egalitarian $\hat{T}$-heads, ${}_1\hat{\mathbf{v}}_{\hat{T}}$ and ${}_1\hat{\mathbf{u}}_{\hat{T}}$, where the constant utility of the former equals that of the latter. By 2U, each such step is indifferent according to R. Hence, by transitivity, we have that

$$({}_1\mathbf{v}_{\hat{T}}, {}_{\hat{T}+1}\mathbf{u})\, I\, ({}_1\hat{\mathbf{v}}_{\hat{T}}, {}_{\hat{T}+1}\mathbf{u}) \text{ and } ({}_1\hat{\mathbf{u}}_{\hat{T}}, {}_{\hat{T}+1}\mathbf{u})\, I\, {}_1\mathbf{u}\,.$$

Since ${}_1\hat{\mathbf{v}}_{\hat{T}} = {}_1\hat{\mathbf{u}}_{\hat{T}}$ and ${}_{\hat{T}+1}\mathbf{v} = {}_{\hat{T}+1}\mathbf{u}$, it follows by transitivity that ${}_1\mathbf{v}\, I\, {}_1\mathbf{u}$.

(2) Consider any ${}_1\mathbf{u}$, ${}_1\mathbf{v}$ such that ${}_1\mathbf{v}\, P^{\mathrm{U}}_{\mathrm{S}}\, {}_1\mathbf{u}$. By definition of $R^{\mathrm{U}}_{\mathrm{S}}$, for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that $\sum_{t=1}^{T} v_t > \sum_{t=1}^{T} u_t$. For this T we can now establish, for each of ${}_1\mathbf{v}_T$ and ${}_1\mathbf{u}_T$, a sequence of steps involving conflicts between two generations. In each step, let the gain for the worse off

[6] For a discussion of the related result in the finite setting, see d'Aspremont [3].

generation be equal to the loss for the better off generation. This procedure leads to two egalitarian T-heads, ${}_1\hat{\mathbf{v}}_T$ and ${}_1\hat{\mathbf{u}}_T$, where the constant utility of the former is higher than that of the latter. By 2U, each such step is indifferent according to R. Hence, by transitivity, we have that

$$({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, I\, ({}_1\hat{\mathbf{v}}_T,\, {}_{T+1}\mathbf{u}) \text{ and } ({}_1\hat{\mathbf{u}}_T,\, {}_{T+1}\mathbf{u})\, I\, {}_1\mathbf{u}\,.$$

Since R satisfies SP, it follows that

$$({}_1\hat{\mathbf{v}}_T,\, {}_{T+1}\mathbf{u})\, P\, ({}_1\hat{\mathbf{u}}_T,\, {}_{T+1}\mathbf{u})\,.$$

Hence, by transitivity, $({}_1\mathbf{v}_T,\, {}_{T+1}\mathbf{u})\, P\, {}_1\mathbf{u}$. Since, for any $\hat{T} \geq 1$, there exists a $T \geq \hat{T}$ such that this holds and R satisfies SPC, it now follows that ${}_1\mathbf{v}\, P\, {}_1\mathbf{u}$.

(Only if) Assume that $R_{\mathrm{S}}^{\mathrm{U}}$ is a subrelation to R. Then it is trivial to establish that R satisfies SP and 2U. Arguments similar to those used in the only-if part of the proof of Prop. 5 to show that R satisfies SPC when $R_{\mathrm{S}}^{\mathrm{L}}$ is a subrelation to R, can be used here to establish that R satisfies SPC. □

Proposition 9. *$R_{\mathrm{W}}^{\mathrm{U}}$ is a subrelation to R if and only if R satisfies Strong Pareto, 2-Generation Utilitarianism, and Weak Preference Continuity.*

Proof. The result follows by trivial modification of the parts of the proof of Prop. 8 that involve SPC. □

Since 2U entails WA, any social preferences to which one of these utilitarian criteria is a subrelation satisfy the Suppes-Sen grading principle.

It is more difficult to establish conditions that guarantee that there exists an optimal (or maximal) path according to the catching up and overtaking criteria, and we leave such a task for another occasion.

5. Conclusion

By introducing two new conditions, Strong and Weak Preference Continuity, we have shown how it is possible to apply interesting equity conditions well-known from the finite setting in order to resolve distributional conflicts between an infinite number of generations. We have applied our analysis to different versions of leximin and utilitarianism, but there are other possibilities worthy of exploration.

References

1. Asheim, G. B. (1991) "Unjust Intergenerational Allocations", *Journal of Economic Theory* 54, 350–371.
2. Asheim, G. B., W. Buchholz. and B. Tungodden (2001) "Justifying Sustainability", *Journal of Environmental Economics and Management* 41, 252–268.
3. d'Aspremont, C. (1985) "Axioms for Social Welfare Orderings". In Hurwicz, L., Schmeidler, D., and Sonnenschein, H., editors, *Social Goals and Social Organizations: Essays in Memory of Elisha Pazner*, pp. 19–76. Cambridge University Press.
4. Hammond, P. (1976) "Equity, Arrow's Conditions and Rawls' Difference Principle", *Econometrica* 44, 793–804.
5. Hammond, P. (1979) "Equity in Two Person Situations - Some Consequences", *Econometrica* 47, 1127–1135.
6. Lauwers, L. (1997) "Rawlsian Equity and Generalised Utilitarianism with an Infinite Population", *Economic Theory* 9, 143–150.
7. van Liedekerke, L. and L. Lauwers (1997) "Sacrificing the Patrol: Utilitarianism, Future Generations and Infinity", *Economics and Philosophy* 13, 159–174.
8. Weiszäcker, C. C. von (1965) "Existence of optimal program of accumulation for an infinite time horizon", *Review of Economic Studies* 32, 85–104.