Price Discrimination and Three Part Tariffs in a Duopoly*

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Abstract

The paper studies how second degree price discrimination can be implemented in a duopoly with differentiated products. Two firms serve consumers having heterogeneous willingness to pay for the good, willingness to pay being private knowledge. Consumers choose from a menu of tariffs and are subsequently billed according to the chosen tariff. Although product differentiation enables the firms to implement price discrimination, it is shown that competition has important effects on the tariff structure. A fully separating equilibrium can only be reached if the firm is allowed to use three part tariffs, i.e., quantity discounts conditional on a certain minimum usage level, in addition to two part tariffs, i.e., quantity discounts on the condition that a fixed fee is paid in advance.

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1 Introduction

We observe nonlinear pricing in many markets, that is, pricing arrangements where payment is not strictly proportional to the quantity of purchases. In the literature, implementation of nonlinear pricing is typically studied as a single two-part tariff or as a menu of two-part tariffs. Further, with a few notable exceptions, the existing literature applies a setting with a monopoly firm where nonlinear pricing is implemented by two-part tariffs. However, it is easy to verify that this does not sufficiently describe the practice of nonlinear pricing. Firstly, nonlinear pricing is a common practice in duopoly and oligopoly markets as well as in monopolies. Secondly, we frequently observe that other tariff arrangements rather than just two-part tariffs are used. The purpose of this paper is to make a contribution in the second part of the gap between theory and practice within the field of nonlinear pricing. We examine whether the fact that there is competition between two firms instead of a monopoly significantly changes the tariff structure. We find that implementation by two-part tariffs may not be a feasible strategy in a duopoly, but if a firm can use a combination of two-part and three-part tariffs, a fully nonlinear pricing schedule can be implemented. Three-part tariffs are used for small quantity purchases while two-part tariffs are used for large quantity purchases. Furthermore, quantity discounts are given for larger purchases only. Finally we show that this is in fact what firms actually do in the telecommunications market, where we observe competition rather than monopoly.

The market perception of what are reasonable tariff structures would vary according to what kind of market one is studying. However, menus of two-part and three-part tariffs are frequently used and it seems natural to restrict the analysis to menus of piecewise linear tariffs. A firm confronts consumers with a menu of tariffs and consumers make their optimal quantity choice subject to the tariff chosen and are also billed according to this tariff. Under two-part tariffs consumers receive larger quantity discounts if they are willing to pay a larger fixed fee in advance. Three-part tariffs can be implemented in two different ways; Consumers may commit to a specific minimum usage level and pay a flat fee until this level is reached. The higher the minimum usage consumers commit to the higher discount they get. Another way to implement a three-part tariff is to apply larger discounts when realized usage exceeds some specific threshold level during a billing period.

1.1 Related literature

In a monopoly context models on optimal nonlinear pricing often assume that it is sufficient to ensure that the individual rationality constraint is satisfied for the worst type only. If the worst type finds it weakly rational to participate, then all types will indeed participate. Under the monotone hazard rate condition, a menu of two-part tariffs is sufficient to implement a fully nonlinear outlay schedule $T(q)$, with complete separation of types. The underlying assumptions behind this result are that the agent's participation decision is deterministic; the reservation utility is independent of consumer type and the private information is single-dimensional. There is an increasing amount of literature that explores how the weakening of the modeling assumptions affects the results. Within the part of incentive theory where an agent contracts with only one principal, i.e., models with only a single principal or models with delegated common agency, richer models incorporate either multi-dimensional types or type-dependent participation constraints. Rochet and Stole (2000) give a review of the literature on multidimensional screening.

Several papers have incorporated nonlinear pricing into models with imperfect competition, but few study tariff design and tariff implementation under asymmetric information about individual quantity-type. The papers by Stole (1995), Armstrong and Vickers (2001) and Rochet and Stole (1999) model nonlinear pricing in a differentiated oligopoly. In Stole's paper the qualitative property of the monopoly model with downward distortion for all types but the highest is kept, while Rochet and Stole (1999) and Armstrong and Vickers (2001) find conditions that imply that efficient two-part tariffs emerge as an equilibrium. The divergence between these two results is partly relying on how transportation costs enter the model. In Stole's model transportation costs depend on the quantities consumed (and on taste) whereas the transportation costs are assumed to be lump-sum costs in the two others. However, Stole (1995) leaves the question of implementation aside.[1] Other papers that study two-part tariffs under competition often do this in a Cournot or Bertrand game, but with focus on two-part tariffs versus linear tariffs rather than on how the informational problem affects the tariff design.[2]

There is literature that deals with multi-dimensional screening where the informational asymmetry relates directly to the variable being contracted upon (e.g., consumers' willingness to pay for different quality attributes, or an agent's efficiency type when performing different tasks for a principal). The work by Armstrong and Rochet (1999), Rochet and Choné (1998) provides an overview of the literature and represents the status on how far the techniques are developed.[3] Another view on multi-dimensionality in mechanism design is taken in Rochet and Stole (1999), who work on a general model of nonlinear pricing where the informational asymmetry is present in the consumers' reservation utility as well

[1]Valletti (1999) derives similar results in a model with discrete types.

[2]Examples of such work are Calem and Spulber (1984), Gasmi, Moreaux and Sharkey (2000), Hayes (1987), Oren, Smith and Wilson (1983). Wilson (1993) provides a comprehensive survey of the literature and the practice of nonlinear pricing, Michell and Vogelsang (1991) provide a survey of the pricing of telecommunications in the U.S. during the 70s and 80s. Stole (1995) also provide a brief overview of the literature.

[3]Literature includes Laffont, Maskin and Rochet (1987), Matthews and Moore (1987), Wilson (1993), Armstrong (1996).

as in their preferences, i.e., with a more general modelling of the participation decision. The methodology developed in Rochet and Stole (1999) paper with randomness in the agents' outside option fits a situation where consumers' location is *not* perfectly known. They demonstrate the difficulties of working on multidimensional problems.

The literature on type-dependent participation constraint includes the work by Lewis and Sappington (1989), Biglaiser and Mezzetti (1993), Ivaldi and Martimort (1994), Maggi and Rodriguez-Clare (1995), Stole (1995) and Jullien (2000). Type-dependent participation constraint may arise in a situation with multiple principals (but where an agent contracts exclusively with one of them e.g., Biglaiser and Mezzetti (1993), Stole (1995)) or it can arise because of other reasons, i.e., it is for some reasons natural to model a type's outside option as a function of the privately known type parameter (e.g., Lewis and Sappington (1989)).[4] Ivaldi and Martimort (1994) provide empirical research that support that nonlinear pricing prevail under oligopolistic competition (energy distribution). Equilibrium pricing schemes are concave and depend on unknown private valuation and on the rivals contract parameters. They restrict the regression of payments to second-order polynomials on quantities. Hence, we cannot rule out a hypothesis that the true outlay schedule has convex parts, although the overall shape is concave.

Insights from these papers show that many of the results achieved earlier in nonlinear pricing are not robust. In models with multi-dimensional screening it is shown that the "no distortion at the top" result may appear together with distortion, no distortion or bunching at the bottom, as opposed to the Mussa and Rosen (1978) result with downward distortions for all types except the highest. The literature on type-dependent participation constraints demonstrates the possibility of a non-monotonic informational rent, i.e., countervailing incentives may arise. The incentive constraint can be downward binding for some types and upward binding for other types.

The model presented in this paper falls into a situation with asymmetric information along a single (vertical) dimension and with a type-dependent participation constraint. The basic model is identical to the model in Stole (1995). But, while he solves for an equilibrium in fully nonlinear tariffs, the model we present here searches for an implementable *tariff structure*. Further, given the difficulties of involving multidimensional screening, we keep the assumption that the agent's participation decision is deterministic. There are no gains from joint consumption and this eliminates the "competitive externality" in the incentive constraint and one source of countervailing incentives.[5] The informational rent

[4]Models on common agency can be found in Stole (1992), Martimort (1992), Martimort (1996), Mezzetti (1997) and Olsen and Osmundsen (1998). These are cases describing a situation where each principal requires that a task be performed by a common agent. The agent's ability or effort in performing the two tasks is unobservable but is privately known by the agent.

[5]A competitive externality exists when the utility from buying q units from firm 0 is evalu-

on the other hand has to be evaluated net of an outside option, which is the maximal utility a consumer gains if he rejects the firm's contract. Generally, it is not sufficient to ensure that the individual rationality constraint is satisfied for the worst type only. A priori, the sign of the marginal information rent can be positive, zero, negative or even change sign over the type space, creating a second source of countervailing incentives.[6] Countervailing incentives do not occur in this model, the participation constraint is binding only in the lower part of the distribution of types (or maybe only for the very lowest type) and the information rent is strictly increasing elsewhere.

2 The model

The model is closely related to Stole (1995). However, the focus is distinctly different from his. The main issue in this paper is implementation of nonlinear prices, an issue not raised in Stole (1995). While Stole in his paper lets consumers buy a single unit of a good but with variable quality, the present paper sets up the alternative quantity framework. However, impose the restriction that a consumer must choose a single tariff. Hence, we exclude the possibility that consumers buy from both firms, and we also exclude the possibility that consumers choose more than one tariff as well.[7]

The model describes a case where two firms, denoted by firm 0 and firm 1, offer one product each and the products are spatially differentiated. The firms are located at the two extremes on a line of length 1, firm 0 at extreme 0 and firm 1 at the other extreme, 1. Each individual's preferences over the two firms are identified according to each individual's location $\gamma \in [0, 1]$ on the interval, referred to as brand preference. Total length of the distance between a consumer and firms 0 and 1 is $|0 - \gamma|$ and $|1 - \gamma|$ respectively. Transportation costs are normalized to unit, hence, the total loss from not being able to buy the ideally preferred product is γ and $(1 - \gamma)$. Brand preferences are common knowledge

ated net of the (foregone) utility from *not* buying the same amount of q from another firm. This will be similar to the models in Lewis and Sappington (1989) and Maggi and Rodriguez-Clare (1995).

[6]This will be similar to the models in Biglaiser and Mezzetti (1993) and Jullien (2000).

[7]This is a simplification to keep the similarity to Stole's model. As pointed out by Stole (1995) it is plausible to restrict a consumer to purchasing from a single firm under the quality framework with unit demand. In the alternative quantity framework it requires additional technical restrictions to ensure that a consumer is not better off by buying two times $q/2$ than one time q. The restriction we impose on consumers' behavior is for instance plausible when we think of telephony, or the mobile phone, industry, where consumers subscribe to a particular tariff option. If they subscribe to more than one option they must also have more than one phone number, which is by most people regarded as undesirable. If this restriction is binding, it indicates that the quantity-quality framework are not as intimately related in the duopoly as in the monopoly framework and that one should be more careful in the modelling and interpretations.

and firms practice first-degree price discrimination over the horizontal dimension. Both firms face constant and identical marginal costs, $c_0(q) = c_1(q) \equiv c$.

Consumers' taste varies over a vertical dimension, which we interpret as a quantity-preference parameter, referred to as quantity-type (θ) subject to private knowledge.[8] The firms have common prior beliefs about the distribution of types $\theta \in [\underline{\theta}, \overline{\theta}]$ described by a cumulative distribution function $F(\theta)$. The corresponding density function $f(\theta)$ is strictly positive on the support. Thus, $F(\theta)$ is the objective distribution over a population of buyers having identical brand preferences γ.[9] We will assume that the distribution satisfies the monotone hazard rate condition.

The first assumption, i.e. about product differentiation, can be justified by considering that identical services – with respect to the communication capabilities they provide – are sold or bundled with different ancillary services or quality levels that consumers value differently. This could for example be differences in billing features (more detailed billing) and in support services, but it could also be features *perceived* as differences in the quality of the service provided.[10] The second assumption can be rationalized by taking into account the fact that consumers have different needs for communication, e.g. residential and business customers.

Consumers' preferences are represented by a utility function $u(q, \theta, \gamma)$ and $u(q, \theta, 1-\gamma)$ when he buys from firm 0 and 1 respectively. If a consumer buys a quantity q and pays an amount T, his net utility is $U = u(q, \theta, \gamma) - T$.

Assumption 1 *The utility function is at least three times continuously differentiable and strictly concave in q. We make the following assumptions about the derivatives of the utility functions $u(q, \theta, \gamma)$ and $u(q, \theta, 1-\gamma)$*

(a)	$u(0, \theta, \cdot) = 0$	*(e)*	$u_{q\theta}(\cdot) > 0$
(b)	$\lim_{q\to 0} u_q(q, \theta, \cdot) = \infty$	*(f)*	$u_{\theta\theta}(\cdot) \leq 0$
(c)	$\lim_{q\to \infty} u_q(q, \theta, \cdot) = 0$	*(g)*	$u_\gamma(q, \theta, \gamma) < 0$
(d)	$u_\theta(\cdot) > 0$	*(h)*	$u_\gamma(q, \theta, 1-\gamma) > 0$

To satisfy sufficient conditions, we will also make assumptions about the third order derivatives, and say that $u_{\theta qq} \leq 0$ and that $u_{\theta\theta q} \leq 0$. Further, we will make use of the following definition on consumers' indirect utility

[8]Since both θ and γ are taken to be continuous, we drop all subscripts for location and consumers' quantity type throughout the paper. However, we use superscript 0 and 1 to denote the location of the two firms.

[9]The distribution over quantity-types θ is independent of γ, i.e., for each γ-value the corresponding density function $f(\theta \mid \gamma) \equiv f(\theta)$ for all possible $\gamma \in [0, 1]$.

[10]Examples on differences in quality may be found in AT&T marketing of "AT&T True Voice". Examples on differences in billing features can be many. Telecom companies undertake large investments to be able to support detailed billing towards business consumers. This can be to break down the cost of telecommunications to different business departments, and/or to different services (fixed link communications, mobile communications, 800-services (Premium Rate Services), etc.

Definition 1 *Let $U^k(\theta,\cdot)$ be the net utility (surplus) for a consumer located at γ, with quantity type parameter θ when he is faced with a general price schedule $T^k(q_k)$ and buys firm k's product. The surplus he obtains is*

(a) $U^0(\theta,\gamma) \equiv \max_q \{u(q,\theta,\gamma) - T^0(q)\}$
(b) $U^1(\theta,1-\gamma) \equiv \max_q \{u(q,\theta,1-\gamma) - T^1(q)\}$

where $T^k(q)$ is a general price schedule $(k = 0,1)$.

Assumptions 1(a)-(c) secure the existence of a unique solution in consumers' choice of consumption q_k as long as there exists a continuous and appropriate outlay schedule $T(q)$.

The necessary single crossing condition together with assumption 1(d), implies that the indifference curves of consumers with different quantity preferences cross at most once, i.e., assumption 1(e). High-quantity type consumers value a marginal quantity increase higher than low-quantity types, regardless of brand preferences. Assumptions 1(g)-(h) follow from the fact that the products are horizontally differentiated.

In a first-best situation consumers would be confronted with prices equal to marginal cost, and under our assumptions this yields unique quantity allocations and consumer surplus.

Definition 2 *The first-best quantity level $\overline{q}_k$ $(k = 0,1)$ is the optimal quantity purchase when consumers buy at marginal cost and the corresponding utility, denoted as first best utility, is given by*

(a) $\overline{q}_k(\theta,\cdot) \equiv \arg\max_{q_k}\{u(q_k,\theta,\cdot) - cq_k\}, \quad k = 0,1$
(b) $\underline{U}^0(\theta,\gamma) \equiv u(\overline{q}_0(\theta,\gamma),\theta,\gamma) - c\overline{q}_0(\theta,\gamma) > 0$
(c) $\underline{U}^1(\theta,1-\gamma) \equiv u(\overline{q}_1(\theta,1-\gamma),\theta,1-\gamma) - c\overline{q}_0(\theta,1-\gamma) > 0.$

It follows from assumptions 1 that the first-best quantity and utility, $\bar{q}(\theta,\cdot)$ and $\underline{U}^k(\theta,\cdot)$ are both increasing in θ.

The two firms' products are perfect substitutes, except that they are of different brands. There are no gains from joint consumption (i.e., utility is not subadditive), and, for $0 < \gamma < 1/2$, the gains from purchasing good q_1 in addition to q_0 will never exceed the surplus from purchasing good q_0. The implication of this is that the quantity purchases of q_0 are always largest when q_0 are bought alone. The opposite apply for $1/2 < \gamma < 1$

According to assumption 1(g), if a consumer chooses to purchase the good from firm 0, utility is decreasing in location, $u_\gamma(q_0,\theta,\gamma) < 0$. Hence, buying from the closest firm will always give largest *first best* utility. For all parameter values $\theta,\gamma \in \left[\underline{\theta},\overline{\theta}\right] \times [0,1/2)$ we have that $\underline{U}^0(\theta,\gamma)$ is strictly larger than $\underline{U}^1(\theta,1-\gamma)$

We will also assume that the first-best utility is convex

$$\frac{\partial^2 \underline{U}^k(\theta,\cdot)}{\partial\theta^2} = \frac{\left[u_{\theta q}(\overline{q}_k,\theta,\cdot)\right]^2}{-u_{qq}(\overline{q}_k,\theta,\cdot)} + u_{\theta\theta}(\overline{q}_k,\theta,\cdot) > 0, \quad k = 0,1. \tag{1}$$

With such characterizations of consumers' preferences, the firm located at 0 has a competitive advantage in serving consumers located in the interval $[0, 1/2]$, whereas the firm located at 1 has a competitive advantage in the interval $[1/2, 1]$. Also, with symmetric marginal costs, price competition between the two firms will force the fixed fee down to zero and the marginal price down to marginal cost toward consumers being indifferent between buying from firms 0 and 1. Also, it is an equilibrium strategy for firm 1 to offer marginal cost pricing towards *every* consumer located in the interval $[0, 1/2]$. The problem is solved within a framework where an agent contracts with a single principal, the other firm's presence does only affect the individual rationality constraint.

At stage one of the game, each firm offers a fully nonlinear tariff with an ordered pair of take-it-or-leave-it contracts. At stage two, consumers make a choice of whether to buy from firm 0 or 1 (or from none) and also a choice of q_k ($k = 0, 1$). This is equivalent to assuming that the firm announces a menu of distinct tariffs at stage one, and letting consumers choose a tariff from this menu at stage two. Then, formally there is a stage three where consumers decide on individual quantity purchase and are billed according to the tariff choice at stage two. As long as the tariffs considered in the second type of game truthfully implement the fully nonlinear tariff in the first game, the two formulations yield identical equilibria. Formally, the solution to the first game is analyzed in section 3, whereas section 4 characterizes the set of tariffs that truthfully implement this solution.

In the game, the firms implement their contracts subject to the incentive compatibility and individual rationality constraints. The consumers' choice of firm and quantity is *de facto* equivalent to announcing a type, which is in line with traditional mechanism-design. Further, since marginal-cost pricing is the single offer from firm 1 inside firm 0's turf (for $\gamma \in [0, 1/2]$), it is only necessary to secure truth-telling mechanisms in a single-dimensional space. That is, we can ignore the complications of a common agency case, in which an agent might misreport his type differently to the two principals. Therefore, we can solve the delegated problem as if it is a single-principal case. Under the single crossing condition, monotonicity is sufficient for local- and global second-order conditions to be satisfied under quasi-linear preferences (Fudenberg and Tirole (1991), theorem 7.1 and 7.2).

2.1 Individual rationality

As a consequence of the existence of a competing firm, consumers in firm 0's turf $[0, 1/2]$ have an outside option. The reservation utility is defined as the maximum utility obtained by not purchasing, which is normalized to zero, and the utility from buying the less preferred good. The latter was in the previous section termed $\underline{U}^1(\theta, 1 - \gamma)$.

Lemma 1 *The individual rationality constraint is given by*

$$U^0(\theta, \gamma) \geq \max\left\{\underline{U}^1(\theta, 1-\gamma), 0\right\}. \tag{2}$$

The proof of Lemma 1 is standard, see for example Fudenberg and Tirole (1991, chapter 7).

Thus, given that the other firm practices marginal cost pricing within firm 0's turf, the individual rationality constraint is a function of consumer type.

Furthermore, since an outside option is of higher valuation for more distant consumers (closer to 1/2), the individual rationality constraint will differ according to consumers' preferences over the two firms' goods. Generally, the value of the outside option is increasing and convex in θ, since the first-best utility is increasing and is assumed to be convex in θ. From (2) we also observe that if $\gamma = 1/2$, the only way to fulfill the *IR* constraint is to offer marginal cost pricing. Otherwise the firms have some market power in their respective market turfs.

2.2 Incentive compatibility

Consumers choose contracts that maximize their net utility. Under a direct-revelation mechanism approach, a consumer of type θ maximizes utility with respect to a type announcement θ'. By definition

$$U^0(\theta, \theta', \gamma) = u(q(\theta', \gamma), \theta, \gamma) - t(\theta', \gamma), \tag{3}$$

$$U^0(\theta, \theta, \gamma) \equiv U^0(\theta, \gamma). \tag{4}$$

Global incentive compatibility requires

$$U^0(\theta, \theta, \gamma) \geq U^0(\theta, \theta', \gamma), \quad \forall \theta', \theta \in [\underline{\theta}, \overline{\theta}]. \tag{5}$$

Hence

$$\begin{aligned} U^0(\theta, \gamma) &= u(q(\theta, \gamma), \theta, \gamma) - t(\theta, \gamma) \\ &= \max_{\theta'}\{u(q(\theta', \gamma), \theta, \gamma) - t(\theta', \gamma)\}. \end{aligned} \tag{6}$$

Lemma 2 *Under the condition of Single Crossing, $u_{q\theta}(\cdot) > 0$, necessary and sufficient conditions for global incentive compatibility are given by*

$$\frac{\partial U^0(\theta, \gamma)}{\partial \theta} = u_\theta(q_0, \theta, \gamma), \tag{7}$$

$$q_0(\theta, \gamma) \text{ nondecreasing.} \tag{8}$$

The proof of Lemma 2 is also standard and is omitted.[11]

Hence, (2), (7) and (8) are necessary and sufficient conditions for implementation. As is usual in the literature, we will ignore (8) at the first stage but subsequently check that it is met.

[11]When the Single Crossing condition is satisfied, local (adjacent) incentive compatibility is also sufficient for global incentive compatibility. See for instance Fudenberg and Tirole (1991).

2.3 Informational rents

Before we proceed it might be convenient to determine the sign on the marginal informational rent to a type θ consumer that truthfully reveal his type.

Lemma 3 *A consumer of type θ that buys exclusively from firm 0, receives an informational rent*

$$R(\theta,\gamma) = U^0(\theta,\gamma) - \underline{U}^1(\theta, 1-\gamma) \geq 0, \tag{9}$$

$$\frac{\partial R(\theta,\gamma)}{\partial\theta} = u_\theta(q,\theta,\gamma) - u_\theta(\overline{q}_1(\theta,1-\gamma),\theta,1-\gamma) \geq 0. \tag{10}$$

When the informational rent is unambiguously increasing in type, we can rule out the presence of countervailing incentives. To see that this is the case consider the following reasoning. When the *IR* constraint is binding in a neighborhood of $+theta$, we have $R(\theta,\gamma) = 0$ and $R'_\theta = 0$. Choosing among the possible solutions in q that meets (10) (if more than one exist) we select the schedule that also satisfy (6). Hence, (10) determine a quantity schedule $q(\theta,\gamma) = \tilde{q}(\theta,\gamma)$.

Hence, if the *IR* constraint is *not* binding, we must follow a quantity schedule satisfying the condition $q(\theta,\gamma) > \tilde{q}(\theta,\gamma)$. Consequently, since $u_{\theta q}(\cdot) > 0$ the information rent is nondecreasing in θ, and $R'_\theta \geq 0$. When the derivatives with respect to θ and the quantity schedule in the equilibrium are continuous, the *IR* constraint can only be binding in the left part of the distribution over θ, (or for $\underline{\theta}$ only), i.e., $U^0(\underline{\theta},\gamma) = \underline{U}^1(\theta,1-\gamma)$ and $U^0\left(\overline{\theta},\gamma\right)$ is free. Note as well that it is sufficient to check whether $\tilde{q}(\theta,\gamma)$ is nondecreasing.

Without loss of generality we normalize the value of an outside option to zero for the *lowest* type, i.e., $\underline{U}^1(\underline{\theta},1-\gamma) = 0$ (in practical terms we subtract this constant from $\underline{U}^1(\theta,1-\gamma)$, which is assumed to be positive). We make the following *redefinition* of the outside option

$$\underline{U}^1(\theta,1-\gamma) \equiv u(\overline{q}_1,\theta,1-\gamma) - c\overline{q}_1 - \underline{U}^1(\underline{\theta},1-\gamma) \geq 0 \tag{11}$$

The justification behind doing so is that the individual rationality constraint is *binding* for the lowest type. Secondly, in this setting we can also compare the strategies of implementing in the duopoly solution and the monopoly solution respectively. In the latter, the value of an outside option is normalized to zero for the lowest type, and for every other type as well.[12] If the reservation utility profile is implementable, i.e., if q is nondecreasing when consumers receive their reservation utility, it might be the case that the individual rationality constraint binds for several types at the low end of the type space.

[12] See also Jullien (2000). If all types are served, the global level of the reservation utility does not really matters, what matter is the slope of reservation utility. If $U^{1*}(\theta,\gamma)$ is the solution to the problem when the reservation utility is U^0, then $U^{1*}(\theta,\gamma)+c$ is the solution to the problem when the reservation utility is $U^0 + c$ for any constant c.

3 Optimal allocations

Firm 0's objective is to maximize profit subject to the individual rationality constraint and the (downward binding) incentive constraint. Profit maximization is a separate problem for each $\gamma \in [0, 1/2]$. The objective is

$$Max \int_{\underline{\theta}}^{\overline{\theta}} [t(\theta,\gamma) - cq(\theta,\gamma)] f(\theta)\, d\theta \tag{12}$$

s.t. IR and IC

We use optimal control to solve the problem, imposing only the first order condition for incentive compatibility at the first stage (8). When we know the sign of the information rent, we are able to state the initial and terminal values of the state variable U^0. From now and onwards, we drop the subscript on q, since the only q we are talking about is q_0 except when we denote the quantity level in the outside option $\overline{q}_1 = \overline{q}$. The objective is

$$\max_{q \geq 0} \int_{\underline{\theta}}^{\overline{\theta}} \left[u(q,\theta,\gamma) - U^0 - c\,q \right] f(\theta) d\theta \tag{13}$$

subject to

$$\begin{aligned} &\partial U^0/\partial\theta = u_\theta(q,\theta,\gamma) \quad (a.e.), \\ &U^0(\underline{\theta},\gamma) = 0, \quad U^0\left(\overline{\theta},\gamma\right) \text{ free}, \\ &U^0(\theta,\gamma) \geq \underline{U}^1(\theta, 1-\gamma), \\ &\qquad \forall\theta \in \left[\underline{\theta},\overline{\theta}\right]. \end{aligned}$$

q is the control variable and U^0 is the state variable. This is a control problem with a pure state constraint.[13]

The *Lagrangian* or *generalized Hamiltonian* L is

$$\begin{aligned} L = &\left[u(q,\theta,\gamma) - U^0 - cq \right] f(\theta) \\ &+ \lambda(\theta)\, u_\theta(q,\theta,\gamma) + \mu(\theta)\left[U^0 - \underline{U}^1\right], \end{aligned} \tag{14}$$

where $L = L(\theta, q, U^0, \lambda, \mu) = H(\theta, q, \gamma, U^0, \lambda) + \mu\left[U^0 - \underline{U}^1\right]$. The costate variable is $\lambda(\theta)$ and $\mu(\theta)$ is the multiplier of the state constraint. The Hamiltonian $H\left(\theta, q, U^{0*}(\theta,\gamma), \lambda(\theta)\right)$ is strictly concave in q and the maximized Hamiltonian, $\widehat{H}\left(\theta, U^0, \lambda(\theta)\right) = \max_{q\geq 0} H\left(\theta, q, U^0, \lambda(\theta)\right)$ is concave in $U^0(\theta,\gamma)$. In addition the state constraint is quasiconcave in U^0.[14]

[13] See Seierstad and Sydsæter (1977) and Seierstad and Sydsæter (1987) for a treatment on optimal control theory with mixed and pure state constraint.

[14] Although γ is certainly an argument in the H and L functions, the parameter is omitted in the writing of these functions as well as the λ and μ functions in order to make the notation easier. As long as $0 \leq \gamma \leq 1/2$, the value of γ has only the effect of shifting the level of the outcome whereas the characterization of the outcome remains the same regardless of γ.

Let $(q^*(\theta,\gamma), U^{0*}(\theta,\gamma))$ be an admissible pair in the problem (13). Further, we assume that there exists a *continuous* function $\lambda(\theta)$ (≤ 0), with a piecewise continuous derivative $\lambda'(\theta)$, and a piecewise continuous function $\mu(\theta) \geq 0$ in the interval $[\underline{\theta}, \overline{\theta})$ Then, we can use the Arrow sufficiency theorem to state the following additional conditions for a solution to the problem[15]

$$(u_q - c) f(\theta) + \lambda(\theta) u_{\theta q}(q,\theta,\gamma) = 0, \tag{15}$$

$$\partial\lambda(\theta)/\partial\theta = -\frac{\partial L}{\partial U^0} = f(\theta) - \mu(\theta), \tag{16}$$

$$\lambda(\overline{\theta}) = 0 \tag{17}$$

$$\partial U^0(\theta,\gamma)/\partial\theta = u_\theta(q,\theta,\gamma), \tag{18}$$

$$\mu(\theta)\left[U^0 - \underline{U}^1\right] = 0, \;\; \mu(\theta) \geq 0, \;\; \left[U^0 - \underline{U}^1\right] \geq 0, \tag{19}$$

A configuration $(U^0(\theta,\gamma), q(\theta,\gamma), \lambda(\theta), \mu(\theta))$ that satisfies (15) – (19), $U^0(\theta,\gamma)$, $q(\theta,\gamma)$ and $\lambda(\theta)$ being continuous and piecewise differentiable, $\mu(\theta)$ piecewise continuous, is also an optimum. In addition we have to allow for optimal configurations in which $\lambda(\theta)$ is only piecewise continuous and has a finite number of jumps in the domain over θ. Under such circumstances we must apply the additional condition

$$\lambda(\theta_i^-) - \lambda(\theta_i^+) = \beta(\tfrac{\partial}{\partial U^0}(U^0 - \underline{U}^1)) = \beta, \tag{20}$$

$$\beta \geq 0 \; (= 0 \text{ if } U^0 > \underline{U}^1) \tag{21}$$

where $\underline{\theta} < \theta_1 < \cdots < \theta_k \leq \overline{\theta}$ are the discontinuity points of $\lambda(\theta)$, and β is a positive number. Since the jump must be from above $(\lambda(\theta^-) - \lambda(\theta^+) \geq 0)$ we can rule out the case that there is a jump at $\theta = \overline{\theta}$, measured by $\lambda(\theta^-) - \lambda(\overline{\theta}) = \lambda(\theta^-) \geq 0$. If we allow $\lambda(\theta^-)$ to be positive it implies that firm 0 sells its' product at a price below marginal cost, since $\lambda(\theta) = -[(u_q - c) f(\theta)]/u_{\theta q}$. But under the assumption that the firms are symmetric with respect to marginal cost the individual rationality constraint can never impose such a strategy. If the *IR* constraint stops binding for some $\theta < \overline{\theta}$, conditions (20)-(21) apply (Seierstad and Sydsæter (1987, theorem 8, p. 380)). Because $R'_\theta \geq 0$ this leaves only one possible discontinuity point, the point where the state constraint stops being binding. If we find a solution with a continuous $\lambda(\theta)$ we focus on this and do not elaborate further on solutions where $\lambda(\theta)$ is not continuous.

First, from the optimality condition (15) the distortion is proportional to $\lambda(\theta)$, which is necessarily negative since setting a price below marginal cost can never be a part of the equilibrium strategy.

[15]See Seierstad and Sydsæter (1977, theorem 7 p. 377) and Seierstad and Sydsæter (1987, chapter 5)

By differentiating the optimality condition with respect to θ we obtain the following condition for the monotonicity constraint to be met

$$\frac{dq}{d\theta} = -\frac{u_{q\theta}(f(\theta) + \lambda') + (u_q - c)f'(\theta) + \lambda u_{q\theta\theta}}{u_{qq}f(\theta) + \lambda u_{qq\theta}} \geq 0. \tag{22}$$

The denominator is negative under the assumption that the Hamiltonian $H(\theta, q, U^{0*}(\theta,\gamma), \lambda(\theta))$ is strictly concave in q. The likelihood of $dq/d\theta$ being positive increases as the slope of $\lambda(\theta)$ increases. When $\lambda'(\theta)$ is negative, there is a chance that the numerator becomes negative. Note that if we assume that third derivatives are indeed small, the slope of $\lambda(\theta)$ rather than $\lambda(\theta)$ itself will be important in the monotonicity constraint. Generally, we need

$$f(\theta) + \lambda'(\theta) \geq -\left[\frac{(u_q - c)}{u_{q\theta}}f'(\theta) + \lambda\frac{u_{q\theta\theta}}{u_{q\theta}}\right]. \tag{23}$$

When third derivatives are zero and θ is uniformly distributed so $f'(\theta) = 0$, the condition can be reduced to

$$\lambda'(\theta) \geq -f(\theta) \tag{24}$$

If the *IR* constraint does not bind, the costate equation states that $\lambda'(\theta)$ is equal to $f(\theta)$ and the monotonicity condition is met when $\mu(\theta) = 0$. On the other hand, if the *IR* constraint is binding we have $\lambda'(\theta) = f(\theta) - \mu(\theta)$, $\mu(\theta) \geq 0$, and therefore $f(\theta) \geq \lambda'(\theta)$. Hence a necessary condition for monotonicity is

$$f(\theta) \geq \lambda'(\theta) \geq -\left[f(\theta) + \left\{\frac{(u_q - c)}{u_{q\theta}}f'(\theta) + \lambda(\theta)\frac{u_{q\theta\theta}}{u_{q\theta}}\right\}\right] \tag{25}$$

Although we will check whether the candidate for a quantity schedule meets the monotonicity constraint, we can tell by now that there is a fairly good chance that it does. The expression in the bracket parenthesis is zero or positive so the condition expresses that the marginal distortions when the *IR* constraint bind can be more than opposite the marginal distortions when the constraint is not binding.

3.1 The IR constraint is not binding

Since λ is continuous at $\overline{\theta}$ we can integrate up the costate equation (16), which gives us $\hat{\lambda}(\theta) = -(1 - F(\theta)) = \lambda$ as a candidate for $\lambda(\theta)$.

A candidate solution for $\hat{q} = q(\theta, \gamma)$ determined by (15) is given by

$$u_q(\hat{q}, \theta, \gamma) = c + \frac{1 - F(\theta)}{f(\theta)}u_{\theta q}(\hat{q}, \theta, \gamma). \tag{26}$$

This is the schedule we know from a monopoly nonlinear pricing problem.

Onwards the notation is simplified by writing the accent (e.g. bar, hat, or tilde) on the symbol for the function to denote that the function is to be evaluated at a point where $q(\theta,\gamma)$ has the relevant accent. Henceforth, $\hat{u} = u(\hat{q},\theta,\gamma)$, $\overline{u} = u(\overline{q},\theta,\gamma)$, and $\widetilde{u} = u(\tilde{q},\theta,\gamma)$. We can then write the slope of the quantity schedule as

$$\begin{aligned} \frac{\partial \hat{q}}{\partial \theta} &= \frac{(\hat{u}_q - c)\,\mathcal{H}' - \hat{u}_{\theta\theta q} + \mathcal{H}\,\hat{u}_{q\theta}}{-[\mathcal{H}\,\hat{u}_{qq} - \hat{u}_{\theta qq}]} \geq 0, \\ \mathcal{H} &= \frac{f(\theta)}{1 - F(\theta)}. \end{aligned} \tag{27}$$

Together with assumptions 1, when the hazard rate $\mathcal{H}$ is increasing in θ, the Hamiltonian $H(\theta, q, U^{0*}(\theta,\gamma), \lambda(\theta))$ is strictly concave in q, $\partial\hat{q}/\partial\theta$ must be positive since our assumptions guarantee that both the numerator and the denominator is positive.

3.2 The IR constraint binds

Since the nonnegativity constraint is binding, we have $\partial U^0/\partial\theta = \partial \underline{U}^1/\partial\theta$, which implies that a candidate for $q(\theta,\gamma)$ is given by

$$u_\theta(\tilde{q},\theta,\gamma) = u_\theta(\bar{q}_1,\theta,1-\gamma). \tag{28}$$

Let (28) determine $\tilde{q}(\theta,\gamma)$, and let (15) define a solution to $\tilde{\lambda}(\theta)$. The solution in $\mu(\theta)$ is determined by the costate equation.

Differentiating (28) yields a solution to $\partial\tilde{q}/\partial\theta$

$$u_{\theta q}(\bar{q}_1,\theta,1-\gamma)\frac{\partial \bar{q}_1}{\partial\theta} + u_{\theta\theta}(\bar{q}_1,\theta,1-\gamma) = u_{\theta q}(\tilde{q},\theta,\gamma)\frac{\partial\tilde{q}}{\partial\theta} + u_{\theta\theta}(\tilde{q},\theta,\gamma),$$

and by Definition 2(a)

$$\frac{\partial \bar{q}_1}{\partial\theta} = -\frac{\overline{u}_{q\theta}}{\overline{u}_{qq}} \geq 0, \tag{29}$$

so

$$\frac{\partial\tilde{q}}{\partial\theta} = \frac{\dfrac{(\overline{u}_{q\theta})^2}{-\overline{u}_{qq}} - [\widetilde{u}_{\theta\theta} - \overline{u}_{\theta\theta}]}{\widetilde{u}_{\theta q}}. \tag{30}$$

For $\tilde{q}(\theta)$ to be an increasing function it is necessary that

$$\left[\frac{(\overline{u}_{q\theta})^2}{-\overline{u}_{qq}} + \overline{u}_{\theta\theta}\right] - \widetilde{u}_{\theta\theta} \geq 0. \tag{31}$$

Because the expression in the bracket is in fact $\partial^2 \underline{U}^1/\partial\theta^2$ and $\underline{U}^1$ is convex, the condition is certainly met when $\widetilde{u}_{\theta\theta} \leq 0$.

Last, the IR constraint binds in the interval $[\underline{\theta}, \theta_1]$ where θ_1 is the solution in θ to the equation $\hat{q}(\theta,\gamma) = \tilde{q}(\theta,\gamma)$ (or equivalently $\hat{\lambda}(\theta) = \tilde{\lambda}(\theta)$), or $\theta_1 = \underline{\theta}$ if a solution in θ to $\hat{q}(\theta,\gamma) = \tilde{q}(\theta,\gamma)$ fails to exist (we can determine θ_1 this way only because we have assumed that $\lambda(\theta)$ is continuous).[16]

The optimal allocation can now be characterized. Quantity-outlay allocations are described by the following characteristics (Stole, 1995)

$$q^*(\theta,\gamma) = \begin{cases} \tilde{q}(\theta,\gamma) & \text{if} \quad \theta \in [\underline{\theta}, \theta_1] \\ \hat{q}(\theta,\gamma) & \text{if} \quad \theta \in [\theta_1, \overline{\theta}] \end{cases}, \tag{32}$$

$$\theta_1 = \{\theta : \tilde{q}(\theta,\gamma) = \hat{q}(\theta,\gamma)\} \tag{33}$$

and finally

$$t^*(\theta,\gamma) = u(q(\theta,\gamma), \theta, \gamma) - U^{0*}(\theta,\gamma), \tag{34}$$

$$\begin{aligned} U^{0*}(\theta,\gamma) &= \underline{U}^1(\underline{\theta},\gamma) + \int_{\underline{\theta}}^{\theta} u_\theta(q(s,\gamma), s, \gamma)\, ds, \\ &= \int_{\underline{\theta}}^{\theta} u_\theta(q(s,\gamma), s, \gamma)\, ds. \end{aligned} \tag{35}$$

This is proved in Stole (1995).

4 Implementation

The outlay function is the upper envelope of a family of indifference curves $u(q,\theta,\gamma) - t = U^0(\theta,\gamma)$. Since $q^*(\theta,\gamma)$ is strictly increasing in θ, there exists an inverse function $\theta^*(q,\gamma)$.[17]

Using (34) we can define the outlay schedule $T(q^*,\gamma)$ by

$$T(q^*(\theta,\gamma), \theta, \gamma) \equiv t^*(\theta^*,\gamma) = u(q,\theta^*,\gamma) - U^0(\theta^*,\gamma), \tag{36}$$

and the slope of the outlay schedule $T(q^*,\gamma)$ is given by

$$\frac{dT}{dq} = u_q + (u_\theta - U^0_\theta)\frac{\partial\theta^*}{\partial q} = u_q(q,\theta^*,\gamma) \geq 0, \tag{37}$$

[16]Using the fact that $\lambda'_\theta = f(\theta) - \mu(\theta)$, $\mu \geq 0$ could lead us to the same conclusion. For $\theta = \overline{\theta}$, $\tilde{\lambda}(\overline{\theta}) - \hat{\lambda}(\overline{\theta}) < 0$ since we cannot have a jump at the right end of the distribution. Since μ is positive if IR binds, we must have $f(\theta) \geq \lambda'_\theta$. Thus, if the IR constraint is binding in any subinterval, this is always in the lower part, for some interval $[\underline{\theta}, \theta_1]$ – either $\tilde{\lambda}(\theta)$ crosses $\hat{\lambda}(\theta)$ once or not at all.

[17]An early paper on implementation is Laffont and Tirole (1986)

which is positive (($u_\theta = U^0_\theta$) by the envelope theorem).

The curvature of $T(q^*, \gamma)$ is given by

$$\frac{d^2T}{dq^2} = u_{qq}\left(q, \theta^*\left(q, \gamma\right), \gamma\right) + u_{q\theta}\left(q, \theta^*\left(q, \gamma\right), \gamma\right)\frac{1}{\partial q^*/\partial\theta}. \tag{38}$$

$u_{qq}(\cdot)$ is negative and the last term is positive, and $T\left(q^*, \gamma\right)$ is concave if

$$\frac{\partial q^*}{\partial\theta} \geq \frac{u_{q\theta}\left(q, \theta^*\left(q, \gamma\right), \gamma\right)}{-u_{qq}\left(q, \theta^*\left(q, \gamma\right), \gamma\right)} \geq 0. \tag{39}$$

Hence, concavity of the outlay schedule imply a stronger restriction than monotonicity with respect to $q^*(\theta, \gamma)$.

When the participation constraint is not binding, substituting $\partial\hat{q}/\partial\theta$ into (39), reorganizing and evaluating the condition for $q^*(\theta, \gamma) = \hat{q}(\theta, \gamma)$ yields

$$\frac{(-\hat{u}_{qq})(\hat{u}_q - c)H' + \hat{u}_{\theta qq}\hat{u}_{qq} - \hat{u}_{q\theta}\hat{u}_{\theta qq}}{(\hat{u}_{qq} - H\,\hat{u}_{\theta qq})\hat{u}_{qq}} \geq 0, \tag{40}$$

and the outlay schedule is certainly concave for $q \in \left[q^*(\theta_1, \gamma), q^*(\overline{\theta}, \gamma)\right]$.

When the participation constraint binds we have to evaluate the condition for $\partial\tilde{q}/\partial\theta$. Rewriting condition (39) for $q^*(\theta, \gamma) = \tilde{q}(\theta, \gamma)$ yields

$$\frac{[u_{q\theta}(\bar{q}_1, \theta, 1-\gamma)]^2}{-u_{qq}(\bar{q}_1, \theta, 1-\gamma)} + u_{\theta\theta}(\bar{q}_1, \theta, 1-\gamma) \geq \frac{[u_{q\theta}(\tilde{q}, \theta, \gamma)]^2}{-u_{qq}(\tilde{q}, \theta, \gamma)} + u_{\theta\theta}(\tilde{q}, \theta, \gamma) \tag{41}$$

The left-hand side in (41) is the second order derivative of the outside option (the first best utility) with respect to θ. This is assumed to be positive. The right-hand side is to be evaluated under a quantity distortion, i.e., $\tilde{q}(\theta, \gamma) \leq \bar{q}(\theta, \gamma)$, but at a more favorable location, i.e., $\gamma \leq (1-\gamma)$, (for $\gamma \leq 1/2$). Hence, it is ambiguous whether the condition is met or not. At $\gamma = 1/2$, the left-hand side equals the right-hand side. If we are able to show that the right-hand side increases when γ decreases, we can conclude that (41) imply a contradiction. Thus, we differentiate the right-hand side at $\gamma = 1/2$ and evaluate the sign of this (the negative of the sign since $d\gamma < 0$)

$$-\frac{\partial}{\partial\gamma}\left\{\frac{[u_{q\theta}(\tilde{q}(\theta, \gamma), \theta, \gamma)]^2}{-u_{qq}(\tilde{q}(\theta, \gamma), \theta, \gamma)} + u_{\theta\theta}(\tilde{q}(\theta, \gamma), \theta, \gamma)\right\}. \tag{42}$$

Hence, if (42) is positive the outlay schedule $T(q^*, \gamma)$ is convex, i.e., if

$$\left\{\frac{2u_{q\theta}\left(u_{qq\theta}\left(-\frac{d\tilde{q}}{d\gamma}\right) + u_{q\theta\gamma}\right)}{u_{qq}} - \left(u_{qqq}\left(-\frac{d\tilde{q}}{d\gamma}\right) + u_{qq\gamma}\right)\left(\frac{u_{q\theta}}{u_{qq}}\right)^2\right\}$$
$$-\left\{u_{\theta\theta q}\left(-\frac{d\tilde{q}}{d\gamma}\right) + u_{\theta\theta\gamma}\right\} \geq 0. \tag{43}$$

We can now formulate the following

Proposition 1 *If (43) is met, the outlay schedule* $T(q^*, \gamma)$ *defined by (36) is strictly convex for any* θ *in the interval* $[\underline{\theta}, \theta_1)$ *and consequently for any* q *in the interval* $[q^*(\underline{\theta}, \gamma), q^*(\theta_1, \gamma))$ *and strictly concave elsewhere, for* $q(\theta, \gamma) > q(\theta_1, \gamma)$. *Otherwise,* $T(q^*, \gamma)$ *is concave everywhere.*

Proposition 1 is proved by the preceding discussion. The sign of the expression in (43) is hard to evaluate using a general utility function. In the case with quadratic utility, $u = \theta(1-\gamma)q - \frac{1}{2}q^2$, (43) reduces to $2(1-\gamma) > 0$. With a logarithmic utility function, $u = \theta(1-\gamma)\ln q$, (43) reduces to $1/\theta > 0$. Hence, for this two important cases, the outlay schedule is convex in the lower part.

Next, we turn to the problem of how to implement the outlay schedule. Instead of announcing the complete set of take-it-or-leave-it contracts, or announcing the fully nonlinear tariff $T(q^*, \gamma)$, the firm try to implement it via a menu of optional tariffs. These are described by the following Lemma

Lemma 4 *If the outlay schedule* $T(q^*, \gamma)$ *is to be implemented by a menu of tariffs defined by* $T_\Lambda(q, \theta, \gamma)$, *these tariffs must meet the following conditions*

$$\begin{array}{ll} (i) & T_\Lambda(q(\theta,\gamma), \theta, \gamma) = T(q^*, \gamma) = t^*(\theta, \gamma), \\ (ii) & T_\Lambda(q, \theta, \gamma) \geq T(q^*, \gamma), \\ (iii) & T_\Lambda(q, \theta, \gamma) \geq 0, \quad \forall q \geq 0. \end{array} \tag{44}$$

The conditions in Lemma 4 follow from the individual rationality constraint and the incentive compatibility constraints. With these characteristics, the outlay function is the lower envelope of the family of tariffs $T_\Lambda(q, \theta, \gamma)$. Implementation requires that type θ (with brand preference γ) finds it optimal to consume an amount $q^*(\theta, \gamma)$, and that he pays an amount $t^*(\theta, \gamma)$ for this consumption. When a consumer of type θ announces a type parameter θ', it is equivalent to selecting a tariff $T_\Lambda(q, \theta', \gamma)$ and purchasing a quantity $q(\theta', \gamma)$. Expected utility is $u(q(\theta', \gamma), \theta, \gamma) - t(\theta', \gamma)$, and by construction of $t(\theta, \gamma)$ this is maximized when $\theta' = \theta$.

If $T(q^*, \gamma)$ is everywhere concave, we know that it can be represented by the lower envelope of its tangents. Hence, a menu of two-part tariffs will meet the incentive compatibility constraint and, of course, by construction, the individual rationality constraint. The following definition characterizes a menu of two-part tariffs.

Definition 3 *A menu of two-part tariffs (subscript* 2P*) is described by*

$$\begin{aligned} T_{2P}(q, \theta, \gamma) &= u(q, \theta^*, \gamma) - U^0(\theta^*, \gamma) + u_q(q, \theta^*, \gamma)(q - q^*(\theta, \gamma)), \\ &= t^*(\theta, \gamma) + u_q(q^*, \theta, \gamma)[q - q^*(\theta, \gamma)]. \end{aligned} \tag{45}$$

If $T(q^*, \gamma)$ is concave the menu of two-part given by definition 3 meet the requirements in Lemma 4. However, if $T(q^*, \gamma)$ is convex, or has convex parts, a two-part tariff that is the tangent to $T(q^*, \gamma)$ at a point $(q^*(\theta, \gamma), t(\theta, \gamma))$ would intersect $T(q^*, \gamma)$ at one or more points and, hence, it would violate part *(ii)* of Lemma 4. Alternatives to pooling tariffs, i.e., tariffs such that different quantity types are confronted with the same tariff, have to involve a more complicated scheme. The following definition characterizes a menu of three-part tariffs.[18]

Definition 4 *A menu of three-part tariffs (subscript* 3P*) is described by*

$$T_{3P}(q, \theta, \gamma) = \begin{cases} t^*(\theta, \gamma) & \text{if} \quad q \leq q^*(\theta, \gamma) \\ t^*(\theta, \gamma) & \\ +u_q(\hat{q}(\theta, \gamma), \theta, \gamma)[q - q^*] & \text{otherwise} \end{cases} . \tag{46}$$

Although $T_{3P}(q, \theta, \gamma)$ is not differentiable at $q = q^*$, it is continuous and both the right side and left side limits are unique and equal to $t^*(\theta, \gamma)$. The menu described by 4 meets the requirements in Lemma 4 given that $u_q(\hat{q}, \theta, \gamma)$ is sufficiently large to satisfy part (ii) in Lemma 4. If not, we can substitute any schedule of marginal prices in the menu three-part tariffs that is decreasing in type.

A three-part includes in the fixed payment $t^*(\theta, \gamma)$ some "free" consumption allowance $q^*(\theta, \gamma)$, subsequent purchases are charged according to a unit price $u_q(\hat{q}, \theta, \gamma)$.

Finally, the following Proposition characterizes the solution in a (possibly) mixed tariff regime.

Proposition 2 *(i) If the outlay schedule has a convex part in the lower quantity end, it can be implemented by a mixed tariff regime with a menu of three-parts and two-part tariffs. A mixed tariff regime is characterized by the following solution*

$$T^*(q, \theta, \gamma) = \begin{cases} T_{3P}(q, \theta, \gamma) & \text{if} \quad \theta \in [\underline{\theta}, \theta_2] \\ T_{2P}(q, \theta, \gamma) & \text{if} \quad \theta \in (\theta_2, \overline{\theta}] \end{cases} , \tag{47}$$

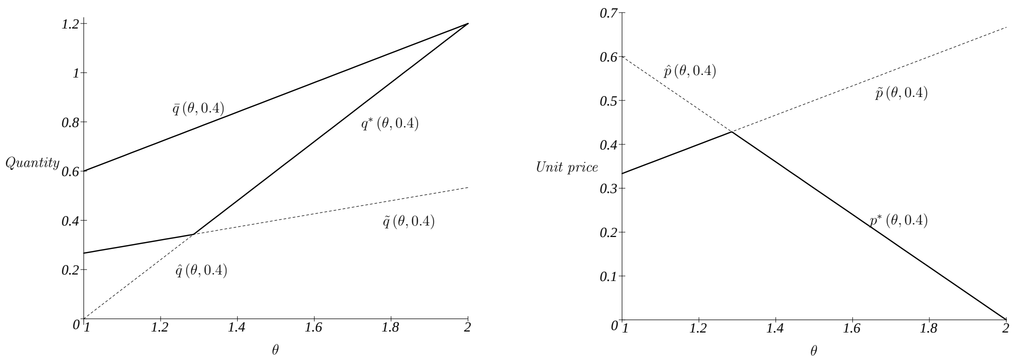
θ_2 *is the minimal solution to* $\{\theta : T_{2P}(q, \theta, \gamma) = t^*(\theta, \gamma)\}$*, which is given by* $\{\theta : T_{2P}(q^*(\underline{\theta}, \gamma), \theta, \gamma) = t^*(\underline{\theta}, \gamma)\}$*. (ii) Otherwise, the outlay schedule is concave everywhere and can be implemented by a menu of two-part tariffs,* $T^*(q, \theta, \gamma) = T_{2P}(q, \theta, \gamma), \forall\theta$*.*

Proposition 2 is proved by the preceding discussion and by applying Lemma 4.

[18] A three-part tariff can be considered as a moderated version of a "knife-edge" mechanism. In the absence of any uncertainty in demand, the allocation can always be implemented by a "knife-edge" mechanism, where a consumer pays $t(\theta, \gamma)$ if he announces θ and consumes $q(\theta, \gamma)$, otherwise he has to pay ∞. But, with even very small demand disturbances present such a mechanism is not implementable. Picard (1987) shows that a menu of quadratic tariffs might implement the optimal solution in a situation where a menu of linear tariffs cannot. See also Laffont and Tirole (1993) pp. 107-109 for a reference to Picard in the case of quadratic transfer schemes in a regulation model. However, quadratic tariffs seem difficult to commercialize, and will therefore be of little interest in this context. Three-part tariffs on the other hand are a fairly good approximation to quadratic tariffs and are sufficiently simple to be understood by the market as well.

5 A numerical example with quadratic utility

Let us consider a numerical example with linear transportation costs and quadratic demand. For reasons of comparison, the assumptions are identical to those used in Stole (1995). The quantity parameter θ is distributed uniformly on the interval $[1, 2]$. Each firm's marginal cost is equal to zero. Utility is specified by the function $u(q, \theta, \gamma) = \theta(1 - \gamma)q - \frac{1}{2}q^2$. The value of an outside option is $\underline{U}^1(\theta, 1 - \gamma) = \frac{1}{2}\theta^2\gamma^2$, when we normalize this to be zero for the very lowest type we get $\underline{U}^1(\theta, 1 - \gamma) = \frac{1}{2}\gamma^2 \left(\theta^2 - 1\right)$. The utility function is linear in quantity type and localization and the reservation utility is convex in θ and γ.



(a) Quantity profile (b) Marginal tariff

Figure 1: *The quantity schedule is implementable but not with two-part tariffs*

Figure 1 represents the solution with respect to q, p, λ and μ. The quantity schedule (the bold upper envelope) in figure 1 is the solution to the problem with direct revelation mechanisms. This is continuous and nondecreasing, and therefore truthfully implementable. In a monopoly context, the dual problem is to implement the allocation by offering the consumers to choose a tariff from a menu of two-part tariffs. But as we see in figure 1, this is not implementable since the outlay schedule is not everywhere a concave function.

The graphics in figure 2 illustrate the allocation in the (t, q) space and what implementation of $T(q^*, \gamma)$ looks like in the numerical example described above, for $\gamma = .4$. The fully nonlinear bold line represents $T(q^*, .4)$, the solid lines are some selected three-part and two-part tariffs, the dotted lines are consumers' indifference curves in the (t, q) space. The three-part tariff $T_{3P}(q, \underline{\theta}, \gamma)$ will truthfully implement $q(\underline{\theta}, \gamma)$, hence it is tangent to the indifference curve $U(\underline{\theta}, \gamma) = t - u(q, \underline{\theta}, \gamma)$ at the point $q = q(\underline{\theta}, \gamma)$. Similarly, a two-part tariff $T_{2P}(q, \theta_2, \gamma)$ will truthfully implement $q(\theta_2, \gamma)$ because it is tangent to the indifference curve $U(\theta_2, \gamma) = t - u(q, \theta_2, \gamma)$ at $q = q(\theta_2, \gamma)$.

Empirical observations do support the theoretical results. The examples

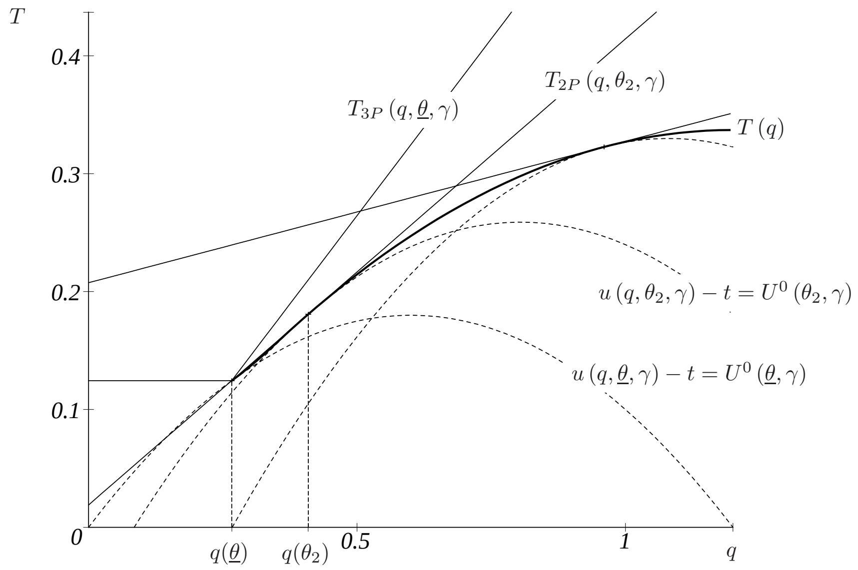


Figure 2: *Utility, outlay, three-part and two-part tariffs,* $\gamma = .4$

drawn in figure 3 seem to represent a trend for tariff arrangements in competitive markets.[19] The first graphic shows examples on tariffs in the US long distance fixed telephony market, represented by some of AT&T's tariff offerings in the residential market, AT&T Basic and AT&T Savings.[20] The latter graphic shows examples on tariffs effective in the Norwegian cellular market, represented by the tariffs of Telenor Mobil.

The idea of using a combination of three-part and two-part tariffs seems more appealing when the outlay schedule is convex for low quantities. Three-part tariffs are communicated to the market as discounts conditional on a minimum usage level and such an idea would be hard to introduce towards high quantity users.

[19]The figures are based on assumptions about daytime-, evening-time and weekend-time usage, as well as usage patterns with respect to distance bands, and are illustrations rather than precise tariff computations.

[20]See also Michell and Vogelsang (1991) and Wilson (1993) for a survey of the practice on telecommunications pricing during the seventies and eighties.

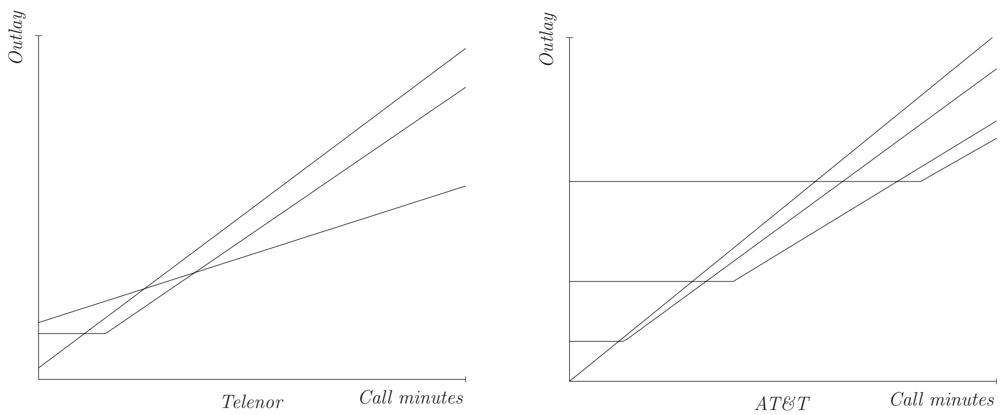


Figure 3: *Pricing of telecom services, AT&T and Telenor Mobil*

6 Concluding remarks

Linear contracts, as two-part tariffs, are attractive because under many conditions they implement the optimal contracts in an easy way. However, the paper shows that the problem is not as straightforward in a duopoly as it is in a monopoly setting. In a monopoly model, the monotone hazard rate condition is sufficient for the payment function to be concave, and hence for a menu of two-part tariffs to implement the outlay function. Although the monotone hazard rate condition is still a necessary condition in our duopoly model, it is shown that under reasonable assumptions two-part tariffs are outruled for low quantity purchases. In a monopoly the firm will balance the magnitude of downward quantity distortions below the first best level in order to reduce the information rent to better types (and all consumer surplus net of the transfer to the firm is informational rent). In the duopoly, however, the existence of an outside option places a restriction on consumers' net surplus. This will in turn change the magnitude of downward quantity distortions. This produces a convexity in the outlay schedule when the individual rationality constraint is binding and prevent the firm from using two-part tariffs for small purchases.

By analyzing the pricing strategies of the firms, one could draw conclusions about the competitiveness in the market. If the firms to a large extent are using three-part tariffs, this indicates that the market is more competitive. Although one should be careful in making comparisons of different markets, the US long distance market seems to be more competitive than the Norwegian cellular market.

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