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Discussion paper

# Making sense of market delineation with the aggregate diversion ratio

BY

**ØYSTEIN DALJORD, LARS SØRGARD, AND ØYVIND THOMASSEN**

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# Making Sense of Market Delineation with the Aggregate Diversion Ratio

Øystein Daljord, Lars Sjørgard & Øyvind Thomassen\*

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## Abstract

The US Merger Guidelines leave it an open question if the SSNIP test requires an increase in one, some or all prices in the candidate market. We argue that the characteristics of the candidate market in question should be decisive for how to perform the SSNIP test. If there are asymmetries between products, increasing one price might be a better procedure in order to identify competitive constraints. Katz & Shapiro (2003) derived a one-price criterion in terms of the aggregate diversion ratio which is applicable for asymmetric candidate markets. Unfortunately, the derivation is incorrect. We derive a corrected criterion.

## 1 Introduction

The SSNIP test was introduced with the 1982 US Merger Guidelines and is widely used by competition authorities to delineate relevant markets. The purpose of market delineation is to identify any competitive constraints. Some economists have argued that market delineation makes little economic sense, and that one should rather focus directly on the anti-competitive effects.<sup>1</sup> We agree that market delineation is not a goal in itself, but rather an input to the analysis of any possible anti-competitive effect. If only for legal reasons, market delineation is and will probably continue to be an important part of antitrust practice. Given that markets are to be delineated, it should be performed in the way that makes the most economic sense.

In line with such a reasoning, when choosing the criterion one should anticipate how market power might be exploited. For example, would a merger result in a symmetric increase in prices on all products controlled by the merged firm or would we expect some asymmetric price increases? If some asymmetries, then one should consider to use a SSNIP test where prices are increased asymmetrically. This is an argument for applying the criterion introduced in Katz & Shapiro (2003). Unfortunately, their criterion is incorrect. In their model they

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\*Contact: oeda@kt.no laso@kt.no and oyvind.thomassen@economics.ox.ac.uk. Affiliations: Norwegian Competition Authority, Norwegian Competition Authority and Norwegian School of Economics and Business Administration, and Norwegian Competition Authority and University of Oxford, respectively. This paper presents our personal views, not necessarily shared by the Norwegian Competition Authority.

<sup>1</sup>See, for instance, Ivaldi and Lörincz (2005).

did not take into account the fact that by increasing the price of one product, some sales is diverted to the other products in the candidate market. We derive the correct criterion, and it turns out that markets are delineated too broadly if one uses the criterion derived in Katz & Shapiro (2003).

In the next section we discuss the choice between two different criteria for performing the SSNIP test. In section 3 we derive the Harris & Simons (1989) critical loss criterion, while we in section 4 derive the Katz & Shapiro criterion. Then we explain the error in the Katz & Shapiro (2003) criterion, and derive the correct criterion in section 5. In section 6 we offer some concluding remarks.

## 2 Increasing any or all prices?

It is not perfectly clear from the wording of the US Merger Guidelines whether the SSNIP test requires a relative increase in the price of one, some or all of the products in the candidate market.<sup>2</sup> The ambiguity of the Guidelines has led to different profitability criteria being used in the literature.<sup>3</sup> Harris & Simons (1989) derived a criterion based on a uniform price increase on all products in the candidate market. Katz & Shapiro (2003) introduced a somewhat different test. They considered the effect of raising the price of only one product and derived a market delineation criterion based on the aggregate diversion ratio.

The basic question that distinguishes the two criteria is whether one in the SSNIP test should assume that one or all prices increase.<sup>4</sup> On p. 54, Katz & Shapiro (2003) argue

“[...] [W]e have followed the letter of the Merger Guidelines in asking whether the hypothetical monopolist would find it most profitable to raise the price of at least one product of the merging parties by some threshold amount above prevailing levels. However, we are aware that the market definition test often employed in practice is slightly different. Specifically, the test often takes the form of asking whether the hypothetical monopolist would find it most profitable to raise the prices of *all* of the products in the candidate market at least 5 percent above prevailing levels” (italics in original)

Although they argue that one should increase only one price, it is no doubt possible to argue that it is consistent with the Merger Guidelines to increase all prices, as is done in Harris & Simons (1989). In fact, one cannot - as noted in Whinston (2007) - from theory alone argue that one of the approaches is better than the other one.

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<sup>2</sup>See, for instance, Whinston (2007).

<sup>3</sup>There is also a debate on whether the price increase is the optimal one, or the price increase is profitable as such. We follow the approaches in Harris & Simons (1989) and Katz & Shapiro (2003), and consider the profitability of a price increase. For a discussion of the optimality criterion, see Bishop & Walker (2002).

<sup>4</sup>Note that Katz & Shapiro caused a controversy over how much structure to impose when deriving the market delineation criterion, see the debate in Scheffman & Simons (2003), Harris (2004), Harkrider (2004) and Katz & Shapiro (2004). The question of structure, though, differs from the question of whether one should increase one, some or all prices.

The basic question is whether some products impose a competitive constraint on the pricing of one or several products, and that should be of importance when choosing the price test in a particular case. In some case there might be important competitive constraints the all-price test misses, but the one-price test captures and vice versa in other cases.<sup>5</sup> In most case studies we observe that a symmetric price increase is imposed on all products in the candidate market. This seems a good idea in a candidate market with symmetric products - for example where all products have equal margins and demand structure - because a hypothetical monopolist is expected to raise prices on all those products symmetrically.

If we observe asymmetries, this may no longer be true. Then a hypothetical monopolist might increase prices more on some products than others. This could be true, for example, if we have a private label with limited sales and small margins that competes (or not) with a national brand with large sales and high margins. Then it might be natural to assume an asymmetric price increase when one examines the market for the private label and the national brand. A uniform price increase may lead to large absolute losses for the national brand compared with the private label. If only the price of the private label is increased, diversion to the national brand may make the price increase jointly profitable. If so, the Katz and Shapiro criterion should be preferred.

This shows that which criterion to use - the one with an asymmetric price increase or the more common one with a symmetric price increase - should be determined by the characteristics of the candidate market in question.<sup>6</sup>

### 3 Harris & Simons (1989) critical loss

The critical loss test of Harris & Simons (1989) is an empirically useful reformulation of the SSNIP test. This test measures the relative reduction in quantity following a given relative price increase for *all* the products in the candidate set that keep their joint profits unchanged. The critical loss defines a profitability threshold to compare with the actual relative quantity reduction of the candidate set of products following the price increase. If the actual loss is smaller than the critical loss, the price increase is jointly profitable and the market is delineated. Starting from the profitability criterion

$$(1) \quad ((1+x)p-c)q(1-CL) - (p-c)q > 0$$

where  $CL$  is the critical loss,  $(p, q)$  are the prevailing price and quantity of the candidate market in question and  $x$  the relative price increase, it is easily seen that the critical loss is given by

$$(2) \quad CL = \frac{x}{x+m}$$

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<sup>5</sup>There might even be cases where a some-prices test is best suited to identify the relevant competitive constraints.

<sup>6</sup>See Daljord, Sørsgard and Thomassen (2007) for an empirical application of these principles.

The actual loss, represented on the left hand below, is smaller than the critical loss, represented on the right, iff

$$(3) \quad x\epsilon < \frac{x}{x+m}$$

where  $\epsilon$  is the elasticity of demand of the products in the candidate market and  $m = \frac{p-c}{p}$  is the price-cost margin. The critical loss test is general in the sense that it does not rely on strong economic assumptions and applicable in the sense that it relates just a few variables: margins and demands sensitivity to price changes. Various versions of the Harris & Simons procedure of increasing all prices in the candidate market have been applied in competition cases.<sup>7</sup>

## 4 The Katz & Shapiro aggregate diversion ratio

Katz & Shapiro (2003) stated their delineating criterion on p. 53:

*“If and only if the aggregate diversion ratio is larger than the critical loss, then the actual loss is less than the critical loss and thus a hypothetical monopolist would find a SSNIP profitable”* (italics in original)

This statement is incorrect. The result refers to a derivation in footnote 25:

Let there be two products 1 & 2 with equal price-cost margins. Let the product specific elasticity of product 1 be related to the margin by the inverse pricing rule as follows:

$$(4) \quad \eta = \frac{1}{m}.$$

$\eta$  is the own-price elasticity of product 1 and  $m = \frac{p-c}{p}$  is the price-cost margin of both products. Then, assuming linear demand, the actual loss of product 1 for an  $x$  percent price increase is:

$$(5) \quad AL = \frac{x}{m}.$$

Let the diversion ratio of product 2 and product 1 be given by:

$$(6) \quad D = \frac{\partial q_2}{\partial p_1} \left( \frac{\partial q_1}{\partial p_1} \right)^{-1}$$

The diversion ratio measures the fraction of sales lost on product 1 that is diverted to product 2 following the price increase. Because the hypothetical monopolist earns a margin on the sales that is diverted from product 1 to product 2, it is *as if* the hypothetical monopolist only lost a fraction  $1 - D$  of sales on product 1. We may rewrite the actual loss in terms of the relative decline in sales of the hypothetical monopolist as:

$$(7) \quad AL_{HM} = \frac{x(1-D)}{m}$$

Katz & Shapiro then stated

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<sup>7</sup>see Kokkoris (2005) and O’Brien & Wickelgren (2003) for a critical review of some applications of the critical loss test.

“Recall that the critical loss is

$$CL = \frac{x}{x+m}$$

With a few steps of algebra, it can be shown that  $AL < CL$  if and only if  $D > CL$ .”

The steps are algebraically correct, but the derivation relies on the false premise that the critical loss when increasing only one price is  $\frac{x}{x+m}$ .<sup>8</sup> Katz & Shapiro apply the critical loss expression of Harris & Simons for the case where *all* prices are increased, to their case, where only one price is increased. The two thresholds are not equal, as we show below.

## 5 The corrected Katz & Shapiro criterion

We first derive the critical loss and the actual loss *of product 1* when increasing the price of only product 1 in the case of two products in the candidate market. All results generalize easily to the case of  $J$  products.

The actual loss of product 1 following an increase of  $p_1$  of  $x$  percent is still given by:

$$(8) \quad x\eta_{11}$$

where  $\eta_{11}$  is the own-price elasticity of demand of product 1. The critical loss of product 1 is given by the relative decline in sales of product 1 that keeps the joint profits of the two products unchanged, i.e. satisfy the equality:

$$(9) \quad \pi_1(p^s) - \pi_1(p) + \pi_2(p^s) - \pi_2(p) = 0$$

where  $p^s = ((1+x)p_1, p_2)$ . Expanding the equation, we get:

$$(10) \quad \begin{aligned} &((1+x)p_1 + c_1)q_1(1 - CL) - (p_1 - c_1)q_1 + \\ &(p_2 - c_2)q_2(1 + x\eta_{21}) - (p_2 - c_2)q_2 = 0 \end{aligned}$$

where  $\eta_{21}$  is the cross-price elasticity of product 2 with respect to  $p_1$ . The criterion reduces to:

$$(11) \quad (x + m_1)R_1(1 - CL) - m_1R_1 + x\eta_{21}m_2R_2 = 0$$

where  $R_j = p_jq_j$ . Solving for  $CL$ , we get:

$$(12) \quad CL = \frac{x}{x+m_1}(1 + \lambda D)$$

where  $\lambda = \frac{p_2 - c_2}{p_1 - c_1}$  measures the relative profitability of the two products. The critical loss of product 1 is the Harris & Simons expression for the critical loss

<sup>8</sup>These are the algebraic steps:

$$AL_{HM} < CL \Rightarrow \frac{x(1-D)}{m} < \frac{x}{x+m} \Rightarrow D > CL$$

In the case of unequal mark-ups, the Katz & Shapiro criterion generalizes to  $\lambda D > CL$ , where  $\lambda = \frac{p_2 - c_2}{p_1 - c_1}$ .

when increasing the price of all products in the candidate market plus a term proportional to  $\lambda D$  accounting for the diversion of sales to product 2. Katz & Shapiro failed to recognize that the critical loss is larger when only one price is increased than if all prices are increased, because the total quantity reduction is attenuated by more sales diverted from product 1 to product 2 than if both prices are increased.

The consequence of the mistake can be seen by setting  $\lambda = 1$ , suppressing the subscripts and taking the difference between the corrected critical loss and the Harris & Simons threshold:

$$(13) \quad \frac{x}{x+m}(1+D) - \frac{x}{x+m} = \frac{x}{x+m}D > 0.$$

The Katz & Shapiro criterion tends to define broader markets than intended since it understates the true critical loss.

Combining (8) and (12), the actual loss of product 1 is smaller than the critical loss of product 1 in the case of increasing only one price if and only if:

$$(14) \quad x\eta_{11} < \frac{x}{x+m_1}(1+\lambda D).$$

Using (4) it simplifies to:

$$(15) \quad x\eta_{11} < \lambda D$$

In the symmetric case of Katz & Shapiro,  $\lambda = 1$  and the criterion is reduced to:

$$(16) \quad AL < D$$

where we have suppressed the subscripts for convenience. Inserting (4) once again in (15), we may compare the corrected criterion to the original:

$$(17) \quad \mathbf{Correct} : \frac{x}{m} < D \mathbf{Wrong} : \frac{x}{x+m} > D.$$

Note that the corrected criterion fortunately is just as simple in terms of information requirements as the incorrect one.

The actual loss of product 1 is smaller than the critical loss if and only if the (aggregate, mark-up weighted) diversion ratio is larger than the actual loss. That is simply another way of stating that the diversion ratio equals the (correct) critical loss. The correction implies a restatement of Katz & Shapiro's delineating criterion:

*If and only if the aggregate diversion ratio is larger than the actual loss, then the hypothetical monopolist would find a SSNIP profitable.*

## 6 Concluding remarks

In Katz & Shapiro (2003) it is shown that the delineating criterion for a one-price test becomes quite simple and easy if one is willing to imply some structure. Unfortunately, there is an error in the derivation. Applying the criterion reported in Katz & Shapiro (2003) tends to delineate too broad markets. The

reason is that the model fails to take into account that by raising only one price some sales are diverted to the other products in the candidate market. The diversion of sales to other products in the candidate market makes a single-price increase more profitable than an all-price increase. There is then more scope for a profitable price increase when one applies the correct criterion than when one applies the erroneous criterion reported in Katz & Shapiro (2003). Fortunately, the correct criterion is just as simple and easy to apply.

In most case studies we observe that a symmetric price increase is imposed on all products in the candidate market. The US Merger Guidelines leave it an open question whether one should increase one, some or all prices in the candidate market when performing a SSNIP test. The ambiguity should be embraced. How to increase prices should be determined by the characteristics of the candidate market in question to more accurately identify the competitive constraints. When products in the candidate market are asymmetric, imposing asymmetric SSNIPs may make more economic sense.

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# NHH

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**Norges  
Handelshøyskole**

Norwegian School of Economics  
and Business Administration

NHH  
Helleveien 30  
NO-5045 Bergen  
Norway

Tlf/Tel: +47 55 95 90 00  
Faks/Fax: +47 55 95 91 00  
[nhh.postmottak@nhh.no](mailto:nhh.postmottak@nhh.no)  
[www.nhh.no](http://www.nhh.no)