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Discussion paper

The Premium of Marine Protected Areas: a Simple Valuation Model

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The Premium of Marine Protected Areas: a Simple Valuation Model ^{*}

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Abstract

Marine Protected Areas are considered as a hedging tool against some of the uncertainties that trouble many fisheries today. Such tools are always connected with a cost; a premium. An optimal harvest rule is combined with a protected area to manage a fishery. The model ignores uncertainty, and is thus ideal to analyze the induced cost arising from protected areas. We address the premium by comparing settings where only an optimal harvesting policy is implemented and where the combined management tool is used. The premium is found to be increasing and convex as a function of the degree of protection. Concepts of *relative growth efficiency*, *relative biological gain*, and *biological efficiency* are all introduced to increase the understanding of the management tool and its effects on the bioeconomic system. Time series solutions show that the net return per unit of fish increases after the protected area is established.

JEL Classification: C61; Q22; Q57.

Keywords: Bioeconomics; Dynamic programming; Fisheries management; Marine protected areas; Migration; Modeling; Optimization; Renewable resources.

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1 Introduction

A marine protected area (MPA) is a geographically defined area, covering parts of a fish stock's habitat, that is closed to fishing (Hannesson 1998). Marine reserves or sanctuaries are other common names for this concept. There exists a vast literature on the biological and economic aspects of MPAs; many authors consider both.

The literature addressing biological issues comes up with many good reasons to consider MPAs as a management tool for fish stocks. Reserves can reduce the variance of a population (Conrad 1999), and of catches (Hannesson 2002), raise the spawning and exploited biomass level (Gell and Roberts 2003) and improve density, diversity (Halpern 2003, Sanchirico 2000) and recovery time for small stock sizes (Grafton, Kompas, and Ha 2005). Further, MPAs can protect against extinction (Flaaten and Mjølhus 2005) and management failure (Lauck, Clark, Mangel, and Munro 1998). “[...] nearly any marine habitat can benefit from the implementation of a reserve.” Halpern (2003, p. 117). When it comes to the economic aspects, the literature is less conclusive. Hannesson (1998) demonstrates that reserves cannot yield economic benefits in a deterministic model and, compared to pure *total allowable catch* (TAC) policies, reserves reduce efficiency (Anderson 2002). However, among others, Grafton, Kompas, and Ha (2005) demonstrates that marine reserves are able to yield large economic payoffs and improve overall harvest levels. Sanchirico (2000) provides a thorough discussion of potential costs and benefits arising from MPAs, and asserts that reserves can increase *nonconsumptive use values* and control fishing effort. There is a wide consensus in the literature that MPAs need to be combined with other management tools to provide positive results, *e.g.* Sanchirico (2000) and Sumaila (2001). We think it is important to study the interplay between a protected area and other management regimes. In this paper, we combine an MPA with a TAC policy enforced on the remaining grounds, *i.e.* grounds open to fishing.

Another issue that is broadly discussed in the literature regarding MPAs is uncertainty. The model in this paper is deterministic, but the paper is all about uncertainty. Or rather, the subject; marine protected areas, is all about uncertainty. Several authors claim that reserves are able to reduce different risks regarding a fishery: *e.g.* poor stock assessment; collapse (Arnason 2001, Lauck, Clark, Mangel, and Munro 1998, Sanchirico 2000); uncontrollability; and economic shortsightedness (Clark 1996). In particular, marine reserves are proposed to address “persistent, *irreducible* scientific uncertainty pertaining to marine ecosystems.” (our emphasis) Lauck, Clark, Mangel, and Munro (1998, p. 72). Terms like insurance value (Sumaila 2001) and bet hedging (Lauck 1996, Sanchirico 2000) are frequently in use in relation to MPAs. “Bet hedging is usually thought to involve a cost, or ‘premium,’ in terms of a decrease in expected benefits, which is accepted in order to achieve a reduction in risk.” Lauck, Clark, Mangel, and Munro (1998, pp. 74-75). This paper addresses that induced cost; we calculate it, and we calculate it with no respect to uncertainty. That is, our model is explicitly devoid of uncertainty. However, as already mentioned, MPAs as a management tool deal with uncertainty. Still, we do, to some extent, investigate results for a range of parameters and thus take into account some uncertainty regarding these and the robustness of the results.

Many authors try to assess the intrinsic value or economic performance of MPAs, *e.g.* Grafton, Kompas, and Ha (2005), Hannesson (1998), Holland and Brazee (1996), and Sumaila (2001), and many consider uncertainty. A problem is that benefits, and in particular, costs are not properly addressed.¹ In this paper, we calculate the cost that arises from the establishment of a marine reserve. Please note that we adopt an industry perspective on this matter. Thus, benefits like *nonconsumptive use value*, biological diversity and similar considerations are not accounted for. Many such benefits are difficult to measure, and we do not include them in our analysis. They should, however, be considered to the costs we calculate in our analysis if MPAs are to be advisable from an

¹Often, the *net* result is considered, with little or no attention to the mechanism that generates the different effects and the relative importance of these.

economic point of view. We address this by formulating a simple bioeconomic model for a fishery, and implement the establishment of an MPA. The fishery is regulated by a TAC *feedback* policy, that is, we maximize returns with respect to the dynamics of the fish stock and the interplay between the protected and the unprotected stock. The resulting policy depends solely on the stock.² We calculate the induced cost by comparing returns from the fishery with and without the MPA. It is important to investigate these issues, because, in most cases, it is different agents, or sectors, in the economy that receive the benefits or have to take on the costs.³ In order to implement MPAs in a proper way, one should have a clear notion of what the potential gains and costs are and, in particular, what the certain costs are. We find the use of *certain* appropriate here, when maximizing returns we use the best case scenario. As already established, in a deterministic model only reductions in returns will arise. To be able to promote MPAs, one must document benefits to cover *at least* the costs documented in this paper. In that sense, our model is a *benchmark model*.

As the trends in the literature indicate, it is difficult to generalize on the subject of marine reserves (Alder, Sumailia, Zeller, and Pitcher 2001). On the other hand, Lauck, Clark, Mangel, and Munro (1998) argues that generality is not always useful, given the variety of ecosystems and possible management regimes.

2 The Model

We will use optimal control theory⁴ to optimize the value of the fishery, that is

$$V(X(t; s); s) = \max_H \int_0^\infty e^{-\delta t} \Pi(X(t; s), H(t; s), t) dt \quad (1)$$

²Refer to Sandal and Steinshamn (1997) for an example on the usefulness of feedback policies.

³The costs calculated here accrue to the fishing industry, as mentioned. Many of the benefits that MPAs are held to generate, will typically accrue to the society as a whole. We will see later, however, that MPAs can increase the gap between the unit harvesting cost and the unit price of fish.

⁴Kamien and Schwartz (1991) gives a good overview and introduction to the field.

subject to constraints on the control, growth in stock and initial conditions,

$$H(t; s) \geq 0, \quad \frac{dX}{dt} = F(X(t; s); K) - H(t; s), \quad X(0; s) = X^0 \quad (2)$$

where $X(t; s)$ is the biomass, $H(t; s)$ is the harvest rate, δ is the rate of discount, t is time, Π is a measure of the profit, F the natural growth in biomass, and K is the carrying capacity of the environment. X^0 is the initial condition of the biomass. We call the optimal value of the fishery $V(X; s)$, omitting the arguments of $X(t; s)$. s is defined as the *degree of protection* and is a number between 0 and 1; it gives the proportion of the total carrying capacity under protection.⁵ As indicated in (1), s is only a parameter in V , X , and H . That is, s is not treated as a control variable, and the concept of an optimal level of protection is thus not an issue as it is in many studies; the level of protection is given exogenously.

Before we move into further technicalities in the model, we will explain the *premium* of the MPA management tool. The induced cost of implementing a marine reserve is simply a result of a reduction in value. Thus, we compare the value of the fishery, when an MPA is established, to the value of the fishery in the no reserve case. We can formulate this idea as follows.

$$\mathcal{P}(X; s) = \frac{V_0 - V(X; s)}{V_0} \quad (3)$$

where $\mathcal{P}(X; s)$ is the premium of the MPA tool and $V_0 = V(X; 0)$. We will use the following notational convention unless otherwise noted: superscripts relate to the time variable and subscripts relate to the protection parameter. The premium is normalized according to V_0 . Thus, the premium is given as a share

⁵When using the model, it is convenient to think of protection in terms of the carrying capacity, which is what matters for the outcome of the model. In the real world, however, it is better to refer to the geographical proportion of the habitat under protection, as MPAs are, indeed, a geographical concept. Under assumptions of *uniformity*; the factors comprising the carrying capacity, *e.g.* food and shelter, are uniformly spread throughout the entire habitat, these two different interpretations of protection coincide. We want to keep both notions of MPAs in mind, as they are useful in different ways. Further, assuming uniformity of the carrying capacity rules out the possibility of a so-called sink-source system (Sanchirico and Wilen 1998).

of the total profits accruing from the fishery with no reserve present. We will sometimes refer to (3) as $\mathcal{P}(s)$ or simply \mathcal{P} .

The rest of this section is devoted to details on the model. First we set up the *reference model*; that is, the model with no reserve. Solving this model yields V_0 in equation (3).⁶ Then we move on to implement the MPA and find $V(X; s)$.

2.1 Reference Model

The model is an aggregated and deterministic formulation in continuous time. We find it instructive to explain the model without introducing the marine protected area first, and then to extend the model to include the MPA case. The simple structure of the reference model will be apparent, and it will be easier to isolate the impact of the MPA on the model and on the results. Considering the simple reference model, the complexity that arises from the idea of MPAs is astonishing.

The model we represent here is *autonomous*, which means that involved functions are independent of time.⁷ A profit maximizing manager considers the problem of maximizing the net value of a renewable resource, specified as follows.

$$\max_H \int_0^\infty e^{-\delta t} \Pi(X, H) dt \quad (4)$$

subject to

$$\left. \begin{aligned} \frac{dX}{dt} &= F(X; K) - H \\ X &\geq 0, \quad H \geq 0 \end{aligned} \right\} \quad (5)$$

where Π is net profit, F is the natural growth of the stock, X is the size of the stock, H is the total harvest, δ is the discount rate, K is the carrying capacity

⁶Putting $s = 0$ in the MPA model yields the same result, but is more complicated and time consuming. Furthermore, comparing the models like this (V_0 stems from the reference model, $V(t; s)$ from the MPA model) indicates that they are consistent with each other.

⁷Time (t) is not an explicit argument of the functions. The discount factor is dependent on time. The problem is still considered autonomous; it can be transformed into an autonomous problem by considering current value functions.

and t is time. The net profit function has the form

$$\Pi(X, H) = P(H) \cdot H - C(X, H)$$

where $P(H)$ is the inverse demand function and $C(X, H)$ is the cost function. This model is considered in Sandal and Steinshamn (1997) and Sandal and Steinshamn (2001), who provide and discuss feedback solutions of the problem. Feedback solutions are a powerful tool to deal with uncertainty; the optimal control only depends on the stock level. Perturbations in the stock level are taken into account as soon as they are discovered, no matter how unexpected.

2.2 Marine Protected Area Model

In the following section, we introduce the idea of an MPA and incorporate it into the model. Let us first examine the biological aspects. As already mentioned, the parameter $s \in (0, 1)$ measures the degree of protection. We find it natural to think of it as a geographical concept, where s simply gives us the share of the entire habitat under protection. As we assume *uniformity* in the carrying capacity of the environment; the means of which the carrying capacity consists of, is uniformly spread throughout the entire habitat,⁸ it follows that s also measures the share of the carrying capacity under protection. The assumption of uniformity distinguishes our model from a sink-source model of the population. The total carrying capacity is given by K , thus the carrying capacity within the reserve is $s \cdot K$ and outside it is $(1 - s) \cdot K$.

The *density of fish* is simply defined as the ratio between actual biomass and carrying capacity, (*i.e.*, the density outside the sanctuary is given by $\frac{X}{(1 - s)K}$, where X now, of course, measures the biomass found outside the sanctuary). It follows that the density is 0 when the stock is extinct, and 1 when the biomass equals the carrying capacity. For all practical purposes, the density is somewhere

⁸Uniformity means that the environment is practically identical everywhere in every way that matters for the stock. This is of course a simplification that yields a situation where the only difference between the protected and the unprotected area is the harvest rate. The harvest rate is zero inside and nonnegative outside the protected area.

between 0 and 1.⁹ We model the migration of fish between the areas according to Hannesson (1998). The idea behind the migration model is inspired by the way the natural sciences consider how substances (typically gas) diffuse through a membrane due to differences in density, based on random movements and the theory of probability. The *net* diffusion always smoothes densities and points into the low-density area.¹⁰ The migration term in our model only depends on the difference in density. We write

$$\phi \left(\frac{Y}{sK} - \frac{X}{(1-s)K} \right)$$

where X and Y and $(1-s)K$ and sK measures the biomass and the carrying capacity in the unprotected and protected area, respectively. ϕ is the *rate of migration* and decides how quickly the difference in density will be smoothed.¹¹ The limits $\phi \rightarrow 0$ and $\phi \rightarrow 1$ have the following implications. The first yields a one dimensional problem (the protected stock is not interesting as there is no interaction) equivalent to the reference model with a reduced carrying capacity. The second yields a system with instantaneous redistribution of densities, which is exactly the reference model, with carrying capacity K . For any positive and finite rate of migration, we have a two dimensional system; that is, two stocks interacting through migration. Note that the migration term is positive whenever the density in the protected area is dominating. A positive term is associated with net migration from the inside to the outside of the reserve. We always refer to the protected habitat as *one* area, but extending the model to considering several protected areas is fairly simple. Then, Y and sK would measure the aggregated biomass and carrying capacity for all the protected areas. Some assumptions regarding the migration rate would take care of the

⁹The upper limit (1) is not actually a limit; the density could, in theory, be infinite. However, a density above 1 implies zero or negative natural growth, and will thus never prevail in an active fishery.

¹⁰The same idea is considered by *inter alios* Conrad (1999), Flaaten and Mjølhus (2005), and Sumaila (2001).

¹¹Extending the diffusion-through-membrane idea, a zero migration rate implies a membrane like an impenetrable brick wall in the sea. An infinite rate would imply no membrane at all.

rest. The way we have formulated the model, ϕ can be understood to depend on the length of the interface between the protected and unprotected area. Keeping ϕ fixed (as we intend to do) then implies that the interface is of the same length for all s .¹² If one thinks this construction is a bit too rigorous or not general enough, there is another way to think of the model: think of a fish stock with a *patchy distribution*, where one or more of the patches are under protection. By patchy distribution, we mean that the fish stock is found at several locations, or areas, and that it travels between these locations. ϕ would then contain information about the distance, current conditions and so on between the patches. These quantities would, for simplicity, be assumed to be constant.¹³

The rest of the model is straightforward from the reference model explained out earlier. We will now specify the model in more detail. We begin with the biological issues. Natural growth obeys the logistic growth law *in each subarea*;

$$F(X; K) = rX \left(1 - \frac{X}{K}\right)$$

where r is the intrinsic growth rate, and X is the biomass and K is the carrying capacity. This means that the aggregated growth will not agree with the logistic growth for the entire area whenever the densities in the subareas are different. There is one simple reason for this, and that is that the logistic growth law is nonlinear in the stock. Thus, in general, one cannot add up two (or more) logistic functions to obtain a new logistic function. This results in a difference

¹²This is feasible: dividing a rectangle into two parts with a line parallel to a given side always produces an interface of equal length, independent of the areas of the parts. As promised, the notion of geography (or rather, geometry) is useful. A rectangle shaped habitat is of course a simplification, however.

¹³Actually, the first notion of the model; one big area divided into two smaller subareas, implies fundamentally different assumptions. As long as $s = 0$ or $s = 1$ there is only one area, and the fish are uniformly dispersed in the entire habitat. Any change in the stock level at one place in the area, *e.g.* harvesting activity, will, with infinite speed, spread throughout the habitat and an instant later, the density of fish is again uniform. But whenever $s \in (0, 1)$, changes in stock level will equalize immediately in each area, but changes across the areas will not happen faster than the migration term allows. This is why the growth in the model does not compare to the growth in the background model, as discussed in the next paragraph. One way to avoid this problem is to motivate the model with the notion of patchy distributions. However, this is more of a philosophical discussion, and we will not go into it any further.

in growth for a given total biomass for the two biological models,¹⁴ something that is widely ignored in the literature. Bischi, Lamantia, and Sbragia (2006) and Flaaten and Mjølhus (2005) recognize and discuss the issue, however. The former paper suggests an extra term in the growth function to correct for the difference, the latter investigates both ideas. Notwithstanding, we find it natural to let the growth in one area depend on local conditions and be independent of conditions in more remote areas. At least, it makes perfect sense with the patchy distribution motivation. Before we move on to the economic aspects of the model, let us describe the growth functions for both the protected and the unprotected area.

$$\left. \begin{aligned} \frac{dX}{dt} &= F(X; (1-s)K) + \phi \left(\frac{Y}{sK} - \frac{X}{(1-s)K} \right) - H \\ \frac{dY}{dt} &= F(Y; sK) - \phi \left(\frac{Y}{sK} - \frac{X}{(1-s)K} \right) \end{aligned} \right\} \quad (6)$$

where F is the logistic function. As earlier, X refers to the biomass in the unprotected area while Y refers to the biomass in the protected area. It is readily seen that the migration term adds to the stock in the low-density area, while reducing the stock in the high-density area. Further, the migration terms cancel in aggregate: *i.e.*, the migration itself does not change the total biomass level. Also note that the carrying capacity is different in the two areas, according to the discussion above and that there is no harvest term in the second equation. ‘Growth’ is perhaps not the best word for these functions now; ‘change’ is better.

Now we turn to the economic aspects of the model. There are many things that influence the economy of a fishery. We simplify and consider two issues: production technology and demand. The production function has the Schaefer form,

$$H = qEX$$

where q is the *catchability coefficient*, and E is the effort. As before, X is

¹⁴The growth in our MPA model will depend on the distribution of fish between the different areas. This is in contradiction to the reference model, where instantaneous redistribution is assumed. We believe that the distribution does matter, and the MPA model can be comprehended as a first approximation to a spatial growth model.

the stock biomass and H is the harvest rate. It simply tells us that a certain amount of effort applied to a certain stock level yields the harvest according to the equation.¹⁵ There is a constant cost per unit effort; the costs are given by $C(X, H) = cE$. If we substitute E according to Schaefer, we obtain

$$C(X, H) = cE = \frac{cH}{qX}$$

We will discuss the interpretation of this expression shortly. There is a downward sloping demand for fish; the inverse demand function has slope $-d$. The price of fish is given by $P(H) = p - dH$, where p is the maximum price the market is willing to pay for the fish. A higher price leads to full substitution. We are now able to describe the net profit function,

$$\Pi(X, H) = (p - dH)H - \frac{cH}{qX} \quad (7)$$

This is the same profit function as suggested by Kugarajh, Sandal, and Berge (2006). The interpretation of the relationship $\frac{c}{pq}$ is important; it is readily seen that if $X = \frac{c}{pq}$, it is not possible to draw rents from the fishery. The relationship arises from price normalization. $\frac{c}{pq}$ is often referred to as the *open-access solution*, which is characterized by zero profits. Further, note that profits depend on X , the biomass in the unprotected area (*i.e.* where fishing activity is allowed), but not on Y , and that there is a *stock effect* on costs (Hannesson 2006). The stock effect represents an economic protection mechanism on the stock and implies that rational fishing activity cannot eradicate the stock.

To make both calculations and later discussions easier, we choose to change the scale of both variables and parameters. A consequence of this transformation is that stocks are measured in *local densities* instead of biomass. The local density is the ratio between the biomass in an area and the local carrying capacity. To avoid confusion and more notation, we use the same symbols as before, now in lower case. The main difference is that we talk about densities

¹⁵Clark (1990) provides a useful discussion of the Schaefer model and its implications.

instead of biomass. The maximization problem now appears as

$$\max_h \int_0^\infty e^{-\gamma\tau} \pi(x(\tau; s), h(\tau; s)) d\tau \quad (8)$$

subject to

$$\left. \begin{aligned} \dot{x} &= \chi(x, y, h) \equiv f(x) + \frac{\omega}{1-s}(y-x) - h \\ \dot{y} &= \psi(x, y) \equiv f(y) - \frac{\omega}{s}(y-x) \\ x(0; s) &= x^0, \quad y(0; s) = y^0, \quad h \geq 0 \end{aligned} \right\} \quad (9)$$

where $\dot{x} \equiv \frac{dx}{d\tau}$, x^0 and y^0 are initial conditions on the densities, and

$$\left. \begin{aligned} \pi(x, h) &= h \left(1 - \frac{x_0}{x} - bh \right) \\ f(x) &= x(1-x) \end{aligned} \right\} \quad (10)$$

where x and y are the local densities of fish in the protected and unprotected area, respectively. $\chi(x, y, h)$ and $\psi(x, y)$ are the right hand sides of the dynamic conditions in (9). Further is $b = (1-s)K \frac{dr}{p} \equiv (1-s)b_0$, and f is the transformed logistic function. $x_0 = \frac{c}{pq} \frac{1}{(1-s)K}$ is the open-access *density* level, $\tau = rt$ is the time variable, $\gamma = \frac{\delta}{r}$ is the discount rate, and $h = \frac{H}{r(1-s)K}$ is the harvest rate. Note that the migration rate ϕ has been replaced with $\omega = \frac{\phi}{rK}$ and a fraction depending on the degree of protection, s . That is, we need to control for the different sizes of the areas when we use densities instead of absolute biomass. We will from now on refer to ω as the rate of migration. The value of the fishery is now given by (8), and we refer to it as $v(x, y; s)$. It relates to the absolute value as $V(X, Y; s) = pK \cdot v(x, y; s)$. Thus, the factor pK will cancel from \mathcal{P} , such that

$$\mathcal{P}(X, Y; s) = \frac{V_0 - V(X, Y; s)}{V_0} = \frac{v_0 - v(x, y; s)}{v_0} = \mathcal{P}(x, y; s) \quad (11)$$

where $v_0 = v(x, y; 0)$. Note that the premium now takes two arguments, as the

model has two state variables and it depends on both.

We should not worry too much about the scaling transformation, the underlying structure of the model is of course the same as before. The relations and effects will hopefully stand out more clearly, however. As an example of the more lucid structure, consider the scaling of the discount rate:

$$\gamma = \frac{\delta}{r}$$

where δ is the absolute discount rate and r is the intrinsic growth rate in the logistic function. The scaling tells us that what matters for the optimal solution of the problem is the ratio between interests from alternative investments and the return from the stock. This is not a particularly deep insight, it is, however, obvious from the scaling transformation.

It might seem unnecessary to state the model in both absolute and relative terms, but the patient reader will discover that both formulations are useful to us.

The *current value Hamiltonian* is a useful tool for these kinds of optimization problems. It is given by

$$\mathcal{H}(x, y, h, m, n) = \pi(x, h) + m \cdot \chi(x, y, h) + n \cdot \psi(x, y) \quad (12)$$

where m and n are the *current value multipliers* associated with the two state variables; x and y , respectively. The multipliers are also known as the *shadow values* or the *costates*. χ and ψ are defined in (9). The first order conditions for solution of (8) and (9) is now given by (Kamien and Schwartz 1991)

$$\left. \begin{aligned} \dot{x} &= \mathcal{H}_m \\ \dot{y} &= \mathcal{H}_n \\ \dot{m} &= \gamma m - \mathcal{H}_x \\ \dot{n} &= \gamma n - \mathcal{H}_y \\ h &= \arg \max_h (\mathcal{H}) \end{aligned} \right\} \quad (13)$$

where subscripts denote partial derivatives. The first two equations in (13) are the dynamic conditions in (9). Remember that the degree of protection, s , is only a parameter in this system. When we produce numerical solutions, a new s poses a new problem, which in turn requires a new optimized solution.

\dot{x} and \dot{y} are given in (9). For the sake of completeness, we write out the expressions for \dot{m} and \dot{n} according to (13).

$$\left. \begin{aligned} \dot{m} &= \gamma m + \frac{x_0}{x^2} h + m \left(1 - 2x - \frac{\omega}{1-s} \right) + n \frac{\omega}{s} \\ \dot{n} &= \gamma n + m \frac{\omega}{1-s} + n \left(1 - 2y - \frac{\omega}{s} \right) \end{aligned} \right\} \quad (14)$$

For *inner solutions*, $h = \arg \max_h (\mathcal{H})$ in (13) yields

$$\mathcal{H}_h = 1 - \frac{x_0}{x} - 2bh - m = 0 \quad (15)$$

Otherwise, we have $h = 0$. Note that when considering the equilibrium solution, that is, putting all dot equations in (13) equal to zero, \dot{y} produces the inequality $y \geq x$. The inequality is strict for any practical purposes ($x, y \in (0, 1)$ and $\omega > 0$).

Before we move on to the results, there are a few things we should be aware of with this model. The first is a technical issue; the two limits $s \rightarrow 1$ and $s \rightarrow 0$. The problem should be clear from the equations in (6); we obtain zero in the numerators. Analyzing what happens with the dynamics in these limits is an interesting discussion in itself, but we are not going into it in full detail. We attend to these limits for the sake of completeness, and only until we have established what we need to. First, we are interested in marine protected areas, which implies $s > 0$. This makes the zero limit uninteresting. Further, with no protected area we are able to substitute our MPA model with the reference model. It also turns out that the models are consistent with each other in this limit. Only variables and structures related to the protected area become ‘singular’, however, the area is nonexistent. Thus, the reference model serves as a form of benchmark for the MPA model. In the opposite situation, where we

approach full protection ($s \rightarrow 1$), we expect the steady state stock to approach the carrying capacity and harvests to drop to zero. This is not always true in our model. That said, a sanctuary covering the entire habitat is not interesting in the context of this paper. Hence, we will focus on $s \in (0, 1)$ and not consider the upper limit.

An important weakness in the model is the lack of knowledge and information about the rate of migration. According to McGarvey and Feenstra (2002), three forms of movement models have been postulated. These are reviewed in Quinn II and Deriso (1999). The diffusion model is one of them (Quinn II and Deriso 1999, pp.402-403). Fournier, Hampton, and Sibert (1998) estimates movement parameters in the diffusion sense for South Pacific Albacore, Their estimates, translated into our migration rate, suggest a rate in the area of 0.1 to 0.3. The estimates are uncertain, however. We investigate a larger range of migration rates. Regarding MPAs, McGarvey (2004) estimates the migration rate of fish stocks from marine sanctuaries. He is, however, not using the diffusion model, and his estimate is difficult to compare with our understanding of the parameter.

We have chosen not to incorporate uncertainty in the model. One motivation for that is that there is very much uncertainty about the uncertainty itself, the probability distribution, the functional form of it, and so on. Additionally, we would be concerned with uncertainty with regard to several aspects of the model. Adding a stochastic term, of which we have little empirical knowledge and know little of its functional form, does not necessarily improve the model.

3 Results

As mentioned earlier, deriving analytic results that are comprehensible is a difficult and often impossible task in models like this.¹⁶ Instead, we resort to numerical investigations of the model. In the next section we briefly explain

¹⁶We are, for example, able to produce an analytical expression for the equilibrium solution of the problem. The expression for one of the variables will then fill several pages, and it is hard to make any sense of it.

how the numerical solutions are obtained, before we turn to the results. Firstly, we address the premium. Moreover, we consider different measures of efficiency and examine some time series solutions.

3.1 The Numerical Analysis

We use optimal control theory to produce the optimal management strategy. Since the problem is *autonomous*, we are able to produce *feedback solutions*, solutions only dependent on state variables, *i.e.* the densities, and thus are independent of time. In particular, we use dynamic programming. The problem is formulated in continuous time, with continuous state variables. We follow the discretization process in Grüne and Semmler (2004) to make state and time variables discrete, and to form the *Hamilton-Jacobi-Bellman equation* for our problem. However, our approach is a slight variation to Grüne and Semmler (2004) when it comes to the maximization procedure, as we assume the control variable, *i.e.*, the harvest rate, to take values in the continuum $[0, \infty)$.¹⁷ This is possible because the control variable enters in the profit function (10) to the second degree. We are thus able to identify a unique control which maximizes π . The method implies computing a fix point of a dynamic programming operator. More technical details are found in the appendix.

The parameter values, found in Table 1, are based on data found in Kugarajh, Sandal, and Berge (2006). Two relevant parameters are not present in the table:¹⁸ ω and s . As mentioned, little data exist on the migration parameter, ω . We will focus on different values, exploring the effects from various levels of migration. We consider a range of protection levels, s .

¹⁷Grüne and Semmler (2004) asserts that the control variable takes values in a predetermined fixed set of discrete values.

¹⁸For parameters for the reference model and the absolute formulation, please refer to Kugarajh, Sandal, and Berge (2006) and the scaling transformation.

Parameter	Value	Explanation
γ	0.1	Relative discount rate
x_0	0.15	Open-access density
b_0	1.5	Relative demand parameter
p	10.	Price parameter (NOK per kilogram)
K	6000.	Carrying capacity (1000 kilograms)

Table 1: Parameters concerning the numerical calculations. The price parameter p and the carrying capacity K are only relevant in the link between the relative and absolute formulation of the premium, appropriate dimensions are given. The other parameters are dimensionless, as are the variables of the transformed maximization problem.

3.2 The Premium of Marine Protected Areas

Figure 1 shows the result from numerical calculations of the premium of an MPA as a function of s for four different values of the migration rate. There are three important things to observe in this figure: (i) the premium is positive and increasing with s ; (ii) increasing the migration rate reduces the premium; and (iii) the premium is smaller than s , which is the same as saying that you have to give up less in profit than what you protect in habitat, $\mathcal{P}(s) < s$. The first observation (i) is a consequence of the fact that the model is deterministic and nonlinear and that we optimize returns in the unprotected area. The second observation (ii) gives us reason to comprehend the migration rate as a degree of exploitation of the stock in the reserve. For higher migration rates, less profits vanish and, as we will see in a moment, harvest is reduced by a smaller amount. The last observation (iii) results from several mechanisms, but the driving force behind all of them is the fact that density levels change. Changing densities influences profits directly through the *stock effect* and indirectly through the *market effect*, demand is elastic and prices change when harvests change. Harvests change due to changes in production. Changes in production relate directly to the changing stock densities. Migration, which in this context can be perceived as a part of the production system, also changes due to changing densities. We shall investigate all of these mechanisms later, but before that, one more observation is necessary in Figure 1.

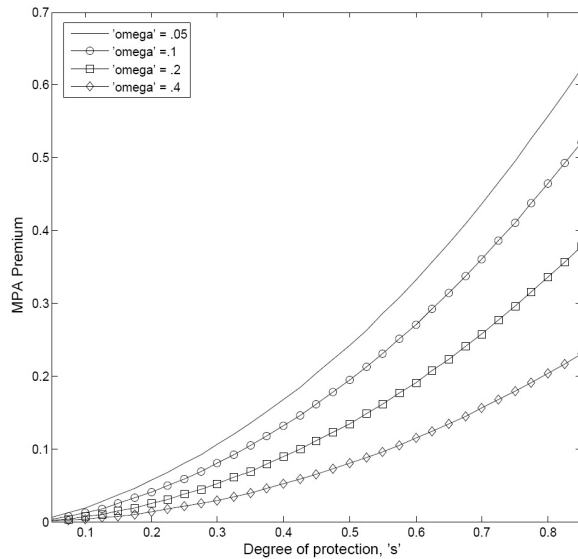


Figure 1: The MPA premium as a function of the degree of protection, $\mathcal{P}(s)$. Parameters are given in Table 1. The different curves correspond to different values for ω .

Note that the premium curves are *convex*. Since the curves are increasing, convexity means that protecting one extra unit of habitat is more costly than the last unit. For example, by trying to make a protectionist twice as happy by doubling the size of the reserve, it more than doubles the pain for the commercial agents in the fishery. This presumably complicates, *e.g.*, the process of deciding on the degree of protection. What makes the premium convex? We observe convexity even after setting a zero migration rate. This suggests that the convex property mainly stems from the nonlinearities in the profit function, *i.e.*, the *market effect* and the *stock effect*.

In Figure 2, we compare the premium from the equilibrium solution in Figure 1 with the premium arising from the dynamic solution for two different initial density levels (.1 and 1).¹⁹ Notice that the equilibrium curve, which is

¹⁹We compare different situations: a situation where the density is uniform at time zero, that is, when the reserve is established; and a situation where the initial density distribution matches the equilibrium distribution. In the former situation, the system evolves and presumably converges towards the equilibrium, *i.e.*, a dynamic situation. The latter situation is a static situation. Such initial conditions are, however, by all practical means impossible. Still, the results indicate that the comparison is justifiable.

based on the assumption that the equilibrium density is the initial density, is found between the curves of the dynamic solutions. This happens as the initial densities are lower (or higher) than the equilibrium density for all degrees of protection. That is, the larger the stock when the MPA is established, the cheaper it becomes. However, we also observe that the gap between the two dynamical solutions is rather small. This indicates that the initial size of the stock is not very important for the premium of an MPA. We should remember that our definition of the premium is a relative measure and that the absolute premium related to highly profitable stock levels, of course, is much higher than that of small stock levels, which are less profitable. Still, our results suggest that using equilibrium solutions when calculating the MPA premium is a good approximation and its simplicity outweighs the error in doing so. Taking into consideration that a lot of the input parameters are only roughly known, we should not worry too much about the use of this simplification. We will use this approximating approach for the rest of the paper, except, of course when dealing with time series solutions. The appendix provides an example of a feedback harvest rule and the corresponding value function for our problem (Figure 9). However, the results in Figure 2 evidently depend on the rate of discounting. We use a rate of 5 % in the calculations reported in the rest of the paper. The equilibrium solution is representative for the dynamic solutions to a decreasing degree for an increasing rate of discounting. Still, after investigating the results for a range of discount rates, we feel confident that our equilibrium analysis gives a good estimation of how these things are connected, at least up to a rate of 20 %.

We shall now investigate the mechanisms behind observation (iii) in Figure 1; $\mathcal{P}(s) < s$. As already stated, the driving force behind the different effects is the changing density levels as the degree of protection changes. Remember again that we are considering the equilibrium solution, so what really happens is that there are different optimal equilibrium positions for different protection levels. Equilibrium density levels and the harvest rate as functions of the degree

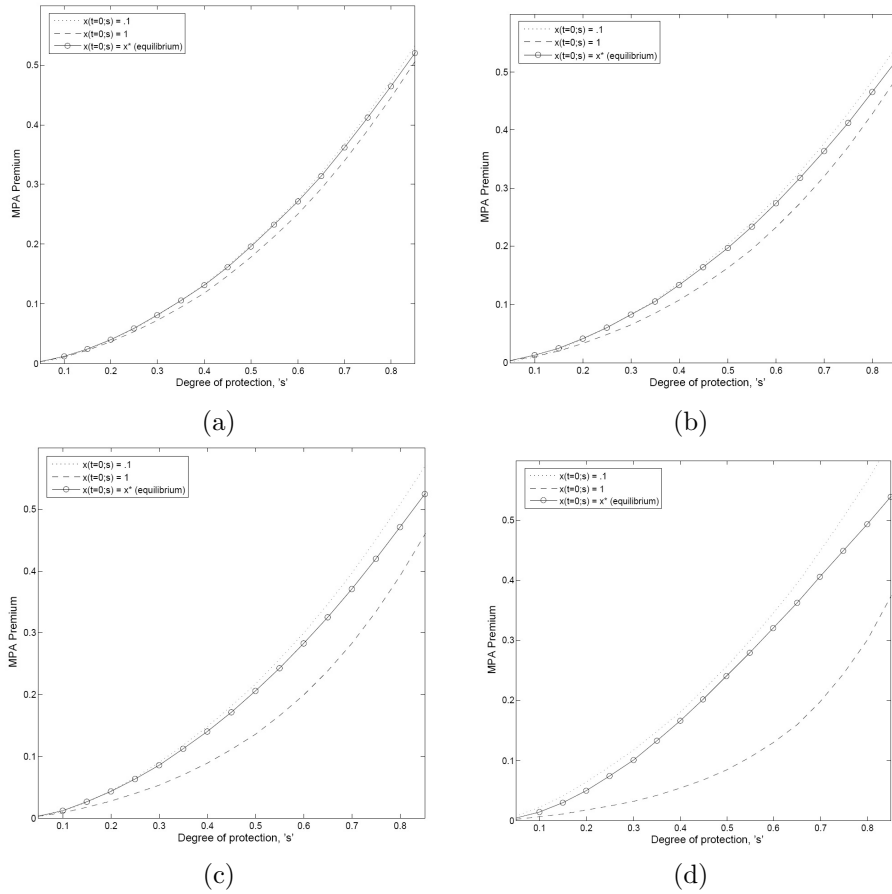


Figure 2: Comparing the static (equilibrium) and dynamic solutions of the premium for two different initial density conditions: $x(t = 0; s) = .1$, and $x(t = 0; s) = 1$. The four panels correspond to four different rates of discounting: (a) 5 %, (b) 10 %, (c) 20 %, and (d) 50 %. These are all per annum rates. Other parameters are given in Table 1. The migration rate is held fixed, $\omega = .1$.

of protection are displayed in Figure 3(a). We observe that the density in the protected area is increasing while decreasing in the rest of the habitat. The increase in density in the protected area reduces production in that area; it moves away from the level of *maximum sustainable yield* (MSY), which is exactly $\frac{1}{2}$.²⁰ The production increases in the remaining grounds as long as the density is above the MSY level, approaching it. As the density falls beyond the MSY level, production also decreases in the unprotected area. In absolute terms, the aggregated production is declining as s is increasing in the entire interval. The falling density in the unprotected area also implies that harvesting costs are increasing. The decline in harvest (Figure 3) induces a higher market value of the fish due to the demand mechanism.²¹ It turns out that the optimal harvesting strategy is to put a stronger pressure on the unprotected stock, and reduce the density. The gain in the unit price of fish is larger than the increase in unit harvesting cost.²² Note that the property $y > x$ is satisfied.

We have identified three mechanisms that have influence on the value of the fishery: rising costs from declining density (*stock effect*); decline in harvest (which we can coin *protection effect*); and rising prices (*market effect*). The first two of these have negative impact on returns; the latter has a positive effect. As already established, the negative effects outweigh the positive effect with increasing strength along s and with decreasing strength along ω . We will later study the interplay between the *stock effect* and the *market effect*, when we turn to time series solutions.

Related *shadow values* (refer equation (12)) are found in Figure 3(b). The shadow values are interpreted as the marginal value of the associated state variables. In the case of a fish stock, it measures the value of fish left unharvested,

²⁰The absolute production may well be rising; as the area size is rising, the actual biomass in it increases faster when both the size and the density are rising. However, we are interested in the densities and how *effective* the production is. When the density level moves away from the MSY level, the production is less effective, or a smaller part of the potential for natural growth is realized.

²¹Note that the scaling of the harvest variable depends on the degree of protection. The scaling transformation is singular in $s \rightarrow 1$, however. We control for the dependence on s in the reported results, which then are easier to interpret.

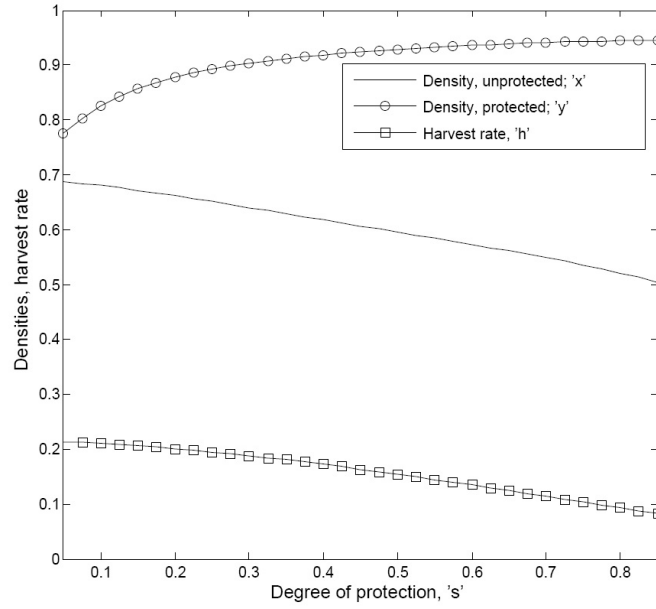
²²That is, for a small agent in the fishery with an individual, absolute quota, the fishery becomes more profitable.

or rather, the value of an additional unit of fish in the sea. Not surprisingly, the value of that extra unit of unprotected fish increases as more of the habitat is protected. As the degree of protection increases, the supply of the resource is reduced. Further, the density in the unprotected area is decreasing. The shadow value of the protected stock displays some interesting behavior. For small reserve sizes, the shadow value is falling, and for large reserve sizes, the value is rising. The increase for large s is hardly visible in the figure. The effect is more pronounced for smaller levels of b ; that is, a weaker *market effect*. However, an additional unit of protected fish has two consequences: production decreases in the protected area since the stock level moves further away from the MSY stock level, and migration increases since the difference in density increases. By studying the shadow price we increase our understanding of the changing densities, and thus the mechanisms behind the premium curves in Figure 1. As observed in Figure 3(a), the stock increases rapidly for small reserve sizes, and slower for large reserve sizes. A possible explanation for the behavior of the shadow value for the protected stock may be that the negative impact on production is stronger than the positive impact on migration for small s , and vice versa for large s .

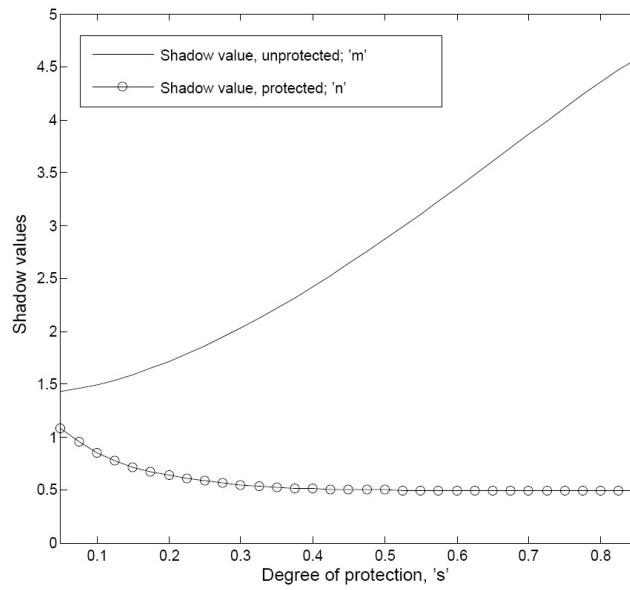
3.3 Measures of Efficiency

To increase the understanding of the dynamics of the model, we construct and compute the *relative growth efficiency* (RGE) for the two areas, which tells us how much of the potential production is reached in each area. We use MSY as a benchmark, hence we have $RGE(\cdot) = 1$ when MSY is attained. Note that we consider densities and consequently the growth function is $f(x) = x(1 - x)$, we have $x_{MSY} = \frac{1}{2} \Rightarrow f(x_{MSY}) = \frac{1}{4}$. The absolute migration between the areas is also calculated. The results are found in Figure 4(a) and (b). The growth efficiency depends on the density level and is given by

$$RGE(x) = \frac{f(x)}{f(x_{msy})} = \frac{f(x)}{1/4}$$



(a)



(b)

Figure 3: (a) Density levels and harvest rate as functions of the degree of protection. The migration rate is $\omega = 0.1$. (b) Corresponding shadow values for the stock levels in (a). Parameters for both figures are given in Table 1.

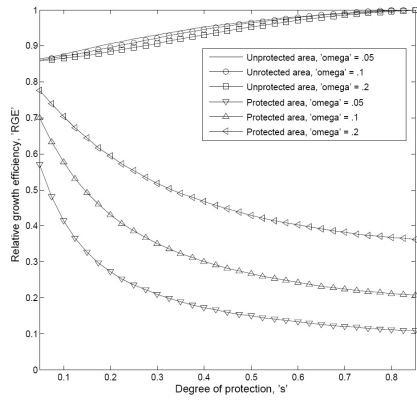
where x is the density in the unprotected area. $RGE(y)$ is defined similarly for the protected area. The argument indicates the area addressed. The absolute growth depends on the actual size of the areas. Observe that even though the density level in the protected area is moving away from the MSY level (Figure 3), that is, the growth efficiency is decreasing, the migration is increasing for all s (Figure 4(b)). The gap between the densities increases, however (Figure 3). The production in the protected area must equal the migration, thus the absolute production is increasing. It becomes less efficient, nevertheless. Moreover, the results in Figure 4(b) suggest that the migration is increasing in the migration rate. It is also interesting that the $RGE(x)$ is more or less unchanged for different migration rates, while $RGE(y)$ changes a lot. Looking further into this, Figures 4(c) and 4(d) compares the migration with the natural growth in the two areas. That is,

$$\frac{\phi(y-x)}{F(X; (1-s)K)}$$

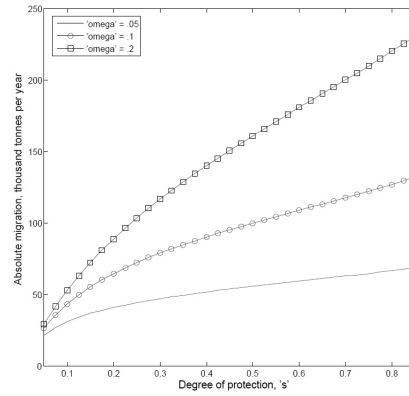
for the open area and correspondingly for the closed area. We find that the relationship responds heavily to changes in the migration rate when it comes to the open area (Figure 4(c)); the production remains more or less the same while the migration changes. The way the migration outperforms the production even for small, closed areas may suggest that the migration rates we have chosen to investigate are too high. As established from the dynamic constraint, there is a one-to-one relationship between the migration and production in the protected area for any rate of migration (Figure 4(d)).

The results reveals a weakness in the model, as the area that ‘receives’ the increasing migration is decreasing with s . This corresponds to the previously discussed issue of the limit $s \rightarrow 1$. However, comparing the migration with the carrying capacity of the ‘receiving’ area, *i.e.*, the grounds open to fishing, and suggesting a condition that migration cannot be larger than the capacity of the ‘receiving’ area,²³ the carrying capacity dominates for the reported results; the

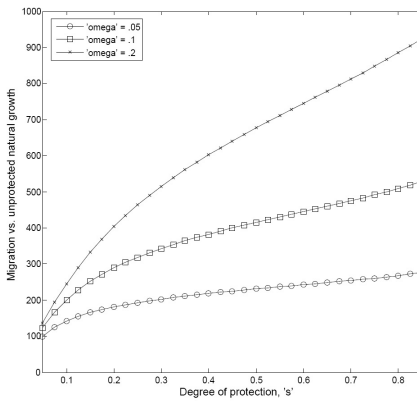
²³That is, if the rate of migration into any one area is smaller than the carrying capacity of that area, the suggested condition would not be binding.



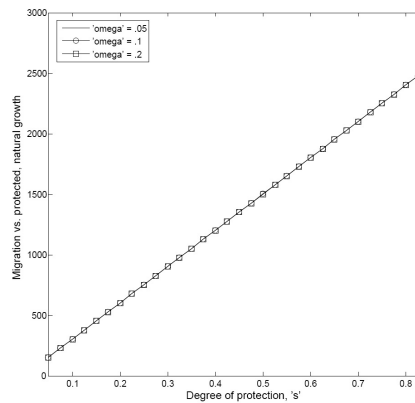
(a)



(b)



(c)



(d)

Figure 4: (a) Relative growth efficiency in the two areas as functions of s for three different rates of migration. (b) Measuring the absolute migration between the two areas, given by $\phi(y - x)$. (c) Migration relative to the growth in the unprotected area. (d) Migration relative to the growth in the protected area. Parameters for all panels are found in Table 1.

condition is met. However, the comparison may not be justifiable in general, comparing a *rate* with a *capacity*. Still, we assert that the comparison yields a useful measure of the validity of the results.

We study two more measures of efficiency. The first is a measure of the biological gain from establishing an MPA, and is simply the increase in standing stock. *E.g.*, Hannesson (1998) calls this measure the *conservation effect*.²⁴ For this measure, we need to consider the absolute values, that is,

$$X(s) + Y(s) - X(0)$$

where $X(s)$ and $Y(s)$ are the absolute standing stock levels for different degrees of protection, and $X(0)$ is the standing stock level for the no protection case, $s = 0$. However, absolute terms are a bit awkward, and, by dividing through by $X(0)$, we convert the measure into a relative measure, the *relative biological gain* (*RBG*).

$$RBG(s) = \frac{X(s) + Y(s)}{X(0)} - 1 \quad (16)$$

The measure compares the size of the equilibrium standing stock under the two different strategies: reserve and no reserve. It turns out that the shape of these curves is similar to the shape of the premium curves (Figure 1). Thus, we find it natural to compare these, that is, in Figure 5 we consider the relationship

$$\frac{RBG(s)}{\mathcal{P}(s)}$$

One could say that this relationship compares the relative increase in biomass (capital) with the relative loss in total profit. Figure 5 shows the result for a range of different values for the migration rate. In order to provide a complete analysis, the corresponding curves for $RBG(s)$ are found in the appendix (Figure 8).

²⁴When we suggest this effect as a gain, it should not be read that any increase to any level of biomass is positive in terms of the total value of the stock. It is, however, a measure of the increase in biomass, or capital, and thus a type of gain.

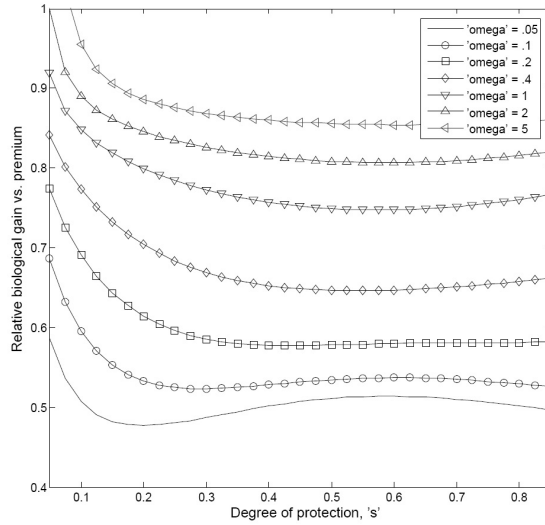


Figure 5: The relative biological gain compared to the premium of MPAs, $\frac{RBG(s)}{\mathcal{P}(s)}$, for different values of the migration rate. Parameters for the calculations are found in Table 1.

The evidence from Figure 5 is that the premium dominates the biological gain for all degrees of protection and for most rates of migration.²⁵ A higher migration rate dilutes the *conservation effect*; that is, the biological gain. This is an important mechanism behind the reduction in the premium from increasing the migration rate. Moreover, the results imply that the premium declines faster than the biological gain with the migration rate. The slope of the curves in Figure 5 is decided by the relationship between the slopes of $RBG(s)$ and $\mathcal{P}(s)$; for small s , the premium curve has the steeper slope and vice versa for large s .

While $RBG(s)$ measures how much the MPA strategy increases the standing stock, the measure we construct next concerns the share of the standing stock that is protected. From the property $y > x$ we know that the density is higher in the reserve, thus the protected share of the total stock is larger than the protected share of habitat, s . What we want to know is how much more of the

²⁵That is, $\frac{RBG(s)}{\mathcal{P}(s)} < 1 \Rightarrow RBG(s) < \mathcal{P}(s)$; the premium dominates.

biomass is found under protection than the share s . The share s of the biomass is found under protection with certainty for any nonnegative migration rate. We can perceive this as a measure of how *effective* the MPA is to protect the stock. We name this measure the *biological efficiency* (BE), biological in the sense that it is a biological quantity we measure. That is,

$$BE(s) = \frac{Y}{X + Y} - s \quad (17)$$

Figure 6 displays $BE(s)$ for different values of the migration rate. It must be zero in $s = 0$ and $s = 1$, and positive for all other $s \in (0, 1)$, given that the rate of migration is positive and finite. This follows from the equilibrium property $y > x$. $BE(s)$ decreases with the migration rate. This is in line with the earlier results. A high rate of migration reduces the *conservation effect* and that, as the migration rate goes to infinity, the density is uniformly dispersed: $BE(s) = 0$. The expression in (17) arises from comparing the two different growth models discussed earlier. The comparison suggests that whenever s equals the share of the stock found in the protected area, implying that the density is uniform in the entire habitat, the growth in the two models compares exactly. Any other s in the MPA model yields a smaller total growth. The ‘lost’ growth is a consequence of the convexity of the logistic function and the dispersed densities. $BE(s)$ assess the *displacement* between the best possible dispersion when it comes to total, natural growth (uniform dispersion) and the equilibrium dispersion. Thus, we can comprehend $BE(s)$ as a measure of *inefficiency* in the production system. The analytical comparison of the two growth models and the derivation of (17) are found in the appendix.

3.4 Time Series Solutions

Up until now, we have studied the equilibrium solution to the problem. We will finally turn to the dynamic solutions. We present a comparison of two time series solutions: one with and one without an MPA. Or rather, we compare

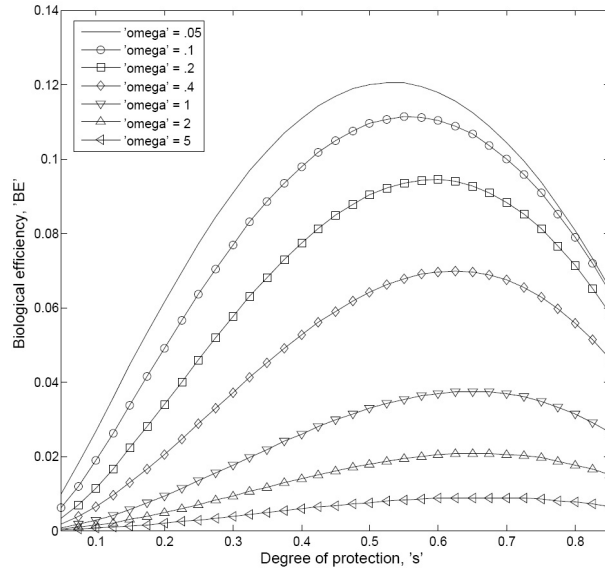


Figure 6: Biological efficiency, $BE(s)$ for different values of the migration rate. Other parameter values are found in Table 1.

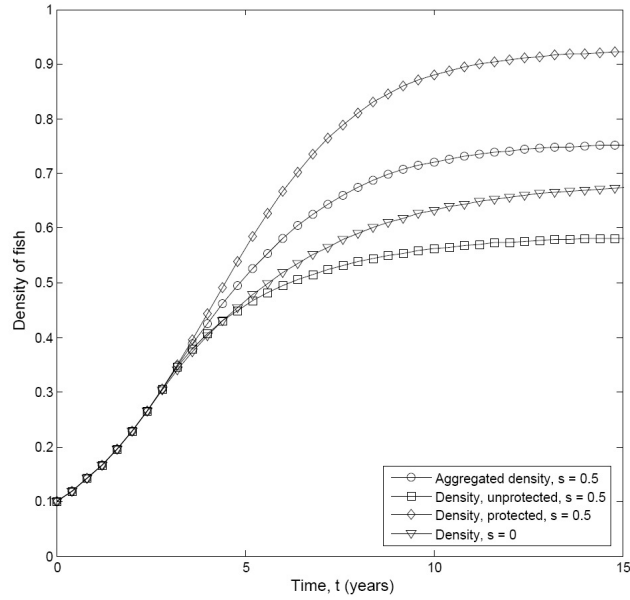
the solution of the reference model with the solution of the MPA model. We investigate the density distribution, the harvest rate and the real unit price of fish as functions of time. The MPA covers half the habitat of the fish stock. Figure 7(a) shows density paths where the initial stock level is only 10 % of the total carrying capacity. The fish are uniformly dispersed in the two areas at time zero; the densities in the two areas are equal. The figure also displays the average density of fish in the MPA case, and the curve is thus comparable to the reference case. Corresponding catch rates are found in Figure 7(b). We observe that the average density is always higher with an established reserve, consequently the total standing stock is larger in the MPA situation. Further, the density in the unprotected area is smaller than the density in the no reserve case; the *stock effect* induces higher harvesting costs with the MPA tool. The catch rate is always higher without the reserve. Further, it seems the system approaches the equilibrium slower when there is no reserve. There is a ‘moratorium’ the first years, until the stock has been rebuilt to levels above the open-access level.

The ‘moratorium’ is lifted earlier in the no MPA case. The two systems (MPA and no MPA) follow the same path as long as the harvest is zero; that is, as long as the density is uniformly dispersed. In Figure 7(b) the real unit price of fish is calculated. The price is always higher in the reserve situation. We have two effects in play here: changes in harvest level change the price of fish in the market, and changing densities influence costs related to catching. A higher real price in the MPA case implies that the *market effect* outruns the *stock effect* and the *net return* per unit of fish increases with the reserve in place. There is typically room for fewer active agents in the fishery, however, as the total catch is smaller. These results depends on the relationship between the price parameter p and the demand parameter d . Certainly, there exist parameters such that the conclusion is reversed; a weaker *market effect* compared to the *stock effect*, and the fishery becomes less profitable for the agents as a consequence of the establishment of a marine reserve.

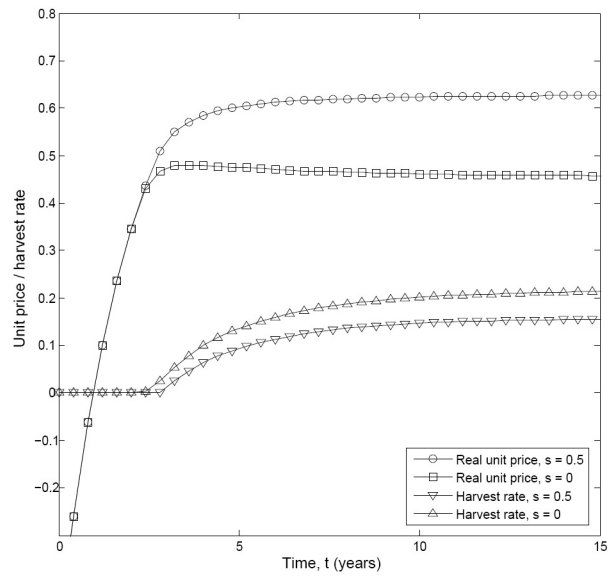
4 Conclusions

Hannesson (2002) is skeptical to advocating marine protected areas as a bet hedging tool, and Sanchirico (2000) dismisses the use of the term *insurance* related to reserves. However, there is a wide consensus that MPAs as a management tool has the potential to reduce uncertainties in several aspects of a fishery. To be able to evaluate the usefulness of this management tool, thorough analyses of economic and biological effects are required, particularly when such instruments are implemented. In this paper, we have gone to new depths in analyzing the cost induced from establishing a protected area. We reveal and discuss how these costs are generated and study the different mechanisms that work on both economic and biological factors. Different measures of efficiency are proposed.

The main finding is that costs are smaller than the share of protected habitat compared to the total profits, and that protecting one more unit is more



(a)



(b)

Figure 7: (a) Densities as functions of time. Two protection scenarios are compared: $s = 0.5$ and $s = 0$. Local densities in the first scenario are also reported. (b) Comparing the harvest rate and the real unit price of fish in the same two scenarios. The migration rate is set to $\omega = .1$ in both panels, further parameter values are found in Table 1.

costly than the last unit protected, *i.e.*, convexity. The convex property of the premium may complicate things when trying to decide on an appropriate degree of protection. In particular, the lack of knowledge on the migration term further entangles the process. The results are based on numerical calculations and the generality depends on the choice of parameter values. However, the model displays mostly monotonic behavior when we alter the different parameters for large intervals around the chosen parameter. We believe that our conclusions are quite general, but that the strength of the different effects observed are more uncertain.

We have investigated different measures of efficiency. We find that a reserve leads to less efficient biological production. Results suggest that the rate of migration is decisive when comparing the relative increase in the standing stock with the premium of the MPA. Further, the share of stock found in the reserve compared to that of protected habitat is investigated. This measure could be understood as the marginal biological value of the protection, it demonstrates the increase in protected biomass relative to the degree of protection.

Our analysis also shows that the fishery potentially becomes more profitable to the active agents. However, this effect is critically dependent on the chosen parameters. Another set of parameters may very well give the opposite conclusion.

All the results aside, we have demonstrated that the perspective on marine protected areas presented here increases the understanding and clarifies the influence and importance of the different effects and mechanisms that are involved. We also suggest there is room for improvements and further investigations within this framework. In particular, we would like to see the results from introducing a stochastic element in the model. Studying more sophisticated biological and economic submodels will potentially provide additional insights.

A Appendix

Comparing Growth Models

In absolute terms, the total natural growth in the entire area in the MPA model less the total natural growth in the reference model is given by

$$F(X; (1-s)K) + F(Y; sK) - F(X+Y; K) = rX \left(1 - \frac{X}{(1-s)K}\right) + rY \left(1 - \frac{Y}{sK}\right) - r(X+Y) \left(1 - \frac{X+Y}{K}\right)$$

This expression should be equal to zero if there is no difference in the two growth models. The first factor in each term and the carrying capacity cancel each other, thus obtain

$$(X+Y)^2 - \frac{X^2}{1-s} - \frac{Y^2}{s} = 0 \tag{A-1}$$

Solving this equation for s yields the double root

$$s_0 = \frac{Y}{X+Y}$$

which shows that the growth is equal in the two models when the fraction of biomass in the two areas is proportional to the fraction of habitat under protection. Whenever that is the case, the density is uniform in the entire habitat; *i.e.*, the situation is identical in the two different models. Equal growth would then be anticipated. The expression for s_0 appears in (17).

Further, s_0 being a double root implies that the expression in (A-1) is either positive or negative for all $s \neq s_0$. It is readily seen that the expression is negative. Hence,

$$F(X; (1-s)K) + F(Y; sK) \leq F(X+Y; K)$$

The natural growth in the reference model is always bigger or equal to the total

natural growth in the MPA model.

Details on the Numerical Solution Scheme

The dynamic programming operator T is given by

$$T_{\Delta\tau}[v_{\Delta\tau}](\mathbf{x}) = \max_{h \in [0, \infty)} \left[\Delta\tau \cdot \pi(\mathbf{x}, h) + \beta \frac{\partial v_{\Delta\tau}(\mathbf{x})}{\partial \mathbf{x}} \right] \quad (\text{A-2})$$

where \mathbf{x} is the discrete state variable, h is the control variable, $\Delta\tau$ is the discrete time step, $v_{\Delta\tau}(\mathbf{x})$ is the value function with respect to the time step and discrete state variable, and $\beta = 1 - \Delta\tau \cdot \gamma$, the discrete discount rate. The discrete variables, time step, and discount rate stem from the discretization procedure given in Grüne and Semmler (2004). Note that we have omitted the s -parameter in the value function. To maximize our problem (8, 9), we approximate a solution to the fixed point equation

$$v_{\Delta\tau}(\mathbf{x}) = T_{\Delta\tau}[v_{\Delta\tau}](\mathbf{x})$$

Solutions are produced from an iterative process (Grüne and Semmler 2004, Bertsekas 2005). The numerical solutions are approximations as we end the iterative process when an error measure satisfies some predetermined condition.

Figures

The relative biological gain, $RBG(s)$ is displayed for two different values of the migration rate in Figure 8. Observe that the *conservation effect* declines with an increasing migration rate.

Figure 9a displays an example of an optimal feedback harvesting rule. The corresponding value function is presented in Figure 9b. The value function yields the value of a fishery for given initial densities.

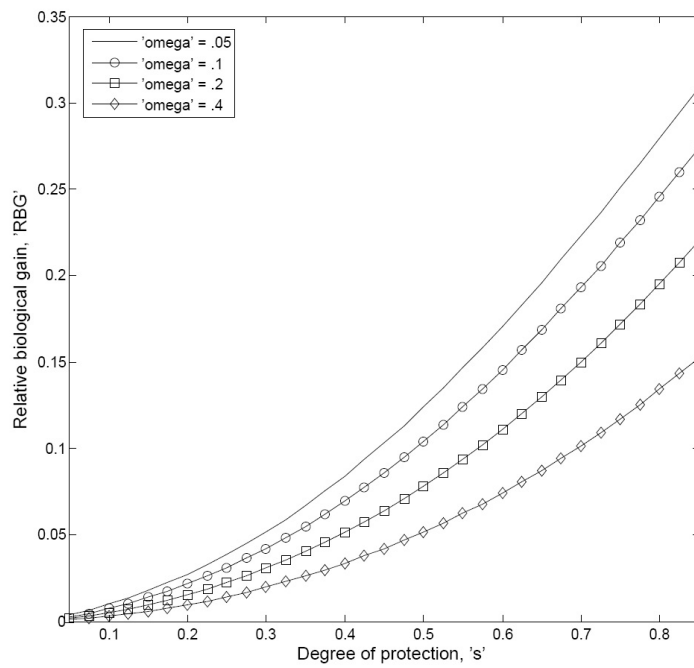
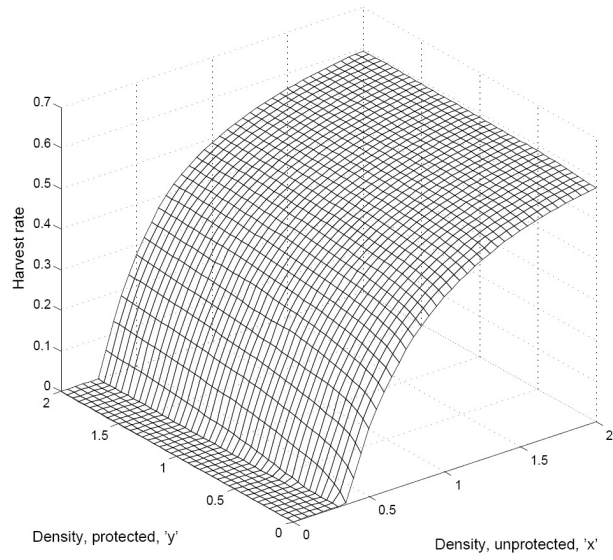
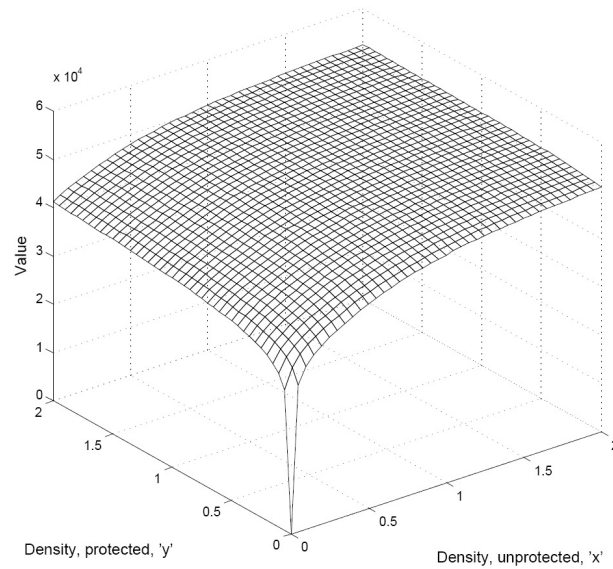


Figure 8: The relative biological gain, $RBG(s)$, for four different values of the migration rate. Further parameters are found in Table 1.



(a)



(b)

Figure 9: (a) Optimal feedback harvesting rule. (b) The value function corresponding to the given harvest rule. The migration rate is set to $\omega = .1$ in both panels, and $s = 0.5$. Further parameter values are found in Table 1.

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