

# Utility analysis, luxuries and risk—a critical comment\*

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**Abstract** This note questions the analysis of Diacon (2006) in the June issue of *Economics Letters*. It points at a misinterpretation and claims that the conclusion—that the presence of luxuries may make people love income gambles—is only true if people love gambles on luxury products.

## 1 Introduction

In a recent paper in *Economics Letters*, Diacon (2006) argues that the risk loving behaviour (accepting unfair gambles) can be due to the presence of luxury commodities in the consumer's basket—goods which take a larger portion of the consumer's budget as she gets richer. Diacon's argument is based on a decomposition of the risk premium.

In this note I argue that Diacon's argument is flawed by a misinterpretation of the coefficient of relative risk aversion and that his conclusion is based on the implicit assumption that the consumer is risk loving when it comes to gambles on the luxury commodity. Risk loving behaviour is therefore not explained, but assumed.

## 2 Analysis of Diacon's arguments

Diacon describes a consumer whose preferences are represented by a strictly quasi concave utility function defined over two goods,  $u(x_1, x_2)$ , of which the maximisation under the budget constraint  $p_1x_1 + p_2x_2 = y$ , results in the maximal utility function  $v(p_1, p_2, y)$ . The second commodity is assumed to be a luxury (having an income elasticity exceeding 1).

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Diacon next derives the central equation in his article, eq (7), by decomposing the elasticity of the marginal utility of income,  $-\frac{v_{yy}}{v_y}y$ , into a cardinal measure for the elasticity of the marginal utility of good 1, and an ordinal term which involves the budget shares of both commodities ( $s_i, i = 1, 2$ ), the income elasticity of commodity 2 ( $\eta_2$ ), and the own Slutsky price elasticity of commodity 1 ( $\widehat{\varepsilon}_{11}$ ):

$$-\frac{v_{yy}}{v_y}y = \left( -\frac{u_{11}}{u_1}x_1 \right) \frac{1}{s_1} - \frac{(s_2)^2(\eta_2)^2}{s_1(-\widehat{\varepsilon}_{11})} \quad (1)$$

The decomposition in equation (1) is not new. It was derived by Drèze and Modigliani (1966, 1972)—in the latter paper as equation (2.9)—in the context of the consumer's two period intertemporal savings problem when she faces an uncertain future income. The first *rhs* term is then the consumer's (relative) risk aversion for a delayed risk (when future consumption is labelled as good 1), while the *lhs* term is the risk aversion for the same risk when timeless, i.e. if the consumer were informed about the draw of future income *before* making her savings decision. The second *rhs* term is then by definition the expected value for perfect information—it is entirely ordinal.

Diacon interprets the *lhs* of (1) as the consumer's coefficient of relative risk aversion and the first *rhs* term as the analogous coefficient "in a one good world" (p 405). Since he does not specify the risks facing the consumer, it is not clear at all why the *lhs* of (1) should be given this interpretation. If the risk is an income risk, then the only correct interpretation of (1) is that the willingness to pay for avoiding a timeless income risk is smaller than the willingness to pay for avoiding a delayed income risk *when part of her income is already committed to commodity 2*. If there is any term in (1) that deserves the interpretation of risk aversion "in a one good world", it is the *lhs* since there is no relative price uncertainty. Eq (1) then tells us that the relative risk premium for gambles on commodity 1 exceeds the relative risk premium for income gambles with the value for perfect information.

Diacon finally makes the point that "in those cases where good two is a bundle of luxuries, the expenditure share of good 1 ( $s_1$ ) becomes small as income increases so that the second term could [on the *rhs* of (1)] dominate the first at high income levels." (pp 404-405). This analysis invites for two comments. *First*, it is silent what happens to the first *rhs* term and to the elasticities as income grows and  $s_1$  falls. It is well known that only Cobb-Douglas preferences yield constant income and price elasticities. But under these preferences the budget shares remain constant as well, not leaving any room for luxury goods.. The inference from (1) is thus a first approximation. *Second*, it begs for the question where the negative risk aversion for timeless income risk comes from. For this purpose it is useful to look at the alternative decomposition of this risk

aversion, now in terms of aversion to gambles on commodity 2:

$$-\frac{v_{yy}}{v_y}y = \left(-\frac{u_{22}}{u_2}x_2\right)\frac{1}{s_2} - \frac{(s_1)^2(\eta_1)^2}{s_2(-\widehat{\varepsilon}_{22})} \quad (2)$$

Keeping elasticities constant *to a first approximation*, (2) tells us that  $-\frac{v_{yy}}{v_y}y$  will approach  $-\frac{u_{22}}{u_2}x_2$ , the relative risk aversion for the luxury good, as  $s_2$  grows. Hence if  $-\frac{v_{yy}}{v_y}y$  becomes negative it must be because the consumer has a negative willingness to pay to avoid gambles on the luxury good. It is then of no surprise that she will also display a preference for timeless income risk as she gets richer. The important issue—an empirical one, on which Diacon remains silent—is then to which extent a consumer is less risk averse towards gambles on luxury goods than towards other commodities.

## References

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- [3] Drèze, J H, Modigliani, F (1972). Consumption decisions under uncertainty. *Journal of Economic Theory* 5, 308-335.