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Foreign aid and sovereign credit worthiness

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FOREIGN AID AND SOVEREIGN CREDIT WORTHINESS

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Abstract[1]**:** International financial markets are far from perfect. Because of problems related to contract enforcement borrowers often end up being rationed; the lenders tend to constrain the amount lent ex ante in order to motivate the borrowers to fulfil their obligations and not default ex post. In this paper we take as our point of departure a relationship like this between a lender (or consortium of lenders) and the government of a poor country and ask: How is this relationship affected by the fact that the borrowing country also receives foreign aid? The answer depends on how the aid is given. If the aid inflow is exogenous we show that some types of aid are effective in the sense that the aid has a positive effect on the credit obtained and aggregate welfare. Other types are directly counterproductive. If the aid inflow is endogenous, supplied by altruistic donors as part of a safety-net, serious incentive distortions arise, crowding out private credit. Such aid may actually be welfare-reducing in the recipient country. The paper also contains a discussion of how aid will influence lenders' incentives to give relief if the initial debt is not sustainable.

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1 Introduction

International financial markets are far from perfect. Problems related to contract enforcement are severe, especially in situations where sovereigns (governments) borrow in private (commercial) financial markets. There exists no international authority capable of enforcing such contracts[2]. Why should the borrowers repay their debt under such circumstances? For them to be willing to repay they must be confronted with some costs in case of default. In the traditional literature at least three types of costs are discussed: bad reputation and lack of access to international capital markets in the future, assets in foreign countries may be seized, and sanctions reducing the benefits from international trade may be imposed. However, any kind default-related costs - also pure domestic costs caused by for example a default-related economic down-turn or collateral being lost in case of default - will have similar disciplining effects. The threats of sanctions, the collateral offered, as well as other default-related costs must be known by both the lenders and the borrowers when the loan contract is signed[3].

Eaton and Gersovitz (1981) were the first to show that in a situation where contract enforcement is difficult, the borrowers' ability to repay is of limited importance; it is their willingness that really matters.

[1]The author wants to thank Rune Jansen Hagen for comments on an earlier draft.

[2]See Rogoff and Zettelmeyer (2002) for a survey of ideas related to bancrupcy procedures for sovereigns - never implemented. See also Sturzenegger and Zettelmeyer (2006) for arguments that the possibilities for creditor legal actions against sovereigns have increased recently.

[3]Credibility may, of course, be a problem here. Ex-post, in case of default, the lenders may end up being better off without imposing the sanctions (actual repayment will be higher if they don't than if they do) and the borrowers will certainly wish to withdraw the collateral offered ex-ante. However, we disregard such credibility problems here.

As a rule the willingness to repay ex-post tends to be lower the higher is the debt obtained ex-ante, i.e., most cost specifications lose their power when the debt becomes very high. As a result, the borrower typically wants to borrow as much as possible and then default. The lender, therefore, will tend to constrain the amount lent ex-ante in order to motivate the lender not to default; some type of credit rationing is likely to result. Obstfeld and Rogoff (1996), chapter 6, contains sketches of some of the most interesting models designed along such lines, one of which is the inspiration for the basic model used in this paper.

Many of the governments borrowing from commercial lenders also borrow from multilateral institutions like the IMF or the WB. Whether multilateral lending leads to more or less privat credit is a question that has been studied by several researchers, for example Cottarelli and Gioanni (2002). That literature is summarized by Hagen (2006) in the following way: "Empirical studies indicate a neutral effect overall, with private agents' negative reactions being approximately cancelled by increased funds from other official sources".

Many of the borrowing governments also receive (bilateral) foreign aid. How such aid (and donors' activities) might affect privat lending, has not been much discussed in the existing literature. This paper offers an analytical approach to that question. We first consider the aid amount obtained by the borrowing government as exogenous and discuss how aid of different kinds affect the relationship between the government and a commercial lender (or consortium of lenders). Some types of aid works well, in the sense that the lender's incentives to lend improves, investment and welfare increase, etc. Other types of aid have the opposite types of effects and are unlikely to benefit the recipient.

Then some of the aid inflow is endogenized, assumed to be supplied by altruistic donors as part of a safety-net. Pedersen (1996) and (2001), among others, has shown in other contexts that altruistic donors may distort incentives in a negative way. Will this be the case in the present context as well? The answer is, unfortunately, yes. We show, among other things, that it may actually be in the borrowing government's own interest to refuse to become a client of altruistic donors and part of their international safety-net.

In addition to the traditional approach, focusing on access to new credit we also show how a simple respecification of the basic model, influenced by Omland (2005), may allow us to throw light on questions related to aid and voluntary debt relief.

1.1 The model structure

The time horizon is two periods, called period 1 and period 2. The sovereign (government) makes productive investments in period 1. The investment strategy chosen depends on whether, in period 2, he plans to repay the debt falling due inclusive of interest (Non-default - N) or not (Default - D). The sovereign knows that if he chooses to default, there will be some costs. However, the lender is bound to lose if the sovereign borrower decides to default and, as a consequence, it is in his interest to keep him from defaulting. That is why the access to credit may be constrained or debt relief given if initial debt is too high.

Considering the aid inflow to be *exogenous* we first analyse399 the situation where the borrowing sovereign has *discretion* over investment, in the sense that he is free to decide how much to invest in the domestic economy *after* an agreement on access to new credit (section 2) or debt relief (section 3) is in place. In section 4 we consider the situation where he is able to *commit* to an investment strategy *before* entering into negotiatios on access to new credit or debt relief. In a world where mechanisms for credible commitments of the type in question typically are in short supply, the discretion case may seem the most realistic. Then, in section 5, the aid inflow is *endogenized*. The basic model used is influenced by a similar model found in Obstfeld and Rogoff (1996), chapter 6[4].

[4]See Dooley & al (2007) for rather critical comments on this type of literature - arguing that countries consciously accumulate foreign reserves and implicitely offer them as collateral to foreign lenders and investors. In our view, however, their comments are not relevant for most poor and middle-income countries.

We let C symbolize aggregate consumption, Y be some exogenous income (domestic value added), A be foreign aid, K be (productive) investment in period 1, and D represent the face value of the debt falling due in period 2. D also represents the amount borrowed in period 1 in a situation where the initial debt is zero. To make things simple we assume that the sovereign has access to some constant-returns technology and express the income generated in period 2 if investment is carried out in period 1 as $(1+\alpha)K$. The interest rate is r and we assume that $\alpha \gg r$, i.e., the marginal (and average) product of capital is higher than the interest paid on foreign debt. To make things simple we assume that the interest rate is exogenous, for example because there is competition among potential lenders who all have the same opportunity cost of capital.

Default will here be interpreted to mean that the sovereign in period 2 refuses to repay the amount falling due as well as interest. Default, however, imlies costs of some type, related to sanctions, domestic economic down-turn, or collateral. We model the costs in a very simple way: $\eta_K (1+\alpha) K + \eta_{\overline{A}} \overline{A}_2^D + \eta_{\overline{Y}} \overline{Y}_2$.

Here $0 \ll \eta_K \ll 1$ works as a "tax" on the return from investment, for example caused by reduced access to the world market resulting in reduced output prices and/or increased input prices, worsened domestic market situation, or delberate offering of a share of output as collateral. $0 \leq \eta_{\overline{Y}} \ll 1$ is an exogenous cost expressed as a share of other types of domestic value added, for example worsened terms of trade for "traditional" export products as a result of trade sanctions, worsened domestic market situation, or collateral. $0 \leq \eta_{\overline{A}} \leq 1$ is the share of the aid donated in case of default which ends up in the lender's pockets - because it has been offered as collateral or because the donor imposes conditions - and/or being withheld [5]. In addition, the access to foreign aid may differ, A_2^N in case of non-default and A_2^D if the result is default. For non-default to be better than default, as perceived by the borrowing sovereign, the following must hold: $\overline{A}_2^N - D(1+r) \geq \overline{A}_2^D - \left(\eta_K (1+\alpha) K + \eta_{\overline{Y}} \overline{Y}_2 + \eta_{\overline{A}} \overline{A}_2^D\right)$ or

$$D \leq \frac{1}{1+r} \left[\eta_K (1+\alpha) K + \eta_{\overline{Y}} \overline{Y}_2 + \eta_{\overline{A}} \overline{A}_2^D + \left(\overline{A}_2^N - \overline{A}_2^D\right)\right] \tag{1}$$

and in the exercises below the lender will typically make sure that this condition holds by not allowing D to become too high.

It follows that we do not necessarily assume that any of the debitor's default costs end up in the hands of the creditor. The " $-$ " means that the variable in question is exogenous.

We have already assumed that it is profitable for the government to borrow and invest one unit in period 1, given that the unit borrowed and interest is repaid in period 2, i.e., $(1+\alpha) - (1+r) = \alpha - r \gg 0$. The arguments below rely, in addition, on the assumption that it is even more profitable not to repay the debt, i.e., $(1+\alpha)(1-\eta_K) \gg \alpha - r$. This is actually a restriction on the severeness of the sancions to be imposed in case of default. Another way of expressing it is $\eta_K (1+\alpha) \ll (1+r)$[6].

2 New loans and discretion over investment

We first discuss the sovereigns investment startegy in case he does not plan to default and then in case he does. Then, given that the creditor does not want the sovereign to default, the optimal amount lent, as perceived by the lender, is determined. Finally, som comparative static analyses are carried out.

2.1 Non-default (N)

Assuming that initial debt in period 1 is zero[7] the two periods' budget equations may be expressed as:

[5]See Rose (2005) for evidence that in practice, default seems to be strongly associated with reduced trade, and Levy-Yeyati and Panizza (2006) for evidence that the costs may be of domestic origin.

[6]Obstfeld and Rogoff (1986) p. 383 argue that this is a very reasonable assumption.

[7]If here was an initial debt, $\overline{D}$, access to new credit, $D - \overline{D}$, would enter period 1's budget constraint instead of D. An increase of $\overline{D}$ works as a reduction of $\overline{A}_1$.

$$C_1 = \overline{Y}_1 + \overline{A}_1 + D - K \text{ and} \tag{2a}$$
$$C_2 = \overline{Y}_2 + \overline{A}_2^N + (1+\alpha) K - (1+r) D \tag{2b}$$

or using period 1's budget equation to express K as a function of C_1, the intertemporal budget equation may be written as

$$C_2 = \overline{Y}_2 + \overline{A}_2^N + (1+\alpha)\left(\overline{Y}_1 + \overline{A}_1 - C_1\right) + (\alpha - r) D \tag{2c}$$

It shows that there is a clear and simple trade-off between consumption in the two periods. For a given level of foreign borrowing, D, the real return to investment/capital determines what will be called the accounting rate of interest, ARI: $\frac{dC_2}{dC_1} = -\left(1 + ARI^N\right) = -(1+\alpha)$.

The sovereign is assumed to maximize the welfare function $U = \ln C_1 + \beta \ln C_2$ where $0 \ll \beta \leq 1$ is the discount factor. The first-order condition may be expressed as

$$\frac{dU}{dC_1} = \frac{1}{C_1} - \beta\frac{1}{C_2}(1 + ARI) = 0 \tag{3}$$

from which the two periods' consumption and the aggregate productive investment may be found. Letting

$$Y^N = \overline{Y}_1 + \overline{A}_1 + \frac{1}{1+\alpha}\left[D(\alpha - r) + \overline{Y}_2 + \overline{A}_2^N\right] \tag{4}$$

be the present value (in period 1) of what will be called the country's disposable income (slightly abusing terms) we have

$$C_1^N = \frac{1}{1+\beta}\left[Y^N\right] \quad , \quad C_2^N = \frac{\beta(1+\alpha)}{1+\beta}\left[Y^N\right] \quad \text{, and} \tag{5a}$$
$$K^N = \frac{\beta}{1+\beta}\left[\overline{Y}_1 + \overline{A}_1 + D\right] - \frac{1}{1+\beta}\left[\frac{\left(\overline{Y}_2 + \overline{A}_2^N\right) - D(1+r)}{1+\alpha}\right] \tag{5b}$$

The welfare level as perceived by the sovereign may now be calculated:

$$U^N = (1+\beta)\ln\left(\frac{1}{1+\beta}\left[Y^N\right]\right) + \beta\ln\left(\beta(1+\alpha)\right) \tag{5c}$$

2.2 Default (D)

In case of default, sanctions of the types discussed above, see (1), are imposed in period 2. Assuming that default means cancelling all debt obligations[8] the two periods' budget equations may now be expressed as:

$$C_1 = \overline{Y}_1 + \overline{A}_1 + D - K \text{ and} \tag{6a}$$
$$C_2 = (1-\eta_{\overline{Y}})\overline{Y}_2 + (1-\eta_{\overline{A}})\overline{A}_2^D + (1-\eta_K)(1+\alpha) K \tag{6b}$$

or using period 1's budget equation to express K as a function of C_1, the intertemporal budget equation may now be written as

$$C_2 = (1-\eta_{\overline{Y}})\overline{Y}_2 + (1-\eta_{\overline{A}})\overline{A}_2^D + (1-\eta_K)(1+\alpha)\left(\overline{Y}_1 + \overline{A}_1 + D - C_1\right) \tag{6c}$$

[8] Maybe repudiation would be a better word than default here. If default meant only a partial cancellation of debt obligations the rest could easily be taken into account as an exogenous cost i period 2. It will work in a similar way as a reduction of $\overline{A}_2^D$.

For a given level of foreign borrowing, D, again the ARI is given by the real return to investment/capital - this time after correcting for the sanction cost: $\frac{dC_2}{dC_1} = -\left(1 + ARI^D\right) = -\left(1 - \eta_K\right)(1 + \alpha)$. The sanction costs makes sure that $ARI^D \ll ARI^N$, i.e. the terms on which period 1 consumption can be transformed into period 2 consumption are better in the non-default case than in the default case.

The sovereign also in this case maximizes the welfare function, see (3) but the ARI differs. Letting

$$Y^D = \overline{Y}_1 + \overline{A}_1 + D + \frac{1}{(1-\eta_K)(1+\alpha)}\left[(1-\eta_{\overline{Y}})\overline{Y}_2 + (1-\eta_{\overline{A}})\overline{A}_2^D\right] \tag{7}$$

be the present value of the country's disposable income, we now have

$$C_1^D = \frac{1}{1+\beta}\left[Y^D\right] \quad , \quad C_2^D = \frac{\beta(1-\eta_K)(1+\alpha)}{1+\beta}\left[Y^D\right] \quad , \tag{8a}$$

$$K^D = \frac{\beta}{1+\beta}\left[\overline{Y}_1 + \overline{A}_1 + D\right] - \frac{1}{1+\beta}\left[\frac{(1-\eta_{\overline{Y}})\overline{Y}_2 + (1-\eta_{\overline{A}})\overline{A}_2^D}{(1-\eta_K)(1+\alpha)}\right], \text{and} \tag{8b}$$

$$U^D = (1+\beta)\ln\left(\frac{1}{1+\beta}\left[Y^D\right]\right) + \beta\ln\left(\beta(1-\eta_K)(1+\alpha)\right) \tag{8c}$$

2.3 The debt limit

Given the way our model has been set up it is clear that the lender wants to avoid default. This means that he must make sure that it is in the borrower's own interest to fullfill his obligations, i.e., that $U^D - U^N \leq 0$. Otherwise the borrower will choose to default. This condition allows the lender to find the upper limit of lending. Actually, given the example with which we are working, this upper limit may be calculated explicitely, see appendix 1:

$$D^* = \Omega\left[\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}}\left(\overline{Y}_1 + \overline{A}_1 + \frac{\overline{Y}_2 + \overline{A}_2^N}{1+\alpha}\right) - \left(\overline{Y}_1 + \overline{A}_1 + \frac{(1-\eta_{\overline{Y}})\overline{Y}_2 + (1-\eta_{\overline{A}})\overline{A}_2^D}{(1-\eta_K)(1+\alpha)}\right)\right] \tag{9}$$

where $\Omega = \frac{1}{1-\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}}\left(\frac{\alpha-r}{1+\alpha}\right)}$. The assumption that $(1+\alpha)(1-\eta_K) \gg \alpha - r$ makes sure that Ω is strictly positive and higher than unity.

There is no uncertainty in this model and because of the lender's rationing of the borrower, setting $D = D^*$, there will actually never be any default. And the level of productive investment can be found from the K^N-function above (5b), given an access to credit equal to D^* :

$$K^* = \frac{\beta}{1+\beta}\left[\overline{Y}_1 + \overline{A}_1 + D^*\right] - \frac{1}{1+\beta}\left[\frac{\left(\overline{Y}_2 + \overline{A}_2^N\right) - D^*(1+r)}{1+\alpha}\right]$$

Given $D = D^*$, $K = K^*$, and non-default the actual welfare level, as perceived by the borrowing and aid-receiving sovereign, will be determined by what we have called the present value of his aggregate income, Y^N(4) and U^N (5c), i.e.:

$$\begin{aligned} Y^* &= \overline{Y}_1 + \overline{A}_1 + \frac{1}{1+\alpha}\left[D^*(\alpha - r) + \overline{Y}_2 + \overline{A}_2^N\right] \text{ and} \\ U^* &= (1+\beta)\ln\left(\frac{1}{1+\beta}[Y^*]\right) + \beta\ln(\beta(1+\alpha)) \end{aligned}$$

2.4 Comparative statics

Even within such a simple framework it is possible to ask interesting questions, for example related to the effectiveness of foreign aid, given that aid will affect the borrower's incentives whether to default or not and, accordingly, the lender's willingness to lend. In the experiments made below all the changes are known ex-ante and there are no surprises once D^* has been determined.

Assume an increase of the expected inflow of aid in case of non-default in perod 2, $\overline{A}_2^N$ by one unit. Now non-default becomes more attractive for the sovereign and, as a result, the lender will increase D^*. He may actually increase his lending by more than one unit. This kind of foreign aid has a clear crowding-in effect on private credit:

$$\frac{dD^*}{d\overline{A}_2^N} = \Omega\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}}\left(\frac{1}{1+\alpha}\right) \gg 0$$

The aid increase in isolation will cause investment to decrease but more loans works in the opposite direction. Since what we have called the present value of disposable incom, Y^*, goes up the welfare level will certainly increase. We have

$$\begin{aligned} \frac{dY^*}{d\overline{A}_2^N} &= \frac{1}{1+\alpha}\left[(\alpha - r)\frac{dD^*}{d\overline{A}_2^N} + 1\right] \\ &= \Omega\frac{1}{1+\alpha} \gg 0 \end{aligned}$$

and the end value of disposable income certainly increases by more than one unit: $(1+\alpha)\frac{dY^*}{d\overline{A}_2^N} = \Omega \gg 1$.

If the expected inflow of aid in case of default, $\overline{A}_2^D$, goes up the sovereign will conclude that default becomes more attractive. As long as the lender is informed about this the amount lent will decrease and, as a result, the equilibrium investment and welfare level must go down. The magnitude of the crowding out effect depends on the share of the aid being lost for the recipient government in case of default. The lower this share the more attractive the default regime becomes and the more crowding out of private credit will be the result.

$$\frac{dD^*}{d\overline{A}_2^D} = \Omega\left(-\frac{(1-\eta_{\overline{A}})}{(1-\eta_K)(1+\alpha)}\right) \ll 0$$

Actually, one unit of such aid may easily end up causing the access to private credit to go down by much more than one unit.

More aid in period 1, $\overline{A}_1$, will actually favour non-default and, accordingly, higher loans:

$$\frac{dD^*}{d\overline{A}_1} = \Omega\left[\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}} - 1\right] \gg 0$$

The reason is that in both regimes the sovereign will want to increase investment in order to enjoy some of the benefits of the extra aid in period 2. However, in the default regime the return (after correcting for the default costs) will be lower than in the non-default regime; one unit of increased consumption in period 2 costs more in the default case. That is why the non-default regime becomes more attractive and the credit worthiness is increased. It is clear that both investment and the welfare level will increase.

What if the amount of foreign aid donated in period 2 is independent of whether the recipient defaults or not, i.e., what if $\overline{A}_2^N = \overline{A}_2^D = \overline{A}_2$? How will a small increase work in this case?

$$\frac{dD^*}{d\overline{A}_2} = \Omega \frac{1}{1+\alpha} \left[\left(\frac{1}{1-\eta_K} \right)^{\frac{\beta}{1+\beta}} - \frac{1-\eta_{\overline{A}}}{1-\eta_K} \right]$$

If no aid is lost for the recipient in case he defaults, i.e., if $\eta_{\overline{A}} = 0$, $\frac{dD^*}{d\overline{A}_2}$ will certainly be negative. A sufficiently high $\eta_{\overline{A}}$ will, however, make $\frac{dD^*}{d\overline{A}_2}$ positive. What about the welfare level? We have

$$\begin{aligned} \frac{dY^*}{d\overline{A}_2} &= \frac{1}{1+\alpha} \left[(\alpha - r) \frac{dD^*}{d\overline{A}_2} + 1 \right] \\ &= \frac{1}{1+\alpha} \left[\Omega \left(\frac{\alpha - r}{1+\alpha} \right) \left(\left(\frac{1}{1-\eta_K} \right)^{\frac{\beta}{1+\beta}} - \frac{1-\eta_{\overline{A}}}{1-\eta_K} \right) + 1 \right] \gg 0 \end{aligned}$$

Again, by inspection, it is clear that if $\eta_{\overline{A}}$ is high enough $\frac{dY^*}{d\overline{A}_2}$ will be positive and welfare must increase. It can actually be shown that even when $\eta_{\overline{A}}$ tends to zero, so that the reduction of D^* is at its maximum, will Y^* increase.

When $\eta_{\overline{A}} = 0$ this experiment represents an unconditional increase of disposable income in period 2. Of course, since some of the benefits will be enjoyed in period 1 there must be a reduction of the investment level in both regimes. One unit of consumption in period 1 costs less in terms of foregone consumption in period 2 in the default regime than in the non-default regime. That is what makes the default regime become more attractive and explains why credit worthiness, and accordingly, access to credit is reduced.

Default costs are very important in the present set-up. We have three types of such costs. By inspection we can see that higher costs, no matter of what type, means reduced attractiveness of the default regime and increased credit worthiness. As a result, the amount lent will increase and so will investment and welfare. Exogenous changes of $\eta_{\overline{A}}$ and $\eta_{\overline{Y}}$ work exactly like changes of $\overline{A}_2^D$ with opposite sign. This means that earmarking aid in ways that favours commercial creditors or providing collateral of some kind in case of default will clearly be advantageous for the sovereign ex ante. A cost increase affecting the return to investment, η_K, negatively will in addition increase the price of future consumption and discriminate against investment in the default regime.

3 New face value (debt forgiveness) and discretion over investment

Assume now that the the country has inherited some debt from earlier periods, falling due in period 2 with an amount so high that it motivates the sovereign to default and not repay anything. In order to motivate the government to choose not to default, the creditor may, in period 1, forgive some of the debt, i.e., cut down on the amount falling due in period 2. How will aid of the types discussed in section 2 affect the creditor's incentives to forgive debt? A small respecification of the model used above (see Omland (2005)) allows us to throw some light on that question. The respecification is simple: No new credit is given in period 1 and the creditor's decision is simply to choose a new face value, D, of outstanding debt in perod 1, falling due in period 2 with the amount $D(1+r)$.

3.1 Non-default (N)

The two periods' budget equations in case of non-default may now be expressed as:

$$C_1 = \overline{Y}_1 + \overline{A}_1 - K \text{ and} \tag{10a}$$

$$C_2 = \overline{Y}_2 + \overline{A}_2^N + (1+\alpha)K - (1+r)D \tag{10b}$$

and the intertemporal budget equation is

$$C_2 = \overline{Y}_2 + \overline{A}_2^N + (1+\alpha)\left(\overline{Y}_1 + \overline{A}_1 - C_1\right) - (1+r)D \tag{10c}$$

Now D only enters as an obligation to pay in period 2. There is no new credit in period 1.

The sovereign is still assumed to maximize the same welfare function. From the first-order condition (3), remembering that the accounting rate of interest is ARI^N, the two periods' consumption and the aggregate productive investment may be found. Letting

$$Y^N = \overline{Y}_1 + \overline{A}_1 + \frac{1}{1+\alpha}\left[\overline{Y}_2 + \overline{A}_2^N - (1+r)D\right] \tag{11}$$

be the present value of the country's disposable income, we have

$$C_1^N = \frac{1}{1+\beta}\left[Y^N\right], \quad C_2^N = \frac{\beta(1+\alpha)}{1+\beta}\left[Y^N\right], \tag{12a}$$

$$K^N = \frac{\beta}{1+\beta}\left[\overline{Y}_1 + \overline{A}_1\right] - \frac{1}{1+\beta}\left[\frac{\left(\overline{Y}_2 + \overline{A}_2^N\right) - D(1+r)}{1+\alpha}\right], \text{ and} \tag{12b}$$

$$U^N = (1+\beta)\ln\left(\frac{1}{1+\beta}\left[Y^N\right]\right) + \beta\ln(\beta(1+\alpha)) \tag{12c}$$

3.2 Default (D)

In case of default the debt's face value is irrelevant and the two periods' budget equations may be expressed as:

$$C_1 = \overline{Y}_1 + \overline{A}_1 - K \text{ and} \tag{13b}$$

$$C_2 = (1-\eta_{\overline{Y}})\overline{Y}_2 + (1-\eta_{\overline{A}})\overline{A}_2^D + (1-\eta_K)(1+\alpha)K \text{ or} \quad 13b$$

$$C_2 = (1-\eta_{\overline{Y}})\overline{Y}_2 + (1-\eta_{\overline{A}})\overline{A}_2^D + (1-\eta_K)(1+\alpha)\left(\overline{Y}_1 + \overline{A}_1 - C_1\right) \tag{13c}$$

Remembering that the accounting rate of investment is ARI^D and letting

$$Y^D = \overline{Y}_1 + \overline{A}_1 + \frac{1}{(1-\eta_K)(1+\alpha)}\left[(1-\eta_{\overline{Y}})\overline{Y}_2 + (1-\eta_{\overline{A}})\overline{A}_2^D\right] \tag{14}$$

be the present value of the country's disposable income we have

$$C_1^D = \frac{1}{1+\beta}\left[Y^D\right] \ , \ C_2^D = \frac{\beta(1-\eta_K)(1+\alpha)}{1+\beta}\left[Y^D\right] \ , \tag{15a}$$

$$K^D = \frac{\beta}{1+\beta}\left[\overline{Y}_1+\overline{A}_1\right] - \frac{1}{1+\beta}\left[\frac{(1-\eta_{\overline{Y}})\overline{Y}_2 + (1-\eta_{\overline{A}})\overline{A}_2^D}{(1-\eta_K)(1+\alpha)}\right] \ \text{, and} \tag{15b}$$

$$U^D = (1+\beta)\ln\left(\frac{1}{1+\beta}\left[Y^D\right]\right) + \beta\ln\left(\beta(1-\eta_K)(1+\alpha)\right) \tag{15c}$$

3.3 The debt limit

The creditor still wants to avoid default and will choose the highest possible face value where $U^D - U^N \leq 0$. Following the same procedures as in section 2 and appendix 1, the cut-off level of the face value, D^*, (the upper limit), as perceived by the creditor can be found:

$$\begin{aligned} D^* = & \frac{1}{\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}}\left(\frac{1+r}{1+\alpha}\right)}\left[\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}}\left(\overline{Y}_1+\overline{A}_1\right)\right] + \frac{1}{1+r}\left[Y_2+\overline{A}_2^N\right] \\ & -\frac{1}{(1+r)\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}}}\left(\frac{(1-\eta_{\overline{Y}})\overline{Y}_2+(1-\eta_{\overline{A}})\overline{A}_2^D}{(1-\eta_K)}\right) \end{aligned} \tag{16}$$

Again, since there is no uncertainty in this model and setting $D = D^*$, there will actually never be any default. And the level of productive investment can be found from the K^N-function (12b), given the face value of outstanding debt D^* :

$$K^* = \frac{\beta}{1+\beta}\left[\overline{Y}_1+\overline{A}_1\right] - \frac{1}{1+\beta}\left[\frac{\left(\overline{Y}_2+\overline{A}_2^N\right) - D^*(1+r)}{1+\alpha}\right]$$

Given $D = D^*$, $K = K^*$, and non-default the actual welfare level (12c), as perceived by the borrowing and aid-receiving sovereign, will be determined by the present value of his aggregate income (11):

$$Y^* = \overline{Y}_1+\overline{A}_1+\frac{1}{1+\alpha}\left[\overline{Y}_2+\overline{A}_2^N-(1+r)D^*\right] \ \text{and}$$

$$U^* = (1+\beta)\ln\left(\frac{1}{1+\beta}\left[Y^*\right]\right)+\beta\ln\left(\beta(1+\alpha)\right)$$

3.4 Comparative statics

In section 2, where the sovereign obtains credit in period 1, increased credit worthiness means access to more credit and higher welfare for the country and increased profits for the lender. When debt forgiveness is on the agenda, however, improved credit worthiness means less forgiveness, tougher debt obligations and often reduced welfare for the sovereign but, of course, higher profits for the creditor. Some of the following results can be found in Omland (2005) in Norwegian.

Assume first an increase of the expected inflow of aid in case of non-default, $\overline{A}_2^N$ by one unit. Now non-default becomes more attractive for the sovereign and, as a result, the creditor will increase D^*, i.e., the face value after forgiveness. There is actually 100% crowding out in this case:

$$\frac{dD^*}{d\overline{A}_2^N} = \frac{1}{1+r} \gg 0$$

As a result, the debitor cum aid recipient is in exactly the same situation as before and

$$\frac{dK^*}{d\overline{A}_2^N} = \frac{dY^*}{d\overline{A}_2^N} = \frac{dU^*}{d\overline{A}_2^N} = 0$$

If the expected inflow of aid in case of default, $\overline{A}_2^D$, goes up default becomes more attractive and credit worthiness goes down. To avoid default the creditor has to reduce the face value of the outstanding debt:

$$\frac{dD^*}{d\overline{A}_2^D} = -\frac{1-\eta_{\overline{A}}}{(1+r)\left(\frac{1}{(1-\eta_K)}\right)^{\frac{\beta}{1+\beta}}} \ll 0$$

Since debt obligations in period 2 are reduced, investment will be cut down. But what we have called disposable income and welfare will certainly increase:

$$\frac{dY^*}{d\overline{A}_2^D} = \frac{1}{1+\alpha}\left[-(1+r)\frac{dD^*}{d\overline{A}_2}\right] \gg 0$$

More aid in period 1, $\overline{A}_1$, will actually favour non-default and, accordingly, cause increased face value:

$$\frac{dD^*}{d\overline{A}_1} = \frac{1+\alpha}{1+r}\left[1-\left(\frac{1}{1-\eta_K}\right)^{-\left(\frac{\beta}{1+\beta}\right)}\right] \gg 0$$

but investment, disposable income, and welfare will increase:

$$\frac{dY^*}{d\overline{A}_1} = \left(\frac{1}{1-\eta_K}\right)^{-\left(\frac{\beta}{1+\beta}\right)} \gg 0$$

What if the amount of foreign aid donated is independent of whether the recipient defaults or not, i.e., what if $\overline{A}_2^N = \overline{A}_2^D = \overline{A}_2$? How will a small increase work in this case?

$$\frac{dD^*}{d\overline{A}_2} = \frac{1}{1+r}\left[1-\left(\frac{1}{1-\eta_K}\right)^{-\left(\frac{\beta}{1+\beta}\right)}\left(\frac{1-\eta_{\overline{A}}}{1-\eta_K}\right)\right]$$

If no aid is lost for the recipient in case he defaults, i.e., if $\eta_{\overline{A}} = 0$, $\frac{dD^*}{d\overline{A}_2}$ will certainly be negative. Default becomes more attractive and the creditor react by cutting down the face value. A sufficiently high $\eta_{\overline{A}}$ will, however, make $\frac{dD^*}{d\overline{A}_2}$ positive, and when $\eta_{\overline{A}} = 1$ this is exactly the same as an increase in $\overline{A}_2^N$ discussed above and the face value is doomed to increase. What about the welfare level?

We have

$$\begin{aligned}\frac{dY^*}{d\overline{A}_2} &= \frac{1}{1+\alpha}\left[1-(1+r)\frac{dD^*}{d\overline{A}_2}\right] \\ &= \left(\frac{1}{1+\alpha}\right)\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}}\left(\frac{1-\eta_{\overline{A}}}{1-\eta_K}\right)\end{aligned}$$

Again, by inspection it is clear that when $\eta_{\overline{A}} = 1$ there will be full crowding out so that $\frac{dY^*}{d\overline{A}_2} = 0$. As long as $\eta_{\overline{A}} \ll 1$, however, the face value is cut down and the sovereign is better off, $\frac{dY^*}{d\overline{A}_2} \gg 0$. The lower $\eta_{\overline{A}}$, the better off the sovereign will tend to be.

Higher default costs, no matter of what type, means reduced attractiveness of the default regime and increased credit worthiness. As a result, forgiveness is reduced and the face value is increased. Higher debt obligations in period 2 means higher investment, but reduced Y^* and welfare. This means that earmarking aid in ways that favour commercial creditors or providing collateral of some kind in case of default will clearly be disadvantageous for the sovereign.

So ex post, in negotiations for debt relief, the situation looks very different from the situation ex ante, negotiating for new credit.

4 Precommitment in investment

So far the sovereign has been free to determine how much to invest (and whether to default or not) *after* beeing informed about the creditor's decision as to new credit or debt relief. In a situation where the sovereign can make credible promises ex ante concerning the investment level, he can use investment as collateral and accordingly, a way of signalling credit worthiness[9]. Sustainable credit obligations in period 2 must still satisfy the condition (1), i.e., if the sovereign chooses default he is hit by costs of the type discussed above. In addition, to the extent the amount of aid given differs between the two regimes, he loses the amount $\overline{A}_2^N - \overline{A}_2^D$.

4.1 New loans

The two periods' budget equations are still given by (2a) and (2b) above, but letting
$D = \frac{1}{1+r}\left[\eta_K (1+\alpha) K + \eta_{\overline{A}}\overline{A}_2^D + \eta_{\overline{Y}}\overline{Y}_2 + \left(\overline{A}_2^N - \overline{A}_2^D\right)\right]$ from (1) they may be expressed as

$$\begin{aligned} C_1 &= \overline{Y}_1 + \overline{A}_1 + \frac{1}{1+r}\left(\eta_{\overline{A}}\overline{A}_2^D + \eta_{\overline{Y}}\overline{Y}_2 + \left(\overline{A}_2^N - \overline{A}_2^D\right)\right) \\ &\quad - \left(\frac{1+r-\eta_K(1+\alpha)}{1+r}\right) K \quad \text{and} \qquad (17a) \\ C_2 &= \overline{Y}_2\left(1-\eta_{\overline{Y}}\right) + \left(1-\eta_{\overline{A}}\right)\overline{A}_2^D + (1+\alpha)(1-\eta_K) K \qquad (17b) \end{aligned}$$

A high $\overline{A}_2^N$ (compared to $\overline{A}_2^D$) means that one consequence of choosing default is losing a lot of aid and works, as a consequence, as collateral for the lender. A high $\overline{A}_2^D$, on the other hand, works in the other direction as long as $\eta_{\overline{A}} = 0$. If $\eta_{\overline{A}}$ is positive, however, the fact that some of the aid obtained in case of default is lost, has a collateral effect, but as long as $\eta_{\overline{A}} \ll 1$, $\overline{A}_2^D$ has a negative effect on the access to credit: $\frac{\partial D}{\partial A_2^D} = \frac{1}{1+r}\left(\eta_{\overline{A}} - 1\right) \ll 0$.

Productive investment has a positive effect on the access to credit and since $0 \ll \frac{\partial D}{\partial K} = \frac{\eta_K(1+\alpha)}{1+r} \ll 1$, we have $-1 \ll \frac{\partial C_1}{\partial K} = -1 + \frac{\partial D}{\partial K} = -\left(\frac{(1+r)-\eta_K(1+\alpha)}{1+r}\right) \ll 0$. The cost in terms of foregone consumption in period 1 per unit invested, is lower than in the non-default case under discretion. The benefit in terms of increased consumtion in period 2 of investing one unit is also lower. Now the reason is that the extra credit obtained in period 1 as a consequence of increase K has to be repaid in period 2. $\frac{\partial C_2}{\partial K} = (1+\alpha) - (1+r)\frac{\partial D}{\partial K} = (1+\alpha)(1-\eta_K)$.

[9] We simply disregard credibility problems here.

Using period 1's budget equation to express K as a function of C_1,

$$K=\frac{1+r}{1+r-\eta_K(1+\alpha)}\left[\overline{Y}_1+\overline{A}_1-C_1+\frac{1}{1+r}\left(\eta_{\overline{A}}\overline{A}_2^D+\eta_{\overline{Y}}\overline{Y}_2+\left(\overline{A}_2^N-\overline{A}_2^D\right)\right)\right]$$

where $\frac{\partial K}{\partial C_1}=-\frac{1+r}{1+r-\eta_K(1+\alpha)}\ll -1$, the intertemporal budget equation may be written as

$$\begin{aligned}C_2 &= \overline{Y}_2(1-\eta_{\overline{Y}})+(1-\eta_{\overline{A}})\overline{A}_2^D \\ &+\frac{(1+\alpha)(1-\eta_K)(1+r)}{1+r-\eta_K(1+\alpha)}\left[\overline{Y}_1+\overline{A}_1-C_1+\frac{1}{1+r}\left(\eta_{\overline{A}}\overline{A}_2^D+\eta_{\overline{Y}}\overline{Y}_2+\left(\overline{A}_2^N-\overline{A}_2^D\right)\right)\right]\end{aligned} \tag{17c}$$

We now have the accounting rate of interest defined by $\frac{dC_2}{dC_1}=\frac{\partial C_2}{\partial K}\frac{\partial K}{\partial C_1}=-\left(1+ARI^P\right)=-\frac{(1+r)(1+\alpha)(1-\eta_K)}{(1+r)-\eta_K(1+\alpha)}\ll -(1+\alpha)$. Since increasing investment means improved credit worthiness and increased access to credit, and since borrowing, investing, and repaying is beneficial, i.e., since $\alpha\gg r$, period 2 consumpiton is "subsidized" compared to the discretion case discussed in section 2.

From the first order condition (3), using ARI^P the two periods' consumption, welfare, aggregate productive investment, and access to credit may be found. Letting

$$\begin{aligned}Y^P &= \overline{Y}_1+\overline{A}_1+\frac{1}{1+r}\left(\eta_{\overline{A}}\overline{A}_2^D+\eta_{\overline{Y}}\overline{Y}_2+\left(\overline{A}_2^N-\overline{A}_2^D\right)\right) \\ &+\frac{1}{\frac{(1+r)(1+\alpha)(1-\eta_K)}{(1+r)-\eta_K(1+\alpha)}}\left(\overline{Y}_2(1-\eta_{\overline{Y}})+(1-\eta_{\overline{A}})\overline{A}_2^D\right)\end{aligned} \tag{18}$$

we can express consumption in the two periods and welfare as

$$\begin{aligned}C_1^P &= \frac{1}{1+\beta}\left[Y^P\right] \quad , \quad C_2^P=\frac{\beta\frac{(1+r)(1+\alpha)(1-\eta_K)}{(1+r)-\eta_K(1+\alpha)}}{1+\beta}\left[Y^P\right] \quad , \text{ and} \\ U^P &= (1+\beta)\ln\left(\frac{1}{1+\beta}\left[Y^P\right]\right)+\beta\ln\left(\beta\left(\frac{(1+r)(1+\alpha)(1-\eta_K)}{(1+r)-\eta_K(1+\alpha)}\right)\right)\end{aligned}$$

and the investment level as

$$\begin{aligned}K^P &= \frac{1+r}{1+r-\eta_K(1+\alpha)}\left[\frac{\beta}{1+\beta}\left(\overline{Y}_1+\overline{A}_1+\frac{1}{1+r}\left(\eta_{\overline{A}}\overline{A}_2^D+\eta_{\overline{Y}}\overline{Y}_2+\left(\overline{A}_2^N-\overline{A}_2^D\right)\right)\right)\right. \\ &\left.-\frac{1}{1+\beta}\frac{1}{\frac{(1+r)(1+\alpha)(1-\eta_K)}{(1+r)-\eta_K(1+\alpha)}}\left(\overline{Y}_2(1-\eta_{\overline{Y}})+(1-\eta_{\overline{A}})\overline{A}_2^D\right)\right]\end{aligned} \tag{19a}$$

Once the investment level has been determined, so has the sovereigns credit worthiness, and the access to credit:

$$D^P=\frac{1}{1+r}\left[\eta_K(1+\alpha)K^P+\eta_{\overline{A}}\overline{A}_2^D+\eta_{\overline{Y}}\overline{Y}_2+\left(\overline{A}_2^N-\overline{A}_2^D\right)\right] \tag{19b}$$

How does aid work in this precommitment case?

If the inflow of aid in period 1, $\overline{A}_1$, goes up the sovereign will want to transfer some of it to period 2, so investment must increase, which again means improved credit worthiness and increased access to credit is:

$$\begin{aligned}\frac{dK^P}{d\overline{A}_1} &= \frac{1+r}{1+r-\eta_K(1+\alpha)}\frac{\beta}{1+\beta} \gg 0\\ \frac{dD^P}{d\overline{A}_1} &= \frac{1}{1+r}\eta_K(1+\alpha)\frac{dK^P}{d\overline{A}_1} \gg 0\end{aligned}$$

Of course, Y^P, consumption in the two periods, and welfare goes up.

Aid expected to come in period 2 may have very different consequences. Assume first that it is unconditional, i.e., that $\overline{A}_2 = \overline{A}_2^D = \overline{A}_2^N$ and that it increases. Now the last term in (1) disappears and $\overline{A}_2$ affects the access to credit directly only to the extent that $\eta_{\overline{A}}$ is positive: $\frac{\partial D}{\partial \overline{A}_2} = \frac{1}{1+r}\eta_{\overline{A}}$. Increased access to credit means higher investment, but the fact that increased aid also means higher income in period 2, has a negative effect on investment.

$$\begin{aligned}\frac{dK^P}{d\overline{A}_2} &= \left(\frac{\beta}{1+\beta}\right)\left(\frac{1+r}{1+r-\eta_K(1+\alpha)}\right)\left(\frac{1}{1+r}\eta_{\overline{A}}\right)\\ &\quad -\left(\frac{1}{1+\beta}\right)\left(\frac{1}{(1+\alpha)(1-\eta_K)}\right)(1-\eta_{\overline{A}})\\ \frac{dD^P}{d\overline{A}_2} &= \frac{1}{1+r}\left[\eta_{\overline{A}}+\eta_K(1+\alpha)\frac{dK^P}{d\overline{A}_2}\right]\end{aligned}$$

If $\eta_{\overline{A}} = 0$, i.e., if aid represents no collateral from the creditor's point of view, an increase of $\overline{A}_2$ simply means higher income in period 2. The sovereign will want to transfer some of it to period 1, so investment has to be reduced and $\frac{dK^P}{d\overline{A}_2} = -\left(\frac{1}{1+\beta}\right)\left(\frac{1}{(1+\alpha)(1-\eta_K)}\right) \ll 0$. As a result, access to credit will also be reduced: $\frac{dD^P}{d\overline{A}_2} = \frac{1}{1+r}\eta_K(1+\alpha)\frac{dK^P}{d\overline{A}_2} \ll 0$. If on the other hand, most of the aid is lost by the sovereign if he defaults, i.e. if $\eta_{\overline{A}} \to 1$, both investment and access to credit will increase.

Y^P, consumption and welfare is bound to increase no matter the level of $\eta_{\overline{A}}$:

$$\begin{aligned}\frac{dY^P}{d\overline{A}_2} &= \frac{1}{1+r}\eta_{\overline{A}} + \frac{1}{\frac{(1+r)(1+\alpha)(1-\eta_K)}{(1+r)-\eta_K(1+\alpha)}}(1-\eta_{\overline{A}})\\ &= \frac{1}{\frac{(1+r)(1+\alpha)(1-\eta_K)}{(1+r)-\eta_K(1+\alpha)}}\left[\frac{(\alpha-r)\eta_{\overline{A}}}{(1+r)-\eta_K(1+\alpha)}+1\right]\end{aligned}$$

However, since $\alpha \gg r$, both from the sovereign's and the creditor's point of view it is best to keep $\eta_{\overline{A}}$ as high as possible.

Assume now that $\overline{A}_2^N$ and $\overline{A}_2^D$ may differ and let $\overline{A}_2^N$ increase. The initial effect will be to increase the access to credit by the entire present value of the aid increase. Investment will certainly increase, increasing the credit worthiness even more:

$$\begin{aligned}\frac{dK^P}{d\overline{A}_2^N} &= \frac{\beta}{1+\beta}\frac{1}{1+r-\eta_K(1+\alpha)} \gg 0\\ \frac{dD^P}{d\overline{A}_2^N} &= \frac{1}{1+r}\left[1+\eta_K(1+\alpha)\frac{dK^P}{d\overline{A}_2^N}\right] \gg 0\end{aligned}$$

Y^P, consumption and welfare is bound to increase.

How will an increase of $\overline{A}_2^D$ work? The main difference from $\overline{A}_2^N$ is that initially the access to credit is bound to go down: $\frac{\partial D}{\partial \overline{A}_2^D} = -\frac{1}{1+r}(1-\eta_{\overline{A}}) \ll 0$. It works as an exogenous transfer of consumption from period 1 to period 2. As a result, investment goes down and access to credit is reduced further. Y^P, and accordingly, consumption in both periods and welfare also will be reduced:

$$\begin{aligned}
\frac{dK^P}{d\overline{A}_2^D} &= -\frac{\beta}{1+\beta}\frac{1}{1+r-\eta_K(1+\alpha)}\left(1-\eta_{\overline{A}}\right)-\frac{1}{1+\beta}\frac{1}{\frac{(1+r)(1+\alpha)(1-\eta_K)}{(1+r)-\eta_K(1+\alpha)}}\left(1-\eta_{\overline{A}}\right)\ll 0 \\
\frac{dD^P}{d\overline{A}_2^D} &= \frac{1}{1+r}\left[-\left(1-\eta_{\overline{A}}\right)+\eta_K(1+\alpha)\frac{dK^P}{d\overline{A}_2^N}\right]\ll 0 \\
\frac{dY^P}{d\overline{A}_2^D} &= -\frac{1}{1+r}\left(1-\eta_{\overline{A}}\right)+\frac{1}{\frac{(1+r)(1+\alpha)(1-\eta_K)}{(1+r)-\eta_K(1+\alpha)}}\left(1-\eta_{\overline{A}}\right) \\
&= -\left(1-\eta_{\overline{A}}\right)\left[\frac{(\alpha-r)}{(1+r)-\eta_K(1+\alpha)}\right]\ll 0
\end{aligned}$$

4.2 New face value (Debt forgiveness)

In a situation where D represents a new, reduced, face value instead of new credit, no new credit is given in period 1, but the face value, i.e., the amount falling due in period 2 is still given by (1). Actually, precommitment (without default) gives exactly the same results as discretion with default, described in section 3.2. The budget equations are the same, see (13c):

$$C_2=\left(1-\eta_{\overline{Y}}\right)\overline{Y}_2+\left(1-\eta_{\overline{A}}\right)\overline{A}_2^D+\left(1-\eta_K\right)(1+\alpha)\left(\overline{Y}_1+\overline{A}_1-C_1\right)$$

so the ARI must also be the same: $\left(1+ARI^P\right)=\left(1-\eta_K\right)(1+\alpha)$. If C_1 is reduced (and K increased) by one unit, the outstanding debt falling due in period 2 increases at exactly the same rate as the sanction costs. It represents a tax om domestic investment, see f.ex. Krugman (1988).

Investment, present value of income, and face value of debt are given by (see (15b), (14), and (1))

$$\begin{aligned}
K^P &= \frac{\beta}{1+\beta}\left[\overline{Y}_1+\overline{A}_1\right]-\frac{1}{1+\beta}\left[\frac{\left(1-\eta_{\overline{Y}}\right)\overline{Y}_2+\left(1-\eta_{\overline{A}}\right)\overline{A}_2^D}{\left(1-\eta_K\right)(1+\alpha)}\right] \\
Y^P &= \overline{Y}_1+\overline{A}_1+\frac{1}{\left(1-\eta_K\right)(1+\alpha)}\left[\left(1-\eta_{\overline{Y}}\right)\overline{Y}_2+\left(1-\eta_{\overline{A}}\right)\overline{A}_2^D\right] \\
D^P &= \frac{1}{1+r}\left[\eta_K(1+\alpha)K^P+\eta_{\overline{A}}\overline{A}_2^D+\eta_{\overline{Y}}\overline{Y}_2+\left(\overline{A}_2^N-\overline{A}_2^D\right)\right]
\end{aligned}$$

How will aid of different types work in this case? Aid earmarked for non-default situations, $\overline{A}_2^N$, represents a loss for the sovereign if he defaults and is a pure gift to the creditor in this case. If this type of aid increases, the face value is increased by exactly the same amount. Aid in case of default, $\overline{A}_2^D$, on the other hand, represents if $\eta_{\overline{A}}=0$ a pure reward in case of default. If it increases, the face value is reduced by exactly the same amount - thereby increasing the amount at the sovereigns disposal in period 2. If $\eta_{\overline{A}}\gg 0$, it tends to reduce the benefit of such aid, as perceived by the sovereign.

Unconditional aid in period 2 means increased face value as long as $\eta_{\overline{A}}\gg 0$ but also increased Y^P as long as it is lower than unity.

5 The endogenization of the aid inflow

So far the inflow of aid has been considered as exogenous. That may be sensible for some types of aid, for example aid given by the creditors' home country. Aid in case of non-default, $\overline{A}_2^N$, could be considered a bonus for good behavior. How to consider this type of aid in case of default, $\overline{A}_2^D$, however, is an open question. If it is low it could represent a punishment for bad behavior but if it is high, especially in combination with a high $\eta_{\overline{A}}$, it could be a way of helping the creditors out in case the borrowing government decides to default.

In reality aid is given for many reasons and it is not necessarily exogenous. Assume, for example, that at least some of the expected aid inflow in period 2 is based on the perception of aid as part of a safety net financed by altruistic donors looking for "needs" to satisfy. In the two-period model used above, it seems natural for the administrator of this safety net to relate "needs" to consumtion in period 2. The lower the consumption in period 2 in the absence of his type of aid, the higher the perceived "need for aid" and the higher the inflow of aid will tend to be. Pedersen (2001) contains one way of representing this safety net, where the aid administrator - not being able to cope with the Samaritan's dilemma (Buchanan (1975)) - ends up being trapped as a Stackelberg follower[10]. The formulation below is based on that approach.

Assume that the administrator of the safety-net has an amount $\overline{A_2}$ at his disposal and that he is responsible for supporting $j = 1, ..., J$ different countries. A number N of these countries do not default while the number D default. If A_{2j} is the amount given to country j, the budget constraint is $\sum_{j=1}^{J} A_{2j} = \overline{A_2}$. Assume, in addition, that the administrator's goal is to maximize the recipient countries' aggregate welfare $W_2 = \sum_{j=1}^{J} \ln C_{2j}$ where $C_{2j} = A_{2j}\left(1-\eta_{Aj}\right) + \widehat{C}_{2j}$ if country j defaults and $C_{2j} = A_{2j} + \widehat{C}_{2j}$ if it does not default. $\widehat{C}_{2j}$ is the administrator's perception of the consumption level in the absence of his type of aid. Being a Stackelberg follower, the administrator considers $\widehat{C}_{2j}$ as given when he determines how much aid to give to country j, A_{2j}. If we let $\overline{C}_2$ be the aid-equivalent of aggregate consumption in period 2 in all aid-receiving countries except country i, i.e., $\overline{C}_2 = \sum_{j\in D, j\neq i} \frac{\widehat{C}_{2j}}{1-\eta_{Aj}} + \sum_{j\in N, j\neq i} \widehat{C}_{2j}$, the resulting amount obtained by country i will be[11]

$$A_{2i}^D = \frac{1}{J}\left[\overline{A}_2 + \overline{C}_2 + \frac{\widehat{C}_{2i}}{1-\eta_{Aj}}\right] - \frac{\widehat{C}_{2i}}{1-\eta_{Aj}} \text{ in case of default and} \tag{20a}$$

$$A_{2i}^N = \frac{1}{J}\left[\overline{A}_2 + \overline{C}_2 + \widehat{C}_{2i}\right] - \widehat{C}_{2i} \text{ in case of non-default} \tag{20b}$$

from which it follows that any effort or activity contributing to consumption in period 2 is "rewarded" with reduced aid, i.e., in reality taxed at the rate

$$t_i^D = -\frac{\partial A_{2i}^D}{\partial \widehat{C}_{2i}} = \frac{1}{1-\eta_{Ai}}\left[1-\frac{1}{J}\right] \text{ in case of default and}$$

$$t_i^N = -\frac{\partial A_{2i}^N}{\partial \widehat{C}_{2i}} = \left[1-\frac{1}{J}\right] \text{ in case of non-default.}$$

The benefits of such activities end up, in reality, being divided between all contries receiving aid from the altruistic donors, i.e., benefitting from the safety-net. The part of the benefits ending up as domestic

[10] There is no obvious way for an altruistic donor to avoid this problem. See, however, Hagen (2006) for a discussion of how delegation may work.

[11] We only consider situations where A_{2i} is strictly positive.

consumption is

$$1-t_i^D = \frac{1-\eta_{Ai}J}{J\left(1-\eta_{Ai}\right)} \text{ and}$$
$$1-t_i^N = \frac{1}{J}$$

Observe that the tax rate in case of default is higher than if non-default is the government's decision as long as some of the aid of the type in question is wasted from the point of view of the donor and the recipient, i.e., $\eta_{Ai} \gg 0$, for example because it ends up in the hands of the lender. This is because the aid equivalent of an exogenous consumption increase in period 2 is higher in that case. The tax rate in case of default could actually exceed unity (100%) unless we make the assumption that $\eta_{Ai} \ll \frac{1}{J}$.

The existence of the safety-net and the resulting aid allocation rules are known to the borrowing governments when they decide whether to default or not and how much to invest in period 1. Also the lender has this knowledge when he decides how much to lend. Therefore, equipped with these aid allocation rules, let us return to relationship between the lender and the government discussed above - in a situation where **access to new credit** is on the agenda and the borrower has **discretion** over investment. We Again we consider the borrowing government first.

5.1 Non-default

From (2b) in section 2.1 above we have period 2 consumption in the absence of the safety-net related aid: $\widehat{C}_2 = \overline{Y}_2 + \overline{A}_2^N + (1+\alpha)K - (1+r)D$. Once we include this type of aid, A_2^N from (20b) above (and skipping the subscript i), actual consumption may be expressed as $C_2 = \widehat{C}_2 + A_2^N = \widehat{C}_2 + \frac{1}{J}\left[\overline{A}_2 + \overline{C}_2 + \widehat{C}_2\right] - \widehat{C}_2$ $= \frac{1}{J}\left[\overline{A}_2 + \overline{C}_2 + \widehat{C}_2\right]$ or

$$C_2 = \frac{1}{J}\left[\overline{Y}_2 + \overline{A}_2^N + (1+\alpha)\left(\overline{Y}_1 + \overline{A}_1 - C_1\right) + (\alpha - r)D + \overline{A}_2 + \overline{C}_2\right] \tag{21}$$

The accounting rate of interest is now $\frac{dC_2}{dC_1} = -\left(1+ARI^N\right) = -\frac{1}{J}(1+\alpha)$ or $-\left(1-t^N\right)(1+\alpha)$, clearly lower than in section 2.1 as long as $J \gg 1$ and, accordingly, $t^N \gg 0$, i.e., as long as the administrator of the safet-net has more than one client country. The benefit from investing is taxed and actually split equally between all the J countries benefitting from the safety-net. This clearly introduces a discrimination against any activities intended to favour investment and future consumption. On the other hand, the country will benefit from investment and other activities contributing to consumption in period 2 in other countries benefitting from the safety-net.

Letting

$$Y^N = \overline{Y}_1 + \overline{A}_1 + \frac{1}{(1+\alpha)\frac{1}{J}}\left[D(\alpha - r) + \overline{Y}_2 + \overline{A}_2^N + \left(\overline{A}_2 + \overline{C}_2\right)\right]\frac{1}{J} \tag{22}$$

be the present value of the country's disposable income we have

$$C_1^N = \frac{1}{1+\beta}\left[Y^N\right] \quad , \quad C_2^N = \frac{\beta(1+\alpha)\frac{1}{J}}{1+\beta}\left[Y^N\right] \quad \text{, and} \tag{23a}$$

$$K^N = \frac{\beta}{1+\beta}\left[\overline{Y}_1 + \overline{A}_1 + D\right] - \frac{1}{1+\beta}\left[\frac{\left(\overline{Y}_2 + \overline{A}_2^N\right) - D(1+r) + \left(\overline{A}_2 + \overline{C}_2\right)}{1+\alpha}\right] \tag{23b}$$

Observe that the total (safety-net) aid budget as well as the period 2 consumption of all countries covered by the safety-net is considered at the government's disposal when the decision on the investment level i period 1 is made. This tends to depress the investment level and planned period 2 consumption without aid dramatically. The benefits of the altruistic donor' activities mainly come in period 1, where consumption certainly goes up when the safety-net is introduced. Consumption i period 2 does not necessarily increase. The welfare level as perceived by the sovereign may now be calculated as

$$U^N = (1+\beta)\ln\left(\frac{1}{1+\beta}\left[Y^N\right]\right) + \beta\ln\left(\beta(1+\alpha)\left(\frac{1}{J}\right)\right) \tag{23c}$$

5.2 Default

We have earlier made the assumption that it is more profitable to borrow and invest one dollar and then default, than to borrow and invest and then repay the loan with interest, i.e., $(1+\alpha)(1-\eta_K) \gg \alpha - r$. This must hold also when we take into account that the aid inflow is affected, i.e., it must hold in an after-tax sense: $(1+\alpha)(1-\eta_K)\left(\frac{1-\eta_{Ai}J}{J(1-\eta_{Ai})}\right) \gg (\alpha - r)\frac{1}{J}$ or $(1+\alpha)(1-\eta_K)\left(1-t_i^D\right) \gg (\alpha - r)\left(1-t_i^N\right)$. Observe that even if it holds in a pre-tax sense it may not hold when the aid inflow is endogenous, because $t_i^D \gg t_i^N$ when $\eta_{Ai} \gg 0$.

In case of default (6.b) in section 2.2 gives us period 2 consumption without aid of the type in question: $\widehat{C}_2 = (1-\eta_{\overline{Y}})\overline{Y}_2 + (1-\eta_{\overline{A}})\overline{A}_2^D + (1-\eta_K)(1+\alpha)K$. With this aid, A_2^D from (20a) (again skipping the subscript i) we have $C_2 = \widehat{C}_2 + A_2^D = \widehat{C}_2 + \frac{1}{J}\left[\overline{A}_2 + \overline{C}_2 + \frac{\widehat{C}_2}{1-\eta}\right] - \frac{\widehat{C}_2}{1-\eta_A} = \frac{1-\eta_A J}{J(1-\eta_A)}\widehat{C}_2 + \frac{1}{J}\left[\overline{A}_2 + \overline{C}_2\right]$ or

$$\begin{aligned} C_2 &= \frac{1-\eta_A J}{J(1-\eta_A)}\left[(1-\eta_{\overline{Y}})\overline{Y}_2 + (1-\eta_{\overline{A}})\overline{A}_2^D + (1-\eta_K)(1+\alpha)\left(\overline{Y}_1 + \overline{A}_1 + D - C_1\right)\right] \\ &\quad + \frac{1}{J}\left[\overline{A}_2 + \overline{C}_2\right] \end{aligned} \tag{24}$$

and the accounting rate of interest is $\frac{dC_2}{dC_1} = -\left(1 + ARI^D\right) = -\left(\frac{1-\eta_A J}{J(1-\eta_A)}\right)(1-\eta_K)(1+\alpha)$ or $-\left(1-t^D\right)(1-\eta_K)(1+\alpha)$ which simplifies to $-\frac{1}{J}(1-\eta_K)(1+\alpha) = -\left(1-t^N\right)(1-\eta_K)(1+\alpha)$ when η_A tends to zero. To avoid anomalies with a tax rate exceeding 100% we assume that $\eta_A \ll \frac{1}{J}$. Letting

$$\begin{aligned} Y^D &= \overline{Y}_1 + \overline{A}_1 + D \\ &\quad + \frac{1}{(1-\eta_K)(1+\alpha)\frac{1-\eta_A J}{J(1-\eta_A)}}\left[\left((1-\eta_{\overline{Y}})\overline{Y}_2 + (1-\eta_{\overline{A}})\overline{A}_2^D\right)\frac{1-\eta_A J}{J(1-\eta_A)} + \frac{1}{J}\left[\overline{A}_2 + \overline{C}_2\right]\right] \end{aligned} \tag{25}$$

be the present value of what is considered as the country's disposable income, we have

$$C_1^D = \frac{1}{1+\beta}\left[Y^D\right], \quad C_2^D = \frac{\beta(1-\eta_K)(1+\alpha)\frac{1-\eta_A J}{J(1-\eta_A)}}{1+\beta}\left[Y^D\right], \tag{26a}$$

$$K^D = \frac{\beta}{1+\beta}\left[\overline{Y}_1 + \overline{A}_1 + D\right] - \frac{1}{1+\beta}\left[\frac{(1-\eta_{\overline{Y}})\overline{Y}_2 + (1-\eta_{\overline{A}})\overline{A}_2^D + \frac{\frac{1}{J}\left[\overline{A} + \overline{C}_2\right]}{\frac{1-\eta_A J}{J(1-\eta_A)}}}{(1-\eta_K)(1+\alpha)}\right], \text{ and} \tag{26b}$$

$$U^D = (1+\beta)\ln\left(\frac{1}{1+\beta}\left[Y^D\right]\right) + \beta\ln\left(\beta(1-\eta_K)(1+\alpha)\left(\frac{1-\eta_A J}{J(1-\eta_A)}\right)\right) \tag{26c}$$

5.3 The debt limit

Again it is possible to find the maximum amount of lending, D^{**}, as perceived by the lender. Using the same procedure as in section 2.3 and appendix 1 we have (27):

$$\begin{aligned} D^{**} &= \Gamma \left[\left(\frac{\frac{1}{J}}{(1-\eta_K)\left(\frac{1-\eta_A J}{J(1-\eta_A)}\right)} \right)^{\frac{\beta}{1+\beta}} \left(\overline{Y}_1 + \overline{A}_1 + \frac{\overline{Y}_2 + \overline{A}_2^N + \left(\overline{A}_2 + \overline{Y}_2\right)}{1+\alpha} \right) \right. \\ & \left. - \left(\overline{Y}_1 + \overline{A}_1 + \frac{(1-\eta_{\overline{Y}})\,\overline{Y}_2 + (1-\eta_{\overline{A}})\,\overline{A}_2^D + \left(\overline{A}_2 + \overline{C}_2\right)\left(\frac{\frac{1}{J}}{\frac{1-\eta_A J}{J(1-\eta_A)}}\right)}{(1-\eta_K)(1+\alpha)} \right) \right] \end{aligned}$$

where $\Gamma = \frac{1}{1-\left(\frac{\frac{1}{J}}{(1-\eta_K)\left(\frac{1-\eta_A J}{J(1-\eta_A)}\right)}\right)^{\frac{\beta}{1+\beta}}\left(\frac{\alpha-r}{1+\alpha}\right)}$ is strictly positive and higher than unity as long as $(1-\alpha)(1-\eta_K)\left(\frac{1-\eta_A J}{J(1-\eta_A)}\right) \gg (\alpha - r)\frac{1}{J}$, i.e., as long as borrowing and investing one dollar with default is more profitable than borrowing and investing with repayment even in an after-tax sense. Γ simplifies to Ω when $\eta_A = 0$.

Again, since there is no uncertainty, the government will never default. The amount lent will be D^{**}, aggregate investment can be found using the K^N function above (23b), aggregate income is given by Y^N in (22)and the welfare level from U^N in (23c):

$$K^{**} = \frac{\beta}{1+\beta}\left[\overline{Y}_1 + \overline{A}_1 + D^{**}\right] - \frac{1}{1+\beta}\left[\frac{\left(\overline{Y}_2 + \overline{A}_2^N\right) - D^{**}(1+r) + \left(\overline{A}_2 + \overline{C}_2\right)}{1+\alpha}\right]$$

$$Y^{**} = \overline{Y}_1 + \overline{A}_1 + \frac{1}{1+\alpha}\left[D^{**}(\alpha - r) + \overline{Y}_2 + \overline{A}_2^N + \left(\overline{A}_2 + \overline{C}_2\right)\right]$$

$$U^{**} = (1+\beta)\ln\left(\frac{1}{1+\beta}\left[Y^{**}\right]\right) + \beta \ln\left(\beta(1+\alpha)\left(\frac{1}{J}\right)\right)$$

5.4 Comparative statics

The comparative static results discussed in section 2.4, related to exogeneous changes of $\overline{A}_1, \overline{A}_2^N, \overline{A}_2^D, \eta_{\overline{A}}, \eta_{\overline{Y}}$, and η_K are still relevant. They follow directly if $\eta_A = 0$ so that $\frac{1-\eta_A J}{J(1-\eta_A)} = \frac{1}{J}$ and $\Gamma = \Omega$ but letting η_A be positive (and $\ll \frac{1}{J}$) does not affect the main results.

How will a marginal increase of the safety-net aid budget, $\overline{A}_2$, work? It means extra consumption i period 2 and reduced investment in both regimes. Both U^N and U^D increase but U^D increases most. The reason is that taking out some of the benefits through increased consumption in period 1 is "cheaper" in the default case. Since the two welfare levels are equal at the outset it follows that given the initial debt level, D^{**}, the government will now choose to default. The lender will therefore cut back on the lending[12]. Letting $\eta_A = 0$ we have:

[12] This is basically the same experiment as the one carried out in section 2 with $\overline{A}_2^N = \overline{A}_2^D = \overline{A}_2$ with $\eta_{\overline{A}} = 0$.

$$\frac{dD^{**}}{d\overline{A}_2} = \Omega\left(\frac{1}{1+\alpha}\right)\left[\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}} - \frac{1}{1-\eta_K}\right] \ll 0$$

I.e., the lender reduces the amount lent. What we have called aggregate disposable income, however, increases, i.e.,

$$\begin{aligned}\frac{dY^{**}}{d\overline{A}_2} &= \frac{1}{1+\alpha}\left[(\alpha - r)\frac{dD^{**}}{d\overline{A}_2} + 1\right] \\ &= \frac{1}{1+\alpha}\left[\Omega\left(\frac{\alpha - r}{1+\alpha}\right)\left(\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}} - \frac{1}{1-\eta_K}\right) + 1\right] \gg 0\end{aligned}$$

so aggregate welfare, U^{**}, must increase. Any increase of aggregate consumption in period 2 in other countries benefitting from the safety-net, $\overline{C}_2$, will have similar consequences - because more aid will be made available for "our" country.

Does the safety-net cause welfare to increase?

We have already seen that endogenous aid (the safety-net) tends to distort incentives and, therefore, lead to underinvestment and inefficient intertemporal distribution of consumption. We have also seen that as long as the safety-net is in place it will be in the recipient government's interest to have the aggregate aid budget at the administrator's disposal as high as possible. We now ask whether it really is in the government's interest to start to recieve safet-net type of aid and become a client of the altruistic donor. To simplify the algebra we stick to the case where $\eta_A = 0$ so that $\Gamma = \Omega$ but similar results follow for $\frac{1}{J} \gg \eta_A \gg 0$.

By comparing D^* in (9) and D^{**} in (27)we see that endogenous aid tends to crowd out commercial lending:

$$D^{**} - D^* = \Omega\left[\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}} - \frac{1}{1-\eta_K}\right]\frac{(\overline{A}_2 + \overline{C}_2)}{1+\alpha} \ll 0$$

However, we already know that $\frac{dY^{**}}{d\overline{A}_2}$ is positive and constant so what we have called disposable income must be higher with the safety-net than without:

$$Y^{**} - Y^* = \left[\Omega\left(\frac{\alpha - r}{1-\alpha}\right)\left(\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}} - \frac{1}{1-\eta_K}\right) + 1\right]\frac{(\overline{A}_2 + \overline{C}_2)}{1+\alpha} \gg 0$$

The benefits of higher disposable income, however, must be weighted against the costs of distorted incentives. What matters is the difference between the welfare level with endogenous aid, U^{**}, and the welfare level without, U^* from section 2.3, i.e., the sign of

$$U^{**} - U^* = (1+\beta)\ln\left(\frac{1}{1+\beta}[Y^{**}]\right) + \beta\ln\left(\beta(1+\alpha)\left(\frac{1}{J}\right)\right) - \left((1+\beta)\ln\left(\frac{1}{1+\beta}[Y^*]\right) + \beta\ln(\beta(1+\alpha))\right)$$

. We have $U^{**} - U^* \gtreqless 0$ for

$$\begin{aligned}(1+\beta)\ln\left(\frac{Y^{**}}{Y^*}\right) &\gtreqless \beta\ln J \quad \text{or} \\ Y^{**} &\gtreqless Y^* J^{\frac{\beta}{1+\beta}}\end{aligned}$$

Using the fact that $Y^{**} = Y^* + \left[\Omega\left(\frac{\alpha - r}{1-\alpha}\right)\left(\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}} - \frac{1}{1-\eta_K}\right) + 1\right]\frac{(\overline{A}_2 + \overline{C}_2)}{1+\alpha}$ it follows that the answer depends on the level of $\overline{A}_2 + \overline{C}_2$, i.e., on the extra income taken into account by the government once it accepts to become a client of the altruistic donor and on the aggregate number of clients.

$$\overline{A}_2 + \overline{C}_2 \gtreqless \left(\frac{1+\alpha}{\Omega\left(\frac{\alpha-r}{1-\alpha}\right)\left(\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}} - \frac{1}{1-\eta_K}\right) + 1} \right) \left(J^{\frac{\beta}{1+\beta}} - 1\right) Y^*$$

As long as there are more than one client country ($J \gg 1$) the right-hand side is strictly positive. As a result, if the safety-net aid budget, $\overline{A}_2$, is very low and/or the other $(J-1)$ client countries indicate very low consumption in period 2 if they do not obtain support from the altruistic donor, $\overline{C}_2$, it may well be that $U^{**} - U^*$ ends up being negative. The benefit resulting from obtaining a small aid inflow is lower than the cost of distorted intertemporal distribution of consumption. However, letting $\overline{A}_2 + \overline{C}_2$ increase, ceteris paribus, means increased aid inflow to "our" country. For some level $U^{**} - U^*$ will change sign and it becomes worth while for the country to become a client of the altruistic donor.

We have in this section only discussed access to credit in a situation where the sovereign has discretion over investment. The main consequenses of the safety-net type of aid is closely related to the consequences of unconditional exogenous aid in period 2. That will be the result in case of debt relief and precommitment in investment as well.

6 Concluding comments

In this paper we consider the relationship between a commercial lender and the government of a poor country as a borrower. Influenced by Obstfeld and Rogoff (1996) we take as our point of departure a model where the borrower ends up being credit rationed - the reason being that the lender wants to motivate him not to default. We introduce foreign aid and ask how aid may affect the incentive structure and, accordingly, the contract designed by the lender (and accepted by the borrower). If the aid flows are exogenous we show that some types of aid work well in the sense that aid leads to more credit, higher investment and welfare, etc. However, other types of aid, making it more favourable for the reciopient to default, is directly counterproductive. Endogenous aid, given by altruistic donors as part of a safety-net, tends to encourage competition among recipient countries and distort incentives and discriminate against investment. Thereby investment and, accordingly, credit worthiness is reduced. We show that such aid may actually lead to reduced welfare in a recipient country. We also discuss how aid of different kinds will affect the lender's willingness to give relief in case the initial debt is unsustainable.

Admittedly, the model on which we base our arguments is simple and many of the assumptions made are not realistic. We believe, however, that the basic incentive structure is captured in a fairly acurate way and that our main conclusions are relatively robust.

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APPENDIX 1
The debt limit: New loans and discretion over investment

From (5c) and (8c) the condition that $U^D \leq U^N$ may be expressed as

$$\begin{aligned}
& (1+\beta)\ln\left(\frac{1}{1+\beta}\left[Y^D\right]\right)+\beta\ln\left(\beta\left(1-\eta_K\right)(1+\alpha)\right) \\
\leq\; & \left[(1+\beta)\ln\left(\frac{1}{1+\beta}\left[Y^N\right]\right)+\beta\ln\left(\beta\left(1+\alpha\right)\right)\right] \\
(1+\beta)\ln\left(\frac{Y^D}{Y^N}\right)+\beta\ln\left(1-\eta_K\right) \;\leq\; & 0 \\
\left(\frac{Y^D}{Y^N}\right)^{1+\beta}\left(1-\eta_K\right)^{\beta} \;\leq\; & 1 \\
Y^D \;\leq\; & \left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}} Y^N
\end{aligned}$$

Since $0 \ll \eta_K \ll 1$ and $\beta \leqq 1$ we know that $\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}} \gg 1$ from which it follows that the present value of the income in case of default will end up being higher than in the non-default case (remembering that the discount rates differ).

Remembering the determinants of Y^D and Y^N from (7) and (4) the last version of the equation may be written as

$$\begin{aligned}
& \overline{Y}_1+\overline{A}_1+D+\frac{1}{\left(1-\eta_K\right)(1+\alpha)}\left[\left(1-\eta_{\overline{Y}}\right)\overline{Y}_2+\left(1-\eta_{\overline{A}}\right)\overline{A}_2^D\right] \\
\leq\; & \left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}}\left(\overline{Y}_1+\overline{A}_1+\frac{1}{1+\alpha}\left[D\left(\alpha-r\right)+\overline{Y}_2+\overline{A}_2^N\right]\right)
\end{aligned}$$

and now it is straight-forward to calculate the cut-off level of lending, D^*, (the upper limit), as perceived by the lender:

$$\begin{aligned}
D \;\leqq\; D^* = \Omega\Bigg[& \left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}}\left(\overline{Y}_1+\overline{A}_1+\frac{\overline{Y}_2+\overline{A}_2^N}{1+\alpha}\right) \\
& -\left(\overline{Y}_1+\overline{A}_1+\frac{\left(1-\eta_{\overline{Y}}\right)\overline{Y}_2+\left(1-\eta_{\overline{A}}\right)\overline{A}_2^D}{\left(1-\eta_K\right)(1+\alpha)}\right)\Bigg]
\end{aligned}$$

where $\Omega = \frac{1}{1-\left(\frac{1}{1-\eta_K}\right)^{\frac{\beta}{1+\beta}}\left(\frac{\alpha-r}{1+\alpha}\right)} \geq 1$ because $(1+\alpha)\left(1-\eta_K\right) \gg \alpha - r$.

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