

SAM 36 2007

ISSN: 0804-6824

NOVEMBER 2007

Discussion paper

Market Definition with Shock Analysis

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This series consists of papers with limited circulation, intended to stimulate discussion.

Market Definition with Shock Analysis

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November 22, 2007

Abstract

The SSNIP test for market definition requires information about demand substitution and profitability. If detailed information about demand is not available, observed effects of a shock in the industry may be an alternative source of evidence. In the existing literature, shock analysis has unfortunately not been clearly linked to the SSNIP test. The lack of a rigorous framework may confuse the interpretation of the effects of shocks. We illustrate how a shock can be evaluated within the SSNIP framework with a minimum of data. We apply our criterion to a capacity expansion in the ferry market in the North Sea.

1 Introduction

Market definition is an important part of almost all competition cases as it identifies the competitive constraints facing the product of interest. Most competition authorities delineate markets using the SSNIP test¹: The relevant market is the smallest group of products where a hypothetical monopolist can profitably raise price(s) by 5-10% above the competitive level.² The SSNIP test provides well-defined principles that guide the delineating process. Harris and Simons [1989] reformulated the SSNIP test in terms of the critical loss - i.e. the loss in own demand after the price change which would leave the joint profits of the included products unchanged. To perform the SSNIP test the critical loss is compared to an estimate of the actual loss following the price increase. While a product's price-cost margin determines the critical loss, an assessment of the actual loss typically requires estimating a demand system. Estimates of demand

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¹Small but Significant and Non-Transitory Increase in Price.

²There is a subtle difference between the SSNIP test of the US Horizontal Merger Guidelines and the EU Notice on market delineation. The European guidelines asks if a hypothetical monopolist *could* profitably increase prices by above some given level, whereas the US version, arguably, asks whether a profit-maximizing hypothetical monopolist *would* increase prices by above some threshold level, see Werden [2002] for a discussion of the tests. The profitability test nevertheless seems to be the most pervasive test applied both in the literature and in US case law. We follow the convention of the simple profitability test rather than the profit maximization test.

systems are unfortunately seldom available in competition cases, often due to lack of relevant data and/or time constraints.

If there is a lack of detailed data on demand, one alternative strategy is to investigate the effects of some unexpected event, a shock, in the industry in the past. A relevant shock can be the introduction of a new product, cost shocks, technological change or a change in relative prices. There is, however, an apparent lack of well-defined principles on how to evaluate the effects of a shock. For instance, the Commission notice on the definition of the relevant market explicitly endorses shock analysis:³

“In certain cases, it is possible to analyse evidence relating to recent past events or shocks in the market that offer actual examples of substitution between two products. When available, this sort of information will normally be fundamental for market definition. [...] Launches of new products in the past can also offer useful information, when it is possible to precisely analyse which products lost sales to the new products.”

In the same spirit, Bishop and Walker [2002], p. 323, state:

“Shock analysis is a way of thinking about what past events in an industry tell us about the form of competition in that industry”.

Neither the Commission notice nor Bishop and Walker provide any standards by which the magnitudes of reactions to the shock should be evaluated. Without a proper framework, there is a risk that measured effects are evaluated against any number of arbitrary standards. Since market delineation is defined by the SSNIP-test, shock analysis is useful for market delineation to the extent it informs on the profitability of a price increase for the hypothetical monopolist.⁴ It might be necessary to interpret the effects of the shock within a structural model to properly address the profitability question. This paper suggests a way of doing that. We derive a threshold level of substitution for when two products belong to the same market from the principles of the SSNIP test. We then evaluate the observed effects of the shock to the criterion and delineate the market accordingly.

In contrast to the critical loss test of Harris and Simons [1989], we don't impose a proportionate price increase on all the products in the candidate market, but follow Katz and Shapiro [2003] in imposing an asymmetric price increase. As the purpose of market delineation is to identify competitive constraints, there may be good reasons to prefer an asymmetric price increase in cases where there are important asymmetries between products.⁵

By imposing a standard pricing rule we simplify the delineating criterion and focus on the diversion ratio, the share of the decrease of sales for product 1 that

³See Commission notice on the definition of the relevant market for the purposes of community competition law (OJ 372, 9/12/1997), paragraph 38.

⁴Note that market delineation and competitive effects analysis are not the same thing. Evidence quite useless for market delineation may be highly useful for competitive effects analysis, and vice versa.

⁵See Daljord, Sjørgard and Thomassen [2007] for a discussion.

is diverted to product 2 following a price increase on product 1.⁶ We show that the only information we then need in addition to the diversion ratio, is the margin of the product in question and the relative mark-ups of the two products. Though the margins can be hard to estimate, we show how it might be possible to establish useful bounds to their relative sizes. Given a margin on product 1, we then have a criterion for the critical diversion ratio.⁷ To check whether the two products belong to the same market, we compare the critical diversion ratio with the actual diversion ratio.

We illustrate how to estimate a diversion ratio from a shock in the ferry industry in the North Sea without estimating a full demand system. The shock we exploit is an expansion of the capacity of one of two ferries in a candidate market and we only have a time-series of passenger data. Theoretical considerations and observed passenger numbers indicate that a change in relative prices has taken place, but prices are unobserved. We estimate a diversion ratio from the observed passenger substitution following the shock. We then compare the actual diversion ratio with the critical diversion ratio, where the latter is given for different combinations of price-cost margins. The application shows that it is possible to properly perform the SSNIP test with only a limited amount of data.

The next section establishes the criterion in terms of a critical diversion ratio which is the link between the shock analysis and the SSNIP criterion. The third section explains how the shock analysis was performed on the ferry market, using the test from section 2. The last section concludes. Full estimation results are in the appendix.

2 The theoretical framework

The market delineating procedure is defined by the 1992 US Horizontal Merger Guidelines on page 6 as

“[...]Begin with each product (narrowly defined) produced or sold by each merging firm and ask what would happen if a hypothetical monopolist of that product imposed at least a “small but significant and nontransitory increase in price, but the terms of sale of all other products remained constant”

Note that the Guidelines are not explicit about whether the test requires an increase in one, some or all of the prices of the products in the candidate market. As noted by Whinston [2007], the choice between the one-price and the multi-price criterion cannot be deduced from neither theory alone, nor the Guidelines, and is apparently left open to the analyst.

We argue that the ambiguity with respect to prices provides valuable scope for the analyst to adopt the profitability criterion that makes most economic

⁶The argument is easily extended to the general case of J products in the candidate market.

⁷Our critical diversion ratio is related to the critical diversion ratios in O’Brien and Wickelgren [2003], but their thresholds are based on increasing all prices in the candidate market. We increase only one price.

sense in light of the purpose of market definition. Consider for instance a candidate market with one product with small sales, low margins and a high degree of price sensitivity that competes with a product with large sales, high margins and low price sensitivity. Then the hypothetical monopolist would prefer to increase the price of the small product more than the price of the large product. If so, the constraints may be more effectively identified by increasing only the price of the small product. Like Katz and Shapiro [2003], we derive a one-price criterion.⁸ As we show in the next section, the industry we analyze displays important asymmetries between firms. We opt for increasing the price of only one product as it likely more effectively identifies the competitive constraints compared to increasing the price of all products proportionately.

The criterion of the Harris and Simons critical loss test (i.e. that a proportionate price increase for all the products increases joint profits) is expressed in terms of prices, costs and features of the demand curve. Katz and Shapiro [2003] further develops the critical loss concept by formulating the criterion in terms of the diversion ratio, relative profitability and the margin on the price-increasing product. We apply the Katz and Shapiro concept to shock analysis, and thereby introduce a direct link between the SSNIP test and shock analysis.

The setting is an industry with differentiated products. We analyse the effects on two products, though we allow for competition from other products. We assume that marginal cost is constant for both firms and that the price of firm 1 has been set to maximize profits given the price of the other firm.⁹

In our practical application the aim is to determine whether product 1 is a market of its own, or whether it is in a market with product 2 (and possibly other products). It is assumed that inspection of the market has revealed that no product can conceivably be a closer substitute to product 1 than product 2 is.¹⁰ The SSNIP test starts by asking whether an increase in the price of product 1 would be profitable for a hypothetical monopolist owning products 1 and 2.

If the increase is profitable, we can draw the conclusion that there is indeed significant substitution from 1 to 2. Product 2 is therefore a competitive constraint on product 1 and the two must be in the same market. If, on the other hand, the price increase is not profitable another step is needed before the test procedure can reach a conclusion. The reason is that there are two possible scenarios which could explain why the SSNIP is not profitable: (a) substitution from product 1 to product 2 is insignificant, so owning both products does not make it profitable to increase the first price; or (b) there is significant substitution from product 1 to product 2, but there is also sufficient substitution to at least one third product to make the price increase unprofitable. In the first case product 1 is in a market of its own, while in the second case the market includes other products in addition to 1 and 2.

⁸But for different reasons. Katz and Shapiro derive a one-price criterion arguing that it is the proper interpretation of the Guidelines.

⁹The optimal pricing assumption is necessary to show that the own-price elasticity of demand $\eta_{jj} = \frac{p_j}{q_j} \frac{\partial q_j}{\partial p_j}$, is equal to minus the inverse of the margin, $-\frac{1}{L_j}$.

¹⁰In the application in section 3, this assumption is based on obvious geographical features of the market.

To distinguish between (a) and (b) it is necessary to look at whether the SSNIP would be profitable if the hypothetical monopolist owned both products 1 and 2 and the second-best substitute to product 1, i.e. product 3. The criterion for the strength of substitution which we develop in this paper can be extended to this multi-product case in a straightforward way. Here we limit ourselves to develop the theory for the two-product case. As discussed in the previous paragraph, if the conclusion of the two-product SSNIP test is that product 2 is indeed a competitive constraint on product 1, there is no need to proceed to a multi-product test: it is then clear that product 1 is in the same market as product 2. We now go on to derive the criterion which will be used to perform the test.

Let α be the percentage increase in the price of product 1 (normally this is 0.05 or 5%). We assume that the price of product 2 is held constant when the price of product 1 increases. We analyse this experiment under the assumption of linear demand.¹¹ In section 3.5, we relax this assumption.

When p_1 increases to $p_1(1 + \alpha)$, the demand for product 2 increases by a percentage $\alpha\eta_{21}$, where η_{ij} is the cross elasticity of demand for product 2 wrt. the price of product 1. Let CL denote the critical loss - i.e. the percentage reduction in own demand which would leave the joint profit from the two products unchanged when the price of product 1 is increased. CL must then satisfy the iso-profit condition

$$(1) \quad ((1 + \alpha)p_1 - c_1)q_1(1 - CL) - (p_1 - c_1)q_1 + (p_2 - c_2)q_2(1 + \alpha\eta_{21}) - (p_2 - c_2)q_2 = 0$$

We can solve for CL as :

$$(2) \quad CL = \frac{\alpha}{\alpha + L_1} \left(1 + \frac{\pi_2}{R_1} \eta_{21} \right)$$

where $L_j = \frac{p_j - c_j}{p_j}$ is the margin and $R_j = p_j q_j$ is revenue. At the profit maximising price, the own price elasticity equals minus the inverse of the margin. The diversion ratio from i to j is defined as $d_{ij} = \frac{\partial q_i}{\partial p_j} \left(\frac{\partial q_j}{\partial p_j} \right)^{-1}$. Using the diversion ratio and the ratio of markups $\lambda_{ij} = \frac{p_i - c_i}{p_j - c_j}$, it is straightforward to show that the expression for the critical loss becomes

$$(3) \quad CL = \frac{\alpha}{\alpha + L_1} \left(1 + \lambda_{21} d_{21} \right)$$

The two products are in the same market if the increase in price increases the joint profits. The joint profits increases if the actual reduction in demand for product 1 (actual loss) is smaller than the critical loss. Using the pricing rule, the actual loss is $\frac{\alpha}{L_1}$ with linear demand. The iso-profit condition in (1) simplifies to a neat expression: the SSNIP is profitable if and only if

$$(4) \quad \frac{\alpha}{L_1} < \lambda_{21} d_{21}$$

¹¹For instance $q_j = ap_j + bp_i + K_j$, where q_j is the quantity of product j and p_j is the price of product j .

that is, if the actual loss is less than the mark-up weighted diversion ratio. As discussed above, if this inequality holds we can conclude that the products are in the same market. If it does not hold, further investigation is needed to see if the market is larger or smaller than the two products. Note that this criterion differs from the one derived in Katz and Shapiro [2003].¹²

The criterion in (4) can be used in a number of different ways, depending on what information is available. We believe that in many competition cases, point estimates can be found for some of the magnitudes in (4), and bounds on plausible values can be found for the remaining ones. Often this will be sufficient to define the market. In the next section we demonstrate the use of this framework by means of an example from an actual competition case, where only limited data were available.

3 An application

This section demonstrates the use of the framework from the previous section to define the relevant market in a specific competition case. The central point in our analysis is to use a shock in the market to estimate a diversion ratio and apply it to the market definition criterion in (4).

3.1 The market

Travel by car from South-Western Norway to the European Continent can take place either by road via eastern Norway, Sweden and Zealand in Denmark, or alternatively by car ferry from a port in the region to Jylland on the Danish mainland. Until the autumn of 2004 there were two car ferry companies sailing from ports in South-western Norway. This is illustrated in Figure 1. The company Color Line operated a ferry from Kristiansand on the southern tip of Norway to Hirtshals on the north-east of Denmark. The ferry of the other company, Fjord Line, sailed from Bergen on the west coast via Egersund further south to Hanstholm in the north-west of Denmark.

Table 1.

	Sailing time to Denmark	Km from Bergen
<i>Bergen</i>	19h	
<i>Egersund</i>	5-8h	490
<i>Kristiansand</i>	4-7h	660

In april 2005, Color Line introduced a ferry sailing from Bergen via Stavanger to Denmark in addition to the Kristiansand line. After this, Fjord Line filed a complaint to the Norwegian Competition Authority alleging that Color Line was engaging in predatory pricing. For the alleged predation to be a breach of competition legislation, a necessary, but not sufficient condition was that Color Line was dominant in a greater south-western Norway ferry market, i.e. including both its own sailings to Denmark from Kristiansand and Fjord Lines sailings from Bergen via Egersund. Therefore, a central issue was to determine whether

¹²Unfortunately, there is an error in the derivation of the delineating criterion in Katz and Shapiro [2003]. We use the corrected criterion of Daljord, Sørsgard and Thomassen [2007].

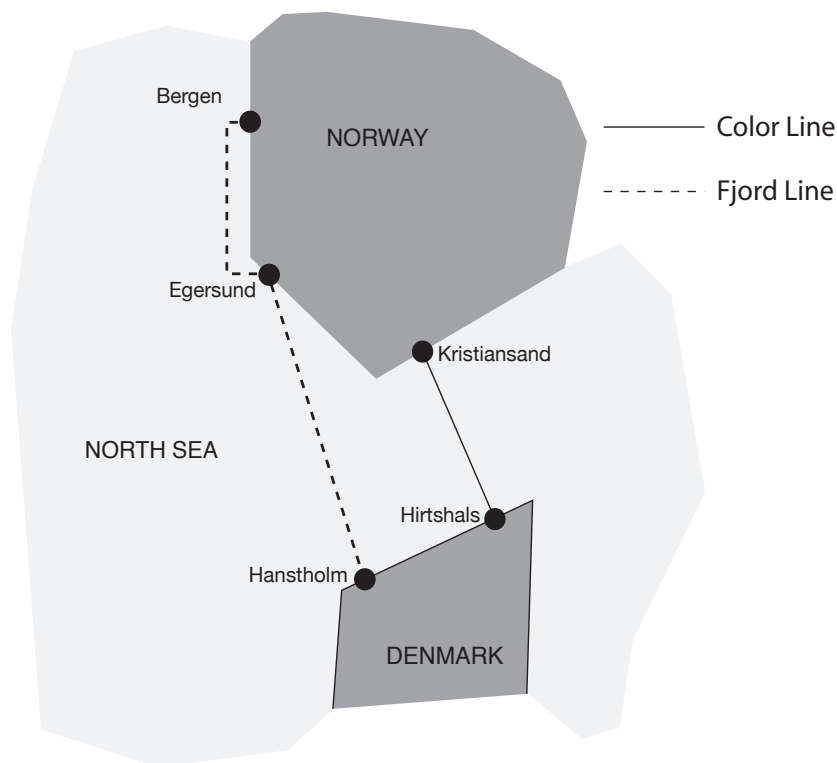


Figure 1: Map of the ferry routes.

ferries from Bergen (via Stavanger or Egersund) to Denmark were in market with the ferry from Kristiansand to Denmark or there were separate markets. The rest of this article address only the question of market delineation.¹³

In section 2 we briefly discussed the choice of whether to increase one or both prices when performing a SSNIP experiment. Some industry characteristics provide an argument for increasing only one price:

- The Kristiansand ferry (from now on called K) has much larger passenger numbers than the ferry from Bergen (B). The negative impact on the profit of K from a proportionate increase in its price may dwarf the positive contribution from diversion from B to K. The test may then fail to identify the asymmetric constraint on B by K.
- East of Kristiansand there are two ports from which car ferries sail to Denmark. It is likely that these ferries exert some competitive constraint on K.

These facts together indicate that joint ownership of the B and K ferries might

¹³At a later stage in the investigation, it was found that no abuse had taken place, and so the questions of dominance and the relevant market were in fact not decisive in this case.

not make it profitable to increase prices on K. This is not because B and K are not in the same market, but because K is part of a greater market. When we want to check only if B is in market with K, it makes more sense to examine the joint profits of B and K by only increasing the price of B.

3.2 The shock

The ideal natural experiment to identify the diversion ratio from B to K would be a unilateral price change by B. Following a price decrease by B alone, assume that B increases the number of passengers with Δq_B while K loses Δq_K passengers. Assuming linear demand, the diversion ratio is then the fraction of the new passengers at K have shifted from B, that is, $d_{KB} = \frac{\Delta q_K}{\Delta q_B}$. The linearity assumption implies there is no difference between the ratio of changes in quantities following from a finite change in prices (which is what we observe) and the ratio of changes in quantities following from an infinitesimal change in prices (which is what the derivative is about). We later relax the linearity assumption and show that in a sense, the results don't critically rely on the linearity assumption.

The shock that we analyse is the introduction of a new ferry by Fjord Line from Bergen in April 2003 that increased its capacity substantially. We do not have adequate price data, but we assume that prices were set optimally prior to the expansion of capacity. If so, Fjord Line must have lowered prices in order to sell out its increased capacity.¹⁴ We regard the introduction of the new ferry on B as equivalent to a change in the relative prices of B and K, with the former decreasing relative to the latter.

The observed shock differs from the natural experiment described above in one important respect. If B lowered its price to attract more passengers and Color Line did not change its price, we could directly measure the diversion ratio. However, it might be profitable for K to respond to Bs price reduction by reducing its own price, but less than B.¹⁵ If so, we will observe a smaller reduction in K passenger numbers than if Ks price had remained fixed and the estimate will be biased downwards. It implies that if any bias, we could from our results conclude that they do not belong to the same market when they in fact do belong to the same market.

There are three clearly distinguishable customer groups: Return travelers without car (going for on-board shopping and entertainment), holiday travelers with a car, and goods transport. There is price discrimination between the three groups of customers, and accordingly they could possibly be in separate markets. Our working hypothesis is that B and K may be in the same market as regards the second group of customers. The first group of customers is most likely in a local market only, while the third could possibly include other ferries. The first and the third group of customers are spread out evenly throughout the year. The second group, however, dominate completely in the Norwegian summer holiday month of July. Since we only have access to total passenger

¹⁴Alternatively, the ferry operator could spur its sales by increasing its sales efforts, for example, advertising outlays. If so, by reinterpreting the relative price change as a relative hedonic price change, the argument is unaffected. We don't observe prices anyhow.

¹⁵See, for instance, Farrell and Shapiro [1990].

numbers every month, we look at the effects of the shock on passenger numbers in July only. These numbers should give a good approximation to the substitution behaviour of holiday travelers, since in July the overwhelming majority of passengers fall in this category. We therefore limit our investigation to the market for holiday travelers with a car.¹⁶

July passenger numbers are higher than those in every other month. It is therefore likely that the capacity is a binding constraint in July, but that it might not be in off-peak months. By focusing on July we therefore measure the effect in the period where a capacity effect would change Bs pricing behaviour.

3.3 Data and estimation

We have 142 observations of monthly total passenger numbers for B and K respectively, going from July 1993 to April 2005.¹⁷ Figure 1 plots the passenger numbers. The most obvious features of the data are

1. Very stable passenger numbers across the years, with regular seasonal variation for both ferries
2. Much higher passenger numbers for K than for B
3. A reduction in the July peaks for K after the shock, and an increase in the July peaks of B after the shock

Our estimate of the diversion ratio will be the effect of the shock on July passenger numbers for K, divided by the effect on the July numbers for B.

Because of (1), seasonal variation appears to explain most of the movement in passenger numbers. We therefore run a regression for each ferry of passenger numbers on eleven month dummies plus a constant. Because of (3), we also include in the regression an extra set of month dummies that are zero before the shock and thereafter the same as the other month dummies. This captures the (seasonally varying) effects of the shock, after controlling for the usual seasonal variation by the month dummies that span the whole sample period. The July effects of the shock that we are looking for will now be the coefficients on the July shock dummies for B and K, respectively. The model is very simple in that no other explanatory variables are included other than the month dummies.¹⁸

3.4 Results and market delineation

The coefficients on the July shock are 25,600 for B and - 20,100 for K, both highly significant in our model, see the appendix for full estimation results. The

¹⁶Since we consider only July, we have to be careful with the interpretation of our results for other months than July. Even those travellers we would label 'holiday travellers with a car' might have another demand pattern when they travel in other months than July.

¹⁷The data are from ShipPax and they are publicly available.

¹⁸We also tried to add lagged values of passenger numbers (both lag 1 and lag 12) to the estimated equation, but found that it hardly changed our results. We also tried to estimate the two equations together with generalised least squares, as well as generalised least squares with four other ferry lines from Southern Norway to Denmark. Neither of these approaches changed our results anything but marginally.

July shock coefficients have the interpretation of average change in passenger numbers in July due to the shock, compared to the average passenger numbers in July before the shock. Since the passenger numbers are very stable before the shock, as can be seen from figure 2, the changes are precisely estimated. The estimated diversion ratio is the absolute value of the ratio for the effects of the shock on K and B, respectively. This number is 0.78, see the Appendix for a discussion on the choice of estimator for this ratio.

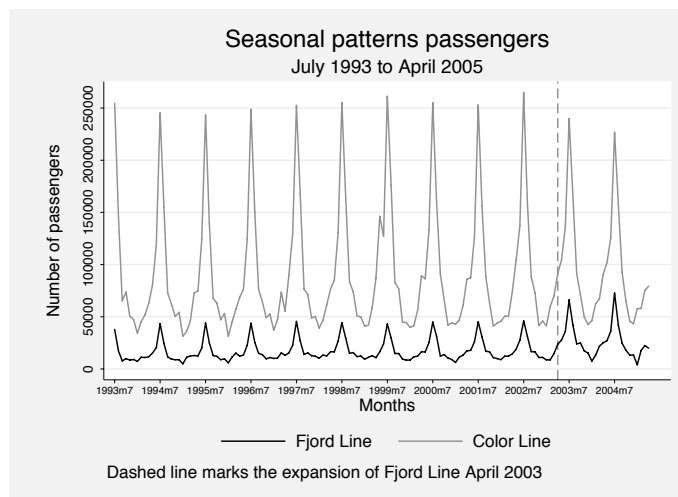


Figure 2: Monthly passenger numbers 1993 to 2005.

This is a high diversion ratio. If we think for a moment about what this number represents, it is not implausible, however. B stops at Egersund which is 170km away from Kristiansand and 490km away from Bergen. The duration of the trip to Denmark is 4-7 hours from Kristiansand, 5-8 hours from Egersund, and 19 hours from Bergen. For customers who live close to Kristiansand, using B would entail both extra driving time and extra sailing time. It seems unlikely that B and K are good substitutes for these customers. For customers living around Egersund, the customer group most likely to substitute when the price changes, the additional driving time to Kristiansand is largely made up for by shorter sailing time. For these customers, it is reasonable to expect that K and B are close substitutes. A diversion ratio of 0.78 is therefore consistent with the story that a large percentage of the additional customers to B after the shock are people from around Egersund, who would have used K before the shock, but who now find B a better alternative. Travelers living around Egersund need to travel even further away than Kristiansand to use an alternative ferry. The diversion ratio of 0.78 therefore implies that 22% of the new customers to B are people who prior to the capacity expansion would not have taken a ferry at all, or who would have gone even further away than Kristiansand. It is not obvious that this percentage should be any higher. We therefore conclude that our estimate of the diversion ratio is plausible.

Before we perform the actual SSNIP test on this market, we recall the conclusions from the discussion in section 2. If the SSNIP is profitable for a hypo-

thetical monopolist who owns both products, we can conclude that the products are in the same market. If it is not profitable, we need to investigate substitution patterns further in order to draw a conclusion. From (4) in section 2, the SSNIP is profitable, and so B and K are in the same market if

$$(5) \quad \frac{\alpha}{L_B} < \lambda_{KB} d_{KB}$$

We now have an estimate of d_{KB} of 0.78, we let $\alpha = 0.05$ and the margin L_B must be between zero and one. This allows us to plot all the combinations of L_B and $\lambda_{KB} = \frac{p_K - c_K}{p_B - c_B}$ that leave the profit unchanged when the price is increased by a percentage (see figure 2). If the actual combination (L_B, λ_{KB}) is above the line, K and B are in the same market.

If we take as a starting point the assumption that the ratio of mark-ups is one, Figure 3 shows that both B and K must have a very low margin (below 10%) for the SSNIP to be unprofitable. Note that we consider the short run margin, where for example costs associated with the ferry as such are not included. In this setting it seems implausible that the margin should be as low as 10%. Mark-ups may differ between B and K. Passengers spend substantially

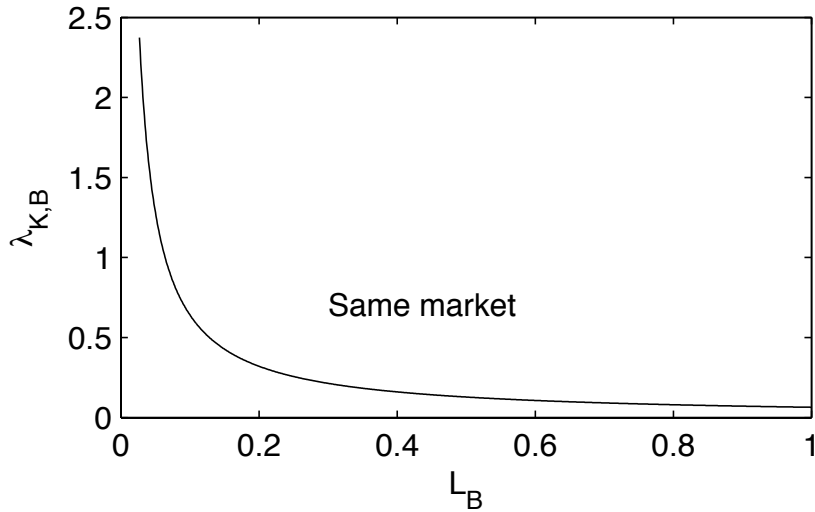


Figure 3: Delineating level set linear demand.

more time on a Fjord Line trip between Bergen and Denmark than a Color Line trip between Kristiansand and Denmark. This may indicate a larger potential for on-board sales for each passenger on B that increases the mark-up $p_B - c_B$. If so, the relative mark-up λ_{KB} is below one. On the other hand, it is not necessarily any positive relationship between on-board sales and the price-cost margin per passenger. For example, it can be argued that K has a better on-board service which leads to higher on-board sales for each passenger. If so, it might be that λ_{KB} is higher and possibly above one. In both scenarios, though, there must be a combination of a low relative markup of K combined with an

implausibly small margin of B to conclude that the SSNIP is not profitable. For example, if $\lambda_{KB} = 0.5$, B's margin must be slightly above 13% and for the price increase to be unprofitable.

All in all it appears highly unlikely that (L_B, λ_{KB}) should be below the line drawn in Figure 3. We can therefore conclude with reasonable certainty that the car ferries to Denmark from Bergen and from Kristiansand are in the same market. Our analysis has the advantage of making explicit the assumptions which allow us to draw this conclusion. To dispute the conclusion, discussion would have to focus on whether mark-ups do not satisfy the bounds that we impose, or possible alternative reasons for the changes in passenger numbers which have not been included as explanatory variables in our analysis.

3.5 A sensitivity analysis with respect to the form of the demand function

Up until now we have assumed that demand functions are linear in both prices. We now briefly discuss the results if we instead assume that demand is iso-elastic.¹⁹

Among commonly used demand functions, linear demand (LD) and constant elasticities demand²⁰ (CE) represents extremes in terms of the curvature of demand, in that for instance both logit and AIDS tend to be more convex than LD, but less convex than CE, see Crooke, Froeb, Tschantz and Werden [1999]. Therefore, if the same market definition is arrived at with both LD and CE, there is a sense in which the conclusion does not rest on strong assumptions about the shape of the demand function.²¹

Figure 4 shows the thresholds for market delineation for both linear and constant elasticities demand. Note that the CE structure require more information. The CE delineating criterion depends on the ratios of prices $\frac{p_K}{p_B}$ between the two products, see the appendix for the derivation. We have therefore plotted the graphs for a range of values of this ratio. Since the trajectory from Bergen is substantially longer, it is unlikely that this ratio should be higher than one. The criterion also depends on the ratio of quantities $\frac{q_K}{q_B}$. This has been set to five the ratio of passengers prior to the shock. The graph shows that the conclusions from the previous section are robust to changes in the assumed form of the demand function.

4 Concluding remarks

We have shown how to delineate markets using shock analysis in accordance with the principles of the SSNIP test without knowledge of demand elasticities

¹⁹The iso-elastic demand structure leads to different expressions for actual and critical loss, see the appendix for derivation of the appropriate criterion.

²⁰For instance $q_j = a_j p_j^{\eta_{jj}} p_i^{\eta_{ji}}$, where q_j is the quantity of product j while p_j is the price of product j .

²¹It is likely that the form of the demand function would matter more for our results if we had increased the price of both products instead of just one, since constant elasticities demand implies smaller own-price effects on quantities.

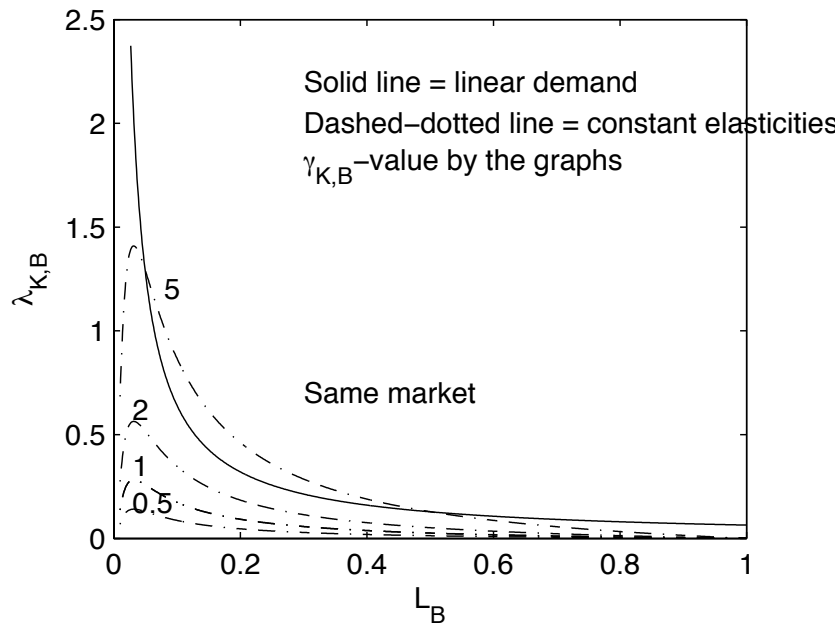


Figure 4: Delineating level set linear and constant elasticity demand.

or exact information on profitability. We show that there are three pieces of information that is needed in a SSNIP test for a candidate market with two products: The diversion ratio, product 1's price cost margin and the relative mark-up of the two products. If evidence of substitution can be defined in terms of diversion ratios, measured substitution can be properly evaluated within the SSNIP framework with a minimum of structure. Even in the absence of precise measurements of product level profitability, it may be possible to establish useful bounds to their sizes.

The application can be regarded either as shock analysis firmly linked to a well defined criterion, or as a SSNIP test performed with very limited data. In any case we believe the approach to be widely applicable and that it can contribute to more informed decisions about market definition where there is limited information about demand.

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A Regression results

The regression results for the OLS-specification are reported below. Both equations regress passenger numbers on eleven monthly dummies, a constant and a dummy for July post-shock.

A.1 OLS regression K passengers

Equation	Obs	Parms	R^2	F-Stat	P
Kpax	142	12	0,9777	470,34	0,0000

	Coef.	Std. Err.	t	p-value
$\hat{\beta}_K^{Feb}$	7701,417	3811,944	2,02	0,044
$\hat{\beta}_K^{Mar}$	20060,75	3811,944	5,26	0,000
$\hat{\beta}_K^{Apr}$	37151,67	3811,944	9,75	0,000
$\hat{\beta}_K^{May}$	53597,48	3897,616	13,75	0,000
$\hat{\beta}_K^{Jun}$	87888,58	3897,616	22,55	0,000
$\hat{\beta}_K^{July}$	212723,9	3998	53,21	0,000
$\hat{\beta}_K^{Aug}$	117270,3	3811,944	30,76	0,000
$\hat{\beta}_K^{Sep}$	40665,67	3811,944	10,67	0,000
$\hat{\beta}_K^{Oct}$	27897,75	3811,944	7,32	0,000
$\hat{\beta}_K^{Nov}$	6109,167	3811,944	1,60	0,110
$\hat{\beta}_K^{Dec}$	6952,667	3811,944	1,82	0,069
$\hat{\beta}_K^{exp}$	-20052,2	7232,655	-2,77	0,006
$\hat{\beta}_B^{cons}$	40697,33	2695,451	15,10	0,000

A.2 OLS regression B passengers

Equation	Obs	Parms	R^2	F-Stat	P
Bpax	142	12	0,8887	85,80	0,0000

	Coef.	Std. Err.	t	p-value
$\hat{\beta}_B^{Feb}$	4325,833	1684,817	2,57	0,011
$\hat{\beta}_B^{Mar}$	7049,667	1684,817	4,18	0,000
$\hat{\beta}_B^{Apr}$	8556,167	1684,817	5,08	0,000
$\hat{\beta}_B^{May}$	10287,1	1722,683	5,97	0,000
$\hat{\beta}_B^{Jun}$	18506,1	1722,683	10,74	0,000
$\hat{\beta}_B^{July}$	36218,72	1767,051	20,50	0,000
$\hat{\beta}_B^{Aug}$	21991,42	1684,817	13,05	0,000
$\hat{\beta}_B^{Sep}$	7825,167	1684,817	4,64	0,000
$\hat{\beta}_B^{Oct}$	7406,25	1684,817	4,40	0,000
$\hat{\beta}_B^{Nov}$	3443,75	1684,817	2,04	0,042
$\hat{\beta}_B^{Dec}$	3233,667	1684,817	1,92	0,056
$\hat{\beta}_B^{exp}$	25632,7	3196,716	8,02	0,000
$\hat{\beta}_B^{cons}$	7617,083	1191,346	6,39	0,000

B Some notes on the diversion ratio estimators

B.1 Asymptotic properties

Under the assumption of linear demand, the diversion ratio is $d_{KB} = \frac{\partial q_K}{\partial q_B} \left(\frac{\partial q_B}{\partial p_B} \right)^{-1}$,

the ratio of changes in demand for the two products following the decrease in p_B . The expansion coefficients in the specification measure the average deviations from the expected volumes, but for the expansion. Our estimator of the diversion ratio is the ratio of expansion coefficients in July

$$(6) \quad \hat{d}_{KB} = \frac{\hat{\beta}_K^{exp}}{\hat{\beta}_B^{exp}}$$

Under the regular assumptions, the OLS estimates of the expansion coefficients are individually unbiased and consistent estimates, however, the ratio of the expansion coefficients is a biased, but consistent estimator of the true ratio. Consistency can be seen from applying Slutskys theorem to the ratio. The bias can be seen from a second-order expansion of the ratio around the true means of the $\hat{\beta}$ s:

$$(7) \quad E\left(\frac{\hat{\beta}_K^{exp}}{\hat{\beta}_B^{exp}}\right) \approx \frac{\beta_K^{exp}}{\beta_B^{exp}} + \left(\frac{1}{\beta_B^{exp}}\right)^2 \left(\sigma_{\hat{\beta}_B^{exp}}^2 \frac{\beta_K^{exp}}{\beta_B^{exp}} - \text{cov}(\hat{\beta}_K^{exp}, \hat{\beta}_B^{exp})\right)$$

where the second term on the right hand side is an approximation to the bias induced by the non-linear entry of $\hat{\beta}_B^{exp}$. The bias is increasing in the variance of $\hat{\beta}_B^{exp}$ due to the disproportional impact of errors of equal absolute value, but different signs. Correlation between the expansion coefficients may exacerbate or attenuate the bias depending on the sign of the covariate. The bias flows from the asymmetric effect of errors in $\hat{\beta}_B^{exp}$ that cause a skewed distribution of the estimator in small samples. The bias is most severe when the denominator is close to zero, where close is understood in terms of its variance. The true values of the first-and second-order moments are unknown. An estimate of the bias is derived by substituting the empirical first-and second-order moments for the true values.

An approximation to the variance of the ratio of coefficients is derived by a first-order expansion of the variance of the ratio around the true means of the coefficients

$$(8) \quad \text{var}\left(\frac{\hat{\beta}_K^{exp}}{\hat{\beta}_B^{exp}}\right) \approx \left(\frac{1}{\beta_B^{exp}}\right)^2 \left(\sigma_{\hat{\beta}_B^{exp}}^2 \left(\frac{\beta_K^{exp}}{\beta_B^{exp}}\right)^2 + \sigma_{\hat{\beta}_K^{exp}}^2 - 2\frac{\beta_K^{exp}}{\beta_B^{exp}} \text{cov}(\hat{\beta}_K^{exp}, \hat{\beta}_B^{exp})\right)$$

where the estimator of the variance is given by substituting the empirical first-and second order moments for their true values.

The small-sample skewness of the distribution of the ratio estimator may render normal-based confidence intervals improper measures of precision. Since the small-sample distribution of the estimator is hard to derive even under strong distributional assumptions of the residuals from the regression, an appropriate alternative is a bootstrap-percentile approach. Bootstrap estimators of standard errors and percentiles are easily implemented in modern statistical packages. Since the bias in our application is small, we don't report bootstrap percentiles.

A caveat to using the OLS-estimator is that the correlation across equations is zero by assumption so the covariates in (7) and (8) drop out. That is fine under the null of independent markets, but more problematic under the alternative hypothesis of substantial substitution. There is an easy fix to the problem. As is well-known, when regressors are equal across equations, the OLS-estimator of the coefficients are the SURE estimators.²² Reinterpreting the coefficients as SURE-estimators, we may calculate the SURE covariance estimator $(X'(\hat{\Sigma}^{-1} \otimes I_2)X)^{-1}$, where $\hat{\Sigma} = \frac{1}{T}\hat{U}'\hat{U}$ and $\hat{U} = (\hat{u}_K, \hat{u}_B)$ while

²²Seemingly Unrelated Regression Equations.

preserving the estimates of the coefficients.²³ The SURE covariance estimator is routinely reported by most statistical packages.

The expansion in (7) suggests a bias-corrected estimator \hat{d}_{KB}^{bc} simply from subtracting the expected bias:

$$(9) \quad \hat{d}_{KB}^{bc} = \frac{\hat{\beta}_K^{exp}}{\hat{\beta}_B^{exp}} - \left(\frac{1}{\hat{\beta}_B^{exp}} \right)^2 \left(\hat{\sigma}_{\hat{\beta}_B^{exp}}^2 \frac{\hat{\beta}_K^{exp}}{\hat{\beta}_B^{exp}} - \text{cov}(\hat{\beta}_K^{exp}, \hat{\beta}_B^{exp}) \right)$$

The estimator can be shown to be consistent. The bias correction comes at the cost of a potential decrease in efficiency due to additional sampling errors in the correction term. We may be trading off bias for inefficiency. Monte Carlo experiments suggests that the loss of efficiency may be severe for imprecise estimates at very non-linear intervals (i.e. close to zero), but the evidence is not clear.²⁴ In some applications, the bias corrected estimator is more efficient. In the latter case, the choice of estimator is unproblematic.

B.2 Concluding remarks on the choice of estimators

Though our reported estimator of the diversion ratio in (6) is biased, the expected bias of the estimator is trivial in our application, mainly due to the precise estimates of the denominators. Hence, the simple ratio of expansion coefficients is a good estimator and is chosen on the virtue of simplicity.

The variance of the estimator may be approximated by the delta-method. Due to the small expected bias in our application, we don't report bootstrap percentiles of the estimator. In our application, allowing for correlation across equations hardly matter for the estimate of the variance of the diversion ratio. In the name of simplicity, we report variance estimates disregarding simultaneous correlations across equations.

The variance of both estimators are for all practical purposes equal. However, since there really doesn't seem to be much to gain from correcting the estimator, we opt for the simple estimator. One should nevertheless be cautious using the ratio of coefficients in small samples if the denominator is either estimated with poor precision or is close to zero, as the bias may be severe.

B.3 Diversion ratio estimates

B.4 Linear demand diversion ratio

	Coef.	Std. Err.	t	p-value
\hat{d}_{KB}	-,7822898	,2985555	-2,62	0,009

B.5 Bias corrected linear demand diversion ratio

	Coef.	Std. Err.	t	p-value
\hat{d}_{KB}^{bc}	-,7701227	,2929343	-2,63	0,009

²³see Davidson & MacKinnon (2003).

²⁴We have performed some illustrative Monte-Carlo experiments on the ratio estimator and their bootstrap percentiles for interested readers. The experiments are available upon request

C The case of constant elasticity

C.1 Actual and Critical Loss

The expressions for Actual and Critical Loss get more complex and require more information under the alternative assumption of demand of the constant elasticity (CE) form. Let the CE demand functions in the two-product case be given by

$$(10) \quad q_1(p) = a_1 p_1^{\eta_{11}} p_2^{\eta_{12}}$$

$$(11) \quad q_2(p) = a_2 p_1^{\eta_{21}} p_2^{\eta_{22}}$$

where the exponents η are the respective elasticities of demand. Then actual loss of product 1 is given by

$$(12) \quad AL_{CE} = (1 + \alpha)^{\eta_{11}} - 1$$

where $p^{ssnip} = ((1 + \alpha)p_1, p_2)$. The relative change in volume of product 2 is similarly given by

$$(13) \quad \frac{q_2(p^{ssnip}) - q_2(p)}{q_2(p)} = (1 + \alpha)^{\eta_{21}} - 1$$

Substituting (13) for $\alpha\eta_{21}$ in (2) and rearranging, the critical loss of product 1 is reduced to

$$(14) \quad CL_{CE} = \frac{\alpha}{\alpha + L_1} \left(1 + \frac{\lambda_{21} L_1 \delta_{21}}{\alpha \gamma_{21}} ((1 + \alpha)^{\eta_{21}} - 1) \right)$$

where $\delta_{21} = \frac{q_2}{q_1}$ and $\gamma_{21} = \frac{p_2}{p_1}$. In lack of data on prices, the case of constant elasticity requires one further assumption on the relative prices. Note that δ_{21} and γ_{21} are evaluated at their pre-shock values. From (12) and (14), the delineating criterion in the CE case becomes

$$(15) \quad (1 + \alpha)^{\frac{1}{\alpha}} - 1 < \frac{\alpha}{\alpha + L_1} \left(1 + \frac{\lambda_{21} L_1 \delta_{21}}{\alpha \gamma_{21}} ((1 + \alpha)^{\eta_{21}} - 1) \right)$$

where we have used the pricing rule to substitute for η_{11} and η_{21} is the parameter of substitution to be estimated.

C.2 CE estimator of substitution

The diversion ratio estimator in (6) relies on the structural assumption of linear demand. Under the assumption of constant elasticity of demand, the coefficients from the regressions require reinterpretation.

From (14), it is clearly an empirically more convenient strategy to estimate the crossprice-elasticity rather than the diversion ratio. Say demand is observed before and after the shock, but prices are unobserved. Let the shock lead to an unobserved decrease in the price of product 1 of κ percent. Pre-shock, the price vector is given by $p = (p_1, p_2)$ and, given the assumption of no price-response of product 2, post-shock the price vector is $p^s = ((1 + \kappa)p_1, p_2)$. As in the case of linear demand, the assumption of no price response of product 2 attenuates

the measured substitution. The bias is towards less substitution, hence broader markets. Under the null of independent markets, however, the bias-causing price response is from small to zero.

Pre-shock, demand is given by $q(p)$ and post-shock by $q(p^s)$. Define the relative post-shock volume of product j as $\Phi_j = \frac{q_j(p^s)}{q_j(p)}$. Taking logs of Φ and inserting in the pricing equation, we have two equations in the two unknowns η_{21} and κ , with (real) solution²⁵

$$(16) \quad \eta_{21} = -\frac{1}{L_1} \frac{\ln(\Phi_2)}{\ln(\Phi_1)}$$

$$(17) \quad \kappa = \Phi_1^{-L_1}$$

The measure of substitution is now a function of observable passenger data q and L_1 .

For comparison with the case of linear demand and assuming a margin of say 0.5, we may back out the diversion ratio from the estimates of the crossprice-elasticity and the pre- and post-shock volumes as

$$(18) \quad d_{21} = \eta_{21} \delta_{21} L_1$$

Note that unlike the case of linear demand, the diversion ratio now varies with relative volumes due to the constancy of the elasticity, so the diversion ratio must be evaluated at some ratio of volumes δ . In the application, we evaluate the diversion ratio at the pre-SSNIP relative passenger volumes.

The Φ s and δ s in the application are simple functions of the coefficients from regressions reported above:

$$(19) \quad \hat{\Phi}_B = \frac{\hat{\beta}_B^{const} + \hat{\beta}_B^{July} + \hat{\beta}_B^{expJuly}}{\hat{\beta}_B^{const} + \hat{\beta}_B^{July}}$$

$$(20) \quad \hat{\Phi}_K = \frac{\hat{\beta}_K^{const} + \hat{\beta}_K^{July} + \hat{\beta}_K^{expJuly}}{\hat{\beta}_K^{const} + \hat{\beta}_K^{July}}$$

$$(21) \quad \hat{\delta}_{KB} = \frac{\hat{\beta}_K^{const} + \hat{\beta}_K^{July}}{\hat{\beta}_B^{const} + \hat{\beta}_B^{July}}$$

The estimators share the property of the linear demand diversion ratio in being biased, but consistent. The expected bias is small in our application and the uncorrected estimators have the virtue of being simpler than their bias corrected alternatives. Below are point estimates of the essential variables with the standard errors calculated by the delta method at a $L = 0.5$.

²⁵There is one additional complex solution we ignore

C.2.1 Constant elasticity estimators evaluated at margin .5

	Coef.	Std. Err.	t	<i>p</i> -value
$\hat{\Phi}_B$	1,584744	,0815944	19,42	0,000
$\hat{\Phi}_K$,920874	,0281762	32,68	0,000
$\hat{\eta}_{KB}^{CE}$,3580712	,1388103	2,58	0,010
$\hat{\kappa}$	-,2056343	,0204499	-10,06	0,000
$\hat{d}_{KB}^{CE}(p)$	1,035031	,4018628	2,58	0,011
$\hat{d}_{KB}^{CE}(p^s)$,6014432	,2259146	2,66	0,008



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