Discussion paper

Competition and quality in regulated markets with sluggish demand

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Competition and quality in regulated markets with sluggish demand

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Abstract

We investigate the effect of competition on quality in regulated markets (e.g., health care, higher education, public utilities), using a Hotelling framework, in the presence of sluggish demand. We take a differential game approach, and derive the open-loop solution (providers commit to an optimal investment plan at the initial period) and the feedback closed-loop solution (providers move investments in response to the dynamics of the states). If the marginal cost of provision is increasing, the steady state quality is higher under the open-loop solution than under the closed-loop solution. Fiercer competition (lower transportation costs and/or less sluggish demand) leads to higher quality in both solutions, but the quality response to increased competition is weaker when players use closed-loop strategies. In both solutions, quality and demand move in opposite directions over time on the equilibrium path to the steady state.

Keywords: Regulated markets; Competition; Quality; Differential game.

JEL: H42; I11; I18; L13.

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1 Introduction

Competition leads to better quality when prices are regulated. This is a fairly robust prediction from economic theory.[1,2] Given that the regulated price is above the marginal costs, firms have an incentive to invest in quality in order to attract (or avoid losing) consumers. Tougher competition – measured for instance by the number of competing firms or by the degree of substitutability among products – amplifies the incentives to invest in quality. A prime example of regulated industries we are thinking of is health care. A further example is education. In both these fields, the consumer choices are mainly driven by considerations on quality rather than price.

In theoretical models, the positive relationship between competition and quality in regulated markets is generally derived within a static framework, neglecting potentially important dynamic issues related to quality. In particular, static models make the following two assumptions: (i) quality can be adjusted instantaneously (and permanently) by the providers; and (ii) demand responds immediately to quality changes. Brekke, Cellini, Siciliani and Straume (2008) address the first assumption by treating quality as a stock variable, moving over time in response to investments. They find that if the marginal provision cost is increasing, competition is less relevant in enhancing quality, as compared to the findings of static models.

In the current paper, we address the second (static) assumption by allowing for sluggish demand responses to (changes in) product quality. If a provider increases quality, sluggish demand implies that it will take some time before the potential demand increase is fully realised. Such demand sluggishness can typically arise for two different kinds of reasons. First, imperfect information on the demand side, which is particularly relevant in markets where quality is the main competition variable: while prices usually are easily and immediately observable, quality is often less readily observable and much more difficult to measure. Second, sticky behaviour of consumers, motivated by (personal or familiar) habits, or by trust or confidence in one specific

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[1] See, for instance, the survey by Gaynor (2006) and references therein. See also the papers by Ma and Burgess (1993), Wolinsky (1997), and Brekke, Nuscheler and Straume (2006). See also Lambertini (2006) for theoretical analyses of vertical differentiation in industrial organization.

[2] If firms can set prices as well, the effect of competition on quality is in general ambiguous. In this case, competition depresses prices and, thus, the marginal revenues from quality investment (see e.g., Economides, 1993).

provider. Let us think, for example, of the cases of persons who choose a dentist simply because relatives went to him/her; or a child going to a specific school (or college) simply because brothers went there. Moreover, the relational content in the service exchange between a provider and a consumer in fields like education or health play an important role in making demand sluggish.

Our basic framework is the widely-used Hotelling model for quality competition with regulated prices.[3] In this model there are two firms offering one product each. We consider the case in which the spatial locations are given, while the products are horizontally and vertically differentiated, and the firms choose quality to maximise profits. First, we derive the static quality equilibrium as a benchmark. Second, we extend the model to a dynamic framework with sluggish demand. Sluggish demand is modelled such that at each point in time only a fraction of consumers respond to quality changes. Thus, it will take some time before potential demand is fully realised. The time it takes for potential and actual demand to align is determined by the degree of demand sluggishness.

We use a differential game approach to analyse dynamic competition between the two firms.[4] There are two main solution concepts. First, we derive the open-loop solution, where each firm commits to an optimal (quality) investment plan at the initial period. This solution is reasonable if it is very difficult or costly to obtain information about competitors and/or the quality variable is subject to some rigidity (e.g., investment regulations). Second, we derive the (feedback) closed-loop solution, where each firm knows the quality of the competitor at each point in time, not just the initial state. Here, firms choose an optimal rule connecting the current value of their choice variable to the current value of states; hence, the feedback closed-loop solution can be interpreted as a more competitive solution in the sense that firms can at each point in time change their investments in response to the dynamics of the states.

A first finding is that the quality steady-state levels under both open-loop and closed-loop are lower than in the static equilibrium. The main reason is that in a dynamic game, firms take future profits into account, resulting in less aggressive competition. In addition, sluggish demand dampens competition even further, since shifting consumers becomes more costly the

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[3]See, for instance, Ma and Burgess (1993), Calem and Rizzo (1995), Wolinsky (1997), Lyon (1999), Del Rey (2001), Beitia (2003), Brekke, Nuscheler and Straume (2006, 2007), and De Fraja and Landeraas (2006).

[4]See Dockner, Jørgensen, Van Long and Sorger (2000) for an introduction.

more sluggish demand responds to quality changes. Moreover, if marginal costs are increasing, we show that quality is lower in the closed-loop than the open-loop solution. This is somewhat surprising, since the closed-loop solution often is considered as the more competitive solution concept, given that firms can set the choice variables in each point in time.[5] The reason is that quality choices are *strategic complements* when marginal costs are increasing, which in a dynamic game provides an incentive to compete less aggressively.[6] Otherwise, if the marginal costs are constant, quality choices become strategically independent, and the two solutions - open-loop and closed-loop - coincide.

Our comparative dynamics results are, however, very much in line with the findings derived within the static framework. We show that both higher prices and lower transportation costs (less horizontal differentiation) lead to higher quality in steady state. In addition, we also find that less sluggish demand increases the steady state quality level. Note that one can interpret lower transportation costs and less sluggish demand as tougher competition. Thus, in line with static models, we report a positive relationship between competition and quality.

A second finding is that demand and quality move in opposite directions over time on the equilibrium path to the steady state, given that marginal costs are *increasing* in production. This result contradicts the static relationship between quality and demand. Consider a situation where actual demand is below the steady-state level. In this situation, the provider will raise quality to a level above the quality steady-state level. However, as demand increases, the marginal profit gain becomes lower due to increasing marginal costs, and the provider will gradually reduce quality until the steady-state is reached. As a result, we obtain a negative relationship between (actual) demand and quality. This result might have implications for empirical studies. Unless sufficient care is taken to account for dynamic adjustment over time, this kind of equilibrium dynamics could potentially lead to spurious relationships between quality and demand.

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[5]In dynamic capital accumulation games the closed-loop solution is typically more competitive (see Dockner, 1992; Dockner, Jørgensen, Van Long and Sorger, 2000). In these models, providers compete a la Cournot but face capacity constraints that can be relaxed by capital accumulation through investments. It turns out that investments under the closed-loop solution is *higher* than under open-loop.

[6]This result has clear parallels to price competition, where a repeated game might result in more collusive outcomes than one-shot games.

Finally, we briefly consider welfare and policy implications. First, we show that first-best quality always can be attained. This is not surprising since we have one instrument (the price) and, due to symmetry, one variable (quality) to manipulate. Second, we show that more high-powered incentives (i.e., higher regulated prices) are required if providers use dynamic rather than static decision rules. Finally, we point out that demand sluggishness might be affected by regulatory policy as well. The regulator might spend resources on publishing quality indicators of the providers. If this is the case, then reducing demand sluggishness is a policy substitute to providing high-powered incentives.

We believe our analysis is relevant for several regulated industries. A prime example is health care. In this industry prices are either set by the insurer (government or private insurer) or settled in negotiations with the providers (hospitals and physicians). Consumers (patients) are insured against medical expenditures, so non-price measures like quality and distance are more relevant for provider choice than price.[7] Since health care providers typically receive payments per patient (or per treatment), they might find it profitable to improve quality to attract (or avoid losing) patients and, in turn, increase revenues.[8]

Another example is the market for (especially higher) education. In most European countries, tuition fees play a negligible role, and funding of educational institutions is to a large extent based on student attendances.[9] A student's choice of school or university is typically based on the quality of the institution, as well as the institution's location (geographically and/or in product space). As for hospitals, universities might find it profitable to invest in quality (new facilities, better laboratories, hiring of top researchers, etc.) in order to attract more students, thereby increasing revenues.[10]

In both health care and education, quality is a major concern. In recent years, most Eu-

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[7] The empirical studies by Kessler and McClellan (2000) and Tay (2003) show that distance and quality are the main predictors of hospital choice. These papers also assess the relationship between competition and quality in the US Medicare hospital market.

[8] Related theoretical studies on competition in health care are, for instance, Calem and Rizzo (1995); Lyon (1999); Gravelle (2000); Gravelle and Masiero (2000); Beitia (2003); Nuscheler (2003); Brekke, Nuscheler and Straume (2006, 2007); and Karlsson (2007).

[9] See Kaiser, Raymond, Koelman and van Vught (1992) for an overview.

[10] For related theoretical studies in education, see papers by Del Rey (2001), De Fraja and Ioassa (2001), and De Fraja and Landeras (2006). For empirical studies on competition and quality in education, see e.g., Dee (1998), Epple and Romano (1998) and Hoxby (2000).

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ropean countries have implemented marked-based reforms exposing providers to competition. In particular, the introduction of provider choice and activity-based payments are aimed at stimulating competition and in turn quality. In both sectors, governments spend resources on collecting information on quality indicators and publishing scores and rankings of institutions (e.g., league tables of hospitals, universities, schools, etc.). Obviously, the purpose of this activity is to make demand more responsive to quality differences. Our purpose is to contribute to the understanding of the impact of competition on quality in regulated markets characterised by demand sluggishness.

The rest of the paper is organised as follows. In section 2, we present the model framework. In Section 3, we briefly present – as a benchmark for comparison – the equilibrium of an equivalent static model, before we derive and characterise the equilibrium quality under the open-loop solution (Section 4) and the feedback closed-loop solution (Section 5). In Section 6 we briefly mention some welfare and policy implications, while Section 7 concludes the paper.

2   Model

In line with previous literature on quality competition in regulated markets, we conduct the analysis within a Hotelling framework (Hotelling, 1929). Consider a market with two providers located (exogenously) at either end of the unit line $S = [0,1]$.[11] On this line segment there is a uniform distribution of individuals, with total mass normalised to 1. We assume unit demand, where each individual demands one unit of output. The utility of an individual who is located at $x \in S$ and chooses provider i, located at z_i, is given by

$$U\left(x, z_i\right) = v + kq_i - \tau\left|x - z_i\right|, \tag{1}$$

where v is the gross valuation from consumption, $q_i \geq \underline{q}$ is the quality at provider i, k is a parameter measuring the (marginal) utility of quality, and τ is a transportation cost parameter.

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[11] S is typically interpreted either as a geographical space or a product (taste) space. Note that we assume exogenous locations, with distance equal to one.

The lower bound $\underline{q}$ on quality represents the minimum quality providers are allowed to offer.[12] For simplicity, we set $\underline{q} = 0$. Moreover, we normalise the marginal utility of quality to one, i.e., $k = 1$, without loss of generality. This implies that τ can be interpreted as the marginal disutility of travelling *relative* to quality. Thus, a low (high) τ means that quality is of relatively more (less) importance to the patient than travelling distance.[13]

Since the distance between providers is equal to one (exogenously fixed), the individual who is indifferent between provider i and provider j is located at D^*, given by

$$v - \tau D^* + q_i = v - \tau \left(1 - D^*\right) + q_j, \tag{2}$$

yielding the *notional* (or *potential) demand* for provider i,

$$D^* = \frac{1}{2} + \frac{q_i - q_j}{2\tau}, \tag{3}$$

implying that the provider with a higher quality has a potential demand in excess of $1/2$. Notice how lower transportation costs make it less costly for consumers to switch between providers, increasing the demand responsiveness to changes in quality.

In the existing literature, it is typically assumed that demand responds instantaneously to quality changes. This is obviously a simplifying assumption. Demand is generally sluggish. If a provider increases quality, sluggish demand responses imply that it will take some time before the potential demand increase is fully realised. Such demand sluggishness can typically arise from imperfect information on the demand side, which is particularly relevant in markets where quality is the main competition variable. While prices usually are easily and immediately observable, quality is often much more difficult to measure and thus less readily observable. For example, assume that, at each point in time, only a proportion $\gamma \in [0, 1]$ of consumers become aware of quality changes in the market. This would imply that, at each point in time, only a

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[12]We can think of $\underline{q}$ as the minimum quality level set by a regulator and/or defined through legislation. If $q < \underline{q}$, the provider might be sued or lose his licence. In health care, we can think of $q < \underline{q}$ as malpractice or failure to meet licence standards.

[13]In the context of hospital competition, there is strong empirical evidence showing that distance and quality are main predictors of patients' choice of hospital, see, e.g., Kessler and McClellan (2000) and Tay (2003).

fraction γ of any potential change in demand is realised. A different set of reasons why demand is sluggish has to do with personal or familiar habits in fields like education or health: people trust in one specific provider, for personal or familiar considerations, apart from the objective quality of the service; sticky behaviour, and in some cases even addiction to a specific provider, lead to sluggish demand.

Define $D(t)$ as the *actual demand* of provider i at time t (as opposed to *potential demand* $D^*(t)$). Analytically, the law of motion of actual demand is given by

$$\frac{dD(t)}{dt} \equiv \dot{D}(t) = \gamma(D^*(t) - D(t)), \tag{4}$$

where $\gamma \in [0, 1]$ is an inverse measure of demand sluggishness. The higher is γ, the less sluggish is demand. If $\gamma = 0$, the demand facing each provider is completely inelastic, as actual demand does not respond to quality changes, while, if $\gamma = 1$, potential demand changes are immediately and fully realised. Such a specification is widely used in theoretical IO models to describe market price stickiness (see, e.g., Fershtman and Kamien, 1987; Cellini and Lambertini 2007; Dockner et al., 2000, for literature reviews).

Since total demand is inelastic, notice that both providers face the same dynamic constraint, given by (4). To see this, notice that *actual demand* for provider j at time t is given by $1 - D(t)$ (as opposed to *potential demand* $1 - D^*(t)$). Analytically, the law of motion of actual demand for provider j is then given by

$$\frac{d[1 - D(t)]}{dt} = \gamma \left[(1 - D^*(t)) - (1 - D(t))\right], \tag{5}$$

which can easily be rewritten as (4). Thus, the dynamics of the demand for provider i automatically determines the demand for provider j.

We assume that providers maximise profits. The instantaneous objective function of provider i is assumed to be given by

$$\pi_i(t) = T + pD(t) - C(D(t), q_i(t)) - F, \tag{6}$$

where p is a regulated price per unit of output provided (for example, a treatment or a patient in the context of health care markets; a student in the context of education markets).[14] T is a potential lump-sum transfer (or a fixed grant/budget) received from a third-party payer.

On the cost-side, each provider i faces a fixed cost F and variable cost $C(\cdot)$ that depends on the quality q_i and the actual demand D. We assume that $C(\cdot)$ is increasing and convex in both quality and output: $C_{q_i} > 0$, $C_{q_i q_i} > 0$, $C_D > 0$ and $C_{DD} > 0$. We also make the simplifying assumption that the cost function is separable in quality and output: $C_{Dq_i} = 0$.[15]

Defining ρ as the (constant) preference discount rate, the provider's objective function over the infinite time horizon is

$$\int_0^{+\infty} \pi_i(t) e^{-\rho t} dt.$$

In reality, providers may not have an infinite-time horizon, but may have reasonably long finite horizons. If the optimal path does not differ significantly from the solution with a very large but finite horizon, the convenience of working with an infinite-horizon model may be worth the loss of realism (see Léonard and van Long, 1992, p. 285). Also, when decision-makers retire, they may well be replaced by other decision-makers with similar utility functions, thus generating an infinite-time horizon.

In this type of dynamic models with strategic interactions – i.e., differential games – there are two main solution concepts: a) *open-loop solution*, where each provider knows the initial state of the system and then nothing else, i.e., each provider knows the initial quality (and thus potential demand) of the other provider, but not in the following periods; b) *closed-loop solution*, where each provider knows the initial state of the system, but also later knows the state variable values, i.e., each provider knows the quality of the other provider, not only in the initial state, but also in all of the subsequent periods. Within the closed-loop solutions, further distinctions can be made: if one assumes that players take into account only the initial

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[14] As long as prices are fixed, whether payments are collected directly from the consumers (as for public utilities) or from a third-party payer (which is more relevant for health care and, to a certain extent, education markets) is immaterial for our results.

[15] The assumption of cost separability between quality and quantity is widely used in the related literature (see, e.g., Economides, 1989, 1993; Calem and Rizzo, 1995; Lyon, 1999; Gravelle and Masiero, 2000; Nuscheler, 2003; Brekke, Nuscheler and Straume, 2006, 2007). Relaxing the cost separability assumption should not qualitatively affect our results as long as $C_{DD} > |C_{Dq_i}|$.

state and the current state, the 'memoryless' closed-loop solution is obtained; if players take into account the whole history of states, the 'perfect state' closed-loop is obtained; finally, if players in each instant take into account the current value of states (i.e., the whole past history is summarised by the current value of states), the feedback rule is obtained. If the rule (i.e., the function) connecting the choice variable to the states is stable over time, the strategy is said to be Markovian. Typically, the feedback closed-loop Markovian solution is obtained based on the Bellman equation.

In order to establish which is the most appropriate solution concept, it is essential to evaluate the relevant information set used by players when they take their decisions. In cases where collection of information over time is difficult, it is reasonable to model the choice according to the open-loop rule;[16] on the contrary, when players can observe the current state of the world and they behave accordingly, the closed-loop rules are more appropriate. Clearly, closed-loop solutions are more appealing, but solving for closed-loop is more difficult. However, in some cases – and health care markets can be a good example – players might have to commit to investment plans and stick to them for long periods of time. In this case, the open-loop solution might be the relevant one. Nevertheless, there is a wide range of problems where the two solutions coincide.[17] Below, we compare the closed-loop and open-loop solutions. After briefly presenting the benchmark static analysis in Section 3, Section 4 provides the open-loop solution, while Section 5 provides the closed-loop one, under the specific rule of feedback behaviour.

3 Benchmark: Static analysis

As a benchmark for comparison, let us briefly derive the Nash equilibrium outcome of a static version of the model. Using the demand function in (3), the profits of provider i are given by

$$\pi_i = T + p\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau}\right) - C\left(\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau}\right), q_i\right) - F. \tag{7}$$

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[16] One example from the education sector could be the Research Assessment Exercise in the UK, that produces quality profiles of higher education institutions every 8 years. Arguably, with a time span of this length, quality becomes observable only quite rarely.

[17] Games where this coincidence arises are presented in Clemhout and Wan (1974); Reinganum (1982); Mehlmann and Willing (1983); Dockner, Feichtinger and Jørgensen (1985). See also Mehlmann (1988), Fershtman, Kamien and Muller (1992), Dockner, Jørgensen, Van Long, Sorger (2000, ch. 7) for review.

The first-order condition for a profit-maximising choice of quality is given by

$$\frac{p - C_D}{2\tau} - C_{q_i} = 0. \tag{8}$$

Suppose that the cost function is quadratic in quality and output:

$$C(D, q_i) = \frac{\theta}{2}q_i^2 + \frac{\beta}{2}D^2. \tag{9}$$

The first-order condition then becomes,

$$\frac{p - \beta\left(\frac{1}{2} + \frac{q_i - q_j}{2\tau}\right)}{2\tau} - \theta q_i = 0, \tag{10}$$

which, in the symmetric equilibrium, yields

$$q^{SA} = \frac{p - \frac{\beta}{2}}{2\tau\theta}. \tag{11}$$

The comparative statics properties of (11) are intuitive, and well known from the theoretical literature: equilibrium quality is increasing in the price, p, and decreasing in the transportation cost parameter, τ. This implies that increased competition, which is typically modelled as a reduction of transportation costs, will increase the supply of quality in equilibrium. Obviously, equilibrium quality is also decreasing in the cost parameters β and θ.

4 Open-loop solution

Consider now the dynamic version of the model, where the providers use open-loop decision rules. Provider i's maximisation problem is given by

$$\text{Maximise} \int_0^{+\infty} \pi_i(t) e^{-\rho t} dt,$$

$$\text{subject to} \quad \dot{D}(t) \quad = \quad \gamma(D^*(t) - D(t)), \tag{12}$$

$$D(0) \quad = \quad D_0 > 0. \tag{13}$$

Let $\mu_i(t)$ be the current value co-state variable associated with the state equation. The current-value Hamiltonian is:[18]

$$H_i = T + pD - C(D, q_i) - F + \mu_i \gamma \left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D \right), \tag{14}$$

The solution is given by (a) $\partial H_i / \partial q_i = 0$, (b) $\dot{\mu}_i = \rho \mu_i - \partial H_i / \partial D$, (c) $\dot{D} = \partial H_i / \partial \mu_i$, or more extensively:

$$\frac{\gamma}{2\tau} \mu_i \quad = \quad C_{q_i}, \tag{15}$$

$$\dot{\mu}_i \quad = \quad \mu_i (\rho + \gamma) - (p - C_D), \tag{16}$$

$$\dot{D} \quad = \quad \gamma \left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D \right), \tag{17}$$

to be considered along with the transversality condition $\lim_{t \to +\infty} e^{-\rho t} \mu_i(t) D(t) = 0$. The second order conditions are satisfied if the Hamiltonian is concave in the control and state variables (Léonard and Van Long, 1992).[19]

Totally differentiating (15) with respect to time we obtain $\frac{\gamma}{2\tau} \dot{\mu}_i = C_{q_i q_i} \dot{q}_i$, or, after substitution, $\frac{\gamma}{2\tau} (\mu_i (\rho + \gamma) - (p - C_D)) = C_{q_i q_i} \dot{q}_i$. Using $\mu_i = C_{q_i} \frac{2\tau}{\gamma}$, we obtain

$$\dot{q}_i = \frac{C_{q_i} (\rho + \gamma) - \frac{\gamma}{2\tau} (p - C_D)}{C_{q_i q_i}}, \tag{18}$$

which, together with (17), describe the dynamics of the equilibrium.

As to possible steady state, setting $\dot{q}_i = 0$ and totally differentiating yields

$$\frac{\partial D}{\partial q_i} \Big|_{\dot{q}_i = 0} = -\frac{C_{q_i q_i} (\rho + \gamma) 2\tau}{\gamma C_{DD}} < 0. \tag{19}$$

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[18] The indication of time (t) is omitted, to ease notation.

[19] This is the case since (a) $H_{q_i q_i} = -C_{q_i q_i} < 0$; (b) $H_{DD} = -C_{DD} < 0$; (c) $H_{DD} H_{q_i q_i} > (H_{D q_i})^2$ or $C_{DD} C_{q_i q_i} > 0$.

The locus of quality, $\dot{q}_i = 0$, is negatively sloped. The second locus, assuming symmetry, is $\dot{D} = 0$, or $D = 1/2$. In the steady state each provider has half of the market since the equilibrium is symmetric.

The dynamics of investment and quality can be represented in matrix form as follows (assuming symmetry and third order derivatives equal to zero):

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\begin{bmatrix} \dot{q}(t) \\ \dot{D}(t) \end{bmatrix} = \begin{bmatrix} (\rho + \gamma) & \frac{\frac{\gamma}{2\tau} C_{DD}}{C_{qq}} \\ 0 & -\gamma \end{bmatrix} \begin{bmatrix} q(t) \\ D(t) \end{bmatrix} + \begin{bmatrix} \frac{-\frac{\gamma}{2\tau} p}{C_{qq}} \\ \frac{\gamma}{2} \end{bmatrix},
\tag{20}
$$

where the 2-by-2 matrix is the Jacobian J of the dynamic system. As for the dynamic properties of the system, suppose that this is evaluated around the steady state. Then, it is immediate to check that the Jacobian matrix J in (20) is such that $tr(J) = \rho > 0$, and $\det(J) = -\gamma (\rho + \gamma) < 0$, implying that the equilibrium is stable in the saddle sense. The solution is described in Figure 1.

[Figure 1 about here]

Let $D^s = 1/2$ be the steady state level of demand. Suppose we start off steady state at a level where the initial demand is low: $D(0) < D^s$. One possible interpretation is the case of a provider who at time 0 enters a previously monopolistic market. The solution is then characterised by a period of increasing demand and decreasing quality. Notice that the optimal solution for the 'incumbent' is precisely the opposite and it is equivalent to the case where the demand is high $(D(0) < 1/2 \iff 1 - D(0) > 1/2)$. For this provider, we should observe a period of decreasing demand and increasing quality. These dynamic patterns establish our first main result:

Proposition 1 *On the equilibrium path to the steady state, demand and quality move in opposite directions over time if the marginal cost of provision is increasing.*

In the next section, we will show that the above result holds also when the players use closed-loop decision rules. Notice that the result holds only if the cost function is strictly convex in output.

To grasp the intuition behind Proposition 1, it is useful to consider, as a benchmark for

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comparison, the special case of constant marginal cost of output, $C_{DD} = 0$, implying that the quality locus is horizontal at the steady state level of quality, q^s. In this case, if $D(0) \neq 1/2$, the two providers will immediately set their qualities at the steady state level, q^s, and maintain this quality level at all times. The demand dynamics, (17), will then eventually bring demand to the steady state level, $D^s = 1/2$, with the speed of adjustment depending of the degree of demand sluggishness. The reason is that, with constant marginal cost of output (and fixed prices), marginal profits $(\partial \pi_i / \partial q_i)$ are independent of output. Thus, the profit-maximising choice of quality is q^s irrespective of demand, and each provider will therefore keep quality at this level at each point in time.

On the other hand, when the cost function is strictly convex in output, $C_{DD} > 0$, marginal profits depend on actual demand. More specifically, for a given level of quality, the marginal profit gain of higher quality is monotonically decreasing in the actual demand facing the provider, since new consumers are increasingly costly to serve. Thus, if a provider faces actual demand $D < D^s$, he will set quality $q > q^s$. As demand increases along the equilibrium dynamic path, the marginal profit gain of quality decreases; consequently, the provider will gradually reduce quality until the steady state level is reached. Obviously, the inverse logic applies for $D > D^s$.

We believe that the result in Proposition 1 has potential implications for empirical analyses of the effect of quality on demand and, in turn, of the relationship between competition and quality. In addition to the opposite movement of quality and demand over time, notice that, at a given point in time, a comparison of the two providers – off the steady state – unambiguously predicts a negative relationship between quality and demand. Thus, it is tempting to speculate that this type of equilibrium dynamics could potentially lead to spurious relationships between quality and demand in empirical studies, unless sufficient care is taken to account for dynamic adjustments over time.

Let us also briefly consider some comparative dynamics in the open-loop solution. Suppose the system is in an initial steady state. Figure 2 shows the effect of an unexpected increase in price or reduction in transportation costs (more competition). The locus $\dot{q}_i = 0$ shifts upwards, generating a positive jump in quality. Notice that there is no overshooting. The quality jumps

to the new steady state value; analytically, this is because the locus $\dot{D} = 0$ is a vertical line, rather than positively/negatively sloped.

[Figure 2 about here]

4.1 Example

Suppose that the cost function is quadratic in quality and demand and given by (9), as in the static analysis benchmark. With this specification, the solution of the Hamiltonian system leads to

$$\dot{q}_i = \frac{\theta q_i \left(\rho + \gamma\right) - \frac{\gamma}{2\tau}\left(p - \beta D\right)}{\theta} \tag{21}$$

In the steady state we have $\dot{q}_i = 0$, which, combined with $D^s = 1/2$, yields

$$q^{OL} = \left(\frac{1}{1 + \frac{\rho}{\gamma}}\right)\left(\frac{p - \frac{\beta}{2}}{2\tau\theta}\right). \tag{22}$$

The results are reasonable and intuitive. If the price is above the marginal cost, then lower transportation costs (τ) or a higher price (p) increase quality. Similarly, a higher marginal cost of quality (β), a higher marginal cost of provision (θ) or a higher time preference discount rate (ρ) reduce quality. Steady state quality is also decreasing in the degree of demand sluggishness (measured by γ^{-1}). Comparing with the equilibrium quality level in the static analysis, (11), we also see that the two solutions coincide if $\rho \to 0$, as expected. For $\rho > 0$, though, the dynamic open-loop solution yields a lower level of steady state quality. Notice also that this difference in equilibrium qualities is increasing with the degree of demand sluggishness.

5 Closed-loop solution

We might intuitively expect that quality under the closed-loop solution will be higher than under the open-loop solution because competition is more intense when the players use closed-loop decision rules. However, this is not the case, as the following will show.

Assuming that the cost function is given by the quadratic form in (9), the dynamic decision

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rules in the closed-loop solution are given by:[20],[21]

$$q_i = \phi_i(D) = \frac{\gamma}{2\tau\theta}\left(\alpha_1 + \alpha_2 D\right) \tag{23}$$

and

$$q_j = \phi_j(D) = \frac{\gamma}{2\tau\theta}\left(\alpha_1 + \alpha_2(1-D)\right), \tag{24}$$

where[22]

$$\alpha_1 = \frac{p + \frac{\gamma\alpha_2}{2}\left(1 - \frac{\gamma\alpha_2}{2\theta\tau^2}\right)}{\gamma + \rho - \frac{\gamma^2}{4\theta\tau^2}\alpha_2} > 0, \tag{25}$$

$$\alpha_2 = -\frac{2\theta\tau^2\left(\psi - 2\gamma - \rho\right)}{3\gamma^2} < 0, \tag{26}$$

and

$$\psi := \sqrt{(2\gamma + \rho)^2 + \frac{3\beta\gamma^2}{\theta\tau^2}}. \tag{27}$$

The quality difference at each point in time is thus given by

$$q_i - q_j = \frac{\gamma\alpha_2}{\theta\tau}\left(D - \frac{1}{2}\right). \tag{28}$$

The first observation we want to highlight is the negative sign of α_2, implying a negative relationship between demand and quality over time along the equilibrium dynamic path. Thus, as previously mentioned, the result reported in Proposition 1 carries over to closed-loop case, and the intuition is equivalent to the one given in the previous section, for the open-loop case. Once more, notice that this result holds only when the cost function is strictly convex in output, as $\alpha_2 = 0$ if $\beta = 0$.

Applying the steady state condition $D^s = 1/2$ to (23)-(24), steady state quality in the closed-loop solution is equal to

$$q^{CL} = \left(\frac{1}{1 + \frac{\rho}{\gamma} - \frac{\gamma\alpha_2}{4\theta\tau^2}}\right)\left(\frac{p - \frac{\beta}{2}}{2\tau\theta}\right). \tag{29}$$

---

[20] To ease notation, we continue the practice of dropping time indications.

[21] The full derivation of the closed-loop solution is given in the Appendix.

[22] The positive sign of α_1 is explicitly confirmed in the Appendix.

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Comparing the steady state equilibria of the open- and closed-loop solutions, we see that, if marginal production costs are constant ($\beta = 0$), implying $\alpha_2 = 0$, the open- and closed-loop solutions coincide: $q^{CL} = q^{OL}$. Otherwise, if the cost function is strictly increasing in output ($\beta > 0$), implying $\alpha_2 < 0$, steady state quality is lower in the closed-loop solution. The reason for the coincidence result is related to the previously discussed implication of constant marginal production costs, namely that the profit margin becomes independent of output. This implies an absence of strategic interaction between the two players that causes the two solution concepts to coincide. From (23)-(24), it is straightforward to verify that the optimal dynamic decision rules imply that each player sets quality at the steady state level at every point in time, irrespective of the quality chosen by the other player.

However, with increasing marginal production costs ($\beta > 0$), a quality increase by provider i will increase the profit margin of provider j, making qualities strategic complements.[23] Under closed-loop feedback behaviour, this creates a dynamic incentive for each provider to reduce quality in order to induce future quality reductions – a strategic response – from the competing provider. Thus, the closed-loop solution yields a less competitive outcome in the steady state.

The comparative statics properties of the closed-loop solution is qualitatively similar to the open-loop case. It is relatively straightforward to show[24] that more competition – measured either by less demand sluggishness or lower transportation costs – will increase steady state quality. However, the strength of the quality responses to increased competition differ. We can measure the relative quality response to an increase in competition by the following elasticities:

$$\eta_\gamma := \left| \frac{\partial q}{\partial \gamma} \frac{\gamma}{q} \right|, \tag{30}$$

$$\eta_\tau := \left| \frac{\partial q}{\partial \tau} \frac{\tau}{q} \right|. \tag{31}$$

---

[23] This can easily be verified by inserting the expression for D into (23)-(24), establishing a positive relationship between q_i and q_j (when $\alpha_2 < 0$) at each point in time in the optimal dynamic decision rules.

[24] See the Appendix for the details of the calculations.

Inserting the steady state levels of quality from the two solutions yields

$$\eta_\gamma^{OL} = \frac{\rho}{\rho + \gamma}; \quad \eta_\gamma^{CL} = \frac{\rho\,(2\gamma + \rho + 5\psi)}{\psi\,(4\gamma + 5\rho + \psi)}; \tag{32}$$

$$\eta_\tau^{OL} = 1; \quad \eta_\tau^{CL} = \frac{(2\gamma + \rho)^2 + (4\gamma + 5\rho)\,\psi}{\psi\,(4\gamma + 5\rho + \psi)}. \tag{33}$$

It is fairly straightforward to verify that $\eta_\gamma^{OL} > \eta_\gamma^{CL}$ and $\eta_\tau^{OL} > \eta_\tau^{CL}$ for all parameter configurations. Thus, an increase in competition – either through less sluggish demand or lower transportation costs – will have a stronger (weaker) impact on quality if the players use open-loop (closed-loop) decision rules.

The following Proposition summarises the most important steady state characteristics of the open- and closed-loop solutions:

Proposition 2 *(i) If the marginal cost of provision is constant, then*

$$q^{SA} > q^{OL} = q^{CL}.$$

(ii) If the marginal cost of provision is increasing, then

$$q^{SA} > q^{OL} > q^{CL}.$$

(iii) Less sluggish demand and/or lower transportation costs will increase the steady state level of quality under both solution concepts:

$$\frac{\partial q^{OL}}{\partial \gamma} > 0; \quad \frac{\partial q^{CL}}{\partial \gamma} > 0; \quad \frac{\partial q^{OL}}{\partial \tau} < 0; \quad \frac{\partial q^{CL}}{\partial \tau} < 0.$$

(iv) The positive impact of increased competition on quality is weaker in the closed-loop equilibrium:

$$\eta_\gamma^{OL} > \eta_\gamma^{CL}; \quad \eta_\tau^{OL} > \eta_\tau^{CL}.$$

6 Welfare and policy implications

First-best quality in the steady state is derived by maximising (instantaneous) aggregate consumer utility net of quality and provision costs. With the quadratic cost function, (9), the maximisation problem is given by

$$
\max_{q_i,q_j} \quad \int_0^{\frac{1}{2}+\frac{q_i-q_j}{2\tau}} (v + q_i - \tau x)\, dx + \int_{\frac{1}{2}+\frac{q_i-q_j}{2\tau}}^1 (v + q_j - \tau(1-x))\, dx
$$
$$
- \sum_{k=1}^2 \frac{\theta}{2} q_k^2 + \frac{\beta}{2}\left(\frac{1}{2} + \frac{q_i-q_j}{2\tau}\right)^2 + \frac{\beta}{2}\left(\frac{1}{2} + \frac{q_j-q_i}{2\tau}\right)^2,
$$

yielding

$$
q_i = q_j = q^{FB} = \frac{1}{2\theta}, \tag{34}
$$

which implies $D = 1/2$.

With competition along only one dimension, namely quality, the first-best steady state level of quality can always be implemented by appropriate choice of the regulated price, p. Since equilibrium quality is monotonically increasing in the price, under all solution concepts, the optimal price in the steady state is such that

$$
p^{SA} = \frac{\beta}{2} + \tau < p^{OL} = p^{SA} + \frac{\tau\rho}{\gamma} < p^{CL} = p^{OL} - \frac{\alpha_2 \gamma}{4\theta\tau}. \tag{35}
$$

Thus, if players use dynamic decision rules of the closed-loop type, more high-powered incentives, in the form of higher regulated prices, are necessary to induce first-best quality in the steady state.

A policy maker could also take measures to reduce demand sluggishness, for example by developing and publishing frequently updated quality indicators that will increase consumers' awareness of quality differences in the market.[25] Notice that this is a policy substitute to high-powered incentive schemes (i.e., high prices). The less sluggish demand is (i.e., the higher γ is), the lower is the optimal price.[26]

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[25] The publication of hospital and school 'League Tables' in the UK are examples of such policy measures.

[26] From (35) we immediately see that p^{OL} is decreasing in γ. It is relatively straightforward to show that the

7 Concluding remarks

In this paper, we have analysed the impact of competition on quality in a market with regulated prices and sluggish demand. The basic model is the widely used Hotelling model where products are horizontally and vertically differentiated. We have considered the case in which the spatial locations are exogenously fixed, while firms choose quality. We first derive the familiar static quality equilibrium as a benchmark. We then extend the model to a dynamic game, where demand responds to quality changes with some degree of sluggishness, implying a divergence between actual and potential demand (out of steady state). We would like to stress that our assumptions fit quite well with the features of markets with regulated price - let us think of education or health: the spatial locations of providers are given; competition among providers is based mainly on the product quality; prices play a limited role in the competitive process; the consumer behavior is characterised by a certain degree of stickiness.

Using a differential game approach, we have derived the open-loop and the closed-loop solutions. In the open-loop solution, each provider knows the quality of the competitor in the initial state, and chooses the time path of quality efforts at the beginning, and then stick to this plan for the whole length of the game. In the closed-loop solution, each provider knows the quality of the competitor, not only in the initial state, but also in all subsequent periods, and thus can choose the quality effort in each point of time, possibly responding to quality changes by the competitor. Specifically, we have found the feedback closed-loop Markovian solution, in which the current choice of each player depends on the current value of the state variables.

The analysis has provided three main findings. First, we showed that if marginal costs are increasing, there is a negative relationship between quality and demand off the steady state, which is contrary to the static relationship. The reason is that the marginal profit gain is decreasing in quality. Second, we found that both the open-loop and the closed-loop steady-state level of quality are lower than the quality level in the static equilibrium. The reason is that firms take into account the impact on future profits, and thus compete less aggressively in a dynamic setting compared to a one-shot game. Third, we showed that if marginal costs are increasing,

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same is true for p^{CL}.

then the closed-loop solution results in lower quality than the open-loop solution. If marginal costs are constant, the two solutions coincide. The reason is that under increasing marginal costs, quality choices are strategic complements, while under constant marginal costs, they are strategically independent. Thus, when firms can observe (and respond to) the competitors' quality at any time period, and quality choices are strategic complements, competition will be less intensive.

Based on the dynamic outcomes, we briefly discussed welfare and policy implications. We showed that the regulator needs to provide more high-powered incentives (higher prices) when firms use dynamic rather than static optimisation rules. More interestingly, forcing firms to stick to time plans (i.e., to adopt open-loop rules in terms of differential game theory) requires smaller incentive efforts to reach first-best quality, as compared to the situation in which firms can set their choice in each point of time, observing the state of the world. Moreover, if demand sluggishness could be affected by the regulator, for instance, by public disclosure of quality indicators, then this would be a policy substitute to high-powered incentives.

We find this analysis relevant for several regulated industries, especially health care and education. In these markets, quality is a major concern, and prices are less crucial for choice of provider. Many European governments have introduced (elements of) competition in health care and education in order to stimulate quality. In the US competition has been in place for many years. Recently, we have seen a trend in both the US and in Europe towards publishing quality rankings (league tables) of hospitals, universities, schools, etc. Obviously, this is done to stimulate demand responses to quality differences. The purpose of our paper has been to analyse the impact of competition on quality in regulated markets when demand is not responding instantaneously to quality differences. Hopefully, our analysis can shed some light on the recent reforms in health care and education.

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Appendix. The closed-loop solution

With the quadratic cost function given by (9), provider i's instantaneous objective function is

$$T + pD - \frac{\theta}{2}q_i^2 - \frac{\beta}{2}D^2 \tag{A1}$$

in which time index is suppressed to ease notation. Eq.(A1), together with the linear dynamic constraint, (4), gives rise to a linear-quadratic problem. Hence, we define the value function of provider i as

$$V^i(D) = \alpha_0 + \alpha_1 D + (\alpha_2/2)D^2, \tag{A2}$$

implying

$$V_D^i(D) = \alpha_1 + \alpha_2 D. \tag{A3}$$

Notice that $\alpha_2 < 0$ is required to ensure concavity of the value function.

The optimal investment strategies are functions of actual demand at each point in time. Thus, we define $q_i = \phi_i(D)$ and $q_j = \phi_j(D)$. We are focusing on Markovian strategies.[27] The value function has to satisfy the Hamilton-Jacobi-Bellman (HJB) equation, which, for provider i, is given by

$$\rho V^i(D) = \max \left\{ T + pD - \frac{\theta}{2}q_i^2 - \frac{\beta}{2}D^2 + V_D^i \gamma \left(\frac{1}{2} + \frac{q_i - q_j}{2\tau} - D \right) \right\}. \tag{A4}$$

Maximisation of the right-hand-side yields $-\theta q_i + V_D^i \frac{\gamma}{2\tau} = 0$, which, after substitution of V_D^i from (A3), yields

$$q_i = \phi_i(D) = \frac{\gamma}{2\tau\theta}\left(\alpha_1 + \alpha_2 D\right). \tag{A5}$$

By symmetry, the optimal investment strategy for provider j is given by

$$q_j = \phi_j(D) = \frac{\gamma}{2\tau\theta}\left(\alpha_1 + \alpha_2(1 - D)\right), \tag{A6}$$

---

[27]The strategy is said to be Markovian since it does not change over time, i.e., it is $q_i(t) = \phi_i(D(t))$. A non-Markovian strategy would appear as $q_i(t) = \phi_i(D(t), t)$.

implying that the quality difference at time t is given by

$$q_i - q_j = \frac{\gamma \alpha_2}{\theta \tau} \left(D - \frac{1}{2} \right). \tag{A7}$$

Notice that quality of provider i is higher than quality of provider j if demand is lower than half of the market (assuming $\alpha_2 < 0$).

Substituting $q_i = \phi_i(D)$, $q_j = \phi_j(D)$ and $V_D^i(D) = \alpha_1 + \alpha_2 D$ into (A4), we obtain

$$\rho V^i(D) = \left\{ \begin{array}{l} T + pD - \frac{\theta}{2} \frac{\gamma^2}{(2\tau\theta)^2} \left(\alpha_1 + \alpha_2 D \right)^2 - \frac{\beta}{2} D^2 \\ + \left(\alpha_1 + \alpha_2 D \right) \gamma \left(\frac{1}{2} + \frac{\gamma \alpha_2}{2\tau^2 \theta} \left(D - \frac{1}{2} \right) - D \right) \end{array} \right\}. \tag{A8}$$

For the above equality to hold, the parameters must satisfy the following equations:

$$\rho \alpha_0 - \frac{\gamma \alpha_1}{2} - T + \frac{\gamma^2 \alpha_1^2}{8\theta\tau^2} + \frac{\gamma^2 \alpha_1 \alpha_2}{4\theta\tau^2} = 0, \tag{A9}$$

$$\left(\gamma \alpha_1 - p - \frac{\gamma \alpha_2}{2} + \rho \alpha_1 + \frac{\gamma^2 \alpha_2^2}{4\theta\tau^2} - \frac{\gamma^2 \alpha_1 \alpha_2}{4\theta\tau^2} \right) D = 0, \tag{A10}$$

$$\left(\frac{\beta}{2} + \gamma \alpha_2 + \frac{\rho \alpha_2}{2} - \frac{3\gamma^2 \alpha_2^2}{8\theta\tau^2} \right) D^2 = 0. \tag{A11}$$

From (A11), solving for α_2, we obtain two candidate solutions:

$$\alpha_2 = \frac{2\tau\theta\tau^2}{3\gamma^2} \left(2\gamma + \rho \pm \sqrt{(2\gamma + \rho)^2 + \frac{3\beta\gamma^2}{\theta\tau^2}} \right). \tag{A12}$$

The condition that the value function be concave leads us to select the negative root. From (A10) we have

$$\alpha_1 = \frac{p + \frac{\gamma \alpha_2}{2} \left(1 - \frac{\gamma \alpha_2}{2\theta\tau^2} \right)}{\gamma + \rho - \frac{\gamma^2}{4\theta\tau^2} \alpha_2}. \tag{A13}$$

In order to establish the sign of α_1, notice that the numerator in (A13) is monotonically increasing in α_2.[28] Furthermore, it is straightforward to verify that $\frac{\partial \alpha_2}{\partial \rho} > 0$ and $\frac{\partial \alpha_2}{\partial \beta} < 0$. An interior solution requires that price is higher than marginal production costs. Thus, α_2 approaches its lowest permissible value if $\rho \to 0$ and $\beta \to 2p$. In this case, $\alpha_2 = 2\theta\tau^2 \frac{2 - \sqrt{4 + \frac{6p}{\theta\tau^2}}}{3\gamma}$, and the numer-

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[28] $\frac{\partial \left(p + \frac{\gamma \alpha_2}{2} \left(1 - \frac{\gamma \alpha_2}{2\theta\tau^2} \right) \right)}{\partial \alpha_2} = \frac{1}{2}\gamma \frac{\theta\tau^2 - \gamma \alpha_2}{\theta\tau^2} > 0.$

ator of (A13) is given by $\frac{1}{9}\left(3p + \theta\tau^2\left(\sqrt{2}\sqrt{2 + \frac{3p}{\theta\tau^2}} - 2\right)\right)$, which is unambiguously positive. Thus, we conclude that α_1 is positive for all permissible parameter configurations.

In the steady state $D^s = 1/2$, so that

$$q_i = \frac{\gamma}{2\tau\theta}\left(\alpha_1 + \frac{\alpha_2}{2}\right),$$

which, after substitution of α_1, leads to

$$q^{CL} = \left(\frac{1}{1 + \frac{\rho}{\gamma} - \frac{\gamma\alpha_2}{4\theta\tau^2}}\right)\left(\frac{p - \frac{\beta}{2}}{2\tau\theta}\right), \tag{A14}$$

where $\alpha_2 < 0$ is given by the negative root in (A12).

The comparative statics properties of (A14) are given by

$$\frac{\partial q^{CL}}{\partial \gamma} = \frac{3\rho\left(2\gamma + \rho + 5\psi\right)\left(p - \frac{\beta}{2}\right)}{(4\gamma + 5\rho + \psi)^2\,\tau\theta\psi} > 0 \tag{A15}$$
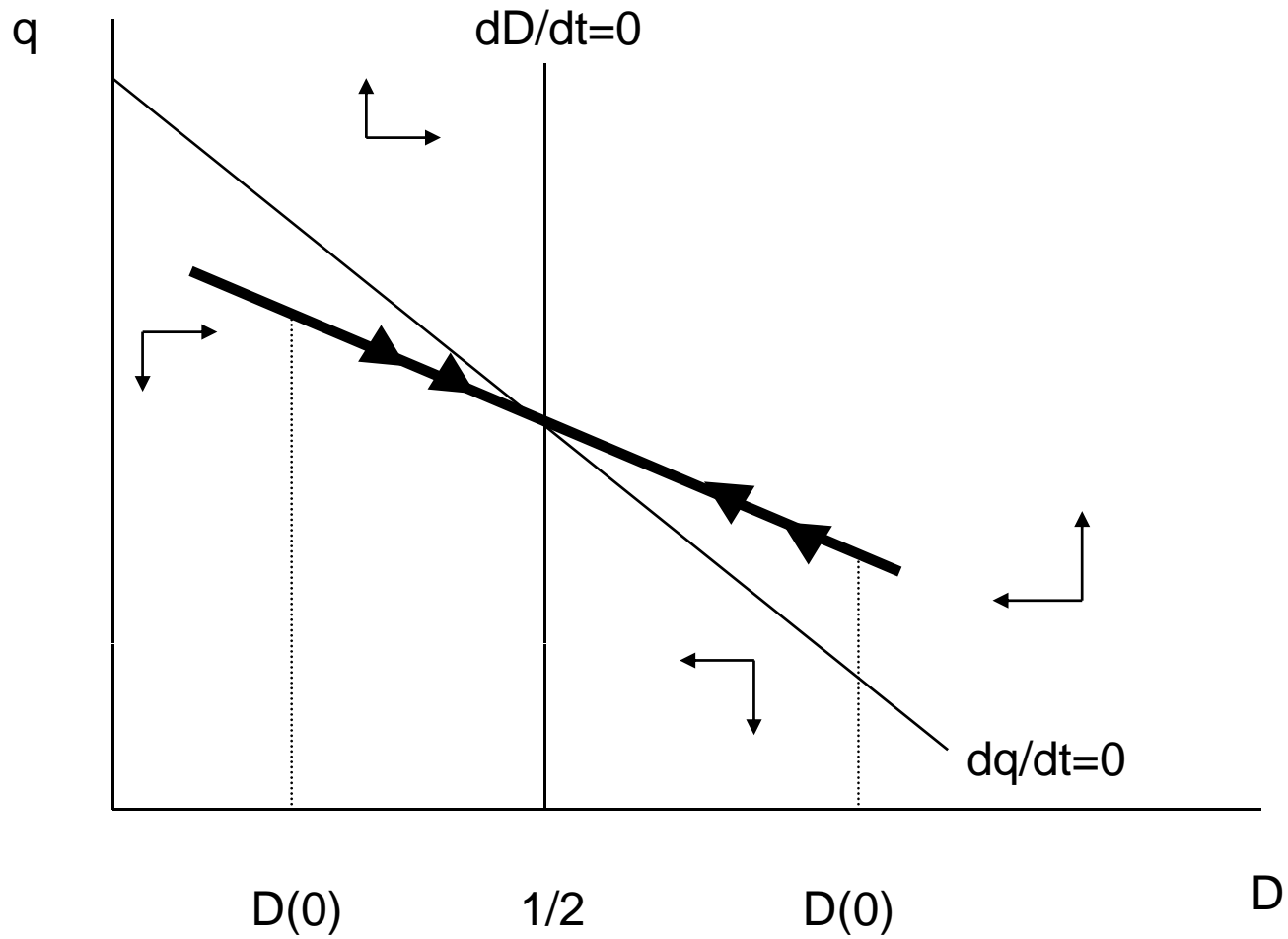
and

$$\frac{\partial q^{CL}}{\partial \tau} = -\frac{\left(p - \frac{\beta}{2}\right)\left[(2\gamma + \rho)^2 + \psi\left(4\gamma + 5\rho\right)\right]3\gamma}{\left[\begin{array}{c}\theta\tau^2\psi\left[(4\gamma + 5\rho)^2 + \psi^2\right] + 24\beta\gamma^3 \\ +2\theta\tau^2\left(16\gamma^3 + 5\rho^3\right) + 24\theta\tau^2\gamma\rho\left(3\gamma + 2\rho\right)\end{array}\right]} < 0, \tag{A16}$$
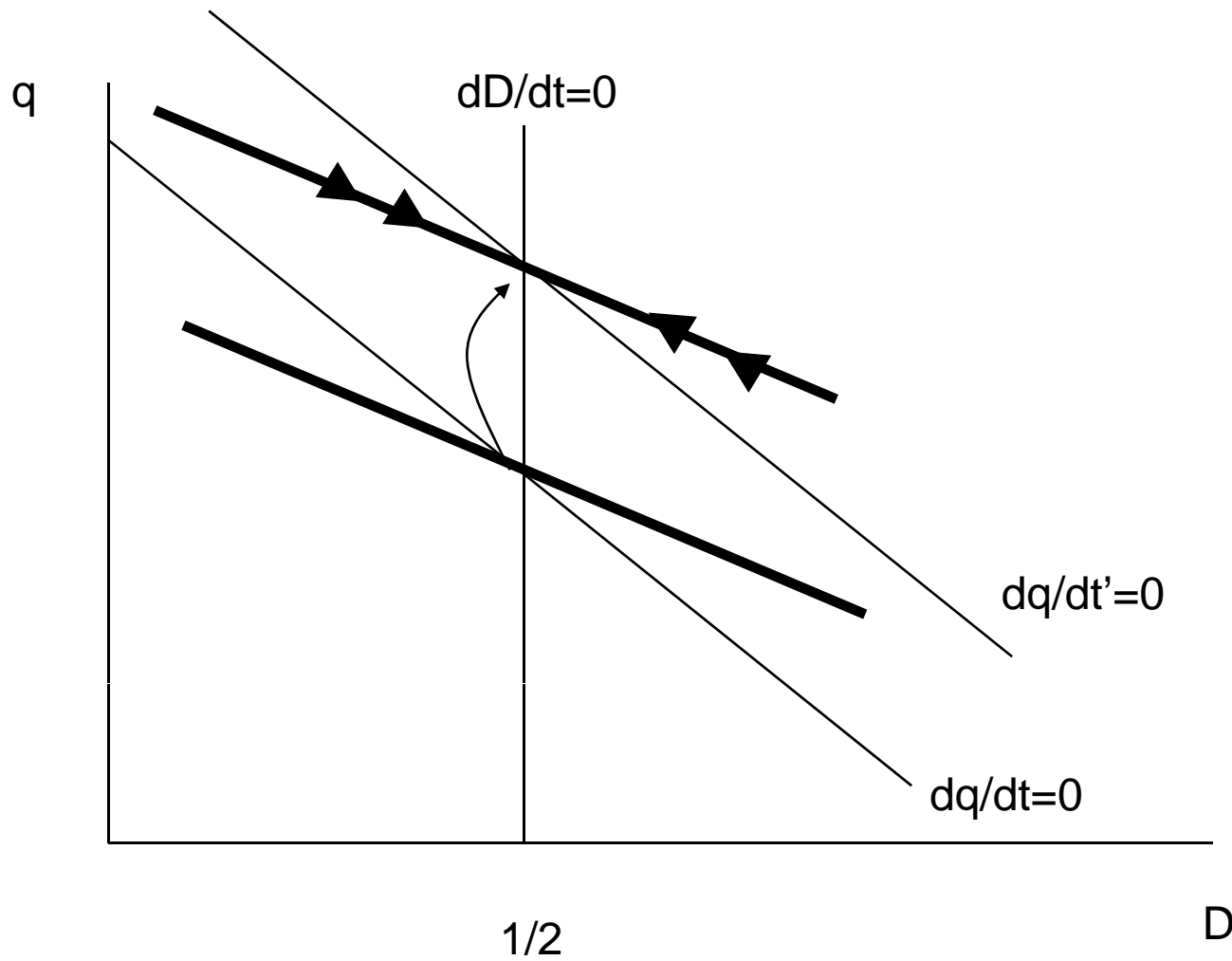
where

$$\psi := \sqrt{(2\gamma + \rho)^2 + \frac{3\beta\gamma^2}{\theta\tau^2}}.$$

Figure 1. Equilibrium is a saddle point

Figure 2. Increase in price or reduction in travel costs (more competition)

q

dD/dt=0

dq/dt'=0

dq/dt=0

1/2

D

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