

**SAM 11 2009**

**ISSN: 0804-6824**

JULY 2009

Discussion paper

# Price and quality in spatial competition

BY  
**KURT R. BREKKE, LUIGI SICILIANI, AND ODD RUNE STRAUME**

This series consists of papers with limited circulation, intended to stimulate discussion.

# Price and quality in spatial competition

Kurt R. Brekke\*      Luigi Siciliani†      Odd Rune Straume‡

30 July 2009

## Abstract

We study the relationship between competition and quality within a spatial competition framework where firms compete in prices and quality. We generalise existing literature on spatial price-quality competition along several dimensions, including utility functions that are non-linear in income and cost functions that are non-separable in output and quality. Our main message is that the scope for a positive relationship between competition and quality is underestimated in the existing literature. If we allow for income effects by assuming that utility is strictly concave in income, we find that lower transportation costs always lead to higher quality. The presence of income effects might also reverse a previously reported negative relationship between the number of firms and equilibrium quality. This reversal result is further strengthened if there are cost substitutabilities between output and quality. Equilibrium quality provision is always less than socially optimal in the presence of income effects.

*Keywords:* Spatial competition; Quality; Income effects.

*JEL Classifications:* D21; L13; L15.

---

\*Department of Economics and Health Economics Bergen, Norwegian School of Economics and Business Administration, Helleveien 30, N-5045 Bergen, Norway. E-mail: kurt.brekke@nhh.no.

†Department of Economics and Related Studies, and Centre for Health Economics, University of York, Heslington, York YO10 5DD, UK; and C.E.P.R., 90-98 Goswell Street, London EC1V 7DB, UK. E-mail: ls24@york.ac.uk

‡Department of Economics/NIPE, University of Minho, Campus de Gualtar, 4710-057 Braga, Portugal; and Department of Economics/HEB, University of Bergen. E-mail: o.r.straume@eeg.uminho.pt

# 1 Introduction

Does more competition induce firms to produce higher-quality goods? If prices are exogenous (e.g., due to price regulation), more competition will increase quality if prices are above marginal costs. This is a well established result in the literature on spatial competition.<sup>1</sup> However, if prices are endogenously set by firms, then the effect of competition on quality incentives is uncertain. While more competition increases the incentives to supply high quality for given prices, more competition also reduces the price-cost margin, which, in turn, reduces the incentives to invest in quality. Thus, the net effect of competition on quality is generally ambiguous under price competition.

Intensity of competition is often measured either as an increase in the number of firms in the market or as a reduction in the degree of horizontal product differentiation (or transportation costs). Using the latter measure, Ma and Burgess (1993) report no effect of less product differentiation on quality incentives. In their paper, the direct effect of more competition on quality incentives is exactly offset by the indirect effect via lower prices. The same result is reported by Gravelle (1999). Using the number of firms as a competition measure, Economides (1993) finds that more firms in the market *reduces* the incentives to invest in quality. Since a higher number of firms reduces the potential demand for each single firm, the returns to quality investments are correspondingly reduced.

In the present paper, we revisit the existing literature on price and quality competition in a spatial framework. We use a Salop-type model where firms have different locations, referring to product space or geographical space. In this set-up, we allow for price-quality competition. For the main part of the analysis, we assume that firms choose price and quality simultaneously. In an extension to the main model, we also allow for sequential choices, where quality is treated more as a long term variable. We take a closer look at the effects of spatial competition on quality and prices by generalising previous work along several dimensions. First, we allow for income effects by assuming that the utility function is concave in the numeraire good. Second, we decompose the transportation

---

<sup>1</sup>See, e.g., Ma and Burgess (1993), Calem and Rizzo (1995), Wolinsky (1997), Brekke, Nuscheler and Straume (2006, 2007).

costs into monetary and non-monetary costs. While non-monetary transportation costs affect utility directly, monetary transportation costs add to the consumption expenditures and affect utility through the budget constraint. This distinction should be particularly relevant with respect to different interpretations of firm location (product space versus geographical space). Third, we apply general benefit and production cost functions where we allow for quality and output to be either cost complements or cost substitutes.

One of our main results is that the relationship between competition and quality depends crucially on the presence of income effects; i.e., whether utility is linear or strictly concave in income. If utility is linear in income, more competition – as measured by lower transportation costs – leads to lower prices but has no effect on quality, since the two aforementioned effects exactly cancel each other out (as in Ma and Burgess, 1993, and Gravelle, 1999). Clearly, this is a special case. If we allow for utility to be strictly concave in income, the dampening effect of competition on quality incentives via a lower price-cost margin is smaller, implying that the net effect is positive: lower transportation costs always lead to *higher* quality in equilibrium. This conclusion holds regardless of whether we are considering monetary or non-monetary transportation costs. In a simplified version of the model, we also show that this conclusion is robust to the case where quality and price choices are made sequentially.

The only qualitative difference between monetary and non-monetary transportation costs is that lower monetary transportation costs (as opposed to non-monetary ones) might lead to *higher*, rather than lower, prices in equilibrium if the degree of cost substitutability between quality and output is sufficiently strong. The degree of cost substitutability is also important in determining the quality effects of a higher number of firms in the market. With constant marginal utility of income and cost independence between quality and output, we replicate the result by Economides (1993) that more firms lead to lower quality. However, we show that this result is reversed for a sufficient degree of cost substitutability (more firms increase quality). Furthermore, with decreasing marginal utility of income we can establish a positive relationship between firm density and equilibrium quality even for

(mild) cost complementarities.

We also derive and characterise the socially optimal quality level, finding that the Nash equilibrium quality level is never socially excessive. If utility is linear in income, equilibrium quality coincides with the socially optimal level. This result is well known from the literature (Ma and Burgess, 1993; Gravelle, 1999) and is due to the marginal utility being equal for the marginal and average consumer.<sup>2,3</sup> However, if utility is strictly concave in income, the marginal utility is higher for the marginal than for the average consumer in the Nash equilibrium, implying that the equilibrium supply of quality is below the socially optimal level.

As indicated above, the papers closest to ours are Ma and Burgess (1993), Economides (1993) and Gravelle (1999), who all studied, in various ways, the effect of spatial competition on prices and quality. While the effect of competition on prices is less ambiguous, and thus perhaps less interesting, the relationship between competition and quality is far from clear-cut. In fact, the existing literature suggests that we cannot expect firms to provide higher quality as a result of more competition. Our main message is that this conclusion is too pessimistic. In a more general framework we show that the special assumptions of linear utility functions and cost separability between quality and output are not innocuous and have led to an underestimation of the scope for competition to improve quality.

In addition to the three key papers cited above, there are also other papers that analyse the relationship between competition and quality using different types of modelling framework. Incorporating product quality into an oligopoly model with a Marshallian-type demand system, Banker et al. (1998) use several different measures of the degree of competition and conclude that the effect of increased competition on quality is generally ambiguous in all cases considered. In a very different setting, where firms and consumers interact repeatedly and quality is only ex-post observable, Kranton (2003) extends the

---

<sup>2</sup>This criterion was first established by Spence (1975).

<sup>3</sup>In a model where quality affects the degree of perceived horizontal differentiation, Degryse and Irmen (2001) show that firms' private incentives for quality provision generally depart from the socially optimal ones. When quality and price decisions are made simultaneously, they find that quality provision is socially excessive if the correlation between quality and horizontal differentiation is negative. The relationship between competition and quality is not an issue in the paper.

previous literature on reputation-based quality incentives (e.g., Klein and Leffler, 1981; Shapiro, 1983; Allen, 1984) to show that competition between firms might eliminate perfect equilibria in which firms produce high-quality goods.

There is also an empirical literature on the relationship between competition and quality, with studies from several different industries. Mazzeo (2003) uses the frequency of on-time flight departures as a measure of quality in the US airline industry and finds a positive correlation between competition and quality. Using questionnaire data from the UK, Domberger and Sherr (1989) show that the introduction of competition for conveyancing services led not only to price reductions, but also to an increase in the quality of the legal services offered. While both of these studies point to a clear-cut positive relationship between competition and quality, a more mixed picture emanates from studies of competition and quality in the banking industry. Dick (2007) finds that quality is higher in more dominant banks, while Cohen and Mazzeo (2007) find that increased competition has different effects on quality, depending on whether the competitors are single-market banks (negative correlation) or multi-market banks (positive correlation). The picture is also mixed for the case of competition in health care markets, where quality is clearly a key issue. For example, Dranove et al. (1992) and Sari (2002) find a positive correlation between hospital competition and quality using US data, while Propper et al. (2004) find a negative correlation using UK data.<sup>4</sup>

Clearly, the spatial competition framework we use in our analysis is relevant for many markets, including the specific ones mentioned above. In retail markets, for example, outlets are spatially differentiated due to different physical locations, and retailers may use price and service (quality) in order to get consumers to buy from them. The assumption that utility is non-linear in income implies that our analysis is particularly relevant for markets where the purchasing decision can be described as a discrete choice with income effects. One example is automobile markets, where the consumer typically buys one car from the most preferred dealer and the purchase expenditures usually constitute a sig-

---

<sup>4</sup>For more references, see the comprehensive survey by Gaynor (2006) on competition and quality in health care markets.

nificant fraction of the consumer's income. While income effects are obviously relevant in the demand for cars, we would expect income effects to be present also in markets for numerous other commodities that are relatively expensive, like TVs, Hi-Fi, furnitures, etc. This will certainly also be the case in private markets for health care and education. In such markets, the quality dimension is also highly important.

The rest of the paper is organised as follows. In the next section we outline the model and derive the equilibrium price and quality under the assumption of simultaneous choices. In Section 3 we analyse the effects of competition on prices and quality, measuring an increase in competition intensity either as a reduction of (monetary or non-monetary) transportation costs or as an increase in the number of firms in the market. In Section 4 we derive the socially optimal level of quality and characterise the welfare properties of the Nash equilibrium. In Section 5 we consider the case of sequential quality and price decisions in a simplified version of the model. The paper is concluded in Section 6.

## 2 Model

There are  $n$  firms equidistantly located on a circle with circumference equal to 1, each offering a product at price  $p_i$ ,  $i = 1, \dots, n$ . Consumers are located on the circle according to a density function  $f(\cdot)$ . We assume that  $f$  is identical and symmetric between any two firms, and the total consumer mass is normalised to 1. Each consumer buys one unit of the product from the most preferred firm. If a consumer buys from Firm  $i$ , her utility is given by a function  $U_i(q_i, d_i, y)$ , where  $q_i$  is the quality of the product sold by Firm  $i$ ,  $d_i$  is the distance between the consumer and Firm  $i$ , and  $y$  is a composite numeraire good. Assuming a separable additive form, we write the utility function as

$$U_i = v + b(q_i) - tg(d_i) + u(y), \quad (1)$$

with

$$y = Y - p_i - \tau h(d_i), \quad (2)$$

where  $Y$  is gross income. The utility derived from product quality is given by the function  $b(q_i)$ , where  $b_q > 0$  and  $b_{qq} \leq 0$ . Transportation costs can be both monetary (e.g., travelling costs) and non-monetary (e.g., time costs or the disutility of consuming a less-than-ideal product variety). The former is captured by  $\tau h(d_i)$ , where  $h_d > 0$  and  $h_{dd} \geq 0$ , while the latter is captured by  $tg(d_i)$ , where  $g_d > 0$  and  $g_{dd} \geq 0$ . We also assume that utility is concave in consumption of the numeraire good:  $u_y > 0$ ,  $u_{yy} \leq 0$ .

The distance between any two firms is equal to  $1/n$ , and we assume that  $v$  is sufficiently large to ensure full market coverage in equilibrium. If we let Firm  $i$  be located at zero and measure distance clockwise, the consumer who is indifferent between buying from Firm  $i$  and Firm  $i + 1$  is located at  $z_+$ , implicitly given by:

$$b(q_i) - tg(z_+) + u(Y - p_i - \tau h(z_+)) = b(q_{i+1}) - tg\left(\frac{1}{n} - z_+\right) + u\left(Y - p_{i+1} - \tau h\left(\frac{1}{n} - z_+\right)\right). \quad (3)$$

An equivalent condition determines the location of the consumer who is indifferent between Firm  $i$  and Firm  $i - 1$ , denoted by  $z_-$ . Total demand for Firm  $i$  is then given by

$$X_i(p_i, p_{i+1}, p_{i-1}, q_i, q_{i+1}, q_{i-1}) = \int_{z_-}^{z_+} f(x) dx. \quad (4)$$

Once we derive the demand function, we can specify Firm  $i$ 's profits as

$$\pi_i = p_i X_i(\cdot) - C(X_i(\cdot), q_i), \quad (5)$$

where  $C_X > 0$ ,  $C_{XX} \geq 0$ ,  $C_{Xq} \geq 0$ ,  $C_q > 0$ ,  $C_{qq} > 0$ . Notice that we allow for both cost complementarity ( $C_{Xq} < 0$ ) and cost substitutability ( $C_{Xq} > 0$ ) between output and quality.

Assume that all  $n$  firms choose price and quality simultaneously. The first-order conditions for Firm  $i$ 's profit-maximising choice of price and quality are then given by<sup>5</sup>

$$\frac{\partial \pi_i}{\partial p_i} = X_i + [p_i - C_X(X_i, q_i)] \frac{\partial X_i}{\partial p_i} = 0, \quad (6)$$

---

<sup>5</sup>The second-order conditions are satisfied if the cost function is sufficiently convex in quality.



$$\frac{\partial \pi_i}{\partial q_i} = [p_i - C_X(X_i, q_i)] \frac{\partial X_i}{\partial q_i} - C_q(X_i, q_i) = 0. \quad (7)$$

By solving (6) for  $(p_i - C_X)$  and substituting into (7), we can express (7) as

$$-\frac{X_i \frac{\partial X_i}{\partial q_i}}{\frac{\partial X_i}{\partial p_i}} - C_q(X_i, q_i) = 0. \quad (8)$$

Since the model is symmetric, all firms will choose the same price and quality in equilibrium. If  $p_{i-1} = p_{i+1}$  and  $q_{i-1} = q_{i+1}$ , total demand for Firm  $i$  is given by

$$X_i(p_i, p_{-i}) = 2 \int_0^{z_+} f(x) dx = 2F(z_+). \quad (9)$$

Given  $p_{i-1} = p_{i+1}$  and  $q_{i-1} = q_{i+1}$ , we can totally differentiate (6)-(7) to find  $\partial z/\partial p_i$  and  $\partial z/\partial q_i$ , and use (9) to calculate the partial derivatives of total demand with respect to price and quality, respectively:

$$\frac{\partial X_i}{\partial p_i} = -\frac{2f(z_+)u_y}{t[g_d(z_+) + g_d(\frac{1}{n} - z_+)] + \tau u_y[h_d(z_+) + h_d(\frac{1}{n} - z_+)]} < 0, \quad (10)$$

$$\frac{\partial X_i}{\partial q_i} = \frac{2f(z_+)b_q}{t[g_d(z_+) + g_d(\frac{1}{n} - z_+)] + \tau u_y[h_d(z_+) + h_d(\frac{1}{n} - z_+)]} > 0. \quad (11)$$

Using (10) and (11), the unique symmetric pure-strategy Nash equilibrium is given by (6) and (8). Setting  $p_i = p$  and  $q_i = q$ ,  $i = 1, \dots, n$ , and noting that  $F(z_+) = z_+ = \frac{1}{2n}$  in the symmetric equilibrium, the equilibrium price and quality are given by the following system of equations:<sup>6</sup>

$$V_p := \frac{1}{n} - \frac{[p^* - C_X(\frac{1}{n}, q^*)] f(\frac{1}{2n}) u_y (Y - p^* - \tau h(\frac{1}{2n}))}{t g_d(\frac{1}{2n}) + \tau h_d(\frac{1}{2n}) u_y (Y - p^* - \tau h(\frac{1}{2n}))} = 0, \quad (12)$$

$$V_q := \frac{b_q(q^*)}{n u_y (Y - p^* - \tau h(\frac{1}{2n}))} - C_q\left(\frac{1}{n}, q^*\right) = 0. \quad (13)$$

---

<sup>6</sup>Equilibrium existence requires that there are no incentives for price undercutting (see D'Aspremont et al., 1979) and that there are no incentives for "ruinous" quality competition, i.e., that the equilibrium candidate  $(p^*, q^*)$  yields non-negative profits (see Brekke et al., 2006). With (weakly) convex transportation cost functions, both requirements are met if the distance between firms is not too small.

### 3 Price and quality effects of competition

In spatial competition models, a standard competition measure is the (inverse of) transportation cost. Lower transportation costs increase the degree of substitutability between the products offered by different firms, which intensifies competition. In a Salop model, we can also use the number of firms as a measure of the intensity of competition. In the following, we will use both of these measures to analyse the effects of increased competition on equilibrium prices and quality.

#### 3.1 Transportation costs

In our model, we have two different measures of transportation costs, where the parameter  $t$  measures the non-monetary costs while the parameter  $\tau$  measures the monetary ones. Using Cramer's rule, the effects of  $t$  on the equilibrium price and quality are given by<sup>7</sup>

$$\frac{\partial p^*}{\partial t} = \frac{[p^* - C_X] f(\frac{1}{2n}) g_d \left[ u_y C_{qq} - \frac{b_{qq}}{n} \right]}{\Delta \phi^2} \quad (14)$$

and

$$\frac{\partial q^*}{\partial t} = \frac{[p^* - C_X] f(\frac{1}{2n}) g_d b_q u_{yy}}{\Delta \phi^2 n u_y}, \quad (15)$$

where  $\phi := t g_d + \tau h_d u_y > 0$  and  $\Delta := V_{pp} V_{qq} - V_{pq} V_{qp} > 0$ .

**Proposition 1** *Lower non-monetary transportation costs affect equilibrium prices and quality as follows:*

- (i) *If utility is linear in income, prices fall while quality is unaffected;*
- (ii) *If utility is strictly concave in income, prices fall while quality increases.*

The result that more competition reduces prices is standard and deserves no further explanation. The effect on quality is less obvious. Increased substitutability implies that demand becomes more responsive to both price and quality, as we can see from (10) and (11). This gives each firm an incentive to reduce the price and increase quality. However,

---

<sup>7</sup>The details of all the comparative statics calculations in this section are given in the Appendix.

a price reduction implies a lower price-cost margin, which reduces the incentive to provide quality, as we can see from (7). Due to these two counteracting effects, the total equilibrium effect of increased substitutability on quality is *a priori* ambiguous. Our results show that the total effect depends crucially on the marginal utility of income. If the marginal utility is constant, the two effects cancel each other out and the equilibrium quality level is independent of  $t$ , as in Ma and Burgess (1993) and Gravelle (1999). However, if utility is strictly concave, the indirect effect on quality incentives through a lower price-cost margin is reduced, implying that lower non-monetary transportation costs will increase the equilibrium supply of quality. Thus, with a decreasing marginal utility of income, consumers benefit from more competition (measured as a reduction of non-monetary transportation costs) along all dimensions as prices fall while quality increases.

Our other (inverse) measure of the degree of substitutability is the monetary transportation costs, reflected by the parameter  $\tau$ . Again, using Cramer's rule, the effects of a marginal change in  $\tau$  on equilibrium price and quality are given by

$$\frac{\partial p^*}{\partial \tau} = \frac{f\left(\frac{1}{2n}\right) h_d}{\Delta \phi u_y} \left[ \frac{C_{Xq} u_{yy} b_q}{n} + \frac{(t g_d u_{yy} + u_y^2) (p^* - C_X) \left(u_y C_{qq} - \frac{b_{qq}}{n}\right)}{\phi} \right], \quad (16)$$

$$\frac{\partial q^*}{\partial \tau} = \frac{[t g_d + \tau h_d u_y + u_y [p^* - C_X]] f\left(\frac{1}{2n}\right) b_q h_d u_{yy}}{\Delta \phi^2 n u_y}. \quad (17)$$

**Proposition 2** *Lower monetary transportation costs affect equilibrium prices and quality as follows:*

- (i) *If utility is linear in income, prices fall while quality is unaffected;*
- (ii) *If utility is strictly concave in income, quality increases and prices may also increase if  $C_{Xq} > 0$  and/or  $t$  is sufficiently high.*

As for the case of non-monetary transportation costs, the price and quality effects of lower monetary transportation costs depend crucially on whether the marginal utility of income is constant or decreasing. As before, the effect on quality is zero in the former

case and positive in the latter.<sup>8</sup> The qualitative difference between monetary and non-monetary costs is the potential effect on equilibrium prices, where a reduction in  $\tau$  might actually lead to higher prices in equilibrium. If  $u_{yy} < 0$ , prices may increase if there is sufficiently strong cost substitutability between quality and output. The reason is that, if  $C_{Xq} > 0$ , a higher quality level increases the marginal cost of production, which puts an upward pressure on prices. Notice, however, that a price increase is only a possibility under decreasing marginal utility of income. If  $u_{yy} = 0$ , quality is unaffected by monetary transportation costs and the above mentioned effect on prices via the cost function is thus absent.

### 3.2 Firm density

In order to simplify the analysis somewhat, we assume here that the distribution of consumers is uniform around the circle, implying that  $f(\cdot) = 1$ . Let us first consider the relationship between  $n$  and  $p$ , which is given by

$$\begin{aligned} \frac{\partial p^*}{\partial n} = & - \left( \frac{C_{qq} - \frac{b_{qq}}{nu_y}}{n^2 \Delta} \right) \left[ 1 + \frac{u_y C_{XX}}{\phi} + \frac{(p^* - C_X) (\tau u_y^2 h_{dd} + t u_y g_{dd} + t g_d \tau h_d u_{yy})}{2\phi^2} \right] \\ & + \frac{C_{Xq} u_y}{n^2 \Delta \phi} \left( C_{Xq} - \frac{b_q}{u_y} - \frac{\tau b_q h_d u_{yy}}{2n u_y^2} \right). \end{aligned} \quad (18)$$

The sign of this expression is generally ambiguous. In the standard versions of the model, where  $u_{yy} = 0$  and  $C_{Xq} = 0$ , we see that the sign is negative and we get the expected result that a higher number of firms leads to lower prices. However, if the marginal utility of income is decreasing, this result might potentially be reversed. We can see this more clearly by considering the special case of constant marginal production and transportation costs, and cost independence between output and quality:  $C_{XX} = h_{dd} = g_{dd} = C_{Xq} = 0$ .

---

<sup>8</sup>Notice that there is a qualitative difference between monetary and non-monetary transportation costs only if utility is non-linear in income. Thus, the first parts of Propositions 1 and 2 are necessarily equal.

In this case, the relationship between  $n$  and  $p$  is given by

$$\frac{dp^*}{dn} = -\frac{(C_{qq} - \frac{b_{qq}}{nu_y})}{n^2\Delta} \left[ 1 + \frac{(p^* - C_X)tg_d\tau h_d}{2\phi^2}u_{yy} \right], \quad (19)$$

which is positive if  $u_{yy}$  is sufficiently large in absolute value. The effect that works in the "counterintuitive" direction is the following: for given (and symmetric) prices and qualities, a higher firm density implies that the net income of the marginal (indifferent) consumer increases due to lower monetary transportation costs. If utility is strictly concave in income, this means that the marginal utility of income decreases, which, in turn, reduces the demand responsiveness to prices (cf. (10)). All else equal, this effect provides an incentive to increase prices.

The effect of a higher number of firms on the equilibrium quality is given by

$$\begin{aligned} \frac{\partial q^*}{\partial n} = & \left( \frac{u_y [p^* - C_X] [2tn_gd(b_q - u_y C_{Xq}) - b_q(tg_{dd} + u_y\tau h_{dd})] - \phi b_q\psi}{2\Delta\phi^2 n^3 u_y^2} \right) u_{yy} \\ & - \left( \frac{b_q - u_y C_{Xq}}{\Delta\phi n^2} \right), \end{aligned} \quad (20)$$

where  $\psi := 2g_{dt} + 3h_d u_y \tau + 2u_y C_{XX} > 0$ . The sign of (20) is generally ambiguous. In the case of constant marginal utility of income ( $u_{yy} = 0$ ), we see that equilibrium quality is increasing in the number of firms if the degree of cost substitutability between output and quality is sufficiently high:  $C_{Xq} > b_q/u_y$ . In the special case of  $C_{Xq} = 0$ , equilibrium quality is inversely related to the number of firms, since more firms reduce the potential demand for each firm, thereby reducing the gain of providing high-quality products. This corresponds exactly to the case analysed by Economides (1993), where  $u_{yy} = C_{Xq} = 0$ . However, if there is cost substitutability between output and quality, a higher number of firms in the market reduces the marginal cost of quality improvements due to the lower level of demand facing each firm. If this second effect is sufficiently strong, the negative relationship between  $n$  and  $q^*$  may be reversed.<sup>9</sup> The sign of  $\partial q^*/\partial n$  is harder to

---

<sup>9</sup>Both effects are present in Gravelle (1999), where  $u_y = b_q = 1$  and  $C(X, q) = \alpha q^2 X$ . With this particular formulation, it turns out that the two effects exactly cancel each other out and quality is independent of the number of firms in the market.

characterise if the marginal utility of income is decreasing ( $u_{yy} < 0$ ) and the only general conclusion that can be drawn is that the relationship between  $q^*$  and  $n$  is ambiguous.

We summarise the above discussion as follows:

**Proposition 3** (i) *If utility is linear in income and the marginal cost of providing quality is independent of output, a higher number of firms leads to lower prices in equilibrium. This relationship might be reversed if utility is strictly concave in income.*

(ii) *If utility is linear in income, a higher number of firms leads to higher quality in equilibrium if the degree of cost substitutability between output and quality is sufficiently high. If utility is concave in income, the relationship between the number of firms and equilibrium quality is generally ambiguous.*

### 3.3 A parametric example

For illustrative purposes, consider the following parametric example where utility is logarithmic in income and linear in quality and distance:  $u(y) = \beta \ln y$ ,  $b(q_i) = bq_i$  and  $h(d) = g(d) = d$ . We also assume a linear-quadratic cost function:  $C(X_i, q_i) = cX + \alpha X_i q_i + \frac{k}{2} q_i^2$ , where  $c > 0$ ,  $k > 0$  and  $\alpha \leq 0$ .

By using these specific functional forms in (12)-(13), we derive the following explicit expressions for equilibrium price and quality:

$$p^* = \frac{(2Yn - \tau)(kt + b\alpha) + 2\beta n(k(\tau + cn) - \alpha^2)}{2(kt + b\alpha + kn\beta)n}, \quad (21)$$

$$q^* = \frac{2nb(Y - c) - 2\alpha(t + n\beta) - 3b\tau}{2(kt + b\alpha + kn\beta)n}. \quad (22)$$

The comparative statics results with respect to the different measures of competition intensity are given by

$$\frac{\partial p^*}{\partial t} = \frac{y^*k}{(kn\beta + kt + b\alpha)} > 0; \quad \frac{\partial q^*}{\partial t} = -\frac{y^*b}{(kn\beta + kt + b\alpha)n\beta} < 0, \quad (23)$$

$$\frac{\partial p^*}{\partial \tau} = \frac{k(2n\beta - t) - b\alpha}{2n(kn\beta + kt + b\alpha)} \leq 0; \quad \frac{\partial q^*}{\partial \tau} = -\frac{3b}{2(kn\beta + kt + b\alpha)n} < 0, \quad (24)$$

$$\frac{\partial p^*}{\partial n} = \frac{-(kt + b\alpha)(2\beta nk(n(Y - c) - \tau) - \tau(kt + b\alpha)) + 2\beta^2 n^2 k(\alpha^2 - k\tau)}{2(kt + b\alpha + kn\beta)^2 n^2} \leq 0, \quad (25)$$

$$\frac{\partial q^*}{\partial n} = \frac{2\beta nk(\alpha(2t + n\beta) + 3b\tau - nb(Y - c)) + (kt + b\alpha)(3b\tau + 2t\alpha)}{2(kt + b\alpha + kn\beta)^2 n^2} \leq 0, \quad (26)$$

where  $y^* = Y - p^* - \frac{\tau}{2n}$  is the net income of the marginal (indifferent) consumers in equilibrium.

The results from this example confirm the analysis of the general model. Here, we see that a higher value of  $\alpha$  increases the parameter space for which equilibrium quality is increasing in the number of firms. However, for certain parameter configurations, a positive relationship between  $q^*$  and  $n$  can also be established even for (mild) cost *complementarity* between output and quality (i.e.,  $\alpha < 0$ ). Numerical simulations also suggest that a price increase due to a higher number of firms appears only for a very restricted parameter configuration.

## 4 Social welfare

Does the market provide the socially optimal level of quality? Suppose that the government can provide output and quality directly, and finance the cost of provision through a lump-sum tax  $T$ . Applying symmetry, the first-best level of quality – equal for all firms – is such that it maximises the utilitarian welfare function

$$W = v + b(q) + 2n \int_0^{\frac{1}{2n}} [u(Y - T - \tau h(x)) - tg(x)] f(x) dx \quad (27)$$

subject to the resource constraint

$$T = nC\left(\frac{1}{n}, q\right). \quad (28)$$

By inserting (28) into (27), yielding

$$W = v + b(q) + 2n \int_0^{\frac{1}{2n}} \left[ u\left(Y - nC\left(\frac{1}{n}, q\right) - \tau h(x)\right) - tg(x) \right] f(x) dx, \quad (29)$$

and maximising with respect to  $q$ , the socially optimal level of quality is implicitly given by

$$\frac{b_q(q^s)}{2n \int_0^{\frac{1}{2n}} u_y \left( Y - nC\left(\frac{1}{n}, q^s\right) - \tau h(x) \right) f(x) dx} = nC_q\left(\frac{1}{n}, q^s\right). \quad (30)$$

Notice that the denominator on the LHS of (30) is the marginal utility of income for the *average* consumer (with the average taken across distance). Thus, the socially optimal level of quality is characterised by the ratio of the marginal utility of quality and the marginal utility of income for the average consumer being equal to the marginal cost of quality provision.

The Nash equilibrium level of quality, on the other hand, is implicitly given by

$$\frac{b_q(q^*)}{u_y \left( Y - p^* - \tau h\left(\frac{1}{2n}\right) \right)} = nC_q\left(\frac{1}{n}, q^*\right), \quad (31)$$

where the denominator on the LHS is the marginal utility of income for the *marginal* consumer, who is indifferent between two firms. Consequently, the difference between the Nash equilibrium level of quality ( $q^*$ ) and the socially optimal level ( $q^s$ ) depends on how the marginal utility of income compares for the *average* and *marginal* consumers, respectively.

**Proposition 4** (i) *If utility is linear in income, the Nash equilibrium level of quality coincides with the socially optimal level.*

(ii) *If utility is strictly concave in income, the Nash equilibrium level of quality is lower than the socially optimal level.*

The first part of the proposition confirms the result reported in Ma and Burgess (1993), and shows that this result generalises beyond specific forms of the transportation and production cost functions. However, this result hinges crucially on the assumption of constant marginal utility of income. Comparing (30) and (31), notice that  $p^* \geq nC\left(\frac{1}{n}, q\right)$ , since, when the population is normalised to one,  $nC\left(\frac{1}{n}, q\right)$  can be interpreted as the average cost of production. Moreover, notice also that  $\tau h\left(\frac{1}{2n}\right) \geq \tau h(x)$  for any  $x$ . Thus, when comparing (30) and (31), we see that the income of the marginal consumer in the Nash



equilibrium is lower than the average income in the first-best solution. With diminishing marginal utility of income, this means that, for  $q = q^*$ , the marginal utility of income for the average consumer is higher than the marginal cost of quality provision, implying an underprovision of quality in the Nash equilibrium.

It follows from Proposition 4 that increased competition affects the welfare properties of the Nash equilibrium only in the case of diminishing marginal utility of income (or, more generally, if utility is non-linear in income). The effects of reduced non-monetary transportation costs are fairly straightforward. Since the first-best level of quality does not depend on non-monetary transportation costs, a reduction in  $t$  will unambiguously improve welfare since equilibrium quality increases towards the first-best level.<sup>10</sup> Monetary transportation costs, on the other hand, affect both  $q^*$  and  $q^s$ . However, notice that a reduction in  $\tau$  reduces the difference between transportation costs for the average and marginal consumers, respectively. If, in addition, a reduction in  $\tau$  also leads to a price reduction, the difference between quality levels in the Nash equilibrium and the first-best solution are unambiguously reduced.

The welfare effect of an increase in the number of firms is considerably more involved and depends, *inter alia*, on the characteristics of the cost function. Using the parametric example from Section 3.3 it can be shown (by numerical simulations) that the effect is generally ambiguous. This naturally reflects that fact that  $\partial q^*/\partial n \leq 0$ .

## 5 Sequential quality and price choices

In this section we extend the main analysis by considering the case where the quality and price choices are made sequentially. More specifically, we consider a game with the following order of moves:

Stage 1: Firms choose qualities ( $q_i$ ) simultaneously and independently.

Stage 2: Firms choose prices ( $p_i$ ) simultaneously and independently.

---

<sup>10</sup>In (31), notice that a reduction in  $t$  affects  $q^*$  through a reduction in  $p^*$ .

By introducing sequential decision making, the analysis is severely complicated. Thus, in order to facilitate analytical tractability, we make a number of simplifying assumptions:  $b(q_i) = bq_i$ ,  $g(d) = d$ ,  $\tau = 0$ ,  $f(x) = 1$ ,  $C_{Xq} = 0$  and  $n = 2$ . This means that we restrict attention to our most important generalisation: allowing utility to be concave in income.

When  $n = 2$ , total demand for Firm  $i$  is given by  $2z_+$ , where  $z_+$  is implicitly given by (3). When  $\tau = 0$ , we can solve (3) explicitly and derive demand for Firm  $i$  as

$$X_i(p_i, p_j, q_i, q_j) = \frac{1}{2} + \frac{b[q_i - q_j] + u(Y - p_i) - u(Y - p_j)}{t}. \quad (32)$$

### 5.1 The price subgame

For a given pair of quality levels,  $(q_i, q_j)$ , the equilibrium in the price subgame is characterised by the first-order condition

$$\frac{\partial \pi_i}{\partial p_i} = X_i(\cdot) - \frac{[p_i - C_X] u_y(Y - p_i)}{t} = 0, \quad (33)$$

from which we can derive the relationships between qualities and prices. Applying Cramer's rule, these comparative statics results are given by

$$\frac{\partial p_i}{\partial q_i} = b \left[ \frac{u_y(Y - p_j) - [p_j - C_X] u_{yy}(Y - p_j)}{\Delta_p} \right] > 0 \quad (34)$$

and

$$\frac{\partial p_i}{\partial q_j} = b \left[ \frac{[p_i - C_X] u_{yy}(Y - p_i) - u_y(Y - p_i)}{\Delta_p} \right] < 0, \quad (35)$$

where  $\Delta_p > 0$  is defined as

$$\begin{aligned} \Delta_p \quad : \quad &= ([p_i - C_X] u_{yy}(Y - p_i) - 2u_y(Y - p_i)) (p_i - C_X u_{yy}(Y - p_i) - 2u_y(Y - p_i)) \\ &\quad - u_y(Y - p_i) u_y(Y - p_j). \end{aligned} \quad (36)$$

## 5.2 Quality choices

Using the equilibrium values of the price subgame, we can express Firm  $i$ 's profits as a function of qualities only. The first-order condition for Firm  $i$ 's profit-maximising choice of quality level is

$$\frac{\partial \pi_i}{\partial q_i} = \frac{\partial p_i}{\partial q_i} X_i + \frac{(p_i - C_X)}{t} \left[ b - u_y(Y - p_i) \frac{\partial p_i}{\partial q_i} + u_y(Y - p_j) \frac{\partial p_j}{\partial q_i} \right] - C_q = 0. \quad (37)$$

Applying symmetry and using (32) and (34)-(35), equilibrium quality is characterised by

$$b \left( \frac{u_y - (p^* - c) u_{yy}}{2\Delta_q} \right) + \left( \frac{p - C_X}{t} \right) \left[ b - \frac{2bu_y [u_y - (p - C_X) u_{yy}]}{\Delta_q} \right] - C_q = 0 \quad (38)$$

where

$$\Delta_q := (p - c)^2 u_{yy}^2 + 3u_y^2 - 4(p - c) u_{yy} u_y > 0. \quad (39)$$

## 5.3 Equilibrium analysis

From (33) we see that, in the symmetric equilibrium, prices do not depend on quality levels. This is due to the assumption of cost independence between quality and output ( $C_{Xq} = 0$ ), and also implies that equilibrium prices are identical in the simultaneous and sequential versions of the game. Comparing (12) and (33), we see that the equilibrium price in both versions of the game is characterised by  $p = C_X + \frac{t}{2u_y}$ . We can use this expression to characterise the equilibrium quality as follows:

$$\frac{b}{2} \left[ \frac{1}{u_y} - \left( \frac{u_y - \frac{t u_{yy}}{2u_y}}{\Delta} \right) \right] = C_q. \quad (40)$$

Comparing (13) and (40), we see that equilibrium quality is lower if price and quality choices are made sequentially. The difference is represented by the second term in the square brackets of (40). This confirms that the "underinvestment" result reported by Ma and Burgess (1993) is robust to the assumption of decreasing marginal utility of income. We already know from Proposition 4 that, if utility is concave in income, quality is below

the socially optimal level in the simultaneous price-quality game. Thus, equilibrium quality is even more suboptimal if quality and price decisions are made sequentially.

In order to examine the effect of competition on prices and quality when these are determined sequentially, we apply the following functional forms:  $u(y) = \beta \ln y$  and  $C(X_i, q_i) = cX_i + \frac{k}{2}q_i^2$ . This allows us to derive closed-form solutions for the equilibrium price and quality:

$$p = \frac{2c\beta + Yt}{t + 2\beta}, \quad (41)$$

$$q = \frac{b(Y - c)(t + 4\beta)}{k(t + 6\beta)(t + 2\beta)}. \quad (42)$$

In this simplified version of the model, the degree of competition is (inversely) measured by the parameter  $t$ . The effects of changes in the degree of competition on equilibrium prices and quality are given by

$$\frac{\partial p}{\partial t} = \frac{2\beta(Y - c)}{(t + 2\beta)^2} > 0 \quad (43)$$

and

$$\frac{\partial q}{\partial t} = -\frac{(20\beta^2 + 8t\beta + t^2)(Y - c)}{(t + 2\beta)^2(t + 6\beta)^2 k} < 0. \quad (44)$$

Thus, the competition effects on prices and quality are in this example qualitatively unaffected by whether the decisions are taken simultaneously or sequentially. As long as utility is strictly concave in income, a more competitive market (measured as a reduction in  $t$ ) produces lower prices and higher quality in equilibrium.

We can also use this parameterisation to say something about how the difference between quality levels under simultaneous and sequential decision making depends on the degree of competition in the market. Denoting equilibrium quality with simultaneous and sequential decisions by, respectively,  $q^*$  and  $q^{**}$ , the degree of "underinvestment" due to sequential decision making is given by

$$q^* - q^{**} = \frac{4(t + 2\beta)k\beta(Y - c)b}{2(t + 6\beta)(t + 2\beta)^2 k^2} > 0, \quad (45)$$

from which we derive

$$\frac{\partial(q^* - q^{**})}{\partial t} = -\frac{4(t + 4\beta)(Y - c)b\beta}{(t + 2\beta)^2(t + 6\beta)^2 k} < 0. \quad (46)$$

Thus, the degree of underinvestment is larger in more competitive markets (lower  $t$ ). This is quite intuitive, since the underinvestment results stems from what Ma and Burgess dub "the price undercutting effect"; i.e., incentives for quality investments at the first stage are dampened by the fact that the rival firm will "compensate" by undercutting its price at the next stage. The incentive for such price undercutting is stronger in more competitive markets, where demand reacts more strongly to price changes.

## 6 Concluding remarks

The relationship between competition and quality is theoretically ambiguous when firms also compete in prices. Within a framework of spatial competition, we have shown in this paper that the effect of competition on quality depends crucially on the presence of income effects on the demand side and cost dependence between output and quality on the supply side. More specifically, if we use transportation costs (i.e., the degree of horizontal differentiation) as an inverse measure of competition intensity, more competition will always increase quality in equilibrium if the marginal utility of income is decreasing. If we measure competition intensity by the number of firms in the market, we find a positive relationship between competition and quality also for the case of constant marginal utility of income, provided that there is a sufficient degree of cost substitutability between output and quality. Thus, when seen in conjunction with existing theoretical literature, our results suggest that the scope for spatial competition to stimulate quality provision is larger than previously thought.

The presence of income effects on the demand side also implies that, from a social welfare perspective, the market provides a sub-optimal level of quality even in the case where prices and quality are chosen simultaneously, a result which is also new to the litera-

ture. More specifically, if utility is strictly concave in income, equilibrium quality is always below the socially optimal level. Thus, although clear-cut and unambiguous conclusions are hard to reach, due to the general nature of our model, our results seem to suggest that the scope for welfare-enhancing competition is larger than previously indicated in the literature on spatial price-quality competition.

## Appendix

Using the notation  $V_{xy} := \frac{\partial V_x}{\partial y}$ , we derive, from (12)-(13), the following expressions:

$$V_{pp} = -\frac{f\left(\frac{1}{2n}\right) [u_y \phi - (p - C_X) u_{yy} t g_d]}{\phi^2} < 0, \quad (\text{A1})$$

$$V_{qq} = \frac{b_{qq}}{n u_y} - C_{qq} < 0, \quad (\text{A2})$$

$$V_{pq} = \frac{C_{Xq} f\left(\frac{1}{2n}\right) u_y}{\phi} \leq 0, \quad (\text{A3})$$

$$V_{qp} = \frac{b_q u_{yy}}{n (u_y)^2} \leq 0, \quad (\text{A4})$$

$$V_{pt} = \frac{(p - C_X) f\left(\frac{1}{2n}\right) u_y g_d}{\phi^2} > 0, \quad (\text{A5})$$

$$V_{qt} = 0, \quad (\text{A6})$$

$$V_{p\tau} = \frac{(p - C_X) f\left(\frac{1}{2n}\right) \left( u_{yy} h_d t g_d + (u_y)^2 h_d \right)}{\phi^2} \leq 0, \quad (\text{A7})$$

$$V_{q\tau} = \frac{b_q h_d u_{yy}}{n (u_y)^2} \leq 0, \quad (\text{A8})$$

$$V_{pn} = -\frac{1}{n^2} \left( 1 + \frac{C_{XX} u_y}{\phi} + \frac{(p - C_X) \left( (u_y)^2 \tau h_{dd} + u_y t g_{dd} + t g_d u_{yy} \tau h_d \right)}{2\phi^2} \right) \leq 0, \quad (\text{A9})$$

$$V_{qn} = -\frac{1}{n^2} \left( \frac{b_q}{u_y} + \frac{b_q u_{yy} \tau h_d}{2n (u_y)^2} - C_{Xq} \right) \leq 0. \quad (\text{A10})$$

The comparative statics results reported in equations (14), (15), (16), (17), (18) and

(20) are then found by using Cramer's rule:

$$\frac{\partial p}{\partial t} = -\frac{\begin{vmatrix} V_{pt} & V_{pq} \\ V_{qt} & V_{qq} \end{vmatrix}}{\begin{vmatrix} V_{pp} & V_{pq} \\ V_{qp} & V_{qq} \end{vmatrix}}; \quad \frac{\partial p^*}{\partial \tau} = -\frac{\begin{vmatrix} V_{p\tau} & V_{pq} \\ V_{q\tau} & V_{qq} \end{vmatrix}}{\begin{vmatrix} V_{pp} & V_{pq} \\ V_{qp} & V_{qq} \end{vmatrix}}; \quad \frac{\partial p^*}{\partial n} = -\frac{\begin{vmatrix} V_{pn} & V_{pq} \\ V_{qn} & V_{qq} \end{vmatrix}}{\begin{vmatrix} V_{pp} & V_{pq} \\ V_{qp} & V_{qq} \end{vmatrix}};$$

$$\frac{\partial q}{\partial t} = -\frac{\begin{vmatrix} V_{pp} & V_{pt} \\ V_{qp} & V_{qt} \end{vmatrix}}{\begin{vmatrix} V_{pp} & V_{pq} \\ V_{qp} & V_{qq} \end{vmatrix}}; \quad \frac{\partial q^*}{\partial \tau} = -\frac{\begin{vmatrix} V_{pp} & V_{p\tau} \\ V_{qp} & V_{q\tau} \end{vmatrix}}{\begin{vmatrix} V_{pp} & V_{pq} \\ V_{qp} & V_{qq} \end{vmatrix}}; \quad \frac{\partial q^*}{\partial n} = -\frac{\begin{vmatrix} V_{pp} & V_{pn} \\ V_{qp} & V_{qn} \end{vmatrix}}{\begin{vmatrix} V_{pp} & V_{pq} \\ V_{qp} & V_{qq} \end{vmatrix}}.$$

## References

- [1] Allen, F., 1984. Reputation and product quality. *RAND Journal of Economics*, 15, 311–27.
- [2] Banker, R.D., Khosla, I., Sinho, K.K., 1998. Quality and competition. *Management Science*, 44, 1179–1192.
- [3] Brekke, K.R., Nuscheler R., Straume O.R., 2006. Quality and locational choices under price regulation. *Journal of Economics and Management Strategy* 15, 207–227.
- [4] Brekke, K.R., Nuscheler, R., Straume, O.R., 2007. Gatekeeping in health care. *Journal of Health Economics*, 26, 149–170.
- [5] Calem, P.S., Rizzo, J.A., 1995. Competition and specialization in the hospital industry: an application of Hotelling's location model. *Southern Economic Journal*, 61, 1182–1198.
- [6] Cohen, A., Mazzeo, M.J., 2007. Investment strategies and market structure: an empirical analysis of bank branching decisions. Mimeo, Kellogg School of Management, Northwestern University.

- [7] D'Aspremont, C., Gabszewicz, J.J., Thisse, J.-F., 1979. On Hotelling's stability in competition. *Econometrica*, 47, 1145–1150.
- [8] Degryse, H., Irmen, A., 2001. Attribute dependence and the provision of quality. *Regional Science and Urban Economics*, 31, 547–569.
- [9] Dick, A.A., 2007. Market size, service quality, and competition in banking. *Journal of Money, Credit and Banking*, 39, 49–81.
- [10] Domberger, S., Sherr, A., 1989. The impact of competition on pricing and quality of legal services. *International Review of Law and Economics*, 9, 41–56.
- [11] Dranove, D.D., Shanley, M., Simon, C., 1992. Is hospital competition wasteful? *Rand Journal of Economics*, 23, 247–262.
- [12] Economides, N., 1993. Quality variations in the circular model of variety-differentiated products. *Regional Science and Urban Economics* 23, 235–257.
- [13] Gaynor, M., 2006. What do we know about competition and quality in health care markets? *Foundations and Trends in Microeconomics*, Vol. 2, Issue 6.
- [14] Gravelle, H., 1999. Capitation contracts: access and quality. *Journal of Health Economics* 19, 1067–1088.
- [15] Klein, B., Leffler, K., 1981. The role of market forces in assuring contractual performance. *Journal of Political Economy*, 89, 615–41.
- [16] Kranton, R.E., 2003. Competition and the incentive to produce high quality. *Economica*, 70, 385–404.
- [17] Ma, C.A., Bruggess Jr, J.F., 1993. Quality competition, welfare and regulation. *Journal of Economics* 58, 153–173.
- [18] Mazzeo, M.J., 2003. Competition and service quality in the U.S. airline industry. *Review of Industrial Organization*, 22, 275–296.



- [19] Propper, C., Burgess, S., Green, K., 2004. Does competition between hospitals improve the quality of care? Hospital death rates and the NHS internal market. *Journal of Public Economics*, 88, 1247–1272.
- [20] Sari, N., 2002. Do competition and managed care improve quality? *Health Economics*, 11, 571–584.
- [21] Shapiro, C., 1983. Premiums for high quality products as returns on reputation. *Quarterly Journal of Economics*, 98, 659–80.
- [22] Spence, M., 1975. Monopoly, quality and regulation. *Bell Journal of Economics* 6, 417–429.
- [23] Wolinsky, A., 1997. Regulation of duopoly: Managed competition vs regulated monopolies. *Journal of Economics & Management Strategy*, 6, 821–847.



# NHH

---

**Norges  
Handelshøyskole**

Norwegian School of Economics  
and Business Administration

NHH  
Helleveien 30  
NO-5045 Bergen  
Norway

Tlf/Tel: +47 55 95 90 00  
Faks/Fax: +47 55 95 91 00  
[nhh.postmottak@nhh.no](mailto:nhh.postmottak@nhh.no)  
[www.nhh.no](http://www.nhh.no)