Discussion paper

Public goods production and private sector productivity

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Abstract: In this paper we study how the use of resources in the public sector affects industrial structure, the size and the productivity in knowledge-intensive clusters in local communities. We also discuss how these considerations should be implemented in cost-benefit assessments of local public goods supply. The topics are studied in a setting where there are gains from agglomeration in knowledge-intensive industries, creating clusters of firms in such industries. We find that the primary effect is a Rybczynski effect: If production in the public sector is knowledge-intensive, the size of the knowledge-intensive private industry declines when the public sector increases its production. If, on the other hand, public sector production uses relatively much unskilled labour, increased public goods production leads to higher production in the knowledge-intensive private industries. Private sector productivity is affected in the same way as production: If production in the knowledge-intensive industry increases, so does its productivity due to agglomeration effects; leading to higher wages for highly skilled labour.

1. Introduction

This paper studies how the use of resources for public sector purposes affects the size, structure and productivity in the private sector of the economy. There is concern in many countries and local communities that public sector activities may crowd out important private sector firms and industries. Some observers and policy-makers, however, argue that public sector jobs may be complementary to private-sector employment, and accordingly that a large public sector, properly designed, can have positive effects on the size and productivity of the private sector. The purpose of the paper is to study the interaction between crowding-out and possible complementarities in a setting where there are agglomeration gains in the private sector.

The analysis is relevant in several contexts. One is the effects related to the problems of an ageing population. Many people fear that the ageing of populations that we are witnessing in a range of countries - illustrated by the old-age dependency ratios in figures 1 and 2 - could have a negative effect on industrial productivity. Undoubtedly, as people grow older they require more care and health services. Many, in fact most, of these services are publicly provided and so an aging population implies a larger public sector. Estimations show that public employment in the Norwegian health sector will more than double between 2020 and 2060 (V.O. Nielsen (2008)). Without growth in the labour stock, increased public sector employment leads to decreased private sector employment. If this reduces industrial productivity, an ageing population is indeed a threat to industrial productivity.
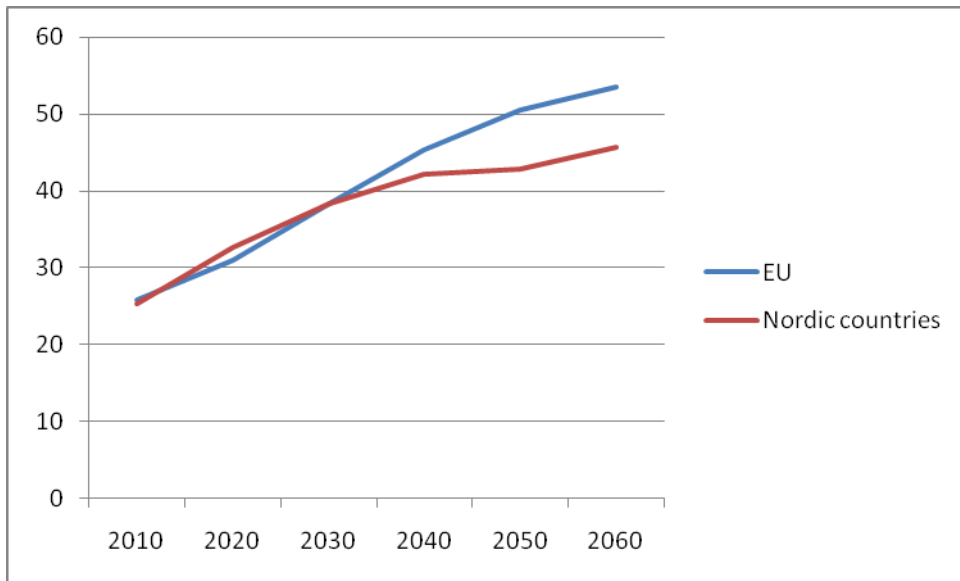
Figure 1: Projected old-age dependency ratio in the EU and the Nordic countries[1] (Eurostat)
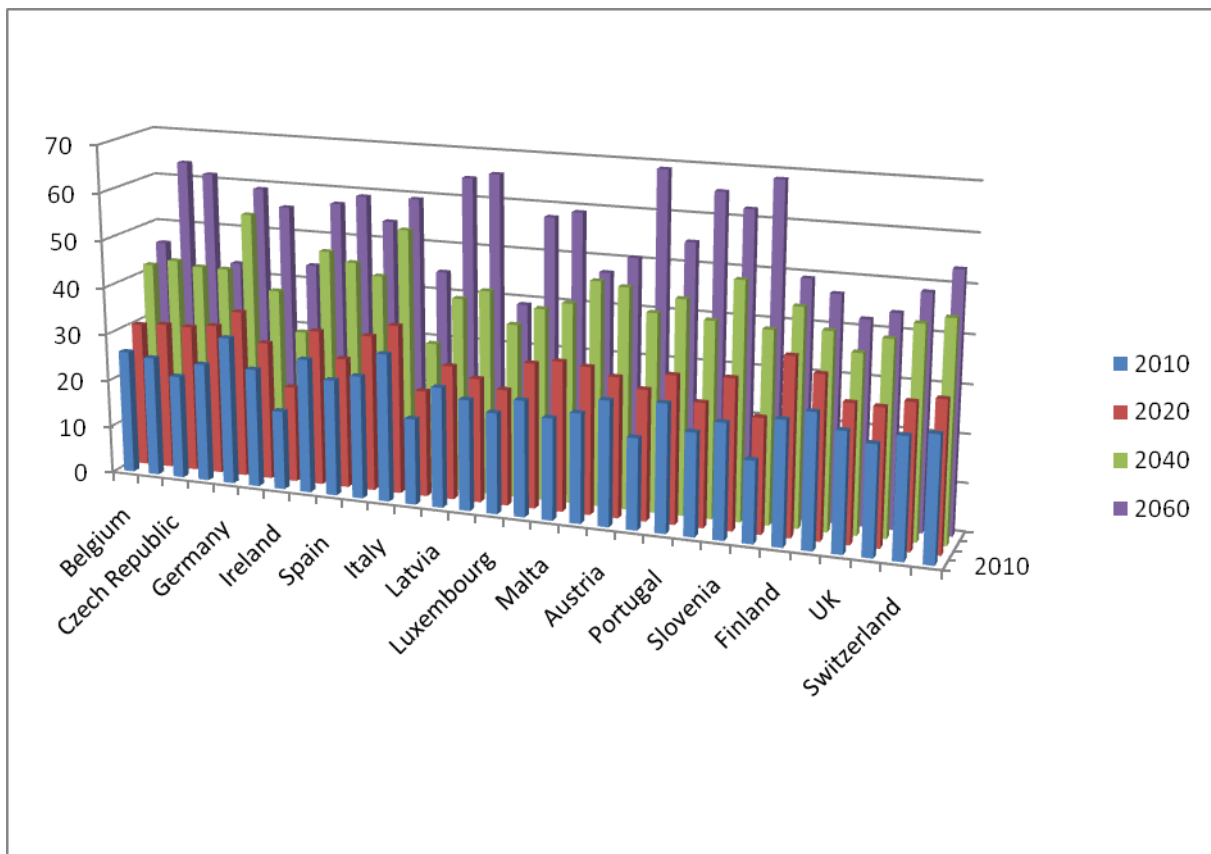

Figure 2[2]: Projected old-age dependency ratio in the EU and the Nordic countries (Eurostat)

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[1] The Nordic countries: Denmark, Finland, Norway and Sweden.
[2] From left to right the countries are Belgium, Bulgaria, Czech Republic, Denmark, Germany, Estonia, Ireland, Greece, Spain, France, Italy, Cyprus, Latvia, Lithuania, Luxembourg, Hungary, Malta, the Netherlands, Austria, Poland, Portugal, Romania, Slovenia, Slovakia, Finland, Sweden, UK, Norway, Switzerland, the EU

Another relevant context is that of regional policy. It is sometimes argued[3] that local governments should try to foster knowledge-intensive industrial agglomerations by establishing more public jobs for highly educated individuals – the presumption being that this will attract more highly educated people to the region and thus benefit the private sector as well.

The paper shows that the intuition is wrong in both cases. If care for the elderly is intensive in the use of low-skilled labour, growth in public care will have a positive effect on knowledge-intensive industrial agglomerations and thus on private-sector productivity. Conversely, if local governments try to attract highly educated people to a region by establishing more public jobs for such people, they will fail. More public jobs for highly skilled workers will lower their wage and thus make it less attractive for such individuals to move to a region.

The reason in both cases is that the first-order effect of public sector resource use on the private sector is a Rybczynski effect (Rybczynski (1955)). If the government hires unskilled workers, and thus reduces the supply of such workers to the private sector, the effect will be a reduction in production and employment in low-skilled private firms and growth in production and employment in high-skilled industries. Conversely, public-sector employment of highly skilled workers will have a negative effect on high-skilled private industry and a positive effect on low-skilled firms. This first-order effect is magnified by industrial agglomeration forces, which also leads to effects on private-sector productivity.

The model used in the paper has two factors of production, skilled and unskilled labour. These are used in three production sectors – one public and two private (a knowledge-intensive and a sector which uses unskilled labour intensively). There are constant returns to scale at the firm level[4], but there are external economies of scale in the knowledge-intensive sector.

In section 2 we develop the basic model. Equilibrium conditions are derived in section 3. In section 3 we also analyse how public sector production affects industrial structure in the private sector. In section 4 we perform a welfare-analysis and discuss how the obtained

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[3] See e.g. St.meld nr 17 (2002-2003), p 17
[4] This implicitly assumes that other factors, such as capital, are freely traded at a fixed price and thus can be netted out.

results should be implemented in cost-benefit analyses of public goods provision. Possible extensions are pointed out in section 5. Section 6 provides a brief summary.

2. The basic model

We consider a small, open economy in which there are three sectors of production: Two private and a public sector.

There are two factors of production - skilled and unskilled labour[5]. Every inhabitant is a worker and each worker (whether skilled or unskilled) inelastically supplies one unit of labour. Total labour supply thus equals the total number of inhabitants in the economy and regional labour supply equals the total number of inhabitants in a region.

Firms in the private industries produce homogenous consumption goods sold in perfectly competitive world markets at given prices. One of the industries is knowledge-intensive *and* experiences external economies of scale (see e.g. Audretsch and Feldman (1996) for the importance of external economies in knowledge-intensive industries and Ottaviano and Puga (1998) or Fujita and Thisse (2002) for overviews of the new economic geography literature). The other industry produces subject to constant returns to scale at both firm and industry levels.

In the public sector, locally consumed goods and services are produced.

Our main question is: How does public sector production affect the private sector; in particular, the industrial structure, the size and the productivity in the private sector?

2.1. The private sector

In the private sector there are two industries, each composed of a large number of firms. We label the two industries 1 and 2. Firms in both industries produce homogenous consumption goods with constant returns to scale at the firm level. There are two factors of production - skilled and unskilled labour. The two industries differ in their skill-intensiveness[6] of production, with industry 1 being the most skill-intensive industry. In the skill-intensive industry 1 there are external economies of scale in production.

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[5] We assume that unskilled workers cannot become skilled or vice versa.
[6] We use the terms skill-intensive and knowledge-intensive interchangeably

Total production in industry 1, x_1, is given by the aggregate production function

(1) $x_1 = \varphi(x_1)\alpha(l_1^s, l_1^u)$,

where l_1^s and l_1^u are the numbers of skilled and unskilled labour, respectively, used to produce good 1. $\varphi(x_1)$ captures the external economies of scale. It is an increasing and concave function; $\varphi'(x_1) > 0$, $\varphi''(x_1) < 0$.

Total production of good 2, x_2, is given by the aggregate production function

(2) $x_2 = \beta(l_2^s, l_2^u)$.

Both $\alpha(l_1^s, l_1^u)$ and $\beta(l_2^s, l_2^u)$ inhibit constant returns to scale.

To simplify the model we assume fixed coefficients in the production of each good, and let the aggregate production functions be

(3) $x_1 = \varphi(x_1)\min(A_1^s l_1^s, A_1^u l_1^u)$,

(4) $x_2 = \min(A_2^s l_2^s, A_2^u l_2^u)$,

where A_i^j are constants, $i = 1,2$; $j = s,u$

Private sector demand for skilled and unskilled labour, l_d^s and l_d^u follows as

(5) $l_d^s = \gamma(x_1)a_1^s x_1 + a_2^s x_2$,

(6) $l_d^u = \gamma(x_1)a_1^u x_1 + a_2^u x_2$; $\qquad a_i^j \equiv \dfrac{1}{A_i^j}$; $\gamma(x_1) \equiv \dfrac{1}{\varphi(x_1)}$; $i = 1,2$; $j = s,u$

Let l^s and l^u be the supplies of skilled and unskilled labour facing firms in the private sector. Equality of labour supply and private sector labour demand is given by

(7) $\quad l^s = \gamma(x_1)a_1^s x_1 + a_2^s x_2$,

(8) $\quad l^u = \gamma(x_1)a_1^u x_1 + a_2^u x_2$.

Skilled and unskilled labour are the only factors of production, and so total costs of producing good i are

(9) $\quad TC_i = w^s l_i^s + w^u l_i^u$,

where w^s and w^u are the wage rates of skilled and unskilled labour, respectively.

Define unit costs of a *single firm* as

(10) $\quad b_i(w^s, w^u) \equiv a_i^s w^s + a_i^u w^u$; $i=1,2$.

Unit costs of the entire *industries* are

(11) $\quad \gamma(x_1)b_1(w^s, w^u)$,

(12) $\quad b_2(w^s, w^u)$

for industry 1 and 2 respectively.

Private firms sell their goods in perfectly competitive world markets with no trade costs. Each firm equates marginal cost to the prevailing market price of the good. There are constant returns to scale at the firm level and hence marginal cost equals unit cost. Equilibria in the goods markets are therefore given by

(13) $\quad \gamma(x_1)b_1(w^s, w^u) = p_1$,

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(14) $b_2(w^s, w^u) = p_2$,

i.e. unit costs equal the prices of the goods, where p_i is the price of good i.

2.2. The public sector

In the public sector locally consumed goods and services are produced by skilled and unskilled labour. The production process is subject to constant returns to scale. We assume constant coefficients in the production process.

Public sector labour demand is

(15) $l_g^s = a_g^s g$,

(16) $l_g^u = a_g^u g$,

where g is the amount of publicly produced goods and services. The amount of publicly produced goods and services, g, is determined by local governments, and the size of g indirectly determines public sector employment, l_g^s and l_g^u.

The supplies of skilled and unskilled labour facing firms in the private sector are

(17) $l^s = n^s - a_g^s g$,

(18) $l^u = n^u - a_g^u g$.

n^s and n^u denote the total supplies of skilled and unskilled labour in the economy.

3. Equilibrium

General equilibrium obtains when labour and goods markets clear.

Labour markets clear when equations (7), (8), (15) and (16) hold. Equations (15) and (16) show how much labour the public sector needs in order to be able to produce the desired amount of public goods. Equations (7) and (8) give the conditions for private sector labour market equilibria (i.e. private firms' labour demand equals the labour supply facing private firms).

Goods market equilibrium is obtained when equations (13) and (14) hold.

We perform a two-fold general equilibrium analysis. First, we assume that there is no production in the public sector. This is done so that we may establish the basic general equilibrium conditions. Thereafter, we introduce public goods production in order to study the effects of public sector production on the productivity and production structure in the private sector.

3.1. Equilibrium with no public sector activity

With no public sector activity, there are firms in two industries producing homogenous consumption goods sold in perfectly competitive world markets. General equilibrium requires that the markets for skilled and unskilled labour clear (as given by equations (7) and (8)), and that the goods markets clear (as given by equations (13) and (14)).

Labour supplies are fixed and with no public sector production the supplies of skilled and unskilled workers facing private firms equal the total number of skilled and unskilled workers in the economy, l^s equals n^s and l^u equals n^u. We may then solve equations (7) and (8) to find the volumes of private goods production

(19) $\quad n^s = \gamma(x_1)a_1^s x_1 + a_2^s x_2$ and $n^u = \gamma(x_1)a_1^u x_1 + a_2^u x_2 \implies (\hat{x}_1, \hat{x}_2)$.

Goods markets equilibria are given by equations (13) and (14). Inserting for the volumes of private goods production from (19) we solve these to find the wage rates of skilled and unskilled labour

(20) $\gamma(x_1)b_1(w^s, w^u) = p_1$ and $b_2(w^s, w^u) = p_2 \Rightarrow w^j(p_1, p_2, \hat{x}_1); j = s, u$.

The equilibrium conditions are illustrated in figures 3 and 4.

$\hat{w}^s$ *gure 3: Equilibrium wage rates*
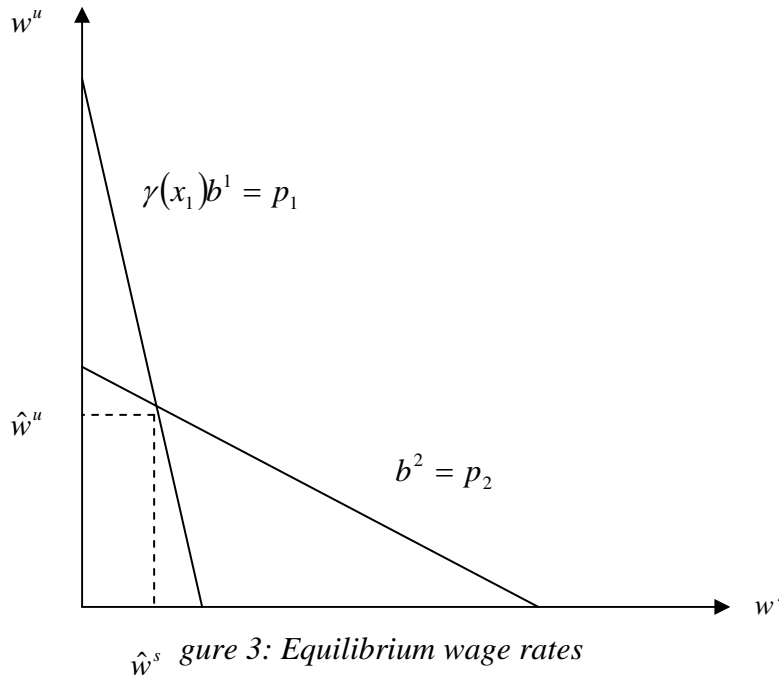
In figure 3 we measure the wage rate of skilled workers along the horizontal axis and the wage rate of unskilled workers along the vertical axis. The curves illustrate the conditions for goods market equilibrium; i.e. unit costs equal goods prices. The slope of the unit cost contour of industry i is $-(A_i^u / A_i^s)$. (The unit cost contours are straight lines due to the assumption of fixed coefficients in production). Equilibrium wage rates, $\hat{w}^s$ and $\hat{w}^u$, are found at the intersection of the two curves.

The location of the unit cost contour of industry 1 depends upon the size of the industry. This contrasts standard Heckscher-Ohlin models and is due to the assumption of external economies of scale in the industry; the larger the industry, the higher the productivity. Thus, unit costs of the industry as a whole decreases with the size of the industry. The unit cost contour of industry 1 moves to the north-east with increased industry 1 production. We see

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that the wage rate of skilled labour is an increasing function of the volume of industry 1 production, the wage rate of unskilled labour is a decreasing one.
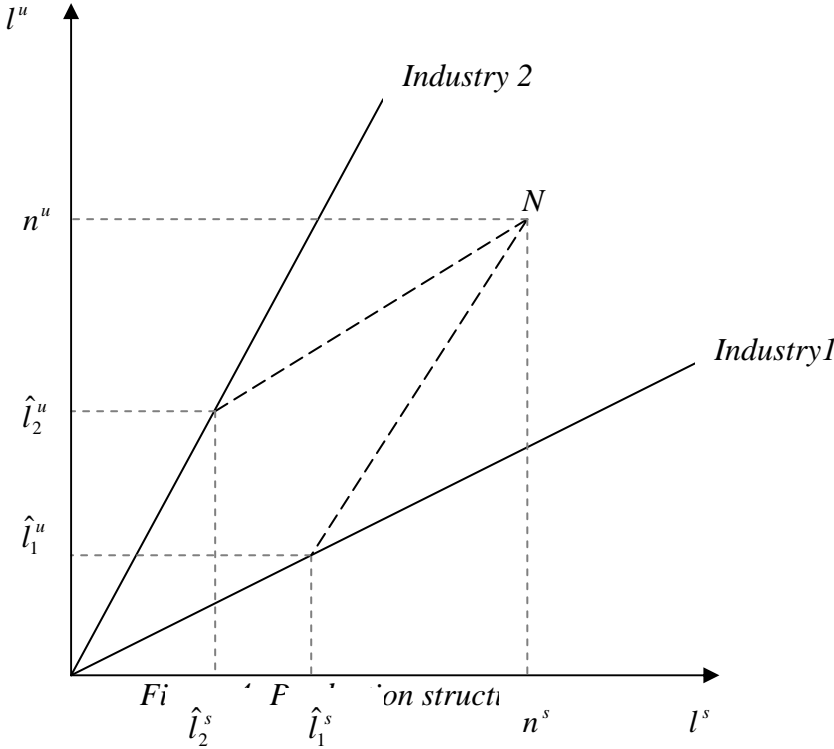
In figure 4 we measure the number of skilled workers along the horizontal axis and the number of unskilled workers along the vertical. The curves show the labour input requirements of the two industries. Total labour supply is given by (the point) N. Industry 1 employs $\hat{l}_1^s$ number of skilled workers and $\hat{l}_1^u$ number of unskilled workers. Industry 2 employs $\hat{l}_2^s$ number of skilled workers and $\hat{l}_2^u$ number of unskilled workers. The size of the two industries (volume of production) follows from the levels of employment.

3.2. Equilibrium with public sector production

In the public sector locally consumed goods are produced by skilled and unskilled labour. We start the analysis by taking the amount of public goods production, g, to be fixed. I.e. we do not consider how the amount of publicly produced goods and services is determined. Public sector labour employment is then given by the volume l^s blic goods production (the size of g). We will return to the question of optimal public goods supply in section 4. Compared to the analysis of section 2.1, the labour supply facing firms in the private sector is reduced by the level of public sector employment.

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Our aim is to study how the structure of production and factor payments (which follow factor productivity) in the private sector are affected by the use of resources for public goods production.

3.2.1. Production structure effects

In order to make the analysis as simple as possible, we define a new variable, z_1, defined as

(21) $z_1 \equiv \gamma(x_1)x_1$,

z_1 shows industry 1 production adjusted for the external economies, i.e. how much firms in industry 1 would have produced if the industry had *not* experienced external economies of scale.

Inserting for z_1 into the equations for labour market equilibrium, equations (7) and (8), gives us

(22) $l^s = a_1^s z_1 + a_2^s x_2$,

(23) $l^u = a_1^u z_1 + a_2^u x_2$.

Manipulating (22) and (23) allow us to express z_1 in terms of production coefficients and labour supply in the private sector

(24) $z_1 = \dfrac{1}{D}\left(a_2^s l^u - a_2^u l^s\right)$; $D \equiv a_1^u a_2^s - a_1^s a_2^u < 0$.

Differentiating (24) with respect to g gives

(25) $\dfrac{dz_1}{dg} = \dfrac{1}{D} a_g^u a_2^u \left(\dfrac{a_g^s}{a_g^u} - \dfrac{a_2^s}{a_2^u}\right)$.

Whether (dz_1/dg) is positive or negative depends on the skill-intensity in public goods production compared to the skill-intensity in industry 2 production (the least skill-intensive private industry)

(26) $\quad \dfrac{dz_1}{dg} > 0$ iff $\dfrac{a_g^s}{a_g^u} < \dfrac{a_2^s}{a_2^u}$

(27) $\quad \dfrac{dz_1}{dg} < 0$ iff $\dfrac{a_g^s}{a_g^u} > \dfrac{a_2^s}{a_2^u}$.

Next, we differentiate z_1 as given by its definition in equation (21) with respect to the amount of public goods production, g:

(28) $\quad \dfrac{dz_1}{dg} = (\gamma + \gamma' x_1) \dfrac{dx_1}{dg}$,

which gives

(29) $\quad \dfrac{dx_1}{dg} = \dfrac{1}{\left(\dfrac{dz_1}{dg} \right)} (\gamma + \gamma' x_1)$

The sign of (dx_1/dg) is equal to the sign of (dz_1/dg):

(30) $\quad \dfrac{dx_1}{dg} > 0$ iff $\dfrac{a_g^s}{a_g^u} < \dfrac{a_2^s}{a_2^u}$

(31) $\quad \dfrac{dx_1}{dg} < 0$ iff $\dfrac{a_g^s}{a_g^u} > \dfrac{a_2^s}{a_2^u}$.

The size of industry 1 is an increasing function of the volume of public goods production if and only if public goods production is less knowledge-intensive than industry 2 production.

This is the Rybczynski effect. If public goods production is less skill-intensive than production in industry 2, increased public goods production leads to increased relative supply of highly skilled labour to private firms. The increased relative supply of highly skilled labour leads to increased production in the knowledge-intensive industry, as predicted by the Rybczynski theorem.

The effect of public goods production on the size of industry 2 is found by performing the same kind of mathematical reasoning as done through equations (22) to (31). This leads to the following results

$$(32) \quad \frac{dx_2}{dg} > 0 \text{ iff } \frac{a_g^s}{a_g^u} > \frac{a_1^s}{a_1^u}$$

$$(33) \quad \frac{dx_2}{dg} < 0 \text{ iff } \frac{a_g^s}{a_g^u} < \frac{a_1^s}{a_1^u}.$$

The size of industry 2 is an increasing function of the volume of public goods production if and only if public goods production is more skill-intensive than the production process in the most skill-intensive private industry. The Rybczynski effect once again.

When we combine the results given by equations (30) to (33), and bear in mind that industry 1 is more skill-intensive than industry 2, we are left with three possible outcomes regarding the impact of public goods production on the production structure of the private sector, depending on the knowledge-intensity in public sector production.

Case 1: Public goods production is the least knowledge-intensive production process

If production of public goods is the least knowledge-intensive production process, increasing these activities leads to increased production in industry 1 and decreased production in industry 2.

$$(34) \quad \frac{a_1^s}{a_1^u} > \frac{a_2^s}{a_2^u} > \frac{a_g^s}{a_g^u} \quad \Rightarrow \quad \frac{dx_1}{dg} > 0, \frac{dx_2}{dg} < 0.$$

The effects are illustrated in figure 5. Total labour supply is given by N, which, with no public goods production, is allocated between the two private industries as illustrated in figure 4. With public sector production, the labour supply facing private firms is reduced to *N'* where the relative supply of unskilled labour facing private firms is reduced. This leads to increased sector 1 production and decreased sector 2 production, as illustrated by the arrows.
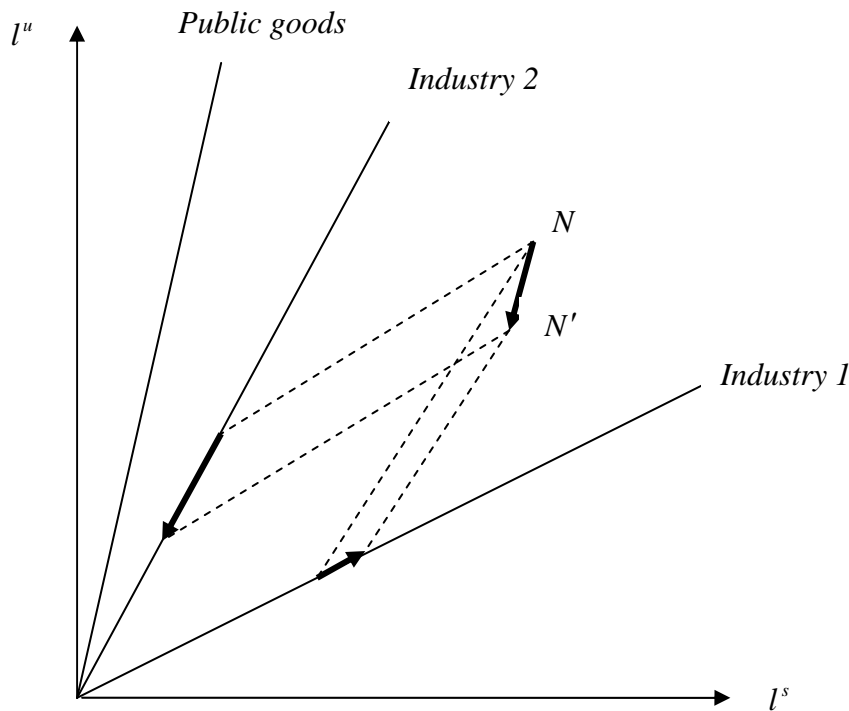

Figure 5: Production structure effects when public sector production is the least skill-intensive

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Case 2: Balanced knowledge-intensity in public sector production

If public sector production is neither extremely skill-intensive nor extremely intensive in the use of unskilled labour, then increasing public goods production leads to an overall reduction in private sector activities:

$$(35) \quad \frac{a_1^s}{a_1^u} > \frac{a_g^s}{a_g^u} > \frac{a_2^s}{a_2^u} \quad \Rightarrow \quad \frac{dx_1}{dg} < 0, \, \frac{dx_2}{dg} < 0.$$

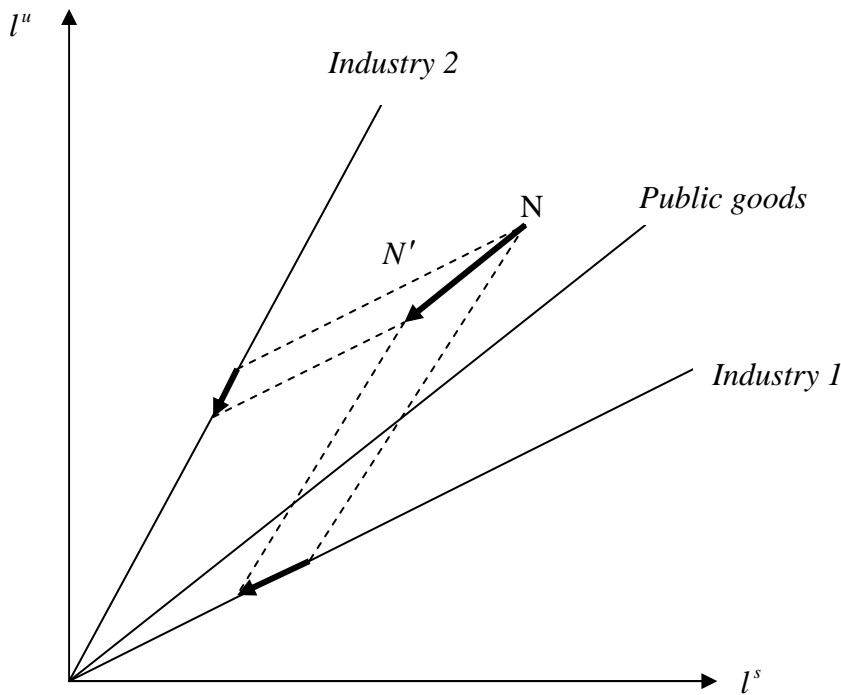This effect is illustrated in figure 6.

Figure 6: Production structure effects with balanced knowledge-intensity in public goods production

Case 3: Public sector production is the most skill-intensive production process

Finally, if public sector production is the most knowledge-intensive production process, increased public goods production leads to a smaller knowledge-intensive and a larger unskilled-intensive private industry:

$$(36) \quad \frac{a_g^s}{a_g^u} > \frac{a_1^s}{a_1^u} > \frac{a_2^s}{a_2^u} \quad \Rightarrow \quad \frac{dx_1}{dg} < 0, \ \frac{dx_2}{dg} > 0.$$
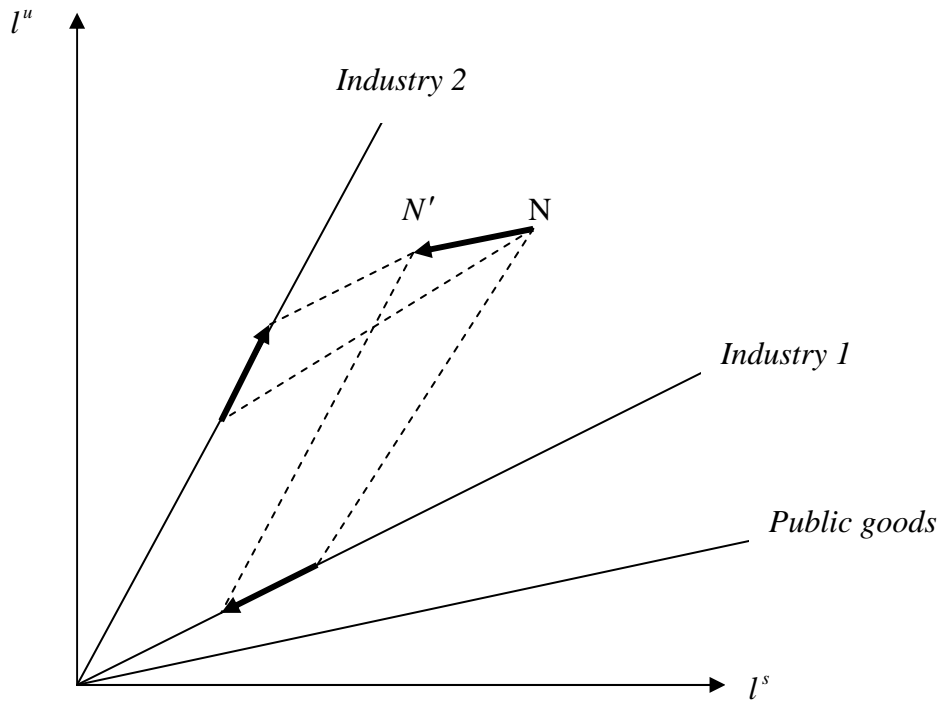

Figure 7: Production structure effects when public sector production is knowledge-intensive

To find the equilibrium wage rates we insert for unit costs into equation (13) and (14), and get

$$(37) \quad \gamma(x_1)\left[w^s a_1^s + w^u a_1^u\right] = p_1 \ ,$$

$$(38) \quad \left[w^s a_2^s + w^u a_2^u\right] = p_2 \ .$$

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Manipulating equations (37) and (38) yields the following expression for the wage rate of skilled labour

$$(39) \quad w^s = \frac{1}{E}\left[\frac{p_2}{a_2^u} - \frac{1}{\gamma(x_1)}\frac{p_1}{a_1^u}\right];$$

Where E is defined as the difference between the knowledge-intensiveness in production of the private goods;

$$E \equiv \frac{a_2^s}{a_2^u} - \frac{a_1^s}{a_1^u} < 0$$
.

Differentiating equation (39) with respect to public sector production, g, yields

$$(40) \quad \frac{dw^s}{dg} = \frac{1}{E}\left[\frac{1}{\gamma^2}\gamma'\frac{p_1}{a_1^u}\frac{dx_1}{dg}\right].$$

We see that

$$(41) \quad \frac{dw^s}{dg} > 0 \text{ iff } \frac{dx_1}{dg} > 0$$

The wage rate of skilled labour is an increasing function of the amount of public goods production if and only if the size of the knowledge-intensive industry increases with increased public goods production. This is due to the agglomeration gains. As the knowledge-intensive industry grows, so does productivity and hence payments to the factor used intensively in production – namely highly skilled labour.

$$(42) \quad \frac{dw^s}{dg} < 0 \text{ iff } \frac{dx_1}{dg} < 0.$$

Conversely, if the size of the knowledge-intensive industry 1 declines as public goods production increases, the wage rate of skilled labour also declines.

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From equations (37) and (38) we find the wage rate of unskilled labour

$$(43) \quad w^u = \frac{1}{F(x_1)} \left[\frac{p_2}{a_2^s} - \frac{1}{\gamma(x_1)} \frac{p_1}{a_1^s} \right]$$

where $F(x_1)$ is defined as

$$F(x_1) \equiv \frac{a_2^u}{a_2^s} - \frac{1}{\gamma(x_1)} \frac{a_1^u}{a_1^s} > 0.$$

In order to find the effect of public goods production on the wage rate of unskilled labour, we differentiate equation (43) with respect to g,

$$(44) \quad \frac{dw^u}{dg} = -\frac{F'}{F^3} \left[\frac{1}{\gamma^2} \gamma' \frac{p_1}{a_1^s} \frac{dx_1}{dg} \right]; \quad F' = \frac{1}{\gamma^2} \gamma' \frac{a_1^u}{a_1^s} \frac{dx_1}{dg},$$

and find

$$(45) \quad \frac{dw^u}{dg} > 0 \text{ iff } \frac{dx_1}{dg} < 0,$$

$$(46) \quad \frac{dw^u}{dg} < 0 \text{ iff } \frac{dx_1}{dg} > 0.$$

Combining the results of equations (41), (42), (45) and (46), we find the total effect of public goods production on the wage rates. Two possibilities arise: Either the wage rate of skilled labour increases and the wage rate of unskilled decreases. This happens if public goods production leads to increased production in the skill-intensive industry. Or the wage rate of skilled workers decreases and the wage rate of unskilled workers increases. This is the case if increased public goods production leads to lower production in the skill-intensive private industry.

3.2.2 Wage effects

As we have seen, the use of resources in the public sector affects not only production structures but also factor payments. To find the link between knowledge-intensiveness of production and the wage effects we combine the above result with equations (34), (35) and (36), resulting in

$$(47) \quad \frac{a_1^s}{a_1^u} > \frac{a_2^s}{a_2^u} > \frac{a_g^s}{a_g^u} \quad \Rightarrow \quad \frac{dw^s}{dg} > 0 , \frac{dw^u}{dg} < 0$$

$$(48) \quad \frac{a_1^s}{a_1^u} > \frac{a_g^s}{a_g^u} > \frac{a_2^s}{a_2^u} \quad \vee \quad \frac{a_g^s}{a_g^u} > \frac{a_1^s}{a_1^u} > \frac{a_2^s}{a_2^u} \quad \Rightarrow \quad \frac{dw^s}{dg} < 0 , \frac{dw^u}{dg} > 0 .$$

If and only if the production process in the public sector is the least skill-intensive of all production processes does an increase in public goods production lead to increased wages of skilled labour – as shown by equation (47). Otherwise, public sector activities will lead to reduced wage rates of highly skilled labour and increased wage rates of low-skilled labour – as shown by equation (48).

The two possibilities of public goods production on the wage rates are illustrated in figures 8 and 9. We measure the wage rate of skilled labour along the vertical axis and the wage rate of unskilled labour along the horizontal axis. The unit cost contours of industries 1 and 2 are drawn.

Figure 8 shows the situation in which public goods production leads to increased production in the skill-intensive industry 1. Public goods production is the least skill-intensive of all production processes. As the volume of industry 1 production goes up, so does the productivity in the industry. The increased productivity moves the unit cost contour to the northeast, leading to an increase in the wage rate of skilled labour and a decrease in the wage rate of unskilled labour.
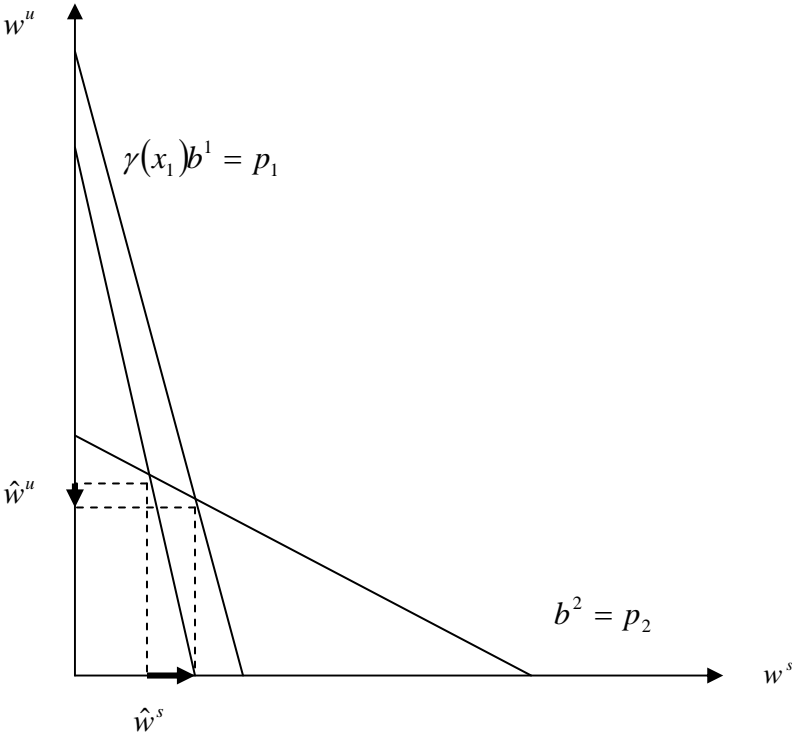

Figure 8: Wage effects of public sector production which is intensive in the use of unskilled labour

Figure 9 illustrates the case in which public goods production leads to a smaller skill-intensive industry 1; which is the case whenever public goods production is more skill-intensive than the least skill-intensive of the private industries. The productivity of industry 1 declines as the volume of production in the industry declines. This leads to a movement of the unit cost contour to the south-west. The wage rate of skilled labour goes down and the wage rate of unskilled labour goes up.
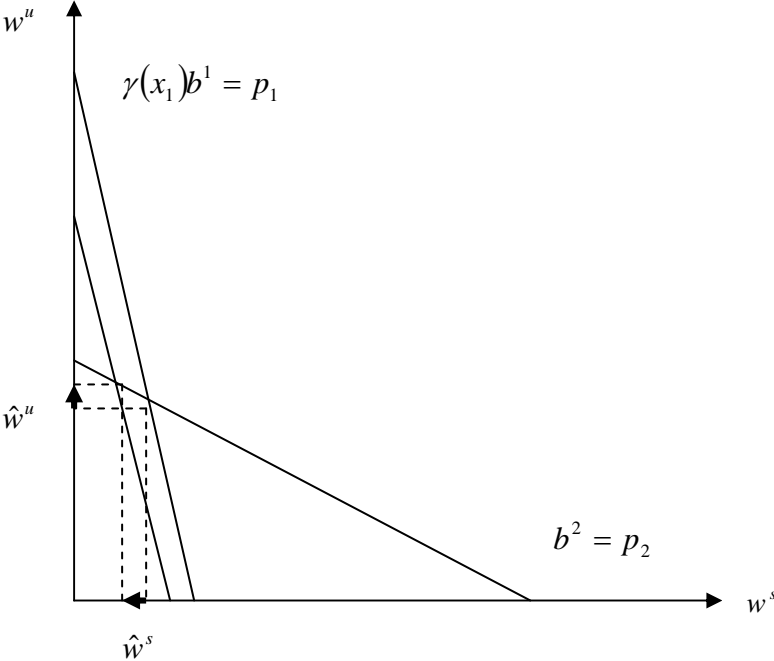

Figure 9: Wage effects of skill-intensive public sector production

4. Utility and welfare

Utility derives from the consumption of private and public goods. The utility of individual k is a function of the amounts consumed of the two private goods and the provision of public goods. We assume that everyone shares the same utility *function*. The utility of individual k is

(49) $\quad U^k = \left(c_1^k, c_2^k, g\right)$,

where c_i^k is individual k's consumption of good i.

Individuals make optimising choices regarding the consumption of private goods. Utility can therefore be expressed by the indirect utility function

(50) $\quad V^k = y^k + f^k(g); \left(f^k\right)' > 0; \left(f^k\right)'' < 0$

Preferences are assumed to be quasi-linear. $f^k(g)$ is the utility person k gets from consuming public goods. Labour income is assumed to be the only source of income, and y^k is the income net of taxes of individual k. We do not consider the question of how public goods production could or should be financed, but simply assume that it is financed through a lumpsum tax.

The optimal amount of public sector production depends on whether the goods produced are pure public goods or publicly provided private goods. In the following analysis we will assume that the goods are pure public goods with no rivalry in consumption, and in chapter 4 briefly discuss how the results would be altered if the goods produced in the public sector were private goods.

Welfare is the sum of all individuals' utility

(51) $\quad W = \sum_{k=1}^{n} V^k = y + \sum_{k=1}^{n} f^k(g); \; y \equiv \sum_{k=1}^{n} y^k \quad .$

Finding the optimal amount of public goods production amounts to choosing the g which maximises equation (51)

$$(52) \quad \max_g W = \max_g \left[y + \sum_{k=1}^{n} f^k(g) \right].$$

The sum of all individuals' labour income must equal the value of production in the private sector,

$$(53) \quad y = p_1 x_1 + p_2 x_2.$$

Differentiating y with respect to g gives

$$(54) \quad \frac{dy}{dg} = p_1 \frac{dx_1}{dg} + p_2 \frac{dx_2}{dg}.$$

Equilibrium wage rates of skilled and unskilled labour equals the value of the marginal product of skilled and unskilled labour, respectively. From equations (1) and (2) we therefore know that

$$(55) \quad p_1 \varphi \alpha'_s = w^s = p_2 \beta'_s,$$

$$(56) \quad p_1 \varphi \alpha'_u = w^u = p_2 \beta'_u,$$

where

$$\alpha'_j \equiv \frac{\partial \alpha(l_1^s, l_1^u)}{\partial l_1^j}; \ j = s, u$$

$$\beta'_j \equiv \frac{\partial \beta(l_2^s, l_2^u)}{\partial l_2^j}; \ j = s, u$$

From equation (1) we also find how the production of goods in the knowledge-intensive sector is affected by public goods production,

$$(57) \quad \frac{dx_1}{dg} = \alpha\varphi' \frac{dx_1}{dg} + \varphi\alpha'_s \frac{dl_1^s}{dg} + \varphi\alpha'_u \frac{dl_1^u}{dg},$$

which gives

$$(58) \quad \frac{dx_1}{dg} = \left(\frac{1}{1-\alpha\varphi'}\right)\left[\varphi\alpha'_s \frac{dl_1^s}{dg} + \varphi\alpha'_u \frac{dl_1^u}{dg}\right].$$

Similarly, from equation (2) we find how the production of goods in industry 2 is affected by public goods production,

$$(59) \quad \frac{dx_2}{dg} = \beta'_s \frac{dl_2^s}{dg} + \beta'_u \frac{dl_2^u}{dg}.$$

Inserting from equations (55), (56), (58) and (59) into (54) gives

$$(60) \quad \frac{dy}{dg} = \left[w^s\left(\frac{dl_1^s}{dg} + \frac{dl_2^s}{dg}\right) + w^u\left(\frac{dl_1^u}{dg} + \frac{dl_2^u}{dg}\right)\right] + \left(\frac{\alpha\varphi'}{1-\alpha\varphi'}\right)\left[p_1\varphi\alpha'_s \frac{dl_1^s}{dg} + p_1\varphi\alpha'_u \frac{dl_1^u}{dg}\right].$$

The total reduction in private sector labour employment must equal the increased employment in the public sector,

$$(61) \quad \frac{dl_g^s}{dg} = -\left(\frac{dl_1^s}{dg} + \frac{dl_2^s}{dg}\right),$$

$$(62) \quad \frac{dl_g^u}{dg} = -\left(\frac{dl_1^u}{dg} + \frac{dl_2^u}{dg} \right).$$

Inserting from (61) and (62) into equation (60) gives

$$(63) \quad \frac{dy}{dg} = -\left[w^s \frac{dl_g^s}{dg} + w^u \frac{dl_g^u}{dg} \right] + \alpha\varphi' \left\{ \left(\frac{1}{1-\alpha\varphi'} \right) \left[p_1 \varphi \alpha_s' \frac{dl_1^s}{dg} + p_1 \varphi \alpha_u' \frac{dl_1^u}{dg} \right] \right\}$$

The term in the last parenthesis equals $p_1(dx_1/dg)$, i.e. the value of the production effects in the knowledge-intensive industry of increased public goods production. Using this fact, we finally arrive at

$$(64) \quad \frac{dy}{dg} = -\left[w^s \frac{dl_g^s}{dg} + w^u \frac{dl_g^u}{dg} \right] + p_1 \alpha\varphi' \frac{dx_1}{dg}$$

Equation (64) shows the marginal cost of public goods production, defined as the amount of foregone consumption of private goods. Total marginal cost is the sum of the direct costs and the indirect costs or cost reductions induced by production of public goods. The direct costs are the wage costs, as shown by the term in parenthesis. The indirect effects on marginal costs arise because of the productivity effects public goods production exert on the private sector. Marginal costs rise if public goods production leads to a smaller knowledge-intensive sector. Whereas they fall if the knowledge-intensive sector grows as public goods production does so. The indirect effect is given by the last term of equation (64).

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Now, return to the welfare maximisation problem of equation (52).

The optimal supply of public goods is such that

$$(65) \quad \frac{dW}{dg} = 0 \Rightarrow -\frac{dy}{dg} = \sum_{k=1}^{n} f^{k'}$$

Equation (65) states the (obvious) condition that the optimal g is such that the total marginal cost of public goods production equals the sum of marginal utilities of public goods consumption.

Inserting for (dy/dg) equation (65) becomes

$$(66) \quad \left[w^s \frac{dl_g^s}{dg} + w^u \frac{dl_g^u}{dg} \right] - \alpha \varphi' p_1 \frac{dx_1}{dg} = \sum_{k=1}^{n} f^{k'}$$,

which is a modified version of the Samuelson rule (Samuelson (1954)). It states that the optimal supply of public goods is such that marginal cost equals the aggregate marginal willingness to pay. Marginal costs are, however, modified to include the indirect costs caused by external economies.

The implications for optimal public goods supply follow: If public goods production leads to decreased productivity in the private sector (the case if knowledge-intensive production is reduced), then the optimal supply of public goods is smaller than in a situation where public goods production has no productivity effect in the private sector. If, on the other hand, public goods production enhances private sector productivity, then optimal public goods supply is larger.

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5. Possible extensions

The model we have developed is a very simplified model. In this chapter we point to some possible extensions and discuss *one* of these in more detail.

In the model there are two factors of production – skilled and unskilled labour. An obvious extension would be to add one or more factors of production, e.g. capital. We have also assumed that local public goods are *consumption* goods. Many local public goods do, however, have direct or indirect production effects, either because the goods are used directly in the production processes of private firms or because they influence the productivity of other production factors. Analysing the case in which the public goods are *production* goods would be another possible extension of the model. Finally, we have assumed that labour is immobile between communities. We now briefly discuss how the results are modified when we allow for migration; i.e. when labour is mobile between communities.

Take the presented model as a starting point, but assume that highly skilled workers are mobile between communities. The supply of highly skilled labour in a community depends on the utility these workers get when living in that specific community as compared to living in other communities. Utility is a function of the consumption of local public goods and the consumption of private goods. The consumption of private goods depends on income/the wage rate, and so utility is a function of the local public goods supply and the regional wage rate,

$$(67) \quad U^s = U^s\left(w^s, g\right).$$

The supply of highly skilled labour in a community, which depends on the difference in utility from living in the specific region compared to living in other regions, may then be written as a function of the regional wage rate and the supply of local public goods:

$$(68) \quad n^s = n^s\left(w^s, g\right);$$

where $n_w^s > 0$ and $n_g^s > 0$.

In order to attract more knowledge-intensive clusters, local governments have *one* policy instrument, namely the provision of local public goods. Increased local public goods supply makes the community more attractive as a place to live, and by doing this governments can attract more highly skilled labour. The change in utility of a highly skilled worker that results from increased public goods supply is

$$(69) \quad \frac{dU^s}{dg} = U_w^s \frac{dw^s}{dg} + U_g^s$$

The first term shows the indirect effect that results because productivity and hence wages are affected. The second term is the direct effect on utility of higher public goods consumption. The optimal supply of local public goods is such that a small increase in local public goods supply will not alter the utility directly, and so the optimum is characterised by

$$(70) \quad U_g^s = 0$$

Inserting into equation (69) gives

$$(71) \quad \frac{dU^s}{dg} = U_w^s \frac{dw^s}{dg}$$

The only effect on utility is the indirect effect which results because wages, and hence private goods consumption, are affected. This means that the results obtained earlier are reinforced.

Assume that public goods production is extremely knowledge-intensive. Then increased public goods production leads to lower wages of highly skilled labour because the productivity in knowledge-intensive private industries falls. The utility of highly skilled workers decreases and some of these will therefore migrate. The total supply of highly skilled labour facing firms in the private industry is further reduced.
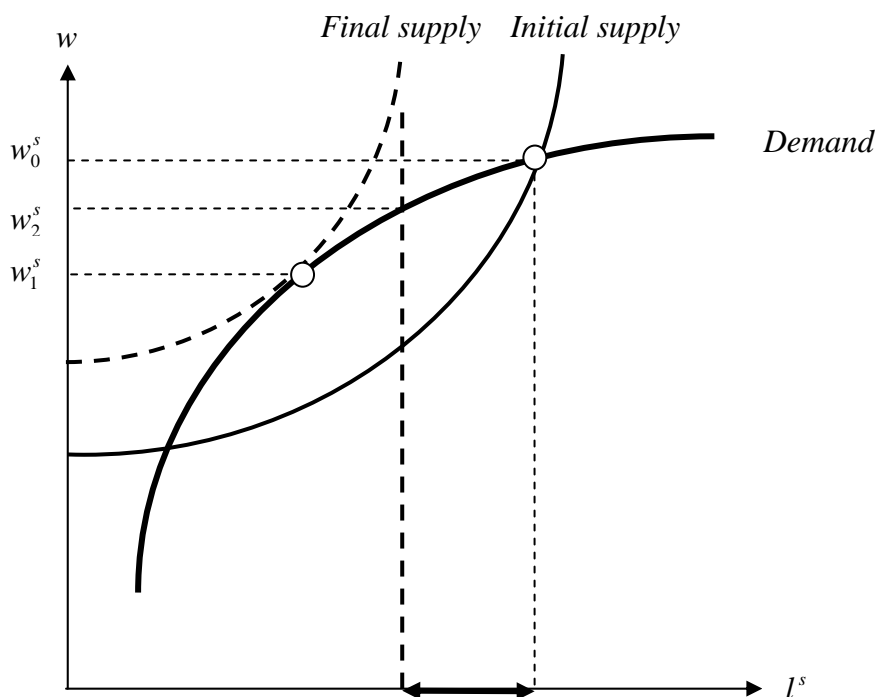
Figure 10: Wage and empl $\bar{l}_1^s$ at ef $\hat{l}_1^s$ of increased public goods production in knowledge-intensive sector when public goods production is knowledge-intensive.

In figure 10 we have illustrated the demand for and supply of highly skilled labour to the knowledge-intensive industry. Demand is an upward-sloping curve due to the agglomeration effects. $\hat{l}_1^s$ is the initial level of employment with the corresponding wage rate w_0^s. If local governments decide on increasing public goods production, the supply of highly skilled labour facing the knowledge-intensive industry is reduced to $\bar{l}_1^s$. The difference between $\hat{l}_1^s$ and $\bar{l}_1^s$ equals the increased need for highly skilled labour in the public sector. If labour supply is given, i.e. labour is immobile, this reduces the wage rate of skilled labour to w_2^s. But when skilled workers are mobile such a reduction in the wage rate induces migration, and so the supply of highly skilled labour is further reduced, as indicated by the shift of the supply curve (from the initial to the final supply curve). The wage rate of highly skilled labour declines further to w_1^s.

6. Summary

In this paper we address the question: How does the use of resources in the public sector affect private goods production, i.e. how does it affect the production structure and productivity? We analyse these questions in a setting where there are gains from geographic agglomeration in knowledge-intensive private industries.

We find that the effect on the production structure is a pure Rybczynski effect: If production of public goods requires much highly skilled labour, then the production of public goods will come at the expense of production of knowledge-intensive private goods. The size of knowledge-intensive clusters in the private sector are reduced. Due to agglomeration effects, the effect on productivity works in the same direction. As the knowledge-intensive industry declines, productivity also declines because the agglomeration gains cannot be exploited to the same degree. The lower productivity is reflected in factor payments; the wage rate of highly skilled labour goes down.

The fact that production of public goods affects productivity in the private sector has implications for the optimal supply of public goods. The Samuelson rule for optimal supply of public goods – marginal costs equals the sum of marginal utilities of public goods – still applies, but the marginal costs of public goods must be modified to account for the productivity costs or gains.

7. References

Audretsch D. B. and M. Feldman (1996). "R&D spillovers and the geography of innovation and production". *American Economic Review* 86, 630-664

Eurostat (2010). http://epp.eurostat.ec.europa.eu

Fujita, M. and J.-F. Thisse (2002), *Economics of agglomeration: Cities, industrial location and regional growth*, Cambridge: Cambridge University Press, 2002

Nielsen, V.O., "Utviklingen i offentlige utgifter til helsetjenester mot 2060", *Økonomiske analyser* 2/2008

Ottaviano, G. I. P. and D. Puga (1998), "Agglomeration in the global economy: A survey of the "new economic geography", *World Economy* 21 (6), August 1998: 707-731

Rybczynski, T. M. (1955), "Factor Endowment and Relative Commodity Prices", *Economica*, November 1955, 336-341

Samuelson, P. A. (1954), "The pure theory of public expenditure" *Review of Economics and Statistics* 36, 387-389

St.meld. nr.17 (2002-2003), "Om statlige tilsyn"

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