Redistributive taxation and the household: the case of individual filings*

Fred Schroyen†

Dept of Economics, Norwegian School of Economics & Business Administration

Helleveien 30, N-5045 Bergen, Norway

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Abstract

In this paper I look at the tax treatment of households under individual filings and characterise the efficiency properties of an income tax schedule that redistributes from rich to poor households. Because tax liabilities are determined on individual incomes but the decision to earn those incomes are made at the household level, the tax liable members of the same household can side trade leisure for net income with one another, and such side trade enables them to carry out tax arbitrage. I analyse the problem for a two class economy both with and without perfect assortative mating. The main conclusion is that the prevention of tax arbitrage imposes structure on the graduation of the tax schedule.

Keywords: individual filing, optimal income taxation, redistribution, tax arbitrage, household production.

JEL-code: H21, H31.

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†E-mail address: fred.schroyen@nhh.no

1 Introduction

Comparative tax studies for advanced economies (OECD: Messere, 1993, EU: O'Donoghue & Sutherland, 1999) show that many countries consider the individual as the tax unit. Most OECD countries apply this principle to some extent, and many recent tax reforms have substituted the family for the individual as the tax unit. What is the position of optimal income taxation theory in this respect? The standard model, as exposed by Mirrlees (1971) or Stiglitz (1982), typically assumes that the agent who makes the decision to earn an income is also the one who bears the legal tax incidence. Thus, it is well suited to discuss the taxation of singles or the taxation of household income. But when labour earnings are the result of a household decision *and* taxed at the individual level, the standard model becomes deficient and needs amendment. In this paper, I take the principle of individual taxation as given, and ask how an income tax system should treat labour income when the earnings decisions are made by a small coalition of people, as in a household with two income earners. In particular, I characterise the efficiency properties of a redistributive income tax system that satisfies such a household incentive compatibility requirement.

Since my aim is not to carry out a comparative analysis of family taxation regimes (joint *vs* individual), I ignore differences in labour supply elasticities of different household members, or the possible divergences between social and household preferences for redistribution.[1] Rather, I want to focus on an incentive problem that has been left out of the discussion so far, namely the tax arbitrage possibilities that an individually based tax system may offer to households.

The mechanism through which such arbitrage can occur is simple: within the household, its members can exchange leisure for consumption (disposable income) by rescheduling duties in housework and by compensating each other for this. Such a side trade in leisure for money can easily be enforced by the presence of trust among the household members. When the government designs the tax schedule, it should take into account that people may engage

[1] Boskin (1975) and Boskin & Sheshinski (1983) were the first to investigate the question whether the individual or the household is the apporpriate tax unit. They provide Ramsey-type arguments against the practice of income splitting where husband and wife face the same marginal tax rate: the higher wage elasticity of the secondary worker warrants a lower marginal tax rate. Apps & Rees (1988, 1997a, 1997b) build on the Boskin-Sheshinki model by introducing redistributional concerns within and across households and by allowing for production within the household of a private household good. In all these models, the marginal income tax rate is taken constant across income levels, but possibly conditioned on gender. Munnell (1980) discusses the incongruence of three basic axioms (progressivity, horizontal equity and neutrality w.r.t. marriage) in tax design.

into this kind of arbitrage. Put differently, it should make the tax system *household* incentive compatible. I will argue that the household compatibility constraints, together with the skewness of the income distribution put structure on the graduation of the optimal marginal tax rate. To formalise my argument, I build on the standard income taxation model for a two-class economy as exposed by Stiglitz (1982), and then amend this model to allow for the fact that within a household people can co-ordinate their labour market and homework decisions.

It is not so difficult to understand why such co-ordination needs to be taken into account. Consider a two-class economy and suppose that the government has designed an individually incentive compatible tax system, so that a high ability person prefers to earn a high rather than low income level and pay the corresponding tax. Now, suppose in addition that two high ability persons form a household. They could agree that one of them takes a part time job on the labour market, earning a low income level with a low tax liability, and that the other earns a high income level with a high tax liability. To the extent that the individual incentive compatibility constraint is slack, the low income earner will initially enjoy a lower utility level. But the fact that both agents have taken a different stance in the labour market makes their marginal rates of substitution between disposable income and leisure differ. The wider this difference, the larger the opportunities for mutually improving side trades. For sufficiently large side trading opportunities, both members earning a high income level (and paying high taxes) will turn out to be a dominated strategy.

There are two ways in which the government can contain side trade opportunities. One is to distort the labour supply of high ability agents downwards, i.e. to impose a positive marginal tax rate on high income levels. A second way is to make the initial welfare loss to a high ability person when earning a low income level sufficiently high, and this can be done by distorting low incomes downwards. The social cost of distorting a bundle is directly proportional to the number of people opting for it. The social benefit, on the other hand, is the reduction in informational rent and is proportional to the number of people that are discouraged from opting for the bundle. This means that the optimal size of the distortion depends on the number of people whose behaviour is distorted *vs* the number who are discouraged. And this relative number depends in turn on the direction of redistribution and the skewness of the ability distribution. That is why household incentive compatibility puts structure on the graduation of the income tax schedule.

The paper relates to the literature on taxation with side trade opportunities. In that literature, one is concerned with the design of a tax system when perfect side trading opportunities exist, either through multilateral coalition formation (Hammond, 1987) or by trading on perfectly competitive side markets (Guesnerie, 1995, ch 1).[2] For both cases, it is shown that the only feasible tax systems that can be designed are linear ones. In an earlier paper (Schroyen, 1997), I characterised an efficient commodity tax system when side trading is limited to take place within bilateral coalitions and investigated how much better this system performs w.r.t. fully linear commodity taxation. The present paper focusses on income taxation and its relation to household production activities. It is organised as follows. In the next section, I describe the basic model. In section 3, I discuss the household incentive compatibility constraints and derive the Pareto efficient income tax schedule for a two class society with perfect assortative mating. The case of imperfect assortative mating is studied in section 4. Section 5 concludes with a discussion of the main assumptions.

2 The model

The framework that I use builds on the standard two-class model for optimal taxation, as exposed by Stiglitz (1982). A large number of citizens populate the economy. A fraction μ_L of them have a low productive ability w_L, while the complementary fraction $\mu_H(= 1 - \mu_L)$ is endowed with a high productive ability w_H ($> w_L$). All citizens dispose of a total time endowment equal to unity, and they all share the same utility function defined over private consumption (x), leisure (ℓ) and a 'public' household commodity (h), $u(x, \ell, h)$, which displays the standard properties of monotonicity and strict concavity. The private consumption good is a normal good that is produced with a linear production technology whose transformation coefficients for both types of labour are normalised to unity. If the consumption good is treated as the *numéraire*, the productive abilities of the citizens also represent their real wage rates. The household commodity cannot be bought on the market, but only produced at home by devoting household time. I assume that g hours spent on household production activity yields $h(g)$ units of the household good, where $h(\cdot)$ is a concave production function measuring the tidiness of the house, the extent to which children are taken care off, etc.

Each citizen in this economy lives with one other citizen in a household. So there are no households with three or more members, and neither are there people living on their own.[3] In a first instance I assume perfect alignment of

[2] In a recent paper, Agell & Persson (2000) describe the implications for labour supply decisions when workers can tax arbitrate via the asset market. That analysis takes the income tax schedule as given and does not derive any normative implications.

[3] What is really necessary is that each household consists of two (and only two) pro-

productive abilities, that is every household consists only of people belonging to the same ability class. In section 4, however, I will relax this assumption and show that the results go through with imperfect assortative mating.

The decision process within each household takes place in a co-operative way. In particular, I use Samuelson's (1956) consensus model and assume that household partners maximise a utilitarian welfare function defined over their individual utilities.[4] Though there is a quickly emerging literature on non-co-operative bargaining within marriage (recently reviewed by Lundberg & Pollak, 1996), I take the household as an institution where trust and reputation work sufficiently well to enforce agreements made by its members. I should hasten to add that the household partners need not be legally linked through marriage. They can be, but they can also be any pair of people willing to assume the shared responsibility for the production of household commodities, be it because they are in love with one another, or because they are good friends and agree to share accommodation. I also note here that the existence of households and their co-operative decision-making is taken as given. No attempt is made to explain why and how households form.

Let us now consider a household of type i $(= L, H)$ composed of Alice (a) and Bert (b) who agree to co-ordinate their consumption, earnings (Y) and housework contributions (g) so as to maximise the sum of their utilities. Allowing also for a (possibly negative) side payment s_i from Bert to Alice, this household solves the following problem in the absence of taxation:

$$\max_{Y_{ia}, Y_{ib}, g_{ia}, g_{ib}, s_i} u\left(Y_{ia} + s_i, 1 - \frac{Y_{ia}}{w_i} - g_{ia}, h(g_{ia} + g_{ib})\right) + u\left(Y_{ib} - s_i, 1 - \frac{Y_{ib}}{w_i} - g_{ib}, h(g_{ia} + g_{ib})\right).$$

Denoting household member m's marginal rate of substitution between gross earnings and private consumption by $MRS^{im} \stackrel{\text{def}}{=} \frac{u_\ell^{im}}{w_i u_x^{im}}$ $(m = a, b)$ (subscripts on the utility function denote partial derivatives), the solution to this problem is characterised by the equalities $MRS^{ia} = MRS^{ib}$, $u_\ell^{ia} = u_\ell^{ib} = (u_h^{ia} + u_h^{ib})\, h'$, and $u_x^{ia} = u_x^{ib}$.[5]

ductive members. The presence of unproductive members does not affect the analysis, provided this presence influences the preferences of the productive household members in the same way in each household.

[4]Put differently, each partner has 'caring' preferences with a 100% degree of caring. Caring preferences invlolve separability, interpersonal utility comparisons and thus cardinalisation (Chen & Wooley, 2001 p 726). By considering 100% caring, each partner internalises the effects of his/her own actions on the other partner's welfare.

[5]One possible solution is where $Y_{ia} = Y_{ib}$, $g_{ia} = g_{ib}$, and $s_i = 0$. But then clearly

I will now assume that the government would like to raise the living standard of low ability households to the level $\overline{W}^L$, say. The government cannot observe individual abilities, neither does it observe the amounts of time supplied on the labour market or used for household work, nor the amount of money household members transfer to each other. However, it is aware of the statistical distribution of abilities, and it can observe the gross income (Y) each citizen earns on the labour market. The revelation principle then says that the government can replicate any redistribution of resources it achieves using an income tax schedule by a direct mechanism where it allocates (*net income, gross income*)-bundles to citizens upon their announcement of their abilities, provided the mechanism satisfies the appropriate incentive compatibility constraints.[6] To formulate these constraints, it is useful to define a household's semi-indirect utility function, giving for any pair of (*net income*, *gross income*)-bundles $[(c_a, Y_a), (c_b, Y_b)]$ the maximal welfare level that household i can achieve, i.e.

$$W^i[(c_a, Y_a), (c_b, Y_b)] \stackrel{\text{def}}{=} \max_{g_{ia}, g_{ib}, s_i} u\left(c_a + s_i, 1 - \frac{Y_a}{w_{ia}} - g_{ia}, h(g_{ia} + g_{ib})\right) + u\left(c_b - s_i, 1 - \frac{Y_b}{w_{ib}} - g_{ib}, h(g_{ia} + g_{ib})\right). \quad (1)$$

The solution to the above problem is characterised by the equalities:

$$u_x^{ia} = u_x^{ib} \text{ and } u_\ell^{im} = [u_h^{ia} + u_h^{ib}] \cdot h'(g_a + g_b) \quad (m = a, b).$$

These may also be combined to give $MRS^{ia} = MRS^{ib}$.

Clearly, when the two household members receive the same (*net income*, *gross income*)-bundle, these conditions can only be verified if both have the same housework load and no money is transferred between them.

To proceed with the analysis, let me first focus on the case where the two household members go for the same (*net income*, *gross income*)-bundle, (c, Y) say—let me call this *uniform labour market behaviour*. Under such behaviour,

$Y_{ia} + \varepsilon w_i, Y_{ib} - \varepsilon w_i, g_{ia} - \varepsilon, g_{ib} + \varepsilon$, and $s_i = -\varepsilon w_i$ also constitutes a solution: since both partners have the same market and household productivities, any reallocation of the optimal amount of total factor earnings and total household activity is optimal as well.

[6] One could argue that in a society with only intra-class household formation and absence of singles, the tax administration could partially solve the incentive problem by imposing a very high tax liability on any household submitting two different income returns, provided it has reliable records on who forms with whom a household. But there are two objections to this argument. First, as I have argued earlier, the household is not necessarily an a priori verifiable coalition. Second, if by law the individual is considered as the tax unit, the use of non-individual information would fall outside the discretion of the the tax authority.

the marginal rate of substitution for a household of type i coincides with the marginal rate of substitution of its members:

$$\frac{\mathrm{d}c}{\mathrm{d}Y}|_{dW^i=0} = MRS^{ia} = MRS^{ib}.$$

I assume that at any (c, Y)-bundle a low ability household will have a larger marginal rate of substitution than a high ability household, i.e. $\frac{\mathrm{d}c}{\mathrm{d}Y}|_{\mathrm{d}W^L=0} > \frac{\mathrm{d}c}{\mathrm{d}Y}|_{\mathrm{d}W^H=0}$ (single crossing).[7] Under uniform labour market behaviour, the incentive compatibility constraint for household i can be written as:

$$W^i[(c_i, Y_i), (c_i, Y_i)] \geq W^i[(c_j, Y_j), (c_j, Y_j)] \ (i, j = L, H). \tag{3}$$

These incentive constraints (which I will refer to as the individual incentive compatibility constraints), together with the single crossing property, imply that both the gross and the net earnings of a high ability individual should be higher than that of a low ability individual: $c_H > c_L$ and $Y_H > Y_L$. Moreover, because the government wants to redistribute from rich to poor, and because it is constrained in this by the budget constraint $\mu_L(Y_L - c_L) + \mu_H(Y_H - c_H) = 0$, the ratio $\frac{c_H - c_L}{Y_H - Y_L}$ should be smaller than 1.

If households for some reason only display uniform labour market behaviour, we know from standard income tax theory that it is efficient to distort the bundle intended for the low ability agents to make the mimicking strategy less tempting for high ability households. This bundle is distorted in the sense that the low ability agent's MRS at this bundle is lower than 1, i.e. that (s)he faces a positive marginal income tax rate.[8] On the other hand, because the low ability person will never have an incentive to choose the bundle intended for a high ability agent, the latter should receive an undistorted bundle; that is, a bundle for which his/her MRS equals unity, or still in other words, one for which the implicit marginal tax rate is zero. See e.g. Stiglitz (1982, p 218).

[7] In the standard model, normality of consumption ($\mathrm{d}(\frac{u_\ell}{u_x})/\mathrm{d}\ell \leq 0$) is a sufficient condition. With the presence of the household good, we need in addition that $\mathrm{d}(\frac{u_\ell}{u_h})/\mathrm{d}\ell \leq 0$, $\mathrm{d}(\frac{u_h}{u_\ell})/\mathrm{d}h \leq 0$ and $\mathrm{d}(\frac{u_\ell}{u_x})/\mathrm{d}h \leq 0$. This is e.g. the case when $u_{h\ell} = 0$ and $u_{xh} \geq 0$.

[8] Facing a marginal tax rate t, a person with wage rate w selects a labour supply in accord with the FOC $\frac{u_\ell}{u_x} = (1-t)w$. $1 - \frac{u_\ell}{wu_x}$ can thus be interpreted as the implicit marginal tax rate.

3 Non-uniform labour market behaviour and household incentive compatibility

From now on, I shall drop the constraint that members of the same household can only make identical labour market choices. First, consider a household of type H and its indifference curve conditioned on uniform labour market behaviour, $\mathrm{d}W^H[(c,Y),(c,Y)] = 0$, passing through (c_H, Y_H). This is drawn as the tin line in figure 1. Next, consider the indifference curve passing through the same bundle but *conditioned* on (c_H, Y_H) being selected by Bert: $\mathrm{d}W^H[(c,Y),(c_H,Y_H)] = 0$. I now claim that when the household good does not affect the marginal willingness to pay for leisure in a too strong way, this second indifference curve is the lower envelope to the first as shown by the bold line in figure 1. This is stated more generally as

Lower Envelope Condition (LE)–The indifference curve $\mathrm{d}W^i[(c,Y),(\overline{c},\overline{Y})]$ $=0$ is the lower envelope to the indifference curve $\mathrm{d}W^i[(c,Y),(c,Y)] = 0$, at bundle $(\overline{c},\overline{Y})$.

Loosely speaking, **LE** follows from the fact that non-uniform labour market behaviour opens up for Pareto improving side trades: if Alice goes for (c_L, Y_L) and Bert for (c_H, Y_H), then they not only are on the same welfare level as when both going for (c_H, Y_H), but they have also ended up with different marginal rates of substitution between leisure and consumption; this then opens up for a Pareto improving side trade. This is a loose argument because leisure cannot be traded directly for money, but only indirectly via the household public good whose quantity in turn may affect the preferences for (x, ℓ) bundles. Lemma 2 in the appendix identifies the restrictions on preferences for **LE** to hold.

When **LE** does not hold, a household's self-selection constraint under non-uniform labour market behaviour will be slack when the corresponding constraint under uniform labour market behaviour binds. In that case we are back to the standard solution discussed in the previous section. On the other hand, when **LE** does hold, it is the self-selection constraints under non-uniform labour market behaviour that take over. This will be the subject of analysis in the remainder of the paper.

The redistribution problem of the government may now be formulated as

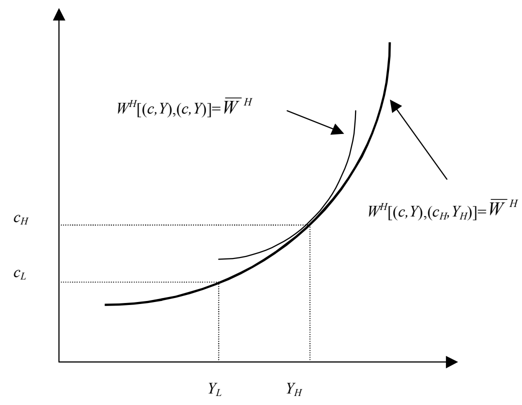

Figure 1. Illustration of the Lower Envelope Condition

Figure 1:

follows:

$$\begin{array}{lll}
\max\limits_{c_L,Y_L,c_H,Y_H} & W^H[(c_H,Y_H),(c_H,Y_H)] & \\
\text{s.t.} & W^L[(c_L,Y_L),(c_L,Y_L)] \geq \overline{W}^L & (\omega) \\
& W^H[(c_H,Y_H),(c_H,Y_H)] \geq W^H[(c_H,Y_H),(c_L,Y_L)] & (\lambda) \\
& \mu_L(Y_L-c_L)+\mu_H(Y_H-c_H) \geq 0 & (\gamma)
\end{array} \quad \text{(P1)}$$

The first constraint ensures that a low ability household obtains at least the living standard $\overline{W}^L$. Constraint (λ) is the *household* incentive compatibility constraint ensuring that a high ability household can never do better by posing as a household with both a high and a low ability member. As I have just argued, if this constraint is verified the individual self-selection constraint will be slack. Thus the set of household incentive compatible allocations is a strict subset of the set of individually incentive compatible allocations.[9] Constraint (γ) rules out that the government runs a deficit. Finally, I mention that the strategy of posing as a household with different abilities is credible in the sense that it pays for *each* member if it does so for the household.[10] No member will therefore refuse to consider this mimicking strategy.

Performing the operations foc(c_L)$\cdot MRS^L$+foc(Y_L) and foc(c_H)$\cdot MRS^H$+ foc(Y^H) and making use of the envelope theorem on problem (1), results in

[9]Theorem 1 in the appendix to section 3 shows that a Pareto efficient tax policy that redistributes from H to L is only constrained by the incentive restriction (λ).

[10]This is shown formally in Theorem 2 in the appendix to section 3.

the characterisation rules for the marginal tax rate on low and high incomes, respectively:

$$1 - MRS^H = \frac{\lambda \widehat{u}_x^H}{\gamma \mu_H}[MRS^H - M\widehat{R}S^H], \text{ and} \tag{4}$$

$$1 - MRS^L = \frac{\lambda \widehat{u}_x^H}{\gamma \mu_L}[MRS^L - M\widehat{R}S^H], \tag{5}$$

where $M\widehat{R}S^H$ ($\widehat{u}_x^H$) denotes the common marginal rate of substitution (marginal utility of consumption) on which the members of a high ability household settle after they have opted for the package $[(c_H, Y_H), (c_L, Y_L)]$ and have concluded an efficient side trade.

The sign of (4)'s *rhs* hinges on the difference between two marginal rates of substitution of a high ability person. The rate of substitution in his or her capacity of a member of a household choosing the package $[(c_H, Y_H), (c_H, Y_H)]$, MRS^H, and the rate of substitution when the same household goes for the package $[(c_H, Y_H), (c_L, Y_L)]$ and subsequently reschedules housework and carries out monetary compensation, $M\widehat{R}S^H$. This difference is positive because the indifference curve $\mathrm{d}W^H[(c, Y), (c_H, Y_H)] = 0$ is convex and $Y_L < Y_H$. Thus, as a husband to a Y_H-earning Alice, Bert tolerates a reduction of MRS^H Euro in net income when presented with the opportunity to earn one Euro less gross income. But if Alice were to earn only Y_L, Bert tolerates a reduction of only $M\widehat{R}S^H$ Euro. This explains why distorting the gross earnings of high ability persons downwards is beneficial: by taxing high incomes at the margin, the gap between the Y_H-earning household member's marginal willingness to pay for leisure and that of the Y_L-earning household member is reduced, side trading opportunities are mitigated and the household incentive compatibility constraint is relaxed.

The marginal tax rate on low incomes is given by (5). Its sign depends on the difference of two marginal rates of substitution: the rate of substitution for an L-household with both members going for the bundle (c_L, Y_L), MRS^L and $M\widehat{R}S^H$. This difference will be positive for an optimal allocation.[11] A one Euro reduction in Y_L, together with a reduction in c_L of MRS^L Euro, leaves household L equally well off. An H-household engaging in tax arbitrage can at most afford a reduction in c_L of $M\widehat{R}S^H$ Euro. So the incentive

[11] We cannot just rely on a single crossing argument to sign this difference, since MRS^L measures the slope of the indifference curve $\mathrm{d}W^L[(c, Y), (c, Y)] = 0$, while $M\widehat{R}S^H$ measures the slope of the indifference curve $\mathrm{d}W^H[(c, Y), (c_H, Y_H)] = 0$, both evaluated at (c_L, Y_L). However, it should be clear that if these two indifference curves cross twice, it will be the right crossing that will be part of the optimal allocation. (A small movement of the bundle at the left crossing along L's indifference curve would increase the government budget and weaken the self-selection constraint for the H-household.)

compatibility constraint for this household is relaxed by distorting labour earnings of L-households downwards.

We may now combine the two tax formulae to obtain

$$\frac{1-MRS^H}{1-MRS^L}=\frac{\mu_L+\frac{\lambda}{\gamma}\widehat{u}_x^H}{\mu_H+\frac{\lambda}{\gamma}\widehat{u}_x^H}. \tag{6}$$

This expression shows that whenever the distribution of abilities is skewed to the right ($\mu_L>\mu_H$), the marginal tax rate on low incomes should be lower than the marginal tax rate on high incomes. Notice that because the *rhs* of (6) is strictly smaller than $\frac{\mu_L}{\mu_H}$, this odds ratio of the ability distribution provides an upper limit on the optimal degree of marginal progressivity. The intuition for marginal progressivity is easy. The benefit of distorting a (*net income, gross income*)-bundle is to discourage dissembling behaviour. It saves the government from leaving too much information rent with the H-household and is thus proportional with the number of H-people. In contrast, the aggregate cost of a distortion is borne in equilibrium and is proportional to the number of people whose decisions are distorted. Therefore, when there are relatively more low ability citizens, an optimal policy should distort the behaviour of these people less than the behaviour of high ability citizens. I summarise this discussion as

Proposition 1 *Suppose condition **LE** holds. In a two-class economy with perfect assortative mating, the marginal tax rate on both low and high incomes is positive at a Pareto efficient allocation. Whenever the distribution of abilities is skewed to the right, the optimal marginal tax rate on high incomes exceeds the marginal tax rate on low incomes.*

4 The case of imperfect assortative mating

A more correct picture of the world recognises the presence of a non-negligible amount of households where the partners' labour market productivities are not aligned (surgeon & nurse, professor & secretary, etc).[12] In this section, I will investigate this case of imperfect assortative mating–some households being formed by one low ability person and one high ability person. I will call such households *mixed* and denote them by index M. So now the economy consists of f_L households of type L, f_H households of type H, and

[12] For empirical evidence on the degree of assortative mating regarding educational attainment, see e.g. Hauser (1982), Hout (1982) and Mare (1991) for the U.S., and Kravdal & Noack (1989) for Norway.

f_M households of type M. We have of course that $f_L + f_M + f_H = 1$, $f_L + \frac{1}{2} f_M = \mu_L$ and $f_H + \frac{1}{2} f_M = \mu_H$; and I still assume an ability distribution skewed to the right ($\mu_L > \mu_H$). For H-households to remain present, f_M should belong to $[0, 2\mu_H)$; I will refer to such f_M values as *feasible*.

A tax system with the individual as the tax unit should now specify *four* (c, Y) bundles. In addition to the bundles (c_L, Y_L) and (c_H, Y_H), two extra bundles are designed: one for the low ability member of a mixed household $((c_{Ml}, Y_{Ml}))$, and one for its high ability member $((c_{Mh}, Y_{Mh}))$.

As will soon become clear, the addition of two new bundles boosts the number of self-selection constraints significantly. In order to ease their manipulation, I assume from now on that preferences are quasi-linear in leisure and strongly separable: $u(x, \ell, h) \stackrel{\text{def}}{=} v(x) + \ell + \varphi(h)$ $(v'', \varphi'' < 0)$. With such preferences, condition **LE** is satisfied, the total optimal amount of household work is fixed at G^* (defined by $\varphi'(h'(G^*)) = \frac{1}{2}$) and net household income is shared equally. Thus, a household of type i with abilities w_{ia} and w_{ib} and having chosen the bundles (c_a, Y_a) and (c_b, Y_b) will in addition to the net utility from the public household good reach a welfare level

$$W^i[(c_a, Y_a), (c_b, Y_b)] = 2v(\frac{c_a + c_b}{2}) - \frac{Y_a}{w_{ia}} - \frac{Y_b}{w_{ib}}.$$

I should stress here that though the *total* amount of household work is fixed to G^*, this does not mean that its allocation over the two partners is. In fact, as I will argue below, variability of the individual levels of housework is essential for the partners to rearrange homework such that none of them becomes worse off under any potential mimicking strategy of the household.[13]

To ease notation in what follows, I refer to the bundles (c_L, Y_L), (c_H, Y_H), $(c_{M\ell}, Y_{M\ell})$ and (c_{Mh}, Y_{Mh}) just as $\mathbf{L}, \mathbf{H}, \mathbf{Ml}$, and $\mathbf{Mh}$. Thus, the three family welfare levels in equilibrium are written as:

$$W^L(\mathbf{L}, \mathbf{L}) = 2v(\frac{c_L + c_L}{2}) - \frac{Y_L}{w_L} - \frac{Y_L}{w_L},$$

$$W^M(\mathbf{Ml}, \mathbf{Mh}) = 2v(\frac{c_{Ml} + c_{Mh}}{2}) - \frac{Y_{Ml}}{w_L} - \frac{Y_{Mh}}{w_H}, \text{ and}$$

$$W^H(\mathbf{H}, \mathbf{H}) = 2v(\frac{c_H + c_H}{2}) - \frac{Y_H}{w_H} - \frac{Y_H}{w_H}.$$

[13] With sufficiently convex preferences, as in the previous section, this is automatically the case (cf footnote 10). But quasi-linear preferences, and their utilitarian summation, however, the household only cares about the total amount of leisure, not its distribution. A shortcut is then to introduce individual rationality constraints when household members consider a potential mimicking strategy–see footnote 15.

Family welfare out-of-equilibrium is written in a similar way. For example

$$W^M(\mathbf{L},\mathbf{H}) = 2v(\frac{c_L + c_H}{2}) - \frac{Y_L}{w_L} - \frac{Y_H}{w_H}$$

is the welfare level that a mixed household would achieve if its low ability member chooses bundle (c_L, Y_L) and its high ability member chooses (c_H, Y_H). Notice that for a mixed household, the order in which the two bundles enter the indirect utility function matters (unlike for uniform households).

The government evaluates its tax policy by means of a weighted utilitarian social welfare function:

$$SW \stackrel{\text{def}}{=} f_L \beta_L W^L(\mathbf{L},\mathbf{L}) + f_M \beta_M W_M(\mathbf{Ml},\mathbf{Mh}) + f_H \beta_H W^H(\mathbf{H},\mathbf{H}),$$

where the weights β_i $(i = L, M, H)$ are normalised in the sense that $f_L \frac{\beta_L}{w_L} + \frac{1}{2} f_M \frac{\beta_M}{w_L} + \frac{1}{2} f_M \frac{\beta_M}{w_H} + f_H \frac{\beta_H}{w_H} = 1$. The following two conditions impose a sufficient willingness to redistribute from rich to poor families (w^{aa} denotes the arithmetic average of the ability distribution):

Assumption R1 $:\frac{\beta_L}{w_L} \geq \frac{\beta_M}{w_L} > \frac{\beta_M}{w_H} \geq \frac{\beta_H}{w_H}$.

Assumption R2 $:\frac{w_L}{w^{aa}} \geq \frac{\beta_M}{w_H}$.

R1 is satisfied by equal (utilitarian) weights or any weight vector that is negatively correlated with ability. It implies that $\frac{\beta_L}{w_L} > 1 > \frac{\beta_H}{w_H}$. Condition **R2** in addition puts an upper bound on $\frac{\beta_M}{w_H}$. For utilitarian social preferences, $\frac{\beta_M}{w_H}$ equals $\frac{w^{ha}}{w_H}$ (where w^{ha} is the harmonic average of the ability distribution), and **R2** satisfied.[14]

The government's problem is then to find four bundles $\mathbf{L}, \mathbf{Ml}, \mathbf{Mh}$ and $\mathbf{H}$ to maximise SW under the budget constraint

$$f_L(Y_L - c_L) + \frac{1}{2} f_M(Y_{Ml} - c_{Ml}) + \frac{1}{2} f_M(Y_{Mh} - c_{Mh}) + f_H(Y_H - c_H) \geq 0,$$

and three sets of self-selection constraints

$$W^L(\mathbf{L},\mathbf{L}) \geq W^L(i,j),\; W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(i,j), W^H(\mathbf{H},\mathbf{H}) \geq W^H(i,j)$$

where $i, j \in \{\mathbf{L}, \mathbf{Ml}, \mathbf{Mh}, \mathbf{H}\}$.

[14] Normalising w_H to 1, this can be written as $\frac{w_L}{\mu_L w_L + (1-\mu_L)} \geq \frac{1}{\mu_L \frac{1}{w_L} + (1-\mu_L)} \iff \mu_L + w_L(1-\mu_L) \geq \mu_L w_L + (1-\mu_L) \iff (2\mu_L - 1) \geq (2\mu_L - 1) w_L$. Since $\mu_L > \frac{1}{2}$, and $w_L < w_H$, **R2** is verified.

There are in total 33 self-selection constraints: 9 of the first and third type, and 15 of the second type. In the appendix it is shown (lemma 8) that these constraints impose the following ranking on the gross and net income levels:

$$Y_{Ml} < Y_L < Y_H < Y_{Mh}, \text{ and } c_{Ml} < c_L < c_H < c_{Mh}.$$

Furthermore, I prove there (theorem 3) that under these monotonicity constraints and the redistribution assumptions **R1** and **R2**, the optimal policy is constrained by the following *three self-selection constraints:*

$$W^H(\mathbf{H},\mathbf{H}) \geq W^H(\mathbf{L},\mathbf{H}), \tag{7}$$

$$W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{L},\mathbf{H}), \text{ and} \tag{8}$$

$$W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{Ml},\mathbf{H}). \tag{9}$$

The first incentive constraint is familiar from the perfect assortative mating case; the second and the third are new and involve the mixed household. Later, I will argue that the third need not bind the optimal solution, and that if it does it implies no cost. Let me therefore assume that all three constraints bind the solution. The last two may then also be written as

$$W^M(\mathbf{Ml},\mathbf{H}) = W^M(\mathbf{L},\mathbf{H}), \text{ and} \tag{10}$$

$$W^M(\mathbf{Ml},\mathbf{Mh}) = W^M(\mathbf{Ml},\mathbf{H}). \tag{11}$$

The advantage of writing the constraints in this way is graphical representation. Expression (10) states that **Ml** and **L** should both lie on the indifference curve $\mathrm{d}W^M(\cdot,\mathbf{H}) = 0$, while according to (11), **Mh** and **H** should both lie on $\mathrm{d}W^M(\mathbf{Ml},\cdot) = 0$. In addition, expression (7) (with equality) then reads that bundles **L** and **H** should both belong to the conditional indifference curve $\mathrm{d}W^H(\cdot,\mathbf{H}) = 0$. Three indifference curves thus connect the four bundles–see figure 2 (the dotted indifference curve will be referred to later).

As I mentioned earlier, in order for no household member to get worse off under mimicking (then when being honest), it is required that they reschedule homework (even though the household is equally well off).[15]

To derive and analyse the optimal solution to the taxation problem, I proceed as Weymark (1986a, 1986b, 1987) and write the income levels in

[15]To illustrate the role of household production, consider (7). Individual utility when the household is honest is given by $v(c_H) - \frac{Y_H}{w_H} + \varphi(h(G^*)) - \frac{G^*}{2}$. If this household chooses $(\mathbf{L},\mathbf{H})$, individual welfares are $v(\frac{c_L+c_H}{2}) - \frac{Y_L}{w_H} + \varphi(h(G^*)) - g_a$, and $v(\frac{c_L+c_H}{2}) - \frac{Y_H}{w_H} + \varphi(h(G^*)) - g_b$, where $g_a + g_b = G^*$. For both members to be willing to consider the

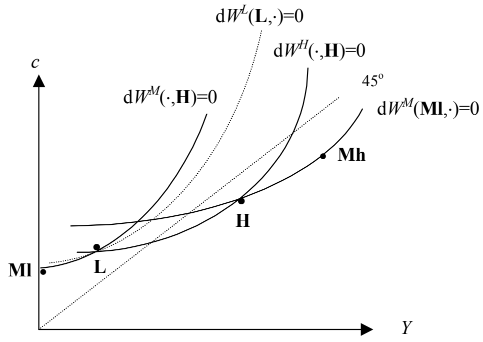

Figure 2. A household incentive compatible allocation without assortative mating.

Figure 2:

terms of differences in the utility of consumption. From (7) (with equality), (10) and (11) it follows that

$$\begin{aligned} Y_H &= Y_{Mh} - 2w_H \Delta_M, \\ Y_L &= Y_{Mh} - 2w_H \Delta_H - 2w_H \Delta_M, \\ Y_{Ml} &= Y_{Mh} - 2w_H \Delta_H - 2w_H \Delta_M - 2w_L \Delta_L, \end{aligned}$$

where $\Delta_M \overset{\text{def}}{=} [v(\frac{c_{Ml}+c_{Mh}}{2}) - v(\frac{c_{Ml}+c_H}{2})]$, $\Delta_H \overset{\text{def}}{=} [v(c_H) - v(\frac{c_H+c_L}{2})]$, and $\Delta_L \overset{\text{def}}{=} [v(\frac{c_L+c_H}{2}) - v(\frac{c_{Ml}+c_H}{2})]$. Substituting Y_H, Y_L and Y_{Ml} for the expressions above in the budget constraint, this can be solved for Y_{Mh}.This relation

mimicking strategy $(\mathbf{L}, \mathbf{H})$ neither should work too much at home:

$$\frac{G^*}{2} + \frac{Y_H}{w_H} - \frac{Y_L}{w_H} - [v(c_H) - v(\frac{c_L + c_H}{2})] \quad \geq \quad g_a, \qquad (\text{IR}_a)$$

$$\frac{G^*}{2} - [v(c_H) - v(\frac{c_L + c_H}{2})] \quad \geq \quad g_b, \qquad (\text{IR}_b)$$

where the last inequality may also be written as

$$g_a \geq \frac{G^*}{2} - [v(c_H) - v(\frac{c_L + c_H}{2})] \qquad (\text{IR}_b')$$

since $g_a + g_b = G^*$. Combining (IR_a) and (IR_b'), it is easy to check that there exists a solution to g_a (and thus to g_b) for both members to be willing to consider mimicking, if and only if $W^H(\mathbf{L}, \mathbf{H}) \geq W^H(\mathbf{H}, \mathbf{H})$. Thus, *unless* the self-selection constraint (7) holds with equality, the individual rationality constraints cannot be verified simultanously. Since the square bracket term in (IR_a) and (IR_b) is positive, it is clear that under mimicking, a does more than half of the homework (and b less than half).

can in turn be used to express the other three income levels in terms of aggregate consumption and the utility differences, and thus in terms of the four consumption levels: $Y_i = Y_i(c_L, c_H, c_{Ml}, c_{Mh})$ $(i = L, H, Ml, Mh)$.The same is true for social welfare: $SW = SW(c_L, c_H, c_{Ml}, c_{Mh})$–for details, see the appendix, section 7.2.2.

The original optimal taxation problem is therefore equivalent to the following reduced form problem:

$$\begin{array}{lll} \max\limits_{c_L, c_H, c_{Ml}, c_{Mh}} & SW(c_L, c_H, c_{Ml}, c_{Mh}) & \\ \text{s.t.} & Y_{Ml}(c_L, c_H, c_{Ml}, c_{Mh}) \geq 0 & (\kappa) \\ & 0 \leq c_{Ml} \leq c_L \leq c_H \leq c_{Mh} & \end{array} \tag{P2}$$

When analysing this taxation problem, I will ignore the monotonicity constraints in the net income levels c_i.

The fact that bundle **Mh** is not part of any mimicking strategy suggests that it should be made available in an undistorted way. This is confirmed by the first order condition w.r.t. c_{Mh}:

$$(1-\kappa)\frac{1}{2}f_M\left[w_H\, v'(\frac{c_{Ml}+c_{Mh}}{2}) - 1\right] = 0.$$

For $\kappa \in [0,1)$, it is clear that a necessary condition for an optimum is that $w_H\, v'(\frac{c_{Ml}+c_{Mh}}{2}) = 1$, or that the marginal tax rate faced by the high ability member of a mixed household is zero.[16] This no-distortion-at-the-top result ties down the value of net income to the mixed household, $c_{Ml} + c_{Mh}$.

The first order condition for c_{Ml} provides the equilibrium value for the Lagrange multiplier κ:

$$\kappa = \frac{\frac{1}{2}f_m(w_H - w_L)}{w_L + \frac{1}{2}f_m(w_H - w_L)}, \tag{12}$$

which is indeed strictly smaller than one, and equal to zero when mixed households are absent. The non-negativity constraint on Y_{Ml} is thus strictly binding. This is not surprising since the low ability member of the mixed household has a comparative advantage in doing the household work and an efficient policy should not make this member participate on the labour market.

The optimal value for c_{Ml} is not unique. For, suppose that (8) is strictly binding. Then we can slightly reduce c_{Ml} and increase c_{Mh} with the same amount. This leaves total net income of the mixed household unaffected, it

[16]The marginal tax rate for member a of household i when receiving the bundles $(c_a, Y_a), (c_b, Y_b)$ is defined as $1 - \frac{\mathrm{d}c_a}{\mathrm{d}Y_a}|_{\mathrm{d}W^i[(c_a,Y_a),(c_b,Y_b)=0} = 1 - \frac{1}{w_{ia}v'(\frac{c_a+c_b}{2})}$.

does not change government revenue, and it turns (8) into a strict inequality. Thus the constraints that really limit the redistribution problem are (7) and (9), and nothing is lost by imposing (8).

The optimality conditions for c_H and c_L are given by

$$(1-\kappa)f_H\left[w_H\, v'(c_H)-1\right] = A\left[w_H v'(\frac{c_L+c_H}{2}) - w_H v'(c_H)\right], \tag{13}$$

$$(1-\kappa)f_L\left[w_L\, v'(c_L)-1\right] = A\left[w_H v'(\frac{c_L+c_H}{2}) - w_L v'(c_L)\right] - B\, w_L v'(c_L), \tag{14}$$

respectively, where

$$A \stackrel{\text{def}}{=} f_H(1-\kappa-\frac{\beta_H}{w_H}) + f_M(1-\kappa-\frac{\beta_M}{w_H}), \text{ and}$$

$$B \stackrel{\text{def}}{=} \frac{1}{2}f_M(\frac{w_H}{w_L}-1)(1-\kappa-\frac{\beta_M}{w_H}).$$

I first comment on (13). The square bracket term on the *lhs* is the loss in tax revenue from an H-household when c_H is lowered marginally and Y_H adjusted downwards with $w_H\, v'(c_H)$ to keep family welfare constant. The square bracket term on the *rhs* is positive due to the monotonicity assumption; it measures by how much mimicking becomes more costly for an H-household. This is multiplied by A, the social benefit of inducing self-selection. First, notice that A reduces to $\mu_H(1-\frac{\beta_H}{w_H}) > 0$ when f_M approaches zero. Thus, with a small fraction of mixed households, the implicit marginal tax rate on Y_H is positive as shown for more general preferences in the perfect stratification case. The formula reveals that the argument to distort Y_H downwards is augmented by the desire to distribute away from the mixed household (to the extent that $\frac{\beta_M}{w_H}$ is smaller than one, which it is under **R2**) but weakened by the non-negativity constraint on Y_{Ml}. (From the expression for Y_{Ml} in appendix 7.2.2., it transpires that it depends heavily on c_H.)

Notice that due to **R1**, a sufficient condition for A to be positive is that $(1-\kappa-\frac{\beta_M}{w_H}) > 0$. In section 7.2.2 of the appendix, I show that **R2** guarantees precisely this. We may therefore conclude that the marginal tax rate on Y_H should be positive for any feasible value of f_M.

The optimality condition for c_L, (14) can be interpreted in a similar way. It balances the budgetary costs with the self-selection benefits of a small reduction in c_L. Since the self-selection constraint $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{L},\mathbf{H})$ does not bind the optimal policy (cf theorem 3 in the appendix), the square bracket term on the *rhs* of (14) is positive:[17] to prevent that a member of

[17] Refer to figure 2 and compare the slope of the indifference curve $dW^L(\mathbf{L},\cdot)=0$ (the dotted line), with that of the indifference curve $dW^H(\mathbf{H},\cdot)=0$ at the bundle $\mathbf{L}$.

an H-household dissembles as L, the **L** bundle should–on balance–be distorted downwards. But when Y_L is reduced, it becomes less costly for an M-household to opt for the package $(\mathbf{L}, \mathbf{H})$ and to the extent that the government wants to redistribute away from mixed households $(1 > \frac{\beta_M}{w_H})$, this has to be taken into consideration (the second term on the *rhs* of (14)). As I just argued, the sign of B is positive and thus there is an argument for distorting Y_L upwards.

Finally, to derive the graduation in the tax schedule I first solve (13) and (14) for $w_H\, v'(c_H)$ and $w_L v'(c_L)$, and then subtract the latter expression from the former. This yields:

$$w_H\, v'(c_H) - w_L v'(c_L) = \tag{15}$$
$$\frac{\frac{A}{1-\kappa}(f_H - f_L)\left[1 - w_H v'(\frac{c_L+c_H}{2})\right] + \frac{B}{1-\kappa}\left[\frac{A}{1-\kappa} w_H v'(\frac{c_L+c_H}{2}) + f_H\right]}{(\frac{A}{1-\kappa} + f_H)(\frac{A}{1-\kappa} + \frac{B}{1-\kappa} + f_L)}.$$

Recall that A and B are positive. Because $w_H v'(\frac{c_L+c_H}{2}) > w_H\, v'(c_H) > 1$, the first square bracket term on the *rhs* is negative. With an ability distribution skewed to the right $(f_L > f_H)$, the entire expression is positive. This means that the marginal tax rate on Y_H is larger than the one on Y_L for any feasible value of f_M.

I now summarise the results on imperfect assortative mating in

Proposition 2 *Consider a two-class economy with imperfect assortative mating, an ability distribution skewed to the right and social preferences favourable to redistribution form rich to poor (in the sense of **R1** and **R2**). The optimal gross income distribution is then as follows:*
• *low ability members of a mixed household should not participate on the labour market;*
• *members of a low ability household should earn a 'low' income level Y_L;*
• *members of a high ability household should earn a 'high' income Y_H; and*
• *high ability members of a mixed household should earn a 'very high' income Y_{Mh}.*
The marginal tax rate on Y_{Mh} should be zero, that on Y_H should be positive, and that on Y_L should be lower than the one on Y_H.

The analysis so far has given unambiguous qualitative results. To gauge the magnitudes of the marginal tax rates on Y_L and Y_H and their difference, I have solved the model for $v(x) = \log x$, $w_L = 5$, $w_H = 10$, $\mu_L = \frac{3}{4}$, and $\beta_L = \beta_M = \beta_H$. Figure 3 shows the optimal marginal tax rates when the fraction of mixed households increases from 0 to .49 (for $f_M \geq \frac{1}{2}$, f_H is zero). The marginal tax rate on Y_H is positive and increasing in f_M,

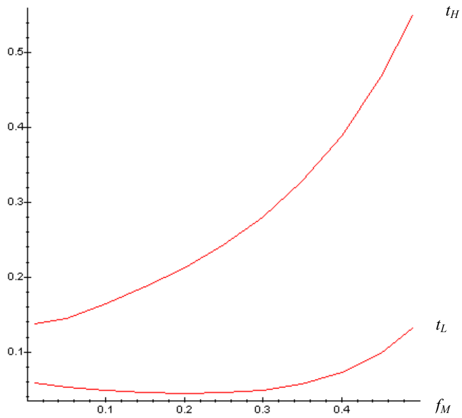

Figure 3. Optimal marginal tax rates on Y_L (t_L) and Y_H (t_H) as a function of f_M.

Figure 3:

while that on Y_L is slightly non-monotonic. The difference between both rates increases the less perfect assortative mating is. Notice that in this experiment, the total endowments in the economy remain the same–only the structure of family composition changes. In figure 4, I present the family welfare levels, as well as social welfare. Here we observe that less assortative mating increases the welfare of L- and M-households, and up to some extent also that of H-households. This indicates that the self-selection constraint on H-households puts a heavier limit on redistribution policy than that on M-households.

5 Concluding remarks

In this paper I have looked at the tax treatment of households under individual filings and characterised the efficiency properties of an income tax schedule that redistributes from rich to poor households. The fact that tax liabilities are determined on individual incomes, but that the decision to earn these incomes are made at the household level makes this a non-trivial problem. This is because the tax liable members of the same household can side trade leisure for net income with one another, and such side trade opportunities enable them to carry out tax arbitrage.

For a two class economy with perfect assortative mating. The main

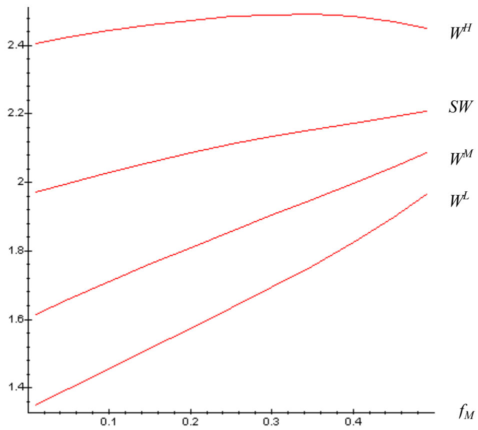

Figure 4. Family welfare (W^i, i=L,M,H) and social welfare (SW) as a function of f_M.

Figure 4:

conclusion is that in order to prevent tax arbitrage, the government imposes a positive marginal tax rate on high incomes and sets the marginal tax rate on low incomes below this rate (when the ability distribution is skewed to the right). Then I showed that these conclusions carry over to a society with imperfect assortative mating. The novel feature that is introduced by imperfect assortative mating concerns the income levels and their tax treatment of the members of a mixed household. Low ability members of such a household should be discouraged from participating in the labour market, while the high ability partner should be given incentives to earn a very high income level which is undistorted at the margin, but taxed on average.

Let me now come back to some of the assumptions made earlier. I have assumed that both household partners are equally efficient in performing housework and that their contributions are perfectly substitutable. If household members differ in their housework productivities as well, the tax system should take this into account by encouraging the more 'house'-productive members to spend less time on the labour market. With imperfect assortative mating w.r.t. both kinds of abilities, there are ten types of households that may form. The design of an optimal income tax system becomes an intricate problem, the more because the labour market decisions of the household members depend on their comparative advantages, while the taxes and transfers that are based on these decisions should correct for differences in absolute advantages.[18]

A second feature of the model was that the time spent by one of the partners in producing the household good is perfectly substitutable with the time input of the other partner. I focused on this case to make my argument in the sharpest possible way. But since side trade opportunities are driven by substitutability, it is clear that the smaller the substitutability between the two inputs, the weaker will be the household incentive compatibility constraints. In the case of zero substitution ($h = \min\{g_a, g_b\}$), the tax arbitrage problem vanishes.[19] It is not difficult to imagine household goods whose production does not allow substitution: a romantic evening together, for example, is by definition of that kind. Related to this discussion is the issue of observability of the household good. Some goods, are in principle observable. One may for example assume that 'caring for the children' could be observed–or at least that the government could make reasonable guesses about how much of this good is produced (because it knows the number of children and their age). But as long as the contributions of the two productive members of the household are not *separately* observable, the arbitrage problem remains. It would remain as well when the household produced good was a private rather than public good; all that is needed then is that this good can be shared among the household members.

Side trading opportunities were in this model maximally exploited by the household because its members were committed to cooperation. Without such cooperation some available options to arbitrate will not be made use of and this will make the incentive constraints less binding. In this sense, non-cooperative household members are a blessing for redistribution policy.

In another paper (Schroyen, 1997), I showed that in an economy with two classes (people poorly and wealthy endowed with the *numéraire*) when side trade in commodities is possible within bilateral coalitions (intra- as well as interclass coalitions), both types of people should face the same marginal commodity tax, but the 'lump sum' part of the tax should be differentiated and a function of the amount traded with the production sector. The reason that we do not get a uniform marginal income tax in the present setting

[18] In a recent paper, Balestrino *et al* (1999) analyse this problem for a society composed of singles (thus avoiding the arbitrage problem).

[19] It was observed in footnote 16 that if the household considers the package $[(c_L, Y_L), (c_H, Y_H)]$, a should perform more than half of the household work in order that b's utility does not drop below the utility he obtains when the household chooses the package $[(c_H, Y_H), (c_H, Y_H)]$. In general, a compensates b by doing more homework.

If a and b are devoted utilitarians, solely concerned with the sum of the two utility levels, tax arbitrage may still occur because it makes available the 'average' bundle $(\frac{c_L+c_H}{2}, \frac{Y_L+Y_H}{2})$. I am grateful to a referee for pointing this out.

with imperfect assortative mating, is that interclass side trading is restricted *within* the institution of the mixed household. Side trade across households is an interesting phenomenon that does take place (low ability persons doing off-the-record housework in the house of high ability spouses). But we are then venturing ourselves on the grounds of tax evasion and 'moonlighting', and should make explicit the audit strategies of the tax administration.

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7 Appendices

7.1 Appendix to section 3

In this appendix, I identify the conditions for the **LE** property to hold (lemma 2), then show that under single-crossing, the **LE** condition, and redistribution from H to L, only the household incentive compatibility constraint for household H may constrain the optimal policy (theorem 1) and finally demonstrate that if a household is indifferent between two packages, then so are both of its members (theorem 2).

The following notation is used: $H = (c_H, Y_H), L = (c_L, Y_L)$, $MRS^H \equiv MRS^H(H,H)$, $MRS^L \equiv MRS^L(L,L)$, $M\widehat{R}S^H \equiv MRS^H(L,H)$, $M\widehat{R}S^L \equiv MRS^L(L,H)$, where $MRS^i(\mathbf{X},\mathbf{Y})$ denotes the common marginal rate of substitution the two members of a household of type i settle on when one member has applied for the bundle $\mathbf{X}$ and the other member has applied for the bundle $\mathbf{Y}$. The incentive compatibility constraints will be referred to as follows:

$$\begin{aligned} W^H(\mathbf{H},\mathbf{H}) &\geq W^H(\mathbf{L},\mathbf{H}): \quad \text{the } HIC_H - \text{constraint} \\ W^L(\mathbf{L},\mathbf{L}) &\geq W^L(\mathbf{L},\mathbf{H}): \quad \text{the } HIC_L - \text{constraint} \\ W^H(\mathbf{H},\mathbf{H}) &\geq W^H(\mathbf{L},\mathbf{L}): \quad \text{the } IIC_H - \text{constraint} \\ W^L(\mathbf{L},\mathbf{L}) &\geq W^L(\mathbf{H},\mathbf{H}): \quad \text{the } IIC_L - \text{constraint} \end{aligned}$$

where HIC stands for household incentive compatibility (under non-uniform labour market behaviour), and IIC stands for individual incentive compatibility (that is, household incentive compatibility, but under the restriction that the two household members must behave identically on the labour market).

The proof of theorem 1 is by contradiction and has the following structure. Lemma 4 provides necessary conditions on the relation between average and marginal tax rates for the HIC_L-constraint to bind policy. Lemma 5 shows that at a Pareto efficient allocation (PEA), the HIC_H- and the HIC_L-constraints cannot bind simultaneously. Lemma 6 characterises for a PEA the distortion in bundle L when the HIC_L-constraint is binding. Using this result, as well as those in lemma 3, theorem 1 then identifies the HIC_H-constraint as the only one binding the PEA when redistribution goes from H to L.

Lemma 1 *The IIC_L and IIC_H constraints imply that $c_L \leq c_H$ and $Y_L \leq Y_H$.*

Proof. Follows immediately from the single crossing property. ■

The next lemma establishes the conditions for the Lower Envelope Condition to hold. When a household where both members have ability w, and both choose

(c, Y) ,it ends up with the welfare level

$$f(c,Y) \overset{\text{def}}{=} \max_{s,g_a,g_b} u[c, 1-\frac{Y}{w}-g_a, h(g_a+g_b)] + u[c, 1-\frac{Y}{w}-g_b, h(g_a+h_b)].$$

When partner b of this same household is assigned the bundle $(\overline{c},\overline{Y})$, maximal household welfare is given by

$$g(c,Y) \overset{\text{def}}{=} \max_{s,g_a,g_b} u[c, 1-\frac{Y}{w}-g_a, h(g_a+g_b)] + u[\overline{c}, 1-\frac{\overline{Y}}{w}-g_b, h(g_a+h_b)].$$

Both $f(\cdot)$ and $g(\cdot)$ are special cases of the welfare function (1). The next lemma compares the curvature of the corresponding indifference curves in the (c, Y)-plane through bundle $(\overline{c},\overline{Y})$.

Lemma 2 *Suppose that $(\overline{c},\overline{Y})$ belongs to the indifference curve $f(c,Y)=\overline{W}$. Then it also belongs to the indifference curve $g(c,Y)=\overline{W}$. When the household good does not affect the marginal willingness to pay for leisure in a too strong way and $u_{hh}-u_{\ell h}< 0$, then $g(c,Y)=\overline{W}$ is the lower envelope of $f(c,Y)=\overline{W}$.*

Proof. The curvatures of the two indifference curves are given by

$$K_f \overset{\text{def}}{=} \frac{\mathrm{d}MRS^a(c,Y)}{\mathrm{d}Y_a}|_{\mathrm{d}f=0}+\frac{\mathrm{d}MRS^a(c,Y)}{\mathrm{d}Y_b}|_{\mathrm{d}f=0}, \text{ and}$$
$$K_g \overset{\text{def}}{=} \frac{\mathrm{d}MRS^a(c,Y)}{\mathrm{d}Y_a}|_{\mathrm{d}g=0}.$$

Since the two indifference curves are tangent at $(\overline{c},\overline{Y})$, I thus need to proof that

$$K_f-K_g=\frac{\mathrm{d}MRS^a(c,Y)}{\mathrm{d}Y_b}|_{\mathrm{d}g=0}> 0.$$

As neither Y_b nor c_b directly enter a's utility function, they affect a's MRS only through the optimal values for s, g_a and g_b. The curvature difference then becomes:

$$K_f - K_g = [\frac{\partial MRS^a}{\partial s}, \frac{\partial MRS^a}{\partial g_a}, \frac{\partial MRS^a}{\partial g_b}] \cdot \begin{bmatrix} \partial\widetilde{s}/\partial Y_b \\ \partial\widetilde{g}_a/\partial Y_b \\ \partial\widetilde{g}_b/\partial Y_b \end{bmatrix}$$

where a tilde ($\tilde{}$) denotes a compensated effect (e.g. $\partial\widetilde{s}/\partial Y_b= \partial s/\partial Y_b+\partial s/\partial c_b \cdot MRS^a$).

For the sake of notational ease, but without loss of generality, I assume from now on that the marginal productivity in household production is fixed at unity. The row vector can then be written as:

$$[\frac{\partial MRS^a}{\partial x_a}, -\frac{\partial MRS^a}{\partial \ell_a}+\frac{\partial MRS^a}{\partial h}, \frac{\partial MRS^a}{\partial h}]$$

Total differentiation of the FOCs w.r.t. the household variables gives:

$$\begin{bmatrix} \mathrm{d}s \\ \mathrm{d}g_a \\ \mathrm{d}g_b \end{bmatrix} = \begin{bmatrix} 2u_{xx} & -u_{x\ell} & u_{x\ell} \\ -u_{x\ell} & u_{\ell\ell}-2u_{\ell h}+2u_{hh} & -2u_{\ell h}+2u_{hh} \\ u_{x\ell} & -2u_{\ell h}+2u_{hh} & u_{\ell\ell}-2u_{\ell h}+2u_{hh} \end{bmatrix}^{-1} \cdot$$
$$\begin{bmatrix} u_{xx} & -\frac{u_{x\ell}}{w} \\ -u_{xh} & \frac{u_{h\ell}}{w} \\ u_{x\ell}-u_{xh} & -\frac{u_{\ell\ell}}{w}+\frac{u_{h\ell}}{w} \end{bmatrix} \begin{bmatrix} \mathrm{d}c_b \\ \mathrm{d}Y_b \end{bmatrix}$$

where use has been made of the full symmetry between a and b at the bundle $(\overline{c},\overline{Y})$.

Straightforward manipulation then gives:

$$\begin{aligned} \frac{\partial \widetilde{s}}{\partial Y_b} &= \frac{1}{2}MRS \\ \frac{\partial \widetilde{g}_a}{\partial Y_b} &= \frac{2\frac{u_{hh}}{w}+\frac{1}{2}\frac{u_{\ell\ell}}{w}-\frac{1}{2}u_x\frac{\partial MRS}{\partial \ell}-u_x\frac{\partial MRS}{\partial h}-2u_{xh}MRS}{u_{\ell\ell}+4u_{hh}-4u_{\ell h}} \\ \frac{\partial \widetilde{g}_a}{\partial Y_b}+\frac{\partial \widetilde{g}_a}{\partial Y_b} &= \frac{2u_x\frac{\partial MRS}{\partial h}-u_x\frac{\partial MRS}{\partial \ell}}{u_{\ell\ell}+4u_{hh}-4u_{\ell h}} \end{aligned}$$

where the denominator $u_{\ell\ell}+4u_{hh}-4u_{\ell h}< 0$ by concavity.

The difference in curvature (times $u_{\ell\ell}+4u_{hh}-4u_{\ell h}$) then becomes:

$$\begin{aligned} &(K_f-K_g)(u_{\ell\ell}+4u_{hh}-4u_{\ell h}) = \\ &\frac{\partial MRS^a}{\partial x_a}\frac{1}{2}MRS^a(u_{\ell\ell}+4u_{hh}-4u_{\ell h})+\frac{\partial MRS^a}{\partial h}u_x\left(2\frac{\partial MRS^a}{\partial h}-\frac{\partial MRS^a}{\partial \ell}\right) \\ &-\frac{\partial MRS^a}{\partial \ell}\left(2\frac{u_{hh}}{w}+\frac{1}{2}\frac{u_{\ell\ell}}{w}-\frac{1}{2}u_x\frac{\partial MRS^a}{\partial \ell}-u_x\frac{\partial MRS^a}{\partial h}-2u_{xh}MRS^a\right) \end{aligned}$$

The conditions under which the *rhs* is negative thus guarantee LEC. Rearrangement of the *rhs* gives:

$$\frac{\mathrm{d}M\widetilde{R}S^a}{\mathrm{d}Y_a}(2u_{hh}-2u_{\ell h})-\frac{1}{2}(MRS^a)^2(u_{xx}u_{\ell\ell}-u_{x\ell}^2)-2u_x\frac{\partial MRS^a}{\partial h}\left(\frac{\partial MRS^a}{\partial \ell_a}-\frac{\partial MRS^a}{\partial h}\right)$$

Note that normality of consumption gives $\frac{\partial MRS^a}{\partial \ell}< 0$, and that by convexity of preferences in the (c,Y)-plane $\frac{\mathrm{d}M\widetilde{R}S^a}{\mathrm{d}Y_a}> 0$, and by concavity of the utility function $u_{\ell\ell}< 0$ and $(u_{xx}u_{\ell\ell}-u_{x\ell}^2) > 0$.

If $\frac{\partial MRS^a}{\partial h}>> 0$, or $\frac{\partial MRS^a}{\partial h}< 0$, but $|\frac{\partial MRS^a}{\partial h}| > |\frac{\partial MRS^a}{\partial \ell}|$ the whole expression may become positive. On the other hand, if $\frac{\partial MRS^a}{\partial h}< 0$, but $|\frac{\partial MRS^a}{\partial h}| < |\frac{\partial MRS^a}{\partial \ell}|$, the whole expression is negative. In words, a modest effect of the public household good on the marginal willingness to pay for leisure ensures that $K_f> K_g$.

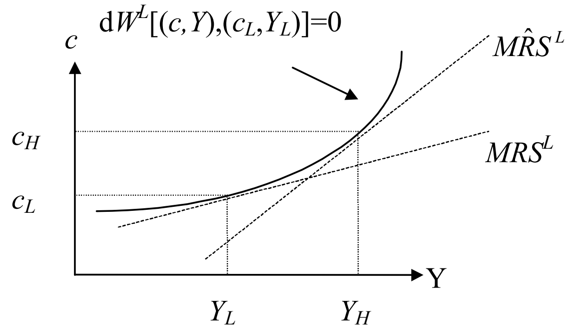

Figure A.1.

Figure 5:

In particular, this is true when the household good is strongly separable from consumption and leisure. Then $u_{xh}= u_{\ell h}= 0$, and the above expression reduces to

$$2u_{hh}\frac{\mathrm{d}M\widetilde{R}S^a}{\mathrm{d}Y_a} - \frac{1}{2}(MRS^a)^2(u_{xx}u_{\ell\ell} - u_{x\ell}^2)$$

which is clearly negative. ■

Lemma 3 *(i) If the HIC_H-constraint is binding, then the IIC_H-constraint is slack. (ii) If the HIC_L-constraint is binding, then the IIC_L-constraint is slack.*

Proof. This follows immediately from the **LE** condition**.** ■

Lemma 4 *A necessary condition for the HIC_L-restriction to constrain policy is that $MRS^L < \frac{c_H - c_L}{Y_H - Y_L} < M\widehat{R}S^L$.*

Proof. Consider the HIC_L- restriction as binding. This means that both the bundles (c_H, Y_H) and (c_L, Y_L) belong to the indifference curve $\mathrm{d}W^L[(\cdot,\cdot),(c_L,Y_L)] = 0$. By the second order envelope theorem, this indifference curve is convex w.r.t. the Y-axis–see figure A.1.

Simple inspection of this figure shows that $MRS^L < \frac{c_H - c_L}{Y_H - Y_L} < M\widehat{R}S^L$. ■

Lemma 5 *At a Pareto efficient allocation, both HIC-constraints cannot be simultaneously binding.*

Proof. Suppose both HIC-constraints are binding. Consider first the case where $\mu_L = \frac{1}{2}$. Suppose we were to carry out the following reform: a one unit reduction in Y_H, accompanied by $\mathrm{d}c_H = -MRS^H$, and a one unit increase in Y_L, accompanied by $\mathrm{d}c_L = MRS^L$. This reform results in a (per household) tax revenue change equal to $MRS^H - MRS^L$ (and by Lemma 1 this change is positive), but will not affect the welfare of either type of household). The maximal reduction in c_H that an H-household can tolerate when one of its members dissembles as L is given by $M\widehat{R}S^H$, so the first part of the reform results in a relaxation of the HIC_H-constraint (measured in terms of the *numéraire*) by $MRS^H - M\widehat{R}S^H$. Likewise, the minimal increase in c_L that an H-household will require for compensation when one of its members dissembles as L is given by $M\widehat{R}S^H$, so the second part of the reform leads to a tightening of the HIC_H-constraint by $MRS^L - M\widehat{R}S^H$. The *net* relaxation of that constraint is therefore equal to $MRS^H - MRS^L$. On the other hand, the first part of the reform tightens the HIC_L-constraint by $M\widehat{R}S^L - MRS^H$ (the minimal increase in c_H that an L-household will require for compensation when one of its members dissembles as H is given by $M\widehat{R}S^L$, the actual increase is MRS^H), while the second part of the reform weakens that constraint by $M\widehat{R}S^L - MRS^L$. On balance, the HIC_L-constraint is thus relaxed by $MRS^H - MRS^L$. To obtain the social desirability of the relaxations of the HIC-constraints, we should multiply these amounts by the respective shadow prices. But the implication is clear: if both HIC-constraints are binding, it is possible to construct a reform which leaves every household equally well off, which results in an increase in tax revenue and which relaxes both HIC-constraints. The original allocation could therefore not have been Pareto efficient. Since the tax revenue is even larger when $\mu_L > \frac{1}{2}$, the result holds for $\mu_L \geq \frac{1}{2}$. ■

Lemma 6 *If only the HIC_L-constraint for a low ability household is binding at an Pareto efficient allocation, then $MRS^L > 1$.*

Proof. Suppose not, i.e. suppose $MRS^L \leq 1$. If we replace bundle (c_L, Y_L) by $(c_L + MRS^L \mathrm{d}Y_L, Y_L + \mathrm{d}Y_L)$, $\mathrm{d}Y_L > 0$, household L is equally well off. The new conditional household indifference curve that is tangent from below to the unconditional indifference curve at the new bundle for L will now lie above the bundle (c_H, Y_H). Thus, after this reform, the HIC_L is slack, and the government budget has increased by $[1 - MRS^L]\mathrm{d}Y_L \geq 0$, contradicting the fact that the original allocation was Pareto efficient. ■

Theorem 1 *Suppose the **LE** condition holds. At an incentive compatible Pareto efficient income tax policy that redistributes income from H to L, at most the HIC_H-constraint will restrict policy.*

Proof. By lemma 3, the *IIC* constraints will never constrain policy, and by Lemma 5, both *HIC* constraints cannot bind at a Pareto efficient policy at the same time. This means that at most one HIC restriction constrains policy at an optimum. Suppose it is the HIC_L-constraint. Lemma 4 and Lemma 6 then imply that $\frac{c_H - c_L}{Y_H - Y_L} > 1$. But this contradicts the fact that redistribution goes from H to L. Hence, at most the HIC_H-restriction can constrain a Pareto efficient redistributive policy. ■

Lemma 7 *When two household members choose different bundles, but both are still active in household work, both end up with the same amount of consumption and leisure and are thus equally well off.*

Proof. The first order conditions w.r.t s, g_a, g_b imply $u_x^a = u_x^b$ and $u_\ell^a = u_\ell^b$. Suppose that (x_a, ℓ_a) is the resulting optimal amount of consumption of leisure for a. Then obviously, $(x_b, \ell_b) = (x_a, \ell_a)$ solves these two equality conditions. The slope of b's *iso* marginal utility curves $u_x(x_b, \ell_b, h) = u_x^a$ and $u_\ell(x_b, \ell_b, h) = u_\ell^a$ in the (x, ℓ)-space are

$$\frac{\mathrm{d}\ell_b}{\mathrm{d}x_b}|_{u_x^b(\cdot)=u_x^a} = \frac{-u_{xx}^b}{u_{x\ell}^b}, \text{and } \frac{\mathrm{d}\ell_b}{\mathrm{d}x_b}|_{u_\ell^b(\cdot)=u_\ell^a} = \frac{-u_{x\ell}^b}{u_{\ell\ell}^b},$$

respectively. By the concavity of the utility function ($u_{xx}u_{\ell\ell} - u_{x\ell}^2 > 0$), the first slope always exceeds (in absolute value) the second. Hence, the two iso marginal utility curves cross only once in the (x_b, ℓ_b)-space, namely at (x_a, ℓ_a). This means that both partners are equally well off. ■

Theorem 2 *Suppose HIC_H is satisfied such that the household is at as well off at $(\mathbf{L}, \mathbf{H})$ and $(\mathbf{H}, \mathbf{H})$, then so are its members. Moreover, at the package $(\mathbf{L}, \mathbf{H})$* a *performs more work at home than* b *such that both end up with the same amount of leisure.*

Proof. This follows immediately from lemma 7. ■

7.2 Appendix to section 4

7.2.1 Results on incentive compatibility

Lemma 8 *Incentive compatibility of the allocation implies that the gross and net income levels are ranked as follows: $Y_{Ml} \leq Y_L \leq Y_H \leq Y_{Mh}$, and $c_{Ml} \leq c_L \leq c_H \leq c_{Mh}$*

Proof.

1. $Y_{Ml} \leq Y_{Mh}$: follows directly from $W^M(\mathbf{Ml}, \mathbf{Mh}) \geq W^M(\mathbf{Mh}, \mathbf{Ml})$

2. $Y_L \leq Y_{Mh}$: follows directly from summing $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{Mh},\mathbf{Ml})$ and $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{L},\mathbf{L})$.

3. $Y_L \leq Y_H$: follows directly from summing $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{H},\mathbf{H})$ and $W^H(\mathbf{H},\mathbf{H}) \geq W^H(\mathbf{L},\mathbf{L})$.

4. $Y_{Ml} \leq Y_H$: follows directly from summing $W^H(\mathbf{H},\mathbf{H}) \geq W^H(\mathbf{Ml},\mathbf{Mh})$ and $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{H},\mathbf{H})$.

5. $c_{Ml} \leq c_{Mh}$: follows from $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{Ml},\mathbf{Ml})$ and $Y_{Ml} \leq Y_{Mh}$.

6. $c_{Ml} \leq c_H \leq c_{Mh}$: follows from summing $W^H(\mathbf{H},\mathbf{H}) \geq W^H(\mathbf{H},\mathbf{Mh})$ and $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{Ml},\mathbf{H})$, from $c_{Ml} \leq c_{Mh}$, and the concavity of $v(\cdot)$.

7. $c_{Ml} \leq c_L \leq c_{Mh}$: follows from summing $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{L},\mathbf{Ml})$ and $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{L},\mathbf{Mh})$, from $c_{Ml} \leq c_{Mh}$, and the concavity of $v(\cdot)$.

8. $c_L \leq c_H$: follows from $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{H},\mathbf{H})$ and $Y_L \leq Y_H$.

9. $Y_{Ml} \leq Y_L$: follows from $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{L},\mathbf{Mh})$, and $c_{Ml} \leq c_L$.

10. $Y_H \leq Y_{Mh}$: follows from $W^H(\mathbf{H},\mathbf{H}) \geq W^H(\mathbf{H},\mathbf{Mh})$, and $c_H \leq c_{Mh}$.

■

Lemma 9 *Suppose that the four bundles* $\mathbf{L},\mathbf{H},\mathbf{Ml}$, *and* $\mathbf{Mh}$ *satisfy the monotonicity constraints* $Y_{Ml} \leq Y_L \leq Y_H \leq Y_{Mh}$, *and* $c_{Ml} \leq c_L \leq c_H \leq c_{Mh}$, *as well as the three self-selection constraints.*

$$W^H(\mathbf{H},\mathbf{H}) = W^H(\mathbf{L},\mathbf{H}), \tag{A1}$$

$$W^M(\mathbf{Ml},\mathbf{Mh}) = W^M(\mathbf{L},\mathbf{H}), \text{ and} \tag{A2}$$

$$W^M(\mathbf{Ml},\mathbf{Mh}) = W^M(\mathbf{Ml},\mathbf{H}), \tag{A3}$$

Suppose in addition that the self-selection constraint

$$W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{L},\mathbf{H}) \tag{A4}$$

is verified. Then the remaining 29 self-selection constraints are verified as well.

Proof.

1. Constraints of the type $W^L(\mathbf{L},\mathbf{L}) \geq W^L(I,J)$, $I,J \in \{\mathbf{Ml},\mathbf{L},\mathbf{H},\mathbf{Mh}\}$ (8 in addition to (A4))

(a) $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{L},\mathbf{Ml})$: from $W^M(\mathbf{L},\mathbf{H}) = W^M(\mathbf{Ml},\mathbf{H})$ and the fact that at bundle $\mathbf{L}$, the slope of $\mathrm{d}W^L(\mathbf{L},\cdot)=0$ $(\frac{1}{w_L v'(c_L)}) \leq$ slope of $\mathrm{d}W^M(\cdot,\mathbf{H})=0$ $(\frac{1}{w_L v'(\frac{c_L+c_H}{2})})$.

(b) $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{L},\mathbf{Mh})$: from (A4) and 3a)

(c) $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{Ml},\mathbf{Ml})$: from $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{L},\mathbf{Ml})$ and the fact that the indifference curve $\mathrm{d}W^L(\cdot,\cdot)=0$ is the upper envelope of the indifference curve $\mathrm{d}W^L(\mathbf{L},\cdot)=0$.

(d) $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{H},\mathbf{H})$: from $W^L(\mathbf{L},\mathbf{L}) > W^L(\mathbf{L},\mathbf{H})$ and the fact that the indifference curve $\mathrm{d}W^L(\cdot,\cdot)=0$ is the upper envelope of the indifference curve $\mathrm{d}W^L(\mathbf{L},\cdot)=0$.

(e) $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{Mh},\mathbf{Mh})$: from $W^{\mathbf{L}}(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{L},\mathbf{Mh})$ and the fact that the indifference curve $\mathrm{d}W^L(\cdot,\cdot)=0$ is the upper envelope of the indifference curve $\mathrm{d}W^L(\mathbf{L},\cdot)=0$.

(f) $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{Ml},\mathbf{H})$: from (A4) and the fact that $W^L(\mathbf{L},\mathbf{H}) \geq W^L(\mathbf{Ml},\mathbf{H})$ follows from $W^M(\mathbf{L},\mathbf{H}) \geq W^M(\mathbf{Ml},\mathbf{H})$.

(g) $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{H},\mathbf{Mh})$: from summing $W^H(\mathbf{H},\mathbf{H}) \geq W^H(\mathbf{H},\mathbf{Mh})$ (proved below, in 3a)) and $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{H},\mathbf{H})$, and adding and subtracting $\frac{Y_{Mh}}{w_L}$ to the *rhs.*

(h) $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{Ml},\mathbf{Mh})$: from 1f), (A3) and that fact that at bundle $\mathbf{H}$ the slope of the indifference curve $\mathrm{d}W^L(\mathbf{Ml},\cdot)=0$ $(\frac{1}{w_L v'(\frac{c_{Ml}+c_H}{2})})\geq$slope of the indifference curve $\mathrm{d}W^M(\mathbf{Ml},\cdot)=0$ $(\frac{1}{w_H v'(\frac{c_{Ml}+c_H}{2})})$.

2. Constraints of the type $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(I,J)$, $I,J \in \{\mathbf{Ml},\mathbf{L},\mathbf{H},\mathbf{Mh}\}$ (13 in addition to (A2) and (A3)).

(a) $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{L},\mathbf{Mh})$: from $W^M(\mathbf{Ml},\mathbf{Mh}) = W^M(\mathbf{Ml},\mathbf{H})$ (from (A2) and (A3)) and the fact that at bundle $\mathbf{Ml}$, the slope of $\mathrm{d}W^M(\cdot,\mathbf{Mh})=0$ $(\frac{1}{w_L v'(\frac{c_{Ml}+c_{Mh}}{2})}) \geq$ slope of $\mathrm{d}W^M(\cdot,\mathbf{H})=0.(\frac{1}{w_L v'(\frac{c_{Ml}+c_H}{2})})$.

(b) $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{Mh},\mathbf{L})$: immediately from 2a)

(c) $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{H},\mathbf{H})$: from $W^M(\mathbf{Ml},\mathbf{Mh}) = W^M(\mathbf{L},\mathbf{H})$, (A1), and the fact that at bundle $\mathbf{L}$, the slope of $\mathrm{d}W^M(\cdot,\mathbf{H})=0$ $(\frac{1}{w_L v'(\frac{c_L+c_H}{2})})\geq$ slope of $\mathrm{d}W^H(\cdot,\mathbf{H})=0$ $(\frac{1}{w_H v'(\frac{c_L+c_H}{2})})$.

(d) $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{H},\mathbf{Mh})$: from 2c), (A3) and the fact that at bundle $\mathbf{H}$, the slope of $\mathrm{d}W^M(\mathbf{H},\cdot)=0$ $(\frac{1}{w_H v'(c_H)})\geq$ slope of $\mathrm{d}W^M(\mathbf{Ml},\cdot)=0$ $(\frac{1}{w_H v'(\frac{c_{Ml}+c_H}{2})})$.

(e) $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{Mh},\mathbf{H})$: immediately from 2d).

(f) $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{Mh},\mathbf{Mh})$: from 2d) and the fact that the indifference curve $\mathrm{d}W^M(\cdot,\cdot) = 0$ tangent to $\mathrm{d}W^M(\cdot,\mathbf{H}) = 0$ at H has more curvature.

(g) $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{H},\mathbf{L})$: from $W^M(\mathbf{Ml},\mathbf{Mh}) = W^M(\mathbf{L},\mathbf{H})$ and the fact that $W^M(\mathbf{L},\mathbf{H}) \geq W^M(\mathbf{H},\mathbf{L})$.

(h) $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{H},\mathbf{Ml})$: from $W^M(\mathbf{H},\mathbf{Ml}) \leq W^M(\mathbf{Ml},\mathbf{H})$ and the fact that $W^M(\mathbf{Ml},\mathbf{Mh}) = W^M(\mathbf{Ml},\mathbf{H})$.

(i) $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{L},\mathbf{L})$: from $W^M(\mathbf{Ml},\mathbf{Mh}) = W^M(\mathbf{L},\mathbf{H})$, (A1) and the fact that at bundle $\mathbf{H}$, the slope of $\mathrm{d}W^H(\cdot,\mathbf{H}) = 0$ $(\frac{1}{w_H v'(c_H)})\geq$ slope of $\mathrm{d}W^M(\mathbf{L},\cdot) = 0$ $(\frac{1}{w_H v'(\frac{c_L+c_H}{2})})$.

(j) $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{Ml},\mathbf{L})$: because $W^M(\mathbf{Ml},\mathbf{Mh}) = W^M(\mathbf{Ml},\mathbf{H})$, $W^H(\mathbf{H},\mathbf{H}) = W^H(\mathbf{H},\mathbf{L})$, and the fact that at $\mathbf{H}$ the slope of $\mathrm{d}W^M(\mathbf{Ml},\cdot) = 0$ $(\frac{1}{w_H v'(\frac{c_{Ml}+c_H}{2})}) \leq$ slope of $\mathrm{d}W^H(\mathbf{H},\cdot)$ $(\frac{1}{w_H v'(c_H)})$=0.

(k) $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{L},\mathbf{Ml})$: immediately from 2j).

(l) $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{Ml},\mathbf{Ml})$: from 2j), (A2) and because at $\mathbf{L}$ the slope of $\mathrm{d}W^M(\cdot,\mathbf{H}) = 0$ $(\frac{1}{w_L v'(\frac{c_L+c_H}{2})}) >$ slope of $\mathrm{d}W^M(\mathbf{Ml},\cdot) = 0$ $(\frac{1}{w_H v'(\frac{c_L+c_{Ml}}{2})})$.

(m) $W^M(\mathbf{Ml},\mathbf{Mh}) \geq W^M(\mathbf{Mh},\mathbf{Ml})$: trivial.

3. Constraints of the type $W^H(\mathbf{H},\mathbf{H}) \geq W^H(I,J)$, $I,J \in \{\mathbf{Ml},\mathbf{L},\mathbf{H},\mathbf{Mh}\}$ (8 in addition to (A1)).

(a) $W^H(\mathbf{H},\mathbf{H}) \geq W^H(\mathbf{H},\mathbf{Mh})$: from (A1), (A3) and the fact that at bundle $\mathbf{H}$, the slope of $\mathrm{d}W^H(\cdot,\mathbf{H}) = 0$ $(\frac{1}{w_H v'(c_H)}) >$ slope of $\mathrm{d}W^M(\mathbf{Ml},\cdot) = 0.(\frac{1}{w_H v'(\frac{c_{Ml}+c_H}{2})})$.

(b) $W^H(\mathbf{H},\mathbf{H}) \geq W^H(\mathbf{L},\mathbf{L})$: from (A1) and the fact that the indifference curve $\mathrm{d}W^H(\cdot,\cdot) = 0$ tangent to $\mathrm{d}W^H(\mathbf{H},\cdot) = 0$ at $\mathbf{H}$ has more curvature

(c) $W^H(\mathbf{H},\mathbf{H}) \geq W^H(\mathbf{L},\mathbf{Ml})$: because $W^M(\mathbf{Ml},\mathbf{H}) = W^M(\mathbf{L},\mathbf{H})$, (A1) and the fact that at bundle $\mathbf{L}$, the slope of $\mathrm{d}W^M(\cdot,\mathbf{H}) = 0$ $(\frac{1}{w_L v'(\frac{c_L+c_H}{2})})>$ the slope of $\mathrm{d}W^H(\cdot,\mathbf{H}) = 0$ $(\frac{1}{w_H v'(\frac{c_L+c_H}{2})})$

(d) $W^H(\mathbf{H},\mathbf{H}) \geq W^H(\mathbf{L},\mathbf{Mh})$: from (A1), $W^M(\mathbf{Ml},\mathbf{H}) = W^M(\mathbf{L},\mathbf{H})$, and the fact that at the bundle $\mathbf{H}$, the slope of $\mathrm{d}W^H(\mathbf{L},\cdot) = 0$ $(\frac{1}{w_H v'(\frac{c_L+c_H}{2})})>$ the slope of $\mathrm{d}W^M(\mathbf{Ml},\cdot) = 0$ $(\frac{1}{w_H v'(\frac{c_{Ml}+c_H}{2})})$.

(e) $W^H(\mathbf{H},\mathbf{H}) \geq W^H(\mathbf{Ml},\mathbf{H})$: from (A1) , $W^M(\mathbf{Ml},\mathbf{H}) = W^M(\mathbf{L},\mathbf{H})$, and the fact that at the bundle $\mathbf{L}$, the slope of $\mathrm{d}W^M(\cdot,\mathbf{H}) = 0$ $(\frac{1}{w_L v'(\frac{c_L+c_H}{2})})>$ the slope of $\mathrm{d}W^H(\mathbf{H},\cdot) = 0$ $(\frac{1}{w_H v'(\frac{c_{Ml}+c_H}{2})})$.

(f) $W^H(\mathbf{H},\mathbf{H}) \geq W^H(\mathbf{Ml},\mathbf{Ml})$: follows from 3e), and the fact that the indifference curve $\mathrm{d}W^H(\cdot,\cdot) = 0$ tangent to $\mathrm{d}W^H(\mathbf{H},\cdot) = 0$ at $\mathbf{H}$ has more curvature.

(g) $W^H(\mathbf{H},\mathbf{H}) \geq W^H(\mathbf{Mh},\mathbf{Mh})$: follows from 3a), and the fact that the indifference curve $\mathrm{d}W^H(\cdot,\cdot) = 0$ tangent to $\mathrm{d}W^H(\mathbf{H},\cdot) = 0$ at $\mathbf{H}$ has more curvature.

(h) $W^H(\mathbf{H},\mathbf{H}) \geq W^H(\mathbf{Ml},\mathbf{Mh})$: follows from 3e), and the fact that $W^H(\mathbf{Ml},\mathbf{H}) = W^H(\mathbf{Ml},\mathbf{Mh})$ follows from (A3).

■

Theorem 3 *Under redistribution assumptions **R1** and **R2**, the optimal taxation policy solves the reduced form problem (P2) subject to (A1),(A2), and (A3) will not be constrained by the self-selection constraint* $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{L},\mathbf{H})$ *(i.e. by (A4))*

Proof. By lemma 9, a tax policy that is monotonic and satisfies (A1)–(A4) will be overall incentive compatible. Since the reduced form problem (P2) is obtained from (P1) by imposing (A1)–(A3) and the budget constraint with equality and eliminating all income levels, it remains to show that the solution to (P2) satisfies (A4). The proof goes by contradiction. Suppose that at the optimal policy described by (P2), the constraint $W^L(\mathbf{L},\mathbf{L}) \geq W^L(\mathbf{L},\mathbf{H})$ is not satisfied. Then it is required that the slope of the indifference curve $\mathrm{d}W^L(\mathbf{L},\cdot) = 0$ at bundle $\mathbf{L}$ $(\frac{1}{w_L v'(c_L)})$ is smaller than the slope of the indifference curve $\mathrm{d}W^H(\mathbf{H},\cdot) = 0$ at the same bundle $(\frac{1}{w_H v'(\frac{c_L+c_H}{2})})$. But this means that the square bracket term on the *rhs* of the optimality rule for c_L (14) is negative, and therefore that $\frac{1}{w_L v'(c_L)} > 1$. Since under R1 and R2 the coefficient A is positive, the optimality rule for c_H (13) tells that the slope of the indifference curve $\mathrm{d}W^H(\mathbf{H},\cdot) = 0$ at bundle $\mathbf{H}$ $(\frac{1}{w_L v'(c_H)})$ is smaller than 1. By convexity of H's preferences, the slope of this indifference curve at $\mathbf{L}$ must be even smaller. Since by assumption the indifference curve $\mathrm{d}W^L(\mathbf{L},\cdot) = 0$ is flatter than the indifference curve $\mathrm{d}W^H(\mathbf{H},\cdot) = 0$ at bundle $\mathbf{L}$, the slope of the former indifference curve at that bundle must be smaller than 1. This gives a contradiction. ■

7.2.2 Solution to the reduced form taxation problem

The income levels can be expressed in terms of the aggregate consumption level and the utility differences:

$$
\begin{aligned}
Y_{Mh} &= C + (f_L + \frac{1}{2} f_M + f_H)\,(2w_H\Delta_M) + (f_L + \frac{1}{2} f_M)\,(2w_H\Delta_H) + \frac{1}{2} f_M (2w_L\Delta_L) \\
Y_H &= C - \frac{1}{2} f_M\,(2w_H\Delta_M) + (f_L + \frac{1}{2} f_M)\,(2w_H\Delta_H) + \frac{1}{2} f_M\,(2w_L\Delta_L) \\
Y_L &= C - \frac{1}{2} f_M\,(2w_H\Delta_M) + (\frac{1}{2} f_M + f_H)\,(2w_H\Delta_H) + \frac{1}{2} f_M\,(2w_L\Delta_L) \\
Y_{Ml} &= C - \frac{1}{2} f_M\,(2w_H\Delta_M) - (\frac{1}{2} f_M + f_H)\,(2w_H\Delta_H) - (1 - \frac{1}{2} f_M)\,(2w_L\Delta_L).
\end{aligned}
$$

where $C \stackrel{\text{def}}{=} (f_L c_L + \frac{1}{2} f_M c_{Ml} + \frac{1}{2} f_M c_{Mh} + f_H c_H)$.

The social welfare function may then be written as

$$
\begin{aligned}
\frac{1}{2} SW &= f_L\beta_L \left(v(c_L) - \frac{Y_L}{w_L} \right) + \frac{1}{2} f_M\beta_M \left(v(\frac{c_{Ml} + c_{Mh}}{2}) - \frac{Y_{Ml}}{w_L} \right) \\
&\quad + \frac{1}{2} f_M\beta_M \left(v(\frac{c_{Ml} + c_{Mh}}{2}) - \frac{Y_{Mh}}{w_H} \right) + f_H\beta_H \left(v(c_H) - \frac{Y_H}{w_H} \right).
\end{aligned}
$$

The derivatives of the Lagrangian to the reduced form problem. The Lagrangian function to problem (P2) is defined as

$$
\begin{aligned}
\mathcal{L} = f_L\beta_L &\left(v(c_L) - \frac{1}{w_L} \left[C + -\frac{1}{2} f_M\,(2w_H\Delta_M) + (\frac{1}{2} f_M + f_H)\,(2w_H\Delta_H) + \frac{1}{2} f_M\,(2w_L\Delta_L) \right] \right) + \\
\frac{1}{2} f_M\beta_M &\left(v(\frac{c_{Ml} + c_{Mh}}{2}) - \frac{1}{w_L} \left[C - \frac{1}{2} f_M\,(2w_H\Delta_M) - (\frac{1}{2} f_M + f_H)\,(2w_H\Delta_H) - (1 - \frac{1}{2} f_M)\,(2w_L\Delta_L) \right] \right) + \\
\frac{1}{2} f_M\beta_M &\left(v(\frac{c_{Ml} + c_{Mh}}{2}) - \frac{1}{w_H} \left[C + (f_L + \frac{1}{2} f_M + f_H)\,(2w_H\Delta_M) + (f_L + \frac{1}{2} f_M)\,(2w_H\Delta_H) + \frac{1}{2} f_M (2w_L\Delta_L) \right] \right) + \\
f_H\beta_H &\left(v(c_H) - \frac{1}{w_H} \left[C + -\frac{1}{2} f_M\,(2w_H\Delta_M) + (f_L + \frac{1}{2} f_M)\,(2w_H\Delta_H) + \frac{1}{2} f_M\,(2w_L\Delta_L) \right] \right) + \\
\kappa &\left(C - \frac{1}{2} f_M\,(2w_H\Delta_M) - (\frac{1}{2} f_M + f_H)\,(2w_H\Delta_H) - (1 - \frac{1}{2} f_M)\,(2w_L\Delta_L) \right).
\end{aligned}
$$

After making use of the two identities $f_L + f_M + f_H = 1$ and $f_L \frac{\beta_L}{w_L} + \frac{1}{2} f_M \frac{\beta_M}{w_L} + \frac{1}{2} f_M \frac{\beta_M}{w_H} + f_H \frac{\beta_H}{w_H} = 1$ the derivatives of the Lagrangian are:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_L} &= -f_L + f_L \frac{\beta_L}{w_L} w_L v'(c_L) - \left[\frac{1}{2} f_M (1 - \frac{\beta_M}{w_L} - \kappa) + f_H (1 - \frac{\beta_H}{w_H} - \kappa) \right] w_H v'(\frac{c_L + c_H}{2}) \\
&\quad - \frac{1}{2} f_M (1 - \frac{\beta_M}{w_L} - \kappa) w_L v'(\frac{c_L + c_H}{2}) - \kappa w_L v'(\frac{c_L + c_H}{2}),
\end{aligned}
$$

$$\frac{\partial \mathcal{L}}{\partial c_{Ml}} = -\frac{1}{2}f_M + \frac{1}{2}f_M w_L v'(\frac{c_{Ml}+c_{Mh}}{2}) + \frac{1}{2}f_M(w_H - w_L)\left[v'(\frac{c_{Ml}+c_{Mh}}{2}) - v'(\frac{c_{Ml}+c_H}{2})\right]$$
$$+\kappa\left(\frac{1}{2}f_M - \frac{1}{2}f_M w_H\left[v'(\frac{c_{Ml}+c_{Mh}}{2}) - v'(\frac{c_{Ml}+c_H}{2})\right] + (1-\frac{1}{2}f_M)w_L v'(\frac{c_{Ml}+c_H}{2})\right),$$

$$\frac{\partial \mathcal{L}}{\partial c_{Mh}} = -\frac{1}{2}f_M + \frac{1}{2}f_M w_H v'(\frac{c_{Ml}+c_{Mh}}{2}) + \kappa\frac{1}{2}f_M\left[1 - w_H v'(\frac{c_{Ml}+c_{Mh}}{2})\right],$$

$$\frac{\partial \mathcal{L}}{\partial c_H} = -f_H + \left[2\frac{1}{2}f_M(1-\frac{\beta_M}{w_H}) + 2f_H - f_H\frac{\beta_H}{w_H}\right]w_H v'(c_H) - \frac{1}{2}f_M(w_H - w_L)v'(\frac{c_{Ml}+c_H}{2}) -$$
$$\left(\frac{1}{2}f_M(w_H + w_L) - \frac{1}{2}f_M 2\beta_M + f_H w_H(1-\frac{\beta_H}{w_H})\right)v'(\frac{c_L+c_H}{2})$$
$$+\kappa\{f_H + \left[w_L + \frac{1}{2}f_M(w_H - w_L)\right]v'(\frac{c_{Ml}+c_H}{2}) - \left[\frac{1}{2}f_M + f_H\right]2w_H v'(c_H)$$
$$+\left[f_H w_H - w_L + \frac{1}{2}f_M(w_H + w_L)\right]v'(\frac{c_L+c_H}{2})\}.$$

The optimality conditions. Equating $\frac{\partial \mathcal{L}}{\partial c_{Mh}}$ to zero immediately gives $w_H v'(\frac{c_{Ml}+c_{Mh}}{2}) = 1$. This result can then be used in $\frac{\partial \mathcal{L}}{\partial c_{Ml}} = 0$ to yield

$$\frac{1}{2}f_M(w_H - w_L)(1-\kappa) = w_L\kappa, \tag{A5}$$

which can be rearranged to give the equilibrium value for κ given in the text. Next, in $\frac{\partial \mathcal{L}}{\partial c_L}$, $w_L\kappa$ can be replaced by the *lhs* of (A5). Equating $\frac{\partial \mathcal{L}}{\partial c_L}$ to zero then gives

$$(1-\kappa)f_L = f_L\frac{\beta_L}{w_L}w_L v'(c_L) - \left[f_M(1-\frac{\beta_M}{w_L}-\kappa) + f_H(1-\frac{\beta_H}{w_H}-\kappa)\right]w_H v'(\frac{c_L+c_H}{2}).$$

Subtracting $(1-\kappa)f_L w_L v'(c_L)$ from both sides, and rearranging gives the first order condition for c_L as stated in the text.

Finally, by making use of (A5) we can eliminate the terms in $v'(\frac{c_{Ml}+c_H}{2})$ from $\frac{\partial \mathcal{L}}{\partial c_H}$. After rearrangement, we get the first order condition for c_L as stated in the text.

To obtain the expression for $w_H v'(c_H) - w_L v'(c_L)$, I first solve (13) and (14) for $w_H v'(c_H)$ and $w_L v'(c_L)$, respectively. This gives

$$w_H v'(c_H) = \frac{(1-\kappa)f_H + A w_H v'(\frac{c_L+c_H}{2})}{(1-\kappa)f_H + A}, \text{and}$$

$$w_L v'(c_L) = \frac{(1-\kappa) f_L + A w_H v'(\frac{c_L + c_H}{2})}{(1-\kappa) f_H + A + B}.$$

Subtracting this second equation from the first and putting the resulting *rhs* on a common denominator, then gives (15).

The expression $(1-\kappa - \frac{\beta_M}{w_H})$ is positive. Since $1-\kappa = \frac{w_L}{w_L + \frac{1}{2} f_M (w_H - w_L)}$, $(1-\kappa - \frac{\beta_M}{w_H})$ positive requires that

$$\frac{w_L}{w_L + \frac{1}{2} f_M (w_H - w_L)} - \frac{\beta_M}{w_H} \geq 0$$
$$\Updownarrow$$
$$\frac{2 w_L \left[(\frac{\beta_M}{w_H})^{-1} - 1 \right]}{(w_H - w_L)} \geq f_M.$$

A sufficient condition for this to hold is that the *lhs* exceeds the largest feasible value for f_M, $2\mu_H$, or

$$\frac{w_L \left[(\frac{\beta_M}{w_H})^{-1} - 1 \right]}{(w_H - w_L)} \geq \mu_H$$
$$\Updownarrow$$
$$w_L (\frac{\beta_M}{w_H})^{-1} \geq w^{aa}$$
$$\Updownarrow$$
$$\frac{w_L}{w^{aa}} \geq \frac{\beta_M}{w_H}$$

which is condition **R2**.