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Discussion paper

Multidimensional screening in a monopolistic insurance market: proofs

BY

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Multidimensional screening in a monopolistic insurance market: proofs

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Abstract: This technical paper contains the proofs of all lemmata, propositions and other statements made in the paper *Multidimensional screening in a monopolistic insurance market*.

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1 Introduction

This technical paper contains the proofs of all lemmata, propositions and other statements made in the paper *Multidimensional screening in a monopolistic insurance market.*[1] For convenience, we reproduce in the next section some of the main definitions, assumptions and notational conventions used in that paper, and restate the main problem. In section 3, we present the proofs of the no-distortion-at-the-top/no-rent-at-the-bottom result (Theorem 1) and the proofs of the optimal contract menu when insurance takers only differ in risk type (Theorem 2), in risk aversion (Theorem 3), and when risk type and risk aversion are perfectly positively correlated (Theorem 4). Section 4 deals with the two-dimensional heterogeneity case: after a reminder of some definitions and assumptions (Section 4.1), we reformulate the main proposition of the paper (Section 4.2), and explain our strategy to prove it (Section 4.3). This strategy consists of four steps; these are dealt with in Sections 5, 6, 7 and 8, respectively. Section 8 concludes with *Theorem 11* which is proven in Appendix A. Appendix B proves the three theorems stated in Section 6.

The results depend on the relationships between a series of critical values for the measure of similarity in risk aversion (defined as x, $x = 1$ corresponding to identical risk aversion). The orderings of these critical values depend on the value for ρ, a measure of correlation between risk type (μ) and risk aversion (ν). Appendix C shows the dependency of these orderings on ρ. In particular, it shows that (almost) all orderings are independent of the exact value of ρ *as long as this value is* non-positive. The exception is given in Lemma C.10.

In the margin of his copy of Diophantus' Arithmetica, Pierre de Fermat wrote: "To divide a cube into two other cubes, a fourth power or in general any power whatever into two powers of the same denomination above the second is impossible, and I have assuredly found an admirable proof of this, but the margin is too narrow to contain it." We have assuredly found a proof of the main proposition of our paper. We doubt that it deserves the label admirable. But that a margin is too narrow to contain it is beyond dispute!

[1] Olivella, P and F Schroyen (2011) "Multidimensional screening in a monopolistic insurance market" (NHH DP 19/2011, CORE DP 21/56)

2 Main notations and assumptions

- $C = (c, P)$, a linear insurance contract with coinsurance rate c and premium P
- $\mu \in \{\mu_L, \mu_H\}$,where $\mu_L < \mu_H$: the expected loss
- $\Delta\mu \stackrel{\text{def}}{=} \mu_H - \mu_L > 0$
- $\nu \stackrel{\text{def}}{=} r\sigma^2$: the product of the coefficient of absolute risk aversion and the variance of the loss
- $\nu \in \{\nu_L, \nu_H\}$, $\nu_L < \nu_H$: the degree of absolute risk aversion (σ^2 normalised to 1)
- $\Delta\nu \stackrel{\text{def}}{=} \nu_H - \nu_L$
- Type ij: a person with characteristics (μ_i, ν_j)
- α_{ij}: the share of ij people in the population ($i, j = H, L$, $\sum_{i,j}\alpha_{ij} = 1$)
- $\alpha_{k\cdot}$: the fraction of people with expected loss μ_k ($\alpha_{k\cdot} = \alpha_{kL} + \alpha_{kH}$)
- $\alpha_{\cdot k}$: the fraction of people with perceived variance ν_k ($\alpha_{\cdot k} = \alpha_{Lk} + \alpha_{Hk}$)
- $R_{ij}(c, P)$: the certainty equivalent rent that the agent enjoys from contract (c, P);

$$R_{ij}(c, P) \stackrel{\text{def}}{=} U^{ij}(c, P) - U^{ij}(1, 0) = -P + (1 - c)\mu_i + \frac{1}{2}(1 - c^2)\nu_j. \quad (1)$$

- $R_{ij} \stackrel{\text{def}}{=} R_{ij}(c_{ij}, P_{ij})$ $(i, j = L, H)$: the rent when truthful
- $\delta(\cdot)$: an auxiliary function to write the rent when mimicking;

$$\delta(c_{kl}, \mu_i - \mu_k, \nu_j - \nu_l) \stackrel{\text{def}}{=} (1 - c_{kl})(\mu_i - \mu_k) + \frac{1}{2}(1 - c_{kl}^2)(\nu_j - \nu_l). \quad (2)$$

- $R_{ij}(c_{kl}, P_{kl})$: the rent when pretending to be of type kl;

$$R_{ij}(c_{kl}, P_{kl}) \stackrel{\text{def}}{=} R_{kl}(c_{kl}, P_{kl}) + \delta(c_{kl}, \mu_i - \mu_k, \nu_j - \nu_l). \quad (3)$$

- monotonicity conditions:

– for incentive compatibility between contracts Hj and Lj ($j = H, L$):

$$c_{Hj} \leq c_{Lj} \tag{4}$$

– for incentive compatibility between contracts iH and iL ($i = H, L$):

$$c_{iH} \leq c_{iL}, \tag{5}$$

- $c = \frac{\Delta\mu}{\Delta\nu}$: the locus of tangency points between HL's and LH's indifference curves in the (c, P)-space
- $D \stackrel{\text{def}}{=} \frac{\Delta\mu}{\nu_L} \in (0, \infty)$: a dimensionless measure of the heterogeneity in μ
- $x \stackrel{\text{def}}{=} \frac{\nu_L}{\nu_H} \in (0, 1]$: a dimensionless measure of the similarity in ν
- $\pi^{ij}(c, P)$: the principal's expected profit when an agent of type ij has accepted contract (c, P);

$$\pi^{ij}(c, P) = P - (1 - c)\mu_i. \tag{6}$$

- Total (or expected) profits are

$$\sum_{i,j} \alpha_{ij} \left[\frac{1}{2}[1 - c_{ij}^2]\nu_j - R_{ij} \right]. \tag{7}$$

- The main problem of the principal/insurance company

$$\max_{\{c_{ij}, R_{ij}\}} \sum_{i,j=H,L} \alpha_{ij} \left[\frac{1}{2}[1 - c_{ij}^2]\nu_j - R_{ij} \right], \text{ s.t.}$$

$$R_{ij} \geq 0 \quad (i, j = L, H), \qquad 0 \leq c_{ij} \leq 1 \quad (i, j = L, H)$$

$$R_{LL} \geq \begin{cases} R_{LH} + \delta(c_{LH}, 0, -\Delta\nu) \\ R_{HL} + \delta(c_{HL}, -\Delta\mu, 0) \\ R_{HH} + \delta(c_{HH}, -\Delta\mu, -\Delta\nu) \end{cases} \qquad R_{LH} \geq \begin{cases} R_{LL} + \delta(c_{LL}, 0, \Delta\nu) \\ R_{HL} + \delta(c_{HL}, -\Delta\mu, \Delta\nu) \\ R_{HH} + \delta(c_{HH}, -\Delta\mu, 0) \end{cases}$$

$$R_{HL} \geq \begin{cases} R_{LL} + \delta(c_{LL}, \Delta\mu, 0) \\ R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu) \\ R_{HH} + \delta(c_{HH}, 0, -\Delta\nu) \end{cases} \qquad R_{HH} \geq \begin{cases} R_{LL} + \delta(c_{LL}, \Delta\mu, \Delta\nu) \\ R_{LH} + \delta(c_{LH}, \Delta\mu, 0) \\ R_{HL} + \delta(c_{HL}, 0, \Delta\nu) \end{cases}$$

The next section provides the solution to this problem.

3 Preliminary results

This section gives the proofs of Theorems 1-4 in the main text.

Theorem 1 *At the optimum solution,(i)* $c_{HH} = 0$ *and (ii)* $R_{LL} = 0$.

Proof. Part (i). Assume, by contradiction, that $c^*_{HH} > 0$. Then let $c'_{HH} = c^*_{HH} - \varepsilon$ for some sufficiently small $\varepsilon > 0$. This still preserves non-negativity of c_{HH}. It also lowers the rents that HL, LH, and LL obtain when mimicking HH, so that none of the IC constraints get more binding. Finally, notice that the objective function decreases in c_{HH}.

Part (ii). Observe first that $R_{ij} \geq R_{LL}$ for all ij. To see this, note that R_{ij} $(ij = HL, LH, HH) \geq R_{LL}$ whenever $c_{LL} \leq 1$. Assume then by contradiction that $R^*_{LL} > 0$. Then the previous observation tells us that $R^*_{ij} > 0$ $(ij = HL, LH, HH)$. Then the alternative rent vector $(R^*_{LL} - \varepsilon, R^*_{LH} - \varepsilon, R^*_{HL} - \varepsilon, R^*_{HH} - \varepsilon)$ does not upset IC and increases the objective function. ■

Theorem 2 *When all agents have the same risk aversion, the optimal menu has* $c_{H\cdot} = 0$ *and* $c_{L\cdot} = \min\{D\frac{\alpha_{H\cdot}}{1-\alpha_{H\cdot}}, 1\}$.

Proof. Since $R_H = \delta(c_{L\cdot}, \Delta\mu, 0)$ and $R_L = 0$, the Lagrange function is

$$\mathcal{L} = \alpha_H\{\frac{1}{2}(1 - c^2_{H\cdot})\nu - \delta(c_{L\cdot}, \Delta\mu, 0)\} + \alpha_L\{\frac{1}{2}(1 - c^2_{L\cdot})\}.$$

The first and second order derivatives are:

$$\frac{\partial}{\partial c_{H\cdot}} = -\alpha_{H\cdot}c_{H\cdot}\nu, \frac{\partial^2}{\partial c^2_{H\cdot}} = -\alpha_{H\cdot}\nu < 0$$
$$\frac{\partial}{\partial c_{L\cdot}} = \alpha_{H\cdot}\Delta\mu - \alpha_{L\cdot}c_{L\cdot}\nu, \frac{\partial^2}{\partial c^2_{L\cdot}} = -\alpha_{L\cdot}\nu < 0$$

Hence, $c^*_{H\cdot} = 0$ and $c^*_{L\cdot}$ is given by $\min\{D\frac{\alpha_{H\cdot}}{1-\alpha_{H\cdot}}, 1\}$. $c^*_{L\cdot}$ becomes 1 when $\alpha_{H\cdot} \geq \frac{1}{1+D}(< 1)$. ■

Theorem 3 *When all agents face the same expected loss, the optimal menu has* $c_{\cdot H} = 0$, *and* $c_{\cdot L} = \begin{cases} 0 \text{ if } x > \alpha_{\cdot H} \\ 1 \text{ otherwise.} \end{cases}$

Proof. With identical risk size but different risk aversion, $R_H = \delta(c, 0, \Delta\nu)$ and $R_L = 0$. The Lagrange function is then

$$\mathcal{L} = \alpha_{\cdot H}\{\frac{1}{2}(1 - c_{\cdot H}^2)\nu_H - \delta(c_{\cdot L}, 0, \Delta\nu)\} + \alpha_{\cdot L}\{\frac{1}{2}(1 - c_{\cdot L}^2)\nu_L\}.$$

The first and second order derivatives are:

$$\frac{\partial}{\partial c_{\cdot H}} = -\alpha_{\cdot H} c_{\cdot H} \nu_H, \frac{\partial}{\partial c_{\cdot H}} = -\alpha_{\cdot H}\nu_H < 0$$
$$\frac{\partial}{\partial c_{\cdot L}} = \alpha_{\cdot H} c_{\cdot L}\Delta\nu - \alpha_{\cdot L} c_{\cdot L}\nu_L = c_{\cdot L}\nu_H[\alpha_{\cdot H} - x], \frac{\partial^2}{\partial c_{\cdot L}^2} = \nu_H[\alpha_{\cdot H} - x]$$

Hence, $c_H^* = 0$ and

$$\begin{aligned} c_{L\cdot} &= 0 \text{ if } \alpha_{\cdot H} - x < 0, \\ &= 1 \text{ if } \alpha_{\cdot H} - x > 0. \end{aligned}$$

■

Theorem 4 *With perfect positive correlation ($\alpha_{HL} = \alpha_{LH} = 0$), the optimal menu has $c_{HH} = 0$ and $c_{LL} = \begin{cases} \min\{D\frac{\alpha_{HH}x}{x-\alpha_{HH}}, 1\} \text{ if } x > \alpha_{HH} \\ 1 \text{ otherwise} \end{cases}$.*

Proof. Since $R_{HH} = \delta(c_{LL}, \Delta\mu, \Delta\nu)$ and $R_{LL} = 0$, the Lagrange function is

$$\mathcal{L} = \alpha_{HH}\{\frac{1}{2}(1 - c_{HH}^2)\nu_H - \delta(c_{LL}, \Delta\mu, \Delta\nu)\} + \alpha_{LL}\{\frac{1}{2}(1 - c_{LL}^2)\}.$$

The first and second order derivatives are:

$$\frac{\partial}{\partial c_{HH}} = -\alpha_{HH} c_{HH}\nu_H, \frac{\partial^2}{\partial c_{H\cdot}^2} = -\alpha_{HH}\nu_H < 0$$
$$\frac{\partial}{\partial c_{LL}} = \alpha_{HH}(\Delta\mu + c_{LL}\Delta\nu) - \alpha_{LL} c_{LL}\nu_L, \frac{\partial^2}{\partial c_{LL}^2} = \alpha_{HH}\Delta\nu - \alpha_{LL}\nu_L$$

Hence, $c_{HH}^* = 0$. Since $\frac{\partial^2}{\partial c_{LL}^2} = \alpha_{HH}\Delta\nu - \alpha_{LL}\nu_L = \nu_H(\alpha_{HH} - x)$, and c_{LL}^* is given by $\min\{D\frac{\alpha_{HH}x}{x-\alpha_{HH}}, 1\}$ if $x > \alpha_{HH}$, and by 1 if $x < \alpha_{HH}$. ■

4 Two-dimensional heterogeneity

4.1 Notation

- Bivariate probability distribution of types:

	ν_L	ν_H	
μ_L	α_{LL}	α_{LH}	$\alpha_{L\cdot}$
μ_H	α_{HL}	α_{HH}	$\alpha_{H\cdot}$
	$\alpha_{\cdot L}$	$\alpha_{\cdot H}$	1

- Correlation between risk (μ) and risk aversion (ν) plays an important role in the analysis;

$$\mathrm{corr}(\mu,\nu) \stackrel{\mathrm{def}}{=} \frac{E(\mu - E\mu)(\nu - E\nu)}{\sigma_\mu \sigma_\nu} = \frac{\alpha_{HH}\alpha_{LL} - \alpha_{LH}\alpha_{HL}}{\sqrt{\alpha_{L\cdot}\alpha_{H\cdot}}\sqrt{\alpha_{\cdot L}\alpha_{\cdot H}}}.$$

- $\rho \stackrel{\mathrm{def}}{=} \alpha_{HH}\alpha_{LL} - \alpha_{LH}\alpha_{HL}$: the numerator of the correlation expression.

- We parameterise the distribution by means of the triplet $(\alpha_{H\cdot}, \alpha_{HH}, \rho)$, and have the remaining fractions determined by

$$\alpha_{HL} = \alpha_{H\cdot} - \alpha_{HH}, \tag{8}$$

$$\alpha_{LH} = \alpha_{HH}\frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}} - \frac{\rho}{\alpha_{H\cdot}}, \text{ and} \tag{9}$$

$$\alpha_{LL} = (\alpha_{H\cdot} - \alpha_{HH})\frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}} + \frac{\rho}{\alpha_{H\cdot}}. \tag{10}$$

- $\overline{\rho} \stackrel{\mathrm{def}}{=} \alpha_{HH}(1-\alpha_{H\cdot})$ and $\underline{\rho} \stackrel{\mathrm{def}}{=} -\alpha_{HL}(1-\alpha_{H\cdot})$: upper and lower bounds on ρ to guarantee α_{LH} and α_{LL} positive

- $\mathcal{A}_0$: the feasible set of distribution parameters;

$$\mathcal{A}_0 \stackrel{\mathrm{def}}{=} \{(\alpha_{H\cdot}, \alpha_{HH}, \rho) \in [0,1]^2 \times R \mid \alpha_{HH} \le \alpha_{H\cdot} \text{ and } \underline{\rho} \le \rho \le \overline{\rho}\}.$$

- $\mathcal{T}_0$: set of admissible values for the parameters x and D;

$$\mathcal{T}_0 \stackrel{\mathrm{def}}{=} \{(D,x) \in R_+ \times (0,1)\}.$$

- $\mathcal{A}_1$: feasible set of distribution parameters when non-positive correlation of characteristics;

$$\mathcal{A}_1 \stackrel{\text{def}}{=} \{(\alpha_{H\cdot}, \alpha_{HH}, \rho) \in \mathcal{A}_0 \text{ and } \rho \leq 0\}.$$

- $\overline{D} \stackrel{\text{def}}{=} \frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}}$: upper bound on D to avoid exclusion of LL types when there is no heterogeneity in risk aversion

- $\mathcal{T}_1$: set of admissible values for the parameters x and D avoid exclusion of LL types when there is no heterogeneity in risk aversion

$$\mathcal{T}_1 \stackrel{\text{def}}{=} \{(D, x) \in \mathcal{T}_0 \mid D \leq \overline{D}\}.$$

- Two possible orderings of coinsurance rates:

$$\text{Order 1: } 0 = c_{HH} \leq c_{HL} \leq c_{LH} \leq c_{LL} \leq 1, \quad (11)$$

$$\text{Order 2: } 0 = c_{HH} \leq c_{LH} \leq c_{HL} \leq c_{LL} \leq 1. \quad (12)$$

Lemma 1 *If order 1 applies with $c_{HH} < c_{LH}$, it is optimal to pool HL with HH if $x > \frac{\alpha_{HH}}{\alpha_{H\cdot}}$. Otherwise, it is optimal to pool HL with LH.*

Proof. With order 1, the only type that may envy the contract for HL is HH. Thus, the choice of c_{HL} is only governed by weighing the profits from these two types. Since they have the same risk size, we may apply Theorem 3 on this sub group. Since the fraction of high risk averse people in this group is $\frac{\alpha_{HH}}{\alpha_{H\cdot}}$, the result follows. ■

4.2 The main result of the paper

Main proposition *Suppose that $(\alpha_{H\cdot}, \alpha_{HH}, \rho) \in \mathcal{A}_1$ and $(D, x) \in \mathcal{T}_1$. Define the following five menus:*

A $c^A_{HH} = c^A_{HL} = 0, c^A_{LH} = c^A_{LL} = D\frac{\alpha_{H\cdot}}{1-\alpha_{H\cdot}}$.

M $c^M_{HH} = 0, c^M_{LL} = 1$, and

$$c^M_{LH} = \begin{cases} D\frac{\alpha_{H\cdot}x}{\alpha_{H\cdot}(1-x)+\alpha_{LH}x} & \text{if } x > \frac{\alpha_{HH}}{\alpha_{H\cdot}}, \quad \textbf{(M1)} \\ D\frac{\alpha_{H\cdot}x}{\alpha_{HL}+\alpha_{LH}} & \text{if } x \leq \frac{\alpha_{HH}}{\alpha_{H\cdot}}, \quad \textbf{(M2)} \end{cases}$$

$$c^M_{HL} = \begin{cases} 0 & \text{if } x > \frac{\alpha_{HH}}{\alpha_{H\cdot}}, \quad \textbf{(M1)} \\ D\frac{\alpha_{H\cdot}x}{\alpha_{HL}+\alpha_{LH}} & \text{if } x \leq \frac{\alpha_{HH}}{\alpha_{H\cdot}}. \quad \textbf{(M2)} \end{cases}$$

B $c^B_{HH} = 0, c^B_{LH} = 2D\frac{x}{1-x} - c^B_{LL}$, and

$$c^B_{LL} = \begin{cases} 1 & \textbf{(BpX)}, \\ D\frac{2\alpha_{LH}+\alpha_{H\cdot}(1-x)}{(1-\alpha_{H\cdot})(1-x)} & \textbf{(B1pI)}, \\ 2D\frac{x}{1-x} & (\textbf{Bf}), \end{cases}$$

$$c^B_{HL} = \begin{cases} 0 & \text{if } x > \frac{\alpha_{HH}}{\alpha_{H\cdot}} & \textbf{(Bf,B1pI,B1PX)}, \\ 2D\frac{x}{1-x} - 1 & \text{if } x \leq \frac{\alpha_{HH}}{\alpha_{H\cdot}} & \textbf{(B2pX)}. \end{cases}$$

C $c^C_{HH} = c^C_{HL} = c^C_{LL} = 0$, and

$$c^C_{LL} = \begin{cases} D\frac{1-\alpha_{LL}}{\alpha_{LL}} & \textbf{(CI)}, \\ 1 & \textbf{(CX)}. \end{cases}$$

E $c^E_{HH} = 0, c^E_{LH} = D\frac{\alpha_{HH}x}{\alpha_{LH}}$, and

$$c^E_{HL} = c^E_{LL} = \begin{cases} D\frac{x\alpha_{HL}}{x-\alpha_{\cdot H}} & (\textbf{EI}), \\ 1 & (\textbf{EX}). \end{cases}$$

*When $\rho < \widehat{\rho}(\alpha_{H\cdot}, \alpha_{HH})$, the solution to the main problem is as depicted in Figure 3, where the functions $x_{BM}(D)$, $\overline{x}^{Bp}(D)$ and $x_{EC}(D)$ are defined in Table 3 below and $\widehat{\rho}(\alpha_{H\cdot}, \alpha_{HH})$ is specified in Theorem 11. Otherwise, the upper bound for the region corresponding to menus **EI** and **EX** will lie in the region corresponding to menus **Bf** and **BpX** (i.e., menus **CI** and **CX** cease to be optimal for any (D, x)).*

–Figure 3 here–

Remark 1. The suffixes to the menu names have the following rationale: "1"("2") stands for HL pooled with $HH(LH)$, in case of order 1; "**I**"("**X**") stands for inclusion (exclusion) of LL; and "**p**"("**f**") stands for partial(full) insurance of LH in case of menu **B**.

Remark 2. Figure 3 shows that no part of $\mathcal{T}_1$ is left unaddressed. The ordering of the critical values on the two axes is valid for any $(\alpha_{H\cdot}, \alpha_{HH}, \rho) \in \mathcal{A}_1$. Hence, the above proposition provides a full characterisation.

Remark 3. The condition on ρ says that this parameter should be sufficiently negative. However, in Theorem 11 we show that $\rho < -0.089$. is a sufficient condition for $\rho < \widehat{\rho}(\alpha_{H\cdot}, \alpha_{HH})$, all $(\alpha_{H\cdot}, \alpha_{HH})$. Hence, Figure 3 is the solution for almost all distributions of μ and ν with non-positive correlation.

In the next subsection, we explain the strategy to prove the main proposition.

4.3 Proof strategy

At a very abstract level, the main problem can be formulated as:

$$\max_{m\in\mathcal{M}^*} F(M), \tag{13}$$

where m is a contract menu $(C_{HH}, C_{HL}, C_{LH}, C_{LL})$ and $\mathcal{M}^*$ is the set of feasible menus satisfying the self-selection and participation constraints. Both $F(\cdot)$ and $\mathcal{M}^*$ depend on $(\alpha_{H\cdot}, \alpha_{HH}, \rho, D, x) \in \mathcal{A}_1 \times \mathcal{T}_1$, but we suppress this in the notation. Problem (13) is complex both due to the number of inequality constraints that define the feasible set, and because this set is beset by non-convexities. To identify the solution for each $(\alpha_{H\cdot}, \alpha_{HH}, \rho, D, x) \in \mathcal{A}_1 \times \mathcal{T}_1$, we proceed as follows.

First, we delineate the set of incentive compatible menus as much as possible by deriving a list of properties that any optimal incentive compatible menu should satisfy. This allows us to restrict the feasible set to a reduced set $\mathcal{M} \subset \mathcal{M}^*$, such that

$$\arg\max_{m\in\mathcal{M}^*(D,x)} F(M; D, x) = \arg\max_{m\in\mathcal{M}(D,x)} F(M; D, x).$$

This is the subject of Section 5.

Second, we identify three subsets $\mathcal{M}_i \subset \mathcal{M}$ $(i = 1, 2, 3)$, with $\cup_i \mathcal{M}_i = \mathcal{M}$ but not necessarily with empty intersections, which allows us to define three sub-problems of the type $m_i = \arg\max_{m\in\mathcal{M}_i} F(M)$ (Section 6). Because the three subsets unite to $\mathcal{M}$, it follows that

$$\arg\max_{m\in\mathcal{M}} F(m) = \arg\max_{m\in\{m_1,m_2,m_3\}} F(m). \tag{14}$$

Third, we solve each of the three sub-problems (Section 7). *Finally*, we perform a comparison to distinguish the global solution from the local ones (Section 8). For this comparison, we make use of the following principle:

Revealed preference principle *Let* $m_i = \arg\max_{m\in\mathcal{M}_i} F(m)$ $(i = 1, 2, 3)$. *If* $m_i \in \mathcal{M}_j$ $(j \neq i)$, *then* $F(m_i) \leq F(m_j)$.

5 Step 1: reduction of the feasible menus set from $\mathcal{M}^*$ to $\mathcal{M}$

We first derive a set of properties that an incentive compatible contract menu (ICM) should satisfy. Next, we derive a set of properties that an optimal contract menu should satisfy. Both sets of properties allow us to divide the main problem into three sub-problems.

We use the following notation:

- $ij \to kl$ stands for "type ij has an incentive to mimic type kl", i.e., $R_{ij} = R_{kl} + \delta(c_{kl}, \mu_i - \mu_k, \nu_j - \nu_l)$;
- $ij \nrightarrow kl$ stands for "type ij has no incentive to mimic type kl", i.e., $R_{ij} > R_{kl} + \delta(c_{kl}, \mu_i - \mu_k, \nu_j - \nu_l)$.

Recall from Section 2 that the monotonicity conditions are necessary for incentive compatibility of the contracts: $c_{Hj} \leq c_{Lj}$ $(j = H, L)$ and $c_{iH} \leq c_{iL}$ $(i = H, L)$.

Lemma 2 *At an ICM, if $HH \to LL$, then $HH \to HL$ and $HH \to LH$.*

Proof. Suppose $HH \to LL$ but $HH \nrightarrow HL$, i.e.,

$$\begin{aligned} R_{HH} &= R_{LL} + \delta(c_{LL}, \Delta\mu, \Delta\nu) \quad &\text{(i)} \\ R_{HH} &> R_{HL} + \delta(c_{HL}, 0, \Delta\nu) \quad &\text{(ii)} \end{aligned}$$

Since $R_{HL} \geq R_{LL} + \delta(c_{LL}, \Delta\mu, 0)$, (i) and (ii) give

$$\begin{aligned} R_{LL} + \delta(c_{LL}, \Delta\mu, \Delta\nu) &> R_{LL} + \delta(c_{LL}, \Delta\mu, 0) + \delta(c_{HL}, 0, \Delta\nu) \\ &\iff \delta(c_{LL}, 0, \Delta\nu) > \delta(c_{HL}, 0, \Delta\nu) \\ &\iff c_{HL} > c_{LL} \end{aligned}$$

contradicting monotonicity. Likewise, suppose $HH \to LL$ but $HH \nrightarrow LH$, i.e.,

$$R_{HH} > R_{LH} + \delta(c_{LH}, \Delta\mu, 0). \quad \text{(iii)}$$

Since $R_{LH} \geq R_{LL} + \delta(c_{LL}, 0, \Delta\nu)$, (i) and (iii) give

$$\begin{aligned} R_{LL} + \delta(c_{LL}, \Delta\mu, \Delta\nu) &> R_{LL} + \delta(c_{LL}, 0, \Delta\nu) + \delta(c_{LH}, \Delta\mu, 0) \\ &\iff \delta(c_{LL}, \Delta\mu, 0) > \delta(c_{LH}, \Delta\mu, 0) \\ &\iff c_{LH} > c_{LL} \end{aligned}$$

contradicting monotonicity. ■

Lemma 3 *At an ICM, if $HH \to LH(HL)$, then $c_{LH} \leq (\geq) c_{HL}$.*

Proof. Incentive compatibility requires

$$\begin{aligned}
\text{(i) } & R_{HH} \geq R_{HL} + \delta(c_{HL}, 0, \Delta\nu) \\
\text{(ii) } & R_{HH} \geq R_{LH} + \delta(c_{LH}, \Delta\mu, 0) \\
\text{(iii) } & R_{HL} \geq R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu) \\
\text{(iv) } & R_{LH} \geq R_{HL} + \delta(c_{HL}, -\Delta\mu, \Delta\nu)
\end{aligned}$$

(i) and (iii) lead to $R_{HH} \geq R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu) + \delta(c_{HL}, 0, \Delta\nu)$. Therefore, if (ii) holds with equality we obtain that

$$\begin{aligned}
R_{LH} + \delta(c_{LH}, \Delta\mu, 0) &\geq R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu) + \delta(c_{HL}, 0, \Delta\nu) \\
&\Longleftrightarrow \delta(c_{LH}, 0, \Delta\nu) \geq \delta(c_{HL}, 0, \Delta\nu)
\end{aligned}$$

and therefore that $c_{LH} \leq c_{HL}$. Similarly, combining (ii) and (iv), and (i) with equality leads to $c_{HL} \leq c_{LH}$. ■

Corollary 1 *At an ICM, if $HH \to LH$ and $HH \to HL$, then $c_{LH} = c_{HL}$ and therefore $LH \to HL$ and $HL \to LH$ hold trivially.*

Corollary 2 *At an ICM, if $HH \to LL$, then $c_{LH} = c_{HL} = c_{LL}$.*

Proof. By Lemma 2, $HH \to LH$ and $HH \to HL$ and by 1 $c_{HL} = c_{LH}$. $c_{HL} = c_{LH} > c_{LL}$ is ruled out by monotonicity. Suppose now that $c_{HL} = c_{LH} < c_{LL}$. Since $HH \to LL$ and $HH \to LH$,

$$\begin{aligned}
R_{LH} + \delta(c_{LH}, \Delta\mu, 0) &= R_{LL} + \delta(c_{LL}, \Delta\mu, \Delta\nu) \\
&\Downarrow \\
R_{LH} &= R_{LL} + \delta(c_{LL}, 0, \Delta\nu) + \delta(c_{LL}, \Delta\mu, 0) - \delta(c_{LH}, \Delta\mu, 0) \\
&= R_{LL} + \delta(c_{LL}, 0, \Delta\nu) + (c_{LH} - c_{LL})\Delta\mu
\end{aligned}$$

Similarly, $HH \to LL$ and $HH \to LH$ imply that

$$R_{HL} = R_{LL} + \delta(c_{LL}, \Delta\mu, 0) + \frac{1}{2}(c_{HL}^2 - c_{LL}^2)\Delta\nu$$

Then by monotonicity, both LH and HL will strictly envy LL's contract, contradicting incentive compatibility. ■

Lemma 4 *At an ICM, if $HH \to LH(HL)$ and $HH \nrightarrow HL(LH)$, then $c_{LH} < (>) c_{HL}$ and HL and LH cannot be pooled.*

Proof. Consider the case where HH has an incentive to mimic LH but not HL: $R_{HH} = R_{LH} + \delta(c_{LH}, \Delta\mu, 0)$ and $R_{HH} > R_{HL} + \delta(c_{HL}, 0, \Delta\nu)$. Using $R_{HL} \geq R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu)$ results in $R_{LH} + \delta(c_{LH}, \Delta\mu, 0) > R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu) + \delta(c_{HL}, 0, \Delta\nu)$ giving $c_{HL} > c_{LH}$. ■

Lemma 5 *At an ICM, either (i)* $\{LH \to LL$ *and* $LH \nrightarrow HL\}$, *or (ii)* $\{HH \to LH$ *and* $HH \nrightarrow HL\}$ *but not both.*

Proof. (i) says $R_{LL} + \delta(c_{LL}, 0, \Delta\nu) > R_{HL} + \delta(c_{HL}, -\Delta\mu, \Delta\nu)$. Adding this to $R_{HL} \geq R_{LL} + \delta(c_{LL}, \Delta\mu, 0)$ gives

$$\delta(c_{LL}, -\Delta\mu, \Delta\nu) > \delta(c_{HH}, -\Delta\mu, \Delta\nu)$$
$$\iff (c_{LL} - c_{HL})\Delta\mu > \frac{1}{2}(c_{LL}^2 - c_{HL}^2)\Delta\nu$$

By monotonicity, this implies that $c_{LL} + c_{HL} < 2\frac{\Delta\mu}{\Delta\nu}$.

On the other hand, adding $R_{HL} \geq R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu)$ to the second part of (i), $R_{LH} > R_{HL} + \delta(c_{HL}, -\Delta\mu, \Delta\nu)$, results in

$$\delta(c_{HL}, \Delta\mu, -\Delta\nu) > \delta(c_{LH}, \Delta\mu, -\Delta\nu)$$
$$\iff (c_{LH} - c_{HL})\Delta\mu > \frac{1}{2}(c_{LH}^2 - c_{HL}^2)\Delta\nu$$

By (ii) and Lemma 4, this inequality implies that $c_{LH} + c_{HL} > 2\frac{\Delta\mu}{\Delta\nu}$. Whence, $c_{LH} > c_{LL}$, contradicting monotonicity. ■

Lemma 6 *If* $HL \to LH$ *and* $LH \to HL$, *then either* $c_{HL} = c_{LH}$ *or* $\{c_{HL} \neq c_{LH}$ *and* $c_{LH} + c_{HL} = 2\frac{\Delta\mu}{\Delta\nu}\}$.

Proof. Adding $R_{LH} = R_{HL} + \delta(c_{HL}, -\Delta\mu, \Delta\nu)$ to $R_{HL} = R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu)$ yields

$$\delta(c_{HL}, \Delta\mu, -\Delta\nu) = \delta(c_{LH}, \Delta\mu, -\Delta\nu)$$
$$\iff (c_{LH} - c_{HL})\Delta\mu = \frac{1}{2}(c_{LH}^2 - c_{HL}^2)\Delta\nu$$

■

Lemma 7 *Consider an ICM. Suppose (i)* $HL \to LH$ *and* $LH \to HL$, *(ii)* $HH \to LH$ *or* $HH \to HL$ *but not both, (iii)* $LH \to LL$. *Then (iv)* $HL \to LL$.

Proof. (ii) and Lemma 4 imply that $c_{LH} \neq c_{HL}$. By (i) and lemma 6, this means that $c_{LH} + c_{HL} = 2\frac{\Delta\mu}{\Delta\nu}$. Now suppose that (iv) is false. Then

$$\begin{aligned} R_{HL} &> R_{LL} + \delta(c_{LL}, \Delta\mu, 0) \\ &= R_{LH} - \delta(c_{LL}, 0, \Delta\nu) + \delta(c_{LL}, \Delta\mu, 0) \\ &= R_{HL} + \delta(c_{HL}, -\Delta\mu, \Delta\nu) - \delta(c_{LL}, 0, \Delta\nu) + \delta(c_{LL}, \Delta\mu, 0) \end{aligned}$$

where the first equality sign follows from (iii). Therefore

$$\begin{aligned} &\delta(c_{LL}, -\Delta\mu, \Delta\nu) > \delta(c_{HL}, -\Delta\mu, \Delta\nu) \\ \iff & c_{LL} > c_{HL} \text{ and } c_{LL} + c_{HL} < 2\frac{\Delta\mu}{\Delta\nu} \end{aligned}$$

But as $c_{LH} + c_{HL} = 2\frac{\Delta\mu}{\Delta\nu}$, we get $c_{LL} < c_{LH}$, contradicting monotonicity. ■

Next, we further delineate the set of incentive compatible contract by eliminating those IC contract that can be improved upon.

Lemma 8 *At an optimal solution, either $HH \to HL$ or $HH \to LH$ or both.*

Proof. Suppose not, i.e. $HH \nrightarrow HL$ and $HH \nrightarrow LH$. Then by lemma 2, $HH \nrightarrow LL$. But this means it is possible to reduce R_{HH} without upsetting incentive compatibility, contradicting optimality. ■

Lemma 9 *At an optimal solution either $HL \to LH$ or $HL \to LL$ or both.*

Proof. Suppose not, i.e., $HL \nrightarrow LH$ and $HL \nrightarrow LL$. We distinguish between two case: (i) $HL \to HH$ and (ii) $HL \nrightarrow HH$.
Case (ii). Then none of the IC constraints for HL are binding and we can decrease R_{HL} by a small amount without violating incentive compatibility, contradicting optimality.
Case (i). Then $R_{HL} = R_{HH} + \delta(c_{HH}, 0, -\Delta\nu)$. By assumption $HL \nrightarrow LH$, i.e., $R_{HL} > R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu)$. Substituting into previous equality gives $R_{HH} > R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu) + \delta(c_{HH}, 0, \Delta\nu)$. By definition of δ, we can rewrite this as $R_{HH} > R_{LH} + \delta(c_{LH}, \Delta\mu, 0) - \delta(c_{LH}, 0, \Delta\nu) + \delta(c_{HH}, 0, \Delta\nu) = R_{LH} + \delta(c_{LH}, \Delta\mu, 0) + \frac{1}{2}\left(c_{LH}^2 - c_{HH}^2\right)\Delta\nu$. The last term is non-negative, since $0 = c_{HH}^2 \leq c_{LH}^2$ by monotonicity. Hence we can write $R_{HH} > R_{LH} + \delta(c_{LH}, \Delta\mu, 0)$, meaning that $HH \nrightarrow LH$. Using this strict inequality with the constraint $R_{LH} \geq R_{LL} + \delta(c_{LL}, 0.\Delta\nu)$ gives $R_{HH} > R_{LL} + \delta(c_{LL}, 0, \Delta\nu) + \delta(c_{LH}, \Delta\mu, 0) = R_{LL} + \delta(c_{LL}, \Delta\mu, \Delta\nu) - \delta(c_{LL}, \Delta\mu, 0) +$

$\delta(c_{LH}, \Delta\mu, 0) = R_{LL} + \delta(c_{LL}, \Delta\mu, \Delta\nu) + \Delta\mu(c_{LL} - c_{LH})$. By monotonicity $c_{LL} \geq c_{LH}$ so last term is non-negative. Hence we can write $R_{HH} > R_{LL} + \delta(c_{LL}, \Delta\mu, \Delta\nu)$, meaning that $HH \nrightarrow LL$ To sum up, we have that.

$$HL \nrightarrow LH,\ HL \nrightarrow LL,\ HH \nrightarrow LH, HH \nrightarrow LL;$$
$$HL \rightarrow HH, \text{ i.e. } R_{HL} = R_{HH} + \delta(c_{HH}, 0, -\Delta\nu); \text{ and}$$
$$R_{HH} \geq R_{HL} + \delta(c_{HL}, 0, \Delta\nu)$$

Consider therefore lowering both R_{HH} and R_{HL} by the *same* small amount. Then, by inspection, none of the above constraints is violated, and profit has increased. This contradicts optimality. ■

Lemma 10 *At an optimal solution either $LH \rightarrow HL$ or $LH \rightarrow LL$ or both.*

Proof. The proof goes along exactly the same lines as the proof for Lemma 9, *mutatis mutandis.* ■

Lemma 11 *At an optimal solution, either $HL \rightarrow LL$ or $LH \rightarrow LL$, or both.*

Proof. From lemma 2, if $HL \nrightarrow LL$ and $LH \nrightarrow LL$, then also $HH \nrightarrow LL$. But then it is possible to increase the profit on LL by lowering c_{LL} and without upsetting incentive compatibility, contradicting optimality. ■

Lemma 12 *Suppose $HH \rightarrow HL$, $HH \nrightarrow LH$, $HL \rightarrow LL$, and $LH \rightarrow HL$. Then profit can be increased by lowering c_{LH} down to c_{HL} without upsetting incentive compatibility.*

Proof. By lemma 4, $c_{HL} < c_{LH}$. Adding $R_{HL} \geq R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu)$ to $R_{LH} = R_{HL} + \delta(c_{HL}, -\Delta\mu, \Delta\nu)$ gives $(c_{LH} - c_{HL})\Delta\mu \geq \frac{1}{2}(c_{LH}^2 - c_{HL}^2)\Delta\nu$. Since $c_{HL} < c_{LH}$, this implies that $c_{LH} + c_{HL} \leq 2\frac{\Delta\mu}{\Delta\nu}$. Whence, $c_{HL} < c_{LH} \leq 2\frac{\Delta\mu}{\Delta\nu} - c_{HL}$. (A requirement is therefore that $c_{HL} < \frac{\Delta\mu}{\Delta\nu}$.). Since $HL \rightarrow LL$, π_{HL} is determined by c_{HL}, c_{LL} and R_{LL}. Since $HH \rightarrow HL$, π_{HH} is determined by c_{HH}, c_{HL}, c_{LL} and R_{LL}. Since $LH \rightarrow HL$, π_{LH} is determined by c_{LH}, c_{HL}, c_{LL} and R_{LL}. Therefore a marginal reduction in c_{LH} will not upset incentive compatibility and will increase the profit from LH without reducing any other profit. ■

Lemma 13 *Suppose $HH \rightarrow HL$, $HH \nrightarrow LH$, $HL \rightarrow LL$, $LH \rightarrow LL$, $LH \nrightarrow HL$, and $HL \nrightarrow LH$. Then profit can be increased by a marginal reduction in c_{LH} without upsetting incentive compatibility.*

Proof. From $R_{HL} > R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu)$, $R_{HL} = R_{LL} + \delta(c_{LL}, \Delta\mu, 0)$ and $R_{LH} = R_{LL} + \delta(c_{LL}, 0, \Delta\nu)$ we obtain that $c_{LH} > 2\frac{\Delta\mu}{\Delta\nu} - c_{LL}$. And from $R_{LH} > R_{HL} + \delta(c_{HL}, -\Delta\mu, \Delta\nu)$ and the same two equalities we obtain that $c_{HL} < 2\frac{\Delta\mu}{\Delta\nu} - c_{LL}$. Whence,$c_{HL} < 2\frac{\Delta\mu}{\Delta\nu} - c_{LL} < c_{LH}$. Since $HL \to LL$, π_{HL} is determined by c_{HL}, c_{LL} and R_{LL}. Since $HH \to HL$, π_{HH} is determined by c_{HH}, c_{HL}, c_{LL} and R_{LL}. Since $LH \to LL$, π_{LH} is determined by c_{LH}, c_{LL} and R_{LL}. A marginal reduction in c_{LH} will then not upset incentive compatibility and will increase the profit from LH without reducing any other profit. ■

Lemma 14 *Suppose $HH \to LH$, $HH \nrightarrow HL$, $LH \to HL$, and $HL \to LH$. Then profit can be increased by lowering c_{HL} without upsetting incentive compatibility.*

Proof. By lemma 4, $c_{LH} < c_{HL}$. And by Lemma 6, $c_{LH} + c_{HL} = 2\frac{\Delta\mu}{\Delta\nu}$. Since $HH \to LH$, π_{HH} is determined by c_{HH}, c_{LH} and R_{LH}. HL can therefore be pooled with LH. This does not upset incentive compatibility. It increases the profit from HL and does not affect the profit from either HH, LH or LL. See figure 2. ■

–Figure 2 here–

Lemma 15 *(suboptimality of full separation under Order 2) Suppose that $HH \to LH$, $HH \nrightarrow HL$, $LH \to HL$, $LH \nrightarrow LL$, $HL \to LL$. Then profit can be increased by pooling HL with LL or with LH. (This lemma was labelled Lemma 2 in the main text.)*

Proof. The situation is depicted in figure 3.

–Figure 3 here–

First note that c_{LL} must exceed $\frac{\Delta\mu}{\Delta\nu}$ for otherwise LH and HL could not have been separated.

The profits from the different types are as follows:

$$\pi_{HH} = \frac{1}{2}(1 - c_{HH}^2)\nu_H - (1 - c_{LH})\Delta\mu + (1 - c_{HL})\Delta\mu - \frac{1}{2}(1 - c_{HL}^2)\Delta\nu - (1 - c_{LL})\Delta\mu$$

$$\pi_{HL} = \frac{1}{2}(1 - c_{HL}^2)\nu_L - (1 - c_{LL})\Delta\mu$$

$$\pi_{LH} = \frac{1}{2}(1 - c_{LH}^2)\nu_H + (1 - c_{HL})\Delta\mu - \frac{1}{2}(1 - c_{HL}^2)\Delta\nu - (1 - c_{LL})\Delta\mu$$

$$\pi_{LL} = \frac{1}{2}(1 - c_{LL}^2)\nu_L$$

Weighing with the respective population proportions, gives the following first derivatives:

$$\frac{\partial \pi_{tot}}{\partial c_{HH}} = -\alpha_{HH} c_{HH} \nu_H, \frac{\partial \pi_{tot}}{\partial c_{LH}} = \alpha_{HH} \Delta\mu - \alpha_{LH} \nu_H c_{LH}$$
$$\frac{\partial \pi_{tot}}{\partial c_{HL}} = -\alpha_{.H} \Delta\mu + \alpha_{.H} c_{HL} \Delta\nu - \alpha_{HL} c_{HL} \nu_L, \frac{\partial \pi_{tot}}{\partial c_{LL}} = (1 - \alpha_{LL})\Delta\mu - \alpha_{LL} c_{LL} \nu_L$$

The solution for c_{LL} is $c_{LL} = \min\{\frac{\Delta\mu}{\nu_L} \frac{1-\alpha_{LL}}{\alpha_{LL}}, 1\}$. The condition that $c_{LL} > \frac{\Delta\mu}{\Delta\nu}$ translates into $x < 1 - \alpha_{LL}$. If this is satisfied, there is room to separate LH from HL. Since

$$\frac{\partial \pi_{tot}}{\partial c_{HL}} = -\alpha_{.H} \Delta\mu + [\alpha_{.H}(1 - x) - \alpha_{HL} x] \nu_H c_{HL}$$

total profit is strictly concave in c_{HL} iff $x \geq \frac{\alpha_{.H}}{1-\alpha_{LL}}$. In that case, the optimal solution for c_{HL} is

$$c_{HL} = \min\{\frac{\Delta\mu}{\nu_L} \frac{\alpha_{.H} x}{\alpha_{.H}(1 - x) - \alpha_{HL} x}, 1\}.$$

By monotonicity, the only chance of full separation is where $c_{HL} = \frac{\Delta\mu}{\nu_L} \frac{\alpha_{.H} x}{\alpha_{.H}(1-x)-\alpha_{HL} x} < 1$. It remains then to check whether $c_{HL} < c_{LL}$. Suppose first that $c_{LL} = \frac{\Delta\mu}{\nu_L} \frac{1-\alpha_{LL}}{\alpha_{LL}} < 1$:

$$c_{HL} < c_{LL} \iff \frac{\Delta\mu}{\nu_L} \frac{\alpha_{.H} x}{\alpha_{.H}(1 - x) - \alpha_{HL} x} < \frac{\Delta\mu}{\nu_L} \frac{1 - \alpha_{LL}}{\alpha_{LL}}$$
$$\iff x < \frac{\alpha_{.H}(1 - \alpha_{LL})}{\alpha_{.H} \alpha_{LL} + (1 - \alpha_{LL})^2}$$

As $\frac{\alpha_{.H}(1-\alpha_{LL})}{\alpha_{.H}\alpha_{LL}+(1-\alpha_{LL})^2} < \frac{\alpha_{.H}}{1-\alpha_{LL}}$, this condition contradicts with the assumption that $x \geq \frac{\alpha_{.H}}{1-\alpha_{LL}}$. Suppose next that $c_{LL} = 1$.

$$c_{HL} < c_{LL} \iff \frac{\Delta\mu}{\nu_L} \frac{\alpha_{.H} x}{\alpha_{.H}(1 - x) - \alpha_{HL} x} < 1 \iff x < \frac{\alpha_{.H}}{1 - \alpha_{LL} + D\alpha_{.H}}.$$

Again, this contradicts with the assumption that $x \geq \frac{\alpha_{.H}}{1-\alpha_{LL}}$. Hence, $c_{HL} = c_{LL}$, meaning that HL is pooled with LL.

On the other hand, if total profit is strictly convex in c_{HL}, it pays to move c_{HL} either down to c_{LH} or up to c_{LL}. Hence, full separation is never optimal. ■

By Lemmas 8, 9 and 10, at least one adjacent IC constraint should be binding for each of the three upper types. This gives 27 possible configurations. But using Lemmas 5, 7, 11, 12, 13, 14, 15 and corollary 1, we can rule out all but six candidates for an optimal contract menu, as shown in the table below. In the next section, we show that these candidates are the solution to three sub-problems.

Table 1. At most 6 configurations of binding and non-binding IC constraints are possible at an optimal solution.

Order			$HL \nrightarrow LL$ $HL \rightarrow LH$	$HL \rightarrow LL$ $HL \rightarrow LH$	$HL \rightarrow LL$ $HL \nrightarrow LH$
		$LH \nrightarrow LL$ $LH \rightarrow HL$	subopt (Lemma 11)	subopt (Lemma 14)	subopt (Lemma 15)
O2	$HH \nrightarrow HL$ $HH \rightarrow LH$	$LH \rightarrow LL$ $LH \rightarrow HL$	not IC (Lemma 7)	subopt (Lemma 14)	**Sub-problem 3**
		$LH \rightarrow LL$ $LH \nrightarrow HL$	not IC (Lemma 5)	not IC (Lemma 5)	not IC (Lemma 5)
		$LH \nrightarrow LL$ $LH \rightarrow HL$	subopt (Lemma 11)	**Sub-problem 2**	not IC (Corollary 1)
	$HH \rightarrow HL$ $HH \rightarrow LH$	$LH \rightarrow LL$ $LH \rightarrow HL$	**Sub-problem 1**	**Sub-problem 1**	not IC (Corollary 1)
O1		$LH \rightarrow LL$ $LH \nrightarrow HL$	not IC (Corollary 1)	not IC (Corollary 1)	not IC (Corollary 1)
		$LH \nrightarrow LL$ $LH \rightarrow HL$	subopt (Lemma 11)	subopt (Lemma 12)	subopt (Lemma 12)
	$HH \rightarrow HL$ $HH \nrightarrow LH$	$LH \rightarrow LL$ $LH \rightarrow HL$	not IC (Lemma 7)	subopt (Lemma 12)	subopt (Lemma 12)
		$LH \rightarrow LL$ $LH \nrightarrow HL$	**Sub-problem 1**	**Sub-problem 1**	subopt (Lemma 13)

6 Step 2: identification of the three sub-problems $\mathcal{M}_i$ $(i = 1, 2, 3)$

By eliminating configurations of binding/non-binding IC constraints, there are three sub-problems that emerge. The first, sub-problem 1, covers four cells in Table 1. Sub-problems 2 and 3 each corresponds to one cell. Both of these cells have open feasible sets because one of the downward adjacent IC constraints is strictly slack. We close the feasible set by allowing the relevant IC constraint to be binding as well. The constraints for the three sub-problems are given in Table 2. In the rest of this section, we will demonstrate why the main problem can be decomposed into these three sub-problems.

Table 2. The constraints of the three sub-problems.

	P1	P2	P3
1	$0 \leq c_{HL}$	$0 \leq c_{HL}$	$0 \leq c_{LH}$
2	$c_{HL} \leq c_{LH}$ (λ)	$c_{HL} = c_{LH}$	$c_{LH} \leq c_{HL}$ (λ_1)
3	$c_{LH} \leq 2\frac{\Delta\mu}{\Delta\nu} - c_{LL}$ (μ_1^a)	$c_{LH} \geq 2\frac{\Delta\mu}{\Delta\nu} - c_{LL}$ (λ_2)	$c_{LH} \geq 2\frac{\Delta\mu}{\Delta\nu} - c_{HL}$ (λ_2)
4	$c_{LH} \leq c_{LL}$ (μ_2)	$c_{LH} \leq c_{LL}$ (λ_1)	$c_{HL} = c_{LL}$
5	$c_{LL} \leq 1$ (μ_1^b)	$c_{LL} \leq 1$ (λ_3)	$c_{LL} \leq 1$ (μ)

We now define each of the three sub-problems.[2]

Sub-problem 1 (P1) Common for four cells in Table 1 is that HH has an incentive to mimic HL, HL has an incentive to mimic LH and LH has an incentive to mimic LL. The last two statements mean that $R_{HL} = R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu)$ and $R_{LH} = R_{LL} + \delta(c_{LL}, 0, \Delta\nu)$. Since HL may or may not envy LL, $R_{HL} \geq R_{LL} + \delta(c_{LL}, \Delta\mu, 0)$. It then follows that

$$R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu) = R_{LL} + \delta(c_{LL}, 0, \Delta\nu) + \delta(c_{LH}, \Delta\mu, -\Delta\nu) \geq R_{LL} + \delta(c_{LL}, \Delta\mu, 0)$$
$$\iff \delta(c_{LH}, \Delta\mu, -\Delta\nu) \geq \delta(c_{LL}, \Delta\mu, -\Delta\nu)$$
$$\iff (c_{LL} - c_{LH})\Delta\mu \geq \frac{1}{2}(c_{LL}^2 - c_{LH}^2)\Delta\nu$$

By the monotonicity condition that $c_{LH} \geq c_{HL}$, we either have $c_{LH} = c_{LL}$, or $c_{LH} > c_{LL}$ and $c_{LL} + c_{LH} \leq 2\frac{\Delta\mu}{\Delta\nu}$. The feasible set in the coinsurance rate space is thus open and non-convex: it consists of the entire 45 line and of the shaded triangle in figure 4.

–Figure 4 here–

We close and convexify it by restricting the feasible set to the shaded area, i.e.,.

$$c_{LH} \geq c_{LL} \text{ and } c_{LL} + c_{LH} \leq 2\frac{\Delta\mu}{\Delta\nu}.$$

In doing so, we forego the possibility to pool LH and LL at a coinsurance rate that exceeds $\frac{\Delta\mu}{\Delta\nu}$. However, below we show that this does not matter for the global analysis.

Since LH may or may not envy HL, we have that

$$R_{LH} \geq R_{HL} + \delta(c_{HL}, -\Delta\mu, \Delta\nu) = R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu) + \delta(c_{HL}, -\Delta\mu, \Delta\nu)$$
$$\iff \delta(c_{LH}, -\Delta\mu, \Delta\nu) \geq \delta(c_{HL}, -\Delta\mu, \Delta\nu)$$
$$\iff (c_{LH} - c_{HL})\Delta\mu \geq \frac{1}{2}(c_{LH}^2 - c_{HL}^2)\Delta\nu$$

Because Order 1 applies, this inequality may hold in two ways. Either $c_{LH} = c_{HL}$, or $c_{LH} > c_{HL}$ and $c_{LH} + c_{HL} \leq 2\frac{\Delta\mu}{\Delta\nu}$. We can now claim that

[2] Alternatively, we could have merged sub-problems P1 and P2 into a single problem by writing the second and third constraints as $c_{HL} \leq c_{LH}$ and $(c_{LH} - c_{HL}) \cdot (2\frac{\Delta\mu}{\Delta\nu} - c_{LL} - c_{LH}) \geq 0$, respectively.

it is sufficient to impose the constraint $c_{HL} \leq c_{LH}$. Indeed, by restricting ourselves to the shaded are in figure 4, we know that $c_{LH} \leq \frac{\Delta\mu}{\Delta\nu}$. Since $c_{HL} \leq c_{LH}$, it follows that $c_{HL} \leq \frac{\Delta\mu}{\Delta\nu}$ and therefore that $c_{LH} + c_{HL} \leq 2\frac{\Delta\mu}{\Delta\nu}$.

By foregoing the possibility of pooling LH and LL at a coinsurance rate above $\frac{\Delta\mu}{\Delta\nu}$, there are two menus that are excluded. The first is where all the three lower types are pooled at a rate above $\frac{\Delta\mu}{\Delta\nu}$. This menu may be optimal when there are a lot of HH people around of which a large rent can be extracted. *However, this menu will be feasible under sub-problem 2 and will therefore be included in the global analysis.* The second possibility that is excluded is sketched in Figure 5. This is a menu where HL is separated from LH and LL. *It is clear that such a menu can never constitute a global optimum*: moving LH from the right hand crossing to the left hand crossing preserves incentive compatibility but raises profits from LH. In sum, nothing is lost by excluding in this part of the analysis pooling of LH and LL at a rate above $\frac{\Delta\mu}{\Delta\nu}$.

–Figure 5 here–

Using the binding rent equations, and the fact that $R_{LL} = 0$, the profits from the four types are as follows

$$\begin{aligned}
\pi_{HH} &= \frac{1}{2}\upsilon_H - \frac{1}{2}[1 - c_{HL}^2]\Delta\nu + \frac{1}{2}[1 - c_{LH}^2]\Delta\nu - (1 - c_{LH})\Delta\mu - \frac{1}{2}[1 - c_{LL}^2]\Delta\nu \\
\pi_{HL} &= \frac{1}{2}[1 - c_{HL}^2]\nu_L + \frac{1}{2}[1 - c_{LH}^2]\Delta\nu - (1 - c_{LH})\Delta\mu - \frac{1}{2}[1 - c_{LL}^2]\Delta\nu \\
\pi_{LH} &= \frac{1}{2}[1 - c_{LH}^2]\nu_H - \frac{1}{2}[1 - c_{LL}^2]\Delta\nu \\
\pi_{LL} &= \frac{1}{2}[1 - c_{LL}^2]\nu_L
\end{aligned}$$

and total profit is

$$\begin{aligned}
\pi_{tot}^{P1} = {} & \frac{1}{2}\nu_L - \alpha_{H.}\Delta\mu + \alpha_{H.}c_{LH}\Delta\mu + \frac{1}{2}(1 - \alpha_{LL})c_{LL}^2\Delta\nu + \frac{1}{2}(\alpha_{HH}\Delta\nu - \alpha_{HL}\nu_L)c_{HL}^2 \\
& - \frac{1}{2}(\alpha_{LH}\nu_H + \alpha_{H.}\Delta\nu)c_{LH}^2 - \frac{1}{2}\alpha_{LL}c_{LL}^2\nu_L.
\end{aligned}$$

The problem is thus to maximise π_{tot}^{P1} s.t. the constraints listed in the first column of Table 2.

Sub-problem 2 (P2) In this sub-problem, HH has an incentive to mimic both HL and LH so that $c_{HL} = c_{LH}$ (Lemma 1). Let us call this common coinsurance rate c_I. Because HL has an incentive to mimic both LH and LL we have $R_{LH} + \delta(c_I, \Delta\mu, -\Delta\nu) = R_{LL} + \delta(c_{LL}, \Delta\mu, 0)$. Since LH does not envy LL at all, $R_{LH} > R_{LL} + \delta(c_{LL}, 0, \Delta\nu)$. From the previous expression we then get that

$$\delta(c_{LL}, \Delta\mu, -\Delta\nu) > \delta(c_I, \Delta\mu, -\Delta\nu)$$
$$\iff (c_{LL} - c_I)\Delta\mu < \frac{1}{2}(c_{LL}^2 - c_I^2)\Delta\nu.$$

Because of the monotonicity condition that $c_I \leq c_{LL}$, the previous inequality can only be satisfied when $c_I < c_{LL}$ and $c_I + c_{LL} > 2\frac{\Delta\mu}{\Delta\nu}$, or $2\frac{\Delta\mu}{\Delta\nu} - c_{LL} < c_I < c_{LL}$. The feasible set for c_I is thus open, but for the purpose of describing the optimal coinsurance rates we close it by including the boundaries. Note that this sub-problem allows for pooling of the three lower types at a coinsurance rate larger than $\frac{\Delta\mu}{\Delta\nu}$, which was excluded from Sub-Problem 1.

The profits from the four types are then

$$\pi_{HH} = \frac{1}{2}v_H - \frac{1}{2}[1 - c_I^2]\Delta\nu - (1 - c_{LL})\Delta\mu$$
$$\pi_{HL} = \frac{1}{2}[1 - c_I^2]\nu_L - (1 - c_{LL})\Delta\mu$$
$$\pi_{LH} = \frac{1}{2}[1 - c_I^2]\nu_H - \frac{1}{2}[1 - c_I^2]\Delta\nu + (1 - c_I)\Delta\mu - (1 - c_{LL})\Delta\mu$$
$$\pi_{LL} = \frac{1}{2}[1 - c_{LL}^2]\nu_L$$

and total profit is

$$\pi_{tot}^{P2} = \frac{1}{2}\nu_L - \alpha_{H.}\Delta\mu + \frac{1}{2}[\alpha_{HH} - (1 - \alpha_{LL})x]c_I^2\nu_H + (1 - \alpha_{LL})c_{LL}\Delta\mu$$
$$- \alpha_{LH}c_I\Delta\mu - \frac{1}{2}\alpha_{LL}c_{LL}^2\nu_L. \quad (15)$$

The problem is thus to maximise π_{tot}^{P2} s.t. constraints 1,3,4 and 5 listed in column P2 in Table 2, (constraint 2 being taken care of by having set $c_{HL} = c_{LH} = c_I$).

Sub-problem 3 (P3) Now, HH has only an incentive to mimic LH and HL has only an incentive to mimic LL. Since LH has an incentive to mimic both HL and LL we have $R_{HL} + \delta(c_{HL}, -\Delta\mu, \Delta\nu) = R_{LL} + \delta(c_{LL}, 0, \Delta\nu)$, and because $R_{HL} = R_{LL} + \delta(c_{LL}, \Delta\mu, 0)$, we obtain that

$$\begin{aligned}
&\delta(c_{HL}, -\Delta\mu, \Delta\nu) = \delta(c_{LL}, -\Delta\mu, \Delta\nu) \\
\iff &(c_{LL} - c_{HL})\Delta\mu = (c_{LL}^2 - c_{HL}^2)\frac{1}{2}\Delta\nu \\
\iff &\left\{ \substack{c_{HL}=c_{LL}, \text{ or} \\ c_{HL} \leq c_{LL} \text{ and } c_{HL}+c_{LL}=2\frac{\Delta\mu}{\Delta\nu}.} \right.
\end{aligned} \tag{16}$$

On the other hand, because HH envies LH but not HL, $c_{LH} < c_{HL}$.

Finally, as HL envies LL but not LH, $R_{LL} + \delta(c_{LL}, 0, \Delta\nu) > R_{LH} + \delta(c_{LH}, \Delta\mu, -\Delta\nu)$. Using the fact that $R_{LH} = R_{LL} + \delta(c_{LL}, 0, \Delta\nu)$ this gives

$$\begin{aligned}
&\delta(c_{LL}, \Delta\mu, -\Delta\nu) > \delta(c_{LH}, \Delta\mu, -\Delta\nu) \\
\iff &(c_{LL} - c_{LH})\Delta\mu < \frac{1}{2}(c_{LL}^2 - c_{LH}^2)\Delta\nu.
\end{aligned}$$

By monotonicity $c_{LH} < c_{HL} \leq c_{LL}$, so that the only way the previous inequality can hold is when

$$c_{LL} + c_{LH} > 2\frac{\Delta\mu}{\Delta\nu}. \tag{17}$$

Since the second line in (16) and (17) would result in $c_{LH} > c_{HL}$, we can conclude that only the first combination in (16), $c_{HL} = c_{LL}$, is feasible. We therefore call this common coinsurance rate for the risk tolerant types $c_{\cdot L}$. We then have: $0 \leq c_{LH} < c_{\cdot L}$ and $c_{LH} > 2\frac{\Delta\mu}{\Delta\nu} - c_{\cdot L}$, or $\max\{0, 2\frac{\Delta\mu}{\Delta\nu} - c_{\cdot L}\} < c_{LH} < c_{\cdot L}$. Clearly, a necessary condition is $c_{\cdot L} > \frac{\Delta\mu}{\Delta\nu}$. The feasible set for c_{LH} is open. For the calculus analysis of the optimal menu, we close the feasible set for c_{LH} as $\max\{0, 2\frac{\Delta\mu}{\Delta\nu} - c_{\cdot L}\} \leq c_{LH} \leq c_{\cdot L}$.

The profit equations are given by **:**

$$\begin{aligned}
\pi_{HH} &= \frac{1}{2}\nu_H - (1 - c_{LH})\Delta\mu - \frac{1}{2}(1 - c_{\cdot L}^2)\Delta\nu \\
\pi_{HL} &= \frac{1}{2}[1 - c_{\cdot L}^2]\nu_L - (1 - c_{\cdot L})\Delta\mu \\
\pi_{LH} &= \frac{1}{2}[1 - c_{LH}^2]\nu_H - \frac{1}{2}[1 - c_{\cdot L}^2]\Delta\nu \\
\pi_{LL} &= \frac{1}{2}[1 - c_{\cdot L}^2]\nu_L
\end{aligned}$$

Hence, total profit is

$$\begin{aligned}
\pi_{tot}^{P3} = &\frac{1}{2}\nu_L - \alpha_{H\cdot}\Delta\mu + (\alpha_{HH}c_{LH} + \alpha_{HL}c_{\cdot L})\Delta\mu - \alpha_{LH}\frac{1}{2}c_{LH}^2\nu_H \\
&+ \frac{1}{2}(\alpha_{HH} + \alpha_{LH} - x)c_{\cdot L}^2\nu_H
\end{aligned} \tag{18}$$

The problem is then to maximise π_{tot}^{P3} s.t. constraints 1,2,3, and 5 listed in column P3 of Table 2 (the second constraint is taken care of by setting $c_{HL} = c_{LL} = c_{\cdot L}$).

7 Step 3: solutions to the three sub-problems

Before presenting the solution to the three sub-problems, we introduce five auxiliary menus.

Menu **PI:** this menu pools HL, LH and LL at the common coinsurance rate larger than $D\frac{x}{1-x}$ but less than 1:

$$c_{HH}^{PI} = 0, c_{HL}^{PI} = c_{LH}^{PI} = c_{LL}^{PI} = D\frac{x\alpha_{H\cdot}}{x - \alpha_{HH}} < 1.$$

Menu **PX**: this menu pools HL, LH and LL at a common coinsurance of 1 (exclusion):

$$c_{HH}^{PX} = 0, c_{HL}^{PX} = c_{LH}^{PX} = c_{LL}^{PX} = 1.$$

Menu $\mathbf{P}\frac{\Delta\mu}{\Delta\nu}$: this menu pools HL, LH and LL at a common coinsurance of $\frac{\Delta\mu}{\Delta\nu}(= D\frac{x}{1-x})$:

$$c_{HH}^{P\frac{\Delta\mu}{\Delta\nu}} = 0, c_{HH}^{P\frac{\Delta\mu}{\Delta\nu}} = 0, c_{HL}^{P\frac{\Delta\mu}{\Delta\nu}} = c_{LH}^{P\frac{\Delta\mu}{\Delta\nu}} = c_{LL}^{P\frac{\Delta\mu}{\Delta\nu}} = D\frac{x}{1-x}$$

Menu **B2pI**: this menu pools HL and LH at the left hand crossing of the indifference curves of HL and LH, and positions LL at the right hand crossing:

$$\begin{aligned}
c_{HH}^{B2pI} &= 0, c_{HL}^{B2pI} = c_{LH}^{B2pI} = 2\frac{Dx}{1-x} - c_{LL} \\
c_{LL}^{B2pI} &= \frac{Dx}{1-x}\frac{2(\alpha_{LH} + \alpha_{HL}) - \alpha_{H\cdot}(1-x)}{x - \alpha_{HH}}
\end{aligned}$$

Menu **SUBI**: this menu is one that LH positions at the left hand crossing of the indifference curves of LH and HL, while HL and LL are positioned at the right hand crossing:

$$\begin{aligned}
c_{HH}^{SUBI} &= 0, c_{HL}^{SUBI} = c_{LL}^{SUBI} = \frac{Dx}{1-x}\frac{(\alpha_{HL} - \alpha_{HH})(1-x) + 2\alpha_{LH}}{x - \alpha_{HH}} \\
c_{LH}^{SUBI} &= 2\frac{Dx}{1-x} - c_{LL}^{SUBI}
\end{aligned}$$

Menu **SUBX**: this menu is similar to **SUBI**, except that the coinsurance rate at which HL and LL are pooled now equals 1 (i.e., these two types are excluded):

$$c_{HH}^{SUBX} = 0, c_{HL}^{SUBX} = c_{LL}^{SUBX} = 1$$
$$c_{LH} = 2\frac{Dx}{1-x} - c_{LL}^{SUBX}$$

Both menu **SUBI** and **SUBX** are globally sub-optimal menu since profits can be unambiguously increased by pooling HL with LH rather than LL (cf Lemma 14).

Table 3. List of employed functions and symbols:

symbol	definition	description	def. on page	(*)
$\overline{D}$	$\frac{1-\alpha_{H.}}{\alpha_{H.}}$	overall upper bound on D	p 6	P1.1
$\underline{D}_{M1}$	$\frac{(1-\alpha_{H.})\alpha_{LL}}{\alpha_{LH}+(1-\alpha_{H.})(1-\alpha_{LL})}$	lower bound on $D$for M1	p 36	P1.3
$\underline{D}_{M2}$	$\frac{\alpha_{HL}(\alpha_{LH}+\alpha_{HL})}{\alpha_{HH}(2\alpha_{LH}+\alpha_{HL})}$	upper bound on D for **B1pX** lower bound on D for **M2**	p 40	P1.7
$\overline{D}_{Bp}$	$\frac{\alpha_{LL}}{1+\alpha_{LH}-\alpha_{LL}}$	upper bound on D for **B1pI** upper bound on D for **B2pI**	p 38	P1.5
$\overline{D}_{C}$	$\frac{\alpha_{LL}}{1-\alpha_{LL}}$	upper bound on D for **CI** lower bound on D for **CX**	p 50	P2.1
$x_{B1pXM1}(D)$	lower root of $f_{P1.3}(x, D)$	upper bound on x for **B1pX** lower bound on x for **M1**	p 36	P1.3
$x_{B2pXM2}(D)$	upper root of $f_{P1.11}(x, D)$	upper bound on x for **B2pX** lower bound on x for **M2**	p 43	P1.11
$x_{BM}(D)$	$\max\{x_{B1pXM1}(D), x_{B2pXM2}(D)\}$		p 26	
$\overline{x}_{Bp}(D)$	$\frac{1-\alpha_{H.}-(1+\alpha_{LH}-\alpha_{LL})D}{1-\alpha_{H.}(1+D)}$	upper bound on x for **B1pI** lower bound on x for **B1pX**	p 38	P1.5
$\overline{x}_{g_{P2.3}}(D)$	upper root of $g_{P2.3}(x, D)$	upper bound on x for **B2pI** lower bound on x for **B2pX**	p 53	P2.3
$\overline{x}_{f_{P3.1}}$	upper root of $f_{P3.1}(x, D)$	upper bound on x for **EI** lower bound on x for **SUBI**	p 67	P3.1
$x_{CE}(D)$		boundary between C and E	p 33	
$\overline{x}_{f_{P3.5}}(D)$	upper root of $f_{P3.5}(x, D)$	upper bound on x for **SUBI** lower bound on x for **SUBX**	p 74	P3.5
$\overline{x}_{f_{P3.2}}(D)$	upper root of $f_{P3.2}(x, D)$	upper bound on x for **EX** lower bound on x for **SUBX**	p 69	P3.2
ρ_E	$\frac{\alpha_{HH}(1-\alpha_{H.})-\alpha_{HL}\alpha_{H.}\alpha_{HH}}{1+\alpha_{H.}}$	critical ρ-value for the description of **E**	p 67	P3.1
$f_{P2.1}(D)$	$\frac{\alpha_{HH}\alpha_{LL}}{(1-\alpha_{LL})^2 D^2 - 2\alpha_{LH}\alpha_{H.}D+\alpha_{LL}}$	lower bound on x for **CI** upper bound on x for **PX**	p 50	P2.1

(*) Sub-problem and configuration in Appendix B.

We can now provide the solutions the the three sub-problems.

Theorem 5 *The solution to sub-problem P1 is as follows:*
*menu **A** if* $1 - \alpha_{LL} < x < 1$, *and* $0 < D < \overline{D}$;

menu **M1** if $\max\{x_{B1pXM1}(D), \frac{\alpha_{HH}}{\alpha_{H.}}\} < x < 1-\alpha_{LL}$ and $\underline{D}_{M1} < D < \overline{D}$;
menu **M2** if $x_{B2pXM2}(D) < x < \frac{\alpha_{HH}}{\alpha_{H.}}$ and $\underline{D}_{M2} < D < \overline{D}$;
menu **B1pI** if $\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} < x < \min\{1-\alpha_{LL}, \overline{x}_{Bp}\}$ and $0 < D < \overline{D}_{Bp}$;
menu **B1pX** if $\max\{\frac{1}{1+2D}, \overline{x}_{Bp}, \frac{\alpha_{HH}}{\alpha_{H.}}\} < x < x_{B1pXM1}(D)$ and $\underline{D}_{M1} < D < \min\{\underline{D}_{M2}, \overline{D}\}$;
menu **B2pX** if $\frac{1}{1+2D} < x < \min\{\frac{\alpha_{HH}}{\alpha_{H.}}, x_{B2pXM2}(D)\}$ and $\frac{1}{2}\frac{\alpha_{HL}}{\alpha_{HH}} < D < \overline{D}$; and
menu **Bf** if $0 < x < \min\{\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}, \frac{1}{1+2D}\}$ and $0 < D < \overline{D}$.

Proof. See appendix B. ■

Remark: menus **M2** and **B2pX** (which have pooling of HL with LH at a strictly positive coinsurance rate) will disappear if $\underline{D}_{M2} \geq \overline{D}$ or $\frac{1}{2}\frac{\alpha_{HL}}{\alpha_{HH}} \geq \overline{D}$, respectively.

Figure 6 sketches the solution to sub-problem P1 and shows that the list in Theorem 5 is exhaustive.

–Figure 6 here–

Theorem 6 *The solution to sub-problem P2 is as follows:*
menu $\boldsymbol{P}^{\frac{\Delta\mu}{\Delta\nu}}$ if $\min\{\frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H.}}, 1\} < x < \frac{1}{1+D}$, and $0 < D < \frac{\alpha_{LL}-\alpha_{HL}-\alpha_{LH}}{2\alpha_{LH}+\alpha_{HL}}$;
menu **B2pI** if $\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} < x < \overline{x}_{gP2.3}$ and $0 < D < \overline{D}_{Bp}$;
menu **B2pX** if $\max\{\overline{x}_{gP2.3}, \frac{1}{1+2D}\} < x < \frac{1}{1+D}$ and $\max\{0, \frac{\alpha_{LL}-\alpha_{HL}-\alpha_{LH}}{2\alpha_{LH}+\alpha_{HL}}\} < D < \overline{D}$
menu **Bf** if $\frac{1-\alpha_{LL}}{1+\alpha_{LL}} < x < \min\{\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}, \frac{1}{1+2D}\}$ and $0 < D < \overline{D}_C$;
menu **CI** if $\begin{cases} \frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H.}^2\alpha_{LL}} < x < \frac{1-\alpha_{LL}}{1+\alpha_{LL}} \text{ and } 0 < D < \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C \\ f_{P2.1} < x < \frac{1-\alpha_{LL}}{1+\alpha_{LL}} \text{ and } \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C < D < \overline{D}_C \end{cases}$;
menu **CX** if $\frac{\alpha_{HH}}{1-\alpha_{LL}+2\alpha_{LH}D} < x < \frac{1}{1+2D}$ and $\overline{D}_C < D < \begin{cases} \overline{D} \text{ if } \alpha_{HH} \leq \alpha_{LH} \\ \min\{\overline{D}, \frac{1-\alpha_{LL}-\alpha_{HH}}{2\alpha_{HH}-2\alpha_{LH}}\} \text{ if } \alpha_{HH} < \alpha_{LH} \end{cases}$;
and
menu **PI** if $\frac{\alpha_{HH}}{1-\alpha_{H.}D} < x < \frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H.}^2\alpha_{LL}}$ and $0 < D < \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C$;
menu **PX** if $0 < x < \begin{cases} \frac{\alpha_{HH}}{1-\alpha_{H.}D} \text{ if } 0 < D < \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C \\ f_{P2.1} \text{ if } \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C < D < \overline{D}_C \\ \frac{\alpha_{HH}}{1-\alpha_{LL}+2\alpha_{LH}D} \text{ if } \overline{D}_C < D < \overline{D} \text{ and } \alpha_{HH} \leq \alpha_{LH} \\ \max\{\frac{\alpha_{HH}}{1-\alpha_{LL}+2\alpha_{LH}D}, \frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H.}}\} \text{ if } \overline{D}_C < D < \overline{D} \text{ and } \alpha_{HH} > \alpha_{LH} \end{cases}$.

Proof. See appendix B. ■

Remark: sub-problem P2 is only defined when $x \leq \frac{1}{1+D}$.

Figure 7 sketches the solution to Sub-problem 2 and shows that the list in Theorem 6 is exhaustive.

–Figure 7 here–

Theorem 7 *The solution to sub-problem P3 is as follows:*
menu $\boldsymbol{P}\frac{\Delta\mu}{\Delta\nu}$ *if* $\max\{\frac{\alpha_{H.}+\alpha_{HH}}{\alpha_{H.}+1}, \frac{2\alpha_{LH}+\alpha_{HL}}{1+\alpha_{HL}-\alpha_{HH}}\} < x < \frac{1}{1+D}$, *and* $0 < D < \frac{\alpha_{LL}-\alpha_{HL}-\alpha_{LH}}{2\alpha_{LH}+\alpha_{HL}}$;
menu ***SUBI*** *if* $\overline{x}_{f_{P3.1}} < x < \min\{\frac{2\alpha_{LH}+\alpha_{HL}}{1+\alpha_{HL}-\alpha_{HH}}, \overline{x}_{f_{P3.5}}(D)\}$, $0 < D < \frac{\overline{x}_{f_{P3.1}}-\alpha_{.H}}{\alpha_{HL}\overline{x}_{f_{P3.1}}}$ *and* $\rho < \rho_E$;
menu ***SUBX*** *if* $\max\{\overline{x}_{f_{P3.2}}(D), \overline{x}_{f_{P3.5}}(D)\} < x < \frac{1}{1+D}$, $\frac{1-2\alpha_{LH}-\alpha_{HH}}{2\alpha_{LH}+\alpha_{HL}} < D < \overline{D}$ *and* $\rho < \rho_E$;
menu ***EI*** *if* $\frac{\alpha_{.H}}{1-\alpha_{HL}D} < x < \min\{\overline{x}_{f_{P3.1}}, \alpha_{.H}+\frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}}\}$ *and* $0 < D < \min\{\frac{\overline{x}_{f_{P3.1}}-\alpha_{.H}}{\alpha_{HL}\overline{x}_{f_{P3.1}}}, \overline{D}\}$;
menu ***EX*** *if* $0 < x < \min\{\frac{\alpha_{.H}}{1-\alpha_{HL}D}, \overline{x}_{f_{P3.2}}\}$ *and* $0 < D < \overline{D}$;
menu ***PI*** *if* $\alpha_{.H} + \frac{\alpha_{HL}\alpha_{LH}}{\alpha_{HH}} < x < \frac{\alpha_{H.}+\alpha_{HH}}{\alpha_{H.}+1}$ *and* $0 < D < \overline{D}$ *and* $\rho > \rho_E$;

Proof. See appendix B. ■

Figure 8.a (8.b) sketches the solution to sub-problem P3 when $\rho > \rho_E$ ($\rho < \rho_E$) and shows that the list in Theorem 7 is exhaustive.

–Figures 8a and b here–

We have now a full characterisation of the solution for each of the three sub-problems. In the next section, we identify the solution to the main problem.

8 Step 4: identification of the global optimum

For each tuple $(D, x) \in \mathcal{T}_1$ we first ask which menu is optimal under Order 1. There are two sub-problems under Order 1, and we can elicit the optimal menu by applying the revealed preference principle stated at the end of Section 4.

Theorem 8 *Under Order 1, the auxiliary menus* $\mathbf{P}\frac{\Delta\mu}{\Delta\nu}$ *and* ***B2pI*** *are always dominated. Moreover, when* $x < \min\{\frac{1-\alpha_{LL}}{1+\alpha_{LL}}, \frac{1}{1+2D}\}$*, the solution prescribed by sub-problem P1 is strictly dominated by that of sub-problem P2.*

Proof. 1. In sub-problem P2, **menu** $\mathbf{P}\frac{\Delta\mu}{\Delta\nu}$ is a menu that pools the three lower types at $D\frac{x}{1-x}$. This menu is feasible as long as $D\frac{x}{1-x} \leq 1$, i.e., $x \leq \frac{1}{1+D}$, and selected when $\min\{\frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H.}}, 1\} < x < \frac{1}{1+D}$, and $0 < D < \frac{\alpha_{LL}-\alpha_{HL}-\alpha_{LH}}{2\alpha_{LH}+\alpha_{HL}}$. But if $x \leq \frac{1}{1+D}$, this menu is also feasible under sub-problem P1. Since it is not selected there, we can conclude that menu $\mathbf{P}\frac{\Delta\mu}{\Delta\nu}$ will be strictly dominated by the solution to sub-Problem P1.
2. In sub-problem P2, **menu B2pI** is chosen when $\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} < x < \overline{x}_{gP2.3}$ and $0 < D < \overline{D}_{Bp}$. This menu is also feasible under sub-problem P1. Since it is not selected there, this menu is strictly dominated by the solution to sub-Problem P1. (Because $\rho \leq 0$, we have $\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} > \frac{\alpha_{HH}}{\alpha_{H.}}$ (cf Lemma C.6 in Appendix C). Hence it is optimal to pool HL with HH rather than with LH (cf Lemma 1)).
3. When $x < \min\{\frac{1-\alpha_{LL}}{1+\alpha_{LL}}, \frac{1}{1+2D}\}$, the solution to sub-problem P1 is given by **menu Bf**. This menu is also available in sub-problem P2, but not chosen there. Hence, for that region, menu **Bf** is strictly dominated by the menu chosen under sub-problem P2. ■

Define

$$x_{BM}(D) \stackrel{\text{def}}{=} \max\{x_{B1pXM1}(D), x_{B2pXM2}(D)\}. \tag{19}$$

Then we can join menus **M1** and **M2** and define menu **M** as $c^M_{LL} = 1, c^M_{HH} = 0$, and

$$c^M_{LH} = \begin{cases} D\frac{\alpha_{H.}x}{\alpha_{H.}(1-x)+\alpha_{LH}x} & \text{if } x > \frac{\alpha_{HH}}{\alpha_{H.}}, \quad \textbf{(M1)} \\ D\frac{\alpha_{H.}x}{\alpha_{HL}+\alpha_{LH}} & \text{if } x \leq \frac{\alpha_{HH}}{\alpha_{H.}}, \quad \textbf{(M2)} \end{cases}$$

$$c^M_{HL} = \begin{cases} 0 & \text{if } x > \frac{\alpha_{HH}}{\alpha_{H.}}, \quad \textbf{(M1)} \\ D\frac{\alpha_{H.}x}{\alpha_{HL}+\alpha_{LH}} & \text{if } x \leq \frac{\alpha_{HH}}{\alpha_{H.}}. \quad \textbf{(M2)} \end{cases}$$

Then sub-problem P1 prescribes the use of menu **M** when $x_{BM}(D) < x < 1-\alpha_{LL}$ and $\underline{D}_{M1} < D < \overline{D}$.

Likewise, we can join menus **B1pX** and **B2pX** as menu **BpX** defined as define $c^{BpX}_{HH} = 0$, $c^{BpX}_{LH} = 2D\frac{x}{1-x} - c^{BpX}_{LL}$, $c^{BpX}_{LL} = 1$ and

$$c^{BpX}_{HL} = \begin{cases} 0 & \text{if } x > \frac{\alpha_{HH}}{\alpha_{H.}} \quad \textbf{(B1PX),} \\ 2D\frac{x}{1-x} - 1 & \text{if } x \leq \frac{\alpha_{HH}}{\alpha_{H.}} \quad \textbf{(B2pX).} \end{cases}$$

Then sub-problem P1 prescribes the use of menu **BpX** when $\max\{\frac{1}{1+2D}, \overline{x}_{Bp}\} < x < x_{BM}(D)$ and $\underline{D}_{M1} < D < \overline{D}$.

Together with Theorems 5, 6, and 8, this leads to

Theorem 9 *If restricted to Order 1, the optimal use of menus is as follows:*
menu ***A*** *if* $1 - \alpha_{LL} < x < 1$, *and* $0 < D < \overline{D}$;
menu ***M*** *if* $x_{BM}(D) < x < 1 - \alpha_{LL}$ *and* $\underline{D}_{M1} < D < \overline{D}$;
menu ***BpI*** *if* $\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} < x < \min\{1 - \alpha_{LL}, \overline{x}_{Bp}\}$ *and* $0 < D < \overline{D}_{Bp}$;
menu ***BpX*** *if* $\max\{\frac{1}{1+2D}, \overline{x}_{Bp}\} < x < x_{BM}(D)$ *and* $\underline{D}_{M1} < D < \overline{D}$;
menu ***Bf*** *if* $0 < x < \min\{\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}, \frac{1}{1+2D}\}$ *and* $0 < D < \overline{D}$.
menu ***CI*** *if* $\begin{cases} \frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H.}^2\alpha_{LL}} < x < \frac{1-\alpha_{LL}}{1+\alpha_{LL}} \text{ and } 0 < D < \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C \\ f_{P2.1} < x < \frac{1-\alpha_{LL}}{1+\alpha_{LL}} \text{ and } \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C < D < \overline{D}_C \end{cases}$;
menu ***CX*** *if* $\frac{\alpha_{HH}}{1-\alpha_{LL}+2\alpha_{LH}D} < x < \frac{1}{1+2D}$ *and* $\overline{D}_C < D < \begin{cases} \overline{D} \text{ if } \alpha_{HH} \leq \alpha_{LH} \\ \min\{\overline{D}, \frac{1-\alpha_{LL}-\alpha_{HH}}{2\alpha_{HH}-2\alpha_{LH}}\} \text{ if } \alpha_{HH} < \alpha_{LH} \end{cases}$;
menu ***PI*** *if* $\frac{\alpha_{HH}}{1-\alpha_{H.}D} < x < \frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H.}^2\alpha_{LL}}$ *and* $0 < D < \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C$; *and*
menu ***PX*** *otherwise.*

Under Order 2, and if $x \leq \frac{1}{1+D}$, the optimal menu is described by the solution to sub-problem P3. If $x > \frac{1}{1+D}$, the solution to sub-problem P3 is empty.

We are now in a position to compare for $x \leq \frac{1}{1+D}$ the optimal solution under Order 1 and Order 2. We start by relying once more on the revealed preference principle:

Theorem 10 *The auxiliary menus* $\mathbf{P}\frac{\Delta\mu}{\Delta\nu}$*,* ***PI****,* ***PX****,* ***SUBI*** *and* ***SUBX*** *are always dominated.*

Proof. 1. In sub-problem P3, **menu** $\mathbf{P}\frac{\Delta\mu}{\Delta\nu}$ is a menu that pools the three lower types at $D\frac{x}{1-x}$. This menu is feasible as long as $D\frac{x}{1-x} \leq 1$, i.e., $x \leq \frac{1}{1+D}$, and selected when $\max\{\frac{\alpha_{H.}+\alpha_{HH}}{\alpha_{H.}+1}, \frac{2\alpha_{LH}+\alpha_{HL}}{1+\alpha_{HL}-\alpha_{HH}}\} < x < \frac{1}{1+D}$, and $0 < D < \frac{\alpha_{LL}-\alpha_{HL}-\alpha_{LH}}{2\alpha_{LH}+\alpha_{HL}}$. But if $x \leq \frac{1}{1+D}$, this menu is also feasible under sub-problem P1. Since it is not selected there, we can conclude that $\mathbf{P}\frac{\Delta\mu}{\Delta\nu}$ will be weakly dominated by the solution to sub-problem P1.
2. The menus **PI** and **PX** are chosen under sub-problem P2 when

$$0 < x < \begin{cases} \frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H.}^2\alpha_{LL}} \text{ if } 0 < D < \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C \\ f_{P2.1} \text{ if } \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C < D < \overline{D}_C \\ \frac{\alpha_{HH}}{1-\alpha_{LL}+2\alpha_{LH}D} \text{ if } \overline{D}_C < D < \overline{D} \text{ and } \alpha_{HH} \leq \alpha_{LH} \\ \max\{\frac{\alpha_{HH}}{1-\alpha_{LL}+2\alpha_{LH}D}, \frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H.}}\} \text{ if } \overline{D}_C < D < \overline{D} \text{ and } \alpha_{HH} > \alpha_{LH} \end{cases} \quad (20)$$

These pooling menus with pooling at a coinsurance rate above $\frac{\Delta\mu}{\Delta\nu}$ are also available under sub-problem P3. **PX** is not chosen under sub-problem P3. Hence it is dominated. **PI** is chosen under sub-problem P3 only if $\alpha_{\cdot H} + \frac{\alpha_{HL}\alpha_{LH}}{\alpha_{HH}} < x < \frac{\alpha_{H\cdot}+\alpha_{HH}}{\alpha_{H\cdot}+1}$ and $0 < D < \overline{D}$ and $\rho > \rho_E$. But $\frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H\cdot}^2\alpha_{LL}} < \alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}}$, and the upper bound in (20) is smaller or equal to $\frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H\cdot}^2\alpha_{LL}}$. Thus in this range, **PI** is suboptimal. Vice versa, **PI** is chosen under sub-problem P3 if $\alpha_{\cdot H} + \frac{\alpha_{HL}\alpha_{LH}}{\alpha_{HH}} < x < \frac{\alpha_{H\cdot}+\alpha_{HH}}{\alpha_{H\cdot}+1}$ and $0 < D < \overline{D}$ and $\rho > \rho_E$; for this range, it is also available under sub-problem P2, but not chosen. Thus, we can conclude that also **PI** will never constitute a global maximum.
3. The menus **SUBI** and **SUBX** in sub-problem P3 are chosen when

$$\max\{\overline{x}_{f_{P3.1}}(D), \overline{x}_{f_{P3.5}}(D)\} < x < \min\{\frac{2\alpha_{LH}+\alpha_{HL}}{1+\alpha_{HL}-\alpha_{HH}}, \frac{1}{1+D}\},$$

and $0 < D < \overline{D}$. Though for this range, the same menus are not available under sub-problem P1 (since that sub-problem has to respect Order 1), these menus are dominated by menus where HL is pooled with LH at the left-hand crossing (cf Lemma 14). Such menus are available under sub-problem P1. Hence, **SUBI** and **SUBX** can be dismissed. ■

We can now conclude that the optimal solution under Order 1 will be strictly dominated by that for Order 2 (sub-problem P3) when (20) holds, and that the optimal solution under Order 2 (sub-problem P3) will be strictly dominated by that for Order 1 if

$$x \geq \min\{\alpha_{\cdot H} + \frac{\alpha_{HL}\alpha_{LH}}{\alpha_{HH}}, \overline{x}_{f_{P3.1}}, \overline{x}_{f_{P3.2}}(D)\}. \tag{21}$$

Therefore, for every value for D, there must be a value for x above the right-hand side of (20) and below the right-hand side of (21) where the optimal solutions under Order 1 and 2 yield the same maximum profit. The final step is to identify the critical value for x at which the optimal menu under Order 1 and menu **E** (**EI and EX,** Order 2) yield the same maximum profit level. The following theorem shows when this critical x-value will be located below $\min\{\frac{1-\alpha_{LL}}{1+\alpha_{LL}}, \frac{1}{1+2D}\}$, the upper bound for menu **C** (**CI** and **CX**):

Theorem 11 *For every pair $(\alpha_{H\cdot}, \alpha_{HH})$ there exists a $\widehat{\rho}(\alpha_{H\cdot}, \alpha_{HH}) \in (\underline{\rho}(\alpha_{H\cdot}, \alpha_{HH}), 0]$ such that for $\rho \leq \widehat{\rho}$, there exists a function $x_{CE}(D; \alpha_{H\cdot}, \alpha_{HH}, \rho)$, non-increasing in D and with a value below $\min\{\frac{1-\alpha_{LL}}{1+\alpha_{LL}}, \frac{1}{1+2D}\}$, the graph of which in the*

*(D, x)-space constitutes a borderline between menus **CI** and **CX** on the one hand, and menus **EI** and **EX** on the other. Above this line, menus **CI** and **CX** dominate menus **EI** and **EX**, and vice versa. A sufficient condition for this to be the case is that $\rho < -0.089$.*

Proof. See appendix A. ■

Proof of the main proposition

This follows immediately from Theorems 7, 9, 10, and 11. If $\rho > \widehat{\rho}(\alpha_{H\cdot}, \alpha_{HH})$, then menus **EI** and **EX** will completely dominate menus **CI** and **CX**. In that case, there will exists for every D a critical value for x, $x_{BE}(D)$, say, such that menu **B** and menu **E** gives the same maximal profit at $(D, x_{BE}(D))$. However, the set of feasible triples $(\alpha_{H\cdot}, \alpha_{HH}, \rho)$ for which $\widehat{\rho}(\alpha_{H\cdot}, \alpha_{HH}) < \rho \leq 0$ is very small.■

Appendix

A Proof of theorem *11*

We start the proof of this theorem by guessing that when Order 1 and Order 2 yield the same profit level, the optimal menu under Order 1 is **CI** (for low D) and **CX** for high D. We will now show when this guess is correct.

In appendix B, it is shown that the four menus referred to in the theorem have the following maximal profit functions (cf (B.11), (B.12), (B.24), (B.25)):

$$\pi_{tot}^{CI} = \nu_L \left\{ \frac{1}{2} - \alpha_{H\cdot} D + \frac{1}{2} D^2 \frac{(1-\alpha_{LL})^2}{\alpha_{LL}} \right\} \tag{A.1}$$

$$\pi_{tot}^{CX} = \nu_L \left\{ \frac{1}{2} + \alpha_{LH} D - \frac{1}{2} \alpha_{LL} \right\} \tag{A.2}$$

$$\pi_{tot}^{EI} = \nu_L \left\{ \frac{1}{2} - \alpha_{H\cdot} D + \frac{1}{2} D^2 x \left(\frac{\alpha_{HH}^2}{\alpha_{LH}} + \frac{\alpha_{HL}^2}{x - \alpha_{\cdot H}} \right) \right\} \tag{A.3}$$

$$\pi_{tot}^{EX} = \upsilon_L \left\{ \frac{1}{2} - \alpha_{HH} D + \frac{1}{2} D^2 \frac{x \alpha_{HH}^2}{\alpha_{LH}} + \frac{1}{2} \frac{\alpha_{HH} + \alpha_{LH} - x}{x} \right\} \tag{A.4}$$

Let T^{CI} and T^{CX} denote the combinations of $(D, x) \in \mathcal{T}_1$ where **CI** and **CX** are optimal under Order 1, i.e.,

$$T^{CI} = \Bigg\{ (D,x) \in \mathcal{T}_1 | \; \frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2 - \alpha_{H\cdot}^2 \alpha_{LL}} < x < \frac{1-\alpha_{LL}}{1+\alpha_{LL}} \text{ and } 0 < D < \frac{\alpha_{HH}}{1-\alpha_{LL}} \overline{D}_C,$$

$$\text{or } f_{P2.1} < x < \frac{1-\alpha_{LL}}{1+\alpha_{LL}} \text{ and } \frac{\alpha_{HH}}{1-\alpha_{LL}} \overline{D}_C < D < \overline{D}_C \Bigg\}$$

$$T^{CX} = \{ (D,x) \in \mathcal{T}_1 | \; \frac{\alpha_{HH}}{1 - \alpha_{LL} + 2\alpha_{LH} D} < x < \frac{1}{1+2D} \text{ and}$$

$$\overline{D}_C < D < \left\{ {}^{\overline{D} \text{ if } \alpha_{HH} \leq \alpha_{LH}}_{\min\{\overline{D}, \frac{1-\alpha_{LL}-\alpha_{HH}}{2\alpha_{HH}-2\alpha_{LH}}\} \text{ if } \alpha_{HH} < \alpha_{LH}} \right\};$$

(cf Theorem 6). Likewise, denote by T^{EI} and T^{EX} the combinations of $(D, x) \in \mathcal{T}_1$ where **EI** and **EX** are optimal under Order 2, i.e.,

$$T^{EI} = \{ (D,x) \in \mathcal{T}_1 | \; \frac{\alpha_{\cdot H}}{1 - \alpha_{HL} D} < x < \min\{ \overline{x}_{f_{P3.1}}, \alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}} \} \text{ and}$$

$$0 < D < \min\{ \frac{\overline{x}_{f_{P3.1}} - \alpha_{\cdot H}}{\alpha_{HL} \overline{x}_{f_{P3.1}}}, \overline{D} \} \}; \text{ and}$$

$$T^{EX} = \{ (D,x) \in \mathcal{T}_1 | \; 0 < x < \min\{ \frac{\alpha_{\cdot H}}{1 - \alpha_{HL} D}, \overline{x}_{f_{P3.2}} \} \text{ and } 0 < D < \overline{D} \}.$$

(cf Theorem 7).

1. Denote by $x_{CIEI}(\alpha_{H\cdot}, \alpha_{HH}, \rho)$ the solution in x to $\pi^{CI}_{tot}(x, D) = \pi^{EI}_{tot}(x, D)$; it is the lower root to a quadratic equation in x. Note that this solution is independent on D. For this to be a valid solution, it must be true that $(D, x_{C1E1}) \in T^{CI} \cap T^{EI}$.

Since for menu **CI** the upper bound for x is $\frac{1-\alpha_{LL}}{1+\alpha_{LL}}$. we can define for each value for ρ (≤ 0) a region $A(\rho)$ in the $(\alpha_{H\cdot}, \alpha_{HH})$-space such that $x_{CIEI}(\alpha_{H\cdot}, \alpha_{HH}, \rho) \geq \frac{1-\alpha_{LL}}{1+\alpha_{LL}}$:

$$A(\rho) = \{(\alpha_{H\cdot}, \alpha_{HH}) \in [0,1]^2 : x_{C1E1}(\alpha_{H\cdot}, \alpha_{HH}, \rho) \geq \frac{1-\alpha_{LL}(\alpha_{H\cdot}, \alpha_{HH}, \rho)}{1+\alpha_{LL}(\alpha_{H\cdot}, \alpha_{HH}, \rho)}\}$$

We can also define a region $R(\rho)$ such that the minimum feasible value for ρ, $\underline{\rho}(\alpha_{H\cdot}, \alpha_{HH})$–cf definition (C.2)–does not exceed ρ:

$$R(\rho) = \{(\alpha_{H\cdot}, \alpha_{HH}) \in [0,1]^2 : -(\alpha_{H\cdot} - \alpha_{HH})(1-\alpha_{H\cdot}) \leq \rho\}$$

(in other words, $R(\rho)$ is a 'slice' out of the three dimensional set of feasible distribution parameters $\mathcal{A}_1$). It can be shown that $A(\rho) \subset R(\rho)$ for all $\rho \leq 0$, and that there exists a critical ρ, $\widehat{\rho} < 0$, such that for all $\rho < \widehat{\rho}$, $A(\rho) = \varnothing$. Figures A.1a-d show $R(\rho)$ and $A(\rho)$ for $\rho = 0, -\frac{1}{30}, -\frac{2}{30}$ and $-\frac{1}{10}$. In the last case, $A(\rho) = \varnothing$. Our calculations show that $\widehat{\rho} \simeq -0.089$.

–Figures A.1a,b,c,d here–

Thus we can state that $(x_{CIEI}, D) \in T^{CI}$ if $(\alpha_{H\cdot}, \alpha_{HH}) \in R(\rho) \backslash A(\rho)$ and $D < \overline{D}_C$. A sufficient condition is that $\rho \leq -0.089$.

When does $(x_{CIEI}, D) \in T^{EI}$? We need to distinguish between two cases.

a. $\rho_E < \rho \leq 0$, in which case we need $\frac{\alpha_{\cdot H}}{1-\alpha_{HL}D} \leq x_{CIEI} < \alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}}$ and $D \leq \frac{x_{CIEI}-\alpha_{\cdot H}}{\alpha_{HL}x_{CIEI}}$.

The equation $x_{CIEI} = \alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}}$ is a 3th degree polynomial in ρ. The roots for ρ are: $\rho_1 = \overline{\rho}(> 0)$ and two nontrivial roots $\rho_2(\alpha_{H\cdot}, \alpha_{HH}), \rho_3(\alpha_{H\cdot}, \alpha_{HH})$ that–if real–are both strictly positive for any feasible pair $(\alpha_{H\cdot}, \alpha_{HH})$. Since $\alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}} > x_{CIEI}$ for any pair $(\alpha_{H\cdot}, \alpha_{HH})$ when $\rho = 0$, this will also be the case for any $\rho \leq 0$.

The inequality $\alpha_{\cdot H} \leq x_{CIEI}$ is always satisfied for a triple $(\alpha_{HH}, \alpha_{H\cdot}, \rho) \in \mathcal{A}_1$. This claim is based on an 3-dimensional implicitplot in $[0,1] \times [0,1] \times [-\frac{1}{4}, 0]$ in Maple of $x_{CIEI}(\alpha_{HH}, \alpha_{H\cdot}, \rho) = \alpha_{HH} + \alpha_{LH}(\alpha_{HH}, \alpha_{H\cdot}, \rho)$ and $\rho =$

$\underline{\rho}(\alpha_{HH},\alpha_{H\cdot})$.[3] Since $\frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}$ is monotonically increasing in D, we can conclude that $(D,x_{CIEI})\in T^{EI}$ for all $0\leq D\leq\frac{x_{CIEI}-\alpha_{\cdot H}}{\alpha_{HL}x_{CIEI}}$.

b. $\rho<\min\{0,\rho_E\}$, in which case we need $\frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}<x_{CIEI}\leq\overline{x}_{f_{P3.1}}$ and $D<\frac{x_{f_{P3.1}}-\alpha_{\cdot H}}{\alpha_{HL}x_{f_{P3.1}}}$.

The first inequality was shown in **a.** to be always satisfied. The second inequality is always satisfied for a triple $(\alpha_{HH},\alpha_{H\cdot},\rho)\in\mathcal{A}_1$. This claim is based on an 3-dimensional implicitplot $[0,1]\times[0,1]\times[-\frac{1}{4},0]$ in Maple of $x_{CIEI}(\alpha_{HH},\alpha_{H\cdot},\rho)=x_{f_{P3.1}}(\alpha_{HH},\alpha_{H\cdot},\rho)$ and $\rho=\underline{\rho}(\alpha_{HH},\alpha_{H\cdot})$. Thus, $(D,x_{CIEI})\in T^{EI}$ if $D\leq\frac{x_{f_{P3.1}}-\alpha_{\cdot H}}{\alpha_{HL}x_{f_{P3.1}}}$.

2. Denote by $x_{CIEX}(\alpha_{H\cdot},\alpha_{HH},\rho,D)$ the solution in x to $\pi^{CI}_{tot}(x,D)=\pi^{EX}_{tot}(x,D)$; it is the lower root to a quadratic equation in x. For this to be a valid solution, it must be true that $(D,x_{CIEX})\in T^{CI}\cap T^{EX}$.

It can be shown that $x_{CIEX}(\alpha_{H\cdot},\alpha_{HH},\rho,D)$ has the following properties:

(2.i) $x_{CIEX}(\alpha_{H\cdot},\alpha_{HH},\rho,0)=\alpha_{\cdot H}$. This follows straightforwardly from solving $\pi^{CI}_{tot}(x,D)=\pi^{EX}_{tot}(x,D)$ for x when $D=0$;

(2.ii) implicit differentiation of $\pi^{CI}_{tot}(x,D)=\pi^{EX}_{tot}(x,D)$ gives $\frac{\partial x_{CIEX}(\alpha_{H\cdot},\alpha_{HH},\rho,D)}{\partial D}|_{D=0}=2\alpha_{\cdot H}\alpha_{HL}>\alpha_{\cdot H}\alpha_{HL}=\frac{\partial(\frac{\alpha_{\cdot H}}{1-\alpha_{HL}D})}{\partial D}|_{D=0}$;

(2.iii) $x_{CIEX}(\alpha_{H\cdot},\alpha_{HH},\rho,D)\gtreqless\frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}\iff D\lesseqgtr\frac{x_{CIEI}-\alpha_{\cdot H}}{\alpha_{HL}x_{CIEI}}$; and

(2.iv) $x_{CIEX}(\alpha_{H\cdot},\alpha_{HH},\rho,\frac{x_{CIEI}-\alpha_{\cdot H}}{\alpha_{HL}x_{CIEI}})=x_{CIEI}(\alpha_{H\cdot},\alpha_{HH},\rho)$

(2.v) $\frac{\partial x_{CIEX}(\alpha_{H\cdot},\alpha_{HH},\rho,D)}{\partial D}<0$ if $D>\frac{x_{CIEI}-\alpha_{\cdot H}}{\alpha_{HL}x_{CIEI}}$

(2.vi) $x_{CIEX}(\alpha_{H\cdot},\alpha_{HH},\rho,\overline{D}_C)>\frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha^2_{\cdot H}\alpha_{LL}}$;

(2.vii) $\frac{x_{f_{P3.1}}-\alpha_{\cdot H}}{\alpha_{HL}x_{f_{P3.1}}}>\frac{\alpha_{LL}}{1-\alpha_{LL}}$ for all $(\alpha_{H\cdot},\alpha_{HH},\rho)\in\mathcal{A}_1$.. Hence: $x_{f_{P3.2}}$ does not matter as upper bound.

Properties (2.iv) and (2.v) show that $x_{CIEX}(\alpha_{H\cdot},\alpha_{HH},\rho,D)$ is smaller than the upper bound defined by T^{CI} under the same conditions than $x_{CIEX}(\alpha_{H\cdot},\alpha_{HH},\rho)$ is. (2.v) and (2.vi) show that $x_{CIEX}(\alpha_{H\cdot},\alpha_{HH},\rho,D)$ exceeds the lower bound defined by T^{CI}. (2.iii) and (2.vii) show that $x_{CIEX}(\alpha_{H\cdot},\alpha_{HH},\rho,D)$ is smaller than the upper bound defined by T^{EX}. It follows that $(D,x_{CIEX}(\alpha_{H\cdot},\alpha_{HH},\rho,D))\in T^{CI}\cap T^{EX}$ for all $D\in[\frac{x_{CIEI}-\alpha_{\cdot H}}{\alpha_{HL}x_{CIEI}},\overline{D}_C]$.

3. Denote by $x_{CXEX}(\alpha_{H\cdot},\alpha_{HH},\rho,D)$ the solution in x to $\pi^{CX}_{tot}(x,D)=\pi^{EX}_{tot}(x,D)$; it is the lower root to a quadratic equation in x. For this to be a valid solution, it must be true that $(D,x_{CXEX})\in T^{CX}\cap T^{EX}$.

[3] Since $\underline{\rho}(\alpha_{HH},\alpha_{H\cdot})=-(\alpha_{H\cdot}-\alpha_{HH})(1-\alpha_{H\cdot})$, the lowest possible value that $\underline{\rho}$ may take is $-\frac{1}{4}$ (when $\alpha_{H\cdot}=\alpha_{HH}=\frac{1}{2}$).

Properties of $x_{CXEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D)$ are:
(3.i) $x_{CXEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, \overline{D}_C) = x_{CIEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, \overline{D}_C)$
(3.ii) $\frac{\partial x_{CIEX}(\alpha_{H\cdot},\alpha_{HH},\rho,D)}{\partial D} < 0$ for all D
(3.iii) $x_{CXEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D) \geq f_{P2.2}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D)$ for $D \in \{\overline{D}_C, \overline{D}\}$
(3.iv) $x_{CXEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D) < f_{P3.2}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D)$ for $D \in \{\overline{D}_C, \overline{D}\}$
(3.v) $\frac{1}{1+2D} \geq x_{CXEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D)$ all D

Properties (3.ii) and (3.iii), together with the fact that $\frac{\partial f_{P2.2}(\alpha_{H\cdot},\alpha_{HH},\rho,D)}{\partial D} < 0$ shows that $x_{CXEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D)$ is larger than the lower bound defined by T^{CX}. (3.i), (3.ii) and (3.v) show that $x_{CXEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D)$ is smaller than the upper bound defined by T^{CX}. (3.i), (3.ii) and (3.iv), together with the fact that $\frac{\partial f_{P3.2}(\alpha_{H\cdot},\alpha_{HH},\rho,D)}{\partial D} < 0$ shows that $x_{CXEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D)$ is smaller than the upper bound defined by T^{EX} in case $\rho < \rho_E$. (3.i), (2.iii) and (2.iv) shows that $x_{CXEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D)$ is smaller than the upper bound defined by T^{EX} in case $\rho > \rho_E$. It follows that $(D, x_{CXEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D)) \in T^{CX} \cap T^{EX}$ for all $D \in [\overline{D}_C, \overline{D}]$.

We can now summarise as follows. Let the locus of (x, D)-values that for which menus **C** and **E** yield the same profit be defined by $x = x_{CE}(D)$. Then $x_{CE}(\cdot)$ is defined as:

$$x_{CE}(D) \overset{\text{def}}{=} \begin{cases} x_{CIEI}(\alpha_{H\cdot}, \alpha_{HH}, \rho) & \text{if } D < \frac{x_{CIEI}-\alpha_{\cdot H}}{\alpha_{HL}x_{CIEI}} \\ x_{CIEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D) & \text{if } \frac{x_{CIEI}-\alpha_{\cdot H}}{\alpha_{HL}x_{CIEI}} < D < \overline{D}_C \\ x_{CXEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D) & \text{if } \overline{D}_C < D < \overline{D} \end{cases} \tag{A.5}$$

with $x'_{CE}(D) \leq 0$ since both $x_{CIEX}(D, \alpha_{H\cdot}, \alpha_{HH}, \rho)$ and $x_{CXEX}(D, \alpha_{H\cdot}, \alpha_{HH}, \rho)$ are strictly decreasing in D.

$x_{CE}(D)$ is depicted in Figure A.2 for $\alpha_{H\cdot} = .6, \alpha_{HH} = .2$ and $\rho = 0$. It consists of the full horizontal line $(x_{CIEI}(\alpha_{H\cdot}, \alpha_{HH}, \rho))$ until this crosses the upward sloping bold line $(\frac{\alpha_{\cdot H}}{1-\alpha_{HL}D})$, the dashed line $(x_{CIEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D))$ until this crosses the vertical bold line $(D = \overline{D}_C)$, and continues as the dotted-dashed line $(x_{CXEX}(\alpha_{H\cdot}, \alpha_{HH}, \rho, D))$.

–Figure A.2–

This completes the proof of theorem 11.

B Solving the sub-problems

B.1 Solution to sub-problem 1

The Lagrangian function associated to this sub-problem is

$$\mathcal{L}_{P1} = \pi_{tot}^{P1} + \lambda \left\{ c_{LH} - c_{HL} \right\} + \mu_2 \left\{ c_{LL} - c_{LH} \right\} + \mu_1^a \left\{ 2\frac{Dx}{1-x} - c_{LH} - c_{LL} \right\} + \mu_1^b \left\{ 1 - c_{LL} \right\}.$$

The K-T conditions are therefore:

$$\frac{\partial \mathcal{L}_{P1}}{\partial c_{HL}} = \left[\alpha_{HH}(1-x) - \alpha_{HL}x \right] c_{HL} - \frac{\lambda}{\nu_H} \leq 0, \frac{\partial \mathcal{L}_M}{\partial c_{HL}} c_{HL} = 0, c_{HL} \geq 0 \quad \text{(B.1)}$$

$$\frac{\partial \mathcal{L}_{P1}}{\partial c_{LH}} = -[\alpha_{H\cdot}(1-x) + \alpha_{LH}] c_{LH} + \alpha_{H\cdot} Dx + \frac{\lambda}{\nu_H} - \frac{\mu_1^a}{\nu_H} - \frac{\mu_2}{\nu_H} = 0 \quad \text{(B.2)}$$

$$\frac{\partial \mathcal{L}_{P1}}{\partial c_{LL}} = (1 - \alpha_{LL} - x) c_{LL} - \frac{\mu_1^a}{\nu_H} - \frac{\mu_1^b}{\nu_H} + \frac{\mu_2}{\nu_H} = 0 \quad \text{(B.3)}$$

P1.1. $\underline{\lambda = 0, \mu_1^a = 0, \mu_2^b = 0, \mu_2 = 0}$. Then (B.3) becomes

$$(1 - \alpha_{LL} - x) c_{LL} = 0$$

So either π^U is increasing in c_{LL} contradicting that $\mu_1^a = \mu_2^b = 0$, or decreasing in c_{LL} contradicting that $\mu_2 = 0$.

P1.2. $\underline{\lambda = 0, \mu_1^a = 0, \mu_1^b = 0, \mu_2 > 0}$. Then (B.1) implies that

$$x \geq \frac{\alpha_{HH}}{\alpha_{H\cdot}}.$$

The reason is that if $x < \frac{\alpha_{HH}}{\alpha_{H\cdot}}$, then π_{tot}^M would be increasing and convex in c_{HL}, contradicting that $\lambda = 0$.

$\mu_2 > 0$ means that $c_{LL} = c_{LH}$ and we denote this common coinsurance rate by $c_{L\cdot}$. (B.3) then gives

$$(1 - \alpha_{LL} - x) c_{L\cdot} = -\frac{\mu_2}{\nu_H}$$

so that $\mu_2 > 0$ requires that

$$x > 1 - \alpha_{LL}.$$

Combining (B.2) and (B.3) gives

$$c_{L\cdot} = D\frac{\alpha_{H\cdot}}{1-\alpha_{H\cdot}}.$$

$\mu_1^a = 0$ then requires that $c_{L\cdot} \leq \frac{Dx}{1-x}$ or

$$x \geq \alpha_{H\cdot}.$$

which is made redundant by the stronger condition that $x > 1 - \alpha_{LL}$.

$\mu_1^b = 0$ requires that

$$D \leq \frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}} = \overline{D}.$$

Since $\rho \leq 0$ is sufficient for $\frac{\alpha_{HH}}{\alpha_{H\cdot}} \leq 1 - \alpha_{LL}$ (cf Lemma C.5 in Appendix C), the condition $x > \frac{\alpha_{HH}}{\alpha_{H\cdot}}$ is redundant.

This menu was defined as menu **A** in the main proposition. We summarise it as

$$\begin{aligned} c_{HL}^A = 0, c_{LH}^A = c_{LL}^A = D\frac{\alpha_{H\cdot}}{1-\alpha_{H\cdot}}, \\ 1 - \alpha_{LL} < x, \\ D \leq \overline{D}. \end{aligned}$$

P1.3. $\lambda = 0, \mu_1^a = 0, \mu_1^b > 0, \mu_2 = 0$**.** Then (B.1) implies that

$$x \geq \frac{\alpha_{HH}}{\alpha_{H\cdot}}.$$

$\mu_1^b > 0$ means that $c_{LL} = 1$. (B.3) then gives

$$(1 - \alpha_{LL} - x) = \frac{\mu_1^b}{\nu_H}$$

so that $\mu_1^b > 0$ requires that

$$x < 1 - \alpha_{LL}.$$

From (B.2) we obtain that

$$c_{LH} = D\frac{\alpha_{H\cdot}x}{\alpha_{H\cdot}(1-x) + \alpha_{LH}}.$$

$\mu_1^a = 0$ requires that $2\frac{Dx}{1-x} - c_{LH} \geq 1$ or

$$f_{P1.3}(x) \stackrel{\text{def}}{=} \alpha_{H\cdot}(1+D)x^2 - [2\alpha_{H\cdot} + \alpha_{LH} + D(\alpha_{H\cdot} + 2\alpha_{LH})]\, x + (1-\alpha_{LL}) \leq 0$$

Since $f_{P1.3}(x)$ is a convex parabola with $f_{P1.3}(0) > 0 > f_{P1.3}(1)$, the condition is that x exceeds the lower root:

$$x \geq \underline{x}_{f_{P1.3}}(D).$$

For this condition to be compatible with $x < 1 - \alpha_{LL}$, we need that

$$D > \underline{D}_{M1} \stackrel{\text{def}}{=} \frac{(1-\alpha_{H\cdot})\,\alpha_{LL}}{\alpha_{LH} + (1-\alpha_{H\cdot})\,(1-\alpha_{LL})}.$$

$\mu_2 = 0$ requires that $c_{LH} \leq c_{LL} = 1$. Using the earlier derived expression for c_{LH}, this is equivalent with

$$x < \frac{1-\alpha_{LL}}{\alpha_{H\cdot}(1+D)}.$$

Since $D \leq \overline{D}$, this condition is weaker than $x < 1 - \alpha_{LL}$. Hence $1 - \alpha_{LL}$ is the proper upper bound on x.

This menu was defined as menu **M1** in the main proposition. We summarise it as:

$$\begin{aligned} c_{HL}^{M1} &= 0, c_{LL}^{M1} = 1, \\ c_{LH}^{M1} &= D\frac{\alpha_{H\cdot}x}{\alpha_{H\cdot}(1-x) + \alpha_{LH}x} \\ \max\{\underline{x}_{f_{P1.3}}(D), \frac{\alpha_{HH}}{\alpha_{H\cdot}}\} &\leq x < 1 - \alpha_{LL} \\ \underline{D}_{M1} &< D < \overline{D} \end{aligned}$$

In the main text, we relabelled $\underline{x}_{f_{P1.3}}(D)$ as $x_{B1pXM1}(D)$.

P1.4. $\lambda = 0, \mu_1^a = 0, \mu_1^b > 0, \mu_2 > 0$**.** Since $\lambda = 0$, (B.1) implies that

$$x \geq \frac{\alpha_{HH}}{\alpha_{H\cdot}}.$$

$\mu_2 > 0$ implies that $c_{LL} = c_{LH}$. We call this common coinsurance rate $c_{L\cdot}$. $\mu_1^b > 0$ then means that $c_{L\cdot} = 1$.

Since $\mu_1^a = 0$, it is required that $1 \leq \frac{Dx}{1-x}$, or

$$x \geq \frac{1}{1+D}.$$

From (B.2), we get that

$$-[\alpha_{H\cdot}(1-x) + \alpha_{LH}] + \alpha_{H\cdot}Dx = \frac{\mu_2}{\nu_H}$$

so that $\mu_2 > 0$ requires that

$$x > \frac{1-\alpha_{LL}}{\alpha_{H\cdot}(1+D)} (> \frac{1}{1+D}).$$

From (B.3), we get that

$$1 - \alpha_{LL} - x = \frac{\mu_1^b}{\nu_H} - \frac{\mu_2}{\nu_H}.$$

Using the earlier derived expression for $\frac{\mu_2}{\nu_H}$, this can also be written as

$$1 - \alpha_{LL} - x - [\alpha_{H\cdot}(1-x) + \alpha_{LH}] + \alpha_{H\cdot}Dx = \frac{\mu_1^b}{\nu_H}$$

or

$$-x\left[1 - \alpha_{H\cdot}(1+D)\right] = \frac{\mu_1^b}{\nu_H}.$$

$\mu_1^b > 0$ then requires that

$$D > \overline{D},$$

contradicting the restriction that $(D, x) \in \mathcal{T}_1$.

<u>**P1.5.** $\lambda = 0, \mu_1^a > 0, \mu_1^b = 0, \mu_2 = 0$</u>**.** Since $\lambda = 0$, (B.1) implies that

$$x \geq \frac{\alpha_{HH}}{\alpha_{H\cdot}}.$$

$\mu_1^a > 0$ means that $c_{LL} = 2\frac{Dx}{1-x} - c_{LH}$. From (B.3),

$$x < 1 - \alpha_{LL} \text{ and } c_{LL} > 0.$$

Combining (B.2) with (B.3) and solving for c_{LH} yields

$$c_{LH} = D\frac{(1 + \alpha_{LH} + \alpha_{LL})x - (1 + \alpha_{LH} - \alpha_{LL})}{(1-x)(1-\alpha_{H\cdot})}$$

This means that

$$c_{LL} = D\frac{2\alpha_{LH}x + \alpha_{H\cdot}(1-x)}{(1-x)(1-\alpha_{H\cdot})} > 0.$$

$\mu_1^b = 0$ means that $c_{LL} \leq 1$, or

$$x \leq \overline{x}_{Bp} \stackrel{\text{def}}{=} \frac{1-\alpha_{H\cdot} - (1+\alpha_{LH}-\alpha_{LL})D}{1-\alpha_{H\cdot}-\alpha_{H\cdot}D},$$

where the denominator is positive since $D \leq \overline{D}$.

$\lambda = 0$ means that $c_{LH} \geq 0$, or

$$x \geq \frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}.$$

For this to be compatible with $x \leq \overline{x}_{Bp}$, it is required that

$$D \leq \overline{D}_{Bp} \stackrel{\text{def}}{=} \frac{\alpha_{LL}}{1+\alpha_{LH}-\alpha_{LL}}(< \overline{D}).$$

$\mu_2 = 0$ requires that $c_{LH} \leq c_{LL}$ or $c_{LH} \leq \frac{Dx}{1-x}$. Using the earlier derived expression for c_{LH}, this is equivalent with

$$x \leq 1 + \alpha_{LH} - \alpha_{LL}.$$

Clearly, this condition is ensured by the stronger $x < 1 - \alpha_{LL}$.

Note that $\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} < 1 - \alpha_{LL}$ (all α_{ij}–cf Lemma C.11 in Appendix C) and that $\rho \leq 0$ is sufficient for $\frac{\alpha_{HH}}{\alpha_{H\cdot}} < \frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}$ (cf Lemma C.6 in Appendix C).

This menu was defined as menu **BpI** in the main proposition. We summarise it as

$$\begin{aligned}
c_{HL}^{B1pI} &= 0, c_{LH}^{B1pI} = D\frac{(1+\alpha_{LH}+\alpha_{LL})x - (1+\alpha_{LH}-\alpha_{LL})}{(1-x)(1-\alpha_{H\cdot})} \\
c_{LL}^{B1pI} &= D\frac{2\alpha_{LH}x + \alpha_{H\cdot}(1-x)}{(1-x)(1-\alpha_{H\cdot})} \\
\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} &\leq x \leq \min\{1-\alpha_{LL}, \overline{x}_{Bp}\} \\
D &\leq \overline{D}_{Bp}.
\end{aligned}$$

P1.6. $\lambda = 0, \mu_1^a > 0, \mu_1^b = 0, \mu_2 > 0$. Since $\lambda = 0$, (B.1) implies that

$$x \geq \frac{\alpha_{HH}}{\alpha_{H\cdot}}.$$

Since $\mu_2 > 0$, $c_{LH} = c_{LL}$; we call this common coinsurance rate $c_{L\cdot}$. Since $\mu_1^a > 0$, $c_{L\cdot} = \frac{Dx}{1-x}$. Because $\mu_1^b = 0$, $c_{L\cdot} \leq 1$, or

$$x \leq \frac{1}{1+D}.$$

(B.2) and (B.3) now become

$$-[\alpha_{H\cdot}(1-x) + \alpha_{LH}]\frac{Dx}{1-x} + \alpha_{H\cdot}Dx = \frac{\mu_1^a}{\nu_H} + \frac{\mu_2}{\nu_H}$$
$$(1 - \alpha_{LL} - x)\frac{Dx}{1-x} = \frac{\mu_1^a}{\nu_H} - \frac{\mu_2}{\nu_H}$$

Solving for $\frac{\mu_2}{\nu_H}$ and $\frac{\mu_1^a}{\nu_H}$ gives

$$\frac{\mu_1^a}{\nu_H} = \frac{1}{2}(\alpha_{H\cdot} - x)\frac{Dx}{1-x}$$
$$\frac{\mu_2}{\nu_H} = \frac{1}{2}(x - \alpha_{H\cdot} - 2\alpha_{LH})\frac{Dx}{1-x}$$

so that $\mu_2 > 0$ requires that $x > \alpha_{H\cdot} + 2\alpha_{LH}$, while $\mu_1^a > 0$ requires that $x < \alpha_{H\cdot}$, a contradiction.

P1.7. $\lambda = 0, \mu_1^a > 0, \mu_1^b > 0, \mu_2 = 0$. Since $\lambda = 0$, (B.1) implies that

$$x \geq \frac{\alpha_{HH}}{\alpha_{H\cdot}}.$$

$\mu_1^a > 0$ means that $c_{LL} = 2\frac{Dx}{1-x} - c_{LH}$, while $\mu_1^b > 0$ means that $c_{LL} = 1$. Therefore $c_{LH} = 2\frac{Dx}{1-x} - 1$. Since $\mu_2 = 0$, $c_{LH} \leq c_{LL}$, requiring that

$$x \leq \frac{1}{1+D}.$$

(B.2) and (B.3) now become

$$-[\alpha_{H\cdot}(1-x) + \alpha_{LH}]\left(2\frac{Dx}{1-x} - 1\right) + \alpha_{H\cdot}Dx = \frac{\mu_1^a}{\nu_H} \tag{B.4}$$

$$(1 - \alpha_{LL} - x) = \frac{\mu_1^a}{\nu_H} + \frac{\mu_1^b}{\nu_H} \tag{B.5}$$

The second expresion means that

$$x < 1 - \alpha_{LL}.$$

Solving for $\frac{\mu_1^b}{\nu_H}$ gives

$$[1 - (1 + D)\alpha_{H\cdot}]\, x^2 - [1 - \alpha_{H\cdot} - (\alpha_{H\cdot} + 2\alpha_{LH})D]\, x = \frac{\mu_1^b}{\nu_H}$$

so that $\mu_1^b > 0$ requires that

$$x > \overline{x}_{Bp}.$$

For this condition to be compatible with $x < 1 - \alpha_{LL}$, we need that

$$D > \underline{D}_{M1}.$$

Using (B.4), it can be shown that $\mu_1^a > 0$ is equivalent with $f_{P1.3}(x) > 0$, or

$$x < \underline{x}_{f_{P1.3}}(\alpha_{H\cdot}, \alpha_{LH}, D)$$

Compatibility with $x > \overline{x}_{Bp}$ requires again that $D > \underline{D}_{M1}$. It can also be shown that $\underline{x}_{f_{P1.3}}(\alpha_{H\cdot}, \alpha_{LH}, D) \gtreqless 1 - \alpha_{LL}$ iff $D \lesseqgtr \underline{D}_M$. Compatibility of $x < \underline{x}_{f_{P1.3}}$ with $x \geq \frac{\alpha_{HH}}{\alpha_{H\cdot}}$ requires that

$$D < \underline{D}_{M2} \stackrel{\text{def}}{=} \frac{\alpha_{HL}(\alpha_{LH} + \alpha_{HL})}{\alpha_{HH}\,(2\alpha_{LH} + \alpha_{HL})}$$

Finally, $c_{LH} \geq 0$ requires

$$x \geq \frac{1}{1 + 2D}.$$

This menu was defined as menu **B1pX** in the main proposition. We summarise it as

$$c_{HL}^{B1pX} = 0, c_{LH}^{B1pX} = 2\frac{Dx}{1 - x} - 1, c_{LL}^{B1pX} = 1$$
$$\max\{\frac{\alpha_{HH}}{\alpha_{H\cdot}}, \overline{x}_{Bp}, \frac{1}{1 + 2D}\} < x < \underline{x}_{f_{P1.3}}(\alpha_{H\cdot}, \alpha_{LH}, D).$$
$$\underline{D}_{M1} < D < \underline{D}_{M2}$$

P1.8. $\lambda = 0, \mu_1^a > 0, \mu_1^b > 0, \mu_2 > 0$. Since $\mu_2 > 0$, $c_{LH} = c_{LL}$–we call this common coinsurance rate $c_{L\cdot}$.

Since $\mu_1^a > 0$, $c_{L\cdot} = \frac{Dx}{1-x}$. And since $\mu_1^b > 0$, $c_{L\cdot} = 1$, or

$$x = \frac{1}{1+D}.$$

We can therefore consider this as an unimportant knife-edge case.

P1.9. $\lambda > 0, \mu_1^a = 0, \mu_1^b = 0, \mu_2 = 0$. $\lambda > 0$ means that $c_{HL} = c_{LH}$–we call this common coinsurance rate c_I.

a. Suppose that $c_I = 0$. Then (B.1) and (B.2) become

$$-\frac{\lambda}{\nu_H} \leq 0,$$
$$\alpha_{H\cdot} Dx + \frac{\lambda}{\nu_H} = 0,$$

a contradiction.

b. Suppose that $c_I > 0$. Then (B.1) and (B.2) become

$$[\alpha_{HH}(1-x) - \alpha_{HL}x]\, c_I - \frac{\lambda}{\nu_H} = 0,$$
$$-[\alpha_{H\cdot}(1-x) + \alpha_{LH}]c_I + \alpha_{H\cdot} Dx + \frac{\lambda}{\nu_H} = 0.$$

Solving for c_I yields

$$c_I = D\frac{x\alpha_{H\cdot}}{\alpha_{HL} + \alpha_{LH}}.$$

On the other hand, (B.3) becomes

$$(1 - \alpha_{LL} - x)c_{LL} = 0.$$

If $x > 1 - \alpha_{LL}$, then profit is strictly decreasing and concave in c_{LL} , which is incompatible with $\mu_2 = 0$. If $x < 1 - \alpha_{LL}$, then profit is strictly increasing and convex in c_{LL} , which is incompatible with $\mu_1^a = 0, \mu_1^b = 0$.

P1.10. $\underline{\lambda > 0, \mu_1^a = 0, \mu_1^b = 0, \mu_2 > 0}$**.** Both $\lambda > 0$ and $\mu_2 > 0$ means that $c_{HL} = c_{LH} = c_{LL}$–we call this common coinsurance rate c_P. The FOCs then become

$$[\alpha_{HH}(1-x) - \alpha_{HL}x]\, c_P - \frac{\lambda}{\nu_H} \leq 0, \frac{\partial \mathcal{L}_{P1}}{\partial c_{HL}} c_P = 0, c_P \geq 0$$
$$-[\alpha_{H\cdot}(1-x) + \alpha_{LH}]c_P + \alpha_{H\cdot}Dx + \frac{\lambda}{\nu_H} - \frac{\mu_2}{\nu_H} = 0$$
$$(1 - \alpha_{LL} - x)c_P + \frac{\mu_2}{\nu_H} = 0$$

Since $\mu_2 > 0$, the last expression requires that

$$c_P > 0 \text{ and } x > 1 - \alpha_{LL}.$$

Then the first FOC tells that

$$[\alpha_{HH}(1-x) - \alpha_{HL}x]\, c_P = \frac{\lambda}{\nu_H}.$$

$c_P > 0$ and $\lambda > 0$ then requires that

$$x < \frac{\alpha_{HH}}{\alpha_{H\cdot}}$$

By assumption, $\rho \leq 0$. This makes $x < \frac{\alpha_{HH}}{\alpha_{H\cdot}}$ incompatible with $x > 1 - \alpha_{LL}$ (cf Lemma C.5 in Appendix C).

P1.11. $\underline{\lambda > 0, \mu_1^a = 0, \mu_1^b > 0, \mu_2 = 0}$**.** $\lambda > 0$ means that $c_{HL} = c_{LH}$ (called c_I). $\mu_1^b > 0$ means that $c_{LL} = 1$. The FOCs then become

$$[\alpha_{HH}(1-x) - \alpha_{HL}x]\, c_I - \frac{\lambda}{\nu_H} \leq 0, \frac{\partial \mathcal{L}_{P1}}{\partial c_{HL}} c_I = 0, c_I \geq 0$$
$$-[\alpha_{H\cdot}(1-x) + \alpha_{LH}]c_I + \alpha_{H\cdot}Dx + \frac{\lambda}{\nu_H} = 0$$
$$(1 - \alpha_{LL} - x) - \frac{\mu_1^b}{\nu_H} = 0$$

The second expression means that

$$c_I > 0 \text{ and } \alpha_{H\cdot}(1-x) + \alpha_{LH}x > 0.$$

This means that the first FOC must hold with equality and

$$x < \frac{\alpha_{HH}}{\alpha_{H\cdot}}.$$

Solving for c_I gives

$$c_I = D\frac{x\alpha_{H\cdot}}{\alpha_{HL}+\alpha_{LH}}$$

$\mu_1^a = 0$ means that $2\frac{Dx}{1-x} - c_I \geq 1$, or

$$f_{P1.11}(x) \overset{\text{def}}{=} -D\alpha_{H\cdot}x^2 - [2D(\alpha_{HL}+\alpha_{LH}) - D\alpha_{H\cdot} + \alpha_{HL}+\alpha_{LH}]\,x + \alpha_{HL}+\alpha_{LH} \leq 0$$

This is a concave parabola with $f_{P1.11}(0) = \alpha_{HL}+\alpha_{LH} > 0$ and $f_{P1.11}(1) = -2D(\alpha_{HL}+\alpha_{LH}) < 0$. Hence x must exceed the upper root, denoted as $\overline{x}_{f_{P1.11}}(D, \alpha_{H\cdot}, \alpha_{HL}+\alpha_{LH})$:

$$x > \overline{x}_{f_{P1.11}}(D, \alpha_{H\cdot}, \alpha_{HL}+\alpha_{LH}).$$

$\mu_2 = 0$ requires that $c_I \leq c_{LL} = 1$, or

$$x \leq \frac{\alpha_{HL}+\alpha_{LH}}{D\alpha_{H\cdot}}$$

Note that

$$\frac{\alpha_{HL}+\alpha_{LH}}{D\alpha_{H\cdot}} \gtrless \frac{\alpha_{HH}}{\alpha_{H\cdot}} \iff D \lessgtr \frac{\alpha_{HL}+\alpha_{LH}}{\alpha_{HH}}$$

Because $\rho \leq 0$ is a sufficient condition for $\overline{D} < \frac{\alpha_{HL}+\alpha_{LH}}{\alpha_{HH}}$, the restriction $D < \overline{D}$ guarantees that $\frac{\alpha_{HH}}{\alpha_{H\cdot}}$ is the relevant upper bound on x, and that the constraint $c_I \leq 1$ will always be slack.

Finally, for $x > \overline{x}_{f_{P1.11}}(D, \alpha_{H\cdot}, \alpha_{HL}+\alpha_{LH})$ to be compatible with $x < \frac{\alpha_{HH}}{\alpha_{H\cdot}}$, we need

$$D \geq \underline{D}_{M2}$$

This menu was defined as menu **M2** in the main proposition. We summarise it as

$$c_{HL}^{M2} = c_{LH}^{M2} = D\frac{x\alpha_{H\cdot}}{\alpha_{HL}+\alpha_{LH}}, c_{LL}^{M2} = 1$$

$$\overline{x}_{f_{P1.11}}(D, \alpha_{H\cdot}, \alpha_{HL}+\alpha_{LH}) < x < \frac{\alpha_{HH}}{\alpha_{H\cdot}}$$

$$\underline{D}_{M2} \leq D < \overline{D}$$

In the main text, we have relabelled $\overline{x}_{f_{P1.11}}(\alpha_{H\cdot}, \alpha_{HL}+\alpha_{LH}, D)$ as $x_{B2pXM2}(D)$.

Remark

This configuration will only exist when $\underline{D}_{M2} < \overline{D}$. This happens when

$$\alpha_{LH}(2-\alpha_{H\cdot})\left[\frac{\alpha_{H\cdot}}{2-\alpha_{H\cdot}} - \frac{\alpha_{HH}}{\alpha_{H\cdot}}\right] < \alpha_{HL}\left[\frac{\alpha_{HH}}{\alpha_{H\cdot}} - \alpha_{H\cdot}\right]$$

If $\frac{\alpha_{HH}}{\alpha_{H.}} > \alpha_{H.} > \frac{\alpha_{H.}}{2-\alpha_{H.}}$, this inequality is always verified. If $\alpha_{H.} > \frac{\alpha_{HH}}{\alpha_{H.}} > \frac{\alpha_{H.}}{2-\alpha_{H.}}$, then the inequality will only be verified if

$$\rho < \overline{\rho} - \alpha_{HL}\alpha_{H.}\frac{\frac{\alpha_{HH}}{\alpha_{H.}} - \alpha_{H.}}{\frac{\alpha_{H.}}{2-\alpha_{H.}} - \frac{\alpha_{HH}}{\alpha_{H.}}}.$$

P1.12. $\underline{\lambda > 0, \mu_1^a = 0, \mu_1^b > 0, \mu_2 > 0}$**.** $\lambda > 0$ and $\mu_2 > 0$ means that $c_{HL} = c_{LH} = c_{LL}$ (called c_P). $\mu_1^b > 0$ means that $c_P = 1$. The FOCs then become

$$[\alpha_{HH}(1-x) - \alpha_{HL}x] = \frac{\lambda}{\nu_H}$$
$$-[\alpha_{H.}(1-x) + \alpha_{LH}] + \alpha_{H.}Dx + \frac{\lambda}{\nu_H} - \frac{\mu_2}{\nu_H} = 0$$
$$(1 - \alpha_{LL} - x) - \frac{\mu_1^b}{\nu_H} + \frac{\mu_2}{\nu_H} = 0$$

The first condition means that

$$x < \frac{\alpha_{HH}}{\alpha_{H.}}.$$

Combining the first two FOC conditions and imposing $\frac{\mu_2}{\nu_H} > 0$ requires that

$$x > \frac{\alpha_{HL} + \alpha_{LH}}{\alpha_{H.}D}.$$

Compatibility of $x < \frac{\alpha_{HH}}{\alpha_{H.}}$ and $x \geq \frac{\alpha_{HL}+\alpha_{LH}}{\alpha_{H.}D}$ requires that

$$D > \frac{\alpha_{HL} + \alpha_{LH}}{\alpha_{HH}}.$$

For this to be compatible with $D < \overline{D}$ it is required that

$$\alpha_{H.}\alpha_{HL} < \rho,$$

which is incompatible with the assumption that $\rho \leq 0$.

P1.13. $\lambda > 0, \mu_1^a > 0, \mu_1^b = 0, \mu_2 = 0$**.** $\lambda > 0$ means that $c_{HL} = c_{LH}$, which we call c_I. $\mu_1^a > 0$ means that $c_{LL} = 2\frac{Dx}{1-x} - c_I$. The FOCs then become

$$[\alpha_{HH}(1-x) - \alpha_{HL}x]\, c_I - \frac{\lambda}{\nu_H} \leq 0$$
$$-[\alpha_{H.}(1-x) + \alpha_{LH}]c_I + \alpha_{H.}Dx + \frac{\lambda}{\nu_H} - \frac{\mu_1^a}{\nu_H} = 0$$
$$(1 - \alpha_{LL} - x)\left(2\frac{Dx}{1-x} - c_I\right) - \frac{\mu_1^a}{\nu_H} = 0$$

a. Suppose that $c_I = 0$. Then the last FOC and $\mu_1^a > 0$ requires that

$$x < 1 - \alpha_{LL}.$$

Combining the 2nd and 3th FOC gives

$$[2(1 - \alpha_{LL} - x) - (1-x)\alpha_{H.}]\, Dx = (1-x)\frac{\lambda}{\nu_H},$$

so that $\lambda > 0$ requires that

$$x < \frac{1 + \alpha_{LH} - \alpha_{LL}}{1 + \alpha_{LH} + \alpha_{LL}},$$

which is a stronger condition than $x < 1 - \alpha_{LL}$ when $\rho \leq 0$ (cf Lemma C.11 in Appendix C).

$\mu_1^b = 0$ requires that $2\frac{Dx}{1-x} \leq 1$ or

$$x \leq \frac{1}{1+2D}.$$

This menu was defined as menu **Bf** in the main proposition. We summarise it as**:**

$$c_{HL}^{Bf} = c_{LH}^{Bf} = 0, c_{LL}^{Bf} = 2\frac{Dx}{1-x},$$
$$x < \min\{\frac{1 + \alpha_{LH} - \alpha_{LL}}{1 + \alpha_{LH} + \alpha_{LL}}, \frac{1}{1+2D}\}.$$

b. Suppose that $c_I > 0$. Then the first FOC holds with equality, and $\lambda > 0$ requires that

$$x < \frac{\alpha_{HH}}{\alpha_{H.}}.$$

Combining all three FOCs gives

$$\frac{Dx}{1-x}\left[(1+\alpha_{LH}+\alpha_{LL})x-(1+\alpha_{LH}-\alpha_{LL})\right]-c_I(x-\alpha_{HH})=0$$

Concavity of π^M in c_I requires that $x>\alpha_{HH}$. Otherwise either $c_I=0$ or $c_I=c_{LL}$ (contradicting $\mu_2=0$). Hence

$$c_I=\frac{Dx}{1-x}\frac{(1+\alpha_{LH}+\alpha_{LL})x-(1+\alpha_{LH}-\alpha_{LL})}{x-\alpha_{HH}}$$

For $c_I>0$, it is required that

$$x>\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}$$

But this requirement is incompatible with $x<\frac{\alpha_{HH}}{\alpha_{H.}}$ if $\rho\leq 0$ (cf Lemma C.6 in Appendix C).

P1.14. $\lambda>0,\mu_1^a>0,\mu_1^b=0,\mu_2>0$. $\lambda>0$ and $\mu_2>0$ mean that $c_{HL}=c_{LH}=c_{LL}$, which we call c_P. $\mu_1^a>0$ means that $c_P=2\frac{Dx}{1-x}-c_P$, so that

$$c_P=\frac{Dx}{1-x}.$$

The FOCs become

$$\left[\alpha_{HH}(1-x)-\alpha_{HL}x\right]\frac{Dx}{1-x}-\frac{\lambda}{\nu_H}=0$$

$$-[\alpha_{H.}(1-x)+\alpha_{LH}]\frac{Dx}{1-x}+\alpha_{H.}Dx+\frac{\lambda}{\nu_H}-\frac{\mu_1^a}{\nu_H}-\frac{\mu_2}{\nu_H}=0$$

$$(1-\alpha_{LL}-x)\frac{Dx}{1-x}-\frac{\mu_1^a}{\nu_H}+\frac{\mu_2}{\nu_H}=0$$

The 1st equation and $\lambda>0$ give that

$$x<\frac{\alpha_{HH}}{\alpha_{H.}}.$$

The 1st and 2nd equations give

$$(\alpha_{HH}-\alpha_{LH}-\alpha_{H.}x)\frac{Dx}{1-x}=\frac{\mu_1^a}{\nu_H}+\frac{\mu_2}{\nu_H},$$

so that $\mu_1^a>0,\mu_2>0$ require that

$$x<\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H.}}.$$

Combining this with the 3th equation gives

$$[\alpha_{HH} + \alpha_{H\cdot} - (1 + \alpha_{H\cdot})x]\frac{Dx}{1-x} = 2\frac{\mu_1^a}{\nu_H},$$

so that $\mu_1^a > 0$ requires that

$$x < \frac{\alpha_{HH} + \alpha_{H\cdot}}{1 + \alpha_{H\cdot}}.$$

We also get

$$[-\frac{1}{2}\alpha_{HL} - \alpha_{LH} + \frac{1}{2}(1 - \alpha_{H\cdot})x]\frac{Dx}{1-x} = \frac{\mu_2}{\nu_H},$$

so that $\mu_2 > 0$ requires that

$$x > \frac{\alpha_{HL} + 2\alpha_{LH}}{1 - \alpha_{H\cdot}}$$

But this condition is incompatible with $x < \frac{\alpha_{HH}}{\alpha_{H\cdot}}$ if $\rho \leq 0$ (cf Lemma C.7 in Appendix C).

P1.15. $\lambda > 0, \mu_1^a > 0, \mu_1^b > 0, \mu_2 = 0$. $\lambda > 0$ means that $c_{HL} = c_{LH}$, which we call c_I. $\mu_1^a > 0$ means that $c_{LL} = 2\frac{Dx}{1-x} - c_I$ and $\mu_1^b > 0$ means that

$$c_I = 2\frac{Dx}{1-x} - 1.$$

a. Suppose that $c_I = 0$. Then $2\frac{Dx}{1-x} = 1$, which can be considered as an unimportant knife-edge case.

b. Suppose that $c_I > 0$. This means that

$$x > \frac{1}{1+2D}.$$

Then the FOCs become

$$[\alpha_{HH}(1-x) - \alpha_{HL}x]\left(2\frac{Dx}{1-x} - 1\right) - \frac{\lambda}{\nu_H} = 0$$

$$-[\alpha_{H\cdot}(1-x) + \alpha_{LH}]\left(2\frac{Dx}{1-x} - 1\right) + \alpha_{H\cdot}Dx + \frac{\lambda}{\nu_H} - \frac{\mu_1^a}{\nu_H} = 0$$

$$(1 - \alpha_{LL} - x) - \frac{\mu_1^a}{\nu_H} - \frac{\mu_1^b}{\nu_H} = 0$$

Since $c_I > 0$, we must have that

$$x < \frac{\alpha_{HH}}{\alpha_{H\cdot}}.$$

Compatibility with $x > \frac{1}{1+2D}$ requires that

$$D > \frac{1}{2}\frac{\alpha_{HL}}{\alpha_{HH}}.$$

Solving for $\frac{\mu_1^a}{\nu_H}$ and $\frac{\mu_1^b}{\nu_H}$ gives

$$\frac{\mu_1^a}{\nu_H} = \frac{1}{1-x} f_{P1.11}(x; \alpha_{HL} + \alpha_{LH}, \alpha_{H\cdot}, D),$$

Since $f_{P1.11}(x; D)$ is concave in x and strictly positive if $x = 0$, $\mu_1^a > 0$ requires that x is smaller than the upper root of $f_{P1.15}(x; \alpha_{H\cdot}, \alpha_{HL} + \alpha_{LH}, D) = 0$:

$$x < \overline{x}_{f_{P1.11}}(D, \alpha_{H\cdot}, \alpha_{HL} + \alpha_{LH}).$$

Solving for $\frac{\mu_1^b}{\nu_H}$ gives

$$(1 - \alpha_{LL} - x) - \frac{1}{1-x} f_{P1.11}(x; \alpha_{HL} + \alpha_{LH}, \alpha_{H\cdot}, D) = \frac{\mu_1^b}{\nu_H},$$

so that $\mu_1^b > 0$ requires that

$$g_{P1.15}(x; D) \overset{\text{def}}{=} (1 - \alpha_{LL} - x)(1 - x) - f_{P1.11}(x; \alpha_{HL} + \alpha_{LH}, \alpha_{H\cdot}, D) > 0.$$

This is a difference of two quadratic forms in x. The first is convex in x, the second concave. Hence the difference is convex in x. Moreover, $g_{P1.15}(0; D) = \alpha_{HH} > 0$, and $g_{P1.15}(1; D) = 2\alpha_{HH} + 2D(\alpha_{HL} + \alpha_{LH}) > 0$. If $g_{P1.15}(x; D) = 0$ has no real roots, then $g_{P1.15}(x; D) > 0$ for all $x \in [0, 1]$. Suppose then that $g_{P1.15}(x; D) = 0$ has two real roots. Let the upper root be given by $\overline{x}_{g_{P1.15}}(\alpha_{H\cdot}, \alpha_{HL}, \alpha_{LH}, D)$. Then it is possible to show that

$$\overline{x}_{g_{P1.15}}(\alpha_{H\cdot}, \alpha_{HL}, \alpha_{LH}, D) \lesseqgtr \frac{1}{1+2D} \Longleftrightarrow D \gtreqless \frac{\alpha_{LL}}{2\alpha_{LH} + \alpha_{H\cdot}}$$

Using (9) and (10), it is possible to show that

$$\frac{\alpha_{LL}}{2\alpha_{LH} + \alpha_{H\cdot}} \gtreqless \frac{1}{2}\frac{\alpha_{HL}}{\alpha_{HH}} \Longleftrightarrow \rho \gtreqless \frac{1}{2}\alpha_{H\cdot}^2 \alpha_{HL}$$

Thus, $\rho \leq 0$ is a sufficient condition for $\frac{\alpha_{LL}}{2\alpha_{LH}+\alpha_{H\cdot}} < \frac{1}{2}\frac{\alpha_{HL}}{\alpha_{HH}}$. It then follows that $D > \frac{1}{2}\frac{\alpha_{HL}}{\alpha_{HH}}$ implies that $D > \frac{\alpha_{LL}}{2\alpha_{LH}+\alpha_{H\cdot}}$ and therefore that $\overline{x}_{g_{P1.15}}(\alpha_{H\cdot}, \alpha_{HL}, \alpha_{LH}, D) < \frac{1}{1+2D}$. Hence, for any pair (x, D) satisfying $x > \frac{1}{1+2D}$ and $D > \frac{1}{2}\frac{\alpha_{HL}}{\alpha_{HH}}$, the expression $g_{P1.15}(x; D)$ will take on a strictly positive value. This means that $\frac{\mu_1^b}{\nu_H} > 0$ is automatically verified.

This menu was defined as auxiliary menu **B2pX**. We summarise it as

$$
\begin{aligned}
c_{HL}^{B2pX} &= c_{LH}^{B2pX} = 2\frac{Dx}{1-x} - 1, c_{LL}^{B2pX} = 1 \\
\frac{1}{1+2D} &< x < \min\{\frac{\alpha_{HH}}{\alpha_{H\cdot}}, \overline{x}_{f_{P1.11}}(D, \alpha_{H\cdot}, \alpha_{HL} + \alpha_{LH})\} \\
\frac{1}{2}\frac{\alpha_{HL}}{\alpha_{HH}} &< D < \overline{D}
\end{aligned}
$$

P1.16. $\lambda > 0, \mu_1^a > 0, \mu_1^b > 0, \mu_2 > 0$**.** This means that $c_{HL} = c_{LH} = c_{LL} = 1$ and $\frac{Dx}{1-x} = 1$. This can be considered as an unimportant knife-edge case.

B.2 Sub-problem 2

The Lagrangian associated to sub-problem 2 is

$$\mathcal{L}_{P2} = \pi_{tot}^{P2} + \lambda_1(c_{LL} - c_I) + \lambda_2(c_{LL} + c_I - 2\frac{\Delta\mu}{\Delta\nu}) + \lambda_3(1 - c_{LL}).$$

The K-T conditions are therefore:

$$\frac{\partial \mathcal{L}_{P2}}{\partial c_{LL}} = (1 - \alpha_{LL})Dx - \alpha_{LL}xc_{LL} + \frac{\lambda_1}{\nu_H} + \frac{\lambda_2}{\nu_H} - \frac{\lambda_3}{\nu_H} = 0 \tag{B.6}$$

$$\frac{\partial \mathcal{L}_{P2}}{\partial c_I} = [\alpha_{HH} - (1 - \alpha_{LL})x]c_I - \alpha_{LH}Dx - \frac{\lambda_1}{\nu_H} + \frac{\lambda_2}{\nu_H} \leq 0, \frac{\partial \mathcal{L}_C}{\partial c_I}c_I = 0, c_I \geq 0 \tag{B.7}$$

P2.1. $\underline{\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0}$. Then (B.6) yields

$$c_{LL} = D\frac{1-\alpha_{LL}}{\alpha_{LL}}.$$

$\lambda_3 = 0$ requires that $c_{LL} \leq 1$, meaning that

$$D \leq \overline{D}_C \overset{\text{def}}{=} \frac{\alpha_{LL}}{1-\alpha_{LL}} (< \overline{D}).$$

(B.7) gives

$$[\alpha_{HH} - (1-\alpha_{LL})x]c_I - \alpha_{LH}Dx \leq 0, \frac{\partial \mathcal{L}_C}{\partial c_I} c_I = 0, c_I \geq 0.$$

If $x \geq \frac{\alpha_{HH}}{1-\alpha_{LL}}$, π^C_{tot} is concave and strictly decreasing in c_I, satsifying the complementary slackness condition $\frac{\partial \mathcal{L}_C}{\partial c_I} \leq 0$ with strict inequality so that $c_I = 0$. If $x < \frac{\alpha_{HH}}{1-\alpha_{LL}}$, π^C_{tot} is strictly convex in c_I and the profit with $c_I = 0$,

$$\pi^{P2}_{tot}|_{(c_I=0, c_{LL}=D\frac{1-\alpha_{LL}}{\alpha_{LL}})} = \nu_L \left[\frac{1}{2} - \alpha_{H.}D + \frac{1}{2}D^2\frac{(1-\alpha_{LL})^2}{a_{LL}}\right] \tag{B.8}$$

has to be compared with the one when c_I is increased to its upper bound, c_{LL}. In that case, $\lambda_1 > 0$, and the analysis below (see configuration P2.5: $\lambda_1 > \lambda_2 = \lambda_3 = 0$) shows that the optimal common coinsurance rate is $D\frac{x\alpha_{H.}}{x-\alpha_{HH}}$, yielding a maximal profit (B.16). The latter profit does not exceed $\pi^{P2}_{tot}|_{(c_I=0, c_{LL}=D\frac{1-\alpha_{LL}}{\alpha_{LL}})}$ iff

$$x \geq \frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2 - \alpha^2_{H.}\alpha_{LL}}. \tag{B.9}$$

Hence the condition $\lambda_1 = 0$ translates as (B.9). From the analysis of configuration P2.5, it also transpires that that configuration is only possible when $D < \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C$. What happens if $D > \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C$? Then the optimal common coinsurance will exceed 1. Hence, we need to compare $\pi^C_{tot}|_{(c_I=0, c_{LL}=D\frac{1-\alpha_{LL}}{\alpha_{LL}})}$ with $\pi^C_{tot}|_{(c_I=c_{LL}=1)} = \frac{1}{2}\alpha_{HH}\frac{\nu_L}{x}$:

$$\begin{gathered}\pi^C_{tot}|_{(c_I=0, c_{LL}=D\frac{1-\alpha_{LL}}{\alpha_{LL}})} \gtrless \pi^C_{tot}|_{(c_I=c_{LL}=1)} \\ \Updownarrow \\ x \gtrless f_{P2.1}(D) \overset{\text{def}}{=} \frac{\alpha_{LL}\alpha_{HH}}{(1-\alpha_{LL})^2D^2 - 2\alpha_{LL}\alpha_{H.}D + \alpha_{LL}}\end{gathered}$$

The function $f_{P2.1}(\cdot)$ has the following properties: (i) $f'_{P2.1}(D) = 0$ iff $D = \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C$, and (ii) $f_{P2.1}(\frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C) = \frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H.}^2\alpha_{LL}}$.

Therefore, in the case where $\frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C < D < \overline{D}_C$, the relevant lower bound on x is $f_{P2.1}(D)$.

Condition $\lambda_2 = 0$ requires that $c_{LL} \geq 2\frac{\Delta\mu}{\Delta\nu}$, and this translates as

$$x \leq \frac{1-\alpha_{LL}}{1+\alpha_{LL}}. \tag{B.10}$$

It can be shown that $\rho \leq 0$ is a sufficient condition for (B.9) and (B.10) to define a non-empty set (cf Lemma C.3 in Appendix C).

This menu was defined as menu **CI** in the main proposition. We summarise it as**:**

$$c_{HL}^{CI} = c_{LH}^{CI} = 0, c_{LL}^{CI} = D\frac{1-\alpha_{LL}}{\alpha_{LL}}$$
$$\frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H.}^2\alpha_{LL}} \leq x \leq \frac{1-\alpha_{LL}}{1+\alpha_{LL}} \text{ if } D < \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C$$
$$f_{P2.1}(D) \leq x \leq \frac{1-\alpha_{LL}}{1+\alpha_{LL}} \text{ if } \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C < D$$
$$D \leq \overline{D}_C$$

For this configuration, the maximal profit is given by

$$\pi_{tot}^{CI} = \nu_L\left\{\frac{1}{2} - \alpha_{H.}D + \frac{1}{2}D^2\frac{(1-\alpha_{LL})^2}{\alpha_{LL}}\right\} \tag{B.11}$$

and we note that it is independent of x.

P2.2. $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 > 0$**.** $\lambda_3 > 0$ means that $c_{LL} = 1$. Then (B.6) yields

$$D > \overline{D}_C.$$

As before, (B.7) now gives

$$[\alpha_{HH} - (1-\alpha_{LL})x]c_I - \alpha_{LH}Dx \leq 0, \frac{\partial\mathcal{L}_C}{\partial c_I}c_I = 0, c_I \geq 0.$$

If $x \geq \frac{\alpha_{HH}}{1-\alpha_{LL}}$, π_{tot}^C is concave and strictly decreasing in c_I and $c_I = 0$, satisfying the complementary slackness condition $\frac{\partial\mathcal{L}_{P2}}{\partial c_I} \leq 0$ with strict inequality, so that $c_I = 0$. If $x < \frac{\alpha_{HH}}{1-\alpha_{LL}}$, π_{tot}^{P2} is strictly convex in c_I and the profit with $c_I = 0$,

$$\pi_{tot}^{P2}|_{(c_I=0,c_{LL}=1)} = \nu_L\left[\frac{1}{2}(1-\alpha_{LL}) + \alpha_{LH}D\right]$$

has to be compared with the one when c_I is increased to its upper bound, $c_{LL} = 1$. In that case, $\lambda_1 > 0$, and the analysis below (see configuration P2.6: $\lambda_1 > 0, \lambda_2 = 0, \lambda_3 > 0$) shows that with a optimal common coinsurance of 1 maximal profit is

$$\pi_{tot}^{P2}|_{(c_I=c_{LL}=1)} = \nu_L \left[\frac{1}{2}\alpha_{HH}\frac{1}{x}\right].$$

It does not exceed $\pi_{tot}^{P2}|_{(c_I=0,c_{LL}=1)}$ iff

$$x \geq f_{P2.2}(D) \stackrel{\text{def}}{=} \frac{\alpha_{HH}}{(1-\alpha_{LL}) + 2\alpha_{LH}D}.$$

For future reference, we note here that (i) $f_{P2.1}(D) \leq f_{P2.2}(D)$ for all D, with equality iff $D = \overline{D}_C$, and (ii) $f'_{P2.1}(\overline{D}_C) = f'_{P2.2}(\overline{D}_C)$.

$\lambda_2 = 0$ requires that $2\frac{Dx}{1-x} - 1 \leq 0$ which is equivalent with

$$x \leq \frac{1}{1+2D}.$$

For this to be compatible with $x \geq \frac{\alpha_{HH}}{(1-\alpha_{LL})+2\alpha_{LH}D}$, we need

$$2(\alpha_{HH} - \alpha_{LH})D < 1 - (\alpha_{HH} + \alpha_{LL})$$

This is trivially satisfied of $\alpha_{HH} \leq \alpha_{LH}$. Otherwise, we need

$$D < \frac{1 - \alpha_{HH} - \alpha_{LL}}{2(\alpha_{HH} - \alpha_{LH})}.$$

We call this menu menu **CX** and summarise it as

$$c_{HL}^{CX} = c_{LH}^{CX} = 0, c_{LL}^{CX} = 1$$

$$\frac{\alpha_{HH}}{(1-\alpha_{LL}) + 2\alpha_{LH}D} \leq x \leq \frac{1}{1+2D}$$

$$\overline{D}_C < D < \min\{\frac{1-\alpha_{HH}-\alpha_{LL}}{2(\alpha_{HH}-\alpha_{LH})}, \overline{D}\} \text{ (if } \alpha_{HH} > \alpha_{LH})$$

$$\overline{D}_C < D < \overline{D} \qquad \text{(otherwise)}$$

For this configuration, the maximal profit is given by

$$\pi_{tot}^{C.2} = \nu_L \left\{\frac{1}{2} + \alpha_{LH}D - \frac{1}{2}\alpha_{LL}\right\} \tag{B.12}$$

and we note that it is independent of x.

P2.3. $\lambda_1 = 0, \lambda_2 > 0, \lambda_3 = 0$**.** $\lambda_2 > 0$ means that $c_I = 2\frac{Dx}{1-x} - c_{LL}$.

a. Suppose that $c_{LL} < 2\frac{Dx}{1-x}$ so that $c_I > 0$. Then (B.7) and (B.6) become

$$[\alpha_{HH} - (1-\alpha_{LL})x]c_I - \alpha_{LH}Dx + \frac{\lambda_2}{\nu_H} = 0$$
$$(1-\alpha_{LL})Dx - \alpha_{LL}xc_{LL} + \frac{\lambda_2}{\nu_H} = 0$$

implying that

$$c_{LL} = \frac{Dx}{1-x}\frac{2(\alpha_{LH}+\alpha_{HL}) - \alpha_{H\cdot}(1-x)}{x-\alpha_{HH}}, \text{ and}$$
$$c_I = \frac{Dx}{1-x}\frac{(1+\alpha_{LH}+\alpha_{LL})x - (1+\alpha_{LH}-\alpha_{LL})}{x-\alpha_{HH}}.$$

For $c_I > 0$, we need

$$x > \frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}.$$

$\frac{\lambda_2}{\nu_H} > 0$ requires that $(1-\alpha_{LL})Dx - \alpha_{LL}xc_{LL} < 0$, or

$$f_{P2.3}(x) \stackrel{\text{def}}{=} [1-\alpha_{LL}+\alpha_{LL}\alpha_{H\cdot}]x^2 - [\alpha_{LL}(\alpha_{HH}-\alpha_{LH}) + (1-\alpha_{LL})^2 + \alpha_{HH}]x$$
$$+\alpha_{HH}(1-\alpha_{LL}) > 0$$

This quadratic form is convex in x. It is possible to show that $f_{P2.3}(x)|_{x=\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}} = \alpha_{LH}(1-x)(x-\alpha_{HH})|_{x=\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}} > 0$ and that $f'_{P2.3}(x)|_{x=\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}} > 0$ if $\rho \leq 0$. This means that $\lambda_2 > 0$ is implied by $x > \frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}$.

$\lambda_1 = 0$ requires that $c_I \leq c_{LL}$, or $c_{LL} \geq \frac{Dx}{1-x}$. This is equivalent with

$$x \leq \frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H\cdot}}.$$

This condition is compatible with $x > \frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}$ because $\rho \leq 0$ is sufficient for $\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} < \frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H\cdot}}$.

Finally, $\lambda_3 = 0$ requires that $c_{LL} \leq 1$. This is equivalent with

$$g_{P2.3}(x,D) \stackrel{\text{def}}{=} (1+\alpha_{H\cdot}D)x^2 + \{[2(\alpha_{LH}+\alpha_{HL}) - \alpha_{H\cdot}]\,D - (1+\alpha_{HH})\}\,x + \alpha_{HH} \leq 0,$$

This is a convex function in x with $g_{P2.3}(0) = \alpha_{HH} > 0$ and $g_{P2.3}(1) = 2(\alpha_{HL}+\alpha_{LH})D > 0$. It can be shown that it always has two real roots if $D \in [0,\overline{D}]$. Thus the requirement is that

$$\underline{x}_{g_{P2.3}}(D) \leq x \leq \overline{x}_{g_{P2.3}}(D).$$

It can be shown that $\underline{x}_{gP2.3}(D) < \frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}$. Hence, the lower root is redundant. For $x \leq \overline{x}_{gP2.3}(D)$ to be compatible with $x > \frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}$ we need that

$$D \leq \overline{D}_{Bp} \stackrel{\text{def}}{=} \frac{\alpha_{LL}}{1+\alpha_{LH}-\alpha_{LL}}.$$

Comparing $\frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H.}}$ with $\overline{x}_{gP2.3}$ shows that

$$\overline{x}_{gP2.3}(D) \gtrless \frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H.}}$$
$$\Updownarrow$$
$$D \lessgtr \frac{\alpha_{LL}-\alpha_{HL}-\alpha_{LH}}{2\alpha_{LH}+\alpha_{HL}}$$

It can be shown that $\rho \leq 0$ is a sufficient condition for $\frac{\alpha_{LL}-\alpha_{HL}-\alpha_{LH}}{2\alpha_{LH}+\alpha_{HL}} \leq \overline{D}_{Bp}$.

We also have that

$$\frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H.}} \gtrless 1$$
$$\Updownarrow$$
$$\frac{\alpha_{LL}-\alpha_{HL}-\alpha_{LH}}{2\alpha_{LH}+\alpha_{HL}} \lessgtr 0$$

This menu was called auxiliary menu **B2pI**. We summarise it as:

$$\begin{aligned}
c_{HL}^{B2pI} &= c_{LH}^{B2pI} = \frac{Dx}{1-x}\frac{(1+\alpha_{LH}+\alpha_{LL})x-(1+\alpha_{LH}-\alpha_{LL})}{x-\alpha_{HH}}\\
c_{LL}^{B2pI} &= \frac{Dx}{1-x}\frac{2(\alpha_{LH}+\alpha_{HL})-\alpha_{H.}(1-x)}{x-\alpha_{HH}}\\
\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} &\leq x \leq \min\{\frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H.}}, \overline{x}_{gP2.3}(D)\}\\
D &\leq \overline{D}_{Bp}
\end{aligned}$$

b. Suppose that $c_{LL} = 2\frac{Dx}{1-x}$ such that $c_I = 0$. Then (B.7) and (B.6) become

$$\frac{\lambda_2}{\nu_H} \leq \alpha_{LH}Dx,$$
$$[(1-\alpha_{LL})(1-x)-2\alpha_{LL}x]\frac{Dx}{1-x} = -\frac{\lambda_2}{\nu_H}.$$

Eliminating λ_2 from these two expressions results in

$$[(1-\alpha_{LL})(1-x)-2\alpha_{LL}x+\alpha_{LH}]\frac{Dx}{1-x}\geq 0$$
$$\Updownarrow$$
$$x\leq \frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}.$$

On the other hand, $\lambda_2 > 0$ requires

$$\frac{1-\alpha_{LL}}{1+\alpha_{LL}} < x.$$

$\lambda_3 = 0$ requires that $2\frac{Dx}{1-x} \leq 1$ or

$$x \leq \frac{1}{1+2D}.$$

For this to be compatible with $\frac{1-\alpha_{LL}}{1+\alpha_{LL}} < x$, we need

$$D < \overline{D}_C.$$

We also have that

$$\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} \gtrless \frac{1}{1+2D}$$
$$\Updownarrow$$
$$D \gtrless \overline{D}_{Bp}$$

This menu was earlier defined as menu **Bf**. We summarise it as

$$c_{HL}^{Bf} = c_{LH}^{Bf} = 0, c_{LL}^{Bf} = 2\frac{Dx}{1-x}$$
$$\frac{1-\alpha_{LL}}{1+\alpha_{LL}} < x \leq \min\{\frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}}, \frac{1}{1+2D}\}$$
$$D < \overline{D}_C$$

P2.4. $\lambda_1 = 0, \lambda_2 > 0, \lambda_3 > 0$. $\lambda_2 > 0$ and $\lambda_3 > 0$ means that $c_{LL} = 1$ and $c_I = 2\frac{Dx}{1-x} - 1$. $\lambda_1 = 0$ requires that $c_I \leq 1$ or

$$x \leq \frac{1}{1+D}.$$

a. Suppose that $c_I = 2\frac{Dx}{1-x} - 1 > 0$, i.e., that

$$x > \frac{1}{1+2D}.$$

Then (B.7) and (B.6) become

$$[\alpha_{HH} - (1-\alpha_{LL})x]\left(2\frac{Dx}{1-x} - 1\right) - \alpha_{LH}Dx + \frac{\lambda_2}{\nu_H} = 0, \tag{B.13}$$

$$(1-\alpha_{LL})Dx - \alpha_{LL}x + \frac{\lambda_2}{\nu_H} = \frac{\lambda_3}{\nu_H}. \tag{B.14}$$

Eliminating $\frac{\lambda_2}{\nu_H}$ results in

$$[2(\alpha_{HL} + \alpha_{LH}) - \alpha_{H\cdot}(1-x)]\frac{Dx}{1-x} - (x - \alpha_{HH}) = \frac{\lambda_3}{\nu_H}. \tag{B.15}$$

The requirement $\lambda_3 > 0$ is then equivalent with

$$g_{P2.3}(x, D) > 0,$$

or

$$x > \overline{x}_{g_{P2.3}}(D).$$

It can be shown that

$$\overline{x}_{g_{P2.3}}(D) > (<)\frac{1}{1+D} > (<)\frac{2\alpha_{LH} + \alpha_{HL}}{1 - \alpha_{H\cdot}}$$

$$\Updownarrow$$

$$D < (>)\frac{\alpha_{LL} - \alpha_{HL} - \alpha_{LH}}{2\alpha_{LH} + \alpha_{HL}}$$

So compatibility of $x > \overline{x}_{g_{P2.3}}(D)$ with $x \leq \frac{1}{1+D}$, requires that

$$D > \frac{\alpha_{LL} - \alpha_{HL} - \alpha_{LH}}{2\alpha_{LH} + \alpha_{HL}}.$$

Notice that in (B.13), $[\alpha_{HH} - (1-\alpha_{LL})x]$ is the coefficient with c_I where the latter is evaluated at $2\frac{Dx}{1-x} - 1$. If $x < \frac{\alpha_{HH}}{1-\alpha_{LL}}$, profit is convex in c_I. The alternative choice for c_I is then not $2\frac{Dx}{1-x} - 1$ but 1. This menu yields a maximal profit of

$$\pi_{tot}^{P2}|_{(c_I=1,c_{LL}=1)} = \nu_L \left[\frac{1}{2}\alpha_{HH}\frac{1}{x}\right].$$

The maximal profit under menu $c_I = 2\frac{Dx}{1-x} - 1, c_{LL} = 1$ is

$$\pi_{tot}^{P2}|_{(c_I=2\frac{Dx}{1-x}-1,c_{LL}=1)} = \nu_L \left\{\frac{1}{2}\alpha_{HH}\frac{1}{x} - \left[2\left(\alpha_{HH}-\alpha_{LH}\right) - 2\alpha_{H\cdot}\right]xD + \left[\frac{\alpha_{HH}-\alpha_{LH}}{x} - (1-\alpha_{LL})\right]\frac{2D^2x^2}{(1-x)^2}\right\}.$$

We then have that

$$\pi_{tot}^{P2}|_{(c_I=1,c_{LL}=1)} \gtrless \pi_{tot}^{P2}|_{(c_I=2\frac{Dx}{1-x}-1,c_{LL}=1)}$$
$$\Updownarrow$$
$$h_{P2.3}(x) \overset{\text{def}}{=} \left[\alpha_{H\cdot} + (1-\alpha_{LL})D\right]x^2 - \left[\alpha_{H\cdot} + \alpha_{HH} - \alpha_{LH} + (\alpha_{HH}-\alpha_{LH})D\right]x + \alpha_{HH} - \alpha_{LH} \gtrless 0$$

This is a convex quadratic form in x with roots: $\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}}$ and $\frac{1}{1+D}$.

Claim 1: $\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}} < \frac{1}{1+D}$.

Proof. This is obvious if $\alpha_{HH} < \alpha_{LH}$. Suppose, on the other hand, that $\alpha_{HH} > \alpha_{LH}$. Then

$$\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}} \lesseqgtr \frac{1}{1+D}$$
$$\Updownarrow$$
$$D \lesseqgtr \frac{1-\alpha_{LL}-\alpha_{HH}}{\alpha_{HH}-\alpha_{LH}}$$

But since $\rho \leq 0$ is a sufficient condition for $\frac{1-\alpha_{LL}-\alpha_{HH}}{\alpha_{HH}-\alpha_{LH}} > \overline{D}$,[4] and since $D \leq \overline{D}$, we have that $\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}} < \frac{1}{1+D}$. ■

[4] We now want to show that $\frac{\alpha_{LH}+\alpha_{HL}}{\alpha_{HH}-\alpha_{LH}} > \overline{D}_A$. Using the fact that $\alpha_{LH} = \alpha_{HH}\frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}} - \frac{\rho}{\alpha_{H\cdot}}$, this inequality can be rewritten as

$$\rho < \alpha_{HH}(1-\alpha_{H\cdot}) - \alpha_{H\cdot}(\alpha_{HH} - \alpha_{H\cdot}^2)$$

Since the *rhs* is strictly positive for all $\alpha_{HH} < \alpha_{H\cdot} < 1$, it follows that $\rho \leq 0$ is a sufficient condition for $\frac{1-\alpha_{LL}-\alpha_{HH}}{\alpha_{HH}-\alpha_{LH}} > \overline{D}_A$.

Claim 2: Since $h_{P2.3}(x)$ is convex in x, we have $\pi^C_{tot}|_{(c_I=1,c_{LL}=1)} < \pi^C_{tot}|_{(c_I=2\frac{Dx}{1-x}-1,c_{LL}=1)}$ iff $\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}} < x < \frac{1}{1+D}$ and $\pi^C_{tot}|_{(c_I=1,c_{LL}=1)} > \pi^C_{tot}|_{(c_I=2\frac{Dx}{1-x}-1,c_{LL}=1)}$ if $x < \frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}}$.

Claim 3: Since we need that $x > \frac{1}{1+2D}$, the above interval $[\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}}, \frac{1}{1+D}]$ is valid if $\frac{1}{1+2D} < \frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}}$, or

$$D > \frac{1-\alpha_{LL}-\alpha_{HH}}{2(\alpha_{HH}-\alpha_{LH})}.$$

So we may conclude as follows: if $\alpha_{HH} < \alpha_{LH}$, then for any $x < \frac{1}{1+D}$ we have $\pi^C_{tot}|_{(c_I=2\frac{Dx}{1-x}-1,c_{LL}=1)} > \pi^C_{tot}|_{(c_I=1,c_{LL}=1)}$. If $\alpha_{HH} > \alpha_{LH}$, then $\pi^C_{tot}|_{(c_I=2\frac{Dx}{1-x}-1,c_{LL}=1)} > \pi^C_{tot}|_{(c_I=1,c_{LL}=1)}$ for $x \in [\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}}, \frac{1}{1+D}]$. The lower bound $\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}}$ below which the ranking of the two profits switches starts to be valid for $D > \frac{1-\alpha_{LL}-\alpha_{HH}}{2(\alpha_{HH}-\alpha_{LH})}$ since for lower levels of D the other lower bound on x, $\frac{1}{1+2D}$, exceeds $\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}}$.

The requirement that $\lambda_2 > 0$ is equivalent with

$$\begin{aligned} k_{P3.2}(x) \stackrel{\text{def}}{=} &\{1-\alpha_{LL}-[\alpha_{LH}-2(1-\alpha_{LL})]\,D\}\,x^2 \\ &+[(\alpha_{LH}-2\alpha_{HH})D-(1-\alpha_{LL})-\alpha_{HH}]\,x+\alpha_{HH} > 0 \end{aligned}$$

This is a convex quadratic form in x with $k_{P3.2}(0) = \alpha_{HH}$ and $k_{P3.2}(1) = 2(\alpha_{LH}+\alpha_{HL})D > 0$.

Claim 4: $\lambda_2 > 0$ is always satisfied.

Proof. Recall that $(1-\alpha_{LL})(D-\frac{\alpha_{LL}}{1-\alpha_{LL}})x+\frac{\lambda_2}{\nu_H} = \frac{\lambda_3}{\nu_H}$. Hence, if $D \leq \overline{D}_C$, $\lambda_3 > 0$ guarantees that $\lambda_2 > 0$.

Suppose now that $D > \overline{D}_C$. Then from (B.14) and (B.15) we get

$$\frac{\lambda_2}{\nu_H} = \alpha_{LH}Dx - [\alpha_{HH}-(1-\alpha_{LL})x]\left(2\frac{Dx}{1-x}-1\right)$$

Assume first that $x \geq \frac{\alpha_{HH}}{1-\alpha_{LL}}$. Then the square bracket term is negative. Since we require that $x > \frac{1}{1+2D}$, the large round bracket term is positive. Therefore $\lambda_2 > 0$.

Assume next that $x < \frac{\alpha_{HH}}{1-\alpha_{LL}}$. Recall from above that the lowest value for x for which this configuration is possible is $\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}}$. Evaluating $k_{P3.2}(x)$ at $\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}}$ gives

$$k_{P3.2}(\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}}) = \frac{\alpha_{LH}(1-\alpha_{LL}-\alpha_{HH})\,[1-\alpha_{HH}-a_{LL}+D(a_{LH}-\alpha_{HH})]}{\alpha^2_{H\cdot}}$$

so that

$$k_{P3.2}(\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}}) > 0$$

$$\Updownarrow$$

$$D < \frac{1-\alpha_{LL}-\alpha_{HH}}{\alpha_{HH}-\alpha_{LH}}$$

Earlier, we argued that $\overline{D} < \frac{1-\alpha_{LL}-\alpha_{HH}}{\alpha_{HH}-\alpha_{LH}}$ if $\rho \leq 0$. Therefore the above inequality is fulfilled for any $D \leq \overline{D}$. We can thus cocnlude that the requirement $\lambda_2 > 0$ is satsified. ∎

This menu was earlier defined as menu **B2pX**. We summarise this menu as:

$$c_{HL} = c_{LH} = 2\frac{Dx}{1-x} - 1, c_{LL} = 1$$

$$\max\{\overline{x}_{g_{P2.3}}, \frac{1}{1+2D}, \frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H\cdot}}\} < x < \frac{1}{1+D}$$

$$\max\{\frac{\alpha_{LL}-\alpha_{HL}-\alpha_{LH}}{2\alpha_{LH}+\alpha_{HL}}, 0\} \leq D \leq \overline{D}$$

b. Suppose that $c_I = 2\frac{Dx}{1-x} - 1 = 0$. Then

$$x = \frac{1}{1+2D},$$

an unimportant knife-edge case.

<u>**P2.5.** $\lambda_1 > 0, \lambda_2 = 0, \lambda_3 = 0$**.**</u> $\lambda_1 > 0$ means that $c_I = c_{LL} \geq \frac{Dx}{1-x}$. Let us call the common coinsurance rate c_P. Since $\lambda_2 = 0$, $c_P \geq \frac{Dx}{1-x} > 0$. Hence (B.7) and (B.6) become

$$[\alpha_{HH} - (1-\alpha_{LL})x]c_P - \alpha_{LH}Dx = \frac{\lambda_1}{\nu_H},$$

$$(1-\alpha_{LL})Dx - \alpha_{LL}xc_P + \frac{\lambda_1}{\nu_H} = 0.$$

This gives

$$\alpha_{H\cdot}Dx - (x - \alpha_{HH})c_P = 0.$$

If $x < \alpha_{HH}$, π^{P2} is strictly increasing and convex in c_P, contradicting that $\lambda_3 = 0$. Hence,

$$x > \alpha_{HH},$$

and

$$c_P = D\frac{\alpha_{H\cdot}x}{x-\alpha_{HH}}.$$

$\lambda_3 = 0$ requires that $c_P \leq 1$ or

$$x \geq \frac{\alpha_{HH}}{1-D\alpha_{H\cdot}}.$$

Note that $\frac{\alpha_{HH}}{1-D\alpha_{H\cdot}} > \alpha_{HH}$, so that $\frac{\alpha_{HH}}{1-D\alpha_{H\cdot}}$ is the relevant lower bound on x.

$\lambda_1 > 0$ requires that $[\alpha_{HH} - (1-\alpha_{LL})x]c_P - \alpha_{LH}Dx > 0$ or

$$x < \frac{\alpha_{HH}(1-\alpha_{LL})}{(1-\alpha_{LL})^2 + \alpha_{LL}\alpha_{LH}}.$$

$\lambda_2 = 0$ requires that $c_P \geq \frac{Dx}{1-x}$ or

$$x \leq \frac{\alpha_{H\cdot} + \alpha_{HH}}{\alpha_{H\cdot} + 1}.$$

It can be shown that $\rho \leq 0$ is sufficient for $\frac{\alpha_{HH}(1-\alpha_{LL})}{(1-\alpha_{LL})^2+\alpha_{LL}\alpha_{LH}} < \frac{\alpha_{H\cdot}+\alpha_{HH}}{\alpha_{H\cdot}+1}$ (cf Lemma C.2 in Appendix C), so $\frac{\alpha_{HH}(1-\alpha_{LL})}{(1-\alpha_{LL})^2+\alpha_{LL}\alpha_{LH}}$ is the relevant upper bound for x. For $x < \frac{\alpha_{HH}(1-\alpha_{LL})}{(1-\alpha_{LL})^2+\alpha_{LL}\alpha_{LH}}$ to be compatible with $x > \frac{\alpha_{HH}}{1-D\alpha_{H\cdot}}$ we need that

$$D < \overline{D}_C.$$

Recall from the discussion of configuration P2.1 that that configuration is dominated by optimal pooling iff

$$x < \frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2 - \alpha_{H\cdot}^2\alpha_{LL}}.$$

Note now that $\frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H\cdot}^2\alpha_{LL}} < \frac{\alpha_{HH}(1-\alpha_{LL})}{(1-\alpha_{LL})^2+\alpha_{LL}\alpha_{LH}}$ so the relevant upper bound on x becomes $\frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H\cdot}^2\alpha_{LL}}$. For this to be compatible with $x \geq \frac{\alpha_{HH}}{1-D\alpha_{H\cdot}}$ we need that

$$D < \frac{\alpha_{HH}\alpha_{LL}}{(1-\alpha_{LL})^2} = \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C (< \overline{D}_{Bp} < \overline{D}_C).$$

This menu was earlier defined as the auxiliary menu **PI**. We summarise it as:

$$c_{HL}^{PI} = c_{LH}^{PI} = c_{LL}^{PI} = D\frac{\alpha_{H\cdot}x}{x-\alpha_{HH}}$$
$$\frac{\alpha_{HH}}{1-D\alpha_{H\cdot}} < x < \frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H\cdot}^2\alpha_{LL}},$$
$$D < \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C$$

For this configuration, the maximal profit is given by

$$\pi_{tot}^{PI} = \nu_L\left\{\frac{1}{2} - \alpha_{H\cdot}D + \frac{1}{2}D^2\frac{x\alpha_{H\cdot}^2}{x-\alpha_{HH}}\right\} \tag{B.16}$$

and we note that it is strictly decreasing in x (as $x > \alpha_{HH}$).

P2.6. $\lambda_1 > 0, \lambda_2 = 0, \lambda_3 > 0$**.** $\lambda_1 > 0$ means that $c_I = c_{LL}$. Let us call the common coinsurance rate c_P. Since $\lambda_3 > 0$, $c_P = 1$. $\lambda_2 = 0$ then requires that $1 \geq \frac{Dx}{1-x}$, or

$$x \leq \frac{1}{1+D}.$$

The first order conditions (B.7) and (B.6) become

$$[\alpha_{HH} - (1-\alpha_{LL})x] - \alpha_{LH}Dx = \frac{\lambda_1}{\nu_H} \tag{B.17}$$
$$(1-\alpha_{LL})Dx - \alpha_{LL}x + \frac{\lambda_1}{\nu_H} = \frac{\lambda_3}{\nu_H}$$

Eliminating $\frac{\lambda_1}{\nu_H}$ gives

$$\alpha_{H\cdot}Dx + (\alpha_{HH} - x) = \frac{\lambda_3}{\nu_H}. \tag{B.18}$$

$\lambda_3 > 0$ then requires that

$$x < \frac{\alpha_{HH}}{1-\alpha_{H\cdot}D}.$$

where the positivity of the denominator is guaranteed by $D < \overline{D}$.

$\lambda_1 > 0$ requires that

$$x < \frac{\alpha_{HH}}{1-\alpha_{LL}+\alpha_{LH}D}.$$

We have that

$$\frac{\alpha_{HH}}{1-\alpha_{H.}D} \gtrless \frac{\alpha_{HH}}{1-\alpha_{LL}+\alpha_{LH}D}$$
$$\Updownarrow$$
$$D \gtrless \overline{D}_C$$

It can also be shown that[5]

$$\frac{1}{1+D} > \frac{\alpha_{HH}}{1-\alpha_{LL}+\alpha_{LH}D} \text{ if } D < \overline{D}$$

so that $x < \frac{1}{1+D}$ is a redundant constraint.

The maximal profit under this configuration is

$$\pi^{P2}(c_I = 1, c_{LL} = 1) = \frac{1}{2}\alpha_{HH}\frac{\nu_L}{x} = \frac{1}{2}\alpha_{HH}\nu_H.$$

I. Consider first the case where $D < \overline{D}_C$. This means that $\frac{\alpha_{HH}}{1-\alpha_{H.}D} < \frac{\alpha_{HH}}{1-\alpha_{LL}+\alpha_{LH}D}$. Then for any feasible $x \leq \frac{\alpha_{HH}}{1-\alpha_{H.}D}$ we have $x < \frac{\alpha_{HH}}{1-\alpha_{LL}+\alpha_{LH}D}$. This means that $\lambda_1 > 0$ and that the constraint $c_I \leq c_{LL}$ is strictly binding.

I.a. If $D \leq \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C$, then

$$\pi^{P2}(c_I = c_{LL} = 1) \gtrless \pi^{P2}(c_I = c_{LL} = D\frac{x\alpha_{H.}}{x-\alpha_{HH}})$$
$$\Updownarrow$$
$$x \lessgtr \frac{\alpha_{HH}}{1-\alpha_{H.}D}$$

Summary:

$$c_I = c_{LL} = 1$$
$$x < \frac{\alpha_{HH}}{1-\alpha_{H.}D}$$
$$D < \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C$$

[5] Since

$$\frac{1}{1+D} \gtrless \frac{\alpha_{HH}}{1-\alpha_{LL}+\alpha_{LH}D}$$
$$\Updownarrow$$
$$(\alpha_{HH}-\alpha_{LH})D \lessgtr \alpha_{LH}+\alpha_{HL}$$

If $\alpha_{HH}-\alpha_{LH} < 0$, it obviously follows that $\frac{1}{1+D} > \frac{\alpha_{HH}}{1-\alpha_{LL}+\alpha_{LH}D}$. Suppose then that $\alpha_{HH}-\alpha_{LH} > 0$. Then $\frac{1}{1+D} > \frac{\alpha_{HH}}{1-\alpha_{LL}+\alpha_{LH}D}$ is equivalent with $D < \frac{\alpha_{LH}+\alpha_{HL}}{\alpha_{HH}-\alpha_{LH}}$. In the previous footnote, we showed that under Assumption N, $\rho \leq 0$ is a sufficient condition for $\frac{1}{1+D} > \frac{\alpha_{HH}}{1-\alpha_{LL}+\alpha_{LH}D}$.

I.b. If $D > \frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C$, then

$$\pi^{P2}(c_I = c_{LL} = 1) \gtrless \pi^{P2}(c_I = 0, c_{LL} = D\frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}})$$
$$\Updownarrow$$
$$x \lessgtr f_{P2.1}(D)$$

Summary:

$$c_I = c_{LL} = 1$$
$$x < f_{P2.1}(D)$$
$$\frac{\alpha_{HH}}{1-\alpha_{LL}}\overline{D}_C < D < \overline{D}_C$$

II. Consider now the case where $D > \overline{D}_C$. This means that $\frac{\alpha_{HH}}{1-\alpha_{H\cdot}D} > \frac{\alpha_{HH}}{1-\alpha_{LL}+\alpha_{LH}D}$. Then for any feasible $x \leq \frac{\alpha_{HH}}{1-\alpha_{LL}+\alpha_{LH}D}$ we have $x < \frac{\alpha_{HH}}{1-\alpha_{H\cdot}D}$. This means that $\lambda_3 > 0$ and that the constraint $c_{LL} \leq 1$ is strictly binding. Notice that in (B.17), $[\alpha_{HH} - (1-\alpha_{LL})x]$ is the coefficient with c_I where the latter is evaluated at 1. If $x < \frac{\alpha_{HH}}{1-\alpha_{LL}}$, profit is convex in c_I.

II.a. If $x < \frac{1}{1+2D}$, the alternative choice for c_I is then not 1 but the lower bound 0. That menu yields a maximal profit of

$$\pi_{tot}^{P2}|_{(c_I=0, c_{LL}=1)} = \nu_L \left[\frac{1}{2}(1-\alpha_{LL}) + \alpha_{LH}D\right]$$

Under configuration P2.2, it was established that $\pi^{P2}(c_I = 1, c_{LL} = 1) > \pi_{tot}^{P2}|_{(c_I=0, c_{LL}=1)}$ iff $x < \frac{\alpha_{HH}}{1-\alpha_{LL}+2\alpha_{LH}D}$. That configuration had $\frac{1}{1+2D}$ as upper bound on x. Since $\frac{\alpha_{HH}}{1-\alpha_{LL}+2\alpha_{LH}D} < \frac{\alpha_{HH}}{1-\alpha_{LL}+\alpha_{LH}D}$, and since $\frac{1}{1+2D} > \frac{\alpha_{HH}}{1-\alpha_{LL}+\alpha_{LH}D}$ iff $D < \frac{1-\alpha_{LL}-\alpha_{HH}}{2(\alpha_{HH}-\alpha_{LH})}$, we can **summarise** as:

$$c_I = c_{LL} = 1$$
$$x < \min\{\frac{\alpha_{HH}}{1-\alpha_{H\cdot}D}, \frac{\alpha_{HH}}{1-\alpha_{LL}+2\alpha_{LH}D}\}$$
$$\overline{D}_C < D < \frac{1-\alpha_{LL}-\alpha_{HH}}{2(\alpha_{HH}-\alpha_{LH})}$$

(Note that $\frac{\alpha_{HH}}{1-\alpha_{LL}+2\alpha_{LH}D} = f_{P2.1}(\overline{D}_C)$.)

II.b. If $x > \frac{1}{1+2D}$, the alternative choice for c_I is then not 1 but the lower bound $2\frac{Dx}{1-x} - 1 > 0$. This menu yields a maximal profit of

$$\pi_{tot}^{P2}|_{(c_I=2\frac{Dx}{1-x}-1, c_{LL}=1)} = \nu_L \left\{ \frac{1}{2}\alpha_{HH}\frac{1}{x} - \left[2\left(\alpha_{HH}-\alpha_{LH}\right) - 2\alpha_{H.}\right] xD + \left[\frac{\alpha_{HH}-\alpha_{LH}}{x} - (1-\alpha_{LL})\right] \frac{2D^2x^2}{(1-x)^2} \right\}$$

We then have that

$$\pi_{tot}^{P2}|_{(c_I=1, c_{LL}=1)} \gtrless \pi_{tot}^{P2}|_{(c_I=2\frac{Dx}{1-x}-1, c_{LL}=1)}$$
$$\Updownarrow$$
$$h_{P2.3}(x) \gtrless 0$$

where $h_{P2.3}(x)$ was defined in the discussion of configuration P2.4.a. That configuration has $\frac{1}{1+2D}$ as lower bound on x. $h_{P2.3}(x)$ is a convex quadratic form in x with lower root $\frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H.}}$ and upper root $\frac{1}{1+D}$.

Hence, $\pi_{tot}^{P2}|_{(c_I=1, c_{LL}=1)} > \pi_{tot}^{P2}|_{(c_I=2\frac{Dx}{1-x}-1, c_{LL}=1)}$ iff $x < \frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H.}}$. For this to be compatible with $x > \frac{1}{1+2D}$, we need

$$D > \frac{1-\alpha_{LL}-\alpha_{HH}}{2(\alpha_{HH}-\alpha_{LH})}$$

Summary:

$$c_I = c_{LL} = 1$$
$$\frac{1}{1+2D} < x < \frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H.}}$$
$$D > \frac{1-\alpha_{LL}-\alpha_{HH}}{2(\alpha_{HH}-\alpha_{LH})}$$

This menu was earlier defined as auxiliary menu **PX**. We summarise it as:

$$c_I^{PX} = c_{LL}^{PX} = 1$$
$$x < \min\{\frac{\alpha_{HH}}{1-\alpha_{H.}D}, f_{P2.1}(D)\} \text{ if } D < \overline{D}_C$$
$$x < \frac{\alpha_{HH}}{1-\alpha_{LL}+2\alpha_{LH}D} \text{ if } D > \overline{D}_C \text{ and } \alpha_{HH} \leq \alpha_{LH}$$
$$x < \max\{\frac{\alpha_{HH}}{1-\alpha_{LL}+2\alpha_{LH}D}, \frac{\alpha_{HH}-\alpha_{LH}}{\alpha_{H.}}\} \text{ if } D > \overline{D}_C \text{ and } \alpha_{HH} > \alpha_{LH}$$
$$D < \overline{D}$$

P2.7. $\lambda_1 > 0, \lambda_2 > 0, \lambda_3 = 0$. $\lambda_1 > 0$ means that $c_I = c_{LL}$. Again, we call this common coinsurance rate c_P. Since $\lambda_2 > 0$, $c_P = \frac{Dx}{1-x}$. $\lambda_3 = 0$ then requires that $\frac{Dx}{1-x} \leq 1$, or

$$x \leq \frac{1}{1+D}.$$

The first order conditions (B.7) and (B.6) become

$$\begin{aligned}[\alpha_{HH} - (1-\alpha_{LL})x]\frac{Dx}{1-x} - \alpha_{LH}Dx &= \frac{\lambda_1}{\nu_H} - \frac{\lambda_2}{\nu_H} \\ (1-\alpha_{LL})Dx - \alpha_{LL}x\frac{Dx}{1-x} &= -\frac{\lambda_1}{\nu_H} - \frac{\lambda_2}{\nu_H}\end{aligned}$$

Adding up and rearranging gives

$$\frac{1}{2}\left[(\alpha_{HH} + \alpha_{H\cdot}) - (1+\alpha_{H\cdot})x\right]\frac{Dx}{1-x} = -\frac{\lambda_2}{\nu_H}.$$

$\lambda_2 > 0$ then requires that

$$x > \frac{\alpha_{HH} + \alpha_{H\cdot}}{1+\alpha_{H\cdot}}.$$

Substituting out $\frac{\lambda_2}{\nu_H}$ in one of the first order conditions then gives

$$\left[-\frac{1}{2}\alpha_{HL} - \alpha_{LH} + \frac{1}{2}(1-\alpha_{H\cdot})x\right]\frac{Dx}{1-x} = \frac{\lambda_1}{\nu_H}$$

$\lambda_1 > 0$ then requires that

$$x > \frac{2\alpha_{LH} + \alpha_{HL}}{1-\alpha_{H\cdot}}$$

Since $\rho \leq 0$ is a sufficient condition for $\frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H\cdot}} > \frac{\alpha_{HH}+\alpha_{H\cdot}}{1+\alpha_{H\cdot}}$, the relevant constraint is $x > \frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H\cdot}}$.

For $x > \frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H\cdot}}$ to be compatible with $x < \frac{1}{1+D}$, we need

$$D < \frac{\alpha_{LL} - \alpha_{HL} - \alpha_{LH}}{2\alpha_{LH} + \alpha_{HL}}.$$

This menu was earlier defined as the auxiliary menu $\frac{\Delta\mu}{\Delta\nu}$. We summarise it as

$$c_I^{P\frac{\Delta\mu}{\Delta\nu}} = c_{LL}^{P\frac{\Delta\mu}{\Delta\nu}} = \frac{Dx}{1-x}$$
$$\frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H\cdot}} < x < \frac{1}{1+D}$$
$$0 < D < \frac{\alpha_{LL}-\alpha_{HL}-\alpha_{LH}}{2\alpha_{LH}+\alpha_{HL}}$$

Remark: this configuration ceases to exist if $\frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H\cdot}} > 1 \iff \frac{\alpha_{LL}-\alpha_{HL}-\alpha_{LH}}{2\alpha_{LH}+\alpha_{HL}} < 0$.

P2.8. $\lambda_1 > 0, \lambda_2 > 0, \lambda_3 > 0$**.** This can be considered as an unimportant knife-edge case.

B.3 Sub-problem 3

The Lagrangian for this sub-problem can be written as

$$\mathcal{L}_{P3} = \pi_{tot}^{P3} + \lambda_1\left\{c_{LH} + c_{\cdot L} - 2\frac{Dx}{1-x}\right\} + \lambda_2\left\{c_{\cdot L} - c_{LH}\right\} + \mu\left\{1 - c_{\cdot L}\right\}$$

The first derivatives w.r.t. $c_{\cdot L}$ and c_{LH} are,

$$\frac{\partial\mathcal{L}_{P3}}{\partial c_{\cdot L}} = \alpha_{HL}Dx + (\alpha_{\cdot H} - x)c_{\cdot L} + \frac{\lambda_1}{v_H} + \frac{\lambda_2}{v_H} - \frac{\mu}{v_H} = 0 \tag{B.19}$$
$$\frac{\partial\mathcal{L}_{P3}}{\partial c_{LH}} = \alpha_{HH}Dx - \alpha_{LH}c_{LH} + \frac{\lambda_1}{v_H} - \frac{\lambda_2}{v_H} \leq 0,\ c_{LH} \geq 0, c_{LH}\cdot\frac{\partial\mathcal{L}_{P3}}{\partial c_{LH}} = 0 \tag{B.20}$$

P3.1. $\lambda_1 = 0, \lambda_2 = 0, \mu = 0$**.** Then $\frac{\partial^2\mathcal{L}_{P3}}{\partial c_{\cdot L}^2} = (\alpha_{\cdot H} - x)\nu_H$.. If $x > \alpha_{\cdot H}$ then π_{tot}^{P3} is strictly concave in $c_{\cdot L}$.and its optimal value is

$$c_{\cdot L} = D\frac{x\alpha_{HL}}{x-\alpha_{\cdot H}}.$$

For $\mu = 0$ we need $c_{\cdot L} \leq 1$ or

$$x \geq \frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}.$$

(Note that $1-\alpha_{HL}D>0$ because by the restriction that $D<\overline{D}$.)

If $x<\alpha_{\cdot H}$ then π^E_{tot} is strictly increasing and convex in $c_{\cdot L}$ whose optimal value is $c_{\cdot L}=1$, contradicting $\mu=0$.

From (B.20) we have that $\frac{\partial^2 \mathcal{L}_{P3}}{\partial c^2_{LH}}=-\alpha_{LH}\nu_H<0$, so that

$$c_{LH}=D\frac{\alpha_{HH}x}{\alpha_{LH}}.$$

For $\lambda_1=0$, we need $2\frac{Dx}{1-x}-c_{\cdot L}\leq c_{LH}$, which translates into

$$f_{P3.1}(x)\stackrel{\text{def}}{=}\alpha_{HH}x^2+[\alpha_{LH}(2+\alpha_{HL})-(1+\alpha_{\cdot H})\alpha_{HH}]x+[\alpha_{\cdot H}\alpha_{HH}-\alpha_{LH}(2\alpha_{\cdot H}+\alpha_{HL})]\leq 0$$

It cannever be a global solution to the main problem to have this inequality constraint binding. The reason is that profits could unambiguously be increased by lowering the coinsurance rate c_{HL} down from $c_{\cdot L}$ to c_{LH} without changing any of the incentive compatibility constraints (cf Lemma 15). The convex quadratic form $f_{P3.1}(x)=0$ has two roots, $\underline{x}_{f_{P3.1}}$ and $\overline{x}_{f_{P3.1}}$ so that the necessary requirement is that

$$\underline{x}_{f_{P3.1}}\leq x\leq\overline{x}_{f_{P3.1}}. \tag{B.21}$$

It can be shown that $\underline{x}_{f_{P3.1}}(\alpha_{H\cdot},\alpha_{HH},\rho)<\alpha_{\cdot H}$ for any $\rho\leq\overline{\rho}$. Hence, $\underline{x}_{f_{P3.1}}$ as a lower bound on x is made redundant by the condition $x\geq\frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}$.

For $\lambda_2=0$, we need $c_{LH}\leq c_{\cdot L}$, which translates into

$$x\leq\alpha_{\cdot H}+\frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}}. \tag{B.22}$$

Moreover, for (λ_1) and (λ_2) to be compatible, we need $2\frac{Dx}{1-x}-c_{\cdot L}\leq c_{\cdot L}$, or $\frac{Dx}{1-x}\leq c_{\cdot L}$. This translates into

$$x\leq\frac{1-\alpha_{LL}}{1+\alpha_{HL}}. \tag{B.23}$$

We have that

$$\alpha_{\cdot H}+\frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}}\gtrless\frac{1-\alpha_{LL}}{1+\alpha_{HL}}\gtrless\overline{x}_{f_{P3.1}}$$
$$\Updownarrow$$
$$\rho\lesseqgtr\rho_E\stackrel{\text{def}}{=}\alpha_{HH}\frac{1-\alpha_{H\cdot}(1+\alpha_{HL})}{1+\alpha_{H\cdot}}$$

So the upper bound $\frac{1-\alpha_{LL}}{1+\alpha_{HL}}$ on x is always redundant.

For $x \geq \frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}$ to be compatible with $x \leq \min\{\alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}}, \overline{x}_{f_{P3.1}}\}$ we need

$$D \leq \min\{\frac{\alpha_{LH}}{\alpha_{HH}^2 + \alpha_{LH}\alpha_{H\cdot}}, \frac{\overline{x}_{f_{P3.1}} - \alpha_{\cdot H}}{\alpha_{HL}\overline{x}_{f_{P3.1}}}\}$$

Note that

$$\frac{\alpha_{LH}}{\alpha_{HH}^2 + \alpha_{LH}\alpha_{H\cdot}} \lesseqgtr \frac{\overline{x}_{f_{P3.1}} - \alpha_{\cdot H}}{\alpha_{HL}\overline{x}_{f_{P3.1}}} \iff \rho \gtreqless \rho_E$$

Since $\rho \leq 0$ is sufficient for $\frac{\alpha_{LH}}{\alpha_{HH}^2+\alpha_{LH}\alpha_{H\cdot}} > \overline{D}$, we can summarise as follows:

We call this menu **EI** and summarise it as

$$c_{\cdot L}^{EI} = D\frac{x\alpha_{HL}}{x - \alpha_{\cdot H}}, c_{LH}^{EI} = D\frac{\alpha_{HH}x}{\alpha_{LH}}$$
$$\frac{\alpha_{\cdot H}}{1 - \alpha_{HL}D} \leq x \leq \min\{\alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}}, \overline{x}_{f_{P3.1}}\}$$
$$D \leq \min\{\overline{D}, \frac{\overline{x}_{f_{P3.1}} - \alpha_{\cdot H}}{\alpha_{HL}\overline{x}_{f_{P3.1}}}\}$$

For this configuration, the maximal profit is given by

$$\pi_{tot}^{EI} = \pi_{tot}^{P3}|_{(c_{LH}=D\frac{\alpha_{HH}}{\alpha_{LH}}x, c_{\cdot L}=D\frac{x\alpha_{HL}}{x-\alpha_{\cdot H}})} = \nu_L\left\{\frac{1}{2} - \alpha_{H\cdot}D + \frac{1}{2}D^2x\left(\frac{\alpha_{HH}^2}{\alpha_{LH}} + \frac{\alpha_{HL}^2}{x - \alpha_{\cdot H}}\right)\right\} \tag{B.24}$$

It can be shown that π_{tot}^{EI} is strictly decreasing and convex in x.

__**P3.2.** $\lambda_1 = 0, \lambda_2 = 0, \mu > 0$__. $\mu > 0$ means $c_{\cdot L} = 1$. The FOCs then become

$$\alpha_{HL}Dx + (\alpha_{\cdot H} - x) = \frac{\mu}{v_H} > 0$$
$$\alpha_{HH}Dx - \alpha_{LH}c_{LH} \leq 0,\ c_{LH} \geq 0, c_{LH} \cdot (\alpha_{HH}Dx - \alpha_{LH}c_{LH}) = 0$$

Thus

$$c_{LH} = D\frac{\alpha_{HH}x}{\alpha_{LH}}.$$

$\mu > 0$ requires that

$$x < \frac{\alpha_{\cdot H}}{1 - \alpha_{HL}D}$$

$\lambda_2 = 0$ requires that $c_{LH} \leq 1$ or

$$x \leq \frac{\alpha_{LH}}{D\alpha_{HH}}.$$

We have that

$$\frac{\alpha_{\cdot H}}{1-\alpha_{HL}D} \gtrless \alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}} \gtrless \frac{\alpha_{LH}}{D\alpha_{HH}}$$
$$\Updownarrow$$
$$D \gtrless \frac{\alpha_{LH}}{\alpha_{HH}^2 + \alpha_{LH}\alpha_{H\cdot}}.$$

Since $\rho \leq 0$ is sufficient for $\overline{D} < \frac{\alpha_{LH}}{\alpha_{HH}^2+\alpha_{LH}\alpha_{H\cdot}}$, $D \leq \overline{D}$, implies that $\frac{\alpha_{\cdot H}}{1-\alpha_{HL}D} < \alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}} < \frac{\alpha_{LH}}{D\alpha_{HH}}$, so that the relevant upper bound so far is $\frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}$.

$\lambda_1 = 0$ requires that $2\frac{\Delta\mu}{\Delta\nu} - 1 \leq c_{LH}$ or

$$D\frac{\alpha_{HH}x}{\alpha_{LH}} \geq 2\frac{Dx}{1-x} - 1,$$

which is a quadratic inequality in x:

$$f_{P3.2}(x) \stackrel{\text{def}}{=} -\alpha_{HH}Dx^2 + (\alpha_{HH}D - 2\alpha_{LH}D - \alpha_{LH})\,x + \alpha_{LH} \geq 0.$$

$f_{P3.2}(x)$ is a concave function with $f_{P3.2}(0) = \alpha_{LH} > 0$ and $f_{P3.2}(1) = -2\alpha_{LH}D < 0$. Hence we must have that x does not exceed the upper root:

$$x \leq \overline{x}_{f_{P3.2}}(\alpha_{HH}, \alpha_{LH}, D).$$

It can be shown that $\lim_{D\to 0}\overline{x}_{f_{P3.2}}(\alpha_{HH}, \alpha_{LH}, D) = 1$, that $\overline{x}_{f_{P3.2}}(\alpha_{HH}, \alpha_{LH}, D)$ falls in D.

Claim: If $\rho > \rho_E$, then $\overline{x}_{f_{P3.2}}(\alpha_{HH}, \alpha_{LH}, D) > \frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}$ for all $D < \overline{D}$.

Proof of claim: Since $\rho_E > \alpha_{HH}(1 - 2\alpha_{H\cdot})$, it follows that $\rho > \alpha_{HH}(1 - 2\alpha_{H\cdot})$ and therefore that $\alpha_{HH} > \alpha_{LH}$. But since

$$\frac{\alpha_{LH}}{\alpha_{HH}D} \gtrless \overline{x}_{f_{P3.2}}(\alpha_{HH}, \alpha_{LH}, D)$$
$$\Updownarrow$$
$$(\alpha_{HH} - \alpha_{LH})\,D \lesseqgtr \alpha_{LH}$$

It follows that

$$\frac{\alpha_{LH}}{\alpha_{HH}D} \gtrless \overline{x}_{f_{P3.2}}(\alpha_{HH}, \alpha_{LH}, D)$$
$$\Updownarrow$$
$$D \lesseqgtr \frac{\alpha_{LH}}{\alpha_{HH} - \alpha_{LH}}$$

Let us now evaluate both $\frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}$ and $\frac{\alpha_{LH}}{\alpha_{HH}D}$ at $D = \frac{\alpha_{LH}}{\alpha_{HH}-\alpha_{LH}}$. Then

$$\frac{\alpha_{LH}}{\alpha_{HH}D}|_{D=\frac{\alpha_{LH}}{\alpha_{HH}-\alpha_{LH}}} \gtrless \frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}|_{D=\frac{\alpha_{LH}}{\alpha_{HH}-\alpha_{LH}}}$$
$$\Updownarrow$$
$$\rho \gtrless \rho_E.$$

Since $\rho > \rho_E$ by assumption, we have that $\overline{x}_{f_{P3.2}}(\alpha_{HH}, \alpha_{LH}, D)|_{D=\frac{\alpha_{LH}}{\alpha_{HH}-\alpha_{LH}}} = \frac{\alpha_{LH}}{\alpha_{HH}D}|_{D=\frac{\alpha_{LH}}{\alpha_{HH}-\alpha_{LH}}} > \frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}|_{D=\frac{\alpha_{LH}}{\alpha_{HH}-\alpha_{LH}}}$. Since $\overline{x}_{f_{P3.2}}(\alpha_{HH}, \alpha_{LH}, D)$ is decreasing in D while $\frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}$ is increasing in D, it follows that $\overline{x}_{f_{P3.2}}(\alpha_{HH}, \alpha_{LH}, D) > \frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}$ for all $D < \frac{\alpha_{LH}}{\alpha_{HH}-\alpha_{LH}}$. Because $\overline{D} < \frac{\alpha_{LH}}{\alpha_{HH}-\alpha_{LH}}$ when $\rho \leq 0$, it follows that $\overline{x}_{f_{P3.2}}(\alpha_{HH}, \alpha_{LH}, D) > \frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}$ for all $D < \overline{D}$. This menu was defined as menu **EX** in the main proposition. We summarise it as

$$\begin{aligned}
c_{LH}^{EX} &= D\frac{\alpha_{HH}x}{\alpha_{LH}}, c_{\cdot L}^{EX} = 1 \\
x &\leq \frac{\alpha_{\cdot H}}{1-\alpha_{HL}D} \text{ if } \rho > \rho_E \\
x &\leq \min\{\frac{\alpha_{\cdot H}}{1-\alpha_{HL}D}, \overline{x}_{f_{P3.2}}(\alpha_{HH}, \alpha_{LH}, D)\} \text{ if } \rho < \rho_E \\
D &< \overline{D}
\end{aligned}$$

For this configuration, the maximal profit is given by

$$\pi_{tot}^{EX} = \pi_{tot}^{P3}|_{(c_{LH}=D\frac{\alpha_{HH}}{\alpha_{LH}}x, c_{\cdot L}=1)} = v_L\left[\frac{1}{2} - \alpha_{HH}D + \frac{1}{2}D^2\frac{x\alpha_{HH}^2}{\alpha_{LH}} + \frac{1}{2}\frac{\alpha_{HH}+\alpha_{LH}-x}{x}\right] \tag{B.25}$$

and we note that is strictly decreasing and convex in x independent of x.

P3.3. $\lambda_1 = 0, \lambda_2 > 0, \mu = 0$. $\lambda_2 > 0$ means that $c_{LH} = c_{\cdot L}$. We call this common coinsurance rate c_P. The FOCs then become

$$\alpha_{HL}Dx + (\alpha_{\cdot H} - x)c_P + \frac{\lambda_2}{v_H} = 0$$
$$\alpha_{HH}Dx - \alpha_{LH}c_P - \frac{\lambda_2}{v_H} \leq 0,\ c_P \geq 0, c_P \cdot \left(\alpha_{HH}Dx - \alpha_{LH}c_P - \frac{\lambda_2}{v_H}\right) = 0 \tag{B.26}$$

From the first FOC, $\lambda_2 > 0$ requires that $x > \alpha_{\cdot H}$ and $c_P > 0$. Hence, the second FOC holds with equality.

Eliminating $\frac{\lambda_2}{v_H}$ gives

$$\alpha_{H.}Dx + (\alpha_{HH} - x)c_P = 0$$

Since $x > \alpha_{.H} > \alpha_{HH}$, profit is strictly concave in c_P. Then

$$c_P = D\frac{\alpha_{H.}x}{x - \alpha_{HH}}.$$

$\lambda_2 > 0$ then requires that

$$x > \alpha_{.H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}}.$$

$\mu = 0$ requires that $c_P \leq 1$ or

$$x \geq \frac{\alpha_{HH}}{1 - D\alpha_{H.}}$$

Note that

$$\frac{\alpha_{HH}}{1 - D\alpha_{H.}} \gtrless \alpha_{.H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}}$$

$$\Updownarrow$$

$$D \gtrless \frac{\alpha_{LH}}{\alpha_{HH}^2 + \alpha_{LH}\alpha_{H.}} (> \overline{D})$$

Hence, the relevant lower bound is $\alpha_{.H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}}$.
$\lambda_1 = 0$ requires that $c_P \geq \frac{Dx}{1-x}$ or

$$x \leq \frac{\alpha_{H.} + \alpha_{HH}}{1 + \alpha_{H.}}$$

For this to be compatible with $x \geq \frac{\alpha_{HH}}{1-D\alpha_{H.}}$, we need that

$$D < \frac{1 - \alpha_{HH}}{\alpha_{H.} + \alpha_{HH}}$$

Finally, for $x \leq \frac{\alpha_{H.}+\alpha_{HH}}{1+\alpha_{H.}}$ to be compatible with $x > \alpha_{.H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}}$ we need

$$\rho > \rho_E.$$

Note that since we assume $\rho \leq 0$, the latter requirement requires that

$$\begin{gathered}\rho_E < 0 \\ \Updownarrow \\ 1 < \alpha_{H\cdot}(1 + \alpha_{H\cdot} - \alpha_{HH}) \\ \Updownarrow \\ \alpha_{H\cdot}^2 > 1 - \alpha_{H\cdot}(1 - \alpha_{HH}) \\ \Updownarrow \\ \alpha_{H\cdot}(1 - \alpha_{HH}) > 1 - \alpha_{H\cdot}^2 = (1 - \alpha_{H\cdot})(1 + \alpha_{H\cdot}) \\ \Updownarrow \\ \frac{1 - \alpha_{HH}}{1 + \alpha_{H\cdot}} > \frac{1 - \alpha_{H\cdot}}{\alpha_{H\cdot}} = \overline{D}\end{gathered}$$

Since $\frac{1-\alpha_{HH}}{\alpha_{H\cdot}+\alpha_{HH}} > \frac{1-\alpha_{HH}}{1+\alpha_{H\cdot}}$, it follows that $\frac{1-\alpha_{HH}}{\alpha_{H\cdot}+\alpha_{HH}} > \overline{D}$. Hence, the relevant upper bound on D is $\overline{D}$.

This menu corresponds to the auxiliary menu **PI**. We summarise it as

$$\begin{gathered}c_{LH}^{PI} = c_{\cdot L}^{PI} = D\frac{\alpha_{H\cdot}x}{x - \alpha_{HH}} \\ \alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}} \leq x \leq \frac{\alpha_{H\cdot} + \alpha_{HH}}{1 + \alpha_{H\cdot}} \\ D < \overline{D} \\ \rho_E < \rho \leq 0\end{gathered}$$

P3.4. $\lambda_1 = 0, \lambda_2 > 0, \mu > 0$**.** $\lambda_2 > 0$ means that $c_{LH} = c_{\cdot L}$. Moreover, $\mu > 0$ means $c_{LH} = c_{\cdot L} = 1$.

The FOCs then become

$$\begin{aligned}\alpha_{HL}Dx + (\alpha_{\cdot H} - x) &= \frac{\mu}{v_H} - \frac{\lambda_2}{v_H} \\ \alpha_{HH}Dx - \alpha_{LH} &= \frac{\lambda_2}{v_H}\end{aligned}$$

$\lambda_2 > 0$ then requires that

$$x > \frac{\alpha_{LH}}{\alpha_{HH}D}.$$

Adding up the two FOCS gives

$$\alpha_{H.}Dx + (\alpha_{HH} - x) = \frac{\mu}{v_H}$$

$\mu > 0$ then requires that

$$x < \frac{\alpha_{HH}}{1 - \alpha_{H.}D}.$$

For this to be compatible with $x > \frac{\alpha_{LH}}{\alpha_{HH}D}$ we need

$$D > \frac{\alpha_{LH}}{\alpha_{HH}^2 + \alpha_{LH}\alpha_{H.}}$$

But if $\rho \leq 0$, this is incompatible with $D < \overline{D}$.

Finally, $\lambda_1 = 0$ requires that $1 \geq \frac{Dx}{1-x}$ or

$$x \leq \frac{1}{1+D}.$$

For this to be compatible with $x > \frac{\alpha_{LH}}{\alpha_{HH}D}$ we need

$$D(\alpha_{HH} - \alpha_{LH}) > \alpha_{LH}$$

requiring that

$$\alpha_{HH} > \alpha_{LH}, \text{ and}$$
$$D > \frac{\alpha_{LH}}{\alpha_{HH} - \alpha_{LH}}$$

Again, if $\rho \leq 0$, this is incompatible with $D < \overline{D}$.

P3.5. $\lambda_1 > 0, \lambda_2 = 0, \mu = 0$. $\lambda_1 > 0$ means that $c_{LH} = 2\frac{Dx}{1-x} - c_{.L}$. $\lambda_2 = 0$ means that $c_{LH} \leq c_{.L}$. The FOCs then become

$$\frac{\partial \mathcal{L}_{P3}}{\partial c_{.L}} = \alpha_{HL}Dx + (\alpha_{.H} - x)c_{.L} + \frac{\lambda_1}{v_H} = 0$$
$$\frac{\partial \mathcal{L}_{P3}}{\partial c_{LH}} = \alpha_{HH}Dx - \alpha_{LH}c_{LH} + \frac{\lambda_1}{v_H} \leq 0, \; c_{LH} \geq 0, c_{LH} \cdot \frac{\partial \mathcal{L}_{P3}}{\partial c_{LH}} = 0$$

The first condition implies that $x > \alpha_{.H}$, for otherwise $\lambda_1 < 0$. For thes ame reason, the second condition implies that $c_{LH} > 0$. Hence the second

FOC must hold with equality. Replacing c_{LH} by $2\frac{Dx}{1-x} - c_{\cdot L}$ and solving the two conditions for $c_{\cdot L}$ and $\frac{\lambda_1}{v_H}$ gives:

$$c_{\cdot L} = D\frac{x}{1-x}\frac{(\alpha_{HL}-\alpha_{HH})(1-x)+2\alpha_{LH}}{x-\alpha_{HH}}$$
$$\frac{\lambda_1}{v_H} = D\frac{x}{1-x}\frac{\left[(\alpha_{HL}-\alpha_{HH})(1-x)+2\alpha_{LH}\right](x-\alpha_{\cdot H}) - \alpha_{HL}(1-x)(x-\alpha_{HH})}{(x-\alpha_{HH})}$$

For $\lambda_2 = 0$, we need that $c_{\cdot L} \geq \frac{Dx}{1-x}$ which requires that

$$x \leq \frac{2\alpha_{LH}+\alpha_{HL}}{1+\alpha_{HL}-\alpha_{HH}}.$$

The condition $\lambda_1 > 0$ is equivalent with $f_{P3.1}(x) > 0$, which we showed earlier to be equivalent with

$$x > \overline{x}_{f_{P3.1}}.$$

For this to be compatible with $x \leq \frac{2\alpha_{LH}+\alpha_{HL}}{1+\alpha_{HL}-\alpha_{HH}}$, we need that $\rho < \rho_E$ (cf Lemma C.10 in Appendix C).

The condition $\mu = 0$ means that $c_{\cdot L} \leq 1$ which is equivalent with

$$f_{P3.5}(x,D) \overset{\text{def}}{=} \left[1-(\alpha_{HL}-\alpha_{HH})D\right]x^2 + \left[D(\alpha_{HL}-\alpha_{HH}+2\alpha_{LH})-1-\alpha_{HH}\right]x + \alpha_{HH} \leq 0.$$

We have that $f_{P3.5}(0) = \alpha_{HH} > 0$ and $f_{P3.5}(1) = 2\alpha_{LH}D > 0$. Hence, $f_{P3.5}$ needs to be sufficiently convex in x for there to exist x-values that make $f_{P3.5}(x)$ negative. Comparison with $g_{P2.3}(x,D)$ shows that $f_{P3.5}(x,D) = g_{P2.3}(x,D) - 2\alpha_{HL}Dx^2$. Since $g_{P2.3}(x,D)$ is convex in x with roots $\overline{x}_{g_{P2.3}}(D)$ and $\underline{x}_{g_{P2.3}}(D)$, it follows that the roots for $f_{P3.5}(x,D)$, $\overline{x}_{f_{P3.5}}(D)$ and $\underline{x}_{f_{P3.5}}(D)$, must satisfy $\overline{x}_{f_{P3.5}}(D) > \overline{x}_{g_{P2.3}}(D)$ and $\underline{x}_{f_{P3.5}}(D) < \underline{x}_{g_{P2.3}}(D)$.

This menu cooresponds to the auxiliary menu **SUBI**. Necessary conditions are

$$c_{\cdot L}^{SUBI} = \frac{Dx}{1-x}\frac{(\alpha_{HL}-\alpha_{HH})(1-x)+2\alpha_{LH}}{x-\alpha_{HH}}$$
$$c_{LH}^{SUBI} = 2\frac{Dx}{1-x} - c_{\cdot L}, c_{HH}^{SUBI} = 0$$
$$\overline{x}_{f_{P3.1}} < x \leq \min\{\frac{2\alpha_{LH}+\alpha_{HL}}{1+\alpha_{HL}-\alpha_{HH}}, \overline{x}_{f_{P3.5}}(D)\}$$
$$0 < D < \frac{\overline{x}_{f_{P3.1}} - \alpha_{\cdot H}}{\alpha_{HL}\overline{x}_{f_{P3.1}}}$$
$$\rho < \rho_E$$

Note that this configuration will never constitute a global optimum, since it would pay off to pool HL with LH rather than with LL (cf Lemma 15).

P3.6. $\underline{\lambda_1 > 0, \lambda_2 = 0, \mu > 0}$**.** $\lambda_1 > 0$ means that $c_{LH} = 2\frac{Dx}{1-x} - c_{\cdot L}$. $\lambda_2 = 0$ means that $c_{LH} \leq c_{\cdot L}$. $\mu > 0$ means that $c_{\cdot L} = 1$. Hence, $c_{LH} = 2\frac{Dx}{1-x} - 1$. The FOCs then become

$$\frac{\partial \mathcal{L}_{P3}}{\partial c_{\cdot L}} = \alpha_{HL} Dx + (\alpha_{\cdot H} - x) + \frac{\lambda_1}{v_H} - \frac{\mu}{v_H} = 0$$
$$\frac{\partial \mathcal{L}_{P3}}{\partial c_{LH}} = \alpha_{HH} Dx - \alpha_{LH}\left(2\frac{Dx}{1-x} - 1\right) + \frac{\lambda_1}{v_H} \leq 0,\ c_{LH} \geq 0, c_{LH} \cdot \frac{\partial \mathcal{L}_{P3}}{\partial c_{LH}} = 0$$

Again, we must have that $c_{LH} = 2\frac{Dx}{1-x} - 1 > 0$ ($= 0$ would contradict $\frac{\lambda_1}{v_H} > 0$) and therefore that the second FOC holds with eqaulity.

Solving the conditions for the two Lagrange multipliers gives

$$\frac{\mu}{v_H} = \frac{Dx}{1-x}\left[(\alpha_{HL} - \alpha_{HH})(1-x) + 2\alpha_{LH}\right] + \alpha_{HH} - x$$
$$\frac{\lambda_1}{v_H} = \frac{Dx}{1-x}\left[(\alpha_{HL} - \alpha_{HH})(1-x) + 2\alpha_{LH}\right] - (\alpha_{LH} + \alpha_{HL} Dx)$$

$\frac{\lambda_1}{v_H} > 0$ turns out to be equivalent with $f_{P3.2}(x) < 0$, which we showed earlier to be equivalent with

$$x > \overline{x}_{f_{P3.2}}$$

$c_{LH} = 2\frac{Dx}{1-x} - 1 > 0$ requires that

$$x > \frac{1}{1+2D}$$

$\lambda_2 = 0$ means that $c_{LH} \leq c_{\cdot L}$ which corresponds to

$$x < \frac{1}{1+D}$$

For this to be compatible with $\overline{x}_{f_{P3.2}} < x$, we need

$$(\alpha_{HH} - \alpha_{LH})\, D < \alpha_{LH}$$

This is always verified if $\rho \leq \alpha_{HH}(1 - 2\alpha_{H\cdot})$. If $\rho > \alpha_{HH}(1 - 2\alpha_{H\cdot})$, the condition above becomes

$$D < \frac{\alpha_{LH}}{\alpha_{HH} - \alpha_{LH}}$$

which can show to be always weaker than $D < \overline{D}$.

$\frac{\mu}{v_H} > 0$ can be shown to be equivalent with equivalent with $f_{P3.5}(x) > 0$, whic requires that $x > \overline{x}_{f_{P3.5}}(D)$. For this to be compatible with $x < \frac{1}{1+D}$, we need

$$\frac{1 - 2\alpha_{LH} - \alpha_{HH}}{2\alpha_{LH} + \alpha_{HL}} < D.$$

And for this to be compatible with $D < \frac{\alpha_{LH}}{\alpha_{HH} - \alpha_{LH}}$ we need $\rho < \rho_E$ (cf Lemma C.10 in appendix C).

This menu corresponds to the auxiliary menu **SUBX**. Necessary conditions are

$$c_{HH}^{SUBX} = 0, c_{LH}^{SUBX} = 2\frac{Dx}{1-x} - 1, c_{\cdot L}^{SUBX} = 1$$

$$\max\{\overline{x}_{f_{P3.2}}(D), \overline{x}_{f_{P3.5}}(D)\} < x < \frac{1}{1+D}$$

$$\frac{1 - 2\alpha_{LH} - \alpha_{HH}}{2\alpha_{LH} + \alpha_{HL}} < D < \overline{D}$$

$$\rho < \rho_E$$

Again, note that this configuration will never consitute a global optimum: pooling HL with LH rather than with LL would pay off (cf Lemma 15).

P3.7. $\lambda_1 > 0, \lambda_2 > 0, \mu = 0$**.** $\lambda_1 > 0$ means that $c_{LH} = 2\frac{Dx}{1-x} - c_{\cdot L}$. $\lambda_2 > 0$ means that $c_{LH} = c_{\cdot L}$. If we call this common coinsurance rate c_P then we have that

$$c_P = \frac{Dx}{1-x}.$$

$\mu = 0$ requires that

$$x \leq \frac{1}{1+D}.$$

The FOCs now become

$$\alpha_{HL}Dx + (\alpha_{\cdot H} - x)\frac{Dx}{1-x} + \frac{\lambda_1}{v_H} + \frac{\lambda_2}{v_H} = 0 \tag{B.27}$$

$$\alpha_{HH}Dx - \alpha_{LH}\frac{Dx}{1-x} + \frac{\lambda_1}{v_H} - \frac{\lambda_2}{v_H} = 0 \tag{B.28}$$

$\frac{\lambda_1}{v_H} + \frac{\lambda_2}{v_H} > 0$ means that

$$x > \frac{1 - \alpha_{LL}}{1 + \alpha_{HL}}.$$

Solving for $\frac{\lambda_1}{v_H}$ and $\frac{\lambda_2}{v_H}$ gives

$$\frac{\lambda_1}{v_H} = -\frac{1}{2}\left[\alpha_{H\cdot}Dx + (\alpha_{HH} - x)\frac{Dx}{1-x}\right]$$
$$\frac{\lambda_2}{v_H} = (\alpha_{HH} - \frac{1}{2}\alpha_{H\cdot})Dx - \left[\alpha_{LH} + \frac{1}{2}(\alpha_{HH} - x)\right]\frac{Dx}{1-x}$$

Then $\frac{\lambda_1}{v_H} > 0$ and $\frac{\lambda_2}{v_H} > 0$ requires that

$$x > \frac{\alpha_{HH} + \alpha_{H\cdot}}{1 + \alpha_{H\cdot}},$$
$$x > \frac{2\alpha_{LH} + \alpha_{HL}}{1 + \alpha_{HL} - \alpha_{HH}},$$

respectively.

Since $\rho < (>)\rho_E \implies \alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}} > (<)\frac{2\alpha_{LH}+\alpha_{HL}}{1+\alpha_{HL}-\alpha_{HH}} > (<)\frac{1-\alpha_{LL}}{1+\alpha_{HL}} > (<)\begin{cases} \overline{x}_{f_{P3.1}} \\ \frac{\alpha_{H\cdot}+\alpha_{HH}}{\alpha_{H\cdot}+1} \end{cases}$ (cf Lemma C.10 in Appendix C), we can ignore $\frac{1-\alpha_{LL}}{1+\alpha_{HL}}$ as a lower bound on x.

This menu correpsonds to the auxiliary menu $\mathbf{P}\frac{\Delta\mu}{\Delta\nu}$. We summarise it as

$$c_{HH}^{\mathbf{P}\frac{\Delta\mu}{\Delta\nu}} = 0, c_{LH}^{\mathbf{P}\frac{\Delta\mu}{\Delta\nu}} = c_{HL}^{\mathbf{P}\frac{\Delta\mu}{\Delta\nu}} = c_{LL}^{\mathbf{P}\frac{\Delta\mu}{\Delta\nu}} = \frac{Dx}{1-x}$$
$$\frac{\alpha_{HH} + \alpha_{H\cdot}}{1 + \alpha_{H\cdot}} < x < \frac{1}{1+D} \text{ if } \rho > \rho_E$$
$$\frac{2\alpha_{LH} + \alpha_{HL}}{1 + \alpha_{HL} - \alpha_{HH}} < x < \frac{1}{1+D} \text{ if } \rho < \rho_E$$
$$D < \begin{cases} \min\{\frac{1-\alpha_{HH}}{\alpha_{HH}+\alpha_{H\cdot}}, \overline{D}\} \text{ if } \rho > \rho_E \\ \min\{\frac{1-2\alpha_{LH}-\alpha_{HH}}{2\alpha_{LH}+\alpha_{HL}}, \overline{D}\} \text{ if } \rho > \rho_E \end{cases}$$

P3.8. $\lambda_1 > 0, \lambda_2 > 0, \mu > 0$. $\lambda_1 > 0$ means that $c_{LH} = 2\frac{Dx}{1-x} - c_{\cdot L}$. $\lambda_2 > 0$ means that $c_{LH} = c_{\cdot L}$. If we call this common coinsurance rate c_P then we have that$c_P = \frac{Dx}{1-x}$. $\mu > 0$ means that $c_P = 1$. This gives $x = \frac{1}{1+D}$, a knife-edge situation.

C Critical ρ-values and ranking of critical x-values

In this section we define a number of critical values for the covariance coefficient, ρ; whether ρ exceeds a critical value or not determines the sequence of treshold values for x.

Recall that, given α_{HH}, $\alpha_{H\cdot}$, and ρ the remaining parameters of the type distribution are given by

$$\alpha_{HL} = \alpha_{H\cdot} - \alpha_{HH} \tag{C.1}$$

$$\alpha_{LH} = \alpha_{HH}\frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}} - \frac{\rho}{\alpha_{H\cdot}}, \text{ and} \tag{C.2}$$

$$\alpha_{LL} = (\alpha_{H\cdot} - \alpha_{HH})\frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}} + \frac{\rho}{\alpha_{H\cdot}}. \tag{C.3}$$

Also recall the maximum and minimum feasible value for ρ which secure that neither α_{LH} nor α_{LL} become negative:

Definition C.1 $\overline{\rho} \overset{def}{=} \alpha_{HH}(1-\alpha_{H\cdot}) > 0$: *maximal feasible value for ρ;*

Definition C.2 $\underline{\rho} \overset{def}{=} -\alpha_{HL}(1-\alpha_{H\cdot}) < 0$: *minimal feasible value for ρ.*

Notice that the lowest possible value for $\underline{\rho}$ is $-\frac{1}{4}$ (when $\alpha_{HH} = 0$ and $\alpha_{H\cdot} = \frac{1}{2}$) and the highest possible value for $\overline{\rho}$ is $\frac{1}{4}$ (when $\alpha_{HH} = \frac{1}{2}$ and $\alpha_{H\cdot} = \frac{1}{2}$).

Next, we define the set of critical $\rho-$values and their properties.

Definition C.3 $\rho_1 \overset{def}{=} \alpha_{HH} - 2\alpha_{HH}\alpha_{H\cdot} + \frac{1}{2}\alpha_{H\cdot}^2(1+\alpha_{HH}),$

Lemma C.1 *$\rho_1 > 0$ for all $\alpha_{HH} \leq \alpha_{H\cdot} \leq 1$ and*

$$\rho > (<)\rho_1$$
$$\Downarrow$$
$$\frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H\cdot}} < (>) \frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+\alpha_{LL}} < (>)\alpha_{HH}$$

Definition C.4 $\rho_2 \overset{def}{=} \frac{\alpha_{HL}\alpha_{H\cdot}^2+\alpha_{HH}(1-\alpha_{H\cdot})}{1+\alpha_{H\cdot}}$

Lemma C.2 $\rho_2 > 0$ *for all* $\alpha_{HH} \leq \alpha_{H\cdot} \leq 1$*, and*

$$\rho > (<) \rho_2$$
$$\Downarrow$$
$$\frac{2\alpha_{LH} + \alpha_{HL}}{1 - \alpha_{H\cdot}} < (>) 1 - \alpha_{LL} < (>) \frac{\alpha_{H\cdot} + \alpha_{HH}}{\alpha_{H\cdot} + 1}$$
$$< (>) \frac{\alpha_{HH}(1 - \alpha_{LL})}{(1 - \alpha_{LL})^2 + \alpha_{LH}\alpha_{LL}} < (>) \frac{\alpha_{HH} - \alpha_{LH}}{\alpha_{H\cdot}}.$$

Definition C.5 $\rho_3 \stackrel{def}{=} \{(\alpha_{H\cdot} - \alpha_{HH})^2 + \alpha_{HH}(\alpha_{H\cdot}^2 - \alpha_{HH}\alpha_{H\cdot} + 1) + \frac{1}{2}\alpha_{H\cdot}^3 - \frac{1}{2}\alpha_{H\cdot}[4\alpha_{HH}^2 + 4\alpha_{H\cdot}^2 + \alpha_{H\cdot}^4]^{\frac{1}{2}}\}/(1 + \alpha_{HH})$,

Lemma C.3 $\rho_3 > 0$ *for all* $\alpha_{HH} < \alpha_{H\cdot} \leq 1$ *and*

$$\rho > (<) \rho_3 \Longrightarrow \frac{1 - \alpha_{LL}}{1 + a_{LL}} < (>) \frac{\alpha_{HH}(1 - \alpha_{LL})^2}{(1 - \alpha_{LL})^2 - \alpha_{H\cdot}^2\alpha_{LL}}$$

Definition C.6 $\rho_4 \stackrel{def}{=} \frac{2\alpha_{HH} + \alpha_{H\cdot}^3 - 3\alpha_{HH}\alpha_{H\cdot}}{2 + \alpha_{H\cdot}}$

Lemma C.4 $\rho_4 > 0$ *for all* $\alpha_{HH} \leq \alpha_{H\cdot} \leq 1$ *and*

$$\rho > (<) \rho_4 \Longrightarrow \frac{\alpha_{HH} - \alpha_{LH}}{\alpha_{H\cdot}} > (<) \frac{1 + \alpha_{LH} - \alpha_{LL}}{1 + \alpha_{LH} + \alpha_{LL}}$$

Definition C.7 $\rho_6 \stackrel{def}{=} \alpha_{HL}\alpha_{H\cdot}$

Lemma C.5 $\rho_6 > 0$ *and*

$$\rho \gtrless \rho_6 \Longleftrightarrow \frac{\alpha_{HH}}{\alpha_{H\cdot}} \gtrless 1 - \alpha_{LL}$$

Definition C.8 $\rho_7 \stackrel{def}{=} \frac{1}{2}\alpha_{HL}\alpha_{H\cdot}$

Lemma C.6 $\rho_7 > 0$ *and*

$$\rho \gtrless \rho_7 \Longleftrightarrow \frac{\alpha_{HH}}{\alpha_{H\cdot}} \gtrless \frac{1 + \alpha_{LH} - \alpha_{LL}}{1 + \alpha_{LH} + a_{LL}}$$

Definition C.9 $\rho_8 \stackrel{def}{=} \frac{1}{2}\alpha_{HL}\alpha_{H\cdot} + \frac{1}{2}\alpha_{HH}(1 - \alpha_{H\cdot})$

Lemma C.7 $\rho_8 > 0$ *and*

$$\rho > (<) \rho_8 \Longrightarrow \frac{\alpha_{HH}}{\alpha_{H\cdot}} > (<) 1 + \alpha_{LH} - \alpha_{LL} > (<) \frac{2\alpha_{LH} + \alpha_{HL}}{1 - \alpha_{H\cdot}}$$

Definition C.10 $\rho_9 \stackrel{def}{=} \frac{\alpha_{HH}}{\alpha_{H\cdot}}\alpha_{HL}(1-\alpha_{H\cdot}) > 0$

Lemma C.8 $\rho \lesseqgtr \rho_9 \iff \frac{\alpha_{LH}}{\alpha_{HH}^2+\alpha_{H\cdot}\alpha_{LH}} \gtrless \frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}} \iff \alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}} \gtrless \frac{\alpha_{HH}}{\alpha_{H\cdot}}$

Definition C.11 $\rho_{10} \stackrel{def}{=} \alpha_{HH}(1-\alpha_{H\cdot})^2 > 0$

Lemma C.9 *Suppose that* $\rho > \alpha_{HH}(1-2\alpha_{H\cdot})$ *such that* $\alpha_{HH} > \alpha_{LH}$. *Then* $\rho \lesseqgtr \rho_{10} \iff \frac{\alpha_{LH}}{\alpha_{HH}-\alpha_{LH}} \gtrless \frac{1-\alpha_{H\cdot}}{\alpha_{H\cdot}}$

Definition C.12 $\rho_E \stackrel{def}{=} \frac{\alpha_{HH}(1-\alpha_{H\cdot})-\alpha_{HL}\alpha_{H\cdot}\alpha_{HH}}{1+\alpha_{H\cdot}}$

Lemma C.10

$$\rho < (>)\rho_E \Longrightarrow \alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}} > (<)\frac{2\alpha_{LH}+\alpha_{HL}}{1+\alpha_{HL}-\alpha_{HH}} > (<)\frac{1-\alpha_{LL}}{1+\alpha_{HL}} > (<)\begin{cases} \overline{x}_{f_{P3.1}} \\ \frac{\alpha_{H\cdot}+\alpha_{HH}}{\alpha_{H\cdot}+1} \end{cases},$$

$$\rho < (>)\rho_E \Longrightarrow \frac{1-2\alpha_{LH}-\alpha_{HH}}{2\alpha_{LH}+\alpha_{HL}} < (>)\frac{\alpha_{LH}}{\alpha_{HH}-\alpha_{LH}}$$

Lemma C.11 *Independent of any feasible value for* ρ*, the following inequalities hold:*

$$\begin{aligned}
\frac{1-\alpha_{LL}}{1+\alpha_{LL}} &< \frac{1+\alpha_{LH}-\alpha_{LL}}{1+\alpha_{LH}+a_{LL}} < 1-\alpha_{LL} < 1+\alpha_{LH}-\alpha_{LL} \\
\alpha_{HH} &< \frac{\alpha_{H\cdot}+\alpha_{HH}}{\alpha_{H\cdot}+1} \\
\frac{\alpha_{H\cdot}-\alpha_{HH}}{1-\alpha_{H\cdot}} &< \frac{2\alpha_{LH}+\alpha_{HL}}{1-\alpha_{H\cdot}} \\
\frac{\alpha_{HH}}{\alpha_{H\cdot}} &\gtrless \frac{\alpha_{H\cdot}+\alpha_{HH}}{\alpha_{H\cdot}+1} \iff \frac{\alpha_{HH}}{\alpha_{H\cdot}} \gtrless \alpha_{H\cdot} \\
\frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H\cdot}^2\alpha_{LL}} &< \frac{\alpha_{HH}(1-\alpha_{LL})}{(1-\alpha_{LL})^2+\alpha_{LL}\alpha_{LH}} \\
\overline{x}_{f_{P3.1}} &> \frac{\alpha_{HH}}{\alpha_{H\cdot}}
\end{aligned}$$

Lemma C.12 *For any non-positive value for* ρ *we have that*

$$\frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H\cdot}^2\alpha_{LL}} < \alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}}.$$

Sketch of proof: for any pair $(\alpha_{H\cdot}, \alpha_{HH}) \in [0,1]$ (i) the numerator of $\frac{\alpha_{HH}(1-\alpha_{LL})^2}{(1-\alpha_{LL})^2-\alpha_{H\cdot}^2\alpha_{LL}} - \left(\alpha_{\cdot H} + \frac{\alpha_{LH}\alpha_{HL}}{\alpha_{HH}}\right)$ is a 3th-degree polynomial in ρ that has three roots which, if real, are all strictly positive; (ii) for $\rho = 0$, this polynomial takes on a negative value. Hence, the difference is always negative for any pair $(\alpha_{H\cdot}, \alpha_{HH}) \in [0,1]$ and for any $\rho \leq 0$.

Lemma C.13 *For any non-positive value for ρ we have that*

$$\alpha_{\cdot H} < \frac{1 + \alpha_{LH} - \alpha_{LL}}{1 + \alpha_{LH} + a_{LL}}.$$

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