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Evaluating Carbon Capture and Storage in a Climate Model with Directed Technical Change

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Evaluating Carbon Capture and Storage in a Climate Model with Directed Technical Change*

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Abstract

Carbon capture and storage (CCS) is considered a critical technology needed to curb CO_2 emissions and is envisioned by the International Energy Agency (IEA) as an integral part of least-cost greenhouse gas mitigation policy. In this paper, we assess the extent to which CCS and R&D in CCS technology are indeed part of a socially efficient solution to the problem of climate change. For this purpose, we extend the intertemporal model of climate and directed technical change developed by Acemoglu et al. (2012, American Economic Review, 102(1): 131–66) to include a sector responsible for CCS. Surprisingly, even for an optimistic cost estimate available for CCS ($60/ton of CO_2 avoided), we find that it is not optimal to deploy CCS or devote resources to R&D in CCS technology either in the near or distant future. Indeed, it is only when the marginal cost of CCS is less than $12/ton that a scenario with an active CCS sector (including R&D) becomes optimal, though not in the near future.

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1 Introduction

In 2010, fossil fuels represented more than 80% of global energy use[1] and are responsible for 65% of global anthropogenic greenhouse gas (GHG) emissions (IEA, 2011, pp. 18–19). Although renewables have significant potential in energy production, fossil fuels will remain the dominant source of energy for decades to come.[2] Without specific actions, atmospheric CO_2 concentration will continue to grow and this may prove disastrous for future generations (UNEP, 2006).

Three main policies have been proposed as possible solutions to the problem of climate change: the more intensive use of renewable and nuclear energy; the more efficient generation of power and end-use of energy carriers; and the development and deployment of technologies to capture and store carbon emissions from fossil fuel use.[3] Carbon capture and storage (CCS) technology can be used by large stationary point sources such as fossil fuel-fired power plants and emission-intensive industrial facilities. Its main purpose is to prevent CO_2 emissions from entering the atmosphere. The rates of carbon captured can be as high as 85–95%, in both the pre- and post-combustion systems.[4]

The development of CCS technologies has been advocated by both individual countries and international organizations. For example, some high-income oil- and gas-producing countries in Europe and North America are strongly committed to the use of resources in the research, development and demonstration (RD&D) of CCS technologies. Using cross-sectional analysis of OECD countries, Tjernshaugen (2008) finds that

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[1] Fossil fuels, renewables, and nuclear electric power respectively account for 83%, 8.3%, and 8.7% of total global energy use (EIA, 2011a, Table 1.1).

[2] According to the U.S. Energy Information Administration, fossil fuels will account for 78% of world energy use in 2035 (EIA, 2011b).

[3] A fourth possible policy solution entering recent debate is geoengineering, the intentional, large-scale manipulation of the earth's climate system. See Rasch et al. (2008), Cicerone (2006), and Barrett (2008).

[4] There are three methods for capturing CO_2. *Post-combustion* carbon capture removes carbon after combustion. Here, CO_2 is separated from the flue gases (whose main constituent is nitrogen) using a liquid solvent. In *pre-combustion* carbon capture, fuel is pretreated and converted into a mix of CO_2 and hydrogen. The hydrogen is then separated from the carbon before being burned to produce electricity. In the *oxy-fuel combustion process*, the fuel is burned using oxygen rather than air. The result is a flue stream of CO_2 and water vapour. Because no nitrogen is present, CO_2 can be easily removed (Golombek et al., 2011; Metz et al., 2005).

fossil fuel reserves and extraction activities are the main variables explaining funding levels for RD&D on CCS. Outstanding examples are Canada and Norway.[5] To give an idea of the orders of magnitude, Tjernshaugen (2008) reports that for these countries the 2005 RD&D budget for CCS normalized by 2002 total government energy-related RD&D expenditures amounted to 6.2% and 38.8% of the total, respectively. The share of Norwegian CCS RD&D clearly stands out and one may wonder why such a relatively small country is so concerned with CCS technology, for which we can offer two reasons. First, with hydropower being historically the main energy carrier in Norway, power generation from fossil fuels was almost absent. When gas-fired power plants were added to the Norwegian energy grid, compliance with domestic emissions targets especially was required, thereby promoting the interest in CSS technology and its potential. The second reason is the large contribution of the oil and gas extraction industry to Norway's GDP. The only way to reconcile a strong commitment to environmental policies alongside Norway being a large exporter of fossil fuels is by producing and making available the know-how to prevent the CO_2 generated by burning fossil fuels entering the environment (Tjernshaugen, 2011).

Several international and intergovernmental agencies, including the International Energy Agency (IEA), the Intergovernmental Panel on Climate Change (IIPC), and the U.S. Energy Information Administration (EIA), also envision an important role for CCS as part of an environmentally sustainable global energy policy, and therefore point to the need for significant R&D efforts today in order to endow the world with an economic carbon capturing and storage technology. For example, in IPCC (2005, p. 12), CCS is shown to have the potential to provide 15% to 55% of the world's cumulative GHG mitigation efforts up to 2100. Further, to bring down GHG emissions to 50% of their 2005 level by 2050, IEA (2008) shows that about 27% of the reductions should come from the extensive use of renewables and nuclear energy, 54% from efficiency enhancement, and 19% from CCS activities. Without access to CCS technology, the same report estimates that the overall cost to achieve these emission reductions increases by 70%. In another report, IEA (2009) sketches a road map for CCS and shows that the technology is required to grow from a handful of existing large-scale projects today to

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[5] In Norway, Technology Centre Mongstad is the world's largest facility for testing and improving CO_2 capture technologies. In Canada, the Boundary Dam Integrated CCS project is planned to be completed and able to commence the capture and storage of carbon by November, 2013.

around 3,000 projects by 2050, to secure the above-mentioned 19% share in emission reductions. However, it is worth noting that CCS is not considered as the only policy for establishing an environmentally sustainable growth path, as the development of renewable energy sources are also given a prominent role.[6] Thus, it appears that the recommended mitigation portfolio is a very balanced one.

Without doubt, these studies have been very useful in informing both policymakers and the general public about the available options and costs involved when directing emissions to a more sustainable trajectory. At the same time, the welfare economic trade-offs underlying the results are not always transparent, i.e., it is not always clear to what extent differences in scenarios are the result of differences in the constraints imposed (emission caps, technological, and economic constraints) or the differences in trade-offs for which the models allow (e.g., between economic growth and environmental quality).

In this paper, we wish to assess the scope for CCS and CCS R&D as part of a socially efficient solution to the climate change problem. The vehicle that we use for this purpose is the intertemporal model of climate and directed technical change developed by Acemoglu, Aghion, Bursztyn and Hemous (2012, AABH hereafter). In this model, final good production requires two inputs, renewable and fossil fuel energy. Both types of energy are produced using labour and capital (and a finite stock of non-renewable fossil fuels) with the help of the latest available technologies. These technologies result from costly R&D efforts, and given a finite number of scientists, faster technological progress in one sector needs to be balanced against slower progress in the other sector. The production of fossil fuel energy increases the stock of CO_2 in the atmosphere, and therefore contributes to a global increase in temperature. This global warming in turn reduces the quality of the environment and with it the welfare of the representative consumer.

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[6] IPCC (2012) reports on 164 commissioned medium- and long-term scenarios from 16 global energy–economic and integrated assessment models. The scenarios range from baseline simulations with CO_2 atmospheric concentrations as high as 1050 ppm by 2100 to those with CO_2 caps as tight as 350 ppm by 2100. The results show that the share of renewable energy varies from today's share of 17% to shares as high as 77% in 2050. But even in some of the baseline scenarios with no CO_2 caps imposed, renewable energy shares can be much higher than those currently, reflecting significant differences regarding the assumptions on the evolution of the energy and abatement technologies (including CCS), energy demand, and prospective fossil fuel availability.

To this model, we append a new sector, that for CCS, which also operates using labour and capital. Like both energy sectors, the CCS technology may be improved by devoting resources to R&D. We calibrate our model using both recent data on world energy production levels and recent estimates of the marginal cost of CCS. We then ask ourselves the following questions: (i) is it socially optimal to include CCS in today's or the near future's mitigation portfolio?; and (ii) is it socially optimal to devote R&D resources to improve CCS technology, such that it becomes part of an optimal mitigation policy in the more distant future? We find that given today's marginal costs of CCS and clean and dirty energy production, the answer to both questions is rather bleak. We then ask by how much the marginal cost of CCS needs to fall such that both CCS and R&D into the CCS technology become socially optimal. Worryingly, we find that the decrease in the cost of CCS must be quite large, at least 80% of the level today.

The remainder of the paper is structured as follows. Section 2 reviews the related literature. Section 3 details the model. In Section 4, we provide the details about the numerical implementation. Section 5 presents the results given the cost estimates for CCS. Section 6 concludes.

2 Related literature

In recent years, a literature has developed that studies the desirability of CCS as part of the first-best or second-best environmental policy portfolio used in combating climate change. This literature has developed in several directions: partial *vs* general equilibrium models, theoretical models *vs* numerical solutions to empirically calibrated models, models encompassing exogenous *vs* endogenous technical progress.

An early contribution to this literature is by Goulder and Mathai (2000) who develop a partial equilibrium model to answer the question about how the endogenization of technological progress affects the optimal trajectories for abatement activity and carbon taxes. They show both analytically and through numerical simulations that endogenous technical progress with respect to (w.r.t) abatement activity (what they term "induced technical change" or the possibility of reducing the cost of abatement through devoting resources to R&D) in general lowers the time profile of optimal carbon taxes, and shifts at least some abatement activity from the present to the future.

However, the more recent literature has often taken a general equilibrium approach. We can discern at least two separate strands in this literature. One is concerned with the characterization of socially efficient environmental policy, and its implementation in a decentralized economy, possibly under some second-best policy restrictions (such as upper bounds on the tax rate set on carbon emissions). Examples include Grimaud and Rouge (2012) and Le Kama et al. (2013). The other strand compares the welfare costs of different (portfolios of) policy instruments when CO_2 stabilization or maximum temperature change targets are imposed. Examples include Gerlagh and van der Zwaan (2006), Grimaud et al. (2011), and Kalkuhl et al. (2012).

Gerlagh and van der Zwaan (2006) use a top-down computable general equilibrium model with an environment module to which they append a CCS sector. Technical progress in this sector stems from learning-by-doing. Assuming a marginal cost of abatement of 45\$/ton CO_2 avoided, they compute the carbon emission trajectories for 30 five-year periods (2000–2150) under five stabilization targets (ranging from 450 to 550 ppm–particles per million) and five policy scenarios in addition to a business-as-usual scenario. Their results reveal that irrespective of the stabilization target, subsidization of renewable energy use is the most expensive policy, while a carbon emission tax in which revenue is recycled as a subsidy for non-fossil energy use represents the least costly policy mix. A carbon tax also dominates policy that charges for fossil fuel use because it incentivizes the use of CCS activity. While CCS activity is low to begin with, about 30–50% of new fossil-fuel capacity from 2050 onwards is complemented with CCS equipment.

Grimaud et al. (2011) extend the Goulder and Mathai (2000) framework to a general equilibrium setting. They model a decentralized market economy where energy, capital, and labour are combined into a final good. Energy is produced from non-renewable fuels and a renewable energy source. Growth is endogenous and depends on R&D investments used to promote the efficiency of use of energy in final good production, the efficiency of producing renewable energy, or the efficiency of CCS in reducing the emissions resulting from the use of fossil fuels. In this market economy, investors are able to capture only a fraction of R&D returns and this motivates the use of (differentiated) R&D subsidies. Assuming a cap on atmospheric carbon concentration (450 or 550 ppm), they then provide a general characterization of the second-best trajectory for the tax on carbon emissions and the three R&D subsidies that maximize social welfare. In particular, the carbon tax is shown to follow an inverted U-shaped trajectory. Their main finding is that both tax and subsidy instruments should be used simultaneously to provide the strongest impact, and that R&D in CCS is warranted in the medium term only if accompanied by the imposition of a ceiling on the stock of atmospheric CO_2.

Grimaud and Rouge (2012) also adopt a general equilibrium approach. In their model, endogenous growth is restricted to the final goods industry. Final output makes use of intermediate goods (embodying technology), labour, and the extracted amounts of a non-renewable energy resource. The use of energy in production causes emissions that can be abated (i.e., captured and stored) using labour. With a constant and inelastic labour supply, the main trade-off in their model is between output production and abatement. The authors first characterize the socially optimal trajectories with and without access to a CCS technology, and then trace out the paths for a decentralized economy when only second-best policy tools are available. They find that the greatest abatement effort should take place in the near future, and thereafter gradually decline over time. Moreover, compared with an economy without CCS technology, the availability of CCS speeds up the optimal extraction rate and lowers output growth as labour is diverted from R&D activities.

Finally, in a static multi-market general equilibrium model for Europe, Golombek et al. (2011) look at the development of CCS in relation to technology-neutral abatement policies (i.e., carbon taxes or tradable permits)[7]. When an uniform tax of $90\$/tCO_2$ is implemented, the results show that new coal power plants with CCS become profitable and replace non-CCS coal power investments and a large share new wind power. For the same tax level, new gas power plants with CCS become profitable and replace almost all non-CCS power investments. Compared to a BAU scenario, this leads to a 90% lower CO_2 emissions in 2030. The results also imply that from a social point of view it is not desirable to retrofit CCS into the existing coal and gas power plants.

Our model shares several aspects with the models described. We employ a global and dynamic general equilibrium setting with four sectors: a "dirty" fossil fuel energy sector, a "clean" renewable energy sector, a CCS sector, and a sector transforming clean and

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[7] Equilibrium is calculated for exogenously taken non-EU parameter values.

dirty energy into a final good that is used for consumption and capital investment. In addition to the standard labour balance constraint, the economy is endowed with a stock of scientists who can be allocated to each of the three lower-level sectors (clean, dirty, and CCS) where their efforts result in efficiency-enhancing innovations. Moreover, rather than imposing exogenous stabilization targets, we let the quality of the environment enter consumer welfare (cf. Smulders and Gradus (1996) and Grimaud and Rouge (2012)). In this regard, we are primarily interested in whether CCS activity and CCS-related R&D effort are part of a first-best policy. We characterize the socially optimal solution, proceed by a numerical calibration of our model in the same vein as AABH, and then optimize as in Gerlagh and van der Zwaan (2006) over a finite but long discrete horizon (thirty 10-year periods). Unlike the models reviewed above, we find little scope for (R&D on) CCS.

3 The model

We specify a four-sector general equilibrium model, which augments the three-sector model in AABH with a fourth sector responsible for CCS activity. As we are primarily interested in socially optimal policy, we discard any issues related to the implementation of this policy in a market economy through taxes and the subsidization of R&D activities. The interested reader is referred to the AABH article.[8]

An infinitely lived representative consumer cares about a final good (c_t) and the quality of the environment (F_t) in each period t of life. The period utility function, $U(c_t, F_t)$, satisfies the standard monotonicity and concavity assumptions. The final good is produced by means of two energy carriers: dirty energy (Y_{dt}) and clean energy (Y_{ct}). The (symmetric) production function is assumed to display a constant elasticity

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[8] See Greaker and Heggedal (2012) for a discussion of the robustness of R&D subsidy policies prescribed by the AABH model w.r.t. the assumptions on the length of the patent period.

of substitution (CES), ε:[9]

$$Y_t = \left(Y_{ct}^{\frac{\epsilon-1}{\epsilon}} + Y_{dt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} . \tag{1}$$

Each of the two energy types j ($j = c, d$) is produced using labour (L_j), of which there is a unit mass available in each period, and capital (x_j), which is available at constant marginal cost ψ. The sector production functions are of Cobb–Douglas form, with technology parameter (sectoral stock of knowledge) A_j ($j = c, d$). The same is true for the CCS sector, which we label with index a (for abatement). Thus:

$$Y_{jt} = A_{jt}^{1-\alpha} L_{jt}^{1-\alpha} x_{jt}^{\alpha} \;\; (j = a, c, d). \tag{2}$$

The stock of knowledge/technology level A_{jt} in sector j is assumed to grow at a rate of $\gamma\eta_j s_{jt}$, where s_j is the number of researchers allocated to sector j ($j = a, c, d$), η_j is the probability that a single researcher is successful in creating an innovation, and γ is the relative increase in knowledge in the case of such an innovation.[10] Subsequently, A_j evolves according to:

$$A_{jt} = (1 + \gamma\eta_j s_{jt})\, A_{jt-1}. \tag{3}$$

There is a unit mass of scientists available in each period and the allocation of a scientist to one sector fully crowds out R&D activity in the other sector/s.[11]

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[9] For a discussion regarding the CES function and the value of the elasticity of substitution between the two energy carriers, we refer the reader to Gerlagh and van der Zwaan (2004). Below, we will follow these authors' suggestion of 3 as a central value for ε. This value implies that the isoquants are tangent with the input axes, but at the same time have endpoints at $y^{\frac{3}{2}}$. Thus, although the CES specification makes it technically feasible to rely solely on renewable energy, such a solution will not be selected as long as the (social) relative price of fossil fuel energy is finite. Alternatively, one could have recourse to a Variable Elasticity of Substitution specification, as in Gerlagh and Lise (2005). The advantage of such specification is that the substitution eleasticity between the two energy carriers falls to 1 if one carrier becomes dominant.

[10] Thus, there are constant returns to scale in research. However, arguments also exist that may provide deviations from this in both directions. For instance, "fishing out" problems, where easy inventions occur sooner with little effort whereas larger technological challenges are solved later and require more effort, infer decreasing returns to scale, while positive spillovers between researchers and/or labs suggest increasing returns to scale. See Mattauch et al. (2012) for a variant of the AABH model with technical progress stemming from learning-by-doing.

[11] Roucade *et al.* (2011) drop the assumption that the pool of scientists differs from the pool of

With an activity level Y_a in the CCS sector, the emissions corresponding to Y_a units of dirty energy input production are captured and stored. The carbon sink (environmental stock) therefore evolves according to the following equation of motion:

$$S_t = -\xi(Y_{dt-1} - Y_{at-1}) + (1+\delta)S_{t-1}, \tag{4}$$

where ξ is the rate of CO_2 emissions from dirty energy production and δ is the regeneration rate of the environment. The size of the carbon sink translates into an environmental quality index $\widetilde{F}(S_t)$ (see p. 15).

As we are primarily interested in optimal policy, we consider the levels of the labour and capital inputs, the level of energy production, the level of CCS activity, and the allocation of scientists that maximize the intertemporal utility of the representative consumer subject to the technology constraints, the equation of motion for the environment and for the sectoral stocks of knowledge, and the balance constraints for labour and scientists.

$$\begin{aligned}
\max_{\{Y_t, Y_{jt},, L_{jt}, x_{jt}, s_{jt}\}_{t=0\ldots\infty}^{j=c,d,a}} \quad & \sum_{t=0}^{\infty} \beta^t U(Y_t - \psi(x_{ct} + x_{dt} + x_{at}), \widetilde{F}(S_t)) \\
& \text{s.t } \; Y_t = \left(Y_{ct}^{\frac{\epsilon-1}{\epsilon}} + Y_{dt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \; (\lambda_t) \\
& Y_{jt} = A_{jt}^{1-\alpha} L_{jt}^{1-\alpha} x_{jt}^{\alpha} \quad (\lambda_{jt}) \quad (j = a, c, d) \\
& A_{jt} = (1 + \gamma \eta_j s_{jt}) A_{jt-1} \quad (\mu_{jt}) \quad (j = a, c, d) \\
& S_t = -\xi(Y_{dt-1} - Y_{at-1}) + (1+\delta)S_{t-1} \quad (\omega_t) \\
& 1 \geq L_{ct} + L_{dt} + L_{at} \quad (w_t) \\
& 1 \geq s_{ct} + s_{dt} + s_{at} \quad (\sigma_t) \\
& Y_{at} \leq Y_{dt} \quad (\phi_t) \qquad .
\end{aligned}$$

In this problem, β is the discount factor, ψ is the amount of final goods necessary to build a machine, and the Lagrange multipliers in brackets following the constraints are all current values (thus the net present value of a marginal unit of labour in period t is

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workers.

$\beta^t w_t$). The final inequality precludes the more than 100% capture of CO_2 emissions (we ignore the fact that existing CCS technology does not allow for capture rates exceeding approximately 90%). In a market economy, these decisions can be decentralized by means of a tax on the dirty energy input production, subsidies to research on clean energy production and CCS technology, subsidies to machine use (to correct for the market power of machine producers), and lump-sum transfers to the representative consumer.

In the sequel, we define the social price of sector j output as $\widehat{p}_{jt} \overset{\text{def}}{=} \frac{\lambda_{jt}}{\lambda_t}$ $(j = c, d, a)$, and the *ad valorem* rate on fossil fuel energy use as $\tau_t \overset{\text{def}}{=} \beta\xi\frac{\omega_{t+1}}{\lambda_{dt}}$. The latter is the social marginal environmental damage of period t emissions $\beta\xi\frac{\omega_{t+1}}{\lambda_t}$, expressed as a fraction of the social price of dirty energy, $\widehat{p}_{dt}$. Here, ω_{t+1}, the shadow value of the environment at time $t + 1$, is the discounted intertemporal sum of marginal disutilities caused by the current dirty input production, which is adjusted for the value of the dirty input and regeneration in every period:

$$\omega_t = \sum_{k=0}^{\infty} \beta^k (1 + \delta)^k U_{sk} \widetilde{F}'_k.$$

In the remainder of this section, we focus on characterizing the optimal policy w.r.t. the CCS sector, in both its level of activity and the efforts directed to R&D. We relegate to the Appendix the solution to the full model.

For both energy carriers, the marginal product in final good production should equal the social price: $MP_{ct} = \widehat{p}_{ct}$ and $MP_{dt} = (1 + \tau_t)\widehat{p}_{dt}$. For abatement activity (i.e., CCS), the optimality condition is:

$$\widehat{p}_{at} \geq \tau_t \widehat{p}_{dt} - \frac{\phi_t}{\lambda_t},$$

with equality whenever $Y_{at} > 0$. The second term on the right-hand side (RHS) is the period t social cost of not being able to capture more CO_2 than the amount emitted by the dirty sector in period t; this cost is obviously zero when $Y_{at} < Y_{dt}$. Thus, when $\widehat{p}_{at} > \tau_t\widehat{p}_{dt}$, any abatement is suboptimal. If partial abatement is optimal, then $\widehat{p}_{at} = \tau_t\widehat{p}_{dt}$, while full abatement requires that $\widehat{p}_{at} \leq \tau_t\widehat{p}_{dt}$.

In the Appendix, we show that allocating a scientist to the R&D department of sector

j yields a marginal social value of:

$$\frac{\mu_{jt}}{\lambda_t}\gamma\eta_j A_{jt-1} = \frac{1}{\lambda_t}\frac{\gamma\eta_j}{1+\gamma\eta_j s_{jt}}(1-\alpha)\sum_{\tau=0}^{\infty}\beta^{\tau}\lambda_{t+\tau}\widehat{p}_{jt+\tau}Y_{jt+\tau}. \tag{5}$$

This value positively depends on (i) the productivity of R&D ($\gamma\eta_j$) and (ii) the discounted social value of the output stream ($\widehat{p}_{jt+\tau}Y_{jt+\tau}, \tau = 0...\infty$) of sector j. If the R&D in sector j is optimal, then this marginal social value should match the social wage of the scientists, $\frac{v_t}{\lambda_t}$. If (5) falls short of $\frac{v_t}{\lambda_t}$, then R&D is not optimal in sector j. It is therefore clear from (5) that substantial CO_2 capture and storage in the near future is a prerequisite for justifying R&D in the CCS sector.[12]

The allocation of labour and capital across sectors should satisfy the standard conditions of equality between the marginal products and the corresponding social prices. In the Appendix, we show how the first-order conditions together with the constraints allow us to reduce the above maximization problem to a simpler model in terms of four sets of decision variables: $\{Y_{at}, \tau_t, s_{ct}, s_{dt}\}_{t=0...\infty}$, subject to the maximum abatement constraints (ϕ_t). This problem is then calibrated and solved (with MATLAB) for a large but finite time horizon. In the next section, we explain the calibration. The optimal solutions are presented and discussed in Section 5.

4 Numerical implementation of the model

To implement the model numerically, we proceed as in AABH. We consider a long but finite horizon (300 years) and let a single period consist of 10 years.[13] The base period ($t = 0$) is 1997–2006. The final period ($T = 30$) is 2297–2306.

We calibrate the model by assuming that in period 0 (the base period) there is no environmental policy. Under this assumption, and using the values for world primary energy production by carrier, we solve for the base period technology efficiency

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[12] In a decentralized equilibrium, this would translate into a high price and/or market size effect for CCS.

[13] AABH take a period to be five years. Because our model has two extra sequences of decision variables (Y_{at} and s_{at}, $t = 1...300$), we double the number of years per period to keep the total number of decision variables in the numerical optimization within limits.

parameters A_{d0} and A_{c0}, as well as their weighted average $B_0 \overset{\text{def}}{=} \left(A_{c0}^{-\varphi} + A_{d0}^{-\varphi}\right)^{-\frac{1}{\varphi}}$ (with $\varphi \overset{\text{def}}{=} (1-\varepsilon)(1-\alpha)$) (see the Appendix): $A_{d0} = 2658$, $A_{c0} = 1072$, and $B_0 = 3232$. Then, using the result that $MC_{j0} = \left(\frac{B_0}{A_{j0}}\right)^{1-\alpha}$ (cf. (21) in the Appendix), we obtain:

$$MC_{d0} = 1.14 \frac{\text{UON}}{\text{QBTU}} \text{ and } MC_{c0} = 2.09 \frac{\text{UON}}{\text{QBTU}},$$

where UON stands for *units of the numeraire* and QBTU are quadrillions (10^{15}) of British Thermal Units.

To convert the value of the *numeraire* to \$ (USD), we take a weighted average of the price of fossil fuels in the base period (EAI (2008), Table 3.1):[14]

$$3.314 \frac{\$}{\text{million BTU}} = 3.314 \times 10^9 \frac{\$}{\text{QBTU}}.$$

Hence, our *numeraire* is worth $\frac{3.314}{1.14} \times 10^9\$ = 2.907 \times 10^9\$$. World carbon dioxide emissions from energy consumption during the base period were 272040(2 × 136020) million tons (EIA (2008), Table 11.19). As $Y_{d0} = 3786 (= 2 \times 1893)$ QBTU, this means an emission rate of

$$\frac{272040}{3786} \frac{\text{million ton CO}_2}{\text{QBTU}} = 71.85 \frac{\text{million ton CO}_2}{\text{QBTU}}.$$

A wide variety of estimates exist for the average cost of CCS, each surrounded by a wide confidence interval. For example, the IEA provides estimates of 55\$/ton CO_2 for a pulverized coal power plant with CO_2 capture and 80\$/ton CO_2 if a natural gas combined cycle is used as a reference (Finkenrath (2011)).[15] As a reference point, we assume a constant marginal cost of 60\$/ton CO_2, but carry out a sensitivity analysis across a wide range. Hence, the reference cost of abating CO_2 when producing one QBTU of dirty energy is:

$$60 \frac{\$}{\text{ton CO}_2} \times 71.85 \frac{\text{million ton CO}_2}{\text{QBTU}} = 4.311 \times 10^9 \frac{\$}{\text{QBTU}}.$$

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[14] Given 1 QBTU= 290×10^9kWh, the average fossil fuel price amounts to $\frac{3.314}{290} \frac{\$}{\text{kWh}} = .0114 \frac{\$}{\text{kWh}}$.

[15] Golombek et al. (2011), making use of the cost parameter estimates reported in IIPC (2005), obtain the following estimates after correcting for differences in fuel costs, the rate of return on capital, and the base year: coal greenfield pre-combustion: 35.6\$/tCO_2; gas greenfield post-combustion: 67.4\$/tCO_2; coal retrofit post-combustion: 73.9\$/tCO_2; gas retrofit post-combustion: 116.6\$/tCO_2.

To express this cost in units of the *numeraire*, we divide by the earlier obtained rate $2.907 \times 10^9 \frac{\$}{\text{UON}}$:

$$MC_{a0} = 1.483 \frac{\text{UON}}{\text{QBTU}}.$$

Thus, in the base period, the reference CCS abatement cost amounts to about 130% of the production cost of dirty energy.

Having found MC_{a0}, we calibrate A_{a0} using the relationship $MC_a = \left(\frac{B_0}{A_{a0}}\right)^{1-\alpha}$ (cf. (21) in the Appendix):

$$A_{a0} = \frac{B_0}{(1.483)^{\frac{3}{2}}} = 1789.6.$$

The quality of the environment, $\widetilde{F}(S_t)$, is modelled as a decreasing and concave function of the rise in temperature since pre-industrial times: $\widetilde{F}(S_t) = F(\Delta t(S_t))$, where

$$F(\Delta t) = \frac{(\Delta t_{dis} - \Delta t)^{\lambda} - \lambda \Delta t_{dis}^{\lambda-1}(\Delta t_{dis} - \Delta t)}{(1-\lambda)\Delta t_{dis}^{\lambda}}.$$

Here, Δt_{dis} is the increase in temperature leading to environmental disaster (taken at 6^oC), and $\lambda = 0.1442$ (see AABH, 2012). This function has the property that $F(0) = 1$ and $F(6) = 0$, so that $F(\Delta t)$ is an index of environmental quality (see solid line in the Figure 1). A λ-value of .1442 amounts to a 1% reduction in environmental quality following a 2^oC Celsius temperature increase. Given this may be considered to be too optimistic, we also consider a λ-value of .3011, which produces 2% damage at the same temperature increase (cf. Weitzman (2010); see the dashed line in Figure 1).

The rise in temperature is a decreasing function of the carbon sink in the atmosphere, S_t:

$$\Delta t = 3 \log\left(\frac{280 \times 2^{\frac{\Delta t_{dis}}{3}} - S_t}{280}\right) / \log(2),$$

where 280 refers to the atmospheric concentration of CO_2, measured in ppm (particles per million by volume) since pre-industrial times, and δ is the regeneration rate of the environment (set at 50% of emissions in the base period), i.e. $\delta = \frac{1}{2}\frac{\text{emissions}_0}{S_0} = .0236$

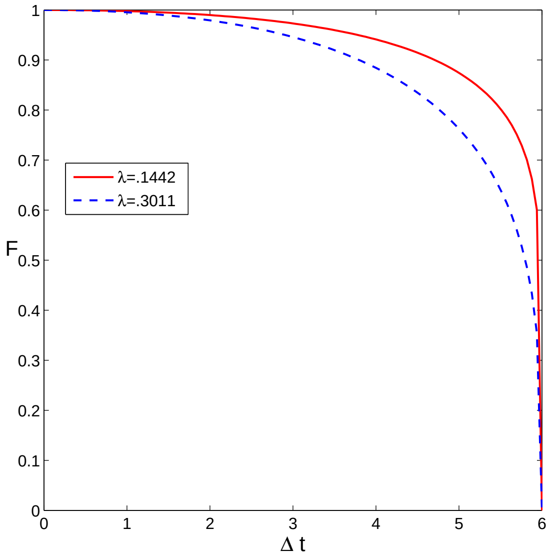

Figure 1: *Damage Function*

$(= 2 \times .0118)$.

As above, the emission rate for the base period was estimated at $71.85 \frac{\text{million ton CO}_2}{\text{QBTU}}$. As 7.78 billion tons of emitted CO_2 give rise to an increase in atmospheric concentration of CO_2 of one ppm, the emission rate as ppm per QBTU is:

$$\xi = 71.85 \frac{\text{million ton CO}_2}{\text{QBTU}} \times \frac{1}{7.78 \frac{\text{billion ton CO}_2}{\text{ppm}}} = .0092 \frac{\text{ppm}}{\text{QBTU}}.$$

S_0 is set at 741 ppm (cf. AABH, 2012). The utility function is assumed to take Cobb–Douglas form $U(c, F) = \frac{[c \cdot F]^{1-\sigma}}{1-\sigma}$, with $\sigma = 2$. However, the use of this particular utility function has been criticized, as it allows for the too easy substitution of consumption for environmental quality (Weitzman, 2010). We also ran simulations using a CES utility function with a substitution elasticity of $\frac{1}{2}$ (cf. Sterner and Persson (2008)): $U(c, F) = \frac{1}{1-\sigma} \left([\frac{1}{2} c^{\frac{\theta-1}{\theta}} + \frac{1}{2} F^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} \right)^{1-\sigma}$ with $\sigma = 2$ and $\theta = \frac{1}{2}$. However, this does not change the qualitative nature of our results (cf. Figure 5 below).

Finally, we follow AABH (and Gerlagh and van der Zwaan (2004)) by setting the elasticity of substitution between clean and dirty energy carriers in energy production

$\varepsilon = 3$ and assuming a discount factor of 0.015 (such that $\beta = .98522$). The technological progress parameters are chosen as follows: $\gamma = 1$, and $\eta_c = \eta_d = \eta_a = .22$ per 10-year period (i.e., 2% per year). We also carry out a sensitivity analysis by assuming a 3% per year probability of succesful research in the renewable energy and CCS sectors (the "infant" sectors) for the first 50 years. This concludes the calibration of our model.

5 Results

We first present the results for $MC_a = 1.47$ (corresponding to $60\frac{\$}{\text{ton CO}_2}$ avoided) when preferences are of Cobb–Douglas form and $\lambda = .1442$. The results are presented in Figure 2. Panel b shows the time path for the optimal tax rate τ_t as well as the cost of CCS relative to the marginal cost of dirty energy. Since $\frac{MC_{at}}{MC_{dt}} \left(= \frac{\hat{p}_{at}}{\hat{p}_{dt}}\right)$ always exceeds τ_t, it is never optimal to have the capture and storage of CO_2 emissions (panel d). Because CCS is never active, there are no scientists allocated to CCS R&D (panel a). Note that the initial R&D activity on "dirty" energy carriers increases the cost of CCS relative to that of Y_d. These trajectories are identical to those depicted in Figure 1 in AABH. In particular, after about 50 years, scientists are relocated from the dirty energy sector in favour of the clean energy sector. Together with the tax on dirty energy, the result is a gradual increase in the intensity of clean energy in final good production (panel e). The temperature continues to increase but stabilizes below the disaster level of a temperature rise of 6^0C. If λ is increased to 0.3011, although deteriorating the environmental impact of a (smaller-than-disaster-level) temperature rise, the overall picture remains almost the same, except that the switch from "clean" to "dirty" R&D takes place a few years earlier. The result is a slightly lower temperature increase to which the climate converges.

–Figure 2a-f here–

As the current estimates for the marginal cost of CCS make neither CCS nor R&D on CCS part of the optimal policy portfolio, we ask by how much this marginal cost must fall before CCS and/or R&D on CCS start to be desirable. When $MC_a = .55$ (corresponding approximately to $22\$/\text{tCO}_2$ avoided, i.e., slightly above $\frac{1}{3}$ of today's (optimistic) reference level), CCS becomes optimal 200 years later. The reason is the steady increase in the tax rate on Y_d, passing the relative cost of abatement around $t = 220$.

From then on, CCS becomes active, but not for long as the use of the dirty input becomes quite minimal. See Figure 3 for details. However, the fact that CCS is only active in the distant future makes it suboptimal to divert any R&D resources to that sector in the near future. We categorize these scenarios—without any R&D on CCS, but possibly with active CCS in the distant future—under *Regime 1.* Our simulations show that Regime 1 continues to hold for MC_a values as low as 0.31, which corresponds to a CCS cost as low as 12\$/tCO$_2$ avoided.[16]

–Figure 3a-f here–

For lower MC_a values, a second regime, *Regime 2*, becomes optimal. We illustrate this regime in Figure 4 under the assumption that $MC_a = .27$ (i.e., 11\$/tCO$_2$). In this regime, CCS becomes active after 50 years, i.e., sooner than in Regime 1 (see panel d). The second difference w.r.t. Regime 1 is that R&D in CCS now becomes part of the optimal research policy (see panel a). Whereas in Regime 1 only clean R&D prevails in the distant future, there is no role at all in Regime 2 for "clean" R&D. Conversely, "dirty" R&D dominates for about 100 years, after which the scientists are shared with the CCS sector.[17]

–Figure 4a-f here–

In panel a in Figure 5 we have plotted the maximal intertemporal welfare against values for MC_{a0}. Recall that $MC_{a0} = .25(2.5)$ corresponds to a marginal cost of about 10(100)\$/tCO$_2$. The maximal value function is drawn for the scenario described above (CD preferences, $\lambda = .1442$) but also for the case of CES preferences and $\lambda = .3011$. For all scenarios, we discern the same pattern: Regime 1 for modest to high CCS marginal cost values and Regime 2 for very low values. In panel b, we draw the same maximal value functions, but now for a disaster temperature increase of 5^{o}C (i.e., $\Delta t_{dis} = 5$) instead of 6^{o}C. The simulations upon which these maximal value functions are based all display the same qualitative features as the simulations reported in Figures 3 and 4.

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[16] At MC$_a$ = 12.5\$/tCO$_2$ avoided, CCS becomes active after 70 years and activity increases to as high as 100% around $t = 220$, after which it begins to decline. These high CCS rates do not necessarily imply a growth in the absolute amounts captured and stored. The reason is the diminishing use of the dirty energy carriers in Regime 1.

[17] The switch from Regime 1 to Regime 2 when MC_a drops below some critical value in [.27, .31] (i.e. [11\$/tCO$_2$, 12\$/tCO$_2$]) points to a non-convexity in the model owing to the endogenous nature of R&D activity.

–Figure 5a-b here–

In all of the above scenarios, the temperature increases above the critical value of 2^oC, and one may wonder whether penalizing temperature rises more heavily could “rescue” CCS as a viable environmental policy instrument. For this reason, we also ran the model under the assumption that environmental disasters are caused by 3^oC or 4^oC rises in the temperature. The implications for lower Δt_{dis} values are substantial decreases in CCS activity and resources dedicated to its R&D in case of Regime 2, or even the disappearance of this regime altogether. For example, when $MC_a = 11\$/\text{tCO}_2$, Regime 2 is unattainable when a disaster occurs with a 4^oC increase in temperature. See Figure 6. The solution is Regime 1, with more resources now devoted to the non-fossil fuel energy R&D (panel a). Moreover, the CCS sector, which once became active after 50 years in Regime 2, now becomes operative only after 130 years and is short-lived owing to the strongly declining use of dirty energy carriers. Another implication is a lower optimal trajectory for the tax rate on Y_d, which is an outcome of a strong bias towards non-fossil fuel energy use and the earlier devotion of resources to related R&D activities.

–Figure 6a-f here–

When $\Delta t_{dis} = 3^o$C, all scientific activity is diverted to the clean sector after 10 years. See Figure 7 (panel a). The share of fossil fuels in the energy mix contracts more sharply and the CCS sector becomes completely idle from the start (see panels d and e). An earlier switch to clean R&D and non-fossil fuel carriers results in an even lower optimal tax trajectory (panel b).

–Figure 7a-f here–

One may object to the assumption that all three sectors share the same rate of success in innovation. In comparison to mature technologies, such as the fossil fuel energy technology, technologies that are in their early stages of development, e.g. the renewable energy and the CCS technologies, may be expected to display higher rates of successful research. To test the implications of such a differentiation, we assumed that the rate of success in innovation in the latter two sectors exceeds temporarily (50 years) the rate in the former sector with 1% ($\eta_c = \eta_a = .03$, $\eta_d = .02$). Figure 8 shows the results for the 'critical' MC_a value of $11\$/\text{tCO}_2$. Compared with Figure 4, we see that Regime 2 is replaced by Regime 1: even though research on CCS technology is potentially more successful, the facts that CCS activity is complementary to the dirty energy production and the latter cannot grow at the same rate reduce the scope for CCS. As a consequence, it becomes optimal to fully allocate the researchers to the clean sector in the long run (despite a short reallocation after 50 years, when all three research sectors face the same potential rate of successful innovation again).

–Figure 8a-f here–

6 Conclusion

In recent decades, carbon capture and storage has been considered as a promising strategy to curb CO_2 emissions and therefore to address the problem of global warming. Given the infancy of CCS technology, and the need for further research, development and demonstration, it is desirable to assess the optimality of this strategy not only on the basis of its current marginal cost, but also on the potential for improvements in cost efficiencies following R&D efforts in dirty energy, clean energy, and CCS sectors.

For this purpose, we utilized the directed technical change model of Acemoglu et al. (2012) by adding a sector responsible for CCS. Assuming that CCS competes for the same R&D resources as the fossil fuel and renewable energy sectors, and that neither sector has any comparative advantage in transforming R&D into technological improvements, we have computed the Pareto–efficient time paths for production and research activity in each sector.

Surprisingly, we found that even for very optimistic estimates for the current marginal cost of CCS (60\$/tCO_2), it is not optimal in either the near or the distant future to deploy this abatement technology and dedicate research efforts to it. It is only when we consider current marginal costs less than 20% of the optimistic reference level, that a regime with CCS and R&D of CCS technology becomes optimal, but even then not in the near future. We also observed that a more stringent environmental constraint (in the form of lower disaster temperature rise) limited the scope for the CCS sector and the corresponding R&D activity.

The stylized model we worked with can be extended in several directions. One dimension is related to the dirty energy carrier, which we assumed to be constrained by the amount of labour and capital devoted to transforming it into energy. Accordingly, we could have introduced a finite fossil fuel resource. However, as CCS depends on fossil fuel use, this will increase neither the scope for CCS nor the R&D devoted to this technology. Similarly, less favourable conditions for CCS (such as technically feasible capture rates below 100%, limited storage possibilities, and the risk of CO_2 leakage), while making the model more realistic, would only reduce the scope for this form of abatement activity and its technology. Conversely, in our model renewable energy is being produced under rather optimistic circumstances, as we have assumed away any problems of intermittency and related problems regarding energy storage. An interesting avenue for future research would be to evaluate the scope for CCS when such favourable conditions are absent.

One can also conjecture that if one energy carrier is or becomes dominant, the ease of substitution with alternative carriers would be reduced. This suggests an inverse-U-shaped relationship between the intensity of, say, fossil fuels, and the elasticity of substitution between fossil fuels and renewable energy (cf. footnote 9 and Gerlagh and Lise (2005, pp. 249)). As it will become more difficult to substitute away from dirty energy when this carrier is dominant, we expect that this would favour the role for CCS in our model. The exploration of these issues is left for future research.

Lastly, we have confined ourselves to search for the first-best policies. With a sufficiently broad set of instruments, these policies should be decentralizable even in imperfect market economies. Some imperfections, though, may be expected to have significant consequences for the results. Hoel and Jensen (2012) recently showed that if policy makers can at best commit to a future climate policy while failing to agree on adoption of a current policy, the reaction of fossil fuel owners (to advance the extraction of fossil fuels in time) may make it more desirable to aim at a faster technical progress in abatement rather than in renewable energy production. The consequences of such restrictions for our model would be worthwhile investigating.

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7 Appendix

7.1 Solution of the model

The Lagrangian function for the planning problem is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t W_t,$$

where

$$\begin{aligned} W_t &= U(Y_t - \psi \left(\sum_{j=a,c,d} x_{jt} \right), \widetilde{F}(S_t)) \\ &+ \lambda_t \left[\left(Y_{ct}^{\frac{\epsilon-1}{\epsilon}} + Y_{dt}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} - Y_t \right] + \sum_{j=a,c,d} \lambda_{jt} \left[L_{jt}^{1-\alpha} A_{jt}^{1-\alpha} x_{jt}^{\alpha} - Y_{jt} \right] \\ &+ w_t \left[1 - \sum_{j=a,c,d} L_{jt} \right] + v_t \left[1 - \sum_{j=a,c,d} s_{jt} \right] + \sum_{j=a,c,d} \mu_{jt} \left[\left(1 + \gamma \eta_j s_{jt}\right) A_{jt-1} - A_{jt} \right] \\ &+ \omega_t \left[-\xi \left(Y_{dt-1} - Y_{at-1} \right) + (1+\delta) S_{t-1} - S_t \right] + \phi_t \left[Y_{dt} - Y_{at} \right]. \end{aligned}$$

Thus W_t is the undiscounted period t welfare, λ_t is the social value of final production in period t, $w_t(v_t)$ is the shadow value of labour (research) in period t, μ_{jt} is the social value of productivity in sector j in period t, and ω_t is the social value of the environment in period t. When writing the quality index of the environment as a function of the stock of CO_2, we have subsumed the relationship through the increase in temperature, Δt.

The first-order condition (FOC) w.r.t. Y_t shows that $\lambda_t = U_{ct}$. The FOC w.r.t. S_t shows that $U_{st}\widetilde{F}_t' = \omega_t - (1+\delta)\beta\omega_{t+1}$. This is a forward-looking equation that can be solved for ω_t, the social value of a one unit improvement of the environment in t, as:

$$\omega_t = \sum_{k=0}^{\infty} \beta^k (1+\delta)^k U_{sk} \widetilde{F}_k'.$$

Improving the environment today thus generates a stream of future benefits.

We now solve for the remaining decision variables. First, note that because of the CES specification, both the clean and dirty inputs will be used in strictly positive quantities.

For the clean input, we obtain the optimality condition:

$$MP_{ct} = \widehat{p}_{ct} \overset{\text{def}}{=} \frac{\lambda_{ct}}{\lambda_t}, \tag{6}$$

i.e., the equality of its marginal product, MP_{ct}, with its social price. A similar condition holds for the dirty input, corrected for the environmental externality:

$$MP_{dt} = \widehat{p}_{dt} + \xi\beta\frac{\omega_{t+1}}{\lambda_t} - \frac{\phi_t}{\lambda_t}, \tag{7}$$

where $\widehat{p}_{dt} \overset{\text{def}}{=} \frac{\lambda_{ct}}{\lambda_t}$, the social price of the dirty input. The term $\xi\beta\frac{\omega_{t+1}}{\lambda_t}$ is equivalent to a tax on the use of the dirty input in a decentralized solution. It ensures a more moderate use of the dirty input than the equality of MP_{dt} with $\widehat{p}_{dt}$ would call for. The extra term $\frac{\phi_t}{\lambda_t}$ is due to abatement. Before interpreting it, we give the FOC w.r.t. Y_{at}:

$$-\lambda_{at} + \beta\omega_{t+1} - \phi_t \leq 0,$$

with equality when $Y_{at} > 0$. Dividing through by λ_t and defining $\widehat{p}_{at} \overset{\text{def}}{=} \frac{\lambda_{at}}{\lambda_t}$, we can write this as:

$$\widehat{p}_{at} \geq \xi\beta\frac{\omega_{t+1}}{\lambda_t} - \frac{\phi_t}{\lambda_t}.$$

If any abatement is suboptimal, $Y_{at} = 0 < Y_{dt}$, and $\widehat{p}_{at} \geq \xi\beta\frac{\omega_{t+1}}{\lambda_t}$; the social marginal cost of abatement is too high compared with its social marginal benefit. However, suppose that abatement is optimal, then either there is partial abatement, $0 < Y_{at} \leq Y_{dt}$, in which case $\widehat{p}_{at} = \xi\beta\frac{\omega_{t+1}}{\lambda_t}$, or there is full abatement, $Y_{at} = Y_{dt}$, in which case $\widehat{p}_{at} \leq \xi\beta\frac{\omega_{t+1}}{\lambda_t}$. In this last case, the social marginal benefit is larger than the social marginal cost, but the welfare programme is constrained by the fact that abatement can only apply to contemporaneous emissions, not to CO_2 emitted in previous periods (i.e., it is not possible to remove previously emitted CO_2 from the atmosphere). If this is the case, then social welfare may be increased by expanding dirty input production beyond the level where $MP_{dt} = \widehat{p}_{dt} + \xi\beta\frac{\omega_{t+1}}{\lambda_t}$. Indeed, then:

$$MP_{dt} = \widehat{p}_{dt} + \widehat{p}_{at}. \tag{8}$$

CO$_2$ abatement is merely an additional social cost. Thus, we can conclude that:

$$\begin{aligned} MP_{dt} &= \widehat{p}_{dt} + \min\{\widehat{p}_{at}, \xi\beta\frac{\omega_{t+1}}{\lambda_t}\}, &(9)\\ &= \widehat{p}_{dt} + \min\{\widehat{p}_{at}, \tau_t^v \widehat{p}_{dt}\}, &(10)\end{aligned}$$

where $\tau_t^v \overset{\text{def}}{=} \xi\beta\frac{\omega_{t+1}}{\lambda_t}\frac{1}{\widehat{p}_{dt}}$, i.e., the *ad valorem* rate that internalizes the externality.

Having determined the optimality conditions for Y_{jt}, we now consider the use of labour and physical capital. Both inputs are required in positive amounts. For labour, the value of the marginal product of labour in the production of sector j must equal the social wage rate $\widehat{w}_t \overset{\text{def}}{=} \frac{w_t}{\lambda_t}$:

$$\begin{aligned} \widehat{p}_{jt} MP_{Ljt} &= \widehat{w}_t, \text{ or} &(11)\\ (1-\alpha)\widehat{p}_{jt}A_{jt}^{1-\alpha}\left(\frac{x_{jt}}{L_{jt}}\right)^{\alpha} &= \widehat{w}_t. &(12)\end{aligned}$$

Likewise, for machines:

$$\begin{aligned} \widehat{p}_{jt} MP_{xjt} &= \psi, \text{ or} &(13)\\ \alpha\widehat{p}_{jt}A_{jt}^{1-\alpha}\left(\frac{L_{jt}}{x_{jt}}\right)^{1-\alpha} &= \psi, &(14)\end{aligned}$$

where ψ is the (exogenously given) amount of final goods necessary to build one machine.

Finally, we determine the allocation of scientists, and the production of knowledge. The FOC w.r.t. s_{jt} is:

$$\frac{\mu_{jt}}{\lambda_t}\gamma\eta_j A_{jt-1} \leq \widehat{v}_t \overset{\text{def}}{=} \frac{v_t}{\lambda_t},$$

with equality whenever $s_{jt} > 0$. The left-hand side (LHS) is the social price of sector j knowledge, $\frac{\mu_{jt}}{\lambda_t}$, times the marginal knowledge production of an additional researcher. The RHS is the social wage rate of a researcher.

The final set of FOCs characterizes the allocation of productivity improvements in the different sectors across time. The FOC w.r.t. A_{jt} reads:

$$\widehat{p}_{jt}(1-\alpha)L_{jt}^{1-\alpha}\left(\frac{x_{jt}}{A_{jt}}\right)^{\alpha} = \frac{\mu_{jt}}{\lambda_t} - \beta\frac{\mu_{jt+1}}{\lambda_{t+1}}\frac{\lambda_{t+1}}{\lambda_t}(1+\gamma\eta_j s_{jt+1}).$$

The LHS is the value of the marginal product of newly acquired knowledge on the use of machines. Using (14), an optimal allocation of knowledge implies that the social price of sector j knowledge, $\frac{\mu_{jt}}{\lambda_t}$, must evolve according to the rule:

$$(1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} L_{jt}\widehat{p}_{jt}^{\frac{1}{1-\alpha}} = \frac{\mu_{jt}}{\lambda_t} - \beta\frac{\mu_{jt+1}}{\lambda_{t+1}}\frac{\lambda_{t+1}}{\lambda_t}(1+\gamma\eta_j s_{jt+1}).$$

Multiplying through by $\lambda_t A_{jt}$ and making use of $A_{jt+1} = (1+\gamma\eta_j s_{jt})A_{jt}$ give:

$$\mu_{jt}A_{jt} = (1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} L_{jt}\lambda_t\widehat{p}_{jt}^{\frac{1}{1-\alpha}}A_{jt} + \beta\mu_{jt+1}A_{jt+1}.$$

The social value of acquired knowledge in sector j at time t is the value of A_{jt} priced at its marginal product plus the "standing on the shoulder of giants" effect (future knowledge builds on today's knowledge). Using the forward operator F, multiplying through by $\gamma\eta_j\frac{A_{jt-1}}{A_{jt}}$ and making use of $A_{jt} = (1+\gamma\eta_j s_{jt+1})A_{jt-1}$ result in:

$$\begin{aligned}\frac{\mu_{jt}}{\lambda_t}\gamma\eta_j A_{jt-1} &= \frac{1}{\lambda_t}\frac{\gamma\eta_j}{1+\gamma\eta_j s_{jt+1}}\frac{1}{1-\beta F}(1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} L_{jt}\lambda_t\widehat{p}_{jt}^{\frac{1}{1-\alpha}}A_{jt}, \\ &= \frac{1}{\lambda_t}\frac{\gamma\eta_j}{1+\gamma\eta_j s_{jt+1}}(1-\alpha)\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}\sum_{\tau=0}^{\infty}\beta^{\tau}L_{jt+\tau}\lambda_{t+\tau}\widehat{p}_{jt}^{\frac{1}{1-\alpha}}A_{jt+\tau},\end{aligned} \tag{15}$$

so that the social value of allocating an extra researcher to sector j is given by the discounted sum of future knowledge levels, appropriately valued and weighted.

Solving (14) for x_{jt} gives:

$$x_{jt} = \left(\alpha\frac{\widehat{p}_{jt}}{\psi}\right)^{\frac{1}{1-\alpha}} A_{jt}L_{jt}, \tag{16}$$

which can be plugged into (12) to yield the social price of sector j output, as a weighted average of the exogenous machine price, ψ, and the shadow price of labour, $\widehat{w}_{jt}$:

$$\widehat{p}_{jt} = \frac{1}{\mathcal{A}}\frac{1}{A_{jt}^{1-\alpha}}\widehat{w}_t^{1-\alpha}\psi^{\alpha}, \tag{17}$$

where $\mathcal{A} \overset{\text{def}}{=} \alpha^{\alpha}(1-\alpha)^{1-\alpha}$. Hence, at an optimum, $\widehat{p}_{jt}$ will equal the social marginal cost of sector j output.

Next, the FOCs for Y_{ct} and Y_{dt} can be used to relate these input levels to aggregate output, Y_t and the shadow prices of the inputs:

$$\begin{aligned} Y_{ct} &= Y_t \widehat{p}_{ct}^{-\varepsilon} && (18) \\ Y_{dt} &= Y_t \left[\widehat{p}_{dt} + \min\{\widehat{p}_{at}, \tau_t^v \widehat{p}_{dt}\}\right]^{-\varepsilon} \\ &= Y_t \widehat{p}_{dt}^{-\varepsilon} \left[1 + \min\{\frac{\widehat{p}_{at}}{\widehat{p}_{dt}}, \tau_t^v\}\right]^{-\varepsilon} \\ &= Y_t \widehat{p}_{dt}^{-\varepsilon} \left[1 + \min\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t^v\}\right]^{-\varepsilon}, && (19) \end{aligned}$$

where the last equality follows from (17). Making use of the final good production function, we obtain:

$$1 = \widehat{p}_{ct}^{1-\varepsilon} + \widehat{p}_{dt}^{1-\varepsilon} \left[1 + \min\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t^v\}\right]^{1-\varepsilon}.$$

As $\widehat{p}_{jt}$ $(j = c, d, a)$ are given by (17):

$$1 = \frac{1}{\mathcal{A}^{1-\varepsilon}} \widehat{w}_t^{\varphi} \psi^{\alpha(1-\varepsilon)} \left(A_{ct}^{-\varphi} + A_{dt}^{-\varphi} \left[1 + \min\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t^v\}\right]^{1-\varepsilon}\right),$$

where $\varphi \overset{\text{def}}{=} (1-\alpha)(1-\varepsilon)$.

Hence, we can solve for the social value of the wage rate:

$$\begin{aligned} \widehat{w}_t &= \mathcal{A}^{\frac{1}{1-\alpha}} \psi^{-\frac{\alpha}{1-\alpha}} \left(A_{ct}^{-\varphi} + A_{dt}^{-\varphi} \left[1 + \min\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t^v\}\right]^{1-\varepsilon}\right)^{-\varphi} \\ &= \mathcal{A}^{\frac{1}{1-\alpha}} \psi^{-\frac{\alpha}{1-\alpha}} B_t, && (20) \end{aligned}$$

thereby implicitly defining the "sector average" productivity parameter B_t as:

$$B_t \overset{\text{def}}{=} \left(A_{ct}^{-\varphi} + A_{dt}^{-\varphi} \left[1 + \min\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t^v\}\right]^{1-\varepsilon}\right)^{-\frac{1}{\varphi}}.$$

From (20) and (17), the social prices of the two inputs as well as the price of abatement are then:

$$\widehat{p}_{jt} = \left(\frac{B_t}{A_{jt}}\right)^{1-\alpha} \quad (j = c, d, a). \tag{21}$$

Machine use in sector j can be obtained from (16) and (21):

$$x_{jt} = \widehat{p}_{jt}^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\psi}\right)^{\frac{1}{1-\alpha}} A_{jt} L_{jt} = \left(\frac{\alpha}{\psi}\right)^{\frac{1}{1-\alpha}} B_t L_{jt}, \tag{22}$$

and therefore the aggregate machine cost (the share of final good production used for capital) is:

$$AMC_t \overset{\text{def}}{=} \sum_{j=c,d,a} \psi x_{jt} = \psi \left(\frac{\alpha}{\psi}\right)^{\frac{1}{1-\alpha}} B_t \sum_{j=c,d,a} L_{jt} = \psi^{-\frac{\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} B_t, \tag{23}$$

where the last equality follows from the normalization of the labour supply to one.

To find the levels of production in the three sectors, we plug the solution for x_{jt} (22) into the production function, yielding:

$$Y_{jt} = A_{jt} L_{jt} \left(\alpha \frac{\widehat{p}_{jt}}{\psi}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} L_{jt} A_{jt}^{1-\alpha} B_t^{\alpha} \qquad (j = c, d, a).$$

Therefore $L_{jt} = \left(\frac{\alpha}{\psi}\right)^{-\frac{\alpha}{1-\alpha}} A_{jt}^{\alpha-1} B_t^{-\alpha} Y_{jt}$, which allows us to write (15) as:

$$\begin{aligned} \frac{\mu_{jt}}{\lambda_t} \gamma \eta_j A_{jt-1} &= \frac{1}{\lambda_t} \frac{\gamma \eta_j}{1 + \gamma \eta_j s_{jt+1}} (1-\alpha) \sum_{\tau=0}^{\infty} \beta^{\tau} A_{jt+\tau}^{\alpha} B_{t+\tau}^{-\alpha} Y_{jt} \lambda_{t+\tau} \widehat{p}_{jt}^{\frac{1}{1-\alpha}} \\ &= \frac{1}{\lambda_t} \frac{\gamma \eta_j}{1 + \gamma \eta_j s_{jt+1}} (1-\alpha) \sum_{\tau=0}^{\infty} \beta^{\tau} \lambda_{t+\tau} \widehat{p}_{jt} Y_{jt}, \end{aligned}$$

where the second equality follows from (21). This is expression (5) in the text.

On the other hand, (18) and (19) together with (21) give:

$$\begin{aligned}
Y_{ct} &= Y_t \left(\frac{B_t}{A_{ct}}\right)^{-\varepsilon(1-\alpha)}, \text{ and} \\
Y_{dt} &= Y_t \left(\frac{B_t}{A_{dt}}\right)^{-\varepsilon(1-\alpha)} \left[1 + \min\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t^v\}\right]^{-\varepsilon}.
\end{aligned}$$

The last three expressions now allow us to write the labour balance equation as:

$$\begin{aligned}
&\Bigg\{A_{ct}^{-\varphi} B_t^{-(1-\varphi)} Y_t + A_{dt}^{-\varphi} B_t^{-(1-\varphi)} \left[1 + \min\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t^v\}\right]^{-\varepsilon} Y_t \\
&+ Y_{at} A_{at}^{-(1-\alpha)} B_t^{-\alpha}\Bigg\} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}}. \qquad (24)
\end{aligned}$$

We now look at the three possibilities. The first is where there is full abatement, $Y_{at} = Y_{dt}$, such that $\min\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t^v\} = \left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}$. In that case:

$$Y_{at} = Y_{dt} = Y_t \left(\frac{B_t}{A_{dt}}\right)^{-\varepsilon(1-\alpha)} \left[1 + \left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}\right]^{-\varepsilon}.$$

Making use of these values for Y_{at} and Y_{dt} in the labour balance equation (24) reduces the latter to:

$$\left(\frac{\alpha}{\psi}\right)^{-\frac{\alpha}{1-\alpha}} B_t^{-(1-\varphi)} B_t^{-\varphi} Y_t = 1,$$

so that

$$\begin{aligned}
Y_t^{FA} &= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} B_t, \\
Y_{ct}^{FA} &= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} B_t^{\varphi+\alpha} A_{ct}^{1-(\varphi+\alpha)}, \\
Y_{dt}^{FA} &= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} B_t^{\varphi+\alpha} A_{dt}^{1-(\varphi+\alpha)} \left[1 + \left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}\right]^{-\varepsilon}, \text{ and} \\
Y_{at}^{FA} &= Y_{dt}^{FA}.
\end{aligned}$$

In the second case, there is partial abatement such that $0 < Y_{at} < Y_{dt}$ and $\min\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t^v\} = \left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha} = \tau_t^v$. Now (24) becomes:

$$\left\{A_{ct}^{-\varphi} B_t^{-(1-\varphi)} Y_t + A_{dt}^{-\varphi} B_t^{-(1-\varphi)} \left[1+\tau_t^v\right]^{-\varepsilon} Y_t + Y_{at} A_{at}^{-(1-\alpha)} B_t^{-\alpha}\right\} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}},$$

yielding:

$$Y_t^{PA} = \left\{\left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} - Y_{at} A_{at}^{-(1-\alpha)} B_t^{-\alpha}\right\} \frac{B_t^{1-\varphi}}{\left[A_{ct}^{-\varphi} + A_{dt}^{-\varphi}\left[1+\tau_t^v\right]^{-\varepsilon}\right]}, \tag{25}$$

$$Y_{ct}^{PA} = Y_t^{PA}\left(\frac{B_t}{A_{ct}}\right)^{-\varepsilon(1-\alpha)}, \tag{26}$$

$$Y_{dt}^{PA} = Y_t^{PA}\left(\frac{B_t}{A_{dt}}\right)^{-\varepsilon(1-\alpha)}\left[1+\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}\right]^{-\varepsilon}. \tag{27}$$

For this to be compatible with partial abatement, we need $Y_{at} \leq Y_{dt}^{PA}$, which can be shown to be equivalent with:

$$Y_{at} \leq \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_{dt}^{\varepsilon(1-\alpha)} B_t^{\alpha}\left[1+\tau_t^v\right]^{-\varepsilon}}{\left\{A_{ct}^{-\varphi} + A_{dt}^{-\varphi}\left[1+\tau_t^v\right]^{-\varepsilon}\left(1+\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}\right)\right\}}. \tag{28}$$

Given partial abatement is optimal, we have $\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha} = \tau_t^v$, and this condition reduces to:

$$Y_{at} \leq \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_{dt}^{\varepsilon(1-\alpha)} B_t^{1-\varepsilon(1-\alpha)}}{\left[1+\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}\right]^{\varepsilon}}. \tag{29}$$

In the third case, there is no abatement: $Y_{at} = 0$ and $\min\{\left(\frac{A_{dt}}{A_{at}}\right)^{1-\alpha}, \tau_t^v\} = \tau_t^v$. The equilibrium value for Y_t is then found by setting $Y_{at} = 0$ in (25):

$$Y_t^{NA} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{B_t^{1-\varphi}}{\left[A_{ct}^{-\varphi} + A_{dt}^{-\varphi}\left[1+\tau_t^v\right]^{-\varepsilon}\right]},$$

and

$$
\begin{aligned}
Y_{ct}^{NA} &= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{B_t^{\alpha} A_{ct}^{\varepsilon(1-\alpha)}}{\left[A_{ct}^{-\varphi} + A_{dt}^{-\varphi}\left[1+\tau_t^v\right]^{-\varepsilon}\right]}, \\
Y_{dt}^{NA} &= \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{B_t^{\alpha} A_{dt}^{\varepsilon(1-\alpha)}}{\left[A_{ct}^{-\varphi} + A_{dt}^{-\varphi}\left[1+\tau_t^v\right]^{-\varepsilon}\right]} \left[1+\tau_t^v\}\right]^{-\varepsilon}, \text{ and} \\
Y_{at}^{NA} &= 0.
\end{aligned}
$$

When solving the model, we search for a sequence $\{\tau_t^v, s_{ct}, s_{dt}, Y_{at}\}_{t=0}^{T}$ (where T is large) that maximizes:

$$
\sum_{t=0}^{T} \beta^t U(Y_t^{PA}(Y_{at}) - AMC_t, \widetilde{F}((1+\delta)S_{t-1} - \xi\left(Y_{dt-1} - Y_{at-1}\right))),
$$

subject to the equality constraints (23), (26), (27), $s_{at} = 1 - s_{ct} - s_{dt}$, $A_{jt} = \left(1 + \gamma\eta_j s_{jt}\right) A_{jt-1}$ (all t and j), the non-linear inequality constraint (28), and with the initial productivity levels A_{j0} given.

7.2 Calibration of the model

Without any policy intervention in the base period, the laissez-faire levels for clean and dirty input production are:

$$
Y_{c0} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{B_0^{\alpha} A_{c0}^{-\varepsilon(1-\alpha)}}{A_{c0}^{-\varphi} + A_{d0}^{-\varphi}}, \text{ and } Y_{dt} = \left(\frac{\alpha}{\psi}\right)^{\frac{\alpha}{1-\alpha}} \frac{B_0^{\alpha} A_{d0}^{-\varepsilon(1-\alpha)}}{A_{c0}^{-\varphi} + A_{d0}^{-\varphi}},
$$

where $B_0 \overset{\text{def}}{=} \left(A_{c0}^{-\varphi} + A_{d0}^{-\varphi}\right)^{-\frac{1}{\varphi}}$, and $\varphi \overset{\text{def}}{=} (1-\varepsilon)(1-\alpha)$. This system can be solved for A_{c0} and A_{d0}:

$$
\begin{aligned}
A_{d0} &= \left(\frac{\alpha}{\psi}\right)^{-\frac{\alpha}{1-\alpha}} Y_{d0} \left[1 + \left(\frac{Y_{d0}}{Y_{c0}}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]^{\frac{\alpha+\varphi}{\varphi}}, \\
A_{c0} &= \left(\frac{\alpha}{\psi}\right)^{-\frac{\alpha}{1-\alpha}} Y_{c0} \left[1 + \left(\frac{Y_{c0}}{Y_{d0}}\right)^{\frac{1-\varepsilon}{\varepsilon}}\right]^{\frac{\alpha+\varphi}{\varphi}}.
\end{aligned}
$$

As in AABH, we have used the values for world primary energy production by energy carrier during the period 2002–2006 (EAI (2008), Table 11.1) and doubled them. Dirty carriers (coal, natural gas, crude oil, and natural gas plant liquids) yield 3786 QBTU, while clean carriers (nuclear electric power, hydroelectric power, geothermal, and others) provide 615 QBTU.[18] Under the assumptions that $\alpha = \frac{1}{3}, \varepsilon = 3$ (and therefore $\varphi = -\frac{4}{3}$), and the normalization $\psi = \alpha^2$, we obtain the following estimates for A_{d0}, A_{c0}, and B_0: $A_{d0} = 2658$, $A_{c0} = 1072$, and $B_0 = 3232$.

---

[18] The corresponding values for 2002–2006, adopted by AABH were 1893.25 and 307.77, respectively.

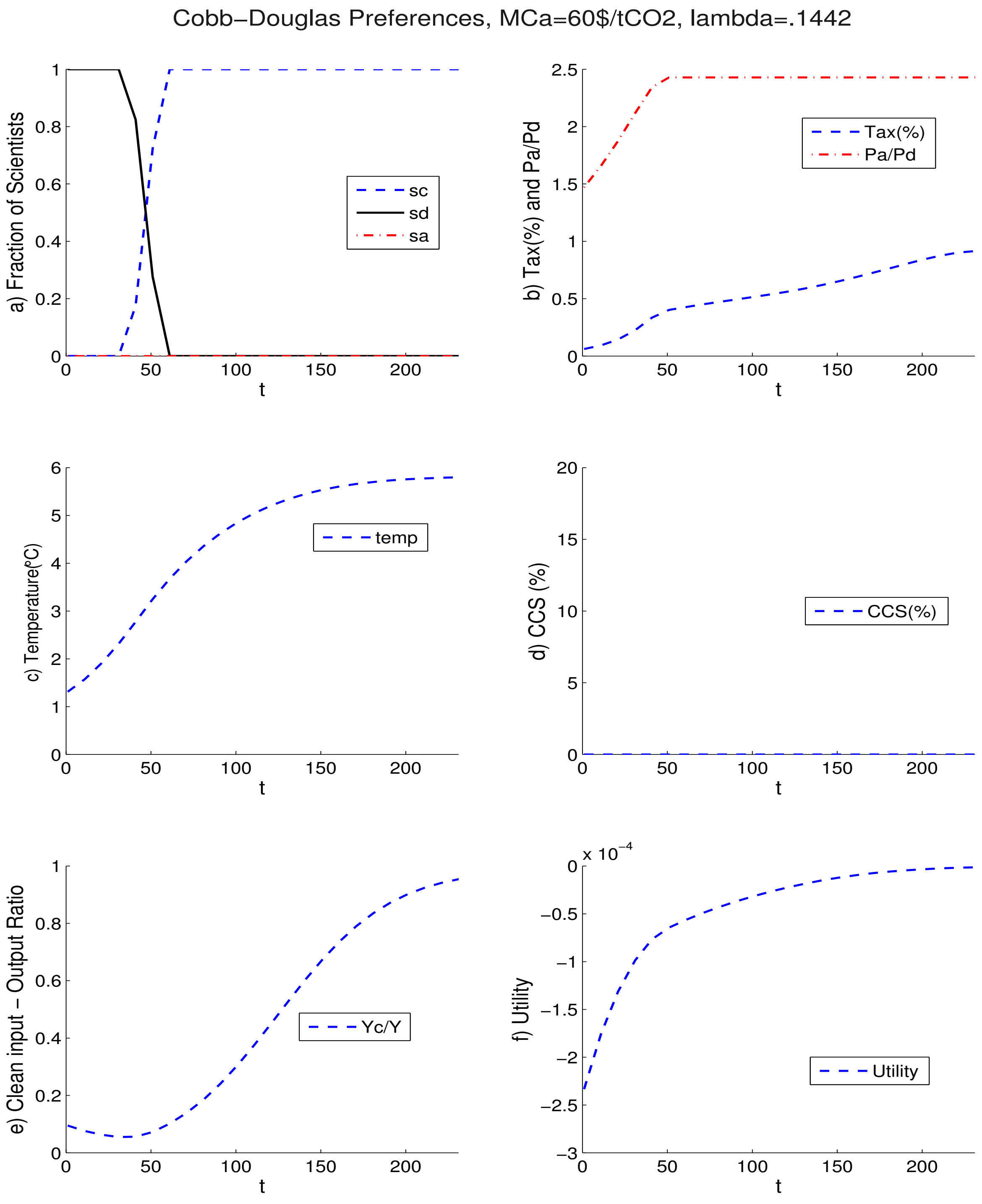

Cobb−Douglas Preferences, MCa=60$/tCO2, lambda=.1442
a) Fraction of Scientists
0
0.2
0.4
0.6
0.8
1
0
50
100
150
200
t
sc
sd
sa
b) Tax(%) and Pa/Pd
0
0.5
1
1.5
2
2.5
0
50
100
150
200
t
Tax(%)
Pa/Pd
c) Temperature(ºC)
0
1
2
3
4
5
6
0
50
100
150
200
t
temp
d) CCS (%)
0
5
10
15
20
0
50
100
150
200
t
CCS(%)
e) Clean input − Output Ratio
0
0.2
0.4
0.6
0.8
1
0
50
100
150
200
t
Yc/Y
f) Utility
x 10^−4
0
−0.5
−1
−1.5
−2
−2.5
−3
0
50
100
150
200
t
Utility

Figure 2

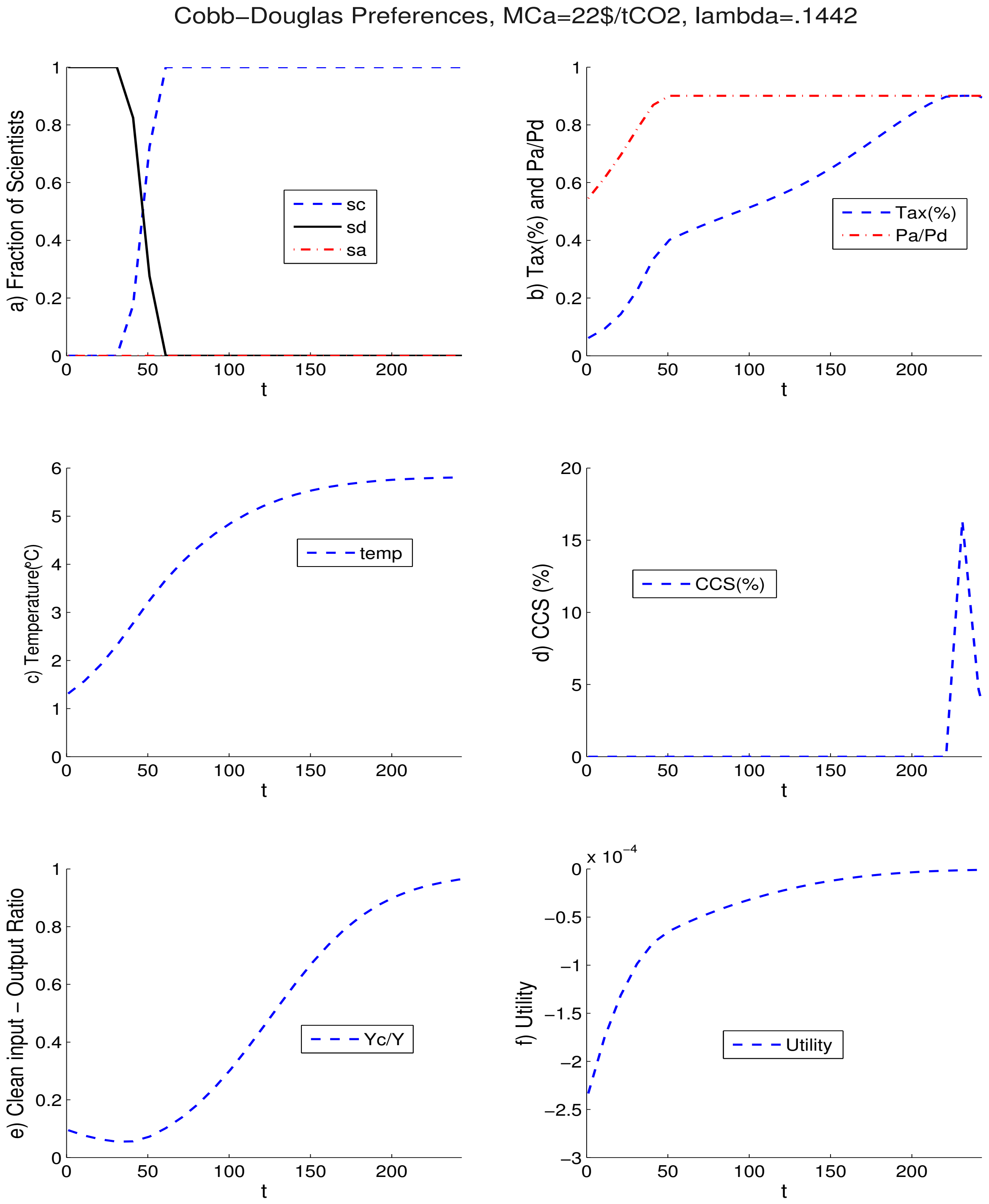
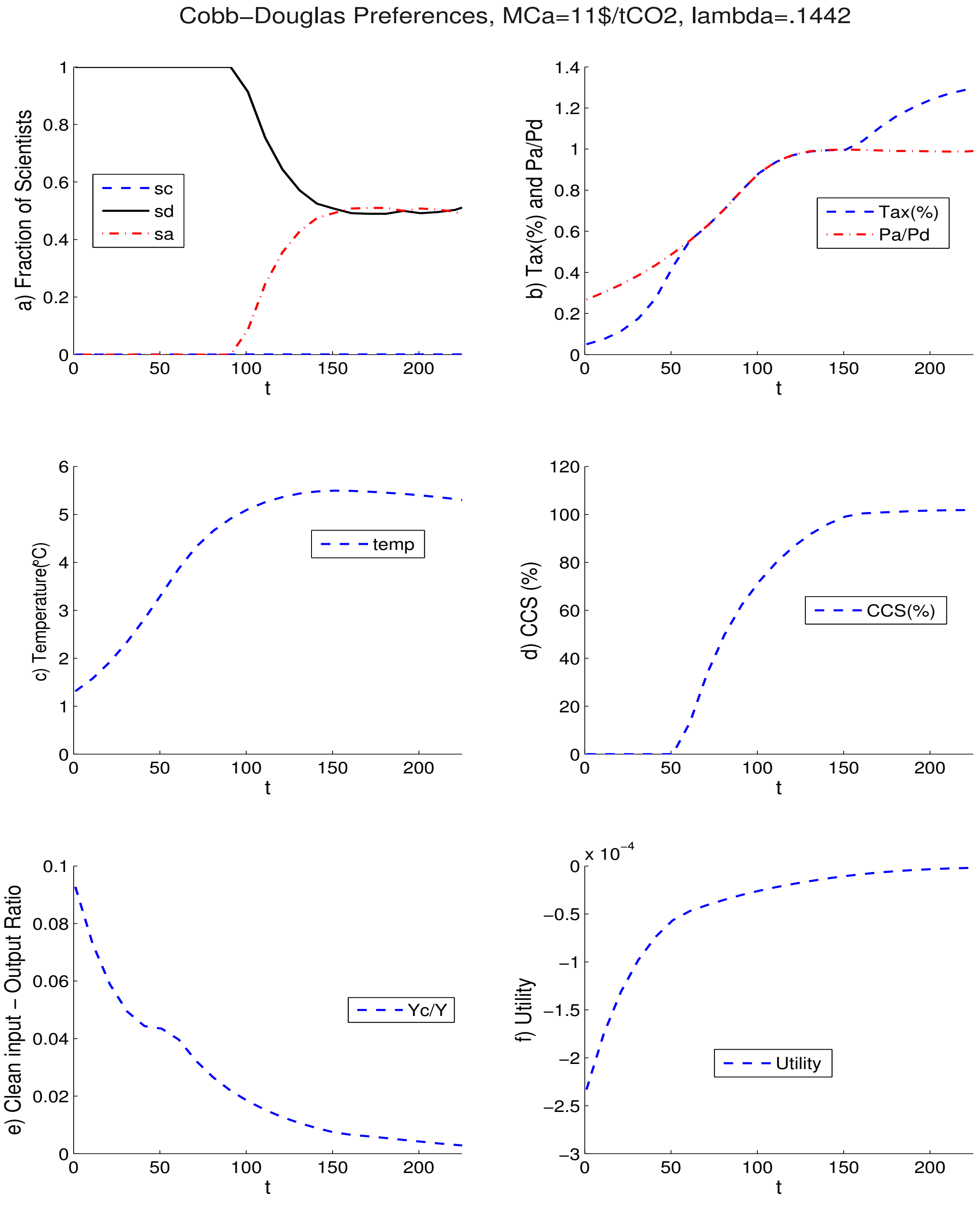

Cobb−Douglas Preferences, MCa=22$/tCO2, lambda=.1442

Figure 3



Cobb–Douglas Preferences, MCa=11$/tCO2, lambda=.1442

Figure 4

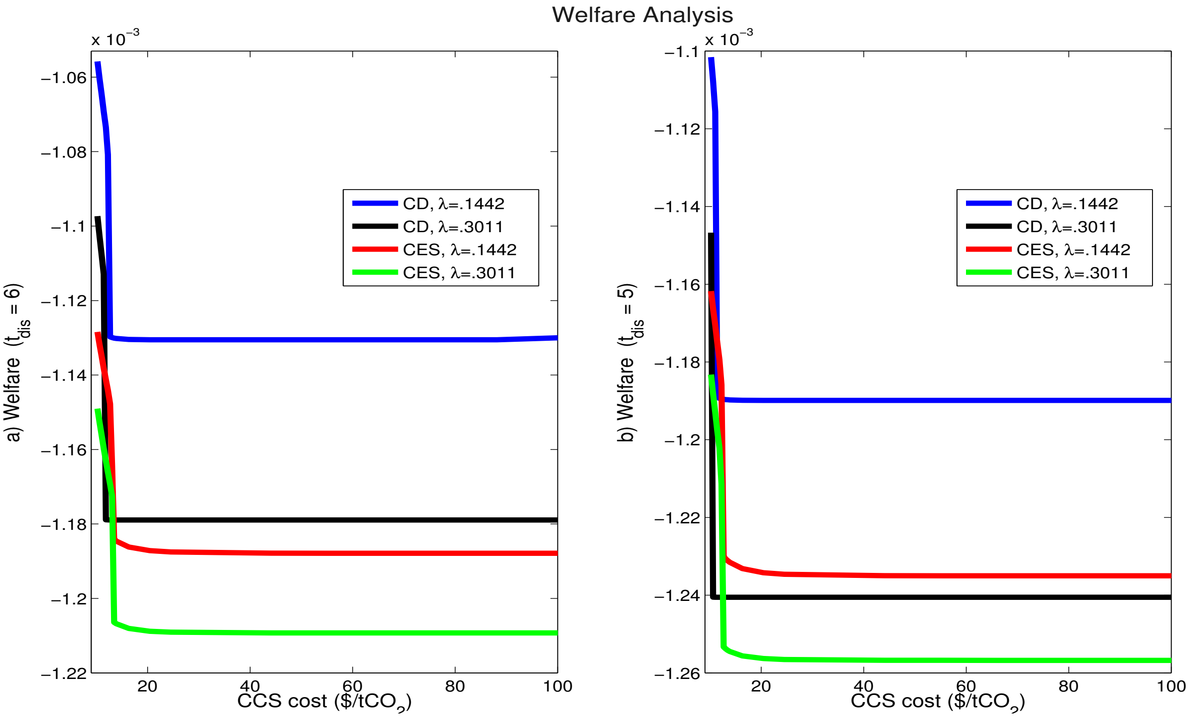


Figure 5: *Maximal intertemporal welfare levels against values for* MC_{a0}

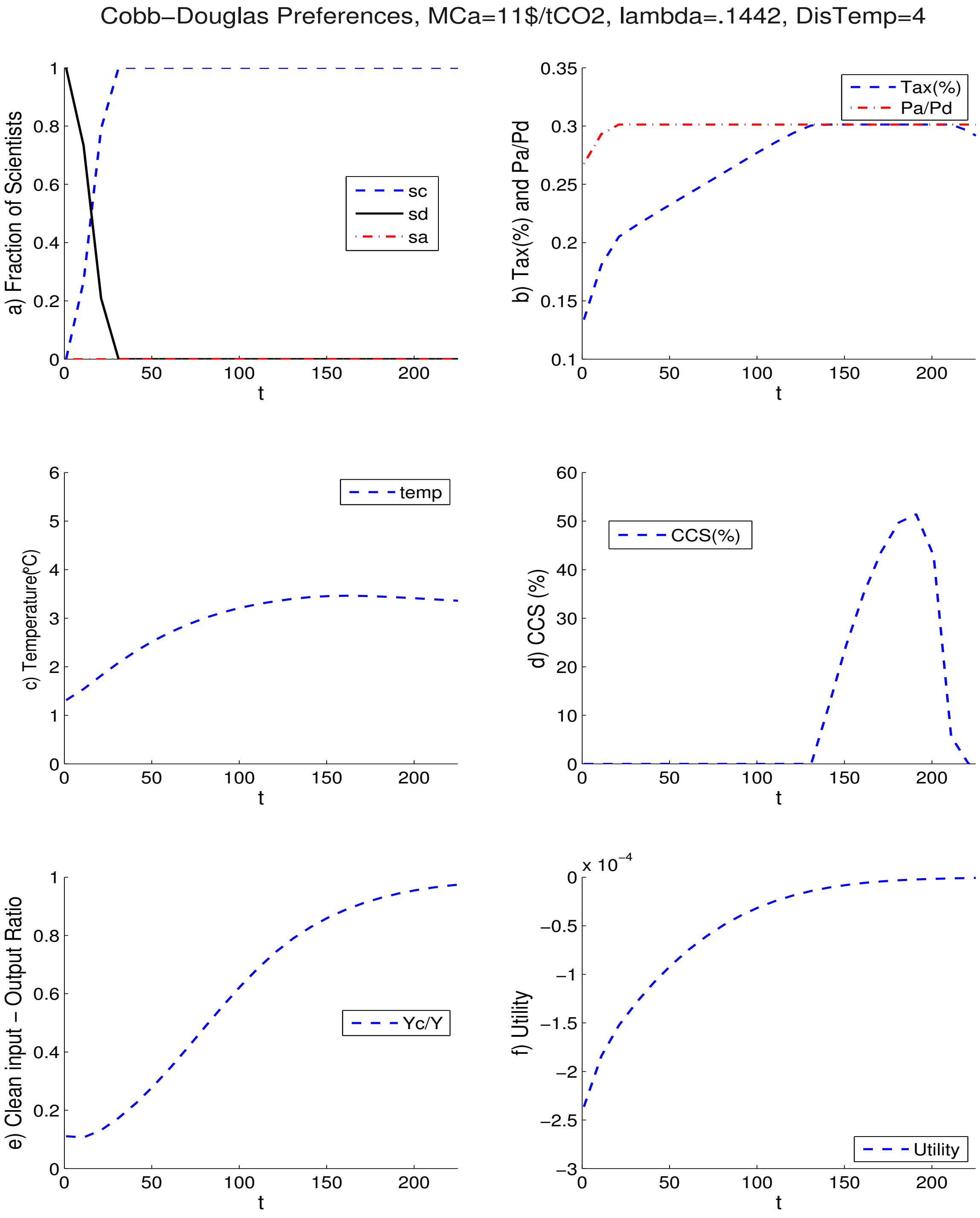
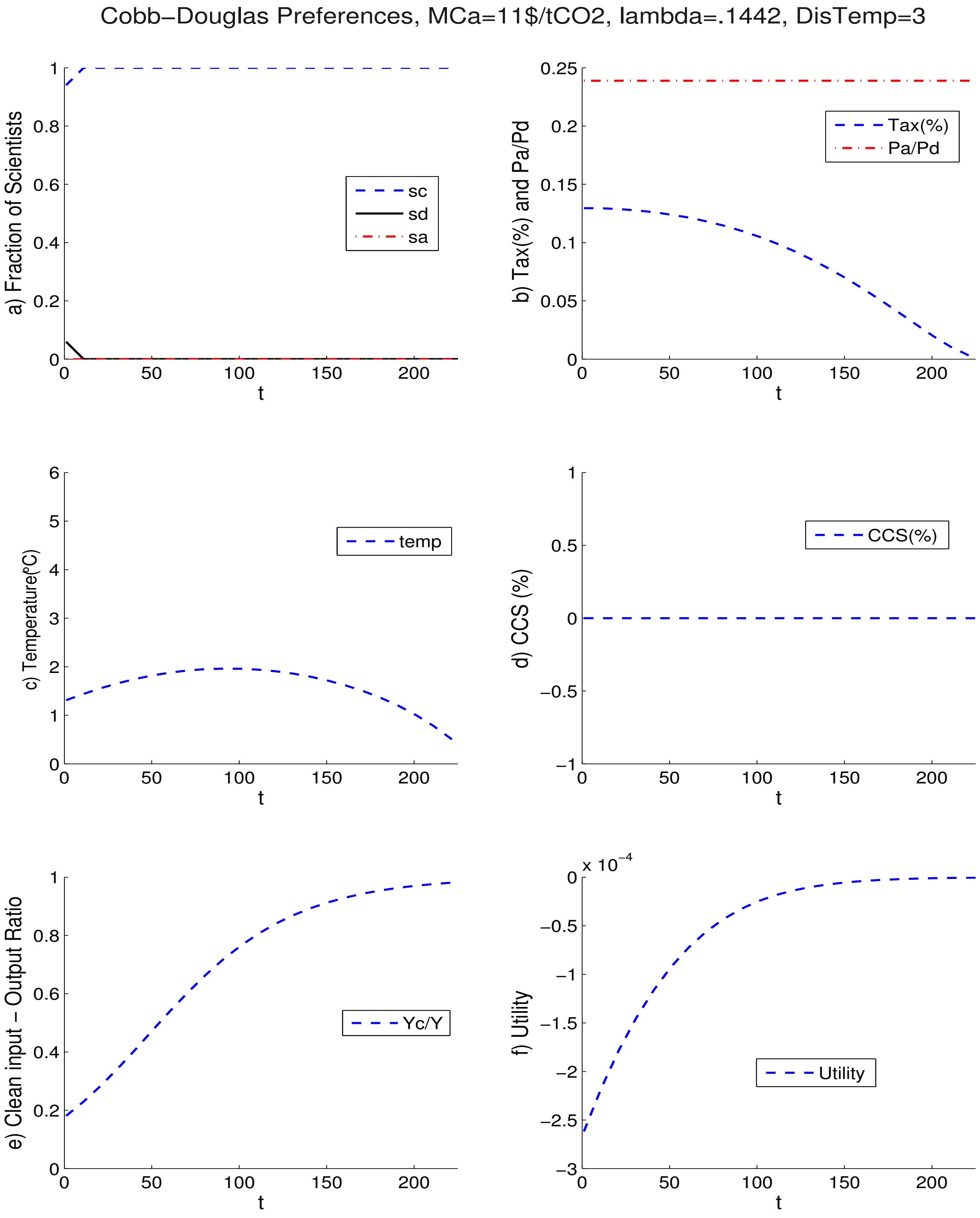

Cobb−Douglas Preferences, MCa=11$/tCO2, lambda=.1442, DisTemp=4
a) Fraction of Scientists
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200
t
sc
sd
sa
b) Tax(%) and Pa/Pd
0.35
0.3
0.25
0.2
0.15
0.1
Tax(%)
Pa/Pd
c) Temperature(ºC)
6
5
4
3
2
1
0
temp
d) CCS (%)
60
50
40
30
20
10
CCS(%)
e) Clean input − Output Ratio
Yc/Y
f) Utility
x 10−4
0
−0.5
−1
−1.5
−2
−2.5
−3
Utility

Figure 6



Cobb−Douglas Preferences, MCa=11$/tCO2, lambda=.1442, DisTemp=3

Figure 7

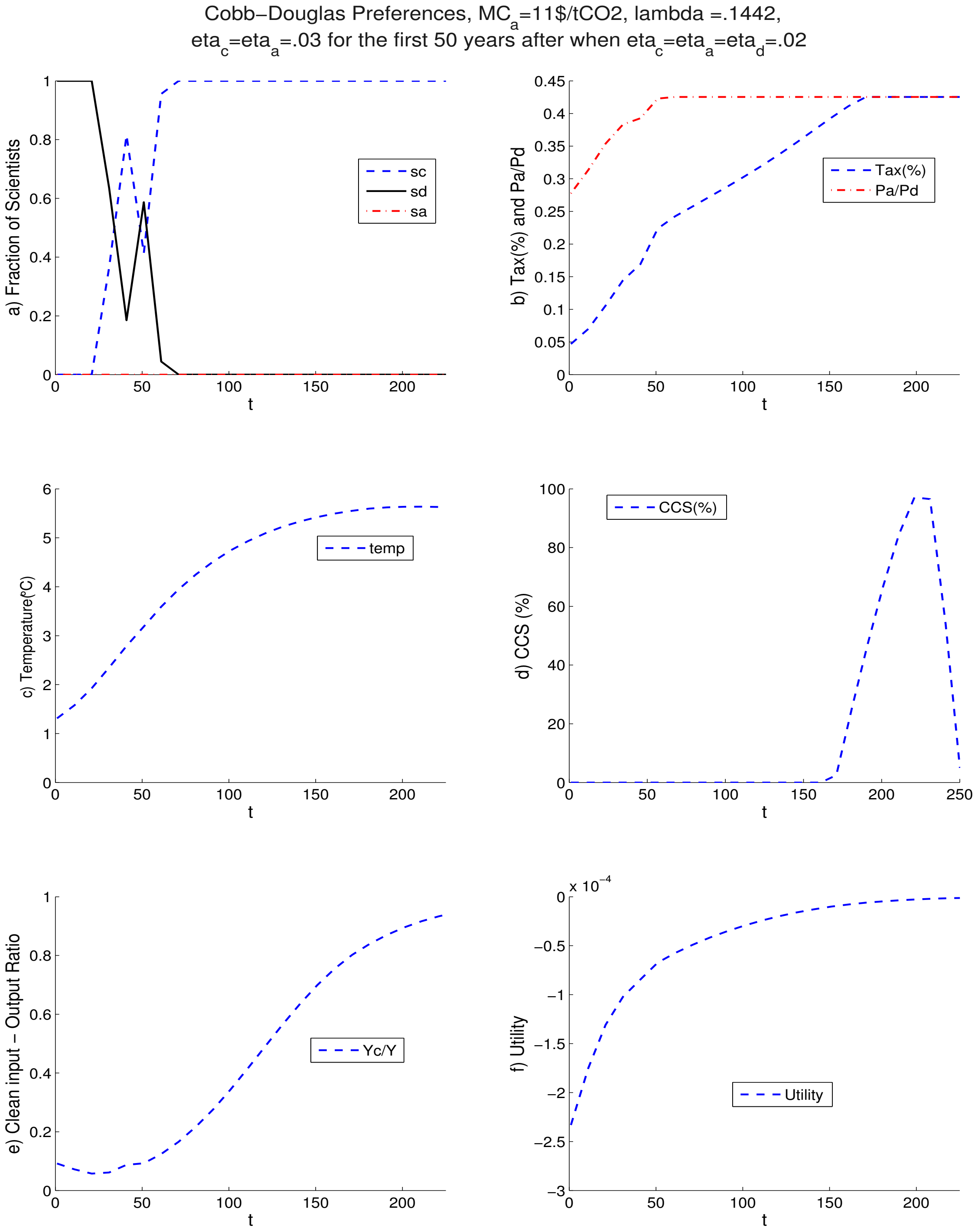

Cobb–Douglas Preferences, MC_a=11$/tCO2, lambda =.1442,
eta_c=eta_a=.03 for the first 50 years after when eta_c=eta_a=eta_d=.02
a) Fraction of Scientists
sc
sd
sa
0
0.2
0.4
0.6
0.8
1
0
50
100
150
200
t
b) Tax(%) and Pa/Pd
Tax(%)
Pa/Pd
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0
50
100
150
200
t
c) Temperature(°C)
temp
0
1
2
3
4
5
6
0
50
100
150
200
t
d) CCS (%)
CCS(%)
0
20
40
60
80
100
0
50
100
150
200
250
t
e) Clean input – Output Ratio
Yc/Y
0
0.2
0.4
0.6
0.8
1
0
50
100
150
200
t
f) Utility
x 10^−4
Utility
0
−0.5
−1
−1.5
−2
−2.5
−3
0
50
100
150
200
t

Figure 8

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